Unit 1

# Scale Drawings

Ratios are all around us. You have used ratios to solve all kinds of real-world problems. But what do ratios have to do with geometry? You will explore how ratios are used to resize images and figures, making them smaller or larger. Resizing can help you make more sense of what you are looking at.

**Essential Questions**

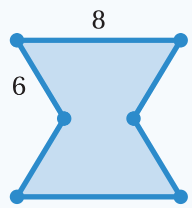
- How can you tell whether a figure is a scaled copy of another figure?
- How are lengths, angles, and areas affected when creating scaled copies?
- How do scales and scale drawings help bring the world to a more manageable size?

## Summary | Lesson 1

**Scaled copies** look similar to their originals. Though their overall size may change, their shape does not. If a shape looks squished or stretched when compared to its original, it's likely not a scaled copy.

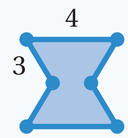
We know that two shapes are scaled copies when the lengths of their matching sides are **proportional** or form *equivalent ratios*. This means that the ratio of two sides on the original shape is equivalent to the ratio of the same two sides on the copy.

**Original**



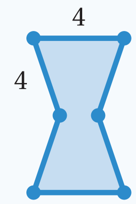

Two of the sides in this figure have a ratio of $\frac{6}{8}$. Scaled copies should have an equivalent ratio.

**Scaled Copy**




The ratio here is $\frac{3}{4}$. This is a scaled copy because the ratio is equivalent to the original.

**Not a Scaled Copy**

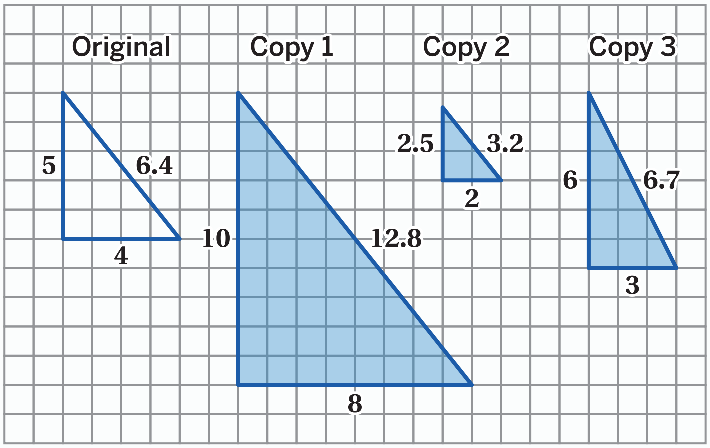



The ratio here is $\frac{4}{4}$. This is not a scaled copy because this ratio is not equivalent to the original.

## Try This

Here is a triangle and three copies.

Select *all* the triangles that are scaled copies of the original.

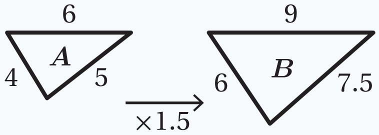



- ☐ **A.** Copy 1
- ☐ **B.** Copy 2
- ☐ **C.** Copy 3

## Summary | Lesson 2

When we create scaled copies, the **scale factor** is the number that every length of the original shape is multiplied by to produce the scaled copy.

For example, the scale factor from triangle $A$ to triangle $B$ is $1.5$.

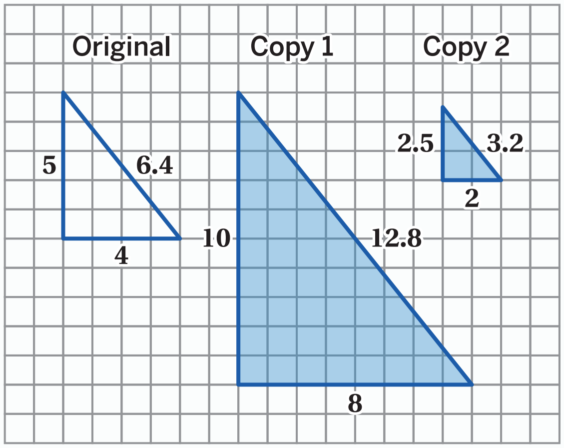



Another way to see the scale factor is to look for the ratio between *corresponding* sides in the two shapes. The ratio of the new length to the original length is the scale factor.

For triangle $A$ and triangle $B$, the ratios are $\frac{9}{6}$, $\frac{7.5}{5}$, and $\frac{6}{4}$. Since those ratios are all equivalent, any one can be used as the scale factor. You can also use another equivalent ratio as the scale factor, like $\frac{3}{2}$ or $1.5$.

## Try This

Here is a triangle and two scaled copies.




What is the scale factor from the original to:

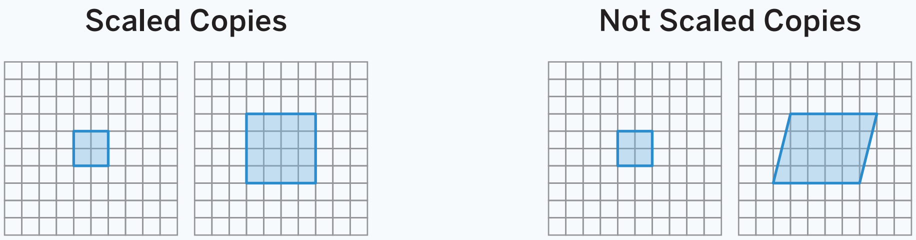
a. Copy 1?

b. Copy 2?

## Summary | Lesson 3

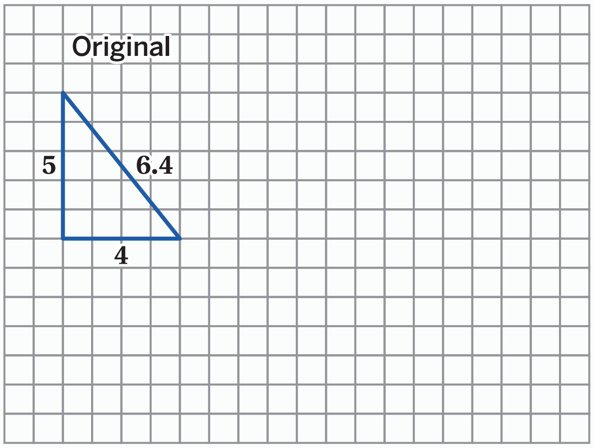
To create a scaled copy, we multiply all the side lengths in a shape by the same *scale factor*. This will create new side lengths, while keeping the angle measures and ratio between sides the same as the original.

To draw an accurate scaled copy, it's helpful to use measuring tools or a grid to make sure your drawing has the correct side lengths and angles.




## Try This

Sketch a scaled copy of the original triangle using a scale factor of $1.5$.

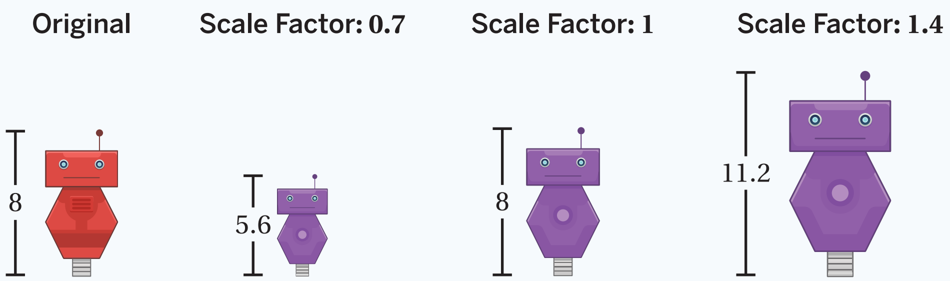


# Summary | Lesson 4

You can use different scale factors to create copies that are smaller, larger, or the same size as the original.

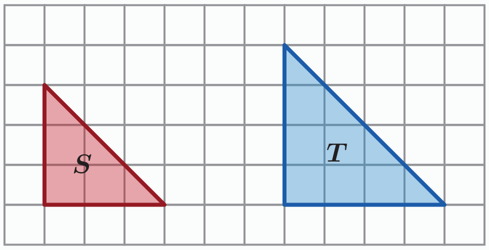
If the scale factor is:

- *Less* than $1$, the copy will be *smaller* than the original.
- *Equal to* $1$, the copy will be *the same size* as the original.
- *Greater* than $1$, the copy will be *larger* than the original.




# Try This

Triangle $T$ and triangle $S$ are scaled copies of one another.

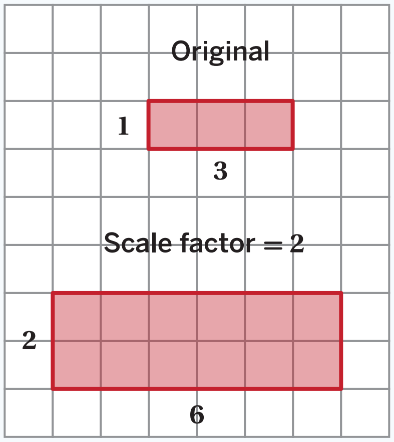



**a** What is the scale factor that takes triangle $S$ to triangle $T$?

**b** What is the scale factor that takes triangle $T$ to triangle $S$?

## Summary | Lesson 5

The scale factor describes how the side lengths of a shape change when it's scaled, but it doesn't directly describe how the area will change. You can use reasoning about the scale factor to find the area of a scaled copy. Here are some strategies to find the area of a scaled copy:




### Scale the Side Lengths

- Multiply each side length by the scale factor.

  $1 \cdot 2 = 2$

  $3 \cdot 2 = 6$

- Calculate the area.

  $2 \cdot 6 = 12$

### Square the Scale Factor

- Determine the area of the original.

  $1 \cdot 3 = 3$

- Square the scale factor.

  $2 \cdot 2 = 4$

- Multiply the original area by the squared scale factor. $3 \cdot 4 = 12$

We can use either strategy to determine that the area of this scaled copy is 12 square units.

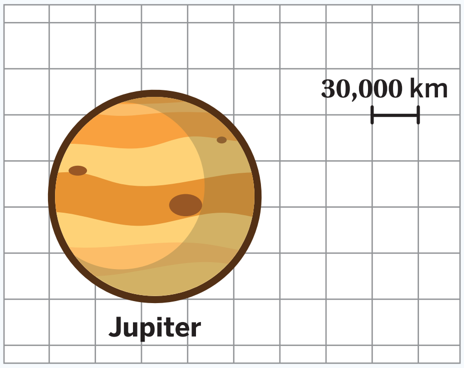
## Try This

A rectangle is 2 units wide and 5 units long. What is the area of the scaled copy of the rectangle that has a scale factor of 3?

# Summary | Lesson 6

Some objects are so big or so small that it's hard to find the right scale factor to represent them in a drawing. Instead, we can use a **scale** to tell how actual measurements are represented in a drawing.

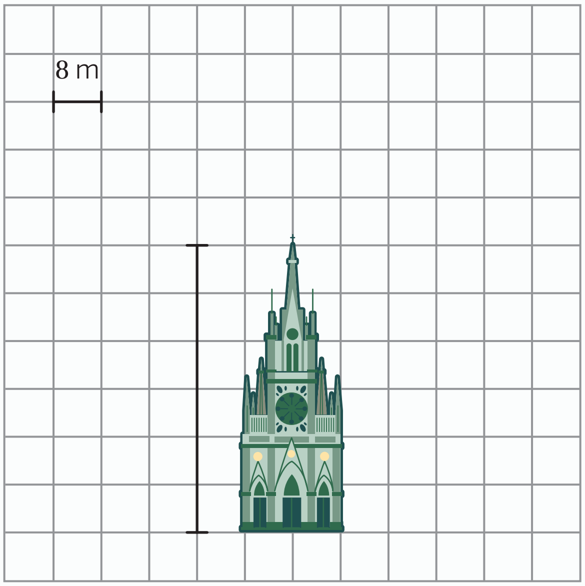
A scale is often shown with a line segment that indicates what distance 1 unit in the drawing represents in the actual object. Scales can also be written in units of measure, like inches or centimeters.




For example, this drawing of Jupiter uses a scale of $1$ unit to $30{,}000$ kilometers. This means that every $1$ unit on the drawing represents $30{,}000$ kilometers. Since the diameter of Jupiter is about $4.5$ units on the grid, the actual diameter of Jupiter is about $30000 \cdot 4.5 = 135000$ kilometers.

## Try This

Here is a scale drawing of Las Lajas Sanctuary in Nariño, Colombia. Use the drawing to estimate the actual height of the building.



# Summary | Lesson 7

**Scale drawings** are two-dimensional representations of actual objects or places. Floor plans and maps are examples of scale drawings you might have seen before.

On a scale drawing:

- Every part of the drawing matches up with a part of the actual object.
- Distances on the drawing are proportional to their matching distances in real life.
- A *scale* tells you how actual measurements are represented on the drawing. For example, if a drawing has a scale of "$1$ inch to $8$ feet," then a $0.5$-inch line segment on that drawing would represent an actual distance of $4$ feet.

# Try This

Xavier drew a floor plan of his classroom using the scale $1$ inch to $6$ feet. Xavier's drawing is $4$ inches wide and $5\frac{1}{2}$ inches long. What are the dimensions of the actual classroom?

## Summary | Lesson 8

When creating scale drawings, you can select any scale that works for the space you have.

No matter what scale you select or the size of the space you are working with, an accurate scale drawing should always have the same shape and angles as the original figure.

When you select a scale, make sure to multiply each length in your drawing by the same scale factor.

Here are two ways to determine the measurements of a scale drawing of New Mexico using the scale 1 inch to 20 miles.


Actual length: 371 miles
$371 \div 20 = 18.55$
Drawing length: 18.55 inches

Actual width: 344 miles
$344 \div 20 = 17.2$
Drawing width: 17.2 inches

$1 \div 20 = 0.05$
The measurements in the scale drawing will be 0.05 times the actual length.

Actual length: 371 miles
$371 \cdot 0.05 = 18.55$
Drawing length: 18.55 inches

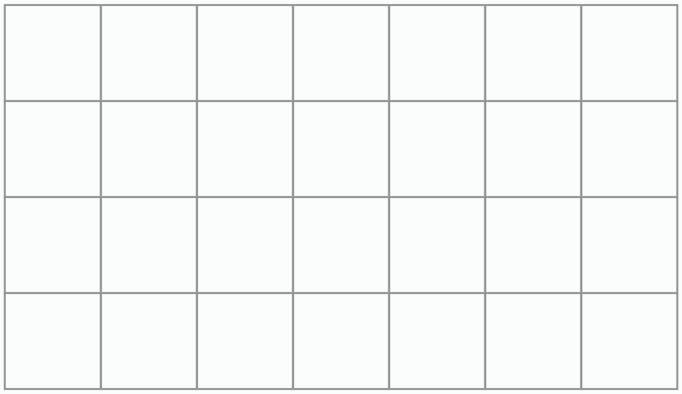
Actual width: 344 miles
$344 \cdot 0.05 = 17.2$
Drawing width: 17.2 inches

## Try This

Xavier drew a floor plan of his classroom using the scale 1 inch to 6 feet. A table in the classroom is 3 feet wide and 6 feet long.

**a** What should the size of the table be on the scale drawing?

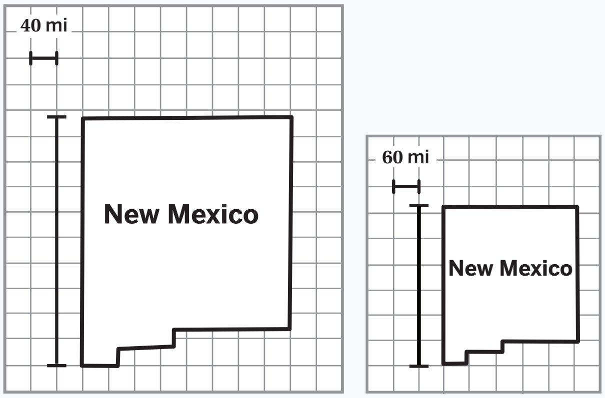
**b** Create a scale drawing of the table using the same scale as Xavier's. Each square on the grid has a side length of $\frac{1}{2}$ inch.



## Summary | Lesson 9

We can represent a place or object using many different scale drawings. Changing the scale will change the size of the scale drawing.

Here are two scale drawings of New Mexico.




The first drawing uses a scale of 1 unit to 40 miles, and the second drawing uses a scale of 1 unit to 60 miles.

When we represent a larger distance for each unit in the scale, the size of the drawing decreases. This is because it takes fewer units to represent the total distance.

For example, in the first drawing, it takes 3 units to represent a distance of 120 miles. In the second drawing, it takes 2 units to represent the same real-life distance.

## Try This

Xavier drew a floor plan of his classroom using the scale 1 inch to 6 feet. He wants to make a larger scale drawing of the same classroom. Which of these scales could he use?

**A.** 2 inches to 12 feet

**B.** 1 inch to 5 feet

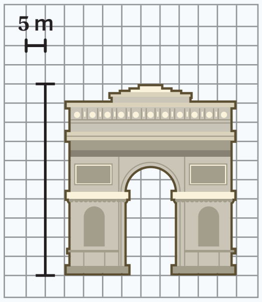
**C.** 2 inches to 15 feet

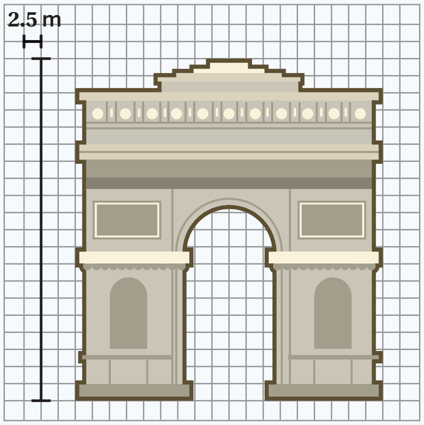
Explain your thinking.

## Summary | Lesson 10

Here is a scale drawing with a scale of $1$ unit to $5$ meters.

If you wanted to change this to a scale of $1$ unit to $2.5$ meters, you could do so in a couple of ways:




### Strategy 1

- Use the original scale to find the actual dimensions.

  The height in the drawing is $10$ units. $10 \cdot 5 = 50$

  The actual height of the building is $50$ meters.
- Use the original dimensions and the new scale to find the dimensions of the new scale drawing. $\frac{50}{2.5} = 20$

The height in the new drawing should be $20$ units.

### Strategy 2

- Determine how the two scales are related. $2.5 \cdot 2 = 5$

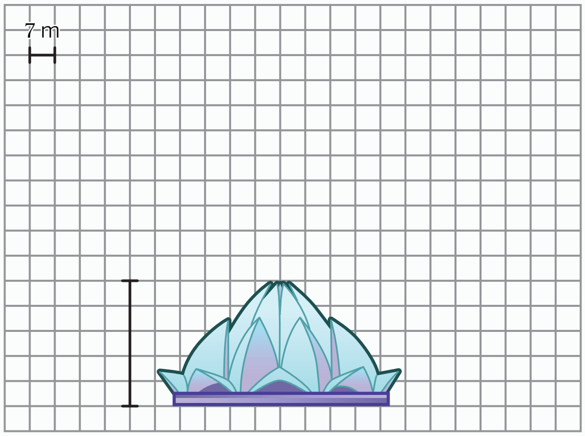
  Because $2.5 \cdot 2 = 5$, each length in the new drawing should be $2$ times as long as they are in the original drawing.
- Use this relationship to calculate the dimensions for the new drawing.

  The height in the original drawing is $10$ units. $10 \cdot 2 = 20$

The height in the new drawing should be $20$ units.

## Try This

Here is a scale drawing of the Lotus Temple in Delhi, India. The scale is $1$ unit to $7$ meters.




Select one of these and write a different scale that will produce it. You do not need to create the drawing.

**a** A larger drawing

**b** A smaller drawing

**c** A drawing that is the same size

## Summary | Lesson 11

When you have a drawing, picture, or piece of art that needs to be a different size, you can communicate the size change with either a scale factor or a scale.

If you're using a scale factor:
- Measure and record relevant dimensions of the original object.
- Choose a scale factor that's appropriate for the situation.
- Multiply all the dimensions by that scale factor.

If you're using a scale:
- Determine the relevant dimensions of the original object in grid units.
- Consider the dimensions of the actual space.
- Choose a scale for the grid that makes the object the right size for the space. This makes a scale drawing.

## Try This

Katsushika Hokusai's famous woodblock print, *Under the Wave off Kanagawa*, measures $10.1$ inches by $14.9$ inches. To create a scale drawing of the print that would fit on a $3$ inch by $5$ inch notecard, what scale could you use?

*Public Domain*

## Lesson 1

Copies 1 and 2 are scaled copies.

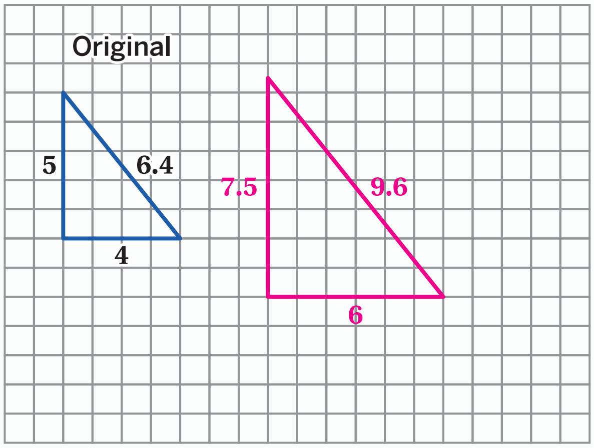
*Caregiver Note: They have the same angles and the same overall shape as the original, and the side lengths have all been multiplied by the same factor. Copy 3 is not a scaled copy because it looks stretched compared to the original. Also, the ratios are not equivalent: on the original, the ratio of two sides is $\frac{5}{4}$, which is not equivalent to $\frac{6}{3}$.*

## Lesson 2

a) 2 since $5 \cdot 2 = 10$, $4 \cdot 2 = 8$, and $6.4 \cdot 2 = 12.8$.

b) 0.5 (or equivalent) since $5 \cdot 0.5 = 2.5$, $4 \cdot 0.5 = 2$, and $6.4 \cdot 0.5 = 3.2$.

## Lesson 3




## Lesson 4

a) The scale factor is $\frac{4}{3}$.

b) The scale factor is $\frac{3}{4}$.

## Lesson 5

90 square units.

*Caregiver Note: With a scale factor of 3, the dimensions of the copy would be $2 \cdot 3 = 6$ units by $5 \cdot 3 = 15$ units. So the area is $6 \cdot 15 = 90$ square units.*

## Lesson 6

*Responses vary.*
*Caregiver Note: The actual height is around 50 meters. On the drawing, the building looks like it's a little more than 6 units tall. Since each unit represents 8 meters, the height is a little more than $6 \cdot 8 = 48$ meters.*

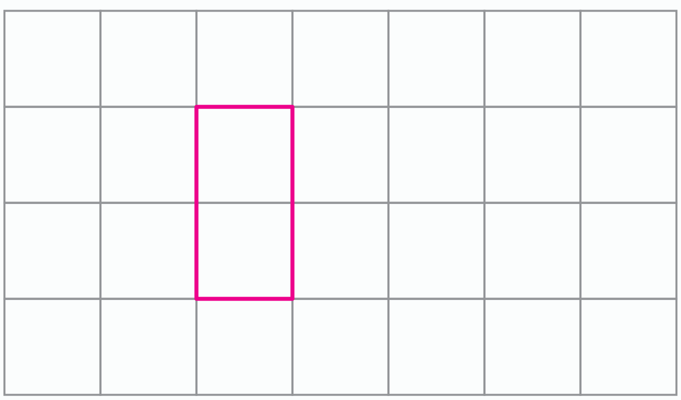
## Lesson 7

The actual classroom is 24 feet wide and 33 feet long.
*Caregiver Note: Since each inch on the drawing represents 6 feet, multiply the dimensions of the drawing by 6 to find the classroom dimensions.* $4 \cdot 6 = 24$ *feet and* $5\frac{1}{2} \cdot 6 = 33$ *feet.*

## Lesson 8

a. On the drawing, the table should be 0.5 inches wide and 1 inch long.

b.



## Lesson 9

B. The scale 1 inch to 5 feet would make a scale drawing that is larger because Xavier would need more inches on the drawing to represent the same actual length.

*Caregiver Note: The scale 1 inch to 6 feet is equivalent to the scale 2 inches to 12 feet because 1 inch would represent the same actual length in both scales.*

*The scale 2 inches to 15 feet would make a scale drawing that is smaller than the scale 1 inch to 6 feet because each inch on the new drawing would represent more actual length.*

## Lesson 10

*Responses vary.*

- **a** One possible scale that would produce a larger drawing is $1$ unit to $2$ meters.
- **b** One possible scale that would produce a smaller drawing is $1$ unit to $15$ meters.
- **c** One possible scale that would produce the same size drawing is $3$ units to $21$ meters.

## Lesson 11

*Responses vary.* $1$ inch to $3.5$ inches.

*Caregiver Note: There are many scales you could use, but try to choose one that will allow the image to fit on the notecard without leaving a lot of extra space.*

*You might start by dividing the dimensions of the original by the dimensions of the notecard to make an estimate for the scale. Since* $\frac{10.1}{3} \approx 3.37$*, let's try the scale* $1$ *inch to* $3.5$ *inches.*

*This would make the dimensions of the scale drawing* $\frac{10.1}{3.5} \approx 2.89$ *inches by* $\frac{14.9}{3.5} \approx 4.3$ *inches, which would fit on the notecard without too much extra space.*