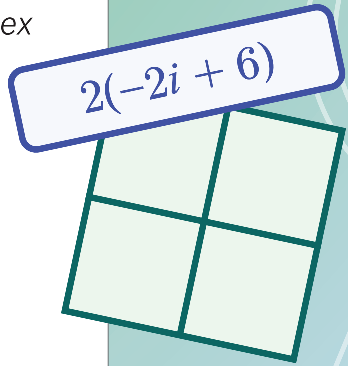
Unit 2

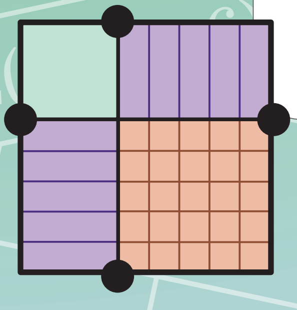
Quadratic Functions and Complex Numbers

Sometimes, the solution to an equation isn't what you expect. It might be an entirely new kind of number! In this unit, you'll rewrite quadratic functions in different forms. You'll also write and solve quadratic equations, sometimes resulting in solutions that can't be described using only real numbers. These numbers are called *complex numbers*, and you'll explore how to add, subtract, and multiply them.



Essential Questions

- How can rewriting quadratic functions in different forms help you determine their key features?
- What strategies can you use to solve quadratic equations?
- What are complex numbers and what can you do with them?



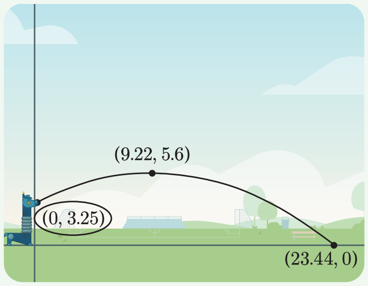
Summary | Lesson 1

You can identify different key features of a quadratic equation from different forms. *Standard form* reveals the y-intercept. *Factored form* reveals the x-intercepts. *Vertex form* reveals the vertex.

Here is the graph of a function, $l(x)$, that represents a llama robot launching a pumpkin. The pumpkin follows the path of the equivalent equations shown.

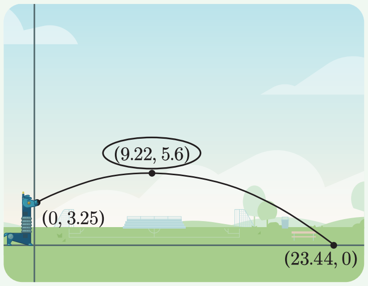
Standard Form

$l(x) = \text{-}0.028x^2 + 0.51x + 3.25$



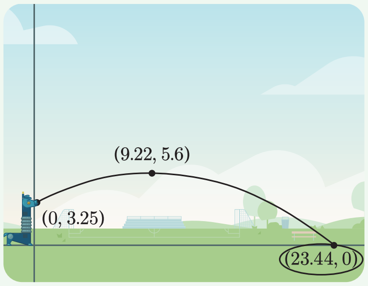
Vertex Form

$l(x) = \text{-}0.028(x - 9.22)^2 + 5.6$



Factored Form

$l(x) = \text{-}0.028(x - 23.44)(x + 5.01)$



Try This

A unicorn robot launched a pumpkin. The function $u(x)$ represents the path of the pumpkin. How high, in feet, did the unicorn's pumpkin go?

$u(x) = \text{-}0.005(x - 20.31)(x + 6.55)$

$u(x) = \text{-}0.005(x - 4.35)^2 + 1.35$

$u(x) = \text{-}0.005x^2 + 0.043x + 1.25$

Summary | Lesson 2

You can use the form of a quadratic equation to determine key features of the equation's graph, without needing to graph it!

- If a quadratic equation is given in standard form, you can rewrite it in factored form and identify the x-intercept(s) from each factor.
- If a quadratic equation is given in factored form, you can multiply to rewrite it in standard form and identify the y-intercept from the constant value.

Let's look at an example using the equation $y = 2x^2 - 5x - 12$.

The constant value is -12, so the y-intercept is $(0, \text{-}12)$.

To rewrite $y = 2x^2 - 5x - 12$ in factored form, you can use a diagram. Start by placing $2x^2$ in the top-left corner of the diagram and -12 in the bottom right. Try out different factors that multiply to $2x^2$ and -12 and write them on the outside of the diagram. Keep going until the inside of the diagram matches the standard form of the equation. Here the equation in factored form is $y = (2x + 3)(x - 4)$, which can be used to determine the x-intercepts of the graph.

	x	
$2x$	$2x^2$	
		-12

$$x - 4 = 0 \text{ and } 2x + 3 = 0$$

$$x = 4 \text{ and } x = \frac{3}{2}$$

So the x-intercepts are $(4, 0)$ and $\left(\frac{3}{2}, 0\right)$.

Try This

a Rewrite $y = 2x^2 - 7x - 15$ in factored form. Use the diagram if it helps with your thinking.

b Identify the x- and y-intercepts of the function.

Summary | Lesson 3

Identities are equations that are always true. We can use identities to rewrite expressions in factored form.

For example, we can factor the expression $y^4 - 6xy^2 + 9x^2$ using the identity $a^2 - 2ab + b^2 = (a - b)^2$. We can rewrite the expression as $(y^2)^2 - 2(y^2)(3x) + 3x^2$, so the a-value in the identity is y^2 and the b-value is $3x$. That means we can factor the expression as $(y^2 - 3x)^2$.

Try This

Rewrite the expression $81x^2 - 121$ in factored form.

Summary | Lesson 4

You can *complete the square* to rewrite the equation of a *parabola* in vertex form and reveal its key features. Take the equation $y = x^2 - 8x + 9$, for example.

$$\begin{aligned}
&y = x^2 - 8x + 9\\
&y = \left(x^2 - 8x + \boxed{}\right) + 9 - \boxed{}\\
&y = (x^2 - 8x + 16) + 9 - 16\\
&y = (x - 4)^2 - 7
\end{aligned}$$

The vertex of the parabola is (4, -7).

Similarly, you can rewrite the equation of a *circle* to reveal its key features. Take a look at this example:

$$\begin{aligned}
(x + 5)^2 + y^2 - 4y &= 5\\
(x + 5)^2 + \left(y^2 - 4y + \boxed{}\right) - \boxed{} &= 5\\
(x + 5)^2 + (y^2 - 4y + 4) - 4 &= 5\\
(x + 5)^2 + (y - 2)^2 - 4 &= 5\\
(x + 5)^2 + (y - 2)^2 &= 9
\end{aligned}$$

The center of the circle is (-5, 2) and the radius is 3.

Try This

Rewrite the equation $y = x^2 + x - 12$ in vertex form.

Summary | Lesson 5

When you're given a quadratic function in vertex form, you can determine its number of x-intercepts by identifying how it has been transformed from the parent function $f(x) = x^2$, which has its vertex at $(0, 0)$ and opens up.

You can then determine the *values* of the x-intercepts by setting the function equal to 0 and solving the quadratic equation.

Take a look at the function $g(x) = -4(x + 1)^2 + 16$, for example.

The -4 indicates that the parabola will open down and the $+ 16$ shows that the parabola is translated up 16 units. This means that the function has two x-intercepts.

You can then determine the values of these x-intercepts by solving algebraically.

$$\begin{aligned}
0 &= -4(x+1)^2 + 16 \\
-16 &= -4(x+1)^2 \\
4 &= (x+1)^2 \\
\pm\sqrt{4} &= x + 1 \\
-1 \pm 2 &= x \\
x = -1 + 2 &\text{ and } x = -1 - 2 \\
x = 1 &\text{ and } x = -3 \\
(1, 0) &\text{ and } (-3, 0)
\end{aligned}$$

Try This

Here is a new function: $h(x) = -3(x + 4)^2 + 27$.

How many x-intercepts does this function have? Circle one.

No x-intercepts   One x-intercept   Two x-intercepts

Determine the x-intercept(s) of this function. If there are no x-intercepts, explain how you know.

Summary | Lesson 6

Some quadratic functions have no x-intercepts, but you can still determine their **zeros** algebraically. We can write these zeros using the imaginary unit i. We define i as $\sqrt{-1}$. This also means that $i^2 = -1$. Any multiple of i is an **imaginary number**.

Let's say you're trying to solve the quadratic equation $-6x^2 = 54$.

You can solve this equation algebraically and use i to express the imaginary solutions.

$$\begin{aligned} -6x^2 &= 54 \\ x^2 &= -9 \\ x &= \pm\sqrt{-9} \\ x &= \pm 3i \\ x = 3i &\text{ and } x = -3i \end{aligned}$$

Try This

Solve $7x^2 + 28 = 0$ algebraically. Use i to express imaginary solutions. Show your thinking.

Summary | Lesson 7

A **complex number** is a number that can be written in the form $a + bi$, where a and b are **real numbers** and i is the imaginary unit. We can call a the real part and bi the imaginary part.

Complex numbers include real numbers (like 12 or π), imaginary numbers (like $-3i$), and numbers like $-2 + 7i$ that have both a non-zero real part and a non-zero imaginary part.

Here are two equations with complex solutions that have both non-zero real parts and imaginary parts. You can solve them in the same way you solve equations with real or imaginary solutions.

$$\begin{aligned} (x+6)^2 - 4 &= -20 \\ (x+6)^2 &= -16 \\ x + 6 &= \pm\sqrt{-16} \\ x &= -6 \pm 4i \\ x = -6 + 4i &\text{ and } x = -6 - 4i \end{aligned}$$

$$\begin{aligned} x^2 - 6x + 12 &= 0 \\ x^2 - 6x + 12 - 3 &= 0 - 3 \\ x^2 - 6x + 9 &= -3 \\ (x-3)^2 &= -3 \\ x - 3 &= \pm\sqrt{-3} \\ x &= 3 \pm i\sqrt{3} \\ x = 3 + i\sqrt{3} &\text{ and } x = 3 - i\sqrt{3} \end{aligned}$$

Try This

Solve $(x-4)^2 - 6 = -42$. Use i to express non-real solutions. Show your thinking.

Summary | Lesson 8

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

You can solve quadratic equations in standard form ($ax^2 + bx + c = 0$) using a few different strategies, such as factoring and completing the square. You can also use the quadratic formula to solve these equations.

Here's an example:

$$2x^2 + 6x + 9 = 0$$

$$x = \frac{-(6) \pm \sqrt{(6)^2 - 4(2)(9)}}{2(2)}$$

$$x = \frac{-6 \pm \sqrt{36 - 72}}{2(2)}$$

$$x = \frac{-6 \pm \sqrt{-36}}{4}$$

$$x = \frac{-6 \pm 6i}{4}$$

Solutions: $x = \frac{-3 \pm 3i}{2}$

You can use the value of the *discriminant*, which is the expression under the radical ($b^2 - 4ac$), to determine the number and type of solutions the quadratic equation will have:

- If the discriminant is positive, there will be two real solutions.
- If the discriminant is negative, there will be two non-real solutions.
- If the discriminant is equal to 0, there will be one real solution.

Try This

Use the quadratic formula to solve the equation $-4x^2 + 3x - 5 = 0$. Use i to express non-real solutions.

The Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Summary | Lesson 9

Complex numbers are numbers that can be written in the form $a + bi$, where a and b are real numbers and the imaginary unit i equals $\sqrt{-1}$.

When adding and subtracting complex numbers, you can rewrite powers of i using equivalent expressions. For example, you can use the relationship $i^2 = -1$ to rewrite any i^2 terms as real numbers.

Think about the expression $i^5 - (3 + 2i^2)$. Here's how we can write an equivalent expression in the form $a + bi$:

$$\begin{array}{rlcc}
i^5 & = i^4 \cdot i & i^5 - (3 + 2i^2) & \\
 & = 1 \cdot i & i^5 - 3 - 2i^2 & \\
 & = i & (i) - 3 - 2(-1) & i^2 = -1 \\
 & & i - 3 + 2 & \\
 & & i - 1 & \\
 & & -1 + i &
\end{array}$$

Try This

Here is an expression: $8 - (11 - 7i) + 5i^2$.

Write an equivalent expression in the form $a + bi$.

Summary | Lesson 10

If you have an expression like $(2-4i)(5+3i)$, you can write an equivalent expression in the form $a+bi$ by multiplying.

When you multiply complex numbers like this, the product can be made up of only real parts, only imaginary parts, or both real *and* imaginary parts.

For example, the product of $(2-4i)(5+3i)$ has both real *and* imaginary parts:

$$(2-4i)(5+3i)$$
$$10-20i+6i-12i^2$$
$$10-14i-12i^2$$
$$10-14i+12$$
$$22-14i$$

	2	$-4i$
5	10	$-20i$
$3i$	$6i$	$-12i^2$

Try This

Write an expression in the form $a+bi$ that is equivalent to $(11+9i)(2-5i)$.

Summary | Lesson 11

Sometimes when you multiply complex expressions, like $(3 - 4i)$ and $(3 + 4i)$, the product is a real number. This happens when the two expressions are almost the same, but one has an addition sign and one has a subtraction sign.

For example, $(3 + 4i)(3 - 4i) = 9 + 12i - 12i - 16i^2$, which is equivalent to $9 + 16$, or 25.

We can also use a similar pattern to rewrite functions without x-intercepts in factored form. Just like when we're multiplying expressions, the two factors are almost the same, but one has an addition sign and one has a subtraction sign.

For example, $j(x) = 16x^2 + 36$ can be rewritten in factored form as $j(x) = (4x + 6i)(4x - 6i)$. $j(x)$ has no x-intercepts, but it does have two zeros: $x = -\frac{3}{2}i$ and $x = \frac{3}{2}i$.

Try This

Rewrite the function $f(x) = 36x^2 + 1$ in factored form.

Try This | Answer Key

Lesson 1

1.35 feet

Lesson 2

a $y = (2x + 3)(x - 5)$

b x-intercepts: $(5, 0)$ and $\left(-\frac{3}{2}, 0\right)$
y-intercept: $(0, -15)$

Lesson 3

$(9x - 11)(9x + 11)$

Lesson 4

$y = (x + 0.5)^2 - 12.25$

Lesson 5

Two x-intercepts
$(-1, 0)$ and $(-7, 0)$

Lesson 6

$x = 2i$ and $x = -2i$. *Work varies.*

$7x^2 = -28$
$x^2 = -4$
$x = \pm\sqrt{-4}$
$x = \pm 2i$
$x = 2i$ and $x = -2i$

Lesson 7

$x = 4 + 6i$ and $x = 4 - 6i$. *Work varies.*

$(x - 4)^2 = -36$
$x - 4 = \pm\sqrt{-36}$
$x - 4 = \pm 6i$
$x = 4 \pm 6i$
$x = 4 + 6i$ and $x = 4 - 6i$

Try This | Answer Key

Lesson 8

$x = \frac{-3 \pm i\sqrt{71}}{-8}$ (or equivalent). *Work varies.*

$x = \frac{-3 \pm \sqrt{(3)^2 - 4(-4)(-5)}}{2(-4)}$

$x = \frac{-3 \pm \sqrt{9 - 80}}{-8}$

$x = \frac{-3 \pm \sqrt{-71}}{-8}$

$x = \frac{-3 \pm i\sqrt{71}}{-8}$

Lesson 9

$-8 + 7i$

Lesson 10

$67 - 37i$

Lesson 11

$f(x) = (6x + i)(6x - i)$

Algebra 2 | Unit 2

Glossary

Note: *Spanish glossary coming fall 2026*

C

circle A shape made out of all the points that are the same distance from a center point. The equation of a circle can be written as $(x-a)^2+(y-b)^2=r^2$.

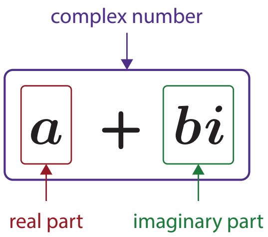
completing the square The process of rewriting a quadratic expression or equation to include a perfect square.

$$y = x^2 - 6x + 5$$
$$y = (x^2 - 6x + \_) + 5 - \_$$
$$y = (x^2 - 6x + 9) + 5 - 9$$
$$\boxed{y = (x-3)^2 - 4}$$

complex conjugates A pair of complex numbers in the form $a+bi$ and $a-bi$. The product of two complex conjugates is a real number, a^2+b^2.

$3+4i$ and $3-4i$ are complex conjugates.
$(3+4i)(3-4i) = 9+12i-12i-16i^2 = 9+16 = 25$

complex number A number that can be written in the form $a+bi$, where a and b are real numbers and i is the imaginary unit. Sometimes a is called the *real part* and bi is called the *imaginary part*.



$-7+11i$ is a complex number with a real part of -7 and an imaginary part of $+11i$.

$-2i$ is a complex number with a real part of 0 and an imaginary part of $-2i$.

13 is a complex number with a real part of 13 and an imaginary part of $0i$.

D

discriminant The quantity b^2-4ac in the quadratic formula. The sign of the discriminant helps us to determine how many real solutions a quadratic equation has.

$$x = \frac{-b \pm \sqrt{\boxed{b^2-4ac}}}{2a} \longleftarrow \text{discriminant}$$

- If the discriminant is positive, then there are two real solutions.
- If the discriminant is zero, then there is one real solution.
- If the discriminant is negative, then there are no real solutions.

F

factor (of a number or expression) A number or expression multiplied with other numbers or expressions to make a product.

1, 2, 4, and 8 are all factors of the number 8 because $1 \cdot 8 = 8$ and $2 \cdot 4 = 8$. Also, 2, $(x+3)$, and $(x-5)$ are factors of $2x^2-4x-30$ because $2(x+3)(x-5) = 2x^2-4x-30$.

factored form When a quadratic or other polynomial expression is written as a product of factors. This is a helpful form for identifying zeros and x-intercepts.

For example, $f(x) = 2(x-1)(x+3)$ and $g(x) = (2x-5)(x^2+1)$ are in factored form.

G

greatest common factor A number or expression that is a common factor between two other numbers or expressions, and is greater than all the other factors they have in common.

The greatest common factor of 48 and 60 is 12.

The greatest common factor of $3(x+5)^2$ and $3x^2+9x-30$ is $3(x+5)$.

I

identity An equation that is true for all defined values of its variables. Identities are often used to rewrite expressions in equivalent forms.

These equations are identities:

- $a^2-b^2 = (a-b)(a+b)$
- $(a+bi)(a-bi) = a^2+b^2$
- $\sin^2(\theta)+\cos^2(\theta) = 1$

imaginary number Any multiple of the number i, which is defined as $i = \sqrt{-1}$ and is sometimes called the *imaginary unit*. The square of an imaginary number is a negative real number.

$6i$ is an imaginary number. Squaring it produces a negative real number: $(6i)^2 = 36i^2 = 36(-1) = -36$.

N

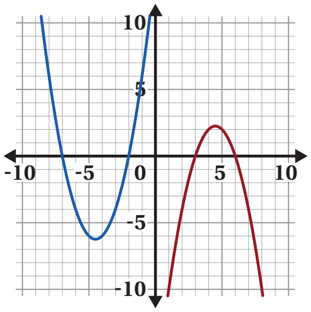
non-real number A number of the form $a+bi$, where b is not zero. A non-real number can be expressed using i.

These are all non-real numbers:

- $3 - i\sqrt{2}$
- $5i$
- $\frac{3}{4} + \frac{1}{4}i$
- $2+i$

P

parabola The graph of a quadratic function, which is a U-shaped curve.



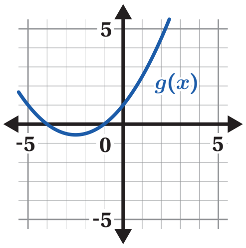
Q

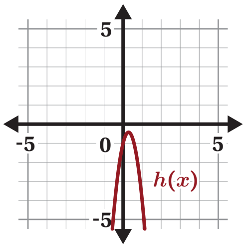
quadratic formula A formula that can be used to determine the solutions of a quadratic equation $ax^2 + bx + c = 0$, where $a \neq 0$.

The quadratic formula is: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

quadratic function A function that is a transformation of $y = x^2$. The graph of a quadratic function is a parabola.

For example, $g(x) = \frac{1}{4}(x + 4)(x + 1)$ and $h(x) = -7x^2 + 4x - 1$ are quadratic functions.





R

real number A number on the real number line. Real numbers include integers, rational numbers, and irrational numbers.

0, -1, 15.3, $\frac{7}{8}$, $\sqrt{2}$, and π are all real numbers.

S

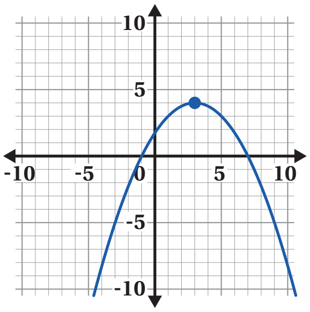
standard form A quadratic or other polynomial expression written as a sum using the fewest number of terms in order of descending powers of x.

$x^2 + 2x - 4x + 8$ and $-4x^6 + x^3 - 7x$ are expressions in standard form.

V

vertex The maximum or minimum point on the graph of a quadratic function. The vertex is also where the function changes from increasing to decreasing, or vice versa.

The vertex of the graph is $(3, 4)$.



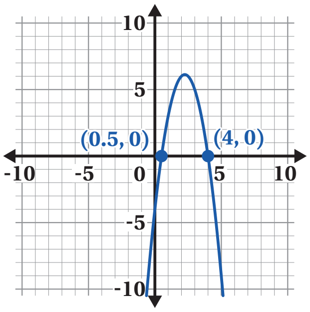
vertex form A quadratic equation in vertex form looks like $f(x) = a(x - h)^2 + k$. This is a helpful form for determining the vertex, line of symmetry, and transformations of the graph.

The equations $f(x) = -(x - 4)^2 + 7$ and $g(x) = 2x^2 - 1$ are in vertex form.

X

x-intercept A point where the graph of an equation or function crosses the x-axis, or when $y = 0$.

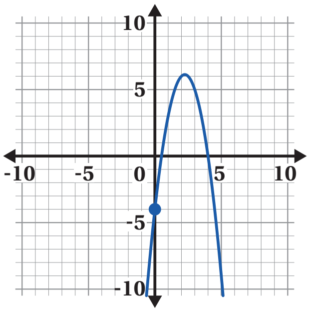
The x-intercepts of the equation $y = -2x^2 + 9x - 4$ are $(0.5, 0)$ and $(4, 0)$, or just $\frac{1}{2}$ and 4.



Y

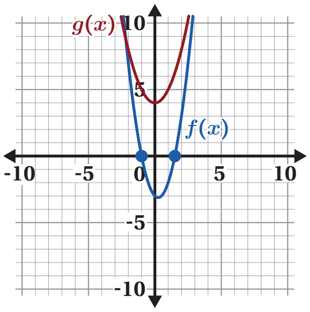
y-intercept A point where the graph of an equation or function crosses the y-axis, or when $x = 0$.

The y-intercept of the equation $y = -2x^2 + 9x - 4$ is $(0, -4)$, or just -4.



Z

zero An x-value that makes an expression or function equal to zero. Zeros are sometimes referred to as *roots* of a function.



The zeros of $f(x) = (2x - 3)(x + 1)$ are $\frac{3}{2}$ and -1. These are also the x-intercepts on the graph of $f(x)$.

The zeros of $g(x) = x^2 + 4$ are $-2i$ and $2i$ because $g(-2i) = 0$ and $g(2i) = 0$. These are not x-intercepts because they are imaginary numbers.