

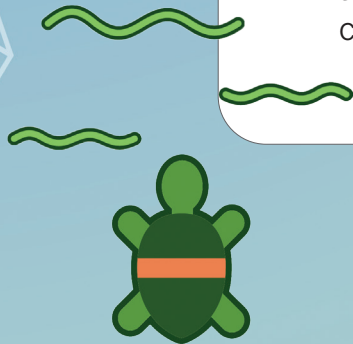
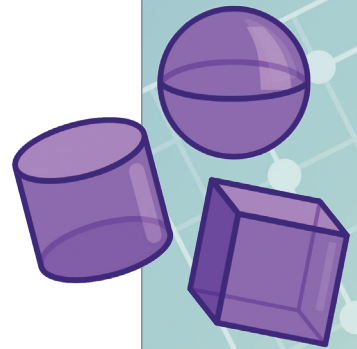
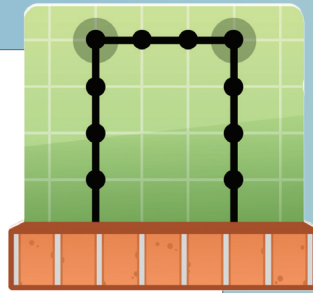
Unit **5**

Functions and Volume

You will learn about functions for the first time. You will analyze representations of functions, and examine functions in the context of the volume of cylinders, cones, and spheres.

Essential Questions

- What makes a relationship a function?
- How are functions useful in representing situations?
- What are some relationships between a cylinder, a cone, and a sphere with common dimensions?

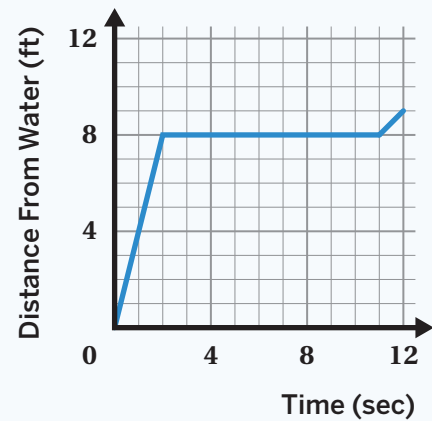


Summary | Lesson 1

You can use a graph to represent a situation. Analyzing a point on a graph or pieces of a graph can help you interpret part of the situation.

For example, this graph represents a turtle's journey across sand. A turtle walks for 2 seconds until it is 8 feet from the water. It stops for 9 seconds and then continues walking away from the water.

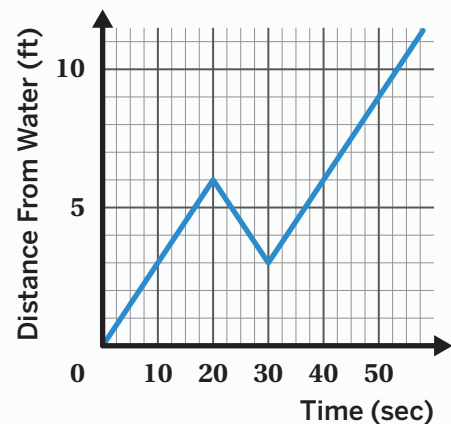
The point $(6, 8)$ represents the turtle's distance of 8 feet from the water after 6 seconds.



Try This

Here is the graph of a turtle's journey away from the ocean.

- a** What story does this graph tell about the turtle's journey?
- b** What is the turtle's distance from the water at 40 seconds?
- c** When is the turtle's distance from the water 3 feet?



A function is a rule that assigns exactly one output for each possible input. Another way to say this is that the output is a function of the input.

Function

Input	Output
15	7
10	7
20	8
5	9

In this function table, each input appears with exactly one output. Even if an input is repeated in the table, the input should give the same output as previously seen within the table.

Not a function

Input	Output
10	6
10	7
20	8
5	9

Notice in this table, the input 10 appears twice with two different outputs.

Try This

Decide if each rule represents a function or not. Explain your thinking.

Rule A

Possible Inputs: Any person

Outputs: The month that person was born in

- a** Does Rule A represent a function? Explain your thinking.

Rule B

Possible Inputs: Any month

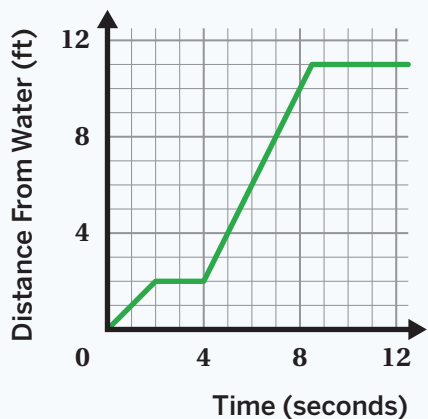
Outputs: A person born in that month

- b** Does Rule B represent a function? Explain your thinking.

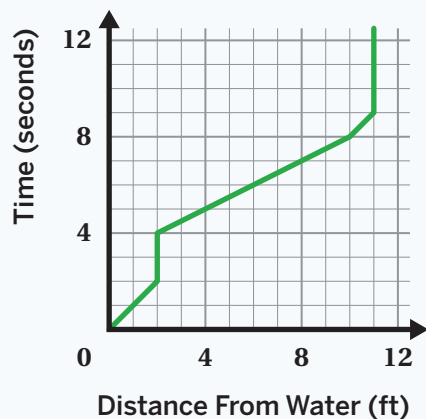
Summary | Lesson 3

A graph represents a function when each x -value, or input, only has one corresponding y -value, or output. If a graph has multiple y -values for the same x -value, it does not represent a function.

Here are two graphs of the same turtle's journey.



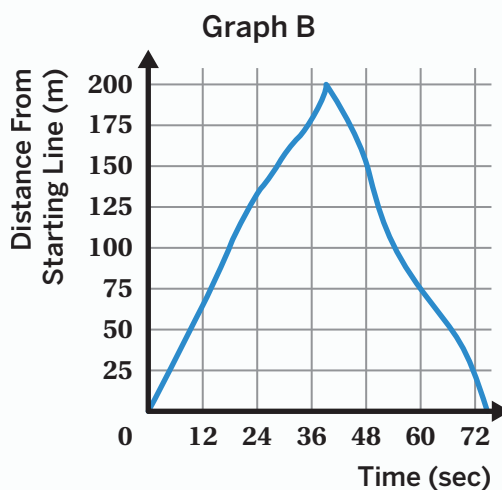
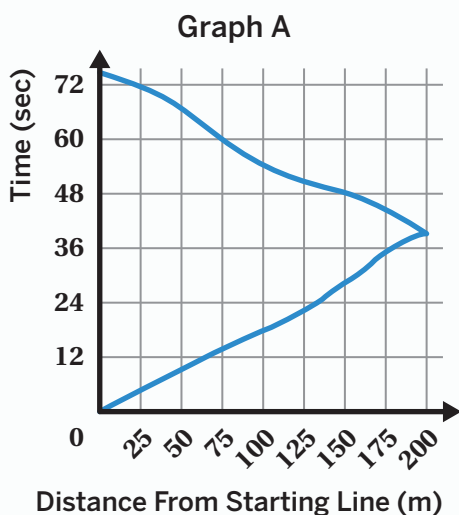
This graph represents a function because for every second, x , the turtle is at only one corresponding distance, y .



This graph does not represent a function because at both 2 feet and 11 feet, the turtle has multiple corresponding times.

Try This

Arianna is running once around the track. The graphs show the relationship between her time and her distance from the starting line.



Which graph represents a function? Explain your thinking.

In a situation represented by a function, the input is often called the **independent variable** and the output is called the **dependent variable**. The independent variable and dependent variable can change depending on the problem you are trying to solve.

The independent variable is an input. The dependent variable, or output, depends on the input.

For example, in this situation, m represents the total number of miles walked and d represents the number of days of walking for someone who walks 2 miles a day.

Question	Independent and Dependent Variable	Equation	Explanation
How many miles have I walked, m , after d days?	Independent: Days, d Dependent: Miles, m	$m = 2d$	The number of miles walked depends on the number of days of walking.
How many days, d , will it take me to walk m miles?	Independent: Miles, m Dependent: Days, d	$d = \frac{m}{2}$	The number of days depends on the number of miles.

Try This

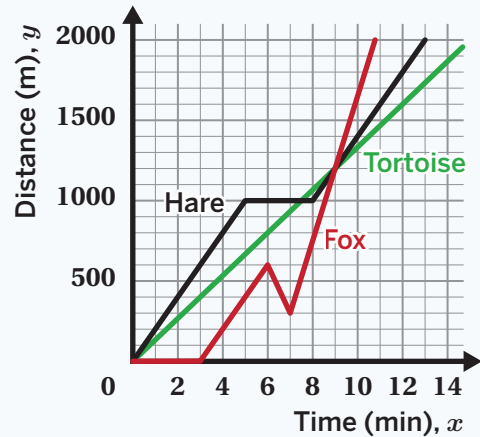
In each situation, complete the table with possible independent and dependent variables.

Question or Equation	Independent Variable	Dependent Variable
How many sandwiches can I make?	Number of bread slices	Number of sandwiches
How much does my ice cream cost if I get different amounts of toppings?		Cost of my ice cream cone
How does sleep affect performance on tests?		
$y = 3x + 5$		

Summary | Lesson 5

A graph can be helpful when comparing multiple functions in a situation, such as by comparing the initial value, slope, and points of intersection.

For example, this graph represents a race between a hare, a tortoise, and a fox. From 0 to 5 minutes, the hare is moving at a steady pace of 200 meters per minute and is in first place. At 9 minutes, the race is tied. The fox does not begin the race until three minutes have passed, but it speeds up at 7 minutes to a pace of 450 meters per minute. The fox wins the race at about 11 minutes.

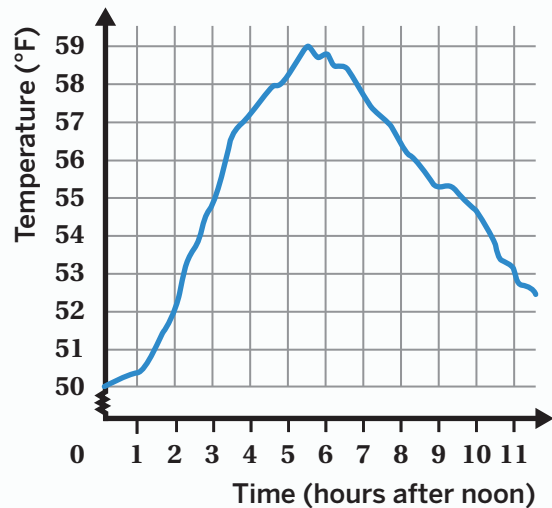


Try This

This graph shows the temperature between noon and midnight on Friday.

- a** Tell a story about the temperature on this day.

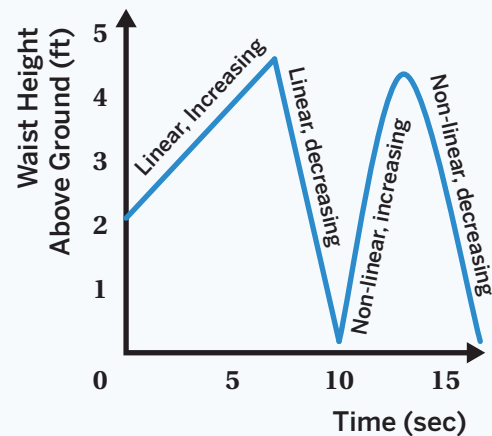
- b** Was it warmer at 3 PM or 9 PM? Explain your thinking.



We can use graphs to represent a story. When drawing a graph, it can be helpful to identify the variables involved so that you can label the axes. Depending on the independent and dependent variables, different graphs can represent distinct details of the same story. It may also be helpful to identify key points in the story according to these chosen variables to help you sketch these features.

The function is:

- **Increasing** when part of the graph is going up from left to right.
- **Decreasing** when part of the graph is going down from left to right.
- **Linear** when part of the graph is a straight line. (Note: A vertical line is not a function.)
- **Non-linear** when part of the graph is not a straight line.

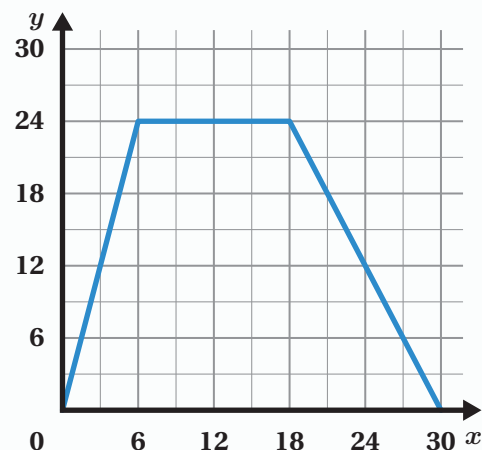


Try This

Using some of the terms below, write a story that can be modeled by the graph.

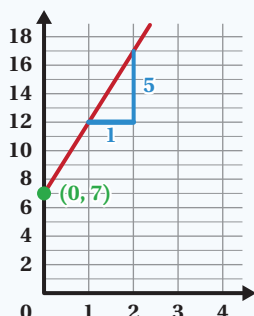
Word Bank

linear	non-linear
increasing	decreasing



Linear relationships that have exactly one output for every possible input are called **linear functions**. All linear functions can be represented with a graph, a table, and an equation in the form $y = mx + b$, where m is the rate of change and b is the initial value.

Graph



Table

x	y
0	7
1	12
2	17

Equation

$$y = 5x + 7$$

The slope, or rate of change, is a ratio between the difference of the y -values and the difference of the x -values. In a graph, you can use slope triangles to find the rate of change. In the equation $y = mx + b$, it is the coefficient of the independent variable. In this example, the slope is $\frac{5}{1} = 5$.

The initial value, or y -intercept, is the dependent value when the independent value is 0. In the equation $y = mx + b$, the y -intercept is the constant, b . In this example, the y -intercept is 7.

Try This

Shanice and Abdul both open bank accounts on the same day.

Shanice's Account

$S = 9w + 60$ models the amount of money in Shanice's bank account, where S is the amount of money and w is the number of weeks since the account opened.

- a** Who starts with more money in their account? Explain your thinking.

- b** Who is saving money at a faster rate? Explain your thinking.

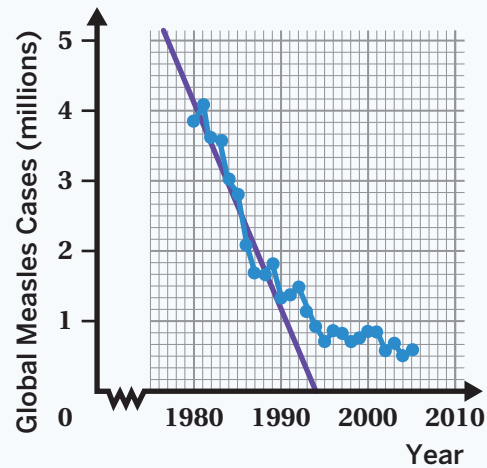
Abdul's Account

Weeks Since Opening Account	Amount in Account (\$)
0	65
2	75
4	85
6	95
8	105

Sometimes a situation can be modeled by a linear function. Even if a function is non-linear, parts of its data can be modeled by a linear function, which can be used to help make predictions.

For example, you can use a linear function to model a section of this data on global measles cases from 1980 to 1990.

You can use this model to estimate that there would have been approximately 2.5 million cases of measles in 1985.



Source: Our World in Data

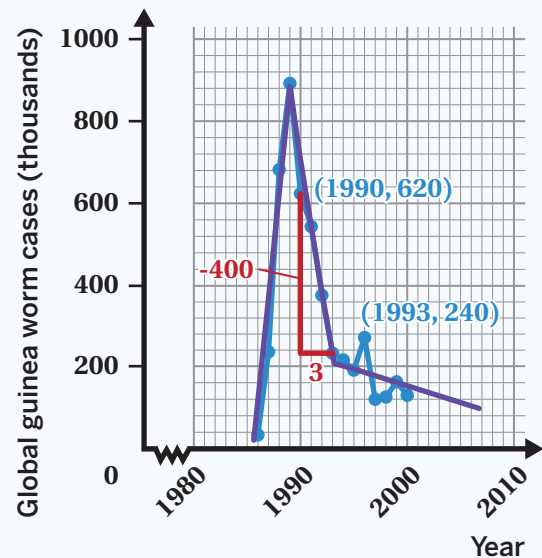
Try This

- a** Select *all* situations that could be modeled by a linear function.
- ☐ **A.** Daeja takes large handfuls of popcorn out of her lunch bag every 5 minutes, and then she starts taking smaller handfuls.
 - ☐ **B.** A plant grows the same amount every week.
 - ☐ **C.** The day started very warm, slowly got colder in the afternoon, and then the temperature dropped suddenly when the sun set.
 - ☐ **D.** A tablet charges at a rate of 5% per minute until it reaches 80%, and then it charges at a slower rate.
 - ☐ **E.** Marco is filling up a water bottle at the fountain.
- b** What are some limitations of using a linear function to model one of the situations you selected?

You can use one or more linear segments to represent a data set. Using multiple linear segments can help you precisely represent a data set.

For example, you can use multiple linear segments to model this data about cases of the Guinea Worm disease over time.

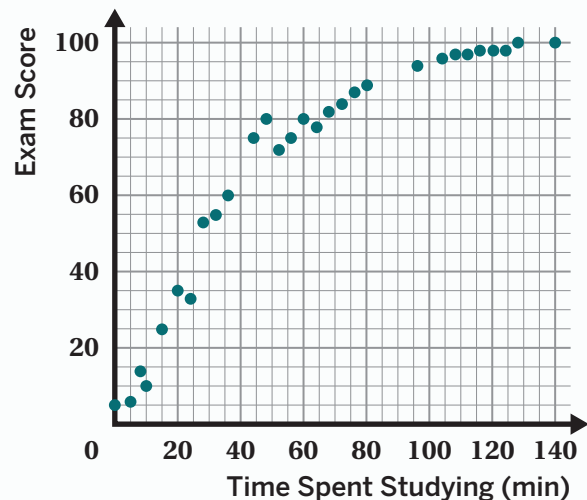
You can use this model to estimate that between 1990 and 1993, cases of Guinea Worm disease were changing at a rate of approximately $-\frac{400}{3}$ cases per year, or dropping by about 133.33 million cases per year.



Try This

Here is a graph that models exam scores and time spent studying.

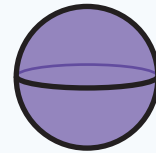
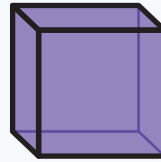
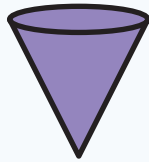
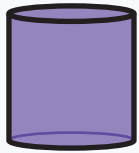
- Use at least three line segments to draw a function on the graph that models the data set.
- A student spent 85 minutes studying. Predict their exam score.



The *volume* of an object is the number of cubic units that fill its three-dimensional region without any gaps or overlaps.

You can often determine relationships between the volumes of different figures with similar measurements. For example, if the base of a **cone** and a **cylinder** have the same *diameter* and height, then the cylinder will have a volume that is three times greater than the cone.

There are also relationships between the volumes of the same figure with different measurements. For example, if the diameter of a **sphere** is doubled, or if the side length of a cube is doubled, the original volume of these figures will be multiplied by 8.



Try This

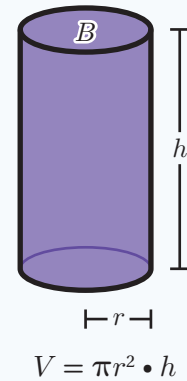
An ice cream cone and a can of soup have the same diameter.



Describe the relationship between the volume of these two items. Explain your thinking.

A prism has two congruent bases connected by perpendicular lines. Its volume can be determined by multiplying the area of its base by its height. A cylinder has two congruent circles for its base and the sides are perpendicular to the bases. This means you can also determine the volume of a cylinder by using the area of its base multiplied by its height.

If you know the radius and height of a cylinder, then you can determine the volume of the cylinder. The base area is determined using the expression $\pi \cdot r^2$. The volume, in cubic units, can be determined by multiplying the base area by the height, h . The formula for the volume of a cylinder is $V = \pi r^2 \cdot h$.

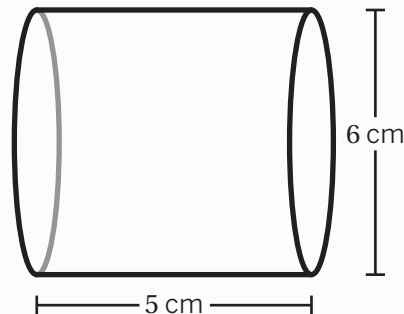


Try This

Here is a cylinder.

a Find the area of the base of the cylinder.

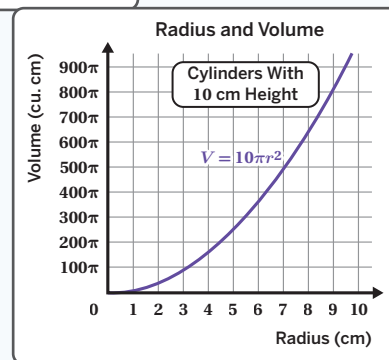
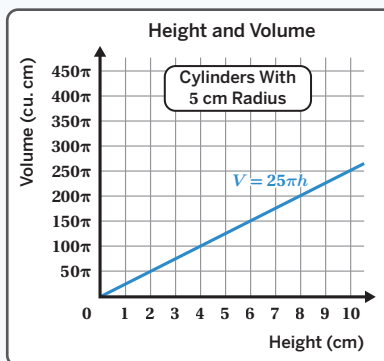
b Find the volume of the cylinder.



The volume of a cylinder depends on the cylinder's radius and height. The formula for the volume of a cylinder is $V = \pi r^2 h$, where r represents the radius and h represents the height.

When a cylinder's height, h , increases at a constant rate, the cylinder's volume, V , also increases at a constant rate. This means there is a proportional linear relationship between the height and volume. That's why we can represent the relationship between volume and height with a straight line.

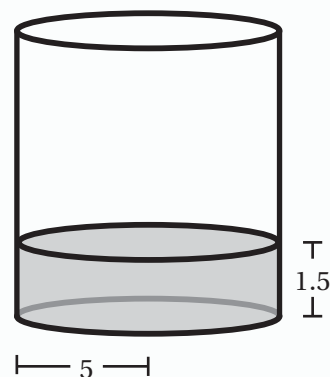
On the other hand, we *cannot* represent the relationship between a cylinder's radius and volume with a line because the ratio of the volume to the radius changes as the radius increases. That's why the graph of the relationship between radius and volume is curved and non-linear.



Try This

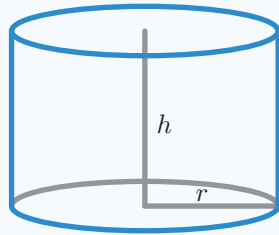
Here is a flower vase that is shaped like a cylinder.

- If you triple the height of the water, will you triple the volume of the water inside the container? Explain your thinking.
- If you triple the radius of the flower vase, will you triple the volume inside the container? Explain your thinking.



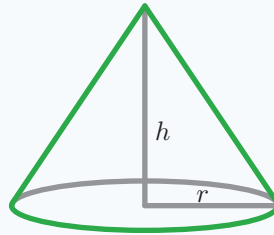
We learned that we can find the volume of a cylinder by calculating $V = \pi r^2 \cdot h$. If a cone and a cylinder have the same base and the same height, then the volume of the cone is one-third the volume of the cylinder.

If the radius and the height are known, we can determine the volume by using this formula for a cone: $V = \frac{1}{3}\pi r^2 \cdot h$.



Volume of a cylinder:

$$V = \pi r^2 h$$



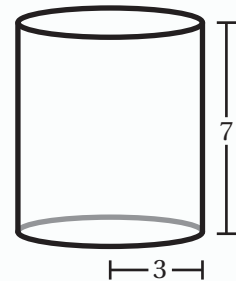
Volume of a cone:

$$V = \frac{1}{3}\pi r^2 h$$

Try This

Here is a cylinder.

Find the volume of a cone with the same dimensions as the cylinder.
Draw the cone if it helps with your thinking.

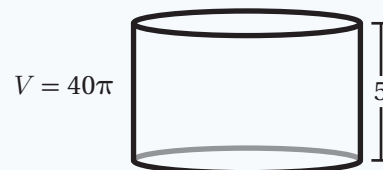


Summary | Lesson 14

The volume of a cylinder and a cone depend on their radius and height. In both cases, if you know the radius and the height, you can determine the volume using the formula $V = \pi r^2 h$ (for cylinders) and $V = \frac{1}{3}\pi r^2 h$ (for cones).

And if you happen to know the volume and either the radius or the height, you can determine the other dimensions, too.

For example, if a cylinder has a height of 5 inches and a volume of 40π , you can calculate the area of the base by dividing the volume by the height: $\frac{40\pi}{5} = 8\pi$.



$$40\pi = B \cdot 5$$

$$8\pi = B$$

Try This

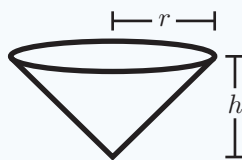
Find the missing dimensions of each object (rounded to the nearest tenth). Use 3.14 as an approximation for π . Draw the figures if it helps with your thinking.

- a** A cylinder has a radius of 4 centimeters and a volume of 80π cubic centimeters.
What is the height of the cylinder?

- b** A cylinder with a volume of 405 cubic inches has a diameter of 10 inches.
What is the height of the cylinder?

- c** A cone with a volume of 135π cubic inches has a height of 5 inches.
What is the radius of the cone?

You can determine the volume of a sphere using the formula $V = \frac{4}{3}\pi r^3$. If the radius and height of a cone, hemisphere, and cylinder are all the same, you can make sense of the volume formulas for each solid by seeing how much of the others they fill.

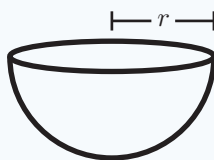


Volume of cone

$\left(\frac{1}{3} V \text{ of cylinder}\right)$

$$V = \frac{1}{3}\pi \cdot r^2 h$$

+

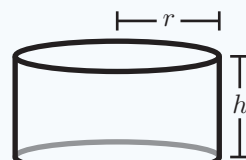


Volume of hemisphere

$\left(\frac{2}{3} V \text{ of cylinder}\right)$

$$V = \frac{2}{3}\pi \cdot r^2 h$$

=



Volume of cylinder

$$V = \pi \cdot r^2 h$$

The volume of the cone is $\frac{1}{3}$ of the volume of the cylinder. Since the volume of the cone and hemisphere together is equal to the cylinder, the volume of the hemisphere must be $1 - \frac{1}{3} = \frac{2}{3}$ of the volume of the cylinder. Then the volume of a sphere is twice the volume of a hemisphere, or $\frac{4}{3}$ the volume of the cylinder.

Try This

A cylinder, a cone, and a sphere all have the same radius of 6 inches. The cylinder and the cone have a height of 5 inches. Calculate the volume of each in terms of π .

- a** What is the volume of the cone?
- b** What is the volume of the cylinder?
- c** What is the volume of the sphere?

Lesson 1

- a *Responses vary.* The turtle started at the edge of the ocean and traveled away from the water for 20 seconds. At 20 seconds, the turtle was 6 feet from the water. The turtle walked 3 feet back toward the ocean between 20 and 30 seconds. After 30 seconds, the turtle started crawling away from the ocean again.
- b At 40 seconds, the turtle is 6 feet away from the water.
- c The turtle is 3 feet away from the water at 10 seconds and at 30 seconds.

Lesson 2

- a *Yes. Explanations vary.* Rule A is a function because each person only has one birth month. So each input has only one possible output.
- b *No. Explanations vary.* Rule B is not a function because many people are born in the same month. So each input has many possible outputs.

Lesson 3

Graph B. *Explanations vary.* Graph B represents a function because every time corresponds to only one distance. Graph A doesn't represent a function because there are some distances that correspond with more than one time. For example, Arianna is 0 meters from the starting line at 0 seconds and at approximately 75 seconds.

Lesson 4

Question or Equation	Independent Variable	Dependent Variable
How many sandwiches can I make?	Number of bread slices	Number of sandwiches
How much does my ice cream cost if I get different amounts of toppings?	Amount of toppings	Cost of my ice cream cone
How does sleep affect performance on tests?	Amount of sleep	Test performance
$y = 3x + 5$	x	y

Lesson 5

- a *Responses vary.* At noon, the temperature is around 50°F and gets warmer. The temperature climbs up to around 59°F . Between 5:30 PM and 6 PM, the temperature starts to decrease. At midnight, the temperature is around 52.5°F .
- b At 9 PM, the temperature is around 55.5°F . That's warmer than the temperature at 3 PM, which is around 54.5°F .

Lesson 6

Responses vary. I fill up a tub with 24 gallons of water in 6 minutes. This can be represented by an increasing linear model. I bathed my dog in the tub for 12 minutes. I unplug the tub, and it takes 12 minutes to drain. This can be represented by a decreasing linear model.

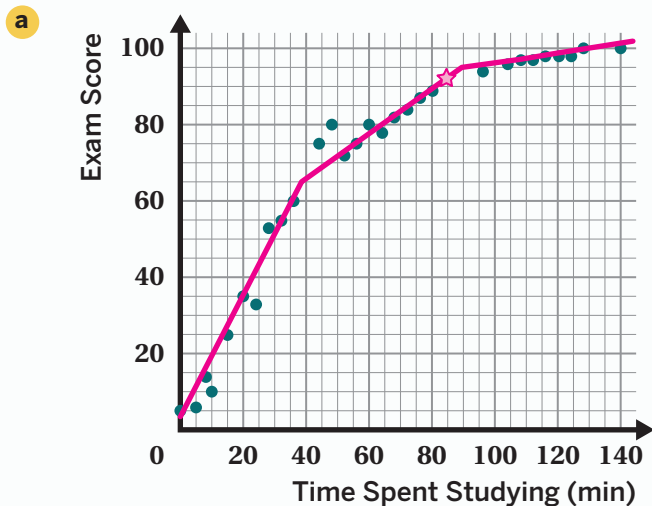
Lesson 7

- a Abdul starts with more money in his bank account. According to the table, Abdul's account has \$65 at week 0. According to the equation, Shanice's account has \$60 at week 0.
- b Shanice is saving money at a faster rate. According to the equation, Shanice's account increases by \$9 every week. According to the table, Abdul's account increases by \$10 every two weeks, or \$5 every week.

Lesson 8

- a B. A plant grows the same amount every week.
E. Marco is filling up a water bottle at the fountain.
- b *Responses vary.*
B: A plant's height may not grow at exactly the same rate for its entire life.
E: The water fountain may stutter or fill at a slower rate when it is first started.

Lesson 9



b Approximately 92

Lesson 10

Responses vary.

- The volume of the can is 3 times the volume of the cone.
- The volume of the cone is $\frac{1}{3}$ the volume of the can.
- The can of soup is a cylinder and the ice cream cone is a cone. The relationship between the volume of a cylinder is 3 times the volume of a cone with the same diameter.

Lesson 11

a $9\pi \text{ cm}^2$

Caregiver Note: The base of the cylinder is a circle. The area of the circle is $\pi \cdot r^2$. This circle has a diameter of 6, so the radius is 3. $\pi \cdot 3^2 = 9\pi$.

b $45\pi \text{ cm}^3$.

Caregiver Note: The volume of the cylinder is the area of the base multiplied by the height. The area of the base is $9\pi \text{ cm}^2$. The height is 5 cm. $9\pi \text{ cm}^2 \cdot 5 = 45\pi \text{ cm}^3$.

Lesson 12

- a Yes. *Explanations vary.* The relationship between the height and the volume of a cylinder is linear, so if you multiply the height by a scale factor, the volume will change by the same factor.
- b No. *Explanations vary.* The relationship between the radius and the volume of a cylinder is not linear, so if you multiply the radius by a scale factor, the volume will not change by the same scale factor.

Lesson 13

21π cubic units.

Caregiver Note: The volume of the cone is $V = \frac{1}{3}\pi \cdot r^2 \cdot h$. The radius is 3 and the height is 7, so $V = \frac{1}{3}\pi \cdot 3^2 \cdot 7 = 21\pi$ cubic units.

Lesson 14

- a 5 centimeters
- b Approximately 5.2 inches
- c 9 inches

Lesson 15

- a 60π cubic inches
- b 180π cubic inches
- c 288π cubic inches