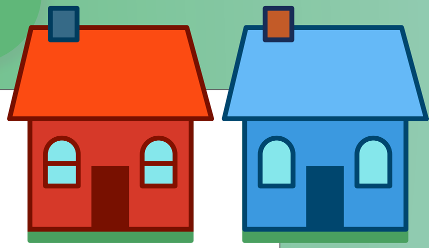
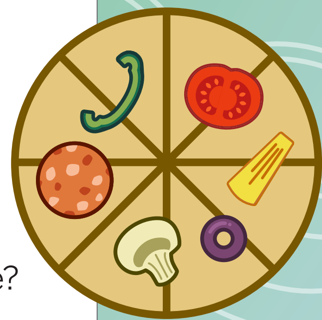


Unit 2

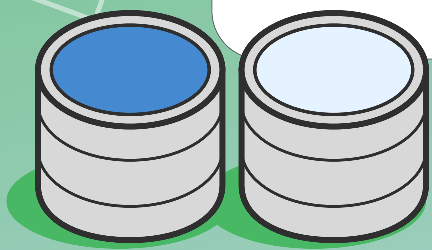
Introducing Ratios

You know how to use math to compare lengths, areas, and temperatures. But what if you wanted to compare price, taste, color, or even fairness? In this unit, you'll use ratios to describe relationships between two quantities and compare them in real-world situations!



Essential Questions

- What does a ratio say about the relationship between quantities?
- How can ratios help you get the same taste, texture, or color every time you make a recipe?
- How can ratios help us consider issues of fairness?



Summary | Lesson 1

You can represent the relationship between different quantities, such as the ingredients in a pizza recipe. Once you create a recipe with your favorite ingredients, you can use the relationship between those ingredients to make multiples of the same pizza.

For example, if your recipe uses 4 mushrooms to make 1 pizza, then you would need 12 mushrooms to make 3 pizzas.

Some quantities do not have this kind of relationship. For example, if you bake one pizza at 800°F in a pizza oven, then you'll need to bake 2 pizzas at about the same temperature. Doubling the number of pizzas does not mean you need to double the temperature.

Try This

a Here are the toppings that a pizza restaurant needs to make 2 of Jaleel's favorite pizzas. How much of each topping do they need to make 8 of these pizzas for Jaleel's birthday party?

	2 pizzas	8 pizzas
Cheese	10 ounces	
Pepperoni	18 slices	
Onion	20 slices	
Pineapple	12 slices	

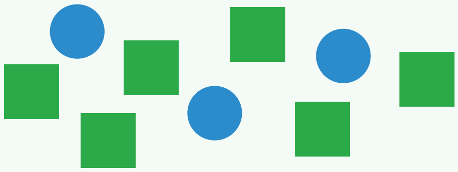
b If Jaleel orders twice as many pizzas, will the cost of the pizzas double? What about the delivery fee? Explain your thinking.

Summary | Lesson 2

A **ratio** is a relationship between two quantities. One way to write a ratio is $a : b$ which means for every a of the first quantity, there are b of the second quantity.

There are many ways to describe a ratio in words.

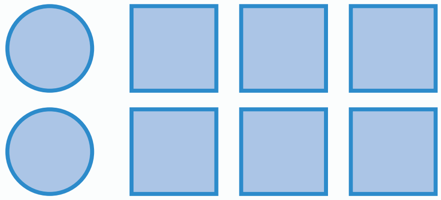
For example, here are some ways you can describe the ratio between circles and squares in this diagram.



- The ratio of circles to squares is 3 to 6.
- There are 6 squares for every 3 circles.
- There are 2 times as many squares as there are circles.
- For every 1 circle, there are 2 squares.

Try This

Here is an image of circles and squares.



a Complete each sentence based on the image.

- The ratio of circles to squares is .... to ....
- The ratio of squares to circles is .... : ....
- For every circle, there are .... squares.

b Make a drawing with a ratio of 3 squares : 2 circles.

Summary | Lesson 3

Recipes can help us understand **equivalent ratios**. Each recipe calls for a specific ratio of ingredients, but you can halve, double, or triple the ratio to make different amounts of the same recipe.

Original Recipe

Boil 3 cups of water for every 2 cups of rice.

3 to 2

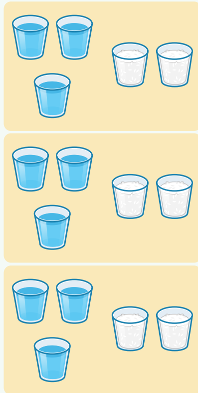
Halved Recipe

$1\frac{1}{2}$ to 1

Doubled Recipe

6 to 4

Tripled Recipe



9 to 6

These ratios are equivalent because they all represent the same recipe. You can multiply or divide each of the values in the first ratio by the same number to get the values in each of the other ratios.

Try This

Rice and peas is a popular side dish from the Caribbean.

One recipe for rice and peas uses 14 ounces of coconut milk and serves 4 people.

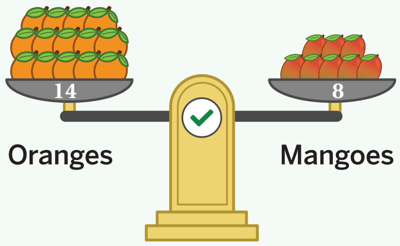
Adriana is making rice and peas for 8 people. She says she needs 18 ounces of coconut milk.

Is Adriana's claim correct? Explain your thinking.

Summary | Lesson 4

We can use balance scales to help us understand equivalent ratios. When both quantities in a ratio are multiplied or divided by the same amount, the ratio relationship remains the same, and the scale stays balanced.

For example, the ratio of oranges to mangoes on this scale is 14 : 8.



You can create an equivalent ratio of 7 : 4 by dividing the number of each fruit by 2. This means that 7 oranges and 4 mangoes will also balance on the scale. 21 oranges to 12 mangoes would also be an equivalent ratio because you can get those values by multiplying 14 and 8 by $\frac{3}{2}$.

You can use a **table** to organize and keep track of equivalent values. Tables organize information into horizontal rows and vertical columns. The first row or column usually tells us what the numbers represent.

Here is a table that represents the different numbers of oranges and mangoes needed to balance the scale.

Number of Oranges	Number of Mangoes
14	8
7	4
21	12

Try This

A scale balances with a ratio of 2 limes to 5 lychees.

a Select *all* the equivalent ratios of limes to lychees.

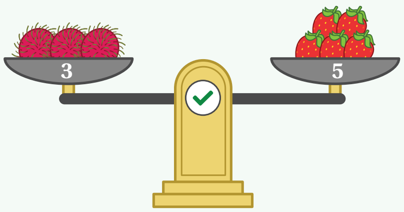
- ☐ **A.** 1 lime to 4 lychees
- ☐ **B.** 4 limes : 10 lychees
- ☐ **C.** 1 lime : 2.5 lychees
- ☐ **D.** 6 limes to 15 lychees
- ☐ **E.** 12 limes to 15 lychees

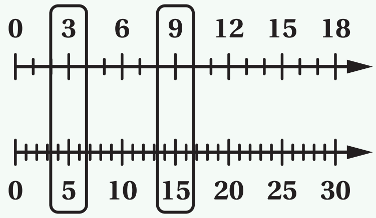
b Choose *one* equivalent ratio from part a. Explain how you know it is equivalent.

Summary | Lesson 5

A **double number line** is another way to represent equivalent ratios. Each double number line is made up of a pair of parallel number lines. The tick marks are labeled so that the numbers that line up vertically make equivalent ratios.

For example, if a ratio of 3 rambutans to 5 strawberries will balance on a scale, you can use a double number line to determine how many strawberries will balance with 9 rambutans.



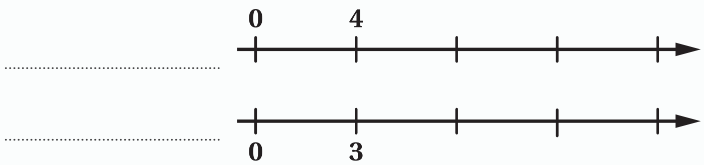


To represent a ratio of 3 : 5, you begin with 3 and 5 in the same location on each number line and then count up by units of 3 on one number line and units of 5 on the other line. In this example, you can determine that it will take 15 strawberries to balance 9 rambutans. Each pair of matching values represents an equivalent ratio to 3 : 5.

Try This

A scale balances with a ratio of 4 dragon fruits to 3 pomegranates.

a Complete this double number line so it shows the ratio of dragon fruits to pomegranates.



b How many dragon fruits will balance with 12 pomegranates?

Summary | Lesson 6

When you're spending your money, you can use the price per item to decide if something is a good deal, or to determine how much different amounts of a product will cost. The word *per* means "for each."

For example, if 3 bottles of juice cost $\$7.50$, you can determine how much 8 bottles of juice will cost.

- The price per bottle is $\$2.50$, because $7.50 \div 3 = 2.50$.
- The price of 8 bottles is 8 times the price per bottle, and $8 \cdot 2.50 = \$20$.

Try This

Adah is shopping at the corner store.

a Four bottles of water cost $\$6$. How much does one bottle cost?

b It costs $\$15$ to buy 12 donuts. How much would 8 donuts cost?

Summary | Lesson 7

A **common multiple** of two numbers is a number that is a multiple of both numbers. Here's a chart that shows some multiples of 2 (marked with squares) and some multiples of 3 (marked with circles). We can see that some of the common multiples of 2 and 3 are 6, 12, and 18.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20

The **least common multiple (LCM)** is the smallest number that is a common multiple of two numbers. In the example of 2 and 3, the LCM is 6. It's helpful to determine the LCM when solving problems like how many packages of tofu dogs and buns you need to buy or how often two trains stop at the same station. For example, if Train A stops at the station every 2 hours, and Train B stops at the station every 3 hours, then both trains will be at the station every 6 hours.

Try This

What is the least common multiple of 9 and 12? Show or explain your thinking.

Use the grid if it helps with your thinking.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Summary | Lesson 8

A *factor* of a number is a whole number that divides evenly into the given number (with no remainder). A **common factor** of two numbers is a number that is a factor of both numbers.

Here's a chart that shows some factors of 8 (marked with squares) and some factors of 12 (marked with circles). We can see that some *common* factors of 8 and 12 are 1, 2, and 4.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20

The **greatest common factor (GCF)** is the largest number that is a common factor of two numbers. In the example of 8 and 12, the GCF is 4.

Try This

a What is the greatest common factor of 24 and 30? Show or explain your thinking.

b What is the greatest common factor of 55 and 75? Show or explain your thinking.

Summary | Lesson 9

You can use different strategies to compare two ratios.

Let's compare the ratios of two cans of paint to see which will make a lighter shade of gray.

Ratio A	**Ratio B**
5 ounces black paint	7 ounces black paint
3 gallons white paint	4 gallons white paint

Strategy 1: Multiply both ratios so they each have the same amount of black paint.

- Multiply both ratios so they each have the same amount of black paint.
- The *LCM* for the number of ounces of black paint for both ratios is 35.
- Multiply Ratio A by 7 to get 35 ounces of black paint and 21 gallons of white paint.
- Multiply Ratio B by 5 to get 35 ounces of black paint and 20 gallons of white paint.

When both ratios have the same amount of black paint, Ratio A has more gallons of white paint, which means it will be a lighter shade of gray.

Strategy 2: Calculate the number of ounces of black paint per gallon of white paint.

- Calculate the number of ounces of black paint per gallon of white paint.
- Ratio A has $\frac{5}{3} = 1\frac{2}{3}$ ounces of black paint for every gallon of white paint.
- Ratio B has $\frac{7}{4} = 1\frac{3}{4}$ ounces of black paint for every gallon of white paint.

Ratio A has less black paint for 1 gallon of white paint, which means it will be a lighter shade of gray.

Try This

Mayra and Juan each mixed teal paint and white paint to make a certain shade of teal paint.

Mayra used a ratio of 4 ounces of teal to 2 gallons of white.

Juan used a ratio of 6 ounces of teal to 4 gallons of white.

Which ratio made a darker teal? Explain your thinking.

Summary | Lesson 10

We can use ratio tables to help make plans for situations that we haven't experienced yet.

Here are some supply recommendations for a 50-person taco party:

- 10 pounds of carnitas
- 15 cups of pinto beans
- 125 tortillas

People	Carnitas (lb)	Pinto Beans (cups)	Tortillas
50	10	15	125
200	40	60	500
10	2	3	25

$\times 4$

$\div 5$

Let's use a table to determine the different amounts of each ingredient we might need for different-sized parties. For example, if we only had 10 people coming to the taco party, we would only need 2 pounds of carnitas, 3 cups of pinto beans, and 25 tortillas. If 200 people were coming to the party, we could multiply the values for the 50-person party by 4 to determine the amount for each ingredient. We just have to multiply or divide all of the values in each row by the same number to preserve each ratio relationship.

Try This

Metropolis Middle School is planning their dances for the year. In the past, they bought 3 quarts of lemonade, 2 bags of ice, and 12 cookies for every 10 people who attended.

This year, they expect 300 people to attend the winter dance and 75 people to attend the 8th-grade dance. Complete the table to predict how much of each item they should buy for each dance.

People	Cookies	Lemonade (qt)	Ice (bags)
10	12	3	2
300			
75			

Summary | Lesson 11

There are a few helpful strategies you can use to determine missing values in equivalent ratios. One strategy is to determine a new ratio where one of the quantities is equal to 1.

For example, if 6 balloons can make 3 marbles float, you can use the ratio $6 : 3$ and equivalent ratios to solve different problems.

To determine the number of balloons that can float 8 marbles:

- Determine the number of balloons that float 1 marble.
- Then you can multiply that ratio by 8 to determine that 16 balloons float 8 marbles.

Number of Balloons	Number of Marbles
6	3
2	1
16	8

To determine the number of marbles that 4 balloons can float:

- Determine the number of marbles that 1 balloon can float.
- Then you can multiply that ratio by 4 to determine that 4 balloons float 2 marbles.

Number of Balloons	Number of Marbles
6	3
1	0.5
4	2

Try This

Zwena is buying apples to bake a pie. 4 apples weigh 16 ounces.

Zwena's pie recipe uses 40 ounces of apples.

How many apples should she buy? Explain your thinking.

Apples	Weight (oz)
4	16

Summary | Lesson 12

We can use equivalent ratios to help solve real-world problems that involve at least two quantities. When working with real-world problems, we may need to round numbers or think about the circumstances of the situation when determining which solutions make sense.

Let's say the Metropolis Delivery Service makes 15 deliveries every 2 hours. They need to make 100 deliveries tomorrow.

You can use a ratio table to determine that it will take approximately $13\frac{1}{3}$ hours, or 13 hours and 20 minutes, to make all 100 deliveries.

Number of Deliveries	Number of Hours
15	2
5	$\frac{2}{3}$
100	$\frac{40}{3}$ or $13\frac{1}{3}$

If you worked for the Metropolis Delivery Service, you might not report the exact value to customers. Instead, you might round to 13.5 or 14 hours to account for traffic and other delays.

Try This

Cho is taking a train to visit their friend. The train travels 45 miles in 60 minutes.

a Cho's stop is 120 miles away. How long will it take the train to reach their stop?

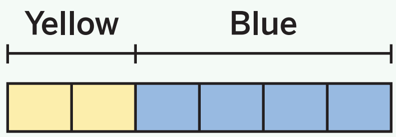
b Cho checked their watch after 12 minutes. How far had the train traveled?

Summary | Lesson 13

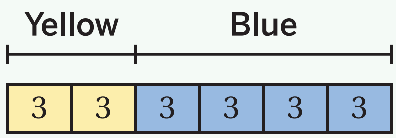
A **tape diagram** is a way to represent relationships between quantities (such as ratios) as lengths of tape. The diagram is divided up to represent the parts. Together, these parts represent the whole.

We can use tape diagrams to represent things like the ratio of different paints in a mixture.

For example, when 2 cups of yellow paint are mixed with 4 cups of blue paint, it creates 6 cups of green paint. Here is a tape diagram representing that ratio, where each part represents 1 cup of paint.



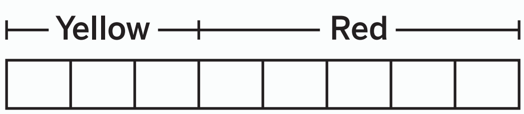
But if each part represented 3 cups of paint, there would be 6 cups of yellow paint, 12 cups of blue paint, and a total of 18 cups of green paint. This is a way to see a ratio that is equivalent to the original ratio.



Try This

DeShawn needs 40 cups of orange paint to paint his room. He plans to mix together 3 parts yellow paint and 5 parts red paint to make orange.

Here is a tape diagram that DeShawn created to represent the ratio of yellow paint to red paint.



a Fill in the tape diagram so that it shows a total of 40 cups of orange paint.

b How much yellow paint does DeShawn need?

c How much red paint does DeShawn need?

Summary | Lesson 14

Ratio tables, tape diagrams, and models can help us determine unknown amounts, which can help us solve real-world problems.

For example, Metropolis has requirements for the ratio of green space to building space in each new neighborhood development. The requirements say that there should be 2 units of green space for every 5 units of building space.

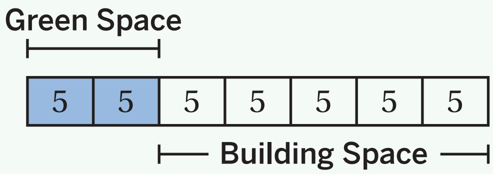
A new development has 35 units of land. Let's use both a ratio table and a tape diagram to determine how many units of building space they can build.

Ratio Table

Green Space	Building Space	Total
2	5	7
10	25	35

$35 \div 7 = 5$

$5 \cdot 5 = 25$



$7 \cdot 5 = 35$

$5 \cdot 5 = 25$

So for 35 total units of land, Metropolis will have 25 units of building space.

Try This

Tyrone makes milk coffee by mixing 4 parts coffee and 3 parts sweetened condensed milk. Then he freezes the mixture to make popsicles. He needs 21 ounces of milk coffee to make popsicles this week.

How much of each ingredient will he need? Explain your thinking.

Summary | Lesson 15

We can use ratios to understand issues in our lives and help develop solutions.

For example, let's say Maria decided to do a food waste experiment at home. She determined that her family threw away 9 pounds of trash in 5 days. Of that 9 pounds of trash, 3 pounds were plastic, 4 pounds were food, and 2 pounds were other waste.

You can use ratios to learn more about her family's waste habits.

- You can determine how much trash Maria's family would throw out in a year (365 days).

365 days is 73 times as long as her experiment.

$9 \cdot 73 = 657$ pounds of trash

- Then you can determine how much of that yearly trash would be plastic, food, and other waste.

Plastics: $\frac{3}{9} \cdot 657 = 219$ pounds

Food: $\frac{4}{9} \cdot 657 = 292$ pounds

Other: $\frac{2}{9} \cdot 657 = 146$ pounds

Try This

a Based on the information from the summary, how much trash would Maria's family throw out in a *month* (30 days)?

b How much of their monthly trash would be plastic, food, and other waste?

Lesson 1

a

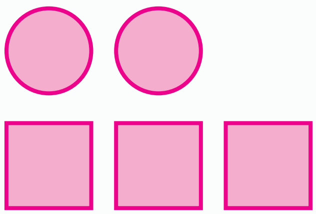
	2 pizzas	8 pizzas
Cheese	10 ounces	**40 ounces**
Pepperoni	18 slices	**72 slices**
Onion	20 slices	**80 slices**
Pineapple	12 slices	**48 slices**

b The cost of the pizzas might double because the restaurant has to use twice as much of each ingredient and do twice as much work. The delivery person still has to make only one trip, so the cost of delivery probably won't double.

Lesson 2

a
- 2 to 6
- 6 : 2
- 3

b *Drawings vary.*



Lesson 3

Adriana's claim is not correct.

Explanations vary. Adriana wants to serve double the number of people, so she should double the amount of coconut milk to get $14 \cdot 2 = 28$ ounces. She might think that she needs to add 4 more ounces of coconut milk to serve 4 more people, but that won't be enough.

Lesson 4

a B. 4 limes : 10 lychees

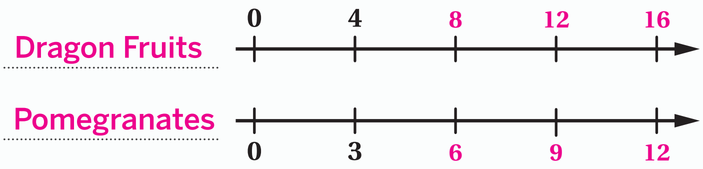
C. 1 lime : 2.5 lychees

D. 6 limes to 15 lychees

b *Responses vary*. The ratio 4 limes : 10 lychees is equivalent because it's double the original ratio.

Lesson 5

a



b 16 dragon fruits

Lesson 6

a $1.50

b $10

Caregiver Note: Students might mention that individual donuts are sometimes priced differently than dozens. The sample response assumed that each donut is the same price and that you don't get a discount for buying a dozen.

Lesson 7

36.

Explanations vary. The first few multiples of 9 are 9, 18, 27, and 36. The first few multiples of 12 are 12, 24, and 36. 36 is the least common multiple because it's the smallest number that is a multiple of both 9 and 12.

Lesson 8

a. 6.

Explanations vary. The factors of 24 are 1, 2, 3, 4, 6, 8, 12, and 24. The factors of 30 are 1, 2, 3, 5, 6, 10, 15, and 30. The largest number that is a factor of both 24 and 36 is 6.

b. 5.

Explanations vary. The factors of 55 are 1, 5, 11, and 55. The factors of 75 are 1, 3, 5, 15, 25, and 75. The largest number that is a factor of both 55 and 75 is 5.

Lesson 9

Mayra's ratio.

Explanations vary. Juan used 6 ounces of teal paint for 4 gallons of white paint. If we double the quantities in Mayra's ratio, we can see that she'd use 8 ounces of teal paint for 4 gallons of white paint. Mayra's ratio used more teal for the same amount of white, which means it made a darker color.

Lesson 10

People	Cookies	Lemonade (qt)	Ice (bags)
10	12	3	2
300	360	90	60
75	90	22.5	15

Lesson 11

10 apples.

Explanations vary. If 4 apples weigh 16 ounces, then 2 apples weigh 8 ounces. I can multiply that by 5 to determine that 10 apples weigh 40 ounces.

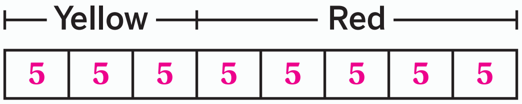
Caregiver Note: Student explanations might acknowledge that apples are not all the same size. Zwena might wish to buy extra apples if some of them are smaller, or fewer apples if some are very large.

Lesson 12

a 160 minutes

b 9 miles

Lesson 13

a 

b 15 cups of yellow paint

c 25 cups of red paint

Lesson 14

12 ounces of coffee and 9 ounces of sweetened condensed milk.

Explanations vary. Tyrone's recipe has 7 parts in total. Since he wants 21 ounces this week, and $7 \cdot 3 = 21$, he should multiply each part of his ratio by 3. This means he'll need $4 \cdot 3 = 12$ ounces of coffee and $3 \cdot 3 = 9$ ounces of sweetened condensed milk, for a total of $12 + 9 = 21$ ounces.

Lesson 15

a 54 pounds of trash

b Plastics: $\frac{3}{9} \cdot 54 = 18$ pounds

Food: $\frac{4}{9} \cdot 54 = 24$ pounds

Other waste: $\frac{2}{9} \cdot 54 = 12$ pounds