

 Amplify Desmos Math CALIFORNIA

Mathematics I

Volume 2: Units 5–7

Student Edition

About Amplify

Amplify is dedicated to collaborating with educators to create learning experiences that are rigorous and riveting for all students. Amplify creates K–12 core and supplemental curriculum, assessment, and intervention programs for today’s students.

A pioneer in K–12 education since 2000, Amplify is leading the way in next-generation curriculum and assessment. All of our programs provide teachers with powerful tools that help them understand and respond to the needs of every student.

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Dear Student,

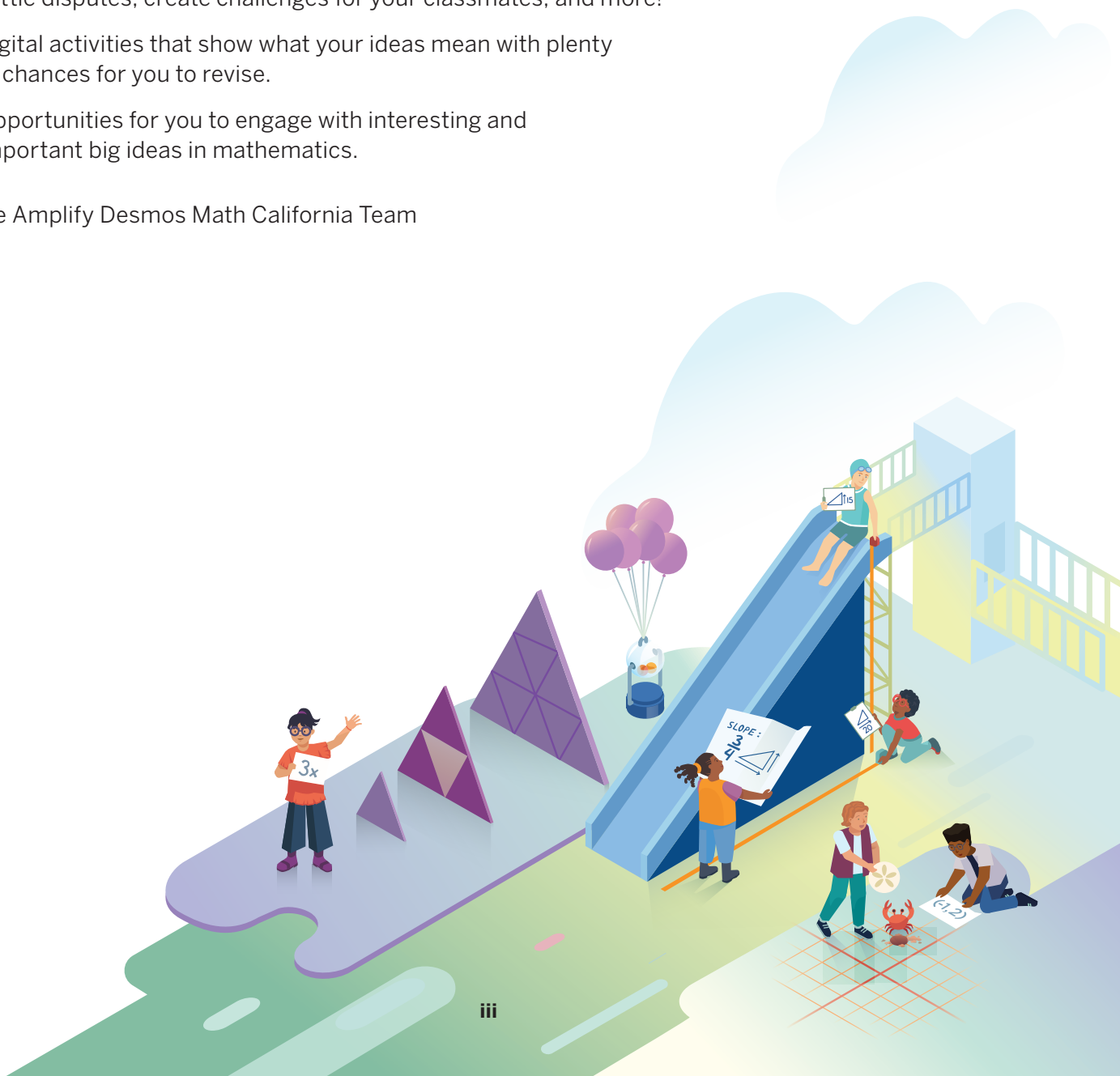
Welcome to Amplify Desmos Math California Algebra 1! We are excited to be partnering with you this year. You play an essential role in math class, so we wanted to reach out to introduce ourselves and tell you a bit about who we are.

Amplify Desmos Math California is a team of math educators on a mission to support you and your classmates in learning math. We hope each lesson inspires you to use your creativity, ask questions, and discover connections between math concepts and the world around us.

Here is what you can expect this year:

- A blend of learning on both paper and devices.
- Interactive lessons that encourage you to ask questions, explore, settle disputes, create challenges for your classmates, and more!
- Digital activities that show what your ideas mean with plenty of chances for you to revise.
- Opportunities for you to engage with interesting and important big ideas in mathematics.


–The Amplify Desmos Math California Team



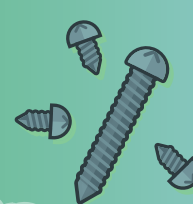
Unit 1 Patterns and Sequences

You will model, compare, and contrast arithmetic and geometric sequences using tables, recursive definitions, expressions, and graphs.



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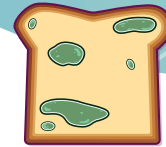
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Unit 5 Exponential Functions

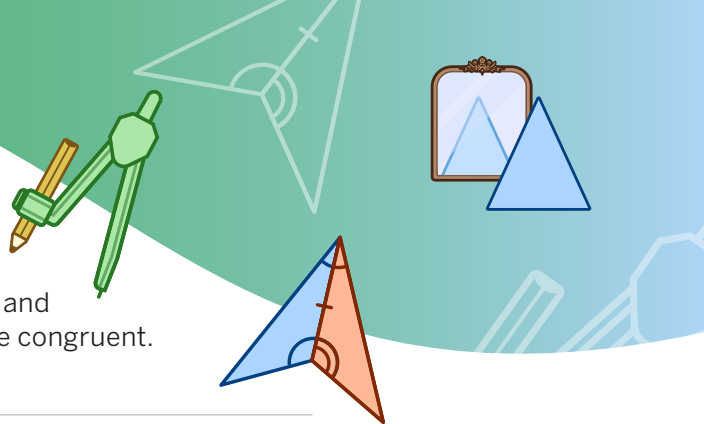
You will use exponential functions to make sense of how changing by a small percentage can make a big impact over time.






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Unit 6 Rigid Transformations and Congruence

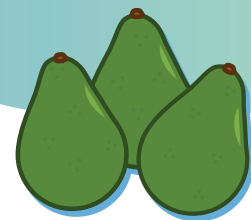
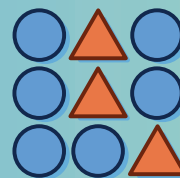
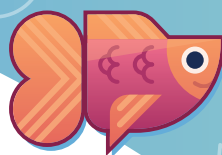
You will construct geometric figures using a variety of tools and use rigid transformations to determine when two figures are congruent.



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Unit 7 Describing Data

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Unit 5

Exponential Functions



Big Ideas in This Unit

CC1 Modeling with Functions Comparing Models CC2 Modeling with Functions
Comparing Models Variability CC3 Composing Functions NS Financial Literacy

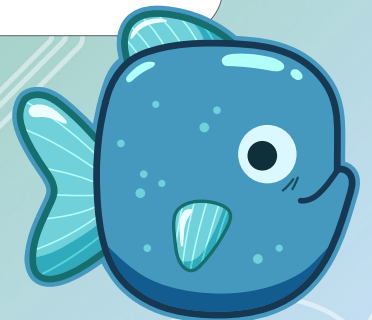
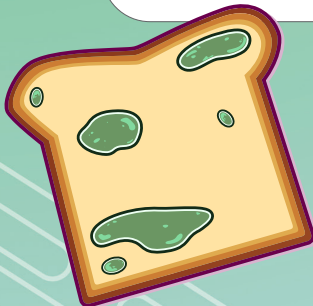
Questions for Investigation

- How do the features of linear and exponential functions compare?
- How can you model situations that grow or decay by a percentage rate per unit interval?
- How can you use exponential models to solve problems?



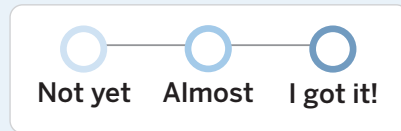
Explore: An Epidemic

How does an epidemic grow?





















Watch Your Knowledge Grow

This is the math you'll explore in this unit. Rate your understanding to see how your knowledge grows!



I can . . .	Before	After
Use equations in the form $y = a \cdot b^x$	<input type="checkbox"/> — <input type="checkbox"/> — <input type="checkbox"/>	<input type="checkbox"/> — <input type="checkbox"/> — <input type="checkbox"/>
Interpret parameters of an exponential equation in context.	<input type="checkbox"/> — <input type="checkbox"/> — <input type="checkbox"/>	<input type="checkbox"/> — <input type="checkbox"/> — <input type="checkbox"/>
Identify functions that grow by a constant difference as a linear functions.	<input type="checkbox"/> — <input type="checkbox"/> — <input type="checkbox"/>	<input type="checkbox"/> — <input type="checkbox"/> — <input type="checkbox"/>
Identify functions that grow by a constant ratio as exponential functions.	<input type="checkbox"/> — <input type="checkbox"/> — <input type="checkbox"/>	<input type="checkbox"/> — <input type="checkbox"/> — <input type="checkbox"/>
Write exponential functions to represent graphs or situations.	<input type="checkbox"/> — <input type="checkbox"/> — <input type="checkbox"/>	<input type="checkbox"/> — <input type="checkbox"/> — <input type="checkbox"/>
Compare and contrast graphs of linear and exponential functions.	<input type="checkbox"/> — <input type="checkbox"/> — <input type="checkbox"/>	<input type="checkbox"/> — <input type="checkbox"/> — <input type="checkbox"/>
Create functions, tables, and graphs of linear and exponential relationships.	<input type="checkbox"/> — <input type="checkbox"/> — <input type="checkbox"/>	<input type="checkbox"/> — <input type="checkbox"/> — <input type="checkbox"/>
Evaluate exponential functions for a specific value.	<input type="checkbox"/> — <input type="checkbox"/> — <input type="checkbox"/>	<input type="checkbox"/> — <input type="checkbox"/> — <input type="checkbox"/>
Describe the domain of exponential functions.	<input type="checkbox"/> — <input type="checkbox"/> — <input type="checkbox"/>	<input type="checkbox"/> — <input type="checkbox"/> — <input type="checkbox"/>
Model repeated percent increase with exponential growth functions.	<input type="checkbox"/> — <input type="checkbox"/> — <input type="checkbox"/>	<input type="checkbox"/> — <input type="checkbox"/> — <input type="checkbox"/>
Model repeated percent decrease with exponential decay functions.	<input type="checkbox"/> — <input type="checkbox"/> — <input type="checkbox"/>	<input type="checkbox"/> — <input type="checkbox"/> — <input type="checkbox"/>

I can . . .	Before	After
Determine the growth factor of an exponential function.		
Use exponential functions to solve problems.		
Describe the effect of vertical and horizontal translations on the graph of an exponential function.		
Describe the connections between simple and compound interest, and linear and exponential functions.		
Write exponential functions that represent situations involving compound interest.		
Calculate account balances given an initial amount, annual interest rate, compounding interval, and period of time.		
Fit an exponential or linear function to data.		
Use linear and exponential models to make predictions in context.		
Assess the fit of a function to data.		

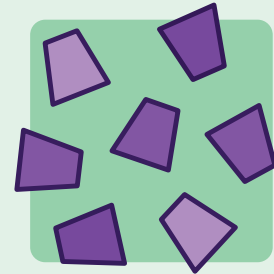
Comparing Linear and Exponential Functions



Explore
An Epidemic



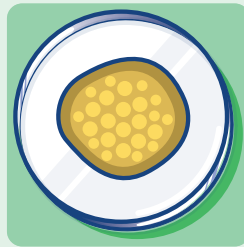
Lesson 1
Carlos's Fish



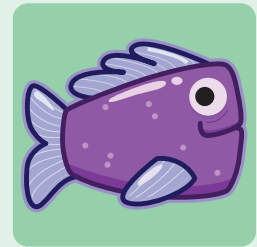
Lesson 2
Growing Globbs



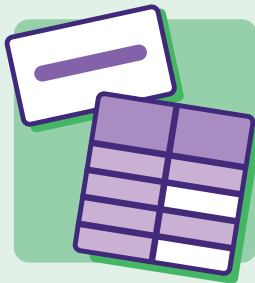
Lesson 3
Going Viral



Lesson 4
Return of the Globbs



Lesson 5
Carlos and Corals



Lesson 6
Differences
and Factors



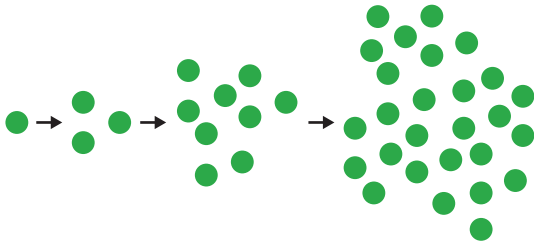
Explore: An Epidemic

How does an epidemic grow?



Warm-up

1. Consider the following image. What do you notice? What do you wonder?



I notice. . .

I wonder. . .



Disease Spread

You will explore how a disease spreads.

2. Complete the data table for each scenario, so that it shows the total number of infected people after each round. Each round, every infected person infects one other person.

Scenario 1:

x	0					
y	2					

Scenario 2:

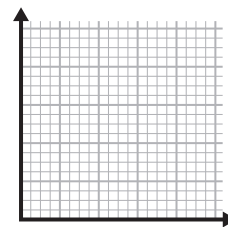
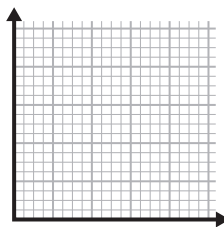
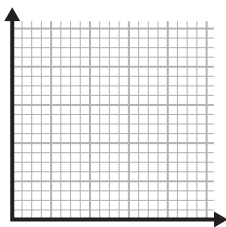
x	0					
y	3					

Scenario 3:

x	0					
y	1					

3. Look at each scenario. What do you notice?

4. Create a graph for each of your tables.





Disease Spread (continued)

5. Look at the graphs from Problem 4.



Discuss: What do you notice? What do you wonder?

6. For each scenario, determine the number of rounds it would take for 1,000 people to be infected.

7. How do you think your graphs would change if every sick person infected 2 new people each round (rather than just 1)?

8. In the real world, infected people might come in contact with other infected people (rather than always infecting someone new). How might this change the outcome of the scenarios?

9. In the real world, some people might be immune. How might this change the outcome of the scenarios?



Building Math Habits of Mind



Discuss:

- Which of these habits of mind did you strengthen during this activity?
- How did you use the one(s) you selected?

I can slow down and first make sense of a challenging problem before trying to solve it.

Not yet Almost I got it!

I can represent real-world problems and interpret their solutions within the context of the problem.

Not yet Almost I got it!

I can justify my thinking and ask questions to help me understand the thinking of others.

Not yet Almost I got it!

I can apply the math that I know to solve real-world problems, make assumptions and revise my thinking as needed.

Not yet Almost I got it!

I can select an appropriate tool to help me solve problems.

Not yet Almost I got it!

I can communicate my thinking and solutions clearly to others.

Not yet Almost I got it!

I can look for structure or patterns to help me solve problems.

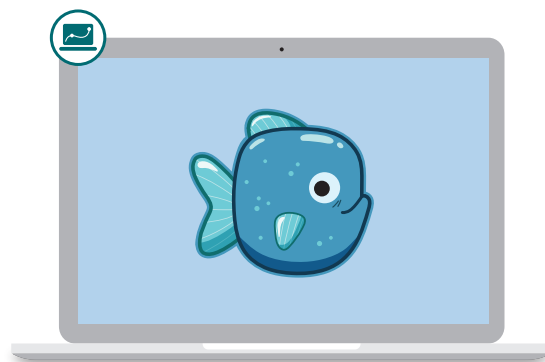
Not yet Almost I got it!

I can look for repeated calculations and other repeated steps to make generalizations.

Not yet Almost I got it!

Carlos's Fish

Let's make connections between exponential equations and the situations they represent.



Warm-Up

1 Carlos's apartment doesn't allow pets, so he decided to buy toy fish.

What do you notice? What do you wonder?



Doubles Every Hour

- 2** Let's watch Carlos's toy fish grow when you drop it in water.

 **Discuss:** What patterns do you see?

Time (hr)	Mass (g)
0	
1	50
2	100
3	200
4	400
5	

- 3** This fish grows by a constant ratio.

What will the mass of the toy fish be after 5 hours?

- 4** What was the mass of the fish before it was in the water?

Doubles Every Hour (continued)

5 Carlos wrote this equation to model the fish's mass: $m = 25 \cdot 2^t$.

He used m for mass and t for time.

Explain what the 25 and 2 mean in this situation.

- The 25 means...
- The 2 means...

Time (hr)	Mass (g)
0	25
1	50
2	100
3	200
4	400
5	800

6 If the fish continues growing this way, what will its mass be after 7 hours?

7 Here is how Angel and Sora figured out the mass of the fish after 7 hours:

Angel

Time (hr)	Mass (g)
1	50
2	100
3	200
4	400
5	800
6	1,600
7	3,200

Sora

$$m = 25 \cdot 2^7$$

$$m = 25 \cdot 2^7$$

$$m = 25 \cdot 128$$

$$m = 3200$$



Discuss: What strategies do you see each student using?

Fish Growing and Shrinking

8 Here is a new toy fish.

Carlos wrote this equation to model the fish's mass:

$$m = 30 \cdot 1.5^t$$

Explain what the 30 and 1.5 mean in this situation.

- The 30 means . . .
- The 1.5 means . . .



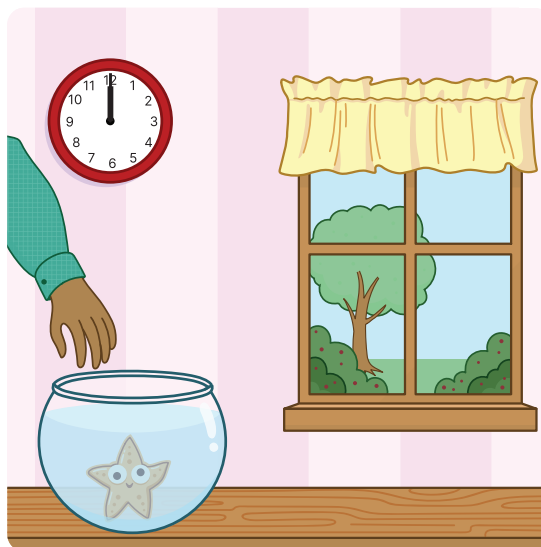
9 What is the mass of the fish when $t = 0$?

10 Carlos wrote this equation to model the starfish's mass:

$$m = 270\left(\frac{1}{3}\right)^t$$

He used m for mass and t for time.

What will its mass be 3 hours after taking it out of the water?



Fish Growing and Shrinking (continued)

11 Match the cards with an equation. Two cards will have no match.

Card A:

This fish's mass is multiplied by $\frac{1}{2}$ each hour.

Card C:

After 2 hours, this fish has a mass of 24 grams.

Card E:

This fish has a mass of $\frac{1}{2}$ gram before it is put in the water.

Card G:

The fish's mass increases by $\frac{1}{2}$ gram every hour.

Card B:

After 2 hours, this fish has a mass of 1.5 grams.

Card D:

After 2 hours, this fish has a mass of 18 grams.

Card F:

This fish has a mass of 6 grams before it is put in the water.

$$m = 6 \cdot \left(\frac{1}{2}\right)^t$$

$$m = \frac{1}{2} \cdot 6^t$$

12 Synthesis

Write a story about a fish whose mass is modeled by the equation $m = 100 \cdot (1.2)^t$.

15 Summary 5.01

Situations that include repeated multiplication can be modeled using an equation of the form $y = a \cdot b^x$. You can use the equation to solve problems about the situation.

For example, Carlos bought a new mega-growing fish. The fish weighed 4 grams when Carlos bought it, and its mass grows 1.5 times greater every hour.

He wrote the equation $m = 4 \cdot 1.5^t$ to represent the relationship, where:

- m represents the mass of Carlos's fish in grams.
- t represents the time in hours.

You can use the equation to determine the mass of the fish after 4 hours by substituting $t = 4$ and solving for m .

$$m = 4 \cdot 1.5^t$$

$$m = 4 \cdot 1.5^4$$

$$m = 4 \cdot 5.0625$$

$$m = 20.25$$

After 4 hours, the fish will have a mass of 20.25 grams.

Practice

5.01

Name: _____ Date: _____ Period: _____

Problems 1–3: Carlos has a pack of toy fish whose mass doubles every hour when you add water. This table shows the mass of a toy fish over time.


Time (hr)	Mass (g)
0	?
1	30
2	60
3	120
4	240
...	...
6	?

1. What was the mass of the toy fish before it was in water?

2. What will the mass of the toy fish be after 6 hours?

3. Wohali buys another brand of toy fish that claims to grow faster than Carlos's fish. Wohali wrote this equation: $m = 10 \cdot 3^t$. He used m for the mass, in grams, and t for time, in hours. Explain what the 10 and 3 mean in this situation.

Problems 4–5: Jamar had 80 followers on social media. His number of followers tripled every month for 4 months.

4.  **Test Practice** Select all the expressions that represent Jamar's followers after 4 months.

- A. $80 \cdot 3 \cdot 3 \cdot 3 \cdot 3$
 B. $80 + 4^3$
 C. $80 \cdot 3^4$
 D. $80 + 3 + 3 + 3 + 3$
 E. $80 \cdot 4 \cdot 4 \cdot 4$

5. Complete the table.

Time (months)	0	1	2	3	4
Followers					

Practice

5.01

Name: _____ Date: _____ Period: _____

Problems 6–8: A group of biologists tracked the number of squirrels in a town. They wrote the equation $n = 40 \cdot 1.5^t$, where n is the total number of squirrels and t is the number of years since the biologists started counting.

6. Explain what the 40 and 1.5 mean in this situation.

7. How many squirrels do the biologists predict there will be 2 years after they started counting?

8. How many squirrels do the biologists predict there will be 10 years after they started counting?

9. A baby chicken weighs 32 grams when it hatches. The chicken's mass increases by 45% each week for the first 12 weeks of its life. Complete the table of the chicken's mass over time.

Time (weeks)	0	1	2	3	4	...	12
Mass (g)						...	

Spiral Review

Problems 10–13: Determine the value of each expression.

10. 3^3

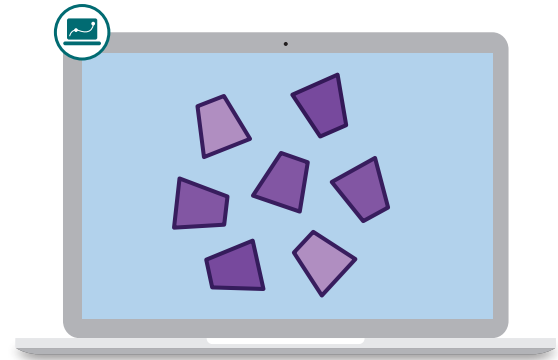
11. $2(3^3)$

12. $3^3 + 4$

13. $2 \cdot 3^3 + 4$

Growing Globs

Let's identify and compare two different patterns of growth.



Warm-Up

1 Let's look at some teal globs.

- a** Watch how the number of globs grows.
- b** Write a story about these globs.

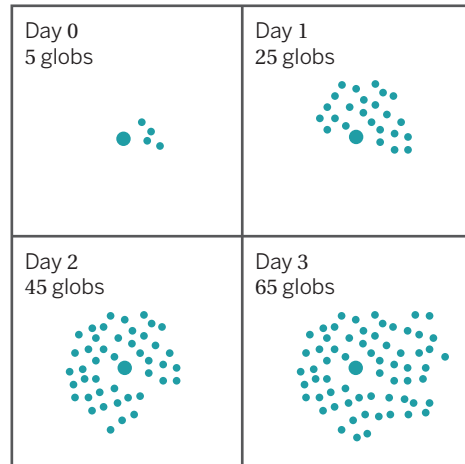
Day 0
5 globs

A diagram showing five teal-colored circles of varying sizes. One is the largest, and four are smaller, arranged in a pattern that suggests a cluster or a small group.

Purple vs. Teal

2 How many teal globs will there be on day 4?

Day	Teal Globs
0	5
1	25
2	45
3	65
4	



3 Here is a new group of globs.

- a** Let's watch how the number of globs grows.
- b** How many purple globs will there be on day 4?

Day	Purple Globs
0	2
1	6
2	18
3	54
4	

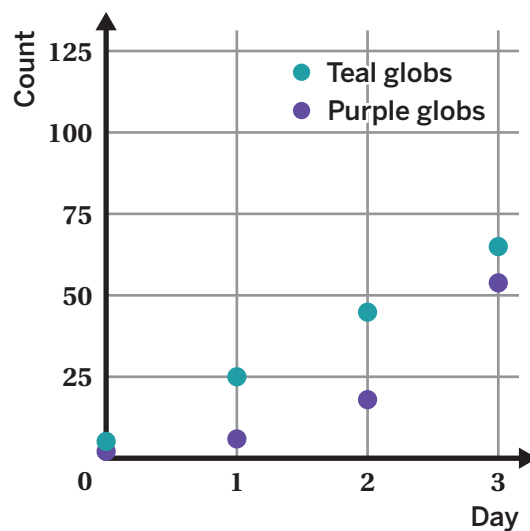
4 The graph shows the number of each type of glob for the first 3 days.

Which do you think there will be more of on day 10? Circle one.

Teal Globs Purple Globs

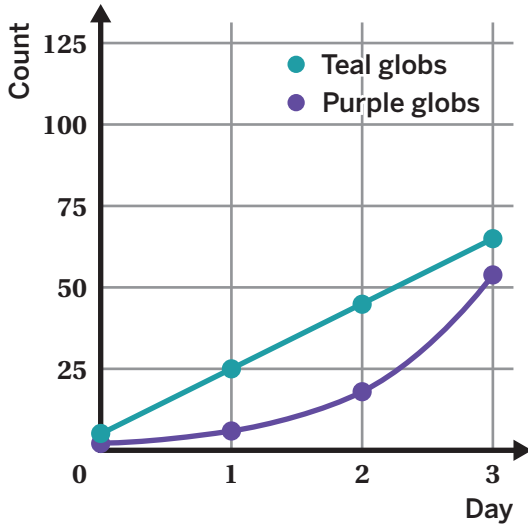
There will be Not enough
the same information

Explain your thinking.

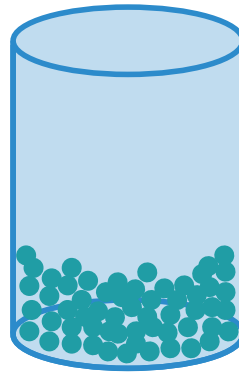


Purple vs. Teal (continued)

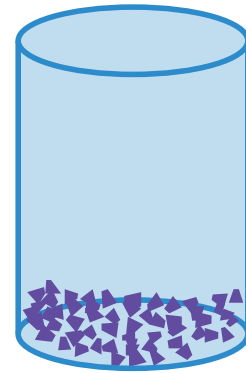
5 Let's watch how the number of teal and purple globs grows.



Day 3



Count: 65



Count: 54

6 Here are tables for the teal and purple growing globs.

- Teal globs' growth is modeled by a **linear function** and has a constant *rate of change*.
- Purple globs' growth is modeled by an **exponential function** and has a constant **growth factor**.

Discuss:

- How are *rate of change* and *growth factor* alike?
- How are they different?

Linear		Constant rate of change
Day	Teal Globs	
0	5	} +20
1	25	
2	45	
3	65	

Exponential		Constant growth factor
Day	Purple Globs	
0	2	} x3
1	6	
2	18	
3	54	

Comparing Growth

7 Let's make two new species of globs and compare their growth.

8 Let's compare globs with different starting amounts and a constant rate of change or a constant growth factor.

Fabiana says: *Globs that grow by a constant growth factor will always eventually outnumber globs that grow by a constant rate of change.*

Lukas says: *If the constant rate of change is large enough, then this won't be true.*

Whose idea do you agree with? Circle one.

Fabiana's

Lukas's

Both

Neither

Explain your thinking.

9 Group these cards by their function type.

Card A		Card B	
x	y	x	y
0	2	0	0
1	4	1	1
2	6	2	4
3	8	3	9
4	10	4	16

Card C		Card D	
x	y	x	y
0	1	0	0
1	2	1	4
2	4	2	8
3	8	3	12
4	16	4	16

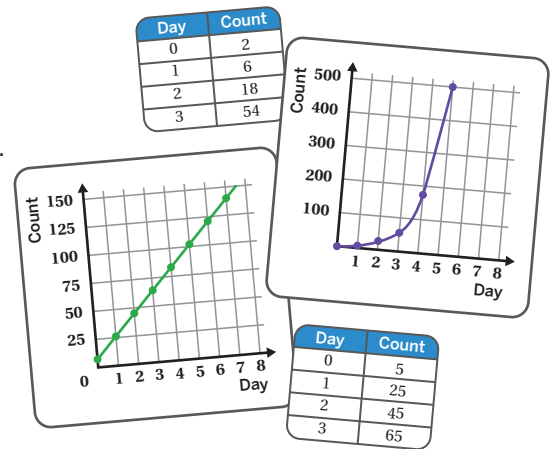
Linear	Exponential	Neither

10 Synthesis

Quantities that grow by a constant rate of change can be modeled by linear functions.

Quantities that grow by a constant growth factor can be modeled by exponential functions.

Discuss: Describe strategies for determining whether a function is linear or exponential.



13 Summary 5.02

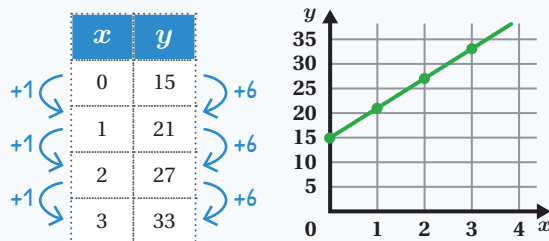
An **exponential function** changes by a constant ratio or **growth factor**. A **linear function** changes by a *constant difference* or *rate of change*.

Here are two examples:

Linear Function

This pattern has a constant difference of 6, so the rate of change is 6.

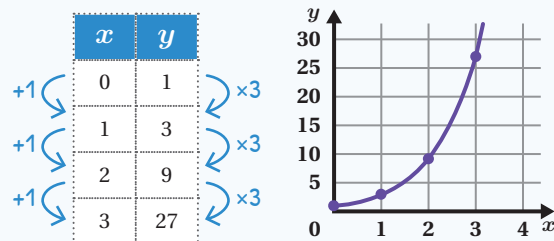
The graph of this linear function is a straight line.



Exponential Function

This pattern has a constant ratio of 3, so the growth factor is 3.

The graph of this exponential function is a curve that gets steeper and steeper.



exponential function A function that increases or decreases by a constant ratio. A function that changes by equal factors over equal intervals. The graph of an exponential function is a curve.

growth factor The constant ratio (or common factor) that each term is multiplied by to generate an exponential pattern.

linear function A function that increases or decreases by a constant rate of change. A function that changes by equal differences over equal intervals. The graph of a linear function is a line.

Practice 5.02

Name: _____ Date: _____ Period: _____

Problems 1–3: These tables show the number of red and yellow globs each day.


Day	0	1	2	3	4
Red Globs	50	70	90	110	

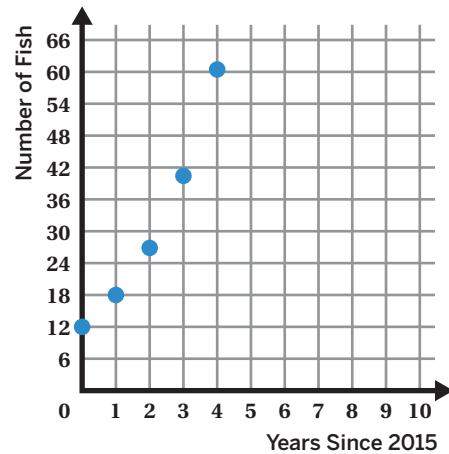
1. How many of each type of glob will there be on day 4?

Day	0	1	2	3	4
Yellow Globs	5	10	20	40	

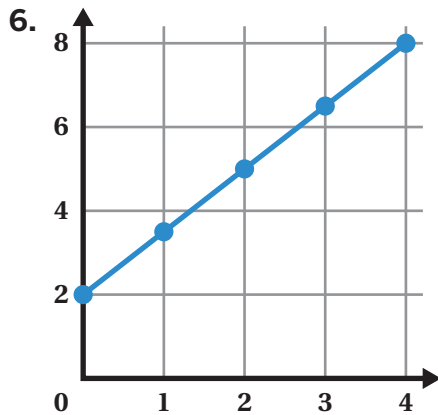
2. Will there be more red or yellow globs on day 10? Show or explain your thinking.
3. Which group of globs changes by a constant growth factor? Show or explain how you know.

Problems 4–5: This graph shows the number of fish in a pond from 2015 to 2019.

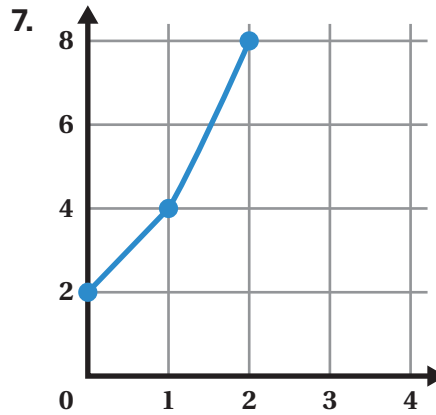
4. How many fish are in the pond in 2015?
5.  **Test Practice** Does the number of fish grow by a *constant difference*? Show or explain how you know.



Problems 6–7: Determine whether each graph shows a constant rate of change or a constant growth rate. Circle your choice.



Constant rate of change Constant growth rate



Constant rate of change Constant growth rate

Practice 5.02

Name: _____ Date: _____ Period: _____

Problems 8–9: Determine whether each table shows a linear or exponential function. Circle your choice.

8.

x	y
0	4
1	8
2	16
3	32

Linear

Exponential

9.

x	y
0	4
1	8
2	12
3	16

Linear

Exponential

10. Pick Problem 8 or Problem 9 and explain how you determined which type of function it was.

Spiral Review

11. Determine the value of each expression when $n = 4$.

a $n^2 - 5$

b $n(n + 6)$

c $3n^2$

12. Approximate the value of each radical expression.

a $\sqrt{10}$

b $\sqrt{54}$

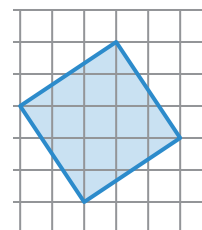
c $\sqrt{41}$

Between _____ & _____

Between _____ & _____

Between _____ & _____

13. Each grid square represents 1 square unit. What is the exact side length of the shaded square?



Unit 5
Lesson
3

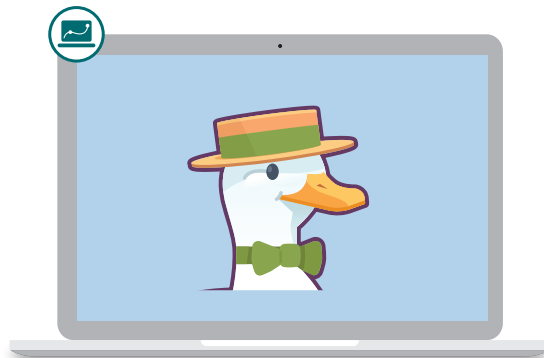
Name: _____ Date: _____ Period: _____

Comparing Models Modeling with Functions

 A-CED.2, F-IF.4, F-LE.1, F-LE.2, F-LE.5, SMP.2, SMP.3, SMP.7

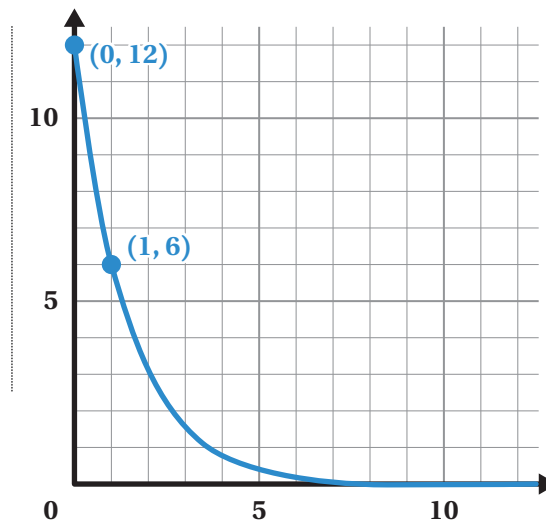
Going Viral

Let's describe connections between graphs and equations, and use graphs to write equations of exponential functions.



Warm-Up

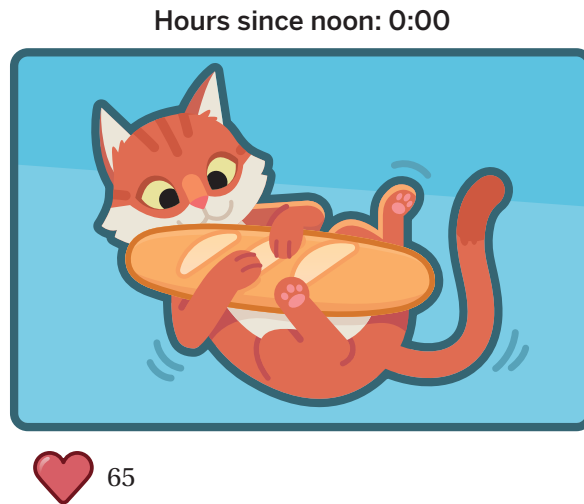
- 1** Here is a graph of an exponential relationship.
 - a** Label the axes with any quantities you'd like.
 - b** Write a story about the quantities based on the graph.



Three Memes

2 Let's watch this meme go viral.

Time (hr)	Likes
0	65
1	130
2	260
3	520
4	1,040
5	2,080

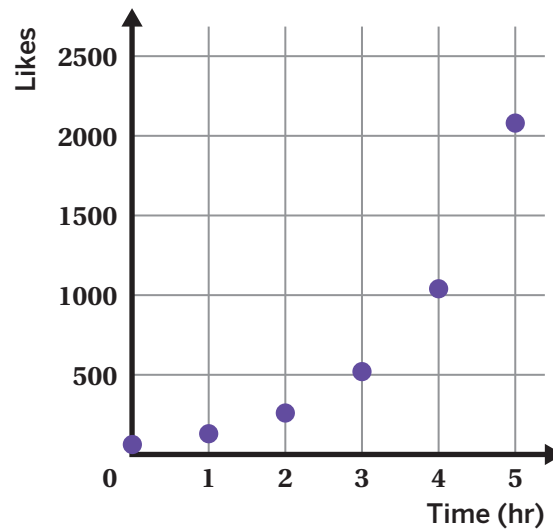


What type of relationship is this?

- A. Linear B. Exponential C. Something else

3 There is an exponential relationship between the number of likes and the hours since noon:

Time (hr), x	Likes, $f(x)$
0	65
1	130
2	260
3	520
4	1,040
5	2,080

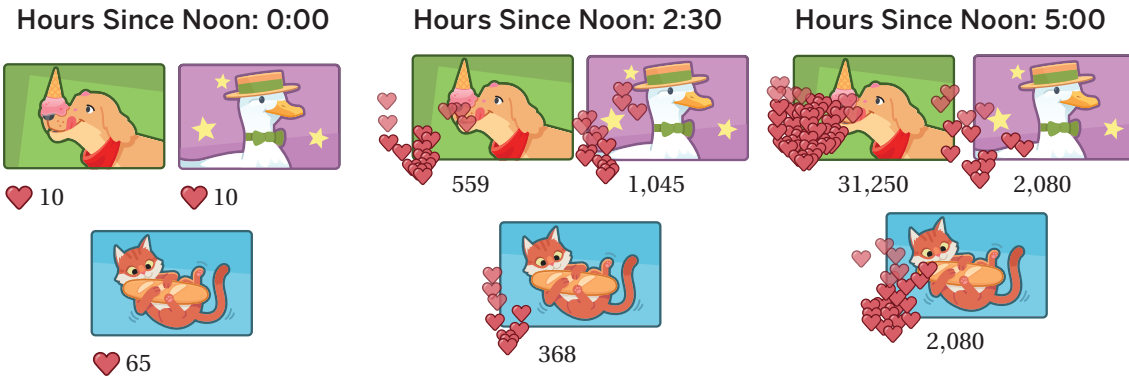


Write an exponential function for this relationship.




$$f(x) = \underline{\hspace{2cm}}$$

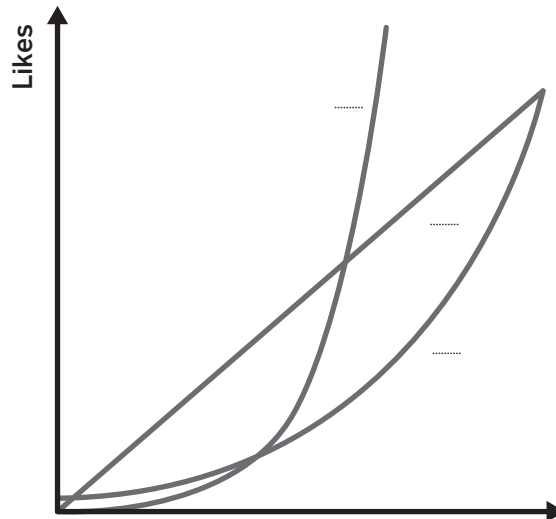
Three Memes (continued)

4 Let's look at the likes for these memes at different times.



Match each meme to the graph that represents it.

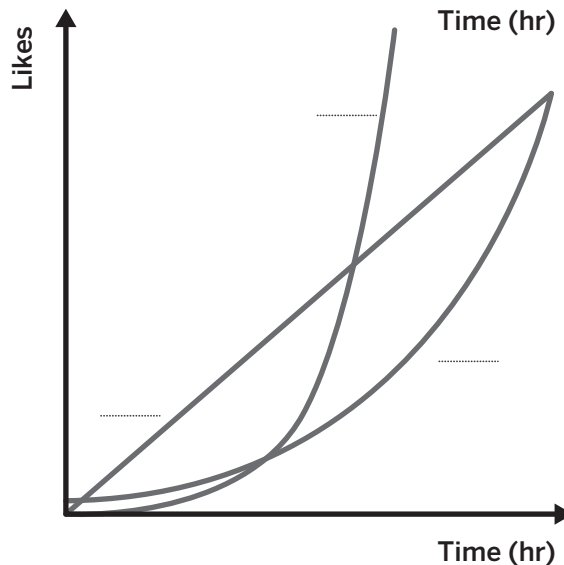
- A. 
- B. 
- C. 



5 Match each function to its graph.

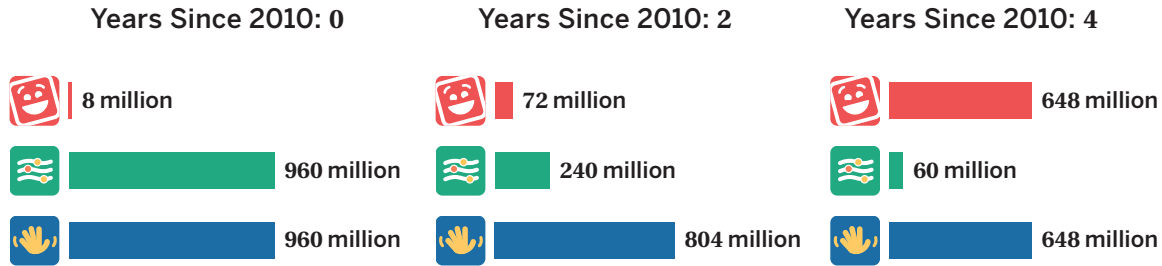
- $f(x) = 10 + 414x$
- $g(x) = 65 \cdot (2)^x$
- $h(x) = 10 \cdot (5)^x$

Explain your thinking.



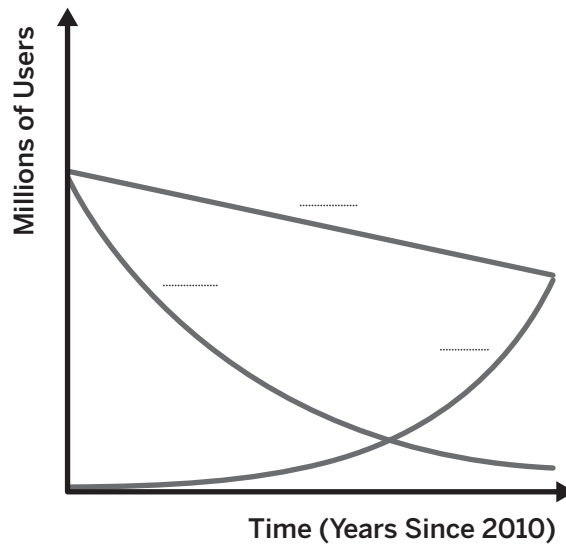
Take It Further

6 Let's look at the number of app users at different times.



Match each app to the graph that represents it.

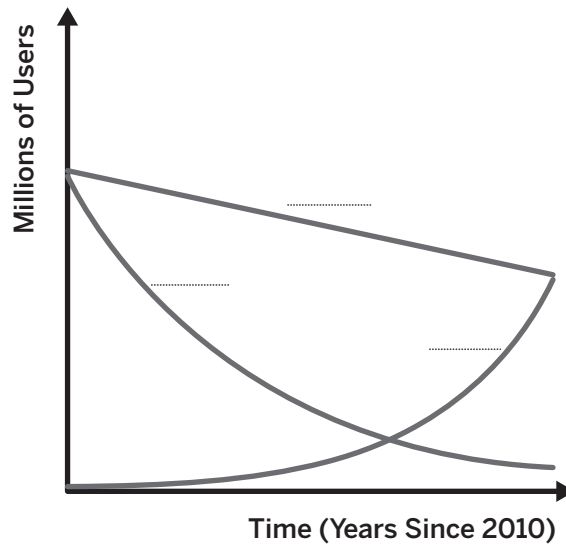
- A.  B.  C. 



7 Match each function to its graph.

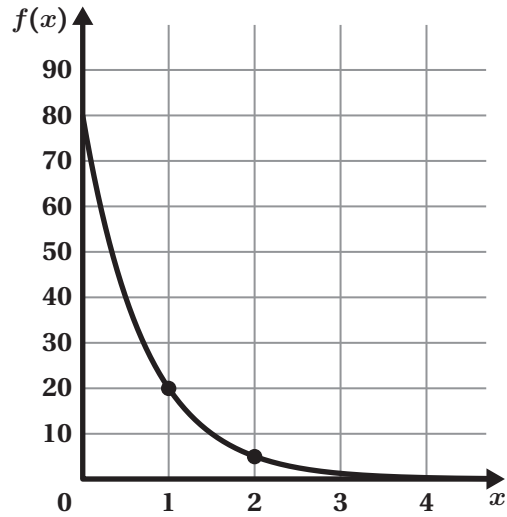
- $m(x) = 960 - 78x$
- $n(x) = 960 \cdot \left(\frac{1}{2}\right)^x$
- $p(x) = 8 \cdot 3^x$

Explain your thinking.

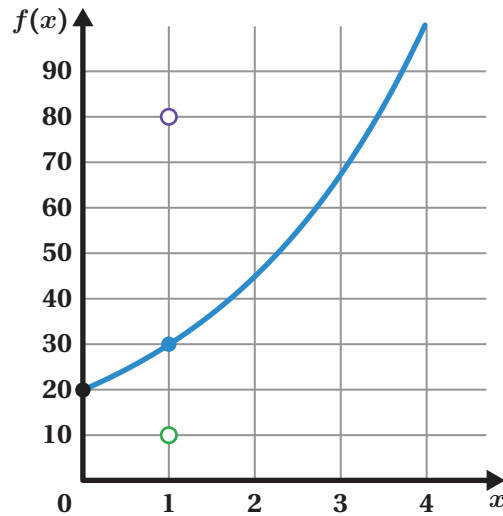


Take It Further (continued)

- 8** Here is the graph of $f(x) = 80 \cdot \left(\frac{1}{4}\right)^x$.
 What might $g(x) = 80 \cdot \left(\frac{1}{2}\right)^x$ look like?
 Show or explain your thinking.



- 9** Here is an exponential relationship.
 The points of two others are given.
 Each relationship includes the point (0, 20)
 and one other point shown on the graph.
 One function has been written for you.
 Write the other two functions.



	Includes the Point	Function
Graph 1	(1, 80)	$f(x) =$
Graph 2	(1, 30)	$g(x) = 20 \cdot \left(\frac{3}{2}\right)^x$
Graph 3	(1, 10)	$h(x) =$

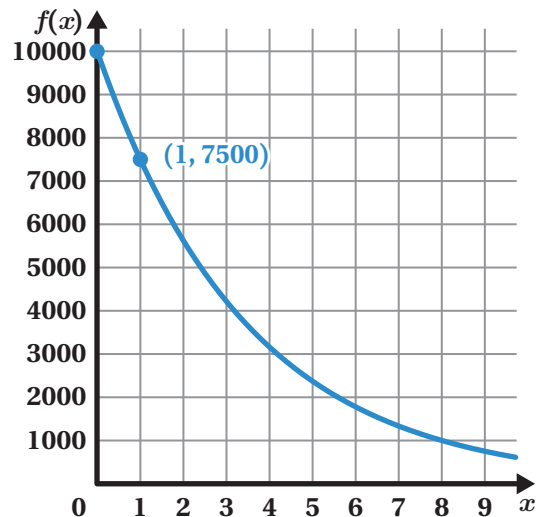
You're invited to explore more.

- 10** Use the You're Invited to Explore More Sheet to answer a question about a pattern.

11 Synthesis

Here is a graph of $f(x) = 10000 \cdot \left(\frac{3}{4}\right)^x$.

Explain where you can see 10,000 and $\frac{3}{4}$ on the graph.



14 Summary 5.03

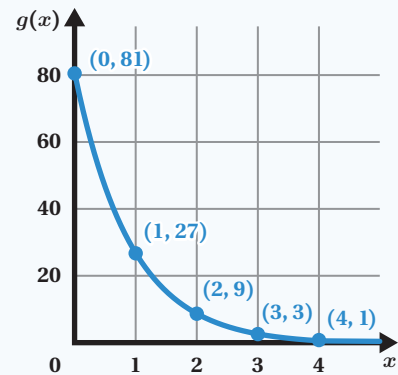
In the exponential function $f(x) = a \cdot b^x$, a represents the y -intercept and b represents the growth factor. Both parts can be seen on a graph.

Let's look at two examples.

Here is the graph of the exponential function

$$g(x) = 81 \cdot \left(\frac{1}{3}\right)^x.$$

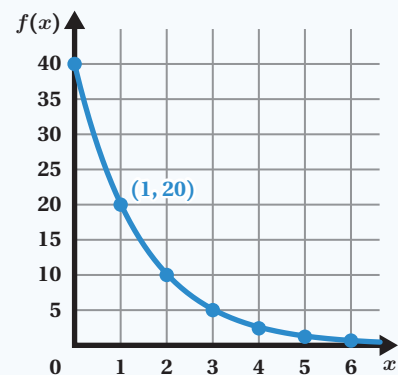
- 81 means that $(0, 81)$ is the y -intercept.
- $\frac{1}{3}$ is the growth factor. As x increases by 1, the y -values are multiplied by a factor of $\frac{1}{3}$.
 $81 \cdot \left(\frac{1}{3}\right) = 27$.



You can also write an exponential equation to represent a graph.

- The y -intercept is $(0, 40)$, so 40 is the a -value.
- The points $(1, 20)$, $(2, 10)$, and $(3, 5)$ are on the graph of the function. Since 10 is $\frac{1}{2}$ of 20 and 5 is $\frac{1}{2}$ of 10, this function has a growth factor of $\frac{1}{2}$, which is the b -value in the equation.

One way to write the equation of this exponential function is $f(x) = 40 \cdot \left(\frac{1}{2}\right)^x$.



Practice 5.03

Name: _____ Date: _____ Period: _____

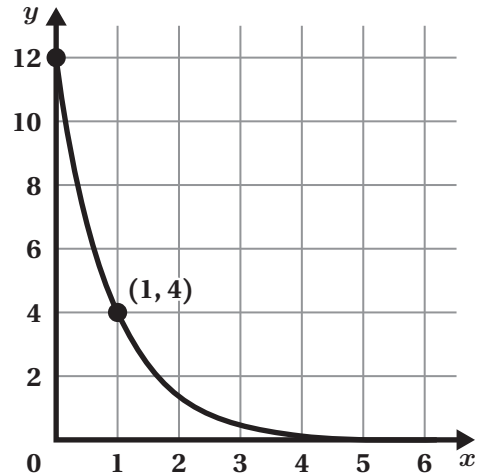
Problems 1–3: Determine the value of each expression when $x = 2$.

1. 4^x

2. $\left(\frac{1}{3}\right)^x$

3. $5(6^x)$

4. Here is a graph of $y = 12 \cdot \left(\frac{1}{3}\right)^x$. Explain where you can see the 12 and the $\frac{1}{3}$ in the graph.



Problems 5–7: Match each equation to the graph that represents it.

Equation A

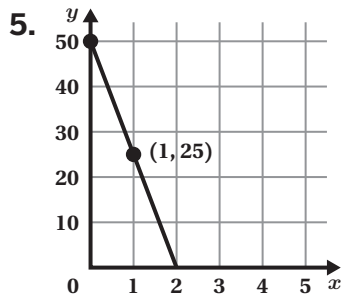
$$y = 50 \cdot \left(\frac{1}{2}\right)^x$$

Equation B

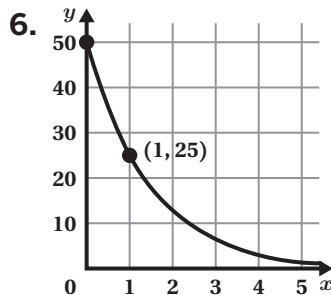
$$y = 50 \cdot 2^x$$

Equation C

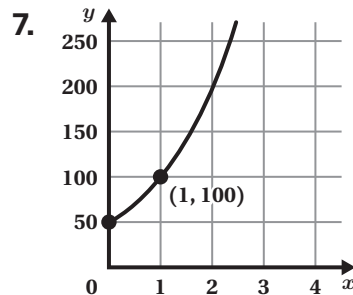
$$y = 50 - 25x$$



Equation _____



Equation _____



Equation _____

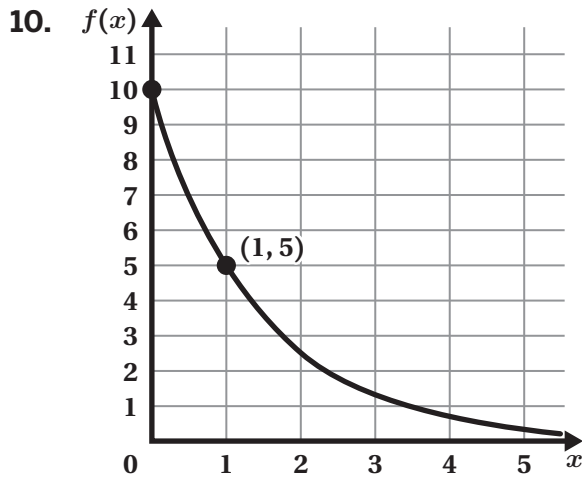
8. Explain how you determined which equation to match with the graph in problem 5.

Practice 5.03

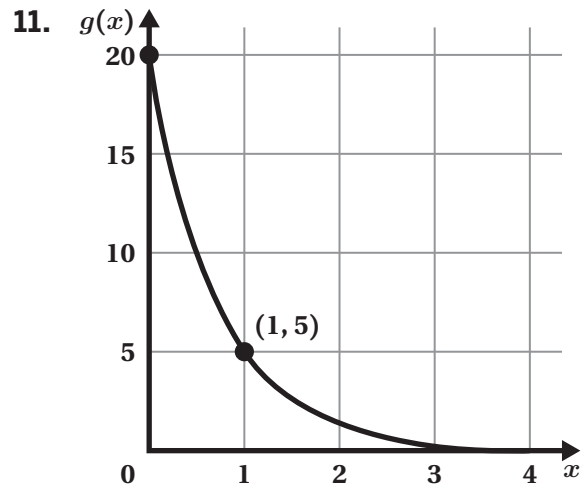
Name: _____ Date: _____ Period: _____

9. Explain how you determined which equation to match with the graph in problem 6.


Problems 10–12: Here are graphs of two different exponential relationships. Write an equation to represent each graph.



Equation _____



Equation _____

12.  **Test Practice** Pick Problem 9 or Problem 10 and explain how you determined the initial value and the growth factor.

Spiral Review

Problems 13–16: Rewrite each expression as a single power.

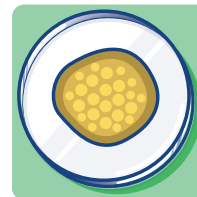
13. $4^4 \cdot 4^8$

14. $\frac{3^5}{3^8}$

15. $(12^3)^5$


16. $\frac{7^3 \cdot 7^4}{7^5}$

Return of the Globs



Let's make connections between different representations of linear and exponential functions.

Warm-Up

1. Let's look at how the number of green and orange globs grows.
2.  **Discuss:** How are these globs growing?

Return of the Globbs

3. For each kind of glob, fill in the situation, table, graph, and equation.

	Green Globbs	Orange Globbs																								
Situation																										
Table	<table border="1"> <thead> <tr> <th>Days, x</th> <th>Number of Globbs, y</th> </tr> </thead> <tbody> <tr><td>0</td><td>4</td></tr> <tr><td>1</td><td>10</td></tr> <tr><td>2</td><td>16</td></tr> <tr><td>3</td><td>22</td></tr> <tr><td>4</td><td>28</td></tr> </tbody> </table>	Days, x	Number of Globbs, y	0	4	1	10	2	16	3	22	4	28	<table border="1"> <thead> <tr> <th>Days, x</th> <th>Number of Globbs, y</th> </tr> </thead> <tbody> <tr><td>0</td><td></td></tr> <tr><td>1</td><td></td></tr> <tr><td>2</td><td></td></tr> <tr><td>3</td><td></td></tr> <tr><td>4</td><td></td></tr> </tbody> </table>	Days, x	Number of Globbs, y	0		1		2		3		4	
Days, x	Number of Globbs, y																									
0	4																									
1	10																									
2	16																									
3	22																									
4	28																									
Days, x	Number of Globbs, y																									
0																										
1																										
2																										
3																										
4																										
Graph																										
Equation	$y =$	$y = 5 \cdot 2^x$																								

4. Parv claims that the Green Globbs are growing linearly.

Support Parv's claim by explaining how the parameters of the equation are shown in the situation, the graph, and the equation.

Return of the Globs (continued)

5. Parv notices the equation given for the Orange Globs is not linear, but exponential.



Discuss: Compare the situation, table, and graph to the equation. How are the numbers 5 and 2 from $y = 5 \cdot 2^x$ evident in each representation?

6. DeAndre discovers another set of globs: Blue Globs. There are 40 blue globs when he first notices them. Every three days, the number of Blue Globs quadruples.

- Parv says: *The Blue Globs will outgrow the Orange Globs because we are starting with so many more.*
- DeAndre says: *The Orange Globs are growing faster. I can tell by the 2 in the equation: $y = 5 \cdot 2^x$.*

a. Whose claim is correct? Explain your thinking.

b. Explain how you could determine if the number of Blue Globs and the number of Orange Globs will ever be equal.

Bacteria

7. For each Petri dish, fill in the table, graph, and equation written in function notation.

	Petri Dish A	Petri Dish B																								
Situation	There are 64 sq. mm of bacteria in a Petri dish. One hour later, there are 96 sq. mm of bacteria in the Petri dish. Two hours later, there are 144 sq. mm of bacteria.	There are 64 sq. mm of bacteria in a Petri dish. Each hour, $\frac{1}{4}$ of the bacteria die.																								
Table	<table border="1"> <thead> <tr> <th>Hours, x</th> <th>Area, $a(x)$</th> </tr> </thead> <tbody> <tr><td>0</td><td>64</td></tr> <tr><td>1</td><td>96</td></tr> <tr><td>2</td><td>144</td></tr> <tr><td>3</td><td></td></tr> <tr><td>4</td><td></td></tr> </tbody> </table>	Hours, x	Area, $a(x)$	0	64	1	96	2	144	3		4		<table border="1"> <thead> <tr> <th>Hours, x</th> <th>Area, $b(x)$</th> </tr> </thead> <tbody> <tr><td>0</td><td></td></tr> <tr><td>1</td><td></td></tr> <tr><td>2</td><td></td></tr> <tr><td>3</td><td></td></tr> <tr><td>4</td><td></td></tr> </tbody> </table>	Hours, x	Area, $b(x)$	0		1		2		3		4	
Hours, x	Area, $a(x)$																									
0	64																									
1	96																									
2	144																									
3																										
4																										
Hours, x	Area, $b(x)$																									
0																										
1																										
2																										
3																										
4																										
Graph																										
Equation	$a(x) =$	$b(x) =$																								

8. **Discuss:** One of these functions shows exponential growth and one shows exponential decay. Which do you think is which, and why?

Synthesis

9. Explain where you see the *growth factor* of an exponential relationship in each representation. Use the examples if they help with your thinking.

Table:

Situation:

Equation:

x	$f(x)$
0	2
1	6
2	18
3	54

$$f(x) = 2 \cdot (3)^x$$

The area of the bacteria triples each day.

Summary 5.04

The *growth factor* is the constant ratio that each term is multiplied by to generate an exponential relationship. You can identify the growth factor of an exponential function from a situation, an equation, a table, or a graph. Here is an example:

	Exponential Function	The growth factor is 2 because . . .								
Situation	There are 5 sq. mm of bacteria in a Petri dish. Each day, the number of cells doubles.	The cells are doubling every day.								
Table	<table border="1"> <thead> <tr> <th>x</th> <th>$g(x)$</th> </tr> </thead> <tbody> <tr> <td>0</td> <td>5</td> </tr> <tr> <td>1</td> <td>10</td> </tr> <tr> <td>2</td> <td>20</td> </tr> </tbody> </table>	x	$g(x)$	0	5	1	10	2	20	$5 \cdot 2 = 10$ and $10 \cdot 2 = 20$.
x	$g(x)$									
0	5									
1	10									
2	20									
Graph		The distance between the points (0, 5) and (1, 10) doubles in the y direction.								
Equation	$g(x) = 5 \cdot (2)^x$	2 is the base of the exponential term and it is raised to the power of x .								

exponential decay An exponential relationship that decreases because it has a growth factor between 0 and 1.

exponential growth An exponential relationship that increases because it has a growth factor greater than 1.

Practice 5.04

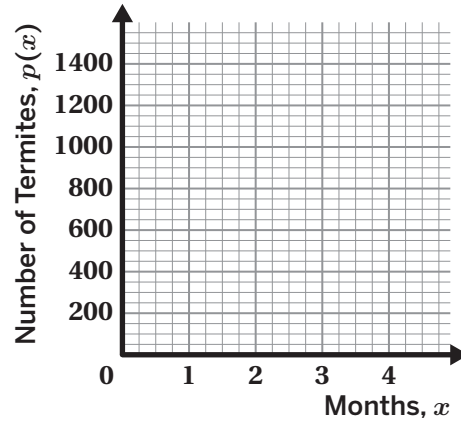
Name: _____ Date: _____ Period: _____

Problems 1–2: A scientist monitors a population of termites that is growing exponentially. She writes the equation $p(x) = 40 \cdot 2.5^x$ to model the total number of termites after x months.

1. Complete the table.

Months, x	Number of Termites, $p(x)$
0	40
1	100
2	
3	
4	

2. Graph the situation.




Problems 3–4: The table models an exponential function with a growth factor of $\frac{1}{3}$.

3. Fill in the table with values for $f(x)$ that could represent this function.

x	0	1	2	3
$p(x)$				

4. Write an equation to model your function.

5.  **Test Practice** There are 50 sq. mm of bacteria in a Petri dish. The bacteria grow exponentially. 1 hour later, there are 70 sq. mm of bacteria in the Petri dish.

What is the growth factor that models the hourly growth of this bacteria?

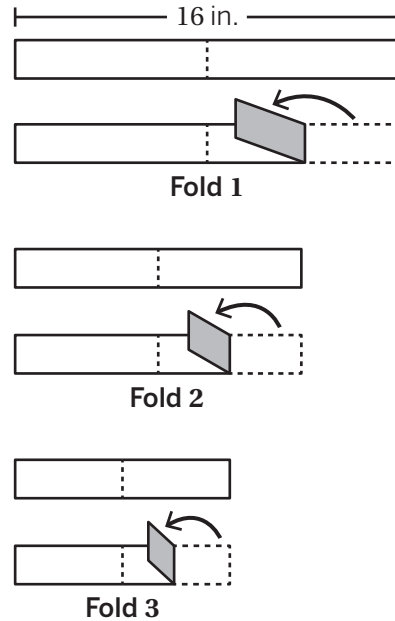
Practice 5.04

Name: _____ Date: _____ Period: _____

6. Neena writes the function $f(x) = 4 \cdot (-0.5)^x$ to model a population that has an initial value of 4 and decreases by half every x days. Is she correct? Explain your thinking.

7. One side of a strip of paper is folded to the middle several times. The new length of the paper is recorded after each fold. Determine the length of the strip of paper after 4 folds. Use the table if it helps your thinking.

Folds	Length (in.)
0	16
1	
2	
3	
4	



Spiral Review

Problems 8–10: Here is a data set: [1, 2, 3, 3, 3, 5, 5, 6, 7, 8, 22].

8. Complete the table. Use a calculator if it helps with your thinking.

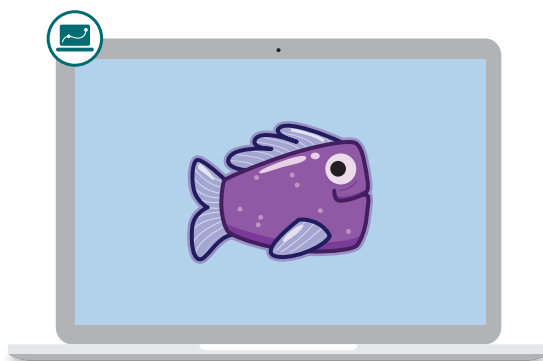
Min.	Q1	Median	Q3	Max.

9. What number is an outlier of the data set?

10. What number could be added to the data set that wouldn't change the 5-number summary?

Carlos and Corals

Let's evaluate exponential functions with inputs that are positive, negative, and zero.



Warm-Up

1 Select *all* the expressions that are equivalent to $2^{(-3)}$.

A. $\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2}$

B. $\frac{1}{2 \cdot 2 \cdot 2}$

C. $2 \cdot (-3)$

D. $(-2) \cdot (-2) \cdot (-2)$

E. $8^{(-1)}$

Carlos's Fish

- 2** Carlos's apartment still does not allow pets, so he decided to buy a new toy fish.

The mass of the fish doubles every hour.

What type of function do you think will model the mass of this fish over time?

Linear Exponential Neither

Explain your thinking.



- 3** Carlos's new toy fish has a constant growth factor when placed in water.

What is the mass of the toy fish after 4 hours?

Time (hr)	Mass (g)
1	20
2	40
3	80
4	

- 4** Carlos writes $m(t)$ to model the fish's mass over time, t .

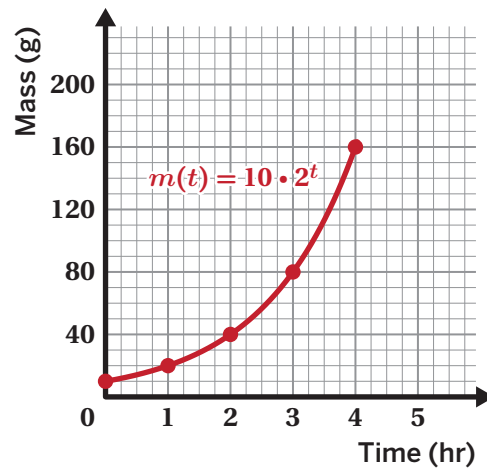
$$m(t) = 10 \cdot 2^t$$

What is the value of $m(0)$? Explain your thinking.

Carlos's Fish (continued)

- 5** Explain where you see the growth factor in the graph, the table, or the function.

Time, t	Mass, $m(t)$
0	10
1	20
2	40
3	80
4	160



- 6** **a** What is the value of $m(5)$?

- b** What is the value of $m(-1)$?

- c**  **Discuss:**

- What does each value say about the fish's mass?
- How would you describe the *domain* of $m(t)$?

Coral Reefs

Screens 7–9: A marine biologist is studying a coral reef. In 2010, she estimated that its volume was 320 cubic meters.

She wrote the function $v(t)$ to represent the volume of the coral reef t years after 2010:


$$v(t) = 320\left(\frac{4}{5}\right)^t$$

7 Based on $v(t)$, what was the reef's volume in 2011?

- A. Less than 320 cubic meters B. Equal to 320 cubic meters C. Greater than 320 cubic meters

Explain your thinking.

8 **a** Determine the value of $v(2)$.

b  **Discuss:** What domain could make sense for $v(t)$?

9 Determine the missing values.

Years Since 2010	-3	-2	-1	0	1	2
Volume (cubic meters)				320	256	204.8

Coral Reefs (continued)

- 10** Here is how Angel and Sora determined the volume of the coral reef in 2007 (3 years before 2010):

Angel

Year Since 2010	Volume (cubic meters)
-3	625 $\cdot \frac{5}{4}$
-2	500 $\cdot \frac{5}{4}$
-1	400 $\cdot \frac{5}{4}$
0	320 $\cdot \frac{5}{4}$


Sora

$$v(-3) = 320 \left(\frac{4}{5} \right)^{-3}$$


$$v(-3) = 320 \cdot \left(\frac{5}{4} \right)^3$$

$$v(-3) = 320 \cdot \frac{125}{64}$$

$$v(-3) = 625$$

 **Discuss:** What is each student's strategy?

- 11** Here is a new function: $f(x) = 18 \cdot 3^x$.

a  **Discuss:** Will $f(-2)$ be less than or greater than 18?

b Determine the value of $f(-2)$.

12 Synthesis

Describe a strategy for evaluating exponential functions for negative inputs.

Use the examples if they help with your thinking.

$$a(-4) = 5 \cdot 10^{(-4)}$$

$$b(-3) = 10 \cdot \left(\frac{1}{2}\right)^{(-3)}$$

15 Summary 5.05

You can evaluate exponential functions for inputs that are positive, negative, or zero.

Let's evaluate the function $f(x) = 9 \cdot 4^x$ for $f(-3)$ using the equation or a table.

You can substitute a value into the *equation*.

Steps	Explanation
$f(-3) = 9 \cdot 4^{(-3)}$	Substitute $x = (-3)$ into the function.
$f(-3) = 9 \cdot \left(\frac{1}{4}\right)^3$	Apply the property of negative <i>exponents</i> to rewrite the expression with a positive exponent.
$f(-3) = 9 \cdot \left(\frac{1}{64}\right)$	Rewrite the expression without any exponents.
$f(-3) = \frac{9}{64}$	Multiply to determine the value of $f(-3)$.

You can move backwards in a *table*.

x	$f(x)$
-3	$\frac{9}{64}$
-2	$\frac{9}{16}$
-1	$\frac{9}{4}$
0	9

$\times \frac{1}{4}$
 $\times \frac{1}{4}$
 $\times \frac{1}{4}$

Multiply 9 by $\frac{1}{4}$ three times to determine the value of $f(-3)$.

Practice 5.05

Name: _____ Date: _____ Period: _____

1. Which equation best models the data in the table?

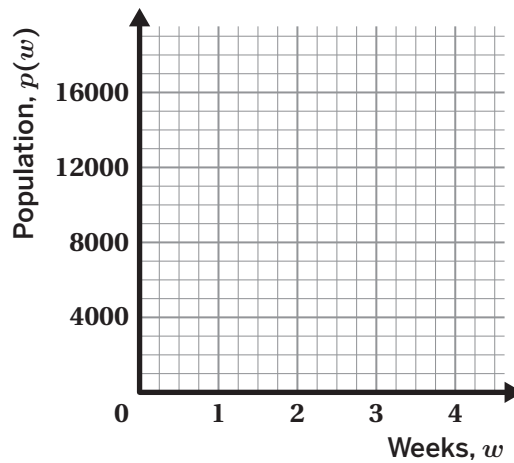
- A. $f(x) = 80(1.25)^x$
- B. $f(x) = 64(1.25)^x$
- C. $f(x) = 64 + 1.25x$
- D. $f(x) = 60 + 20x$

x	$f(x)$
1	80
2	100
3	125
4	156.25

Problems 2–4: The equation $p(w) = 1000 \cdot 2^w$ models a population of mosquitos, $p(w)$, where w is the number of weeks after the population was first measured.

2. Complete the table and plot the values on the graph.

Weeks, w	Population, $p(w)$
0	
1	
2	
3	
4	




3. Where on the graph do you see the 1,000 from the equation?

4. Determine the value of $p(-2)$ and explain what it means in this situation.

Problems 5–7: The equation $f(t) = 800 \cdot \left(\frac{1}{2}\right)^t$ models a fish population, $f(t)$, where t is time in years since the beginning of 2015.

5. What is the population of fish at the beginning of 2015?

6. What is the population of fish at the beginning of 2018?

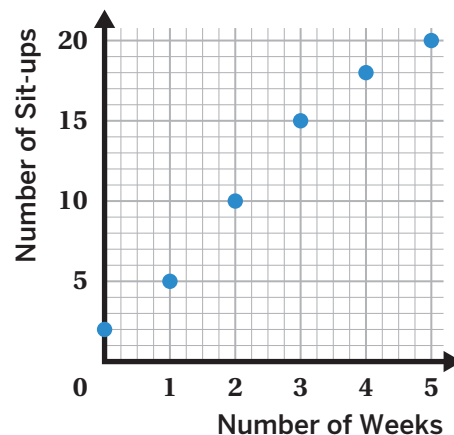
7.  **Test Practice** What is the population of fish at the beginning of 2012? Do you think this makes sense in this situation?

Practice 5.05

Name: _____ Date: _____ Period: _____

Problems 8–9: Dalia created this graph to represent her progress with her fitness goal.

8. What is the domain of the function graphed?
9. What would be a reasonable estimate for the number of sit-ups Dalia could do by week 6?



Spiral Review

Problems 10–11: Charlie's gaming club wants to make at least \$300 selling boxes of cookies and pies. They make \$9 for each box of cookies and \$15 for each pie.

10. Write an inequality to represent the number of boxes of cookies, c , and pies, p , the team can sell to make at least \$300.
11. If the team sells 5 boxes of cookies, what is the minimum number of pies they need to sell in order to meet their goal?

Differences and Factors

Let's generalize how linear and exponential functions change over equal intervals.



Warm-Up

1. Rewrite each expression using a single exponent.

a $3^2 \cdot 3^5$

b $\frac{4^5}{4^2}$

c $\frac{5^{x+2}}{5^x}$

2. Rewrite this expression using as few terms as possible.

$$(4(n + 1) + 5) - (4n + 5)$$

Linear Functions

3. Complete the table for the function $f(x) = 2x + 5$.

x	$f(x)$
0	5
1	7
2	
...	...
7	
8	

4. What happens to $f(x)$ when x increases by 1?

5. Precious says: *Without calculating, I know that $f(5009) - f(5008)$ must be 2.*

Is this correct? Explain how you know.

6. What happens to $f(x)$ when x increases to $x + 1$? In other words, what is $f(x + 1) - f(x)$?

7. Precious wrote this expression to help answer the previous problem. Rewrite it using as few terms as possible.

$$(2(x + 1) + 5) - (2x + 5)$$

x	$f(x)$
x	$2x + 5$
$x + 1$	$2(x + 1) + 5$

8.  **Discuss:**

- Why does Precious's expression have the same value as your answer to Problem 4?
- What does this tell you about any linear function?

9. What do you think happens to $f(x)$ when x increases to $x + 3$?

10. Write an expression like Precious's expression in Problem 7. Then rewrite your expression using the fewest terms and check if it matches your prediction.

Exponential Functions

11. Complete the table for the function $g(x) = 3^x$.

12. What happens to $g(x)$ when x increases by 1?

13. Precious says: *Without calculating, I know that $\frac{g(5009)}{g(5008)}$ must be 3.*

Is this correct? Explain how you know.

x	$g(x)$
0	1
1	3
2	
...	...
6	
7	

14. What happens to $g(x)$ when x increases to $x + 1$? In other words, what is $\frac{g(x+1)}{g(x)}$?

15. Precious wrote this expression to help answer the previous problem. Rewrite it using a single exponent.

$$\frac{3^{x+1}}{3^x}$$

x	$g(x)$
x	3^x
$x + 1$	3^{x+1}


16.  **Discuss:**

- Why does Precious's expression have the same value as your answer to Problem 12?
- What does this tell you about any exponential function?

17. What do you think happens to $g(x)$ when x increases to $x + 3$?

18. Write an expression like Precious's expression in Problem 15. Then rewrite your expression using a single exponent and check if it matches your prediction.

Synthesis

19.  **Discuss:** Precious says: *If intervals are equal, then linear functions grow by constant differences and exponential functions grow by constant growth factors.*

Explain what this means in your own words.

Summary 5.06

A linear function always increases (or decreases) by equal differences over equal *intervals*, and an exponential function increases (or decreases) by equal *factors* over equal intervals.

Here are two examples:

	Linear function $g(x) = 2x + 3$	Exponential function $h(x) = 3^x$
Describe how the function changes when x grows by 1	<p>When x grows by 1, $g(x)$ will always increase by $2(1) = 2$:</p> $\begin{aligned} g(x+1) - g(x) &= (2(x+1) + 3) - (2x + 3) \\ &= (2x + 2 + 3) - (2x + 3) \\ &= 2x + 5 - 2x - 3 \\ &= 5 - 3 \\ &= 2 \end{aligned}$	<p>When x grows by 1, $h(x)$ will always multiply by a factor of $3^1 = 3$.</p> $\begin{aligned} \frac{h(x+1)}{h(x)} &= \frac{3^{(x+1)}}{3^x} \\ &= \frac{3^x \cdot 3^1}{3^x} \\ &= 3 \end{aligned}$
Describe how the function changes when x grows by k	<p>When x grows by k, $g(x)$ will always increase by $2(k)$.</p> $\begin{aligned} g(x+k) - g(x) &= 2(x+k) + 3 - (2x + 3) \\ &= 2x + 2k + 3 - 2x - 3 \\ &= 2k \end{aligned}$	<p>When x grows by k, $h(x)$ will always multiply by a factor of 3^k.</p> $\begin{aligned} \frac{h(x+k)}{h(x)} &= \frac{3^{(x+k)}}{3^x} \\ &= \frac{3^x \cdot 3^k}{3^x} \\ &= 3^k \end{aligned}$

Practice

5.06

Name: _____ Date: _____ Period: _____


Problems 1–2: Here are two functions. $f(x)$ is a linear function and $g(x)$ is an exponential function.


$f(x) = 2x + 1$						
x	0	1	2	3	4	5
$f(x)$	1	3	5	7	9	11


$g(x) = 2 \cdot 3^x$						
x	0	1	2	3	4	5
$g(x)$	2	6	18	54	162	486

- How does $f(x)$ change when x grows by 1?
- How does $g(x)$ change when x grows by 1?

Problems 3–4: Here is a function: $h(x) = 4^x$.

3.  **Test Practice** Calculate $\frac{h(x+1)}{h(x)}$.

4.  **Test Practice** Calculate $\frac{h(x+2)}{h(x)}$.

5.  **Test Practice** Here is a function: $g(x) = 2^x$. Select *all* of the true statements.

- A. When the input x is increased by 1, the value of $g(x)$ increases by 2.
- B. When the input x is increased by 1, the value of $g(x)$ multiplies by 2.
- C. When the input x is increased by 3, the value of $g(x)$ increases by 8.
- D. When the input x is increased by 3, the value of $g(x)$ multiplies by 8.
- E. When the input x is increased by 3, the value of $g(x)$ multiplies by 6.

6. Whenever the x -value increases by 1, the value of $j(x)$ increases by 5. Which of these functions could be $j(x)$?

- A. $j(x) = 3x + 5$ B. $j(x) = 5x + 3$ C. $j(x) = 5^x$ D. $j(x) = x^5$

7. Whenever the x -value increases by 2, the value of $m(x)$ multiplies by 9. Which of these functions could be $m(x)$?

- A. $m(x) = 3x + 9$ B. $m(x) = 9x + 3$ C. $m(x) = 3^x$ D. $m(x) = 9^x$

Practice 5.06

Name: _____ Date: _____ Period: _____

8. Match each description to its function.

- a Decreases by 19% when x increases by 1. $f(x) = 0.19^x$
- b Increases by 81% when x increases by 1. $g(x) = 1.81^x$
- c Increases by 19% when x increases by 1. $h(x) = 0.81^x$
- d Decreases by 81% when x increases by 1. $j(x) = 1.19^x$

9. Determine *all* of the situations that are changing in a linear fashion.

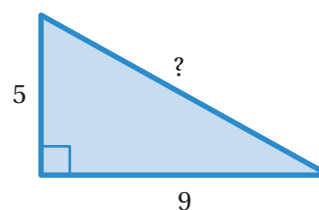
- A. A student adds \$20 to their piggy bank each month.
- B. A student adds \$20 to their bank account each month. The account earns 1.5% interest compounded monthly.
- C. Every week, 85% of a radioactive substance remains from the beginning of the week.
- D. Every day, 8 gallons of water evaporate from a swimming pool.
- E. The value of a car depreciates by 15% each year.

Spiral Review

10. Determine the value of $f(x) = 15x + 4$ for each function statement.

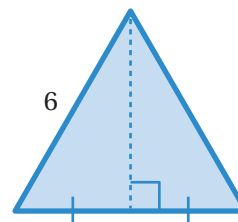
$f(3) =$ $f(-1) =$ $f\left(\frac{1}{3}\right) =$

11. Here is a right triangle. What is the *exact* length of the hypotenuse?



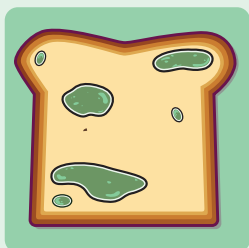
12. Here is an equilateral triangle. The length of each side is 6 units.

Find the *exact* height of the triangle.



Notes:

Exponential Growth and Decay



Lesson 7
Growing Mold



Lesson 8
At a Loss



Lesson 9
Lake Desmosia



Lesson 10
Marbleslides:
Exponentials

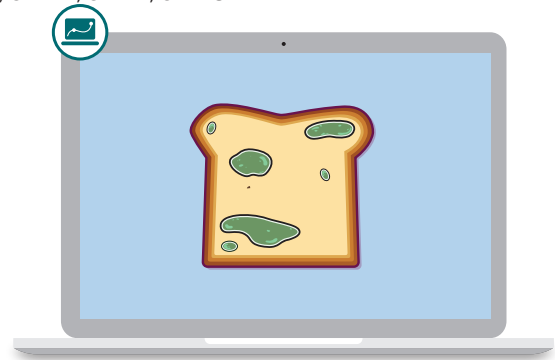
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Comparing Models Modeling with Functions Composing Functions

A-SSE.1.a, A-SSE.1.b, F-BF.1, F-LE.1.c, F-LE.2, SMP.2, SMP.7, SMP.8

Growing Mold

Let's explore how to model situations that change by a percent increase with exponential functions.



Warm-Up

Determine the value of each statement.

1 10% of 30

2 100% of 30

3 110% of 30

4 110% of 50

Growing Mold

5 A piece of bread is left out on a counter.

Let's watch an animation to see what happens over time.

What do you notice? What do you wonder?



6 This mold grows by 75% each day. How much mold will there be on day 4?

Days	Area of Mold (sq. cm)
3	16
4	
5	
6	

7 How much mold will there be on days 5 and 6?

Growing Mold (continued)

- 8** Arnav made a table to help him write a function to represent the area of mold, $m(x)$, after x days.

How do you see the 75% increase represented in the function's equation? In the table?

Equation:

Table:

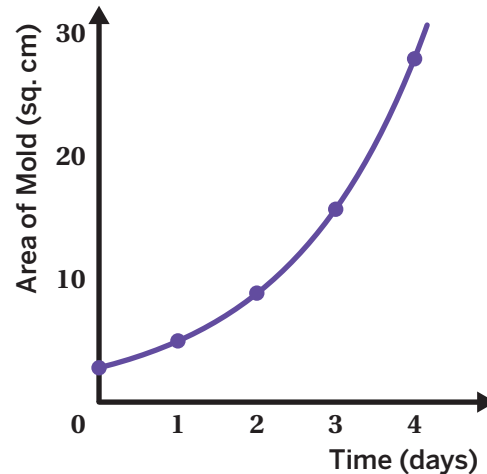
$$m(x) = 2.985(1.75)^x$$

Days	Area of Mold (sq. cm)
0	2.985
1	5.224
2	9.14
3	16
4	28

Discuss: How might Arnav have found the entries in the table for 2 days, 1 day, and 0 days?

- 9** Here is the graph of $m(x) = 2.985(1.75)^x$.

- a** **Discuss:** What does 2.985 represent in Arnav's equation?
- b** Determine how much mold there will be after 10 days.




Growing with Percents

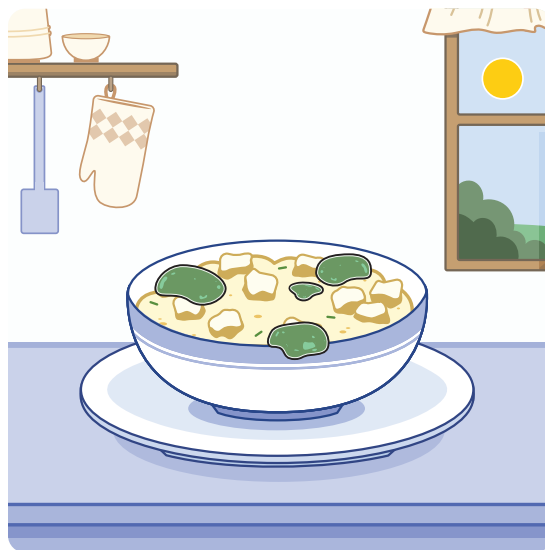
- 10** Tyler made potato salad and forgot to put it in the refrigerator. The amount of bacteria (in microns) in the potato salad increases by 4% every minute.


Tyler wrote a function to represent the amount of bacteria in the potato salad:

$$p(t) = 5 \cdot 0.04^t.$$

 **Discuss:** In Tyler's equation, what do each of these represent?

- $p(t)$
- 5
- 0.04
- t



 **Discuss:** Tyler made a mistake when writing this equation. What was Tyler's mistake? Explain your thinking.

- 11** Which equation (or equations) correctly represents the amount of bacteria growing in the potato salad? Select *all* that apply.

- | | |
|---------------------------|----------------------------------|
| A. $p(t) = 5(1 + 0.04)^t$ | B. $p(t) = 5 \cdot (1 - 0.04)^t$ |
| C. $p(t) = 5 + 1.04t$ | D. $p(t) = 5 \cdot 1.04^t$ |

Growing with Percents (continued)

12 Let's look at which function Tyler selected.

What does the 1 represent in this situation?

13 Match each function with the situation that represents the same relationship. One function will have no match.

Functions**Situations**

a $a(x) = 20 \cdot 0.85^x$

..... A population of bacteria starts with 20 cells and grows by 85%.

b $b(x) = 20 \cdot 1.85^x$

..... A population of frogs starts with 85 frogs and grows by 20%.

c $c(x) = 85 \cdot 1.2^x$

d $d(x) = 85 \cdot 1.02^x$

Days	Amount of Money (\$)
0	85
1	86.70
2	88.43

You're invited to explore more.

14 Heat and humidity can cause some types of bacteria to grow quickly. Imagine that in a humid room the amount of bacteria in a potato salad *triples* every hour.

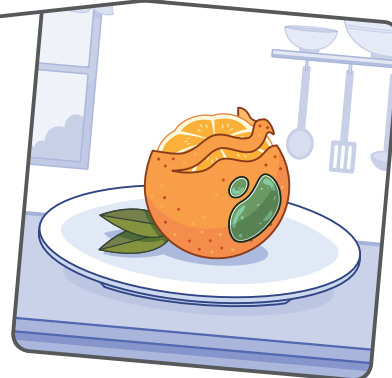
By what percent is the bacteria growing per hour? Explain your thinking.

15 Synthesis

How do you write an exponential function in the form $f(x) = a \cdot b^x$ that represents growing by a percentage?

Use the example if it helps with your thinking.

An orange has 3 sq. mm of mold.
The mold grows by 90% each day.



17 Summary 5.07

Exponential functions can be written in the form $f(x) = a \cdot b^x$, where a is the *initial value* and b is the growth factor. Exponential functions represent repeated *percent increase* when the growth factor is larger than 1.

In situations that model repeated percent increase, the growth factor b can be written as 1 (representing 100%) plus the percent increase in decimal form.

For example, the value of a baseball card collection increases by 4% every year. In 2020, the collection was valued at \$500. Let $f(x)$ represent the value of the collection and x represent the years since 2020.

- At first, the collection was valued at \$500. The initial value, or a , is 500.
- The value increases by 4% every year. Because the value of the cards increases by a repeated percent each year, we can represent the growth by changing the percent to a decimal (4% to 0.04) and adding 1. The growth factor, or b , will be 1.04.

We can write the exponential function that represents this situation as $f(x) = 500 \cdot (1.04)^x$.

Practice

5.07

Name: _____ Date: _____ Period: _____

Problems 1–3: A group of biologists tracked the number of deer in a forest over several years. There were 600 deer when they first counted. The population has increased by 15% each year.


1. How many deer are in the forest 1 year after the biologists first counted?

2. Write a function that represents the deer population after 3 years.

$$f(3) =$$

3. Write a function that represents the deer population after t years.

$$f(t) =$$

4.  **Test Practice** Sothy's family paid \$1,300 in property tax last year. This year, the county will increase the property tax by 2.1%.

Select *all* the expressions that represent Sothy's family's property taxes this year.

A. $1300 + (1.021)$

B. $1300(1.21)$

C. $1300(1.021)$

D. $1300(1.0021)$

E. $1300 + 1300(0.021)$

Problems 5–6: Sai gets a \$500 loan from the bank with an annual interest rate of 6%.

5. Write a function, $f(t)$, to represent the amount Sai will owe, in dollars, after t years.

6. Complete the table to determine how much money Sai will owe over time if no payments are made.

Time (yr)	Amount Owed (\$)
0	500
1	
2	
3	
4	

Practice 5.07

Name: _____ Date: _____ Period: _____

Problems 7–9: Three cities have the same initial population and different percent increases each year. Match each function $p(t)$, representing the population after t years, with its correct description.

$$p(t) = 5000 \cdot (1.20)^t$$

$$p(t) = 5000 \cdot (1.02)^t$$

$$p(t) = 5000 \cdot (1.002)^t$$

- 7.** City A has a 0.2% annual increase in population.
.....
- 8.** City B has a 20% annual increase in population.
.....
- 9.** City C has a 2% annual increase in population.
.....

Spiral Review

- 10.** Order these values from least to greatest.

75% of 12

25% of 32

50% of 20

10% of 95

--	--	--	--

Least

Greatest

Problems 11–12: A shoe store decreased the prices 16% compared to last year.

- 11.** If x is the price before the decrease and y is the price after the decrease, which equations correctly represent this situation?

- A. $y = x - 0.16$
- B. $y = x - 0.16x$
- C. $y = 16 - x$
- D. $y = (1 - 0.16)x$
- E. $y = 0.84x$


- 12.** A pair of designer boots was \$325 last year. What is the price this year?

At a Loss

Let's make connections between different representations of exponential decay functions.



Warm-Up

1.  **Discuss:** How do you think each of these might change over time?

- The trade-in value of a cell phone
- The amount of caffeine in the body after drinking coffee
- The resale value of a laptop

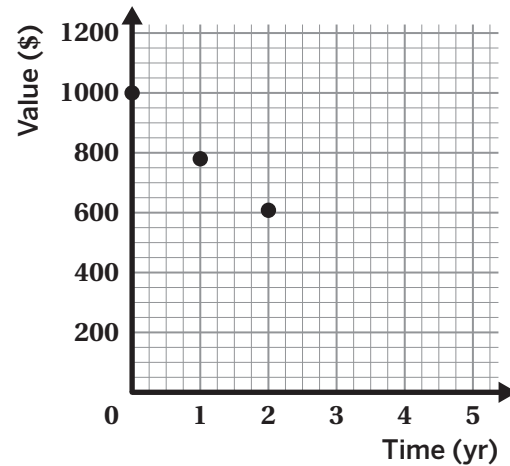


Des-Phone

Aaliyah bought a Des-Phone for \$1,000. Typically, the value of phones decays exponentially. Every year, she checks the trade-in value to see how much money her phone is worth.

2. Complete the table and graph to represent Aaliyah's situation.

Time (yr)	Value (\$)
0	1,000
1	780
2	608.40
3	
4	



Aaliyah and Taylor wrote different functions to represent this situation.

Aaliyah

$$f(x) = 1000(0.78)^x$$

Taylor

$$f(x) = 1000(1 - 0.22)^x$$

3.  **Discuss:**

- How are the functions alike?
- How are they different?

4. Choose one function. Show or explain what each number represents about the situation.

Des-Phone (continued)

5. Use any strategy to calculate the trade-in value of the phone after 10 years.
6. Select *all* the statements that are true about this situation.
- A. The phone retains 78% of its value each year.
 - B. The value of the phone is decreasing by 78% each year.
 - C. The trade-in value of the phone is decreasing by 22% each year.
 - D. The value of the phone will decrease by \$220 each year.

Sorting Percents

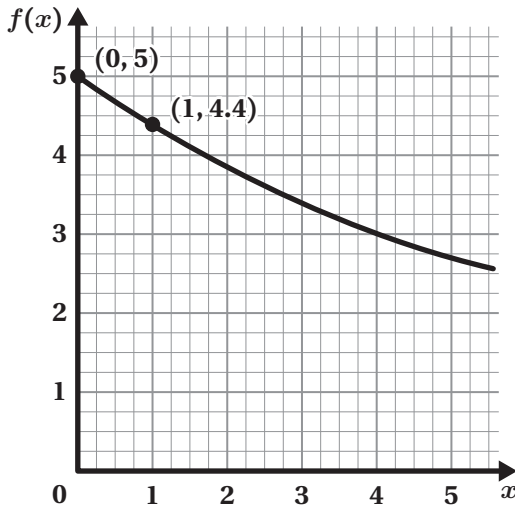
7. You will use a set of cards for this activity.

Use the cards to complete each row. Create your own situation for the last row.

	Situation	Percent Decrease	Functions
Medicine	<p>Victor takes 800 mg of an antiviral medicine. The amount of medicine left in his body is reduced by 25% every hour.</p> <p>Write a function to represent the amount of medicine remaining in Victor's body after x hours.</p>		
Laptops	<p>A laptop that was originally \$800 loses 75% of its value each year.</p> <p>Write a function to represent the value of the laptop after x years.</p>		
Coffee	<p>Aditi drinks coffee with 200 mg of caffeine. Typically, the amount of caffeine in the body decreases by 10% each hour.</p> <p>Write a function to represent the amount of caffeine remaining in the body after x hours.</p>		
Your Situation			$j(x) = 200(0.1)^x$

Graphs to Functions

This graph represents an exponential function. Diya tried to write its function and made an error.



Diya

$$(0, 5)$$

$$(1, 4.4) \frac{5}{4.4} = 1.14 \rightarrow \boxed{f(x) = 5(1.14)^x}$$

I used the y-values of two points to calculate the growth factor. Then I wrote the function using my growth factor and the y-intercept.


8. What did Diya do correctly?

9. What error did Diya make?

10. Write a correct function.

11. By what percent is the function decreasing when x increases by 1?
12. Show or explain where you see your answer to Problem 11 in your equation from Problem 10.

Synthesis

13.  **Discuss:** Compare writing equations with percent increase to writing equations with percent decrease.

How are they alike? How are they different?

Use the examples if they help with your thinking.

Example 1

A person has 200 mg of caffeine in their body. Typically, the amount of caffeine remaining in their body falls by 10% each hour.

$$f(x) = 200(1 - 0.1)^x$$

Example 2

An orange has 3 sq. mm of mold. The mold grows by 90% each day.

$$g(x) = 3(1 + 0.9)^x$$

Summary 5.08

Exponential functions with a growth factor between 0 and 1 represent exponential decay. You can describe a relationship that models exponential decay as a *percent decrease*.

In situations that model exponential decay, the growth factor b can be written as 1 (representing 100%) minus the percent of decay in decimal form.

For example, a potted plant is given 24 milliliters of fertilizer. The amount of fertilizer decreases by 1% every hour. We can write a function to represent this situation in the form $f(x) = a \cdot b^x$. Let $f(x)$ represent the amount of fertilizer left in the potted plant and x represent the time in hours since the potted plant received the fertilizer.

- At first, the amount of fertilizer is 24 milliliters. The *initial value*, or a , is 24.
- The amount of fertilizer decreases by 1% every hour. Because the fertilizer decreases by a constant percent, we can represent the decay by changing the percent to a decimal (1% to 0.01) and subtracting it from 1. The growth factor, or b , will be $1 - 0.01$ or 0.99.

We can write the exponential function that represents this situation as $f(x) = 24 \cdot (0.99)^x$.

Practice

5.08

Name: _____ Date: _____ Period: _____

Problems 1–3: Determine the percent decrease for each function.

1. $f(x) = 20(0.9)^x$ 2. $g(x) = 45(1 - 0.75)^x$ 3. $h(t) = 0.65\left(1 - \frac{25}{100}\right)^t$

4. Select *all* the functions with a 30% decrease.

A. $g(t) = 22(0.7)^t$

B. $h(x) = 22(0.3)^x$

C. $f(x) = 30(1 - 0.7)^x$

D. $r(t) = 30(1 - 0.3)^x$

E. $v(x) = 70\left(1 - \frac{30}{100}\right)^x$

5. There are 1,024 players in a tennis tournament. After each round, *half* of the players are eliminated. Which function represents the number of players remaining after x rounds?


A. $f(x) = 1024(1.50)^x$

B. $f(x) = 1024(1.05)^x$

C. $f(x) = 1024(0.50)^x$

D. $f(x) = 1024(0.05)^x$

6. A cell phone is worth \$800 when it is first made, but loses 25% of its value after each year. Write an equation to represent the value, $v(t)$, of the cell phone, t years after it is made.

7.  **Test Practice** Jamir says that $r(x) = 26(0.3)^x$ and $r(x) = 26\left(1 - \frac{7}{10}\right)^x$ are equivalent equations.

a Is Jamir correct? Explain your thinking.

b Write another equivalent equation that uses fractions.

Practice 5.08

Name: _____ Date: _____ Period: _____

8. Fill in each blank using the digits 0 to 9 only once each to create the smallest possible value.

$$\square\square(\square.\square\square)^3$$

Spiral Review

Problems 9–10: A factory makes boxes. Each box's height, $h(x)$, is 2 inches more than 3 times its width, x .

9. Complete the table

x	1	2	3	4
$h(x)$				

10. Write an equation for the function $h(x)$.

Problems 11–13: Fill in the blanks to complete each sequence. Each sequence has a constant ratio.

11. 2, 4, _____, _____, 32

12. _____, 9, -27, _____, _____

13. 0.8, -4, _____, -100, _____

Lake Desmosia

Let's use exponential functions to solve problems about situations that grow or decay by a given percent.



Warm-Up

Alisha and Mohamed are park rangers who work year-round at Lake Desmosia. Park rangers have many responsibilities, such as monitoring the health of the lake and keeping park visitors safe.

1. Here is Lake Desmosia.



Discuss:

- What are some ways the community can help keep the lake healthy?
- What are some activities that community members might enjoy doing at Lake Desmosia?



Lake Desmosia Closure

Algae are important for a healthy lake, but too much can be harmful! When algae grows out of control (also called an *algae bloom*), Alisha and Mohamed close the lake to protect the community.

Last year, the algae bloom covered 85 square meters of the lake. Alisha and Mohamed were able to reduce the algae bloom size with a water treatment.

2. The size of the algae bloom decreased at a rate of 12% per week with treatment.

Show or describe where you see 12% in the table.

Weeks Since Treatment	Algae Bloom Size (sq. m)
0	85
1	74.8
2	65.8
3	57.9

3. Alisha and Mohamed each wrote a function to represent the size of the algae bloom after x weeks.

Explain why each function is incorrect.

Alisha

$$f(x) = 85 \cdot (1.12)^x$$

Mohamed

$$f(x) = 85 \cdot (0.12)^x$$

4. Write a correct function to represent the size of the algae bloom, in square meters, after x weeks.

Graph your equation on screen 3 of the digital companion to make sure your function matches the data.

5. Fishing and water sports aren't allowed on Lake Desmosia while the algae covers more than 20 square meters. How many weeks until the lake can reopen for water sports?

Use a graphing calculator to support your thinking.

Spring Algae Bloom at Lake Desmosia

In the spring, Alisha and Mohamed care for Lake Desmosia by managing the yearly algae bloom, which can negatively impact amphibians, fish, and other wetland animals.

You will use a set of cards for this activity.

6. Sort the situation cards based on whether they represent exponential growth or exponential decay.

Exponential Growth	Exponential Decay

7. Read the prompts on the question cards. Use the space below to write your responses. Use Screens 5–8 of the digital companion to help you with your thinking.

Card 1

a

b

Card 2

a

b

Card 3

a

b

Card 4

a

b

Comparing Algae Treatments


Alisha and Mohamed are researching two different treatment options for a larger algae bloom. These functions model the size of the algae bloom, in square meters, after x weeks with each treatment.

Treatment A

$$f(x) = 350 \cdot (0.72)^x$$

Treatment B

$$g(x) = 340 \cdot (0.82)^x + 10$$

8. Graph each equation on Screen 10 of the digital companion.
- How are the graphs alike?
 - How are they different?
9.  **Discuss:** What do the graphs tell you about each treatment method?
10. Let's take a closer look at Treatment B's function: $g(x) = 340 \cdot (0.82)^x + 10$.
- a** What do the numbers 0.82 and 10 mean in the situation?
- 0.82 represents . . .
 - 10 represents . . .
- b** Why is the y -intercept for $g(x)$ *not* at $(0, 340)$?
11. Mohamed knows that some amount of algae is important for the health of Lake Desmosia and doesn't want to eliminate all of it. He thinks that Treatment B is the better proposal because it will never get rid of all of the algae. Explain what he means by this.

Synthesis

12. When you add a constant to a function, what changes? What stays the same?

Use the examples if they help with your thinking.

Changes:

Stays the same:

$$f(x) = 350 \cdot (0.85)^x$$

$$g(x) = 350 \cdot (0.85)^x + 50$$

Summary 5.09

You can interpret exponential functions in the form $f(x) = a \cdot b^x$ and $f(x) = a \cdot b^x + k$ to help understand the situations they represent.

Here are functions that describe two treatment methods for algae blooms. Each function models the size of an algae bloom, in square meters, x weeks after using the treatment in the lake.

$$h(x) = 250 \cdot (0.95)^x$$

- The initial size of algae bloom on the lake is 250 square meters.
- The growth factor is 0.95. The percent decrease each week is 5% because $1 - 0.95 = 0.05$.
- Over a long period, the algae bloom will eventually decrease in size to cover very close to 0 square meters.

$$j(x) = 250 \cdot (0.95)^x + 20$$

- The initial size of the algae bloom on the lake is 270 square meters because $250 \cdot (0.95)^0 + 20 = 250 \cdot 1 + 20 = 270$.
- The value of b is 0.95, which means the amount of algae bloom is decreasing since the value is less than 1.
- Over a long period, the algae bloom will eventually decrease in size to cover very close to 20 square meters.

Practice 5.09

Name: _____ Date: _____ Period: _____

Problems 1–4: Determine if each scenario represents exponential growth or exponential decay. Circle your choice.

1. $f(x) = 100(2)^x$

Exponential growth

Exponential decay

2. $f(x) = 100(0.17)^x$

Exponential growth

Exponential decay

3. A plant receives 18 milliliters of fertilizer. It will lose 8% of the fertilizer every day after treatment.

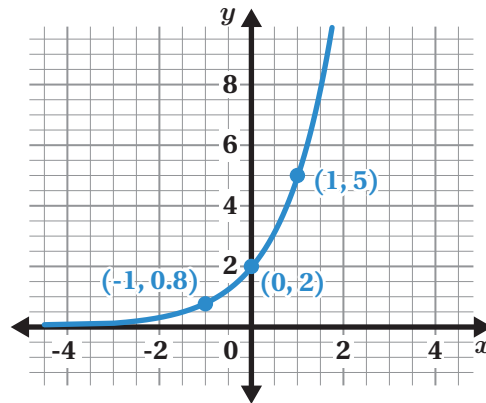
The function represents the amount of fertilizer left in the plant x days since receiving the fertilizer.

$f(x) = 18(0.92)^x$

Exponential growth

Exponential decay

4.



Exponential growth

Exponential decay

Problems 5–8: Match each equation to the description of its graph.

5. $f(x) = 15(1.04)^x$ _____

Description A

- An initial value of 15
- Exponential decay
- The y -values will eventually reach 0 as the x -values get bigger

6. $f(x) = 15(0.04)^x$ _____

Description B

- An initial value of 6
- Exponential decay
- The y -values will eventually reach 9 as the x -values get bigger

7. $f(x) = 6(1.04)^x + 9$ _____

Description C

- An initial value of 15
- Exponential growth
- The y -values will eventually get very close to 0 as the x -values get smaller

8. $f(x) = 6(0.04)^x + 9$ _____

Description D

- An initial value of 6
- Exponential growth
- The y -values will eventually get very close to 9 as the x -values get smaller


Practice 5.09

Name: _____ Date: _____ Period: _____

Problems 9–10: A group of scientists records the population of frogs in a lake over time.

9. Write a function of the form $f(x) = a \cdot b^x$ to represent the number of frogs, $f(x)$, in the lake after x years.
10. What percent increase does this function represent? Explain your thinking.

Time (yr)	Number of Frogs
0	80
1	120
2	180
3	270

11.  **Test Practice** The table shows the value in dollars of a motorcycle at the end of x years.

Which exponential function models this situation?

- A. $v(x) = 12000(1.10)^x$
B. $v(x) = 12000(0.90)^x$
C. $v(x) = 10800(1.10)^x$
D. $v(x) = 10800(0.90)^x$

Number of years, x	Value in dollars, $v(x)$
0	12,000
1	10,800
2	9,720
3	8,748

Spiral Review


12. Here is an inequality: $2x + 1 > 7x + 6$. Select *all* of the values that are solutions.

- A. $x = 8$ B. $x = 1$ C. $x = -1$ D. $x = -2$ E. $x = -8$

Unit 5
Lesson
10

Name: _____ Date: _____ Period: _____

Comparing Models Modeling with Functions Composing Functions

 A-CED.2, F-BF.3, F-IF.4, F-IF.7.e, SMP.1, SMP.7, SMP.8



Marbleslides: Exponentials

Let's practice translating exponential functions by playing a game.



Warm-Up

You'll use the digital activity for the Warm-Up.

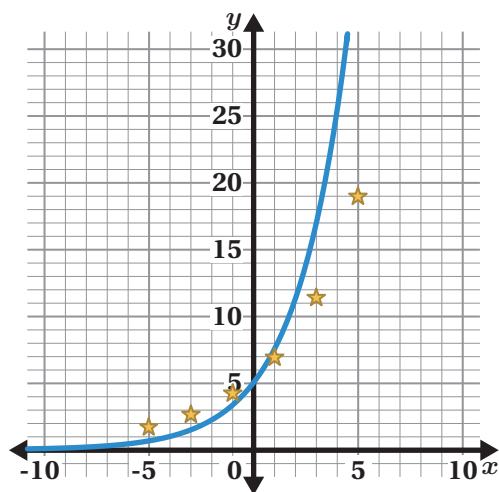
1 Your goal is to capture all the stars.

Change the function to capture all the stars.

Original function:

$$f(x) = 5 \cdot 1.5^x$$

Your function:

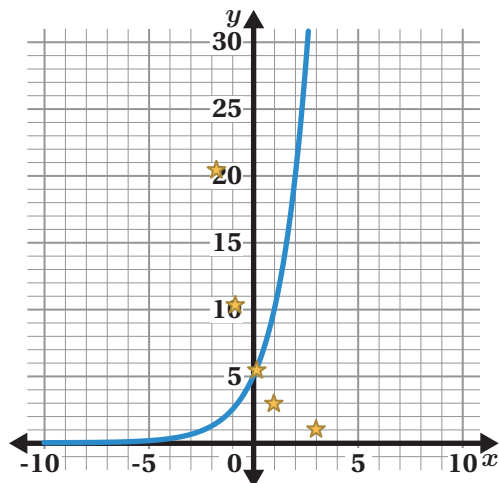


2 Change the function to capture all the stars.

Original function:

$$f(x) = 5 \cdot 2^x$$

Your function:



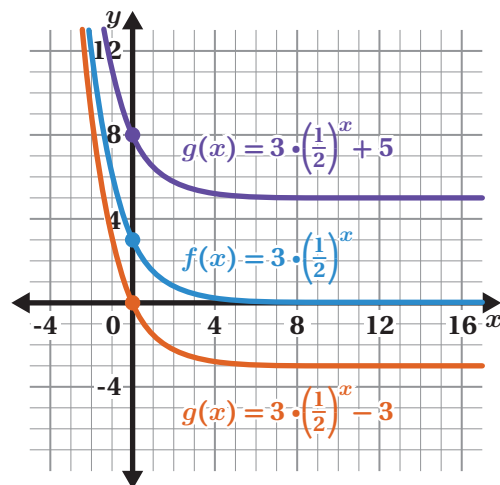
Translations

You'll use the digital activity for Problems 3–6.

- 3** To capture all of the stars, you may want to translate your function up or down.

Use the activity to translate $f(x)$ vertically.

What do you notice? What do you wonder?

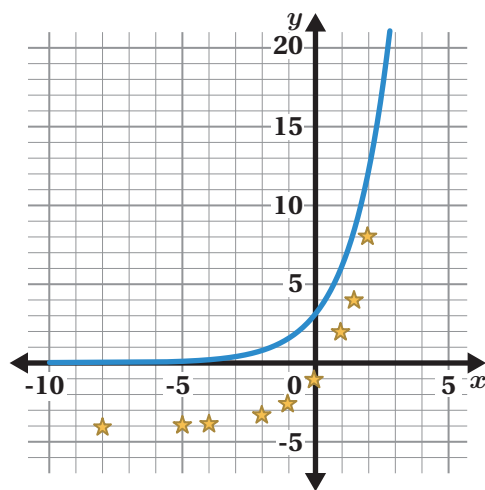


- 4** Change the function to capture all the stars.

Original function:

$$f(x) = 3 \cdot 2^x$$

Your function:



Translations (continued)

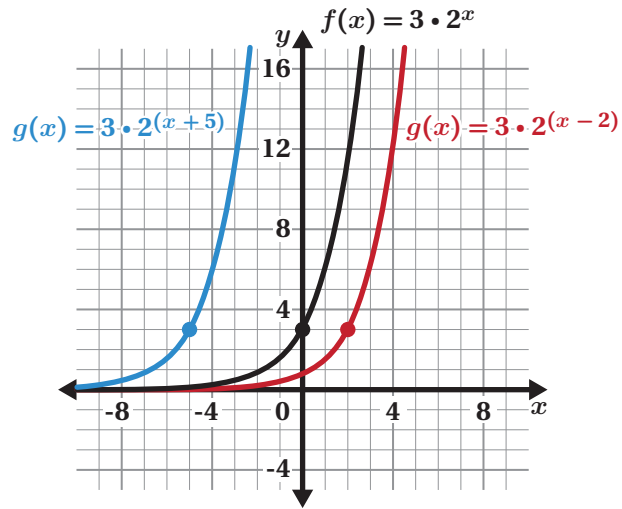
5 To capture all of the stars, you may want to translate your function left or right.

Use the activity to translate $f(x)$ horizontally.

Which function will translate $f(x)$ 12 units to the right?

- A. $g(x) = 3 \cdot 2^x - 12$
- B. $g(x) = 3 \cdot 2^{(x-12)}$
- C. $g(x) = 3 \cdot 2^{(x+12)}$
- D. $g(x) = 3 \cdot 2^x + 12$

Explain your thinking.

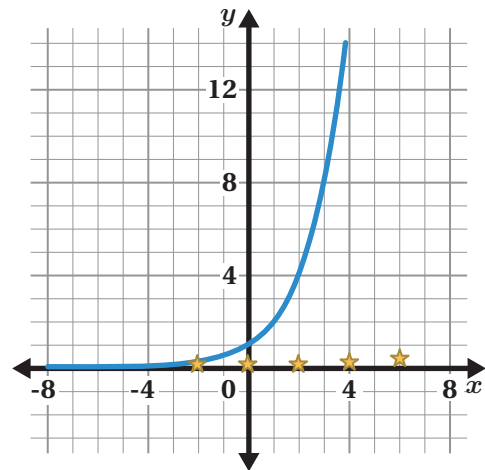


6 Change the function to capture all the stars.

Original function:

$$f(x) = 2^{(x)}$$

Your function:



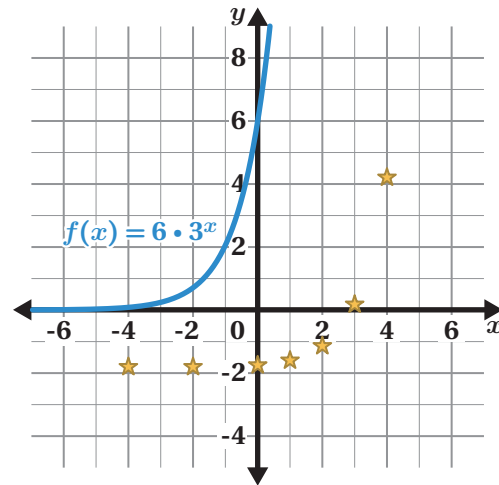
Challenges

- 7** Kiri wrote the function
 $g(x) = 6 \cdot 3^{(x+4)} - 2$ to capture all the stars.

Will Kiri's function capture all of the stars?
 Circle one.

Yes No I'm not sure

Explain your thinking.



You'll use the digital activity for Problems 8–12.

- 8** Here is Kiri's function from the previous screen. Change the function to capture all the stars.

$$g(x) = 6 \cdot 3^{(x+4)} - 2$$

- 9** Create as many exponential functions as you need to capture all the stars.

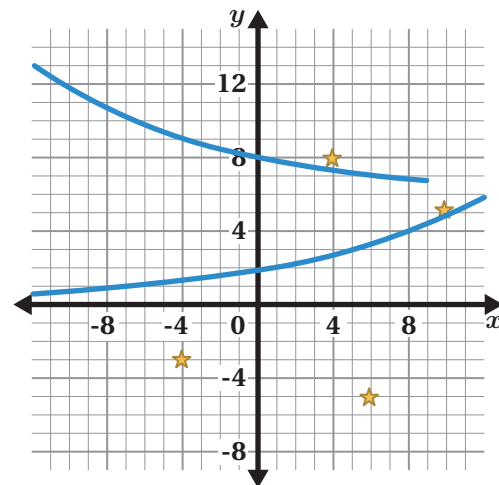
We have included functions that may help you start.

Original functions:

$$f(x) = 2 \cdot (0.9)^x + 6 \{x < 9\}$$

$$g(x) = 3 \cdot (1.1)^{(x-5)}$$

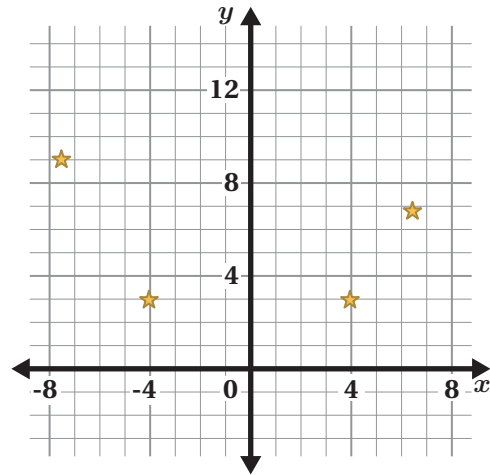
Your functions:



Challenges (continued)

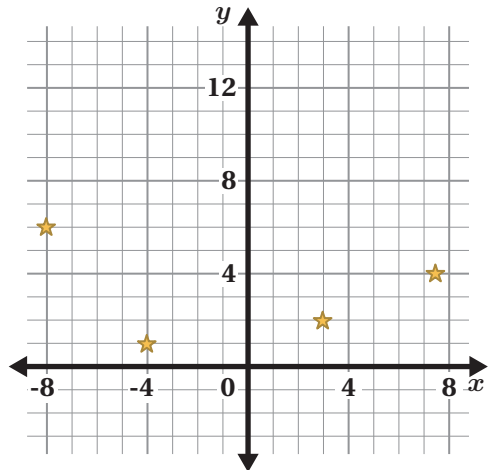
10 Create as many exponential functions as you need to capture all the stars.

Your functions:



11 Create as many exponential functions as you need to capture all the stars.

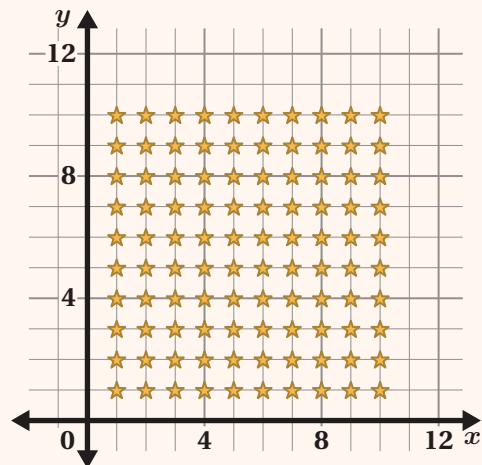
Your functions:



You're invited to explore more.

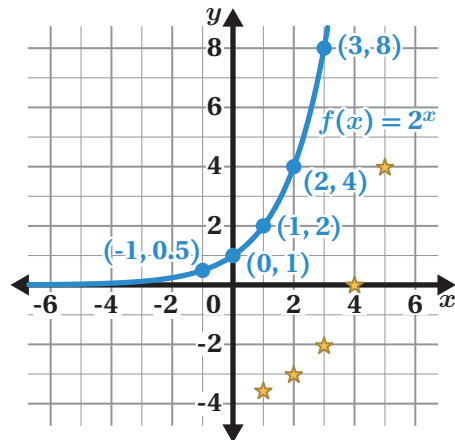
12 Challenge yourself to capture as many stars as you can!

Your functions:



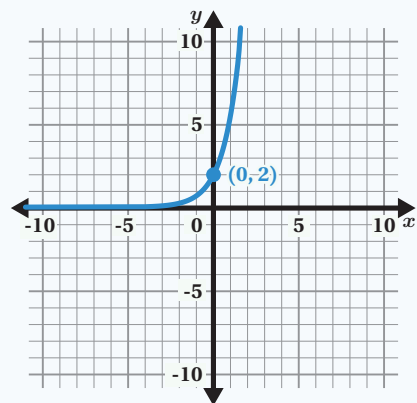
13 Synthesis

Discuss: Describe how to use vertical and horizontal *translations* to write a function that would capture all the stars.



16 Summary 5.10

Here is the graph of $f(x) = 2 \cdot 3^x$:

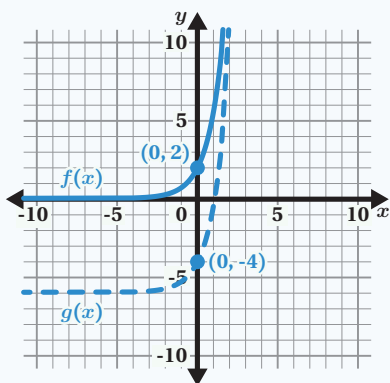


Functions can be translated horizontally and vertically.

Here are two examples of *translations* of $f(x)$:

Vertical Translations

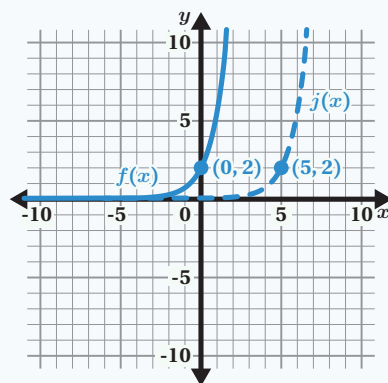
The equation $f(x) = 2 \cdot 3^x + k$ represents a vertical translation by k units.



Here $f(x)$ was translated 6 units down and can be represented with the new equation: $g(x) = 2 \cdot 3^x - 6$.

Horizontal Translations

The equation $f(x) = 2 \cdot 3^{(x-h)}$ represents a horizontal translation by h units.



Here $f(x)$ was translated 5 units to the right and can be represented with the new equation: $j(x) = 2 \cdot 3^{(x-5)}$.

Practice 5.10

Name: _____ Date: _____ Period: _____

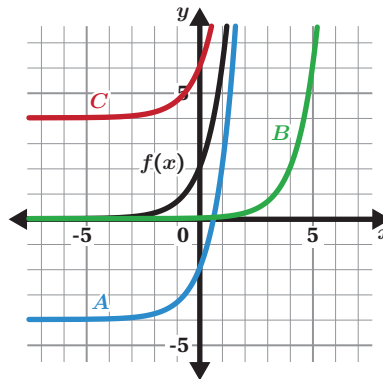
1. Here is the graph of $f(x) = 2 \cdot 3^x$.

Match each function with its graph.

$f(x) = 2 \cdot 3^x + 4$

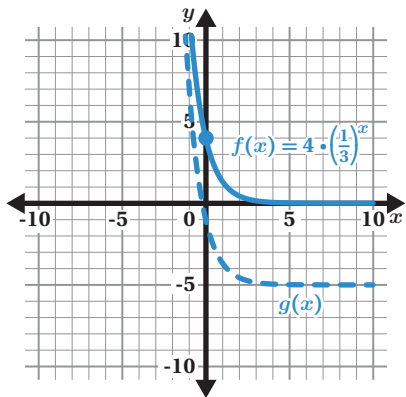
$f(x) = 2 \cdot 3^x - 4$

$f(x) = 2 \cdot 3^{(x-4)}$



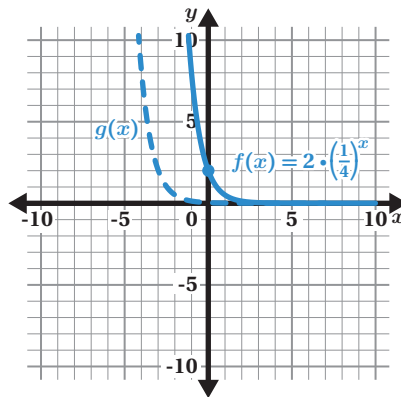
Test Practice Problems 2–3: For each set of graphs shown, write an equation for the dotted curve, $g(x)$.

2. $f(x) = 4 \cdot \left(\frac{1}{3}\right)^x$



$g(x) =$

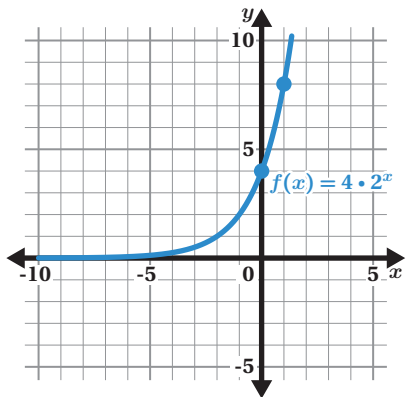
3. $f(x) = 2 \cdot \left(\frac{1}{4}\right)^x$



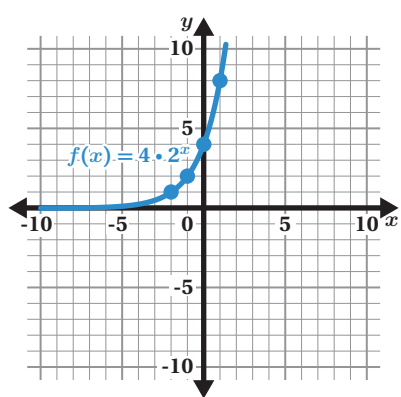
$g(x) =$

Problems 4–5: The function $g(x)$ is a transformation of $f(x) = 4 \cdot 2^x$.

4. Graph $g(x) = 4 \cdot 2^x + 1$.



5. Graph $g(x) = 4 \cdot 2^{(x-3)}$.



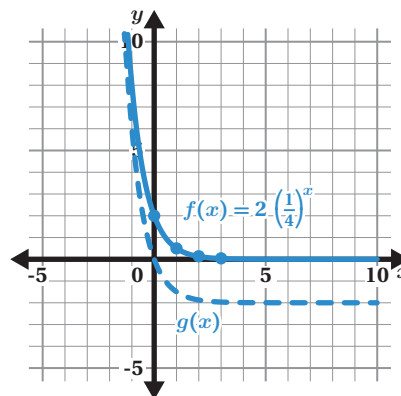
Practice

5.10

Name: _____ Date: _____ Period: _____

Problems 6–8: The function $g(x)$ is a transformation of $f(x) = 2 \cdot \left(\frac{1}{4}\right)^x$.

6. Ariana says $g(x) = 2 \cdot \left(\frac{1}{4}\right)^x - 4$ because the y -intercept of $f(x)$ shifted down 4 units. Explain why Ariana's thinking is incorrect.



7. Kyrie says $f(x)$ transformed using both vertical and horizontal translations. Explain why Kyrie's thinking is incorrect.
8. Write the correct equation for $g(x)$.

Spiral Review

9. What is the solution to this system of equations?

$$5x + y = 18$$

$$x - 3y = 10$$

- A. (-2, -4)
B. (4, -2)
C. There is no solution.
D. There are an infinite number of solutions.

10. Solve this system of equations. Write the solution as a coordinate pair.

$$y = -3x + 4$$

$$y = 3x - 2$$

Practice Day 1

Let's practice what you've learned so far in this unit!



You will use problem cards for this Practice Day. Record all of your responses here.

Card 1

- a** Circle the correct answer:

Kala's Remy's

Explanation:

b

Card 2

- a** Circle the correct answer:

$f(x)$ $g(x)$

Explanation:

b

Card 3

Card 4

Card 5

Card 6

- a** Circle the correct answer:

$m(x)$ $n(x)$

b

- a** Circle the correct answer:

$f(x)$ $g(x)$

b

Practice Day 1 (continued)

Card 7

a Circle the correct answer:

Growing Decaying

Explanation:

b

c

Card 9

a

b

Card 11

a

b

Card 8

a Circle the correct answer:

Growing Decaying

Explanation:

b

c

Card 10

a

b

Card 12

a $j(x)$: $k(x)$:

b

x	$j(x)$	x	$k(x)$
2	7	0	12
3	17.5	1	33.25
4	43.75	2	54.5
5		3	
6		4	

Modeling With Exponentials



Lesson 11
Bank Accounts



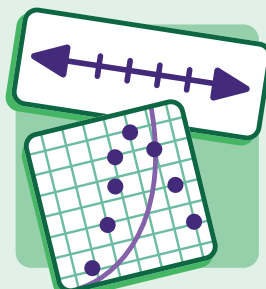
Lesson 12
Payday Loan



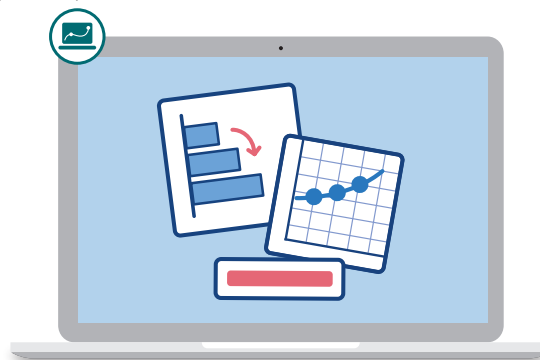
Lesson 13
Credit Card
Compounding



Lesson 14
Detroit's Population,
Part 1



Lesson 15
Detroit's Population,
Part 2

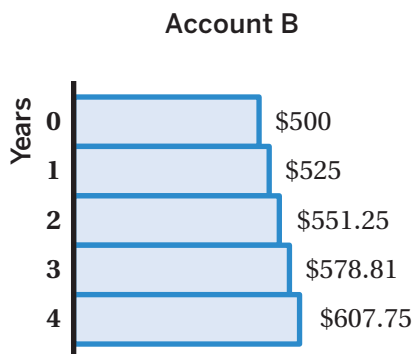
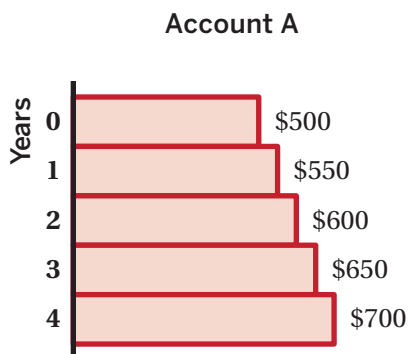


Bank Accounts

Let's learn how to model situations involving simple and compound interest.

Warm-Up

1-2 Mauricio has \$500 to invest. He is researching different kinds of investment accounts. Here are the values of Accounts A and B over time:

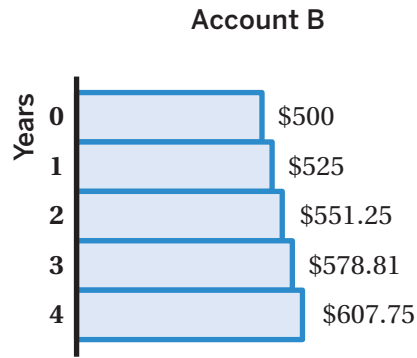
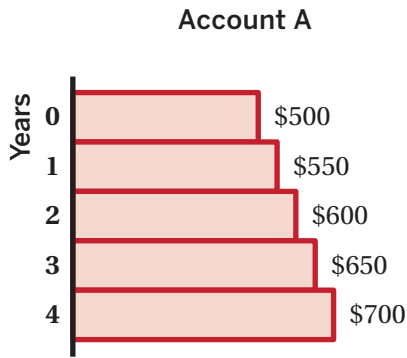


Show or describe how the value of each account grows over time.

Earning Interest

3 Here is Mauricio's work to show how each account is growing.

Discuss: Describe his work to a partner. Which account would you recommend he invest in?

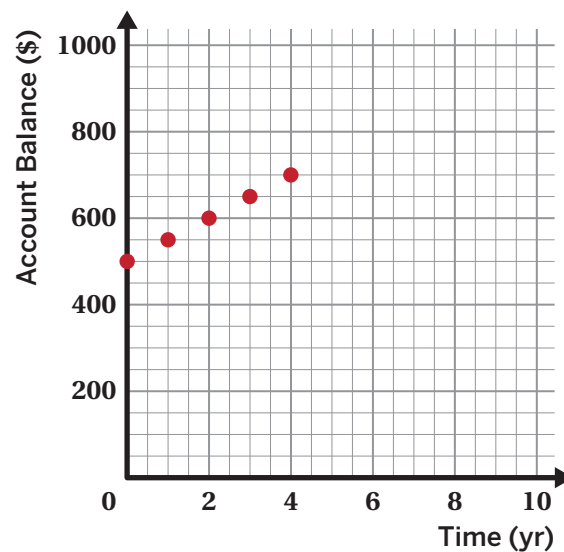


4 Account A earns 10% simple interest per year.

a **Discuss:** How do you think *simple interest* works?


b Determine the account balance after 5 years.

Time (yr)	Account Balance (\$)
0	500
1	550
2	600
3	650
4	700
5	



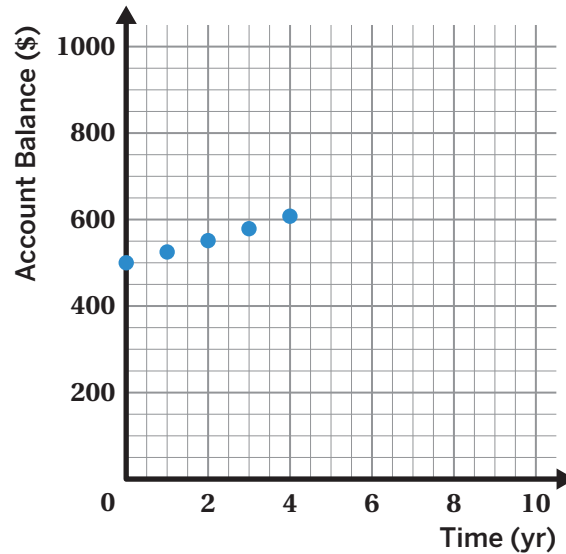
Earning Interest (continued)

5 Account B earns 5% **compound interest** per year.

a  **Discuss:** How do you think *compound interest* works?

b Determine the account balance after 5 years.

Time (yr)	Account Balance (\$)
0	500
1	525
2	551.25
3	578.81
4	607.75
5	

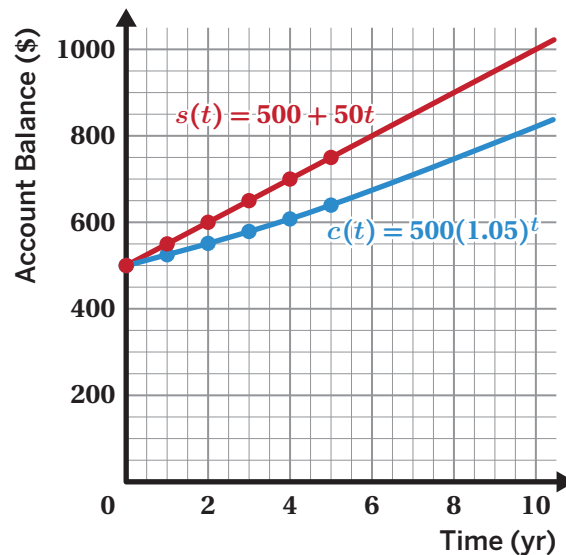


6 We can use functions to describe the account balances after t years.


- Simple interest: $s(t) = 500 + 50t$
- Compound interest: $c(t) = 500(1.05)^t$

How are these functions alike?

How are they different?



7 a Let's watch the account balances grow.

b  **Discuss:** Which account would you recommend Mauricio invest in? Why?

Simple and Compound Interest

- 8** Mauricio decided to invest in an account that offers 6% compound interest per year.

$a(t) = 500(1.06)^t$ represents its balance after t years.

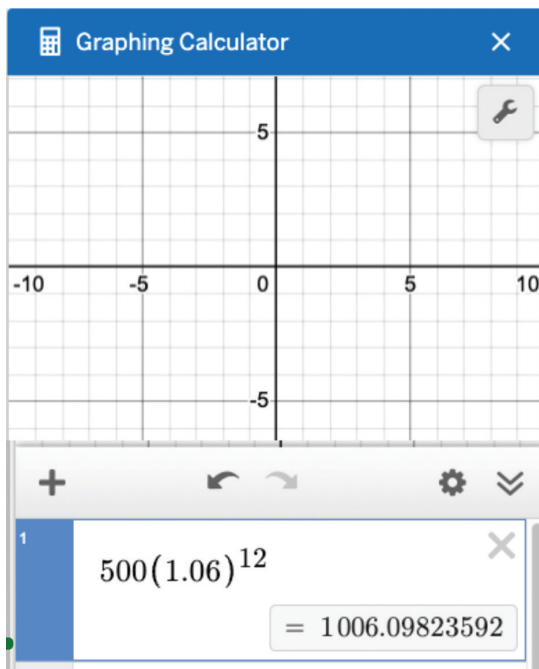
- a** Write an equation to represent how many years it will take for the balance to reach \$1,000.
- b** About how many years will it take for the balance to reach \$1,000? Explain your thinking.

Use this space or the Desmos Graphing Calculator to help with your thinking.

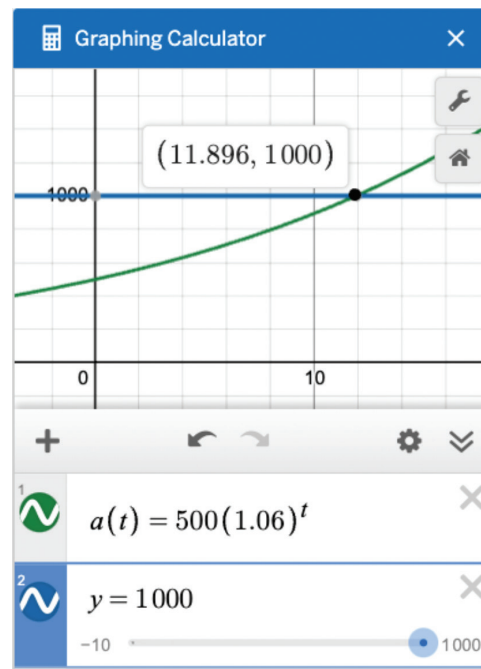
- 9** Let's watch each video to see how two students determined the time it would take to reach \$1,000.

 **Discuss:** Which method helps get a more precise answer?

Fabiana



Antwon



Simple and Compound Interest (continued)

10 Solve as many challenges as you have time for.

- a** A \$1,000 investment earns 4% compound interest.

The function $f(t) = 1000(1.04)^t$ gives the account balance after t years.

About how many years will it take for the balance to reach \$2,500?

- b** A \$200 investment earns 7% compound interest.

The function $f(t) = 200(1.07)^t$ gives the account balance after t years.

About how many years will it take for the balance to reach \$450?

- c** A \$1,700 investment earns 3% compound interest.

The function $f(t) = 1700(1.03)^t$ gives the account balance after t years.

About how many years will it take for the balance to reach \$4,150?

- d** A \$1,150 investment earns 2% compound interest.

The function $f(t) = 1150(1.02)^t$ gives the account balance after t years.

About how many years will it take for the balance to reach \$2,700?

You're invited to explore more.

- 11** Use the You're Invited to Explore More Sheet to answer questions about an account balance.

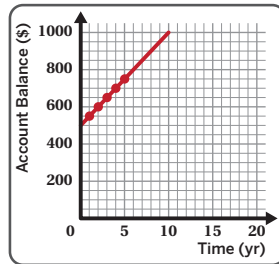
12 Synthesis

Here are some examples of simple and compound interest:

Simple Interest

$$s(t) = 500 + 50t$$

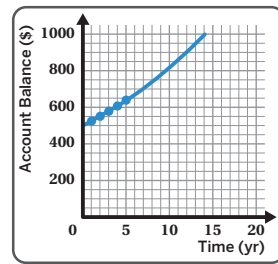
Time (yr)	Account Balance (\$)
0	500
1	550
2	600
3	650
4	700



Compound Interest

$$c(t) = 500(1.05)^t$$

Time (yr)	Account Balance (\$)
0	500
1	525
2	551.25
3	578.81
4	607.75



How do investments grow with simple interest?

How do investments grow with simple interest?

15 Summary 5.11

You can invest money in accounts that earn **simple interest** which are modeled by linear functions or accounts that earn **compound interest** which are modeled by exponential functions. Which account earns the most interest depends on how much time there is to invest and other variables. Let's look at an example.

Simple Interest	Compound Interest
Adah could invest \$100 in an account that earns 10% simple interest annually. The function $a(t) = 100 + 10t$ models the account balance after t years.	Adah could invest \$100 in an account that earns 10% compound interest annually. The function $b(t) = 100 \cdot (1.10)^t$ models the account balance after t years.
To determine the balance of the account after 12 years, substitute $t = 12$ into each function and solve for $a(t)$.	
$a(12) = 100 + 10(12)$ $a(12) = 220$ After 12 years, the account balance will be \$220.	$b(12) = 100 \cdot (1.10)^{12}$ $b(12) = 313.84$ After 12 years, the account balance will be about \$313.84.

Adah may choose to invest in the account that earns compound interest because it earns more money over 12 years.

compound interest Interest calculated based on the initial amount and the interest from previous periods. It is calculated at regular intervals (daily, monthly, annually, etc.). The balance in an account that earns compound interest can be modeled by an exponential function.

simple interest Interest that is calculated solely based on the initial amount. The balance in an account with simple interest is modeled by a linear function.

Practice 5.11

Name: _____ Date: _____ Period: _____

Problems 1–4: Determine if each equation or table represents simple or compound interest. Circle your choice.

1. $b(t) = 1000(1.03)^t$

Simple

Compound

2. $b(t) = 1000 + 30t$

Simple

Compound

3.

Time (yr)	Account Balance (\$)
0	300
1	330
2	360

Simple

Compound

4.

Time (yr)	Account Balance (\$)
0	200
1	230
2	264.50

Simple

Compound

Problems 5–7: Jin invests \$4,000 in an account that earns 5% compound interest per year.

5. Complete the table.

6. Which function represents the amount of money in Jin's account after x years?

A. $f(x) = 4000 + 1.05x$

B. $f(x) = 4000(1.05)^x$

C. $f(x) = 4000(0.05)^x$

D. $f(x) = 4000 + (1.05)^x$

Time (yr)	Account Balance (\$)
0	
1	4,200
2	4,410
3	
4	

7. What will the balance of the account be after 10 years?

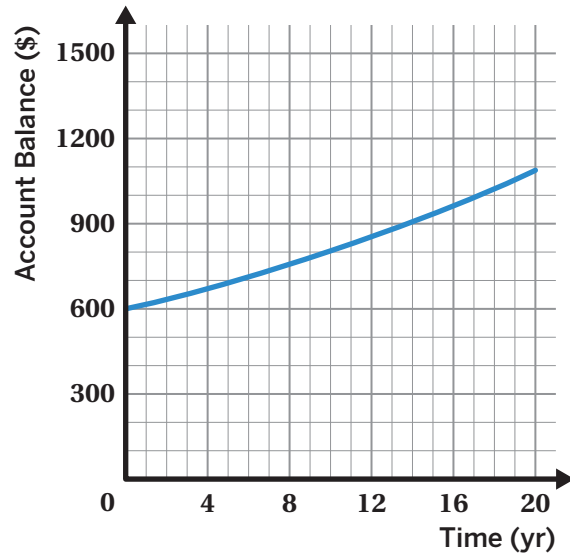
Practice

5.11

Name: _____ Date: _____ Period: _____

Test Practice Problems 8–9: Keya invests \$600 in an account that earns 3% compound interest per year. The graph shows the function $f(t) = 600(1.03)^t$, which gives Keya's account balance after t years.

- About how many years will it take for her account balance to reach \$1,000?
- Use the graph to determine the value of $f(14)$.



What does that tell you about the situation?

- You just won a contest and have two prize options.
 - Option A:** One payment of \$20 million
 - Option B:** 2 cents on day one, 4 cents on day two, 8 cents on day three, and so on, for 30 days

Which option would you choose? Explain your choice.

Spiral Review

Problems 11–14: Determine whether each function is linear, exponential, or something else. Circle your choice.

- | | | | |
|----------------------|--------|-------------|----------------|
| 11. $f(x) = x^2 + 5$ | Linear | Exponential | Something else |
| 12. $g(x) = 2x + 5$ | Linear | Exponential | Something else |
| 13. $h(x) = 2^x + 5$ | Linear | Exponential | Something else |
| 14. $m(x) = x^3 + 1$ | Linear | Exponential | Something else |

Unit 5
Lesson
12

Name: _____ Date: _____ Period: _____

Comparing Models Modeling with Functions  A-SSE.1.a, F-BF.1, N-Q.1, SMP.1, SMP.2, SMP.7

Payday Loan

Let's analyze exponential functions that represent different compound interest scenarios.



Warm-Up

1 Zola says that $x^{12} = (x^4)^3$.

The diagram shows why that is true.


$$\underbrace{(x \cdot x \cdot x \cdot x)}_{x^4} (x \cdot x \cdot x \cdot x) (x \cdot x \cdot x \cdot x)$$

What are three other ways to rewrite x^{12} ?

Payday Loan

- 2** A payday loan is a short-term loan designed to be paid back within a month.

Here is an advertisement for a payday loan.

 **Discuss:** What do you notice? What do you wonder?



FASTCASH
PAYDAY LOAN

Settle Up by Your Next Payday

Bad credit? No problem! **\$ 100**

Up to \$1000 now! **\$ 500**

Immediate approval! **\$ 1,000**

APPLY NOW **GET APPROVED** **GET CASH FAST**

*Proof of regular paychecks required.

NO CREDIT CHECK!

- 3** FastCash offers payday loans that charge 15% compound interest per month.

Marc borrows \$100 to help pay his heating bill.

How much will Marc owe after one month?

Explain your thinking.

Payday Loan (continued)

- 4** The function $f(m)$ represents how much Marc will owe if he doesn't pay back the loan for m months. Write an equation to represent $f(m)$.

$$f(m) = \underline{\hspace{2cm}}$$

Months, m	Amount Owed, $f(m)$
0	100
1	115
2	132.25
3	152.09
4	174.90

- 5** Marc wonders how much money he would owe if he doesn't pay back the loan after 3 years.

He wrote two expressions to represent this situation.

Expression A

$$100 \cdot 1.15^{36}$$

Expression B

$$100 \cdot (1.15^{12})^3$$

 **Discuss:** How are the expressions alike? How are they different?

- 6** Marc wrote a third expression to represent this situation.

Expression C

$$100 \cdot (5.35)^3$$

What interest rate does the 5.35 represent?

- A. 435% per year B. 535% per year C. Neither

Explain your thinking.

Credit Cards and Other Loans

- 7** Marc sees an advertisement for a credit card that charges a 2% monthly interest rate.

How much would he owe for a \$100 charge on the credit card after 3 years of no payments?

Payday Loan

- \$100 loan
 - 15% monthly interest
- Amount owed after
3 years of no payments:
\$15315.19

Credit Card

- \$100 charge
 - 2% monthly interest
- Amount owed after
3 years of no payments:
?

- 8** Here are three functions that represent the amount owed on a credit card charge of \$100 after t years of 2% monthly interest:

- $g(t) = 100 \cdot 1.02^{12t}$
- $g(t) = 100 \cdot (1.02^{12})^t$
- $g(t) = 100 \cdot 1.2682^t$

Use one or more of the functions to determine the interest rate *per year*.


Explain your thinking.

Comparing Rates

9 Marc wants to compare interest rates on different types of loans.

a Complete the table.

	Monthly Interest Rate (%)	Monthly Growth Factor	Growth Factor per Year	Interest Rate per Year (%)
Payday Loan	15.00	1.15	5.3503	435.03
Credit Card	2.00	1.02	1.2682	
Private Loan	1.21	1.0121		
30-year Mortgage	0.53			
Federal Student Loan	0.41			

b  **Discuss:** In what situations might people take out each of these different types of loans?

Comparing Rates (continued)

- 10** Here's the information about federal student loans that another student entered on the previous screen.

Monthly Interest Rate (%)	Monthly Growth Factor	Growth Factor per Year	Interest Rate per Year (%)
0.41	1.0041	1.0503	5.03

Annika takes out a \$20,000 federal student loan.

Write a function, $h(t)$, to calculate the amount Annika owes after making no payments for t years.


You're invited to explore more.

- 11** Tyler charges \$3,000 to a credit card with a 2% monthly interest rate.

Many credit cards require a monthly minimum payment.

- a** Let's see how long will it take Tyler to pay off the charges with different monthly payment amounts.
- b** What do you think is important to remember when getting and using a credit card?

12 Synthesis

 **Discuss:** What do different parts of an expression reveal about a situation involving compound interest?

See the examples of the different expressions if it helps with your thinking.

Expression A

$$100 \cdot 1.15^{36}$$

Expression B

$$100 \cdot (1.15^{12})^3$$

Expression C

$$100 \cdot (5.35)^3$$

15 Summary 5.12

You can write exponential expressions representing compound interest in multiple ways to reveal different information about the account and situation.

Here is an example:

The amount owed on a \$400 loan has a monthly interest rate of 3%. Let t represent the number of years since taking out the loan if no payments are made.


- The expression $400 \cdot 1.03^{12t}$ represents a 3% monthly interest rate 12 times each year, t .
- The expression $400 \cdot (1.03^{12})^t$ helps us think about the interest rate for every t year.
- Since $1.03^{12} = 1.4258$, the expression $400 \cdot (1.4258)^t$ shows the annual interest rate of 42.58%.

Each expression reveals different information about the monthly and annual interest rates applied to the account. While a monthly interest rate of 3% may not seem like it impacts the account balance much, the annual interest rate reveals that the loan amount is increasing by 42.58% each year, and that really adds up!

Practice 5.12

Name: _____ Date: _____ Period: _____

Problems 1–2: Alina takes out a \$1,000 loan with a monthly interest rate of 5%. She makes no additional payments, deposits, or withdrawals.

1.  **Test Practice** Write an expression that could be used to calculate her balance after t years.

2. What is the interest rate per year for this loan?

Problems 3–7: Alejandro invests money into a college savings account. He writes the expression $750(1.006^{12})^3$ to help him calculate what the account balance will be in 3 years.

3. Explain what 750 represents in the expression.

4. Explain what 1.006 represents in the expression.

5. Explain what 12 represents in the expression.

6. Explain what 3 represents in the expression.

7. Write an equivalent expression that could represent Alejandro's account balance in 3 years.

Practice 5.12

Name: _____ Date: _____ Period: _____

Problems 8–9: Rebecca is considering taking out a payday loan that has a 17% monthly interest rate.

8. Complete the table.

Monthly Interest Rate	17%
Monthly Growth Factor	
Growth Factor per Year	
Interest Rate per Year	

9. Write an expression that could be used to calculate how much Rebecca would owe on a \$300 payday loan after 2 years if she made no additional payments.
10. Wohali takes out an \$8,000 federal student loan with a monthly interest rate of 0.38% for professional trade school. Write a function, $g(t)$, to calculate the amount Wohali owes after making no payments for t years.

Spiral Review

Problems 11–13: Determine the value of each function when $n = 2$.

11. $f(n) = 4 \cdot 2^n$

12. $g(n) = 2 \cdot 4^n$

13. $h(n) = 8 + 2^n$

14. Using the digits 0 to 9, without repeating, fill in each blank to create four equivalent expressions.

$$7^{\square} = 7^{\square} \times 7^{\square} = 7^{\square} \times 7^{\square} \times 7^{\square} = \left(7^{\square}\right)^{\square}$$

Credit Card Compounding

Let's explore how to calculate and compare account balances with interest rates that compound at different intervals.



Warm-Up

1 Group each card with the word that it describes.

Card A 2 times per year	Card B 4 times per year	Card C $\frac{1}{12}$ of a year	Card D $\frac{1}{4}$ of a year
Card E Every 3 months	Card F Every 6 months	Card G Every 12 months	

Monthly	Quarterly	Semi-Annually	Annually

PayLater

- 2** Alejandro is considering charging \$1,000 to this credit card.

He wrote $1000(1 + 0.24)^5$ to determine the balance after 5 years with no payments or additional charges.

Explain what each part of the expression means.

1000:

$1 + 0.24$:

5:

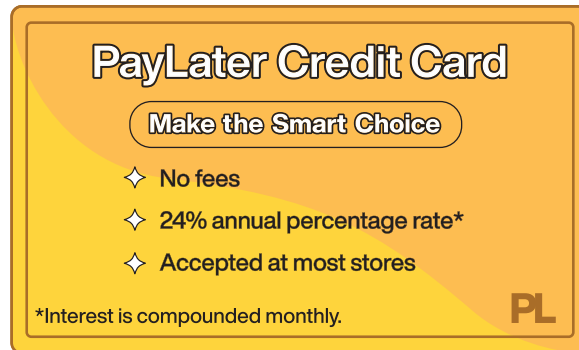
- 3** The fine print says interest is compounded monthly.

This means the interest is $\frac{24}{12} = 2$, or 2% per month.

Compared to compounding annually, how do you think compounding monthly will affect the total Alejandro owes after 5 years? Circle one.

- A.** He will owe more **B.** He will owe less **C.** He will owe the same

Explain your thinking.



PayLater (continued)

- 4** Alejandro is considering charging \$1,000 to this credit card.

If the interest is compounded at 2% monthly, how much would he owe after 5 years?

PayLater Credit Card

Make the Smart Choice

- ◆ No fees
- ◆ 24% annual percentage rate*
- ◆ Accepted at most stores

*Interest is compounded monthly.

PL

- 5** Alejandro wrote $1000\left(1 + \frac{0.24}{12}\right)^5$ to determine the balance after 5 years, but he made an error.

Find the error and explain why it is incorrect.

PayLater and Flash Bucks

- 6** Alejandro is considering charging \$1,000 to a different credit card.

PayLater Credit Card

Make the Smart Choice

- ◆ No fees
- ◆ 24% annual percentage rate*
- ◆ Accepted at most stores

*Interest is compounded monthly.

PL


Flash Bucks

Make the Smart Choice


- No fees
- 24% annual percentage rate*
- Accepted at most stores

*Interest is compounded daily.

FB

 **Discuss:** Compared to compounding monthly, how do you think compounding daily will affect the total amount owed?

- 7** Alejandro is considering charging \$1,000 to this credit card.

a  **Discuss:** How would you determine the daily interest rate?

Flash Bucks

Make the Smart Choice

- No fees
- 24% annual percentage rate*
- Accepted at most stores

*Interest is compounded daily.

FB

b If interest is compounded daily, how much would Alejandro owe after 5 years with no payments or additional charges?

Compounding Differently

- 8** Here are some expressions to calculate the total amount for \$800 and a 12% annual interest rate compounded using different *intervals*.

Match each expression with its compounding period and length. One card will have no match.

Card A $800\left(1 + \frac{0.12}{4}\right)^{(4 \cdot 3)}$	Card B $800(1 + 0.01)^{24}$	Card C $800\left(1 + \frac{0.12}{12}\right)^{(12 \cdot 2)}$
Card D $800(1 + 0.04)^{(3 \cdot 2)}$	Card E $800(1 + 0.03)^{12}$	

Compounded Quarterly for 3 Years	Compounded Monthly for 2 Years

- 9** Compound interest expressions can be represented using this formula:

$$P\left(1 + \frac{r}{n}\right)^{nt}$$

Circle one variable and describe what it represents.

P r n t

Compounding Differently (continued)

10 Solve as many challenges as you have time for.

- a** A person puts \$500 into an account with a 10% annual interest rate compounded quarterly.

What is the balance in the account after 4 years?

- b** A person puts \$800 into an account with a 5% annual interest rate compounded daily.

What is the balance in the account after 3 years?

- c** A person puts \$3,000 into an account with a 5% annual interest rate compounded daily.

What is the balance in the account after 7 years?

- d** A person puts \$1,000 into an account with a 20% annual interest rate compounded yearly.

What is the balance in the account after 8 years?

11 Synthesis

How can you use this formula to calculate the total value of an account or loan with compound interest?

$$P\left(1 + \frac{r}{n}\right)^{nt}$$

14 Summary 5.13

When you take out a loan on a credit card, the annual interest rate may be compounded at different *intervals*, or different lengths of time.

You can use the formula $P\left(1 + \frac{r}{n}\right)^{nt}$ to calculate the total amount in an account that accrues compound interest.

- P represents the initial amount of the loan. In finance this is often called the *principal*.
- r represents the interest rate in decimal form.
- n represents the number of compounding intervals in a year.
- t represents the time in years.

Common compounding periods:

Annually	Semi-annually	Quarterly	Monthly	Daily
$n = 1$	$n = 2$	$n = 4$	$n = 12$	$n = 365$

Let's look at the impact of compound interest applied at different intervals on a loan for \$1,000.

Interest	Owed in	Compounded Monthly	Compounded Quarterly	Compounded Annually
15% annually	5 years	$1000\left(1 + \frac{0.15}{12}\right)^{12 \cdot 5}$ $\approx \$2,107.18$	$1000\left(1 + \frac{0.15}{4}\right)^{4 \cdot 5}$ $\approx \$2,088.15$	$1000(1 + 0.15)^5$ $\approx \$2,011.36$

Practice 5.13

Name: _____ Date: _____ Period: _____

1. Tyrone puts \$2,500 into a savings account with a 1.2% annual interest rate, compounded semi-annually. He makes no additional payments, deposits, or withdrawals.

Write an expression that could be used to calculate his balance after 3 years.


Problems 2–4: Maneli wants to take out a \$5,000 loan to help pay for a new washing machine and dryer. The bank offers her the loan with an 18% annual interest rate, compounded quarterly.

Maneli wrote this expression to calculate the balance of the loan in 2 years, but she made an error.

$$5000\left(1 + \frac{0.18}{2}\right)^{(4 \cdot 2)}$$

2. Find the error and explain why it is incorrect.
3. Write a correct expression to represent Maneli's balance after 2 years.
4. What will her balance be in 2 years?

A payday loan company offers a \$1,000 loan with a 25% annual interest rate.

5.  **Test Practice** If no other charges or payments are made, what will the balance of the loan be after 1 year at each compounding period?

Compounding Period	Balance (\$)
Annually	
Monthly	
Daily	

Practice 5.13

Name: _____ Date: _____ Period: _____

Problems 6–7: Xavier has \$5,000 to invest and has to choose between three investment options.

- Option A: 2.25% interest applied each quarter
- Option B: 3% interest applied every 4 months
- Option C: 4.5% interest applied twice each year

6. Write an expression for each account to represent Xavier's balance after 5 years.

7. Which option will have the largest balance after 5 years?

Spiral Review

Problems 8–9: Irene needs to make at least 25 dinners for a party, including chicken dinners and vegetarian dinners. She has \$250 to spend. Chicken dinners cost \$8.75 each and vegetarian dinners cost \$5.50 each.

- c represents the number of chicken dinners.
- v represents the number of vegetarian dinners.

8. Write a system of inequalities that represents Irene's constraints.

9. Can Irene make 5 chicken dinners and 20 vegetarian dinners? Explain your thinking.

Detroit's Population, Part 1

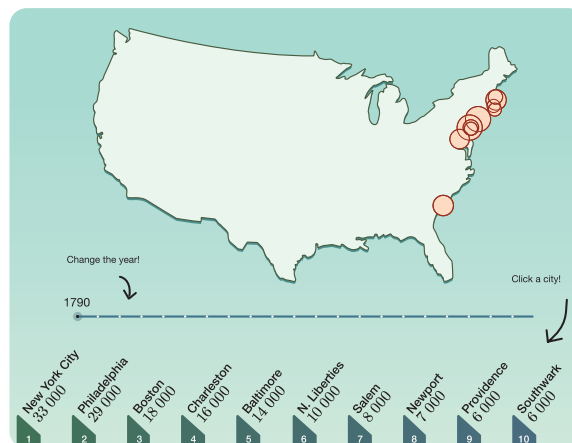
Let's use functions to model the population growth of Detroit.



Warm-Up

1 Use the digital activity to see the ten U.S. cities with the largest populations from 1790–2000.

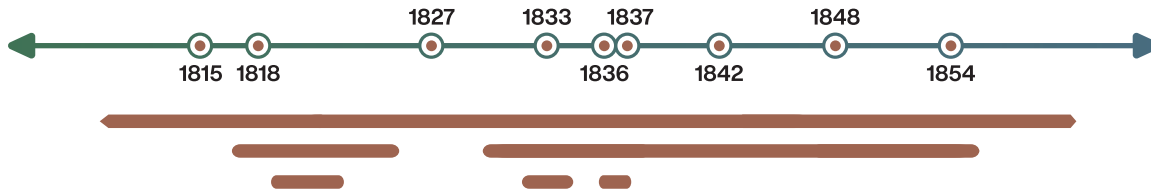
Discuss: What do you notice? What do you wonder?



Early History of Detroit

You'll use the digital activity for Problems 2–6.

2 Explore the timeline to learn about one of the largest U.S. cities from 1815 to 1855.



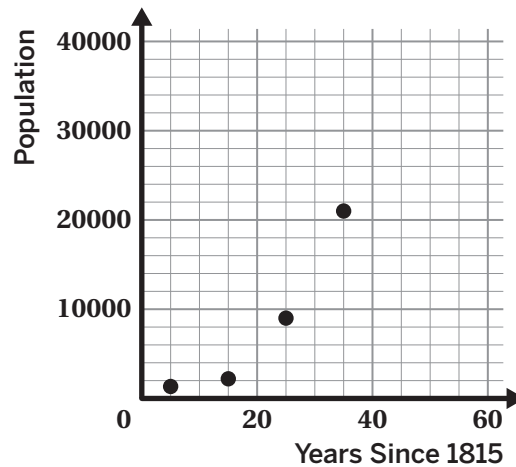
Discuss: How do you think the population of Detroit changed during that time?

3 Detroit became a U.S. city in 1815. This scatter plot shows the census data for Detroit's population from 1820 to 1850.

a Which type of function do you think better fits the data?

Linear

Exponential



b Use the digital activity to fit the function to the data.

c Explain how you decided which function to use.

Early History of Detroit (continued)

- 4** Write the function you created to model the population of Detroit x years since 1815.

$$p(x) = \dots\dots\dots$$

Explain what each part of the *model* represents.

- 5 a** Based on the graph's scale, which is the best approximation for $p(30)$?

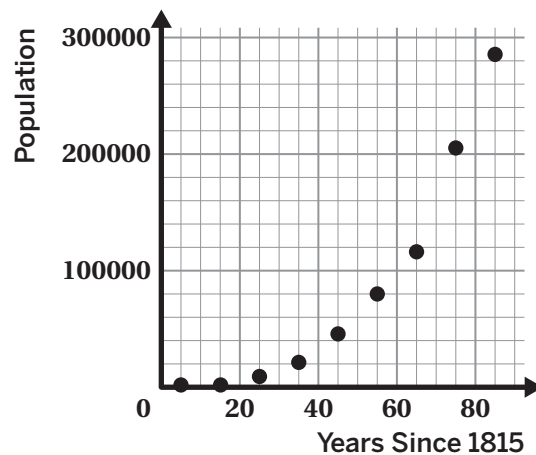
- A. 12000
- B. 14000
- C. 13470
- D. 12500

- b** What does $p(30)$ represent in this situation?

- 6** The U.S. census data for Detroit's population from 1860 to 1900 has been added to the graph.

Discuss:

- What do you notice? What do you wonder?
- How does your model compare to the actual data?



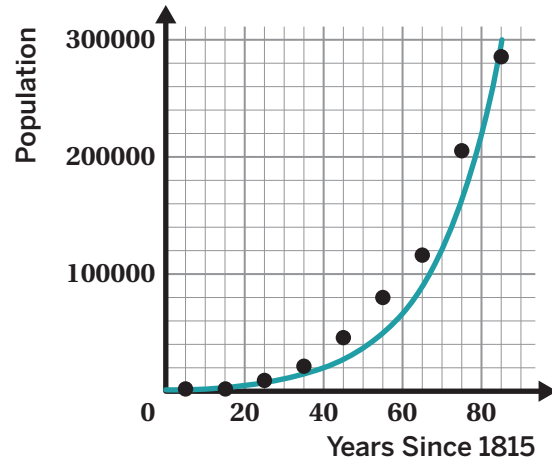
Predicting the Future

- 7** Zwena chose to revise her model to better fit the data from 1820 to 1900.

$$q(x) = 1904(1.0611)^x$$

According to Zwena's model, by what percent is Detroit's population increasing each year?

Explain your thinking.



- 8** Use the model to predict Detroit's population in 1910.

Years Since 1815, x	Population, $q(x)$
95	

- 9** Zwena wondered what the model would predict for Detroit's population in 2000, 185 years after 1815.

She used her model to calculate that the population of Detroit was about 111 million people in 2000.

Do you think this number is realistic? Explain your thinking.

Zwena

$$q(x) = 1904(1.0611)^x$$

$$q(185) = 1904(1.0611)^{185}$$

$$q(185) = 110811576$$

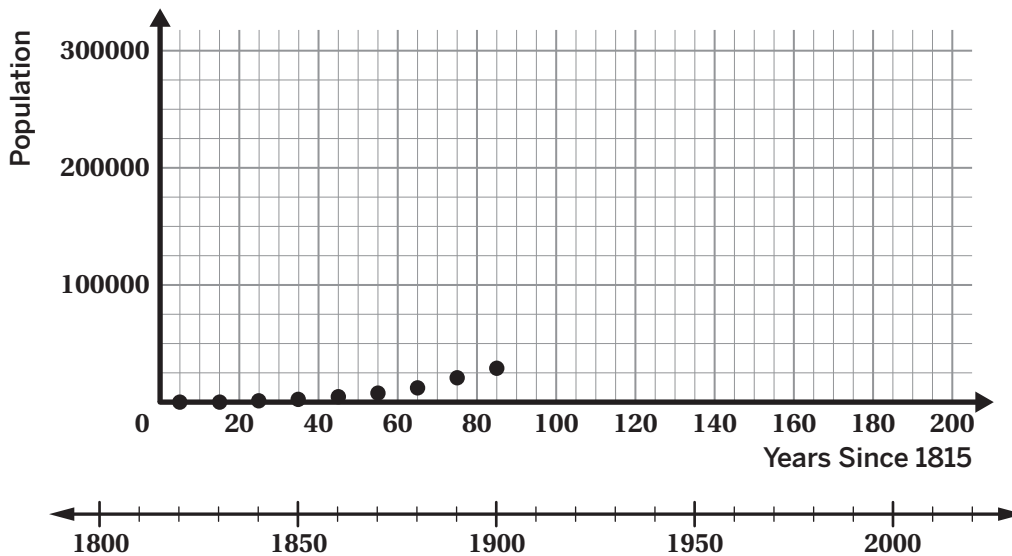
Based on Historical Events

You'll use the digital activity for Problems 10–13.

- 10** Examine the timeline in the digital activity to learn more about industry and migration throughout Detroit's history.

Discuss: How would you describe the change in population of Detroit in the years since 1900?

- 11** Sketch a prediction for the population of Detroit in the years after 1900.



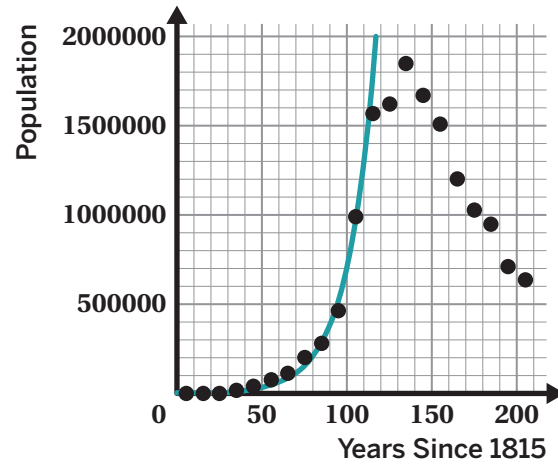
- 12** Use the digital activity to reveal the population data after 1900.

Discuss: What do you notice? What do you wonder?


Based on Historical Events (continued)

13 Here is Zwena's model.

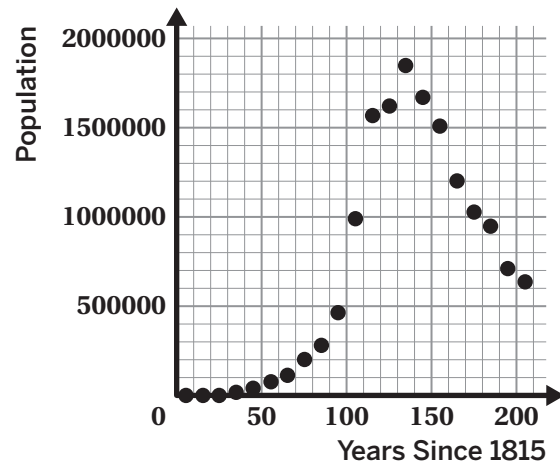
- a** Use the digital activity to highlight a *domain* in which this model would be useful for making predictions.
- b** What might be some issues with using this model outside of the highlighted domain?



14 Synthesis

 **Discuss:** Select one of the questions to discuss.

- A. What is something you learned about using exponential functions to model population change over time?
- B. What is a question you have about the population or history of Detroit?



17 Summary 5.14

Exponential and linear functions are often used to model the population growth of a city or country. *Models* can help us predict unknown data values, including future values. While some models can be useful, they also have limitations.

- Exponential functions increase toward infinity, but populations are limited by space, time, and available land.
- Linear models may generate a negative y -intercept, but populations do not have less than 0 people.
- Models may only be useful for predicting unknown values within a specific *domain*.

Let's look at an example. Here is data about the population of a city since 1880.

An exponential model is a better fit than a linear model. The data fits the shape of a curve more closely than it follows a line.

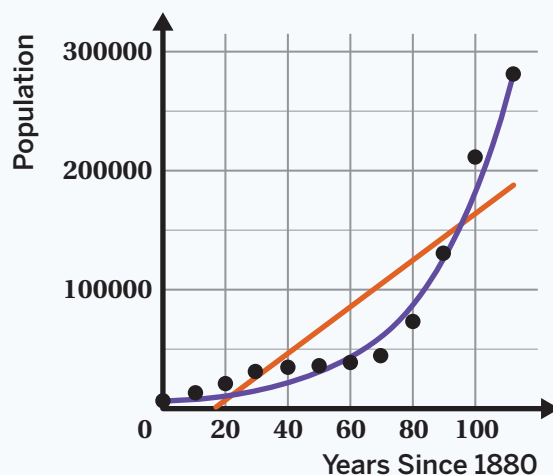
You can use the exponential function $p(t) = 3992(1.0397)^x$ to model the number of people in 1945.

Since 1945 is 65 years after 1880, substitute 65 into $p(t)$ to estimate the unknown value.

$$p(65) = 3992(1.0397)^{65}$$

$$p(65) = 50143$$

The model estimates that there were 50,143 people in 1945.

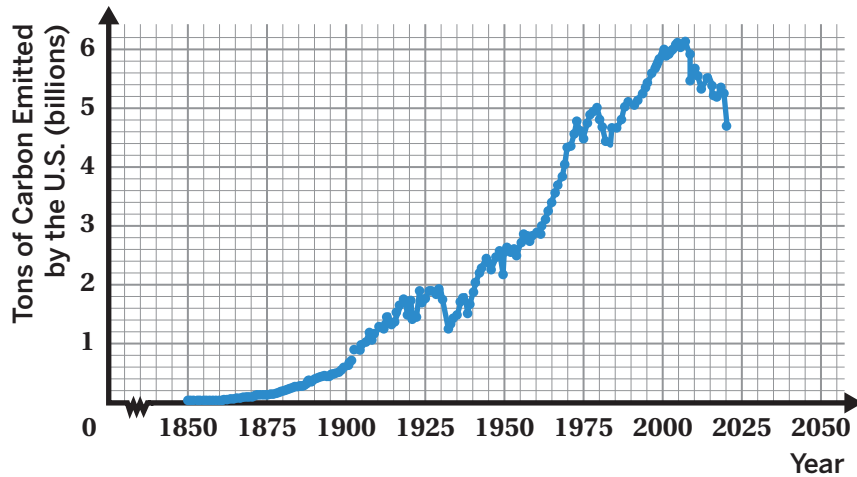


Practice

5.14

Name: _____ Date: _____ Period: _____

Problems 1–5: The graph shows the number of tons of carbon, in billions, that the United States emitted from 1850 to 2020.

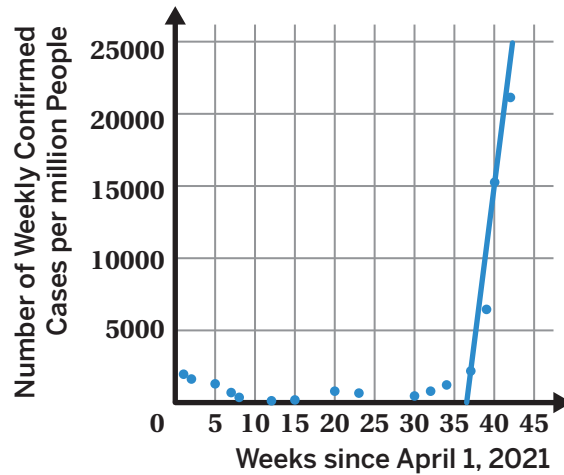


1. How would you describe the data in this graph?
2. What change occurs in the data around 2005? What do you think may have caused that change?
3. Sketch a line or exponential curve to model the data from 1850 to 2005.
4. Sketch a line or exponential curve to model the data after 2005.
5. Use your model to predict how many tons, in billions, the United States will emit in 2050.

Practice 5.14

Name: _____ Date: _____ Period: _____

Problems 6–8: The graph shows the number of weekly confirmed cases of COVID-19 per million people in Italy. Emma modeled the given data with a linear function.



6. Describe an advantage of using Emma's model.
7. Describe a disadvantage of using Emma's model.
8. Provide a suggestion for Emma to improve her model.

Spiral Review

Problems 9–10: Determine whether each table shows a linear or exponential function. Circle your choice.

9.

x	y
0	3
1	6
2	12
3	24

Linear

Exponential

10.

x	y
0	3
1	6
2	9
3	12

Linear

Exponential

Problems 11–12: A car costs \$24,000 when you buy it, but loses 15% of its value after each year.

11. **Test Practice** Write an equation to represent the value, $v(t)$, of the car t years after you buy it.
12. What will be the value of the car after 3 years?

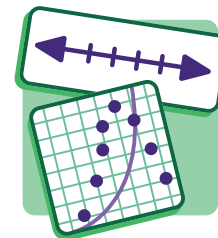
Unit 5
Lesson
15

Name: _____ Date: _____ Period: _____

Comparing Models Modeling with Functions Variability

 F-IF.4, F-LE.1, N-Q.3, S-ID.6, S-ID.6.a, SMP.4, SMP.5


Detroit's Population, Part 2



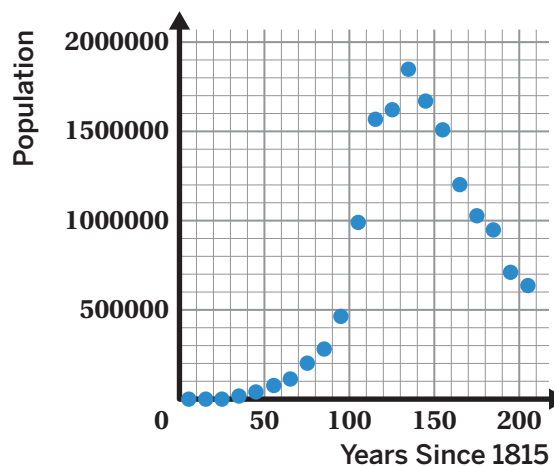
Let's use functions to model the population of Detroit.

Warm-Up

Here is the data for the population of Detroit from 1815–2020.

1.  **Discuss:** How would you describe the graph using some of the following vocabulary?

exponential	decreasing	percent change
linear	growth factor	domain
increasing	decay	range



Analyzing Data

Use Screen 2 of the digital companion to answer the following questions:

- Look closely at the data from 1950–2020. How would you describe the population of Detroit during that time period?
- Generate a linear *and* an exponential *model* for the data. Hint: For a linear model, type $y_1 \sim ax_1 + b$ in Row 3. For an exponential mode, type $y_1 \sim a \cdot b^{x_1}$ in Row 5.
- Record the slope of the linear model and the growth factor of the exponential model. Then explain what each value says about how the population of Detroit is changing each year.

Linear Model

Slope:

This means . . .



Exponential Model

Growth Factor:

This means . . .

- Which model do you think better fits the data for the population of Detroit from 1950–2020? Explain your thinking.

Analyzing Data (continued)

6.  **Discuss:** Why might city planners want to predict the population of Detroit in 2050?
7. Imagine it is your job to assist the city planners with these predictions.
- a. Use each model to predict the population of Detroit in 2050.
- Linear model prediction: Exponential model prediction:
- b.  **Discuss:** What level of accuracy makes sense to use when you are making a prediction so far into the future?
- c. Based on your agreed upon level of accuracy, what would you tell the city planners the predicted population will be in 2050?
8. Do you think either model should be used to predict the population of Detroit in 2075? Explain your thinking.

A Closer Look at Detroit

You will use the Activity 2 Sheet.

9. Choose several dates.



Discuss: How might each event have impacted Detroit's population?

10. Use the timeline, graph, and vocabulary from this course to describe the changes in Detroit's population over the past 200 years.

11. Suppose you continue working with Detroit city planners and community leaders to better understand the causes of Detroit's population increase and decrease.

What questions might you investigate to help you learn more?

You're invited to explore more.

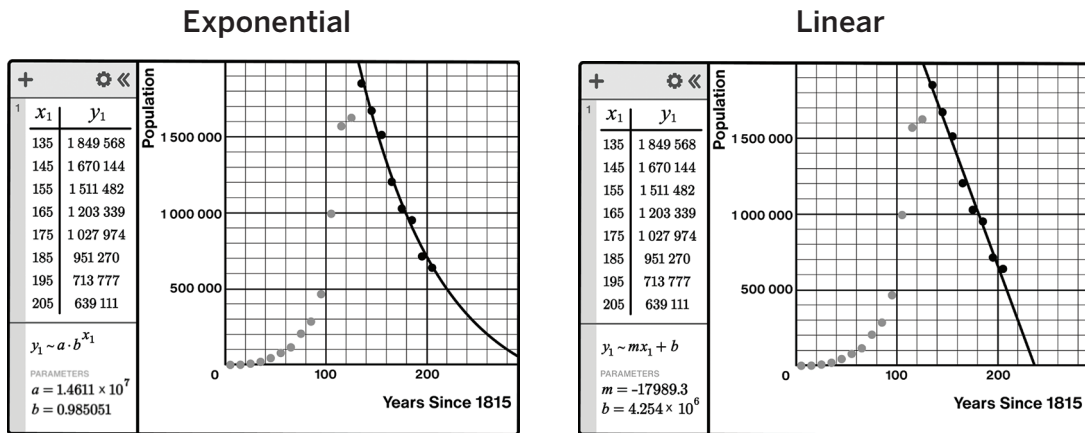
12. Let's consider our own communities!

- a What do you want to know about the population in your community?
- b Search the Internet for population [your community] [year] to see data about the population in your community from different years. For example: population Brooklyn, NY 2000 and population Brooklyn, NY 2020.
- c Describe how the population of your community has changed over time.
- d What events in your community's history might have impacted the changes in population?

Synthesis

13. Here's a quote about modeling that you may remember: *All models are wrong, but some are useful.*

Choose a model we explored in this lesson.



Explain how that model is both wrong and useful.

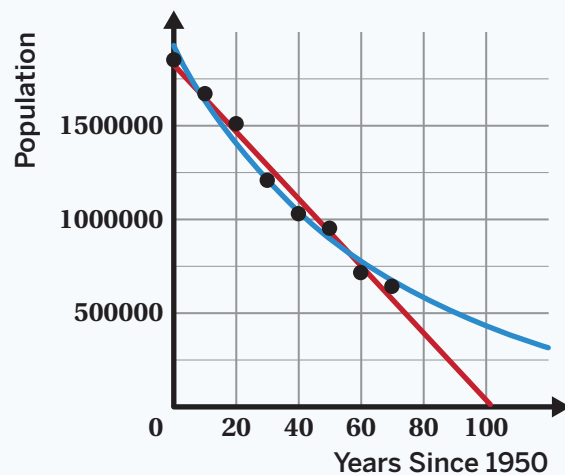
Summary 5.15

You can use a graphing calculator to generate a *line of best fit* or an exponential curve of best fit to model data. Here is an example of data about the population of Detroit from 1950–2020 that was used to generate a line and exponential curve of best fit.

The slope of the linear *model* means for every 1 year, the population of Detroit is predicted to decrease by about 17,989 people.

The growth factor of the exponential model means for every 1 year, the population of Detroit decreases by about 1.50%.

Which model is used depends on how well the function fits the data. In this case, both models fit the data well and can be used to predict unknown values accurately.



$$y_1 \sim mx_1 + b$$

Parameters
 $m = -17989.3$
 $b = 1825460$

$$y_1 \sim a \cdot b^{x_1}$$

Parameters
 $a = 1912612$
 $b = 0.985051$

Practice 5.15

Name: _____ Date: _____ Period: _____

Problems 1–6: Esi looked at data on the population of Boston for a history project. She graphed the population each decade from 1800 to 1940, then generated the line and exponential curve of best fit.

$$y_1 \sim mx_1 + b$$

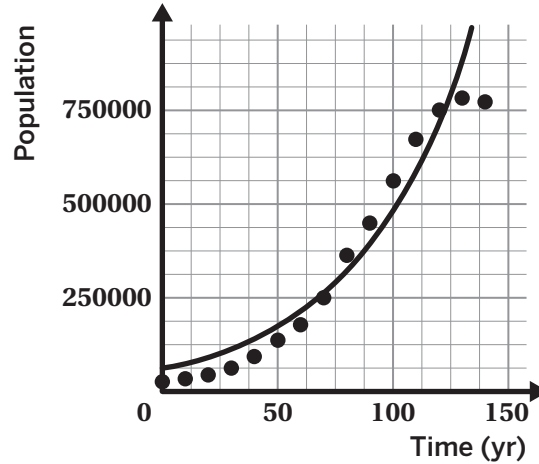
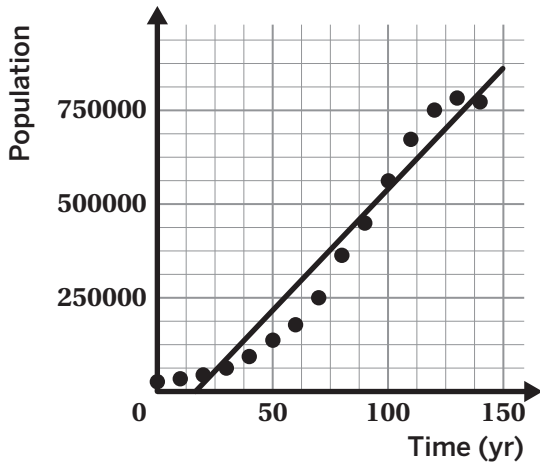
$$m = 6452$$

$$b = -105514$$

$$y_1 \sim a \cdot b^{x_1}$$

$$a = 63130.9$$

$$b = 1.02055$$




1. Describe the data using vocabulary from this unit.
2. Identify the slope for the line of best fit. Describe what the slope means in this situation.
3. Identify the growth factor for the exponential curve of best fit. Describe what the growth factor means in this situation.
4. Use the linear model to predict the population of Boston in 2000.
5. Use the exponential model to predict the population of Boston in 2000.
6. Which model do you think fits the data better? Explain your thinking.


Practice 5.15

Name: _____ Date: _____ Period: _____

Problems 7–9: The function $f(x) = 789(1.033)^x$ models the population of a city x years after 1900.

7.  **Test Practice** What is the meaning of the value 789 in the equation?

- A. The rate by which the population is growing
- B. The factor by which the population is growing
- C. The maximum population of the city
- D. The population of the city in 1900

8.  **Test Practice** What is the meaning of the value 1.033 in the equation?

- A. The rate by which the population is growing
- B. The factor by which the population is growing
- C. The maximum population of the city
- D. The population of the city in 1900

9. Use the exponential model to predict the population of the city in 2030.

Spiral Review

10. Use digits from 1 to 9, without repetition, to fill each blank so that the graphs of the equations never intersect where $x \geq 0$.

$$y = \square \cdot \square^x$$

$$y = \square \cdot \square^x$$

$$y = \square \cdot \square^x$$

Practice Day 2



Let's practice what you've learned so far in this unit!

You will use task cards for this Practice Day. Record all of your responses here.

Task A: Certificate of Deposit

1.

2. **a**

b

c

3. Circle one: Save-A-Lot CD Smart Wealth CD

Explanation:

Task B: National Debt

1. Circle one: Linear Exponential

Explanation:

2.

3.

4.

5.

Practice Day 2 (continued)

Task C: Used Car

-
- Circle one: A B C D
- a
 b
-
-

Task D: A Tale of Two Tables

- a
 b
 c
 d
- a
 b
 c
 d
-

You're invited to explore more.

- a
 b
 c
 d
 e
- a
 b
 c
 d
 e

Notes:

Career Connection

Why might a medicine label advise waiting a certain number of hours before taking the next dose?

Exponential decay functions can model how the amount of a medicine decreases in the human body over time. The *half life* is the time it takes for the initial amount to decrease in the body to half its starting dose. After one half life, 50% of the initial amount remains. Different medicines have different half lives. Waiting to take the next dose of a medicine allows the initial amount of time to decrease so that it is safe to take more.



leungchopan/Shutterstock.com.

Medicinal chemists develop, test, and improve medicines to help them be safe and effective. They might use exponential functions to model how medicines in the human body decrease over time.



Meet Colton Sawyer

Colton Sawyer is an Associate Professor of Mathematics at Regis University in Denver, Colorado. His research focuses on modeling biological systems, such as the impacts of Vitamin D in healthy patients compared to patients with chronic kidney disease. Beyond his research, Colton Sawyer teaches and develops new math courses and he works to increase student interest in and access to mathematics.

Are you interested in studying medicinal chemistry? What can you do to learn more?

Math in the World

Caffeine is a substance found in the beans of coffee trees. Its half-life is about 5 hours. If a person drinks a cup of green tea that contains 30 milligrams of caffeine, about how much remains in their body after 15 hours?



New Africa/Shutterstock.com.

Math Mindset

How can you use the structure of an exponential function to determine the initial value and growth factor?

Unit 6

Rigid Transformations and Congruence

Big Ideas in This Unit

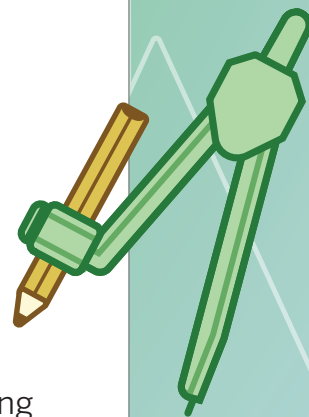
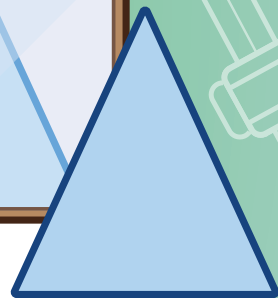
CC4 Shapes in Structures Transformations and Congruence

Questions for Investigation

- How can we construct precise geometric figures using circles, lines, and tools such as geometry software?
- How can we describe transforming geometric figures using reflections, rotations, and translations?
- How can we define congruence in terms of rigid transformations?



Explore: Making Precise Symbols How are geometric shapes and concepts used in art and culture?

























Watch Your Knowledge Grow

This is the math you'll explore in this unit. Rate your understanding to see how your knowledge grows!

Not yet
 Almost
 I got it!

I can . . .	Before	After
Use precise definitions for the terms angle, circle, perpendicular line, parallel line, and line segment.	<input type="radio"/> — <input type="radio"/> — <input checked="" type="radio"/>	<input type="radio"/> — <input type="radio"/> — <input checked="" type="radio"/>
Represent transformations in the plane using tracing paper and geometry software.	<input type="radio"/> — <input type="radio"/> — <input checked="" type="radio"/>	<input type="radio"/> — <input type="radio"/> — <input checked="" type="radio"/>
Describe transformations as functions that take points in the plane as inputs and give other points as outputs.	<input type="radio"/> — <input type="radio"/> — <input checked="" type="radio"/>	<input type="radio"/> — <input type="radio"/> — <input checked="" type="radio"/>
Compare transformations that do and do not change distance and angle measures.	<input type="radio"/> — <input type="radio"/> — <input checked="" type="radio"/>	<input type="radio"/> — <input type="radio"/> — <input checked="" type="radio"/>
Describe rotations and reflections that carry a given figure onto itself.	<input type="radio"/> — <input type="radio"/> — <input checked="" type="radio"/>	<input type="radio"/> — <input type="radio"/> — <input checked="" type="radio"/>
Define rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.	<input type="radio"/> — <input type="radio"/> — <input checked="" type="radio"/>	<input type="radio"/> — <input type="radio"/> — <input checked="" type="radio"/>
Draw a figure after it has been transformed by a rotation, reflection, or translation.	<input type="radio"/> — <input type="radio"/> — <input checked="" type="radio"/>	<input type="radio"/> — <input type="radio"/> — <input checked="" type="radio"/>
Describe a sequence of transformations that will take one figure onto another.	<input type="radio"/> — <input type="radio"/> — <input checked="" type="radio"/>	<input type="radio"/> — <input type="radio"/> — <input checked="" type="radio"/>
Transform figures when given descriptions of rigid transformations and predict how a rigid transformation will affect a figure.	<input type="radio"/> — <input type="radio"/> — <input checked="" type="radio"/>	<input type="radio"/> — <input type="radio"/> — <input checked="" type="radio"/>
Determine if two figures are congruent by using rigid transformations.	<input type="radio"/> — <input type="radio"/> — <input checked="" type="radio"/>	<input type="radio"/> — <input type="radio"/> — <input checked="" type="radio"/>
Use rigid transformations to show two triangles are congruent if and only if their corresponding sides and corresponding angles are congruent.	<input type="radio"/> — <input type="radio"/> — <input checked="" type="radio"/>	<input type="radio"/> — <input type="radio"/> — <input checked="" type="radio"/>

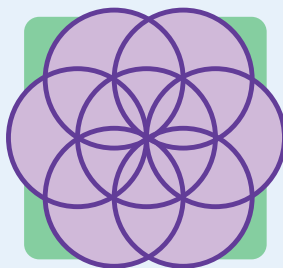
I can . . .	Before	After
Explain how rigid transformations can be used to prove the ASA, SAS, and SSS triangle congruence theorems.		
Construct a copy of a segment using a variety of tools and methods.		
Construct a copy of an angle using a variety of tools and methods.		
Bisect a segment using a variety of tools and methods.		
Bisect an angle using a variety of tools and methods.		
Construct perpendicular lines using a variety of tools and methods.		
Construct the perpendicular bisector of a line segment using a variety of tools and methods.		
Construct a line parallel to a given line that goes through a point not on the line using a variety of tools and methods.		
Construct an equilateral triangle inscribed in a circle.		
Construct a square inscribed in a circle.		
Construct a regular hexagon inscribed in a circle.		

Constructions



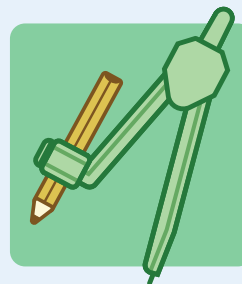
Explore

Making Precise Symbols



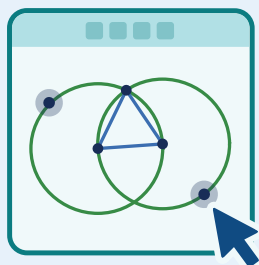
Lesson 1

Circles and Segments



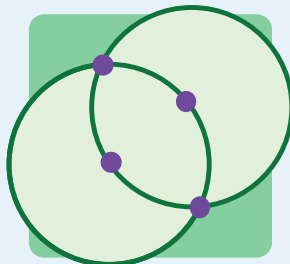
Lesson 2

Compass Constructions



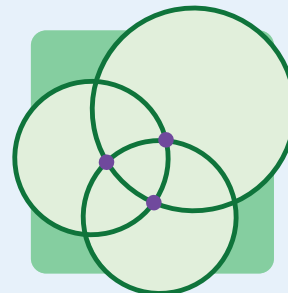
Lesson 3

Constructing Digitally



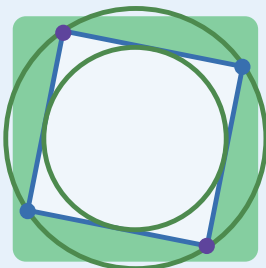
Lesson 4

Right in the Middle



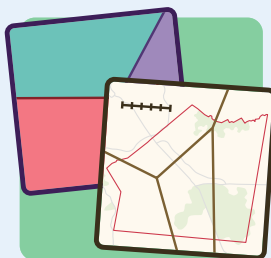
Lesson 5

Lines of Construction



Lesson 6

Square Up



Lesson 7

Community Constructions



Explore: Making Precise Symbols

Let's explore some ways geometric shapes and concepts are used in art and culture.



Warm-Up

Indigenous Australian art, made by Aboriginal Australians, has been created using many different methods, including rock painting, dot painting, carving, sculpting, and string art. While many artistic traditions have existed for long periods of time, Australian Aboriginal Art is one of the oldest continuing traditions in the world. Consider the image of an Australian Aboriginal dot painting.



Pilbara/Pixabay



1. What do you notice? What do you wonder?



Drawing with Precision (continued)

4. Whether it was the dot painting you saw in the Warm-Up or the symbols you drew freehand, it can be challenging to draw geometric shapes and concepts with complete precision. Create the figure you were assigned by using any of the tools provided. You may attempt to create your shape or concept multiple times, but be sure to label the most precise drawing.

Study the figures that you and your group members created. Select the geometric shape or concept that you believe is the closest to “perfect,” or the most precise.

5.  **Discuss:** What method(s) and tools did you use to create your most precise drawing in Problem 4?
6.  **Discuss:** What about the geometric shape or concept selected makes it the most precise?
7. What does it mean for a geometric figure to be precise?



Building Math Habits of Mind



Discuss:

- Which of these habits of mind did you strengthen during this activity?
- How did you use the one(s) you selected?

I can slow down and first make sense of a challenging problem before trying to solve it.

Not yet Almost I got it!

I can represent real-world problems and interpret their solutions within the context of the problem.

Not yet Almost I got it!

I can justify my thinking and ask questions to help me understand the thinking of others.

Not yet Almost I got it!

I can apply the math that I know to solve real-world problems, make assumptions and revise my thinking as needed.

Not yet Almost I got it!

I can select an appropriate tool to help me solve problems.

Not yet Almost I got it!

I can communicate my thinking and solutions clearly to others.

Not yet Almost I got it!

I can look for structure or patterns to help me solve problems.

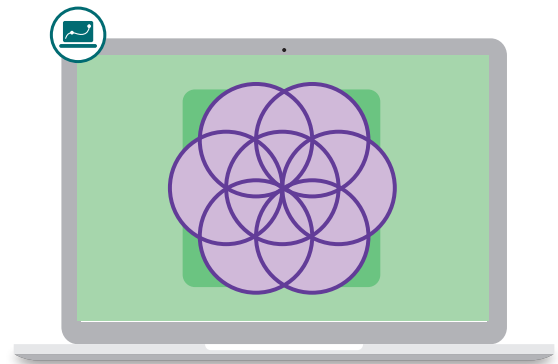
Not yet Almost I got it!

I can look for repeated calculations and other repeated steps to make generalizations

Not yet Almost I got it!

Circles and Segments

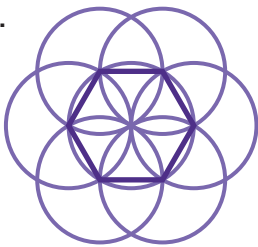
Let's construct designs using circles and line segments.



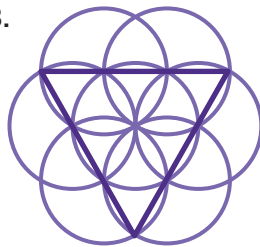
Warm-Up

1 Which one doesn't belong?

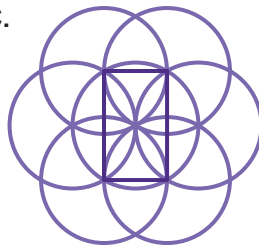
A.



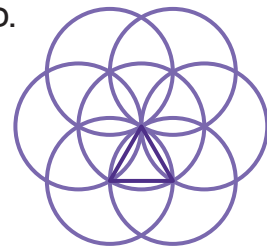
B.



C.



D.

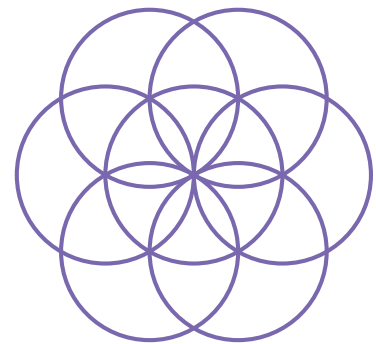


Design Matcher

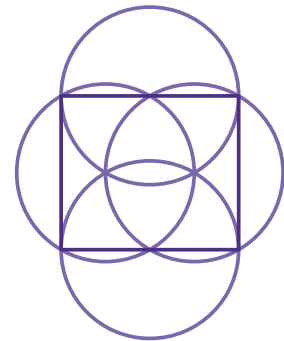
For Problems 2–4, you will use the digital activity to support your thinking.

- 2** **a** Let's use the design tool to make a design.
- b** What do you notice about how the tool works?

- 3** Describe how you used the tool to make this design.



- 4** Describe how you used the tool to make this design.

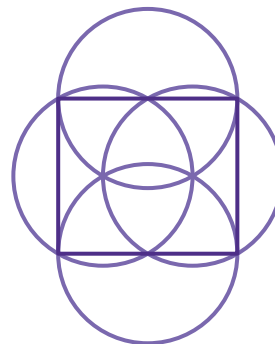


Under Construction

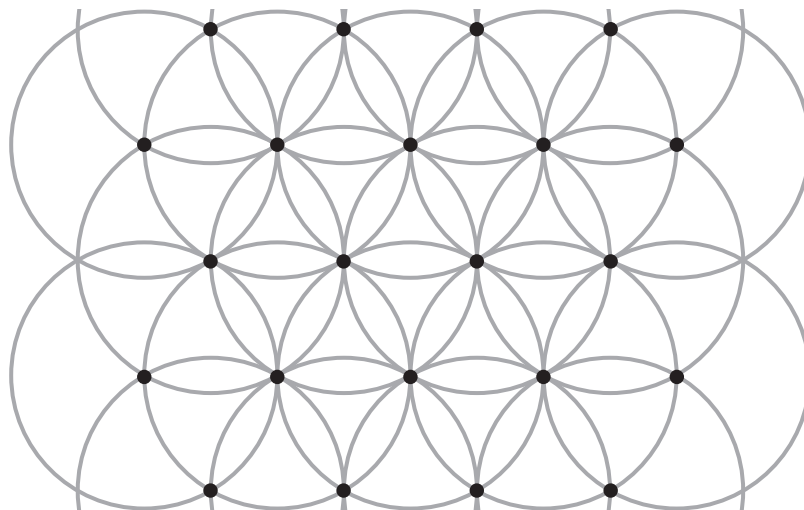
- 5** Thiago says that this **construction** is a square. Is this a square? Circle one.

Yes No Not enough information

Show or explain your thinking.



- 6** Construct two different equilateral triangles.



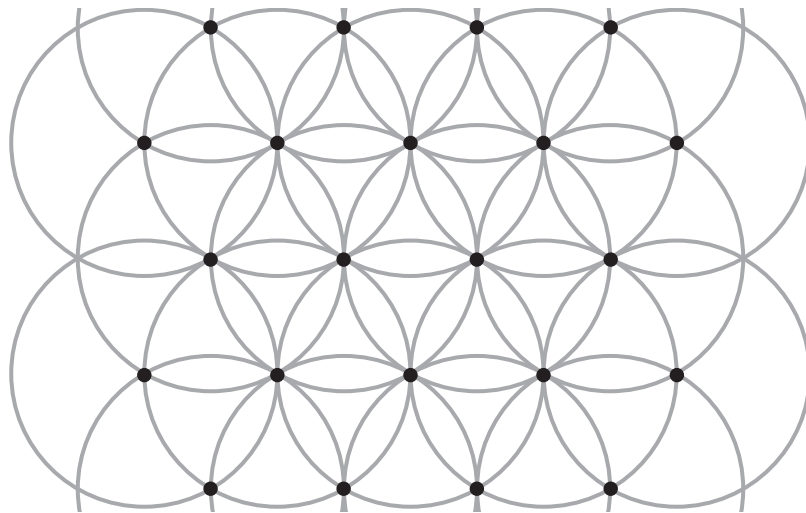
- 7** The design tool makes *line segments* and **circles**. How is a line segment . . .

a The same as a line? **b** Different from a line?

Challenge Creator

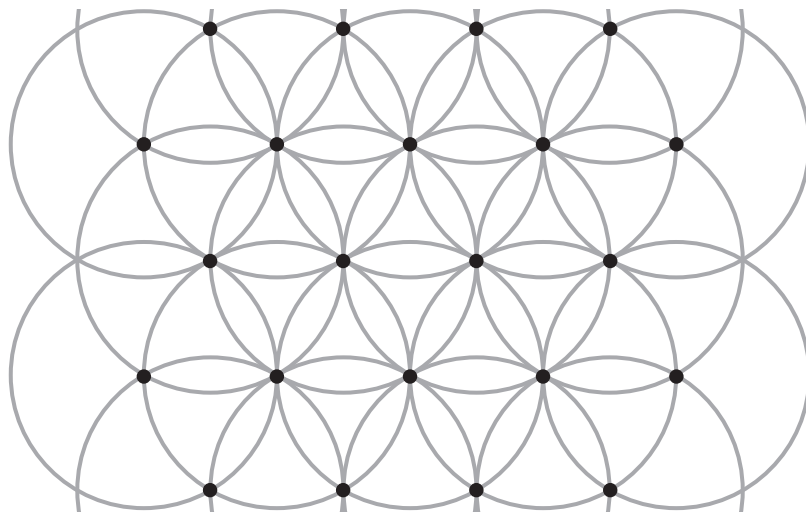
8 Construct a design by connecting the intersection points using line segments and circles.

a **Make It!** My design:



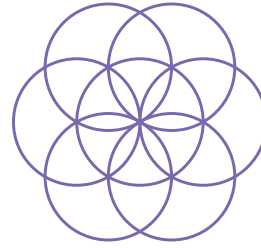
b **Swap It!** Trade your design with a partner. Match your partner's design.

_____ 's design:



9 Synthesis

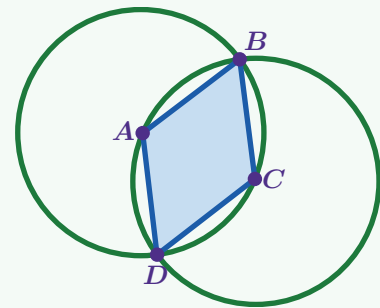
How can **circles** help to make segments that are the same length?



12 Summary 6.01

Because **circles** have a known radius, you can use them to construct *line segments* of equal length. When circles overlap with their centers on either end of the same radius, you can construct line segments with the same length.

For example, you know that each side length of quadrilateral $ABCD$ is the same because each side is a radius of one of two circles that are the same size.



circle A shape formed by the set of all points in a plane that are the same distance from a fixed point called the center.

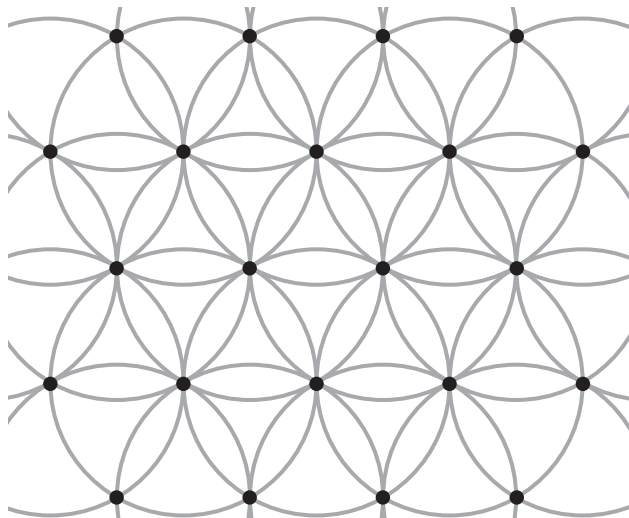
construction A geometric drawing made by using precise tools.

Practice 6.01

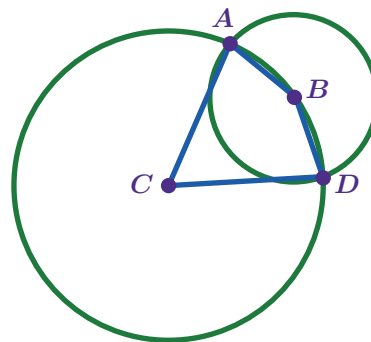
Name: _____ Date: _____ Period: _____


Problems 1–4: Use line segments and intersection points to construct each polygon.
Note: Each circle is the same size.

1. Equilateral triangle
2. Equilateral triangle with different side lengths from Problem 1
3. Hexagon
4. Rectangle

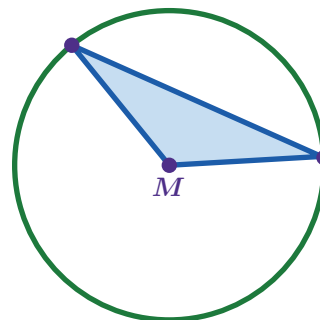


5. Points C and B are the centers of these circles. Highlight two segments that you know are the same length.



6.  **Test Practice** A triangle is drawn in a circle. Point M is the center of the circle. Which type of triangle is shown?

- A. Equilateral: three sides are the same length
- B. Right: one 90° angle
- C. Scalene: no sides are the same length
- D. Isosceles: two sides are the same length



Practice 6.01

Name: _____ Date: _____ Period: _____

7. Describe how you could construct a line segment that's exactly 4 times the length of this line segment using only circles and line segments.



Spiral Review

Problems 8–10: Here is a circle with center H .

8. Highlight a line segment that is a radius.

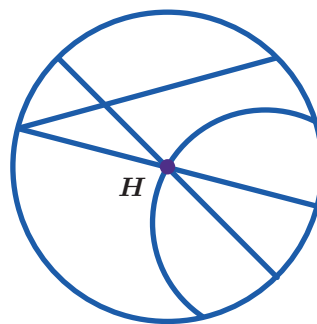
Explain how you know it's a radius.

9. Highlight a line segment that is a diameter.

Explain how you know it's a diameter.

10. Highlight a line segment that is neither a radius nor a diameter.

How do you know it's neither?



Problems 11–12: Determine the distance between each set of points.

11. $A(2, 1)$ and $B(1, 5)$

12. $C(-2, 5)$ and $D(2, 2)$

Compass Constructions

Let's use a compass to make precise designs on paper.

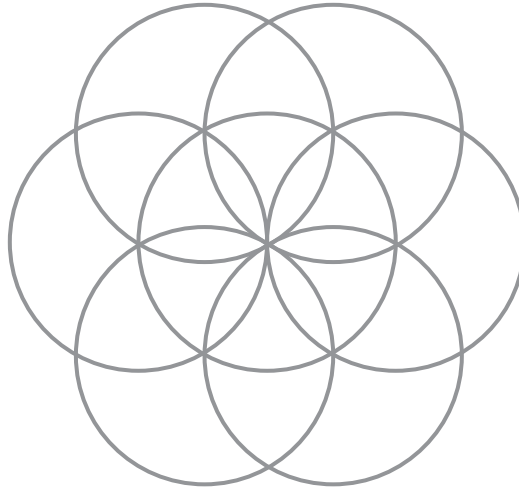


Warm-Up

1. Experiment with using a compass to construct circles.

Circles

This design is made of circles that are all the same size.



2. Use a compass to recreate this design. You can make the design as large or as small as you want.

3. Write one suggestion for how to make your design more precise. Discuss your thinking with a classmate. Write their suggestions below.

My Suggestion

Other Suggestion(s)

Circles (continued)

4. Use a compass to construct the same design more precisely.

Regular polygons have sides that are the same length and angles that are the same measure. A regular triangle can also be called an **equilateral triangle**.

When all the vertices of a polygon lie on a circle, the polygon is **inscribed** in the circle.

5. Use your design from Problem 4 to help you with the following constructions.
- a Construct an equilateral triangle inscribed in a circle by connecting three points in your design.

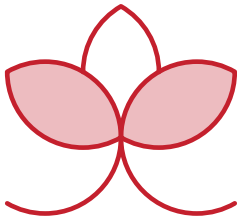
 - b Construct a regular hexagon inscribed in a circle by connecting six points in your design.

 - c How do you know that your hexagon and triangle each have sides that are the same length?

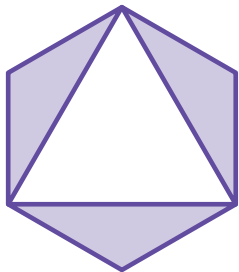
Compass Constructions

All of these designs use the pattern from Activity 1 as an underlying construction.

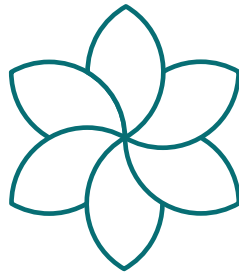
Design A



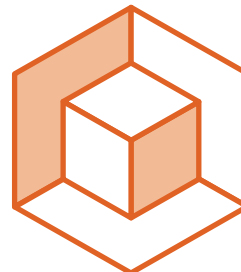
Design B



Design C



Design D




6. Select a design and construct it.

Selected Design: _____

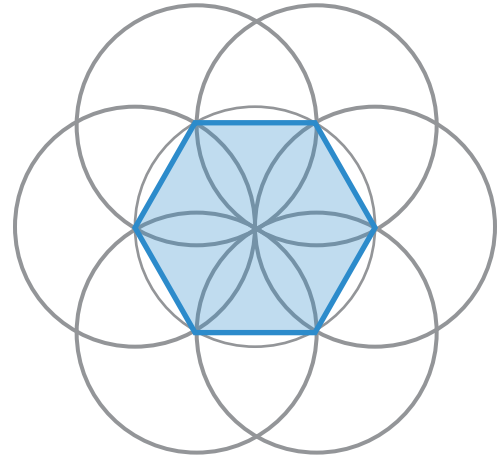
Compass Constructions (continued)

7. Write instructions for how to construct your design.

Synthesis

8.  **Discuss:** How can using a compass help you construct a hexagon with six equal sides?

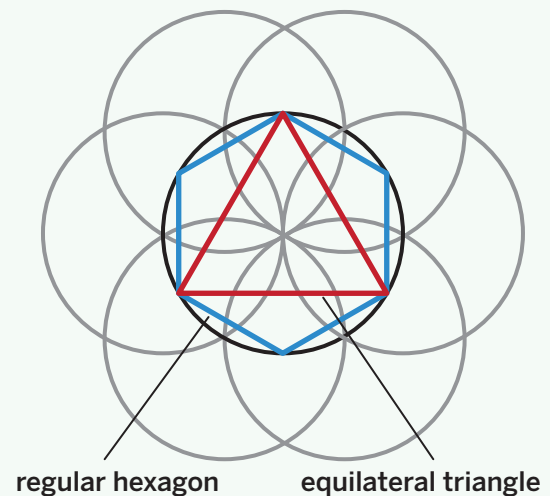
Use the example if it helps with your thinking.



Summary 6.02

You can use a *compass* to make circles that are all the same size. This is a helpful measurement tool when constructing **regular polygons inscribed** in a circle.

The intersections of lines and circles define points that help you replicate constructions and write instructions. You can use these points to reason about distances, precisely construct designs, and communicate about how to recreate your constructions.



equilateral triangle A triangle with three equal side lengths and three equal angle measures.

inscribe To draw a figure within another such that the inner figure intersects but never crosses the sides or boundary of the outer figure.

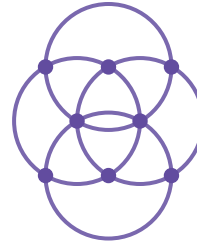
regular polygon A polygon where all sides are the same length and all angles are the same measure.

Practice

6.02

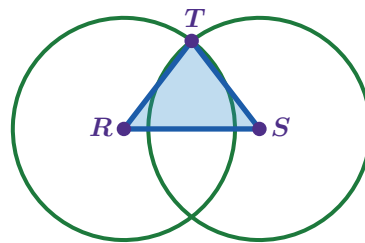
Name: Date: Period:

Problems 1–3: Here is a circle design:




1. Construct this design.
2. Construct a polygon that has at least four sides using intersection points on the design.
3. How many equilateral triangles could be created using intersection points on the design? Show or explain your thinking.

4. How could you change this design so you could use it to construct an equilateral triangle?

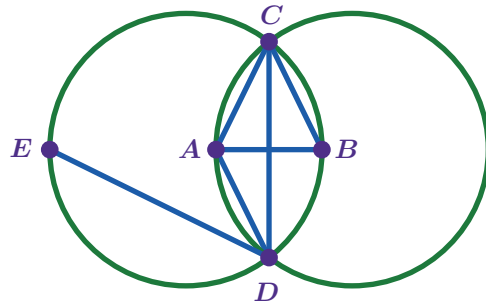


Practice 6.02

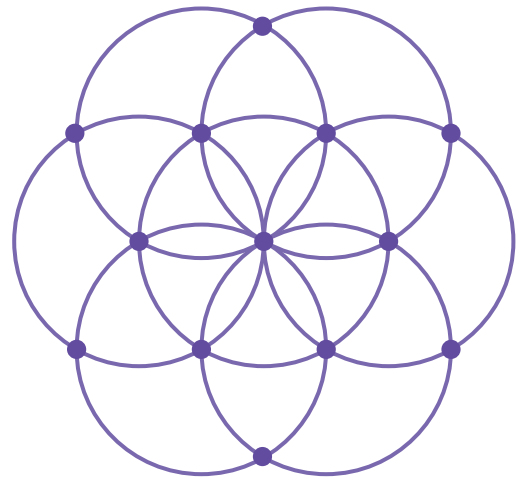
Name: _____ Date: _____ Period: _____

5.  **Test Practice** Point A is the center of one circle. Point B is the center of the other. Select *all* the segments that have the same length as segment AB .

- A. Segment AC
- B. Segment AD
- C. Segment BC
- D. Segment CD
- E. Segment DE



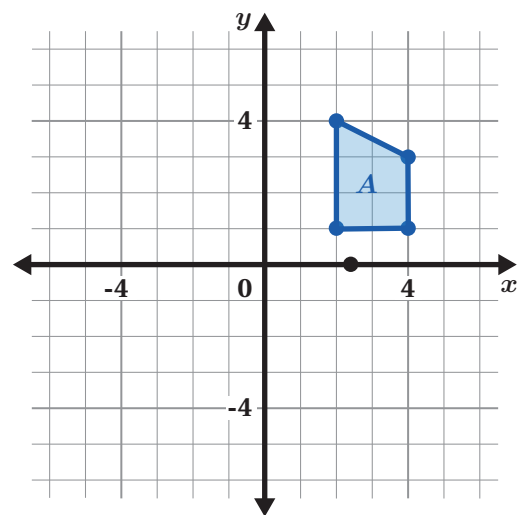
6. Use intersection points in the design to construct a polygon with at least seven sides. All of the sides should be the same length.



Spiral Review

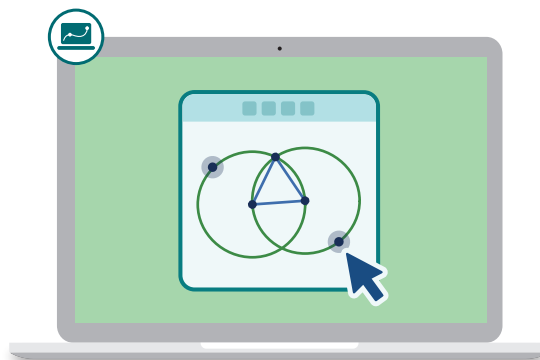
Problems 7–9: Transform polygon A based on each set of directions.

7. Reflect polygon A over the y -axis. Label the image B .
8. Translate polygon A down 7 units. Label the image C .
9. Translate polygon A down 3 units and left 4 units. Label the image D .



Constructing Digitally

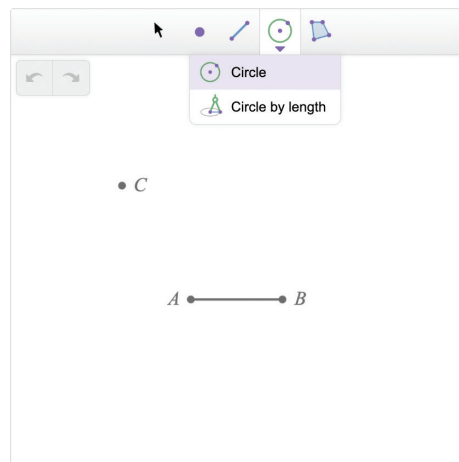
Let's explore how to communicate to each other about constructions.



Warm-Up

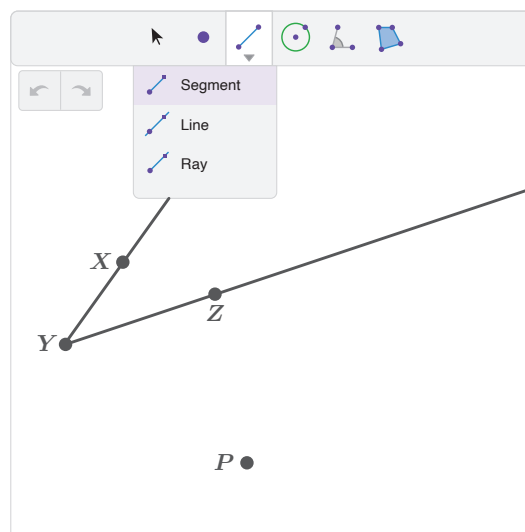
1 Use the digital activity to explore some digital construction tools.

- a** Try these tools and see how they work.
- b** Explain how you can create a copy of \overline{AB} with one of its endpoints as C .



2 Use the digital activity to explore how to copy an angle.

Explain how you can create a copy of $\angle XYZ$ that has a vertex at point P .



Digital Equilaterals

- 3** Use the digital activity to construct an equilateral triangle using segment AB as one of the sides.

Drag points A and B to different locations.

Is your triangle still equilateral?

- 4** We have made constructions on paper and digitally.

What are some advantages and disadvantages to constructing with digital tools?

Advantages:



Disadvantages:


Construction Instructions

5 Aaliyah wrote these steps to construct a triangle.

- a** Use the digital construction tools to follow her instructions.

Step 1 Draw a circle centered at A.
Step 2 Draw a circle centered at B.
Step 3 Draw a polygon from points A and B to an intersection point of the two circles.



- b**  **Discuss:** Do you think that everyone who follows Aaliyah's instructions will make the same triangle?

6 You will use the digital activity to create your own construction challenge and solve your classmates' challenges.

Construction Instructions (continued)

- 7 How would you revise your instructions after seeing the designs your classmates constructed?

8 Synthesis

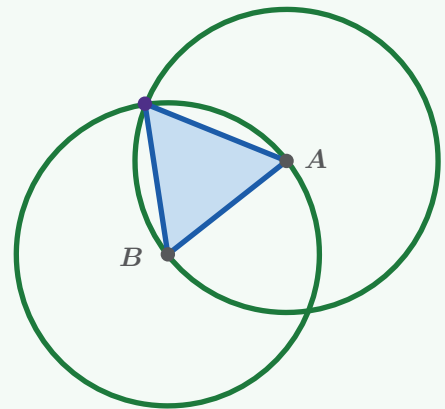
What are things to keep in mind when writing instructions for constructions?

11 Summary 6.03

Digital tools are helpful in making precise constructions. For circles, the radius lengths will always be exactly what you define (and your compass won't slip like it can on paper).

You can define a center and a point on a circle to construct line segments of exactly the same length.

When using digital tools to construct, you can also drag points to see what relationships hold true about the constructions.



Using precise language can help you describe digital constructions. It's helpful to:

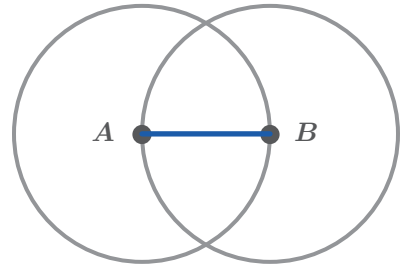
- Clearly define the center and radius length of a circle.
- Use points on constructed objects, like circles, line segments, or **angles**.
- Label intersection points to have clear ways to describe constructed points.

angle A geometric figure made up of 2 rays that share the same endpoint.

Practice 6.03

Name: _____ Date: _____ Period: _____

Problems 1–3: Here is a diagram.



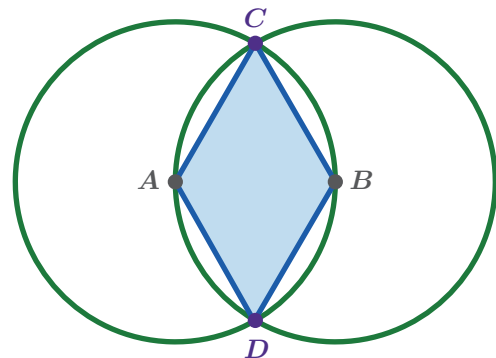
1. Construct an equilateral triangle using points A and B as vertices.
2. Construct two more circles centered at the intersections of the original circles. All of the circles should be the same size.
3. Use the intersections to construct an equilateral triangle that is larger than your first one.
4. Here are instructions for constructing an equilateral triangle.

Which step should be revised? Explain your thinking.

Step 1 Construct a circle with center A
Step 2 Mark a point on the circle and label it B .
Step 3 Construct a circle with center B .
Step 4 Mark a point at one of the intersection points of the circles and label it C .
Step 5 Construct segments AB , BC , and CA .

5. Roberto started with points A and B , and created quadrilateral $ABCD$.

What steps could he write so that someone else can construct the exact same quadrilateral?



Practice 6.03

Name: _____ Date: _____ Period: _____

Problems 6–7: Here is line segment GH .

6. Create a design that takes *exactly* 5 steps to complete.



7. List your instructions.

Step 1:

Step 2:


Step 3:

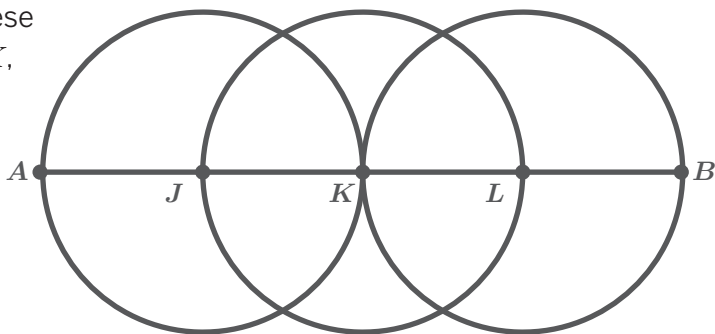
Step 4:

Step 5:

Spiral Review

8. What information is helpful to include when writing precise directions for constructing a circle with a compass?

 **Test Practice Problems 9–10:** These three circles are centered at points J , K , and L . The length of segment LB is 3 centimeters.

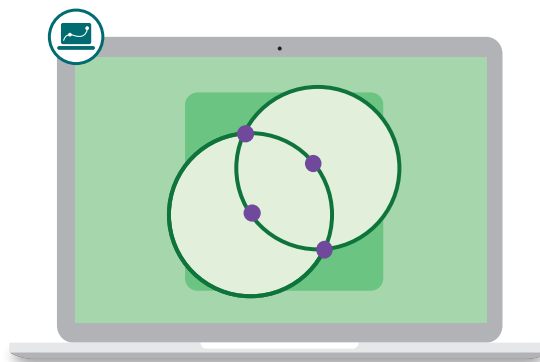


9. What is the length of segment AJ ?

10. What is the length of segment AL ?

Right in the Middle

Let's explore equal distances through constructions.

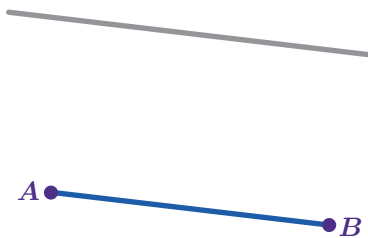


Warm-Up

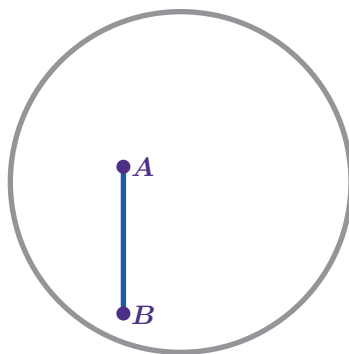
1 For each challenge:

- Plot a point on the gray path that is an equal distance from point A and point B .
- If you think there is more than one possible place, try to plot them all.

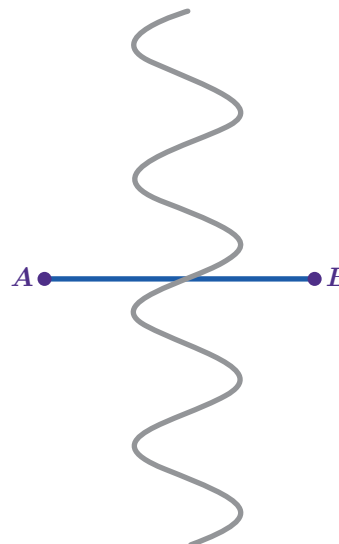
Challenge #1



Challenge #2



Challenge #3

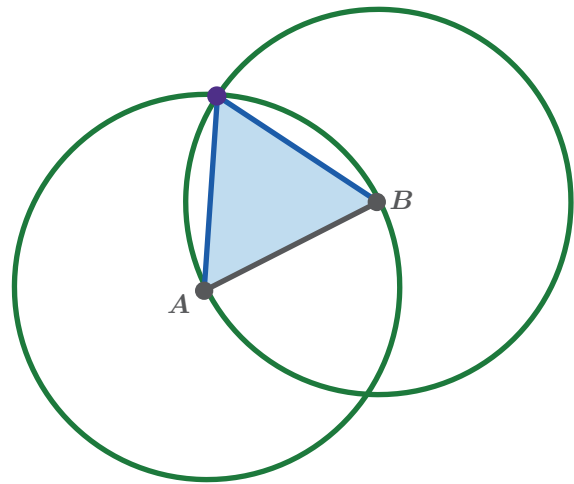


Plotting Points

2 Here is a construction of an equilateral triangle with base \overline{AB} .

Plot some points in places that are **equidistant** (an equal distance) from point A and point B .

Try to find places none of your classmates will think of.



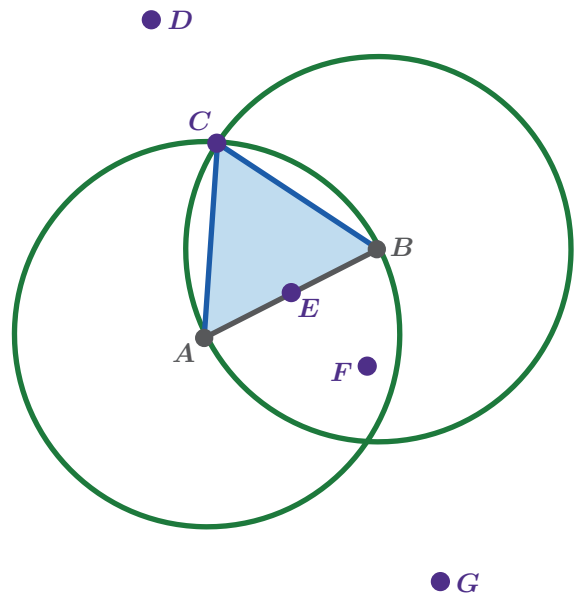
3 Here are Eliza's points from the previous problem.

Which point is not equidistant from points A and B ? Circle one.

Use construction tools if that helps with your thinking.

C D E F G

Explain your thinking.



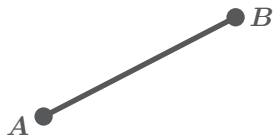
4 Let's look at how Bao and Zahra identified points that are not equidistant from points A and B .

Discuss: What was each student's strategy?

Lining It Up

- 5** The line that passes through all the points that are equidistant from point A and point B is perpendicular to \overline{AB} .

Construct this line. Use the example if it helps with your thinking.



Lining It Up (continued)

6 Line m is the **perpendicular bisector** of \overline{JK} .

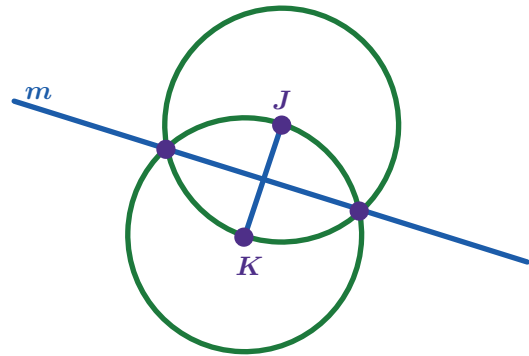
Zahra says perpendicular bisectors pass through a segment's **midpoint**.

Is this statement *always*, *sometimes*, or *never* true?

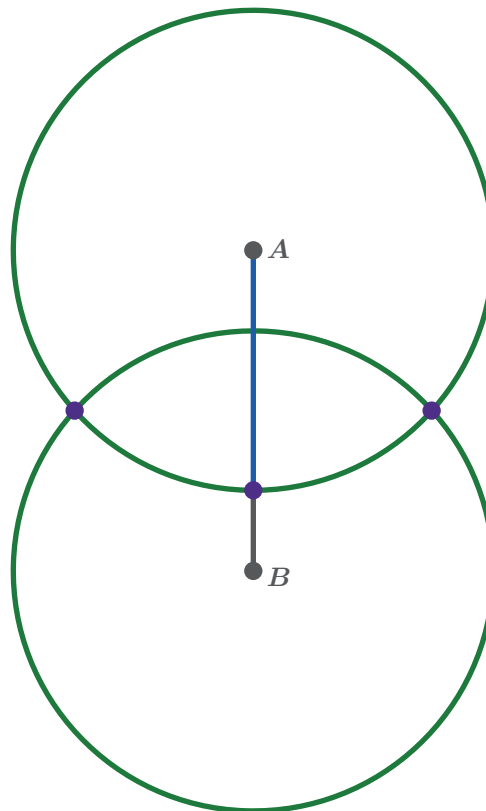
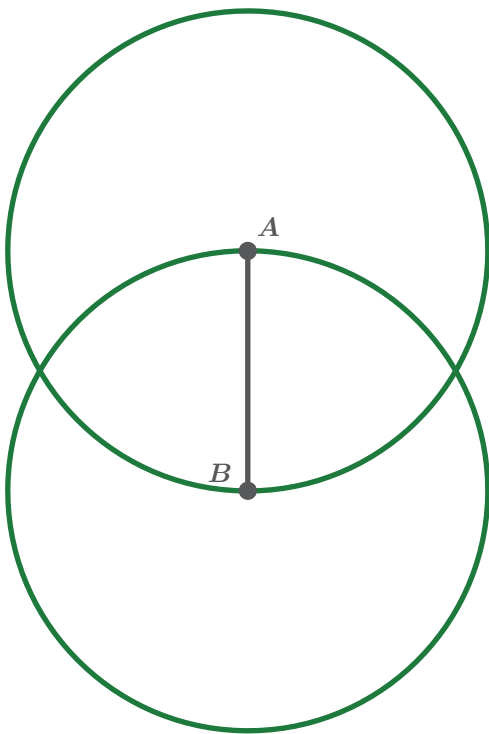
Circle one.

Always Sometimes Never

Explain your thinking.



7 Zahra ran out of space constructing a perpendicular bisector, so she used the “circle by length” digital tool instead.



Discuss: Why does Zahra's strategy work?

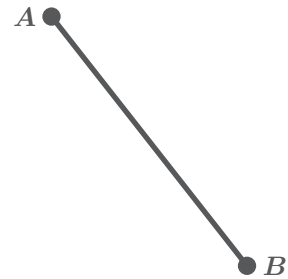
Center Line

8 For each challenge, construct the perpendicular bisector of segment AB .

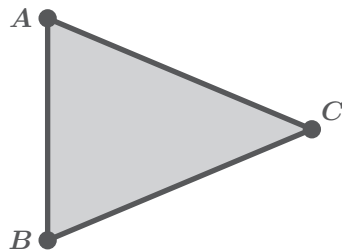
Challenge #1



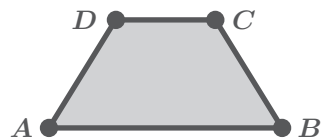
Challenge #2



Challenge #3

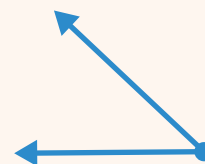


Challenge #4



You're invited to explore more.

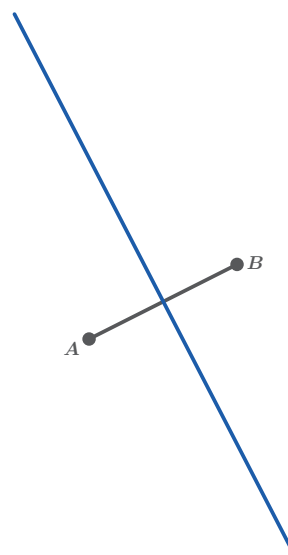
9 Construct a line that divides the angle into two angles with equal measures.



10 Synthesis

Discuss: Describe the relationship between the endpoints of a line segment and the perpendicular bisector of that segment.

Use segment AB and its perpendicular bisector if it helps with your thinking.



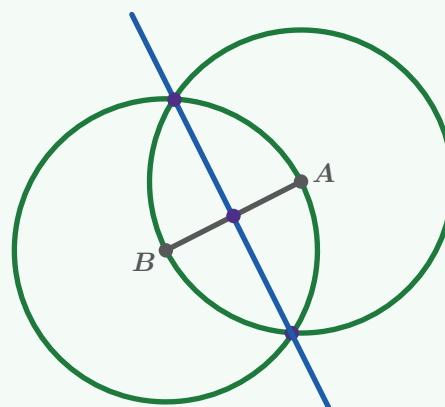
13 Summary 6.04

A **perpendicular bisector** passes through a segment's **midpoint**. Any points on the perpendicular bisector are **equidistant** from the endpoints of that line segment.

To construct a perpendicular bisector:

- You can construct two circles centered at each endpoint of a line segment, using the segment as the radius of each circle.
- You can construct two circles with the same radius centered at the endpoints of the segment.

The line that passes through the intersection points of the circles is the perpendicular bisector.



equidistant At equal distances from a given reference point.

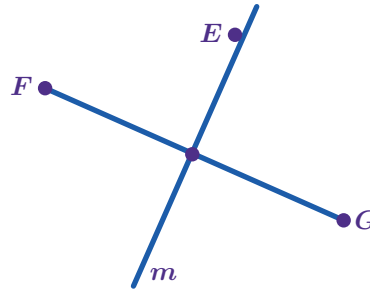
midpoint The point that divides a line segment into two equal parts.

perpendicular bisector A line, segment, or ray that is made of the set of points equidistant from two endpoints of a line segment. Perpendicular bisectors pass through the midpoint of a line segment.

Practice 6.04

Name: _____ Date: _____ Period: _____

Problems 1–2: Line m is the perpendicular bisector of \overline{FG} .




- Plot a point that is the same distance from F as it is from G .
- Choose the phrase that best completes the statement: *Point E is _____*.
Circle one.

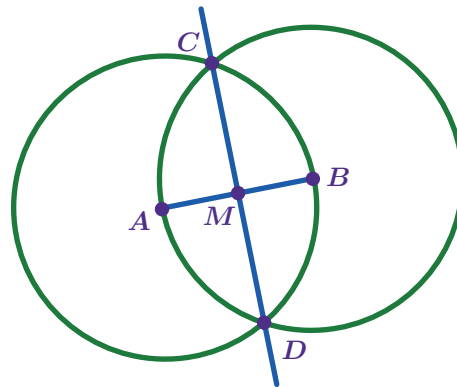
Closer to point F

Closer to point G

The same distance from point F as it is from point G

-  **Test Practice** In the diagram, point A is the center of one circle and point B is the center of another. Select *all* the true statements.

- A. Line CD is perpendicular to segment AB .
- B. Point M is the midpoint of segment AB .
- C. The length of segment AB is equal to the length of segment CD .
- D. Segment AM is perpendicular to segment BM .
- E. The length of segment CB is equal to the length of segment DB .



Problems 4–5: Here is \overline{TS} .

- Construct the perpendicular bisector of \overline{TS} .
- Using \overline{TS} as a side, construct a triangle with *only* two sides of the same length.



Practice 6.04

Name: _____ Date: _____ Period: _____

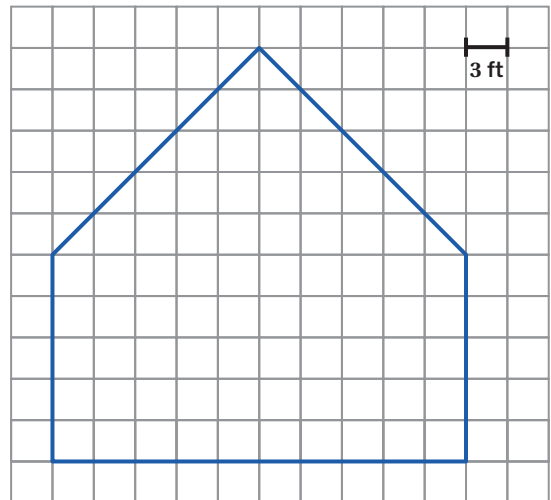
6. Construct an equilateral triangle with a side that is half the length of \overline{LM} .



Spiral Review

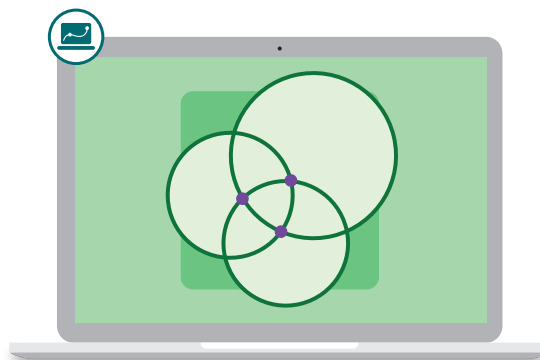
Problems 7–9: Here is a scale drawing of the front of a shed

7. What is the height of the shed, in feet?
8. What is the width of the shed's base at its widest point, in feet?
9. Find the total area, in square feet.



Lines of Construction


Let's develop strategies to construct perpendicular and parallel lines.

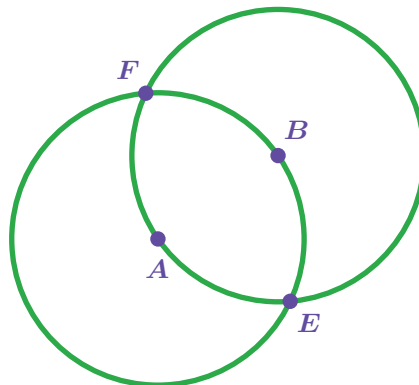


Warm-Up

1 Use a compass and straightedge, as needed, to construct each of these objects in the diagram.

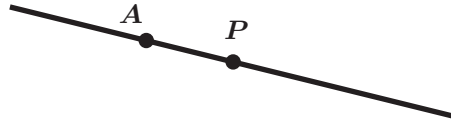
- Radius
- Equilateral triangle
- Perpendicular bisector
- Midpoint

 **Discuss:** How did you know where to add each object in the diagram?

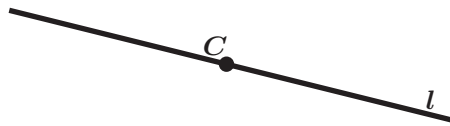


Construction Challenges

- 2** Construct point B so that point P is the midpoint of segment AB .



- 3** Construct line m so it is perpendicular to line l through point C .

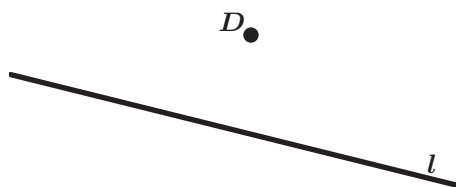


Construction Challenges (continued)

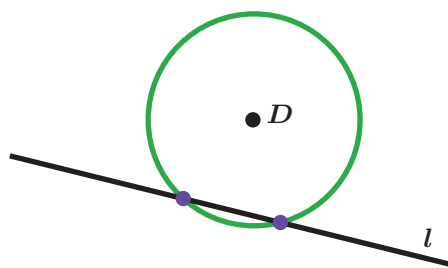
4 Let's watch Neo's strategy for the previous problem.

Discuss: How can Neo improve the construction so that the perpendicular line always goes through point C ?

5 Construct line m through point D so it is perpendicular to line l .



6 Here is Raven's first step to construct the challenge in the previous problem. Why is this the first step in her construction?



Can You Construct It on Paper?

7 Use a compass and straightedge to make each construction and write the steps you took.

- a** A line perpendicular to line l that passes through *either* point A or point B .

Construction



Steps

- b** The line parallel to line l that passes through point A .

Construction



Steps

You're invited to explore more.

- 8** In the digital lesson or on a separate piece of paper, use construction tools to create a pattern of lines that is parallel and perpendicular to a given line.

9 Synthesis

What strategies are helpful for constructing parallel or perpendicular lines?

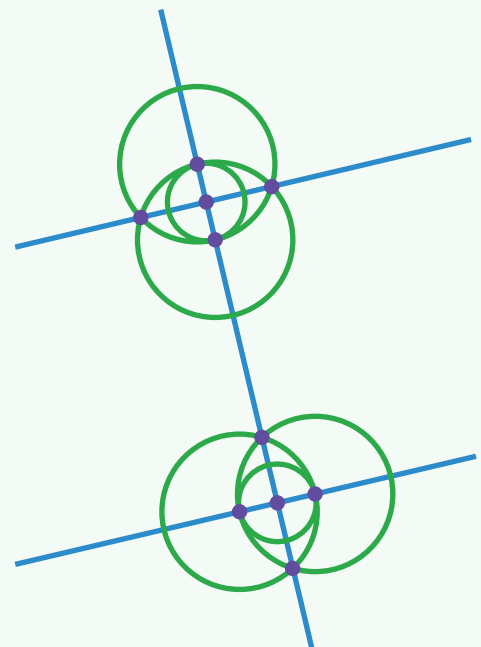
12 Summary 6.05

You can use perpendicular bisectors to construct *perpendicular lines* and *parallel lines* with precision.

To construct a line through a specific point that's perpendicular to a given line:

- Construct a circle using the specific point as its center, making sure that it intersects the given line at two points.
- Use the two intersection points as endpoints of a line segment and then construct the perpendicular bisector of that segment.

One technique for constructing a parallel line through a specific point is to use the perpendicular bisector construction twice. If two lines are perpendicular to a given line, then those two lines are parallel.

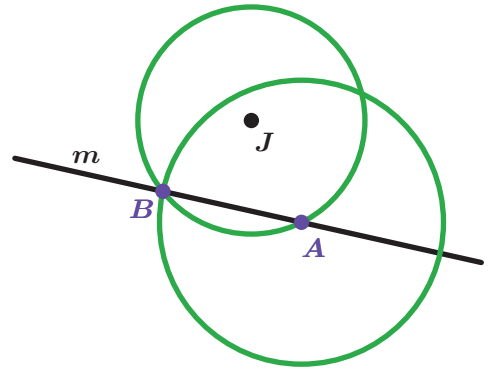


Practice 6.05

Name: _____ Date: _____ Period: _____

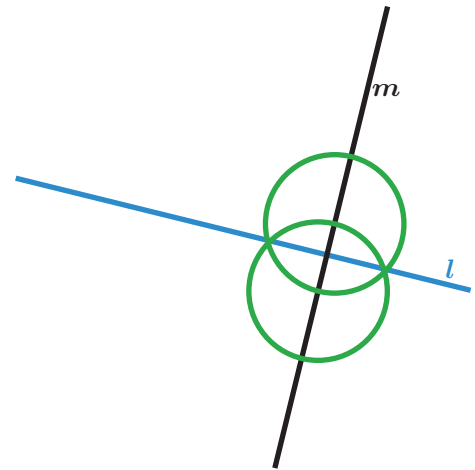
Problems 1–2: Mauricio started to construct the line perpendicular to line m that passes through point J .

1. Describe the next step in his construction.
2. Complete the construction.



Problems 3–4: Yolanda started to construct a line parallel to line m .

3. Describe possible remaining steps to complete her construction.



4. Complete the construction.

5.  **Test Practice** Lines q and r are perpendicular to line p . Select *all* the true statements.

- A. Line q is perpendicular to line r .
- B. Line q is perpendicular to line p .
- C. Line r is perpendicular to line p .
- D. Line q is parallel to line r .
- E. Line q is parallel to line p .

Practice 6.05

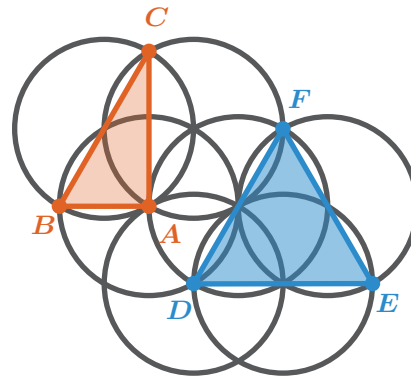
Name: _____ Date: _____ Period: _____

6. Construct a quadrilateral with exactly one pair of parallel sides and a side perpendicular to both parallel sides. Use segment GH as one of the four sides.



Spiral Review

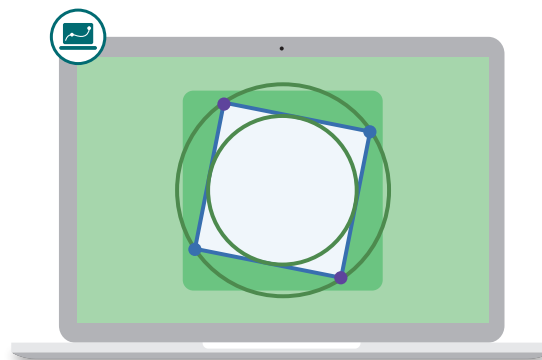
7. Here are two constructed triangles. Which statement is true?
- A. The length of segment AB is $\frac{1}{2}$ the length of segment DE .
 - B. The length of segment AC is equal to the length of segment DE .
 - C. The perimeter of triangle ABC is equal to $\frac{1}{2}$ the perimeter of triangle DEF .
 - D. Triangles ABC and DEF are both equilateral.



8. Select *all* properties that are true for squares.
- A. A square has four pairs of perpendicular sides.
 - B. A circle centered at one of the vertices of a square, with a radius equal to the length of the square's side, will go through exactly two other vertices.
 - C. The length of one diagonal of a square is less than the length of the other diagonal.
 - D. The lengths of all four sides of a square are equal.
 - E. The sides of a square are not parallel.

Square Up

Let's make squares using circles.



Warm-Up

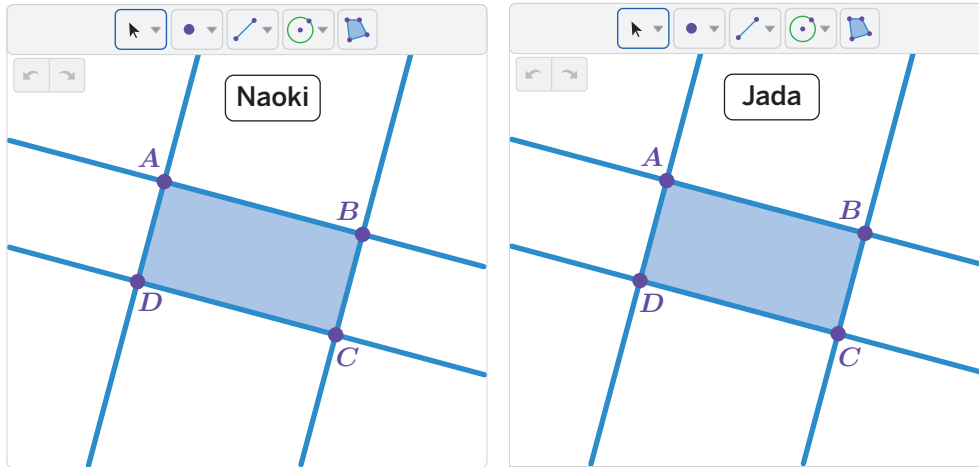
1 You have some new geometry tools!

- a** Use the digital activity to click on the dropdown arrows, and tell a classmate which new tools you notice.

- b** Construct a rectangle on Screen 1.

Square or Not?

- 2** a Use the digital activity to see how Naoki's and Jada's constructions work.



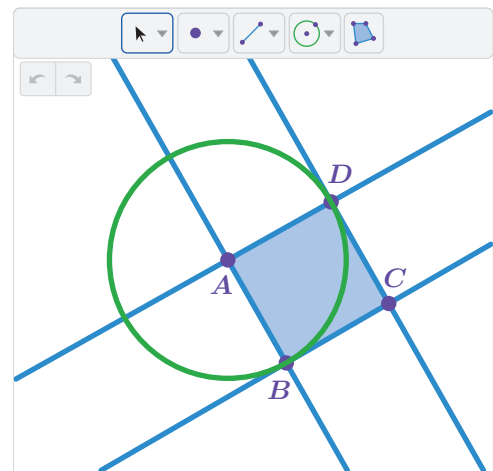
- b** **Discuss:** Which constructions are rectangles? How do you know?

- 3** a Use the digital activity to see how Sam's construction works.

- b** Did Sam construct a square? Circle one.

Yes No I'm not sure

Explain your thinking.



Circles and Squares

You'll use the digital activity for Problems 4–7.

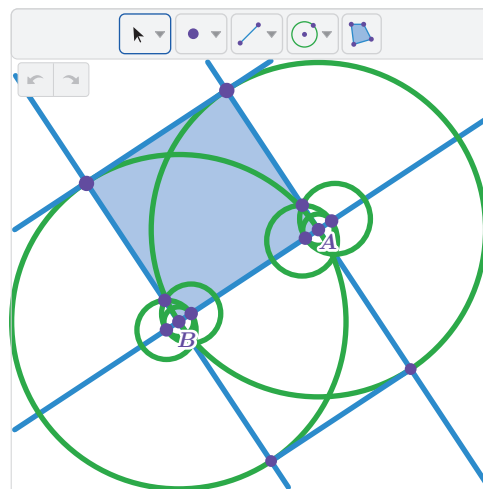
4 **a** Use the digital activity to construct a square that has points A and B as vertices.

b **Discuss:** How many different squares are possible?

5 Dylan constructed a square, but it's hard to see with all the objects used to construct it.


a Use the digital construction tool to hide or delete objects so that the square is easier to see.

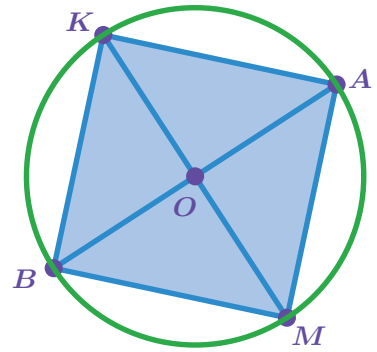
b What is the difference between hiding and deleting an object in a construction?



Circles and Squares (continued)

6 Santiago's design shows a square inscribed in a circle and the square's *diagonals* AB and KM .

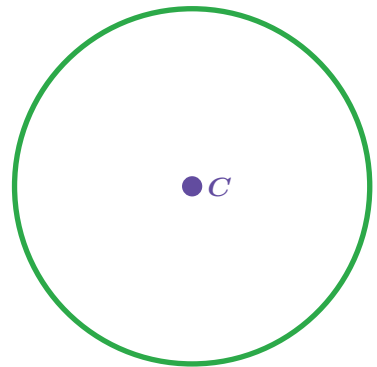
- a** Use the digital activity to recreate Santiago's design.
- b**  **Discuss:** What do you notice and wonder about point O and triangles AOK , AOM , MOB , and BOK ? Use the construction tools in the digital activity to test your thinking.




7 Here is a circle with center C .

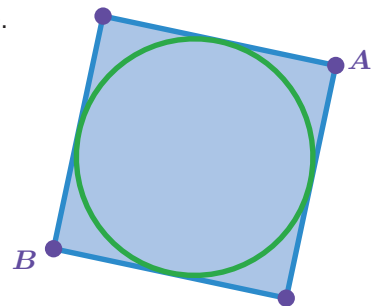
- a** Use the digital activity to inscribe a square in the circle.
- b** Use the construction tools given in the digital activity to verify that you constructed a square. Explain your process.

Hide objects in your construction if it helps with your thinking.



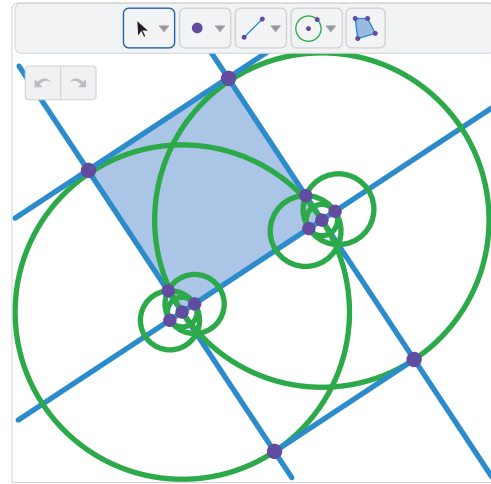
8 Santiago created a new design with a circle inscribed in a square.

- a**  **Discuss:** What do you think it means for a circle to be inscribed in a polygon?
- b** Use the digital activity to recreate Santiago's design.



9 Synthesis

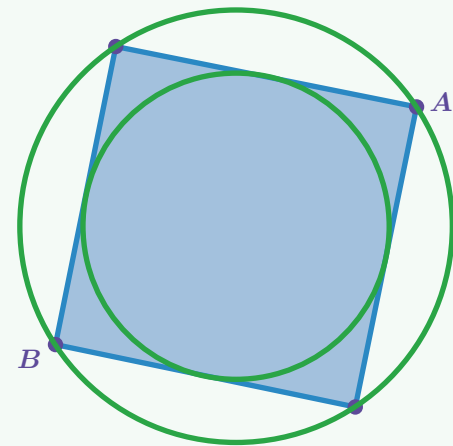
Discuss: How is constructing a square different from sketching one?



10 Summary 6.06

Parallel lines, perpendicular lines, and midpoints are helpful when constructing a square. Midpoints are also helpful when constructing a square inscribed in a circle and a circle inscribed in a square.

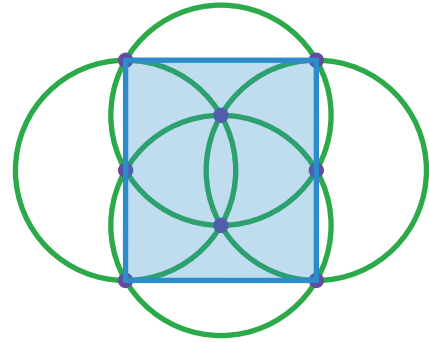
For a circle to be inscribed in a square (or any polygon), the circle must fit inside the square and the circle must intersect each side of the square at a single point. When you construct either a square inscribed in a circle or a circle inscribed in a square, it's helpful to determine midpoints and perpendicular lines to identify the center of the circle.



Practice 6.06

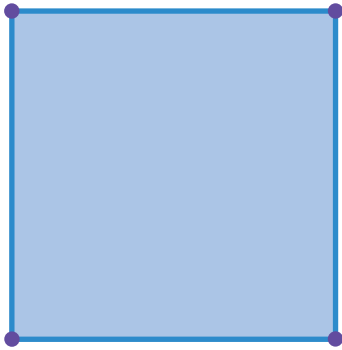
Name: _____ Date: _____ Period: _____

1. Is this construction a square? Explain your thinking.

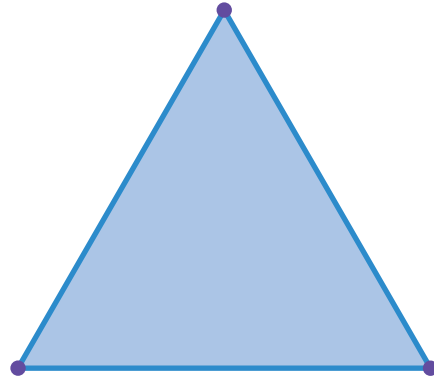


Problems 2–3: Here are regular polygons. Construct a circle so each polygon is inscribed in the circle.

2.



3.



4. Construct a square using segment BC as a side.

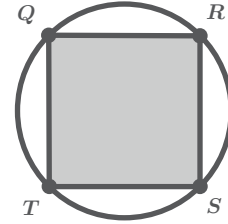
5. Construct a circle with the square from the previous problem inscribed in the circle.



Practice 6.06

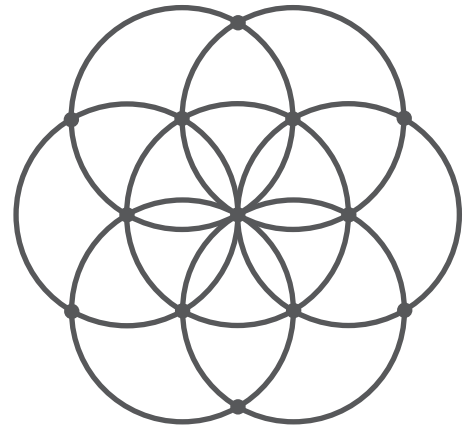
Name: Date: Period:


6. Square $QRST$ is inscribed in a circle. Construct a regular octagon that is inscribed in the circle.

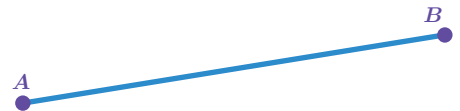


Spiral Review

7. Construct an equilateral triangle using only the intersections of the circles. Explain how you know the triangle is equilateral.



8.  **Test Practice** Explain how to construct a line segment that is half the length of segment AB .



9. Describe the domain and range of the function $f(x) = -2x + 5$.

Domain: Range:

Community Constructions

Let's use constructions to help us make decisions.



Warm-Up

- 1** Hospitals are an important community resource.

In emergency situations, emergency medical services (E.M.S.) teams need to quickly identify the closest hospital.

Use geometry tools to divide the map into regions that are closest to each hospital in Springtown.



- 2** Let's say a resident of Springtown needs to go to the hospital. Which hospital should the E.M.S. team take them to? Circle one.

Hospital 1 Hospital 2 I'm not sure

Explain your thinking.

Mapping Matters

3 Let's watch two videos to see how two students created dividing lines to help them split the map into regions for each hospital.

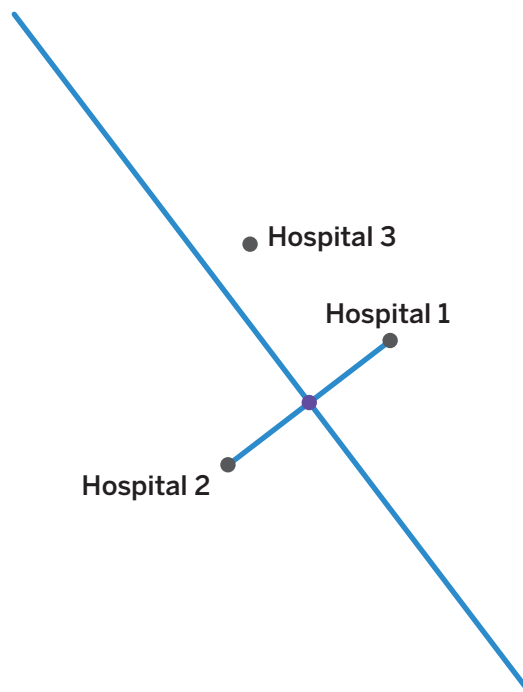
Discuss:

- How did each student create their line?
- Which diagram is more accurate?

4 Springtown is building a third hospital and the E.M.S. team needs to update their map.

Here is one possible location for the third hospital.

Continue Sol's work. Divide the map into three regions, where each region shows all the locations closest to each hospital.



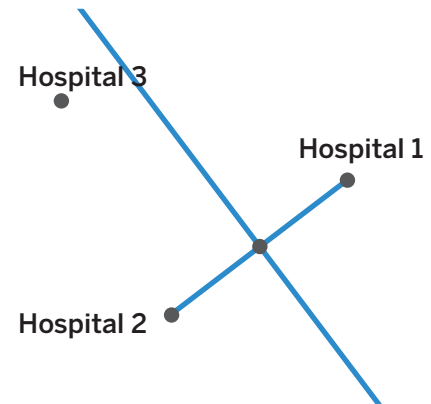
City Construction

Use the digital activity to complete Activity 2.

5 Let's watch a video. The map that Sol created for the E.M.S. team is called a **Voronoi diagram**. Voronoi diagrams use perpendicular bisectors to create different regions. Each region is created based on the closest distance to a given point or object. How would the diagram change if Hospital 3 were in a different location?

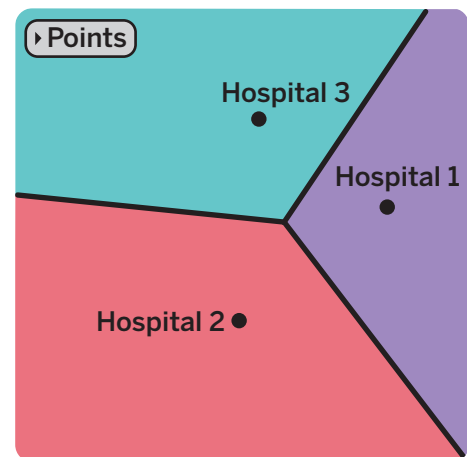
6 Here is another possible location for the third hospital.

Create a Voronoi diagram based on the new location of Hospital 3.



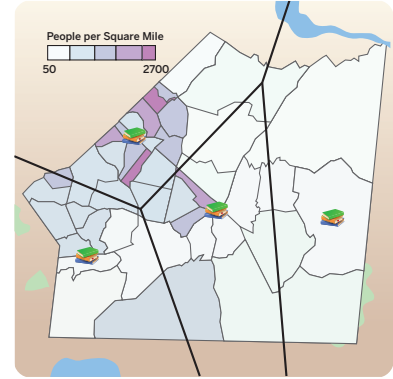
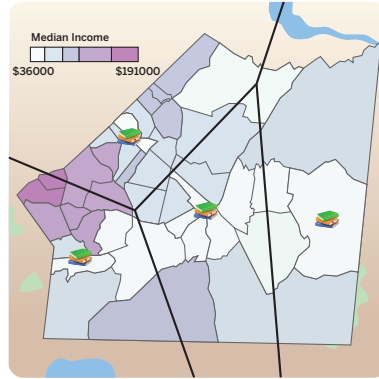
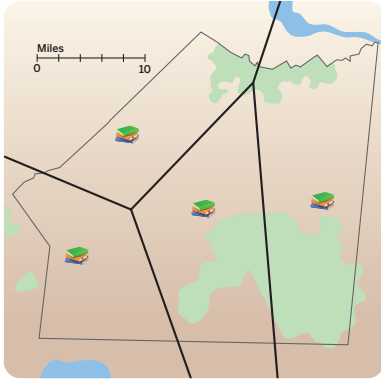
7 Let's think about more complex Voronoi diagrams.

- a** Predict: How will the diagram change if a new point is added?
- b** Let's drag a new point into the diagram.
💬 **Discuss:** What do you notice?



Library Location

8 Libraries are also important community resources. Metropolis County currently has four libraries and wants to build a new one. Here is information about the median income and population in Metropolis County.



What is one thing you notice about these maps? What is one thing you wonder?

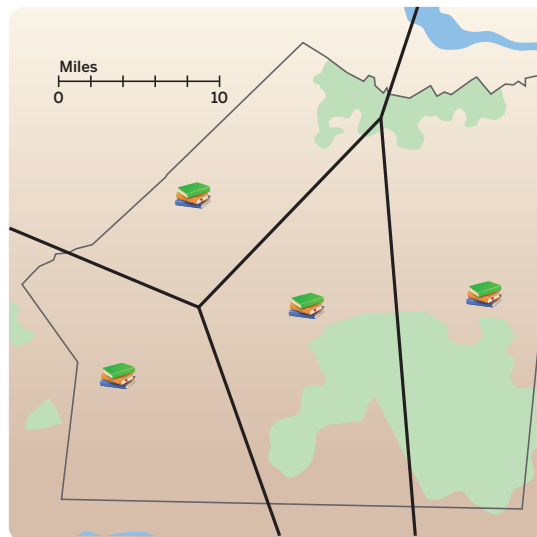
9 Here are some criteria Metropolis might use to help them decide on the new library's location. Order them according to how important they are to you.

- A. Distance from the library to the most number of residents
- B. Distance from the library to public transportation
- C. Distance from the library to low-income communities
- D. Distance from the library to other community resources
- E. Distance from the library to green space

	Most Important
	Least Important

Library Location (continued)

- 10** **a** Plot a point to propose a location for a new library based on your criteria.
- b** Explain why you chose that location.

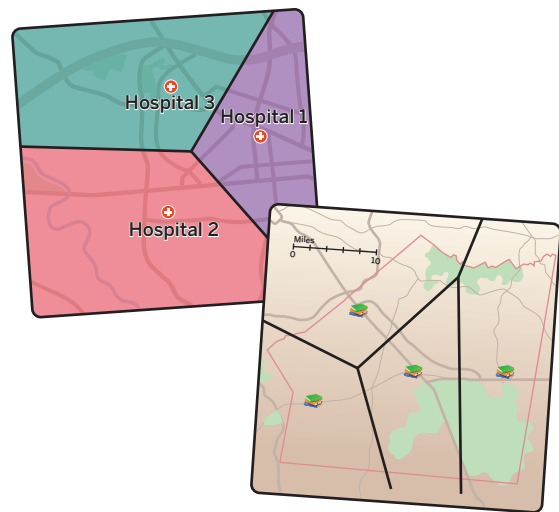
**You're invited to explore more.**

11-12 You will create your own Voronoi diagram.

- a** Think of something you'd like to create a diagram for. It can be for a community resource or something else.
- b** Find and print a map or image. Determine what the points will represent (e.g., hospitals) and create a Voronoi diagram.
- c** After you create your diagram, think about:
- Who might be interested in this information?
 - How would you describe what your Voronoi diagram shows?
 - What other information could you research in order to revise your diagram?

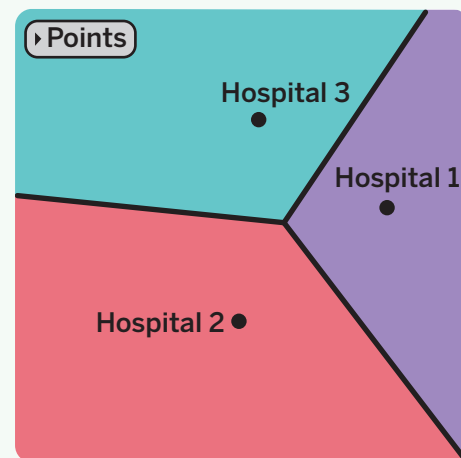
13 Synthesis

Describe how **Voronoi diagrams** can be a tool for making decisions.



16 Summary 6.07

You can use constructions to solve real-world problems. **Voronoi diagrams** are constructed with perpendicular bisectors and are tools that help us make decisions in community planning. They provide evidence to support where specific services might be located and how these locations might impact a community.



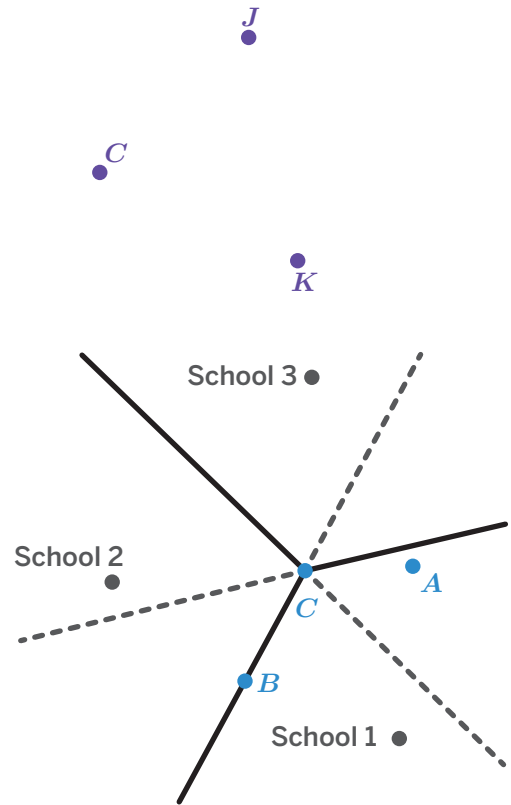
Voronoi diagram A diagram that uses perpendicular bisectors to create different regions based on the closest distance to a given point or object.


Practice 6.07

Name: _____ Date: _____ Period: _____

- How can you use geometry tools to determine if point C is closer to point J or point K ?

Use the diagram if it helps to show your thinking.



-  **Test Practice** This is a Voronoi diagram based on three different schools. Select *all* the statements that are true.

- A. School 1 is the closest school to Location A .
- B. School 3 is the farthest school from Location A .
- C. Location B is equidistant from Schools 2 and 3.
- D. Location B is closer to School 1 than to School 2.
- E. Location C is equidistant from Schools 2 and 3.

Problems 3–4: Points A , B , and C in the diagram represent the homes of three students who need to meet after school for a project.



- Plot a point where you think the students located at point A and point B should meet. Explain your thinking.
- Plot a point at a location that is an equal distance from all three students' homes. Show or explain your thinking.

Practice 6.07

Name: _____ Date: _____ Period: _____

5. Create a Voronoi diagram based on these three points. Show your thinking.

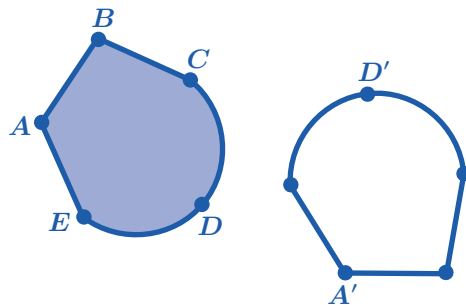


6. What are ways Voronoi diagrams could be used in your community to help make decisions?

Spiral Review

Problems 7–8: The unshaded figure is a rotation of figure $ABCDE$.

7. Label the points corresponding to B , C , and E as B' , C' , and E' .
8. Construct segment AD and segment $A'D'$. What is the relationship between the lengths of these two line segments?



Practice Day 1



Let's practice what you've learned so far in this unit!

You will use task cards for this Practice Day. Record all of your responses here.

Task A: Equilateral Triangle

1. Statement 1:

Statement 2:

Statement 3:

2. Circle one: Yes No Not enough information

Explain your thinking.

3. Equilateral triangles:

Task B: Square It

1.

2. Two parallel sides:

3. Two perpendicular sides:

Practice Day 1 (continued)

Task C: Statements and Lines

1. Lines or segments:

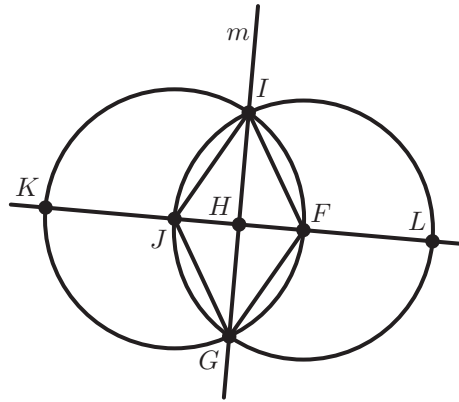
2. **a** Triangle _____ is equilateral.
- b** Points _____, _____, and _____ are equidistant from point J .
- c** Line m bisects segment _____.

3. Statement:

4.

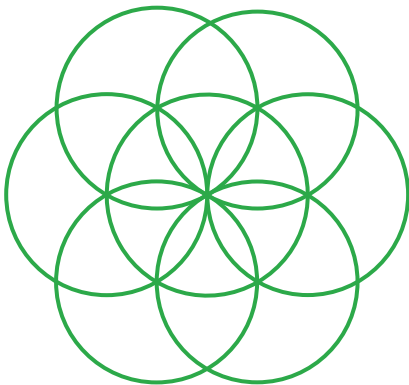
.....

5.



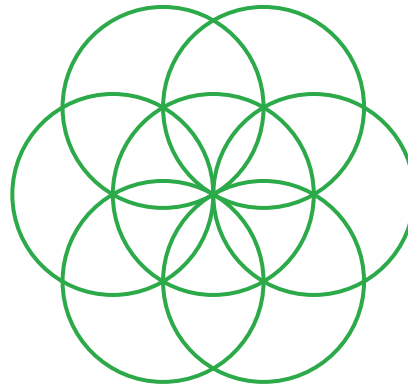
Task D: Inscribing

1. **a**



b

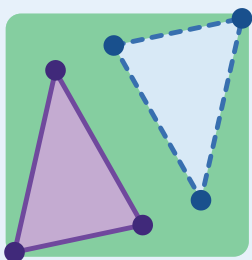
2. **a**



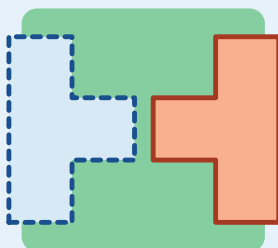
b

Notes:

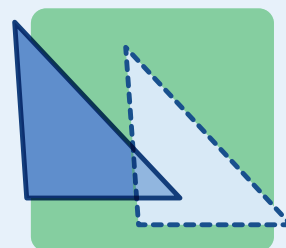
Transformations



Lesson 8
Going Off the
Grid, Part 1



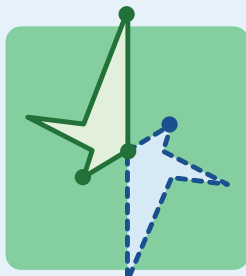
Lesson 9
Time to Reflect



Lesson 10
Translation
Information



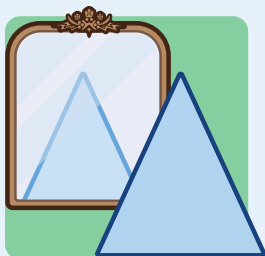
Lesson 11
Rotation Devices



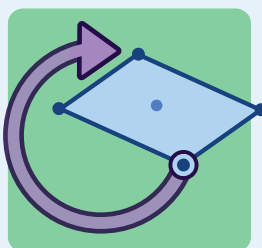
Lesson 12
Going Off the Grid,
Part 2



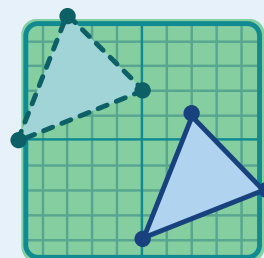
Lesson 13
Tiles and
Patterns



Lesson 14
Reflection Symmetry



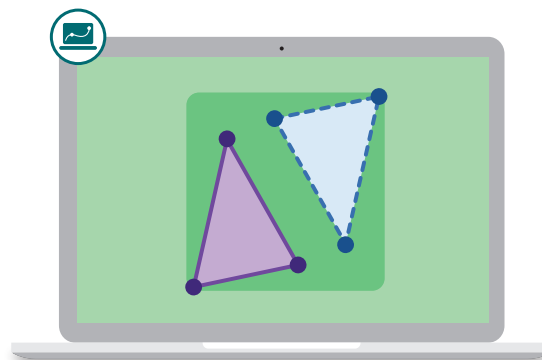
Lesson 15
Rotational Symmetry



Lesson 16
Transformations
as Functions

Going Off the Grid, Part 1

Let's explore how we can describe sliding, flipping, and spinning shapes.



Warm-Up

1 **Discuss:** How did the original figure transform into figures A, B, C, and D?

Original

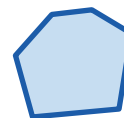


Figure A

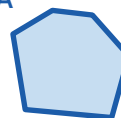


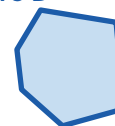
Figure B



Figure C



Figure D



2 A *rigid transformation* is a transformation that doesn't change the side lengths or angle measures of a figure.

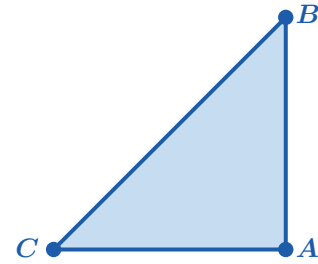
Select *all* the figures that show a rigid transformation of the original figure.

- A. Figure A
- B. Figure B
- C. Figure C
- D. Figure D

Tracing and Placing

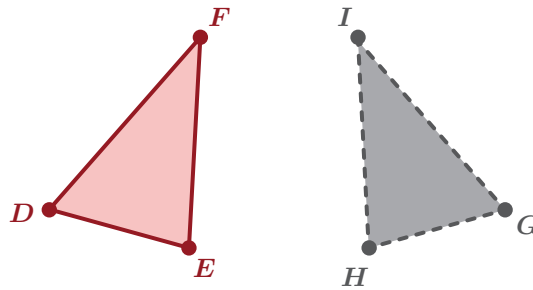
3 Discuss:

- What do you know about *translations*, *rotations*, and *reflections*?
- How can tracing paper help you translate, rotate, or reflect a figure?

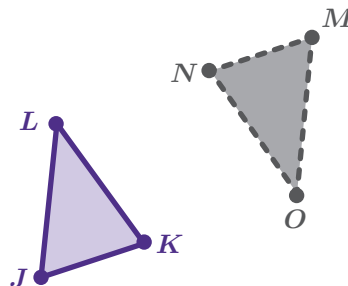


4 Translations, reflections, and rotations are all rigid transformations that you can use to transform one figure onto another.

Use tracing paper to transform triangle DEF onto triangle GHI .



5 a Use tracing paper to transform $\triangle JKL$ onto $\triangle MNO$.

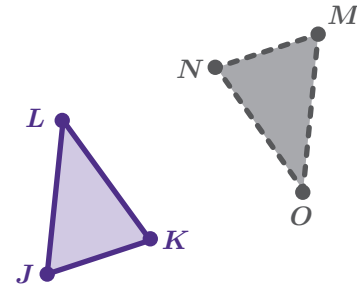


- b Describe the transformations you performed.

Tracing and Placing (continued)

6 Here is how some students described transforming $\triangle JKL$ onto $\triangle MNO$.

- Rotate $\triangle JKL$. Then translate it over a lot and up a little.
- Translate $\triangle JKL$ so that point L moves onto point O . Then rotate around point O so that the image of point J moves onto point M .
- Move $\triangle JKL$ so that it's on top of $\triangle MNO$.
- Translate $\triangle JKL$ right and up. Then rotate it so it's flipped upside down.



Order the descriptions from *most helpful* to *least helpful*.

	Most Helpful
	Least Helpful

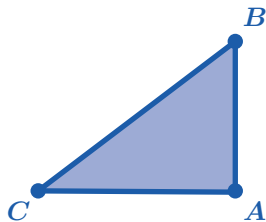
Trace and Transform

- 7** When a figure is transformed, the vertices of the *image* can be labeled using **prime notation**. The image of $\triangle ABC$ after one transformation is $\triangle A'B'C'$. After two transformations the image is $\triangle A''B''C''$.

Use tracing paper to transform $\triangle ABC$ based on this description:

Reflect $\triangle ABC$ vertically, translate it up, and then rotate it a quarter turn clockwise.

Then record the location of $\triangle A'''B'''C'''$ on this page.



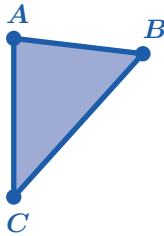
Trace and Transform (continued)

8 You will use blank paper to create your own transformation challenge.

a Make It!

- Start with $\triangle ABC$. Create $\triangle XYZ$ that is congruent to $\triangle ABC$ in this box.
- On a separate sheet of paper, describe the transformation(s) that take(s) $\triangle ABC$ onto $\triangle XYZ$.

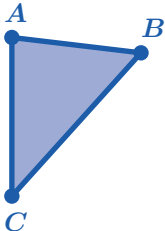
Your Transformation



b Swap It! Swap your description with one or more partners and keep your transformation(s) a secret. Determine the location of $\triangle XYZ$ using the transformation(s) your classmate described. Then record the location of the triangle.

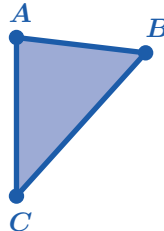
Partner 1's Transformation

Transformation



Partner 2's Transformation

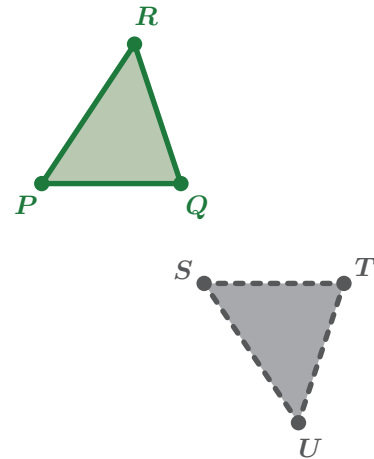
Transformation



9 Synthesis

Discuss: How do you know when to use translations, rotations, and reflections when transforming a figure onto another figure?

Use $\triangle PRQ$ and $\triangle SUT$ if they help with your thinking.



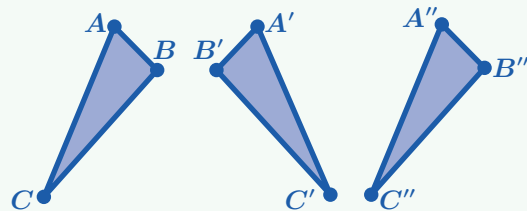
12 Summary 6.08

You can use tracing paper to perform *rigid transformations*, which don't change the side lengths or angle measures of a figure. With tracing paper, you can perform:

- *Translations*, which slide the figure up, down, left, or right.
- *Reflections*, which flip the figure.
- *Rotations*, which spin the figure.

Using tracing paper can help you make sense of how to use rigid transformations to move one figure on top of another. Sometimes you only need one transformation, but other times you may need to use multiple transformations to perfectly align one figure to another.

When performing one or more transformations on a figure, also known as a *pre-image*, it's helpful to label the vertices of the *image* with **prime notation**, like A' or B' , which are read out loud as *A prime* or *B prime*. One prime symbol is used for every transformation that is performed on a pre-image.



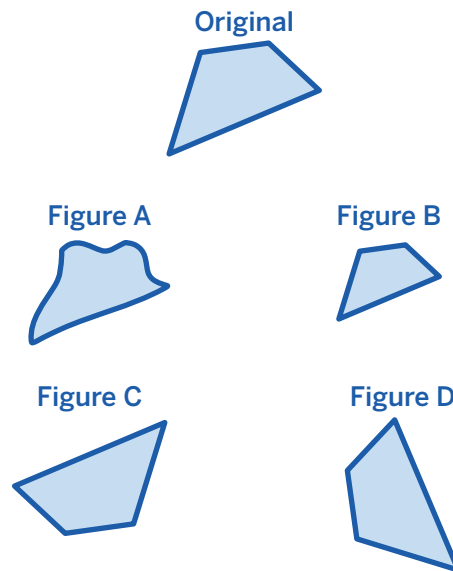
prime notation Using the prime symbol (') to name points in an image that correspond to points in the pre-image.

Practice 6.08

Name: _____ Date: _____ Period: _____

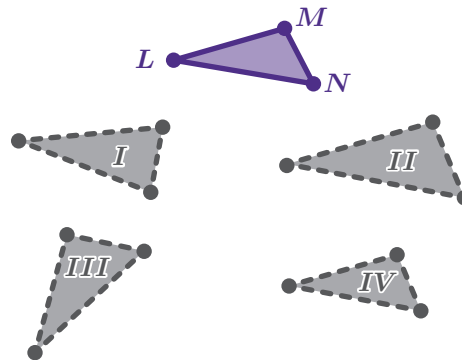
1. Select *all* the figures that show a rigid transformation of the original figure.

- A. Figure A
- B. Figure B
- C. Figure C
- D. Figure D



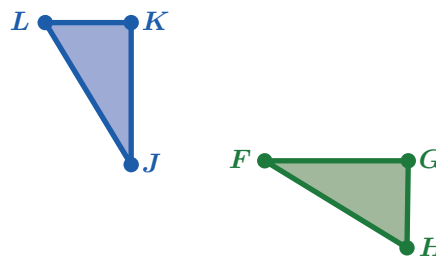
2. Which triangle could be the result of a single translation of $\triangle LMN$?

- A. *I*
- B. *II*
- C. *III*
- D. *IV*



Problems 3–4: Here are two triangles.


3. Describe a sequence of rigid transformations that transforms $\triangle FGH$ onto $\triangle JKL$.



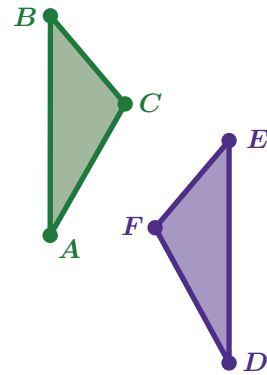
4. Describe a *different* sequence of rigid transformations that transforms $\triangle FGH$ onto $\triangle JKL$.

Practice 6.08

Name: _____ Date: _____ Period: _____

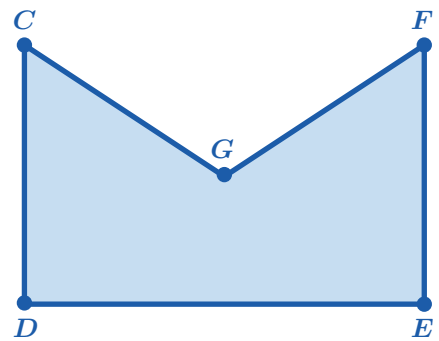
5.  **Test Practice** Select *all* the sequence(s) of transformations that could transform $\triangle ABC$ onto $\triangle DEF$.

- A. A rotation and a reflection
- B. A translation
- C. A reflection and a translation
- D. A rotation and a translation
- E. A reflection



6. This polygon was made by overlapping only a triangle and its reflection.

Name one of the triangles.



Spiral Review

Problems 7–8: Here is \overline{JK} .

7. Construct the perpendicular bisector of \overline{JK} .



8. Which point is closer to the perpendicular bisector you just constructed? Circle one.

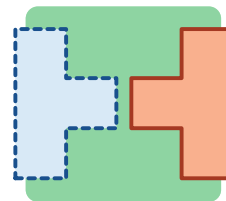
Point J

Point K

They're the same distance from the perpendicular bisector.

Time to Reflect

Let's get precise about reflections.

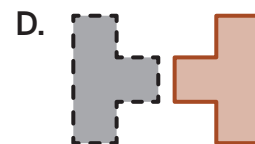
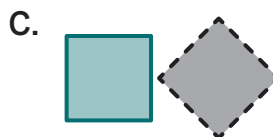
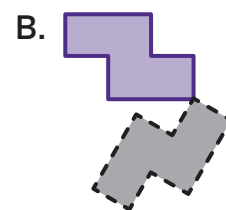
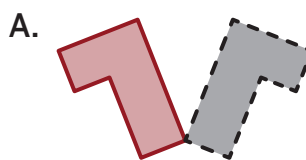


Warm-Up

1. Here are four polygons (in color) and their images (in gray) after transformations.

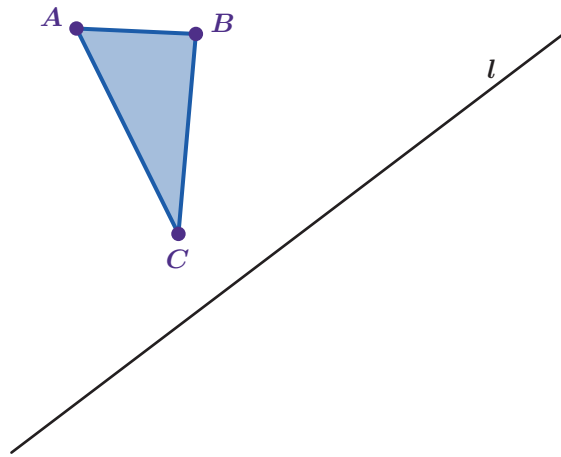
Which of these *must* include a reflection?

Explain your thinking.

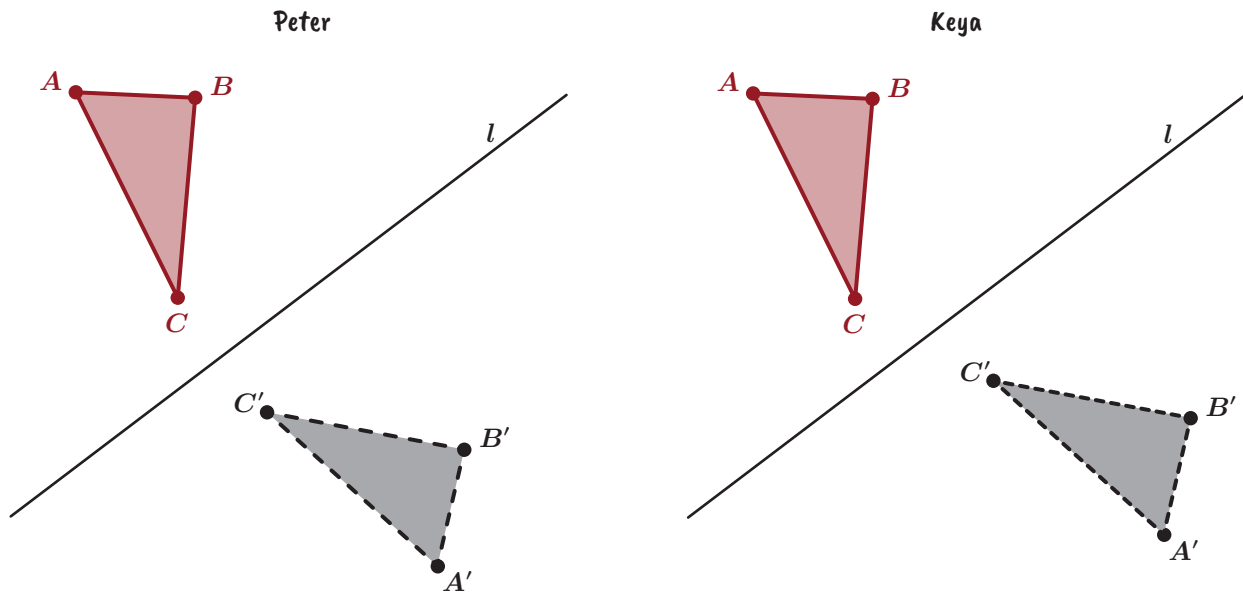


Accurate Reflections

2. Use tracing paper to reflect the pre-image, $\triangle ABC$, over line l . Label the image $\triangle A'B'C'$.



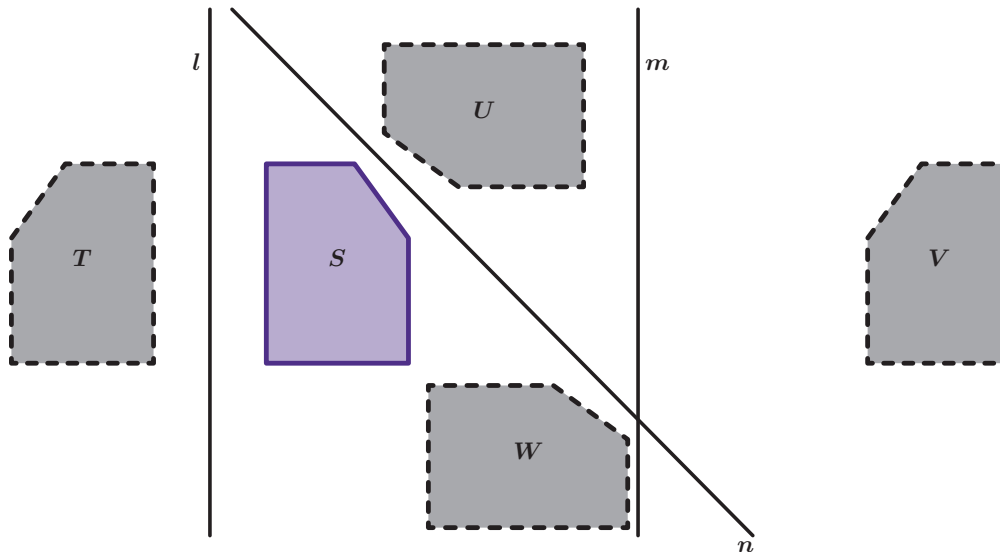
3. Peter and Keya reflected $\triangle ABC$ over line l .



Which student's reflection is more accurate? Explain your thinking.

Reflecting Pentagons

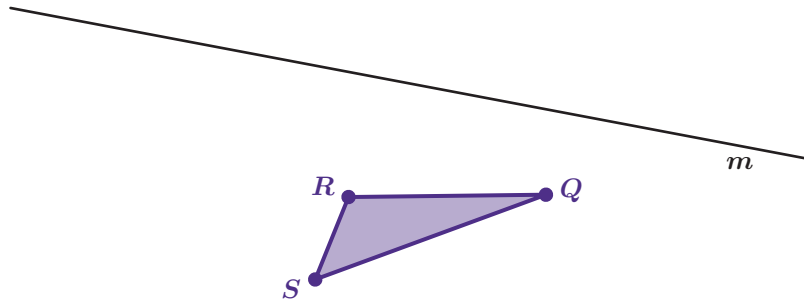
This diagram shows four reflections of pentagon S .




4. Determine which reflected image completes each statement.
- Pentagon S reflects over line l to image
 - Pentagon S reflects over line m to image
 - Pentagon S reflects over line n to image
5. There is one reflection of pentagon S whose line of reflection is missing.
- Which image is it?
 - Use any tool to construct the line of reflection for that pentagon.
6. Wohali says: *If you connect a point and its reflection using a line segment, then the line segment is perpendicular to the line of reflection.*
- Is Wohali's statement *always*, *sometimes*, or *never* true? Circle one.
 Always true Sometimes true Never true
 Explain your thinking.
 - Describe how to construct a line of reflection between a point and its reflected image.

Encompassing Reflections

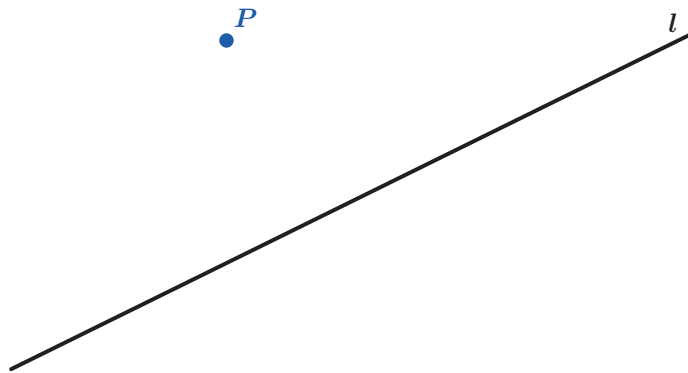
7. Given $\triangle QRS$ and line m :




- a** Reflect $\triangle QRS$ over line m . Label the vertices Q' , R' , and S' .
- b**  **Discuss:** Describe what happens to the pre-image, $\triangle QRS$, when it is reflected over line m .

Encompassing Reflections (continued)

8. Reflect point P over line l using a compass and straightedge. Label the image P' .

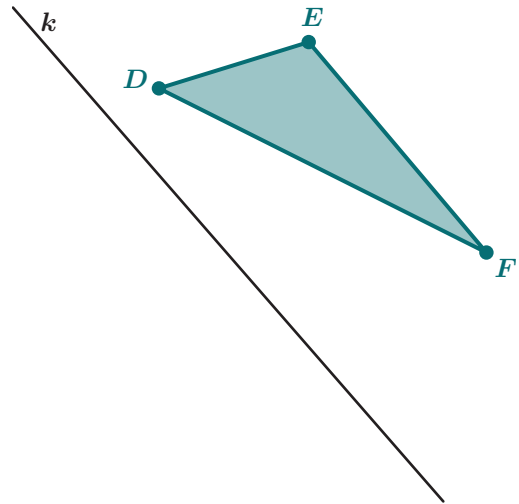


9.  **Discuss:** Describe your strategy for reflecting a point over a line using a compass and straightedge.

Synthesis

Discuss: What would you tell a classmate who was absent about the relationship between a reflected image, its pre-image, and the line of reflection that created it?

Use triangle DEF and any tools you'd like if they help with your thinking.

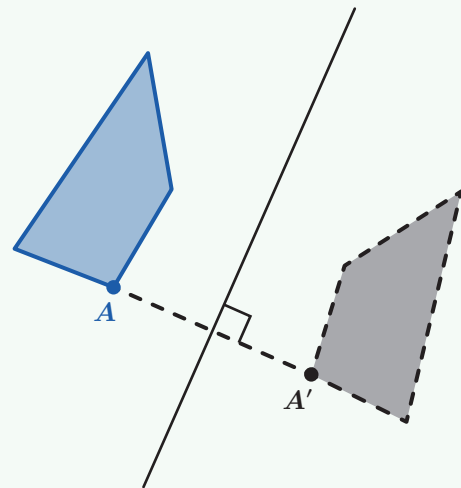


Summary 6.09

You used tracing paper to explore the relationship between a **line of reflection** and the reflected image. By doing so, you developed a precise definition for reflections.

A reflection is a rigid transformation that moves every point on a figure to a point directly on the opposite side of the line of reflection.

- As a rigid transformation, the reflected image will be the same size and shape as the pre-image.
- The image point is the same distance from the line of reflection as its corresponding pre-image point.
- If you connect a point to its reflected image point using a line segment, the line segment will be perpendicular to the line of reflection.
- The orientation of the vertices is not preserved.

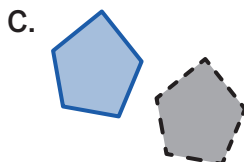
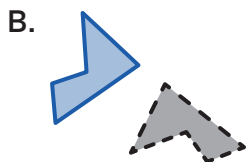
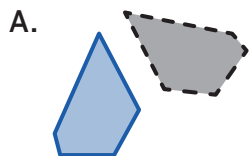


line of reflection The line across which reflections are performed. When a point is reflected, the new point (or image) is the same distance from the line as the original (or pre-image).

Practice 6.09

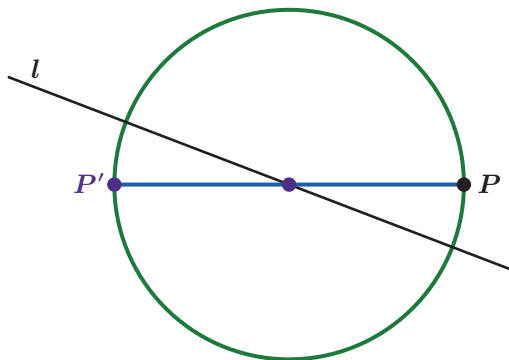
Name: _____ Date: _____ Period: _____

1. Which of the transformed images *must* include a reflection?



Problems 2–3: Hailey tried to reflect point P over line l .

2. What is one thing Hailey did correctly?



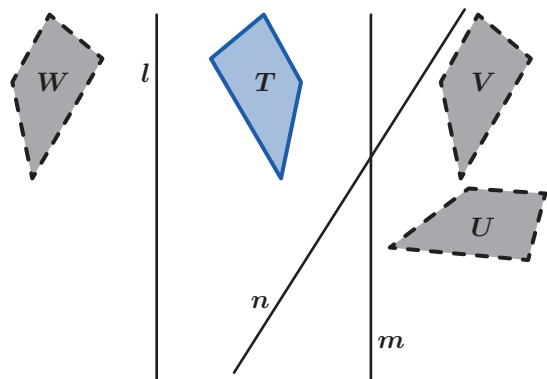
3. How would you improve Hailey's reflection?

Problems 4–6: **Test Practice** Determine which reflected image completes each statement.

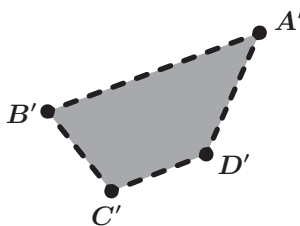
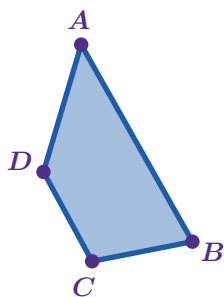
4. Trapezoid T reflects over line l to image _____.

5. Trapezoid T reflects over line m to image _____.

6. Trapezoid T reflects over line n to image _____.



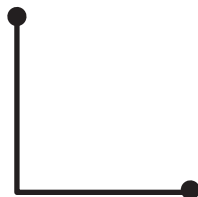
7. Construct the line of reflection.



Practice 6.09

Name: _____ Date: _____ Period: _____

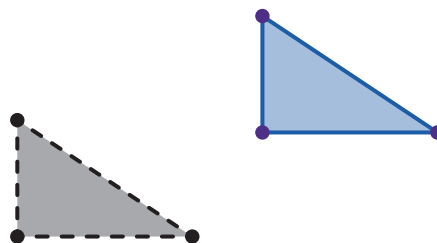
8. Construct a line of reflection that transforms this figure onto itself.



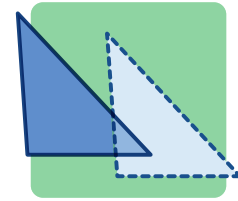
Spiral Review

9. If circle O is reflected over line l and every point on circle O is taken onto another point on circle O , what must be true about line l ?
10. Select *all* the statements that are always true about an image after a sequence of rigid transformations.
- A. The angle measures of the image and pre-image are the same.
 - B. The orientation of the image and the pre-image are the same.
 - C. The side lengths of the image and the pre-image are the same.
 - D. The side lengths of the image are smaller than in the pre-image.
 - E. The side lengths of the image are greater than in the pre-image.
11. Which type of single transformation is shown?

- A. Translation
- B. Reflection
- C. Rotation
- D. Dilation



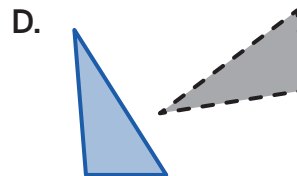
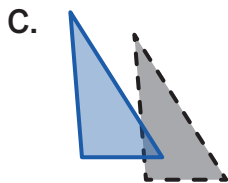
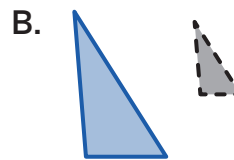
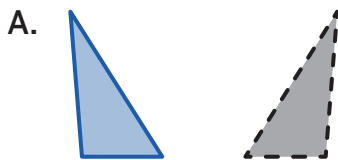
Translation Information



Let's get precise about translations.

Warm-Up

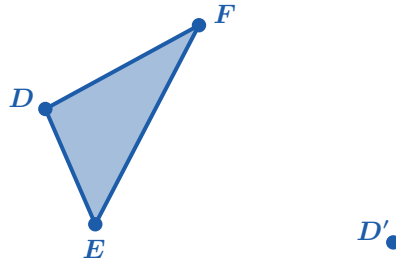
1. Which one doesn't belong?



Explain your thinking.

Translate This!

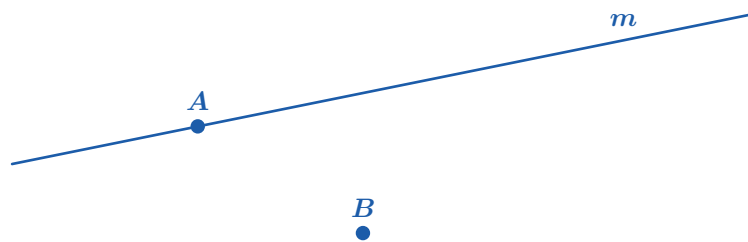
2. Translate $\triangle DEF$ so that point D goes to point D' . Label the vertices in the image using prime notation.



3. Nakia says that you can tell one figure is a *translation* of another figure because they are both the same size, but in different places. Is Nakia's thinking correct?

Explain your thinking.

4. Translate line m so that point A goes to point B .



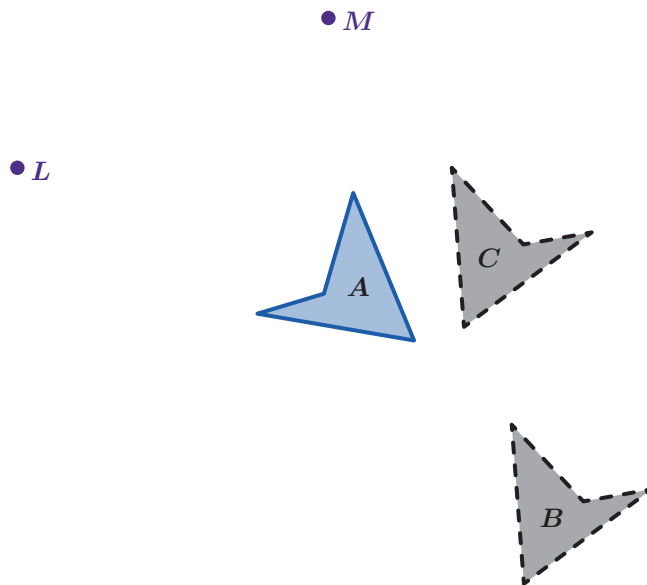
5. Nakia says: *When a line is translated, the line and its image are parallel.* Is this statement *always*, *sometimes*, or *never* true? Circle one.

Always Sometimes Never

Explain your thinking.

Translate This! (continued)

Here is quadrilateral A and some transformations.



- Which of these transformed quadrilaterals could be the result of a *reflection* only?
- Which of these quadrilaterals could *not* be the result of a single reflection?
- Translate quadrilateral A using the same translation that moves point L onto point M .

Transformation Communication

Your partner will describe a transformation for each polygon. Construct the result.

9. Card:



F ● ● E

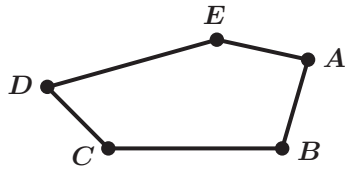
10. Card:



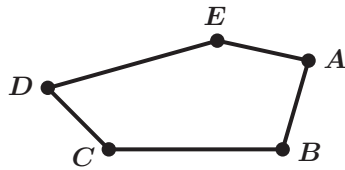
F ● ● E

Transformation Communication (continued)


11. Card:



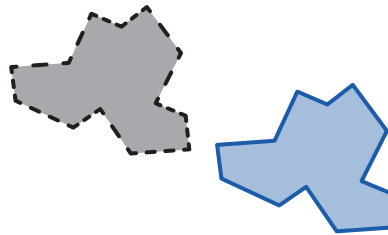
12. Card:



Synthesis

13.  **Discuss:** What information is important to include when describing a translation?

Use the figures if they help with your thinking.

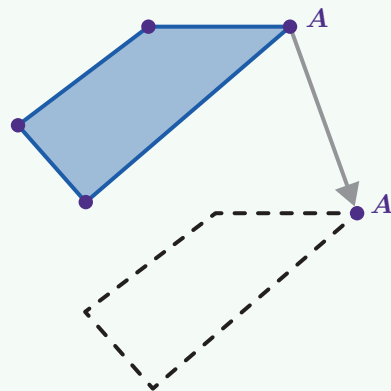


Summary 6.10

You developed a precise definition for translation.

A *translation* is a type of rigid transformation that moves every point of the figure the same distance in the same direction.


- The movement is defined by two points — a starting point and an ending point.
- When translating a line or line segment, the image is parallel to the pre-image.
- The orientation of the vertices is preserved.



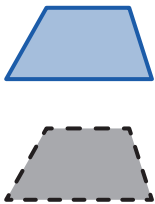
Practice

6.10

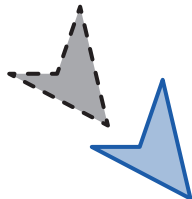
Name: _____ Date: _____ Period: _____

1.  **Test Practice** Which one of these transformations cannot represent only a single translation?

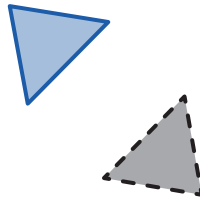
A.



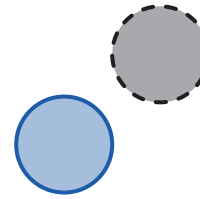
B.



C.

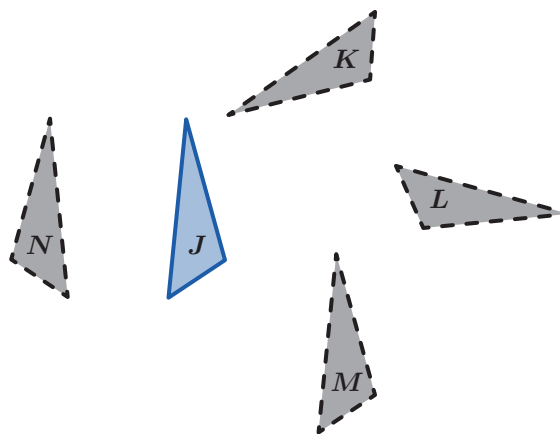


D.



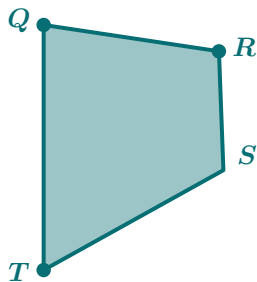
Problems 2–5: Here are four transformations of triangle J . Which of these images is the result of:

2. A single *translation* only?
3. A single *reflection* only?
4. *Both* a single translation and a single reflection?
5. A single transformation that is not a translation or reflection?

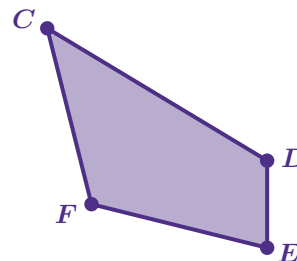


Problems 6–7: Translate each quadrilateral so that:

6. Point Q goes to point S .



7. Point F goes to point F' .

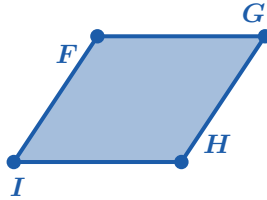


F'

Practice 6.10

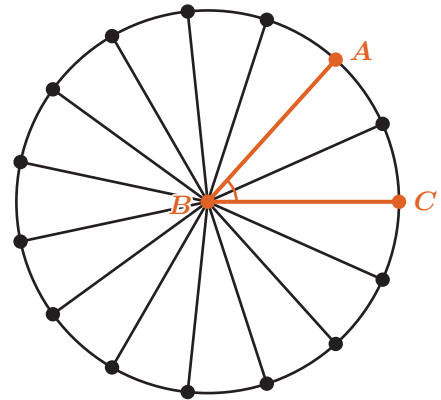
Name: _____ Date: _____ Period: _____

8. Construct translations of parallelogram $FGHI$ to create a final design that looks the same when rotated but not reflected.

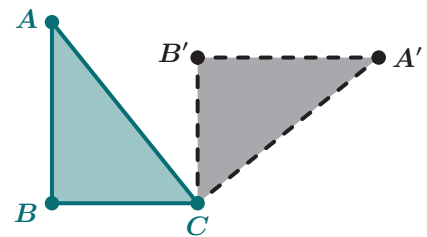


Spiral Review

9. A circle is divided into 15 equal pieces. What is the measure of angle ABC in degrees?



10. Triangle ABC was rotated 270° counterclockwise. If angle $ABC = 90^\circ$, what is the measure of angle $A'B'C'$?



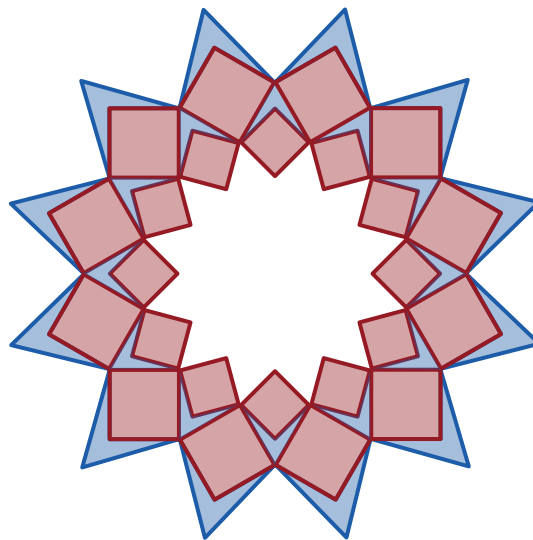
Rotation Devices

Let's get precise about rotations.



Warm-Up

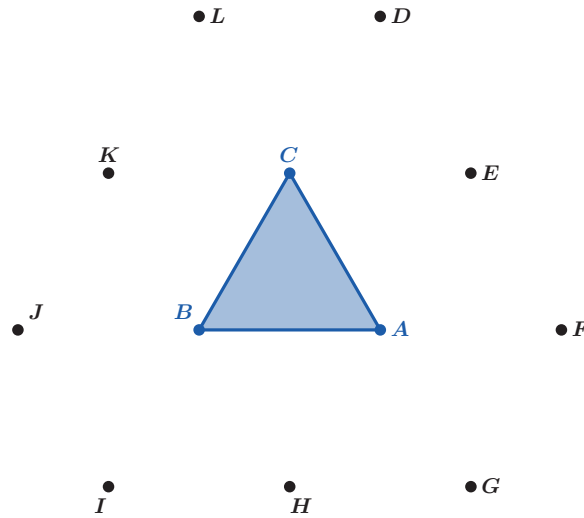
Let's watch how this figure was created.



1. Pick one *rotation* you noticed and describe it.

Equilateral Images

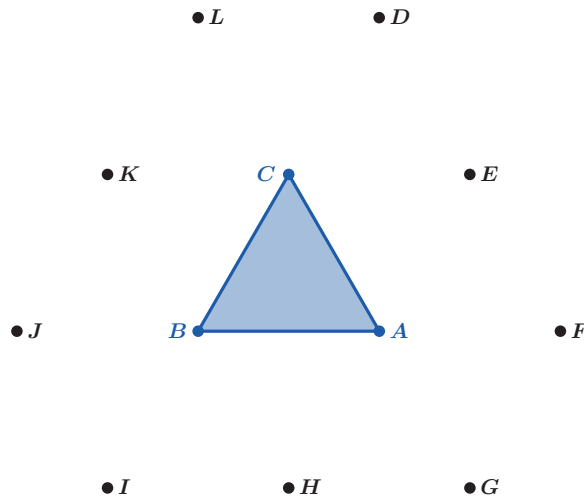
2. Rotate $\triangle ABC$ around point A , point B , or point C .



What's the relationship between the *center of rotation*, a point, and that point's image after the rotation?

3. Write a first draft of a description of your rotation.

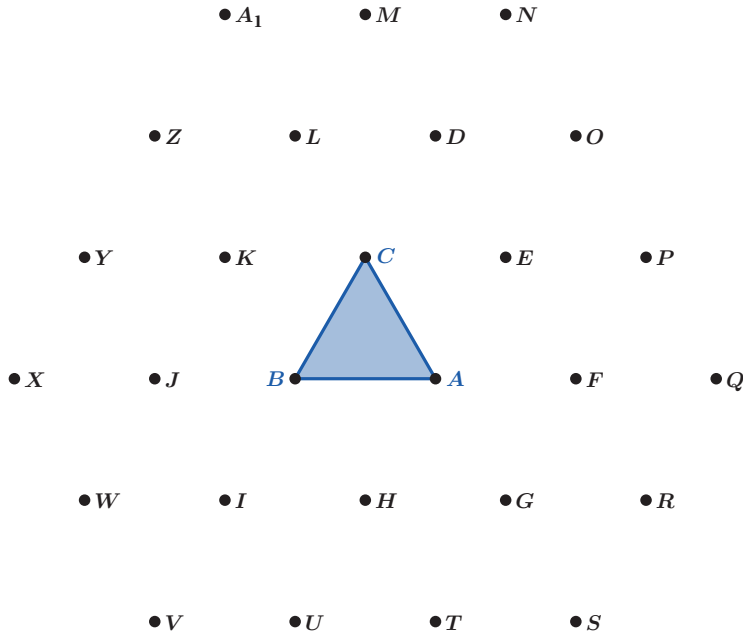
4. Meet with a partner and read your description. Do not show them your paper! Perform the rotation that matches your partner's description.



5. Write a second draft that is stronger and clearer.

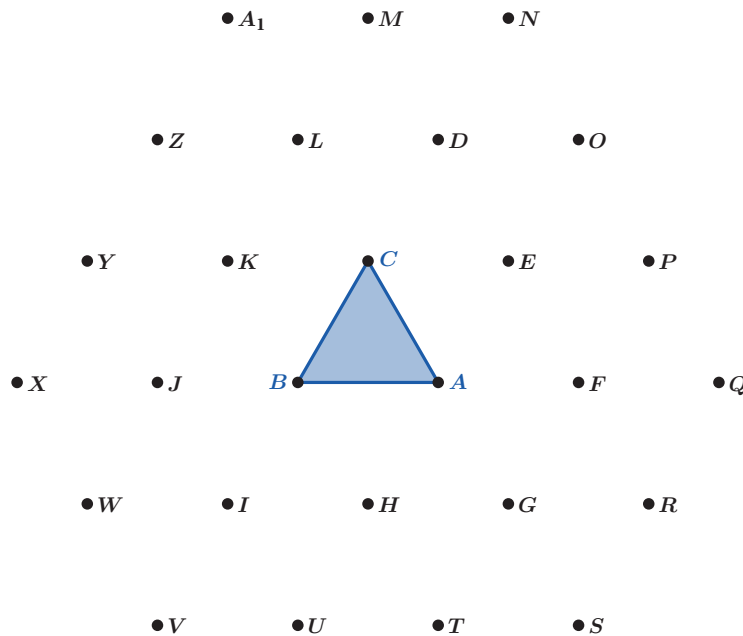
Equilateral Images (continued)

6. Rotate $\triangle ABC$ around any point except points A , B , or C . Make sure every vertex in the image is on a labeled point.



7. Describe your rotation.

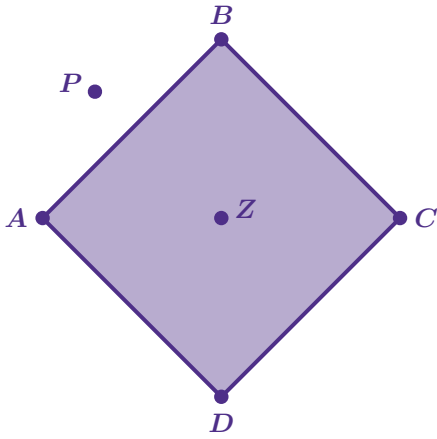
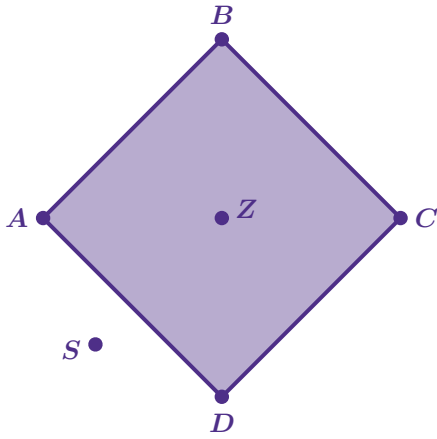
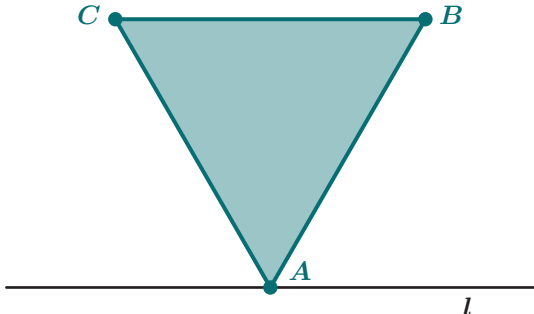
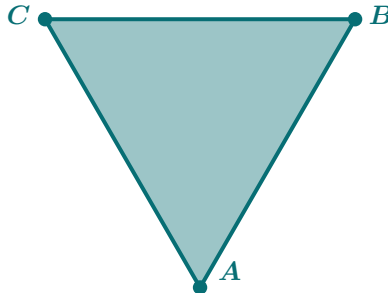
8. Meet with a partner and read your description. Do not show them your paper! Perform the rotation that matches your partner's description.



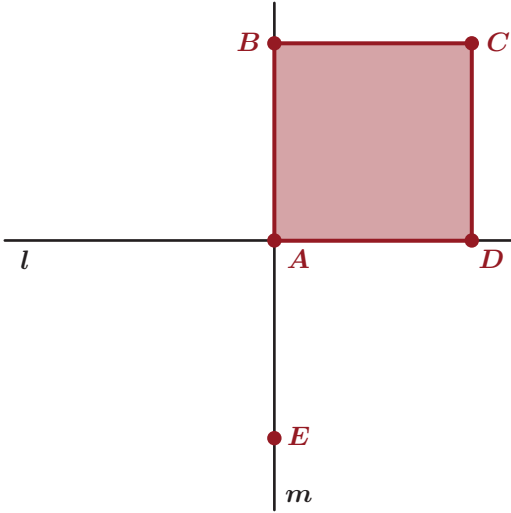
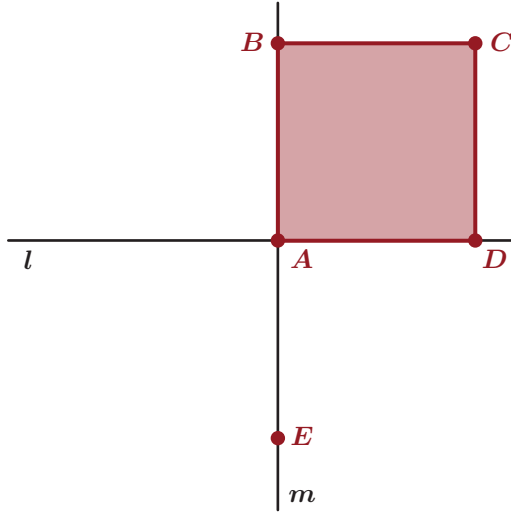
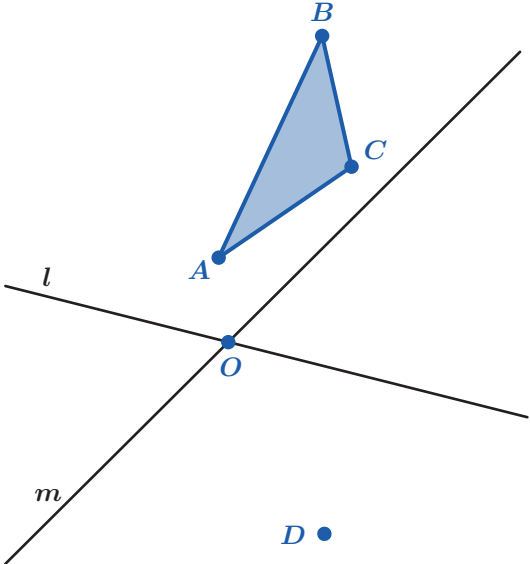
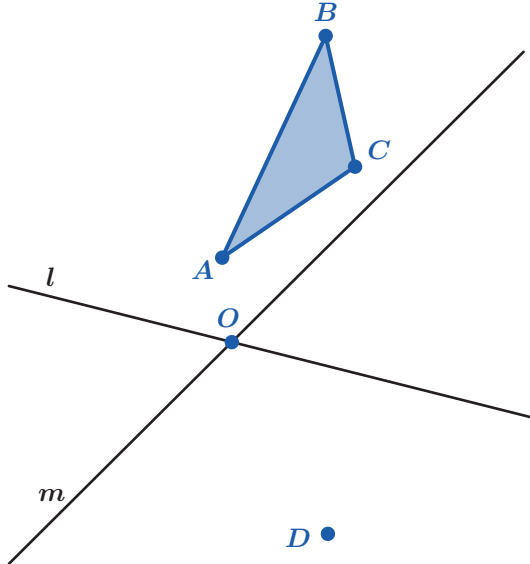
Partner Problems

Decide with a partner who will complete Column A and who will complete Column B.

After each problem, compare your images. Discuss whether the final transformed figures are in the same location.

Column A	Column B
<p>9. Rotate square $ABCD$ clockwise around point Z by $\angle AZP$ so that point A goes to point P.</p> 	<p>Rotate square $ABCD$ clockwise around point Z by $\angle DZS$ so that point D goes to point S.</p> 
<p>10. Reflect $\triangle ABC$ over line l.</p>  <p>D •</p>	<p>Rotate $\triangle ABC$ counterclockwise around point A by $\angle BAD$.</p>  <p>D •</p>

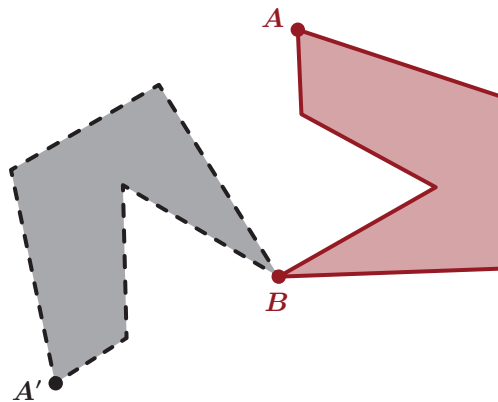
Partner Problems (continued)

Column A	Column B
<p>11. Rotate quadrilateral $ABCD$ <i>counterclockwise</i> around point A by $\angle BAE$.</p> 	<p>Reflect quadrilateral $ABCD$ over line m, then over line l.</p> 
<p>12. Reflect $\triangle ABC$ over line m, then over line l.</p> 	<p>Rotate $\triangle ABC$ <i>clockwise</i> around point O by $\angle COD$.</p> 

Synthesis

14. What information is important to include when describing a rotation?

Use the figures if they help with your thinking.



Summary 6.11

You used circles to visualize the path of a point during a rotation and developed a precise definition for rotations.

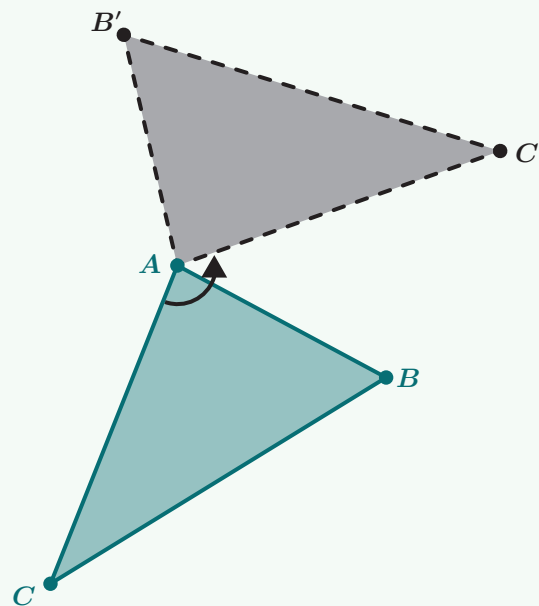
A *rotation* is a rigid transformation that moves all of the points in a figure around a given point (*center of rotation*), by a certain angle and in a specific direction (*clockwise* or *counterclockwise*).

- The pre-image points and their images are always the same distance from the center of rotation.
- The orientation of the vertices is preserved.

We can describe the rotation shown by saying:


- Rotate triangle ABC counterclockwise around point A by $\angle CAC'$.

One way to identify and name the angle of rotation is by identifying corresponding points and connecting them to the center of rotation. In this example, the angle of rotation can be identified as $\angle CAC'$ or $\angle BAB'$. Notice the center of rotation, A , is the vertex of the angle of rotation.

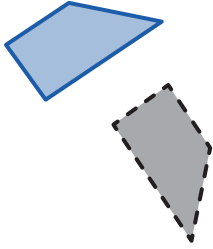


Practice 6.11

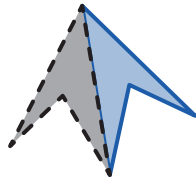
Name: _____ Date: _____ Period: _____

1.  **Test Practice** Which of these transformations could *not* represent a single rotation?

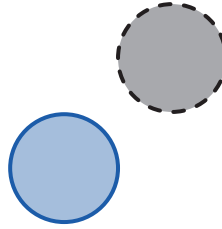
A.



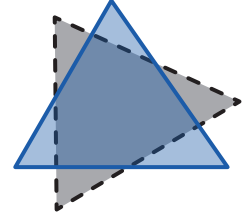
B.



C.

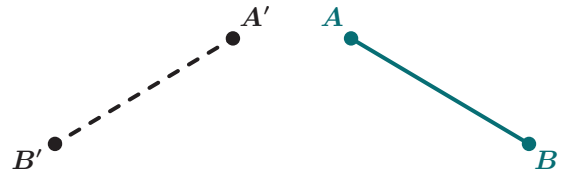


D.



Problems 2–3: Rudra tried to rotate \overline{AB} around point Z .

2. What is one thing Rudra did well?

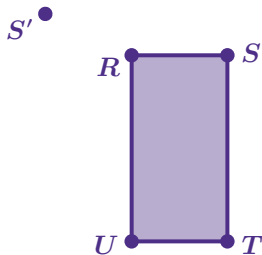


3. How could Rudra improve his rotation?

• Z

Problems 4–5: Rotate each quadrilateral around point R so that point S goes to S' . Identify the direction and angle of rotation.

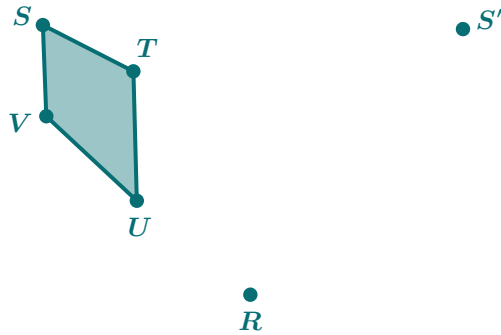
4.



Direction:

Angle of rotation:

5.



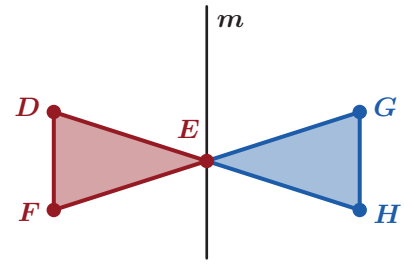
Direction:

Angle of rotation:

Practice 6.11

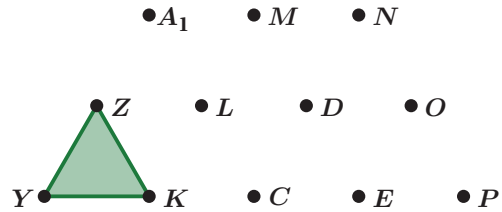
Name: _____ Date: _____ Period: _____

6. Describe two different single transformations that move one of these triangles onto the other.



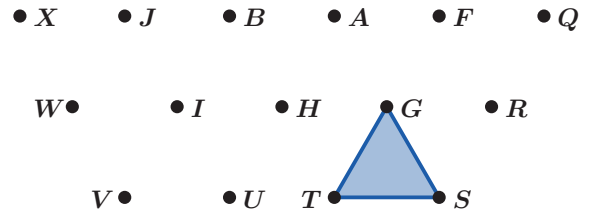
Problems 7–8: Use the grid.

7. Rotate each triangle *exactly once* to create the outline of a parallelogram.



8. Circle the direction and fill in the blanks to describe your rotations.

Rotate $\triangle YZK$ counterclockwise /clockwise around point _____ by \angle _____ until point Z goes to point _____.

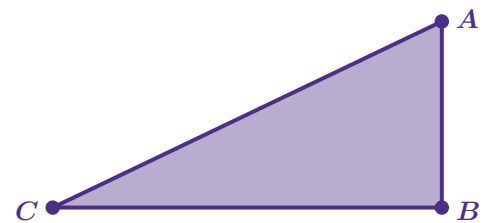


Rotate $\triangle GST$ counterclockwise/ clockwise around point _____ by \angle _____ until point T goes to point _____.

Spiral Review

Problems 9–10: Here is right triangle ABC . Describe how to use only rotations or reflections to create an image that is the outline of a rectangle when combined with triangle ABC .

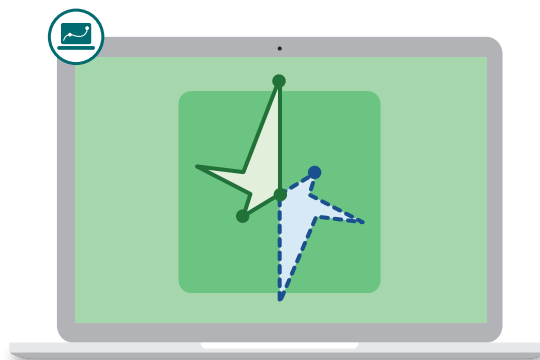
9. Description using only rotations:



10. Description using only reflections:

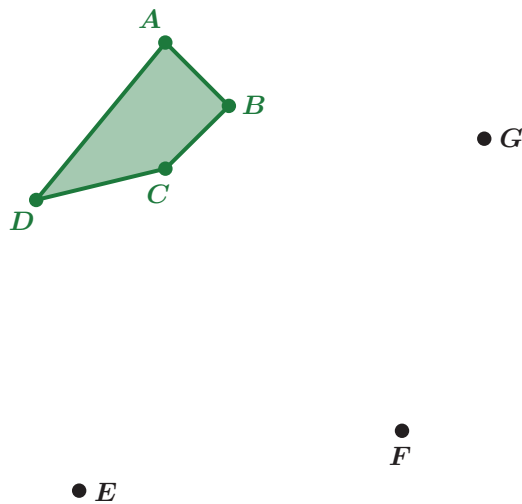
Going Off the Grid, Part 2

Let's use geometry tools to perform sequences of rigid transformations that move one figure onto another.



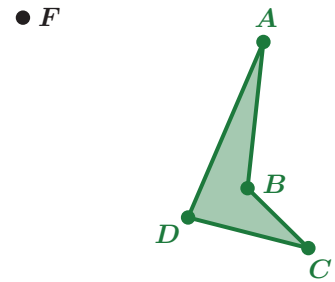
Warm-Up

- 1**
 - a** Let's watch how to translate quadrilateral $ABCD$ so that point A goes to point G .
 - b** Use the geometry tools to translate quadrilateral $ABCD$ so that point C goes to point F . Describe how you used the tools to translate quadrilateral $ABCD$.



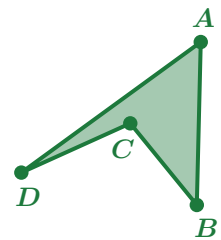
Digital Transformations

- 2** Reflect quadrilateral $ABCD$ over line EF . Describe how you used the tools to reflect quadrilateral $ABCD$.



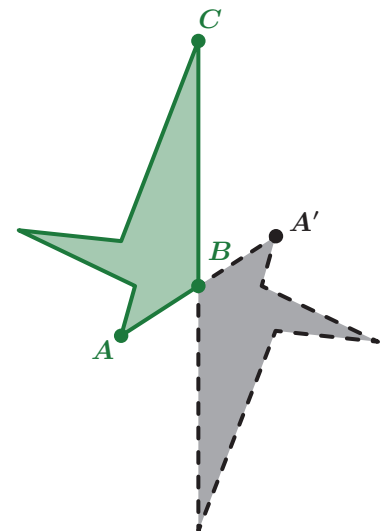
• E

- 3** Rotate quadrilateral $ABCD$ counterclockwise around point D by $\angle ADE$ so that point A goes to point E . Describe how you used the tools to rotate quadrilateral $ABCD$.



• E

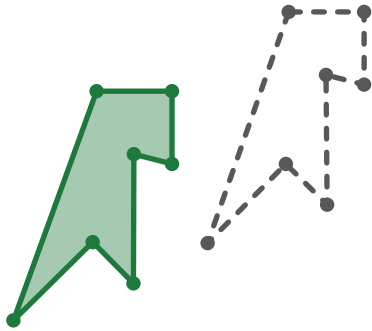
- 4** Which description will transform the solid polygon onto the dashed polygon?
- Translate so that point A goes to point A' .
 - Reflect over segment BC .
 - Reflect over segment AB .
 - Rotate counterclockwise around point B by $\angle ABA'$ so that point A goes to point A' .



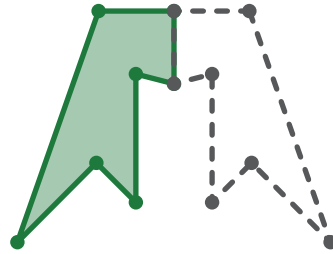
Digital Transformations (continued)

5 Transform the solid polygon onto the dashed polygon.

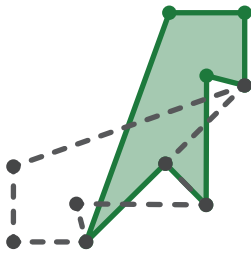
a



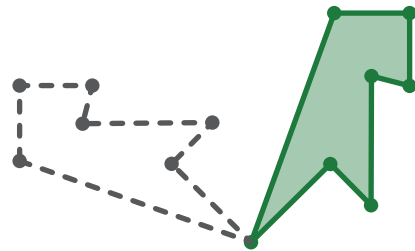
b



c



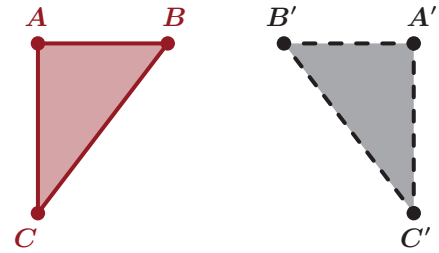
d



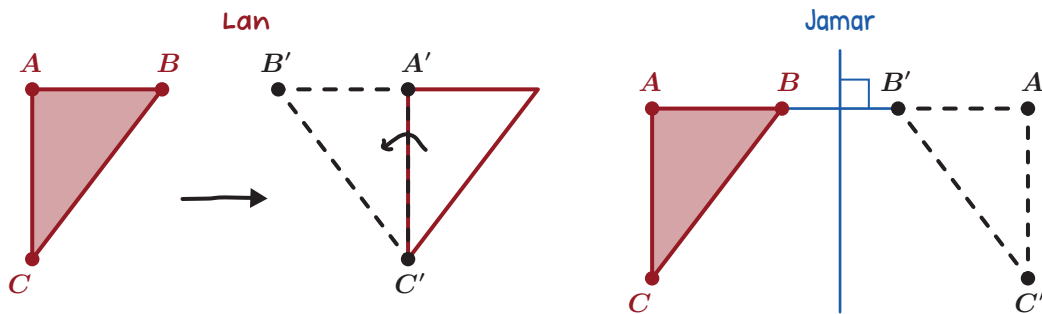
Sequences of Transformations

6 Describe how you can transform triangle ABC to triangle $A'B'C'$.

Use this diagram if it helps with your thinking.

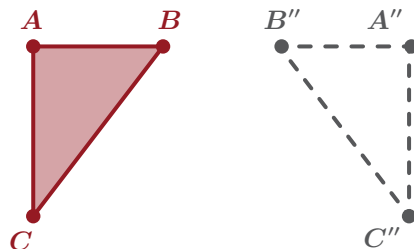


7 Here are Lan's and Jamar's sketches showing how they would transform triangle ABC onto triangle $A'B'C'$.



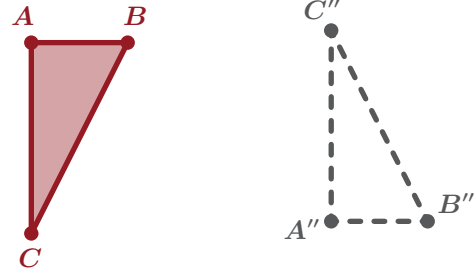
Discuss: What transformations do you see each student using?

8 Transform triangle ABC onto triangle $A''B''C''$. Show or explain your thinking.

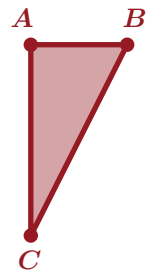
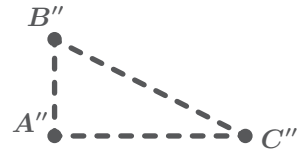


Sequences of Transformations (continued)

- 9** **a** Transform triangle ABC onto triangle $A''B''C''$.
b Describe the transformations you performed.



- 10** **a** Transform triangle ABC onto triangle $A''B''C''$.
b Describe the transformations you performed.

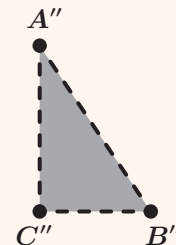
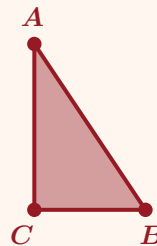


You're invited to explore more.

- 11** Determine a sequence of transformations that moves triangle ABC onto triangle $A''B''C''$ using *only reflections* or *only rotations*.

Put a check next to each rule that is possible.

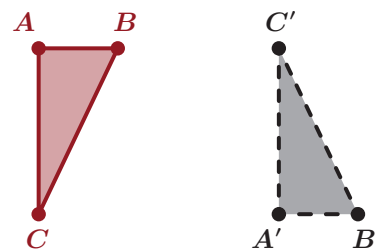
- Rule #1: Using only reflections.
 Rule #2: Using only rotations.



12 Synthesis

How do you decide what to do first when transforming one figure onto another?

Use the example if it helps with your thinking.

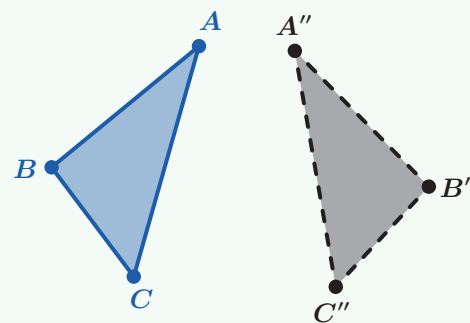


15 Summary 6.12

There are many strategies to transform one figure onto another.

Here are three ways you could transform triangle ABC onto triangle $A''B''C''$:

1. Translate triangle ABC so that point B goes to B'' . Rotate the image clockwise around B'' by $\angle C'B''C''$, and then reflect triangle $A'B''C''$ over segment $B''C''$.
2. Reflect triangle ABC over the perpendicular bisector of segment AA'' .
3. Identify the intersection point D between line AB and line $A''B''$ then rotate triangle ABC counterclockwise around D by $\angle ADA''$ until segment AB aligns with segment $A''B''$. Then reflect triangle $A'B''C''$ over segment $A''B''$.

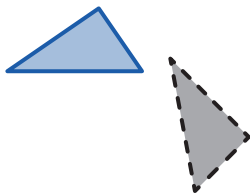


Practice 6.12

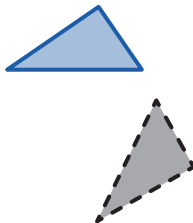
Name: _____ Date: _____ Period: _____

1. Which set of figures does *not* show a rigid transformation?

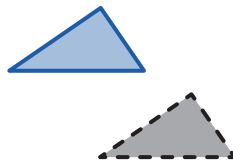
A.



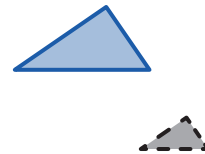
B.




C.



D.



2.  **Test Practice** Select *all* the transformations that could move point A onto point B .

• B

A. Reflect point A over \overline{AB} .

B. Reflect point A over the perpendicular bisector of \overline{AB} .

• A

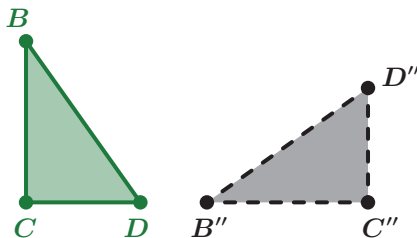
C. Translate point A so that it moves onto point B .

D. Rotate point A around point B .

E. Rotate point A clockwise around the midpoint of \overline{AB} .

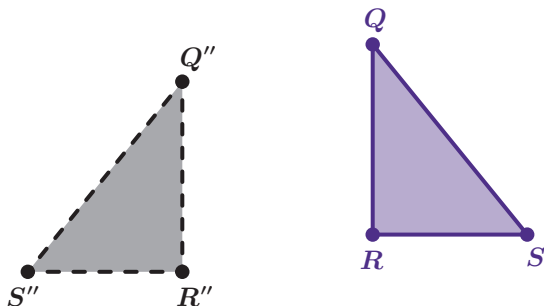
Problems 3–4: Transform the solid triangle onto the dashed triangle. Then describe the transformations you performed.

3.



Description:

4.

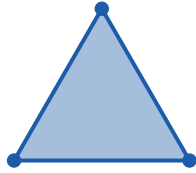


Description:

Practice 6.12

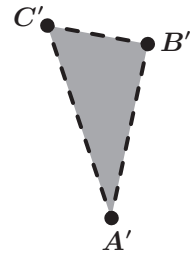
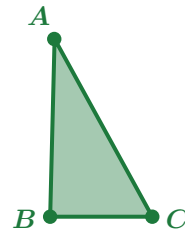
Name: _____ Date: _____ Period: _____

5. Here is an equilateral triangle. Create a larger triangle using only reflections. Show or explain your thinking.



Spiral Review

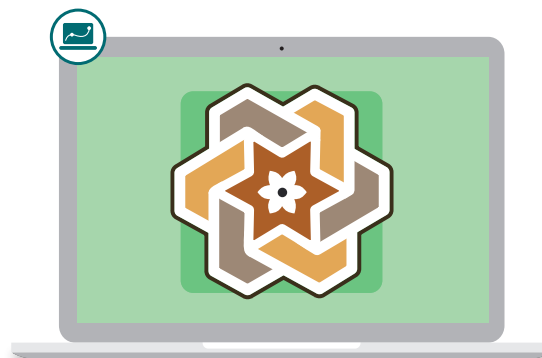
6. Describe a strategy for determining the center of rotation for the transformation that would move triangle ABC onto triangle $A'B'C'$.



7. A triangle has one angle that measures 55° . What are possible values for the other two angles?

Tiles and Patterns

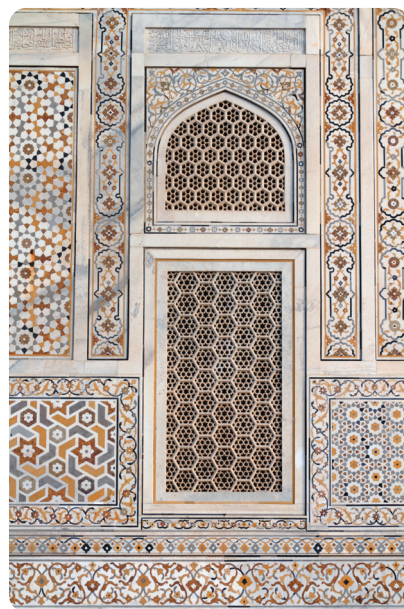
Let's use transformations to describe and create patterns.



Warm-Up

- 1** Let's look at some images of architecture from different countries. They contain examples of geometric patterns from Islamic cultures.

What do you notice? What do you wonder?



"Tomb of I'timād-ud-Daulah artwork 8"
by Swapnil.Karambelkar via Wikimedia
Commons, CC-BY-SA 4.0

- 2** You can use a compass and straightedge to create geometric patterns. Spend some time researching and learning more about how artists like Samira Mian recreate geometric patterns found in architecture.

As you explore, consider discussing:

- What is a pattern that stands out to you? Why?
- What are some characteristics of these patterns?

Pattern Play

3 Here is a geometric pattern found in the Tomb of I'timād-ud-Daulah.

Use at least two of these words to describe the geometry in this pattern.

- Translate
- Rotate
- Reflect
- Transformation

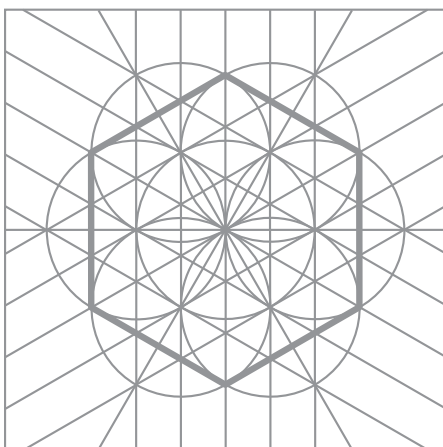


"Decoration on the wall of the mausoleum of Itmad-ud-Daulah's tomb"
by Royroydeb via Wikimedia Commons, CC-BY-SA 4.0

4 Let's watch an animation of the underlying construction that Samira Mian used to recreate the pattern from the Tomb of I'timād-ud-Daulah.

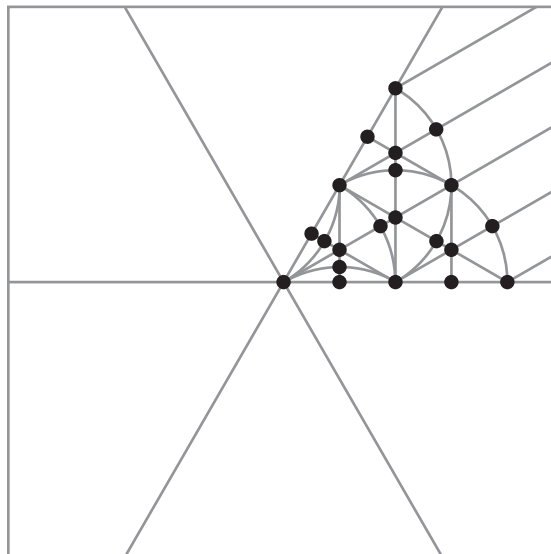
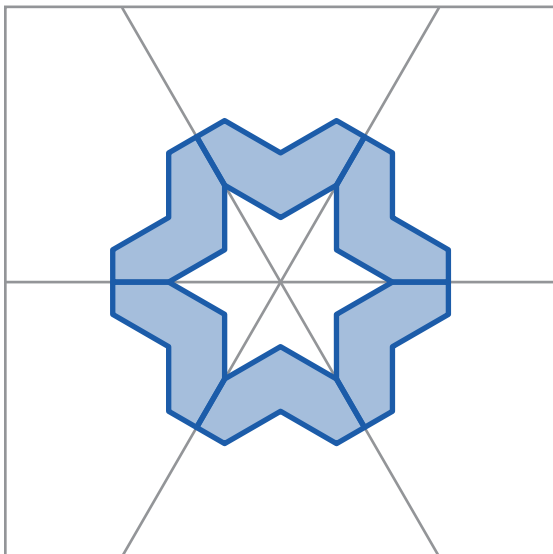
 **Discuss:** How is this related to constructions you've done? How is it different?

5 Use the underlying construction to outline the pattern.



Pattern Play (continued)

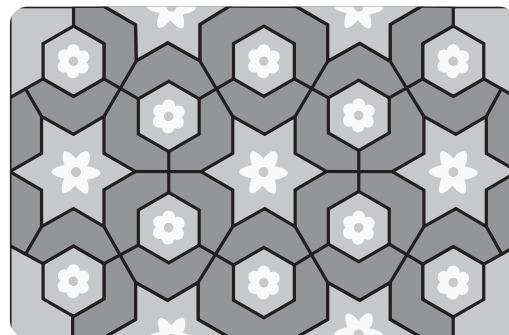
- 6** Here is one design inspired by the Tomb of I'timād-ud-Daulah. Connect points to create a polygon that, when rotated, will make the complete design.



- 7** Let's see how this design is related to the geometric pattern found in the Tomb of I'timād-ud-Daulah.

Select *all* transformations that could be used on this design to recreate the pattern.

- A. Translation
- B. Rotation
- C. Reflection
- D. Other



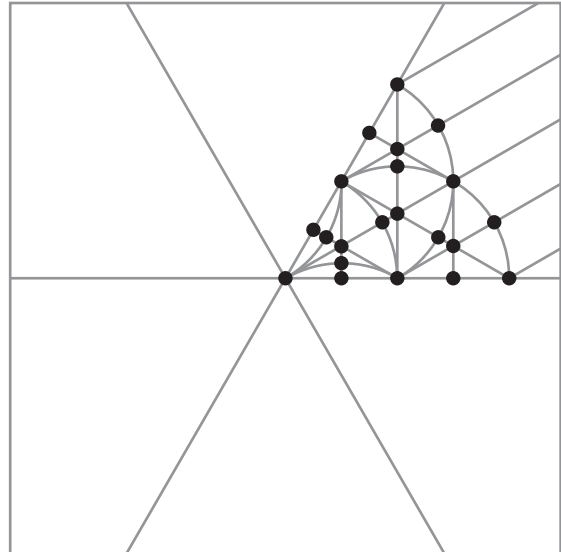
Explain your thinking.

Challenge Creator

8 You will use the Activity 2 Sheet to create your pattern challenge.

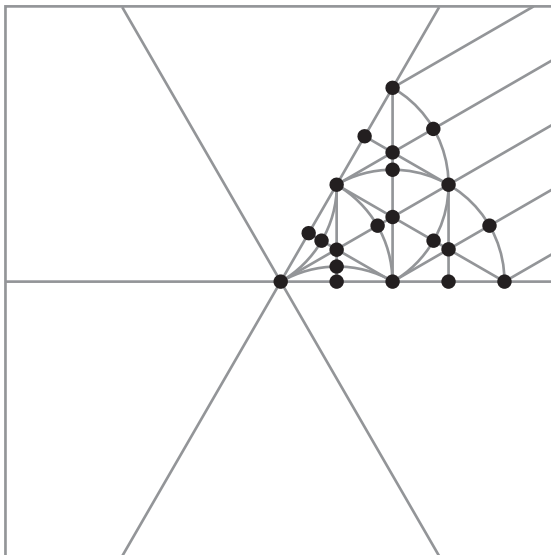
a Make It!

- On this page, connect points to create a polygon that, when rotated, will make your design.
- Use tracing paper to replicate your design. Then complete your pattern on the Activity 2 Sheet.

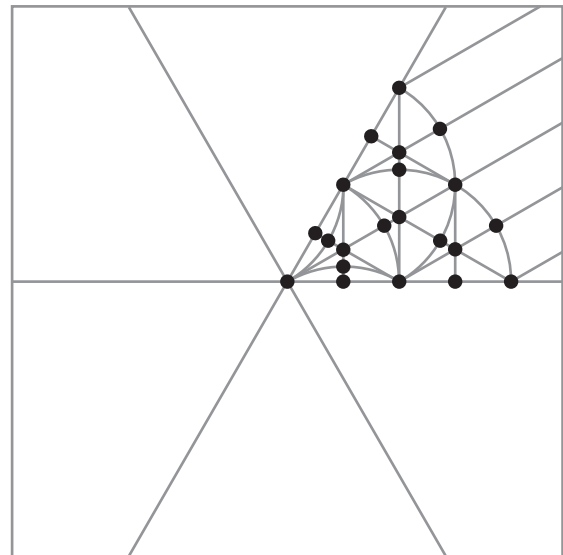


- b Swap It!** Swap your challenge on the blank paper with one or more partners. For each partner's challenge, create a polygon that, when rotated, will create the design. Then use tracing paper to replicate that design and complete the pattern.

Partner 1

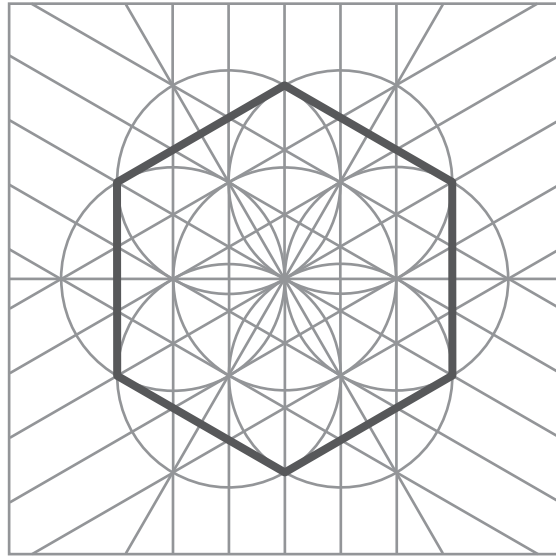


Partner 2



9 Synthesis

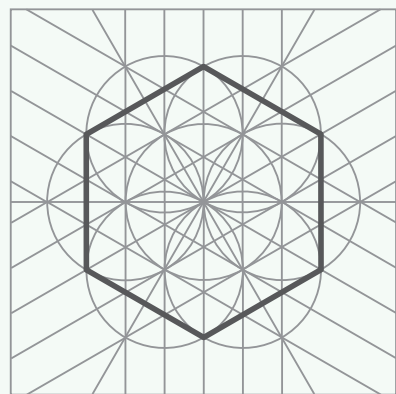
How can transformations and constructions help you create and recreate patterns?



12 Summary 6.13

Artists and mathematicians throughout history have developed strategies for making geometric patterns. Here is an example of a geometric pattern from the Tomb of I'timād-ud-Daulah in India.

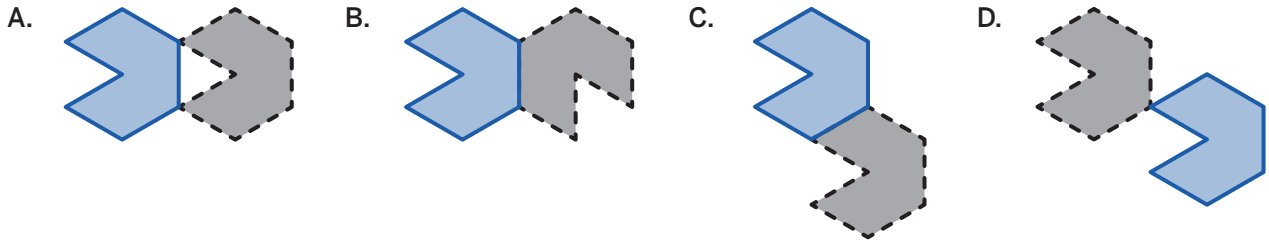
There are many strategies to construct and recreate designs. You can use circles, intersection points, and line segments to precisely construct figures. You can transform those figures using translations, rotations, and reflections. By combining constructions and transformations, you can create complex designs made up of beautiful shapes and patterns.



Practice 6.13

Name: _____ Date: _____ Period: _____

1. Which transformation *cannot* be achieved with a translation?



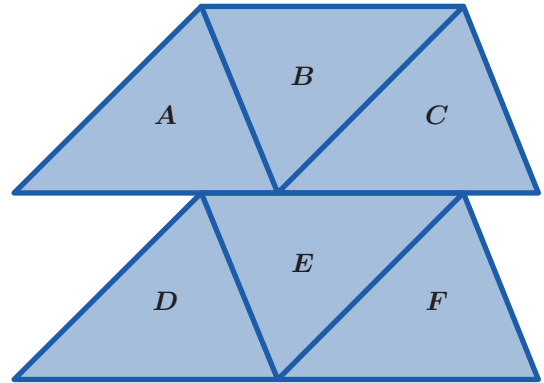
Problems 2–3: These triangles are rigid transformations of triangle A .

2. Select *all* the triangles that could be a translation of triangle A .

- B C D
 E F

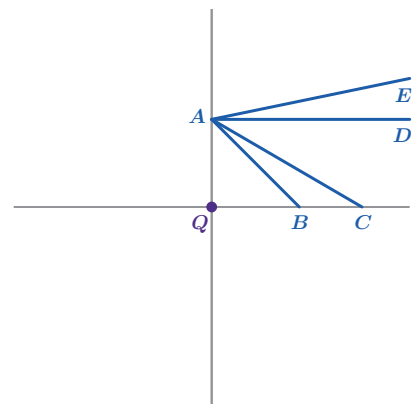
3. Select *all* the triangles that could be a rotation of triangle A .

- B C D
 E F

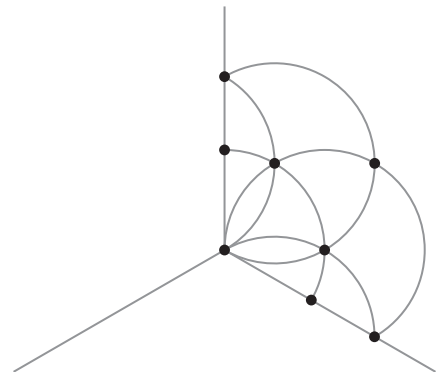


4. **Test Practice** Which segment, when rotated around point Q into each quadrant, will form a square?

- A. Segment AB
 B. Segment AC
 C. Segment AD
 D. Segment AE



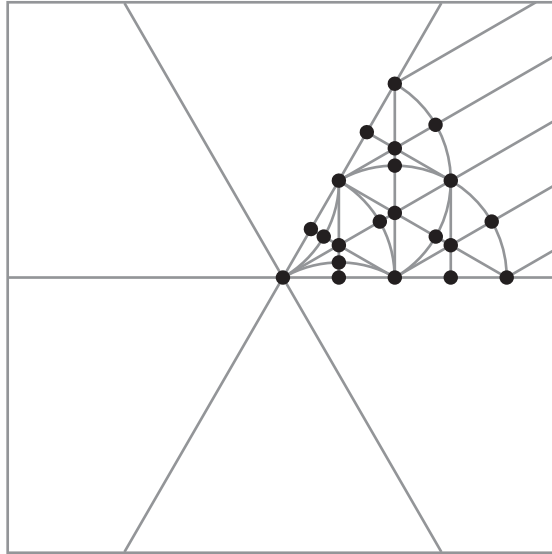
5. Use the intersection points to construct a line segment that will form an equilateral triangle when rotated.



Practice 6.13

Name: _____ Date: _____ Period: _____

6. Connect points to create a polygon that, when rotated, will make a design that also includes at least one translation and at least one reflection. Use tracing paper to replicate your polygon to complete the design.

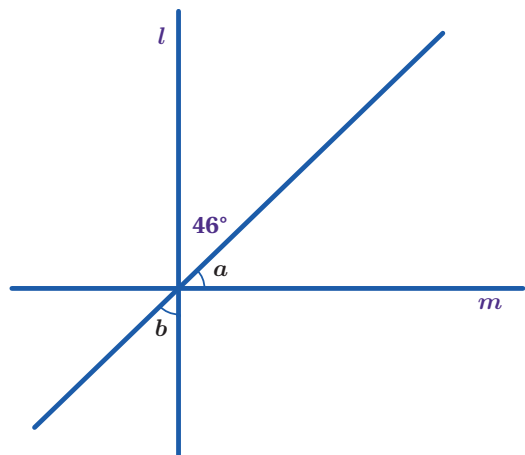


Spiral Review

Problems 7–8: Line l is perpendicular to line m . Determine the values of a and b .

7. $a =$

8. $b =$

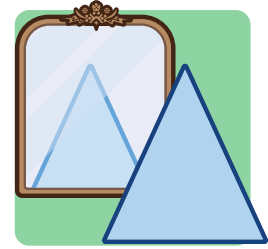


9. Construct the perpendicular bisector of segment AB .



Reflection Symmetry

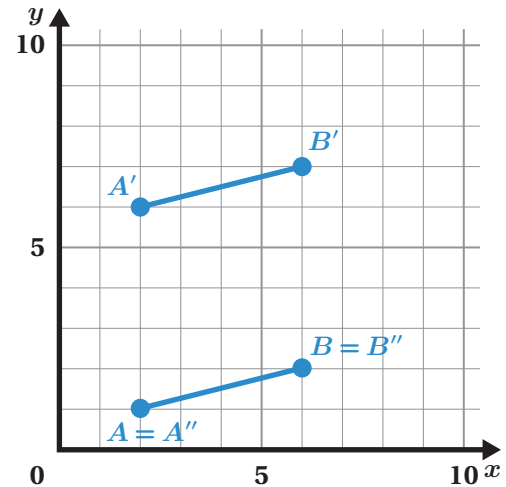
Let's explore what it means for a figure to have reflection symmetry.



Warm-Up

Here is segment AB . If the segment is translated up 5 units and then down 5 units, the image fits exactly over the original segment.

1. What other two-step rigid transformations create an image that fits exactly over the original segment?


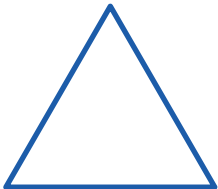


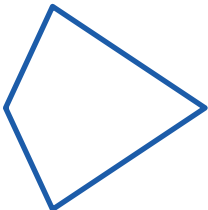


2. What single rigid transformations, if any, do the same thing?


Self-Reflection

3. You will use the cutouts from the Activity 1 Sheet to complete the table.

- Use the cutouts to determine the **lines of symmetry** for each shape. Then sketch the lines of symmetry in the first column of the table.
- In the second column, describe the lines of symmetry that each shape has. If there are none, write *no lines of symmetry*.
- In the third column, describe a non-example of a line of symmetry.

Lines of Symmetry	Descriptions of Lines of Symmetry	Non-Example
		
		
		
		
		

Self-Reflection (continued)

4. What do the lines of symmetry for the trapezoid, triangle, and rectangle have in common?
5.  **Discuss:** What properties can you look for to help you determine if a polygon has reflection symmetry? Explain your thinking.

6. Describe a reflection that takes a square onto itself.

Use the image if it helps with your thinking.

**You're invited to explore more.**

Look at all of the shapes the class explored and focus on those which had more than one line of symmetry.

7. What is true for all the lines of symmetry in these shapes?
8. Give an example of a figure that has two or more lines of symmetry that do not intersect at the same point.

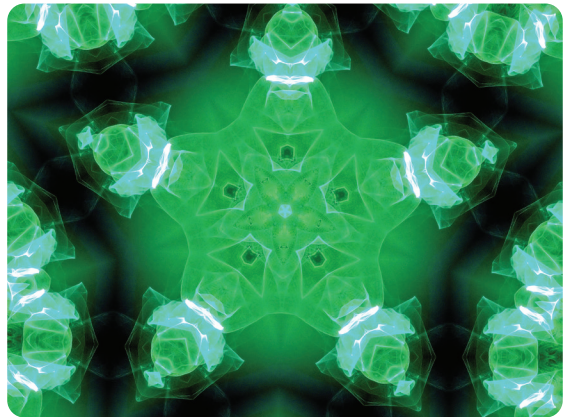
Kaleidoscopes

A kaleidoscope uses mirrors tilted at different angles to create symmetrical patterns. Often, a kaleidoscope is made by enclosing mirrors in a tube and placing various shapes at one end that result in intricate and symmetrical designs.

9. Alina created a kaleidoscope that produced the image shown. What do you notice? What do you wonder?

I notice:

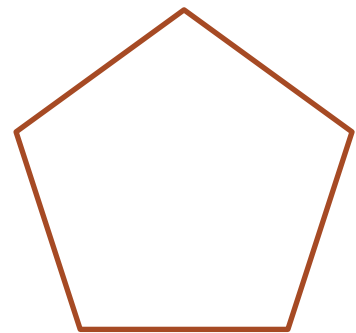
I wonder:



Georgy Shafeev/Shutterstock.com

10. Determine how many lines of symmetry there are for the main figure in the center of the kaleidoscope. Give a description for each line of symmetry.

11. Determine and draw the lines of symmetry for the regular pentagon. Compare this to the lines of symmetry for the kaleidoscope image. What do you notice?




Create a Kaleidoscope Image

- 12.** In Activity 2, you saw how kaleidoscope images use symmetry. Now, you will create your own kaleidoscope image.

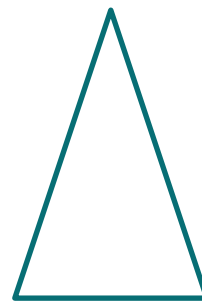
Your image should be meet the following criteria:

- Have more than one line of symmetry.
- Use at least one polygon.

Synthesis

13.  **Discuss:** How can you determine whether a figure has reflection symmetry?

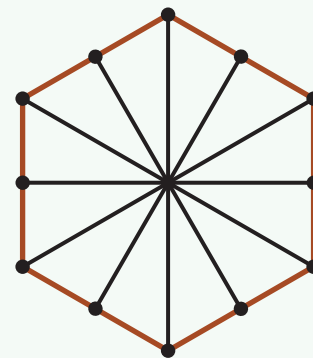
Use the image of the isosceles triangle if it helps with your thinking.



Summary 6.14

Reflection symmetry is one type of symmetry that a figure can have. Identifying congruent sides and congruent angles in a figure can be helpful when trying to determine lines of symmetry.

For example, a regular hexagon has 6 lines of symmetry.



line of symmetry A line, which could be imaginary, that divides a shape into two identical halves.

reflection symmetry A figure exhibits reflection symmetry if a line of symmetry exists such that when the figure is reflected over that line, the figure is taken to itself.

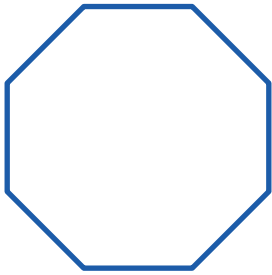
symmetry A property of a figure that exists when a rigid transformation creates an image that takes the original figure onto itself.

Practice 6.14

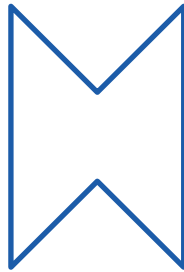
Name: _____ Date: _____ Period: _____

Problems 1–6: Does each figure have reflection symmetry? If yes, draw the lines of symmetry and describe one reflection that will take the figure onto itself.

1.



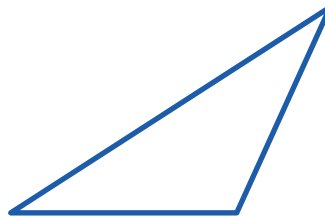
2.



3.



4.



5.




6.

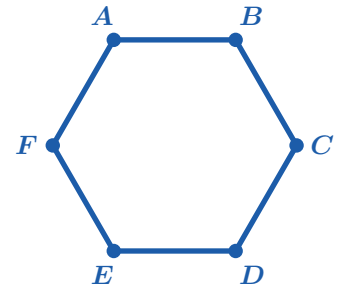


Practice 6.14

Name: _____ Date: _____ Period: _____

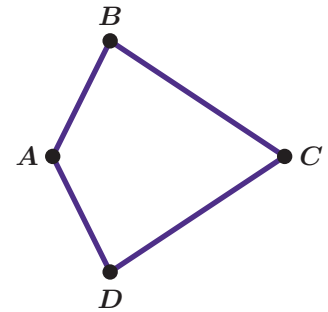
7.  **Test Practice** Select *all* the transformations that will take this regular hexagon onto itself.

- A. A reflection over line segment AD
- B. A reflection over line segment FC
- C. A reflection over line segment DF
- D. A reflection over the line that passes through the midpoints of line segments AB and ED
- E. A reflection over the line that passes through the midpoints of line segments BC and ED



Problems 8–9: In quadrilateral $BADC$, segments AB and AD have the same length, and segments BC and DC have the same length. Line AC is a line of symmetry for this quadrilateral.

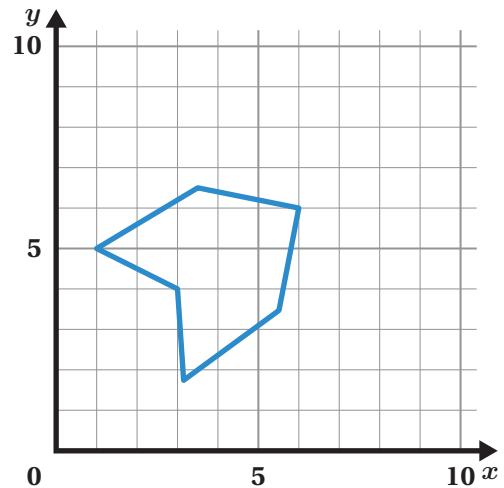
8. Based on the line of symmetry, explain why the diagonals (segments AC and BD) are perpendicular.



9. Based on the line of symmetry, explain why angles ABC and ADC have the same measure.

10. Here is a figure that has reflection symmetry. Write an equation to complete the following statement:

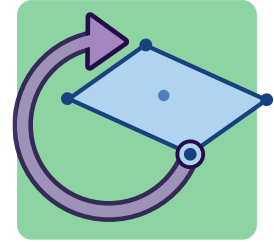
A reflection over the line _____ will take this figure onto itself.



Spiral Review

11. Calculate the exact distance between the points $A(2, -5)$ and $B(-7, 4)$.

Rotational Symmetry

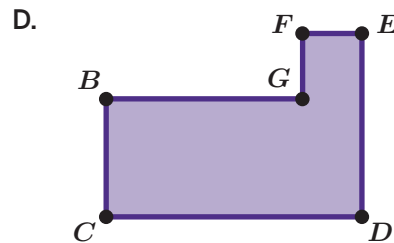
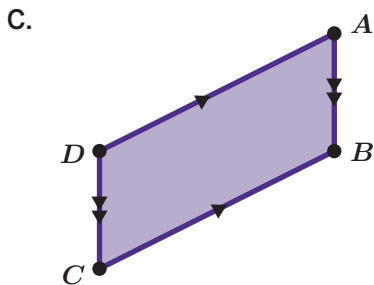
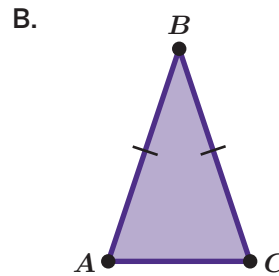
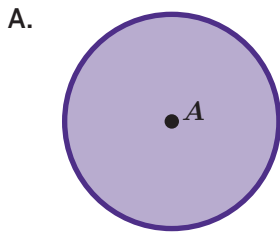


Let's explore what it means for a figure to have rotational symmetry.

Warm-Up

Each figure can be taken onto itself.



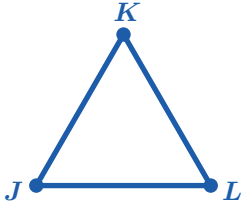

1. Which one doesn't belong? Explain your thinking.



Self-Rotation


2. You will use tracing paper to complete the table.

- Use tracing paper to determine whether each shape has **rotational symmetry**. If it does, then in the first column, use arrows to show each rotation that will take the shape onto itself. Include the point(s) each shape is being rotated around.
- In the second column, describe the rotations that will take each shape onto itself. If there are none, write *no rotational symmetry*.
- In the third column, describe a rotation that does not take the shape onto itself.

Rotations	Descriptions of Rotations	Non-Example
		
		
		
		

Self-Rotation (continued)

3. Consider the shapes in the table that have rotational symmetry versus those that do not.
- a For the shapes that have rotational symmetry, what do they have in common?

 - b What is different about the shape(s) that *does not* have rotational symmetry?
4.  **Discuss:** What properties can you look for to help you determine if a polygon has rotational symmetry? Explain your thinking.

You're invited to explore more.

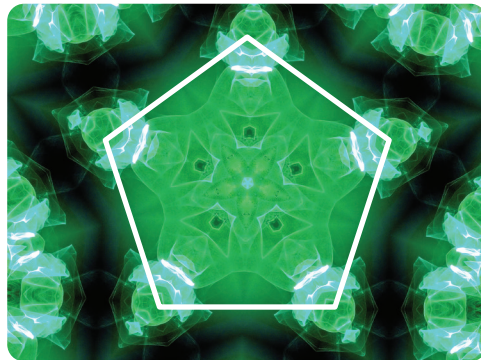
5. Can you think of a figure that has translation symmetry? Explain your thinking.

Rotating Kaleidoscopes

In the previous lesson, you saw that kaleidoscopes create intricate patterns using reflections from rotatable mirrors in tubes.

Alina created a kaleidoscope that produced the image shown. The shape of the image can be approximated with a regular pentagon.

Alina and Mauricio want to use the regular pentagon to help them determine what rotations will create symmetry for the kaleidoscope image.

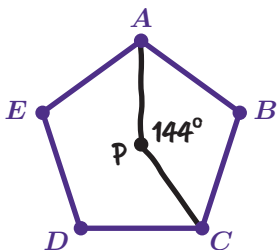


Georgy Shafeev/Shutterstock.com

6. Here are Alina's and Mauricio's strategies for describing a rotation that will take pentagon $ABCDE$ onto itself.

Alina

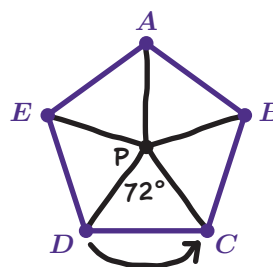
After estimating the center P of the pentagon, I used my protractor to measure angle APC .



A rotation of 144° clockwise around center P will take the pentagon onto itself.

Mauricio

$$\frac{360}{5} = 72$$




A rotation of 72° counterclockwise around center P will take the pentagon onto itself.



Discuss: How are Alina's and Mauricio's strategies alike? How are they different?

7. Use Mauricio's method to determine a rotation that will take a regular hexagon onto itself.

Synthesis

8.  **Discuss:** How can you determine whether a figure has rotational symmetry?

Use the image of the parallelogram if it helps with your thinking.

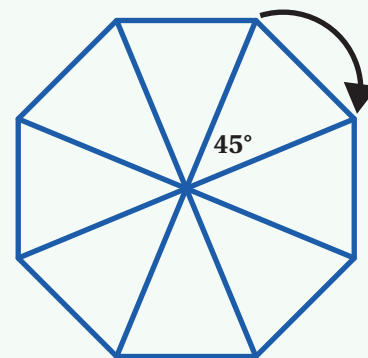


Summary 6.15

Another type of symmetry that a figure can have is called rotational symmetry. When describing rotational symmetry, the center, angle, and direction of the rotation must be specified.

For example, a regular octagon can be rotated 45° clockwise around the center of the octagon, resulting in an octagon that is perfectly overlaid with the original.

A regular octagon can also be taken onto itself by being rotated any multiple of 45° , such as 90° , 135° , and 180° , around its center either clockwise or counterclockwise.



$$\frac{360}{8} = 45$$

rotational symmetry A figure exhibits rotational symmetry when a rotation between 0° and 360° exists that takes the figure onto itself.

Practice 6.15

Name: _____ Date: _____ Period: _____

1. Select *all* the angles of rotation that produce symmetry for the figure.

A. 45°

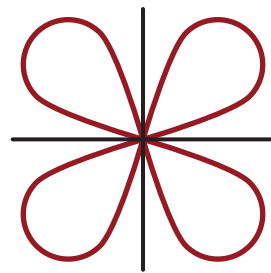
B. 90°

C. 135°

D. 180°

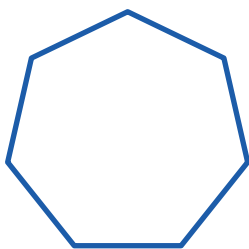
E. 225°

F. 270°

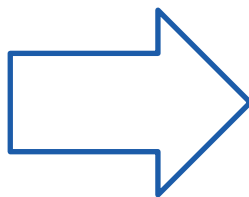


Problems 2–7: Determine whether each figure has rotational symmetry. If so, describe one rotation that will take the figure onto itself. If not, write *no rotational symmetry* and explain your thinking.

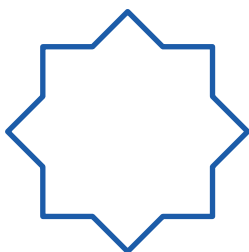
2.



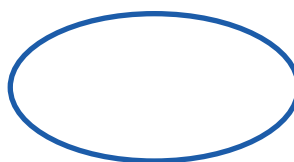
3.



4.



5.



6.




7.



Practice 6.15

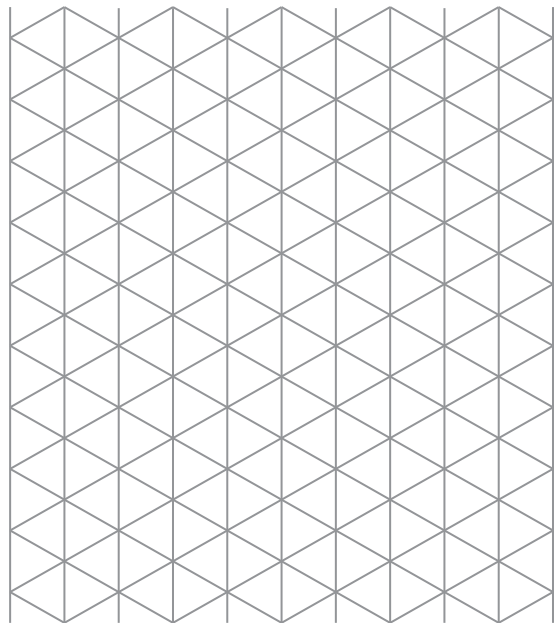
Name: _____ Date: _____ Period: _____

8.  **Test Practice** Suppose that a triangle has rotational symmetry that can take any of its vertices to any of its other vertices. Determine whether each statement about the triangle is true or false.

	True	False
All sides of the triangle have the same length.		
All angles of the triangle have the same measure.		
The triangle is obtuse.		

9. Sketch a figure that has rotational symmetry but not reflection symmetry.

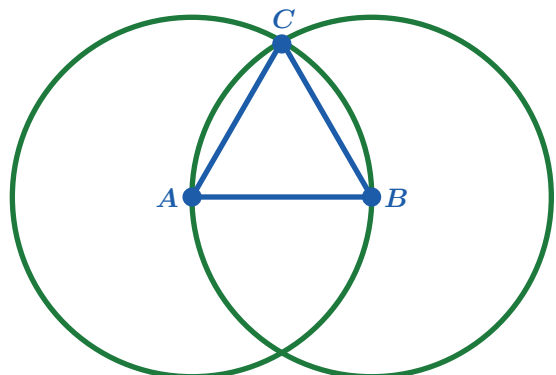
Use the grid if it helps with your thinking.



Spiral Review

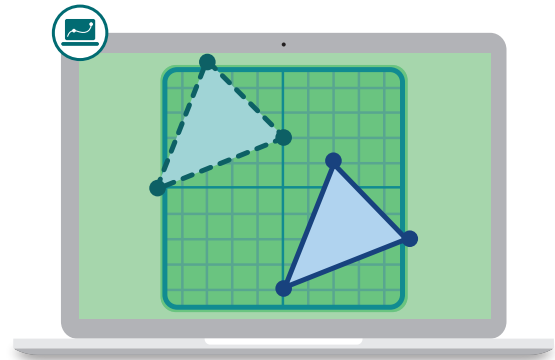
10. Determine the percent decrease for the function $f(x) = 50(0.75)^x$.

11. What type of triangle is triangle ABC given that points A and B are the centers of the two circles?




Transformations as Functions

Let's explore how we can represent transformations as functions.

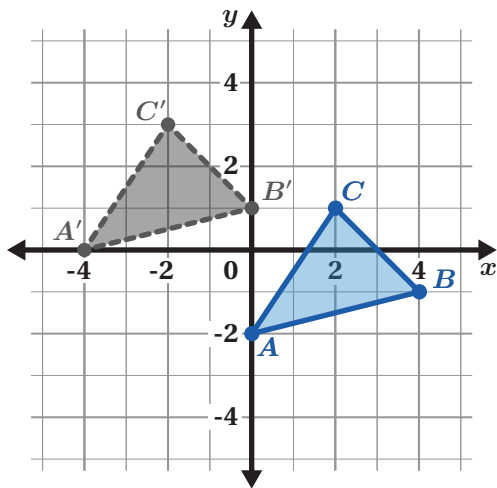


Warm-Up

1 Here are two transformations.

 **Discuss:** What do you notice? What do you wonder?

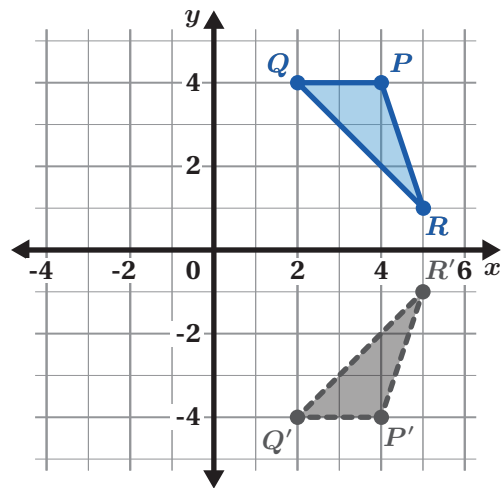
Translation left 4 units and up 2 units



$$(x, y) \rightarrow (x - 4, y + 2)$$

I notice:

Reflection over the x -axis



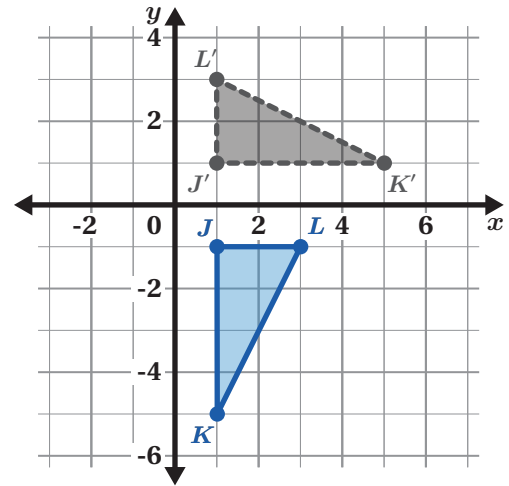
$$(x, y) \rightarrow (x, -y)$$

I wonder:

Inputs and Outputs

2 Here is the graph of triangle JKL and its image after a transformation.

Describe the transformation that created triangle $J'K'L'$.



3 Here are the coordinates of the vertices for triangle JKL and its image.

a **Discuss:** What happens to the pre-image coordinates to create the image coordinates?

b Represent the transformation using *coordinate transformation notation*.

$(x, y) \rightarrow$

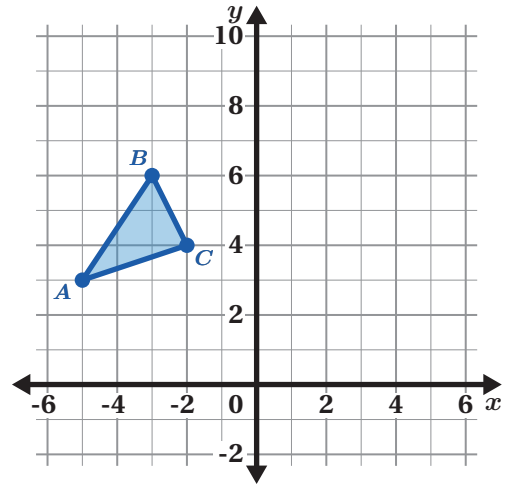
Pre-Image Coordinates	Image Coordinates
$J(1, -1)$	$J'(1, 1)$
$K(1, -5)$	$K'(5, 1)$
$L(3, -1)$	$L'(1, 3)$

4 Describe the transformation that is represented by the notation $(x, y) \rightarrow (x + 6, y - 2)$. Explain your thinking.

Inputs and Outputs (continued)

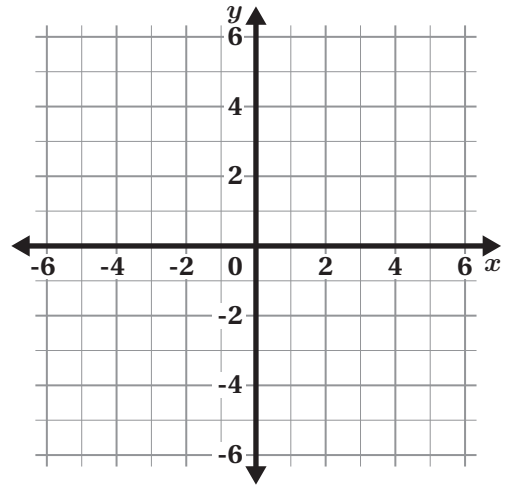
5 Determine the coordinates for the image of triangle ABC after the transformation $(x, y) \rightarrow (x + 6, y - 2)$. Then graph the image.

(x, y)	$(x + 6, y - 2)$
$A(-5, 3)$	A' _____
$B(-3, 6)$	B' _____
$C(-2, 4)$	C' _____



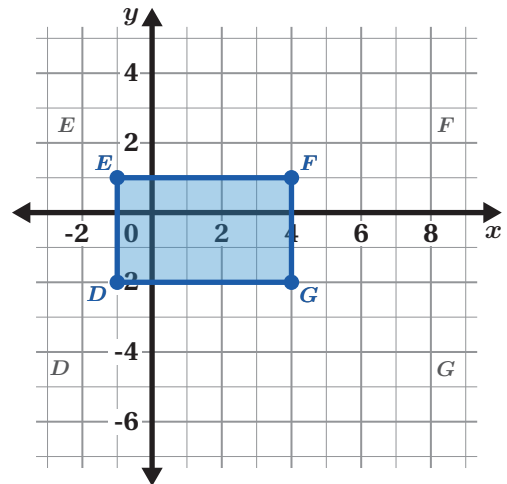
6 **Discuss:** Describe the transformation that is represented by the notation $(x, y) \rightarrow (2x, 2y)$.

Use the coordinate plane if it helps with your thinking.



7 Determine the coordinates for the image of quadrilateral $DEFG$ after the transformation $(x, y) \rightarrow (2x, 2y)$. Then graph the image.

(x, y)	$(2x, 2y)$
$D(-1, -2)$	D' _____
$E(-1, 1)$	E' _____
$F(4, 1)$	F' _____
$G(4, -2)$	G' _____



What Does It Do?

- 8** Here are some statements Taylor and Mia made about the transformation represented by $(x, y) \rightarrow (2x, y)$.

Taylor

This transformation makes every point on an image twice as far away horizontally from the origin, but it does not change anything vertically.

Mia

This transformation would stretch a figure horizontally and move points away from the y -axis by a factor of 2.

 **Discuss:** Whose claim is correct? Circle one.

Taylor's

Mia's

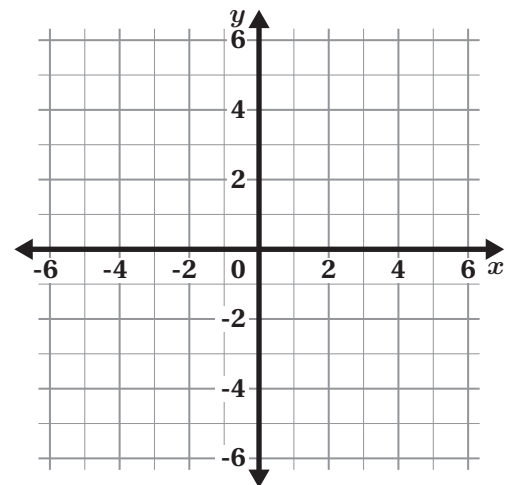
Both

Neither

Explain your thinking.

- 9** Describe the transformation represented by $(x, y) \rightarrow (-x, y)$.

Use the coordinate plane if it helps with your thinking.



**Activity
3**

Name: Date: Period:

Challenge Creator

10 You will use the Activity 3 Sheet to create your own transformation challenge.

- On the Activity 3 Sheet, choose one of the given polygons and draw its image after a transformation. Label the vertices of the image.
- On this page, describe your transformation and represent it using coordinate transformation notation.

My Transformation Description	My Coordinate Transformation Notation

- Swap your challenge with one or more partners. Describe the transformation that was applied and represent it with coordinate transformation notation.

	Transformation Description	Coordinate Transformation Notation
Partner 1		
Partner 2		
Partner 3		

11 Synthesis

Discuss: When given an example of coordinate transformation notation, how can you determine what transformation it represents?

$$(x, y) \rightarrow \left(\frac{1}{2}x, \frac{1}{2}y\right)$$

$$(x, y) \rightarrow (x - 6, y + 8)$$

$$(x, y) \rightarrow (-x, y)$$

15 Summary 6.16

Transformations can be represented with *coordinate transformation notation*. This notation takes the points of an original figure as inputs and produces the points of the image as outputs, similar to a function. Examples of coordinate transformation notation are shown in the table.

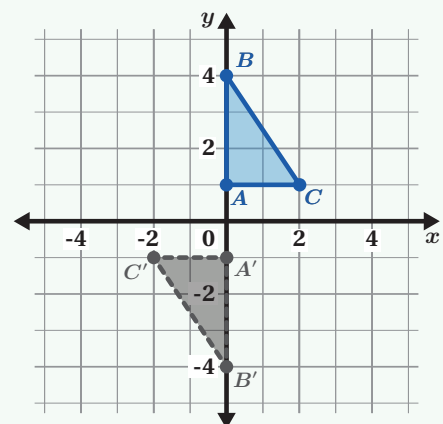
Coordinate Transformation Notation	Transformation Description
$(x, y) \rightarrow (x + h, y + k)$	A translation of h units horizontally and k units vertically, where h and k can be positive or negative values
$(x, y) \rightarrow (kx, ky)$	A dilation with scale factor k and center $(0, 0)$
$(x, y) \rightarrow (x, -y)$	A reflection over the x -axis

Consider triangle ABC and the coordinate transformation notation $(x, y) \rightarrow (-x, -y)$.

Triangle ABC has the coordinates $A(0, 1)$, $B(0, 4)$, and $C(2, 1)$. These points are used as inputs for $(x, y) \rightarrow (-x, -y)$.

The outputs created by the notation are $A'(0, -1)$, $B'(0, -4)$, and $C'(-2, -1)$.

This coordinate transformation notation represents a rotation 180° counterclockwise around center $(0, 0)$.



Practice 6.16

Name: _____ Date: _____ Period: _____

1. Match each transformation represented with coordinate transformation notation to the corresponding description.

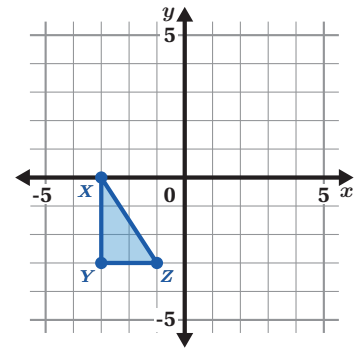
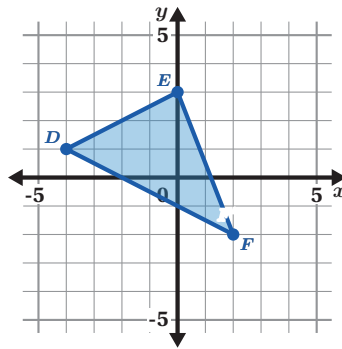
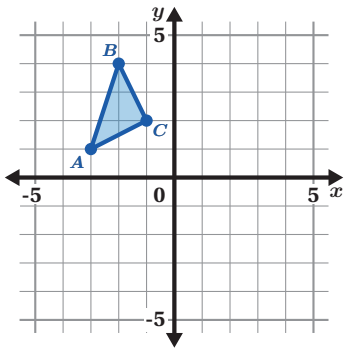
Coordinate Transformation Notation	Description
a. $(x, y) \rightarrow (x + 3, y)$ Translation 4 units up
b. $(x, y) \rightarrow (4x, 4y)$ Translation 3 units right
c. $(x, y) \rightarrow (x, y + 4)$ Dilation with a scale factor of 4 that uses the origin as the center
d. $(x, y) \rightarrow (x, y - 4)$ Translation 4 units down
e. $(x, y) \rightarrow (x - 3, y + 4)$ Translation 3 units left and 4 units up

Problems 2–4: Precisely describe each transformation based on the coordinate transformation notation. Then graph the image of each triangle after the given transformation.

2. $(x, y) \rightarrow (x, -y)$

3. $(x, y) \rightarrow (0.5x, 0.5y)$

4. $(x, y) \rightarrow (y, -x)$



Problems 5–7: Think about the transformation represented by each coordinate transformation notation. Determine whether each transformation is a *rigid transformation*, *dilation*, or *neither*.

5. $(x, y) \rightarrow (x - 2, y - 7)$

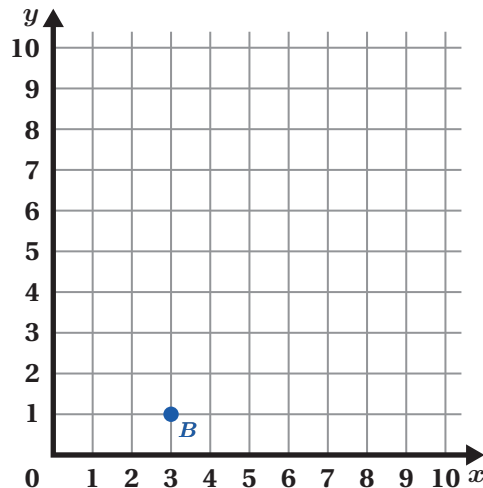
6. $(x, y) \rightarrow (2x, 3y)$

7. $(x, y) \rightarrow (5x, 5y)$

Practice 6.16

Name: _____ Date: _____ Period: _____

Problems 8–12: Here is point B .



- 8.** Complete the coordinate transformation notation so that it represents a dilation with a scale factor of 3 using the origin as the center.

$(x, y) \rightarrow$ _____

- 9.** Plot point D as the image of point B after the transformation given in Problem 8.

- 10.** Complete the coordinate transformation notation so that it represents a dilation with a scale factor of $\frac{1}{3}$ using the origin as the center.

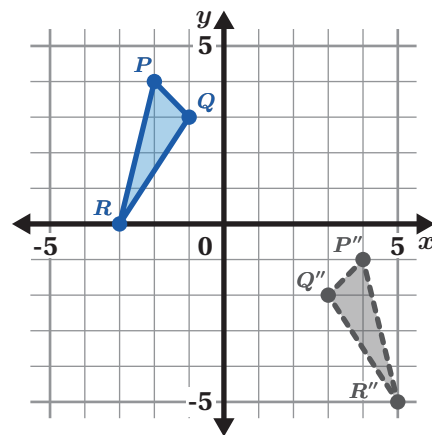
$(x, y) \rightarrow$ _____

- 11.** Plot point E as the image of point B after the transformation given in Problem 10.

- 12.** Complete the coordinate transformation notation so that it represents the dilation that will take point E to point D .

$(x, y) \rightarrow$ _____

- 13.** Melissa says that if the transformation represented by $(x, y) \rightarrow (x, -y)$ is applied to triangle PQR followed by the transformation represented by $(x, y) \rightarrow (x + 2, y - 5)$, then the final image will be triangle $P''Q''R''$. Do you agree with Melissa? Explain your thinking.



Spiral Review

- 14. Test Practice** Kwame invests \$5000 in an account that earns 4.5% interest every year. Select the expression that can be used to find the balance of the account after t years.

A. $5000 \cdot (1.045)^t$

B. $5000 \cdot (1.45)^t$

C. $5000 \cdot (0.045)^t$

D. $5000 + (1.45)^t$

Practice Day 2

Let's practice what you've learned so far in this unit!



You will use task cards and the digital companion. Record your responses here.

Task A: My Favorite Tools

1. Choose one to construct below and one to construct using the digital companion.

- | | | |
|--------------------------|--|--|
| a Right triangle | <input type="checkbox"/> Constructed below | <input type="checkbox"/> Digital companion |
| b Regular hexagon | <input type="checkbox"/> Constructed below | <input type="checkbox"/> Digital companion |

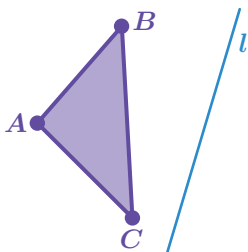
2. For geometric constructions, what is one advantage of using:

- | | |
|-------------------------|-----------------------------|
| a Digital tools? | b Paper-based tools? |
|-------------------------|-----------------------------|

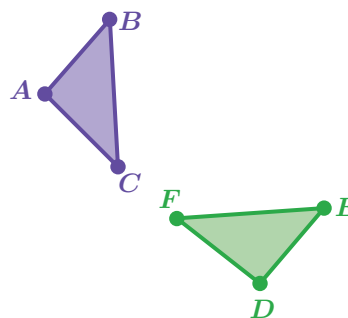
Task B: Reflected Triangles

1.

2.



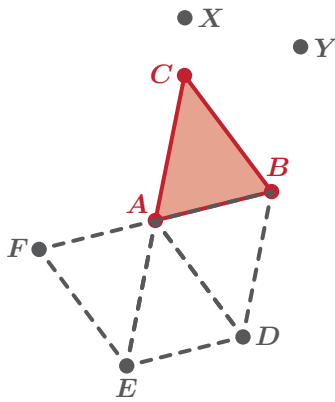
3.



Practice Day 2 (continued)

Task C: Translated Triangles

1.



2.

3. Circle one: Yes No

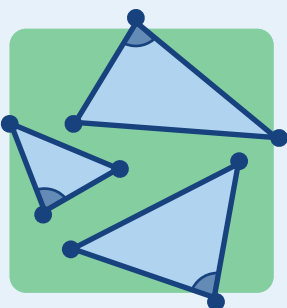
Explanation:

Task D: Reflection and Rotational Symmetry

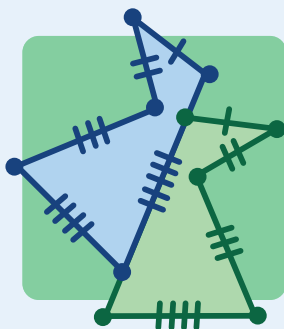
	Rotational Symmetry	Reflection Symmetry
<p>Figure A</p>	Yes or No	Yes or No
<p>Figure B</p>	Yes or No	Yes or No
<p>Figure C</p>	Yes or No	Yes or No

Notes:

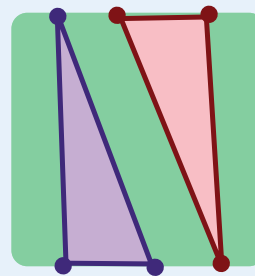
Congruence



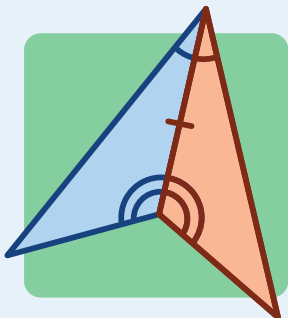
Lesson 17
Congruent Parts, Part 1



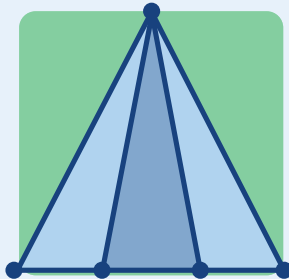
Lesson 18
Congruent Parts, Part 2



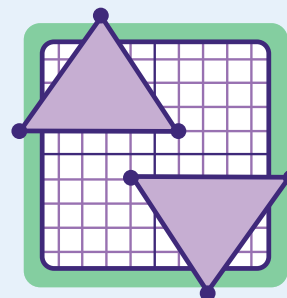
Lesson 19
Congruent Triangles,
Part 1



Lesson 20
Congruent Triangles,
Part 2



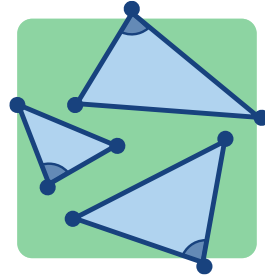
Lesson 21
Triangle Palindromes



Lesson 22
Another Triangle
Congruence Theorem

Congruent Parts, Part 1

Let's inspect what corresponding sides and angles in figures have to do with congruence.

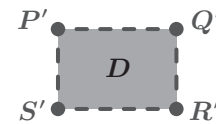
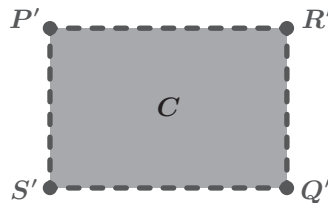
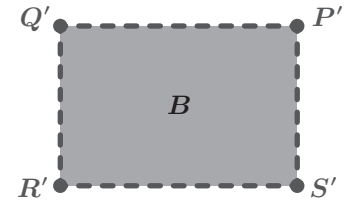
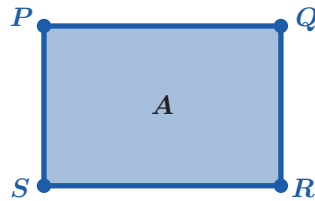


Warm-Up

1. Three students think they performed a rigid transformation of figure *A*.

Which figure(s) correctly show a rigid transformation of figure *A*?

- A. Figure *B*
- B. Figure *C*
- C. Figure *D*
- D. Figure *B* and Figure *C*

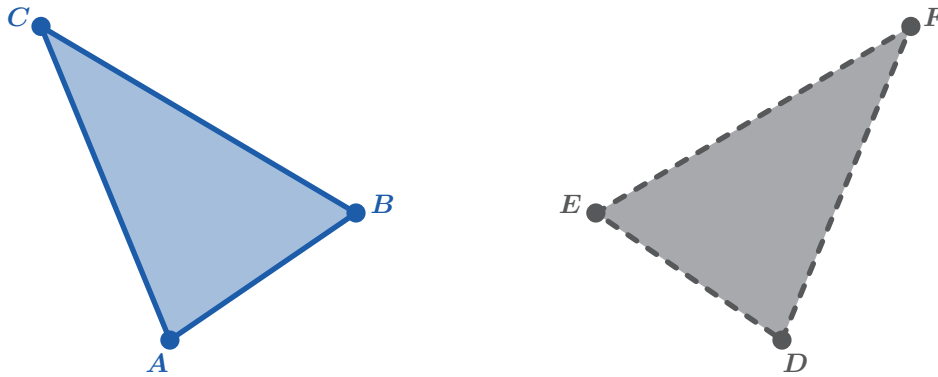


Explain your thinking.

If We Know This, Then We Know That...

For this activity, you will use either the Digital Companion or physical geometry tools.

Triangle ABC is congruent to Triangle DEF .



2. Determine a sequence of rigid transformations that takes Triangle ABC to Triangle DEF .

3. What segment in triangle DEF is segment BC taken to by the transformation?

4. Explain how you know that segment BC and the segment you identified in Problem 3 are congruent.

5. Explain how you know angle ABC is congruent to angle DEF .

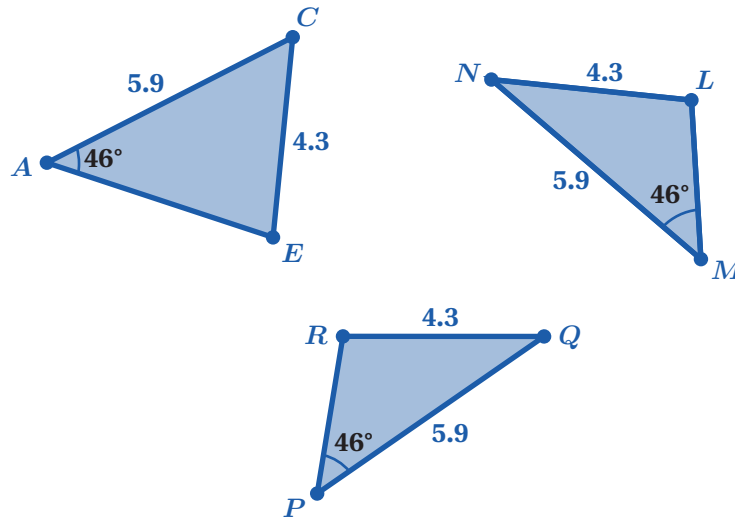
6. Are there other angles and sides that are congruent?


Explain how you know.

Which Triangles are Congruent?

For this activity, you will use either the Digital Companion or physical geometry tools.

Consider the three triangles. They are drawn to scale.

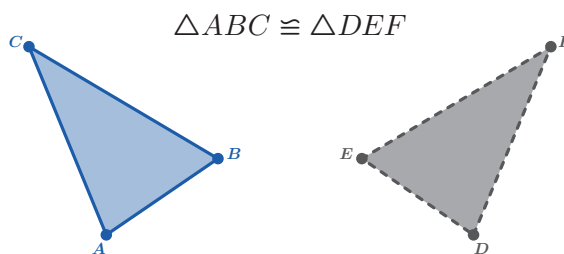


7.  **Discuss:** What do you notice about each triangle? What do you wonder?
8. Which triangle is congruent to triangle PQR ? Which triangle is *not* congruent to triangle PQR ?
 Explain your thinking.
9. Describe a sequence of rigid transformations that takes Triangle PQR to the congruent triangle you identified in Problem 8.
10. Explain why there is not a rigid transformation that could take triangle PQR to the triangle you determined not congruent in Problem 8.

Synthesis

11. Explain why corresponding parts of congruent figures must be congruent.

Use the example if it helps with your thinking.



Summary 6.17

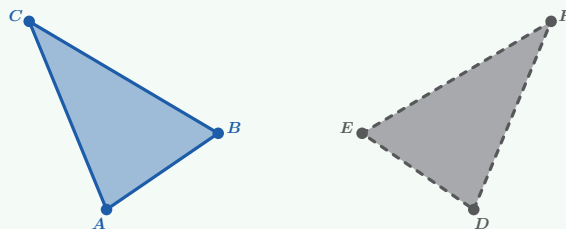
Two figures are *congruent* if there is a sequence of rigid transformations that takes one figure onto the other.

If two figures are congruent, then the corresponding parts of these figures must also be congruent.

For example, $\triangle ABC$ is taken to $\triangle DEF$ by a reflection across the perpendicular bisector of segment BE .

This takes every side and angle from $\triangle ABC$ to its corresponding part of $\triangle DEF$:

- Segment $AB \cong$ Segment DE
- Segment $AC \cong$ Segment DF
- Segment $BC \cong$ Segment EF
- Angle $ABC \cong$ Angle DEF
- Angle $BCA \cong$ Angle EFD
- Angle $CAB \cong$ Angle FDE

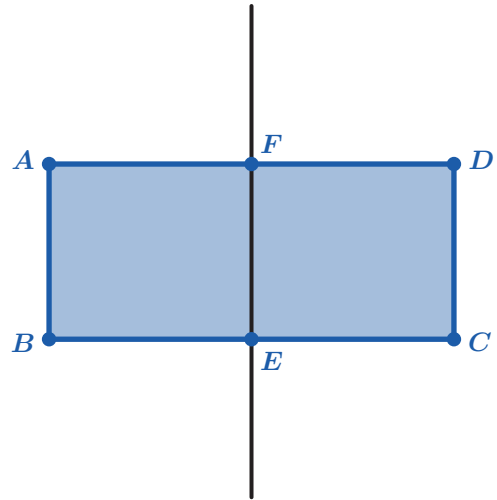


Two figures cannot be congruent if any pair of their corresponding parts are not congruent.

Practice 6.17

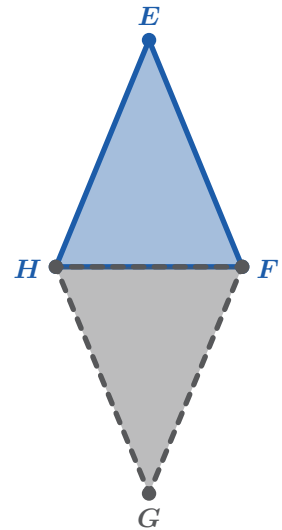
Name: _____ Date: _____ Period: _____


- Rectangle $DCBA$ is the image of rectangle $ABCD$ after a reflection over line EF . How do you know that segment AB is congruent to segment DC ?
 - A rectangle has 2 pairs of parallel sides.
 - Any 2 sides of a rectangle are congruent.
 - Congruent parts of congruent figures are corresponding.
 - Corresponding parts of congruent figures are congruent.



Problems 2–4: Triangle FGH is the image of isosceles triangle FEH after a reflection over line HF .

- What segment in triangle FGH is segment EH taken to by the transformation?
- What angle in triangle FGH is angle HEF taken to by the transformation?



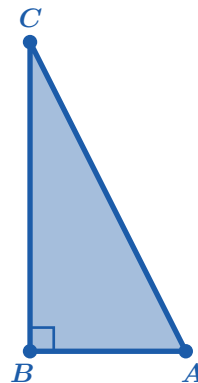
-  **Test Practice** Select *all* the statements that are true because triangles FGH and FEH are congruent.
 - A. Quadrilateral $EFGH$ is a rectangle.
 - B. Quadrilateral $EFGH$ has 4 congruent sides.
 - C. Diagonal FH bisects angles EFG and EHG .
 - D. Diagonal FH is perpendicular to side FE .
 - E. Angle FEH is congruent to angle FGH .

Practice 6.17

Name: _____ Date: _____ Period: _____

5. Triangle ABC is reflected over line BC . Classify triangle ACA' according to its side lengths.

Explain your thinking.



Spiral Review

Problems 6–7: The equation of line a is $2x + 5y = 3$.

6. Write an equation of a line that passes through $(4, -9)$ and is parallel to line a . 7. Write an equation of a line that passes through $(4, -9)$ and is perpendicular to line a .

Problems 8–9: Imagine a set of line segments measuring 12 units, 6 units, 4 units in length. Amir says that this set of line segments will form a triangle.

8. Is Amir correct?

Yes

No

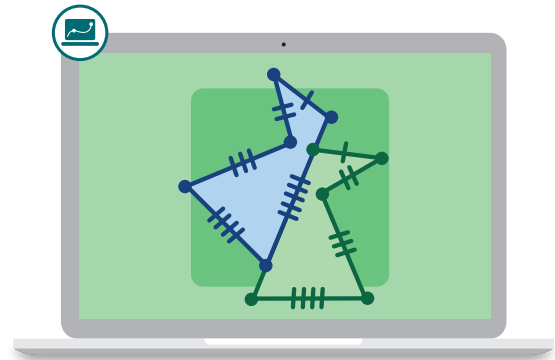
Explain your thinking.

9. Revise one of the side lengths. Explain how you know this will form a triangle.

10. Create a triangle with side lengths 2 centimeters, 5 centimeters, and 6 centimeters.

Congruent Parts, Part 2

Let's name figures in ways that help us see corresponding parts.



Warm-Up

1 Polygon $HIJKL$ is a reflection and translation of polygon $GFONM$.

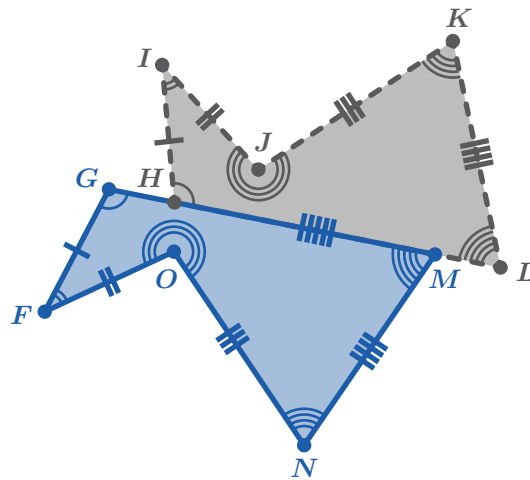
Here are some of the congruent corresponding parts in the polygons:

segment $HI \cong$ segment GF

segment $JI \cong$ segment OF

angle $KLH \cong$ angle NMG

angle $JKL \cong$ angle ONM



What do you notice? What do you wonder?

I notice:

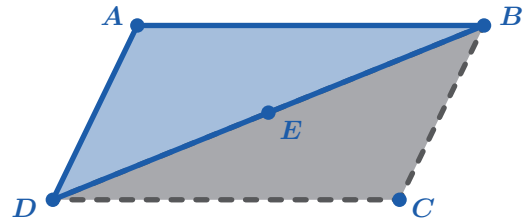
I wonder:

Congruent Polygons

- 2** $\triangle ABD$ was rotated 180° clockwise around point E to map onto $\triangle CDB$. This means the two figures are congruent.

We can represent this in a **congruence statement**:
 $\triangle ABD \cong \triangle CDB$.

 **Discuss:** How can you identify the corresponding parts?



- 3** Quadrilateral $PQRS \cong$ Quadrilateral $ZYWX$. Does angle QRS correspond to angle ZYW ?

Yes

No

I'm not sure

Explain your thinking.

- 4** The polygons are congruent. Determine if this congruence statement is true or false.

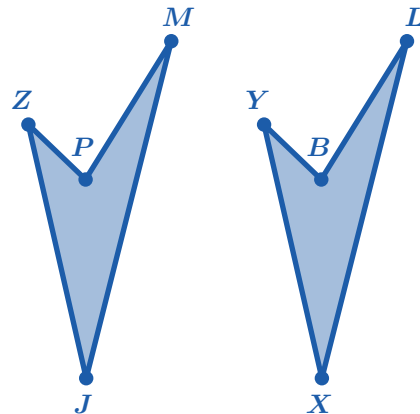
True

or

False

Explain your thinking.

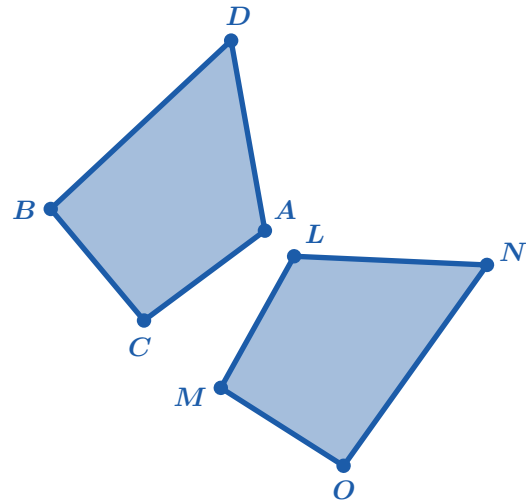
polygon $PZJM \cong$ polygon $YXLB$



Congruent Polygons (continued)

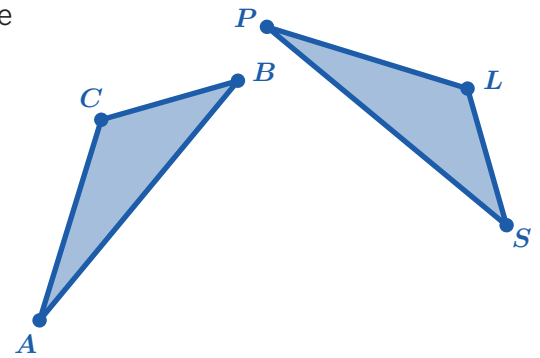
5 The two polygons are congruent. Which congruence statement is correct?

- A. quadrilateral $LNOM \cong$ quadrilateral $DACB$
- B. quadrilateral $LNOM \cong$ quadrilateral $CBDA$
- C. quadrilateral $LNOM \cong$ quadrilateral $BDAC$
- D. quadrilateral $LNOM \cong$ quadrilateral $ADBC$



6 The two triangles are congruent. Complete the congruence statement.

\triangle \cong \triangle



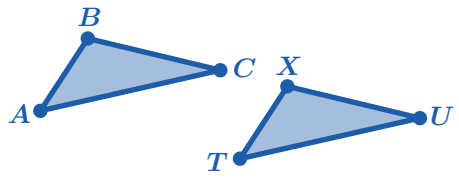
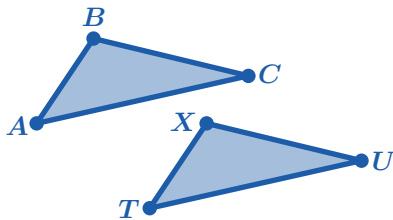
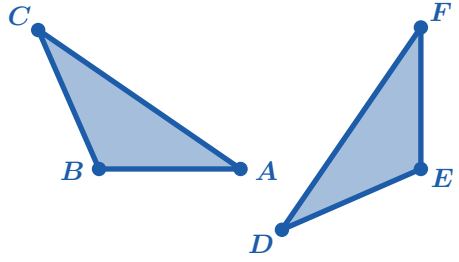
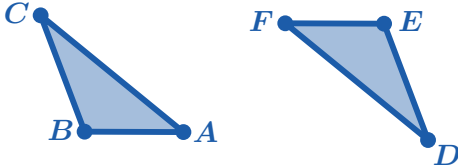
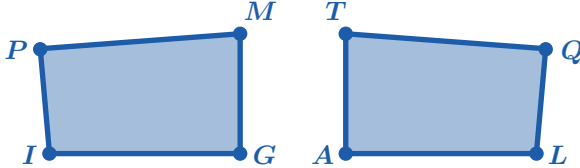
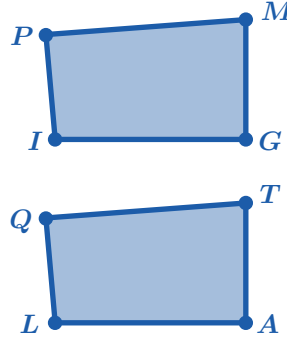
7 Use your congruence statement from Screen 6 to identify these corresponding parts.

- a** Segment BC corresponds to segment
- b** Angle CAB corresponds to angle
- c** Segment AB corresponds to segment

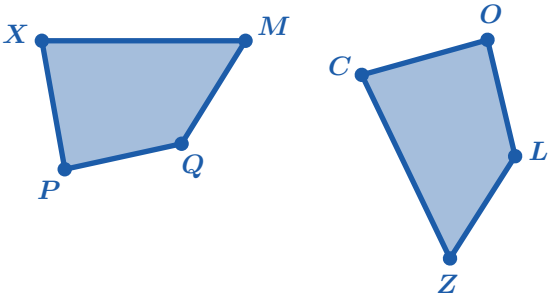
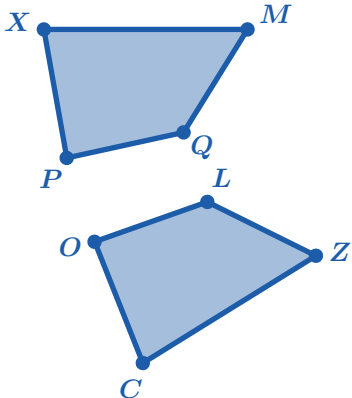
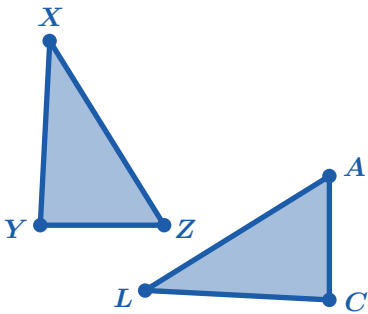
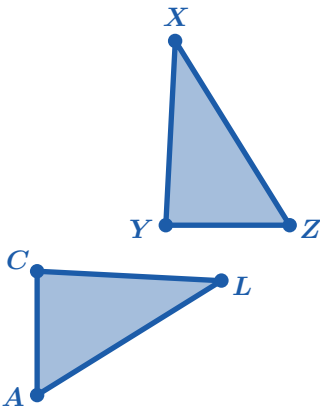
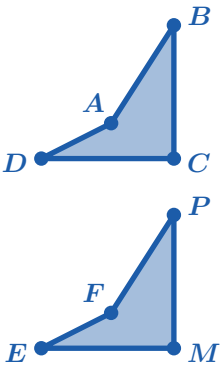
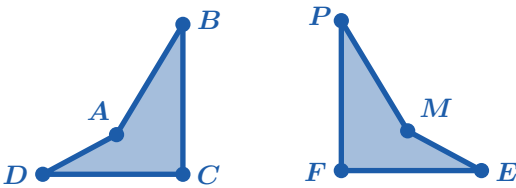
Partner Problems

8 For each of the challenges:

- Decide with your partner who will complete Column A and who will complete Column B.
- Each pair of figures is congruent. Complete as many congruence statements as you have time for.
- The solutions in each row should be the same. Compare your solutions, then discuss and resolve any differences.

Column A	Column B
 <p>$\triangle ACB \cong$ _____</p>	 <p>$\triangle ACB \cong$ _____</p>
 <p>$\triangle ACB \cong$ _____</p>	 <p>$\triangle ACB \cong$ _____</p>
 <p>polygon $PMGI \cong$ _____</p>	 <p>polygon $PMGI \cong$ _____</p>

Partner Problems (continued)

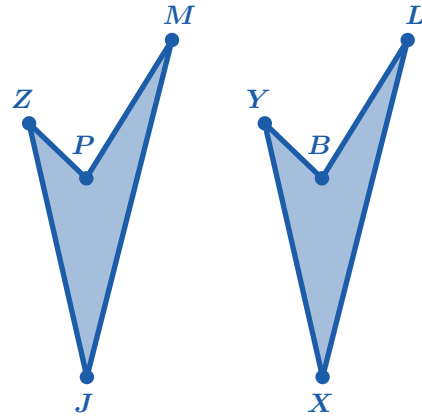
Column A	Column B
 <p>quadrilateral $XMQP$ and quadrilateral $CLOZ$</p> <p>polygon $XMQP \cong$ _____</p>	 <p>quadrilateral $XMQP$ and quadrilateral $OCLZ$</p> <p>polygon $XMQP \cong$ _____</p>
 <p>triangle XYZ and triangle LCA</p> <p>$\triangle XYZ \cong$ _____</p>	 <p>triangle XYZ and triangle CAL</p> <p>$\triangle XYZ \cong$ _____</p>
 <p>triangle $ABCD$ and triangle $EPFM$</p> <p>polygon $ABCD \cong$ _____</p>	 <p>triangle $DCBA$ and triangle $FMEP$</p> <p>polygon $CBAD \cong$ _____</p>

9 Synthesis

What is a congruence statement and what information does it provide?

Use the example if it helps with your thinking.

polygon $PZJM \cong$ polygon $BYXL$



12 Summary 6.18

Congruence statements are helpful when determining corresponding parts of congruent figures.

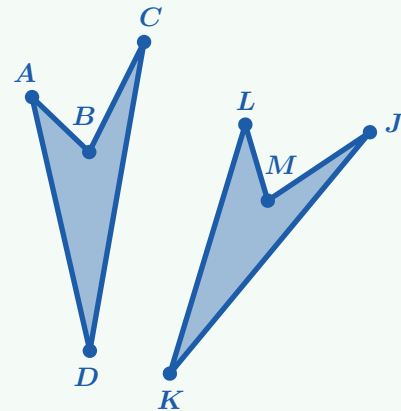
The order in which the vertices are named matters because the congruence statement should match corresponding parts of each figure.

For example, when determining the congruence statement for the two quadrilaterals shown, some possible congruence statements are:

- polygon $BCDA \cong$ polygon $MJKL$
- polygon $ADCB \cong$ polygon $LKJM$

From this statement, corresponding parts can be identified by looking at the ordering of the letters:


- Side AD corresponds with side LK
- Angle DCB corresponds with angle KJM



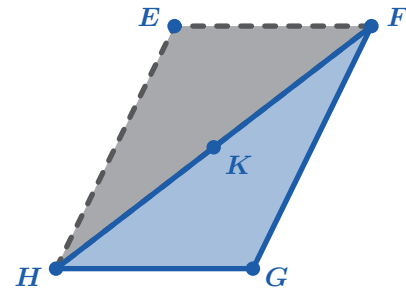
congruence statement A statement that shows the correspondence between the corresponding parts (vertices, sides, and angles) of two congruent polygons. For example, if $\triangle ABC \cong \triangle DEF$, then vertex A corresponds to vertex D , vertex B corresponds to vertex E , and vertex C corresponds to vertex F .

Practice 6.18

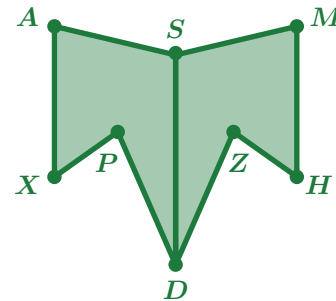
Name: _____ Date: _____ Period: _____

1.  **Test Practice** Triangle HEF is the image of triangle FGH after a 180 degree rotation about point K . Select *all* the statements that must be true.

- A. Triangle FGH is congruent to triangle FEH .
- B. Triangle EFH is congruent to triangle GFH .
- C. Angle KHE is congruent to angle KFG .
- D. Angle GHK is congruent to angle KHE .
- E. Segment EH is congruent to segment GF .
- F. Segment GH is congruent to segment EF .



Problems 2–3: Segment DS is a line of symmetry for figure $AXPDZHM$. Arnav says that figure $AXPDS$ is congruent to figure $HMZDS$ because sides AX and HM are corresponding.

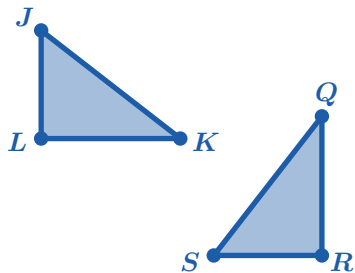


2. Why is Arnav's congruence statement incorrect?

3. Write a correct congruence statement for the polygons.

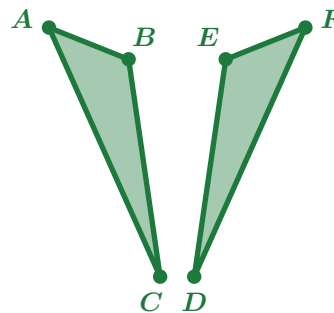
Problems 4–5: Each pair of figures is congruent. Determine whether each congruence statement is true or false.

4. $\triangle JKL \cong \triangle SQR$



True or False

5. $\triangle ABC \cong \triangle EFD$

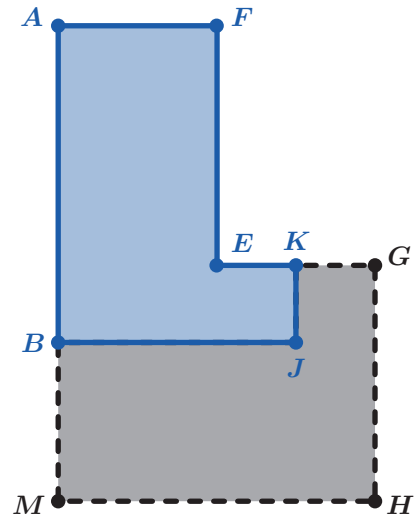


True or False

Practice 6.18

Name: _____ Date: _____ Period: _____

Problems 6–7: Figure $MBJKGH$ is the image of figure $AFEKJB$ after being rotated counterclockwise around point K by 90 degrees.

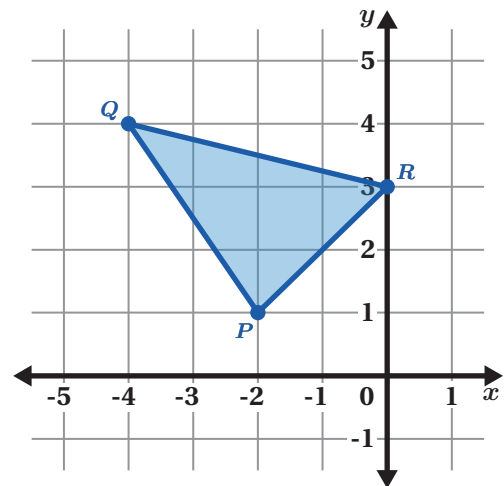


6. Draw a segment in figure $AFEKJB$ to create a quadrilateral. Then draw the image of the segment when rotated counterclockwise around point K by 90 degrees.
7. Write a congruence statement for the quadrilateral you created in figure $AFEKJB$ and the image of the quadrilateral in figure $MBJKGH$.

Spiral Review

Problems 8–9: $\triangle PQR$ has vertices with coordinates $P(-2, 1)$, $Q(-4, 4)$, and $R(0, 3)$.

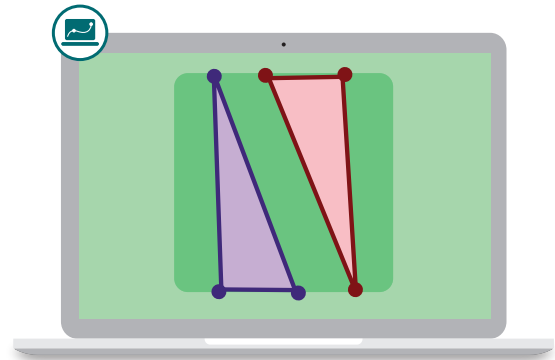
8. Determine the perimeter of $\triangle PQR$. Round your answer to the nearest tenth.



9. Determine the area of PQR .
10. Solve the formula $T = mg + ma$ for a .

Congruent Triangles, Part 1

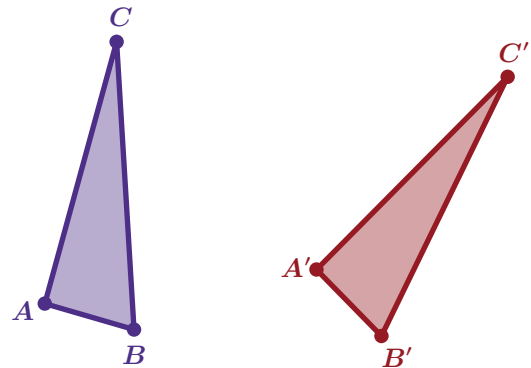
Let's use rigid transformations to determine whether two triangles are congruent.



Warm-Up

1. What *must* be true?

$$\triangle ABC \cong \triangle A'B'C'$$



2. What definitely is *not* true?

Invisible Triangles

3. You will be given either a transformer card or 3 cards with congruent triangles. Do not show your card(s) to your partner.

If you are given the <i>transformer card</i> :	If you are given the <i>triangle cards</i> :
<p>1. Listen to your partner.</p> <ul style="list-style-type: none"> Record the information you receive so that you know which parts of the triangles correspond. 	<p>1. Select a triangle card.</p> <ul style="list-style-type: none"> Silently study the diagram to determine which parts of the triangles correspond. Tell your partner which parts of the triangles correspond.
<p>2. Determine a sequence of rigid transformations that your partner can perform to take one of the triangles onto the other.</p> <ul style="list-style-type: none"> Share the sequence of rigid transformations with your partner. Use precise language. Hint: The notes on your card can help with this. 	<p>2. Listen to the rigid transformation instructions from your partner.</p> <ul style="list-style-type: none"> Use tracing paper to follow their instructions. Sketch the image after each step in the space below. Let your partner know when they have lined up 1, 2, or all 3 vertices of the triangles.

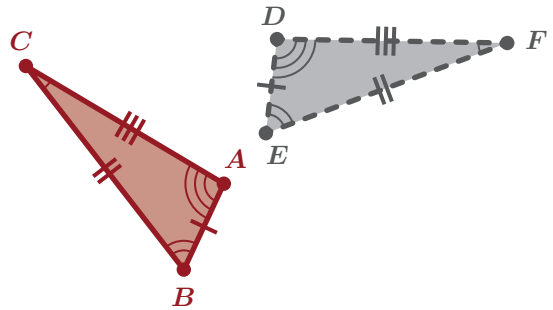
You're invited to explore more.

- 4.** Is it possible to use only one of the three types of rigid transformations to transform triangle ABC onto triangle DEF from Card C in the playing cards? Explain your thinking.

Why Do They Coincide?


Noah and Priya are playing “Invisible Triangles” from Activity 1. Here is the sequence of rigid transformations Noah told Priya to use to take triangle ABC onto triangle DEF .

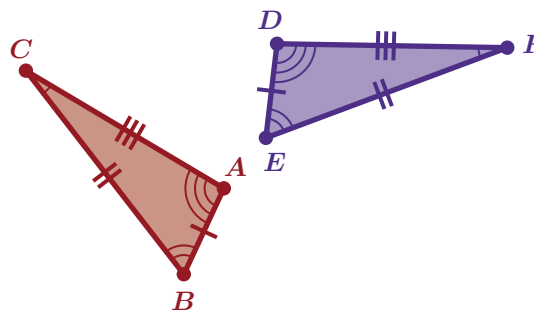
- Step 1:** Translate triangle ABC so that point A goes to point D .
- Step 2:** Rotate triangle $A'B'C'$ counterclockwise around point D by angle $B'DE$.
- Step 3:** Reflect triangle $A''B''C''$ over line DE .



5. After a translation of triangle ABC , Noah and Priya know that point A' will coincide with point D , because that is how the translation was defined. They know that rays DB' and DE can be drawn and will line up after triangle $A'B'C'$ is rotated around point D by angle $B'DE$. Why do points B'' and E have to be in the exact same place?
6. After a reflection of triangle $A''B''C''$ over the line DE . . .
- Priya and Noah know that rays $A'''C'''$ and DF can be drawn. How do they know that ray $A'''C'''$ and ray DF will line up?
 - How do Priya and Noah know point C''' and point F will line up exactly?

Synthesis

7.  **Discuss:** Explain how to use rigid transformations to show two triangles are congruent when all their corresponding parts are congruent.

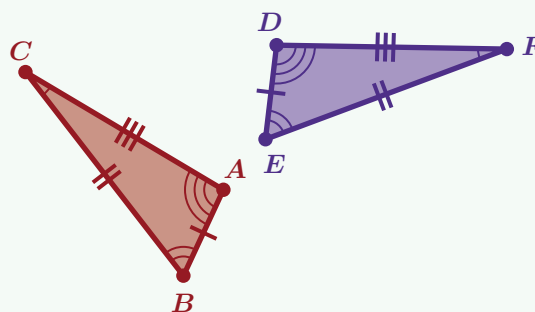


Summary 6.19

You used the definition of congruence in terms of rigid transformations to show that two triangles are congruent **if and only if** corresponding pairs of sides and corresponding pairs of angles are congruent.

If corresponding pairs of sides and corresponding pairs of angles of two triangles are congruent, then the triangles are congruent. This is because one triangle can be taken exactly onto the other triangle using a sequence of translations, rotations, and reflections.

If two triangles are congruent, then the corresponding pairs of sides and corresponding pairs of angles of the two triangles are congruent. This can be justified using properties of rigid transformations to show how corresponding points and rays coincide.



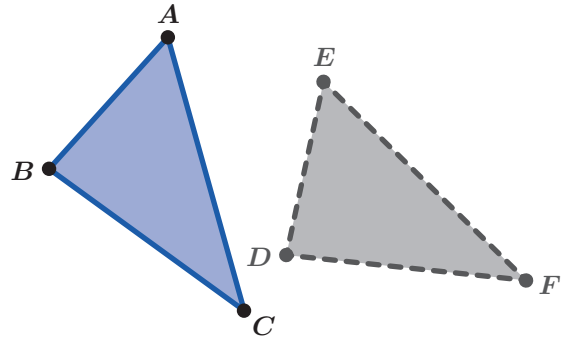
if and only if A phrase commonly added to a geometry statement that means both the statement and its converse are true. For example, for statements A and B , the statement “ A if and only if B ” means that if A is true, then B must be true, and if B is true, then A must be true.


Practice 6.19

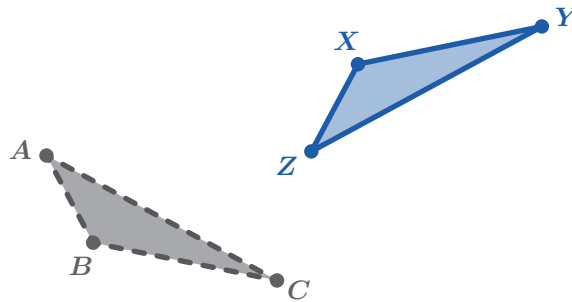
Name: _____ Date: _____ Period: _____

1. Triangle ABC is congruent to triangle EDF , so Na'ilah knows that there is a sequence of rigid transformations that takes triangle ABC to triangle EDF . Select *all* the true statements after the transformations.

- A. Angle A coincides with angle F .
- B. Angle B coincides with angle D .
- C. Segment AC coincides with segment EF .
- D. Segment BC coincides with segment ED .
- E. Segment AB coincides with segment ED .



2.  **Test Practice** The triangles are congruent. Which sequence of rigid transformations will take triangle XYZ onto triangle BCA ?

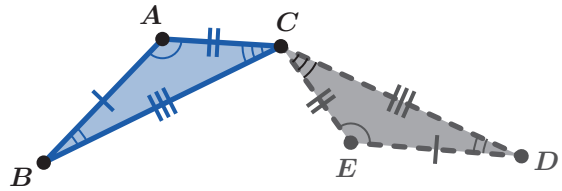


- A. Translate triangle XYZ so that point Y goes to point C . Rotate triangle $X'Y'Z'$ clockwise around point C by angle $X'CB$. Reflect triangle $X''Y''Z''$ over line CB .
- B. Translate triangle XYZ so that point Y goes to point C . Rotate triangle $X'Y'Z'$ clockwise around point C by angle $X'CB$. Reflect triangle $X''Y''Z''$ over line AC .
- C. Translate triangle XYZ so that point Y goes to point C . Rotate triangle $X'Y'Z'$ clockwise around point C by angle $X'CA$. Reflect triangle $X''Y''Z''$ over line CB .
- D. Translate triangle XYZ so that point Y goes to point C . Rotate triangle $X'Y'Z'$ clockwise around point C by angle $X'CA$. Reflect triangle $X''Y''Z''$ over line AC .

Practice 6.19

Name: _____ Date: _____ Period: _____

Problems 3–5: A rotation counterclockwise around point C by angle ACE takes triangle CBA onto triangle CDE .



3. Explain why the image of ray CA lines up with ray CE .
4. Explain why the image of point A coincides with point E .
5. Is triangle CBA congruent to triangle CDE ?

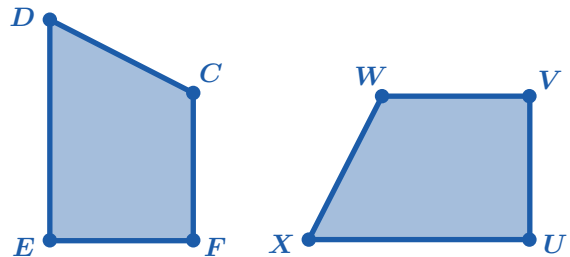
Yes No

Explain your thinking.

Spiral Review

Problems 6–9: These figures are congruent.

6. Write a congruence statement.
7. Write a different congruence statement.
8. Segment CF corresponds to segment
9. Angle VUX corresponds to angle
10. Two triangles have angle measures of 40° , 50° , and 90° . Are two triangles with these measurements *always*, *sometimes*, or *never* congruent?

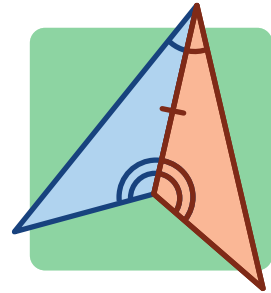


Always Sometimes Never

Explain your thinking.

Congruent Triangles, Part 2

Let's explore shortcuts for determining whether two triangles are congruent.



Warm-Up

Draw triangle ABC with the given measurements. As you create your triangle, circle each necessary measurement you use.

Angle measures	Side lengths
Angle A is 40 degrees.	Segment AB is 5 cm.
Angle B is 20 degrees.	Segment AC is 2 cm.
Angle C is 120 degrees.	Segment BC is 3.7 cm.

Be prepared to explain which given measurements you used to draw your triangle and why you chose those measurements.

Create It!

1. You will use four sets of cards for this activity. Each set has a problem card and a corresponding support card.
 - Decide with a partner who will have the problem card and who will have the support card.
 - Switch roles after each round.

Problem Card Instructions	Support Card Instructions
<ul style="list-style-type: none">• Read the problem aloud.• Respond to the questions your partner asks.• Use the information obtained to create a congruent triangle.	<ul style="list-style-type: none">• Help your partner by asking the questions on the card and following the other instructions.• Check your partner's work to see if they created the triangle. If not, offer an additional piece of information.

Use a protractor and ruler to create your triangles here.

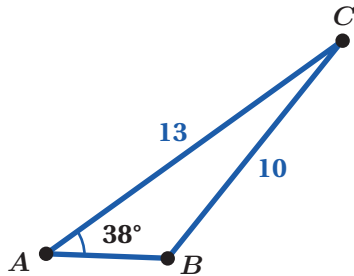
Too Little Information?

Alexis and Keya were working on a problem together. They know:

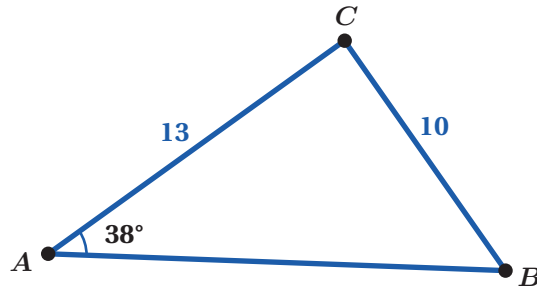
- The length of AC is 13 units.
- The length of BC is 10 units.
- The measure of angle A is 38 degrees.

Here is their work:

Alexis's work



Keya's work



Compare Alexis's and Keya's triangles.  **Discuss:**

- Did Alexis or Keya make a mistake using the measurements they were given?
- Are the two triangles congruent? Explain your thinking.

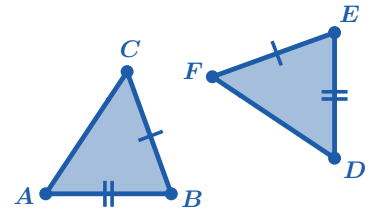
- What information would Alexis and Keya need so that they could be guaranteed to create congruent triangles?

Synthesis

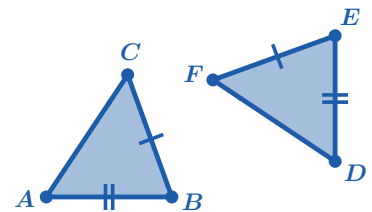
3. Choose one of the following to answer.

What additional information is needed to show two triangles are congruent if . . .

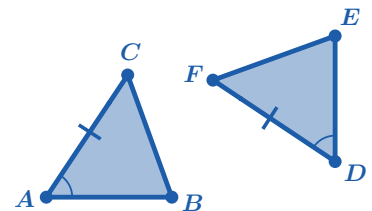
- a** You know two pairs of corresponding sides are congruent?



- b** You know two pairs of corresponding angles are congruent?

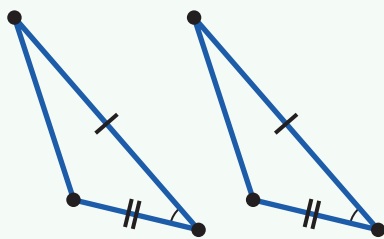


- c** You know one pair of corresponding angles and one pair of corresponding sides are congruent?

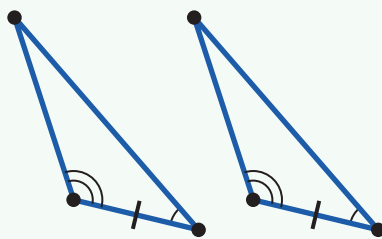


Summary 6.20

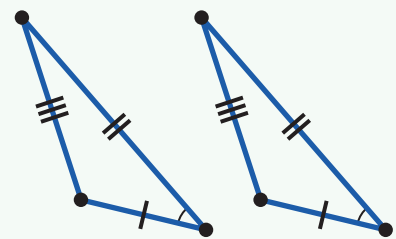
To know that two triangles are congruent, you do not need to know every pair of corresponding parts is congruent. Two triangles are congruent if:



Two pairs of corresponding sides are congruent and the angles between those sides are congruent.



Two pairs of corresponding angles are congruent and the sides between those angles are congruent.



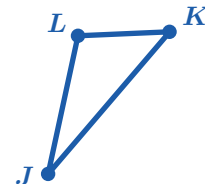
Three pairs of corresponding sides are congruent.

Practice

6.20

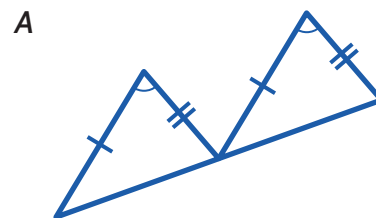
Name: _____ Date: _____ Period: _____

1. What is the least amount of information that you need to construct a triangle congruent to this one?

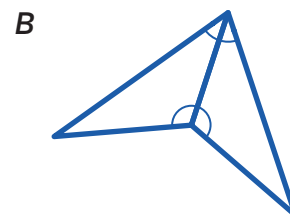


Problems 2–4: Match each figure using only the information given in the pairs of congruent triangles.

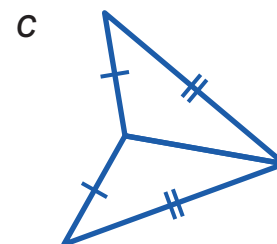
2. In the two triangles, there are three pairs of congruent sides.



3. The two sides and the included angle of one triangle are congruent to two sides and the included angle of another triangle.




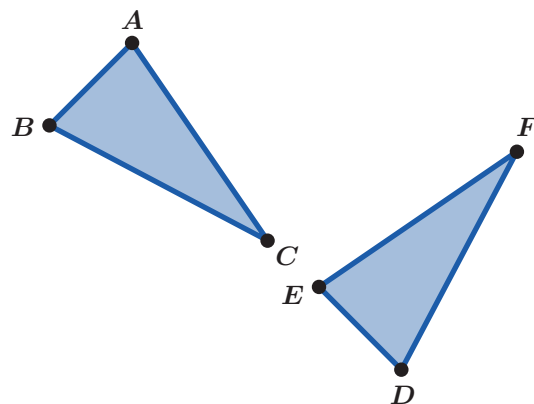
4. The two angles and the included side of one triangle are congruent to two angles and the included side of another triangle.



5. Sketch the unique triangles that can be made with angles measures 40° and 100° and side length 3. Explain how you know you have sketched all possibilities.

Spiral Review

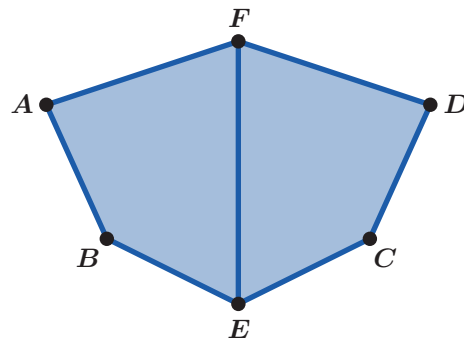
6.  **Test Practice** Triangle ABC is congruent to triangle EDF . There is a sequence of rigid transformations that takes triangle ABC to triangle EDF . Select *all* true statements after the transformation.



- A. Angle A coincides with angle E .
- B. Angle B coincides with angle F .
- C. Segment AB coincides with segment EF .
- D. Segment BC coincides with segment DF .
- E. Segment AC coincides with segment ED .

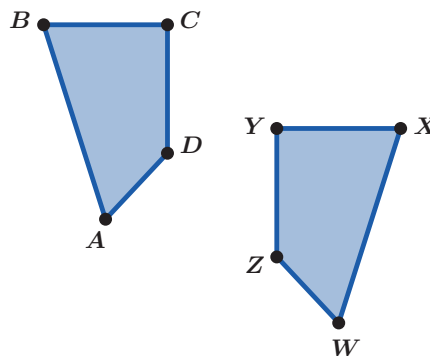
Problems 7–8: Line EF is a line of symmetry for figure $ABECDF$. Ava says quadrilateral $ABEF$ is congruent to quadrilateral $CDFE$ because sides AB and CD are corresponding.

7. Why is Ava's congruence statement incorrect?



8. Write a correct congruence statement for the quadrilaterals.

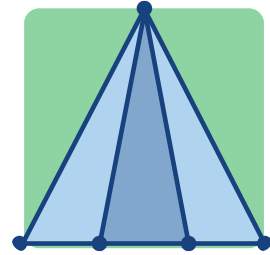
9. Describe a sequence of rigid motions that will take quadrilateral $ABCD$ onto quadrilateral $WXYZ$.



10. Describe the transformation that is represented by $(x, y) \rightarrow (-x, -y)$.

Triangle Palindromes

Let's explore two ways to show triangle congruence.



Warm-Up

1. Using a ruler and protractor draw and label each of the following with measurements:

- Draw a 3 in. line segment and label it AB .
- Draw a 4 in. line segment and label it AC .
- Include a 30° angle between AB and AC .

2. Complete the triangle by drawing line segment BC .

3.  **Discuss:**

- a** Compare your triangle with a partner's. What do you notice?
- b** Is there another triangle you can draw based on the original instructions? Explain your thinking.

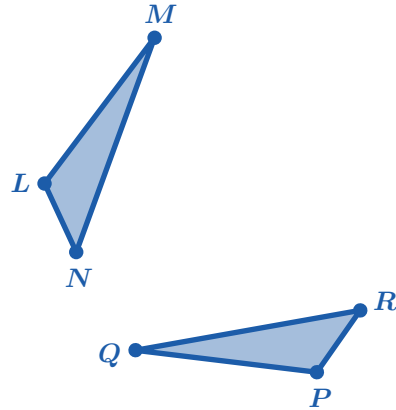
4. What conjecture can you make about the triangles you and your partner created and your responses to Problem 3?

Side-Angle-Side Triangle Congruence

Triangle LMN and triangle PQR have the following properties:

- Segment LM is congruent to segment PQ .
- Segment LN is congruent to segment PR .
- Angle L is congruent to angle P .

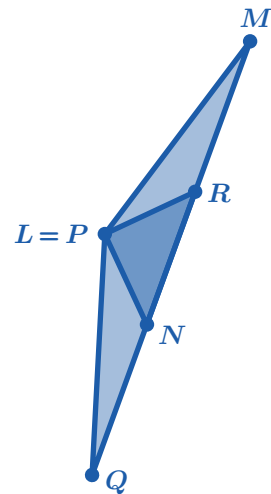
5. Describe a sequence of rigid transformations that takes $\triangle LMN$ onto $\triangle PQR$.



6. What do you know about the relationship between sides MN and QR ?

7. What do you know about the relationship between angle M and angle Q ?

8. Suppose triangles LMN and PQR are drawn as shown. Explain how you know $\triangle LMN$ is congruent to $\triangle PQR$ in this image.



9. Is it possible to draw two triangles with the given information that are not congruent to each other? Explain your thinking.

Angle-Side-Angle Triangle Congruence

10. Using a ruler and a protractor, complete the construction moves to create a triangle with the measurements labeled.

- Construct a 3 in. line segment and label the segment AB .
- Construct a 45° angle with its vertex at point A .
- On the same side of line segment AB , construct a 30° angle with its vertex at point B .

11.  **Discuss:**

- a** Compare your triangle with a partner. What do you notice?

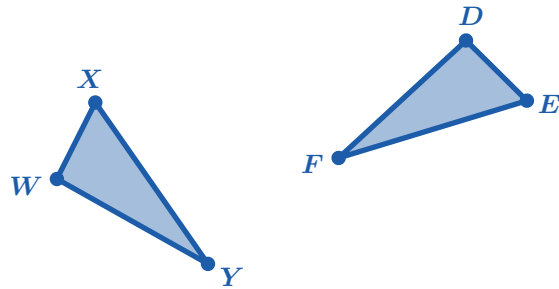
- b** If you used the same information to construct other triangles, how many unique triangles can you create? Explain your thinking.

12. What conjecture can you make based on the triangle you created and your answers to Problem 11?

Angle-Side-Angle Triangle Congruence (continued)

Triangle WXY and triangle DEF are drawn so that they have the following properties:

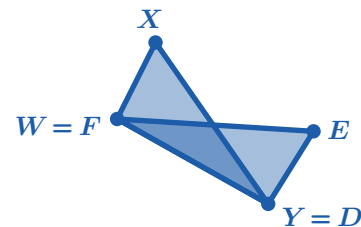
- Angle W is congruent to angle D .
- Angle X is congruent to angle E .
- Side WX is congruent to side DE .



13. Describe a sequence of rigid transformations that takes $\triangle WXY$ onto $\triangle DEF$. Explain your thinking.

14. What must be true about sides XY and EF ? Explain your thinking.

15. Suppose triangles DEF and WXY are drawn as shown. Explain how you know $\triangle DEF \cong \triangle WXY$ in this image.




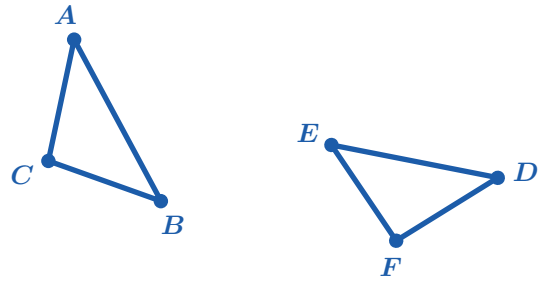
16. Is it possible to draw $\triangle DEF$ and $\triangle WXY$ so that they are not congruent? Explain your thinking.

Synthesis

17. $\triangle ABC$ and $\triangle DEF$ are shown.

- $\angle A$ is 44° , $\overline{AB} = 2.6$, and $\overline{AC} = 1.8$.
- $\angle D$ is 44° , $\overline{DE} = 2.6$, and $\overline{DF} = 1.8$.

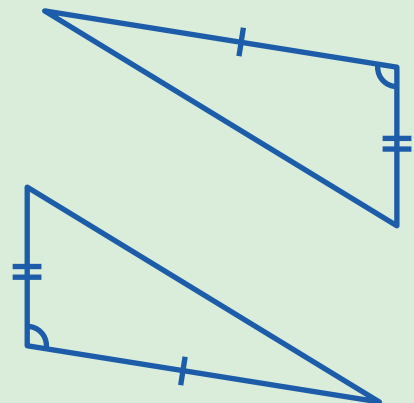
 **Discuss:** Are $\triangle ABC$ and $\triangle DEF$ congruent? Explain your thinking.



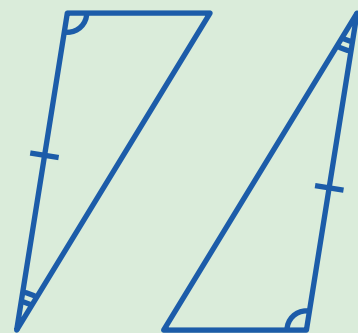
Summary 6.21

Any sequence of rigid transformations that take one triangle onto the other can prove the two triangles are congruent. Rigid transformations can be used to show the following:

side-angle-side triangle congruence theorem Two triangles are congruent when two pairs of corresponding congruent sides and the corresponding angle between them are congruent.



angle-side-angle triangle congruence theorem Two triangles are congruent when two pairs of corresponding angles and the side between them are known to be congruent.

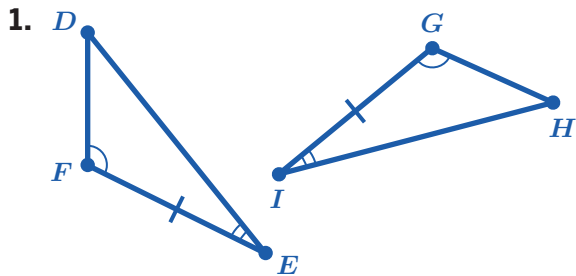


Practice

6.21

Name: _____ Date: _____ Period: _____

Problems 1–3: Determine if the two triangles are congruent based upon the information provided.



- A. Not enough information is provided.
- B. Congruent by side-angle-side congruence theorem.
- C. Congruent by angle-side-angle congruence theorem.
- D. Not congruent.

2. Triangle ABC and triangle XYZ
 Angle A is congruent to angle X .
 Angle B is congruent to angle Y .
 Angle C is congruent to angle Z .

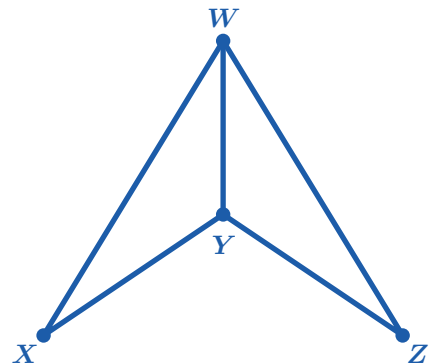
- A. Not enough information is provided.
- B. Congruent by side-angle-side congruence theorem.
- C. Congruent by angle-side-angle congruence theorem.
- D. Not congruent.

3. Triangle ABC and triangle XYZ
 Angle B is congruent to angle Y .
 Segment BC is congruent to segment YZ .
 Segment AB is congruent to segment XY .

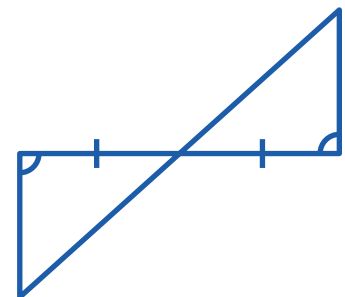
- A. Not enough information is provided.
- B. Congruent by side-angle-side congruence theorem.
- C. Congruent by angle-side-angle congruence theorem.
- D. Not congruent.

4. **Test Practice** What must be congruent to prove $\triangle WXY \cong \triangle WZY$ by SAS ?

- A. $\angle X \cong \angle Z$ and $\overline{XW} \cong \overline{ZW}$
- B. $\angle X \cong \angle Z$ and $\overline{XY} \cong \overline{ZY}$
- C. $\angle XYW \cong \angle ZYW$ and $\overline{XW} \cong \overline{ZW}$
- D. $\angle XWY \cong \angle ZWY$ and $\overline{XW} \cong \overline{ZW}$



5. Determine if these triangles are congruent. Explain how you know.

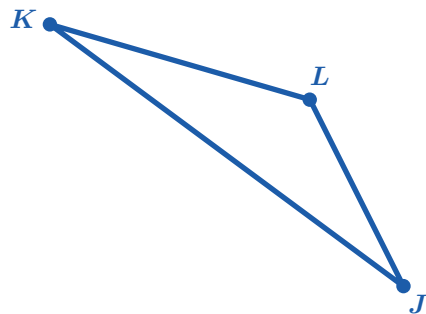


Practice

6.21

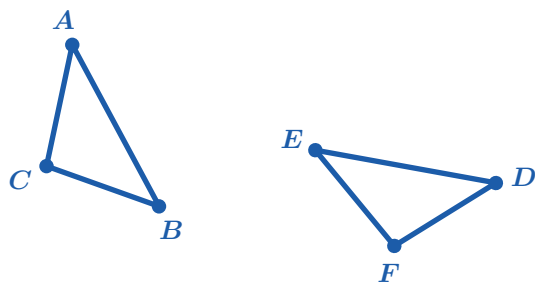
Name: _____ Date: _____ Period: _____

6. Zahra is drawing a triangle that is congruent to this one. She begins by constructing an angle congruent to $\angle LKJ$. What is the least amount of additional information that Zahra needs to construct a triangle congruent to this one?



Spiral Review

7. Triangle ABC is congruent to triangle EDF . So, Vincente knows that there is a sequence of rigid transformations that takes $\triangle ABC$ to $\triangle EDF$. Select *all* true statements after the transformations:



- A. \overline{AB} coincides with \overline{EF} .
- B. \overline{BC} coincides with \overline{DF} .
- C. \overline{AC} coincides with \overline{ED} .
- D. $\angle A$ coincides with $\angle E$.
- E. $\angle C$ coincides with $\angle F$.

8. Match the coordinate transformation to its corresponding description.

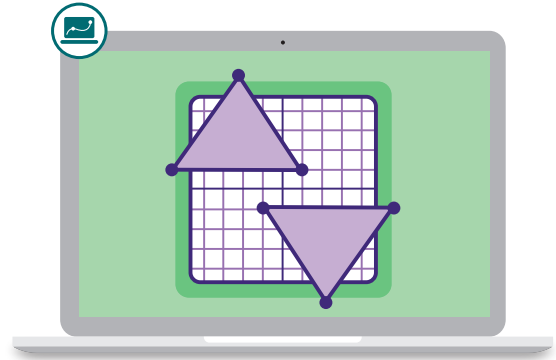
Description

Coordinate Transformation Notation

- | | |
|---|------------------------|
| a. A translation 2 units to the right and 2 units up | $(x, -y)$ |
| b. A dilation with center at $(0, 0)$ and a scale factor of 2 | $(2x, 2y)$ |
| c. A reflection over the x -axis | $(-x, y)$ |
| d. A reflection over the y -axis | $(x + 2, y + 2)$ |

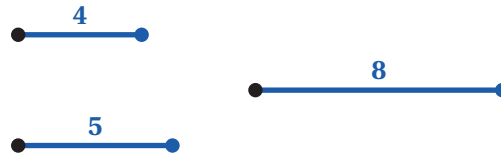
Another Triangle Congruence Theorem

Let's explore one more congruence theorem.



Warm-Up

- 1** Construct a triangle using the given line segments as the 3 side lengths.

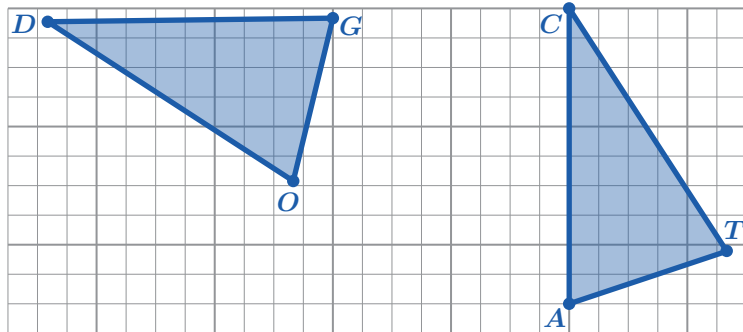


Compare your triangle with a partner. What do you notice?

Three Sides to Every Story?

Triangles CAT and DOG are drawn so that they have the following properties:

- Side CA is congruent to side DO .
- Side AT is congruent to side OG .
- Side CT is congruent to side DG .



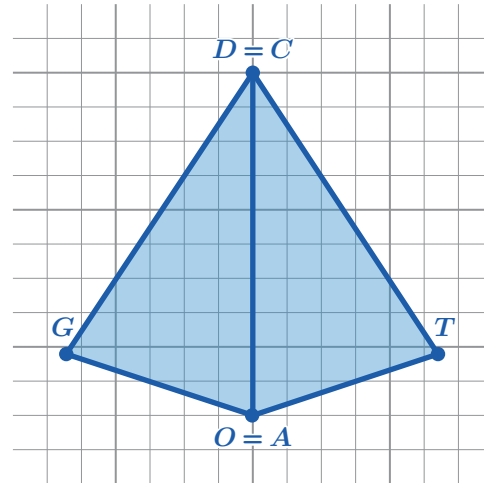
2 Describe a sequence of rigid transformations that takes $\triangle CAT$ onto $\triangle DOG$.

3 What do you know about the relationship between angle C and angle D ?

Three Sides to Every Story? (continued)

4 Here are triangles CAT and DOG .
Which will transform triangle CAT onto triangle DOG ?

- A. Rotate triangle CAT counterclockwise around point C by angle ACG .
- B. Translate triangle CAT along segment OG until point A is taken to point G . Then rotate $C'A'T'$ clockwise around point G by angle OGT .
- C. Reflect triangle CAT across line DO .
- D. Rotate triangle CAT clockwise around point A by angle CAG .



Discuss: Explain how you know these triangles are congruent to each other.

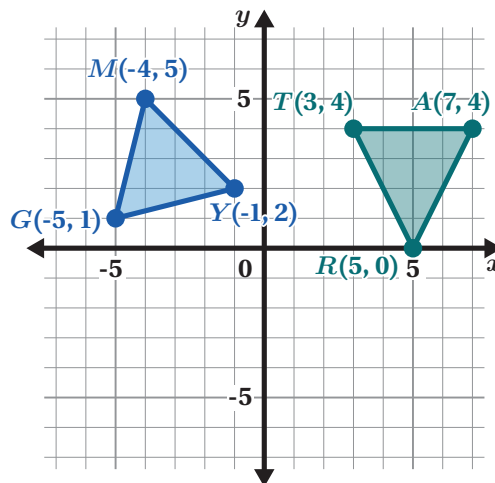
5 Is it possible to draw two triangles with the given information that are not congruent to each other? Explain your thinking.

Which Triangles Are Congruent?

Ava incorrectly believes triangle ART is congruent to triangle GYM .

6 Discuss:

- Why might Ava have thought these triangles are congruent?



- What question could you ask Ava to help her understand that $\triangle ART$ is not congruent to $\triangle GYM$?

7 Use the coordinates to justify that $\triangle ART$ and $\triangle GYM$ are not congruent triangles.

Which Triangles Are Congruent? (continued)

Determine if enough information is provided to prove the pairs of triangles are congruent. Explain your thinking.

8 A rigid transformation moves $\triangle PQR$ so that side $P'Q'$ aligns with side TU , side $Q'R'$ aligns with side UV , and side $P'R'$ aligns with side TV . Is triangle PQR congruent to $\triangle TUV$?

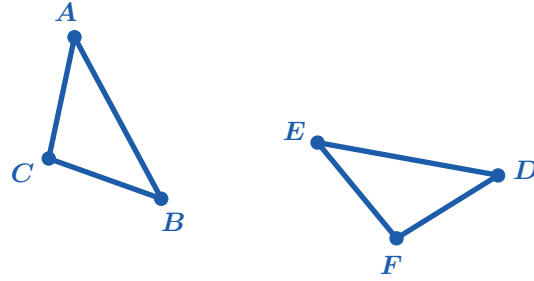
9 Side AB is congruent to side LM . Side BC is congruent to side LN . Is $\triangle ABC \cong \triangle LMN$?

10 Angle F is congruent to angle J . Side DF is congruent to side HJ . Side EF is congruent to side IJ . Is $\triangle DEF \cong \triangle HIJ$?

11 Synthesis

Here is $\triangle ABC$ and $\triangle DEF$.

- The length of segment AB is 2.6 units, and the length of segment BC is 1.8 units.
- The length of segment DE is 2.6 units, and the length of segment EF is 1.8 units.

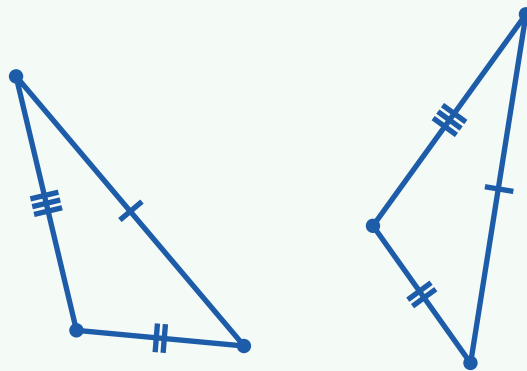


Discuss: What additional information is needed to prove $\triangle ABC$ and $\triangle DEF$ are congruent?

14 Summary 6.22

To prove that two triangles are congruent, look at the diagram and given information and think about whether it will be easier to find pairs of corresponding angles that are congruent or pairs of corresponding sides that are congruent. Then check to see if the information matches the angle-side-angle, side-angle-side, or **side-side-side triangle congruence theorem**.

When we do not have any information about corresponding pairs of angle measures in triangles, see if the side-side-side triangle congruence theorem can be used.



side-side-side triangle congruence theorem Two triangles are congruent when all three pairs of corresponding sides are congruent.

Practice

6.22

Name: _____ Date: _____ Period: _____

Problems 1–3: Determine whether the pair of triangles is *always congruent*, *never congruent*, or *possibly congruent*. Explain your thinking.

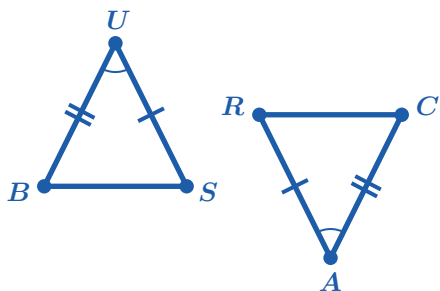
1. Triangle ABC and triangle XYZ


- Segment AB is congruent to segment XY .
- Segment BC is congruent to segment YZ .
- Segment AC is congruent to segment XZ .

2. Triangle ABC and triangle XYZ

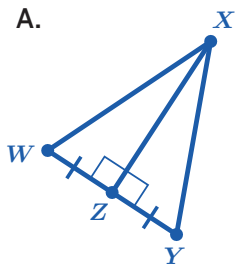
- Angle B is congruent to angle Y .
- Segment BC is congruent to segment YZ .
- Segment AC is congruent to segment XZ .

3.

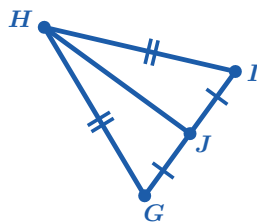


4.  **Test Practice** Which pair of triangles could you prove is congruent using the side-side-side triangle congruence theorem?

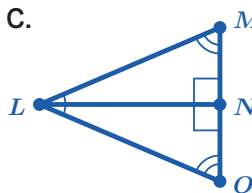
A.



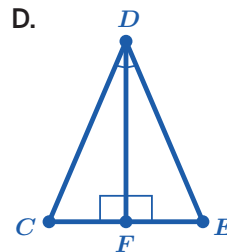
B.



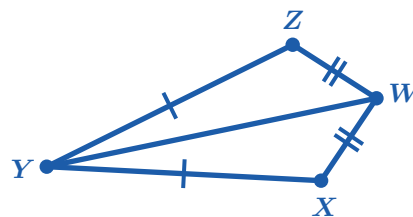
C.



D.



5. Tasia has proven that triangle WYZ is congruent to triangle WYX using the side-side-side triangle congruence theorem. Why can she now conclude that diagonal WY bisects angles ZWX and ZYX ?



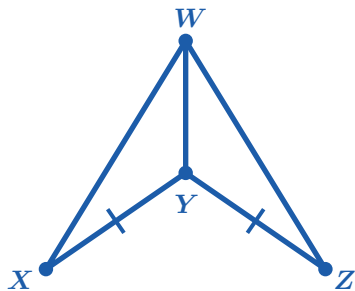
Practice

6.22

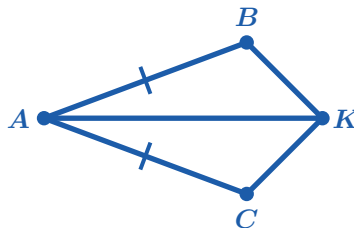
Name: _____ Date: _____ Period: _____

Problems 6–8: What additional information do you need to determine whether these triangles are congruent, using the given theorems?

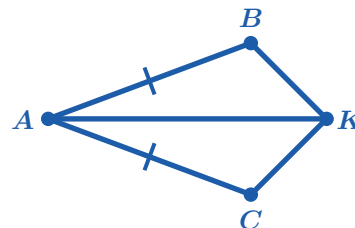
6. SSS



7. SAS

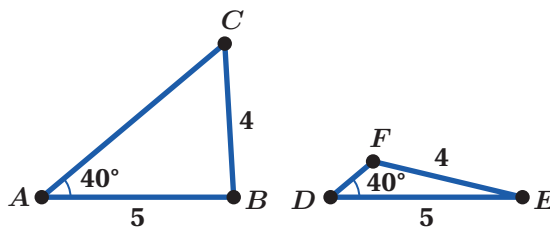


8. SSS



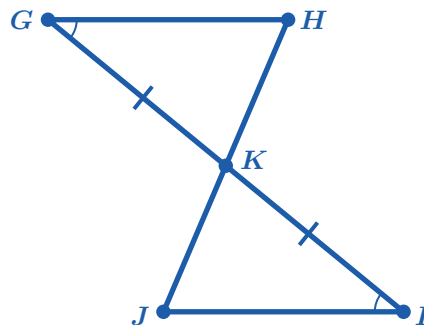
Spiral Review

9. Triangle ABC and triangle DEF have two pairs of congruent corresponding sides and one congruent angle. Why can't you use the side-angle-side triangle congruence theorem prove that these two triangles are congruent?



10. Complete each statement about this figure.

- a Angle GKH is congruent to angle _____ because they are vertical angles.
- b If two corresponding angles and the side between them are congruent, we have the information to use the _____ theorem.
- c Triangle GKH is congruent to triangle _____.



Practice Day 3

Let's practice what you've learned so far in this unit!

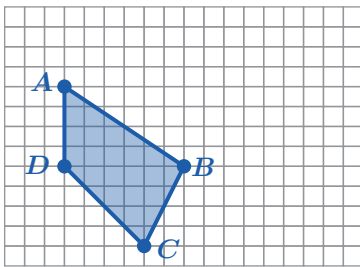


You will use problem cards for this Practice Day. Record all of your responses here.

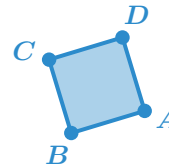
Card 1

a

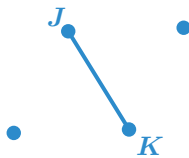
b Here is figure $ABCD$.



Card 2



Card 3



Card 4

Here is a regular 10-sided decagon.

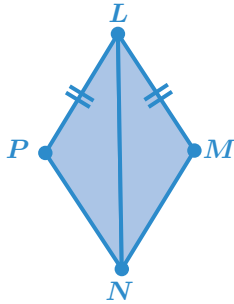
a

b



Practice Day 3 (continued)

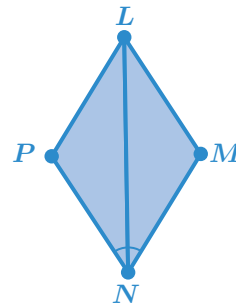
Card 5



a

b $\triangle LMN \cong$ _____

Card 6



a

b $\overline{LP} \cong$ _____

Card 7

Card 8

You're invited to explore more.

Draw and label one pair of congruent triangles with congruence markings, using *at least three* of these statements.

(Note: Not all these statements can be used in the same triangle pair.)

- Point R can be rotated counterclockwise by $\angle RTC$ using point T as the center to move to point C .
- $\angle CAT \cong \angle RTA$
- $\angle CAT \not\cong \angle RTA$
- $\overline{RA} \cong \overline{CA}$
- $\overline{RA} \cong \overline{CT}$

Career Connection

What do you notice about this pattern?



Kozlik/Shutterstock.com

This is an example of a Frieze pattern, which is a repeating pattern that extends horizontally, vertically, or at an angle. All Frieze patterns have translation symmetry in common. They can also have other kinds of symmetries. Most mathematicians agree that there are 7 types of Frieze groups, which describe their possible symmetries. One type of Frieze group is a translation only. Another type is a horizontal mirror.

Architectural designers create designs and plans for construction projects, such as buildings, bridges, and more. They might use Frieze patterns and geometric transformations in their design concepts.



"John Horton Conway" by Thane Plambeck, courtesy of Flickr, licensed under the Creative Commons Attribution 2.0 Generic license.

Meet John Horton Conway

John Horton Conway was a British mathematician and professor of mathematics at both Cambridge University and Princeton University. He was known for his fun-loving attitude toward mathematics. He made significant contributions to the fields of group theory, number theory, algebra, game theory, geometry, and physics. Among his many contributions, he provided playful names for the 7 Frieze groups, including *hop*, *step*, and *sidle*.

Are you interested in exploring Frieze patterns further or studying architectural design? What can you do to learn more?

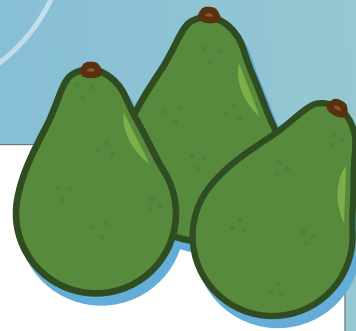
Math in the World

Built mostly between 1238 and 1358, the Alhambra is a palace in Granada, Spain. Many of the walls of the Alhambra are tiled using Frieze patterns. Research the patterns used in the Alhambra walls. What geometric transformations do you see?

Math Mindset

Why might it be important for an architectural designer to understand how rigid transformations preserve congruence?

Unit 7



Describing Data

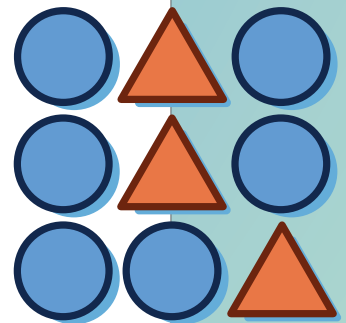
Big Ideas in This Unit

CC1 Comparing Models Correlation and Causation Modeling with Functions

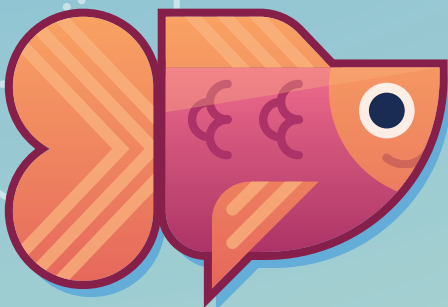
Variability

Questions for Investigation

- What can you use to determine if there is an association in a set of data?
- How can you compare two data sets?
- How can you use statistical tools to explore relationships in real-world data?

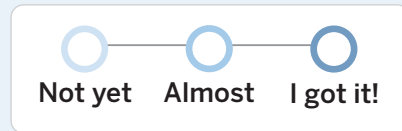


Explore:
A Statistical Question
What is a statistical question?

















Watch Your Knowledge Grow

This is the math you'll explore in this unit. Rate your understanding to see how your knowledge grows!



I can . . .	Before	After
Explain the difference between quantitative and categorical data.		
Represent categorical data in a two-way table and a relative frequency table.		
Calculate values in a two-way table and a relative frequency table and analyze the table for associations in data.		
Use technology to represent data with a dot plot, histogram, or box plot.		
Describe the shape of a data set using terms like bimodal, bell-shaped, skewed, symmetric, and uniform.		
Use technology to calculate the measures of center (mean and median) and measures of spread (IQR and standard deviation).		
Compare and contrast two or more data sets using measures of center and spread that are appropriate for the shape of the data.		
Determine whether or not a data point is an outlier and explain the data point's impact on the mean or median of a data set.		
Use technology to calculate the correlation coefficient of data on a scatter plot.		

I can . . .	Before	After
Use the correlation coefficient to describe the strength and direction of a linear association between two variables on a scatter plot.		
Estimate unknown values using a linear model for data on a scatter plot.		
Describe what the slope and y-intercept for a line of fit mean in a situation.		
Use residuals and residual plots to understand what it means for a line to fit data well.		
Use technology to find the line of best fit for data on a scatter plot.		
Use the equation of a line of best fit to make predictions.		
Determine if the relationship between two variables represents correlation or causation.		

Representing Categorical Data



Explore
A Statistical Question



Lesson 1
Survey Says



Lesson 2
Hear Here



Lesson 3
School Choice



Explore: A Statistical Question

What is a statistical question?



Warm-Up

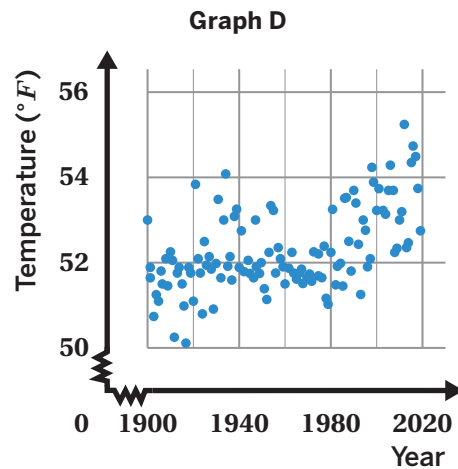
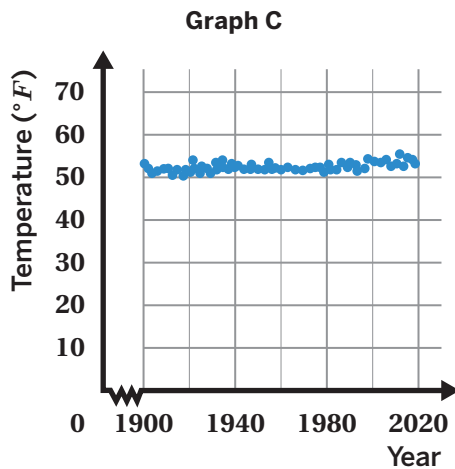
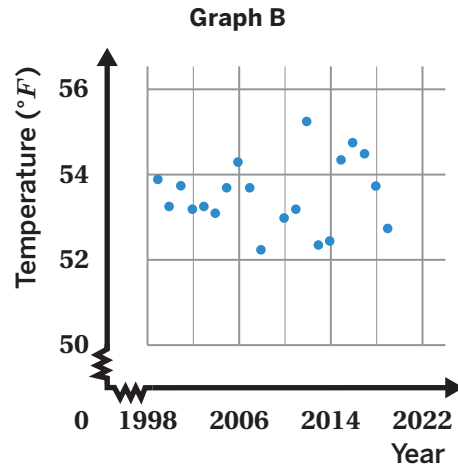
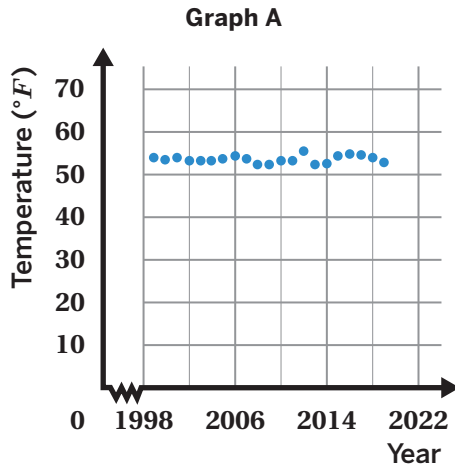
1. You will be shown three different displays of the same data set. For each display, what do you notice? What do you wonder?

	I notice...	I wonder...
Display 1		
Display 2		
Display 3		



Weather vs. Climate

The following four graphs all show the annual average U.S. temperature, but with different scales on the axes.



- While all four graphs show the same data, how do they appear different?
- Which graph(s) support the statement, “The U.S. average temperature is rising”? Explain your thinking.
- If you wanted to convince someone the average U.S. temperature was *not* rising, which graph would you show them? Explain your thinking.



Weather vs. Climate (continued)

Rising temperatures cause changes all over the world, like in Greenland. In recent years, the Greenland ice sheet has been melting due to rising temperatures. This melting has been the focus of much scientific research. Consider these three statements about Greenland's temperatures:

Statement 1: From 1991 to 2019, the winter temperatures of Greenland's coastal region increased by 7.9° F.

Statement 2: From 1991 to 2019, the summer temperatures of Greenland's coastal region increased by 3.1° F.

Statement 3: On January 2, 2020, Greenland recorded its lowest temperature ever, -86° F.

- How are the statements alike? How are they different?
- Assume that each statement is an answer to a question that a researcher asked. Write a possible question the researcher may have asked for each statement.

A *statistical question* is a question that can only be answered using data that varies, meaning there are multiple data points that are not all the same.

- Which of your questions in Problem 6 are statistical questions? Explain your thinking.
- After reading Statement 3, a blogger says that Greenland's ice sheet cannot be getting warmer because Greenland recently recorded its coldest temperature. How would you respond to this blogger?



Building Math Habits of Mind

Discuss:

- Which of these habits of mind did you strengthen during this activity?
- How did you use the one(s) you selected?

I can slow down and first make sense of a challenging problem before trying to solve it.

— —
 Not yet Almost I got it!

I can represent real-world problems and interpret their solutions within the context of the problem.

— —
 Not yet Almost I got it!

I can justify my thinking and ask questions to help me understand the thinking of others.

— —
 Not yet Almost I got it!

I can apply the math that I know to solve real-world problems, make assumptions and revise my thinking as needed.

— —
 Not yet Almost I got it!

I can select an appropriate tool to help me solve problems.

— —
 Not yet Almost I got it!

I can communicate my thinking and solutions clearly to others.

— —
 Not yet Almost I got it!

I can look for structure or patterns to help me solve problems.

— —
 Not yet Almost I got it!

I can look for repeated calculations and other repeated steps to make generalizations.

— —
 Not yet Almost I got it!

Survey Says

Let's make sense of the kinds of data that can be collected and write questions to get to know each other better.



Warm-Up

1 Every year, the U.S. government sends the American Community Survey (ACS) to thousands of people.

The goal is to gather “vital information about our nation and its people.”

What would you like to know about our nation and its people? List at least three questions.

Types of Data

2 Here are the initials of seven people who took the American Community Survey and their responses.

a Here are the responses to each question.

Anonymous People	What is your age?	Do you have health insurance?	What is your main occupation?	How did you get to work or school last week?	What is the monthly rent where you live?
M.A.	19	No	Cook	Bicycle	\$1,200
P.Y.	25	Yes	Construction worker	Car	\$900
Z.M.	53	Yes	Nurse	Walked	\$1,700
B.H.	62	Yes	I don't work	–	–
J.S.	44	Yes	Accountant	Worked from home	\$1,250
O.L.	12	I'm not sure	Student	Car	–
A.B.	29	No	Carpenter	Bus	\$1,050

b  **Data Talk!** What do you notice about the responses? What do you wonder?

I notice:

I wonder:

Types of Data (continued)

3 Asking different types of questions can produce different types of data.

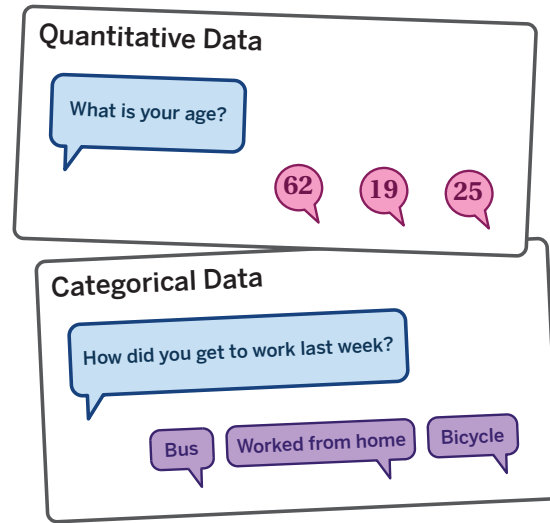
What is your age? produces **quantitative data** (also called *numerical data*).

How did you get to work last week? produces **categorical data**.

Describe what you think these terms mean.

Quantitative data:

Categorical data:



4 Which type of data would each of these questions produce?

Question	Categorical or Quantitative?
Do you live within two miles of a grocery store?	
What type of fuel is used to heat your home?	
What is the monthly rent where you live?	
How old is the youngest person in your family?	
How many minutes does it take you to get to work or school?	
What type of building do you live in?	

5 Convince someone about which type of data this question would produce: *Do you live within two miles of a grocery store?*

Crafting and Asking Questions

- 6** Jalen is making a survey that includes a question about people's television-watching habits.

He wrote three different ways to ask the question.

- Do you watch a lot of television?
- How many hours of TV did you watch last night?
- How much TV did you watch last month?

- a**  **Discuss:** What are some advantages and disadvantages of each question?

- b** If you were making the survey, how would you write the question?

- 7** Think of something you want to learn about your classmates.

- a** Write a question for your classmates to answer. Will the data you collect be quantitative or categorical?

Question:


Circle one: Categorical Quantitative

- b** Collect responses to your question.

Initials	Response	Initials	Response


Crafting and Asking Questions (continued)

8 Look at your classmates' responses to your question.

a  **Data Talk!** Summarize the data in some way.

b Now that you've seen the responses, how would you change your survey question?

9 Synthesis

 **Discuss:** How can you decide whether a question will produce **quantitative data** or **categorical data**?

How many minutes does it take you to get to work or school?

What is your age?

What type of fuel is used to heat your home?

12 Summary 7.01

Different types of questions lead to different types of data, such as **quantitative data** and **categorical data**.

Questions for Quantitative Data	Questions for Categorical Data
How many pets do you have?	What's your favorite animal?
What is your age?	In what month were you born?
How many minutes does it take you to get to work or school?	How did you get to work last week?
How many hours do you spend on hobbies weekly?	What hobbies do you have?

It's important to be specific when writing survey questions, so you can gather the exact type of data you need.

quantitative data Data represented as numbers, quantities, or measurements that can be meaningfully compared is called quantitative data, or *numerical data*.


categorical data Data with values that are represented as categories, such as colors, words, or ZIP Codes, instead of values that represent a quantity or measurement.

Practice 7.01

Name: _____ Date: _____ Period: _____

- Which would *not* be a good survey question?
 - What grade are you in?
 - How many books did you read last year?
 - How many inches are in 1 foot?
 - How many pets do you have?
- Determine which type of data these questions produce.

Question	Categorical or Quantitative?
How old are you?	
Do you have any pets?	
How many siblings do you have?	

-  **Test Practice** Select *all* the questions that would produce quantitative data.
 - How many people live in your home?
 - What is your favorite breakfast food?
 - How did you travel to school this morning?
 - How many minutes did it take you to get ready this morning?
 - What is the last thing you ate or drank?
- Callen claims that players on the school basketball teams are taller than players on the soccer teams. Write two survey questions that Callen could ask to investigate the claim.
 - _____
 - _____

Problems 5–6: Nikolai wants to know about the types of food his classmates prefer.

- Write a survey question that would give him *categorical* data about his classmates' food preferences.
- How might you change your question from Problem 5 so that it gives *quantitative* data?
- Think about your community. What information would you like to know about the people in it? Write an example of one question that will produce quantitative data and one question that will produce categorical data about your community.

Quantitative	Categorical

Practice 7.01

Name: _____ Date: _____ Period: _____

Problems 8–9: Here are some responses to the question: *What is your birthday?*

January 7 March 18 December 23

8. Jaylin is not sure whether the data is categorical or quantitative. Explain why this type of data is unclear.

9. What is another question that might generate data that is unclear?

Spiral Review

Problems 10–12: A scientist is studying how two different types of bacteria grow. This table represents the number of bacteria cells in the days since her experiment began.

	Day 0	Day 1	Day 2	Day 3	Day 4
Bacteria A (number of cells)	1	3	9	27	?
Bacteria B (number of cells)	80	100	120	140	?

10. If the patterns continue, which type of bacteria will there be more of after 4 days?
11. If the patterns continue, which type of bacteria will there be more of after 10 days?
12. The scientist begins looking at a third bacteria for her experiment. Bacteria *C* has 50 cells on Day 0 and follows the same growth pattern as Bacteria *A*. Write an equation for the function $c(x)$ that represents the number of cells for Bacteria *C* after x days.

Problems 13–14: Tasia is organizing two sets of marbles.

13. How many marbles are in each set?

Set A: _____

Set B: _____

	Green Marbles	Marbles That Are Not Green
Set A	17	23
Set B	13	12

14. Across both sets, how many marbles does Tasia have in total that are *not* green?

Hear Here

Let's use categorical data to make an argument.



Warm-Up

1. **a** Order these sounds from *quietest* to *loudest*.

- Traffic noise inside a car
- Siren
- Sporting event
- Washing machine
- Leaf blower
- Rock concert

Quietest

Loudest

b Which of these sounds do you think could cause hearing loss?

Hearing Loss

People in Metropolis are worried that having trains pass through their town might be associated with hearing loss. Trains have been running in the nearby city of Springtown for four years.


They compare hearing loss in their town to hearing loss in Springtown to see if there is an **association** between train noise and hearing loss.

2. Here is a **two-way frequency table**.

- a Complete the table.

	No Hearing Loss	Hearing Loss	Total
People in Metropolis	228,816		269,196
People in Springtown	34,929	9,852	
Total	263,745		313,977

- b Write a statement that is true based on the data in the table.

- c  **Data Talk!** Make a claim about whether hearing loss is more common in Springtown or in Metropolis based on the data in the table.

- d What *grand total* is given in the table? Interpret what this total means in the context of the situation.


Hearing Loss (continued)

Rishi notices that the population of Metropolis is larger than the population of Springtown.

He says: *Comparing the total number of people with hearing loss in each city doesn't make sense.*

He uses the values in the table to calculate what percent of people in Metropolis have no hearing loss, and then makes a **conditional relative frequency table**.

	No Hearing Loss	Hearing Loss	Total
People in Metropolis	$\frac{228816}{269196} = 85\%$		100%
People in Springtown			

3. **a**  **Discuss:** Explain how Rishi calculated the percent of people in Metropolis who have no hearing loss.
- b** Complete the table.
- c** If you're concerned about hearing loss in your community, does it matter whether you live in Metropolis or Springtown? Explain your thinking.

Hearing Loss and Age

Tyani says: *I don't have enough information to decide whether there is an association between hearing loss and the city you live in because age might also be relevant to hearing loss.* She decides to explore data by age for both cities.

4. Complete the tables.

Metropolis Population

	No Hearing Loss	Hearing Loss	Total
18 or Younger	53,669		
Over 18		40,110	
Total	228,816	40,380	269,196

Springtown Population

	No Hearing Loss	Hearing Loss	Total
18 or Younger		179	8,956
Over 18	26,152		
Total			44,781

5. Construct a conditional relative frequency table for Metropolis and Springtown by age groups. Round your answers to the nearest tenth of a percent.


Metropolis Population

	No Hearing Loss	Hearing Loss	Total
18 or Younger			100%
Over 18			100%

Springtown Population

	No Hearing Loss	Hearing Loss	Total
18 or Younger			100%
Over 18			100%

Hearing Loss and Age (continued)

6. Adhira says: *Out of all the people who are over the age of 18 in Metropolis, about 19% of them already have hearing loss, so the trains aren't going to make a big difference.* Do you agree or disagree with Adhira? Explain your thinking.
7.  **Data Talk!** Based on this data, do you think there is an association between the city that people live in and hearing loss? Explain your thinking.
8. Is the association strong enough to cause the people of Metropolis to act?
9. Hearing loss is one impact that trains might have on the population of Metropolis. What are some other impacts that trains might have on the population of Metropolis?

Synthesis

10. When might a **two-way frequency table** be useful? When might a **conditional relative frequency table** be useful?

Summary 7.02

It can be helpful to represent categorical data in a **two-way frequency table** or a **conditional relative frequency table**. A two-way frequency table shows exact counts for different variables while a conditional relative frequency table shows percents. You can use either table to see if the data presents evidence that there is an **association** between the two variables.

Arts Studio Students

	Takes Dance Class	Does Not Take Dance Class	Total
13 or Older	105	121	226
Under 13	89	62	151
Total	194	183	377

Arts Studio Students

	Takes Dance Class	Does Not Take Dance Class	Total
13 or Older	≈46.5%	≈53.5%	100.0%
Under 13	≈58.9%	≈41.1%	100.0%

In these tables, there is evidence of an association in the data between age and whether a student takes dance class. This association is suggested because the students under 13 are much more likely (≈58.9%) to take dance class than the students 13 or older (≈46.5%).

two-way frequency table A table used to compare two categorical variables that shows one of the variables across the top of the table and the other down one side. Each entry in the table is the frequency of the category shown by the column and row headings.

conditional relative frequency table A table used to compare data across two categorical variables. It presents the fraction or percent of the data that is in each category, instead of the actual number of data points, with either both rows or both columns totaling 100% based on the condition of interest.

Practice 7.02

Name: _____ Date: _____ Period: _____

Problems 1–4: This two-way frequency table shows student responses to the question: *Do you prefer to have gym class in the morning or in the afternoon?*

	Morning	Afternoon	Total
6th Grade	15	8	23
8th Grade	18	21	39
10th Grade	12	26	38
Total	45	55	100

- How many students participated in the survey?
- How many 8th grade students prefer to have gym class in the morning?
- How many 10th grade students participated in the survey?
- How many students prefer to have gym class in the afternoon?
- A random sample of teenagers were asked about their preferences when organizing a birthday party for a friend. Use this information to complete the two-way table:
 - 122 people responded to the survey.
 - 50 of the people who said they prefer surprise parties also prefer that guests bring individual gifts.
 - 68 people prefer organizing one group gift.
 - 56 people prefer not to have a surprise party.

	Surprise Party	No Surprise	Total
Group Gift			
Individual Gifts			
Total			


Problems 6–7: Students in the 7th, 8th, and 9th grade were asked whether they prefer to write in pen or pencil. 40 students prefer to write in pen and 60 students prefer to write in pencil.

- Create values that could represent the number of students in the 7th, 8th, and 9th grade that responded to the survey.
- Write a statement about the data in your table.

	Pen	Pencil
7th Grade		
8th Grade		
9th Grade		
Total		

Practice 7.02

Name: _____ Date: _____ Period: _____

8.  **Test Practice** This conditional relative frequency table shows data on whether students listen to music while they do their math and Spanish homework. Based on this data, is there an association between the homework subject and whether students listen to music?

Explain your thinking.

	Music	No Music	Total
Math Homework	61%	39%	100%
Spanish Homework	28%	72%	100%

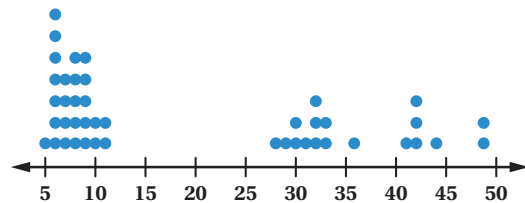
9. 1,430 people are surveyed about whether they take daily vitamins and whether they eat breakfast. The table shows the results. Create a relative frequency table based on the grand total that shows the percentage of the entire group that is in each cell.

	Take Daily Vitamins	Don't Take Daily Vitamins
Eat Breakfast	384	476
Don't Eat Breakfast	268	302

	Take Daily Vitamins	Don't Take Daily Vitamins
Eat Breakfast		
Don't Eat Breakfast		

Spiral Review

Problems 10–11: This dot plot represents the ages of people who watched a movie at a movie theater on a Saturday.



10. What kind of movie do you think this is?
Explain your thinking.

11. How many people over 40 years old watched the movie on Saturday?


School Choice

Let's use data to make decisions and write arguments.



Warm-Up

Benjamin Banneker Secondary School won a grant to remodel their outdoor space.

1.  **Discuss:** If your school won a similar grant, what are some options you would suggest?
2. The principal thinks it would serve the school best if the grant money were spent on new basketball courts or a new picnic area. Which option would you choose to build for your school? Circle your choice.

Basketball Courts

Picnic Area

Explain your thinking.

Courtyard Questions

The principal surveyed the students to help the school decide whether to build basketball courts or a picnic area. The school has students in middle school (Grades 6–8) and high school (Grades 9–12).

The principal collected some data and displayed it in the *total relative frequency table* shown. The table contains **joint relative frequencies** and **marginal relative frequencies**. The joint relative frequencies are the entries in the body of the table where the different categorical variables intersect. The marginal relative frequencies are the totals for the rows and columns.

3. Complete the table.

	Basketball Courts	Picnic Area	Total
Middle School	30%		37%
High School		41%	
Total	52%		

4. Does the school have more students in middle school or high school? Explain how you know.
5. Kadeem says: *The grade level with the largest number of students should get to decide what we do.*

Do you agree or disagree? Explain your thinking.

Courtyard Questions (continued)


Problems 6–9: Take a closer look at the survey results.

6. What percent of the total school population chose basketball courts?

7. What does the 41% mean in this situation?

8. Which choice was more popular for each group of students? Place a check mark in the correct column.

Group of Students	Basketball Courts	Picnic Area
Middle School		
High School		
All Students		

9.  **Discuss:** Is there an association between grade level and whether the students want to build basketball courts or a picnic area? Use data from the table to support your claim.

Making Decisions

Sothy wanted to know how many people voted for each category. He knows that the school has 800 students total and that all students participated in the survey.

	Basketball Courts	Picnic Area	Total
Middle School	30%	7%	37%
High School	22%	41%	63%
Total	52%	48%	100%

10. Complete the table with the number of students in each category.

	Basketball Courts	Picnic Area	Total
Middle School			
High School			
Total			800

11. Write a true statement about the situation using each fraction.

a $\frac{240}{416}$

b $\frac{416}{800}$

c $\frac{328}{800}$

12. Demetrius says: *About 58% of people who want to build basketball courts are middle school students.*

Nikolai says: *81% of middle school students want to build basketball courts.*

Who do you agree with? Circle one.


Demetrius Nikolai Both Neither

Closing Arguments

One middle schooler and one high schooler decided to talk to the principal about what the school should choose.


Kayleen (a middle schooler) said: *The school should build basketball courts because it's the most popular across the whole school.*

Mariam (a high schooler) said: *High school students have been here the longest, and we think a picnic area would benefit the entire school the most.*

13.  **Data Talk!** Imagine you want to convince the principal which choice the school should make. Write an argument using data, reasoning, and your personal preference about what choice the school should make.

14. What is a decision that your school is trying to make? How might a two-way frequency table help you make this decision?

Synthesis

15.  **Discuss:** Describe a strategy for recognizing whether there is an association in a two-way table. Use the example if it helps with your thinking.

	Basketball Courts	Picnic Area	Total
Middle School	30%	7%	37%
High School	22%	41%	63%
Total	52%	48%	100%

Summary 7.03

Two-way tables can be helpful when making a decision. The distribution of data in a *total relative frequency table* can indicate whether there are associations in the data or other information that might support you in making a fair and informed decision.

The table shows the data gathered by Tyani's principal about which field trip the students would prefer to take: going whale watching or going to an art museum.

Total Relative Frequency Table

	Whale Watching	Art Museum	Total
Tyani's Homeroom	7%	5%	12%
Other Homerooms	41%	47%	88%
Total	48%	52%	100%

The **joint relative frequencies** show that Tyani's homeroom has a stronger preference for whale watching because $7% > 5%$. Other homerooms have a preference for going to the art museum because $41% < 47%$.

The **marginal relative frequencies** for the columns show that there are more students overall who would prefer to go to the art museum.

joint relative frequency The ratio of a frequency in the body of a two-way frequency table to the total number of data points, or the grand total. The sum of the joint relative frequencies is 100%.

marginal relative frequency The ratio of a row or column total in a two-way frequency table to the total number of data points, or the grand total. The sum of the marginal relative frequencies for each categorical variable is 100%.

Practice 7.03

Name: _____ Date: _____ Period: _____

1. A group of student athletes set personal goals for a big race. After the race, they were asked whether they achieved their goal and whether they practiced daily.

This table shows the results. What does 35% represent?

	Daily Practice	No Daily Practice
Achieved Goal	35%	14%
Did Not Achieve Goal	11%	40%

2. A group of people were asked whether they have any siblings and whether they have any pets.

What percent of people have no siblings and no pets?

	Have Siblings	Have No Siblings
Have Pets	20.5%	26%
Have No Pets	35.5%	?

3. A fashion magazine asked 250 people whether they wear a belt and a watch each day. Use the data in the total relative frequency table to complete the two-way frequency table.

Total Relative Frequency

	Watch	No Watch
Belt	44%	22%
No Belt	20%	14%

Two-Way Frequency

	Watch	No Watch
Belt		55
No Belt		

Problems 4–5: Scientists gave two different treatments to people who have the flu, and then determined whether their health improved. This two-way frequency table shows the results.

	Treatment 1	Treatment 2	Total
Improved Health	28	30	58
No Improvement	23	38	61
Total	51	68	119

4. What percent of people receiving Treatment 1 had improved health?
5. What percent of people receiving Treatment 2 had improved health?

Practice 7.03

Name: _____ Date: _____ Period: _____

Test Practice Problems 6–7: A random sample of people were asked about their preferences in home decoration and their interest in fashion.

6. Use this information to complete the two-way frequency table.

- 150 people completed the survey.
- 70% of the people do not enjoy fashion.
- 33 of the people who prefer neutral decorations also do not enjoy fashion.
- 20 of the people who enjoy fashion also prefer colorful decorations in their homes.

	Prefer Colorful Decorations	Prefer Neutral Decorations	Total
Enjoy Fashion			
Don't Enjoy Fashion			
Total			150

7. Create a total relative frequency table for this data.

	Prefer Colorful Decorations	Prefer Neutral Decorations	Total
Enjoy Fashion			
Don't Enjoy Fashion			
Total			100%

Spiral Review

Problems 8–10: Complete the coordinate transformation notation so it represents the given description.

8. Dilation with a scale factor of $\frac{1}{2}$ that uses the origin as the center.
 $(x, y) \rightarrow$

9. Translation 6 units left and 4 units down.
 $(x, y) \rightarrow$

10. Reflection over the x -axis
 $(x, y) \rightarrow$

11. What are the solutions to $|-3x - 4| \geq 11$?

Notes:

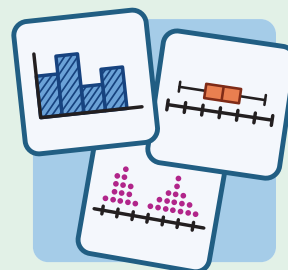
Summarizing One-Variable Data



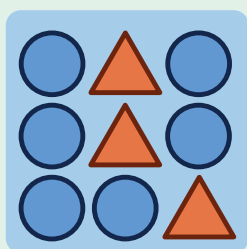
Lesson 4
Love It or Leave It



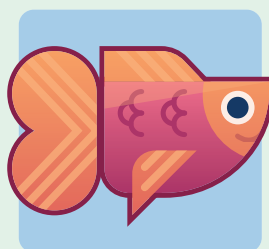
Lesson 5
Better Weather?



Lesson 6
Shapes of Data



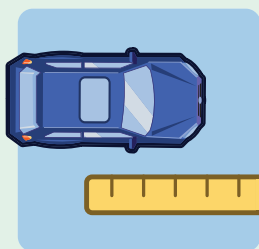
Lesson 7
Quick Pick



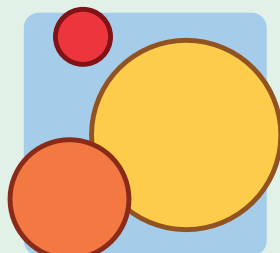
Lesson 8
Finding Desmo



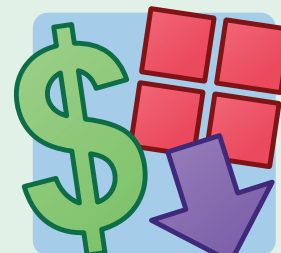
Lesson 9
Wavering Weather



Lesson 10
Race Car



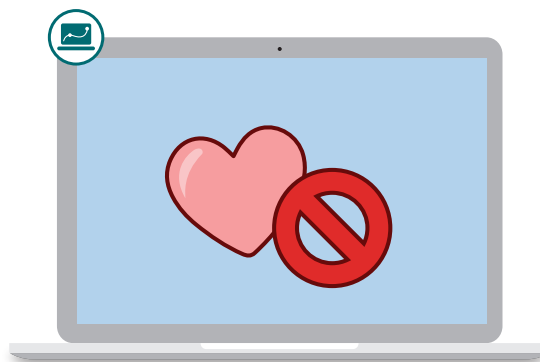
Lesson 11
Far Out



Lesson 12
Dynamic Decades

Love It or Leave It

Let's make sense of dot plots and histograms as ways to visualize one-variable data.



Warm-Up

1 This is a game called *Love It or Leave It*.

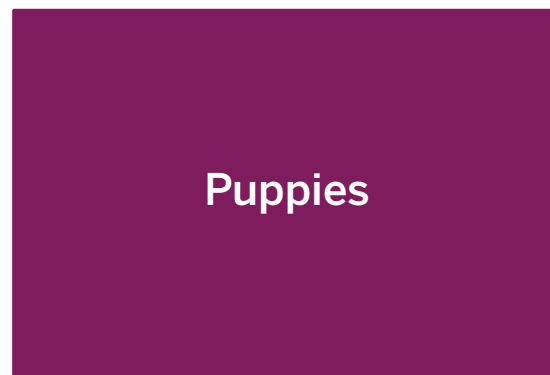
A word or phrase will appear on the screen.

Rate it on a scale of 0–10.

10 = Love it!

0 = Leave it!

Use the digital activity to play several rounds of this game.



Leave it!

Love it!

Dot Plots

You'll use the digital activity for Problems 2–4.

2 Look at some ratings from *Love It or Leave It* in a dot plot.

What surprises you?

3 Look at another dot plot showing ratings from your class.

Which of these ratings is represented by the dot plot? Circle one.

Hiking


The zoo

Traffic

Saturday

Explain your thinking.

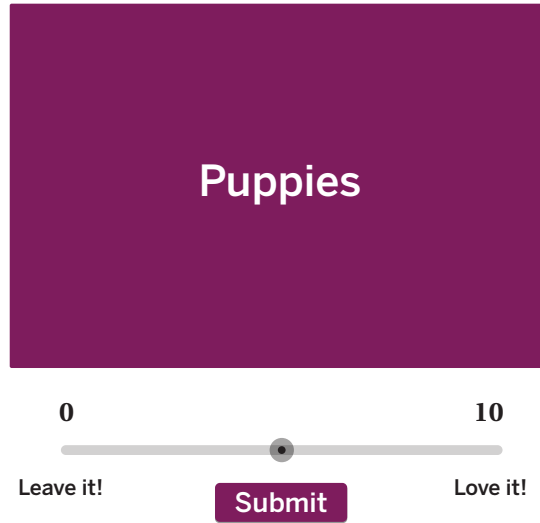
4 **a** In row 3, replace P with C or H to see the dot plot for those data sets.

b  **Data Talk!** What ratings do you think data set H could represent?

Histograms

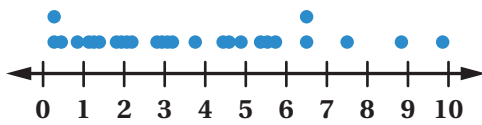
5 The makers of *Love It or Leave It* are considering changing the way people rate words.

- a** Use the digital activity to play several rounds of the new version.
- b** How would the new version change the data that gets collected?



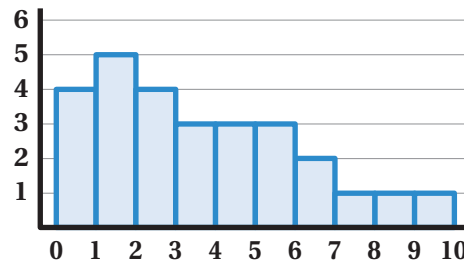
6 This dot plot and *histogram* show many students' ratings from the new version.

Dot Plot



Ratings for Snakes

Histogram



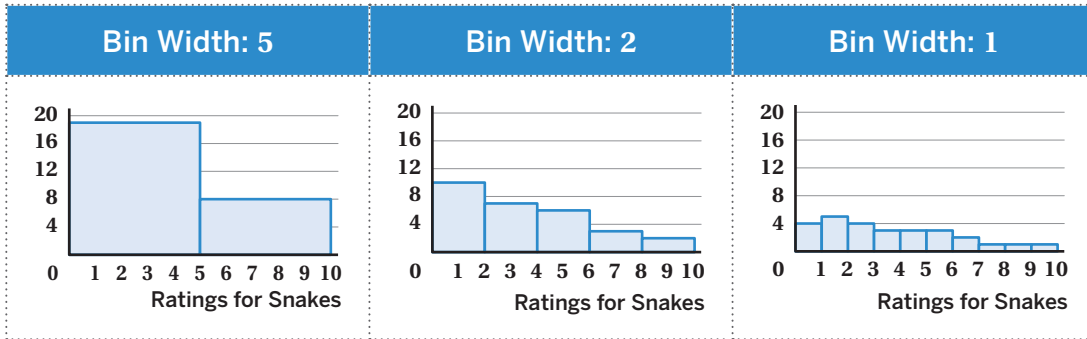
Ratings for Snakes

- a** In the digital activity, watch a histogram get made.
- b** How many students gave snakes a rating between 2 and 3?

Histograms (continued)

7 Here is the data for snakes from the previous problem.

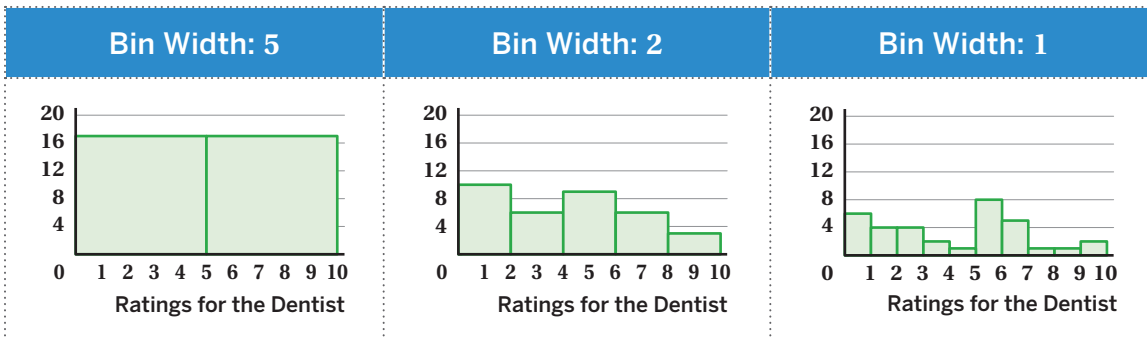
a Observe how the bin width changes the histogram.



b **Discuss:**

- What do you think *bin width* means?
- What stays the same when the bin width changes? What does not?

8 **Data Talk!** Here are ratings for the dentist from Kwame's class.



Kwame says: *Based on the data, people seem to like and dislike the dentist equally.*

What is true about Kwame's claim? What is misleading?

It's true that . . .

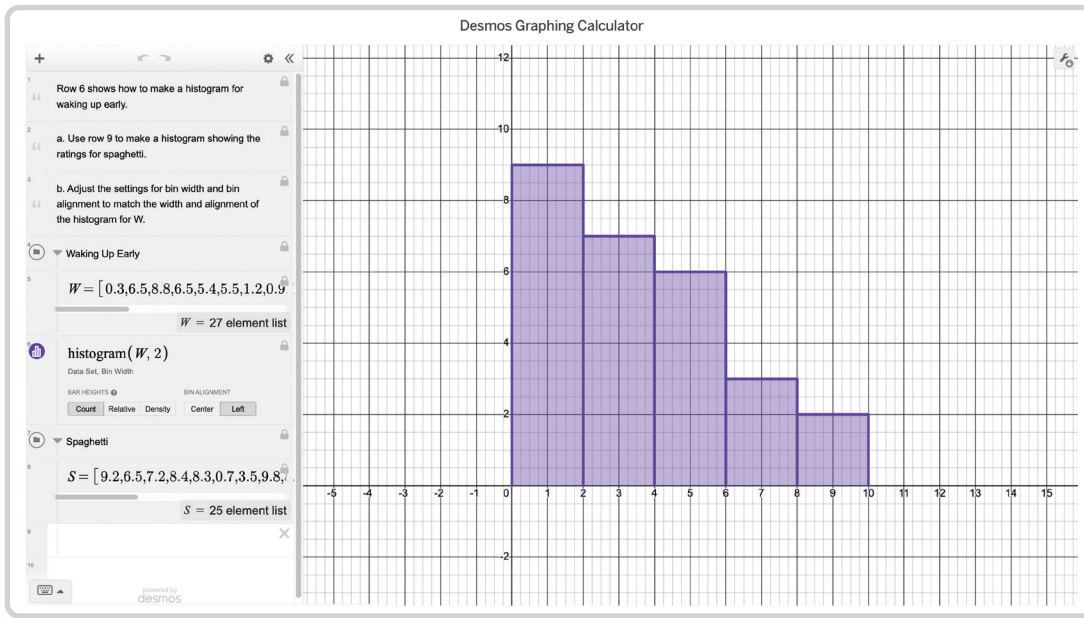
It's misleading that . . .

Activity 3

Name: _____ Date: _____ Period: _____

Making Histograms

- 9 a** Use the digital activity to make a histogram showing the ratings for spaghetti.



- b** Adjust the settings for bin width and bin alignment to match the width and alignment of the histogram for W .

You're invited to explore more.

- 10 a** Ask people to rate something on a 0–10 scale. Collect your responses in the table.

Initials	Rating	Initials	Rating	Initials	Rating

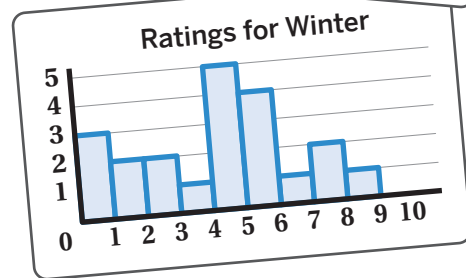
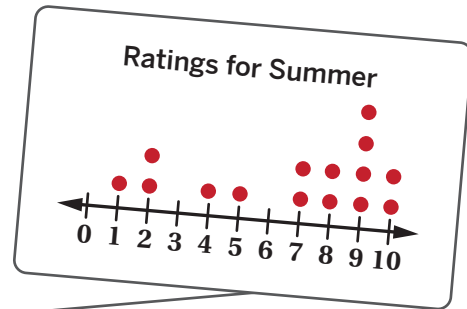
- b** Use the digital activity to make a histogram and/or dot plot of the data.

11 Synthesis

In this lesson, we examined two ways of visualizing data.

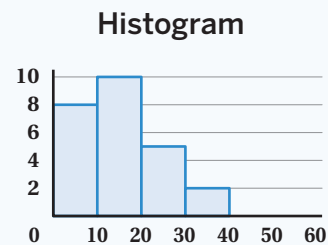
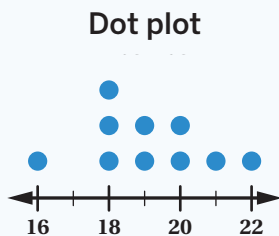
What are the advantages of a dot plot?

What are the advantages of a histogram?



14 Summary 7.04

It can be helpful to visualize data that's presented in lists or tables using *dot plots* or *histograms*.

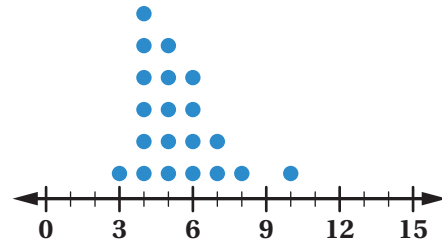


A dot plot is useful for observing the frequency of individual data, and the number of points in a data set. Histograms are useful for representing large data sets and observing the shape of a distribution. Changing the bin width of a histogram can affect how someone interprets the data being shown.

Practice 7.04

Name: _____ Date: _____ Period: _____

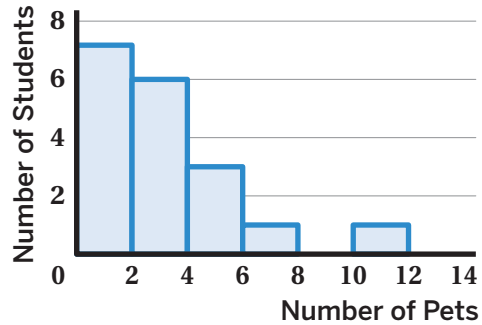
Problems 1–2: A class made this dot plot to explore how many letters are in their first names.



- How many letters are in the shortest first name?
- Dyani says that most students have 10 letters in their first name.

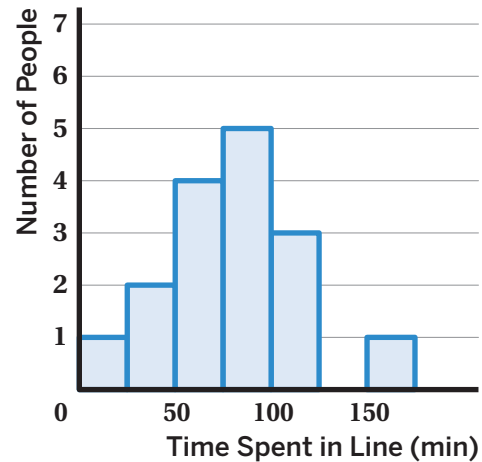
Is Dyani correct? Explain your thinking.

Problems 3–5: Jayden created a histogram about the number of pets students have.



- What is the bin width of the histogram?
- How many students have less than 4 pets?
- What is a number of pets that no students have?

6. Test Practice A group of 15 friends went to a theme park for the day. Each person kept track of how many minutes they spent waiting in line. This histogram shows their data. Select *all* the statements that must be true.



- A. 1 person was in line for at least 150 minutes.
- B. 3 people were in line for less than 50 minutes.
- C. Most people spent over 100 minutes in line.
- D. 1 person spent 0 minutes in line.
- E. No one spent exactly 130 minutes in line.

7. Imani collected this data about the outdoor temperature, in degrees Fahrenheit, at noon over the last two weeks:

50.0	55.4	57.2	60.8	64.4	59.0	60.9
68.9	60.8	60.8	64.4	55.9	55.9	57.9

Would this data be best displayed using a dot plot or histogram? Explain your thinking.

Spiral Review

Problems 8–9: Vihaan is investigating whether there is an association between wearing sneakers and participating on an athletic team. This table summarizes the data he collected from a survey of students.

	On Athletic Team	Not on Athletic Team	Total
Wear Sneakers	16%	17%	33%
Don't Wear Sneakers	32%	35%	67%
Total	48%	52%	100%

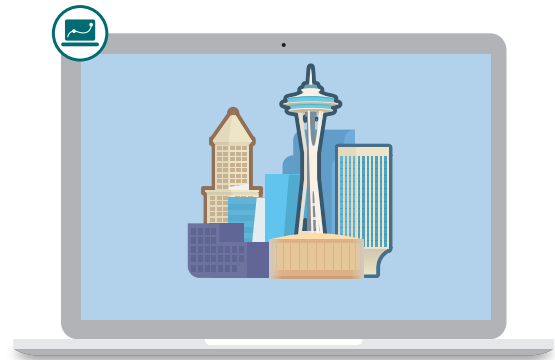
8. Interpret what 32% means in this situation.
9. Is there evidence of an association between wearing sneakers and participating on an athletic team?
10. Does a regular octagon have rotational symmetry? Circle one.
 Yes No
- If so, describe one rotation that will take the octagon onto itself. If not, explain your thinking.

Problems 11–12: Here is a data set: [2, 4, 5, 5, 7, 9, 10, 11, 11, 11, 21]

11. What is the median of this data set?
12. What is the mean of this data set? Round your answer to the nearest tenth.

Better Weather?

Let's use box plots to visualize and compare weather data.



Warm-Up

1 Mia lives in Seattle, Washington. Her best friend Bao lives in Charleston, South Carolina.

Mia and Bao are debating whose city has better weather.

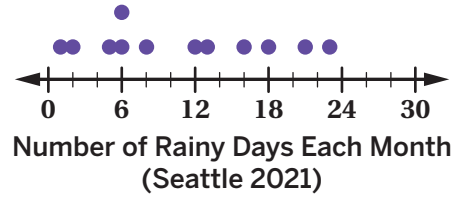
How could they compare the weather in Seattle and Charleston?



Rainy Days

2 Mia and Bao wondered: *Which city has rainier weather?*

Mia made a dot plot of the number of rainy days for each month in Seattle in 2021.



- a** What was the *greatest* number of rainy days in a month?
- b** What was the *fewest* number of rainy days in a month?

3 Here are some **statistics** of rainy days for each month in Seattle in 2021. A statistic is a single number that measures something about a data set.

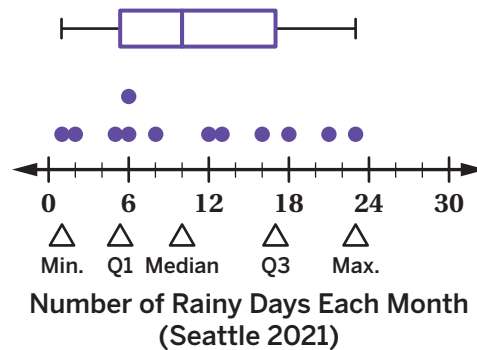
Minimum	Q1	Median	Q3	Maximum
1	5.5	10	17	23

Discuss: What do you think each of these statistics tells us about the data?

4 The five statistics on the previous problem split the data into quarters, which are called *quartiles*.

A **box plot** is one way to represent quartiles.


- a** Use the digital activity to see how a box plot for the Seattle data gets made.
- b** **Discuss:**
 - Why do you think this is called a box plot?
 - What percentage of the data is inside the box? How do you know?

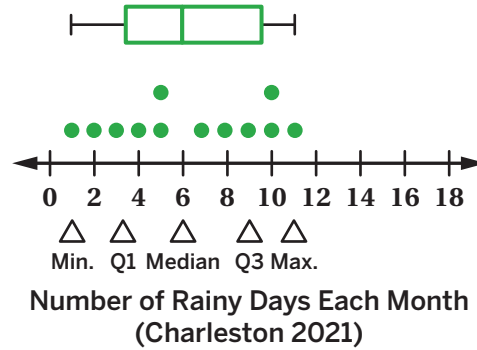


Comparing

5 Here is a dot plot and a box plot of the number of rainy days for each month in Charleston in 2021.

Bao says: *In half of the months of the year, Charleston had at least 6 rainy days.*

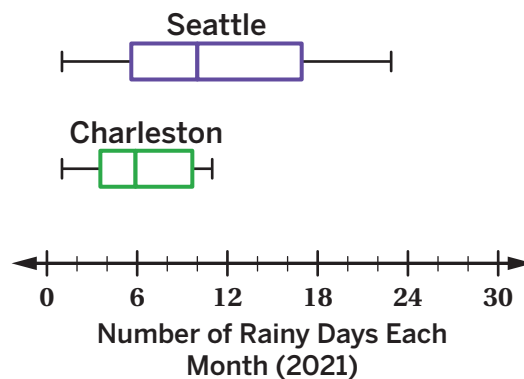
a  **Discuss:** How can the box plot help you know that Bao's statement is true?



b Write two more true statements that can be determined from the dot plot or box plot.

6 Here are two box plots showing the number of rainy days for each month in Seattle and Charleston in 2021.

Use the box plots to help Bao convince Mia that Seattle is the rainier city.



Temperature

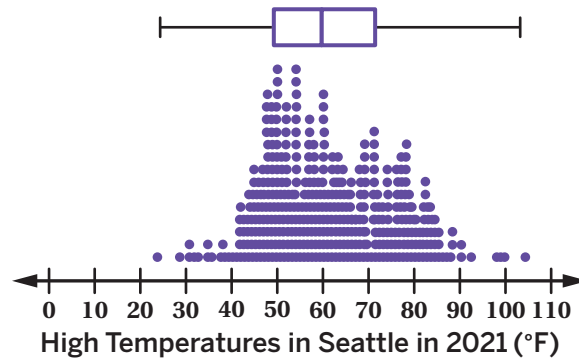
7 Mia and Bao also wondered: *Which city has hotter weather?*

Here are the high temperatures in Seattle for each day of 2021.

Which representation do you prefer for making sense of the data? Circle one.

Dot plot Box plot

Explain your thinking.



8 Use the digital activity to create a box plot for the Charleston data.

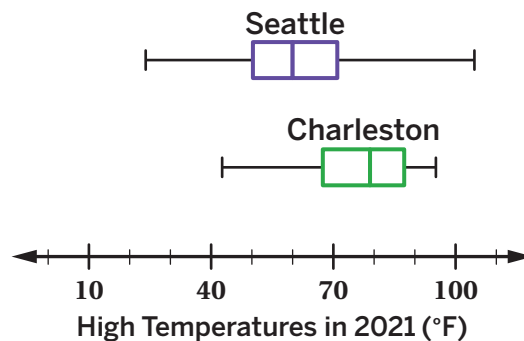
Discuss: How does the temperature in Seattle compare to the temperature in Charleston?

9 Here are two box plots showing the high temperatures in Seattle and Charleston.

What is the *median* of each data set?

Seattle:

Charleston:



Activity
3

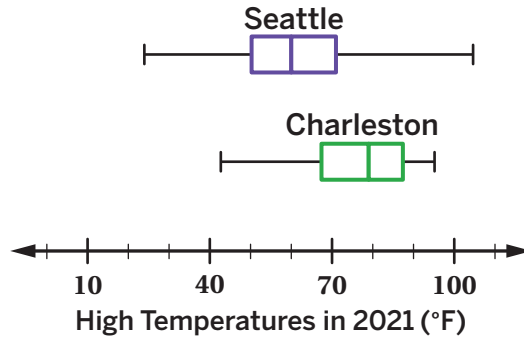
Name: Date: Period:

Temperature (continued)

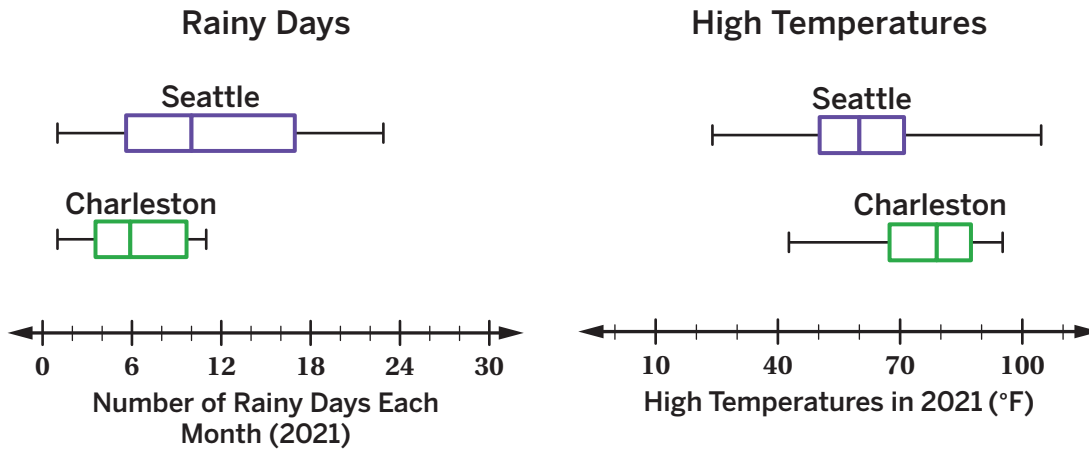
10 Here are the box plots for Seattle and Charleston.

Select *all* the statements that are true.

- A. Seattle had at least one day over 100°.
- B. Charleston had exactly 25 days with a high temperature over 87°.
- C. For about half of the days, Seattle's high temperature was between 50° and 72°.



11 Here are two ways that we compared the weather in Seattle and Charleston in this lesson.




Which city's weather do you prefer? Circle one.

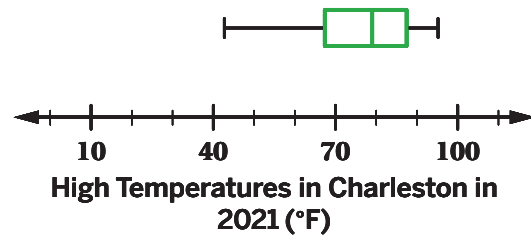
Seattle Charleston Neither

Use information from the box plots to support your reasoning.

12 Synthesis

 **Discuss:** Describe what each part of a box plot tells you about a data set.

Use the box plot if it helps with your thinking.

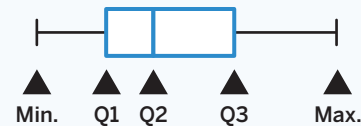


Summary 7.05

Boxplots are useful for representing large data sets, especially those with extreme values. They are less helpful for seeing the shape of the data.

To make a box plot:

- Calculate the values for Q1, Q2, and Q3.
 - Quartile 1 (Q1) is the median of the lower half of the data.
 - Quartile 2 (Q2) is also the *median* of the entire data set, which divides the data into two halves.
 - Quartile 3 (Q3) is the median of the upper half of the data.
- Mark the values for the minimum, maximum, Q1, Q2, and Q3.
- Draw a box between Q1 and Q3.
- Draw a vertical line inside the box at Q2 to represent the median.
- Draw horizontal lines from the edges of the box to both the minimum and maximum values.



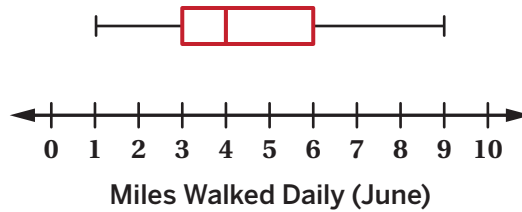
box plot One way to visualize quantitative data. The data is divided into four sections using five values: the *minimum*, the *maximum*, and three quartiles.

statistic A single number that measures something about a data set. The minimum, maximum, median, and quartiles are all examples of **statistics**.

Practice 7.05

Name: _____ Date: _____ Period: _____

Problems 1–2: Ricardo used a fitness app to track how many miles he walked each day in June. Here is a box plot of Ricardo’s data.



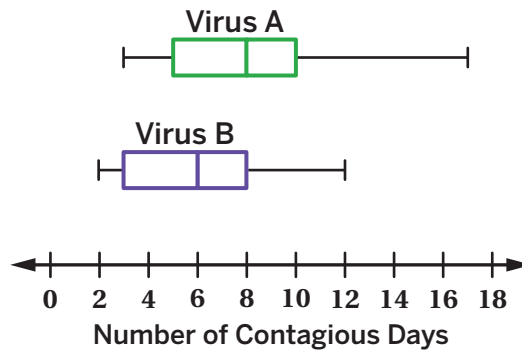
1. Determine each of these statistics:

Min	Q1	Median	Q3	Max

2. Select *all* of the questions that the box plot can answer.

- A. Did Ricardo walk more than 8 miles any day in June?
- B. How many times did Ricardo walk more than 8 miles in a day?
- C. Did Ricardo walk more on weekends or weekdays?
- D. About how often did Ricardo walk 4 miles or more in a day?
- E. Did Ricardo walk exactly 5 miles on any day in June?

Problems 3–5: A team of scientists wanted to know how long people sick with certain viruses were contagious. They studied 2 groups of 500 people and measured how many days each person was contagious.



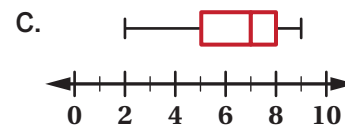
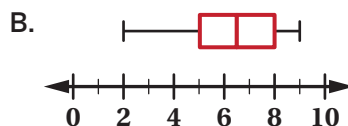
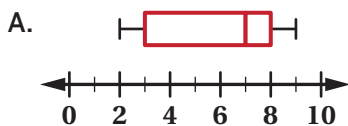
3. Approximately what percentage of people with Virus B were contagious for more than 8 days?

- A. 25% B. 50% C. 75% D. 33%

4. How do the two viruses compare?

5. Adah says that the box plot for Virus A tells us that at least one person was contagious for exactly 14 days. Is Adah correct? Explain your thinking.

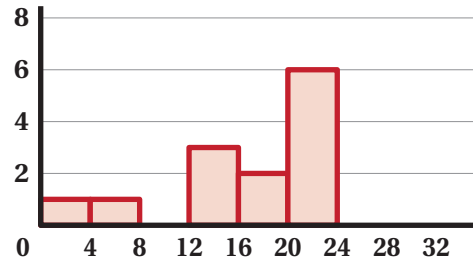
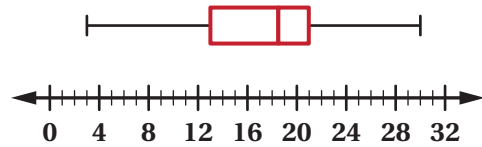
6. Determine which box plot represents the data set: 2, 3, 5, 5, 6, 6, 6, 7, 7, 7, 8, 8, 8, 9



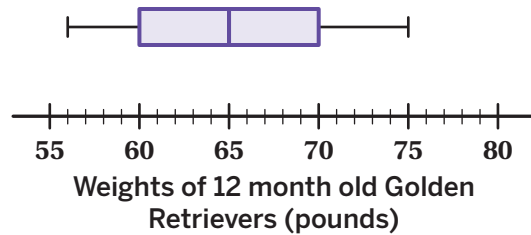
Practice 7.05

Name: _____ Date: _____ Period: _____

7. Here is a box plot and a histogram. Could these two representations have come from the same set of data? Explain your thinking.



8. **Test Practice** A healthy male twelve-month-old golden retriever should weigh between 60 and 70 pounds. A veterinarian weighed 24 male twelve-month-old golden retrievers and found this distribution:



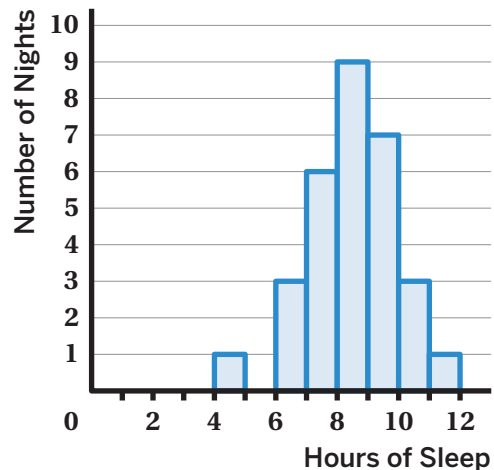
How many golden retrievers weighed at least 60 pounds?

- A. 6 B. 12 C. 15 D. 18

Spiral Review

9. This histogram shows how many hours of sleep Dakota got each night in September.

How many nights did Dakota sleep less than 7 hours?



10. Select *all* the ordered pairs that are solutions to the inequality, $2x - 3y > 18$.

- A. (6, -4) B. (-4, 6) C. (6, -2)
 D. (0, 0) E. (0, -8)



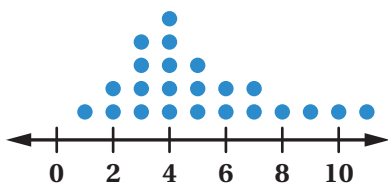
Shapes of Data

Let's describe different shapes of data.

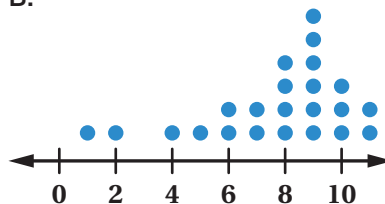
Warm-Up

1 Which one doesn't belong? Explain your thinking.

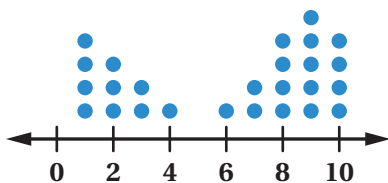
A.



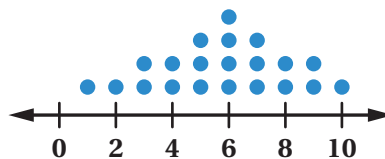
B.



C.



D.



Polygraph

2 Play a few rounds of Polygraph with your classmates!

You will use the Activity 1 Sheet. For each round:

- You and your partner will take turns being the Picker and Guesser.
- Picker: Select a scatter plot from the Activity 1 Sheet. Keep it a secret!
- Guesser: Ask the Picker yes-or-no questions, eliminating scatter plots until you're ready to guess which scatter plot the Picker chose.


Record helpful questions from each round in the this workspace:

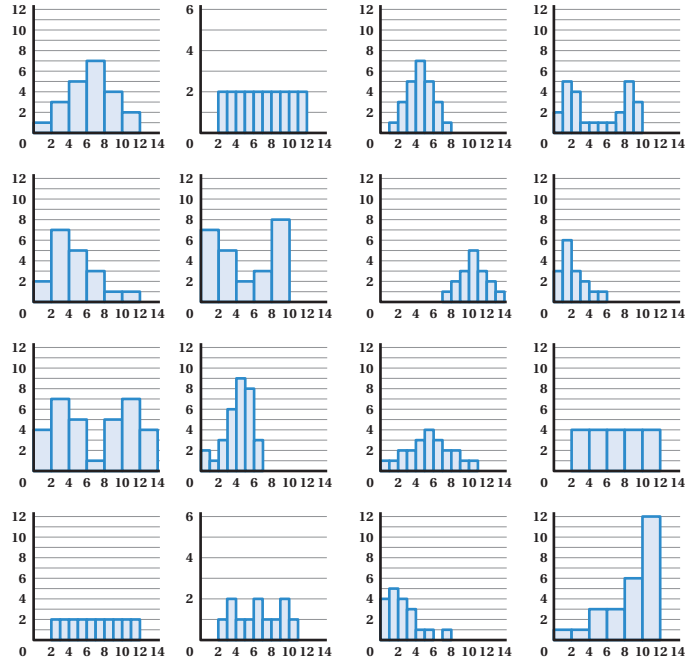
Shapes of Data Sets

3 Here are some terms that describe the *shape of a data set*:

- Skewed
- Uniform
- Symmetric
- Bimodal
- Bell-shaped

a Let's see which histograms each term describes.

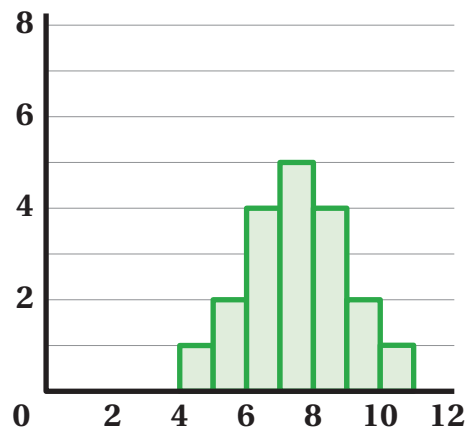
b  **Discuss:** What does each term mean?



4 Here is a histogram.

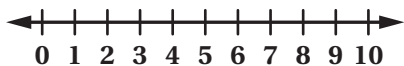
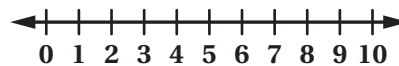
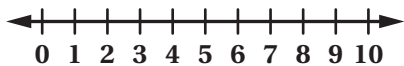
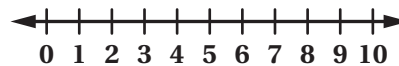
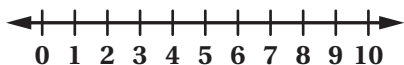
Select *all* the terms that describe the shape of this histogram.

- A. Skewed
- B. Uniform
- C. Symmetric
- D. Bimodal
- E. Bell-shaped



Shapes of Data Sets (continued)

5 Make a dot plot that matches each shape.

Skewed**Uniform****Symmetric****Bimodal****Bell-shaped**

All the Representations

6 You will use a set of cards to complete this activity.

- Match each card with one of the data sets.
- Select the terms that describe the shape of each data set. Share your reasoning with a partner.

	Dot Plot	Histogram	Box Plot	Description
Data Set 1				<input type="checkbox"/> Skewed <input type="checkbox"/> Symmetric <input type="checkbox"/> Bimodal <input type="checkbox"/> Bell-shaped <input type="checkbox"/> Uniform
Data Set 2				<input type="checkbox"/> Skewed <input type="checkbox"/> Symmetric <input type="checkbox"/> Bimodal <input type="checkbox"/> Bell-shaped <input type="checkbox"/> Uniform
Data Set 3				<input type="checkbox"/> Skewed <input type="checkbox"/> Symmetric <input type="checkbox"/> Bimodal <input type="checkbox"/> Bell-shaped <input type="checkbox"/> Uniform
Data Set 4				<input type="checkbox"/> Skewed <input type="checkbox"/> Symmetric <input type="checkbox"/> Bimodal <input type="checkbox"/> Bell-shaped <input type="checkbox"/> Uniform

7 Synthesis



Discuss: Choose one statement to explain why it is true.

- a. *Bimodal* and *uniform* cannot describe the same distribution.
- b. *Bimodal* and *symmetric* could describe the same distribution.
- c. *Bell-shaped* and *skewed* cannot describe the same distribution.

10 Summary 7.06

Here are some terms that can help us describe the *shape* of a data set.

bell-shaped Most of the data is at the center and there are fewer points farther from the center. In a dot plot or histogram, the data looks like a bell.

bimodal Two very common data values that appear in a dot plot or histogram as two peaks.

skewed More values are concentrated on one end of the data than the other.

symmetric You can draw a vertical line of symmetry through it.

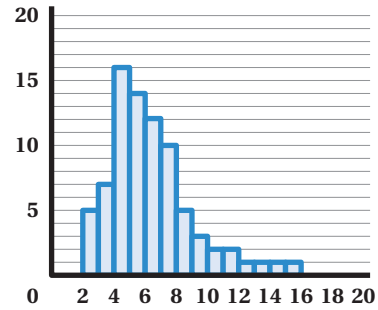
uniform The data values are evenly distributed.

Practice 7.06

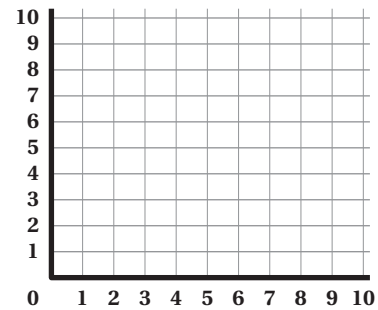
Name: _____ Date: _____ Period: _____

1. Which term best describes this histogram?

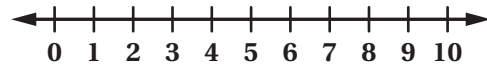
- A. Symmetric
- B. Skewed
- C. Uniform
- D. Bimodal



2. Create a histogram that is bell-shaped.

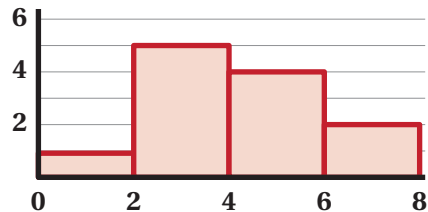
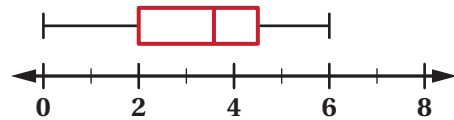
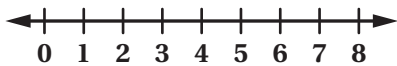


3. Create a dot plot that is *symmetric but not uniform*. Plot at least eight points.



Problems 4–5: Here is a box plot and a histogram of the same data set.

4. Create a dot plot that could also represent this data set.

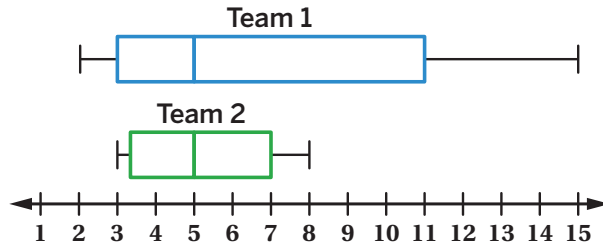


5. What is the value of Q1?

Practice 7.06

Name: _____ Date: _____ Period: _____

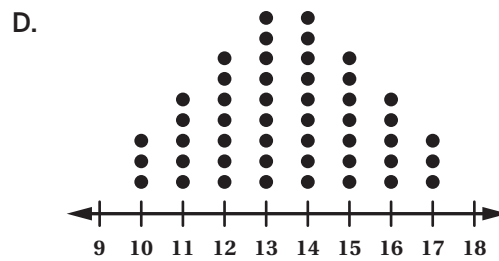
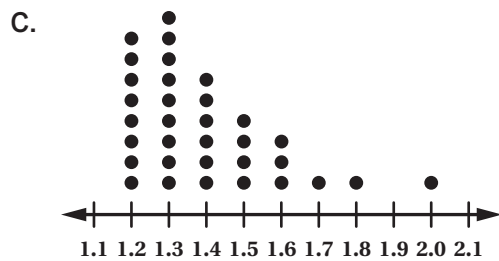
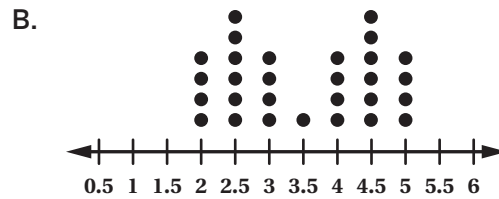
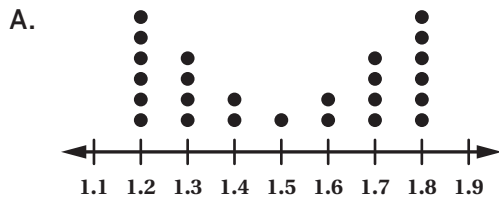
Problems 6–7: The players on two different football teams were surveyed about the number of hours they spend in the weight room each week. Here are the box plots for each team's data.



6. What is one *similarity* between the two data sets?

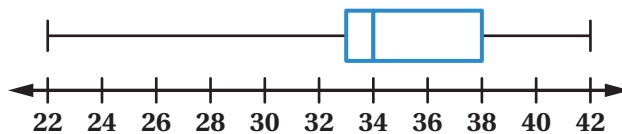
7. What is one *difference* between the two data sets?

8. **Test Practice** Which of these dot plots shows a skewed distribution?



Spiral Review

9. This box plot shows the number of points scored by a cross country team at 12 different meets.

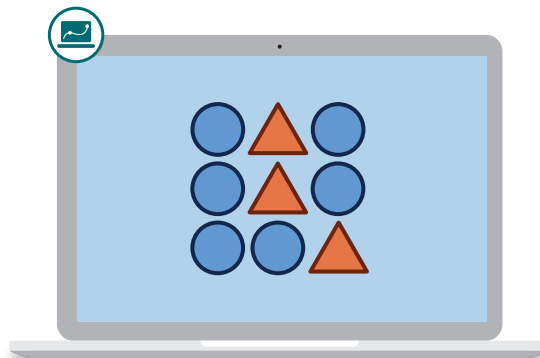


What percentage of meets did the team score between 33 and 38 points?

10. Solve $5x + 3y = 30$ for y .

Quick Pick

Let's explore how extreme values impact mean and median.



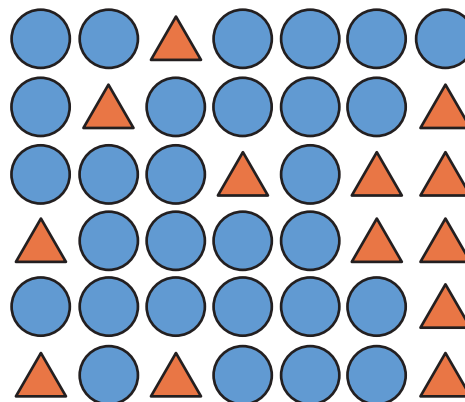
Warm-Up

1 This game is called *Quick Pick*. Here's how it works:

- You pick different shapes to get points.
- Triangles are 3 points. Circles are 2 points.
- A five-second timer starts after your first pick.

You will use the digital activity to play up to 15 times.

Score
0



Measures of Center


- 2** In the digital activity, look at your scores and the scores of three students from another class.

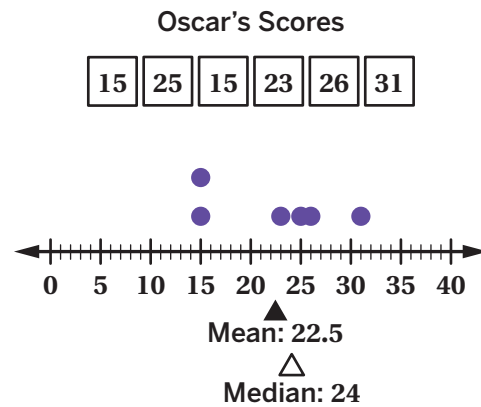
Choose a student and describe their scores.

- 3** A **measure of center** is a single number that represents a central value in a data set.


Mean and *median* are two measures of center.

Here are the mean and median for Oscar's scores.

 **Data Talk!** What are some things you know about calculating the mean and median?



- 4** Here is the work Oscar did to calculate the median and the mean.

 **Discuss:** How did Oscar calculate each measure of center?

Oscar

~~15~~, ~~15~~, 23, 25, ~~26~~, ~~31~~

∨
24

The median is 24.

$15 + 25 + 15 + 23 + 26 + 31 = 135$

$135 \div 6 = 22.5$

The mean is 22.5.

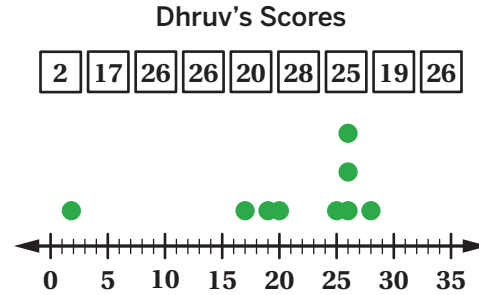
Shape and Center

5 Here are Dhruv's scores from the game. Determine the median and mean.

Use the Desmos Graphing Calculator if it helps with your thinking.

Median: _____

Mean: _____



6 In the digital activity, look at the mean and median for Dhruv's scores.

a **Discuss:** Why do you think the mean and median are far apart?

b Which measure of center would you use to represent Dhruv's typical score? Circle one.

Mean

Median

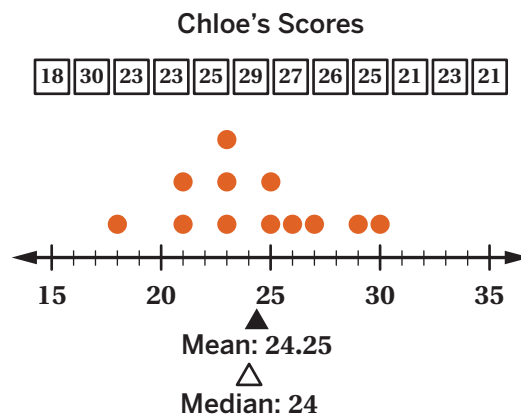
Explain your thinking.

7 Here are Chloe's scores from the game.

Imagine that her highest score was 300 instead of 30.

Which measure(s) of center would increase? Circle one.

Mean Median Both Neither

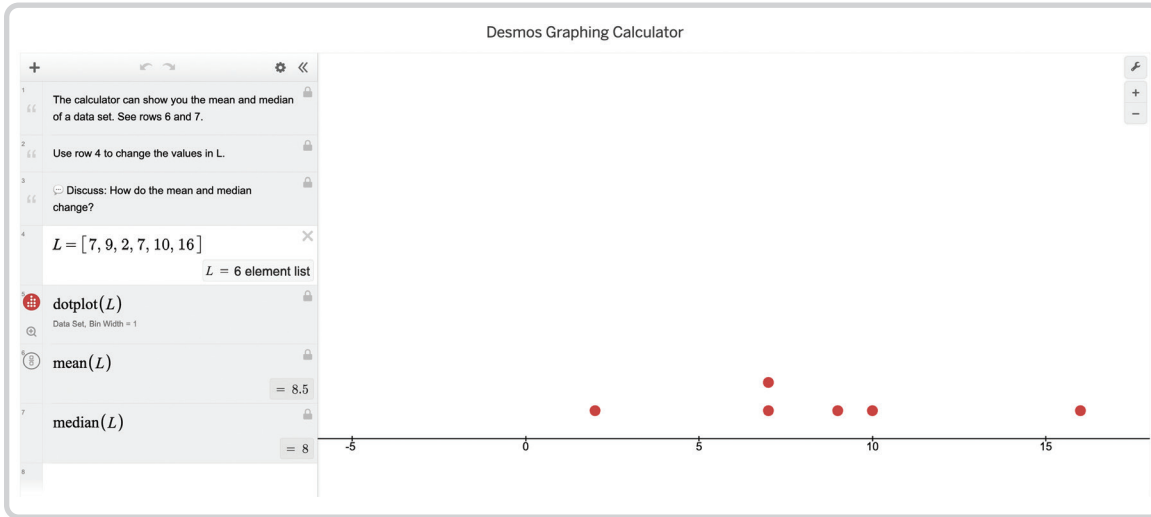


Activity 3

Name: Date: Period:

Calculate and Create

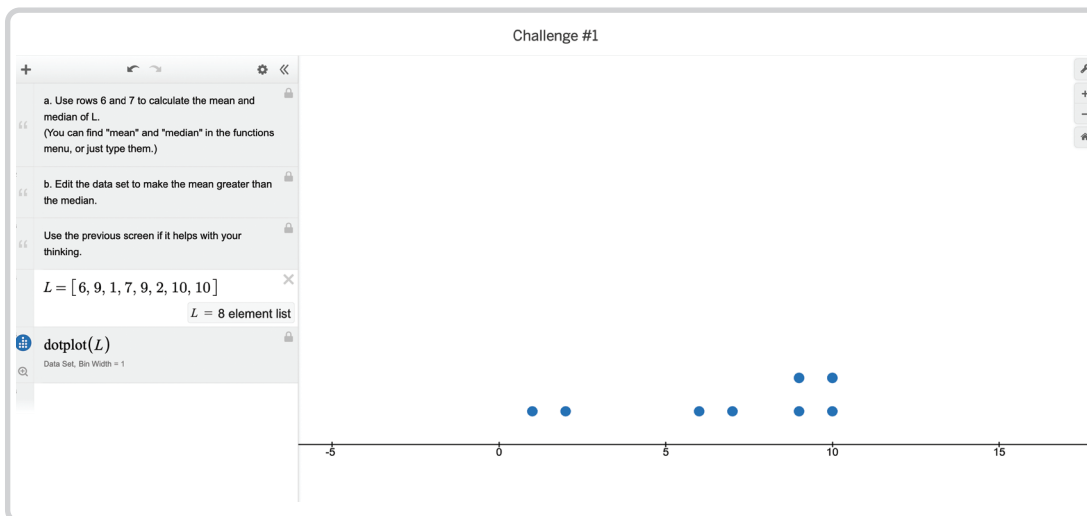
8 The Desmos Graphing Calculator can show you the mean and median of a data set.



Use the digital activity to change the values in L .

Discuss: How do the mean and median change?

9 a In the digital activity, use rows 6 and 7 to calculate the mean and median of L . (You can find “mean” and “median” in the functions menu, or just type them.)



b Edit the data set to make the mean greater than the median.

Activity 3

Name: _____ Date: _____ Period: _____

Calculate and Create (continued)

- 10** **a** In the digital activity, use row 3 to create a data set that has a median of 6 and a mean less than 6.

Challenge #2

a. Use row 3 to create a data set that has a median of 6 and a mean less than 6.

b. Discuss: How did you solve this challenge?

$L = []$

$L = 0$ element list

-5 0 5 10 15

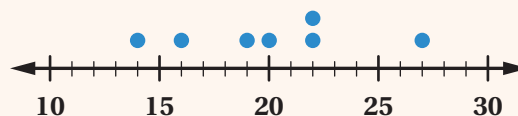
- b** **Discuss:** How did you solve this challenge?

You're invited to explore more.


- 11** Here is a dot plot.

The mean and median are currently equal.

Add data points to make the mean and median as far apart as you can.



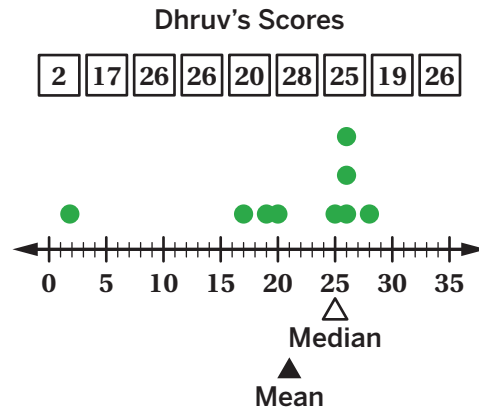
12 Synthesis

 **Discuss:** How are median and mean *alike*?
How are they *different*?

Use the data set if it helps with your thinking.

Alike:

Different:



15 Summary 7.07

Mean and *median* are measures of center that are used to describe a typical value of a data set.

- The mean is also called the average of a data set. To calculate the mean, you can add up all the data values, and divide by the number of data points.
- The median is the middle value of a data set when the values are in numerical order. If there are two values in the middle of the data set, then the median is the middle of those two values.

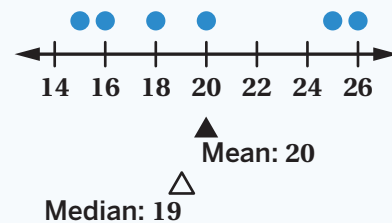
Here are the steps you can use to determine the mean of the data in this dot plot.

$$15 + 16 + 18 + 20 + 25 + 26 = 120$$

$$120 \div 6 = 20$$

The median is the value halfway between 18 and 20, so the median is 19.

When a data set includes extreme values that are much larger or smaller than most of the data, the value of the mean and median will be very different. Extreme values impact the mean more than they impact the median.



measure of center A single number that summarizes all of its values. It is usually a typical value for a data set. Mean and median are measures of center.

Practice 7.07

Name: _____ Date: _____ Period: _____

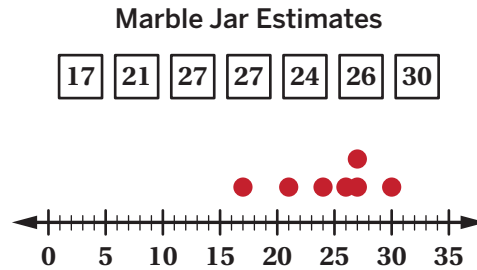
1. Determine the mean and median of each data set.

	Set A [27, 30, 33]	Set B [0, 100, 100, 100, 100]	Set C [3, 5, 7, 15]
Mean			
Median			

2. Seven people estimated how many marbles there were in a jar. Determine the mean and median of the estimates.

Mean: _____

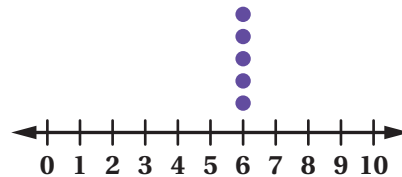
Median: _____



3. **Test Practice** Here is a dot plot. If you added 4 and 9 to the data set, which statistic would change? Circle one.

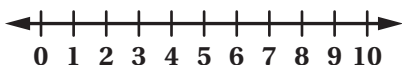
Mean Median Both Neither

Explain your thinking.

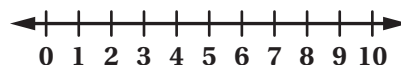


Problems 4–5: Create a dot plot that matches each description.

4. A median of 3 and a mean that is greater than the median.



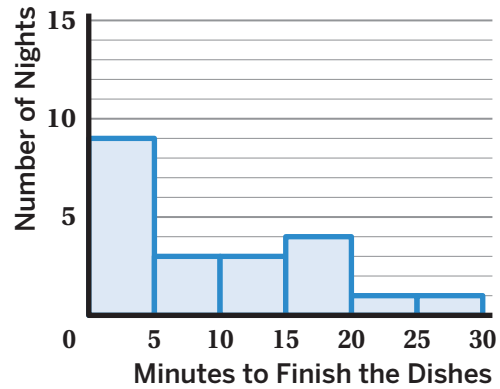
5. Bell-shaped, with a median of 6.



Practice 7.07

Name: _____ Date: _____ Period: _____

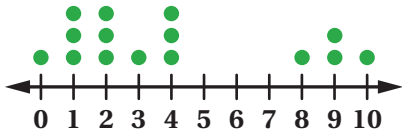
Problems 6–7: For the past few weeks, Anand kept track of how long it took him to do the dishes each night. This histogram shows the results organized in 5-minute bins.



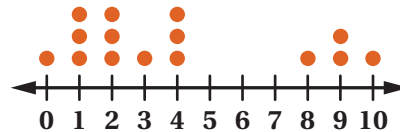
6. Circle the bin that contains the median.

7. Explain how you know.

8. Remove one value to change the *mean* but not the median.



9. Remove one value to change the *median* but not the mean.



Spiral Review

10. Select all the expressions that are equivalent to $2(x + 3)$.

A. $2x + 6$

B. $2 \cdot x + 3$

C. $x + 6$

D. $(x + 3) \cdot 2$

E. $2 \cdot x + 3 \cdot 2$

Problems 11–14: Evaluate each expression. Write your solution in scientific notation.

11. $(2 \cdot 10^4)(6 \cdot 10^5)$

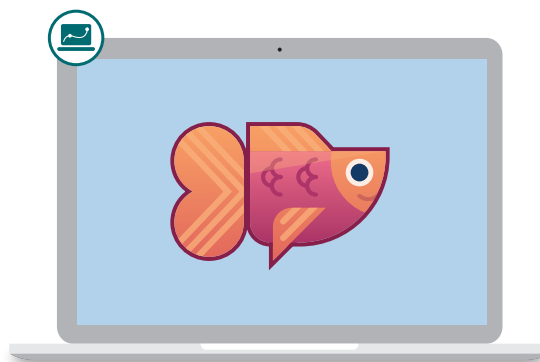
12. $\frac{3 \cdot 10^{-8}}{2 \cdot 10^{-3}}$

13. $(5 \cdot 10^{-1})(4 \cdot 10^5)$


14. $\frac{8 \cdot 10^{-6}}{4 \cdot 10^{-3}}$

Finding Desmo

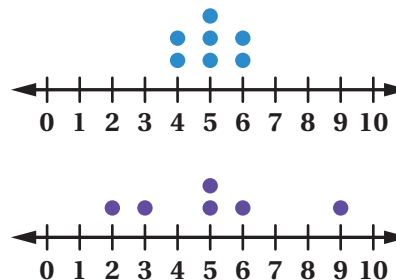
Let's explore what standard deviation describes about a data set.



Warm-Up

1  **Data Talk!** Here are two data sets.

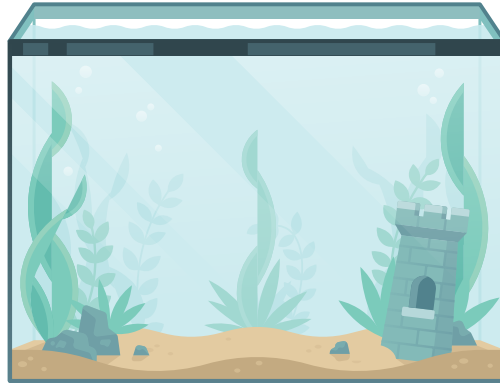
How are they alike? How are they different?



Introduction to Standard Deviation

2 You are getting a new fish tank.

a In the digital activity, add up to 10 fish to your tank.

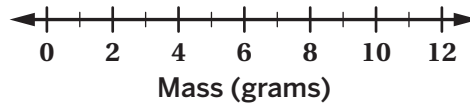


b Tell a partner how you decided which fish you wanted and how many.



3 A customer at the pet store says: *I would like fish that are close to the same size.*

a In the digital activity, build them a fish tank you think they would like.

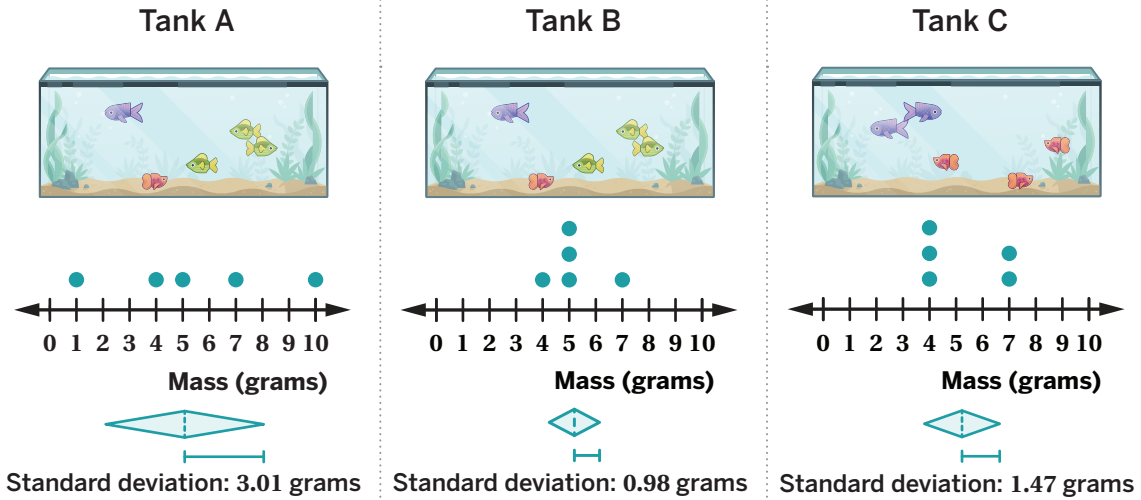


b  **Discuss:** What do you notice and wonder about the dot plot?

Introduction to Standard Deviation (continued)

4 One way to determine the consistency of data is to calculate the standard deviation, which is a measure of spread.

Here are three different fish tanks.



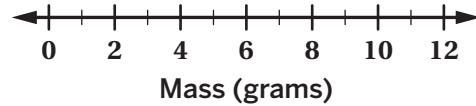
Explain what you think standard deviation measures.

Standard Deviation Challenges

You'll use the digital activity for Problems 5–7.

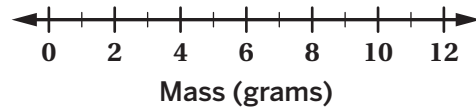
- 5** Make a fish tank with a large standard deviation.
Discuss your strategy with a partner.

Note: You can add up to 10 fish to your tank.




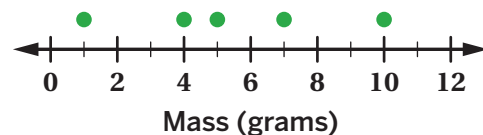
- 6** Make a fish tank with a small standard deviation that is greater than 0. Discuss your strategy with a partner.

Note: You can add up to 10 fish to your tank.



- 7** Add fish to this tank to increase the standard deviation.

 **Discuss:** Is there anything you find surprising?



Original standard deviation: 3.01 g

Calculating Standard Deviation

- 8** Here are the masses, in grams, of the fish in three new tanks.

Tank D	13, 13, 13, 13, 14, 15
Tank E	3, 4, 6, 7, 7, 9
Tank F	7, 7, 7, 7, 7, 9

Which tank do you think has the largest standard deviation? Circle one.

Tank A

Tank B

Tank C

I'm not sure

Explain your thinking.

- 9** The Desmos Graphing Calculator can help calculate standard deviation.

- a** In the digital activity, watch an animation to see how to calculate the standard deviation of 1, 2, 3, 4, 5.

- b** Use the Desmos Graphing Calculator to answer:

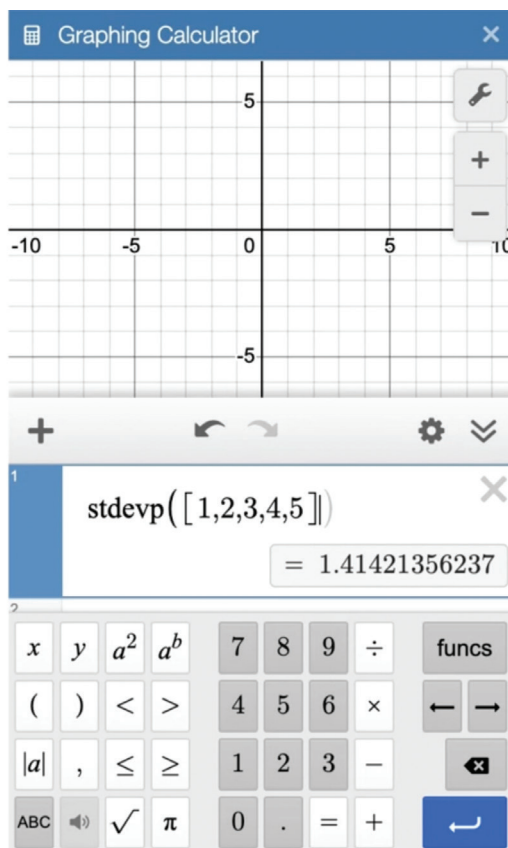
What is the standard deviation of 3, 4, 6, 7, 7, 9? Circle one.

1.75

2

3.14

6



Calculating Standard Deviation (continued)

10 Order the fish tanks from smallest to largest standard deviation.

Use the Desmos Graphing Calculator to help with your thinking.

Tank A	13, 13, 13, 13, 14, 15
Tank B	3, 4, 6, 7, 7, 9
Tank C	7, 7, 7, 7, 7, 9

--	--	--

Smallest Standard Deviation

Largest Standard Deviation

11 Here is a data set:

1, 2, 3, 4, 5, 6, 7

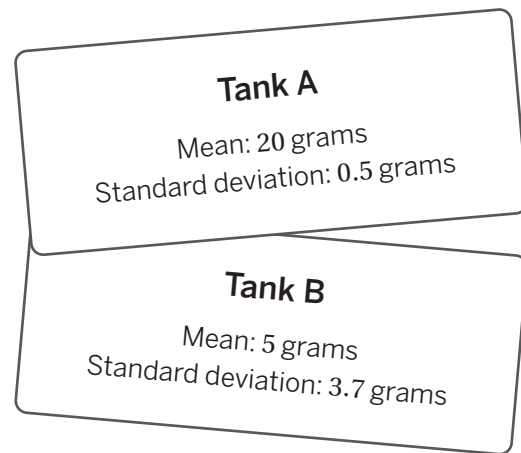
Select *all* the moves that would make the standard deviation *larger*.

Use the calculator to help you with your thinking.

- A. Removing the 7 from the data set.
- B. Adding a 0 to the data set.
- C. Removing the 3 from the data set.
- D. Adding a 4 to the data set.
- E. Increasing each value by 1.

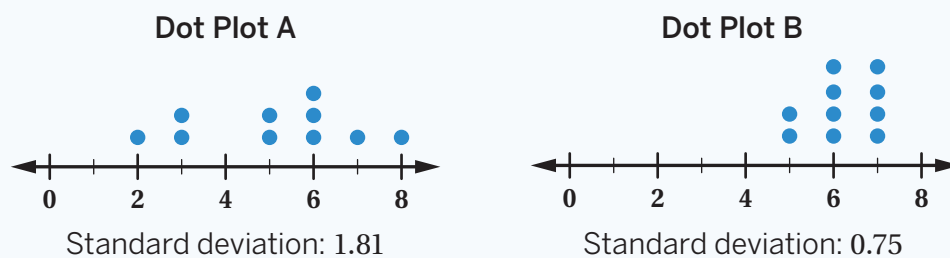
12 Synthesis

Describe what the mean and standard deviation tell us about the fish in Tank A and Tank B.



15 Summary 7.08

A **measure of spread** can help you describe how consistent a data set is. If a data set is very consistent and much of the data clustered together, the **standard deviation** will be small. If the data is spread out, the standard deviation will be larger. For example, dot plot A has a greater standard deviation than dot plot B because the data in A is more spread out than the data in B. Since the data values in dot plot B are more consistent, B has a lower standard deviation.



measure of spread A statistic that measures the variability within a data set. The more consistent the data is within a data set, the less variability there is within that data set, and the smaller the measure of spread. Range, interquartile range, and standard deviation are measures of spread.

standard deviation A statistic that measures how spread out the values in a data set are from its mean. The larger the standard deviation, the further the data points within a data set are from its mean. Standard deviation is calculated by a method similar to calculating the MAD (mean absolute deviation). The exact method is studied in more advanced courses.

Practice 7.08

Name: _____ Date: _____ Period: _____

Problems 1–2: Use the Desmos Graphing Calculator to determine the mean and standard deviation of each data set. Round your answer to the nearest hundredth.

1. 14, 14, 15, 16, 16, 18, 18, 19

Mean: _____

Standard Deviation: _____

2. 2, 3, 6, 6, 8, 8, 10, 11

Mean: _____


Standard Deviation: _____

Problems 3–4: Titus and Alejandro are both on the track team. Each day in May, they recorded the average distance they threw the javelin.

	Mean (ft)	Standard Deviation (ft)
Alejandro	128	1.2
Titus	130	2.6

3. Based on the data, who is the more consistent thrower?

4. If a throw of 160 feet was added to each data set, whose standard deviation would change more?

5.  **Test Practice** Determine whether each statement about data sets A, B, and C are true or false.

	True	False
Set C has the smallest standard deviation.		
Set A has the largest standard deviation.		
Set B has a standard deviation of 10.6		

Set A	[7, 10, 12, 14, 17, 19]
Set B	[2, 2, 5, 22, 25, 25]
Set C	[4, 8, 9, 10, 12, 13]

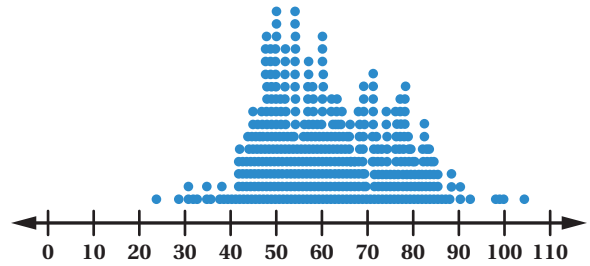
Practice 7.08

Name: _____ Date: _____ Period: _____

6. Here are the high temperatures in Seattle for each day of 2021. If the data points for the hottest and coldest temperatures were removed, would the standard deviation increase, decrease, or stay the same? Circle one.

Increase Decrease Stay the same

High Temperatures in Seattle
in 2021 (°F)

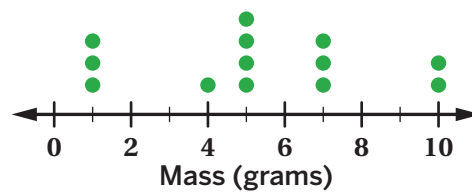


7. Here is a data set: 4, 5, 5, 6, 7, 9.

Select *all* of the changes that will make the standard deviation *smaller*.

- A. Add a 2 to the data set. B. Add a 6 to the data set.
 C. Remove the 4 from the data set. D. Remove the 9 from the data set.
 E. Increase each value by 10.

Problems 8–9: Here is a dot plot showing the weights of fish.



8. Circle the weight of fish that would change the standard deviation the *most* if added to the data set.

1 gram 4 grams 5 grams 7 grams 10 grams

9. Circle the weight that would change the standard deviation the *least* if added to the data set.

1 gram 4 grams 5 grams 7 grams 10 grams

Spiral Review

Problems 10–13: Rewrite each expression as a single power.

10. $(12^3)^5$

11. $\frac{6^4}{6^5}$

12. $5^7 \cdot 5^2 \cdot 5$

13. $\frac{4^7}{4^2}$


Wavering Weather



Let's use mean and standard deviation to compare the temperatures in different cities.

Warm-Up

1. You will use Screen 1 of the digital activity to see what the high temperatures were on June 28th, 2021 in 11 cities across the U.S.

 **Discuss:** What do you notice? What do you wonder?

Portland vs. Phoenix

Use Screen 1 of the digital activity to help you answer Problems 2–3.

2. List the three hottest cities on June 28th, 2021.

.....

3. Jacy says: *Portland had the highest temperature, so it must be the hottest city overall.*

What would you say to help Jacy understand why this might not be true?

Jacy and Neo wondered which city is generally hotter: Portland or Phoenix. They decided to compare the high temperatures on each day in 2021.

4. Use Screen 2 of the digital activity to help you complete the table.

	Minimum (°F)	Maximum (°F)	Median (°F)	Mean (°F)	Standard Deviation (°F)
Portland, OR	30	116			15.93
Phoenix, AZ			88	88.23	

5. Which city is generally hotter: Portland or Phoenix? Use the statistics above to support your argument.
6. Which city has a more consistent temperature? Use the statistics above to support your argument.

Phoenix vs. Orlando

It is sometimes hotter in Portland than in Phoenix. However, Phoenix has the hotter weather in general. Jacy and Neo wondered how Phoenix's weather compared to another hot city: Orlando.

They decided to compare the mean and standard deviation of the temperatures in Phoenix and Orlando.

7. Do you agree with their choice of statistics? Why or why not?

8. Use Screen 3 of the digital activity to help you calculate these statistics for Orlando:

	Mean ($^{\circ}\text{F}$)	Standard Deviation ($^{\circ}\text{F}$)
Phoenix, AZ	88.22	15.11
Orlando, FL		

9.  **Data Talk!** How do the temperatures in Phoenix compare to the temperatures in Orlando?

Use statistics to support your argument.

10. Neo noticed that the standard deviation in Orlando is lower than the standard deviation in Phoenix. Does this surprise you? Why or why not?

Make dot plots or box plots of the data in the digital activity if it helps with your thinking.

Comparing Temperatures

11. Pick two cities to compare from Screen 4 of the digital activity.

City A: City B:

12. Circle 2–3 statistics that would help you compare the temperatures in these places.

Minimum Maximum Median Mean Standard Deviation

13. Label each column with the statistics you chose. Then use Screen 5 to help you calculate the statistics for each city you chose.


	Statistic 1:	Statistic 2:	Statistic 3:
City A:			
City B:			

14. How do the temperatures in City A compare to the temperatures in City B?
Use statistics to support your argument.

You're invited to explore more.

15. An extreme heat wave affected much of western North America from late June through mid-July 2021. On June 28, 2021, Portland, Oregon, recorded its hottest-ever temperature of 116°F. The average high temperature in Portland during the month of June was 82°F.
What might the impacts of this extreme heat wave be on people in western North America?

Synthesis

16.  **Discuss:** Select *one* question and answer it.

- A. How can the mean/median help us compare temperatures in different places?
- B. How can the standard deviation help us compare temperatures in different places?
- C. What new questions do you have about temperatures around the United States?

Summary 7.09

You can use statistics, like *mean*, *median*, or standard deviation, to help you compare data sets and make conclusions about the real world. Different statistics can reveal different kinds of information about the data sets you are comparing. Comparing the mean of two data sets can help you determine which one generally has higher or lower values. Comparing the standard deviation of two data sets can help you determine which one is more consistent.

For example, here are statistics about the high temperatures in Metropolis and Springtown over one week.

When you compare the means, you can see that both Metropolis and Springtown had similar high temperatures in that week. But when you compare the standard deviations, you can see that the temperatures in Springtown were more consistent than the temperatures in Metropolis because the data set has a lower standard deviation

	Mean (°F)	Standard Deviation (°F)
Metropolis	74	6
Springtown	74.43	2.06

Practice 7.09

Name: _____ Date: _____ Period: _____

1. Create three data sets, each with 5 numbers and a mean of 10.

.....

2. Here is some data about the yearly salaries for a group of dental hygienists and helicopter pilots. Select *all* the true statements.

	Mean (\$)	Standard Deviation (\$)
Dental Hygienist	96,257	15,000
Helicopter Pilot	81,241	12,000

- A. In general, dental hygienists make more money than helicopter pilots.
- B. A typical salary for a dental hygienist is \$15,000.
- C. The salaries for dental hygienists vary more than the salaries for helicopter pilots.
- D. In general, helicopter pilots make more money than dental hygienists.
- E. The maximum pay for a helicopter pilot is \$93,241.

Problems 3–4: A sports magazine asked several professional athletes: *How many bones have you broken?* Here are the responses from two groups of athletes.

	Skateboarders	Rock Climbers
Number of Broken Bones	3, 0, 5, 8, 15, 25, 0, 6, 12, 20	1, 0, 10, 3, 5, 5, 5, 6, 12, 0

3. Which data set has a higher mean?
4. Which data set has a greater standard deviation?


Problems 3–4: DesGoods and DesMarket both sell a variety of shampoos. Here is the mean and standard deviation for shampoo prices at each store.

DesGoods	DesMarket
Mean: \$8.49	Mean: \$8.53
Standard deviation: \$3.82	Standard deviation: \$1.13

5. Which store has more consistent prices?
6. Which store is likely to have the shampoo with the highest price?

Practice 7.09

Name: _____ Date: _____ Period: _____

7.  **Test Practice** Think about the data set: 2, 3, 3, 4, 4, 4, 4, 5, 5, 6, 7. If the 7 is changed to 70, what would happen to the *mean*, *median*, and *standard deviation* of the data set? Select all the statements that are correct.

- A. The mean will decrease.
- B. The median will stay the same.
- C. The standard deviation will increase.
- D. The median will decrease.
- E. The mean will increase.

Spiral Review

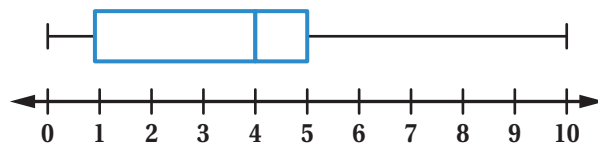
Problems 8–9: Think about the equation $5x + 3y = 60$.

8. For each value of x , determine the value of y .

x	y
6	
12	

9. Solve the equation for y .

Problems 10–12: Here is a box plot.

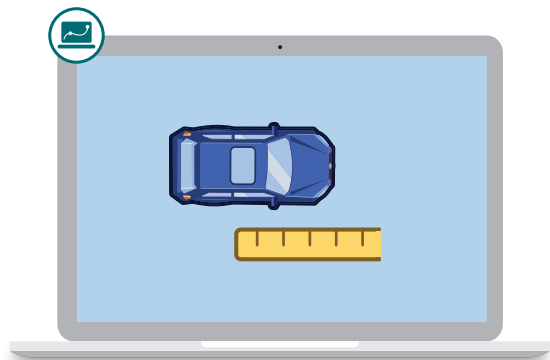


10. Select *all* the statements that must be true.

- A. The mean of this data set is 4.
 - B. Most of the data in this data set is between 0 and 5.
 - C. 3 is not in this data set.
 - D. There are no negative values in this data set.
 - E. The maximum of this data set is 10.
11. What is the median for this data set?
12. Write a question that could have produced this data set.

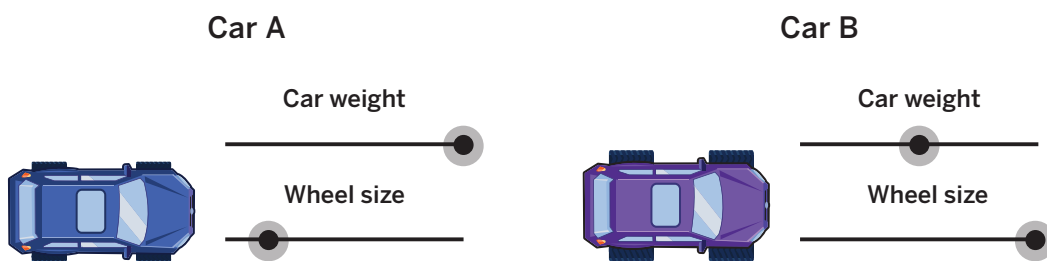
Race Car

Let's compare measures of spread in skewed data sets.



Warm-Up

1 Here are two toy race cars with different colors, weights, and wheel sizes.



2 a Each car was launched 12 times. Compare their results.



b  **Discuss:** Which car generally travels farther? How do you know?

Box Plots and Races

3 Abena launched her car 12 times.

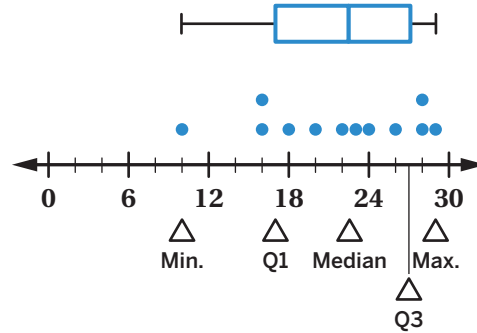
Let's watch how the dot plot of Abena's data is made into a box plot.

What do you think are the advantages of each representation?

Dot plot:

Box plot:

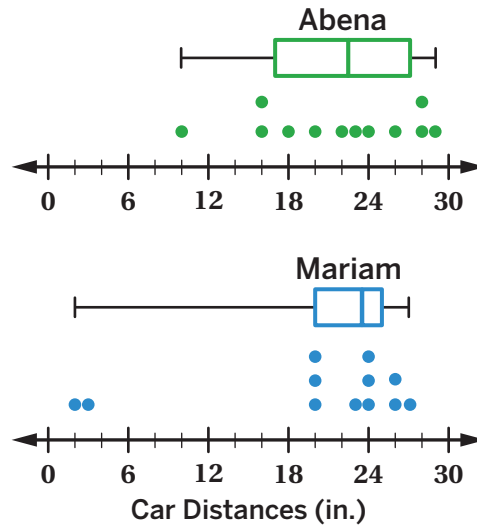
Abena's Car Distances (in.)



4 Professor Cho is giving out some racing awards to the class.

Discuss:

- What awards do you think should be given out?
- For each award you come up with, who would win between Abena and Mariam?



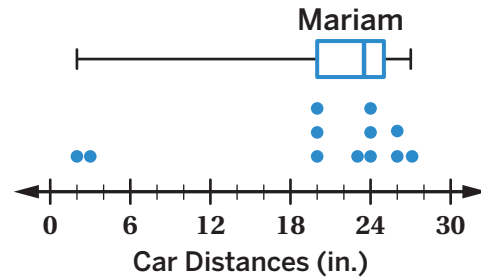
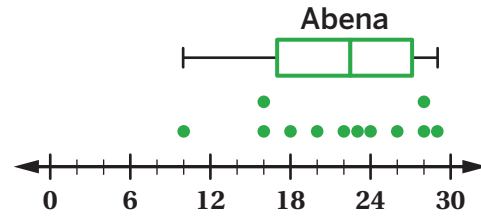
Box Plots and Races (continued)

- 5** Professor Cho wants to give an award for Most Consistent Race Car.

To do this, the professor looks at the middle half of each car's data.

Select one question to answer:

- Do you think Abena's or Mariam's race car is more consistent? Why?
- Why might Professor Cho look at only the middle half of the data?
- How might Professor Cho measure the middle half of the data?



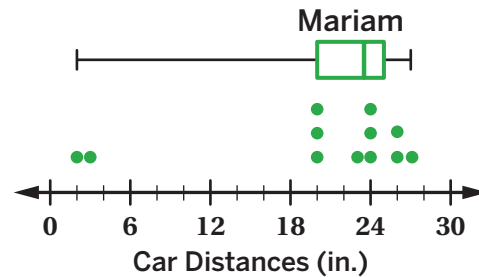
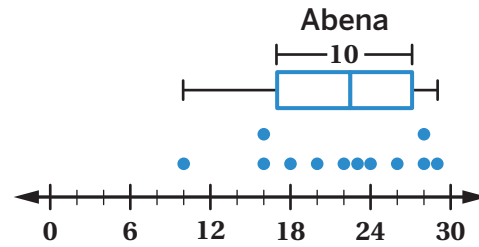
Interquartile Range

6 One way to measure the middle half of a data set is to use the *interquartile range* (or *IQR*).

IQR is a statistic that measures the spread of a data set.

The middle half of Abena's distances are between 17 and 27 inches, so the IQR of her data is 10 inches.

What is the interquartile range (IQR) of Mariam's data?

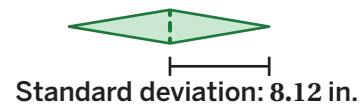
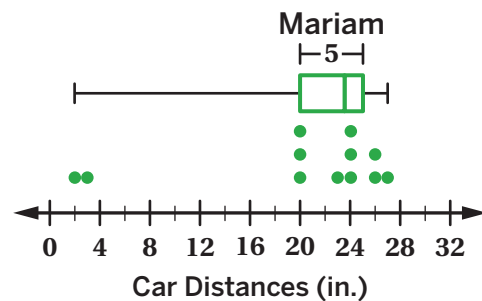


7 Standard deviation and IQR are statistics that measure the spread of a data set.

Which statistic is a more appropriate measure of the spread of Mariam's data?
Circle one.

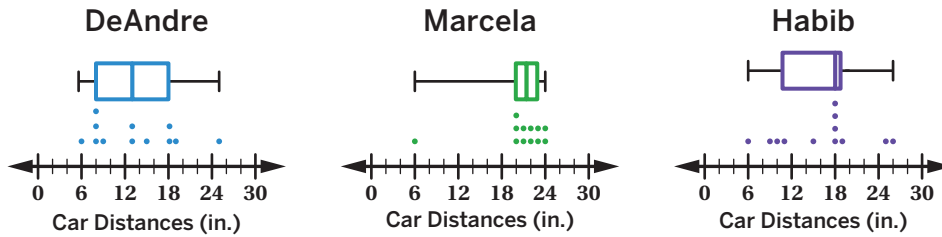
IQR Standard deviation

Explain your thinking.



Interquartile Range (continued)

8 Help Professor Cho give out the award for Most Consistent Race Car.



a Determine the IQR for each student's data.

Student	IQR (in.)
Abena	10
Mariam	5
DeAndre	
Marcela	
Habib	

b Who do you think Professor Cho will give the award to? Circle one.

Abena Mariam DeAndre Marcela Habib

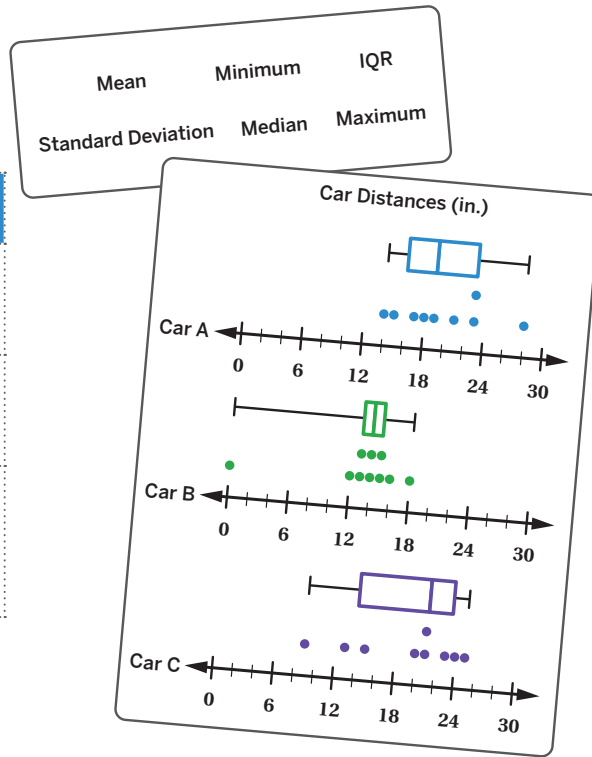
Explain your thinking.

Which Statistic?

9 Here is the data for three new cars.

Which statistic would you use to answer each of the following questions?

Question	Statistic
Which car had the best individual launch?	
Which car was the most consistent?	
Which car typically traveled the farthest distance?	



10 Which car would you give the Best Overall Car award to? Circle one.

Car A Car B Car C

Use vocabulary from this unit to justify your thinking.

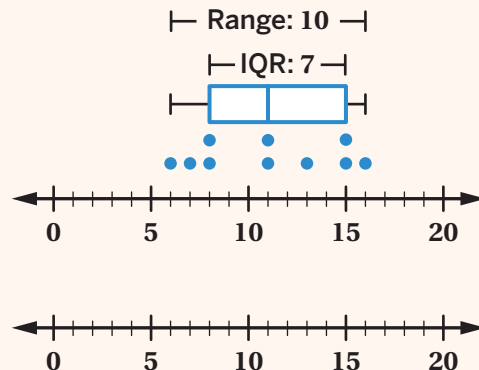
You're invited to explore more.

11 Range and IQR are both measures of spread.

Here is a box plot with an IQR of 7 and a range of 10.

Can you make a second box plot with the following features? Select *all* the box plots that are possible to create.

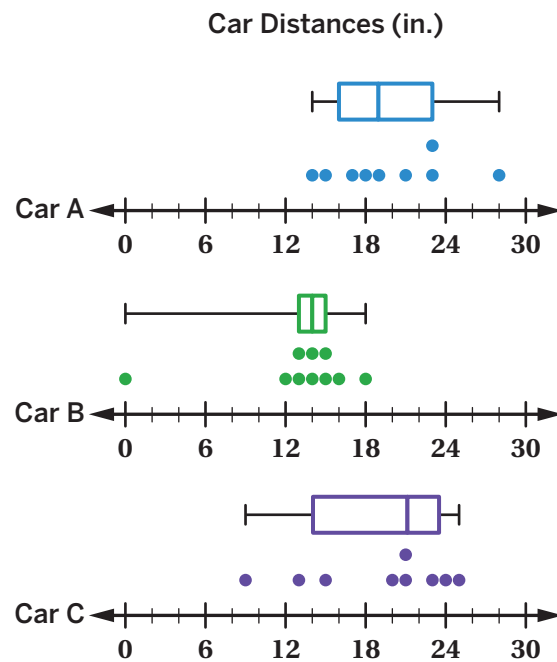
- A. A smaller IQR and larger range
- B. A larger IQR and smaller range
- C. An IQR of 10 and a range of 7
- D. An IQR of 0 and a large range



12 Synthesis

How can the interquartile range help you compare data sets?

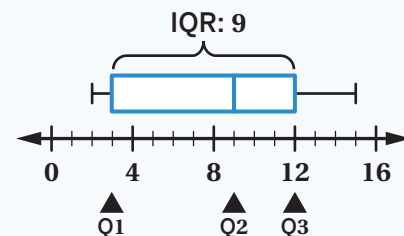
Use the example data if it helps with your thinking.



15 Summary 7.10

The *interquartile range* (IQR) is a statistic that measures the spread of a data set. Measures of spread, like IQR, help us determine how consistent the data within a set is. The IQR represents the middle half of the data set, and we calculate it by determining the distance from Q1 to Q3. In a box plot, the IQR is the width of the box.

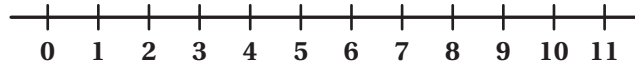
Here is an example of a data set where $Q1 = 3$ and $Q3 = 12$. The IQR of this data set is 9, because $12 - 3 = 9$.



Practice 7.10

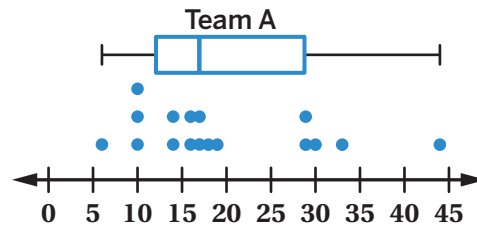
Name: _____ Date: _____ Period: _____

1. Create a box plot to represent the data set : {2, 2, 3, 4, 4, 4, 5, 6, 6, 6, 7, 9}

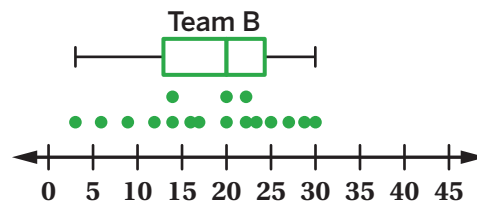


Problems 2–4: These box plots represent the points scored per game for three different football teams over the course of a season.

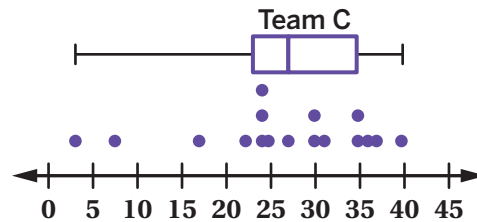
2. Which team generally scored the most points?



3. Which team scored the most points in a single game?

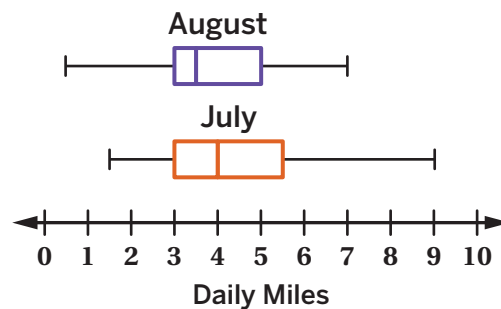


4. Which team was the most consistent? Explain your thinking.



Problems 5–6: Mariana used a fitness app to track how many miles she walked each day in July and August. Here is a box plot of her data.

5. Which month has a smaller IQR?




6. What does the IQR tell you about Mariana's walking habits in July and August?

Practice

7.10

Name: _____ Date: _____ Period: _____

7.  **Test Practice** Here is a data set: [2, 2, 4, 4, 5, 5, 6, 7, 9, 15].

If you added 24 as a new data point, how would the IQR change?
Explain your thinking.

8. Here is a different data set: [3, 6, 7, 23].

Determine a number you could add to make an IQR of 15.

Spiral Review

Problems 9–11: Write each equation in slope-intercept form.

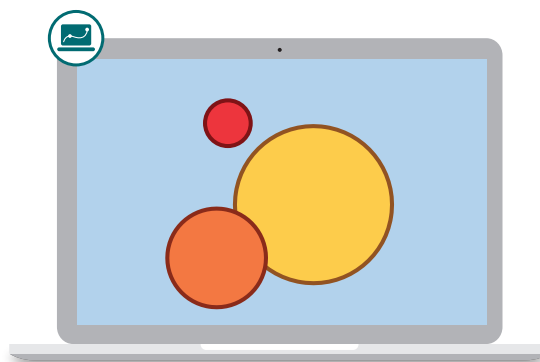
9. $150x + 50y = 400$

10. $y = 3(x - 4)$

11. $16y - 32 = 20x$

Far Out

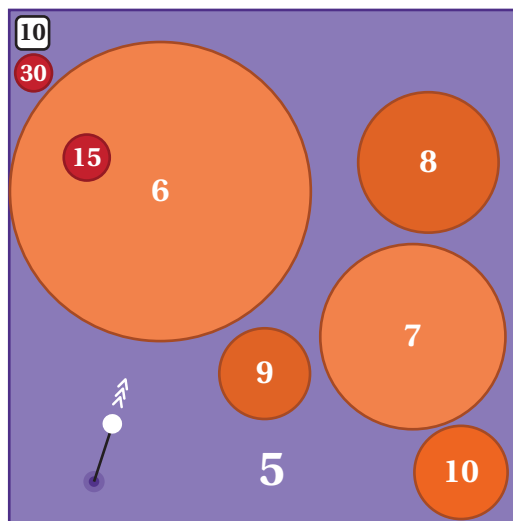
Let's determine whether a data point is an outlier and consider its effect on the mean and median.



Warm-Up

1 Use the digital activity to play a game.

Play the game as many times as you want.



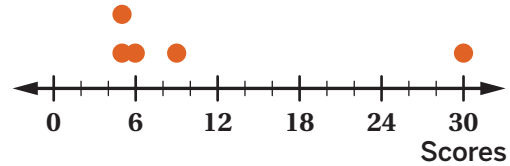
Outliers and Their Effects

2 Koharu played the game 5 times.

Here are her scores.

What do you think is her typical score?

5 6 30 9 5



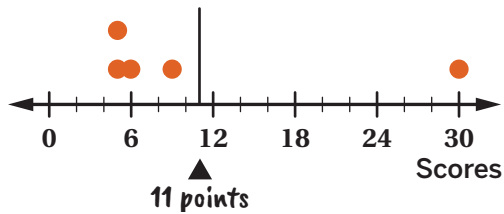
3 Here are two different strategies for determining Koharu's typical score.

Student 1

5 6 30 9 5

$$\frac{5 + 6 + 30 + 9 + 5}{5} = 11$$

Koharu's typical score is 11.

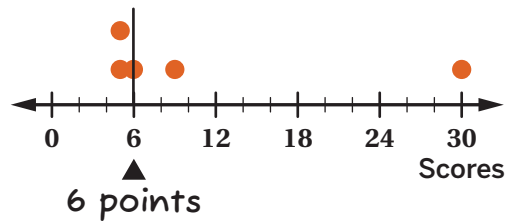


Student 2

5 6 30 9 5

5, 5, 6, 9, 30

Koharu's typical score is 6.



Discuss: How are Student 1's and Student 2's strategies alike?
How are they different?

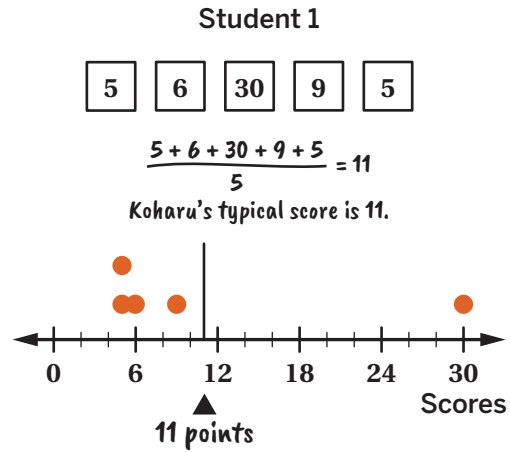
Outliers and Their Effects (continued)

- 4** Koharu says: *You shouldn't use the mean for the typical score because the 30 messes it up.*

Do you agree? Circle one.

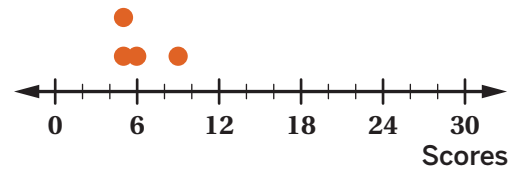
Yes No I'm not sure

Explain your thinking.

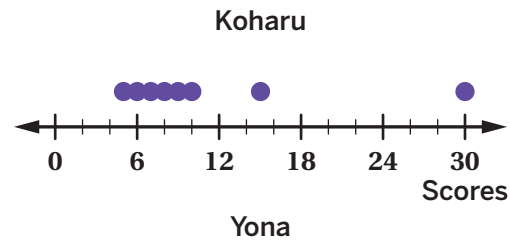


- 5** In Koharu's data, 30 is an **outlier** because it's far from other values in the data set.

a **Data Talk!** Discuss the impact an outlier has on the *mean* of a data set.



b Circle the point(s) in Yona's data that you think are outliers.



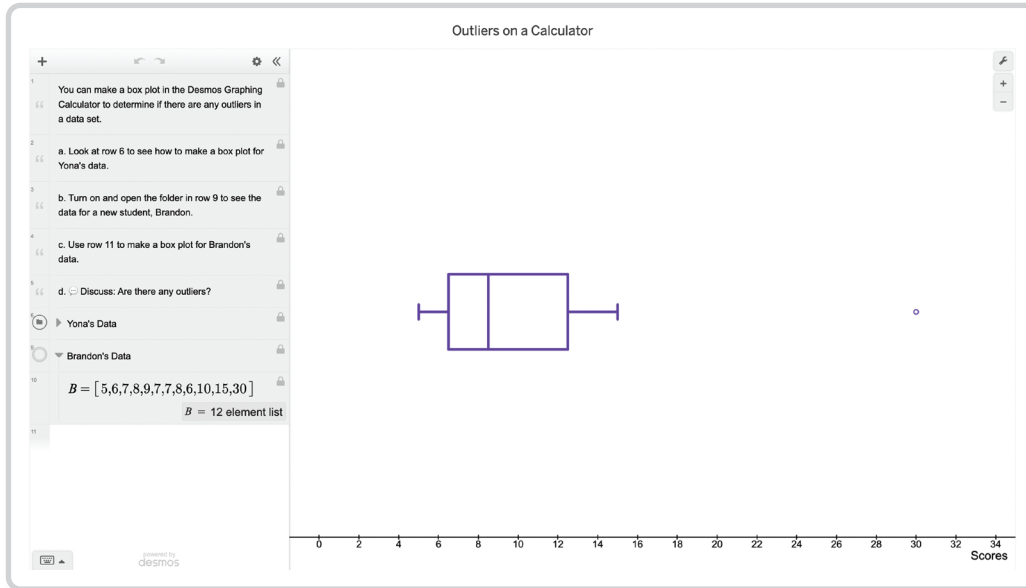
- 6** Let's look at a dot plot and a box plot for Yona's data.

a Watch what happens when we click and unclick the "Exclude outliers" checkbox.

b **Discuss:** How does the checkbox change the box plot?

Outliers and Box Plots

- 7** You can make a box plot in the Desmos Graphing Calculator to determine whether there are any outliers in a data set.



- Look at row 6 to see how to make a box plot for Yona's data.
- Turn on and open the folder in row 9 to see the data for a new student, Brandon.
- Use row 12 to make a box plot for Brandon's data.
- Discuss:** Are there any outliers?

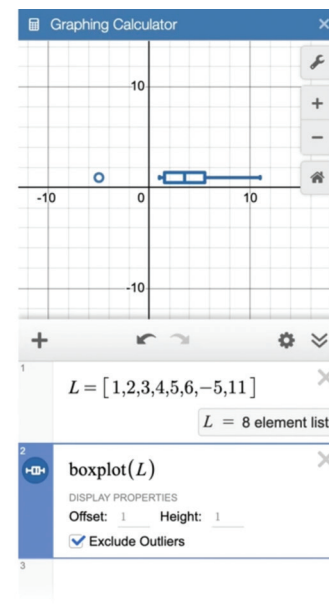
- 8** Making a box plot can help us determine the outliers in any data set.

- Let's watch an animation.
- Use the Desmos Graphing Calculator to answer:


Which value is an outlier in this data set?

$[1, -1, 0, 0, 1, -7, 2, 3, 7]$

- 7
- 1
- 3
- 7



Outliers and Box Plots (continued)

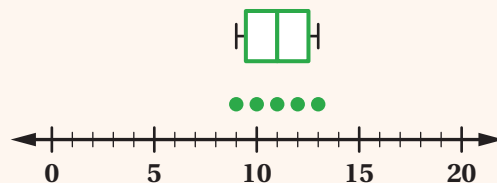
- 9** **a** In the digital activity, drag the point to see when a value becomes an outlier.
- b** How do you think the calculator determines if a value in a data set is an outlier?
- 10** One way the Desmos Graphing Calculator decides if a value is an outlier is by looking at its distance from *quartiles* 1 or 3.
- a** Let's watch an animation to see what we mean.
- b**  **Discuss:** How would you describe this strategy for determining whether a point is an outlier?

You're invited to explore more.


- 11** Use the digital activity to add points to the dot plot.

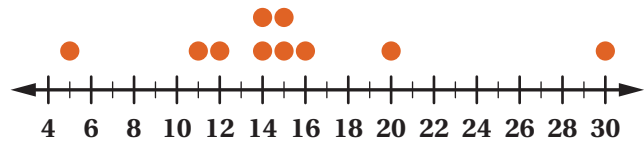
Can you make a dot plot that has:

- One outlier?
- Two outliers?
- Three outliers?
- Four outliers?
- More than four outliers?



12 Synthesis

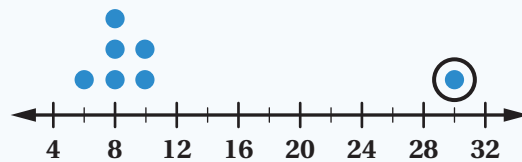
 **Discuss:** Describe how to use the Desmos Graphing Calculator to determine which values in a data set are outliers.



Use the Desmos Graphing Calculator if it helps with your thinking.

15 Summary 7.11

In this data set, the circled data point is an **outlier**.



You can identify outliers using dot plots, box plots, and technology tools such as graphing calculators.

- You can also identify outliers using the *IQR*. Outliers are values further than 1.5 times the *IQR* below *Q1* or above *Q3*.

When deciding which measure of center is appropriate to represent a data set, it's important to identify any outliers.

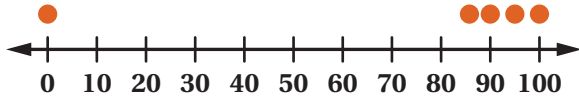
- Outliers have a big impact on the *mean*, but they don't impact the *median* very much.

Outlier a data value that is far from the other values in the data set.

Practice 7.11

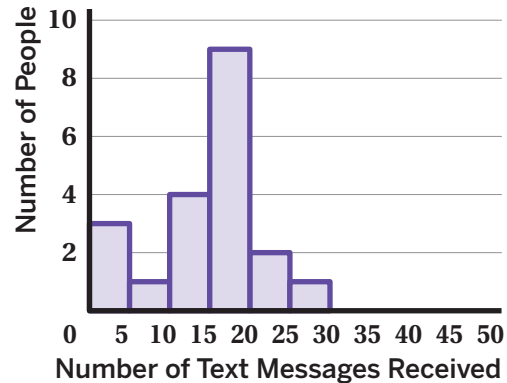
Name: _____ Date: _____ Period: _____

Problems 1–2: Ricardo got the following scores on five class assignments: 87, 90, 0, 95, and 100. His teacher lets students decide whether their final score will be the mean or the median of those five scores.



1. Would using the mean or the median give Ricardo the higher final score?
2. Explain your thinking.

Problems 3–4: This histogram represents the number of text messages that 20 people received in one day.

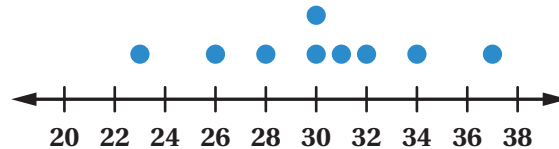


3. If you removed one person who received 3 text messages from the data set, would the median increase, decrease, or stay the same?
4. If you added a person who received 52 text messages to the data set, would the mean or median change more? Circle one.

Mean Median

Explain your thinking.


5. Here is a data set:
26, 30, 31, 32, 28, 30, 34, 37, 23.

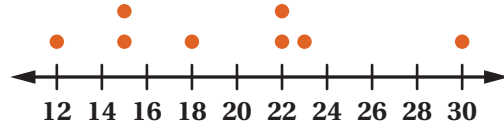


Julian says that 37 is an outlier because it is the maximum value. What could you say to Julian to help him understand his mistake?

Practice 7.11

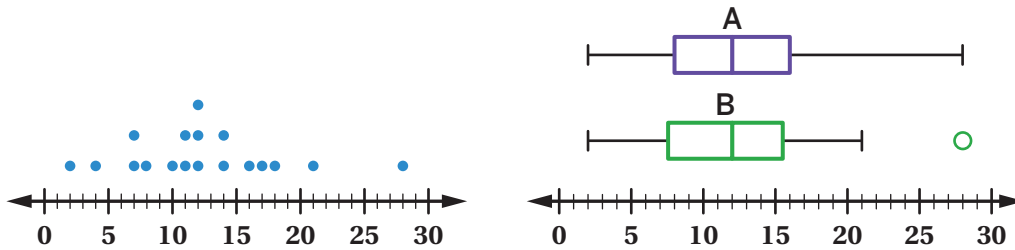
Name: _____ Date: _____ Period: _____

6.  **Test Practice** This dot plot represents the ages of different people in a bike shop. Which data point would be an outlier if it were added to the data set?



- A. 44 B. 8 C. Both D. Neither

7. The dot plot represents the data in box plot A. Circle one point to remove from the dot plot so that it represents box plot B instead.



Spiral Review

Problems 8–10: Evaluate each expression. Write your answer as a fraction.

8. $\frac{2}{3} + \frac{1}{5}$

9. $\frac{2}{5} - \frac{3}{8}$

10. $\frac{6}{15} + \frac{8}{20}$

11. 620 people were surveyed about whether they pour dressing on their salad or put it on the side. They were also asked whether they pour shampoo directly on their hair or pour it in their hand first. The table shows the results.

	Dressing on Salad	Dressing on Side
Shampoo in Hair	116	28
Shampoo in Hand	392	84

What is the relative frequency of people in this group who pour dressing on their salad and pour shampoo in their hand first? Round your answer to the nearest tenth.

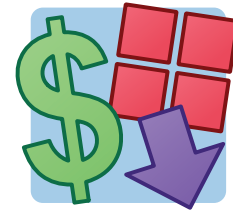
Unit 7
Lesson
12

Name: _____ Date: _____ Period: _____

Variability 🔄 S-ID.1, S-ID.2, S-ID.3, SMP.1, SMP.3, SMP.4, SMP.5

Dynamic Decades

Let's use measures of center and spread to describe how aspects of the United States have changed over time.



Warm-Up

You will use Screen 1 of the digital activity to see the minimum wages for each U.S. state in 2024.

1. Click on a state to see its name and minimum wage.

2.  **Discuss:**

- What do you notice? What do you wonder?
- Why might someone want to compare the minimum wage in different states?

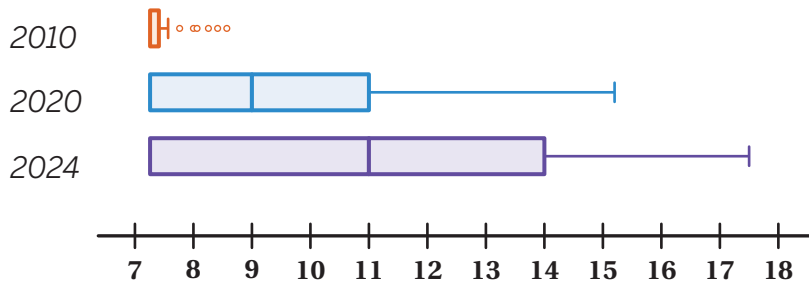
Minimum Wages


3. Think about the minimum wage data from the Warm-Up. Write a question you could answer using that data. Then write one you could *not* answer.

Question You Could Answer

Question You Could Not Answer

Omari wondered: *How has the minimum wage in the United States changed over time?* He used the Desmos Graphing Calculator to create boxplots for the minimum wage in 2010, 2020 and 2024.




4.  **Data Talk!** Compare the distributions. How are they alike? How are they different?


Minimum Wages (continued)

5. You will use Screen 3 of the digital activity to complete the table for 2020 and 2024. You can see how Omari completed the row for 2010 in the folder “Omari’s Work for 2010.”

Year	Minimum	Maximum	Median	IQR	Mean	Standard Deviation
2010	\$7.25	\$8.55	\$7.25	\$0.15	\$7.43	\$0.36
2020	\$7.25	\$15				
2024	\$7.25					

6.  **Data Talk!** Help Omari answer: *How has the minimum wage in the United States changed over time?*

Use statistics about center and spread to support your ideas.

7.  **Discuss:** Did the minimum wage change in the ways you expected or were you surprised?

Change in the 2010s

8. Circle one question to investigate.
- A. How has the median rent changed from 2010 to 2019?
 - B. How has the percentage of the population between the ages of 15 and 17 years old changed from 2010 to 2019?
 - C. How has the percentage of people with internet access changed from 2013 to 2019?
9. You will use Screens 4, 5, or 6 of the digital activity to make a dot plot, histogram, or box plot for both years. Describe the shape of the data for each year (symmetrical, skewed, etc.).

Year	Shape of Data

10. Which measure of center and spread will best help you answer your question? Explain your thinking.

Change in the 2010s (continued)

- 11.** Make a poster of your work to answer your question. Your poster should include:
- The question you are trying to answer.
 - A data visualization of each data set (dot plot, histogram, or box plot).
 - A description of the shape of the distribution (bell-shaped, skewed, bimodal, etc.).
 - Whether or not there are any outliers in the data sets.
 - Measures of center and spread (mean/standard deviation or median/IQR) for each year.
 - Your answer to the question.
 - Two new questions that you have after analyzing the data.

Synthesis

Circle *one* question and answer it.

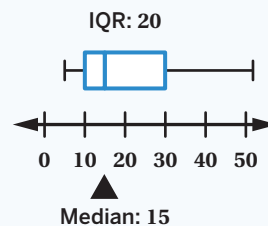
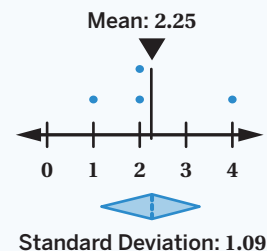
- What statistics did you find most helpful for comparing data sets? Why?
- Why might it be important to study how things like the minimum wage or median rent are changing over time?
- After seeing the work of other groups, what new questions do you have about how things have changed in the United States in the last 10–15 years?

Summary 7.12

It can be helpful to use measures of center and measures of spread to compare data sets. You can choose which measure of center or spread to use based on the shape of the data:

- When data distributions are symmetric or bell-shaped, you can use the mean and standard deviation to compare.
- When data distributions are skewed or contain outliers, you can use the median and IQR because the mean and standard deviation are both affected by extreme values.

Comparing different data sets can reveal important information about change over time in different situations, like minimum wage or median rent. Comparing measures of center, like mean or median, can help you determine if the data has increased, decreased, or stayed the same. Comparing measures of spread, like standard deviation or IQR, can help you determine if the data has become more or less consistent over time.



Practice 7.12

Name: _____ Date: _____ Period: _____

Problems 1–3: Yona is interested in the lengths of online videos. She found this data about the lengths of thousands of videos, organized by category.

	Mean Length (min)	Standard Deviation (min)
Music	4.15	0.6
Gaming	6.38	2.1

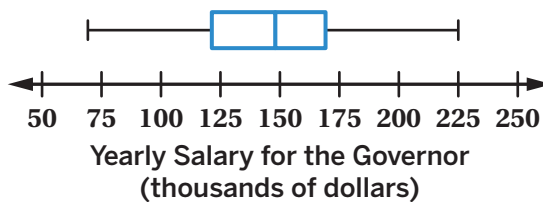
- Write a question you could answer using this data.
- Write a question you could *not* answer using this data.
- How do the lengths of the music videos and the gaming videos compare? Use statistics to support your answer.

4. The table shows some statistics about the points per game scored by the Chicago Bulls in 1978 and 2020.

	Median (pts)	Mean (pts)	Standard Deviation (pts)	IQR (pts)
1978	102	104.9	10.99	15
2020	110	110.70	11.36	17.5

How has the number of points per game changed from 1978 to 2020? Use statistics to support your answer.

Problems 5–6: The box plot represents the yearly salary in 2022 for each governor in all 50 states.



- What do you notice about the data?
- Test Practice** Unlike governors, United States senators all have the same salary. In 2022, senators made \$174,000 per year. Which job generally has a higher salary? Explain your thinking.

Practice 7.12

Name: _____ Date: _____ Period: _____

Spiral Review

Problems 7–9: Use mental math to determine each value.

7. 50% of 120

8. 10% of 120

9. 90% of 120

Problems 10–11: How many solutions does each equation have? Circle one.

10. $6t + 2 = -3 + 6t$

A. One solution

B. No solution

C. Infinitely many solutions

11. $15 - 3t - 15 = -3t$

A. One solution

B. No solution

C. Infinitely many solutions

Problems 12–14: Here is a two-way frequency table with data about people who live on two different islands and whether they prefer to wear a hat outside.

12. Complete the two-way frequency table and the conditional relative frequency table about the islands.

	Hat	No Hat	Total
Island A	2,394	6,128	
Island B	7,911		
Total		39,446	

13. What does 6 128 mean in this situation?

14. Is there an association between which island a person lives on and whether they prefer to wear a hat? Explain your thinking.

	Hat	No Hat	Total
Island A			100.0%
Island B			100.0%

Practice Day 1

Let's practice what you've learned so far in this unit!



You will use task cards and the digital activity for this Practice Day. Record all of your responses here.

Task A: Constitutions

1. Description:

2. Circle one: Yes or No

Outliers or Explanation:

3. IQR: _____

4. **a** Mean: _____

Explanation:

Median: _____

4. **b** Explanation:

4. **c** Measure of center:

Explanation:

Task B: Caps or Visors?

1. Caps: _____

Visors: _____

Explanation:

2. Explanation:

3. Circle one: People who buy caps or People who buy visors

Explanation:

4. Explanation:

Practice Day 1

Task C: Home Run King

1. Circle *all* the true statements: A B C
2. Circle one: 1 or 5

Explanation:

3. Circle one: Aaron or Bonds
4. Circle one: Aaron or Bonds

Explanation:

Explanation:

Task D: Hot Summer Nights

1. 1960s: _____ 2010s: _____

2. Explanation:

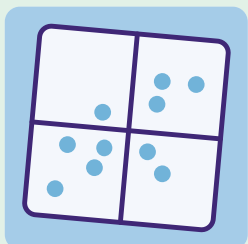
3. Circle one: Yes or No

Explanation:

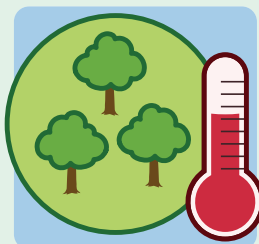
You're invited to explore more.

1. _____
2. Circle one: A B C
3. Circle one: A B C
4. Explanation:

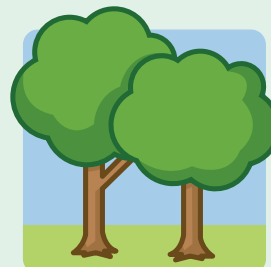
Summarizing Two-Variable Data



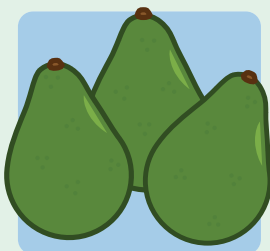
Lesson 13
Correlation Coefficient



Lesson 14
How Hot Is It?



Lesson 15
City Slopes



Lesson 16
Residual Fruit



Lesson 17
Penguin Populations



Lesson 18
Behind the Headlines



Lesson 19
City Data

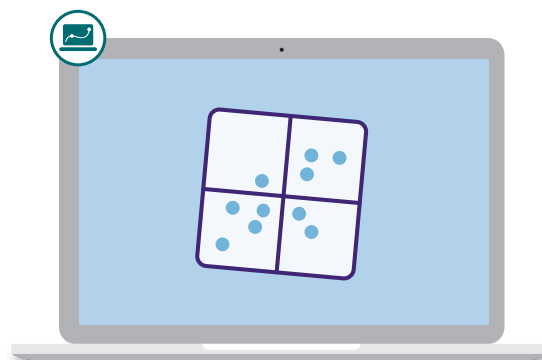
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Comparing Models, Correlation and Causation 🕒 S-ID.8, SMP.1, SMP.6, SMP.7

Correlation Coefficient

Let's learn about the correlation coefficient (r -value) as a way to measure the strength and direction of a linear relationship.



Warm-Up

1 Play a few rounds of Polygraph with your classmates!

You will use a Warm-Up Sheet with **scatter plots**. For each round:

- You and your partner will take turns being the Picker and the Guesser.
- Picker: Select a scatter plot from the Warm-Up Sheet. Keep it a secret!
- Guesser: Ask the Picker yes-or-no questions, eliminating scatter plots until you're ready to guess which scatter plot the Picker chose.

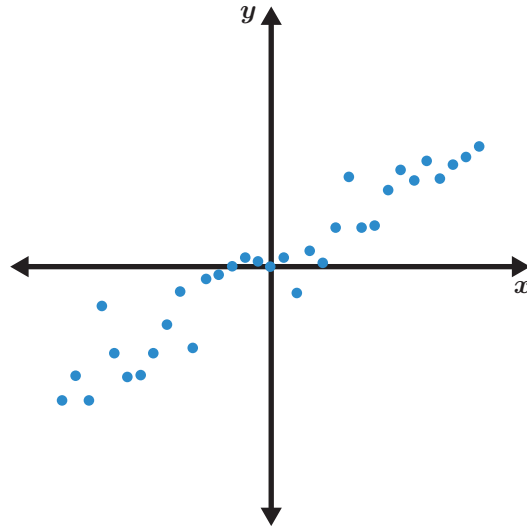
Record helpful questions from each round in the space below.

Linear Associations and Scatter Plots

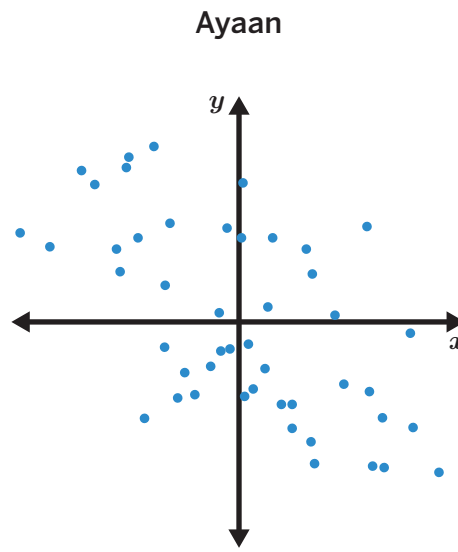
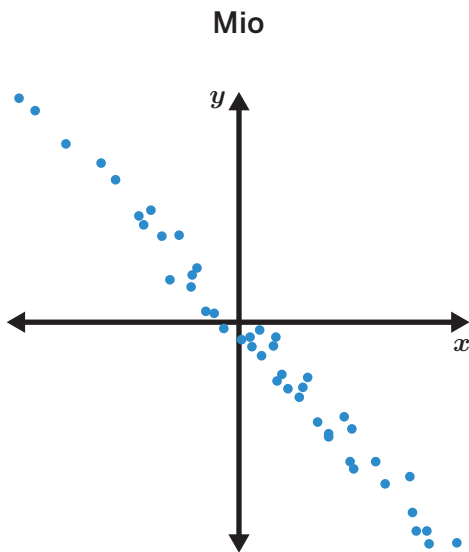
- 2** In Polygraph, you looked at different scatter plots.

Now let's use the digital activity to make your own!

- a** Drag the sliders in the digital activity to make a scatter plot you like.
- b** Describe your scatter plot.



- 3** Here are the scatter plots Mio and Ayaan made.




Data Talk! How are they alike? How are they different?

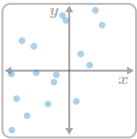
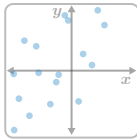
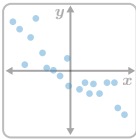
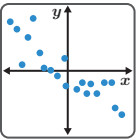
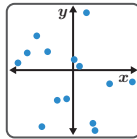
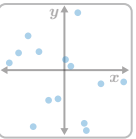

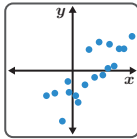
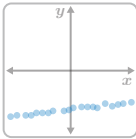
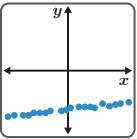
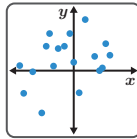
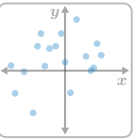
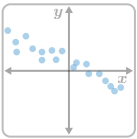
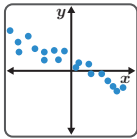
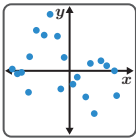

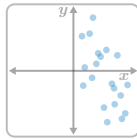
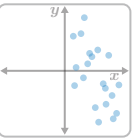
Linear Associations and Scatter Plots (continued)

4 When the points on a scatter plot follow a line, we say there is a *linear association* between x and y .

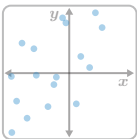
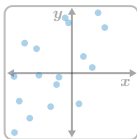
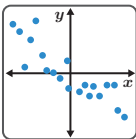
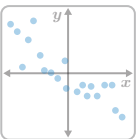
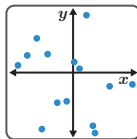
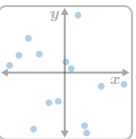
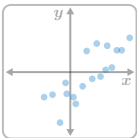
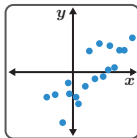
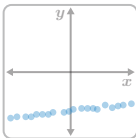
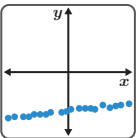
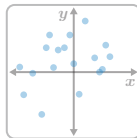
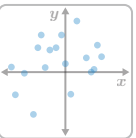
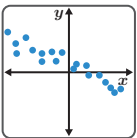
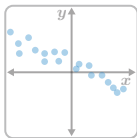
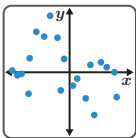

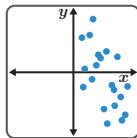
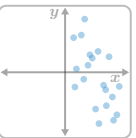
a Here are some terms that describe linear associations.

b  **Discuss:** What does each term mean?

Strength

Weak	Strong
	
	
	
	
	
	
	
	
	

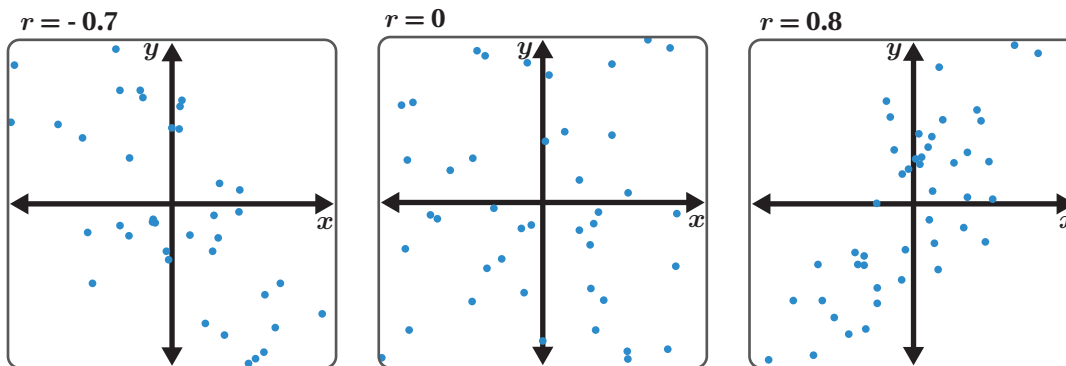
Direction

Negative	Positive
	
	
	
	
	
	
	
	
	

The r -value

5 The r -value is a number that measures the strength and direction of a linear association.

a Here are three different r -values:



b What do you notice and wonder about the r -value?

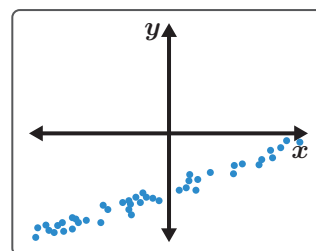
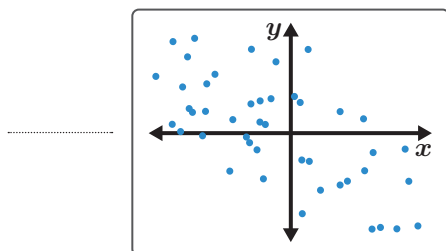
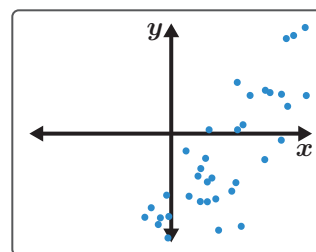
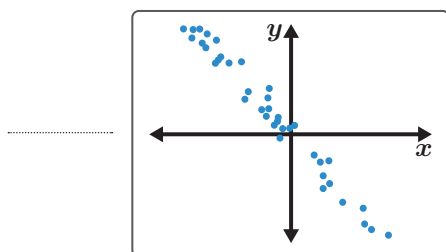
6 Match each scatter plot to its r -value.

-0.99

-0.65


0.86

0.99

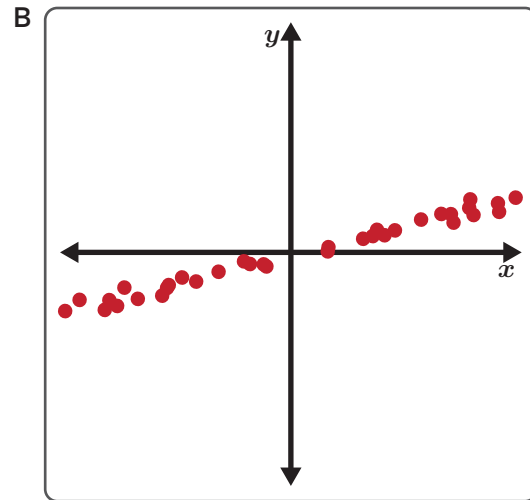
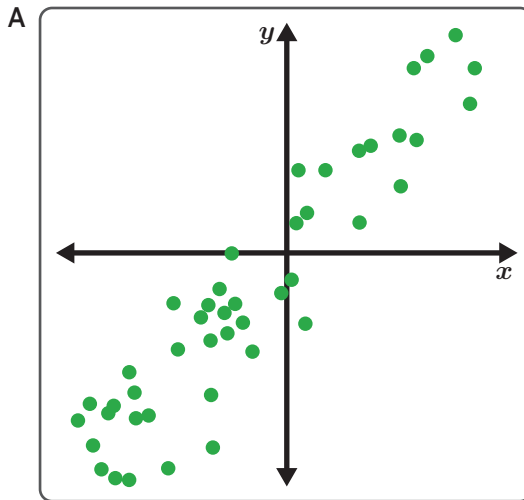


Correlation Coefficient

7 The r -value is also known as the correlation coefficient.

a  **Discuss:** What do you think *correlation* means?

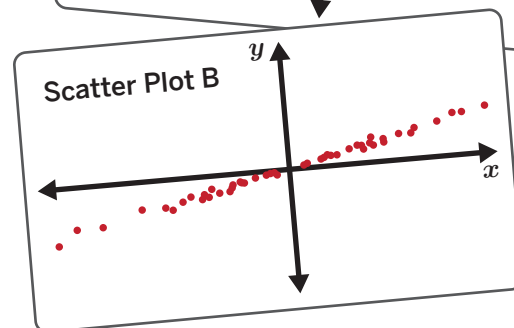
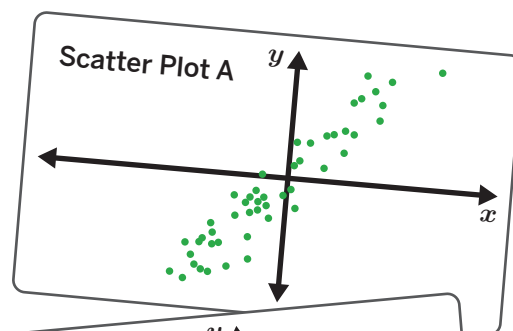
b Circle the scatter plot that has a greater correlation coefficient. Revisit Problem 5 if it helps with your thinking.



Explain your thinking.

8 Mio says Scatter Plot A has a greater correlation coefficient because the slope of its line is larger.

What would you say to Mio to help her understand her mistake?

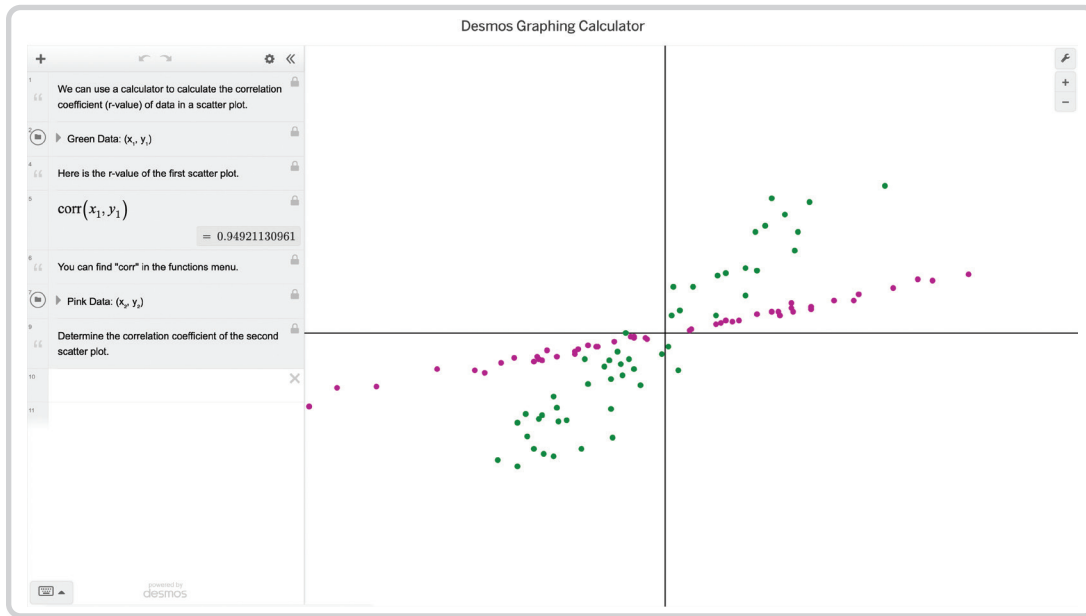


Activity 3

Name: Date: Period:

Correlation Coefficient (continued)

- 9** We can use the Desmos Graphing Calculator to calculate the correlation coefficient (r -value) of data in a scatter plot.



Determine the correlation coefficient of the second scatter plot.

You're invited to explore more.

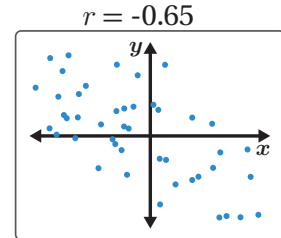
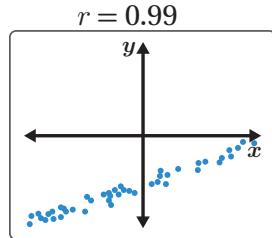
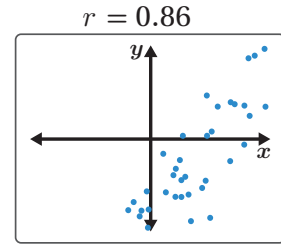
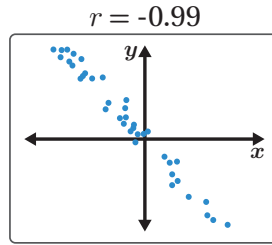
- 10** Use the digital activity to drag the sliders and see how the scatter plot changes.
- a** What does the top slider control?

 - b** What does the bottom slider control?

11 Synthesis

Discuss: What does the correlation coefficient tell us about the data in a scatter plot?

Use these scatter plots if they help with your thinking.

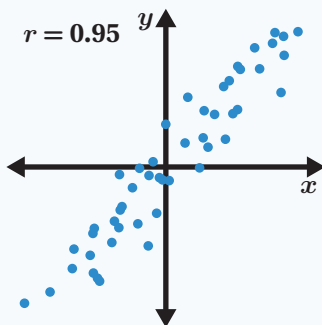


14 Summary 7.13

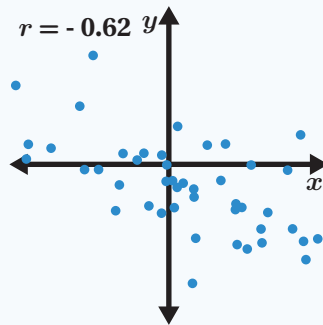
You can describe the strength and direction of a linear association shown on a **scatter plot** using the **correlation coefficient** (also called the **r-value**). The closer the r-value is to 0, the weaker the linear association. The closer the r value is to -1 or 1, the stronger the linear association.

These graphs show examples of different correlation coefficients.

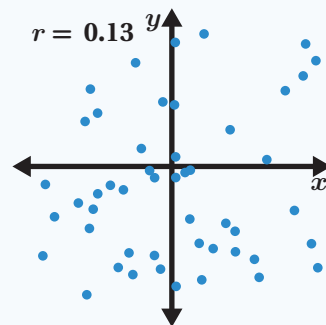
Positive Association
Strong Association



Negative Association



Positive Association
Weak Association



correlation coefficient A number between -1 and 1 that describes the strength and direction of a linear association between two numerical variables.

r-value A value that indicates the correlation coefficient.

scatter plot A graph of plotted points that show the relationship between two variables.

Practice 7.13

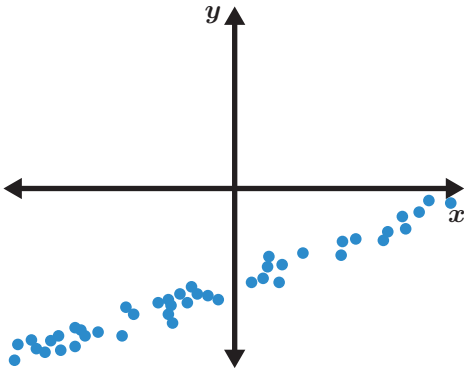
Name: _____ Date: _____ Period: _____

Problems 1–4: Determine whether each scatter plot has a strong linear relationship, weak linear relationship, or no linear relationship.

1. Strong

Weak

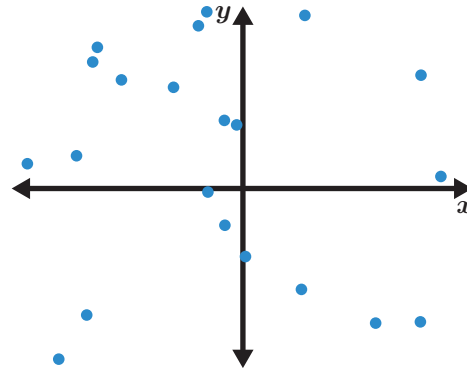
None



2. Strong

Weak

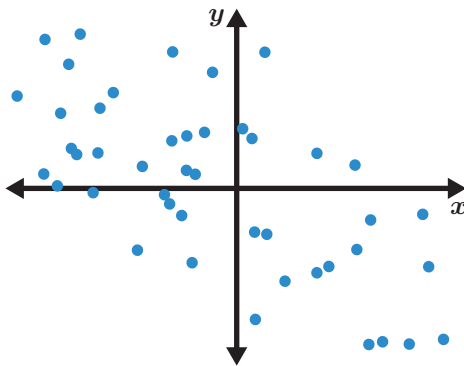
None



3. Strong

Weak

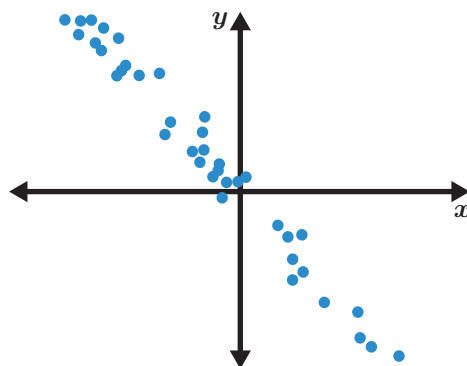
None




4. Strong

Weak

None



5.  **Test Practice** Which number could be the correlation coefficient for this scatter plot?

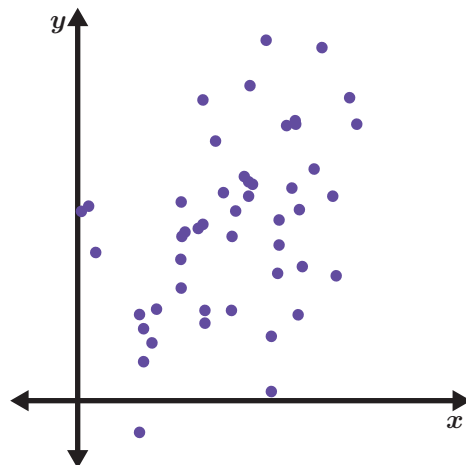
A. 0.4

B. -0.4

C. 0.9

D. -0.9

Explain your thinking.



6. A scatter plot has a correlation coefficient of $r = 0.85$. What does this tell you about the data?

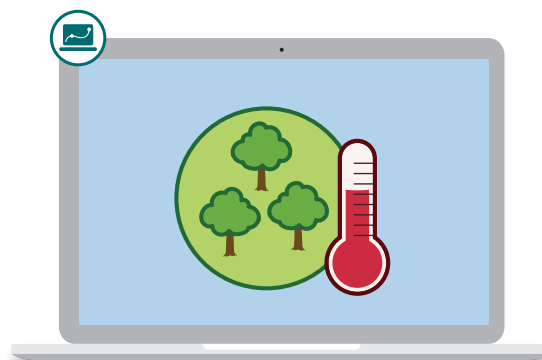
Unit 7
Lesson
14

Name: _____ Date: _____ Period: _____

Comparing Models, Variability, Correlation and Causation N-Q.2, S-ID.6, S-ID.8, SMP.1, SMP.4

How Hot Is It?

Let's use correlation coefficients to analyze relationships between income, tree cover, and average temperature.



Warm-Up

1 Here is a map of Philadelphia, PA.

Let's look at a few neighborhoods.

What do you notice? What do you wonder?

I notice:




I wonder:


Tree Cover vs. Temperature

2 Laila talked to people in different parts of Philadelphia to learn more about tree cover and temperature.

a Let's watch the data get collected.

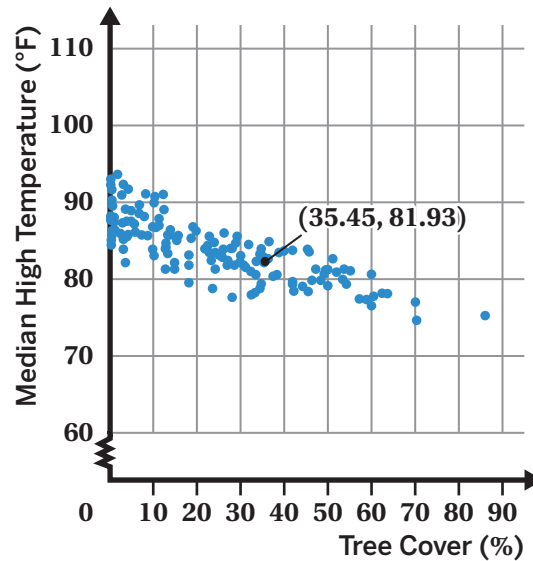
b  **Discuss:** Based on this data, do you think there is a relationship between tree cover and temperature?

Tree Cover (%)	Median High Temperature (°F)
43	78
88	74
20	88
27	81
36	80
78	75
4	90
8	85

3  **Data Talk!** Here is some data from 150 blocks in Philadelphia.

One of the coordinates is shown.

Describe what the coordinates tell you about that block.



Tree Cover vs. Temperature (continued)

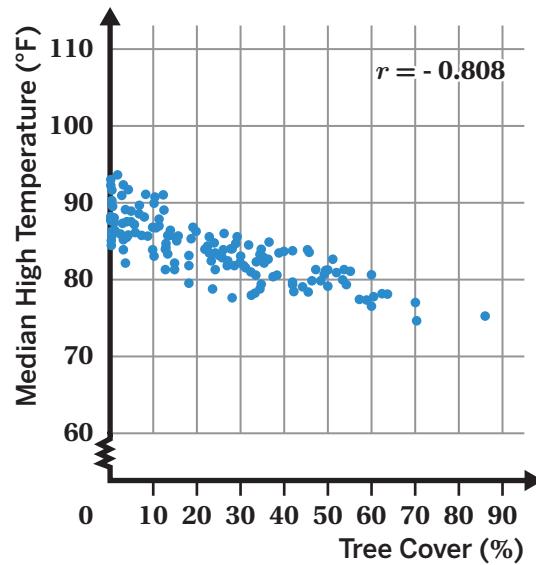
4 Here is the correlation coefficient (the r -value) of the data.

- a** Based on the r -value, what kind of relationship is there between tree cover and temperature? Circle one.

Positive Negative No association

- b** What is the strength of the relationship? Circle one.

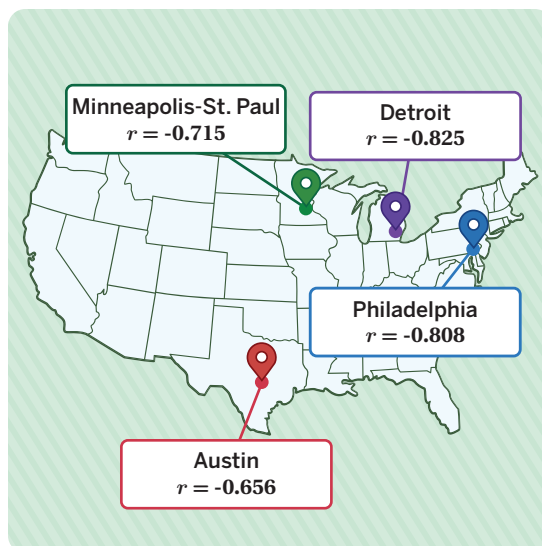
Weak Strong



5 Laila wonders how other cities compare to Philadelphia.

Let's look at other city data on the map.

Discuss: What do you notice? What do you wonder?



Including Income

6 Temperature is one of many variables associated with tree cover.

What other variables do you think could be associated with tree cover?

7 Laila wonders: *Is there an association between income and tree cover?*

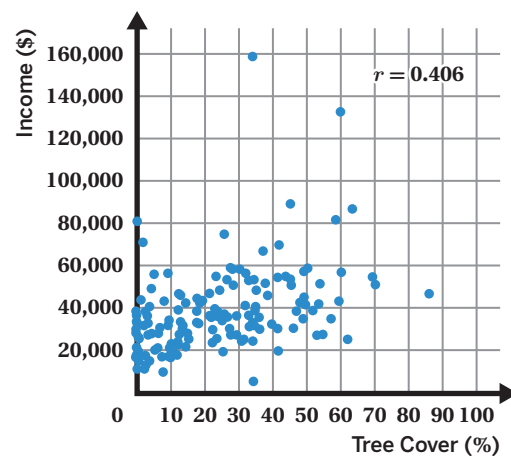
a Make a prediction: What kind of association do you expect between these variables? (E.g., weak positive or strong negative.)

b Let's look at the data.

 **Discuss:** Was your prediction correct?

8 This graph shows the average income and the percentage of tree cover for 150 blocks in Philadelphia.

What does the r -value say about the association between income and tree cover?



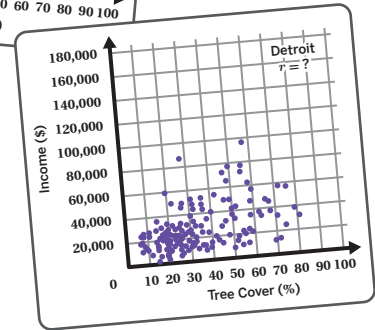
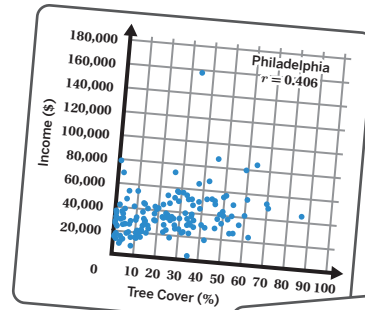
Including Income (continued)

- 9 The Philadelphia data has a correlation coefficient of 0.406.

Here is some data about income and tree cover in Detroit.

Which could the r -value for Detroit be?

- A. $r = 0.35$
 B. $r = 0.85$
 C. $r = -0.35$
 D. $r = -0.85$

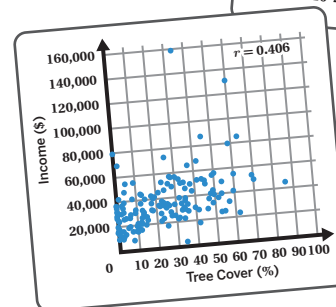
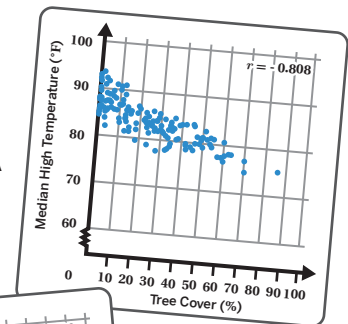


- 10 At a neighborhood meeting, someone said:

It is unfair that lower income neighborhoods are hotter in Philadelphia.

- a Does the data support the statement that lower income neighborhoods are hotter?

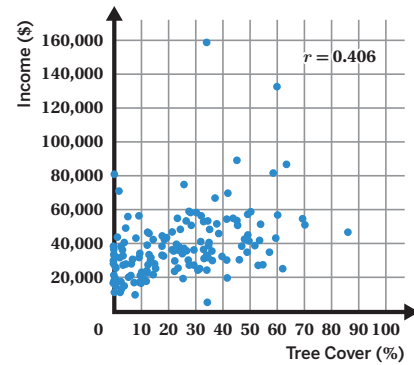
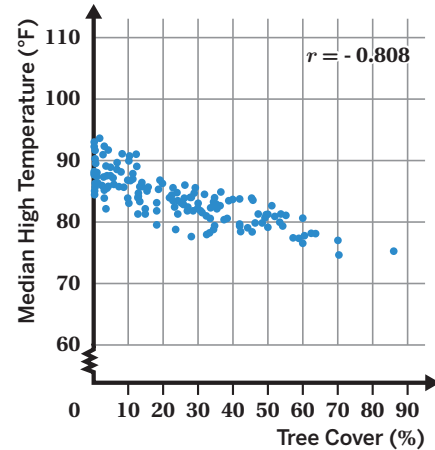
Philadelphia, PA



- b What changes do you think should be made?
- c How might lower income neighborhoods and their communities benefit from more trees being planted?

11 Synthesis

How can correlation coefficients help us describe the relationship between two variables in the real world?



14 Summary 7.14

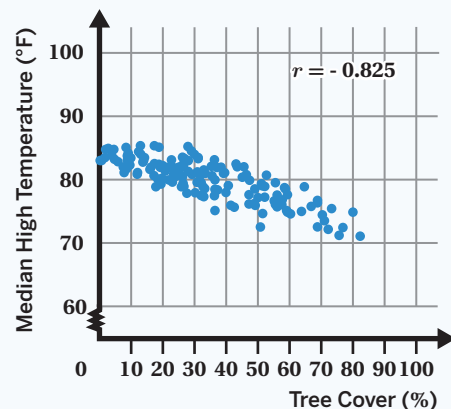
You can use a correlation coefficient to analyze the relationship between two variables and determine whether there is an association between them. The correlation coefficient, or r -value, describes the strength and direction of the relationship that may exist between two variables.

- A positive r -value means that as one variable increases, the other variable also increases.
- A negative r -value means that as one variable increases, the other variable decreases.
- The closer the r -value is to 1 or -1, the stronger the correlation.

People may use correlations in data to understand and address issues in their community.

For example, this scatter plot shows data on tree cover and temperature for 150 blocks in Detroit, Michigan.

The r -value is -0.825. This means there is a negative and strong relationship between the amount of tree cover and median high temperature in Detroit neighborhoods. Community members may use this correlation to advocate for more trees to be planted in different neighborhoods across the city.



Practice 7.14

Name: _____ Date: _____ Period: _____

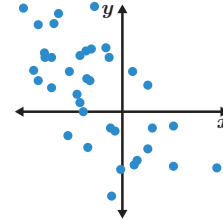
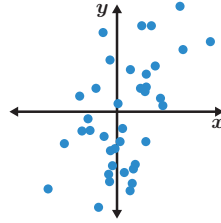
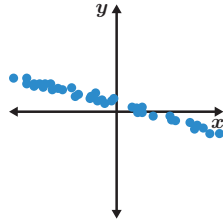
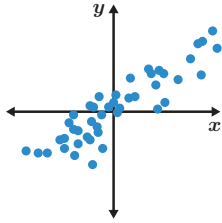
1. Match each scatter plot to its r -value.

A. $r = -0.7$

B. $r = -0.99$

C. $r = 0.7$

D. $r = 0.9$

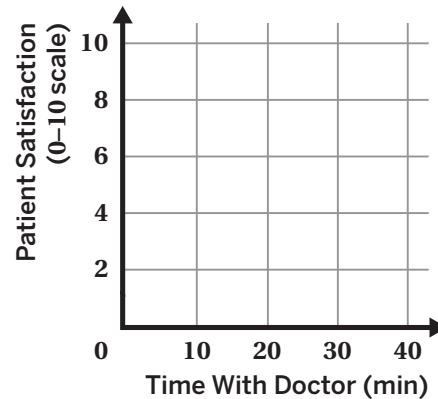


2. A medical clinic wanted to know about the experiences of their patients. They looked at the following variables:

- The number of minutes spent with a doctor.
- Patient satisfaction (on a 0-10 scale).

They found the variables have a *weak, positive* relationship.

Make a scatter plot that could represent this data.



3. Noah creates a scatter plot showing the relationship between the number of free throws taken in a basketball game and the final score. The correlation coefficient for the line of best fit is 0.76.

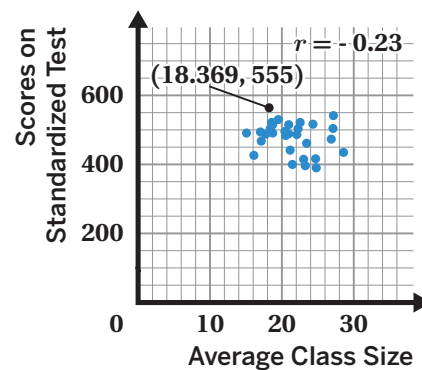
Are the number of free throws and the final score correlated? Explain your thinking.

Problems 4–6: Martina is interested in learning about different education systems around the world. She found data about two variables:

- The average class size.
- The scores on a standardized test.

Here is a scatter plot for the data.

4. The point (18.369, 555) represents Slovakia. What do the coordinates tell you about Slovakia?




5. Based on the r -value, what relationship is there between the variables? Circle one.

Positive Negative None

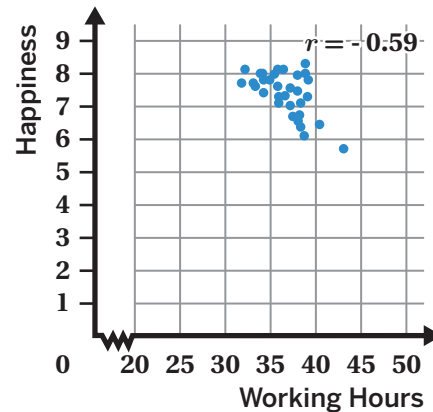
6. What is the strength of the relationship? Circle one. Weak Strong

Practice 7.14

Name: _____ Date: _____ Period: _____

7.  **Test Practice** The correlation coefficient for a given set of data is $r = 0.962$. Select *all* of the conclusions you can make about the data.
- A. There is a strong association between the two variables.
 - B. There is a weak association between the two variables.
 - C. There is no association between the two variables.
 - D. As one variable *increases*, the other variable *increases*.
 - E. As one variable *increases*, the other variable *decreases*.

8. Saanvi is interested in learning more about life in different countries. She found data about two variables:
- Average happiness (on a 1–10 scale).
 - The average number of hours people worked in a week.
- What does the r -value tell you about the relationship between average happiness and hours worked?



Spiral Review


Problems 9–11: This table shows the statistics for Fabiana’s quiz scores in math class and science class.

	Median (%)	Mean (%)	Standard Deviation (%)	IQR (%)
Math Class	85	85	7.5	6
Science Class	80	85.5	13.4	30

9. Were Fabiana’s quiz scores more consistent in math or science class? Explain your thinking.
10. If Fabiana scored 65% on a math quiz, how would the standard deviation change? Circle your choice and explain your thinking.
 Increase Decrease No change
11. Why do you think the mean and median are far apart for Fabiana’s science class quiz scores?

Unit 7
Lesson
15

Name: _____ Date: _____ Period: _____

Comparing Models, Variability  F-LE.5, S-ID.6.a, S-ID.6.c, S-ID.7, SMP.1, SMP.4, SMP.6

City Slopes

Let's use a line of fit to describe the relationship between two variables and make predictions.



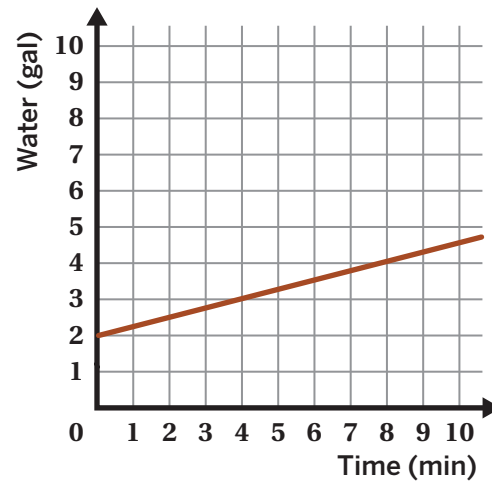
Warm-Up

1 An equation for this line is $y = \frac{1}{4}x + 2$.

Show or explain where you see $\frac{1}{4}$ and 2 in the graph.

$\frac{1}{4}$:


2:

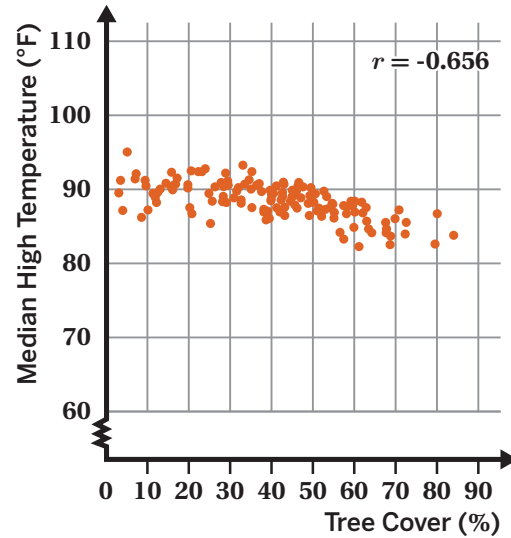


Lines of Fit

- 2** Here's the graph of temperature and tree cover for Austin, Texas.

Mathematicians use a *line of fit* to describe relationships and make predictions.

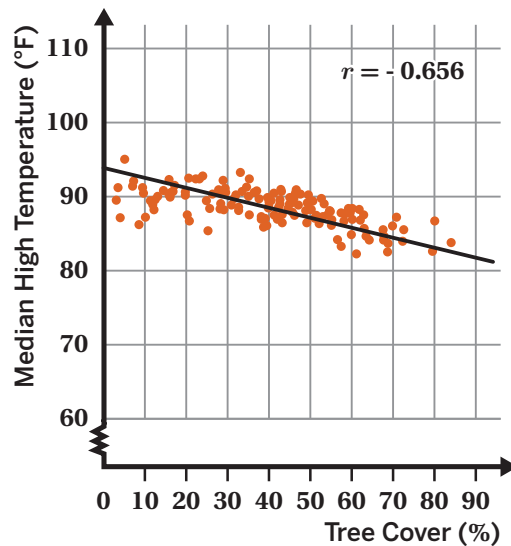
- a**  **Data Talk!** Why is a line a good fit for this data?
- b** Draw a line of fit for the data.



- 3** Here is the line of fit a student drew.

Jamal lives in Austin, on a block that has 75% tree cover.

What might the median high temperature be on his block?

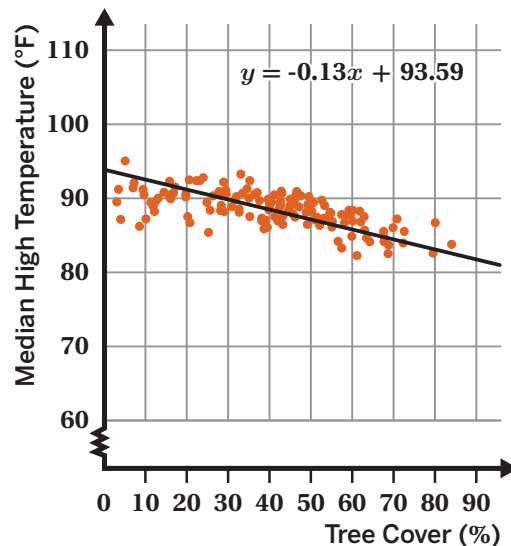


- 4** Here is an equation for this line of fit.

What does the *slope* mean for the relationship between temperature and tree cover? Circle one.

The slope means that when the tree cover increases by 1%, the temperature decreases by 0.13°F.

The slope means that when the tree cover increases by 1%, the temperature decreases by 93.59°F.



Interpreting in Context

5  **Data Talk!** Let's compare some cities.

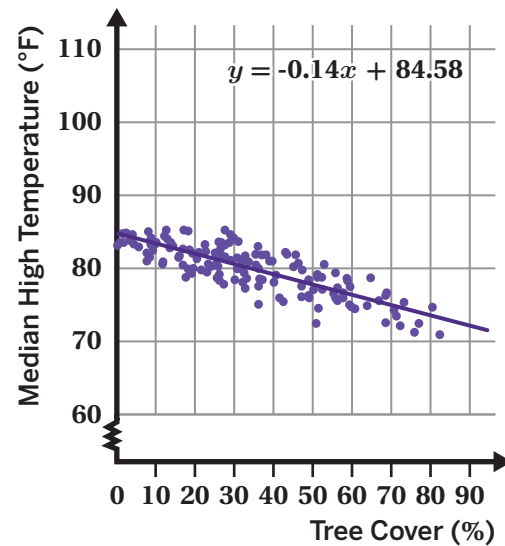
Discuss the following: What do you notice? What do you wonder?

6 Here is an equation for Detroit's line.

What do the -0.14 and 84.58 mean about the relationship between temperature and tree cover?

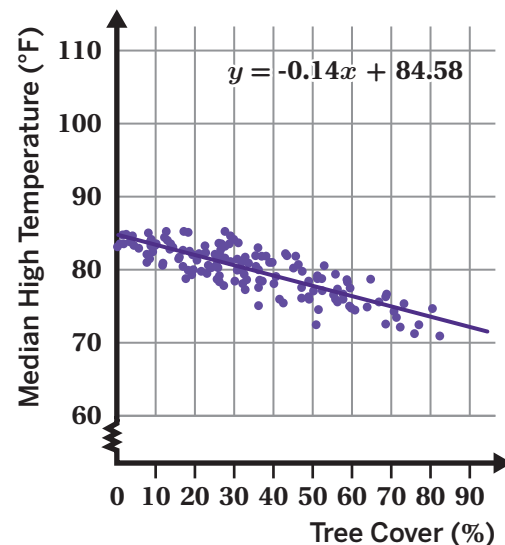
-0.14 :

84.58 :



7 A community in Detroit wants to build a park.

If the park has 80% tree cover, what might its median high temperature be?



Activity
2

Name: Date: Period:

Interpreting in Context (continued)

8  **Discuss:**

- a** How would building a new park impact the amount of tree coverage and temperature in the community?
- b** What might be some other impacts of building a new park for Detroit and its surrounding area?




9 Some community members want to tear down an existing park to build more housing.

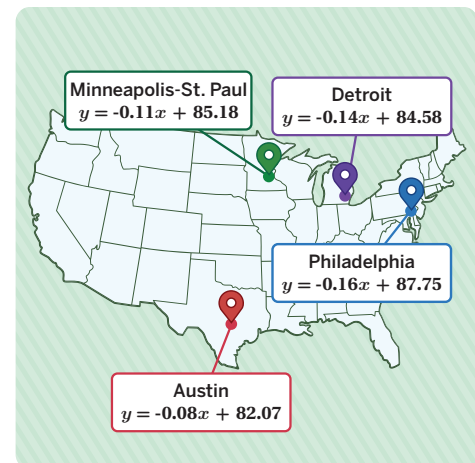
 **Discuss:**

- a** How might this affect tree coverage and temperature for the community?
- b** How might tearing down an existing park impact the community in Detroit more generally?


10 **a** Order the cities according to where tree cover has the greatest impact on temperature.

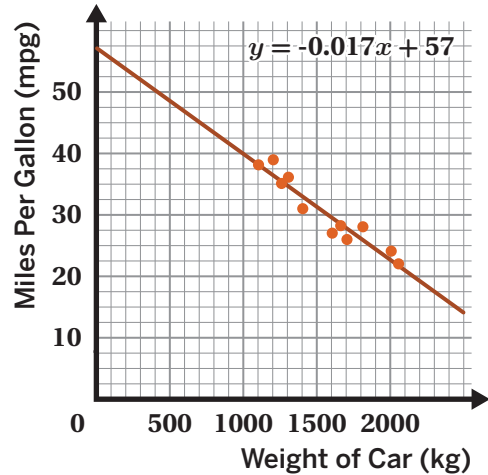
	Greatest
	Least

- b**  **Discuss:** How did you order each city?



11 Synthesis

 **Discuss:** How can a line help us make predictions about data?



14 Summary 7.15

While the correlation coefficient can help you understand the general relationship between two variables, a *line of fit* can help you make predictions about specific values in a data set. Points along the line of fit represent the likely value of unknown data in the data set.

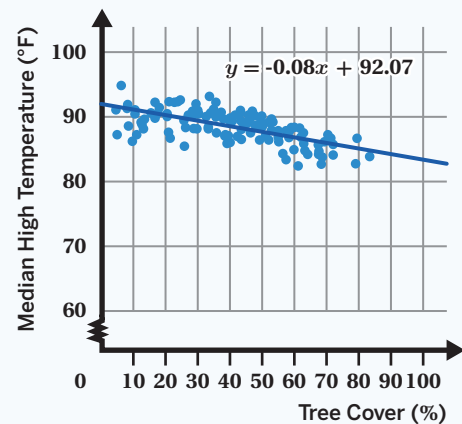
You can use the equation of the line of fit to better understand the data. The y -intercept of the line represents a potential initial value and the slope describes the rate that the variables change in relationship to one another.

For example, this scatter plot shows data on tree cover and temperature for 150 blocks in Austin, Texas. The equation of the line of fit is $y = -0.08x + 92.07$.

The slope is -0.08 . This means that when the tree cover increases by 1% in Austin, the predicted temperature decreases by 0.08°F .

The y -intercept is 92.07 . This means that if the tree cover in Austin is 0%, the predicted temperature is 92.07°F .

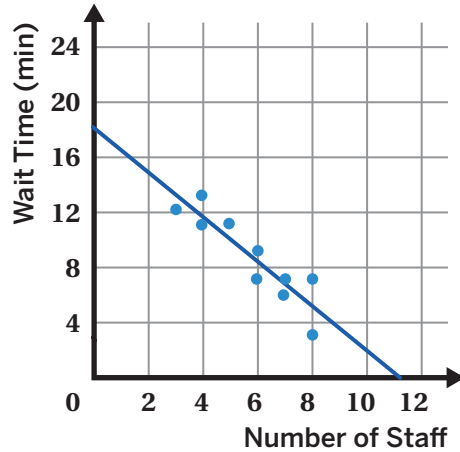
You can use the line to predict that if a block in Austin has 80% tree cover, the temperature will be about 85°F .



Practice 7.15

Name: _____ Date: _____ Period: _____

Problems 6–9: A restaurant gathered data about how long customers had to wait and how many staff members were working. The slope of the line of fit is -1.62 . The r -value is -0.9 .



6. What does the slope of -1.62 mean in this situation?

7. What does the y -intercept of 18 mean in this situation?

8. What does the r -value of -0.9 mean in this situation?

9. Madison says that for any scatter plot, if the r -value is negative, then the slope must also be negative. Is this true or false? Circle one.

True False

Explain your thinking.

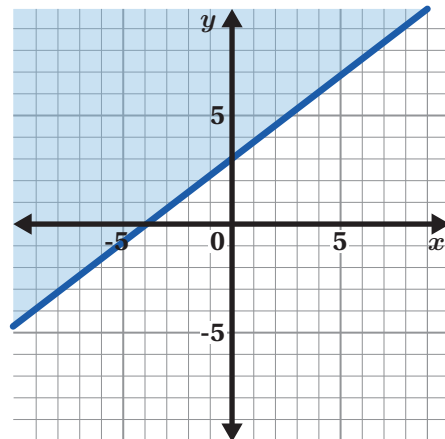
Spiral Review

10. Select *all* the coordinate pairs that are solutions to $y = -\frac{1}{2}x + 10$.

- A. $(0, \frac{2}{3})$ B. $(-10, 15)$ C. $(4, 12)$ D. $(2, 9)$ E. $(5, 8)$

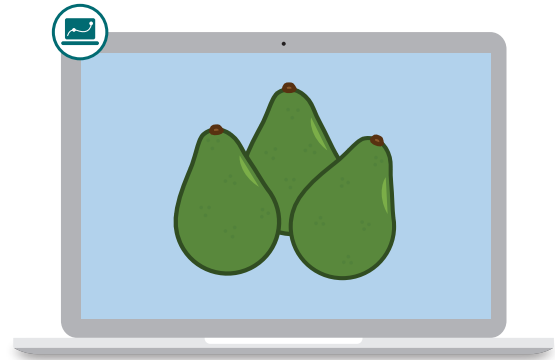
11. Here is the graph of the inequality $y \geq 0.75x + 3$. Select *all* the coordinate pairs that are solutions to this inequality.

- A. $(-9, -5)$
 B. $(-3, 2)$
 C. $(0, 3)$
 D. $(2, -3)$
 E. $(6, 8)$



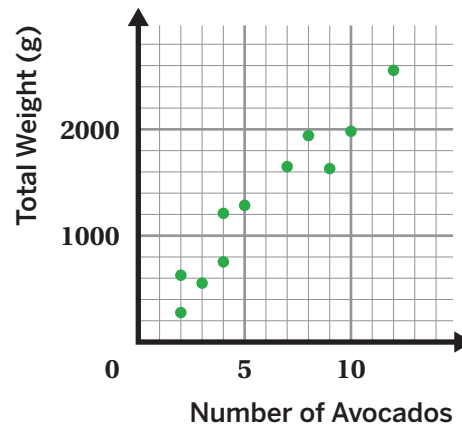
Residual Fruit

Let's use residual plots to determine how well a line fits data.



Warm-Up

1 Brianna has a business that ships different kinds of fruit.



a Let's watch orders being weighed.

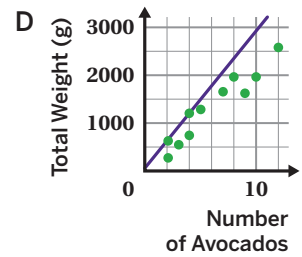
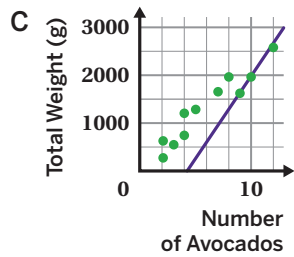
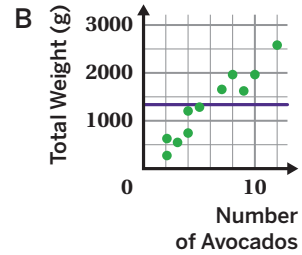
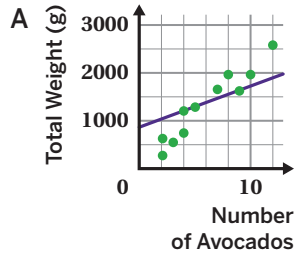
b  **Data Talk!** Discuss how much you think `11` avocados will weigh.

Predicting With Lines


2 Lines that fit the data well can help us make predictions.

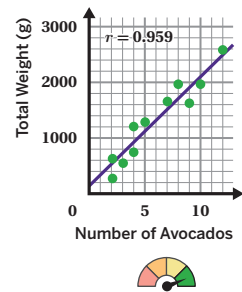
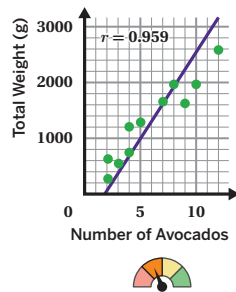
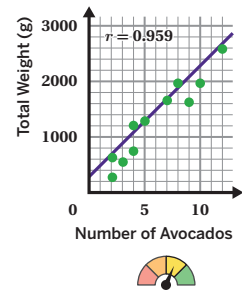
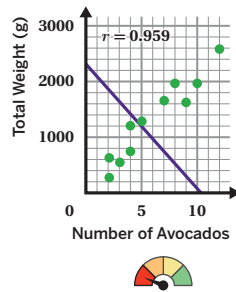
None of these lines fit the data well.

Circle a scatter plot. Explain why the line does not fit the data well.



3 a Take a look at these scatter plots. The meters show how well each line fits the data.

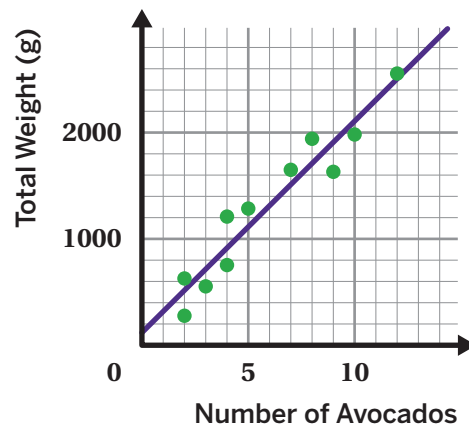
b  **Data Talk!** Explain to a classmate how to get a high score on the meter.



Predicting With Lines (continued)

4 Brianna has an order to ship 6 avocados.

How could you use this line to predict the weight of the order?



5 The line predicts that 6 avocados will weigh 1,316 grams, but 6 avocados actually weigh 1,740 grams.

What is the difference, in grams, between the predicted weight and the actual weight?

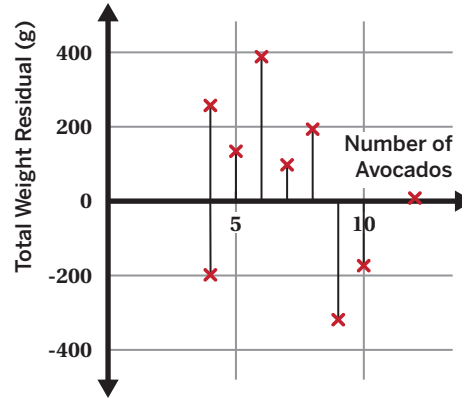
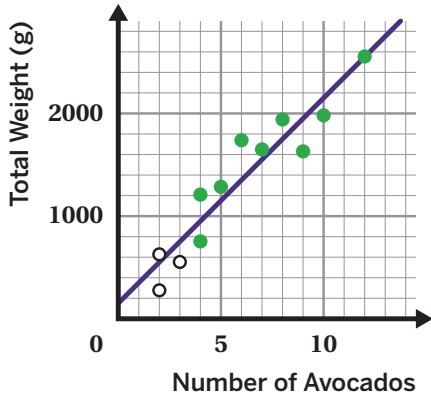
Activity 2

Name: Date: Period:

Residual Plots

6 A **residual** is the difference between the predicted and measured weight.

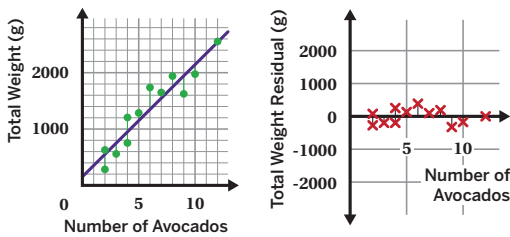
a Let's take a look at the residuals for the solid green points.



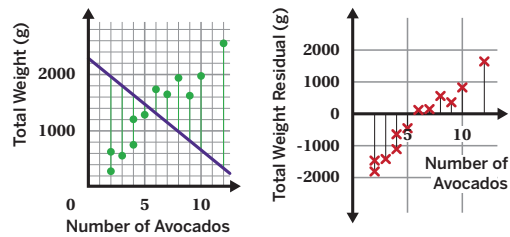
b Plot the residuals for the open black points to complete the residual plot.

7 A **residual plot** shows how far each point is from the line of fit.

Fits the Data Well



Doesn't Fit the Data Well

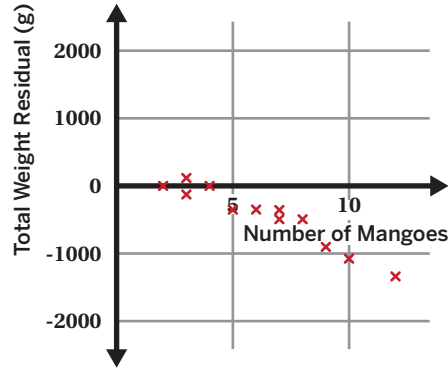
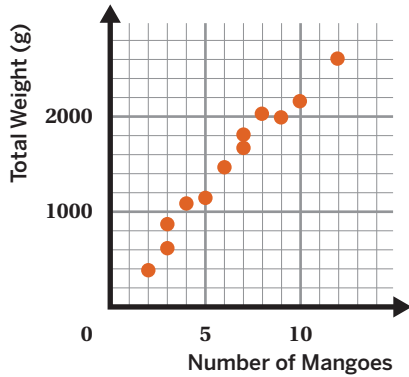


What does the residual plot look like when the line fits the data well? What about when it doesn't fit the data well?

Residual Plots (continued)

8 Here is the residual plot for a line of fit Brianna created.

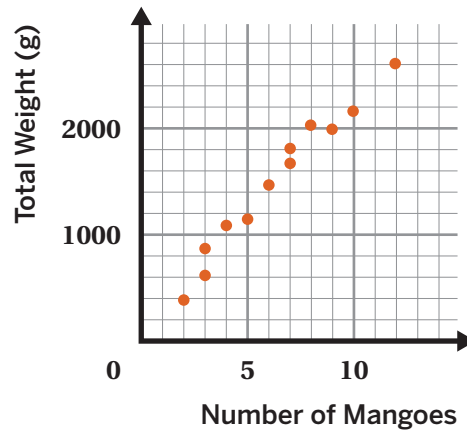
Sketch the line of fit you think Brianna made.



9 Let's improve on your sketch of Brianna's line.

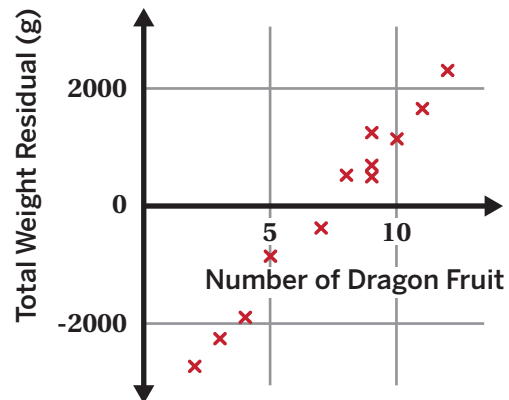
a Now, draw a line that is a better fit for the data.

b **Discuss:** How will the residual plot change once the line is a better fit for the data?



10 Aditi made a line of fit for this data showing the total weight of different numbers of dragon fruit.

Here is the graph of the residuals from Aditi's line. How well do you think Aditi's line fits the data?

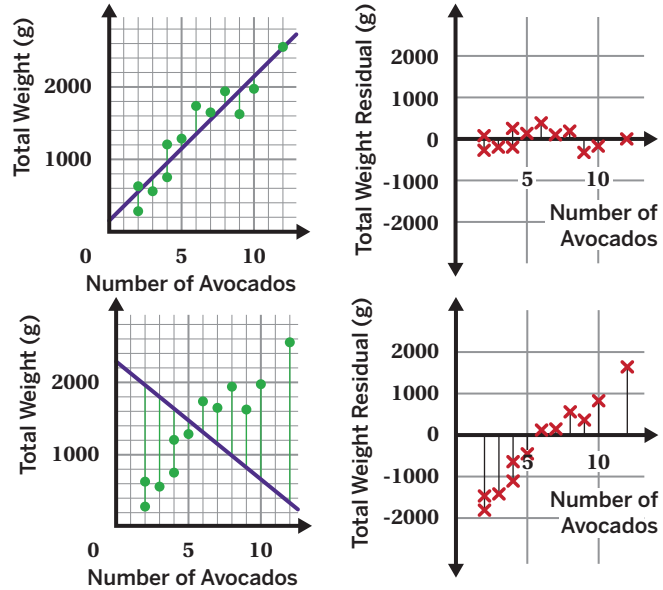


You're invited to explore more.

11 Use the You're Invited to Explore More Sheet to draw a line of fit.

12 Synthesis

How can you use a residual plot to determine if a line fits the data well?



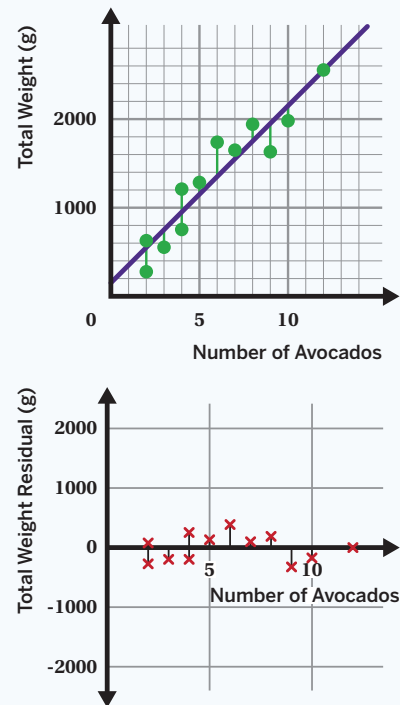
15 Summary 7.16

You can use **residuals** to determine how well a line fits a data set.

Here is a scatter plot with data on the number of avocados and their weights. The residuals are represented with dashed lines connecting each point to the line of best fit.

You can also create a **residual plot** to analyze how well a line fits a data set.

Here is the residual plot of the graph of avocado weights. The closer a point is to the x -axis, the closer that point is to the line of best fit.



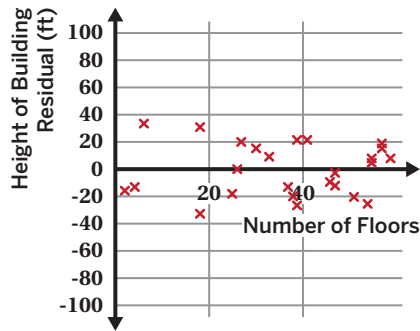
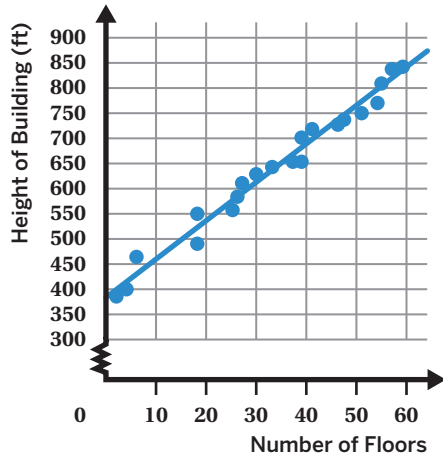
residual The difference between the y -value for a point in a scatter plot and the value predicted by the line of best fit.

residual plot A scatter plot of residual values for a data set. The x -axis represents the value predicted by the line of best fit, and the y -value of each point represents the value of the residual.

Practice 7.16

Name: _____ Date: _____ Period: _____

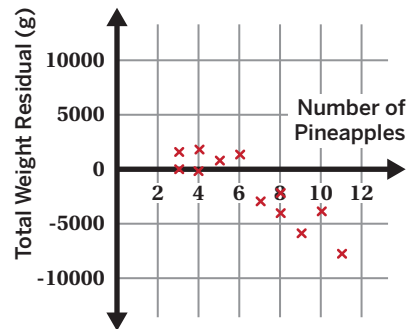
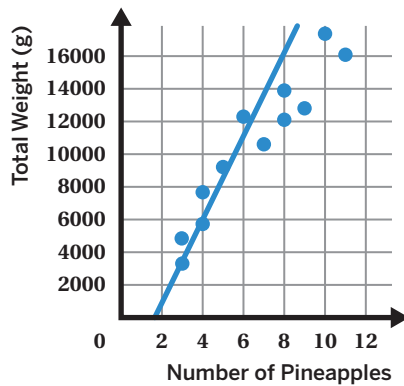
Problems 1–2: The scatter plot shows the heights of a group of buildings, the number of floors in each building, and a line that best fits the data. The residual plot is also shown.



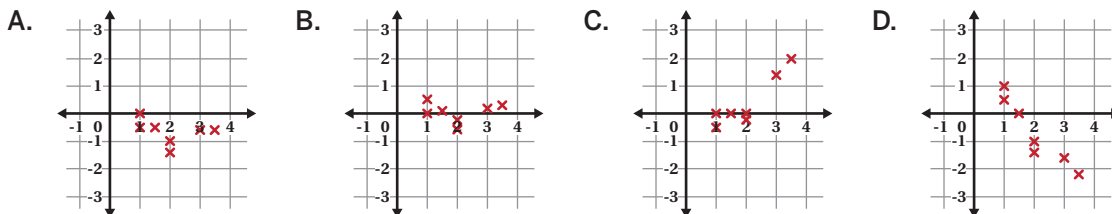
- Predict the height of a building that has 50 floors.
- How can you tell that the graphed line is a good fit for the data? Use the residual plot if it helps with your thinking.

- Here is a scatter plot and its corresponding residual plot.

Draw a better line of fit on the scatter plot.



- These residual plots are from the same set of data, but each one represents a different line of fit. Which residual plot shows the best line of fit?

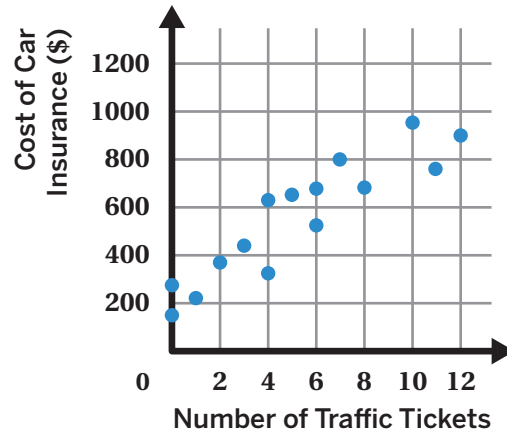


Explain your thinking.

Practice 7.16

Name: _____ Date: _____ Period: _____

Test Practice Problems 5–6: This scatter plot shows the number of traffic tickets and the cost of car insurance for 16 people.



5. Which r -value could represent the correlation coefficient for this data?

- A. 0.25 B. -0.25
C. 0.9 D. -0.9

6. Which equation could represent the line of best fit?

- A. $y = -62x + 220$ B. $y = 62x + 220$ C. $y = -220x + 62$ D. $y = 220x + 62$

Explain your thinking.

Spiral Review

Problems 7–8: Polygon $BCDA \cong$ Polygon $MJKL$. Complete each statement.

7. Segment $CB \cong$ Segment _____ 8. Angle $DAB \cong$ Angle _____

Problems 9–10: Here is a data set: [1, 1, 1, 1, 2, 2, 3, 3, 4, 4, 5, 5, 5, 6, 20]


9. Are there any outliers in this data set? Use a calculator if it helps with your thinking.

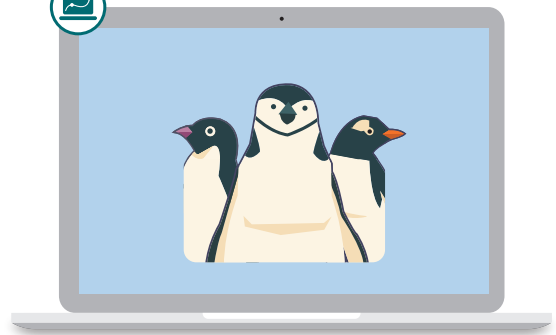
Circle one. Yes No

Explain your thinking.

10. Is the interquartile range or the standard deviation a more appropriate measure of the spread of this data set?

Modeling with Functions, Comparing Models, Variability

 F-LE.5, S-ID.6.a, S-ID.6.c, S-ID.7, S-ID.8, SMP.4, SMP.6



Penguin Populations

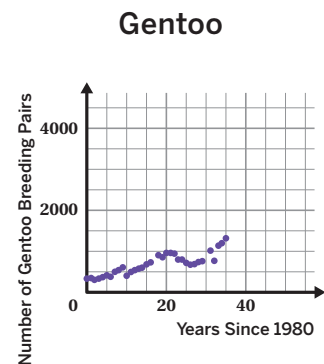
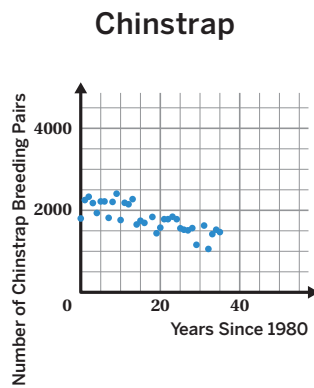
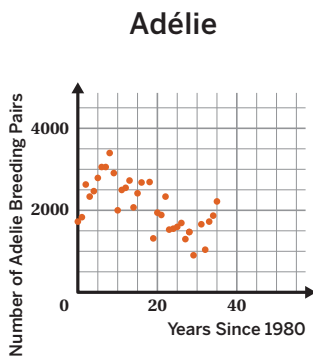
Let's generate and analyze lines of best fit to explore how penguin populations have changed over time.

Warm-Up

1-2 Researchers are conducting a long-term study on the South Orkney Islands in Antarctica.

The goal of this study is to understand how the populations of three species of penguins have changed over the last 40 years.

Here is the study's data for each of the penguin species.

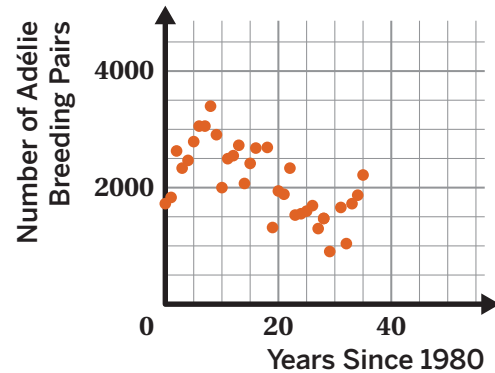


Data Talk! How is the population of each species changing over time? What might be affecting these changes?

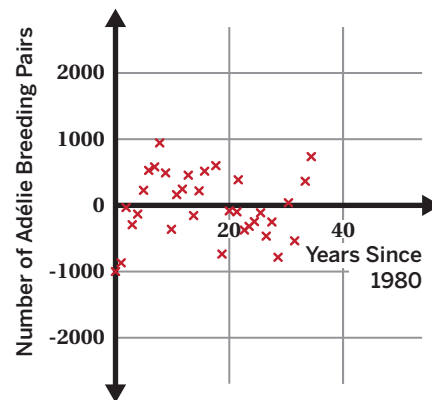
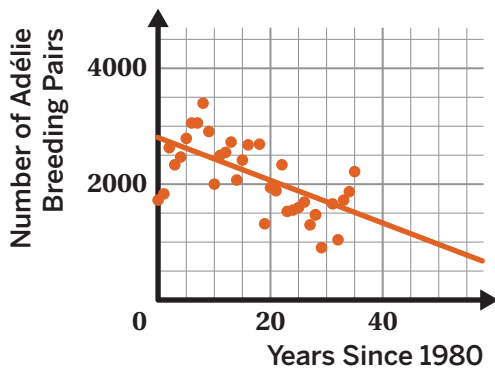
Using the Line of Best Fit

3 Let's explore how the Adélie penguin population on the South Orkney Islands has changed over time.

- Draw a line that fits the data.
- What does the line tell you about the penguin population?



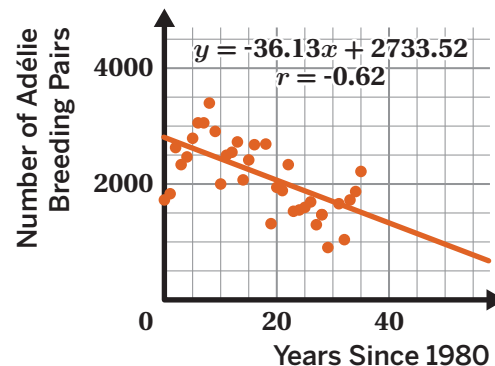
4 Here is the **line of best fit** computed by a calculator for the Adélie's penguin population.



Discuss: What do you notice about the residual plot?

5 Let's look at the equation of the line of best fit for the Adélie penguin data.

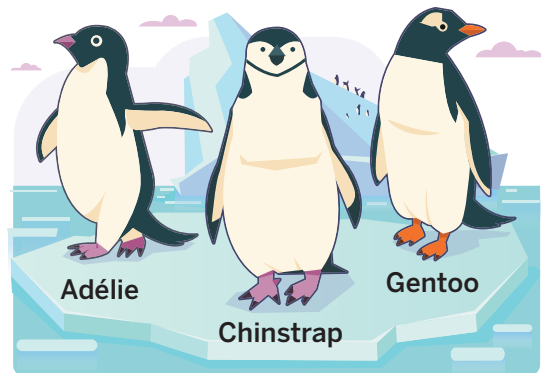
- Discuss:** What year would 2030 be on the x -axis?
- Use the line of best fit to predict how many breeding pairs of penguins there will be in 2030.





Generating a Line of Best Fit

6-7 In the digital activity, take a look at the table of data showing the Adélie penguin population over time.

Then use the calculator to generate the line of best fit for this data.



a  **Discuss:** What does each part of $y_1 \sim mx_1 + b$ represent?

b  **Discuss:** How are the parts of the equation in row 9 related to the information in row 8?

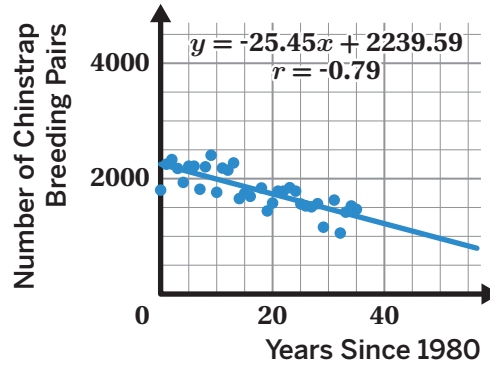
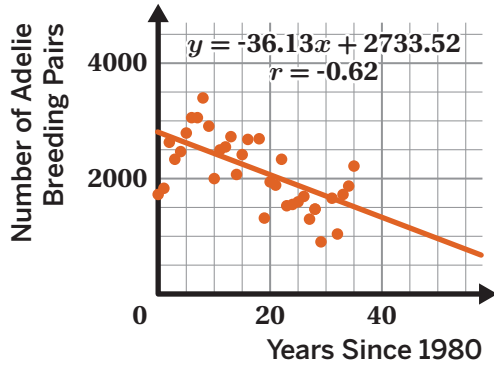
8 Use the digital activity to analyze the Chinstrap data.

a Use row 7 to generate the line of best fit for the Chinstrap data ($y_2 \sim mx_2 + b$).

b Use row 8 to write an equation for the line of best fit, in the form $y = mx + b$.

Generating a Line of Best Fit (continued)

9 Compare the data of these two penguin populations.



Discuss: How are they alike? How are they different?

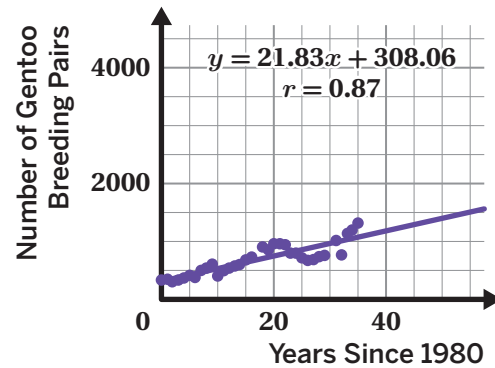
Making Predictions

10 Use the digital activity to analyze the Gentoo data.

- a** Use rows 5 and 6 to generate the line of best fit and write the equation for the Gentoo data.

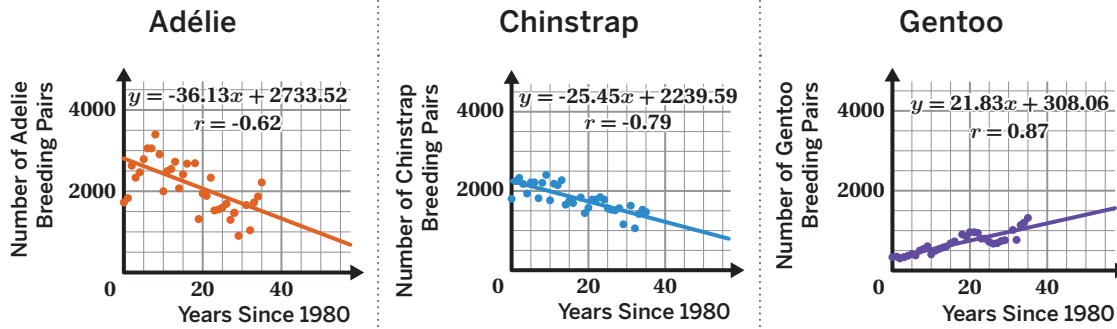
11 Here is the line of best fit for the Gentoo data.

How many breeding pairs of penguins does the line of best fit predict there will be in 2030?



Making Predictions (continued)

- 12** What questions might researchers have after analyzing the data from the Adélie, Chinstrap, and Gentoo penguin populations?



- 13** Here's a quote about modeling that you may remember:

All models are wrong, but some are useful.

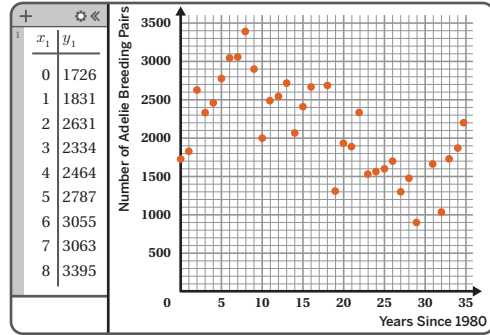
- Select a model we've explored today.
- Explain how that model is wrong and how it is useful.

The model is wrong because . . .

It is useful because . . .

14 Synthesis

Discuss: How can you use the Desmos Graphing Calculator to generate the line of best fit for data?

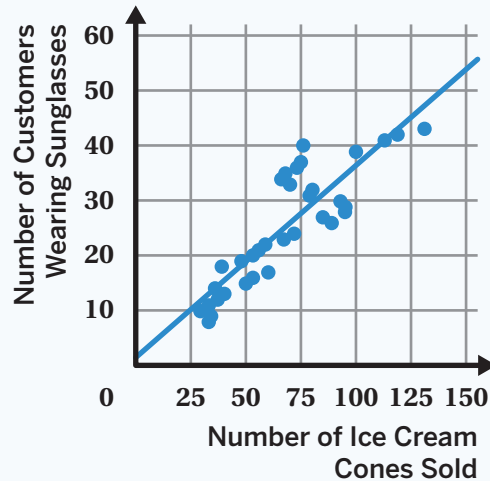


17 Summary 7.17

Instead of sketching a line of fit, you can use a graphing calculator to precisely generate the equation of the **line of best fit** from a scatter plot. The equation of the line of best fit can help you interpret information about a situation, or allow you to substitute values into the equation to make predictions. You can also use a graphing calculator to calculate the correlation coefficient for a given data set.

Here is an example of a line of best fit generated by a graphing calculator, and the information about the line that a graphing calculator will show you.

$y_1 \sim mx_1 + b$	
STATISTICS	PARAMETERS
$r_2 = 0.7642$	$m = 0.351312$
$r = 0.8742$	$b = 1.31984$

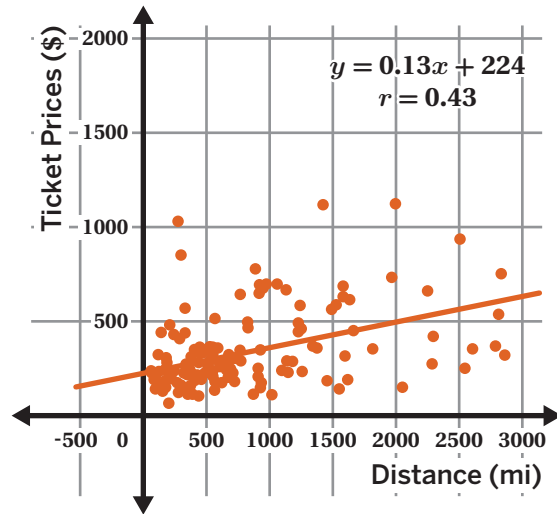


line of best fit The line on a scatter plot that best represents the trend created by the points in a data set. There are many lines of fit for a single data set, but there is only one line of best fit.

Practice 7.17

Name: _____ Date: _____ Period: _____

Problems 1–3: This scatter plot shows the distances of one way flights and their ticket prices.



1. The equation for the line of best fit is $y = 0.13x + 224$.

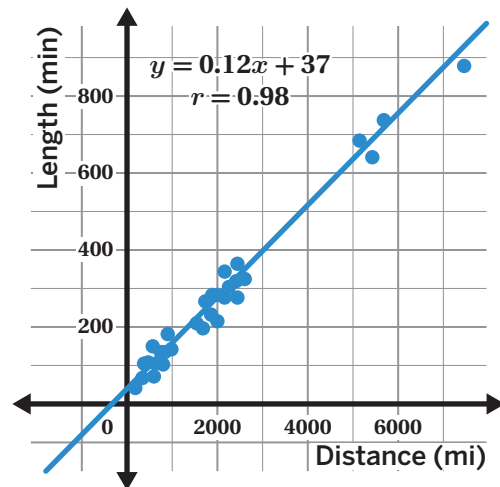
What does 0.13 mean in this situation?

2. The distance from Phoenix, Arizona, to Jacksonville, Florida, is 1,795 miles.

Use the equation of line of best fit to predict the cost of a plane ticket from Phoenix to Jacksonville.

3. Do you think the prediction is accurate? Use the r -value to explain your thinking.

Problems 4–6: This scatter plot shows the distances of non-stop flights and their lengths in minutes.



4. The equation for the line of best fit is $y = 0.12x + 37$.

What does 0.12 mean in this situation?

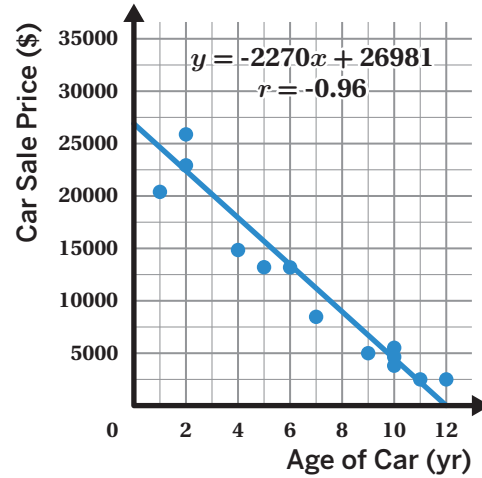
5. Use the equation of line of best fit to predict the length, in minutes, of a direct flight from Phoenix to Jacksonville (1,795 miles).

6. Do you think the prediction is accurate? Use the r -value to explain your thinking.

Practice 7.17

Name: _____ Date: _____ Period: _____

Test Practice Problems 7–9: Kwasi wanted to know the relationship between the ages of cars and their values. He found data on the ages of several cars (in years) and their sale prices (in dollars).



- Describe the relationship between the age of a car and its sale price.
- Do you think one of the variables causes the other? Explain your thinking.
- What else might affect this relationship? Explain your thinking.

Spiral Review

Problems 10–12: Here is a data set: [1, 1, 2, 2, 3, 3, 7, 8, 9, 10, 11, 35]

10. Complete the table. Use the Desmos graphing calculator if it helps with your thinking.

Min.	Q1	Median	Q3	Max.

- What is the interquartile range of this data set?
- Are there any outliers in this data set? Circle one. Yes No

Problems 13–15: Solve each equation for y .

13. $7 = 6x - y$

14. $3y + 15x = 24$

15. $4y - x = 44$


Behind the Headlines

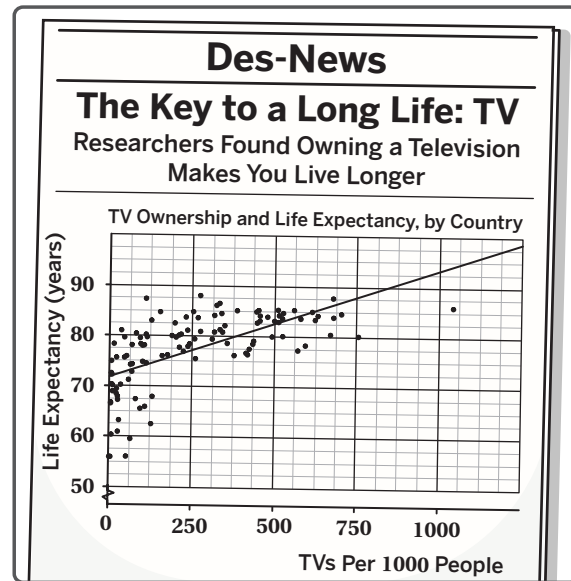


Let's consider the differences between correlation and causation.

Warm-Up

1. Let's look at a headline.

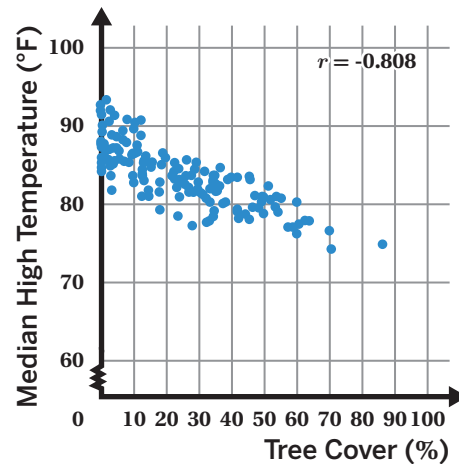
 **Discuss:** What do you notice? What do you wonder?



Correlation, Yes. But Causation?

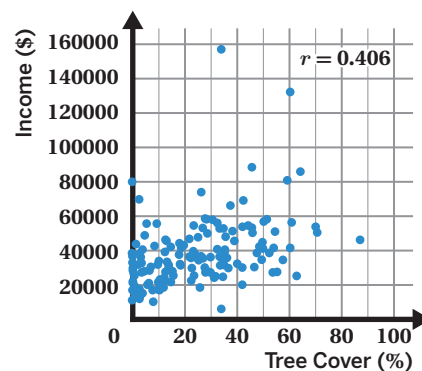
Here is data about Philadelphia neighborhoods that we studied in a previous lesson.

- Describe the relationship between tree cover and median high temperature.
- Do you think one of the variables *causes* the other? If not, what else could be affecting the relationship? Explain your thinking.




- Here is a headline: *Increasing Tree Cover Lowers Temperatures on City Blocks*. Does the headline suggest **causation**? Explain your thinking.

- Describe the relationship between tree cover and income.
- Do you think one of the variables *causes* the other? If not, what else could be affecting the relationship? Explain your thinking.



Correlation, Yes. But Causation? (continued)

7. Here is a headline: *Planting More Trees Can Increase Your Wealth.*
How could you rewrite the headline to suggest **correlation** but *not* causation?

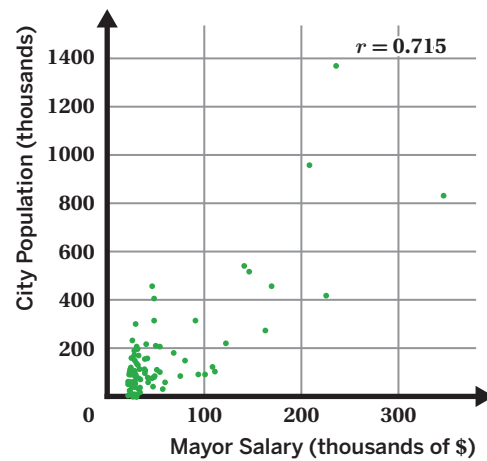
8.  **Data Talk!** Here are two headlines for the same data.

A: *Population Increases When Mayors Earn More*

B: *Mayor Salary Associated With City Population*

Which headline do you think is more accurate?

Explain your thinking.



Card Sort

You will use a set of cards. Each card has a headline about the relationship between two variables.

9. Sort the cards based on the type of relationship the headline is suggesting.

Causation	Correlation but Not Causation

10. Select two cards. Read the article summaries for your headlines. Then answer:

- Does the information in the summary support the headline? Explain your thinking.
- How could you rewrite the headline to make it more accurate?

Card

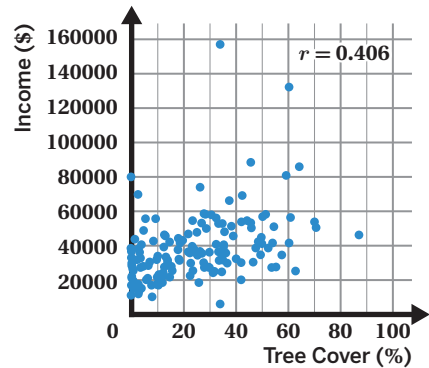
Card

Synthesis

11. How can you tell if a headline is suggesting *causation* or *correlation but not causation*?

Use the example if it helps with your thinking.

Money Grows on Trees!



Summary 7.18

Correlation and **causation** are types of relationships that can exist between variables.

It is important to know that not every correlation is a *causal relationship*. For example, a third variable can cause the relationship between two variables to change. Correlation can even be caused by coincidence: If lots of variables are considered, then it's very likely that at least two of them will be somewhat correlated.

Media, such as article headlines, might present relationships that are correlated as causal. To be a critical consumer of information, check the sources behind claims and ask questions about what conclusions can be made from a data set.

Statements that include words such as "lowers" or "causes" suggest causation while terms such as "associated with" or "is connected to" suggest correlation but not causation.

Example of correlation: *Positive Association Found Between Tree Cover and Income*

Example of causation: *Tree Cover Lowers Temperatures on City Blocks*

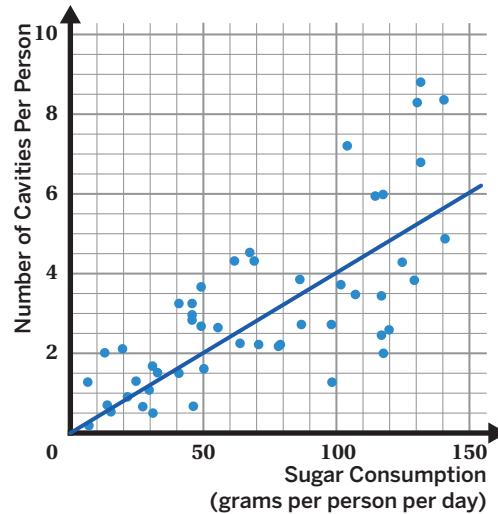
causation Describes a relationship between two variables in which a change in one variable causes a change in the other variable. A special type of correlation.

correlation A statistical relationship between two or more variables. Also called an association.

Practice 7.18

Name: _____ Date: _____ Period: _____

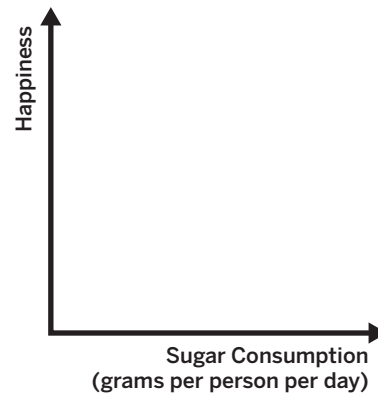
Problems 1–2: This scatter plot shows data from different countries about average sugar consumption and tooth cavities.



1. Which statement best describes the data in the graph?
 - A. The rate of tooth cavities has increased in many countries.
 - B. The rate of sugar consumption has increased in many countries.
 - C. People who eat more sugar are likely to have more cavities.
 - D. People brush their teeth more frequently in some countries than others.

2.  **Test Practice** Describe the relationship between sugar consumption and teeth cavities.

Problems 3–4: A group of scientists researched sugar consumption and happiness levels. Their research data showed a negative linear correlation.



3. Make a scatter plot that fits this description.
4. A news website presents the research with the headline, *Eating Sugar Causes Happiness to Decrease!*

Is this an accurate claim? Circle one.

Yes

No

Explain your thinking.

Practice 7.18

Name: _____ Date: _____ Period: _____

Problems 5–8: Circle the type of relationship each headline suggests.

5. Brushing Twice a Day Can Prevent Gum Disease

Causation Correlation only

6. Owning a Pet Is Linked to Positive Benefits for Your Health

Causation Correlation only

7. Life Satisfaction Increases Along With Time Spent on Physical Activity

Causation Correlation only

8. Sitting Too Much Decreases Life Expectancy

Causation Correlation only

9. Lukas is not sure about the difference between correlation and causation. What could you say to Lukas to explain the difference?

Spiral Review

10. Select *all* of the equations that are equivalent to $20 + 2m - 8 = m + 17$.

A. $20 - 8 + 17 = -m$

B. $2m + 12 = m + 17$

C. $3m = 17 - 20 + 8$

D. $m = 5$

E. $2m = m + 5$

11. Write an equation to match this table.

x	0	1	2	3	4
y	5	15	45	135	405

12. Researchers found a strong positive correlation between the number of hours of sleep a person gets per night and their overall well-being. Which of the following correlation coefficient values best matches the researchers' results?

A. $r = -0.94$

B. $r = 0$

C. $r = 0.51$

D. $r = 0.89$

13. A scatter plot has a correlation coefficient of $r = -0.3$. What does this tell you about the data?

Unit 7
Lesson
19

Name: _____ Date: _____ Period: _____

Comparing Models, Correlation and Causation, Variability

 S-ID.6.a, S-ID.6.c, S-ID.7, S-ID.8, SMP.1, SMP.4

City Data

Let's apply what we've learned about analyzing two-variable data to explore relationships between variables in society.



Warm-Up

Here are some variables about cities around the U.S. that communities collect data about.

- Annual days of sunshine
- Homelessness (%)
- People in poverty (%)
- Median rent
- Unemployment (%)
- Green space (%)
- Median household income
- Mean commute time

1. Circle a variable that interests you.

2.  **Discuss:**

- Why do you think it would be useful to collect data about this variable?
- How do you think the data for this variable is collected?

Analyzing Data

3. Choose two variables from the Warm-Up that you think might be related.

Variable 1:

Variable 2:

4. What do you predict the relationship between these variables might be?

5. Select your chosen variables on Screen 2 of the digital activity.

6.  **Data Talk!** On Screen 3 of the digital activity, generate the line of best fit for your data.


Equation of the Line of Best Fit

r -value (Correlation Coefficient)

7. Describe the relationship between the two variables you chose.

Use the scatter plot, equation, and r -value to support your reasoning.

Analyzing Data (continued)

8. How does the relationship between the variables compare to your prediction?
9.  **Data Talk!** Write a headline that could represent the relationship between your variables.
10. How might the city or town you live in use the information about the relationship you studied?
11. Did a linear function fit your data well? Would a non-linear function fit your data better?


Interview a Classmate

11. Find a classmate who chose different variables than you did.

Here are a few questions you can ask them. Use these questions to interview your classmate about their work. Take notes on their responses.

- What variables did you choose? What about those variables is interesting?
- What relationship did you see? What statistics support that?
- Were the results what you expected or were you surprised?
- Do you think one of your variables *causes* the other? Are there other variables that might be affecting this relationship?
- How might your city or town use the information about the relationship you studied?
- What other questions do you have about the data?

Synthesis

12.  **Discuss:** How can scatter plots, lines of best fit, and correlation coefficients help us describe relationships between two variables?

Summary 7.19

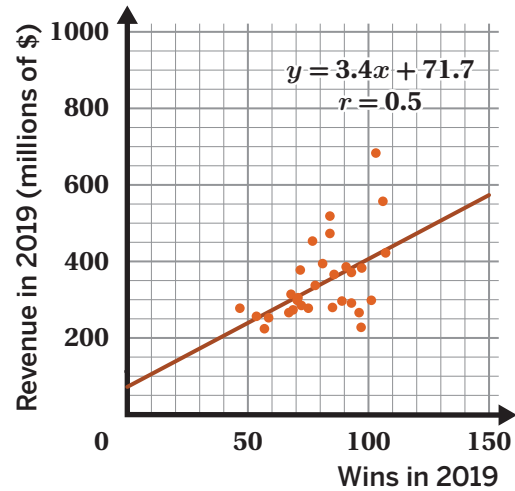
We can use statistical tools to help us better understand our communities and the world. Scatter plots help us visualize large amounts of data, lines of best fit help us see trends and relationships or associations in that data, and correlation coefficients (r -values) help us describe the strength and direction of any relationships that we find.

While all of these tools help us describe associations and test predictions, we need to explore the variables and relationships further to determine things like the cause and the effect on real people. Statistical tools are powerful, but they are just one step in understanding issues that affect real communities.

Practice 7.19

Name: _____ Date: _____ Period: _____

Problems 1–3: This scatter plot shows the revenue (earnings) of different Major League Baseball teams in 2019, as well as their number of wins that year.



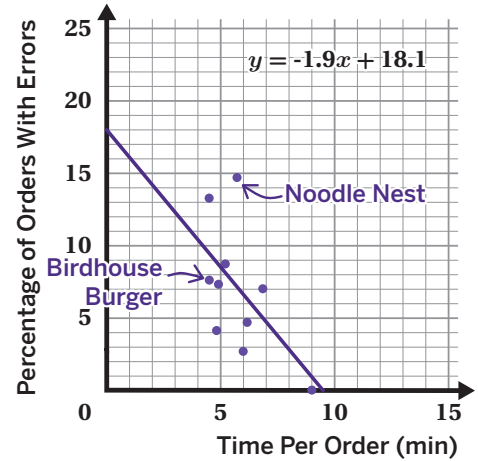
1. What kind of association is there between wins and revenue? Circle one.
Positive Negative No association
2. The equation for the line of fit is $y = 3.4x + 71.7$. What does the number 3.4 mean in this situation?
3. Do you think one of the variables causes the other? Explain your thinking.
4. Liam gathered data on the relationship between listening to music while studying and test scores. His data has a correlation coefficient of $r = 0.41$. Based on this data, Liam believes that his school should introduce a music-listening program to improve test scores. Do you agree? Explain your thinking.
5. Daeja is interested in finding out if there is an association between which homeroom a student is in and their mood at the beginning of the school day. Select *all* the possible tools and data representations it could make sense for Daeja to use in her study.
 - A. Survey
 - B. Scatter plot
 - C. Two-way frequency table
 - D. Total relative frequency table
 - E. Residual plot


Practice 7.19

Name: _____ Date: _____ Period: _____

Problems 6–9: A study gathered data about drive-throughs at two restaurants. This scatter plot shows the average time per order and the percentage of orders with errors for each restaurant.

6. Describe the relationship between the two variables. Use the scatter plot and equation to support your reasoning.



7.  **Test Practice** Which restaurant has better drive-through service: Noodle Nest or Birdhouse Burger? Explain your thinking.
8. Which value is likely the correlation coefficient (r -value) for this data?
 A. -1.9 B. 18.1 C. -0.6 D. 0.6
9. A restaurant sets a goal to serve customers in 2 minutes. Use the equation for the line of best fit to predict what percent of orders served in 2 minutes would have errors.

Spiral Review

Problems 10–12: Determine the value of n in each equation.

10. $\frac{4n}{3} = 12$

11. $2n + 6 = 11$

12. $12 - 10n = 7$

Problems 13–14: Alma wants to find out more about the health of the students at her school.

13. Write a survey question on this topic that would generate *quantitative* data.
14. Write a survey question on this topic that would generate *categorical* data.

Practice Day 2

Let's practice what you've learned so far in this unit!



You will use task cards for this Practice Day. Record all of your responses here.

Task A: Pepperoni and Pineapple

- Median: _____ IQR: _____
- Circle one: A B C D
- Circle one: Desminos or Pizzamos
Explanation:

Task B: Take Me Out to the Ball Game

- $r =$ _____
- Description:
- Equation: _____
- Interpretation:
- Cost: _____
- Circle one: Yes or No
Explanation:

Practice Day 2

Task C: A Bit Scattered

1. Explanation:

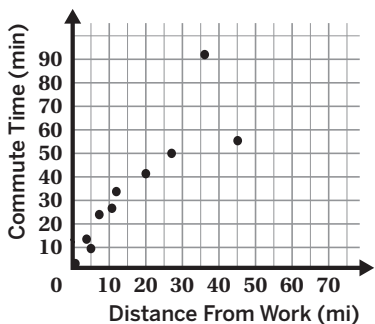
2. **a** $r =$ _____

b $r =$ _____

c $r =$ _____

3. **a**

b Description:



Task D: Movies

1. Critic ratings: _____

Audience ratings: _____

2. Explanation:

3. Explanation:

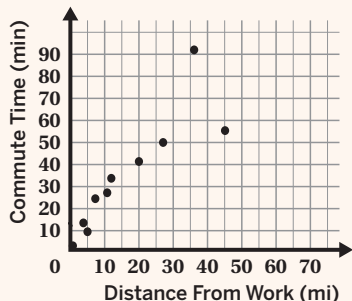
4. Explanation:

5. Circle one: mean/standard deviation or median/IQR

Explanation:

You're invited to explore more.

1.



2. Explanation:

Career Connection

From Pole to Pole – the same or different?

Both the North and South Poles are very cold because they do not get as much direct sunlight as the rest of Earth. But the South Pole averages much colder temperatures! Why do you think that might be?

	Mean Temperature	
	North Pole	South Pole
Summer	32° F	-18° F
Winter	-40° F	-76° F

Source: NASA

Climate scientists or *climatologists* study the long-term pattern of weather conditions for a particular area or areas. They use climate models to analyze large data sets which can be used to help make future predictions.



Oregon State University/Flickr.
(CC BY-SA 2.0)

Meet Warren Washington

A Nobel Prize winner and presidential advisor, Warren Washington, was among the first to develop atmospheric computer models to help scientists understand Earth's climate patterns and help make future predictions. *Climate modeling* combines mathematics and physics to make sense of weather related phenomena. Washington's climate models have provided data for climate risk-related decisions being made by many policymakers and businesses.

Are you interested in learning more about climate modeling? What can you do to learn more?

Math in the World

Suomi-NPP is a NASA satellite that orbits Earth at a distance of 512 miles. The data collected include land cover, ice cover, and surface temperatures. Just think about how large *these* data sets are!



GizemG/Shutterstock.com

Math Mindset

Was there a time during this unit when a classmate helped you make sense of a data set, data distribution, or scatter plot? How did they help you?