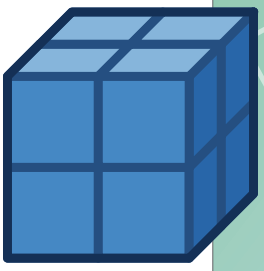
Unit 4

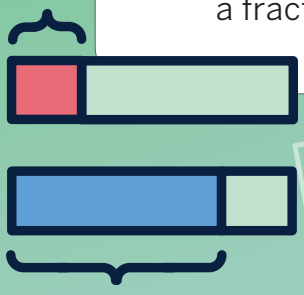
Dividing Fractions

Division is useful for answering questions like "How many groups?" or "How many in 1 group?" You've already divided using whole numbers, but how do you divide when the number of groups is a fraction? Let's find out!



Essential Questions

- What are two ways to think about dividing by a fraction?
- How are division and multiplication related to each other?
- In what situations would there be fraction-sized groups or a number of groups that is a fraction?



Summary | Lesson 1

You can use the parts of an expression to estimate the size of its *quotient*. In a division expression, the *dividend* is the number being divided. The *divisor* is the number of equal-sized groups or the size of each group being created. Take a look at these examples:

$4 \div 1\frac{1}{2}$	$4 \div 4$	$4 \div 4\frac{1}{2}$
The quotient is greater than 1.	The quotient is equal to 1.	The quotient is less than 1.
This expression represents 4 cookies separated onto $1\frac{1}{2}$ plates. Here, the *dividend* is greater than the *divisor*. This means each plate would have *more than* 1 cookie.	This expression represents 4 cookies separated onto 4 plates. Here, the dividend is equal to the divisor. This means each plate would have *exactly* 1 cookie.	This expression represents 4 cookies separated onto $4\frac{1}{2}$ plates. Here, the dividend is less than the divisor. This means each plate would have *less than* 1 cookie.

Try This

Group these expressions based on the size of their quotient.

$20 \div 200$	$\frac{4}{5} \div 2$	$\frac{9}{3} \div 3$
$20 \div \frac{1}{2}$	$5 \div 2\frac{1}{2}$	

Greater Than 1	Equal to 1	Less Than 1

Summary | Lesson 2

You can interpret division expressions in two ways, both of which involve thinking about equal-sized groups. One way answers "How many groups?" and the other answers "How many in each group?"

Let's use the division expression $35 \div 7$, where 35 represents the number of bagels. The quotient, 5, could answer two different questions: "How many boxes?" or "How many in each box?" Here's what the situation would be for each question.

How many boxes?

35 bagels are placed in boxes so that there are 7 bagels in each box.

Quotient: 5 boxes

How many in each box?

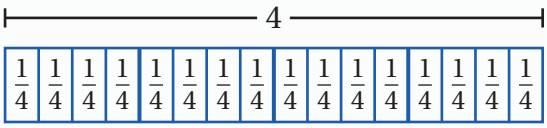
35 bagels are divided equally among 7 boxes.

Quotient: 5 bagels in each box

Try This

Takeshi bought a 4-pound bag of dog food. His dog eats $\frac{1}{4}$ of a pound per day.

a What does this tape diagram tell you about the number of days until Takeshi runs out of dog food?



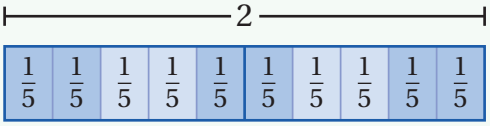
b Write *two* equations that could be used to represent this situation.

Summary | Lesson 3

You can answer the question "How many groups?" using different representations that include both whole numbers and fractions.

Here's the problem "How many $\frac{2}{5}$s are in 2?" represented using a tape diagram, a multiplication equation, and a division equation.

Tape Diagram



Multiplication Equation

$\frac{2}{5} \cdot ? = 2$

Division Equation

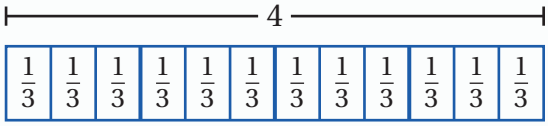
$2 \div \frac{2}{5} = ?$

Because there are 5 groups of $\frac{2}{5}$ in 2, the value 5 makes both equations $\frac{2}{5} \cdot 5 = 2$ and $2 \div \frac{2}{5} = 5$ true.

Try This

Maneli needs 4 cups of flour. She has a $\frac{1}{3}$-cup measuring scoop.

Maneli drew this diagram to determine how many scoops to use.



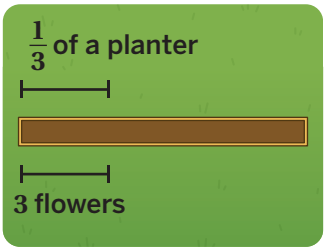
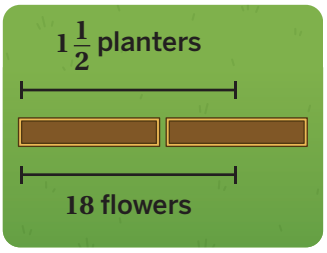
a Write *at least* one equation to represent Maneli's diagram.

b How many scoops should Maneli use?

Summary | Lesson 4

You can answer "How many are in *one* group?" by:

- Evaluating division and multiplication expressions.
- Using tape diagrams that represent division and multiplication expressions.

Situation	Diagram	Expressions	Number of Flowers in 1 Planter
3 flowers fill $\frac{1}{3}$ of a planter.		$3 \div \frac{1}{3} = ?$ or $\frac{1}{3} \cdot ? = 3$	9
18 flowers fill $1\frac{1}{2}$ planters.		$18 \div 1\frac{1}{2} = ?$ or $1\frac{1}{2} \cdot ? = 18$	12

Try This

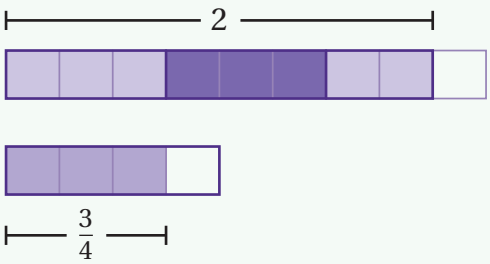
Caasi picked 12 strawberries, which filled $\frac{3}{4}$ of her basket.

How many strawberries will fill her whole basket? Draw a diagram if it helps with your thinking.

Summary | Lesson 5

You can use division to determine how many groups fit into a whole. For example, the expression $2 \div \frac{3}{4}$ can represent how many $\frac{3}{4}$-foot-long bricks fit along a 2-foot garden wall. You can use tape diagrams or reasoning about equal groups to determine how many groups (bricks) fit into the whole (along the garden wall).

Tape Diagram



$$2 \div \frac{3}{4} = 2\frac{2}{3}$$

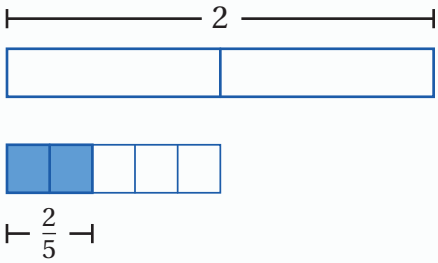
Reasoning About Equal Groups

- To calculate how many lengths of $\frac{3}{4}$ fit into 2, it would help to determine how many $\frac{1}{4}$s there are in 2 wholes.
- I can rewrite 2 as $\frac{8}{4}$.
- There are two groups of $\frac{3}{4}$ in $\frac{8}{4}$, with $\frac{2}{4}$ left over.
- The leftover $\frac{2}{4}$ has 2 of the 3 parts needed to complete a whole group of $\frac{3}{4}$. That means there are $2\frac{2}{3}$ groups of $\frac{3}{4}$ in 2.

Try This

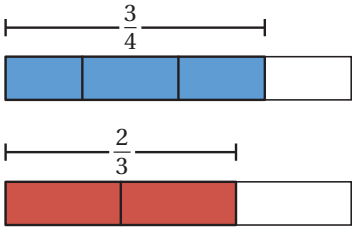
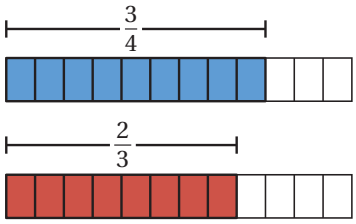
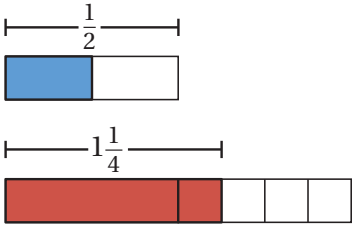
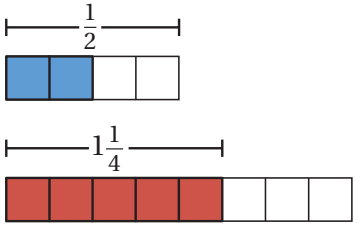
Determine the value of $2 \div \frac{2}{5}$, then explain your thinking.

Use the tape diagram if it helps with your thinking.



Summary | Lesson 6

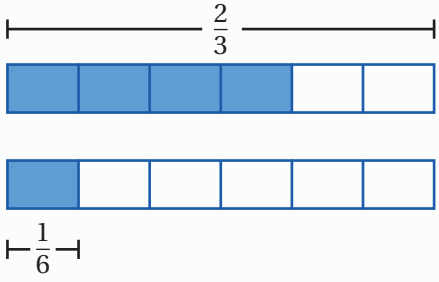
Creating equal-sized pieces, or using a *common denominator*, is a helpful strategy for calculating quotients involving fractions and determining when there is more or less than 1 group.

	Expression and Tape Diagram	Expression and Tape Diagram Using a Common Denominator
More Than 1 Group	$\frac{3}{4} \div \frac{2}{3}$ 	$\frac{9}{12} \div \frac{8}{12} = \frac{9}{8}$  A common denominator of 4 and 3 is 12.
Less than 1 Group	$\frac{1}{2} \div 1\frac{1}{4}$ 	$\frac{2}{4} \div \frac{5}{4} = \frac{2}{5}$  A common denominator of 2 and 4 is 4.

Try This

Determine the value of $\frac{2}{3} \div \frac{1}{6}$.

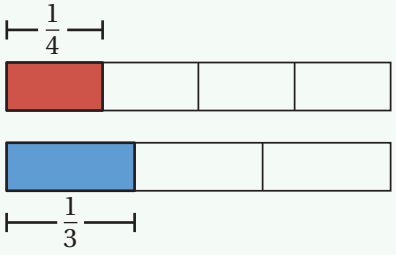
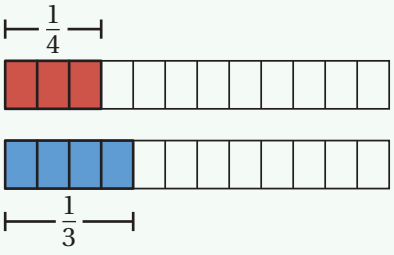
Use the diagram if it helps with your thinking.



Summary | Lesson 7

You can use common denominators to determine quotients involving fractions.

For example, in $\frac{1}{4} \div \frac{1}{3}$, you can use 12 as a common denominator of 4 and 3. Then you can rewrite the division expression as $\frac{3}{12} \div \frac{4}{12}$. This helps you determine that there are $\frac{3}{4}$ groups of $\frac{4}{12}$ in $\frac{3}{12}$.

Tape Diagram of Original Problem	Tape Diagram With Common Denominator	Equivalent Fractions With Common Denominator
		$\frac{1}{4} \div \frac{1}{3}$ $\frac{3}{12} \div \frac{4}{12}$ $3 \div 4$ $\frac{3}{4}$

Try This

Determine the value of each expression.

a. $\frac{2}{5} \div \frac{3}{5}$

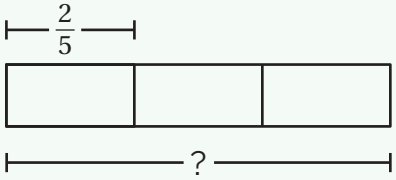
b. $1\frac{1}{3} \div \frac{3}{5}$

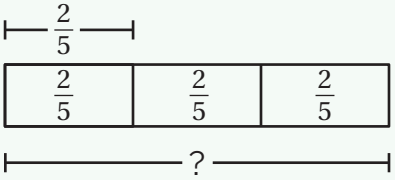
Summary | Lesson 8

When you divide a number by a unit fraction $\frac{1}{b}$, it's generally the same as multiplying the number by b.

For example, think about the expression $\frac{2}{5} \div \frac{1}{3}$. In our planter and soil situation, this means it takes $\frac{2}{5}$ bags of soil to fill $\frac{1}{3}$ of a planter.

To fill the entire planter, you would need 3 times $\frac{2}{5}$ bags of soil, or $\frac{2}{5} \cdot 3$.





$$\frac{2}{5} \div \frac{1}{3}$$
$$= \frac{2}{5} \cdot 3$$
$$= \frac{6}{5}$$
$$= 1\frac{1}{5}$$

Try This

a It takes $\frac{1}{4}$ of a box of dried pasta to make $2\frac{1}{2}$ cups of cooked pasta. How many cups of cooked pasta could the whole box make?

b Calculate $\frac{3}{4} \div \frac{1}{5}$.

Summary | Lesson 9

You don't have to use tape diagrams to determine the quotient of two fractions!

Here are two ways to calculate the quotient of the expression $\frac{9}{10} \div \frac{3}{4}$: by using common denominators and by simplifying numerators.

Common Denominators

- Rewrite the expression using common denominators.
  $\frac{18}{20} \div \frac{15}{20}$
- Then divide the numerator of the first fraction by the numerator of the second fraction.
  $18 \div 15 = \frac{18}{15}$ or $\frac{6}{5}$

Simplifying Numerators

- Divide the first fraction by the numerator of the divisor to create a unit fraction.
  $\frac{3}{10} \div \frac{1}{4}$
- To divide by the unit fraction, multiply the dividend by the denominator of the divisor.
  $\frac{3}{10} \cdot 4 = \frac{12}{10}$ or $\frac{6}{5}$

Try This

Determine the value of each expression using any strategy. Show or explain your thinking.

a $4 \div \frac{1}{5}$

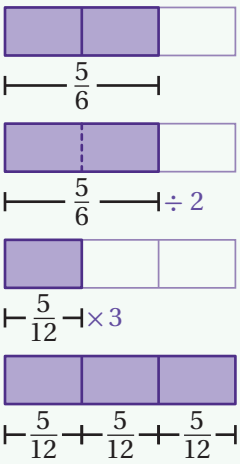
b $\frac{3}{2} \div \frac{4}{5}$

Summary | Lesson 10

In general, when you divide a number by a unit fraction, $\frac{1}{b}$, it's the same as multiplying the number by b (which is the **reciprocal** of $\frac{1}{b}$).

Here are three strategies you can use to solve the problem $\frac{5}{6} \div \frac{2}{3}$.

Tape Diagram



Simplifying Numerators

Divide by the numerator, a, then multiply the result by the denominator, b.

$$\frac{5}{6} \div \frac{2}{3}$$
$$= \frac{5}{12} \div \frac{1}{3}$$
$$= \frac{5}{12} \cdot 3$$
$$= \frac{15}{12}$$
$$= \frac{5}{4}$$

Multiplying by the Reciprocal

Multiply by the reciprocal of the fraction $\left(\frac{b}{a}\right)$.

$$\frac{5}{6} \div \frac{2}{3}$$
$$= \frac{5}{6} \cdot \frac{3}{2}$$
$$= \frac{15}{12}$$
$$= \frac{5}{4}$$

Try This

a Select *all* the expressions that are equivalent to $\frac{3}{4} \div \frac{2}{3}$.

- ☐ A. $\frac{3}{4} \cdot \frac{3}{2}$
- ☐ B. $\frac{3}{8} \cdot 3$
- ☐ C. $\frac{3}{4} \cdot \frac{2}{3}$
- ☐ D. $\frac{3}{4} \div \frac{1}{3}$
- ☐ E. $\frac{9}{8}$

b Write a multiplication expression that is equivalent to $\frac{1}{8} \div \frac{4}{5}$.

Summary | Lesson 11

There are many real-life situations where you can use fraction division.

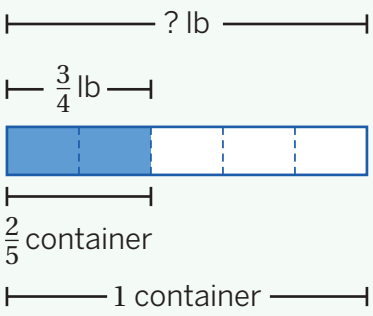
For example, let's say $\frac{3}{4}$ pounds of rice fills $\frac{2}{5}$ of a container.

There are two possible questions you can ask:

- How many pounds fill 1 container?
- How many containers for 1 pound?

Here's how you can use different division expressions and tape diagrams to answer each question.

How many pounds fill 1 container?

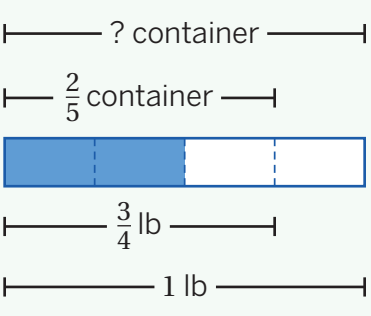


$1\frac{7}{8}$ pounds fill 1 container.

$\frac{3}{4} \div \frac{2}{5}$

$= \frac{15}{8}$ or $1\frac{7}{8}$

How many containers for 1 pound?



1 pound would fill $\frac{8}{15}$ of a container.

$\frac{2}{5} \div \frac{3}{4}$

$= \frac{8}{15}$

Try This

Marquis walked $\frac{3}{5}$ of a mile, which is $\frac{2}{3}$ of the distance between his home and school.

a Write an expression to represent the total distance between Marquis's home and school.

b Calculate the total distance.

Summary | Lesson 12

You can use division to determine how many times as large one quantity is compared to another.

For example, let's say a song is $1\frac{1}{2}$ minutes long, and another song is $3\frac{3}{4}$ minutes long. You can compare the lengths of the two songs by answering either of these questions:

How many times longer is the second song than the first song?

$$3\frac{3}{4} \div 1\frac{1}{2} = ?$$
$$= \frac{15}{4} \div \frac{3}{2}$$
$$= \frac{15}{4} \cdot \frac{2}{3}$$
$$= \frac{30}{12} \text{ or } 2\frac{1}{2}$$

The second song is $2\frac{1}{2}$ times as long as the first song.

What fraction of the second song is the first song?

$$1\frac{1}{2} \div 3\frac{3}{4} = ?$$
$$= \frac{3}{2} \div \frac{15}{4}$$
$$= \frac{6}{4} \div \frac{15}{4}$$
$$= \frac{6}{15} \text{ or } \frac{2}{5}$$

The first song is $\frac{2}{5}$ as long as the second song.

Try This

Each episode of Kiri's favorite show is $\frac{3}{4}$ of an hour long. Her favorite movie is $2\frac{1}{4}$ hours long.

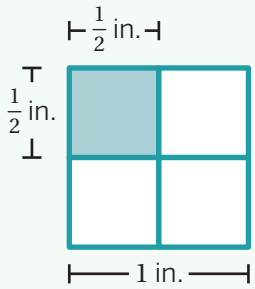
a What fraction of the length of the movie is an episode of the show?

b How many episodes could Kiri watch in the time it takes to watch the movie?

Summary | Lesson 13

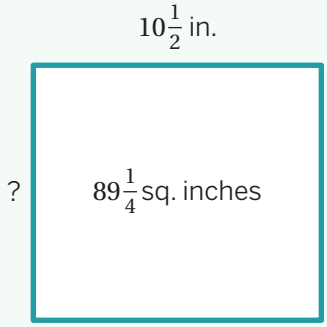
You can determine the area of a polygon that has fractional side lengths just like you would a polygon that has whole-number side lengths.

For example, you can calculate the area of the shaded square using the formula $A = l \cdot w$. The area is equal to $\frac{1}{2} \cdot \frac{1}{2}$, or $\frac{1}{4}$ square inches.



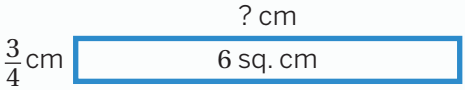
You can also use area formulas to determine an unknown length. If you know the area and one side length of a rectangle, you can divide to determine the other side length.

For example, to determine the missing side length of this rectangle, you can calculate $89\frac{1}{4} \div 10\frac{1}{2} = 8\frac{1}{2}$. The missing side length is $8\frac{1}{2}$ inches.



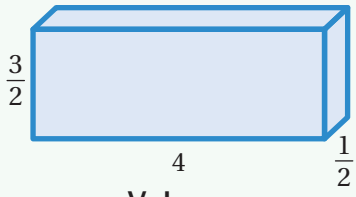
Try This

Use any strategy to determine the unknown value.



You can determine the *volume* of a prism by multiplying its dimensions.

For example, here is a rectangular prism with side lengths measuring 4 units, $\frac{3}{2}$ units, and $\frac{1}{2}$ units.

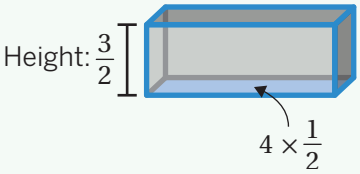


Volume

$$4 \cdot \frac{3}{2} \cdot \frac{1}{2}$$

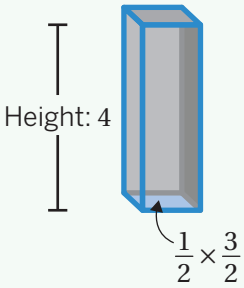
$= 3$ cubic units

You can also calculate the volume of a prism as the product of its base area and the height. You can choose any of the rectangles as the base.



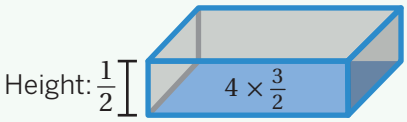
If you choose the 4-by-$\frac{1}{2}$ rectangle as the base, then the base area will be 2 square units.

The volume is $2 \cdot \frac{3}{2} = 3$ cubic units.



If you choose the $\frac{3}{2}$-by-$\frac{1}{2}$ rectangle as the base, then the base area will be $\frac{3}{4}$ square units.

The volume is $\frac{3}{4} \cdot 4 = 3$ cubic units.

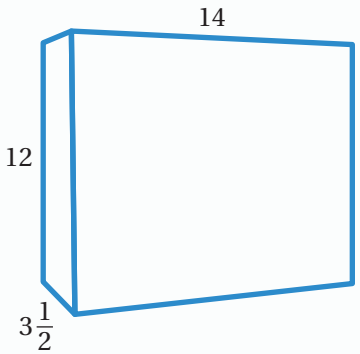


If you choose the 4-by-$\frac{3}{2}$ rectangle as the base, then the base area will be 6 square units.

The volume is $6 \cdot \frac{1}{2} = 3$ cubic units.

Try This

What is the volume of a box that measures 14 inches by 12 inches by $3\frac{1}{2}$ inches?



Summary | Lesson 15

Take a moment to review what you know about fractions and operations.

To multiply fractions . . .

- Multiply the denominators as a way to determine a common denominator and make same-sized parts.
- Then multiply the numerators to determine how many of those parts exist.

Example:

$$\frac{3}{8} \cdot \frac{5}{9}$$

$$= \frac{3 \cdot 5}{8 \cdot 9}$$

$$= \frac{15}{72}$$

$$= \frac{5}{24}$$

To divide by a fraction, $\frac{a}{b}$ . . .

Multiply the dividend by the reciprocal, $\frac{b}{a}$.

Example:

$$\frac{4}{7} \div \frac{5}{3}$$

$$= \frac{4}{7} \cdot \frac{3}{5}$$

$$= \frac{12}{35}$$

Try This

Angela is growing potatoes in a planter that is 4 feet wide and $3\frac{3}{4}$ feet long. Each potato plant needs $\frac{1}{4}$ square foot of space.

a How many potato plants can Angela grow in the planter?

b The soil in the planter is $\frac{2}{3}$ of a foot deep. How many cubic feet of soil are in the planter?

Lesson 1

Greater Than 1	Equal to 1	Less Than 1
$20 \div \frac{1}{2}$ $5 \div 2\frac{1}{2}$	$\frac{9}{3} \div 3$	$20 \div 200$ $\frac{4}{5} \div 2$

Lesson 2

a. *Responses vary*. The diagram shows 16 groups of $\frac{1}{4}$, which means Takeshi has enough dog food for 16 days.

b. *Responses vary*. Two possible equations are $4 \div \frac{1}{4} = 16$ and $\frac{1}{4} \cdot 16 = 4$.

Lesson 3

a. *Responses vary*. Some possible equations are $4 \div \frac{1}{3} = 12$ and $\frac{1}{3} \cdot 12 = 4$.

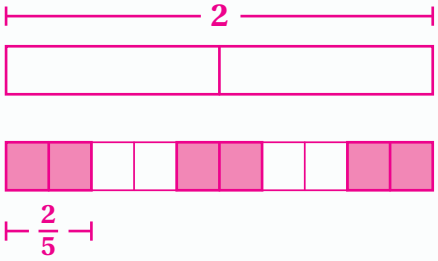
b. 12 scoops

Lesson 4

16 strawberries

Lesson 5

5. *Explanations vary.* 2 equals $\frac{10}{5}$. There are 5 groups of $\frac{2}{5}$ in $\frac{10}{5}$.



Lesson 6

4. According to the diagram, it would take four $\frac{1}{6}$ pieces to equal $\frac{2}{3}$.

Lesson 7

a $\frac{2}{3}$.

Caregiver Note: When dividing fractions with the same denominator, you can just divide the numerators and $2 \div 3 = \frac{2}{3}$.

b $\frac{20}{9}$.

Caregiver Note: One strategy is to find a common denominator. $1\frac{1}{3} \div \frac{3}{5}$ *is equivalent to* $\frac{20}{15} \div \frac{9}{15}$. *Then you can divide the numerators.* $20 \div 9 = \frac{20}{9}$.

Lesson 8

a 10 cups.

Caregiver Note: Since $\frac{1}{4}$ *of the box makes* $2\frac{1}{2}$ *cups, the whole box would make 4 times as much and* $2\frac{1}{2} \cdot 4 = 10$.

b $\frac{15}{4}$.

Caregiver Note: Since $\frac{1}{5}$ *is a unit fraction, you can multiply by the denominator.* $\frac{3}{4} \div \frac{1}{5}$ *is equivalent to* $\frac{3}{4} \cdot 5 = \frac{15}{4}$.

Lesson 9

a 20. *Explanations vary.* Since $\frac{1}{5}$ is a unit fraction, one strategy is to multiply by the denominator. $4 \div \frac{1}{5}$ has the same value as $4 \cdot 5 = 20$.

b $\frac{15}{8}$. *Explanations vary.* One strategy is to find a common denominator. $\frac{3}{2} \div \frac{4}{5}$ is equivalent to $\frac{15}{10} \div \frac{8}{10}$. Then you can divide the numerators and $15 \div 8 = \frac{15}{8}$.

Lesson 10

a. A. $\frac{3}{4} \cdot \frac{3}{2}$, B. $\frac{3}{8} \cdot 3$, and E. $\frac{9}{8}$

b. *Responses vary.* Two possible expressions are $\frac{1}{8} \cdot \frac{5}{4}$ and $\frac{1}{32} \cdot 5$.

Lesson 11

a. $\frac{3}{5} \div \frac{2}{3}$

b. $\frac{9}{10}$ of a mile

Lesson 12

a. $\frac{1}{3}$ (or equivalent).

Caregiver Note: Here is one strategy:

$$\frac{3}{4} \div 2\frac{1}{4}$$
$$= \frac{3}{4} \div \frac{9}{4}$$
$$= \frac{3}{9}$$
$$= \frac{1}{3}$$

b. 3 episodes.

Caregiver Note: Since an episode of the show is $\frac{1}{3}$ as long as the movie, Kiri could watch 3 episodes in the same amount of time.

Lesson 13

8 centimeters.

Caregiver Note: One strategy is to divide the area by the given dimension and $6 \div \frac{3}{4} = 8$.

Lesson 14

588 cubic inches

Lesson 15

a. 60 potato plants.

Caregiver Note: The area of the planter is $4 \cdot 3\frac{3}{4} = 15$ square feet, and $15 \div \frac{1}{4} = 60$ plants.

b. 10 cubic feet of soil.

Caregiver Note: One strategy is to multiply the area from part a by the depth of the soil and $15 \cdot \frac{2}{3} = 10$ cubic feet.