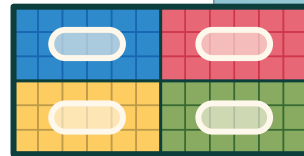


Unit 5

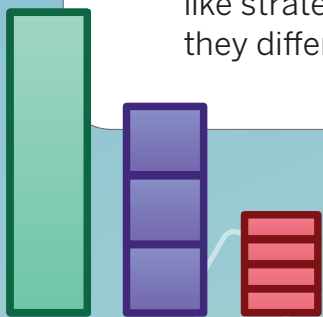
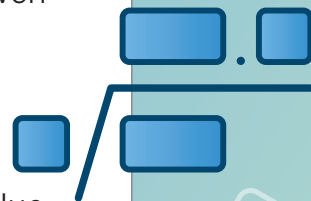
Decimal Arithmetic



When you add, subtract, multiply, and divide numbers with decimal places, instead of just estimating, you can get far more precise answers. Doing so can help you make sense of all sorts of real-world situations — like working with money, comparing grocery prices, and even buying a car!

Essential Questions

- How do the place values in each decimal number in a calculation affect the place value of the result?
- How are strategies for multiplying decimals like strategies for dividing decimals? How are they different?



One of the most common, everyday places you might see decimals is at the grocery store. While you can use all sorts of strategies to calculate exact decimal values, estimation and rounding can help you do quick calculations in your daily life.

Let's see how estimation and rounding can help Adnan shop within his budget.

Adnan has \$15.00 to spend on groceries and wants to know if he can afford all of the ingredients for his favorite recipe.

- Spaghetti: \$1.34
- Chicken thighs: \$4.99
- Diced tomatoes: \$0.79
- Parmesan cheese: \$4.38

First, Adnan can round each price to a value that is easier to add using mental math.

- Spaghetti: \$1.50
- Chicken thighs: \$5
- Diced tomatoes: \$1
- Parmesan cheese: \$4.50

Then Adnan can add the rounded values to determine an estimate for the total cost.

$$1.5 + 5 + 1 + 4.5 = \$12$$

The ingredients will cost about \$12, so Adnan knows he can make his favorite recipe tonight!

Try This

Fatima is making cheesy potatoes.

She needs 5 russet potatoes and 1 container of Parmesan cheese.

Ingredient	Cost
Russet potato	\$0.94
Parmesan cheese	\$4.38

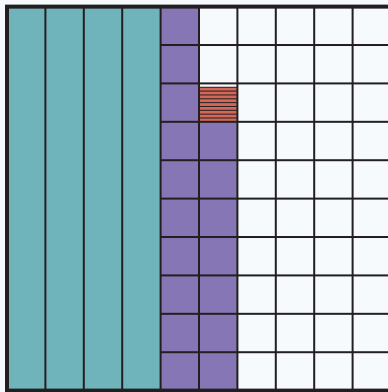
- About how much will it cost to buy these ingredients?
- If Fatima pays with \$10.00, will she have money left over? Show or explain your thinking.

You can represent decimals in more than one way using words, diagrams, and decimal points. For example, six tenths, 0.6, sixty hundredths, and 0.60 all represent the same quantity.

Using multiple representations can help when you're adding or subtracting decimals.

Let's say we're calculating $0.189 + 0.39$.

Hundredths Chart



Vertical Calculation

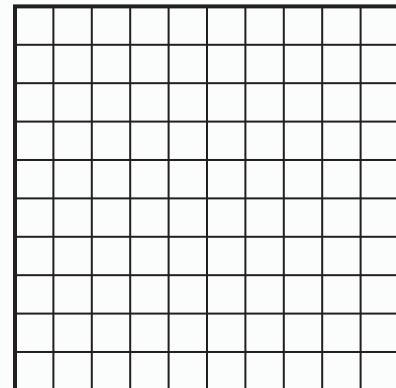
$$\begin{array}{r} 1 \\ 0.189 \\ + 0.390 \\ \hline 0.579 \end{array}$$

Both the hundredths chart and vertical calculation show a total of 4 tenths, 17 hundredths, and 9 thousandths. 10 hundredths equals 1 tenth, so the final answer is 5 tenths, 7 hundredths, and 9 thousandths, or 0.579.

Try This

Calculate the value of $0.472 - 0.081$.

Use the diagram or a vertical calculation if it helps to show your thinking.



When you're adding and subtracting decimals, it's helpful to rewrite them in different ways.

Let's say a lemon and a strawberry together weigh 0.35 pounds, and the lemon on its own weighs 0.235 pounds.

How much does the strawberry weigh?

$$\begin{array}{r} 4 10 \\ 0. \cancel{5} \\ - 0. 2 5 \\ \hline 0. 1 5 \end{array}$$

You can use vertical calculations to rewrite the total weight, 0.35, as 3 tenths, 4 hundredths, and 10 thousandths. Now you can subtract the weight of the lemon from the total to determine the weight of the strawberry.

Try This

Calculate the value of $3.725 - 1.14$.

Use a vertical calculation if it helps to show your thinking.

One strategy that can help you make sense of decimal addition and subtraction is vertical calculations.

To use a vertical calculation, you just align numbers by place value so that you're adding or subtracting ones with ones, tenths with tenths, hundredths with hundredths, and thousandths with thousandths.

Here's a vertical calculation. To check if the calculation is correct, you could either estimate or use the opposite operation (addition).

$$\begin{array}{r} 6.2 \\ - 2.5 \\ \hline 3.7 \end{array}$$

You could estimate that $6.2 - 3 = 3.2$, so your difference should be larger than 3.2.

You could also use addition to check your work, adding 2.5 to 3.7 to get 6.2.

Try This

Here is a subtraction problem.

- a** Determine the missing digits.

$$\begin{array}{r} 8.8 \\ - \square.2\square \\ \hline 4.\square4 \end{array}$$

- b** Use addition to check your work.

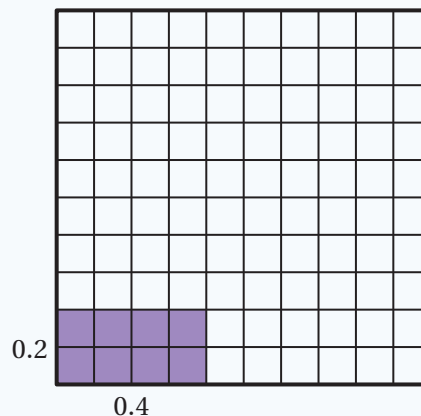
Two strategies for multiplying decimals are:

- Using an area model.
- Writing the decimals as equivalent fractions.

One advantage to using an area model is that you can visualize the product. For example, you can represent $0.4 \cdot 0.2$ as a rectangle with a length of 0.4 and a width of 0.2 . On a hundredths chart, you can count the shaded boxes, each representing $\frac{1}{100}$, to determine the product.

It can, however, be challenging to use an area model to represent decimals smaller than tenths or hundredths.

Your other option is to convert decimals to equivalent fractions. $0.4 \cdot 0.2$ can be written as $\frac{4}{10} \cdot \frac{2}{10}$, which equals $\frac{8}{100}$ or 0.08 .

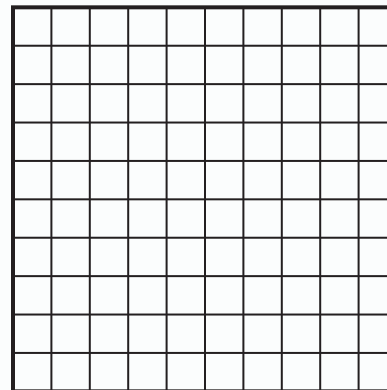


$$\begin{aligned} \text{Area} &= \text{length} \cdot \text{width} \\ &= 0.4 \cdot 0.2 \\ &= 0.08 \end{aligned}$$

Try This

Calculate the value of $0.8 \cdot 0.05$.

Use the hundredths chart or equivalent fractions if it helps to show your thinking



We can use common factors to create equivalent expressions.

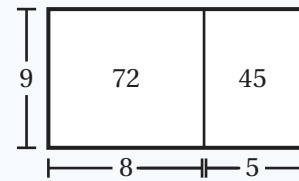
For example, since 7 is a common factor of each term in the expression $21 + 14$, you can create the equivalent expression $7(3 + 2)$. This is an example of the **distributive property**.

The distributive property tells us that $a(b + c) = ab + ac$. This means that multiplying a number by the sum of two or more terms is equal to multiplying the number by each term separately before adding the products together.

We can use area models to determine equivalent expressions.

Let's write two expressions for this area model.

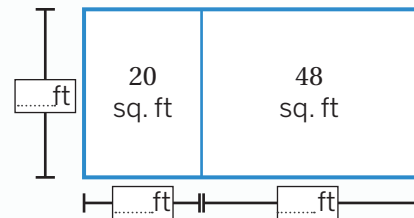
- $72 + 45$ represents the sum of the areas of the smaller rectangles.
- $9(8 + 5)$ represents the length of the whole rectangle multiplied by the width. This expression shows us that 9 is a common factor of 72 and 45.



Try This

Here is an area model.

- a** Determine the missing lengths and width.



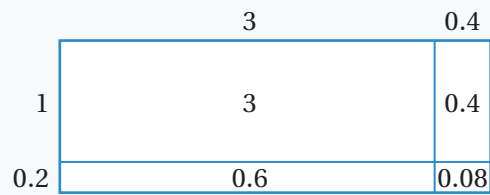
- b** Write two other expressions to represent the total area of $20 + 48$.

Summary | Lesson 7

One way to multiply two decimals is to use an area model.

To use an area model, separate the decimals into parts. This rectangle has side lengths measuring 3.4 and 1.2 units. Each side length has been split apart by place value: 3.4 has been split into $3 + 0.4$ and 1.2 has been split into $1 + 0.2$.

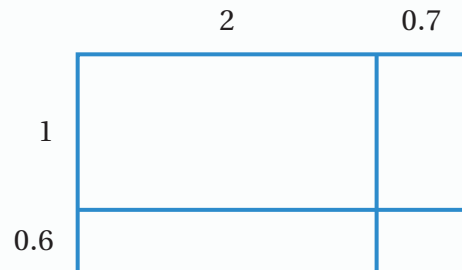
The total area of the rectangle is equal to the sum of the areas of the four smaller rectangles: $3.4 \cdot 1.2 = 3 + 0.4 + 0.6 + 0.08 = 4.08$.



Try This

Here is an area model for $2.7 \cdot 1.6$, split into parts.

- a** Calculate the area of each part.



- b** Use your area model to calculate $2.7 \cdot 1.6$.

There are several different strategies you can use to multiply decimals, such as area models and fractions. You can even convert the decimals to whole numbers and then use place value reasoning. Depending on the problem, one strategy might be more helpful than another.

Let's solve $2.4 \cdot 0.03$ using two strategies: *converting fractions* and *using whole numbers with place value reasoning*.

Strategy 1:

Converting Fractions

Rewrite each value as an equivalent fraction.

$$2.4 \cdot 0.03 = \frac{24}{10} \cdot \frac{3}{100}$$

Multiply the fractions.

$$\frac{24}{10} \cdot \frac{3}{100} = \frac{72}{1000}$$

Use the denominator to determine the place value.

$$\frac{72}{1000} \text{ is 72 thousandths.}$$
$$\frac{72}{1000} = 0.072$$

Strategy 2: Whole Numbers
With Place Value Reasoning

Think of each term as a whole number, then multiply.

$$2.4 \cdot 0.03 \rightarrow 24 \cdot 3$$

$$24 \cdot 3 = 72$$

Think about the place value of each term.

2.4 is 24 tenths.

0.03 is 3 hundredths.

Determine the appropriate place value of the product.

tenths times hundredths = thousandths

$$2.4 \cdot 0.03 = 72 \text{ thousandths}$$

$$2.4 \cdot 0.03 = 0.072$$

Try This

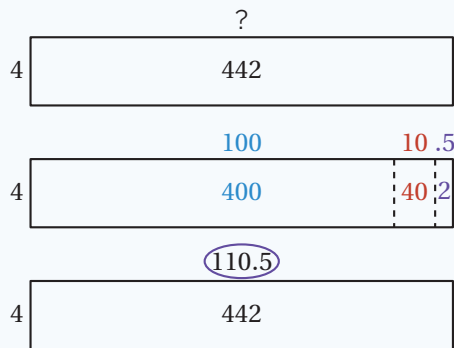
Calculate the value of $1.5 \cdot 0.023$.

Use any strategy to show your thinking.

There are many ways you can divide numbers. Here are two strategies you can use to calculate $442 \div 4$.

Area Model

Draw a diagram with a width of 4 and an area of 442. Cut the diagram into smaller areas, such as 400, 40 and 2, to help find the missing lengths. Determine the sum of the missing lengths to determine the total quotient.



$$442 \div 4 = 110.5$$

Partial Quotients

Starting with the hundreds place, determine the size of each group if 400 is divided by 4. Then subtract 400 from 442 to find out how much is remaining in the tens place, and determine the size of each group if it's divided by 4. Repeat with the ones place.

$$\begin{array}{r}
 \boxed{110.5} \\
 0.5 \\
 10 \\
 + 100 \\
 4 \overline{) 442} \\
 \underline{-400} \quad \leftarrow 400 \text{ is 4 groups of } 100 \\
 42 \\
 \underline{-40} \quad \leftarrow 40 \text{ is 4 groups of } 10 \\
 2 \\
 \underline{-2} \quad \leftarrow 2 \text{ is 4 groups of } 0.5 \\
 0
 \end{array}$$

$$442 \div 4 = 110.5$$

Try This

Calculate the value of $723 \div 6$.

Use any strategy to show your thinking.

When you get a remainder in a division expression, you can just continue to divide.

Here is how you can calculate $86 \div 4$ using **long division**.

In this strategy, you break down the remaining 2 ones into 20 tenths by writing the dividend of 86 as 86.0.

This allows you to bring a 0 down to the right of the remaining 2 ones.

Then you add a decimal point to the right of the 1 in the quotient, to show that the resulting 5 is in the tenths place.

$$\begin{array}{r} 21.5 \\ 4 \overline{) 86.0} \\ \underline{-8} \\ 6 \\ \underline{-4} \\ 2 \\ \underline{-2} \\ 0 \end{array}$$

So $86 \div 4 = 21.5$.

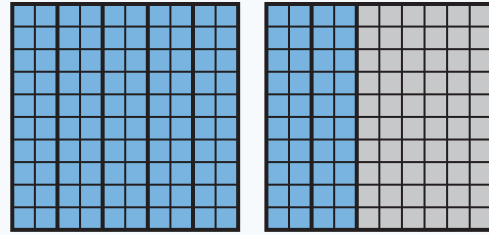
Try This

Use long division to calculate the value of $855 \div 6$.

We can use hundredths charts, thousandths charts, and fractions to help us visualize and divide decimals.

This diagram represents the expression $1.4 \div 0.2$.

- Using the hundredths chart, you can count the number of groups of 2 tenths needed to fill the 1 whole and 4 tenths. It takes 7 groups, so $1.4 \div 0.2 = 7$.
- You can rewrite each decimal as an equivalent fraction, so 1.4 becomes $\frac{14}{10}$ and 0.2 becomes $\frac{2}{10}$. Now you can use your knowledge of fraction division to calculate the quotient.



$$\begin{aligned} 1.4 \div 0.2 &= \frac{14}{10} \div \frac{2}{10} \\ &= 14 \div 2 \\ &= 7 \end{aligned}$$

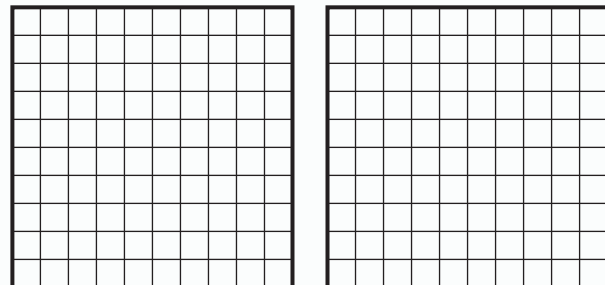
Sometimes you will need to use common denominators to solve expressions with fractions. For example,

$$1.5 \div 0.03 = \frac{15}{10} \div \frac{3}{100} = \frac{150}{100} \div \frac{3}{100} = 150 \div 3 = 50.$$

Try This

Calculate $1.08 \div 0.09$.

Use the diagram if it helps to show your thinking.



Summary | Lesson 12

When dividing by a decimal, it can be helpful to rewrite the expression using whole numbers by multiplying by a power of 10.

For example, you can rewrite $7.65 \div 1.2$ as $765 \div 120$.

$$\begin{aligned} 7.65 \div 1.2 &= \frac{765}{100} \div \frac{12}{10} \\ &= \frac{765}{100} \div \frac{120}{100} \\ &= 765 \div 120 \end{aligned}$$

Once you have an expression with whole numbers, you can use long division to calculate the quotient.

$$\begin{array}{r} \textcolor{blue}{6.375} \\ 120 \overline{) 765.000} \\ \underline{-720} \downarrow \\ 450 \downarrow \\ \underline{-360} \downarrow \\ 900 \downarrow \\ \underline{-840} \downarrow \\ 600 \\ \underline{-600} \\ 0 \end{array}$$

Try This

Calculate $5.62 \div 0.05$.

Use any strategy to show your thinking.

Summary | Lesson 13

To determine how long it will take to play a movie at a certain speed, you can divide the length of the original movie by the playback speed.

Let's say a movie is 5.6 minutes long.

- When the playing speed is doubled, it will take 2.8 minutes to watch the movie: $5.6 \div 2 = 2.8$.
- When the playing speed is halved, it will take 11.2 minutes to watch the movie: $5.6 \div 0.5 = 11.2$.

Try This

Ichiro wants to play a 20-second movie at different speeds.

- Write an expression he could use to calculate how long it would take to play the movie at $4x$.
- How long would it take to play the movie at $0.8x$?

Decimals show up in all sorts of real-world problems. We can use decimal operations to make well-informed decisions in these situations.

You might start by writing an equation to represent the situation, then determine its solution.

Problem	Equations
Apples cost \$0.75 per pound. I have \$4.00 to spend on apples. I need to know how many pounds of apples I can buy.	$4.00 \div 0.75 = 5\frac{1}{3}$ $5\frac{1}{3}$ pounds of apples
Callen is making a wheelchair ramp that is 5.7 feet long with a nail placed every 6 inches. How many nails will Callen use?	$5.7 \cdot 12 = 68.4$ $68.4 \div 6 = 11.4$ 11 or 12 nails
Marco works at the grocery store and earns \$11.50 per hour. Ren works at a pet store and earns \$12.85 per hour. How much more money will Ren earn than Marco for a 5.5-hour shift?	$12.85 - 11.50 = 1.35$ $1.35 \cdot 5.5 = 7.425$ \$7.43

Try This

Zoe's family is planning to buy a DesMobile. Here is some information they used to help make their decision.

- a** How much would it cost to fill a tank of gas in the DesMobile?

- The DesMobile can hold 12.5 gallons of gas.
- The DesMobile can travel 500 miles on a full tank of gas.
- Zoe's family drives about 12,000 miles a year.
- Gas in Zoe's town costs \$3.20 per gallon.

- b** How much would it cost to drive 1 mile based on the cost of gas and the range of the DesMobile?

- c** If Zoe's family drives the DesMobile, about how much would they spend on gas each year?

We can convert between percentages, decimals, and fractions to solve problems, including problems related to money.

We've learned that percent means "out of 100." This means that:

- We can write percentages as fractions with a denominator of 100. For example, 13% is equivalent to $\frac{13}{100}$.
- We can think of percentages as a number of hundredths, then determine an equivalent decimal value. For example, 13% is 13 hundredths, or 0.13.

It's important to think about place value when converting percentages into fractions and decimals. For example, 2.5% is equivalent to 2.5 hundredths ($\frac{2.5}{100}$) or 25 thousandths ($\frac{25}{1000}$), both of which can be represented by the decimal 0.025.

Try This

Determine the missing values in each row.

Percent (%)	Decimal	Fraction
6		
170		
	0.24	
		$\frac{300}{100}$

You've used ratios, double number lines, and tape diagrams to model and solve percent problems. Another strategy you can use is to convert the percent to a decimal, then multiply or divide.

Let's say BZ's family spent 4% of their monthly income on groceries last week. Their monthly income is \$4,000.00. How much money did they spend on one week's worth of groceries?

To determine the answer, first write 4% as a decimal: $4\% = 0.04$.

Then multiply by the total monthly income: $0.04 \cdot 4000 = 160$.

That means BZ's family spent \$160.00 on their groceries last week.

Try This

In the United States, the cost of food for one person is about \$340 per month.

- a** Tyler spends around \$68 on salad ingredients each month.

What percent of the average monthly food cost is this?

- b** Fruit makes up 6% of Tyler's monthly food cost.

How much money is that?

Lesson 1

- a** Responses vary. $1 + 1 + 1 + 1 + 1 + 4.50 = \9.50
- b** Yes. Explanations vary. I estimated Fatima's cost by rounding up each cost and determining the sum. The actual items will cost less than the sum of \$9.50.

Lesson 2

0.391

Lesson 3

2.585

Lesson 4

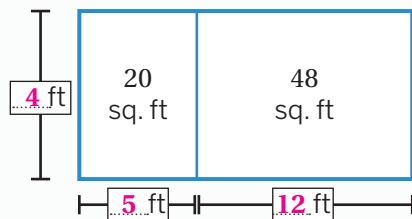
- a**
- $$\begin{array}{r} 8.8 \\ - 4.26 \\ \hline 4.54 \end{array}$$
- b** $4.26 + 4.54 = 8.8$

Lesson 5

0.04

Lesson 6

- a** Responses vary.



- b** Responses vary. $4(5 + 12)$, $4(5) + 4(12)$, $2(10 + 24)$, $2(10) + 2(24)$, 68 (or equivalent)

Lesson 7

a

	2	0.7
1	2	0.7
0.6	1.2	0.42

b 4.32

Lesson 8

0.0345

Explanations vary. One strategy is to use the converting fractions model to calculate $1.5 \cdot 0.023$.

- Rewrite each value as an equivalent fraction. $1.5 \cdot 0.023 = \frac{15}{10} \cdot \frac{23}{1000}$
- Multiply the fractions. $\frac{15}{10} \cdot \frac{23}{1000} = \frac{345}{10000}$
- Use the denominator to determine the place value. $\frac{345}{10000}$ is 345 ten-thousandths, which is written as 0.0345 in decimal form.

Lesson 9

120.5

Explanations vary. One strategy is to use the partial quotients model to calculate $723 \div 6$.

- Start with the hundreds place. There are 6 groups of 100 in 700.
- Then subtract 600 from 723 to determine that 123 remains.
- Next go to the tens place to determine that there are 6 groups of 20 in 120.
- Subtract 120 from 123 to determine that 3 remains.
- There are 6 groups of 0.5 in 3.
- Add all the partial quotients to determine that $723 \div 6 = 120.5$.

$$\begin{array}{r}
 \boxed{120.5} \\
 0.5 \\
 20 \\
 100 \\
 6 \overline{)723} \\
 \underline{-600} \longleftarrow 6 \text{ groups of } 100 \\
 123 \\
 \underline{-120} \longleftarrow 6 \text{ groups of } 20 \\
 3 \\
 \underline{-3} \longleftarrow 6 \text{ groups of } 0.5 \\
 0 \\
 723 \div 6 = 120.05
 \end{array}$$

Lesson 13

a $20 \div 4$ (or equivalent)

b 25 seconds

Caregiver Note: One strategy is to divide the length of the movie by the playback speed.

$$20 \div 0.8 = 25$$

Lesson 14

a \$40

b \$0.08

Caregiver Note: One strategy is to divide \$40 by 500 to determine the unit rate.

$$40 \div 500 = 0.08$$

c \$960

Caregiver Note: One strategy is to multiply the cost per mile by the number of miles.

$$0.08 \cdot 12000 = 960$$

Lesson 15

Percent (%)	Decimal	Fraction
6	0.06	$\frac{6}{100}$ (or equivalent)
170	1.7	$\frac{170}{100}$ (or equivalent)
24	0.24	$\frac{24}{100}$ (or equivalent)
300	3.0	$\frac{300}{100}$

Lesson 16

a 20%

Caregiver Note: One strategy is to divide the part by the whole. $68 \div 340 = 0.2 = 20\%$

b \$20.40

Caregiver Note: One strategy is to change the percent to a decimal, then multiply.

$$6\% \text{ of } \$340 \text{ is } 0.06 \cdot 340 = 20.40.$$