Amplify Desmos Math CALIFORNIA

Mathematics I

Volume 1: Units 1–4

Student Edition

About Amplify

Amplify is dedicated to collaborating with educators to create learning experiences that are rigorous and riveting for all students. Amplify creates K–12 core and supplemental curriculum, assessment, and intervention programs for today's students.

A pioneer in K–12 education since 2000, Amplify is leading the way in next-generation curriculum and assessment. All of our programs provide teachers with powerful tools that help them understand and respond to the needs of every student.

Amplify gratefully acknowledges the work of distinguished program advisors from English Learners Success Forum (ELSF), who have been integral in the development of Amplify Desmos Math. ELSF is a 501(c)(3) nonprofit organization whose mission is to expand educational equity for multilingual learners by increasing the supply of high-quality instructional materials that center their cultural and linguistic assets. ELSF does not rate or endorse materials.

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Dear Student,

Welcome to Amplify Desmos Math California Algebra 1! We are excited to be partnering with you this year. You play an essential role in math class, so we wanted to reach out to introduce ourselves and tell you a bit about who we are.

Amplify Desmos Math California is a team of math educators on a mission to support you and your classmates in learning math. We hope each lesson inspires you to use your creativity, ask questions, and discover connections between math concepts and the world around us.

Here is what you can expect this year:

- A blend of learning on both paper and devices.
- Interactive lessons that encourage you to ask questions, explore, settle disputes, create challenges for your classmates, and more!
- Digital activities that show what your ideas mean with plenty of chances for you to revise.
- Opportunities for you to engage with interesting and important big ideas in mathematics.

–The Amplify Desmos Math California Team

Unit 1 Patterns and Sequences

You will model, compare, and contrast arithmetic and geometric sequences using tables, recursive definitions, expressions, and graphs.

Unit 2 Linear Equations and Inequalities

You will revisit strategies for solving one-variable equations and inequalities and extend your knowledge to make sense of multi-variable equations and two-variable inequalities.

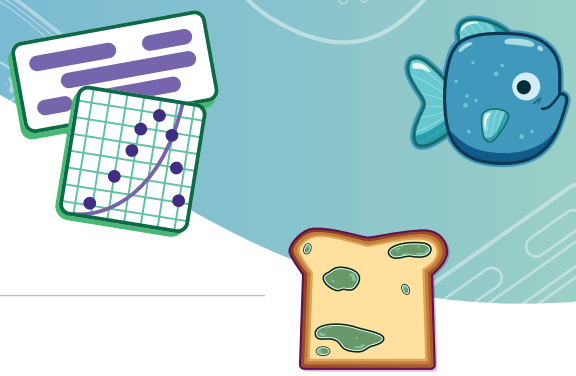


Unit 3 Systems of Linear Equations and Inequalities

You will write and solve systems of linear equations and inequalities, interpreting their solutions in context, and use coordinates to write equations of line and calculate measures.

Unit 4 Describing Functions

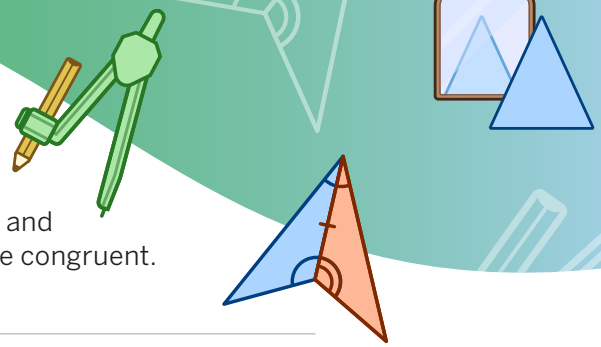
You will model situations with functions and use function notation to describe key features of functions, compare different functions, and define functions.



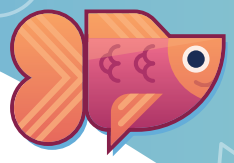
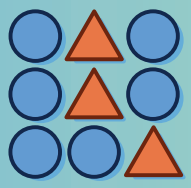
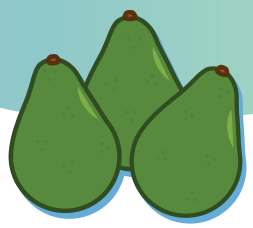
Unit 5 Exponential Functions

You will use exponential functions to make sense of how changing by a small percentage can make a big impact over time.

Unit 6 Rigid Transformations and Congruence



You will construct geometric figures using a variety of tools and use rigid transformations to determine when two figures are congruent.

Unit 7 Describing Data

You will analyze, describe, and compare one- and two-variable data sets.

Unit 1

Patterns and Sequences

Big Ideas in This Unit

CC2 Comparing Models | Modeling with Functions

NS Parallels Between Numbers and Functions

Questions for Investigation

- What strategies can be used to make predictions about sequences?
- How can sequences be used to investigate and model situations that involve linear and non-linear relationships?

Explore: Patterns Found in Nature
What mathematical patterns can be seen in nature?

Watch Your Knowledge Grow

This is the math you'll explore in this unit. Rate your understanding to see how your knowledge grows!

I can . . .	Before	After
Determine a process for finding figures in a pattern.		
Determine steps for calculating missing terms in a sequence.		
Recognize situations in which a quantity changes by a constant difference or a constant ratio.		
Determine a recursive definition for a sequence.		
Create arithmetic and geometric sequences when given a graph.		
Create arithmetic and geometric sequences when given a description.		
Create arithmetic and geometric sequences when given input-output pairs in a table.		
Explain what an explicit expression and its parts represent in a real-world situation.		
Write arithmetic sequences using recursive rules and explicit expressions.		
Write geometric sequences using recursive rules and explicit expressions.		

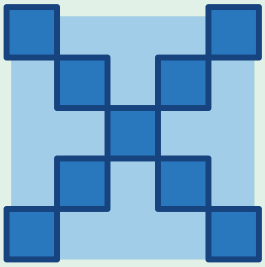
I can . . .	Before	After
Translate between recursive rules and explicit expressions.		
Use arithmetic and geometric sequences to model real-world situations.		
Describe the limitations of a model and how these limitations affect the accuracy of predictions.		

Sub-Unit

1

Sequences

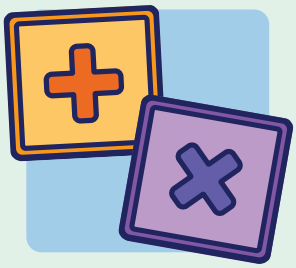
Explore
Patterns Found in Nature



Lesson 1
Visual Patterns

Lesson 2
Sequence Carnival

Lesson 3
Recursion Machine



Lesson 4
See the Sequence

Name: ..... Date: ..... Period: .....

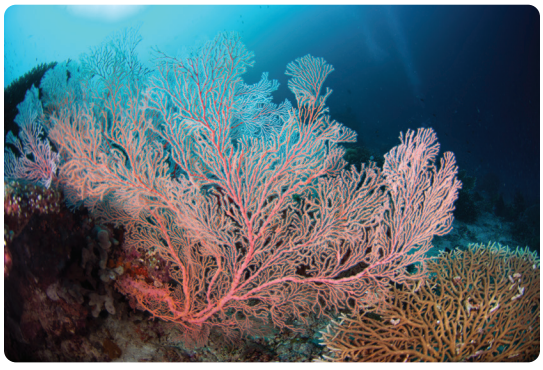
Comparing Models | Building Toward F–BF.1.a, SMP.6, SMP.7

Explore: Patterns Found in Nature

What mathematical patterns can be seen in nature?

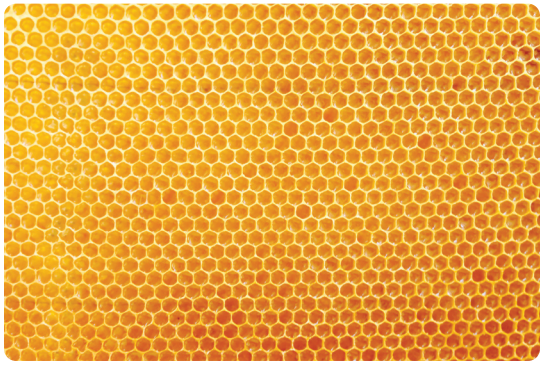
Warm-Up

Here are several images from nature.



Bill45/Shutterstock.com

pixelgerm/Shutterstock.com



Inna Novogel/Shutterstock.com

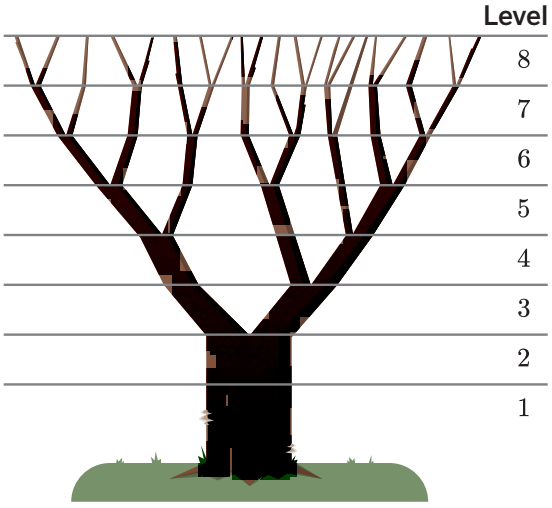
Alex Borderline/Shutterstock.com

1. **Discuss:** What do you notice? What do you wonder?

Name: ..... Date: ..... Period: .....

A Natural Pattern

When a tree begins to grow, it starts off with a single trunk that eventually splits into two separate branches: a main branch and a smaller branch. While the smaller branch is maturing, the main branch will split into two branches. Once the smaller branch has grown enough, it will also split into two branches. This process continues and results in the pattern shown.



2. Complete the table by determining the number of branches at each level on the tree. Then determine how many branches the tree is gaining each time the level increases.

Level	1	2	3	4	5	6	7	8
Number of Branches								
	↗	↗	↗	↗	↗	↗	↗	
	.....	.....	.....	.....	.....	.....	.....	

3. What pattern do you notice between the branches of the tree? How did you determine this pattern?

The number of branches at the different levels of the tree are part of a famous pattern of numbers known as the *Fibonacci sequence*.

Name: ..... Date: ..... Period: .....

A Natural Pattern (continued)

The Fibonacci sequence can also be seen in spiral patterns that occur in nature.

4. Determine how many clockwise and counterclockwise spirals the pine cone has. Sketch on the images if it helps with your thinking.

Clockwise

xpixel/Shutterstock.com

The pine cone has ..... clockwise spirals.

Counterclockwise

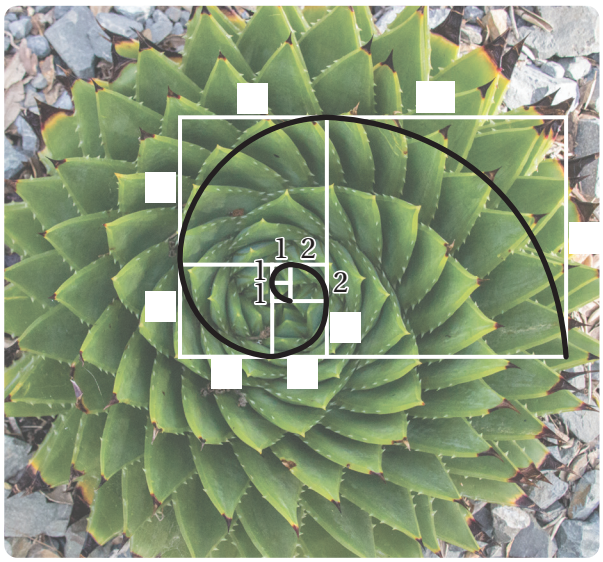
xpixel/Shutterstock.com

The pine cone has ..... counterclockwise spirals.

5. What do you notice about the number of clockwise and counterclockwise spirals?

A geometric representation of the Fibonacci sequence is shown on top of a spiral aloe plant. It is created using squares and can be used to make a spiral that is often seen in nature.

6. Complete the image by determining the side length of each square.



pixelgerm/Shutterstock.com

7. **Discuss:** How is the geometric representation and the spiral created?

Name: ..... Date: ..... Period: .....

Building Math Habits of Mind

Discuss:

- Which of these habits of mind did you strengthen during this activity?
- How did you use the one(s) you selected?

I can slow down and first make sense of a challenging problem before trying to solve it.

Not yet — Almost — I got it!

I can represent real-world problems and interpret their solutions within the context of the problem.

Not yet — Almost — I got it!

I can justify my thinking and ask questions to help me understand the thinking of others.

Not yet — Almost — I got it!

I can apply the math that I know to solve real-world problems, make assumptions, and revise my thinking as needed.

Not yet — Almost — I got it!

I can select an appropriate tool to help me solve problems.

Not yet — Almost — I got it!

I can communicate my thinking and solutions clearly to others.

Not yet — Almost — I got it!

I can look for structure or patterns to help me solve problems.

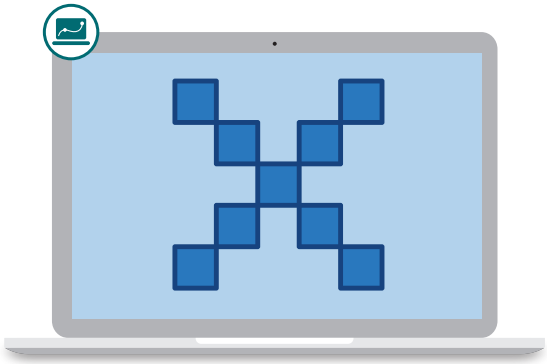
Not yet — Almost — I got it!

I can look for repeated calculations and other repeated steps to make generalizations.

Not yet — Almost — I got it!

Name: ..... Date: ..... Period: .....

Comparing Models F–BF.1.a, SMP.3, SMP.7

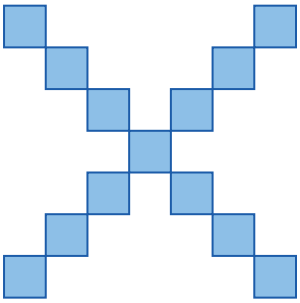


Visual Patterns

Let's explore visual patterns.

Warm-Up

1 Without counting one by one, determine how many tiles are in this figure.



Show or explain your thinking.

Name: ..... Date: ..... Period: .....

Pattern A

2 The figure in the Warm-Up is part of a visual pattern.

Figure 1

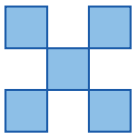


Figure 2

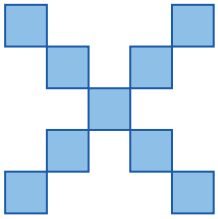
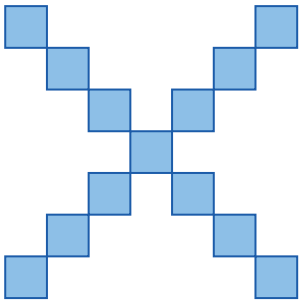


Figure 3



a **Discuss:** What do you think Figure 4 will look like?

b How many tiles will there be in Figure 4?

3 This table shows the number of tiles in Figures 1–3.

How many tiles will there be in Figure 10?

Figure	Number of Tiles
1	5
2	9
3	13

Name: ..... Date: ..... Period: .....

Pattern B

4 Here is a new visual pattern.

Figure 1 **Figure 2** **Figure 3**

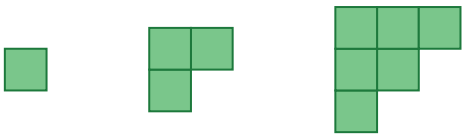


Figure	Number of Tiles
1	1
2	3
3	6

Matias says Figure 4 will have $6 + 4$ tiles.

Do you agree? Circle one.

Yes No I'm not sure

Explain your thinking.

5 Draw Figure 5 of the pattern.

6 How many tiles will there be in Figure 10?

Name: ..... Date: ..... Period: .....

Pattern C

7 Here are the two visual patterns we've seen.

Pattern A

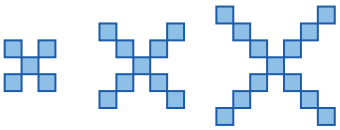


Figure 1 Figure 2 Figure 3

Figure	Number of Tiles
1	5
2	9
3	13
4	17

Pattern B

Figure 1 Figure 2 Figure 3

Figure	Number of Tiles
1	1
2	3
3	6
4	10

How are these patterns alike? How are they different?

Alike	Different

8 Here is a new visual pattern.

Figure 1

Figure 2

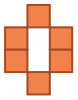


Figure 3

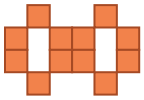


Figure 4

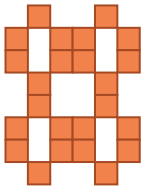


Figure	Number of Tiles
1	3
2	6
3	12
4	24

How many tiles will there be in Figure 7?

Name: ..... Date: ..... Period: .....

Pattern C (continued)

9 Here are the number of tiles in Figures 1–3 of another new visual pattern.

Figure	Number of Tiles
1	4
2	7
3	10

a Draw three figures to match the pattern in the table.

Figure 1	Figure 2	Figure 3

b How many tiles will there be in Figure 8?

You're invited to explore more.

10 There are many possibilities for Figure 3 in this visual pattern.

Make two different versions that each continue the pattern in some way.

Figure 1	Figure 2	Figure 3
		

Figure 1	Figure 2	Figure 3
		

11 Synthesis

Discuss: What is *at least one* strategy for determining the number of tiles in Figure 7 of a visual pattern? Use these examples if they help with your thinking.

Pattern B

Figure 1 Figure 2 Figure 3

Figure	Number of Tiles
1	1
2	3
3	6
4	10

Pattern C

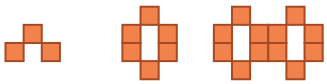


Figure 1 Figure 2 Figure 3

Figure	Number of Tiles
1	3
2	6
3	12
4	24

14 Summary 1.01

When you're trying to visualize how a pattern will grow, it helps to have a toolbox of different strategies.

Here are the first three figures in a pattern and a table showing the number of tiles at each stage. You could determine how many tiles there will be in Figure 7 by:

- Drawing the next four figures and adding a row of 3 each time.
- Continuing the table, increasing the number of tiles by 3 in each new row.
- Noticing that the number of tiles is 3 times the figure number, plus 2, and then calculating the tiles for Figure 7.

Figure 1 Figure 2 Figure 3

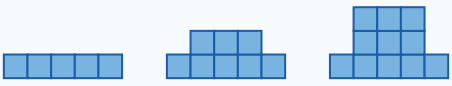


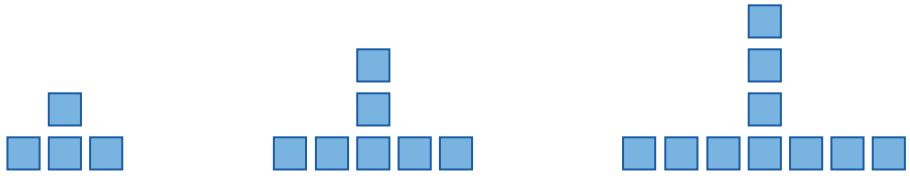
Figure	Number of Tiles
1	5
2	8
3	11

Practice
1.01

Name: ..... Date: ..... Period: .....

Problems 1–3: Here is a visual pattern.

Figure 1 **Figure 2** **Figure 3**



1. Draw Figure 4.

2. Complete the table with the number of tiles in each figure.

Figure	1	2	3	4	…	10
Number of Tiles					…	

3. Which expression represents the number of tiles in Figure 11?

A. $1 + 3(11)$

B. $1 + 4(11)$

C. $4 + 3(11)$

D. $4 + 1(11)$

For Problems 4–5: Create your own pattern.

4. Draw Figures 1 and 3. Then complete the table.

Figure 1 **Figure 2** **Figure 3**

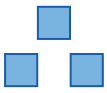


Figure	Number of Tiles
1	
2	3
3	

5. How many tiles will Figure 6 of your pattern have?

Name: ..... Date: ..... Period: .....

6. **Test Practice** Here is a visual pattern. How many tiles will there be in Figure 10?

Figure 1

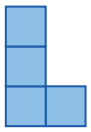


Figure 2

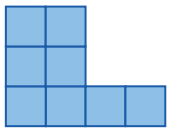
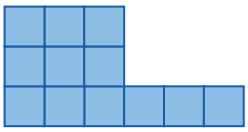


Figure 3



A. 16 **B.** 40 **C.** 49 **D.** 110

Spiral Review

7. Create two different patterns that begin with the numbers 5 and then 10.

Pattern A: 5, 10, ..., ..., ...

Pattern B: 5, 10, ..., ..., ...

Describe how each pattern is changing.

Pattern A: Pattern B:

8. Create two different patterns that each include the numbers 2 and 12.

Pattern C:

Pattern D:

9. Determine the value of each expression.

Expression	Value
$4 + 4 + 4 + 4$	
$4 \cdot 4 \cdot 4$	
$4(3)$	
3^4	

10. Select *all* the expressions that are equal to 16.

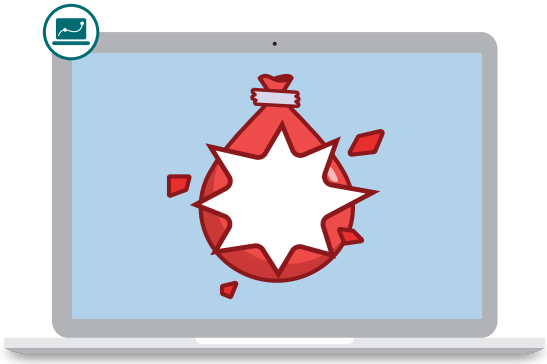
☐ **A.** 8^2 ☐ **B.** 2^4 ☐ **C.** 2^8 ☐ **D.** 4^2 ☐ **E.** 16^1

Name: ..... Date: ..... Period: .....

Comparing Models F–BF.1.a, SMP.3, SMP.8

Sequence Carnival

Let's explore sequences.



Warm-Up

1 Here is a **sequence**: a list of numbers in a particular order.

Let's watch an animation to see how the sequence is made.

What do you notice? What do you wonder?

I notice . . .

I wonder . . .

Name: ..... Date: ..... Period: .....

Seeing Sequences

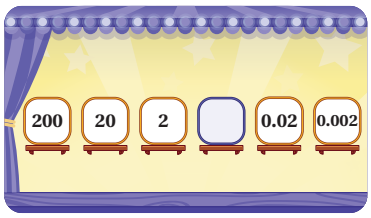
2 Each sequence follows a pattern and has a missing term. Write the missing term for each sequence.

Sequence A

Sequence B

Sequence C

Sequence D



Sequence E

Sequence F

3 Here are two sequences.

A **5, 8, 11, 14, 17, . . .**

B **5, 15, 45, 135, 405, . . .**

How are they alike? How are they different?

Alike	Different

4 Sequence A changes by a **constant difference**. Sequence B changes by a **constant ratio**.

What type of change does Sequence C show? Circle one.

C **40, 20, 10, 5, . . .**

Constant difference Constant ratio Neither

Explain your thinking.

Activity 2

Name: ..... Date: ..... Period: .....

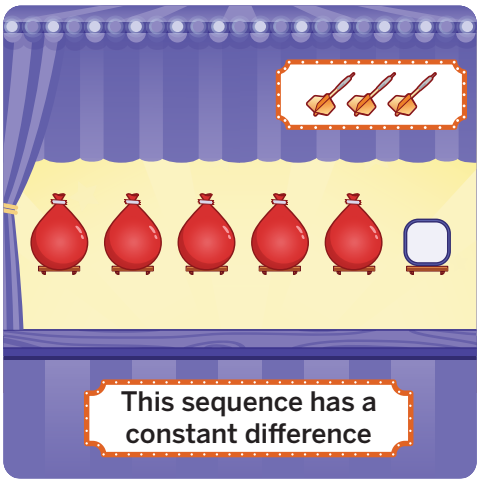
Sequence Challenges

5 Let's look at a new sequence.

This sequence has a constant difference. The terms are hidden by balloons.

As a class, decide which balloons to pop. You can pop up to *three* balloons.

What is the missing term?

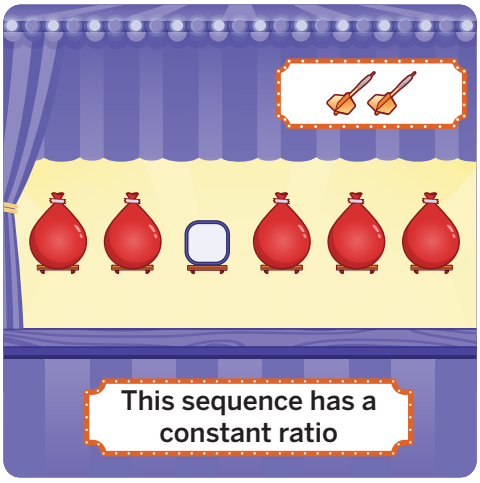


6 Let's look at a new sequence.

This sequence has a constant ratio.

As a class, decide which balloons to pop. You can pop up to *two* balloons.

What is the missing term?



7 Marc and Gabriel looked at this sequence.

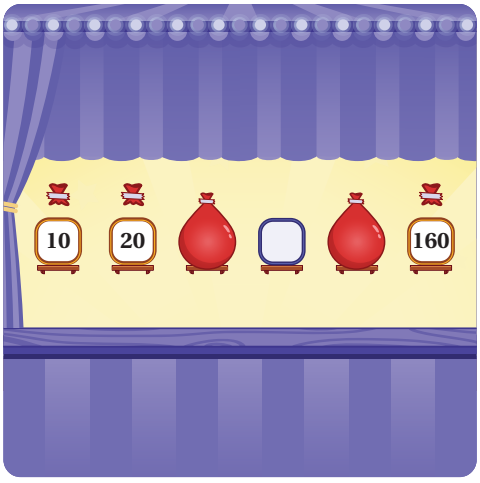
Marc said: *This sequence has a constant difference of* 10.

Gabriel said: *This sequence has a constant ratio of* 2.

Who is correct? Circle one.

Marc Gabriel Both Neither

Explain your thinking.



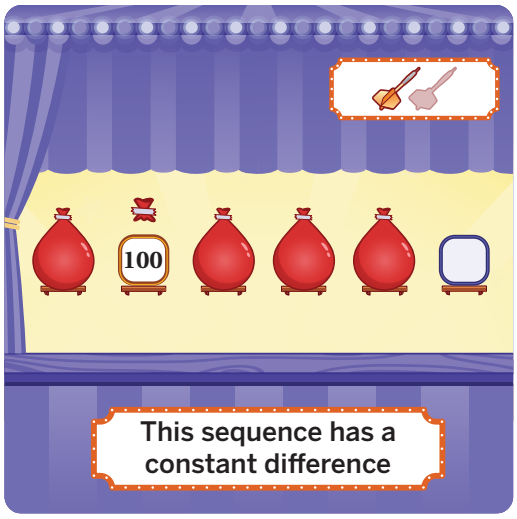
Name: ..... Date: ..... Period: .....

Sequence Challenges (continued)

8 Laila is working on this sequence, which has a constant difference.

She can pop *one more* balloon.

Which balloon do you think she should pop? Explain your thinking.

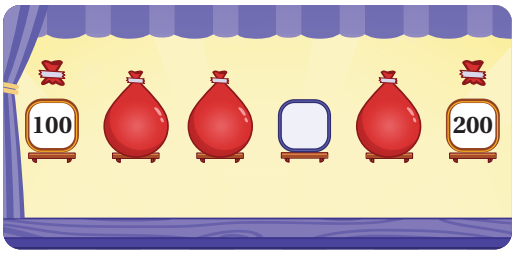


9 Let's see which balloon Laila chose to pop.

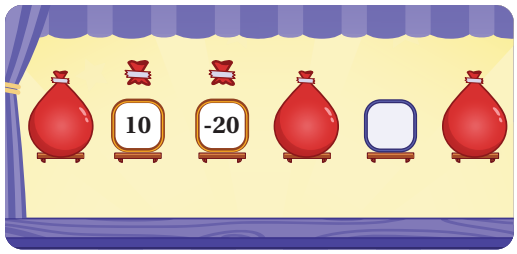
What is the missing term?

10 What is the missing term in each sequence?

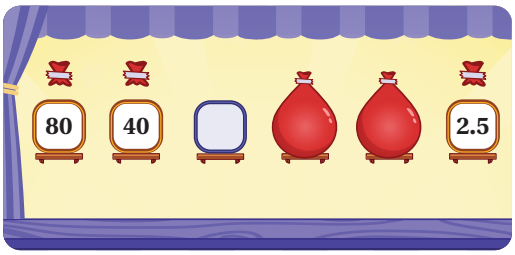
a This sequence has a constant difference.



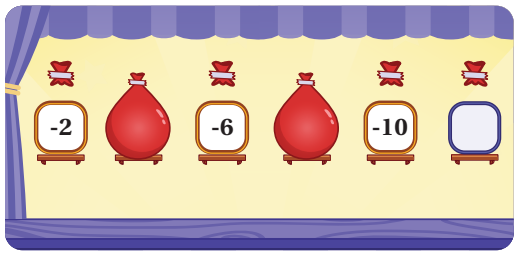
b This sequence has a constant ratio.



c This sequence has either a constant ratio *or* a constant difference.



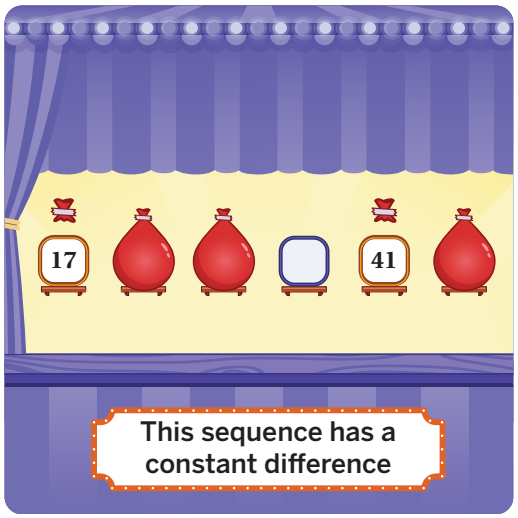
d This sequence has either a constant ratio *or* a constant difference.



11 Synthesis

Describe a strategy for finding a missing term in a **sequence**.

Use the example if it helps with your thinking.



14 Summary 1.02

Sequences can change in predictable ways. Two examples of predictable change are **constant differences** and **constant ratios**. Sequences can also change in predictable ways that are neither a constant difference or ratio.

Constant Difference

$$5,\ 11,\ 17,\ 23 \quad (+6,\ +6,\ +6)$$

Constant Ratio

$$2,\ 8,\ 32,\ 128 \quad (\times 4,\ \times 4,\ \times 4)$$

Something Else

$$13,\ 12,\ 10,\ 7 \quad (-1,\ -2,\ -3)$$

Once you know how a sequence changes, you can use that change to determine unknown terms.

Here is an example:

- The known terms in this sequence have a constant ratio of 0.5.
- The value of the unknown term should be 0.5 times the term before, 40, or 2 times the term after, 10.
- That means the unknown term is 20.

$$80,\ 40,\ ?,\ 10,\ 5 \quad (\times 0.5,\ \times 0.5)$$

constant difference When the difference between any two consecutive values in a pattern is the same, there is a constant difference.

constant ratio When the ratio between any two consecutive values in a pattern is the same, there is a constant ratio.

sequence A list of numbers in a particular order.

Practice
1.02

Name: ..... Date: ..... Period: .....

1. Determine the *constant ratio* for each sequence.

Sequence	Constant Ratio
256, 128, 64	
18, 54, 162	
0.8, 0.08, 0.008	

2. Determine the *constant difference* for each sequence.

Sequence	Constant Difference
12, 17, 22	
102, 85, 68	
$\frac{1}{4}$, 1, $\frac{7}{4}$	

3. Select *all* the sequences that have a constant difference.

☐ **A.** 8, 13, 18, 23, . . . ☐ **B.** 5, 10, 15, 75, . . . ☐ **C.** 5, 10, 15, 20, . . .

☐ **D.** 100, 50, 25, 12.5, . . . ☐ **E.** 100, 50, 0, -50, . . .

4. Here's the start of a sequence: 1, -1, . . . Write two different rules the sequence could follow and determine the next three terms for each rule.

Rule 1:

1, -1, ..., ..., ...

Rule 2:

1, -1, ..., ..., ...

Problems 5–8: Fill in the blanks to complete each sequence. Each sequence has a constant difference.

5. -3, -2, ..., ..., 1

6. ....., 13, 25, ..., ...

7. 1, 0.25, ..., -1.25, ...

8. 92, ..., ..., ..., 80

9. **Test Practice** A sequence has a first term of 6 and a constant ratio of -2. What are the first four terms of the sequence?

A. 6, 4, 2, 0 **B.** -2, 4, 10, 16 **C.** 6, 8, 10, 12 **D.** 6, -12, 24, -48

Problems 10–11: Here are Sequence A and Sequence B. One sequence has a constant difference and one has a constant ratio.

10. Complete the table.

Sequence A	70, 90, 110, . . .	Constant ratio or Constant difference	Fourth term:
Sequence B	10, 20, 40, . . .	Constant ratio or Constant difference	Fourth term:

11. Which sequence will have the greater 10th term? Show or explain your thinking.

Spiral Review

12. Square A has an area of 64 square feet. Select *all* the expressions that are equal to the side length of this square in feet.

- ☐ **A.** $\sqrt{8}$
- ☐ **B.** $\sqrt{64}$
- ☐ **C.** 4
- ☐ **D.** 8
- ☐ **E.** $\frac{64}{2}$

13. The points $(7, 21)$ and $(-5, 17)$ lie on a line. What is the slope of the line?

14. Determine the value of each expression when $n = 4$.

Expression	$n^2 - 5$	$n(n + 6)$	$3n^2$
Value When $n = 4$			

Name: ..... Date: ..... Period: .....

Parallels Between Numbers and Functions | F–BF.1.a, F–BF.2, F–LE.1.b, SMP.1, SMP.3

Recursion Machine

Let's write recursive definitions of sequences to meet certain requirements.

Warm-Up

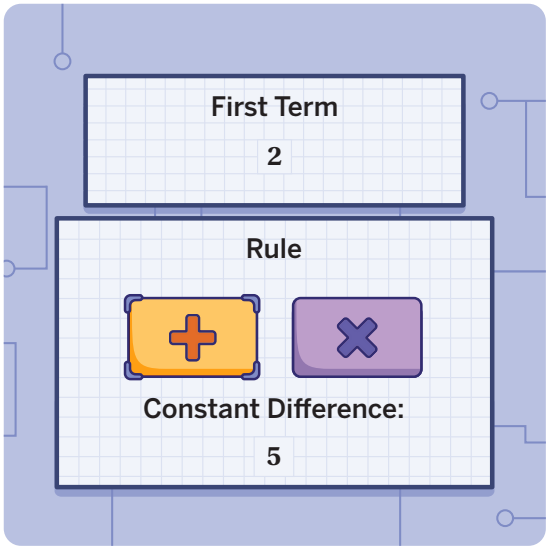
1 a. Let's watch an animation to see how a machine created this sequence.

b. How would you describe what the machine is doing?

Name: ..... Date: ..... Period: .....

Recursive Definitions

2 The machine creates a sequence using a first term and a rule. Together, a first term and a rule make a **recursive definition**.



a Take a look at these sequences that were made with different *recursive definitions*.

Sequence	First Term	Rule
2, 7, 12, 17, 22	2	Constant difference: 5
3, 6, 12, 24, 48	3	Constant ratio: 2
48, 24, 12, 6, 3	48	Constant ratio: $\frac{1}{2}$

b Create as many sequences as you can that include the number 12.

Sequence	First Term	Rule
....., ..., ..., ..., ...		
....., ..., ..., ..., ...		
....., ..., ..., ..., ...		

Name: ..... Date: ..... Period: .....

Recursive Challenges

3 In this sequence, the first term is 1,600. Create a rule that will produce this sequence.

Sequence	First Term	Rule
1600, 400, 100, 25, 6.25	1600	

4 In this sequence, the first term is 3. Create a rule that will make the fourth term 24.

Sequence	First Term	Rule
3, ..., ..., 24, ...	3	

5 Create a recursive definition for a sequence that makes the second term 25 and the fourth term 1.

Sequence	First Term	Rule
....., 25, ..., 1, ...		

Name: ..... Date: ..... Period: .....

Recursive Challenges (continued)

6 Troy made a mistake when he created a recursive definition on the previous problem.

a **Discuss:** How do you think Troy created this recursive definition?

Troy

....., 25, ..., 1, ...

First term: 49

Rule: Constant difference of −24

b What is something Troy can improve on?

7 Create a recursive definition for a sequence that makes the fourth term -40. Try to complete this challenge in different and interesting ways!

Sequence	First Term	Rule
....., ..., ..., -40, ...		
....., ..., ..., -40, ...		
....., ..., ..., -40, ...		

Activity 3

Name: ..... Date: ..... Period: .....

Challenge Creator

8 Now it's your turn to design your own sequence challenge.

a Make it!

- Write up to three terms anywhere in the sequence.
- Determine the first term and a rule that will complete your sequence.

My Sequence	First Term	Rule
....., ..., ..., ..., ...		

b Swap it!

- Share your three terms and where they are in the sequence with a partner. Keep your recursive definition a secret!
- Create a recursive definition that completes your partner's sequence.

Partner's Sequence	First Term	Rule
....., ..., ..., ..., ...		

Partner's Sequence	First Term	Rule
....., ..., ..., ..., ...		

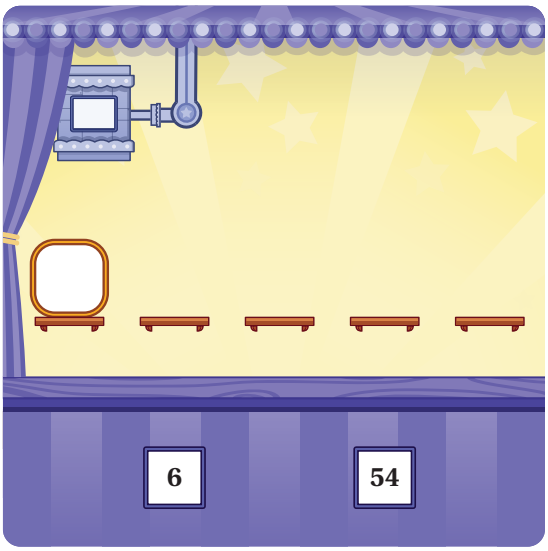
Partner's Sequence	First Term	Rule
....., ..., ..., ..., ...		

9 Synthesis

We learned that a **recursive definition** of a sequence includes a first term and a rule.

Discuss: What's a strategy for determining the recursive definition of a sequence that meets certain requirements?

Use the example if it helps with your thinking.



12 Summary 1.03

There are several ways to define, or describe, a sequence. When you define a sequence recursively, you're determining each term using the previous term.

Here are some examples of **recursive definitions** for this sequence: $32, 16, 8, 4, 2, 1, 0.5$.

First term: 32

Rule: Half of the previous term

First term: 32

Rule: Constant ratio of $\frac{1}{2}$

First term: 32

Rule: Multiply the previous term by 0.5

recursive definition Must include at least the first term of the sequence and a rule for determining each term that follows.

Practice
1.03

Name: ..... Date: ..... Period: .....

Problems 1–2: Use the sequence $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \ldots$

1. Write a recursive definition for this sequence.

First term:

Rule:

2. Write the next three terms of the sequence.

3. Complete the recursive definition for each sequence.

	Sequence A	Sequence B	Sequence C	Sequence D
Sequence	$2, 4, 6, 8, \ldots$	$5, 7, 9, 11, \ldots$	$50, 25, 0, -25, \ldots$	$\frac{1}{3}, 1, 3, 9, \ldots$
First Term				
Constant Difference or Constant Ratio?				
Rule				

4. This sequence has a constant difference of 5. Fill in the missing terms.

....., ..., 7, ..., ...

Problems 5–6: Here is the start of a sequence: $1, 5, \ldots$

5. Write a rule and the next three terms the sequence could follow.

Rule: Terms:

6. Write a *different* rule and the next three terms the sequence could follow.

Rule: Terms:

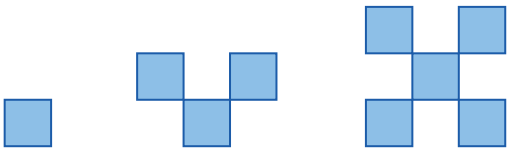
7. Remy is studying this sequence: 20, ..?.., 80. Remy thinks the missing term could be less than the first and third terms. Is Remy correct? Explain your thinking.

Name: ..... Date: ..... Period: .....

8. A swimming pool is filling with water at a rate of 17 gallons per minute. The pool started with 100 gallons of water. Write a recursive definition to model the situation.

First term: Rule:

9. **Test Practice** Here are the first three stages in a floor tile pattern. Select the recursive definition that best models the situation.

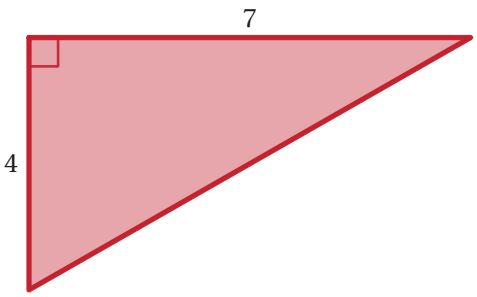


A. First term: 4 Rule: Constant ratio of 3
B. First term: 1 Rule: Constant ratio of 2
C. First term: 4 Rule: Constant difference of 3
D. First term: 1 Rule: Constant difference of 2

Spiral Review

10. Explain how you know that $\sqrt{42}$ is between 6 and 7.

11. Determine the exact length of the unlabeled side.

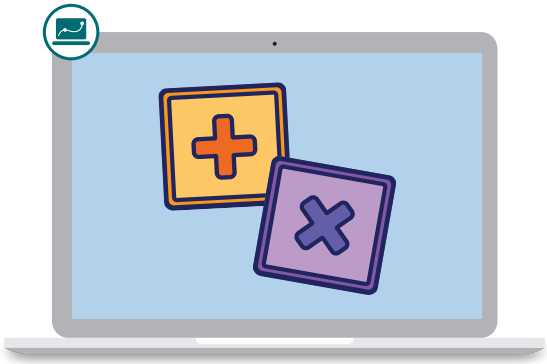


12. Write each expression as a single power of 10.

Expression	$\frac{10^7}{10^3}$	$10^2 \cdot 10^5$	$10^0 \cdot 10^9$	$\frac{10^5}{10^0}$
Single Power of 10				

Name: ..... Date: ..... Period: .....

Comparing Models F–LE.1.b, F–LE.2, SMP.3, SMP.5, SMP.8



See the Sequence

Let's compare sequences using tables and graphs.

Warm-Up

1 Here are two sequences.

a Let's watch an animation to see how the machines create the first four terms of each sequence.

b What do you notice? What do you wonder?

I notice:

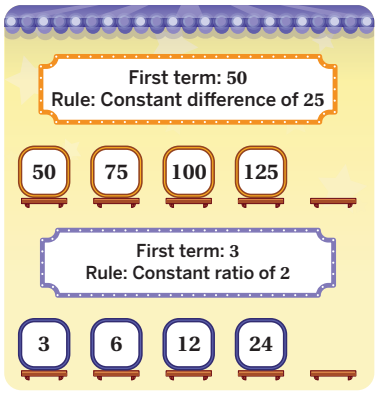
I wonder:

Activity 1

Name: ..... Date: ..... Period: .....

Sequence Types

2 The top sequence changes by a constant difference. We call that an **arithmetic sequence**. The bottom sequence changes by a constant ratio. We call that a **geometric sequence**. Which do you think will have a greater 10th term?



A. The arithmetic sequence

B. The geometric sequence

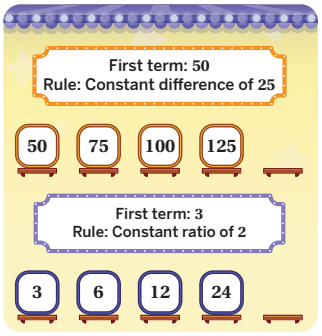
C. They will be the same

D. Not enough information

Explain your thinking.

3 Sequences can be represented in multiple ways.

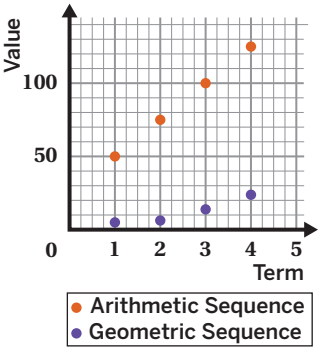
Recursive Definition



Table

Term	Arithmetic Sequence	Geometric Sequence
1	50	3
2	75	6
3	100	12
4	125	24

Graph



a **Discuss** What are the advantages and disadvantages of each representation?

b Choose one representation. Explain how it could be used to help determine which sequence has the greater 10th term.

Name: ..... Date: ..... Period: .....

Sequence Types (continued)

4 **a** Let's see which sequence has the greater 10th term.

b **Discuss** What do you notice about the graphs of the two sequences?

5 **a** Group together the cards that represent the same sequence.

Card A

Term	Value
1	3
2	1
3	-1
4	-3
5	-5

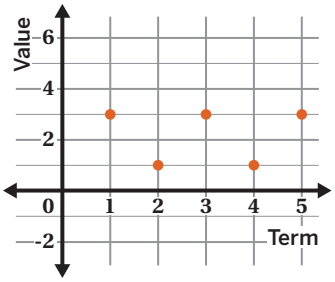
Card B

Term	Value
1	3
2	6
3	12
4	24
5	48

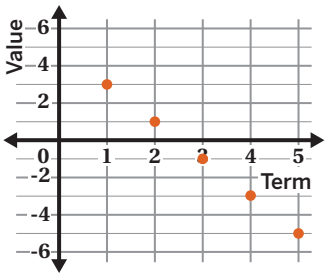
Card C

Term	Value
1	3
2	1
3	3
4	1
5	3

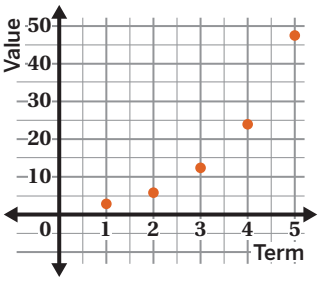
Card D



Card E



Card F



Sequence 1	Sequence 2	Sequence 3
First term: 3 **Rule:** Multiply the previous term by 2	**First term:** 3 **Rule:** Add -2 to the previous term	**First term:** 3 **Rule:** Alternate between 3 and 1

b **Discuss:** Is each sequence arithmetic, geometric, or neither?

Activity 2

Name: ..... Date: ..... Period: .....

Make Your Own Sequences

6 You will use Screen 6 to complete this activity.

a Create several different sequences and compare their tables and graphs. Then describe something that you found interesting or surprising.

b Decide whether each statement is *always*, *sometimes*, or *never* true. Explain your thinking.

Statement	Always, Sometimes, Never	Explanation
When you graph an arithmetic sequence, the points lie on a line.		
The graph of a geometric sequence curves upward.		
The graph of a geometric sequence with a positive first term will stay above the x-axis.		

c Write a recursive definition of a sequence that meets each set of criteria.

Criteria	Recursive Definition
Its graph lies on a horizontal line.	
It approaches 0 but never reaches it.	
Its 6th term is negative and 7th term is positive.	

Name: ..... Date: ..... Period: .....

Make Your Own Sequences (continued)

You're invited to explore more.

7 Malik notices he can make an arithmetic sequence and a geometric sequence that have the same first two terms.

a In the table, continue the sequence in two ways: assuming it is *arithmetic* and assuming it is *geometric*.

b How long do you think it would take each sequence to reach 1,000?

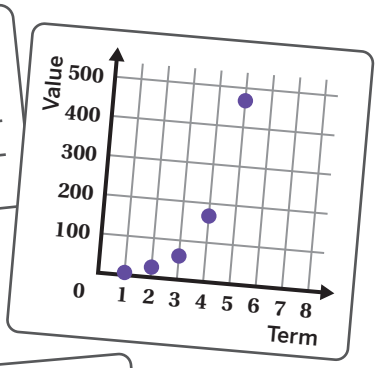
Term	Arithmetic Sequence	Geometric Sequence
1	8	8
2	12	12
3		
4		
5		

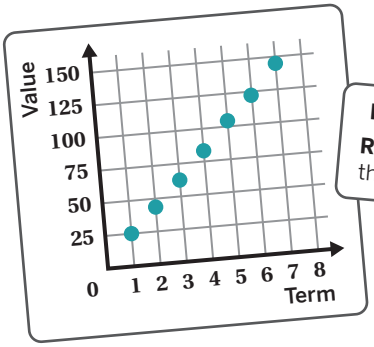
8 Synthesis

Sequences can be represented in multiple ways.

Discuss: What are some clues that a sequence might be arithmetic? Geometric?

Term	Value
1	6
2	18
3	54
4	162





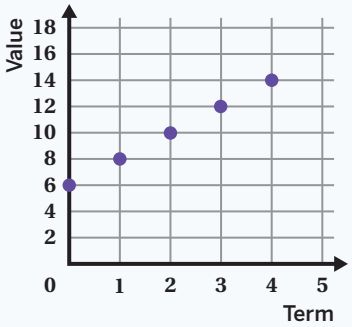
First Term: 25
Rule: Add 20 to the previous term.

11 Summary 1.04

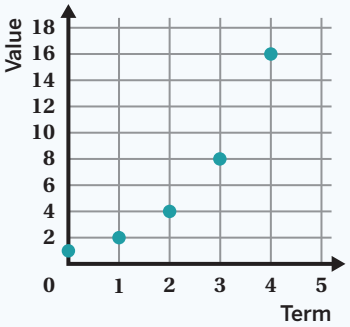
Sequences can be represented in multiple ways: as a list of numbers, in a table, on a graph, or with a recursive definition. In each representation, there are ways to identify whether the sequence is arithmetic, geometric, or neither.

- In a recursive definition, **arithmetic sequences** involve adding or subtracting and **geometric sequences** involve multiplying or dividing.
- In a table, arithmetic sequences have a constant difference and geometric sequences have a constant ratio.
- On a graph, arithmetic sequences are linear and geometric sequences are often curved, as shown below.

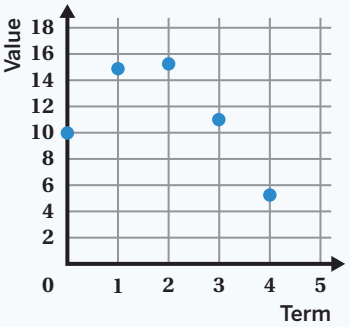
Arithmetic



Geometric



Neither



arithmetic sequence A sequence that changes by a constant difference.

geometric sequence A sequence that changes by a constant ratio.

Practice
1.04

Name: _____ Date: _____ Period: _____

1. Determine whether each sequence is arithmetic, geometric, or neither.

Sequence	1000, 200, 40, 8	2, 4, 16, 256	10, 20, 30, 40	500, 100, 20, 4
Arithmetic, Geometric, or Neither				

2. Complete each arithmetic sequence with its missing terms.

-2, 4, _____, 16, _____

11, 111, _____, _____, 411

_____, 7.5, 10, _____, _____

5, _____, -13, -22, _____

3. Complete each geometric sequence with its missing terms.

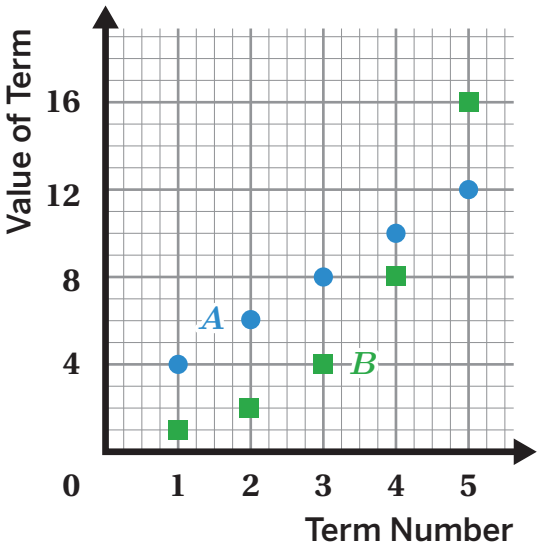
_____, 5, 25, _____, 625

-1, _____, -36, 216, _____

10, 5, _____, _____, 0.625

_____, _____, -36, 108, _____

Problems 4–6: Here are the graphs of two sequences.



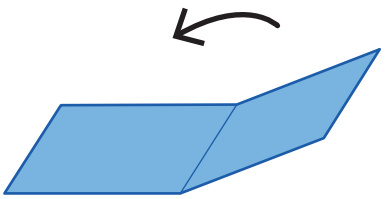
4. For Sequence A, describe a way to produce a new term from the previous term.

5. For Sequence B, describe a way to produce a new term from the previous term.

6. **Test Practice** Which of these is a geometric sequence? Explain your thinking.

Name: ..... Date: ..... Period: .....

7. Ahmed is trying to see how many times he can fold a piece of paper in half. The thickness of the piece of paper he starts with is 0.001 centimeters. How thick will the paper be after 10 folds?



Spiral Review

8. Select *all* the sets of side lengths that form a right triangle.

- ☐ **A.** $2, 3, 5$
- ☐ **B.** $\sqrt{7}, 9, \sqrt{88}$
- ☐ **C.** $\sqrt{12}, 6, \sqrt{48}$
- ☐ **D.** $4, 5, \sqrt{41}$
- ☐ **E.** $4, 5, 9$

9. If a line has a negative slope and contains the point $(4, 6)$, which of these points could it also contain?

A. $(7, 6)$ **B.** $(3, 1)$ **C.** $(6, 5)$ **D.** $(5, 8)$

10. Evaluate each expression.

Expression	$\left(\frac{1}{3}\right)^2$	$\left(\frac{1}{3}\right)^{-2}$
Value		

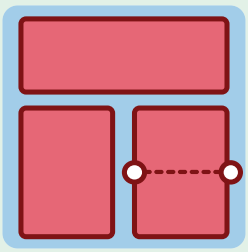
11. What two whole numbers is $\sqrt{60}$ between?

Explain your thinking.

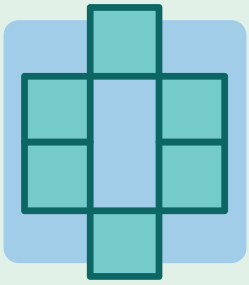
Sub-Unit

2

Variables and Relationships



Lesson 5
Paper Patterns



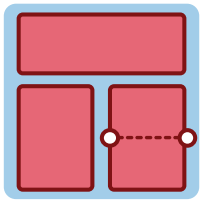
Lesson 6
More Visual Patterns

Lesson 7
Tree-Mendous Models

Name: ..... Date: ..... Period: .....

Modeling with Functions A–SSE.1, A–SSE.1.a, F–BF.1.a, F–BF.2, F–LE.2, SMP.1, SMP.7, SMP.8

Paper Patterns



Let's represent situations in different ways.

Warm-Up

1. Match each *expression* with an equivalent one.

a $10 + 3 + 3 + 3 + 3 + 3$ ..... $10 \cdot 3^5$

b $10 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$ ..... $10 \cdot \left(\frac{1}{3}\right)^5$

c $10 - 3 - 3 - 3 - 3 - 3$ ..... $10 + 5(-3)$

d $10 \div 3 \div 3 \div 3 \div 3 \div 3$ ..... $10 + 5 \cdot 3$

2. **Discuss:** Why might it be useful to have multiple forms of an expression?

Activity 1

Name: ..... Date: ..... Period: .....

Paper Cutting

Each of these patterns starts with a piece of paper that measures 8-by-10 inches.

3. Complete each representation.

Pattern 1

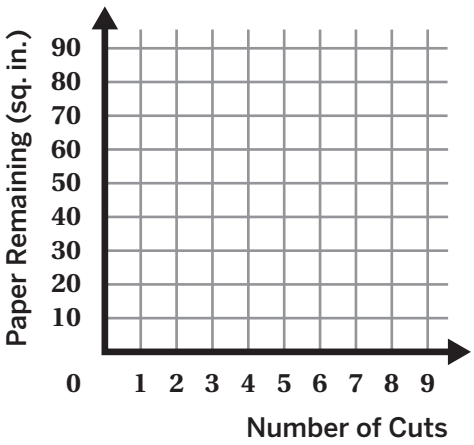
Situation

Cut off half of the paper and discard. Repeat.

Table

Number of Cuts	Paper Remaining (sq. in.)
0	80
1	
2	
3	

Graph



Pattern 2

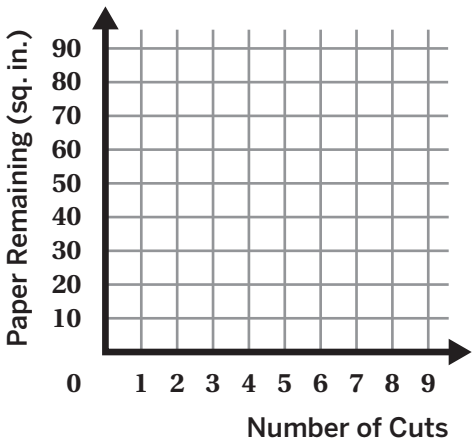
Situation

Cut a 1-by-8-inch strip off the paper and discard. Repeat.

Table

Number of Cuts	Paper Remaining (sq. in.)
0	80
1	
2	
3	

Graph



4. For each pattern, what is the area of the paper remaining after 9 cuts?

Name: ..... Date: ..... Period: .....

Explicit Expressions

5. Let's look at Wohali's and Kadeem's strategies for Pattern 1. Wohali created a rule for his strategy and Kadeem created an expression for his strategy.

Wohali's Rule:	**Kadeem's Expression:**
Starting amount: 80 Rule: Multiply the previous amount by $\frac{1}{2}$.	$80\left(\frac{1}{2}\right)^4$

a How are Wohali's and Kadeem's strategies alike? How are they different?

b Where do you see the same information in Wohali's rule and Kadeem's expression?

6. Kadeem wants to write an expression that represents the area of the paper remaining after n cuts. This expression is an example of an **explicit definition**.

a Write Kadeem's expression.

b What does each part of his expression represent in this situation?

80 represents . . .

$\frac{1}{2}$ represents . . .

n represents . . .

Name: ..... Date: ..... Period: .....

Explicit Expressions (continued)

7. Let's think about Pattern 2.

a Select *all* the expressions that could be used to determine the area of the paper remaining after 9 cuts.

- ☐ **A.** $80 - 8 - 8 - 8 - 8 - 8 - 8 - 8 - 8 - 8$
- ☐ **B.** $80 + 9(8)$
- ☐ **C.** $80 \cdot (-8)^9$
- ☐ **D.** $80 - 9(8)$
- ☐ **E.** $80 \cdot 0.5^9$

b Write a recursive rule to determine the area of the paper remaining after n cuts.

c Write an explicit expression to determine the area of the paper remaining after n cuts.

d What does each part of your explicit expression represent in this situation?

8. How are the explicit expressions for the two patterns alike? How are they different?

Name: ..... Date: ..... Period: .....

More Representations

9. You will use a set of cards to complete this activity. Group the cards based on whether they have a constant difference, a constant ratio, or neither.

Constant Difference	Constant Ratio	Neither

10. Choose one card with a *constant difference*.

a Create the matching representations.

Situation

Recursive Rule

Explicit Expression

Table

0	
1	
2	
3	
...	...
10	

b What does each part of your explicit expression represent in this situation?

Name: ..... Date: ..... Period: .....

More Representations (continued)

11. Choose one card with a *constant ratio*.

a Create the matching representations.

Situation

Recursive Rule

Explicit Expression

Table

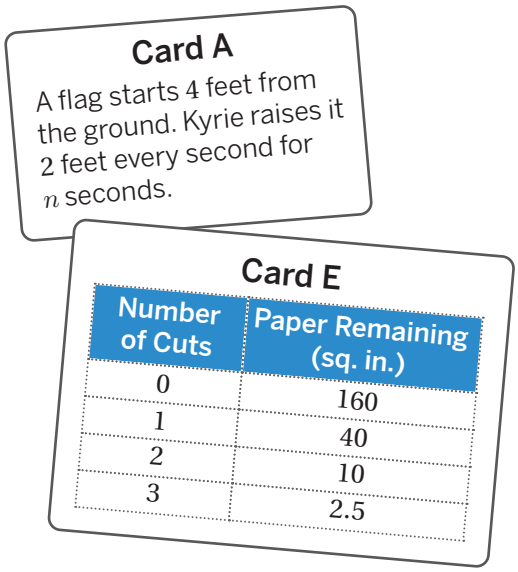
0	
1	
2	
3	
...	...
10	

b What does each part of your explicit expression represent in this situation?

Synthesis

12. What information from a situation can help you write an explicit expression?

Use the cards if they help with your thinking.



Summary 1.05

An **explicit definition** of a sequence can be written as an *expression* or in words. Information from recursive rules can be used to write an explicit expression. Here are two situations with the recursive rules and explicit expressions that define them.

Situation: Julian's family is 250 miles away from home. They drive 50 miles toward their home for every n hours.

Recursive Rule:
Starting distance from home: 250
Rule: Subtract 50 from the previous amount for every hour.

Explicit Expression: $250 - 50n$
The expression represents the number of miles Julian's family is from home after n hours.

- 250 represents the starting distance.
- -50 represents the distance traveled each hour.
- $-50n$ represents the total distance traveled.
- n represents the number of hours driven.

Situation: A scientist tracks a group of 250 termites in a mound. Each month, the number of termites increases by 1.5 times, for n months.

Recursive Rule:
Starting number of termites: 250
Rule: Multiply by 1.5 from the previous amount for each month.

Explicit Expression: $250 \cdot 1.5^n$
The expression represents the number of termites in the mound after n months.

- 250 represents the initial number of termites.
- 1.5 represents the ratio that the termites grow by each week.
- n represents the number of weeks.

explicit definition A formula that determines the value of any term in a sequence using the term number and can be written as an expression, an equation, a function, or in words.

Practice
1.05

Name: ..... Date: ..... Period: .....

1. Jamar had 80 followers on social media. His number of followers *tripled* every month for 4 months. Select *all* the expressions that represent Jamar's followers after 4 months.

- ☐ **A.** $80 \cdot 3 \cdot 3 \cdot 3 \cdot 3$
- ☐ **B.** $80 + 4^3$
- ☐ **C.** $80 \cdot 4 \cdot 4 \cdot 4$
- ☐ **D.** $80 + 3 + 3 + 3 + 3$
- ☐ **E.** $80 \cdot 3^4$

2. The weather forecast predicts a snowfall rate of 0.75 inches of snow per hour overnight. There are already 9 inches of snow on the ground in Anushka's neighborhood.

Complete the table to show the amount of snow on the ground, in inches, after n hours.

Hours, n	0	1	2	3	…	8
Amount of Snow (in.)					…	

3. The population of a city was $100{,}000$ in 1970 and has doubled 3 times since then. Select *all* the expressions that represent the population today.

- ☐ **A.** 300000
- ☐ **B.** 800000
- ☐ **C.** $100000 \cdot 2 \cdot 2 \cdot 2$
- ☐ **D.** $100000 \cdot 3^2$
- ☐ **E.** $100000 \cdot 2^3$

Problems 4–5: Here is a table representing a pattern.

x	y
0	60
1	30
2	15
3	7.5
4	3.75

4. Circle the equation that represents the table.

- **A.** $y = 60 + \frac{1}{2}x$
- **B.** $y = 60 \cdot \left(\frac{1}{2}\right)^x$
- **C.** $y = 60 \cdot 2^x$
- **D.** $y = 60 - \frac{1}{2}x$

5. Explain your thinking.

Name: ..... Date: ..... Period: .....

Problems 6–7: Maia earns money by shoveling snow in her neighborhood. She starts the winter with $52 in her bank account and will deposit $10 for every driveway she shovels.

6. Complete the table with Maia's account balance, in dollars, after she shovels n driveways.

Number of Driveways, n	0	1	2	3
Account Balance ($)	52			

7. Write an explicit expression for this situation.

Test Practice Problems 8–9: A group of biologists tracked the number of squirrels in a town. They represented this number using the equation $n = 40 \cdot 1.5^t$, where n is the total number of squirrels and t is the number of years since the biologists started counting.

8. Explain what the 40 and 1.5 mean in this situation.

9. How many squirrels do the biologists predict there will be 2 years after they started counting?

Spiral Review

Problems 10–11: Write each expression using an exponent.

10. $\left(\frac{4}{5}\right) \cdot \left(\frac{4}{5}\right) \cdot \left(\frac{4}{5}\right)$

11. $9.3 \cdot 9.3 \cdot 9.3 \cdot 9.3 \cdot 9.3$

12. An arithmetic sequence starts with $10, 5, \ldots$
Explain how you would calculate the value of the 100th term.

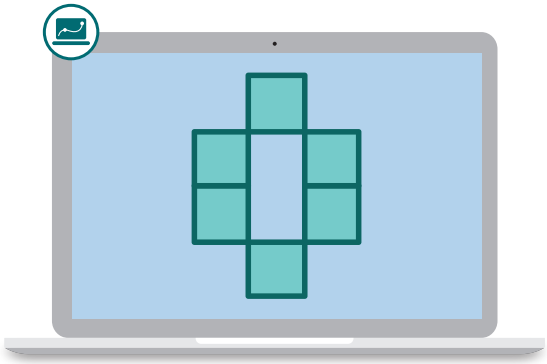
13. Write a recursive definition for the sequence: $\frac{1}{3}, \frac{2}{9}, \frac{4}{27} \ldots$

First Term:

Rule:

Name: ..... Date: ..... Period: .....

Modeling with Functions A–SSE.1, A–SSE.1.a, F–BF.1.a, F–BF.2, SMP.2, SMP.3, SMP.7



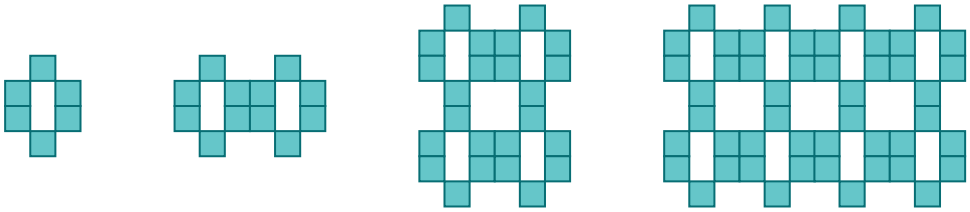
More Visual Patterns

Let's write explicit expressions for arithmetic and geometric sequences.

Warm-Up

1 How do you see this pattern growing?

Figure 1 Figure 2 Figure 3 Figure 4



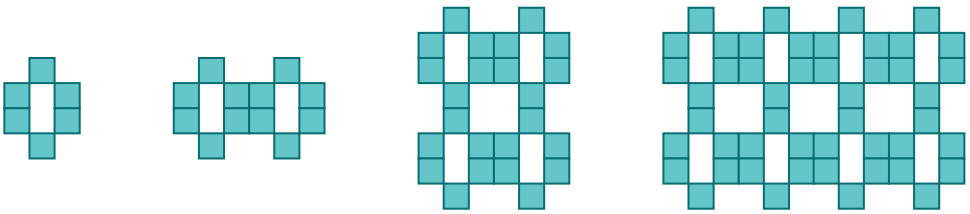
Activity 1

Name: ..... Date: ..... Period: .....

How Many Tiles?

2 Here is the pattern from the Warm-Up.

Figure 1 **Figure 2** **Figure 3** **Figure 4**

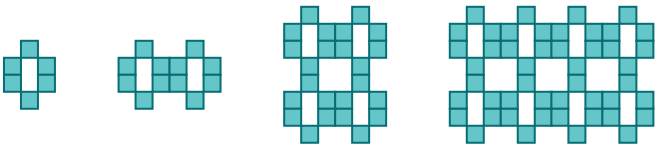


a How many tiles will there be in Figure 7?

b **Discuss** How could you determine the number of tiles in Figure 15?

3 Here is Zoe's strategy for determining the number of tiles in Figure 15.

Figure 1 **Figure 2** **Figure 3** **Figure 4**



6 tiles — 6

12 tiles — 6·2

24 tiles — 6·2·2

48 tiles — 6·2·2·2

Figure 15: $6 \cdot 2^{15}$ tiles

a What did Zoe do well?

b What was Zoe's mistake?

4 Amir and Maria want to write an explicit expression for the number of tiles in Figure n.

Amir writes: $6 \cdot 2^{(n-1)}$ Maria writes: $3 \cdot 2^n$

Which explicit expression is correct? Explain your thinking.

A. $6 \cdot 2^{(n-1)}$ **B.** $3 \cdot 2^n$ **C.** Both **D.** Neither

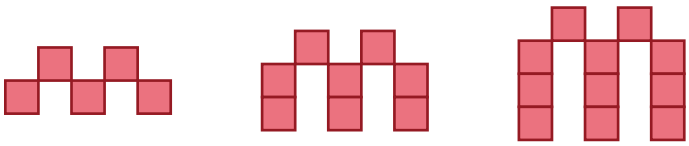
Activity 2

Name: ..... Date: ..... Period: .....

A New Pattern

5 Here is a new pattern.

Figure 1 **Figure 2** **Figure 3**



How many tiles will there be in Figure 4 and Figure 15?

Figure	Number of Tiles
1	5
2	8
3	11
4	
...	...
15	

6 Here are Omari's and Ivory's responses for Figure 15.

Figure	Number of Tiles (Omari)	Number of Tiles (Ivory)
15	$5 + 3(14)$	$2 + 3(15)$

Discuss:

- Where did the numbers in Omari's and Ivory's work come from?

- How might each student determine the number of tiles in Figure 50?

7 Write an explicit expression for the number of tiles in Figure n.

Activity 3

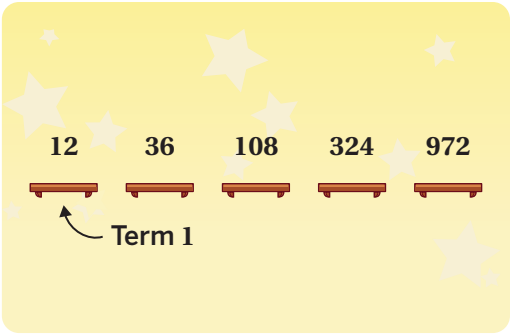
Name: ..... Date: ..... Period: .....

Sequences Only

8 Let's look at a sequence *without* a visual pattern.

Here are the first five terms of the sequence.

Write an explicit expression for Term n.

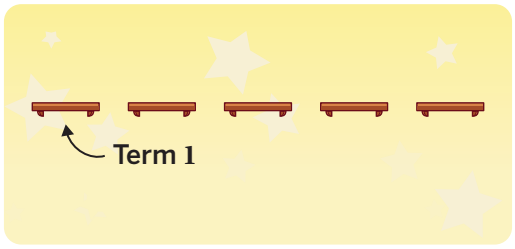


9 You will be designing a challenge for your classmates to solve.

a Make your sequence.

- Write the first five terms of your arithmetic or geometric sequence.
- Write an explicit expression for Term n of your sequence.

My Challenge **Explicit Expression**



b Solve your classmates' sequence challenges.

- Share your sequence with a classmate. Keep your explicit expression a secret!
- Write an explicit expression for Term n of their sequence.

.....'s Challenge **Explicit Expression**

....., ..., ..., ..., ...

.....'s Challenge **Explicit Expression**

....., ..., ..., ..., ...

.....'s Challenge **Explicit Expression**

....., ..., ..., ..., ...

10 Synthesis

Describe one strategy for writing an explicit expression for Term n of a sequence.

27, 34, 41, 48, ...

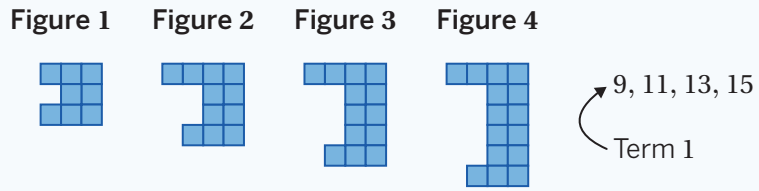
Term 1

Use the example if it helps with your thinking.

13 Summary 1.06

You can write an explicit expression for a pattern or sequence in multiple equivalent ways, just by referencing different term numbers.

Here's an example of a pattern and its matching sequence.



$7 + 2(1)$ $7 + 2(2)$ $7 + 2(3)$ $7 + 2(4)$ $9 + 2(n - 1)$

One expression that represents this sequence is $9 + 2(n - 1)$; this expression uses Term 1 as the starting value and $n - 1$ to calculate the change.

The expression $7 + 2n$ also represents this sequence, using Term 0 as the starting value, and n to calculate the change.

Practice
1.06

Name: ..... Date: ..... Period: .....

1. Match each sequence to its explicit expression.

Sequence	Explicit Expression
a 4, 10, 16, 22	..... $320 \cdot \left(\frac{1}{2}\right)^n$
b 4, 12, 36, 108	..... $4 + 6(n - 1)$
c 160, 80, 40, 20	..... $320 + \frac{1}{2}(n - 1)$
d 320, 320.5, 321, 321.5	..... $4 \cdot 3^{(n - 1)}$

2. Select *all* the expressions that could represent the number of tiles in Figure n of this pattern.

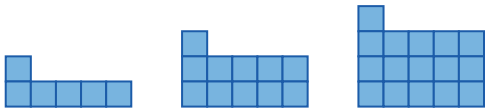


Figure 1 Figure 2 Figure 3

- ☐ **A.** $5 + n$
- ☐ **B.** $5n + 1$
- ☐ **C.** $6n - 1$
- ☐ **D.** $6 + 5(n - 1)$
- ☐ **E.** $10n - 4$

Problems 3–4: Here is a visual pattern.

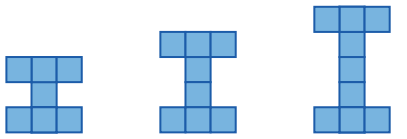


Figure 1 Figure 2 Figure 3

3. Complete the table with the number of tiles in each figure.

4. Write an explicit expression for the number of tiles in Figure n.

Figure	# of Tiles
1	
2	
3	
4	
…	…
15	

5. **Test Practice** The first four terms of a geometric sequence are: 6, 18, 54, 162. Which explicit expression can be used to describe this sequence?

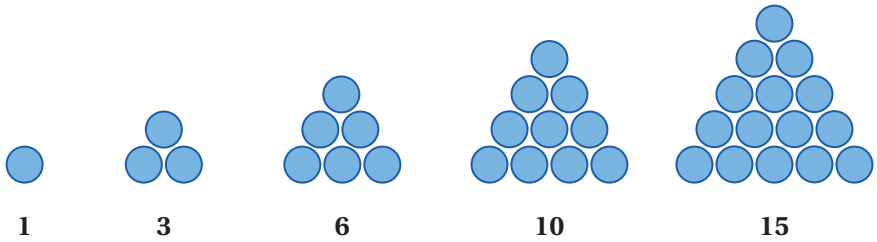
A. $2 + 3n$ **B.** 3^n

C. $2 \cdot 3^n$ **D.** $6 \cdot 3^n$

Name: ..... Date: ..... Period: .....

6. The first four terms in a sequence are $15, 22, 29, 36$. Write two explicit expressions for Term n of this sequence.

7. A *triangular number* can be represented as the number of dots in an equilateral triangle.



What is the 8th triangular number?

Spiral Review

Problems 8–10: Determine the value of each expression when $x = 3$.

8. $7x - 8$

9. $5(x + 9)$

10. $\frac{x - 6}{4x}$

Problems 11–13: Decide whether each sequence is arithmetic, geometric, or neither. Circle one.

11. $25, 5, 1, \ldots$ Arithmetic Geometric Neither

12. $25, 19, 13, \ldots$ Arithmetic Geometric Neither

13. $25, 52, 25, 52, \ldots$ Arithmetic Geometric Neither

14. Complete the recursive definition for this sequence.

Sequence	First Term	Constant Difference or Constant Ratio?	Rule
$\frac{1}{2}, 2, 8, 32, \ldots$			

Name: ..... Date: ..... Period: .....

Modeling with Functions A–SSE.1.a, F–BF.1.a, F–BF.2, F–LE.2, N–Q.3, SMP.1, SMP.2, SMP.3, SMP.4

Tree-Mendous Models

Let's model proposals mathematically and consider how they might affect a community.

Warm-Up

1. What are some advantages of living in a place with a lot of trees? What are some disadvantages?

Advantages:

Disadvantages:

Activity 1

Name: Date: Period:

Tree Proposals

In 2022, the fictional city of Metropolis had 125,000 trees. The Metropolis Council's plan is to have 152,000 trees by 2032 to help reduce extreme heat and air pollution.

Council Members Adachi and Nassar need help completing their proposals for how to meet this goal.

2. Work with your group to complete each proposal.

Table

Adachi's Proposal

Years Since 2022, n	Total Number of Trees
0	
1	
2	
3	
4	

Nassar's Proposal

Years Since 2022, n	Total Number of Trees
0	125000
1	127500
2	130050
3	132651
4	

Recursive Definition

Starting number of trees: 125,000

Rule: Add 2,700 trees each year.

Starting number of trees:

Rule:

Explicit Expression

Proof the Proposal Will Meet the Goal

Planting Locations

New trees should be divided equally across all neighborhoods in the city.

New trees should be planted in neighborhoods that have the fewest number of trees.

Name: ..... Date: ..... Period: .....

Tree Proposals (continued)

3. **Discuss:** What are some possible advantages of each proposal? Possible disadvantages?

Use the table if it helps with your thinking.

	Adachi's Proposal	Nassar's Proposal
Advantages		
Disadvantages		

Activity 2

Name: ..... Date: ..... Period: .....

Models in Real Life

The Metropolis Council selected a third proposal by Council Member Garcia, which uses the expression $125000 + 3000x$ to model the number of trees in Metropolis x years since 2022.

A **model** is a mathematical representation (such as a graph, equation, or relationship) of a situation. You can use a model to make predictions or decisions.

4. What does each part of Garcia's model represent in this situation?

Garcia's Proposal

Explicit Expression	$125000 + 3000x$
Planting Locations	New trees should all be planted in areas that have the most homes.

5. **Discuss:** Do you think the model will predict the exact number of trees in Metropolis each year? Why or why not?

Several community members shared additional information about planting trees.

6. For each situation, describe how you might revise Garcia's proposal to better meet the community's needs.

a The landscapers planting the trees report that their work will take more time than planned. Metropolis decides to extend the timeline to 12 years instead of 10 but keep the same total.

b An arborist (tree specialist) says that a percentage of trees die each year.

c Some homeowners say they cannot maintain trees and do not want trees planted on their land.

d An accountant reports that after the third year, the cost of purchasing and planting trees will increase significantly.

Activity 3

Name: Date: Period:

Your Community

Think about the land around *your* school and community.

7. **Discuss:** Where do you see trees? Where do you *not* see trees?

8. What information could you gather to better understand how trees (or the lack of trees) are impacting your community?

9. What experts, organizations, or community members could you talk to in order to learn more?

10. Do you think your community would benefit from planting more trees? Why or why not?

Synthesis

11. The British statistician George Box once wrote, "All **models** are wrong, but some are useful."

Discuss: What does this quote mean in your own words?

Use Counsel Member Garcia's model, $125000 + 3000x$, if it helps with your thinking.

Summary 1.07

You can use a **model** to make predictions or decisions.

For example, let's say a group of scientists track the amount of algae on the surface of a lake, in square feet, over n weeks.

Weeks (n)	0	1	2	3	4
Amount of Algae (sq. ft)	2	2.8	3.92	5.49	7.68

The scientists model this information with the expression $2 \cdot 1.4^n$, which can be used to predict how much algae there will be in the upcoming weeks. Predictions might not be exactly accurate because factors like weather or human action can affect algae growth. The model might also be limited by the size of the lake.

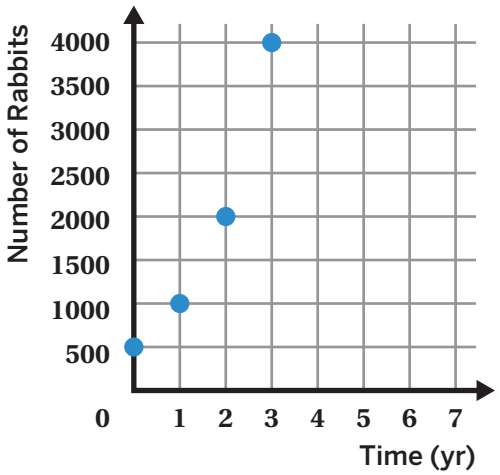
No mathematical model applies perfectly to real life, but a model can still be a powerful and useful tool. We can even improve models when we take into account more information.

model A mathematical representation of real-world data, such as a graph, equation, or relationship that you can use to solve problems or make predictions and decisions.

Practice
1.07

Name: ..... Date: ..... Period: .....

Problems 1–3: A group of scientists are tracking a population of rabbits. They count the number of rabbits each year.



1. How many rabbits were there when the scientists began counting?

2. After how many years will the population of rabbits reach $4{,}000$?

3. Write an explicit expression to represent the number of rabbits after n years.

Problems 4–5: A skateboarder scrapes their arm. 1 hour after they apply an antibiotic, the population of bacteria is $1{,}000{,}000$. After every hour, the population decreases by half.

Number of Hours, n	Population of Bacteria
1	1000000
2	50000
3	
4	
5	

4. Complete the table with the population of bacteria that is present after n hours.

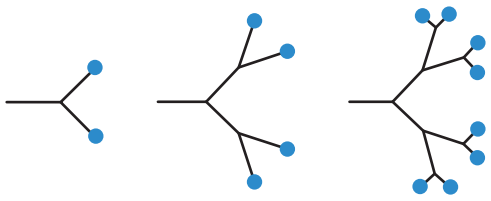
5. Is the population of bacteria arithmetic, geometric, or neither? Circle one.

Arithmetic Geometric Neither

Explain how you know.

Problems 6–7: Here are the first three figures in a pattern.

Figure 1 Figure 2 Figure 3



6. Is the number of dots arithmetic, geometric, or neither? Circle one.

Arithmetic Geometric Neither

Explain how you know.

7. Write an explicit expression to determine the number of dots in Figure n.

Practice
1.07

Name: ..... Date: ..... Period: .....

Test Practice Problems 8–10: A paper clip weighs 0.5 grams and an empty envelope weighs 6.75 grams. Nikhil adds paper clips one at a time to an empty envelope.

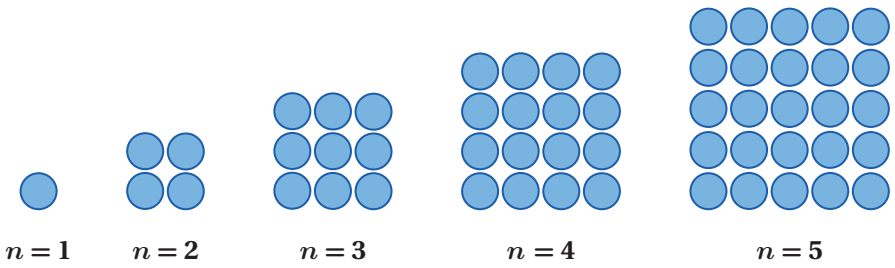
8. Complete the table with the weight of the envelope, in grams, after Nikhil adds n paper clips.

9. Does it make sense to have a row in the table where $n = 10.25$? Explain your thinking.

Number of Paper Clips, n	Weight (g)
0	6.75
1	
2	
3	

10. How many paper clips will be in the envelope when the weight is 9.75 grams?

Problems 11–12: A *square number* can be represented as the number of dots in a square.



11. Complete the table for square numbers 1 to 5.

n	1	2	3	4	5
Number of Dots					

12. Write an expression for the number of dots in square number n.

Spiral Review

Problems 13–14: Determine the value of each expression.

13. $2(3^3)$

14. $2 + 3^3$

Problems 15–16: Determine the value of each expression.

15. $20\left(\frac{1}{2}\right)^3$

16. $\frac{1}{2}(20)^3$

Name: _____ Date: _____ Period: _____

Comparing Models | Modeling with Functions | Parallels Between Numbers and Functions

A–SSE.1.a, F–BF.1.a, F–BF.2, F–LE.1.b, F–LE.2, SMP.2, SMP.7, SMP.8

Practice Day

Let's practice what you have learned so far in this unit!

You will use task cards for this Practice Day. Record all of your responses here.

Task A: Visual Patterns

1. Figure 4:

2. Figure 10:

3. Expression:

4.

Figure 1	Figure 2	Figure 3

5. Circle one: Pattern A Pattern B

Explanation:

Task B: Sequences

1. _____, _____, _____, _____, _____, . . .

2. _____, _____, 20, _____, 30, . . .

3. _____, 12, 36, _____, _____, . . .

4. _____, _____, _____, _____, _____, . . .

5. First term:

6. First term:

	Sequence	Recursive Definition	Explicit Expression
7.	30, _____, _____, _____, . . .	First term: 30 Rule: Constant ratio of 1.5	
8.	_____, _____, _____, _____, . . .	First term: 64 Rule:	$64 \cdot \left(\frac{1}{2}\right)^{n-1}$
9.	40, 35, 30, 25, . . .	First term: Rule:	

Name: ..... Date: ..... Period: .....

Practice Day (continued)

Task C: Constant Ratio, Constant Difference, or Neither?

1. Representation A Circle one: Constant ratio Constant difference Neither

2. Representation B Circle one: Constant ratio Constant difference Neither

3. Representation C Circle one: Constant ratio Constant difference Neither

4. Representation D Circle one: Constant ratio Constant difference Neither

5. Representation E Circle one: Constant ratio Constant difference Neither

6. Representation:

Expression:

Variable:

7.

8. Representation:

Expression:

Variable:

9.

Task D: Lily Pads

1. Circle one: Constant ratio Constant difference

Explanation:

2. ..... lily pads

3. Expression:

4. Explanation:

5. Circle one: The pond The lake

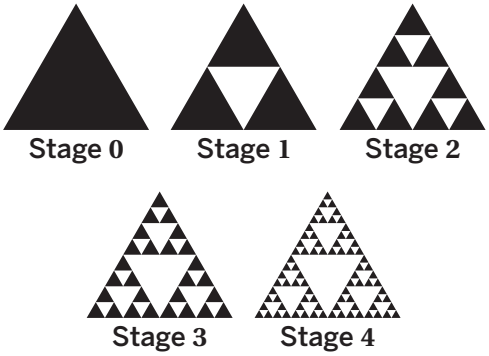
Explanation:

Notes:

Career Connection

Did you know patterns and sequences are found in major motion pictures?

Filmmakers who use CGI (computer-generated imagery) to create visual effects actually use complex math patterns. Fractals, such as the Sierpiński triangle shown, are patterns that show self-similarity. When applied to CGI, fractals can be used to create realistic landscapes in movies.



CGI Animators use computer-generated technology to create these visual effects for movies, T.V., and even video games. Their work involves computer programs and mathematical algorithms.

"Wacław Sierpiński"
Courtesy National Digital Archive, Poland.

Meet Wacław Sierpiński

Polish mathematician Wacław Sierpiński (1882–1969) is perhaps best known for three fractals that bear his name: the Sierpiński triangle, the Sierpiński carpet, and the Sierpiński curve. While the triangle is named after Wacław Sierpiński who described many of its properties, it appeared in decorative mosaics many centuries earlier. Now that you know what a Sierpiński triangle looks like, research what a Sierpiński carpet and Sierpiński curve look like!

Are you interested in studying fractals and their applications to filmmaking? What can you do to learn more?

Math in the World

The Fibonacci sequence is a famous sequence whose pattern can be found in seashells, flower petals, sunflowers, pine cones, and more!

stasB/Shutterstock.com

The first eight terms in the sequence are $0, 1, 1, 2, 3, 5, 8, 13, \ldots$ What are the next 5 terms?

Math Mindset

Create your own sequence of numbers or figures with a specific pattern. Share your sequence with a partner and ask them to determine or draw the next term.

Unit 2

Linear Equations and Inequalities

Big Ideas in This Unit

CC2 Comparing Models | Modeling with Functions | Systems of Equations

NS Financial Literacy

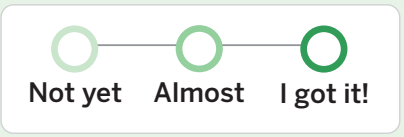
Questions for Investigation

- How do you determine, graph, and interpret the meaning of the solutions to an equation or an inequality?
- How can the different representations of linear equations or inequalities be used to model linear situations?

Explore: Planning for Homecoming
How can different conditions affect preparation for homecoming?

Watch Your Knowledge Grow

This is the math you'll explore in this unit. Rate your understanding to see how your knowledge grows!



I can . . .	Before	After
Use inverse operations to solve linear equations in one variable.		
Explain each step in solving a simple equation.		
Justify whether an equation has one solution, no solution, or infinitely many solutions.		
Create equations and use them to solve problems.		
Interpret solutions of one- and two- variable equations that represent a real-world situation.		
Rearrange equations to highlight a specific variable.		
Write the equation of a line in point-slope form.		
Interpret the properties of a line when given its equation in point-slope form.		
Rewrite an equation in standard and slope-intercept form.		
Graph linear equations on coordinate axes.		

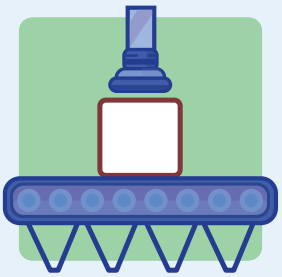
I can . . .	Before	After
Create inequalities and use them to solve problems.		
Represent constraints using equations or inequalities.		
Solve one-variable linear inequalities.		
Graph solutions to one-variable linear inequalities.		
Solve absolute value equations and inequalities.		
Graph solutions to absolute value equations.		
Interpret solutions to absolute value equations in context.		
Determine solutions to a two-variable inequality by finding points that make the inequality true.		
Graph the solution region to a two-variable linear inequality.		

Sub-Unit

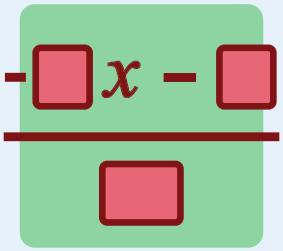
1

One-Variable Equations

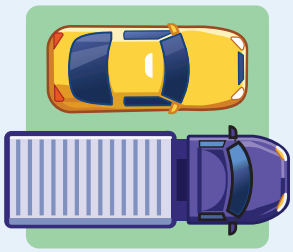
Explore
Planning for Homecoming



Lesson 1
Working Backwards



Lesson 2
Solving Strategies



Lesson 3
Same Position

Name: ..... Date: ..... Period: .....

Financial Literacy N–Q.1, SMP.1, SMP.2

Explore: Planning for Homecoming

How can different conditions affect preparation for homecoming?

Warm-Up

Let's think about big events at your school.

1. What school event are you most looking forward to this year? Why?

2. Do you want to go to a school dance? If so, what specifically are you looking forward to? If not, what would you like to do instead?

3. In many schools in the U.S., the homecoming dance is a major event. If you were to attend, what would you want the experience to be like?

4. What are some things that need to be considered when planning a major event like a homecoming dance?

Name: ..... Date: ..... Period: .....

Planning the Dance

The homecoming committee at Desmond High School is beginning to plan this year's homecoming dance. The committee has a budget of $4500 and can spend the money on five total items, each from a different category: venue, music, food, decorations, and photography. The committee gives each item "preference points" based on what they like. The committee would like to maximize their preference points while staying within the $4500 budget.

You will use a sheet and a set of cards with items to choose from for the committee to complete Problem 5. To help with planning, the committee's teacher advisor created the following table with five items that show their cost and preference points.

Category	Item/experience	Cost ($)	Preference points
Venue	Event center	1200	25
Music	DJ	1100	50
Food	Appetizers/snacks	1100	50
Decorations	Lights	450	25
Photography	Selfie station	400	25
Total		4250	175

5. Make choices for the committee that stay within the $4500 budget, but align more with the committee's preferences than the teacher advisor's choices.

Category	Item/experience	Cost ($)	Preference points
Venue			
Music			
Food			
Decorations			
Photography			
Total			

Name: ..... Date: ..... Period: .....

Planning the Dance (continued)

The PTA, Parent-Teacher Association, of Desmond High School has offered to contribute an additional $2000 to the budget for the homecoming dance. That brings the total budget to $6500. The homecoming committee also wants the student council to help with planning the dance. The two groups decide that they will spend the $6500 budget together. The two groups will need to agree on the same venue and music option. They will only need to pay for each of these items one time. The items and experiences for the remaining categories are paid for separately unless both groups agree on the same option.

You will use a sheet and a set of cards with each group's options, including how much they cost and their number of preference points, to complete Problem 6.

6. Determine a combination of options that results in at least 750 total preference points, while making sure each group has at least 325 preference points (so that both groups are happy with the final decisions).

	Homecoming committee	Student council		
Category	**Item/experience**	**Item/experience**	**Combined cost ($)**	**Combined preference points**
Venue				
Music				
Food				
Decorations				
Photography				
Total				

7. Explain your method for choosing options for both groups.

Name: ..... Date: ..... Period: .....

Building Math Habits of Mind

Discuss:

- Which of these habits of mind did you strengthen during this activity?
- How did you use the one(s) you selected?

I can slow down and first make sense of a challenging problem before trying to solve it.

Not yet — Almost — I got it!

I can represent real-world problems and interpret their solutions within the context of the problem.

Not yet — Almost — I got it!

I can justify my thinking and ask questions to help me understand the thinking of others.

Not yet — Almost — I got it!

I can apply the math that I know to solve real-world problems, make assumptions, and revise my thinking as needed.

Not yet — Almost — I got it!

I can select an appropriate tool to help me solve problems.

Not yet — Almost — I got it!

I can communicate my thinking and solutions clearly to others.

Not yet — Almost — I got it!

I can look for structure or patterns to help me solve problems.

Not yet — Almost — I got it!

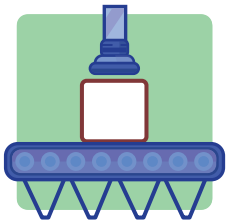
I can look for repeated calculations and other repeated steps to make generalizations.

Not yet — Almost — I got it!

Name: ..... Date: ..... Period: .....

Systems of Equations A–REI.3, SMP.1, SMP.2, SMP.7

Working Backwards

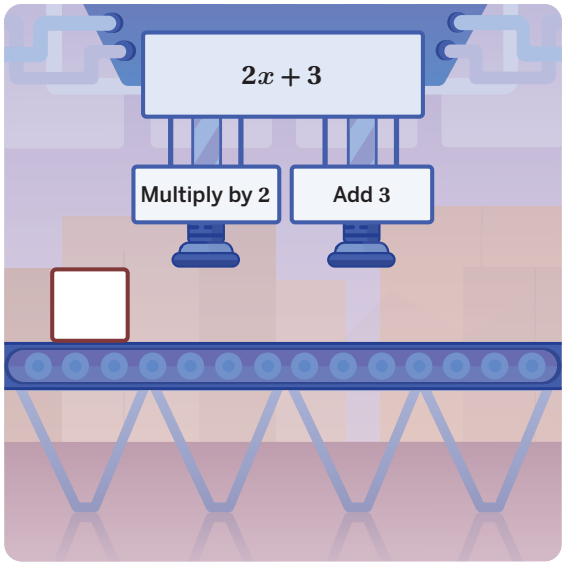


Let's solve equations by working backwards.

Warm-Up

1. Let's look at a number machine.

Complete the table for different values of x.



x	$2x$	$2x + 3$
5	10	13

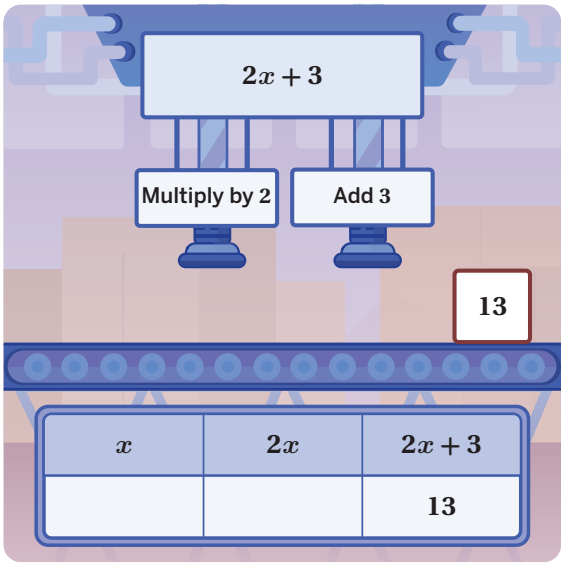
Name: ..... Date: ..... Period: .....

Number Machines

2. Alina put a number into this machine, and 13 came out.

What number did Alina put in?

Use the table if it helps with your thinking.



x	$2x$	$2x + 3$
		13

3. Alina's situation is represented by the equation $2x + 3 = 13$.

A **solution** to an equation is a value that makes the equation true.

Explain how you know that your answer from the previous problem is a solution to $2x + 3 = 13$.

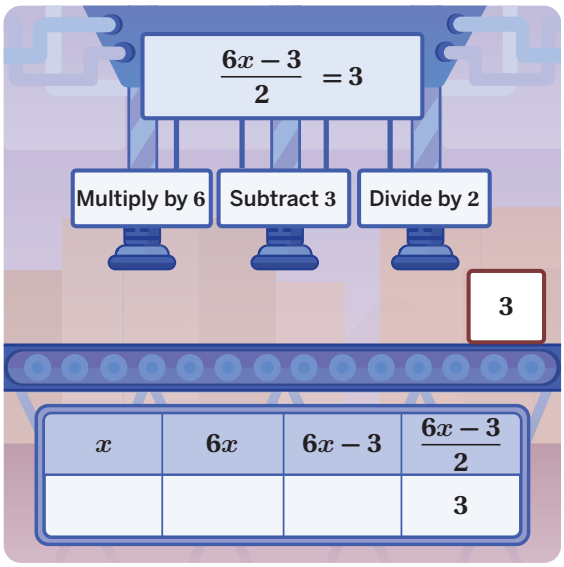
Name: ..... Date: ..... Period: .....

Number Machines (continued)

4. Here is a machine for the equation $\frac{6x-3}{2} = 3$.

Determine the solution to the equation.

Use the table if it helps with your thinking.



x	$6x$	$6x-3$	$\frac{6x-3}{2}$
			3

5. Let's look at Jin's and Nasir's strategies for determining what number has to go into the machine for 3 to come out.

Discuss: How are their strategies alike and how are they different?

Name: ..... Date: ..... Period: .....

Solve It

6. Dakota says the solution to $2(3x - 9) = -6$ is $x = \frac{1}{2}$.
How can you convince Dakota that this is incorrect?

7. Solve $2(3x - 9) = -6$.

8. For each of the challenges:

- Decide with your partner who will complete Column A and who will complete Column B.
- Solve as many equations as you have time for.
- The solutions in each row should be the same. Compare your solutions, then discuss and resolve any differences.

Column A	Column B
$x + 3 = 7$	$x - 3 = 1$
$12 = 2x - 4$	$6 = 3x - 18$
$-4(x + 2) = 20$	$5(x + 15) = 40$
$\frac{2}{3}x - 2 = 5$	$2x - 6 = 15$
$-5x + 5 = -8x + 11$	$5x + 3 = 2x + 9$

Synthesis

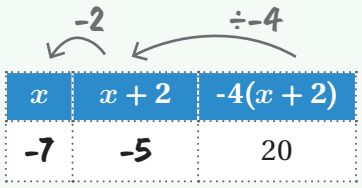
What are some first steps you could take to determine the solution to $10 - 6 = -2(x + 4)$?

Summary 2.01

Solving an equation means taking steps to determine a **solution**.

There are many ways to solve an equation, including working backwards, inverse operations, and moves that keep the equation balanced.

Here are two strategies for solving the equation $-4(x + 2) = 20$:



$$\frac{-4(x+2)}{-4} = \frac{20}{-4}$$

$$x + 2 = -5$$

$$-2 \quad -2$$

$$x = -7$$

You can check that the value you determined is a solution to an equation by substituting the value back into the equation to see if it makes the equation true. The solution the equation $-4(x + 2) = 20$ is $x = -7$ because $-4(-7 + 2) = 20$ is a true statement.

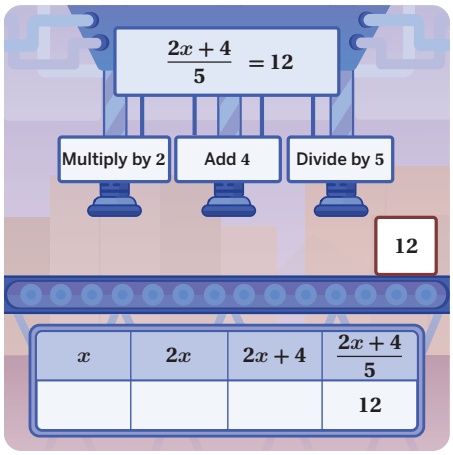
solution A value that makes an equation true.

Practice
2.01

Name: ..... Date: ..... Period: .....

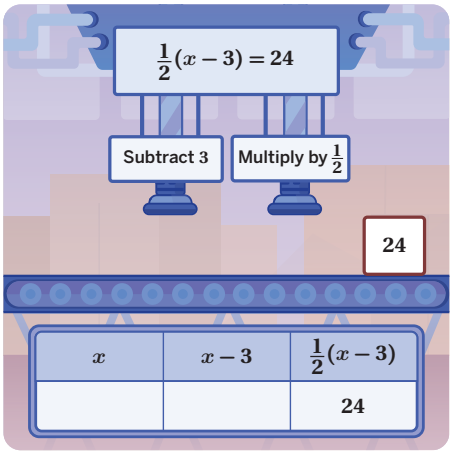
Problems 1–2: Solve each equation for x. Use the tables if they help with your thinking.

1. Here is a machine for the equation $\frac{2x+4}{5} = 12$.



x	$2x$	$2x+4$	$\frac{2x+4}{5}$
			12

2. Here is a machine for the equation $\frac{1}{2}(x-3) = 24$.



x	$x-3$	$\frac{1}{2}(x-3)$
		24

Problems 3–4: Solve each equation for x.

3. $5(3x-2) = -55$

4. $\frac{6x-2}{2} = x+9$

Problems 5–6: Zwena made a mistake when solving $3(x-4) = 5x$ for x.

5. Show or explain her mistake.

Zwena

Step 1: $3(x-4) = 5x$

Step 2: $3x - 12 = 5x$

Step 3: $8x - 12 = 0$

Step 4: $8x = 12$

Step 5: $x = 1.5$

6. What is the correct solution to this equation?

Name: ..... Date: ..... Period: .....

Problems 7–10: Solve each equation for x.

7. $3x + 19 = 40$

8. $5 = x + 1 + 3x$

9. $4x + 18 = 6x$

10. $3x + 5 = 4x + 1$

11. **Test Practice** Determine the value of x that makes the given equation true: $2(5x + 6) = 17$.

Spiral Review

12. Select *all* the equations where $x = 0.5$ is a solution.

- ☐ **A.** $8 = 4x$
- ☐ **B.** $13x = 6.5$
- ☐ **C.** $3x + 8 = 9.5$
- ☐ **D.** $3 = 2x + 1$
- ☐ **E.** $5 - x = 5.5$

13. Here is a table representing a pattern. Choose the equation that represents the table.

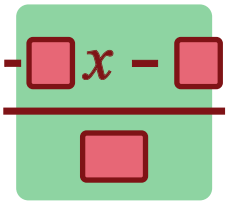
x	y
0	48
1	24
2	12
3	6

- **A.** $y = 48 - \frac{1}{2}x$
- **B.** $y = 48 + \frac{1}{2}x$
- **C.** $y = 48 \cdot 2^x$
- **D.** $y = 48 \cdot \left(\frac{1}{2}\right)^x$

Name: ..... Date: ..... Period: .....

Systems of Equations A–REI.1, A–REI.3, SMP.3, SMP.6, SMP.7

Solving Strategies



Let's explore strategies for solving equations.

Warm-Up

Equivalent equations have the exact same solution.

1. **Discuss:** Which of these equations are equivalent to each other? How do you know?

Equation A	Equation B
$-2(2x + 3) = 10$	$2x + 3 = -5$
Equation C	**Equation D**
$2x + 3 = 12$	$-4x - 6 = 10$

Activity 1

Name: ..... Date: ..... Period: .....

Equivalent or Not?

For each pair of equations, determine whether they are equivalent.

- If they're equivalent, explain how to get from one equation to the other.
- If they're not equivalent, explain how you know.

2. $5x = 24 + 2x$

$3x = 24$

Are the equations equivalent? .....

Explanation:

3. $-3(2x + 9) = 12$

$-4 = 2x + 9$

Are the equations equivalent? .....

Explanation:

4. $\frac{1}{2}x - 8 = 9$

$x - 8 = 18$

Are the equations equivalent? .....

Explanation:

5. **a** Write one equation that is equivalent to $12 = 5x - 30 + x$ and one equation that is not.

.....

.....

b Trade papers with a classmate. Then circle the equation they wrote that is equivalent to $12 = 5x - 30 + x$. Show or explain your thinking on their worksheet.

Activity 2

Name: Date: Period:

Step It Up

Problems 6–7: Here are Sadia's and Amir's steps for correctly solving the same equation.

Sadia

$$6 - 7x = \frac{-15x - 12}{3}$$
$$6 - 7x = -5x - 4$$
$$10 - 7x = -5x$$
$$10 = 2x$$
$$5 = x$$

Amir

$$6 - 7x = \frac{-15x - 12}{3}$$
$$18 - 21x = -15x - 12$$
$$18 - 6x = -12$$
$$-6x = -30$$
$$x = 5$$

6. **Discuss:** How did each student solve the equation?

7. How did Amir and Sadia take different first steps but have the same solution?

8. Caleb and Roberto also tried to solve the equation but made some errors. For each student's work:

a **Discuss:** What is correct? What is incorrect?

b Write a question to help each student see how they could revise their work.

Caleb

$$6 - 7x = \frac{-15x - 12}{3}$$
$$-1x = -5x - 4$$
$$4x = -4$$
$$x = -1$$

Roberto

$$6 - 7x = \frac{-15x - 12}{3}$$
$$6 - 7x = -5x - 4$$
$$2 - 7x = -5x$$
$$2 = 2x$$
$$1 = x$$

Activity 3

Name: ..... Date: ..... Period: .....

The Choice Is Yours

Equation A

$2(2a + 1.5) = 17 - 3a$

Equation B

$-\frac{1}{2}(b + 3) - 5 = -\frac{7}{2}$

Equation C

$\frac{-6 + 4c}{2} = 3(2c + 1)$

Equation D

$4d + 1 = -2(d - 5)$

Equation E

$\frac{x}{4} - 2 = 2x + 5$

Equation F

$9f + 3 - (f - 1) = 2(3f + 1)$

Equation G

$g - 4 = -\frac{8 + 4g}{8}$

Equation H

$h + 5h + 20 = h - 6 + h$

9. Examine these equations. Organize the equations into two or three groups based on patterns you notice.

Group A	Group B	Group C

10. **Discuss:** How did you group the equations?

11. Choose *three* equations to solve. (Choose at least one equation from each group.) Show your thinking.

You're invited to explore more.

12. Two of the equations in this activity are equivalent. Identify the two equivalent equations and explain your thinking.

Synthesis

13. **a** Write an equation that you would consider challenging to solve.

b **Discuss:** What makes your equation challenging to solve?

c **Discuss:** What are some strategies or tips for solving equations like this?

Summary 2.02

You can solve one-variable equations by creating **equivalent equations**. To create equivalent equations, use solving moves that keep the equation balanced, such as combining like terms or using inverse operations to move a variable from one side of the equation to the other.

Here is an example of a set of solving moves that keep an equation balanced:

$-3m + 5 + m = 2(6m + 3)$	This is the original equation.
$-2m + 5 = 12m + 6$	We combined like terms on the left and distributed on the right.
$5 = 14m + 6$	We added $2m$ to each side of the equation.
$-1 = 14m$	We subtracted 6 from each side of the equation.
$\frac{-1}{14} = m$	We divided each side of the equation by 14.

All of the equations created at each step of this solution process are *equivalent equations* and they have the same solution $m = \frac{-1}{14}$.

equivalent equations Equations that have the exact same solution(s).

Practice
2.02

Name: ..... Date: ..... Period: .....

1. Which equation is equivalent to $6x + 9 = 12$?

A. $x + 9 = 6$

B. $2x + 3 = 4$

C. $3x + 9 = 6$

D. $6x + 12 = 9$

2. Write another equation that is equivalent to $6x + 9 = 12$.

3. Select *all* the equations that are equivalent to $\frac{-8x - 6}{2} = 15$.

☐ **A.** $4x + 3 = 15$

☐ **B.** $\frac{1}{2}(-4x - 3) = 15$

☐ **C.** $-4x - 3 = 15$

☐ **D.** $-8x - 6 = 30$

☐ **E.** $8x + 6 = 30$

Problems 4–6: Solve each equation.

4. $26 - 2x = 3(x + 2)$

5. $\frac{4x - 6}{2} = x - 8$

6. $\frac{1}{4}x - 5 = x - 14$

Problems 7–8: Polina made a mistake when solving $-3(x + 7) = 24$ for x.

Polina

Step 1: $-3(x + 7) = 24$

Step 2: $x + 7 = 27$

Step 3: $x = 20$

7. What is one thing that Polina did well?

8. What is one thing that she did incorrectly?

Name: ..... Date: ..... Period: .....

9. **Test Practice** Which equation is equivalent to $0.05n + 0.1d = 3.65$?

A. $5n + d = 365$

B. $0.5n + d = 365$

C. $5n + 10d = 365$

D. $0.05d + 0.1n = 365$

10. Select *all* the moves that could be the first step to solving the equation $4(x + 3) = 8x - 4 + 12x$.

- ☐ **A.** Divide each side by 4.
- ☐ **B.** Take away $12x$ on the right side.
- ☐ **C.** Distribute 4 on the left side.
- ☐ **D.** Combine like terms on the right side.
- ☐ **E.** Add 4 to the right side.

11. Determine a value for x that makes this equation true: $9x - 4(x - 3) = 27 + 2x$. Show or explain your thinking.

Spiral Review

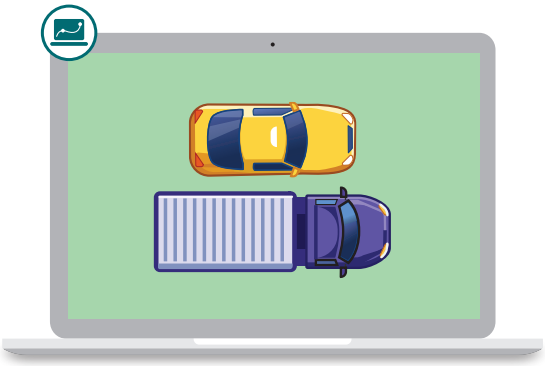
12. Renata scored 409 points in a video game. This was 223 more points than Sadia scored, s. Which equation does *not* represent this situation?

A. $223 = 409 - s$ **B.** $s = 409 - 223$ **C.** $s = 409 + 223$ **D.** $223 + s = 409$

13. What is the side length of a square that has an area of 64 square centimeters?

Name: ..... Date: ..... Period: .....

Systems of Equations A–CED.1, A–REI.3, SMP.1, SMP.3, SMP.7



Same Position

Let's explore how many solutions are possible for a one-variable equation.

Warm-Up

1 Let's watch the animation.

Write a story about what you see.

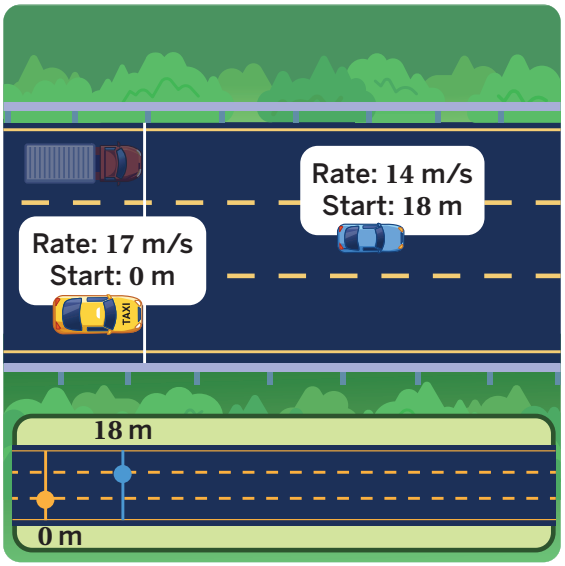
Activity 1

Name: ..... Date: ..... Period: .....

Same Position

2 The car and the taxi are moving at constant speeds.

Car Position Expression	Taxi Position Expression
$14t + 18$	$17t$

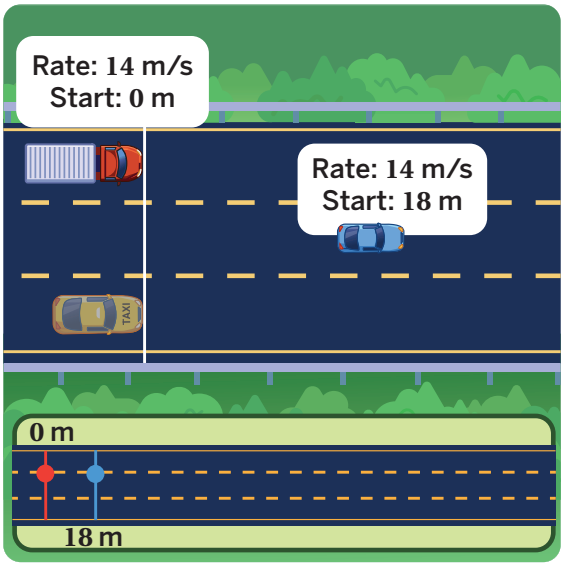


a Write an equation to represent when the car and the taxi will be in the same position.

b At what time, t, will the car and the taxi be in the same position?

3 The truck and the car are moving at constant speeds.

Truck Position Expression	Car Position Expression
$14t$	$14t + 18$



a Let's see what happens at different times.

b **Discuss:** What do you know about when the truck and the car will be in the same position?

4 Here is Antwon's work on the previous problem.

What does his work say about the time, t, when the truck and the car will be in the same position?

Antwon

$$14t = 14t + 18$$
$$-14t \quad -14t$$
$$0 = 0 + 18$$

Name: ..... Date: ..... Period: .....

Same Position (continued)

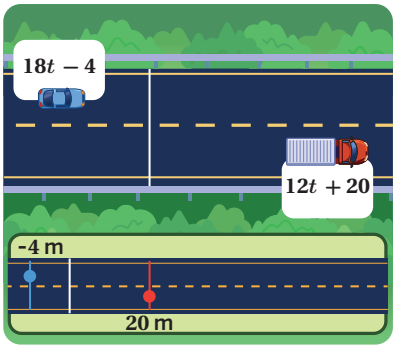
5 The following equations represent when these vehicles will be in the same position.

a $18t - 4 = 12t + 20$

How often will they be in the same position? Circle one.

Once Never Always

If once, then after how many seconds? .....

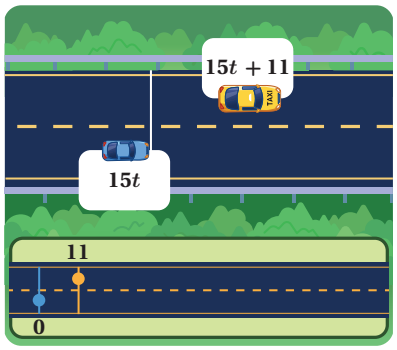


b $15t + 11 = 15t$

How often will they be in the same position? Circle one.

Once Never Always

If once, then after how many seconds? .....

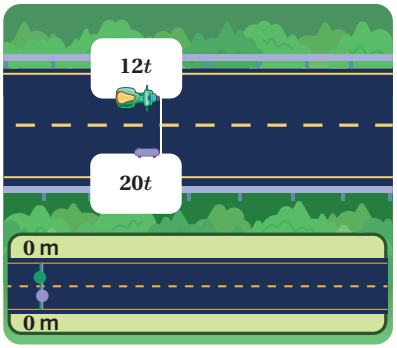


c $12t = 20t$

How often will they be in the same position? Circle one.

Once Never Always

If once, then after how many seconds? .....

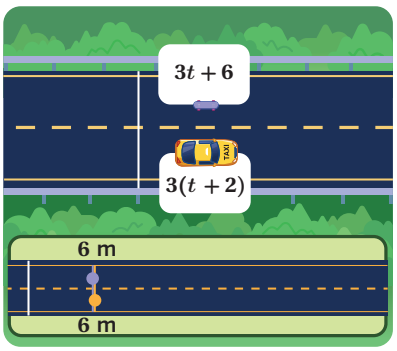


d $3t + 6 = 3(t + 2)$

How often will they be in the same position? Circle one.

Once Never Always

If once, then after how many seconds? .....



Activity 2

Name: Date: Period:

Once, Never, Always

6 Here are Ava's and Nikhil's strategies for solving a challenge from the previous activity.

Ava

$12t = 20t$

$-12t \quad -12t$

$0 = 8t$

$0 = t$

they will meet when $0 = t$

Nikhil

$\frac{12t}{t} = \frac{20t}{t}$

$12 = 20$

they will never meet

Discuss: Is each strategy correct?

7 Each equation represents the time, t, when two vehicles will meet.

$12 - t = t - 12$	$t + 1 = t + 1$	$t = t + 2$
$2t + 6 = 2(t + 3)$	$2t = 8t$	$8(t + 1) = 8t - 8$

Sort the six equations based on how often the vehicles will be in the same position.

Once	Never	Always

Name: ..... Date: ..... Period: .....

Once, Never, Always (continued)

8 Darryl and Jasmine solved $t + 1 = t + 1$ and got $0 = 0$.

- Darryl says the vehicles will never be in the same position.
- Jasmine says the vehicles will always be in the same position.

Who is correct? Explain your thinking.

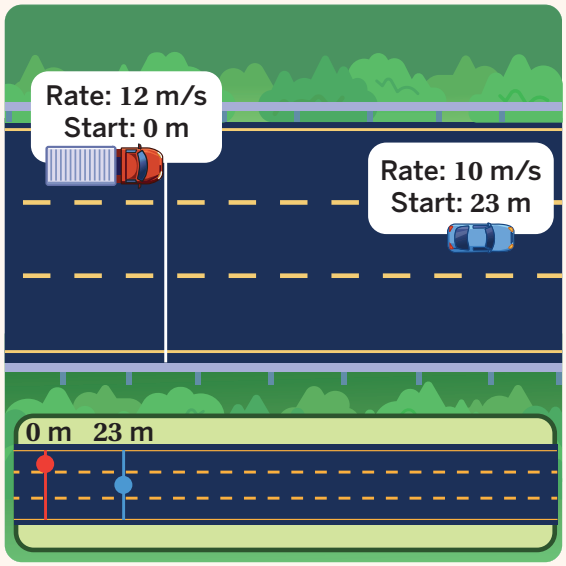
You're invited to explore more.

9 The truck and the car are moving at constant speeds.

Write an expression, in terms of t, for the position of a taxi that makes *both* of these statements true:

- The taxi will never be in the same position as the truck.
- The taxi will be in the same position as the car when $t = 8$.

Vehicle	Position Expression
Truck	$12t$
Car	$10t + 23$
Taxi	

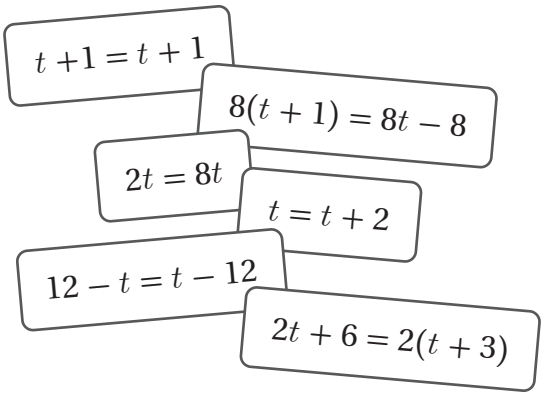


10 Synthesis

How can you tell whether an equation will have:

- No solution?

- Infinitely many solutions?



13 Summary 2.03

Not all one-variable linear equations have a single solution. Some linear equations have **infinitely many solutions**, and some have **no solution**.

Here are examples of equations with one solution, no solution, and infinitely many solutions.

One Solution	No Solution	Infinitely Many Solutions
$3x + 4 = 2x + 10$	$2x + 4 = 2x + 10$	$2(x + 5) = 2x + 10$
$3x = 2x + 6$	$4 = 10$	$2x + 10 = 2x + 10$
$x = 6$	This is *never* true!	$10 = 10$
		This is *always* true!

If the variable in an equation is eliminated during the solving process, that tells you that the equation has either no solution or infinitely many solutions. If the statement remaining is false, the equation has no solution. If the statement remaining is true, the equation has infinitely many solutions.

infinitely many solutions An equation that is true for any value of the variable.

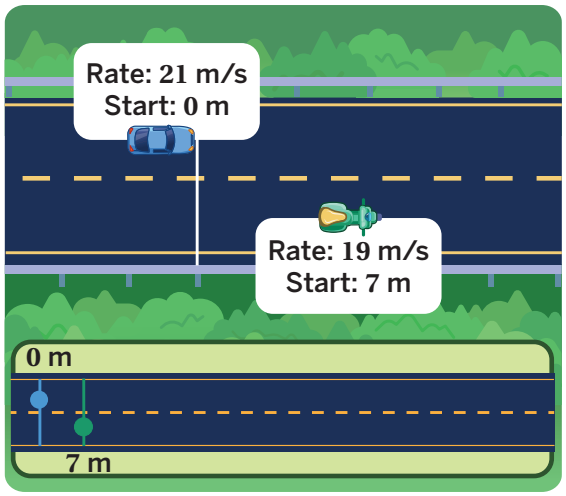
no solution An equation where no value of the variable will make the equation true.

Practice
2.03

Name: ..... Date: ..... Period: .....

1. The car and scooter are moving at constant speeds. The equation $21t = 19t + 7$ represents the time, t, when they will be in the same position.

When will the car and scooter be in the same position?



Problems 2–3: The equation $10t = 2.5t$ represents the time, t, when two vehicles will be in the same position.

2. When will these two vehicles be in the same position? Circle one.

Once Never Always

3. Explain how you know.

4. Here is Kiandra's work to solve $16x = 10x$. She says there is no solution.

Is this correct? Show or explain your thinking.

Kiandra

$\frac{16x}{x} = \frac{10x}{x}$

16 = 10

There is no solution.

5. Group the equations based on their number of solutions.

A. $5t = 3t$ **B.** $2t = 10 - 2t$ **C.** $15 - 3(t + 5) = -3t$

D. $4t + 7 = 4(t + 2)$ **E.** $6t + 2 = -3 + 6t$

One Solution	No Solution	Infinitely Many Solutions

Name: ..... Date: ..... Period: .....

6. Which equation has infinitely many solutions?

A. $3x + 9 - 3 = 4x - x$

B. $3x + 9 - 3 = 5x - x + 6$

C. $3x + 9 - 3 = 4x - x + 6$

D. $3x + 9 - 3 = 4x - x + 7$

Problems 7 and 8: Create two different equations that each have a solution of $x = 1$.

7. Fill in each blank using the digits 0 to 9 only once each.

$\square x + \square = \square x + \square$

$\square x + \square = \square x + \square$

8. Explain what you notice about your equations.

Spiral Review

9. Select *all* the equations where $x = 2$ is a solution.

☐ **A.** $\frac{x}{4} = 8$

☐ **B.** $19 = 2(x + 6) + 3$

☐ **C.** $2x + 10 = 2x + 8$

☐ **D.** $5 - 3x = -1$

☐ **E.** $4 - x = x$

10. Select *all* the expressions that are equivalent to $2(x + 3)$.

☐ **A.** $(x + 3) \cdot 2$

☐ **B.** $2x + 5$

☐ **C.** $2x + 3 \cdot 2$

☐ **D.** $2x + 3$

☐ **E.** $2x + 6$

11. **Test Practice** Select the expression that is equivalent to $6 - 2(x + 1)$.

A. $4(x + 1)$

B. $7 - 2x$

C. $4 - 2x$

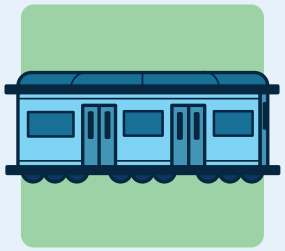
D. $4x + 4$

Notes:

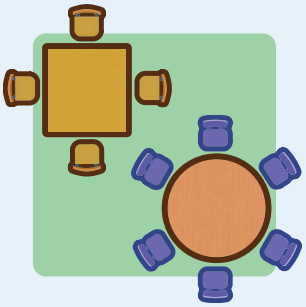
Sub-Unit

2

Multi-Variable Equations

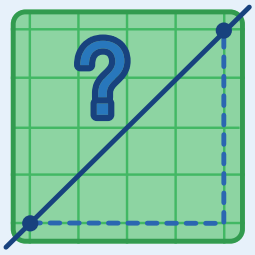


Lesson 4
Subway Seats

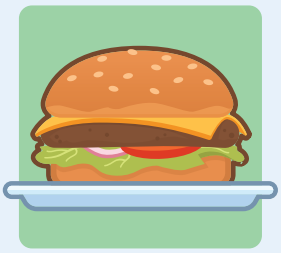


Lesson 5
Various Variables

Lesson 6
Shelley the Snail



Lesson 7
Equations of Lines



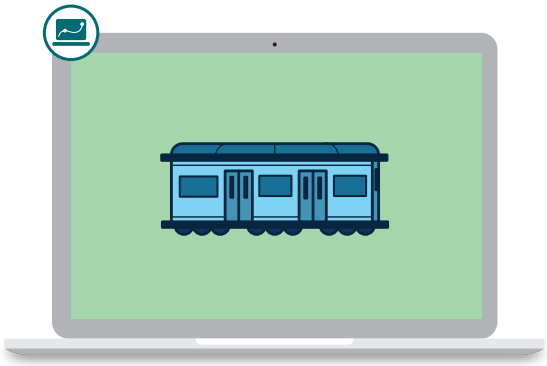
Lesson 8
Five Representations

Name: ..... Date: ..... Period: .....

Modeling with Functions A–CED.3, A–CED.4, SMP.2, SMP.6, SMP.8

Subway Seats

Let's explore what different forms of linear equations reveal about a situation.



Warm-Up

1 Which one doesn't belong? Explain your thinking.

Equation A	Equation B	Equation C	Equation D
$x + y = 5$	$x + y - 5 = 0$	$x = 5 - y$	$5 + x = y$

Name: ..... Date: ..... Period: .....

Crowded Subways

2 Some subway cars can be crowded.

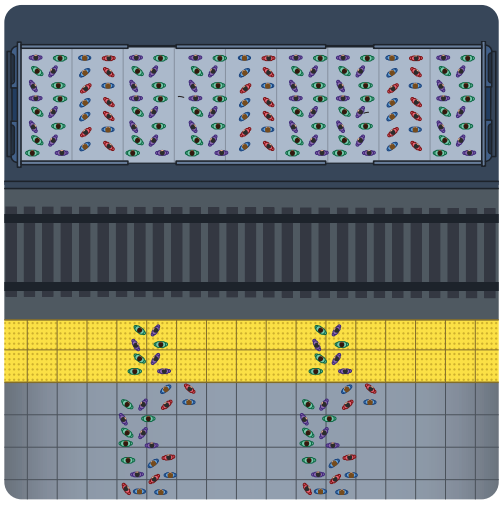
In order to fix this, the transit authority decided to remove seats to fit more people.

Some residents feel that this isn't fair. Why might they feel this way?

3 Some subways removed all of the seats to allow more room for people to stand.

- A subway car has about 600 square feet.
- A standing passenger requires 2 square feet.

What is the *standing capacity* on this subway car with no seats?



4

- A subway car has about 600 square feet.
- A seat requires 6 square feet.

What is the *seating capacity* on this subway car with no room to stand?

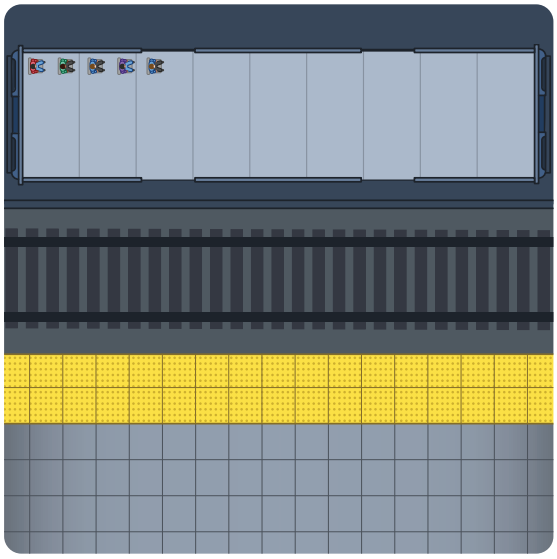
Name: ..... Date: ..... Period: .....

Crowded Subways (continued)

5 The capacity of this subway is modeled by $6t + 2d = 600$, where t is the seating capacity and d is the standing capacity.

For each number of seats, determine how many standing passengers can fit.

Seating Capacity, t	Standing Capacity, d
5	
10	
15	



6 Here is Tiam's strategy for determining the number of standing passengers that can fit when you know the number of seats.

- t is the seating capacity.
- d is the standing capacity.

What do 300 and -3 mean in this situation?

Tiam

$6t + 2d = 600$

$-6t \quad -6t$

$\frac{2d}{2} = \frac{600 - 6t}{2}$

$d = 300 - 3t$

300:

-3:

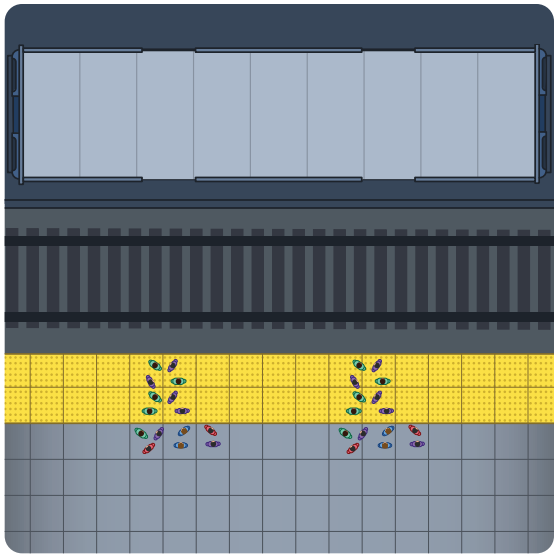
Name: ..... Date: ..... Period: .....

Standing and Sitting

7 The capacity of this subway is modeled by $6t + 2d = 600$, where t is the seating capacity and d is the standing capacity.

For each number of standing passengers, determine the seating capacity.

Seating Capacity, t	Standing Capacity, d
	30
	150
	240



8 Solve for t so that the transit authority can calculate the seating capacity for any standing capacity.

9 Group the equations that represent the same relationship. One equation will have no match.

$x = \frac{50 - 3y}{2}$ $x = \frac{50 - 2y}{3}$ $y = \frac{50 - 3x}{2}$ $y = \frac{50 - 2x}{2}$

$2x + 3y = 50$	$3x + 2y = 50$

10 Synthesis

Here are two equations that we considered in this lesson about subway capacity.

$$6t + 2d = 600$$
$$t = 100 - \frac{1}{3}d$$

- t is the seating capacity.
- d is the standing capacity.

Discuss: Pick *two* numbers and explain what they mean in this situation.

6:

2:

600:

100:

$-\frac{1}{3}$:

13 Summary 2.04

Two-variable linear equations can be represented in different forms. Sometimes the different forms of an equation can reveal information that is useful for solving problems. Depending on what information you are looking for, you might choose to use one form or the other.

Here is an example of two equivalent equations. They are each represented in different forms, and they each reveal different information about seating and standing capacity. In each equation, t is the seating capacity and d is the standing capacity.

$$4t + 2d = 300$$

- Each seated passenger requires 4 square feet.
- Each standing passenger requires 2 square feet.
- The total area of the subway car is 300 square feet.

$$d = 150 - 2t$$

- When there are no seats ($t = 0$), 150 passengers can stand in the car.
- For every seat that is added, the standing capacity decreases by 2 square feet.

Practice
2.04

Name: ..... Date: ..... Period: .....

Problems 1–4: Adriana spent $\$24$ on fruit punch and apple juice. Fruit punch costs $\$3$ per bottle. Apple juice costs $\$2$ per bottle.

1. How many bottles of *fruit punch* could Adriana buy if she didn't get any apple juice?

2. How many bottles of *apple juice* could Adriana buy if she did not get any fruit punch?

Adriana wrote the equation $3f + 2a = 24$ to represent the situation.

3. Use the equation to help you complete the table.

f	a
2	
6	

4. Which equation represents the same relationship?

A. $a = 8 - \frac{2}{3}f$

B. $a = 8 - \frac{3}{2}f$

C. $a = 12 - \frac{2}{3}f$

D. $a = 12 - \frac{3}{2}f$

Problems 5–7: Here is an equation: $2x + 4y = 80$.

5. Use the equation to help you complete the table.

x	y
6	
12	

6. Solve the equation for x.

7. Which equation represents the same relationship?

A. $y = 20 - 2x$

B. $y = 40 - 2x$

C. $y = 20 - \frac{1}{2}x$

D. $y = 40 - \frac{1}{2}x$

Problems 8–9: Nia is buying bananas and apples for her family. Bananas cost $\$0.50$ each. Apples cost $\$1.00$ each.

8. Select *all* the combinations of bananas and apples that Nia could buy for exactly $\$3.50$.

- ☐ **A.** 1 banana and 3 apples
- ☐ **B.** 5 bananas and 1 apple
- ☐ **C.** 1 banana and 2 apples
- ☐ **D.** 3 bananas and 2 apples
- ☐ **E.** 5 bananas and 2 apples

9. The equation $0.5b + 1a = 3.50$ represents the number of bananas and apples that Nia can buy for $\$3.50$. Solve this equation for a.

Practice
2.04

Name: ..... Date: ..... Period: .....

Spiral Review

10. Select *all* the expressions that are equivalent to $8 - 12 - (6 + 4)$.

- ☐ **A.** $(6 + 4) - 8 - 12$
- ☐ **B.** $8 - 6 - 12 + 4$
- ☐ **C.** $8 - 12 - 6 - 4$
- ☐ **D.** $8 - (6 + 4) - 12$
- ☐ **E.** $(8 - 12) - 6 + 4$

11. Which equation is equivalent to $\frac{1}{3}m + \frac{1}{2}n = 9$?

A. $3m + 2n = 9$

B. $2m + 3n = 54$

C. $m + 3n = 27$

D. $2m + n = 18$

12. **Test Practice** Explain how you know that equation A and equation B are equivalent.

Equation A	Equation B
$48 - 5x = 13$	$5x = 35$

13. Create two equivalent equations by filling in the blanks using the digits 0 to 9 only once.

$$\square x + \square y = \square$$

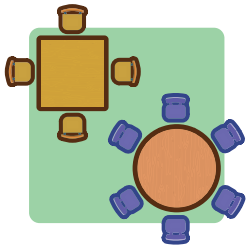
$$y = \square - \square x$$

Name: ..... Date: ..... Period: .....

Systems of Equations A–CED.4, A–REI.3, SMP.8

Various Variables

Let's rearrange equations with multiple variables.



Warm-Up

1. Which equations are equivalent? How do you know?

Equation A	Equation B	Equation C	Equation D
$g = 3 - f$	$3 - g = f$	$3 = f - g$	$f + g = 3$

Activity 1

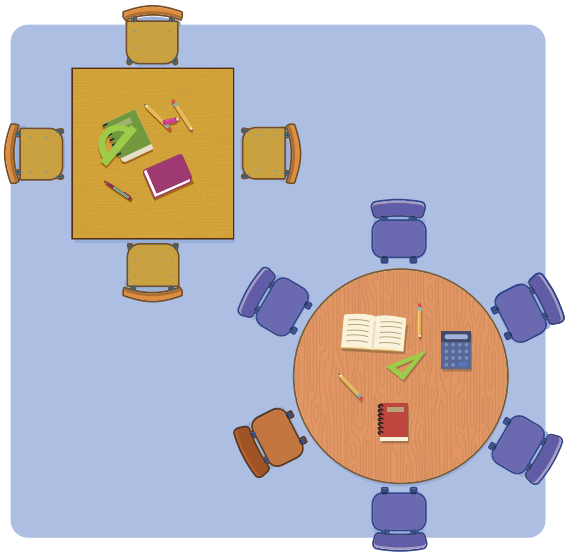
Name: ..... Date: ..... Period: .....

Sol's Strategies

Sol is planning a party.

Sol can use round tables and square tables.

- Round tables seat 6 people.
- Square tables seat 4 people.



2. Sol wrote $6r + 4s = 240$ to represent this situation. Explain what each number and variable in Sol's equation means.

3. How many square tables does Sol need if there are 16 round tables? Show or explain your thinking.

4. Sol solved the equation for s and got a new equation. How can Sol use this new equation?

Sol

$$6r + 4s = 240$$
$$-6r \qquad -6r$$
$$\frac{4s}{4} = \frac{240 - 6r}{4}$$
$$s = 60 - 1.5r$$

Name: Date: Period:

Sol's Strategies (continued)

5. Use Sol's new equation, $s = 60 - 1.5r$, to decide how many square tables are needed if 10 round tables are used. Show or explain your thinking.

6. How many round tables does Sol need if 18 square tables are used? Show or explain your thinking.

7. Solve Sol's original equation $6r + 4s = 240$ for r. Show or explain your thinking.

Activity 2

Name: ..... Date: ..... Period: .....

Equations and Formulas

Solve each equation on the left. Show or explain your thinking. Then use the same strategy to solve each equation on the right.

8. Solve for t. $12 = 3t$ $D = rt$

9. Solve for b. $20 = b \cdot 5$ $A = bh$

10. Solve for x. $2x + 5 = 12$ $mx + b = y$

11. **Discuss:** How is solving the equations on the left like solving the equations on the right?

Name: ..... Date: ..... Period: .....

Equations and Formulas (continued)

Solve each equation on the left. Show or explain your thinking. Then use the same strategy to solve each equation on the right.

12. Solve for x. $-5x - 2 = 13$ $-kx - p = w$

13. Solve for a. $\frac{a}{5} - 18 = 2$ $\frac{a}{g} - r = c$

14. Solve for h. $289 = \pi(2)^2 \cdot h$ $V = \pi r^2 \cdot h$

Synthesis

15. **Discuss:**

a How is solving an equation with many variables like solving an equation with one variable?

$2x + 5 = 12$

$mx + b = y$

b How is it different?

Summary 2.05

You can solve equations that contain multiple variables using some of the same strategies for solving equations with one variable. These strategies include working backwards, using inverse operations, and keeping the equation balanced.

When solving problems that model real-world situations, it can be helpful to rearrange an equation to highlight a variable of interest. Rearranging equations can also reveal different relationships between the variables and the quantities that they represent.

Here is how you could rearrange the equation $y = mx + b$ to solve for m or b.

Solving for m

$$y = mx + b$$
$$y - b = mx + b - b$$
$$\frac{y - b}{x} = \frac{mx}{x}$$
$$\frac{y - b}{x} = m$$

Solving for b

$$y = mx + b$$
$$y - mx = mx + b - mx$$
$$y - mx = b$$

Rearranging equations for a specific variable can help make some calculations easier. For example, if you know the values of r and s for $6r + 4s = 240$, solving the equation for s can make it simpler to test different values of r and s to see how they affect the value of s.

Practice
2.05

Name: ..... Date: ..... Period: .....

Problems 1–2: Solve each equation for y.

1. $6(2.5) - 4y = 11$

2. $6x - 4y = 11$

Problems 3–4: Here is an equation: $2x - 4y - 31 = 123$.

3. Solve for x.

4. Solve for y.

5. Abdel has \$12 to spend on beans and rice.

- Beans, b, cost \$5 per pound.
- Rice, r, costs \$2 per pound.

Abdel wrote $5b + 2r = 12$ to represent this relationship. Use the equation to complete the table.

Beans (lbs), b	Rice (lbs), r
2	
0.6	
b	

6. Solve for b.

$\frac{bh}{2} = A$

7. Solve for w.

$2\ell + 2w = P$

8. Solve for r.

$C = 2\pi r$

Name: ..... Date: ..... Period: .....

9. Here is an equation: $x + 3y = 6$.

Use the whole numbers 0 to 6 without repeating to create two pairs of x- and y-values that are solutions and one pair that is not.

	x	y
Solution		
Solution		
Not a Solution		

Spiral Review

10. Deven ran 27 miles last week, which was 3 times as far as Hailey ran. Select *all* the equations that represent the number of miles Hailey ran, h.

- ☐ **A.** $h = \frac{1}{3} \cdot 27$
- ☐ **B.** $\frac{1}{3} \cdot h = 27$
- ☐ **C.** $3 \cdot h = 27$
- ☐ **D.** $h = 3 \cdot 27$
- ☐ **E.** $h = 27 \div 3$

11. **Test Practice** Which equation has the same solution as $2(x - 5) - 6 = 0$?

- **A.** $2(3x + 8) = 0$
- **B.** $\frac{1}{2}(x + 1) + 1 = 0$
- **C.** $2x + 8 = 0$
- **D.** $3(x - 4) - 12 = 0$

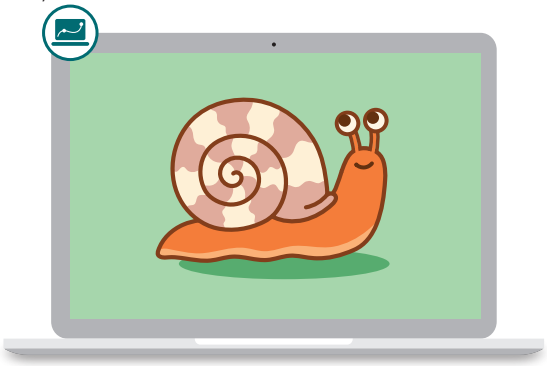
12. Match each equation to its number of solutions.

	One Solution	No Solutions	Infinitely Many Solutions
$3t = 11 + 3t$			
$12 = -6t + 3(2t + 4)$			
$2t = 7t$			

Name: ..... Date: ..... Period: .....

Modeling with Functions | Systems of Equations

A–CED.2, A–CED.3 A–CED.4, A–REI.10, N–Q.2, SMP.1, SMP.2



Shelley the Snail

Let's connect graphs, tables, and equations to the situations they represent.

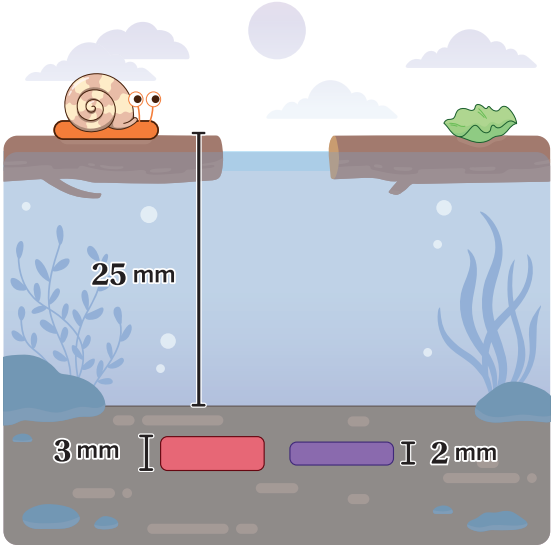
Warm-Up

1 Shelley the Snail loves eating lettuce leaves.

She needs to cross the gap to get them.

Write different combinations of blocks that fill the gap.

Number of 3 mm Blocks	Number of 2 mm Blocks

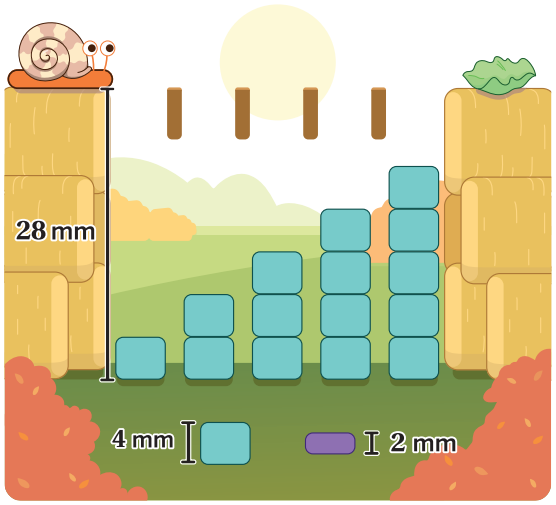


Activity 1

Name: ..... Date: ..... Period: .....

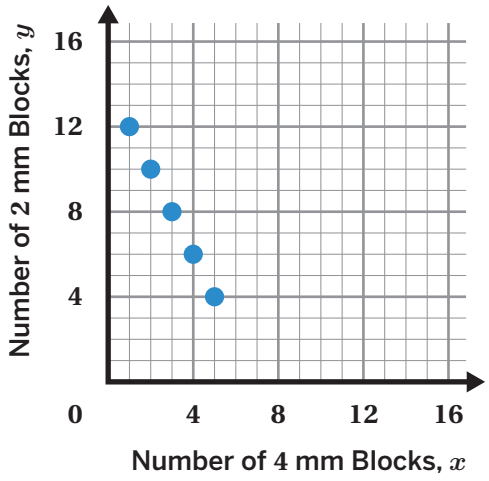
Mind the Gap

2 Complete the table to help Shelley get to the lettuce.



Number of 4 mm Blocks, x	Number of 2 mm Blocks, y
1	
2	
3	
4	
5	

3 Shelley has to cross a gap that is 28 mm deep using 4 mm and 2 mm blocks. This graph shows some of the possible combinations of blocks. Complete the table.



Number of 4 mm Blocks, x	Number of 2 mm Blocks, y
0	
6	
7	

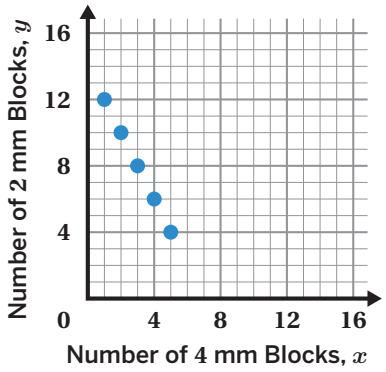
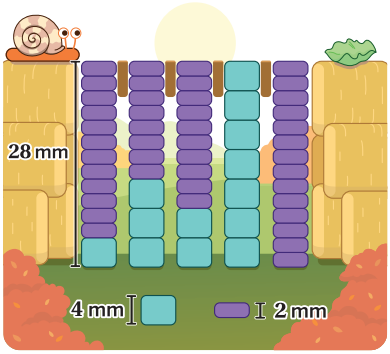
4 Here is Tyrone's work from the previous problem. Write an equation for the number of 2 mm blocks, y, needed for any number of 4 mm blocks, x.

Number of 4 mm Blocks, x	Number of 2 mm Blocks, y
0	14 − 2(0)
6	14 − 2(6)
7	14 − 2(7)
x	?

Name: ..... Date: ..... Period: .....

Mind the Gap (continued)

5 The x-intercept of the graph is $(7, 0)$. The y-intercept of the graph is $(0, 14)$.

Graph	Situation	Equation
		$y = 14 - 2x$

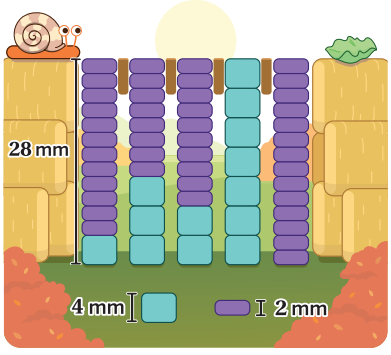
a Select *one* representation.

b Show or explain where you see the intercepts.

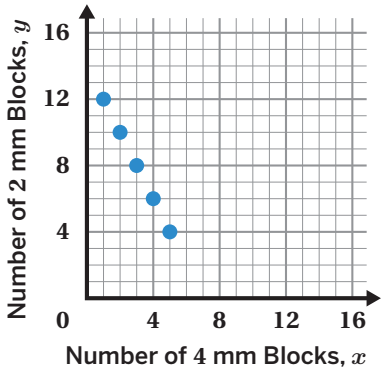
x-intercept: y-intercept:

6 Here is the same relationship represented in two different ways.

$4x + 2y = 28$



$y = 14 - 2x$



Discuss:

- How do you see the equation in each representation?
- Are these equations equivalent? Why or why not?

Activity 2

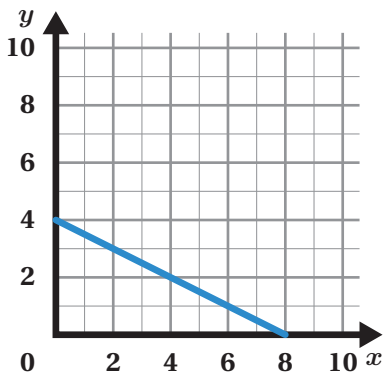
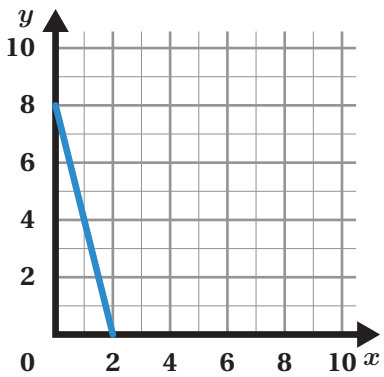
Name: ..... Date: ..... Period: .....

Rearrange It

7 Solve the equation $4x + 2y = 28$ for y to show that it is equivalent to $y = 14 - 2x$. Show or explain your thinking.

8 Match each graph with two equations. Two equations will have no match.

$2x + 8y = 24$	$y = 8 - 2x$	$2x + 4y = 16$
$y = 4 - \frac{1}{2}x$	$8x + 2y = 16$	$y = 8 - 4x$

Graph		
Equations		

9 Rearrange each equation to solve for y.

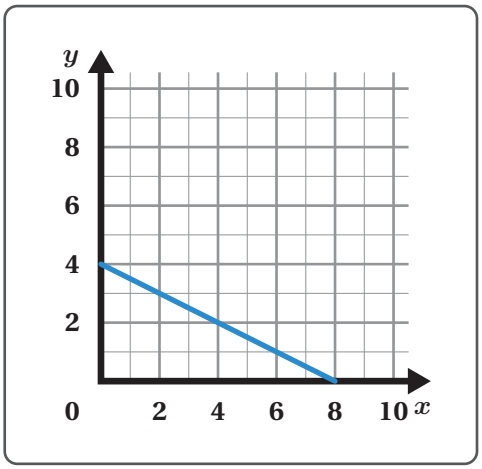
$6x + 2y = 34$	$5x + 2y = 46$
$y =$	$y =$
$2x + 4y = 26$	$3x + 4y = 40$
$y =$	$y =$

10 Synthesis

Discuss: How would you convince a classmate that these three cards represent the same situation?

$2x + 4y = 16$

$y = 4 - \frac{1}{2}x$



13 Summary 2.06

Equations, tables, and graphs are all different ways to model a situation. The graph of a linear equation represents all the pairs of values that are solutions to the equation and make the equation true.

Linear equations can be written in different but equivalent forms. Rearranging equations into different but equivalent forms helps reveal new information such as slope or intercepts.

For example: Let's say a lemonade stand sold lemonade for $3 per cup and cookies for $2 each. The stand made $12. l represents the number of cups of lemonade sold and c represents the number of cookies sold. This situation can be represented in different ways:

Equation

$3\ell + 2c = 12$

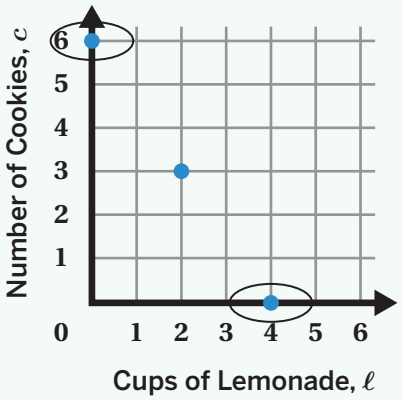
Equation Solved for c

$c = 6 - \frac{3}{2}\ell$

Table

ℓ	0	2	4
c	6	3	0

Graph



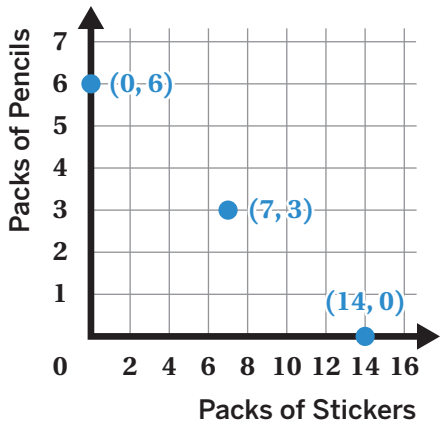
x-intercept A point where the graph of an equation or function crosses the x-axis or when $y = 0$

y-intercept A point where the graph of an equation or function crosses the y-axis or when $x = 0$.

Practice
2.06

Name: ..... Date: ..... Period: .....

Problems 1–2: Here is a graph about school supplies.



1. A teacher spent \$21 on packs of stickers and packs of pencils for her class.

- Stickers cost \$1.50 per pack.
- Pencils cost \$3.50 per pack.

Show or explain how you know that this graph represents this situation.

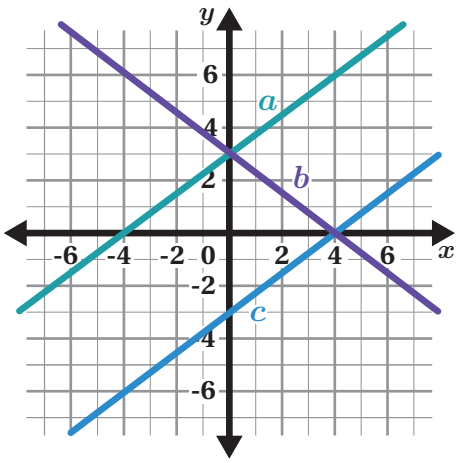
2. Circle a coordinate pair and explain what it means in this situation.

$(0, 6)$ $(7, 3)$ $(14, 0)$

3. Circle the line that represents $12 = 3x + 4y$.

Line a Line b Line c

Show or explain how you know.



4. Solve $15x + 3y = 2$ for y.

5. Match each equation with its equivalent equation by solving for y.

a $4x + 6y = 20$ **b** $3x - 6y = 16$ **c** $2x - 3y = 10$ **d** $-3x + 6y = 16$

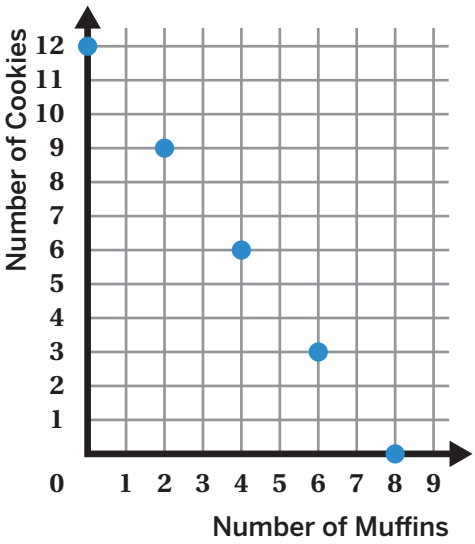
..... $y = \frac{8}{3} + \frac{1}{2}x$ ..... $y = -\frac{10}{3} + \frac{2}{3}x$ ..... $y = \frac{10}{3} - \frac{2}{3}x$ ..... $y = -\frac{8}{3} + \frac{1}{2}x$

Name: _____ Date: _____ Period: _____

6. Jordan had a bake sale. Muffins were $3 each and cookies were $2 each. Jordan earned $24.

Here is a graph of Jordan's situation.

Select *all* the combinations of muffins and cookies Jordan could have sold.



- ☐ **A.** 0 muffins and 8 cookies
- ☐ **B.** 9 muffins and 2 cookies
- ☐ **C.** 2 muffins and 9 cookies
- ☐ **D.** 6 muffins and 4 cookies
- ☐ **E.** 4 muffins and 6 cookies

7. Solve $-3x + 4y = 28$ for y.

8. Solve $6x - 3y = 36$ for y.

9. **Test Practice** Is the given equation equivalent to $4x - 2y = 12$? Select *Yes* or *No* for each equation.

	Yes	No
$-2y = -4x + 12$		
$y = 2x - 6$		
$2y = 12 + 4x$		

Spiral Review

10. Makayla is trying to predict the price of her next taxi ride.

She recorded her last three rides in the table.

How much can she expect to pay to travel 10 miles?

Distance (mi)	Price ($)
2	10
4	17
7	27.50

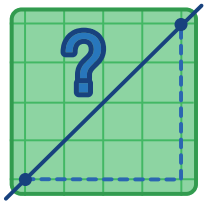
11. Select *all* the equations where $x = -2$ is a solution.

- ☐ **A.** $4x = 4 + 2x$
- ☐ **B.** $2(x + 5) = x + 8$
- ☐ **C.** $3x - 5 = 1$
- ☐ **D.** $19 = 2(x - 6) + 3$
- ☐ **E.** $5 + 3x = -1$

Name: ..... Date: ..... Period: .....

Comparing Models | Systems of Equations | A–CED.2, A–SSE.1.a, SMP.3, SMP.6, SMP.7

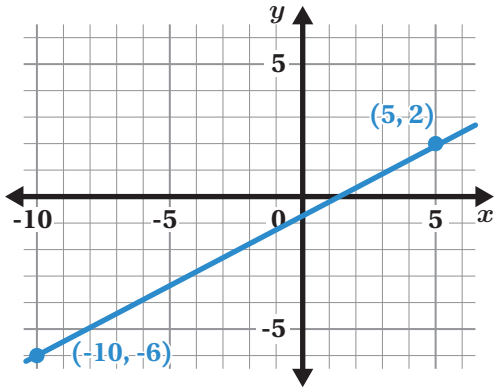
Equations of Lines



Let's explore what information is needed to determine a point-slope form equation of a line.

Warm-Up

1. The slope of the line shown is $\frac{8}{15}$. Explain why this is true.

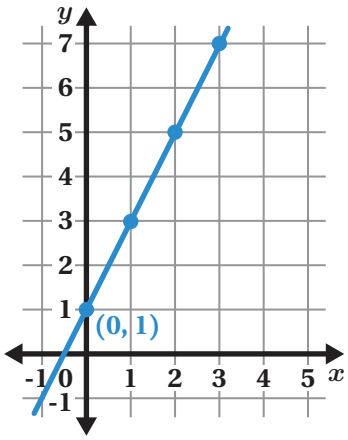


Activity 1

Name: ..... Date: ..... Period: .....

Building an Equation of a Line

Consider the line shown that passes through the point $(0, 1)$.



2. Determine the slope between each set of points. Write each slope in lowest terms.

Points	Calculation	Slope
$(0, 1)$ and $(1, 3)$		
$(0, 1)$ and $(2, 5)$		
$(0, 1)$ and $(3, 7)$		

3. Will this line pass through the point $(4, 9)$? Circle one.

Yes No

Explain your thinking.

4. Based on your responses to Problems 2 and 3, what must be true for *any* point (x, y) that lies on this line?

5. Using the point $(0, 1)$, write an equation that you can use to verify whether *any* point (x, y) is on this line.

Explain your thinking.

Name: Date: Period:

Building an Equation of a Line (continued)

6. In this table, each row displays information about a different line.

Point on the Line	Slope Equation	Point-Slope Equation
$(0, 4)$	$\frac{y-4}{x-0}=3$	$y-4=3(x-0)$
$(1, 2)$	$\frac{y-2}{x-1}=4$	$y-2=4(x-1)$
$(3, -1)$	$\frac{y+1}{x-3}=5$	$y+1=5(x-3)$
$(0, 1)$		

Compare the slope equations to the point-slope equations. How are these two equations related?

7. Complete the last row of the table by inserting your slope equation from Problem 5 and writing the corresponding point-slope equation.

8. **Discuss:** Mateo incorrectly thinks that the equation $y-7=-\frac{1}{2}(x-5)$ represents a line that passes through the point (-5, -7) and has a slope of $-\frac{1}{2}$.

a Describe what mistake Mateo might have made.

b Correctly describe the line that is represented by the given equation.

9. Suppose a line passes through the point (x_1, y_1) and has a slope of m. Write the point-slope equation for this line.

Name: ..... Date: ..... Period: .....

Using Point-Slope Form

10. You will use a set of four cards that have equations in **point-slope form**. You and your partner will take turns choosing a card and describing the properties of the line that has the given equation. Decide who will be Partner A and who will be Partner B.

- For Round 1, Partner A will choose a card. Without telling their partner what the equation is, they will describe the slope of the line and a point that the line passes through.
- Partner B will use the information they are told to write the equation of the line in point-slope form.
- Partner B will check their equation with Partner A. If the equations do not match, then the partners will need to work together and come to an agreement about the equation and its properties.
- For each round, switch roles. Record the information you are given and what you determine in the table.

	Point and Slope	Equation in Point-Slope Form
Round 1		
Round 2		
Round 3		
Round 4		

Name: ..... Date: ..... Period: .....

Using Point-Slope Form (continued)

11. Write an equation of the line that goes through the points $(-2, -11)$ and $(3, 4)$.

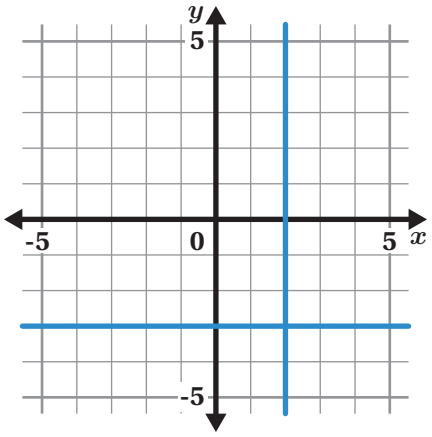
Explain your thinking.

12. Amari says that point-slope form can be used to write the equation of a horizontal line but not a vertical line.

Do you agree or disagree with Amari? Circle one.

Agree Disagree I'm not sure

Explain your thinking. Use the graph if it helps with your thinking.

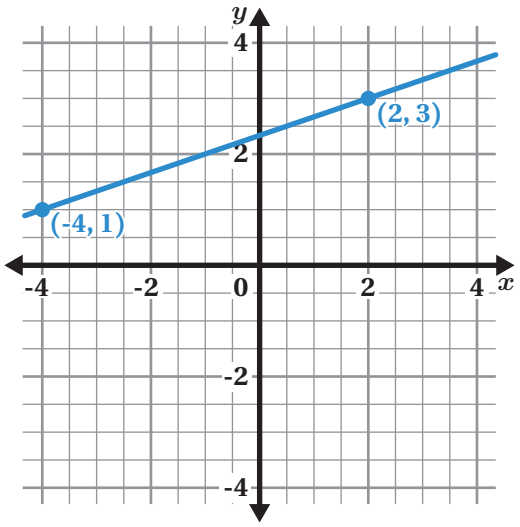


13. What are the equations of the horizontal and vertical lines?

Synthesis

14. **Discuss:** How can you write the equation of a line using **point-slope form**?

Use the graph if it helps with your thinking.



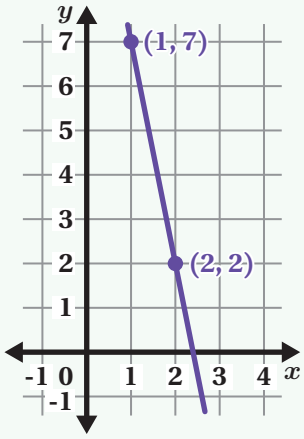
Summary 2.07

Any linear equation can be written in **point-slope form**.

You can use the point-slope form of a line to write an equation when:

- Any point and slope are given.
- Two points are given.
- The x- and y-intercepts are not integer values or are unknown.

For example, the line shown has a slope of -5 and passes through the points (1, 7) and (2, 2). By substituting the slope for m and the x- and y-coordinates of one of the points into $y - y_1 = m(x - x_1)$, the line can be represented by $y - 7 = -5(x - 1)$ or $y - 2 = -5(x - 2)$. These two equations are equivalent, represent the same line, and can both be rewritten as $y = -5x + 12$.



point-slope form The form of a linear equation written as $y - y_1 = m(x - x_1)$ for a line with slope m that passes through the point (x_1, y_1).

Practice
2.07

Name: ..... Date: ..... Period: .....

Problems 1–3: Think about the equation of each line. Determine the slope of the line and a point that the line passes through.

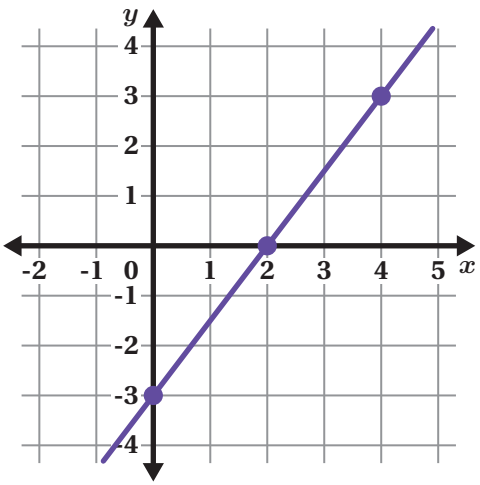
1. $y - 8 = \frac{1}{2}(x - 4)$

2. $y + 3 = -5(x - 7)$

3. $y - 1 = 6x$

4. A line with slope $\frac{3}{2}$ passes through the point $(1, 3)$. Is the point $(13, 22)$ on this line? Explain your thinking.

5. **Test Practice** Select *all* the equations that represent this graph.



- ☐ **A.** $3x - 2y = 6$
- ☐ **B.** $y = \frac{3}{2}x + 3$
- ☐ **C.** $y = \frac{3}{2}x - 3$
- ☐ **D.** $y - 3 = \frac{3}{2}(x - 4)$
- ☐ **E.** $y - 6 = \frac{3}{2}(x - 2)$

Problems 6–9: Write an equation in point-slope form for the line with the given properties.

6. Passes through $(12, 3)$ and has a slope of -1.

7. Passes through $(-4, 6)$ and has a slope of $\frac{3}{5}$.

8. Passes through $(-7, 9)$ and $(5, 5)$.

9. Passes through $(5, 10)$ and $(11, -2)$.

Name: ..... Date: ..... Period: .....

10. A line that passes through the point (17, -8) has the equation $y - n = -2(x - 7)$. Determine the value of n.

Spiral Review

11. How many solutions does the equation $4(x - 3) = -12 + 4x$ have? If it has one solution, what is the solution?

12. Adriana is working at a local restaurant to save money for a spring break trip. She starts with $\$350$ in her bank account and adds $\$100$ to the account every month. Let A represent the amount of money in Adriana's account after she's been saving money for x months.

Write an equation to represent this situation.

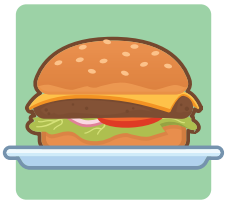
Name: ..... Date: ..... Period: .....

Comparing Models | Modeling with Functions | Systems of Equations

A–CED.2, A–CED.4, F–IF.7, F–LE.5, N–Q.1, SMP.1

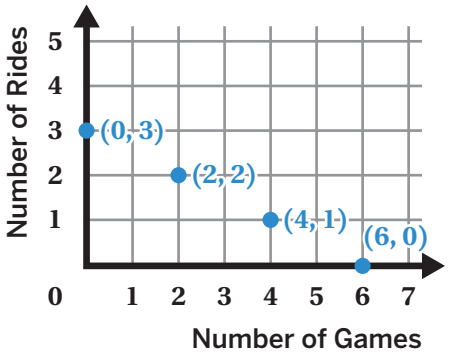
Five Representations

Let's explore different ways to represent linear relationships.



Warm-Up

1. **Discuss:** Describe as many connections as you can between the representations.

Description	Graph
Caasi has \$12 to spend on games and rides at a fair. Games cost \$2 each. Rides cost \$4 each.	
Equation in standard form	**Equation in slope-intercept form**
$2x + 4y = 12$	$y = 3 - \frac{1}{2}x$
x represents the number of games. y represents the number of rides.	x represents the number of games. y represents the number of rides.

Name: ..... Date: ..... Period: .....

Match It

2. You will use a set of three cards: two graph cards and one equation card.

- Match each card to the correct description.
- Create the missing representation(s) in each column.

	Burgers	Bracelets
Description	A family bought 2 hamburgers and 4 salads at the fair. Their total was \$32. Use x for the price of a hamburger. Use y for the price of a salad.	Vihaan sold 4 bracelets at the fair. To make the bracelets, he bought 8 packs of beads. Vihaan earned \$32 in total. Use x for the price of a bracelet. Use y for the cost of a pack of beads.
Graph		
Equation in Standard Form		
Equation in Slope-Intercept Form		

Name: ..... Date: ..... Period: .....

Make It

3. Situation 1: Complete all the representations for this situation.

Variables

- x represents the number of bus rides Kiana took last month.
- y represents the number of train rides Kiana took last month.

Description

- Each bus ride costs $\$2$.
- Each train ride costs $\$2.50$.
- Kiana spent $\$40$ riding the bus and the train last month.

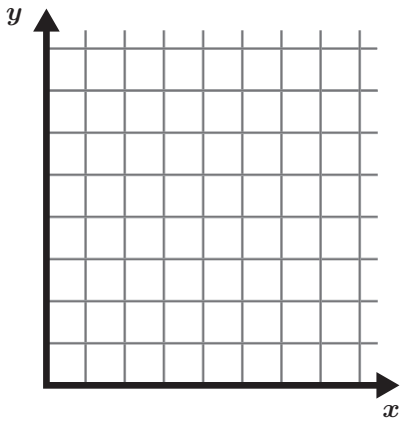
Equation in Standard Form

Equation in Slope-Intercept Form

Table

Number of Bus Rides, x	Number of Train Rides, y
0	
	12
10	
	4
20	0

Graph



Name: ..... Date: ..... Period: .....

Make It (continued)

4. Situation 2: Complete all the representations for this situation.

Variables

- x represents .....
- y represents .....

Description

Equation in Standard Form

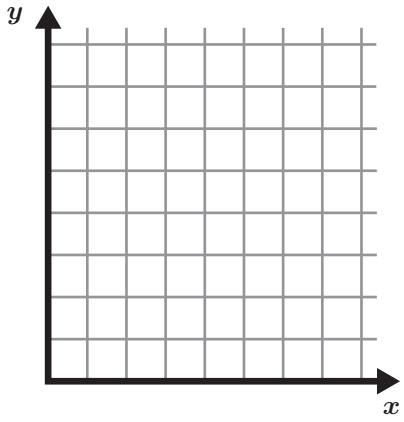
$$2x + y = 40$$

Equation in Slope-Intercept Form

Table

x	y
0	
5	30
	20
15	

Graph



5. What does each parameter in your equation in slope-intercept form represent in this situation?

Synthesis

6. Choose *one* representation on the right.

Explain how you could use your representation to create the other three.

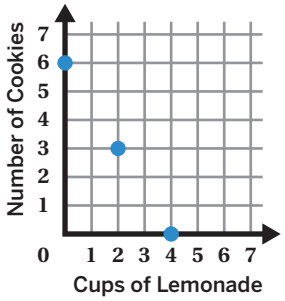
Equation in Standard Form

$3x + 2y = 12$

Equation in Slope-Intercept Form

$y = 6 - \frac{3}{2}x$

Graph



Description

A lemonade stand sold lemonade for $3 per cup and cookies for $2 each. The stand made $12.

x represents the number of cups of lemonade sold.

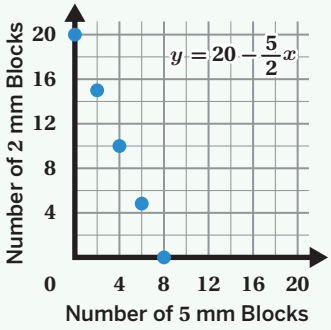
y represents the number of cookies sold.

Summary 2.08

There are different ways to represent situations that involve two variables. These representations can be descriptions, tables, or graphs.

A description explains how the quantities are related verbally or in writing.
For example: A snail needs to cross a gap that is 40 mm tall using 5 mm and 2 mm blocks.

A graph reveals the *slope* and *y-intercept*. This graph shows a slope of $-\frac{5}{2}$ and a y-intercept of 20.



A table reveals several combinations of 5 mm and 2 mm blocks that let the snail cross the gap.

x	0	2	4	6
y	20	15	10	5

The ordered pairs can be used to make a graph or find the rate of change (slope).

Different forms of an equation reveal unique information. In this situation, standard form reveals the relationship between combinations and slope-intercept form reveals how the blocks are changing in relationship with one another.

standard form (of a linear equation) Linear equations that are written in the form $ax + by = c$, where a, b, and c are constants and x and y are variables, are written in standard form.

slope-intercept form A way to write a linear equation that highlights the slope and the y-intercept of the line it represents. Slope-intercept form equations are written as $y = mx + b$, where parameter m represents the slope, parameter b represents the y-intercept of the line, and x and y are variables.

Practice
2.08

Name: ..... Date: ..... Period: .....

Problems 1–3: A school is putting on a play. Adult tickets cost \$8 each and student tickets cost \$4 each. The school collected \$320 total.

- a represents the number of adult tickets sold.
- s represents the number of student tickets sold.

1. Which equation represents this relationship?

A. $a + s = 320$

B. $8a + 4s = 320$

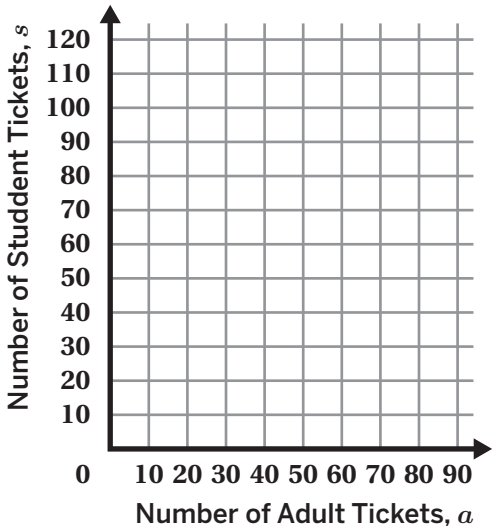
C. $8a - 4s = 320$

D. $8s + 4a = 320$

2. Complete the table representing this relationship.

Number of Adult Tickets, a	Number of Student Tickets, s
0	
20	

3. Make a graph showing this relationship.

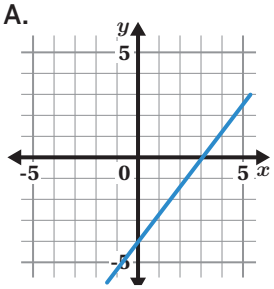


Problems 4–5: Here is the equation $-x + 4y = 12$.

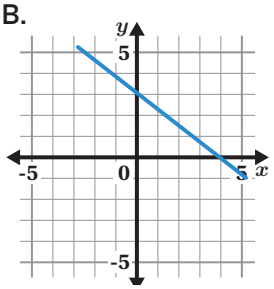
4. Explain how you can determine that $-x + 4y = 12$ is equivalent to $y = 3 + \frac{1}{4}x$.

5. Which graph matches the equations $-x + 4y = 12$ and $y = 3 + \frac{1}{4}x$?

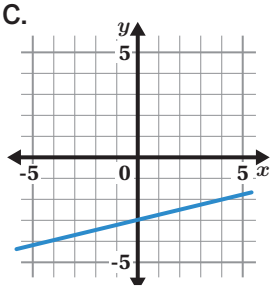
A.



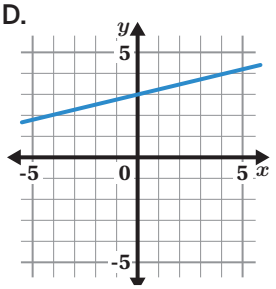
B.



C.



D.



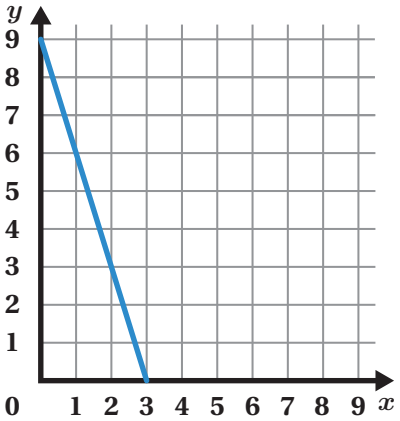
Practice
2.08

Name: ..... Date: ..... Period: .....

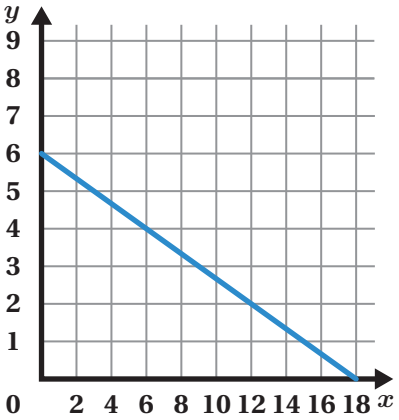
Problems 6–7: Match each graph to two equations. One equation will have no match.

Equation A	Equation B	Equation C	Equation D	Equation E
$y = 6 - \frac{1}{3}x$	$2y + 6x = 18$	$y = 9 - 3x$	$6y + 2x = 36$	$3y + 6x = 18$

6. Equation ..... Equation .....



7. Equation ..... Equation .....



Spiral Review

8. **Test Practice** Nyanna is selling cupcakes and lemonade to raise money for a bike. The bike she wants costs \$100. She is selling each cupcake for \$2 and each cup of lemonade for \$1.

x represents the number of cupcakes and y represents the number of cups of lemonade.

Write an equation in standard form that represents Nyanna's situation.

9. Write at least *three* different equations that are equivalent to $\frac{1}{2}(-10x + 2) = 6$.

Equation 1	Equation 2	Equation 3
.....	.....	.....

10. Select *all* the equations that have no solution.

- ☐ **A.** $2t + 1 = 1 + 2t$
- ☐ **B.** $2t + 1 = 2t$
- ☐ **C.** $2t + t = 2t + 1$
- ☐ **D.** $2t = 1$
- ☐ **E.** $2(t + 1) = 2t + 1$

11. Write an equation equivalent to $x = \frac{10 - 5y}{3}$ in the form $Ax + By = C$.

Name: ..... Date: ..... Period: .....

Comparing Models | Modeling with Functions | Systems of Equations

A–CED.2, A–CED.3, A–CED.4, A–REI.1, A–REI.3, A–REI.10, A–SSE.1, SMP.2, SMP.3, SMP.7

Practice Day 1

Let's practice what you've learned so far in this unit!

You will use task cards for this Practice Day. Record all of your responses here.

Task A: Equations and Their Solutions

1. Step: .....

Explanation:

2. Step: .....

Explanation:

3. Solution: .....

4. Solution: .....

5. Solution: .....

6. Solution: .....

7. Solution: .....

Task B: Snack Solving

1.

Pounds of Almonds, a	Pounds of Raisins, r
1	
2	
3	

2. Circle one: A B C D

Explanation:

3. Explanation:

4. Solution: .....

Name: ..... Date: ..... Period: .....

Practice Day 1

Task C: Very Many Variables

1. Solution: .....

2. Solution: .....

3. Solution: .....

4. Solution: .....

5. Explanation:

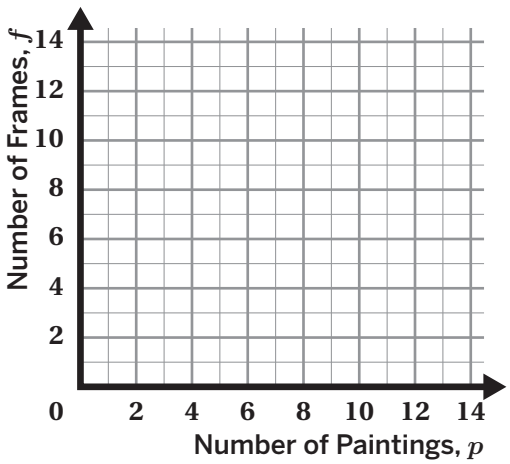
6. Explanation:

7. Solution: .....

Task D: Equations, Tables, Graphs, and Solutions

1. Equation: .....

2.



3. **a** Point: .....

b Slope: .....

c Explanation:

d Equation: .....

4. Circle one: Yes No

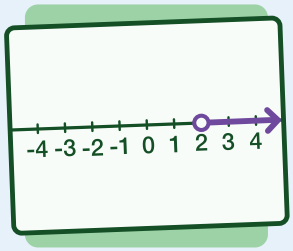
Explanation:

Sub-Unit

3

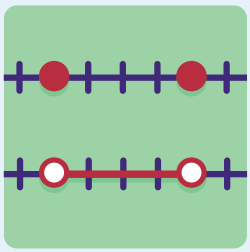
One-Variable and Two-Variable Inequalities

Lesson 9
Pizza Delivery

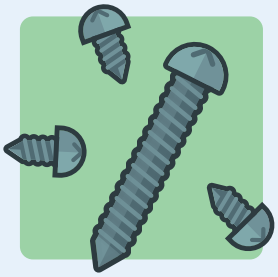


Lesson 10
Graphing Inequalities

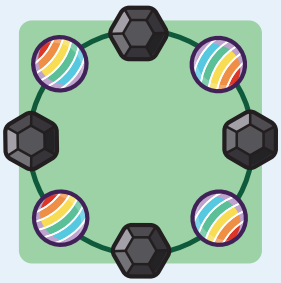
Lesson 11
Solutions and Sheep



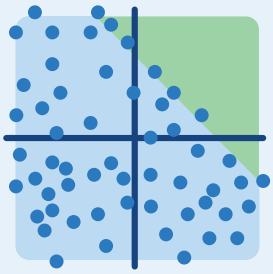
Lesson 12
Absolute Value Solutions



Lesson 13
All Sorts of Screws

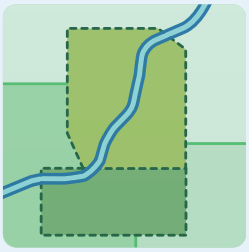


Lesson 14
Bracelet Budgets



Lesson 15
All of the Solutions

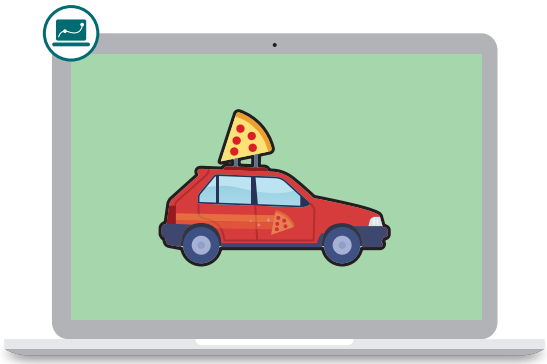
Lesson 16
Charity Concerts



Lesson 17
Water Way

Name: ..... Date: ..... Period: .....

Modeling with Functions | Systems of Equations | A–CED.1, A–CED.3, N–Q.1, SMP.1, SMP.2, SMP.3



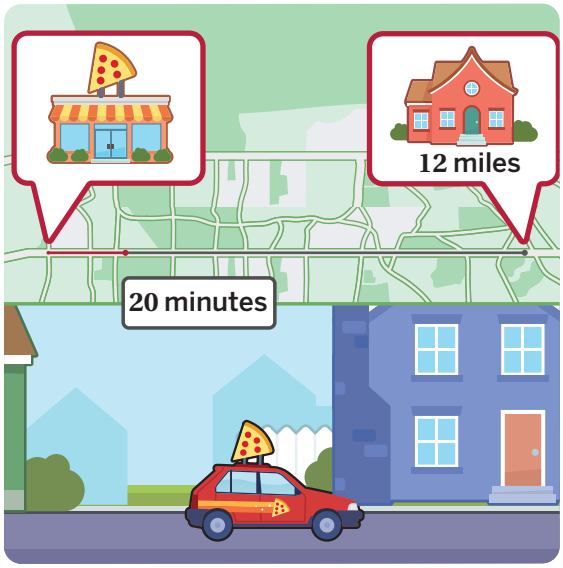
Pizza Delivery

Let's write inequalities to represent constraints.

Warm-Up

1 Let's watch an animation.

Write a story about what you see.



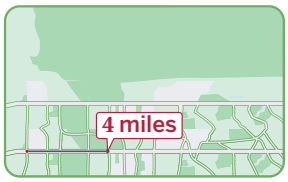
Name: ..... Date: ..... Period: .....

Order Up!

2 Desmos Pizza is now offering delivery!

It takes 15 minutes to prepare an order and 3 minutes to drive each mile.

How long would it take to deliver each order?

Distance (mi)			
Time (min)			

3 Here is Mariana's work from the previous problem.

Write an expression for the number of minutes it would take to deliver a pizza x miles away.

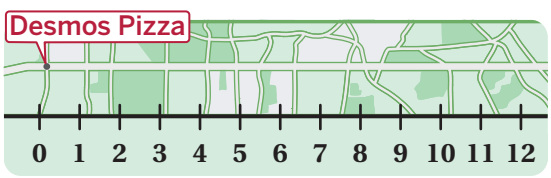
Distance (mi)	Time (min)
2	15 + 6
4	15 + 12
8	15 + 3(8)
x	?

4 Desmos Pizza wants to deliver orders in 30 minutes or less.

a Which inequality represents this situation?

A. $15 + 3x \leq 30$ **B.** $15 + 3x \geq 30$ **C.** $15 + 3x > 30$ **D.** $15 + 3x < 30$

b Show or describe all the distances they can deliver to in 30 minutes or less.



Name: ..... Date: ..... Period: .....

Order Up! (continued)

5 Here is how Mariana and Camila figured out all of the distances in which pizza could be delivered in 30 minutes or less.

Mariana

Miles, x	Minutes
1	18
2	21
3	24
4	27
5	30
6	33

$x \leq 5$

You have to live 5 miles away or less.

Camila

$15 + 3x = 30$

$-15 \quad -15$

$3x = 15$

$x = 5$

5 miles is the farthest you can live from Desmos Pizza.

Discuss: Why is each strategy helpful? How could each strategy be improved?

6 A large pizza is \$12 and toppings are \$2 each. Mariana can spend as much as \$20 on a pizza.

a If x is the number of toppings, which of these inequalities represents the situation?

A. $12 + 2x \geq 20$ **B.** $12x + 2 \geq 20$ **C.** $12 + 2 \leq 20x$ **D.** $12 + 2x \leq 20$

b Show or describe all the numbers of toppings, x, that Mariana can get for \$20 or less.

7 A **constraint** is a limitation on possible values in a model. Here are some examples of constraints:

- Desmos Pizza wants to deliver in 30 minutes or less.
- Mariana can spend as much as \$20 on a pizza.
- Desmos Pizza wants to sell more than 50 pizzas per day.

What is a constraint in *your* life?

Name: ..... Date: ..... Period: .....

Trampoline World

8 Jamir is planning to host a party at Trampoline World.

Discuss: What constraints might Jamir think about when planning this party?

9 Hosting a party at Trampoline World costs a flat fee of $80, plus $30 per hour for the small room or $50 per hour for the large room.

Match each constraint to an inequality, where x represents the number of hours for the party. One inequality will have no match.

$30 + 80x \geq 140$	$80 + 30x \leq 140$	$80 + 50x \geq 140$	$80 + 80x \leq 140$

Mariana's party in the small room costs at most $140.	**The owner wants to earn at least $140 for a party in the large room.**	**Amoli can spend up to $140 for a party that uses both the big and small rooms.**

Name: ..... Date: ..... Period: .....

Trampoline World (continued)

10 Jamir can spend as much as $155 for a party in the large room.

Write an inequality to match this new constraint, where x represents the number of hours for the party.

11 *Select all* the possible numbers of hours, x, that could work with Jamir's constraints.

- ☐ **A.** 0.75 hours
- ☐ **B.** 1 hour
- ☐ **C.** 1.5 hours
- ☐ **D.** 2.25 hours
- ☐ **E.** 3 hours

12 Synthesis

Discuss: What are some suggestions you have for writing a constraint as an inequality?

15 Summary 2.09

We can use *inequalities* to model situations with **constraints**.

Here is an example of a situation, the constraint, and the inequality that models them.

Situation	Constraint	Inequality
Tasia is ordering pizza for a party. Each plain pizza costs $12 and there is a delivery fee of $8.	Tasia can spend *up to* $140.	$12p + 8 \leq 140$ Where p represents the number of pizzas.

When writing inequalities to model situations, you can use the symbols $>$, $\geq$, $<$, and $\leq$ to represent the nature of the constraint. Terms like *greater than, less than, at most, at least*, or *up to* can help you determine which inequality symbol to use.

constraint A limitation on what values are possible in a model or situation.

Practice
2.09

Name: ..... Date: ..... Period: .....

1. For each constraint, write the letter of the matching inequality.

A. $x \leq 10$ **B.** $x > 10$ **C.** $x \geq 10$

..... x is less than or equal to 10.

..... x is at most 10.

..... x is greater than or equal to 10.

..... x is greater than 10.

..... x is at least 10.

2. Marquis wants to work at least 20 hours a week to earn enough money to go to a concert. Which inequality represents x, the number of hours Marquis wants to work?

A. $x > 20$ **B.** $x < 20$ **C.** $x \leq 20$ **D.** $x \geq 20$

3. Demetrius can spend as much as $\$50$ on shirts. Shirts, s, cost $\$16$ each at a nearby store. Which inequality represents this situation?

A. $16s \geq 50$ **B.** $16s \leq 50$ **C.** $50s \geq 16$ **D.** $50s \leq 16$

Explain your thinking.

4. List *three* values for x that would make $8 + 2x \leq 20$ true.

$x =$ $x =$ $x =$

Problems 5–7: Write an inequality for each constraint. Use t for time (in hours).

5. Trevon practices his clarinet at least 1 hour each day.

6. At some colleges, students must work 20 hours or less per week.

7. The American Academy of Pediatrics recommends teenagers play video games for no more than 2 hours each day.

Name: ..... Date: ..... Period: .....

Problems 8–9: Tell a story about each inequality. Specify the constraint and what the variable represents.

8. $x \geq 3$

9. $5 > y$

10. **Test Practice** Fatima makes $\$9.25$ per hour, h, plus $\$150$ in commissions. She created an inequality that could be used to find the number of hours she needs to work to make at least $\$510$ on her next paycheck.

Her inequality is $9.25 + 150h \leq 510$. Find the mistakes in Fatima's inequality and correct it.

Spiral Review

Problems 11–13: Solve each equation.

11. $4x - 6 = 12 - 2x$

12. $\frac{1}{3}x - 8 = 12 - 3x$

13. $2x + 7 - 3x = \frac{5}{2}$

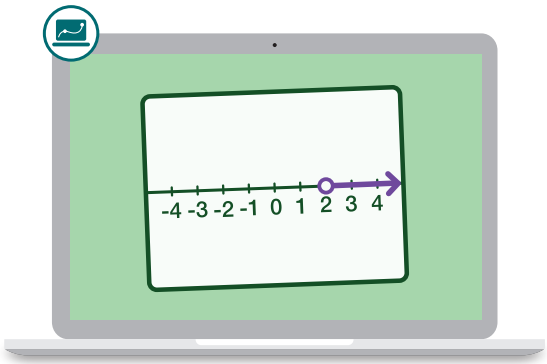
14. Solve the equation for b: $\frac{b}{c} + g = e$

Name: ..... Date: ..... Period: .....

Systems of Equations A–REI.3, SMP.1, SMP.2, SMP.3

Graphing Inequalities

Let's represent solutions to inequalities on a number line.

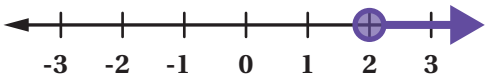


Warm-Up

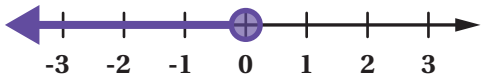
1 Here are four different inequalities.

Discuss: What do you notice?

$x \geq 2$



$x \leq 0$

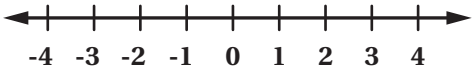


$x < 2$

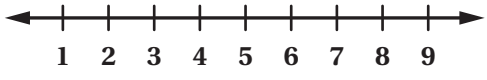
$x > -3$

2 Make a graph of *all* the solutions to each inequality.

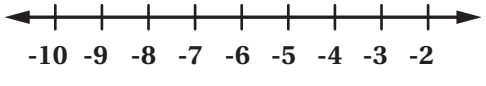
$x > -1.5$



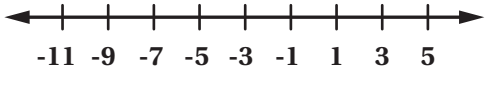
$x \leq 7$



$x < -5$



$x \leq -6$



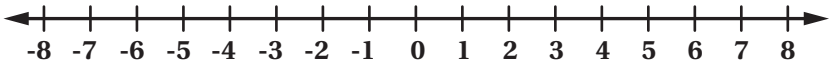
Activity 1

Name: ..... Date: ..... Period: .....

Show a Solution

3 **a** Plot a solution to this inequality.

$2x - 8 > 0$

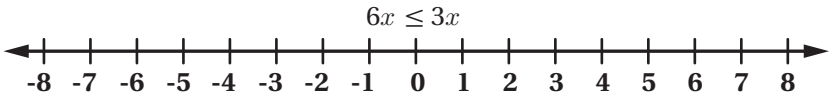


Share your response with your classmates.

b **Discuss:** Are any of the points incorrect? How do you know?

4 **a** Plot a solution to this inequality.

$6x \leq 3x$



b Explain how you know that your point is a solution (or why there is no solution).

5 Lan explains that $6x \leq 3x$ does not have a solution because *6 of something is always more than 3 of the same thing*.

Is Lan's statement correct? Circle one.

Yes No I'm not sure

Show or explain your thinking.

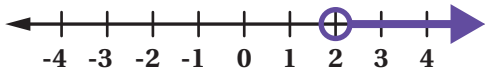
Activity 2

Name: ..... Date: ..... Period: .....

Solution Sets

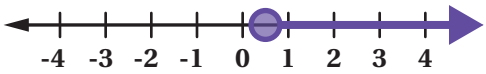
6 How can you check that this inequality and graph represent the solutions to $\frac{1}{4}x > \frac{1}{2}$?

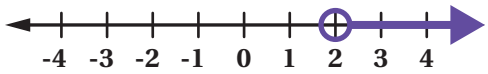
$x > 2$



7 Match each inequality to a number line that represents its solutions.

$5x + 4 \geq 7x$	$3 - x < 1$	$2(x + 3) \geq 7$	$8x - 2 < 4x$



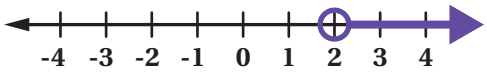


Name: Date: Period:

Solution Sets (continued)

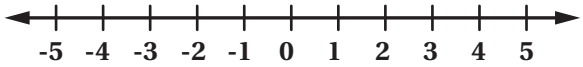
8 Lan matched this inequality and number line.

$$5x + 4 \geq 7x$$



How could you convince Lan that this inequality and number line don't match?

9 Create a graph of the solutions to the inequality $15 - x < 11$.

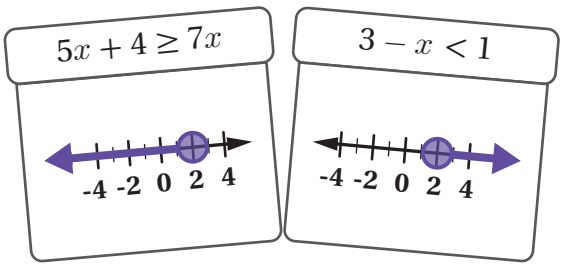


Explain your thinking.

10 Synthesis

Explain how you can determine whether a number line represents the solutions to an inequality.

Use these examples if they help with your thinking.



13 Summary 2.10

A **solution set** can be represented on a number line by marking the *boundary point* and then shading the **region** of values that make the inequality true. To identify the boundary point, you can solve the equation that corresponds to the inequality and then you can test one or more values to determine the *solutions to an inequality*.

Here is an example of how you can determine and represent the solution set for $2x - 4 \geq 8$:

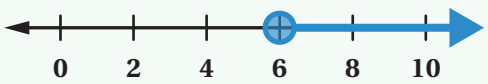
Determine the boundary point:

$$2x - 4 = 8$$
$$2x = 12$$
$$x = 6$$

Determine the region:

$$2x - 4 \geq 8$$
$$2(0) - 4 \geq 8$$
$$4 \geq 8 \quad \textit{False!}$$

- 6 is the boundary point and the $\geq$ symbol means it is included in the solution set.
- Since this statement is false when we substitute 0, we know 0 is not in the solution set.
- An open circle is used on the number line when a boundary point is not included in the solution set.



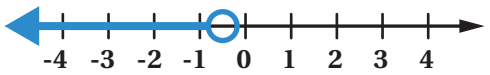
region The set of points on a number line or in a coordinate plane that are solutions to one or more inequalities. Shading is often used to indicate the location of a solution region.

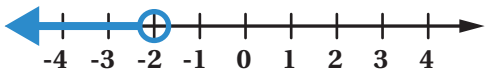
solution set The set of all values that makes an inequality true. Solution sets are described symbolically with inequalities and are described graphically by shading a portion of a number line or a region of the coordinate plane.

Practice
2.10

Name: ..... Date: ..... Period: .....

1. Which graph represents the solutions to $2x < -4$?

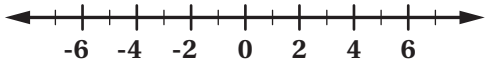
A.


B.


C.


D.

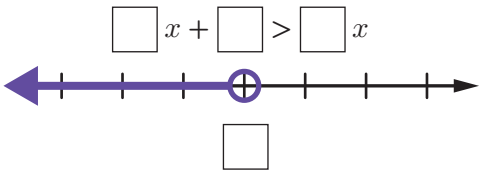

2. Create a graph of the solutions to $2x \leq 10$.



3. Leo is trying to solve $-15 + x < -14$. He knows the boundary point is at $x = 1$. How can he determine whether the solutions are $x < 1$ or $x > 1$?

4. Diego says that $x = 5$ is a solution to $-3x > 9$ because when you divide both sides by -3, you get $x > -3$. Is this correct? Explain your thinking.

5. Create an inequality and the graph of its solutions by filling in each blank using the numbers 0 to 9 only once.



6. **Test Practice** What are the solutions to $-2(2 + c) + 4 < 6$?

A. $c > -3$

B. $c < -3$

C. $c > -2$

D. $c < -2$

Name: ..... Date: ..... Period: .....

7. Write an inequality where all the solutions are $x > 2$.

Spiral Review

Problems 8–10: A community pool offers two different membership plans.

Plan A	Plan B
$4 per visit	An initial $12 fee, then $2 per visit

Brielle wants to spend no more than $48 at the community pool this month.

8. How many times could she visit the pool with Plan A?

9. How many times could she visit the pool with Plan B?

10. After how many visits will the cost of both plans be the same?

Problems 11–12: Desmo Flowers is offering deliveries to residences in 40 minutes or less. It takes 10 minutes to prepare a flower order and 2 minutes to drive each mile. Let x represent the number of miles Desmo Flowers needs to drive to a residence.

11. Which inequality represents this situation?

A. $10 + 2x > 40$

B. $10 + 2x < 40$

C. $10 + 2x \leq 40$

D. $10x + 2 \leq 40$

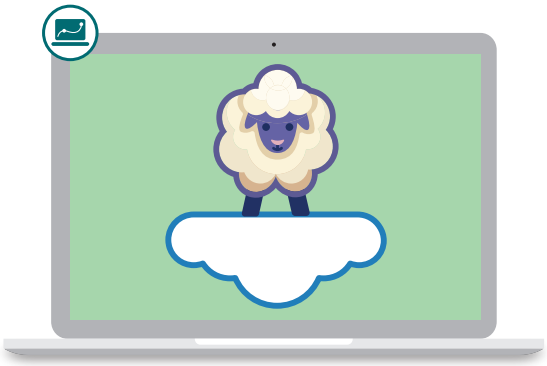
12. Write an inequality that represents all the distances that Desmo Flowers can deliver in 40 minutes or less.

Name: ..... Date: ..... Period: .....

Systems of Equations A–REI.3, SMP.1, SMP.3

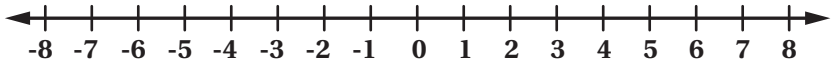
Solutions and Sheep

Let's make connections between solving one-variable equations and solving inequalities.



Warm-Up

1 **a** Plot *three* different solutions to the inequality $10 - 5x < 0$.



Share your response with your classmates.

b **Discuss:** Are any of the points incorrect? How do you know?

2 Let's look at all the correct solutions to $10 - 5x < 0$.

Kayleen and Leo are discussing how to write the solutions to this inequality.

Kayleen says the solution is $x < 2$.
Leo says the solution is $x > 2$.

Who is correct? Circle one.

Kayleen Leo Both Neither

Explain your thinking.

Activity 1

Name: ..... Date: ..... Period: .....

Feed the Sheep

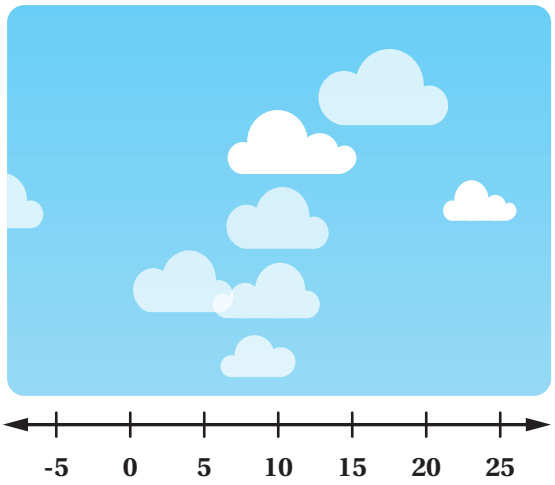
3 Shira the Sheep loves eating grass. She does *not* like water.

a Let's watch what happens when we try out different inequalities.

b **Discuss:** What do you notice?

4 Here is an inequality: $\frac{x}{3} \geq 5$

Solve the inequality to help Shira eat all the grass.

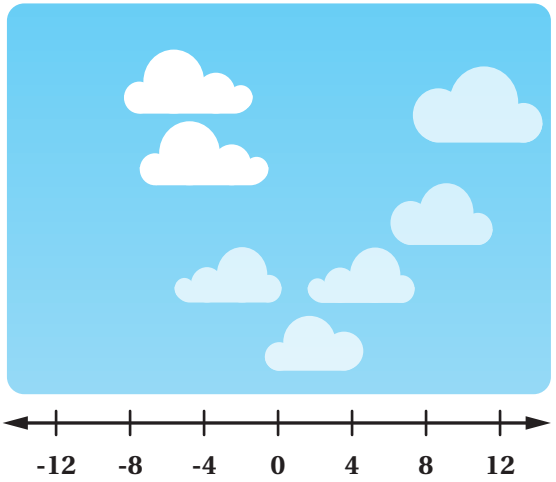


Name: ..... Date: ..... Period: .....

Feed the Sheep (continued)

5 Here is a new inequality: $9 > 2x - 4$

Solve the inequality to help Shira eat all the grass.



6 Here is some of Kayleen's work from the previous problem.

$9 > 2x - 4$

$9 = 2x - 4$

$13 = 2x$

$6.5 = x$

Test

$x = 6$	$x = 7$
$9 > 2(6) - 4$	$9 > 2(7) - 4$
$9 > 8$	$9 > 10$
✓	False!

a **Discuss:** What do you notice and wonder about Kayleen's strategy?

b Describe how Kayleen's work can help her decide which way to shade the solution set on the number line.

Name: ..... Date: ..... Period: .....

Solving for Sheep

7 Here is a new inequality: $8x - 4 < 10x + 2$.

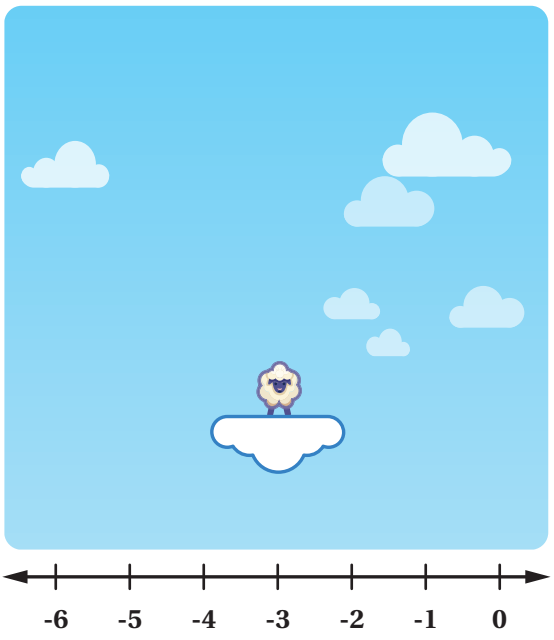
Solve the inequality to help Shira eat all the grass.

8 Leo was trying to solve the previous inequality: $8x - 4 < 10x + 2$.

He knew the sheep needed to land at -3, but didn't know if the grass was to the *right* or the *left*.

He wrote $8(0) - 4 < 10(0) + 2$.

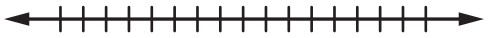
How might that help Leo decide where the grass is?



Name: Date: Period:

Solving for Sheep (continued)

9 Here is a new inequality: $-2(x + 2) \leq -23$



Solve the inequality to help Shira eat all the grass.

10 For each of the challenges:

- Decide with your partner who will complete Column A and who will complete Column B.
- Solve as many inequalities as you have time for.
- The solutions in each row should be the same. Compare your solutions, then discuss and resolve any differences.

Column A	Column B
$2x + 10 > 20$	$\frac{x}{5} + 9 > 10$
$\frac{x}{5} + 10 \leq 12$	$5x - 10 \leq 40$
$-10 \geq 4x + 4$	$14 \geq 2x + 7$
$3 - 2x > 3$	$8 > 4x + 8$
$\frac{x + 3}{5} < 2$	$\frac{x + 8}{5} < 3$
$5x + 10 \geq 3x + 12$	$2x - 4 \geq x - 3$

11 Synthesis

Discuss: Describe a strategy for solving any inequality.

Use the examples if they help with your thinking.

$10 - 5x < 0$

$9 > 2x - 4$

$8x - 4 < 10x + 2$

14 Summary 2.11

One strategy that you can use to solve any one-variable inequality is to:

- Solve the equation that corresponds with the inequality to determine the boundary point.
- Then test value(s) greater than or less than the boundary point to determine where the solution region is located.

Here is an example of how you could solve the inequality $10 > -3x - 2$.

Solve the corresponding equation:

$$10 = -3x - 2$$
$$12 = -3x$$
$$-4 = x$$

The boundary point is $x = -4$.

Test $x = -5$

$10 > -3(-5) - 2$

$10 > 13$

False!

Test $x = 0$

$10 > -3(0) - 2$

$10 > -2$

True!

When $x = -5$, the inequality is false, so the solutions are greater than -4.

The solution to an inequality does not always have the same inequality symbol as the original inequality. Since the value that is *greater than* -4 is true, the solution set to the inequality can be written as $-4 < x$ or $x > -4$.

Practice
2.11

Name: ..... Date: ..... Period: .....

Problems 1–4: Solve each inequality.

1. $4x + 5 \geq 37$

2. $-3x + 4 \geq 12$

3. $-6 + \frac{x}{2} < 7$

4. $-8x - 6 > 2x - 26$

5. **Test Practice** Here is an inequality: $7x + 6 < 3x + 2$. Select *all* values that are solutions.

- ☐ **A.** $x = 1$
- ☐ **B.** $x = 0$
- ☐ **C.** $x = -1$
- ☐ **D.** $x = -2$
- ☐ **E.** $x = -8$

6. Write three values of n that make this inequality true: $-5(n + 1) > 10$.

7. Make the two inequalities equivalent by filling in each blank using the numbers 0 to 9 only once.

$$\square x + \square < \square x + \square$$

$$x > \square$$

Name: ..... Date: ..... Period: .....

8. Which inequality is represented by the graph?

A. $2x + 6 < 10$

B. $2x + 6 \geq 10$

C. $-2x + 6 \geq 10$

D. $-2x - 6 \geq -10$

Spiral Review

Problems 9–10: Rewrite each expression as a single power of 10.

9. $\frac{10^2 \cdot 10^5}{10^4}$

10. $(10^4)^2 \cdot (10^2)^2$

11. Imani is going shopping with a budget of $125. Which inequality represents the amount of dollars, x, that Imani can spend while shopping?

A. $x \leq 125$ **B.** $x \geq 125$ **C.** $x > 125$ **D.** $x < 125$

12. Solve for x.

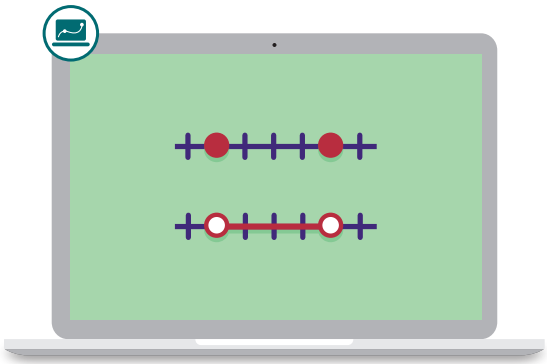
$\frac{x}{4} - y = 5$

Name: ..... Date: ..... Period: .....

Modeling with Functions | Systems of Equations | A–REI.3.1, A–CED.1, SMP.2, SMP.7

Absolute Value Solutions

Let's solve absolute value equations and inequalities.

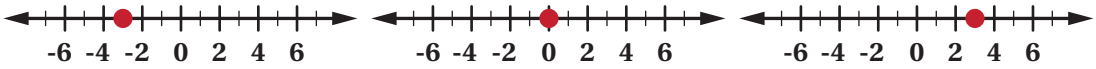


Warm-Up

1 $|x|$ is pronounced "the **absolute value** of x."

a Here are three different values of $|x|$.

$|-3| = 3$ $\quad$ $|0| = 0$ $\quad$ $|3| = 3$



b How would you explain to someone else what $|x|$ means?

Activity 1

Name: ..... Date: ..... Period: .....

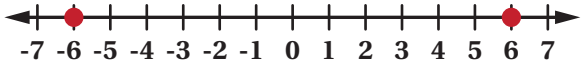
Showing Solutions

2 Here is one solution to $|x| = 4$.

Show another solution to $|x| = 4$.

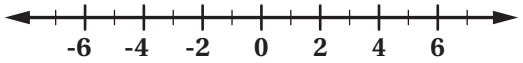
Explain your thinking.

3 Here are the solutions to an absolute value equation.



Write an absolute value equation that has the solutions shown on the number line.

4 **a** Show a solution to $|x| < 4$.



Share your response with your classmates.

b **Discuss:**

- Are any points incorrect?
- Are any solutions missing?

Name: ..... Date: ..... Period: .....

Showing Solutions (continued)

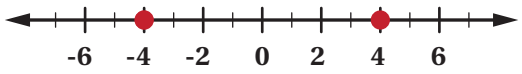
5 Here are all the solutions to $|x| = 4$ and $|x| < 4$.

How are the solutions alike? How are they different?

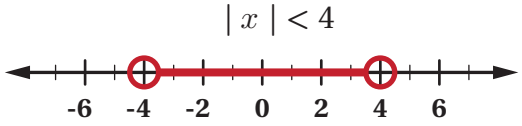
Alike:

Different:

$| x | = 4$

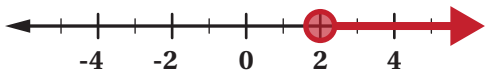


$| x | < 4$



6 Which number line represents the solutions to $|x| \geq 2$?

A.



B.

C.

D.

7 Felipe is graphing the solutions to $2.5 \leq |x|$.

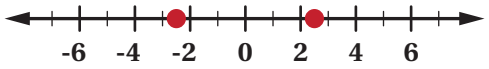
Here is some of his work.

Describe how Felipe's work can help him decide where to shade the solution set on the number line.

Felipe

$2.5 \leq | x |$

$2.5 \leq |-5|$ $2.5 \leq |0|$ $2.5 \leq |5|$



✓ ✗ ✓

Activity 2

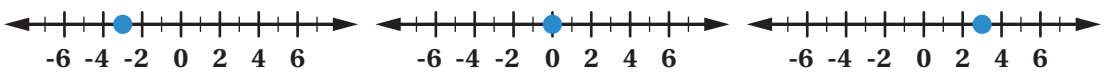
Name: ..... Date: ..... Period: .....

Solving Strategies

8 Polina says the value of $|x - 2|$ is the distance between any number and 2.

a Here are three different values of $|x - 2|$.

$|-3 - 2| = 5$ $|0 - 2| = 2$ $|3 - 2| = 1$

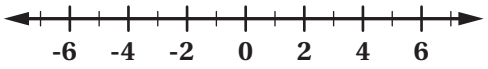


b **Discuss:** Do you agree with Polina?

9 **a** Show or describe all the solutions to $|x - 2| = 4$.

$|x - 2| = 4$

$|x - 2| \leq 4$



b Show or describe all the solutions to $|x - 2| \leq 4$.

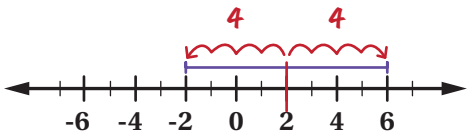
Name: ..... Date: ..... Period: .....

Solving Strategies (continued)

10 Two students graphed the solutions to $|x - 2| \leq 4$.

Esi

$|x - 2| \leq 4$

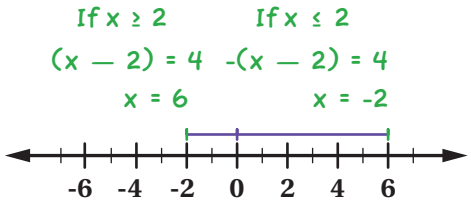


Deja

$|x - 2| \leq 4$

If $x \geq 2$: $(x - 2) = 4$, $x = 6$

If $x \leq 2$: $-(x - 2) = 4$, $x = -2$

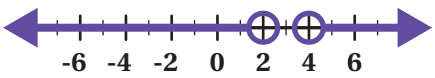


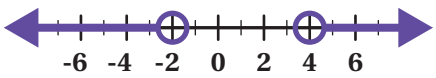
$|0 - 2| \leq 4$ ✓

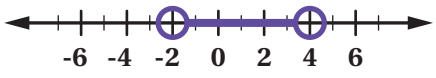
Discuss: How would you describe each students' strategy?

11 Match each inequality to the number line that represents its solutions.

$|x - 3| > 1$ $|x - 3| < 1$ $|x - 1| > 3$ $|x - 1| < 3$







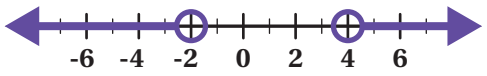
Activity 3

Name: ..... Date: ..... Period: .....

Repeated Challenges

12 Raven wants to use the graph of $|x - 1| > 3$ to help her make a graph of $|x + 1| > 3$.

$| x - 1 | > 3$



a **Discuss:**

- How would the solutions be alike?
- How would they be different?

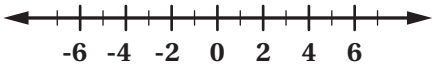
b Graph all the solutions to $|x + 1| > 3$.

$| x + 1 | > 3$

13 Graph all the solutions to each inequality or equation.

$| x | \leq 2$

$| x | \geq 5$



$| x - 3 | = 1$

$| x - 2 | > 4$

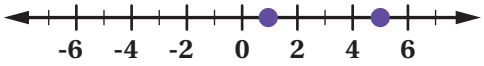
$4 \geq | x |$

$| x - 2 | > 2$

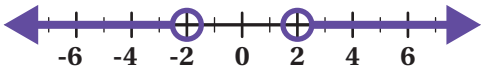
14 Synthesis

Discuss: Describe a strategy you would use to graph the solutions to absolute value equations and inequalities.

$|x - 3| = 2$



$|x| > 2$



17 Summary 2.12

We can use the definition of **absolute value** to help us understand how to solve absolute value equations and inequalities. For an absolute value equation, there are usually two input values with the same output. This means there are often two solutions that make an absolute value equation true.

When solving and graphing absolute value inequalities, we can use the solve-and-test strategy we used when solving one-variable inequalities.

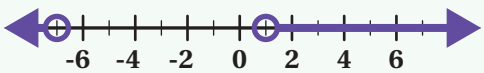
- First, solve the two corresponding absolute value equations to determine the boundary points.
- Then decide whether the boundary points are included in the solution set. 1 and -7 are not included because of the inequality symbol >. Draw open circles on the number line to represent these boundary points.
- Next, test a value between the two boundary points to decide which region of values makes the inequality true.
- Finally, graph the solution region.

$|x + 3| > 4$

If $x > -3$	If $x < -3$
$x + 3 = 4$	$-(x + 3) = 4$
$x = 1$	$x = -7$

$|(0) + 3| > 4$

False!



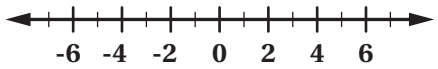
absolute value The distance a number is from 0 on the number line.

Practice
2.12

Name: ..... Date: ..... Period: .....

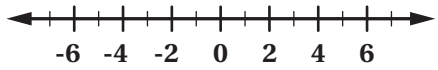
Problems 1–4: Graph all the solutions to each inequality or equation.

1. $|x| \leq 3$

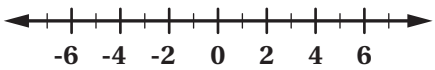


2. $|x + 2| > 4$

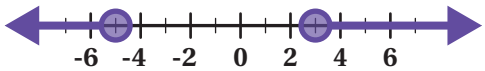
3. $|x - 5| = 1$



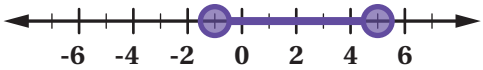
4. $2 > |x - 3|$



5. What value of p would make $|x - p| \geq 4$ match the graph?



6. **Test Practice** Which inequality's solutions are shown by the graph?



A. $|x + 5| < -1$

B. $|x - 2| \leq 3$

C. $|x - 2| \geq 3$

D. $|x + 3| \leq 2$

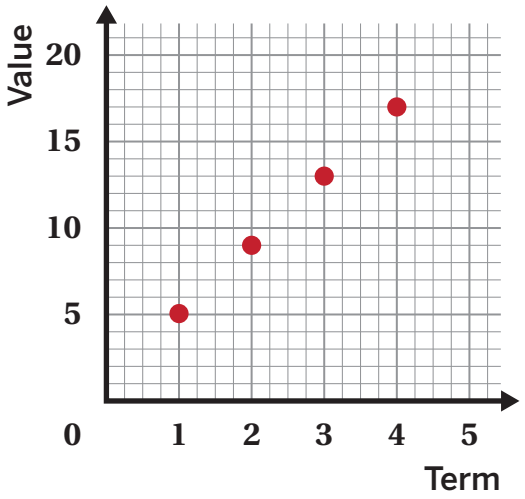
7. Omar is thinking of a number somewhere on this number line. Here are some guesses from his friends. No guess was more than 5 away. Only one guess was correct. What is Omar's number? Explain your thinking.

Name: ..... Date: ..... Period: .....

8. List three numbers that are not solutions to $|x + 1| \geq 2$.

Spiral Review

Problems 9–11: Here is the graph of a sequence.



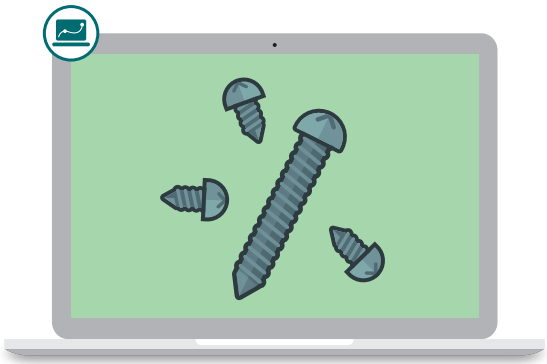
9. What is the value of the 5th term?

10. Is this sequence arithmetic, geometric, or something else? Explain your thinking.

11. What is the value of the nth term?

Name: ..... Date: ..... Period: .....

Modeling with Functions | Systems of Equations | A–CED.1, A–REI.3.1, SMP.1, SMP.3, SMP.7

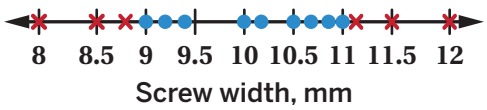


All Sorts of Screws

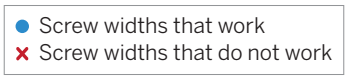
Let's create and solve absolute value equations and inequalities.

Warm-Up

1 A bookcase is designed for 10 mm screws, but screws that are a little bigger or smaller will also work.

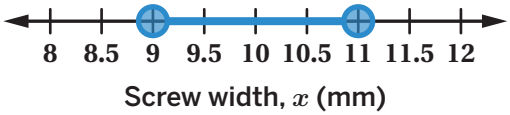


a Here are some screw widths that work and do not work.



b **Discuss:** What do you notice?

2 Here is a graph of all the screw widths that work. x represents the width of a screw in mm.



Arjun says this is the graph of $|x - 1| \leq 10$.

Eliza says this is the graph of $|x - 10| \leq 1$.

Who is correct? Circle one.

Arjun Eliza Both Neither

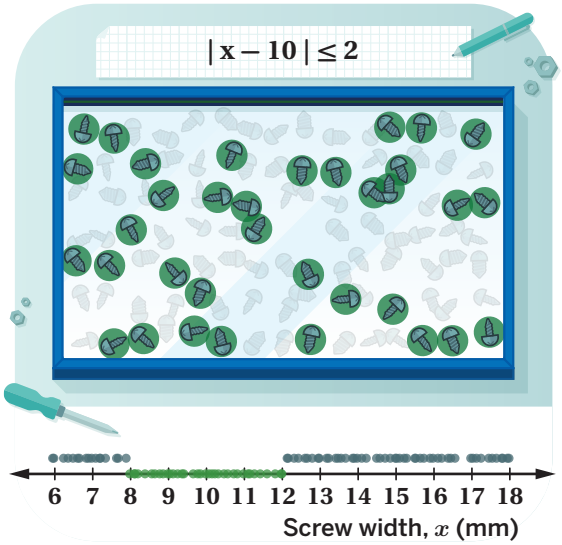
Explain your thinking

Activity 1

Name: ..... Date: ..... Period: .....

Screw Sorter

3 This machine uses algebraic rules to select screws that are x mm wide.



a Let's watch what happens.

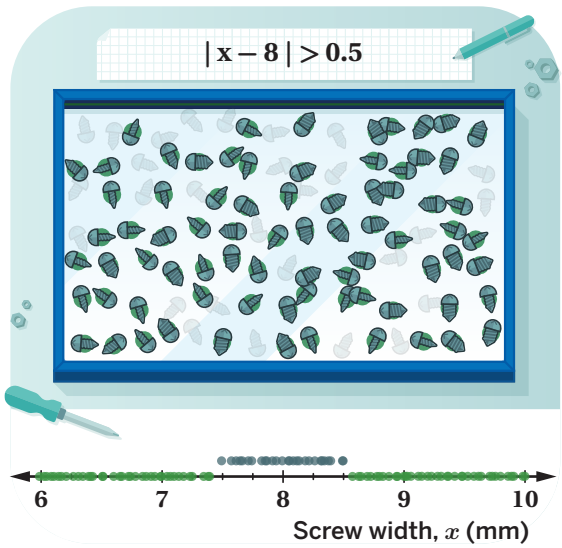
b **Discuss:** What do the numbers in each rule tell you about how the machine selects screws?

4 **a** A chair is designed for 8 mm screws. Screws that are up to 0.5 mm bigger or smaller will also work. Write a rule to select the right screws.

b A table is designed for 13 mm screws. Screws that are up to 1.2 mm bigger or smaller will also work. Write a rule to select the right screws.

c A stool is designed for 7.2 mm screws. Screws that are up to 0.25 mm bigger or smaller will also work. Write a rule to select the right screws.

5 Here is Miko's inequality for the chair from the previous problem.



What do the solutions to Miko's inequality represent?

Activity 2

Name: ..... Date: ..... Period: .....

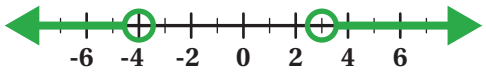
Compound Inequalities

6 Here are the graphs of the solutions to two inequalities.

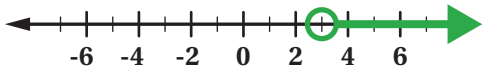
Discuss:

- How are they alike?
- How are they different?

$|2x + 1| > 7$



$2x + 1 > 7$



7 Diya started graphing all the solutions to $|2x + 1| > 7$.

a Examine Diya's work.

b What should Diya do next?

$|2x + 1| > 7$

if $x < -\frac{1}{2}$

$-(2x+1) = 7$

$2x+1 = -7$

$2x = -8$

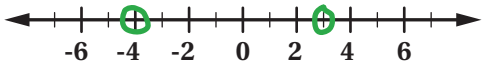
$x = -4$

if $x > -\frac{1}{2}$

$(2x+1) = 7$

$2x = 6$

$x = 3$



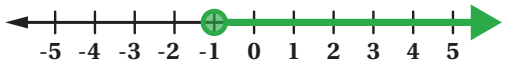
8 A **compound inequality** joins two or more inequalities with the word *and* or *or*.

a Here are two compound inequalities.

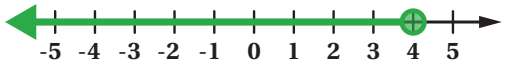
b **Discuss:**

- What do you notice about the compound inequalities?
- What do you wonder?

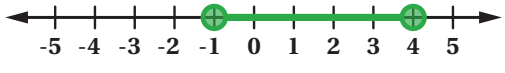
$x \geq -1$



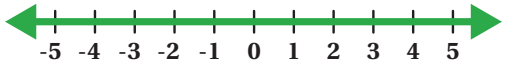
$x \leq 4$



$x \geq -1$ **and** $x \leq 4$



$x \geq -1$ **or** $x \leq 4$

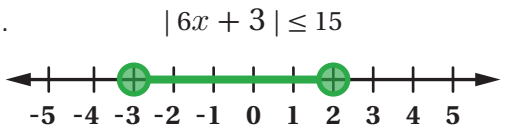


Activity 3

Name: Date: Period:

Describing Solutions

9 Here is the graph of the solutions to $|6x + 3| \leq 15$.



Which compound inequality describes the graph?

A. $x \geq -3$ or $x \leq 2$

B. $x \geq -3$ and $x \leq 2$

C. Both

D. Neither

Explain your thinking.

10 Match each absolute value inequality with its solutions. One solution will have no match.

$x > -2$ and $x < 1$	$x < -2$ or $x > 1$	$x < -1$ or $x > 2$	$x > -1$ and $x < 2$

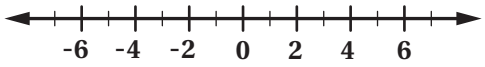
$\|4x + 2\| > 6$	$\|4x + 2\| < 6$	$\|4x - 2\| < 6$

11 Synthesis

Describe a strategy you would use to algebraically solve an absolute value inequality.

Use the example if it helps with your thinking.

$|3x - 6| \leq 9$



14 Summary 2.13

We can write the solution to an absolute value inequality using a **compound inequality** that joins two or more inequalities. Using the word "and" communicates that the solutions are located between the two boundary points on the number line where the graphs of the inequalities overlap. Using the word "or" communicates that the solutions are located to the left of one boundary point and to the right of the other.

Here is an example of absolute value inequalities with their solutions written as compound inequalities.

And

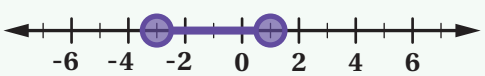
$|x + 1| \geq -2$

$x + 1 = -2 \qquad -(x + 1) = -2$

$x = -3 \qquad x = 1$

$|0 + 1| \geq -2$ ***True!***

$x \geq -3$ **and** $x \leq 1$



Or

$|x + 1| \geq 2$

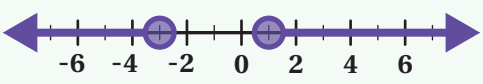
If $x \geq -1$ If $x \leq -1$

$x + 1 = 2 \qquad -(x + 1) = 2$

$x = 1 \qquad x = -3$

$|0 + 1| \geq 2$ ***False!***

$x \leq -3$ **or** $x \geq 1$

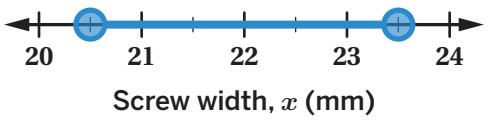


compound inequality Two or more inequalities joined together. A compound inequality can be written using symbols or the words "and" or "or".

Practice
2.13

Name: ..... Date: ..... Period: .....

1. A chair is designed for 22 mm screws. Screws that are up to 1.5 mm bigger or smaller will also work.



Screw width, x (mm)

Tasia says this is the graph of $|x - 1.5| \leq 22$.

Maria says this is the graph of $|x - 22| \leq 1.5$.

Who is correct? Explain your thinking.

2. Which of the following values are solutions to the equation $|-2x + 3| < 5$? Select *all* that apply.

- ☐ **A.** $x = -2$
- ☐ **B.** $x = 0$
- ☐ **C.** $x = 2$
- ☐ **D.** $x = 4$
- ☐ **E.** $x = 6$

3. Match each compound inequality with an absolute value inequality. One compound inequality will have no match.

$x \geq -3$ and $x \leq 2$	$x \leq -3$ or $x \geq 2$	$x \leq -2$ or $x \geq 3$	$x \geq -2$ and $x \leq 3$

$\|4x - 2\| \leq 10$	$\|4x - 2\| \geq 10$	$\|4x + 2\| \leq 10$

4. What are the solutions to $\left|\frac{1}{2}x - \frac{2}{3}\right| \geq 2$?

- **A.** $x \geq -\frac{8}{3}$ and $x \leq \frac{16}{3}$
- **B.** $x \leq -\frac{8}{3}$ or $x \geq \frac{16}{3}$
- **C.** $x \geq -\frac{16}{3}$ and $x \leq \frac{8}{3}$
- **D.** $x \leq -\frac{16}{3}$ or $x \geq \frac{8}{3}$

5. **Test Practice** An archery target is a circle where the center is elevated 120 centimeters off the ground. The bullseye in the center of the target is 14 centimeters in diameter. Write an absolute value inequality that represents the vertical height above the ground, x, that would result in a bullseye. Explain your thinking.

Name: ..... Date: ..... Period: .....

6. Tiara is trying to find the boundary points for the equation $|3x - 5| \leq 5$. She sets up the two equations $3x - 5 \leq 5$ and $3x - 5 \leq -5$. Are her equations correct? Explain your thinking.

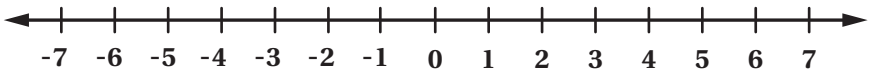
7. Create an inequality and its solutions by filling in each blank using the numbers 0 to 9 only once.

$\left|\square x - \square\right| < \square$

$x > \square$ and $x < \square$

Spiral Review

8. Graph all the solutions to the inequality $|x + 1| \geq 2$.



Problems 9–10: Here is a pattern: 3, 6, 12, 24, 48. . .

9. Determine the next two terms in the pattern.

10. What is the explicit formula for the pattern?

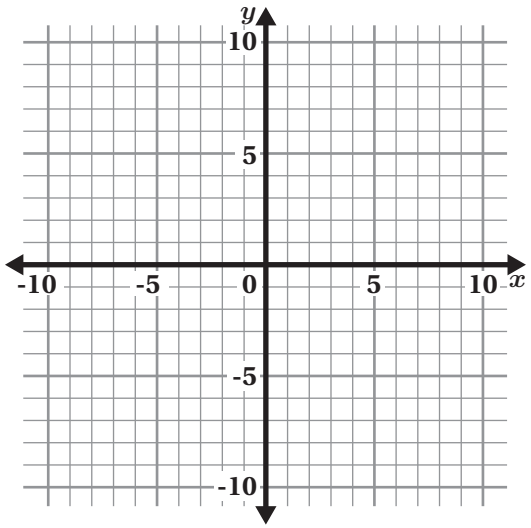
A. $a_n = 3(2)^n$

B. $a_n = 3(2)^{n-1}$

C. $a_n = 6(2)^n$

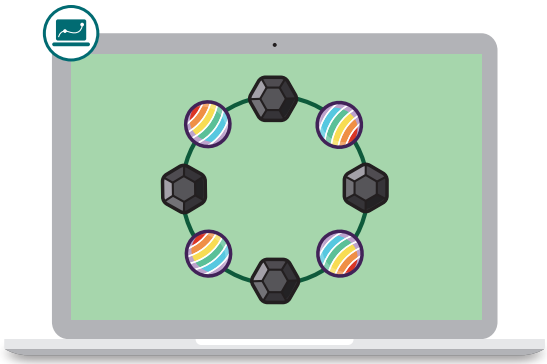
D. $a_n = 3 + (2)^n$

11. Graph the equation $2x + 3y = 6$.



Name: ..... Date: ..... Period: .....

Modeling with Functions | Systems of Equations | A–CED.3, A–REI.10, N–Q.2, SMP.1, SMP.2, SMP.3, SMP.4



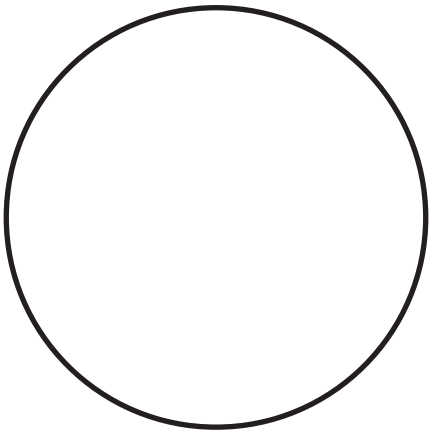
Bracelet Budgets

Let's explore solutions to two-variable inequalities graphically and symbolically.

Warm-Up

1 Draw or describe a bracelet. You can use any combination of the beads shown.

Tell us about your bracelet design.



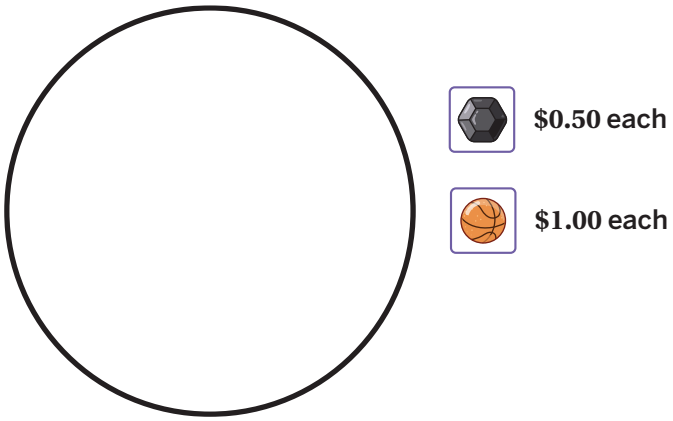
Name: ..... Date: ..... Period: .....

Modeling with Inequalities

2 Here are two types of beads:

- Black beads are $0.50 each.
- Basketball beads are $1 each.

Draw or describe a $5 bracelet.



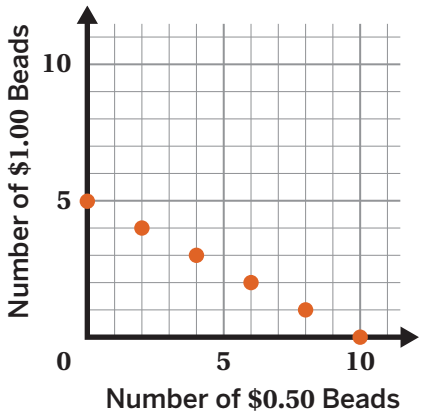
3 Each of these points represents a $5 bracelet.

- x is the number of $0.50 beads.
- y is the number of $1.00 beads.

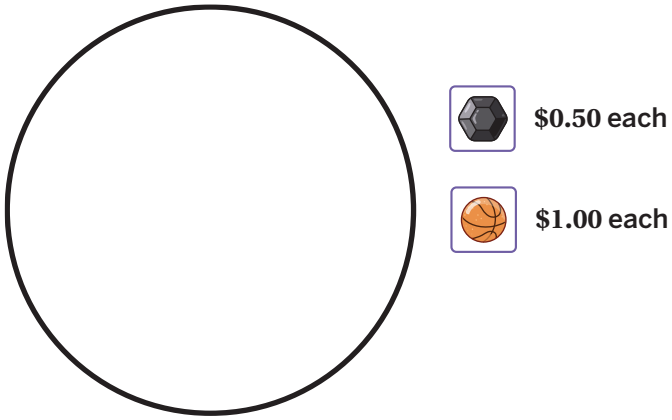
Which equation represents all the $5 bracelets? Circle one.

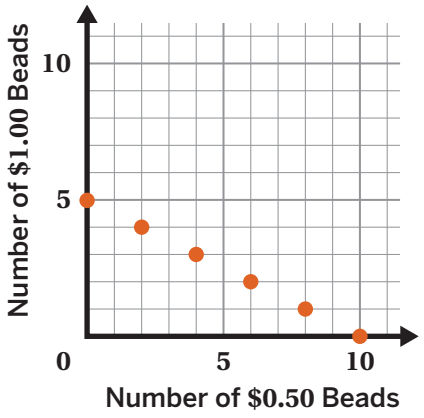
$x + y = 5$ $0.5x + y = 5$ $y = 0.5x + 5$

Explain your thinking.



4 Binta can spend $5 or less on a bracelet. Graph some bracelets that Binta could buy. Draw or describe them if it helps with your thinking.

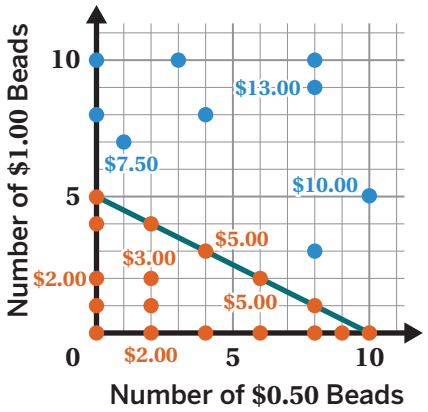




Name: ..... Date: ..... Period: .....

Modeling with Inequalities (continued)

5 Here are some bracelets that Binta could buy and some she could not buy for $5 or less.



What do you notice? What do you wonder?

I notice . . .

I wonder . . .

6 Binta can spend $5 or less on a bracelet.

- x is the number of $0.50 beads.
- y is the number of $1.00 beads.

Which statement describes all the bracelets that Binta can buy? Circle one.

$0.5x + y \leq 5$ $0.5x + y \geq 5$ $0.5x + y = 5$

Explain your thinking.

Activity 2

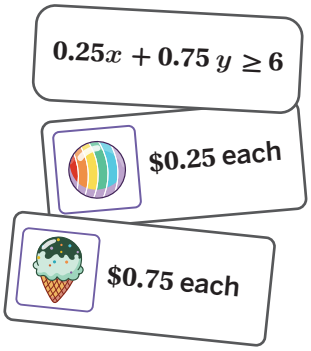
Name: Date: Period:

Solutions to Inequalities

7 Caleb loves Binta's bracelet and wants to make his own.

The inequality represents all the bracelets he can make.

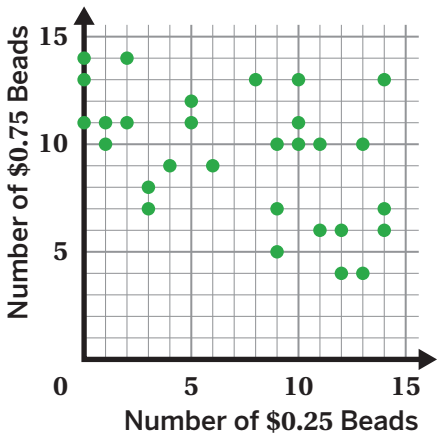
Discuss: What does each number and variable in the inequality represent about Caleb's bracelet?



8 Each of these points is a solution to $0.25x + 0.75y \geq 6$.

a Let's look at several points to see what this means.

b In your own words, what is a solution to an inequality with two variables?



9 Is $(2, 10)$ also a solution to $0.25x + 0.75y \geq 6$? Circle one.

Solution Not a solution I'm not sure

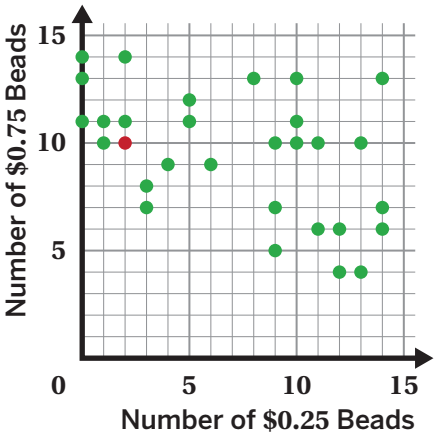
Explain your thinking.

Name: ..... Date: ..... Period: .....

Solutions to Inequalities (continued)

10 Here is how two students determined that $(2, 10)$ is a solution to $0.25x + 0.75y \geq 6$.

Marco



Jada

$0.25x + 0.75y \geq 6$

$0.25(2) + 0.75(10) \geq 6$

$0.50 + 7.50 \geq 6$

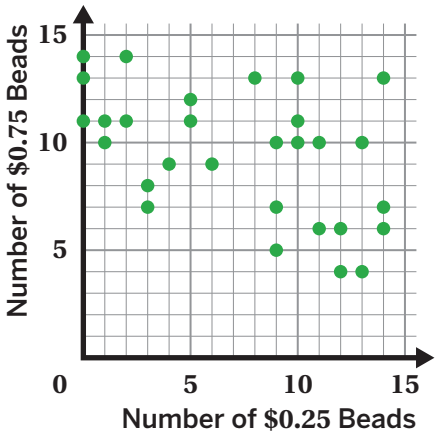
$8 \geq 6$

Explain each person's strategy to a partner.

11 The graph shows some solutions to $0.25x + 0.75y \geq 6$.

Select *all* of the other points that are also solutions.

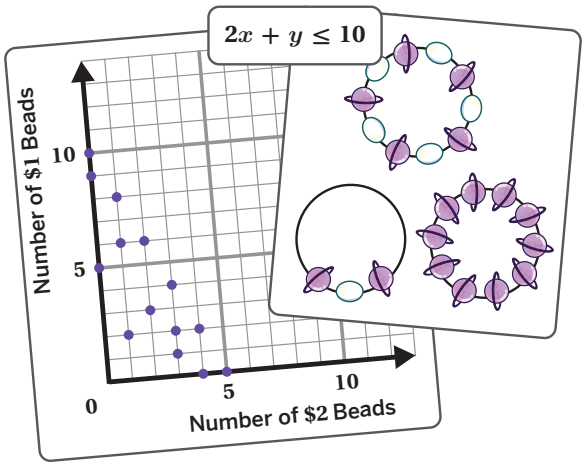
- ☐ **A.** $(14, 8)$
- ☐ **B.** $(13, 1)$
- ☐ **C.** $(7, 5)$
- ☐ **D.** $(1, 13)$
- ☐ **E.** $(0, 8)$



12 Synthesis

Discuss: How can you tell if a point is a solution to a two-variable inequality?

Use the example if it helps with your explanation.



15 Summary 2.14

The solutions to a two-variable inequality are all of the ordered pairs that make the inequality true. Graphs can help us visualize all of these solutions. To check if an ordered pair is a solution symbolically, you can substitute the x- and y-values into the inequality to see if it makes the inequality true.

Here is an example of how you can determine if an ordered pair is a solution to an inequality:

Marco is making bracelets. Each bracelet needs to cost no more than $\$10$. Planet beads cost $\$1$ and oval beads cost $\$2$. Marco wants to know if he can make a bracelet with 3 planet beads and 4 oval beads.

To check, Marco looked at the graph of this situation to see if the point $(4, 3)$ was in the solution region, but he wasn't sure. Marco substituted 4 and 3 into the inequality $2x + y \leq 10$:

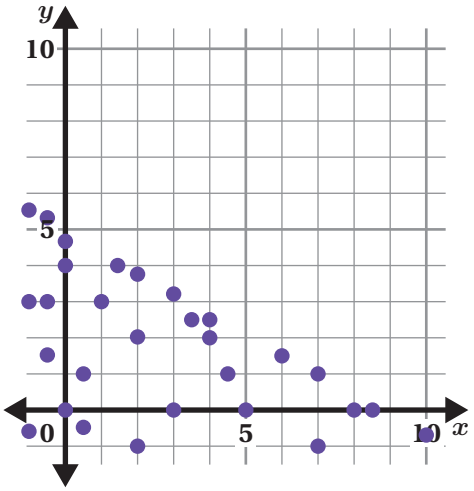
$$\begin{aligned} 2(4) + (3) &\leq 10 \\ 8 + (3) &\leq 10 \\ 11 &\leq 10 \quad \textit{False}! \end{aligned}$$

That means Marco *cannot* make a bracelet with 3 planet beads and 4 oval beads while staying within his budget.

Practice
2.14

Name: ..... Date: ..... Period: .....

1. **Test Practice** This graph shows some solutions to $5x + 9y < 45$. Select *all* of the points that are also solutions.



- ☐ **A.** $(1, 1)$
- ☐ **B.** $(4, 0)$
- ☐ **C.** $(10, 4)$
- ☐ **D.** $(0, 10)$
- ☐ **E.** $(6, -1)$

2. Write at least *three* coordinate pairs that are solutions to the inequality $x \leq y$.

Problems 3–4: Tyler can spend up to $\$45$ on hats and socks. A hat costs $\$10$ and a pair of socks costs $\$2.50$.

- h is the number of hats.
- s is the number of pairs of socks.

3. **Test Practice** Which inequality represents this situation?

A. $10h + 2.50s > 45$

B. $10h + 2.50s < 45$

C. $10h + 2.50s \geq 45$

D. $10h + 2.50s \leq 45$

4. Explain how you know that $h = 2$ and $s = 1$ are solutions to this situation.

5. Marc is making a bracelet for his sister using beads that cost $\$0.50$ and $\$0.75$. He cannot spend more than $\$8.00$ on the bracelet.

- x is the number of $\$0.50$ beads.
- y is the number of $\$0.75$ beads.

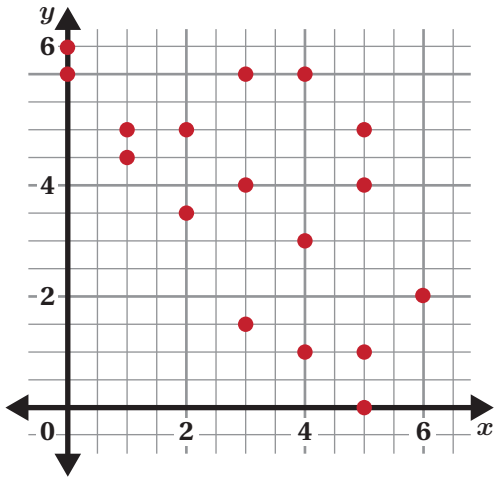
Marc says that the inequality $0.5x + 0.75y \geq 8$ represents all the bracelets he can make. Do you agree with him? Explain your thinking.

Name: ..... Date: ..... Period: .....

6. Write an ordered pair for a point that is *not* a solution to $x + 3y \geq 24$.

7. This graph shows some solutions to $3x + 2y \geq 12$.

Write an ordered pair for *another* point that is a solution to this inequality.



Spiral Review

Problems 8–9: Write an inequality for each situation.

8. Duri will stay warm in their sleeping bag when the temperature is at least 30°F. Use t to represent temperatures at which Duri will stay warm.

9. Duri wants their backpack to weigh less than 45 pounds. Use w to represent the weight of the backpack.

Problems 10–11: Here is an equation: $6x + 2y = 36$.

10. For each value of x, determine the value of y.

x	y
2	
4	

11. Which equation represents the same relationship?

A. $y = 6 - 3x$

B. $y = 18 - 3x$

C. $y = 18 - \frac{1}{3}x$

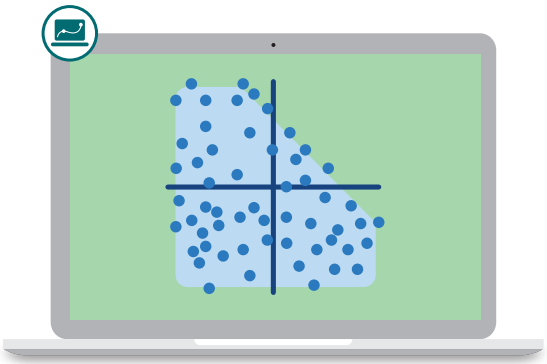
D. $y = 6 - \frac{1}{3}x$

Name: ..... Date: ..... Period: .....

Systems of Equations A–REI.12, F–IF.7, SMP.3, SMP.6, SMP.8

All of the Solutions

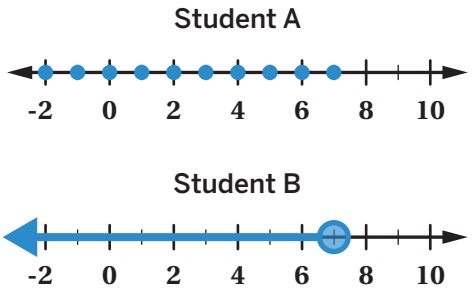
Let's represent all of the solutions to two-variable inequalities graphically.



Warm-Up

1 Two students created graphs of the *solutions* to $x \leq 7$.

How are their graphs alike? How are they different?



Alike:

Different:

Activity 1

Name: ..... Date: ..... Period: .....

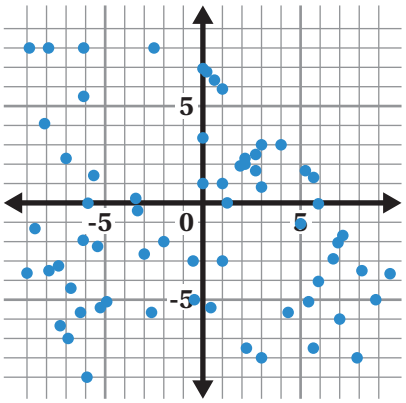
Some to All

2 Write three solutions to $x + y \leq 7$. Try thinking of x- and y-values that no one else will!

Solution 1 (x, y)	Solution 2 (x, y)	Solution 3 (x, y)

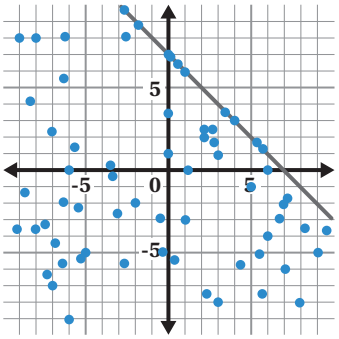
3 Here is a graph of some of the solutions to $x + y \leq 7$.

Sketch what you think the graph of *all* the solutions to $x + y \leq 7$ looks like.

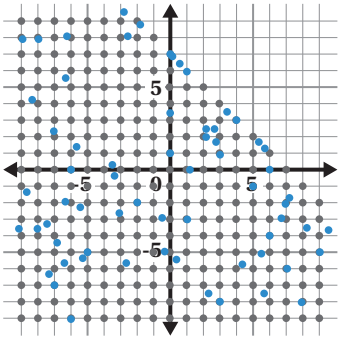


4 Here are Pablo's, Deven's, and Adhira's sketches of *all* the solutions to $x + y \leq 7$.

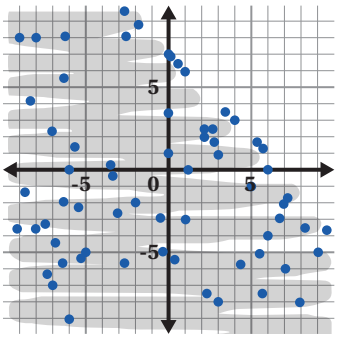
Pablo



Deven



Adhira



a. Select *one* sketch.

b. **Discuss:** What do you like about this sketch? What would you change?

Name: ..... Date: ..... Period: .....

Shading the Solutions

5 The Environmental Club is raising money for additional waste and recycle bins around campus. A few students are making "school spirit" bracelets with beads in school colors that cost \$1 each and some additional donated clear beads. It must cost less than \$7 to make each bracelet for the fundraiser to be profitable.

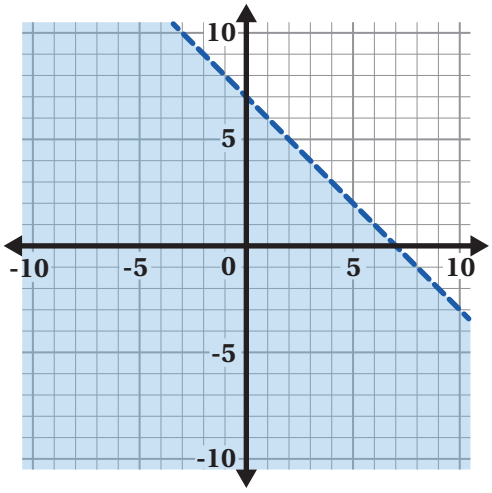
a Let's watch an animation to see what a graph of all solutions looks like.

b How does this graph represent all the solutions to $x + y \leq 7$?

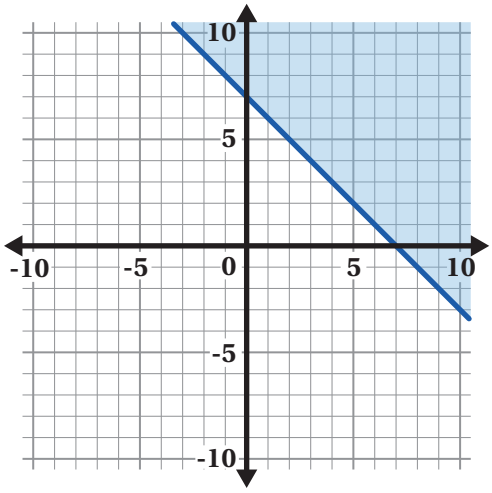
c Explain why some of the solutions do not make sense in terms of the bracelets?

6 Here are the graphs of two other inequalities.

$x + y < 7$



$x + y \geq 7$



Discuss:

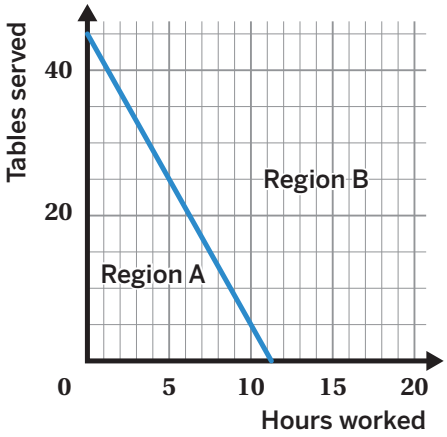
- How are the graphs of the two inequalities different?
- What do you think the **boundary line** represents?

Activity 3

Name: Date: Period:

Which Region?

7 Rebecca is working as a server at a local cafe after school. She needs to earn enough money next week to pay her monthly cell phone bill of $180. Rebecca makes $16 per hour of work and an average of $4 in tips per table served. Here is the graph of $16x + 4y = 180$.



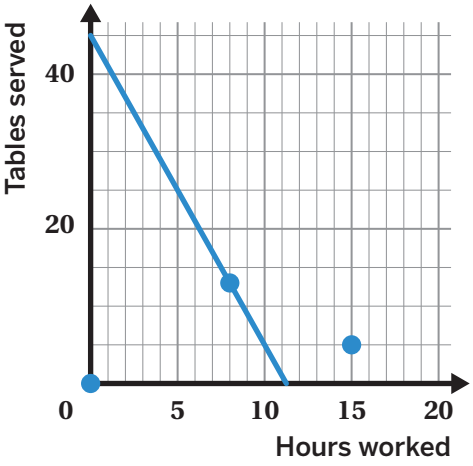
Where will the solutions to $16x + 4y \geq 180$ be? Circle one

Region A Region B I'm not sure

Explain your thinking.

8 Rebecca thought that checking points would help her decide how to graph the solutions to $16x + 4y \geq 180$.

She chose the points $(0, 0)$, $(8, 13)$, and $(15, 5)$.

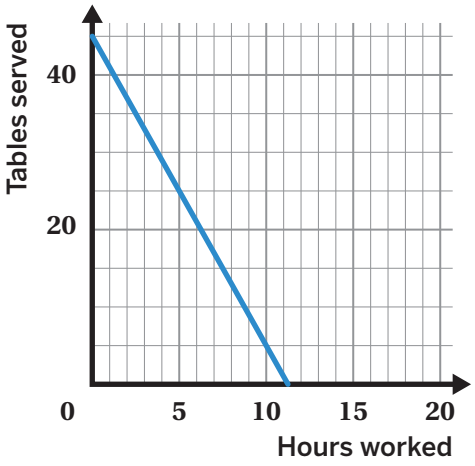


a. **Discuss:** Why do you think Rebecca chose these points?

b. Select *all* the points that are solutions to $16x + 4y \geq 180$.

☐ A. $(0, 0)$ ☐ B. $(8, 13)$ ☐ C. $(15, 5)$

9 Rebecca is trying to determine how many hours she needs to work.



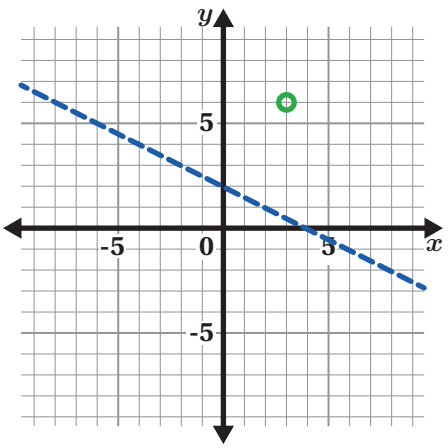
a. Graph *all* the solutions to $16x + 4y \geq 180$.

b. If Rebecca works 5.5 hours, how many tables does she need to serve to make at least $180?

Name: ..... Date: ..... Period: .....

Which Region? (continued)

10 Nathan is graphing the solutions to $x + 2y < 4$.



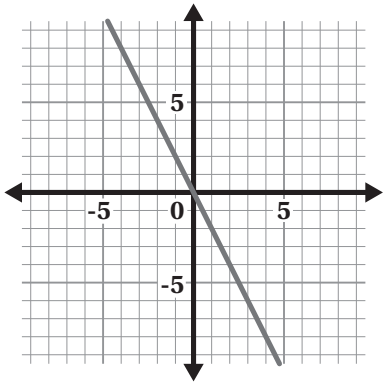
a. **Discuss:** Why is his line dashed?

b. Nathan determined that $(3, 6)$ is *not* a solution.

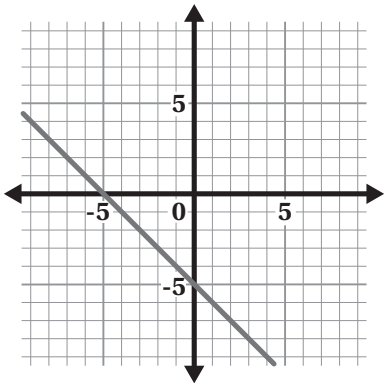
Does he have enough information to graph all the solutions?

11 Graph all the solutions to the following inequalities. The graph of each corresponding equation has been given to you.

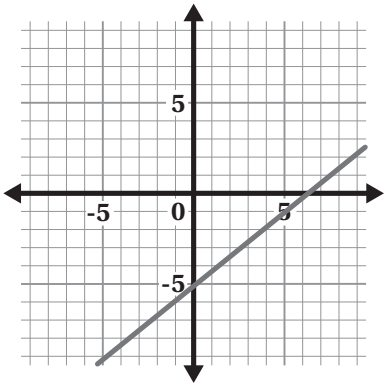
$2x + y \leq 0$



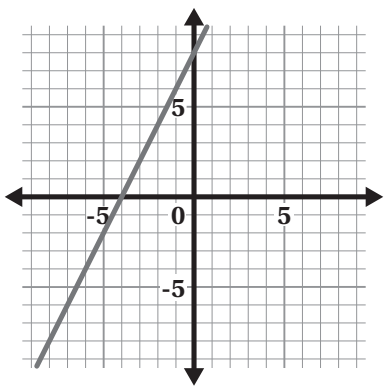
$x + y > -5$



$5x - 6y < 30$



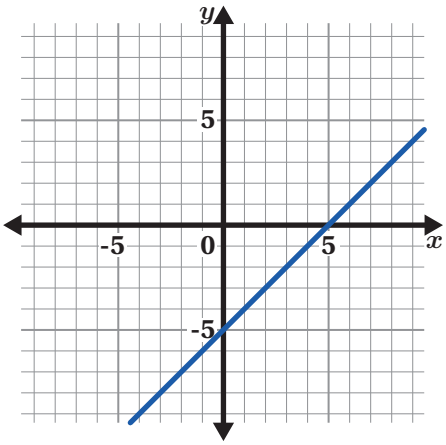
$-2x + y \geq 8$



12 Synthesis

Discuss: Here is the graph of $x - y = 5$

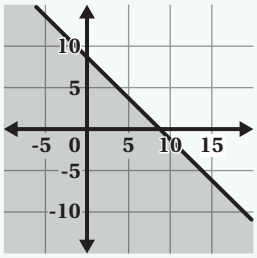
Describe a strategy for graphing all the solutions to $x - y < 5$.



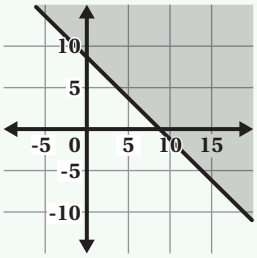
15 Summary 2.15

The solutions to a two-variable linear inequality can be represented on a graph as a *half-plane*. A **boundary line** separates the plane into the region that contains solutions and the region that does not. The shaded area represents all of the solutions, which are the values of (x, y) that make the inequality true.

$x + y \leq 9$

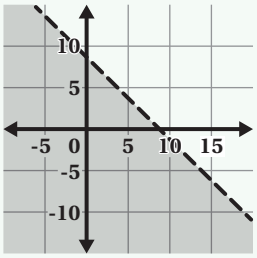


$x + y \geq 9$

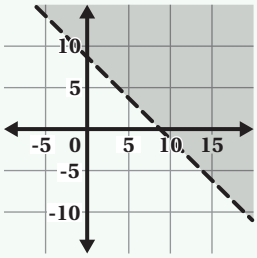


A solid line means that the points on the boundary line *are* included in the solutions. This is represented by the ≤ and ≥ symbols.

$x + y < 9$



$x + y > 9$



A dashed line means the points on the boundary line are *not* included in the solutions. This is represented by the < and > symbols.

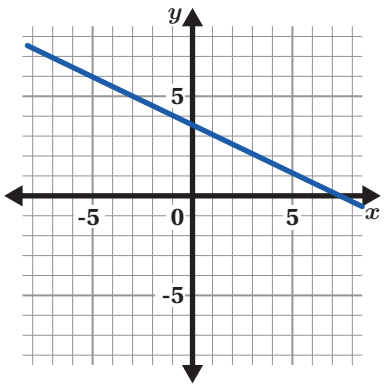
To determine which of the half-planes is the *solution region*, you can test points on either side of the *boundary line* to see whether they make the inequality true or false.

boundary line The line that separates the solution region of a linear inequality from non-solutions. A linear inequality (e.g., $y < 2x + 5$) has a boundary line that is represented symbolically by the corresponding equation (e.g., $y = 2x + 5$). A solid boundary line indicates that these points are included in the solution set (e.g., $y \leq x$). A dashed boundary line indicates that they are not (e.g., $y < x$).

Practice
2.15

Name: ..... Date: ..... Period: .....

1. Here is a graph of the equation $x + 2y = 7$.



Which of these points is a solution to the inequality $x + 2y < 7$?

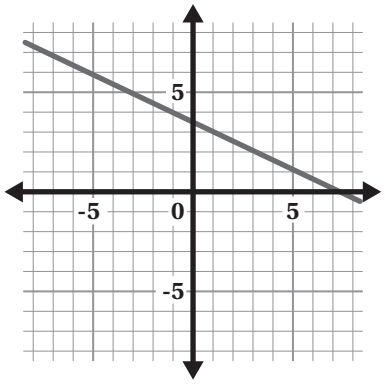
A. $(0, 0)$

B. $(10, 0)$

C. $(7, 0)$

D. $(0, 7)$

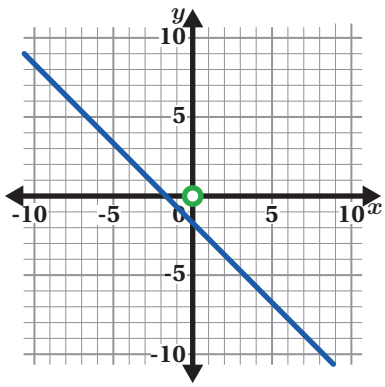
2. Graph all the solutions to $x + 2y < 7$. Explain your thinking.



3. Here is an inequality: $x + y \leq -2$.

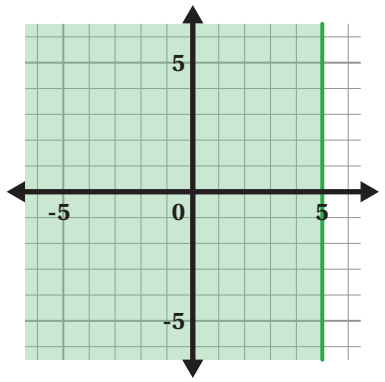
- Ada graphed the equation $x + y = -2$.
- Ada noticed that $(0, 0)$ is not a solution to $x + y \leq -2$.

How can Ada use this information to graph the solutions to this inequality?



4. Brianna is creating a graph for the inequality $2x - 5y > 10$. She says that since the inequality has a greater-than symbol, she should shade the region above the line $2x - 5y = 10$. Is Brianna correct? Explain your thinking.

5. **Test Practice** Which inequality is shown on the graph?



A. $y \leq 5$

B. $y \geq 5$

C. $x \geq 5$

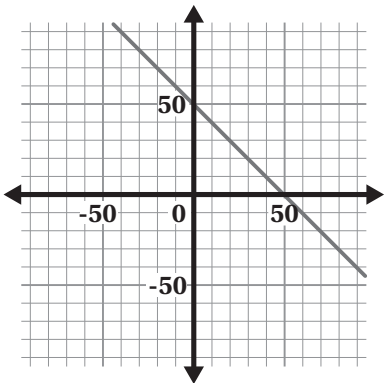
D. $x \leq 5$

Practice
2.15

Name: ..... Date: ..... Period: .....

Problems 6–7: A food truck only sells hot dogs and hamburgers. They want to sell 50 items or more each day.

- x represents the number of hot dogs sold.
- y represents the number of hamburgers sold.



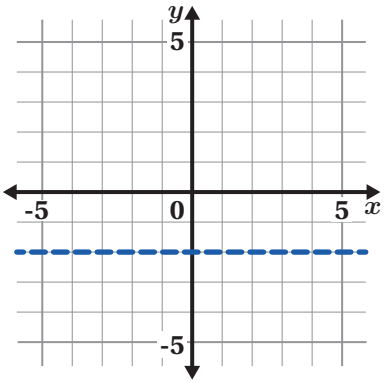
6. Which inequality represents this situation?

A. $x + y > 50$ **B.** $x + y < 50$

C. $x + y \geq 50$ **D.** $x + y \leq 50$

7. Complete the graph so that it represents all the solutions to the inequality for this situation.

8. Lucy started to graph the inequality $y > -2$ by graphing a dashed line at $y = -2$. How might Lucy decide where to shade?



Spiral Review

9. The equation of line a is $y = -2x - 1$. Select all the points that are on line a.

☐ **A.** $(-2, 3)$ ☐ **B.** $(0, 4)$ ☐ **C.** $(1, -3)$

☐ **D.** $(-1, -3)$ ☐ **E.** $(2, 5)$

10. Lola can spend up to \$15 on pens and notebooks. A pen costs \$2 and a notebook costs \$1.50. Using p for the number of pens and n for the number of notebooks, write an inequality that represents this situation.

11. A golf ball weighs 1.6 ounces and an empty metal bucket weighs 12 ounces. Neel adds golf balls one at a time to the empty metal bucket. How many golf balls will be in the bucket when the total weight is 20 ounces?

12. Solve for y in the equation $5x + 4y = 100$.

Name: ..... Date: ..... Period: .....

Modeling with Functions | Systems of Equations | A–CED.3, A–REI.10, A–REI.12, F–IF.7, N–Q.1, N–Q.2, N–Q.3, SMP.1, SMP.4, SMP.7

Charity Concerts

Let's represent constraints by graphing two-variable inequalities in context.

Warm-Up

Here is a situation with hidden information. Let's make sense of it together as a class.

The Funk-tions want to raise at least $..... for charity by holding multiple concerts at SoundZone. Each concert at SoundZone costs the band $..... They typically sell tickets for $..... each.

1. Read this information three times.
2. For each blank, choose a value that makes sense. (You can be as accurate as you want to be.) Record your values in this table.

Amount to Raise for Charity ($)	Cost of SoundZone Venue ($)	Price of Each Ticket ($)

3. For the values you chose, will the band reach their goal if they hold 2 concerts and sell 100 total tickets? Explain your thinking.

Activity 1

Name: Date: Period:

SoundZone Concerts

The Funk-tions want to raise at least $\$2{,}000$ for charity by holding multiple concerts at SoundZone.

- Each concert at SoundZone costs the band $\$500$.
- They typically sell tickets for $\$20$ each.

4. Will the band reach their goal if they hold 2 concerts and sell 100 tickets total?

5. The band manager wrote the equation $-500c + 20t = 2000$. Explain what each part of the equation represents.

c:

t:

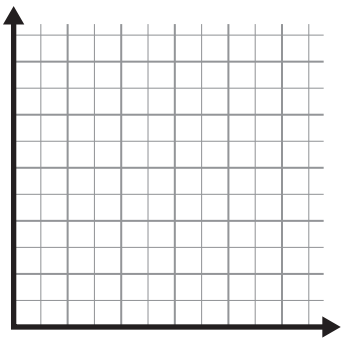
-500:

20:

$2{,}000$:

6. Graph the band manager's equation. Be sure to label and scale each axis. Complete the table of values for the band manager's equation if it helps with your thinking.

Number of Concerts, c	Number of Tickets Sold, t



7. Write an inequality that represents all the combinations of concerts and tickets that would raise at least $\$2{,}000$ for charity.

8. Shade the region on the graph that represents all the solutions to the inequality you wrote. What does the solution mean in this situation?

9. Are all the solutions to the inequality realistic in this context? Explain your thinking.

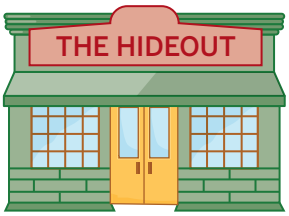
10. Write a question that the Funk-tions could answer using the graph.

Activity 2

Name: ..... Date: ..... Period: .....

Which Venue?

The Funk-tions are considering two other venues for their charity concerts.



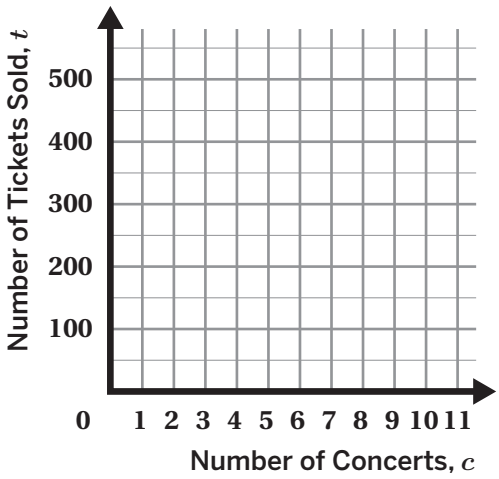
Cost per concert: $250
Ticket price: $10

Cost per concert: $4,000
Ticket price: $40

In this activity, you will choose one venue to explore. Your partner will choose the other.

11. Write the name of the venue you chose. Then write an inequality that represents the number of concerts and tickets sold that would raise at least $2,000 at that venue.

12. Graph the solutions to your inequality.



13. Explain how you determined the boundary line and the shading.

14. **Discuss:** Consider your and your partner's graphs. How are they alike? How are they different?

Name: ..... Date: ..... Period: .....

Which Venue? (continued)

15. For each situation, work with your partner to determine which concert venue you would recommend. Explain your thinking.

a 1 concert and 200 tickets total.

The Hideout

Palace Arena

b 5 concerts and 400 tickets total.

The Hideout

Palace Arena

c 2 concerts and 250 tickets total.

The Hideout

Palace Arena

16. Victor knows that $(2.5, 500)$ represents a solution to both the Hideout inequality and the Palace Arena inequality but says it is not realistic. Is Victor correct? How do you know?

Synthesis

17. A group of students is installing a garden at their school.

- A vegetable bed will cost $\$15$ per square foot to install.
- A flower bed will cost $\$12$ per square foot to install.
- Their budget for the project is $\$300$.

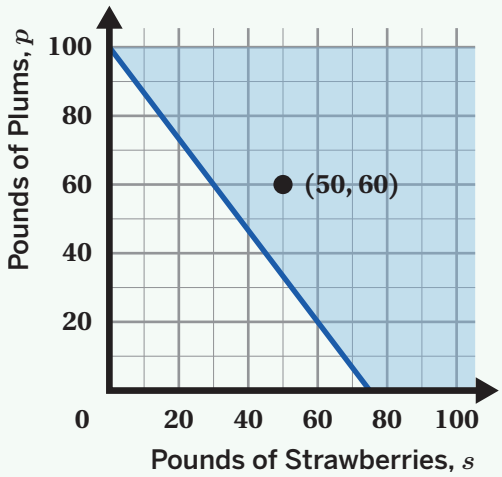
Discuss: Describe how to define variables and write an inequality that represents this context.

Summary 2.16

Looking at solutions to two-variable inequalities on a graph can help us make sense of different situations.

Here is an example: Angel makes $\$4$ per pound of strawberries and $\$3$ per pound of plums that are sold. The inequality $4s + 3p \geq 300$ represents the pounds of strawberries, s, and pounds of plums, p, that Angel needs to sell to meet a goal of making at least $\$300$.

- To determine the solutions to the inequality, graph the corresponding equation
- $4s + 3p = 300$. Decide whether the points on the line will reach the goal by looking at the original inequality symbol.
- Then test a value, such as $(50, 60)$, to identify the solution region. $4(50) + 3(60) \geq 300$ ***True!***



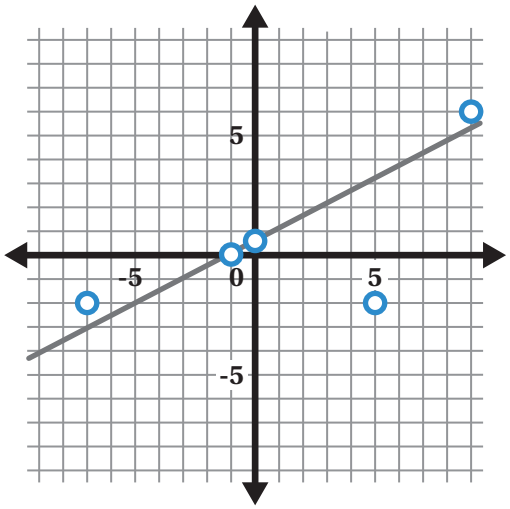
Because the point $(50, 60)$ makes the inequality true, the *half-plane* that includes $(50, 60)$ is the solution region. So any combination of strawberries and plums in the shaded region, including those on the line, would meet Angel's goal.

But not all solutions to the inequality will make sense for the situation. For example, the point $(90, -20)$ makes the inequality true, but it doesn't make sense for Angel to sell -20 pounds of plums.

Practice
2.16

Name: ..... Date: ..... Period: .....

Problems 1–2: Here is an inequality: $2y - x > 1$.



1. Select *all* the points that are solutions to this inequality.

- ☐ **A.** $(-7, -2)$
- ☐ **B.** $(-1, 0)$
- ☐ **C.** $(0, 0.5)$
- ☐ **D.** $(5, -2)$
- ☐ **E.** $(9, 6)$

2. Graph all the solutions to $2y - x > 1$.

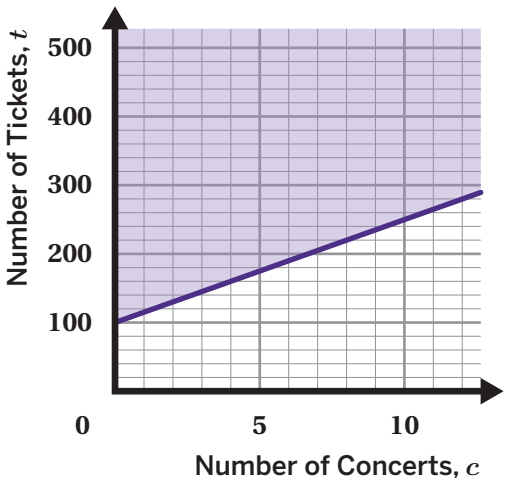
3. Jasmine says that $(3, 10)$ is a solution to $-2x + 4y < 15$. Is she correct? Explain how you know.

4. Mia is buying snacks for a movie night with her friends. Her budget is $\$60$.

- A package of snack mix costs $\$5.00$.
- A package of popcorn costs $\$2.50$.

Write an inequality that represents all the snacks that Mia can buy for $\$60$ or less. Use s for packages of snack mix and p for packages of popcorn.

Problems 5–6: The Funk-tions are looking into one more venue for their charity concerts. The Depot charges $\$300$ per concert and tickets cost $\$20$ each. The Funk-tions want to raise at least $\$2{,}000$. The graph shows the solution set of $-300c + 20t \geq 2000$.



5. Write *one* solution to the inequality.

6. If the Funk-tions have 4 concerts and sell 200 tickets, will they reach their fundraising goal? Explain how you know.

Practice
2.16

Name: ..... Date: ..... Period: .....

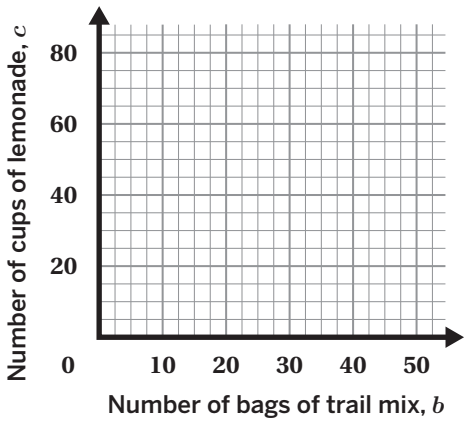
Problems 7–9: Aditi's soccer team is selling bags of trail mix for \$3 each and cups of lemonade for \$2 each. To make a profit, they need to earn more than \$120.

7. **Test Practice** b represents the number of bags of trail mix and c represents the number of cups of lemonade. Which inequality describes all the ways Aditi's soccer team can make a profit?

A. $3b + 2c > 120$ **B.** $3b + 2c < 120$ **C.** $3b + 2c \geq 120$ **D.** $3b + 2c \leq 120$

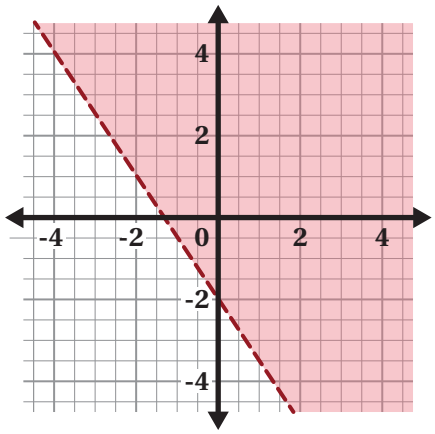
8. Graph all of the solutions to the inequality you chose.

9. Explain how you could check if the boundary is included or not included from the solution region.



Spiral Review

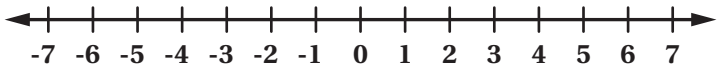
10. The graph shows the solution set to an inequality. Select *all* the inequalities that produce this solution set.



- ☐ **A.** $y < -1.5x - 2$
- ☐ **B.** $y > -1.5x - 2$
- ☐ **C.** $1.5x + y > -2$
- ☐ **D.** $y > 1.5x + 2$
- ☐ **E.** $y \leq 1.5x - 2$

11. Create an inequality using x that has the solution set shown on the graph.

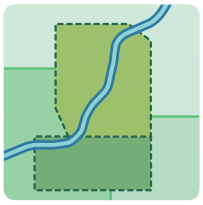
12. Graph all the solutions to the inequality $|x + 2| < 3$.



Name: ..... Date: ..... Period: .....

Modeling with Functions | Systems of Equations | A–CED.3, A–REI.12, F–IF.7, SMP.1, SMP.3, SMP.4

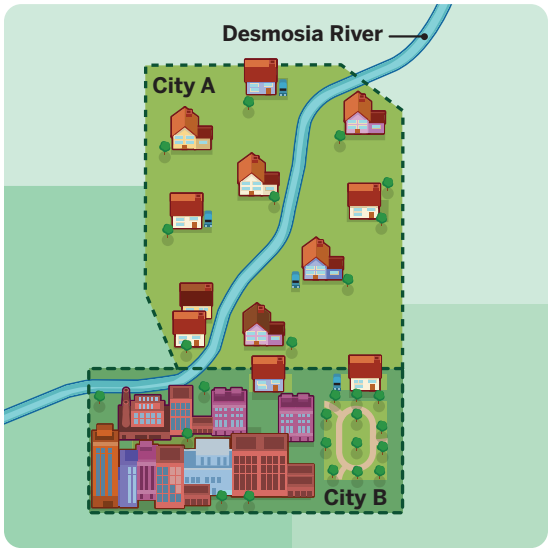
Water Way



Let's use two-variable inequalities to make decisions.

Warm-Up

1. Study the map. What do you notice? What do you wonder?



I notice . . .

I wonder . . .

Activity 1

Name: ..... Date: ..... Period: .....

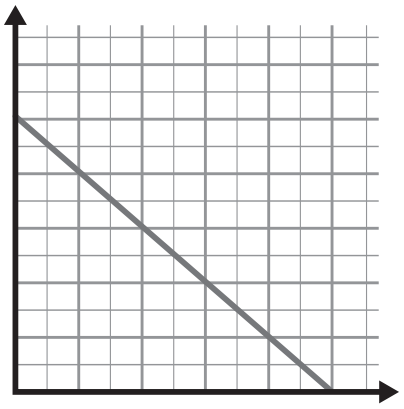
The Desmosia River Agreement

Each year, two cities share up to 500 billion gallons of water that flow from the Desmosia River. As the head of water management, your task is to decide how to fairly split the water between City A and City B next year.

2. Write an inequality that represents all the ways you could distribute the water.

- Use x to represent the billions of gallons of water for City A.
- Use y to represent the billions of gallons of water for City B.

3. Graph the solutions to your inequality. Be sure to label and scale your axes.



4. How would you fairly distribute the water?

Water for City A (billions of gallons)	Water for City B (billions of gallons)

Explain your thinking.

5. **Discuss:** Does each combination represent a realistic distribution of water?

A	B	C	D	E
(0, 500)	(200.5, 200.5)	(-100, 500)	(250, 150)	(150, 250)

6. Which distribution of water is most fair? Explain your thinking.

7. What information would help you make a more fair decision?

Name: ..... Date: ..... Period: .....

Information Cards

8. Each person in your group will use an information card to learn more about the situation from Activity 1.

a. Take turns reading the information on your card to your group.

b. **Discuss:** How could this help you make a more fair decision?

When you have finished reading and discussing your cards, answer the following questions.

9. Is your original decision still fair for both cities? Explain your thinking.

10. As a group, decide the most fair way to distribute the water.

Water for City A (billions of gallons)	Water for City B (billions of gallons)

Explain your thinking.

Activity 3

Name: ..... Date: ..... Period: .....

Perspectives

City A wants to build a dam that will create electricity for both cities and store up to 150 billion gallons of water to help with future droughts.

Here are two people that live in City A and City B.

Dalia — City A	Ricardo — City B
Dalia is a farmer in City A. She grows crops on a farm that borders the Desmosia River. Most of her crops are sold to restaurants and grocery stores in City B. 	Ricardo lives in a large apartment building in City B. His water bill has been increasing each year. If City B receives more water, the cost of water for all its residents will go down. 

11. Given that City A wants to build a dam that will support both cities, how do you think Dalia and Ricardo may feel about your group's decision from Activity 2? Explain your thinking.

12. How do you think the environment and animals in the area could be affected by the construction of the dam?

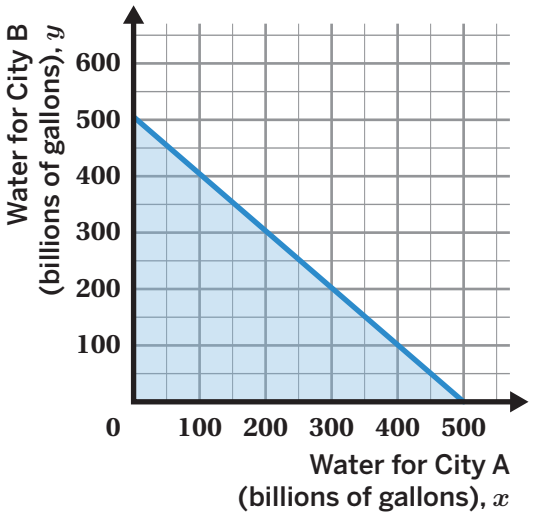
13. Based on how the dam will impact the two cities and the environment, would you recommend that City A builds the dam? Explain your thinking.

Synthesis

This graph shows all the solutions to $x + y \leq 500$.

14. Plot a solution on the graph that is *not* a realistic way that City A and City B could distribute the water.

15. Explain why your point is not realistic in this situation.



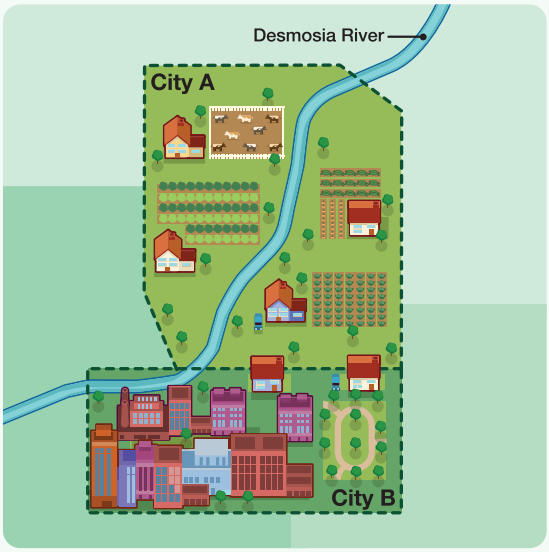
Summary 2.17

We can use two-variable linear inequalities to model real-world constraints and make sense of issues in society, such as fair water distribution.

When modeling, we often make an initial decision based on the information and variables we're given. But we might find additional variables to consider when we look deeper.

In the example of fair water distribution, some additional variables to consider are:

- The population of each city.
- The amount of land in each city.
- The amount of water each city uses.
- Any future events that will change the river's flow.

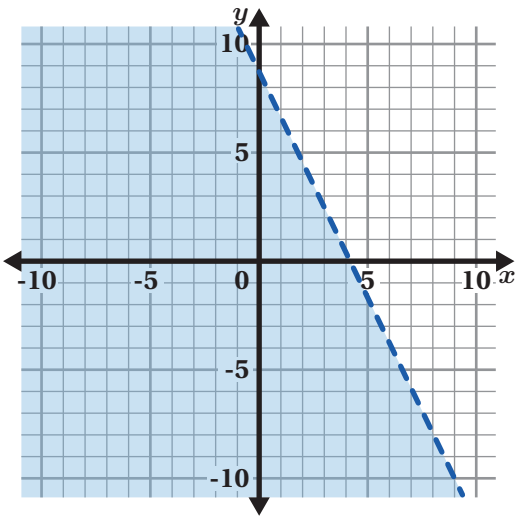


When we consider additional variables, we have the opportunity to revise our initial model to make it better and more precise. Additional variables, along with empathy, personal experiences, listening to the experiences of others, and considering the environmental impact can help us make better models that meet the needs of the diverse groups of individuals who make up a community.

Practice
2.17

Name: ..... Date: ..... Period: .....

1. Here is the graph of $y < 8 - 2x$. Select *all* the points that are solutions to the inequality.



- ☐ **A.** $(-5, 0)$
- ☐ **B.** $(10, 2)$
- ☐ **C.** $(4, 0)$
- ☐ **D.** $(-2, -6)$
- ☐ **E.** $(5, 5)$

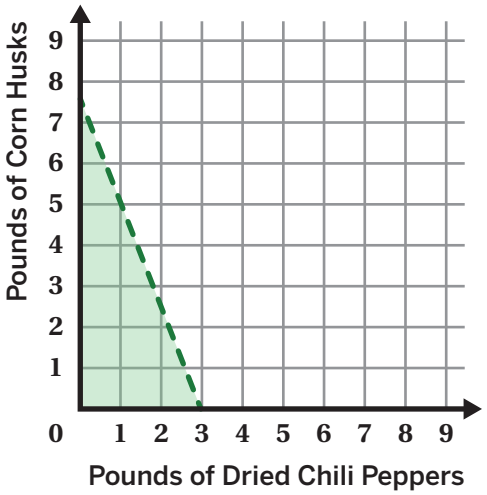
Problems 2–3: Gabriel is selling tickets to a fundraising concert for the school band. His goal is to raise more than $\$145$. Student tickets, s, cost $\$3$ each. Adult tickets, a, cost $\$5$ each.

2. **Test Practice** Which inequality represents the number of tickets Gabriel could sell to achieve his goal?

A. $3s - 145 > 5a$ **B.** $3s + 5a \geq 145$ **C.** $3s + 5a \leq 145$ **D.** $3s + 5a > 145$

3. Is $s = 10000$ and $a = 30000$ a reasonable solution in this situation? Explain your thinking.

Problems 4–6: Ramon is buying ingredients to make tamales. This graph represents the pounds of dried chili peppers and pounds of corn husks he can buy for less than $\$50$.



4. Can Ramon buy 2 pounds of dried chili peppers and 4 pounds of corn husks for less than $\$50$?

5. Can Ramon buy 1.5 pounds of dried chili peppers and 3 pounds of corn husks for less than $\$50$?

6. Explain how you determined your choices.

Practice
2.17

Name: ..... Date: ..... Period: .....

Problems 7–8: Naoki and Tariq rented an electric scooter for a total of 60 minutes to explore the city they are visiting. Let n represent the number of minutes Naoki rides the scooter and t represent the number of minutes Tariq rides the scooter.

7. Write an inequality that represents all the ways Naoki and Tariq could share the total time they have to ride the scooter.

8. Determine a solution to your inequality that you think is fair to Naoki and Tariq. Explain your thinking.

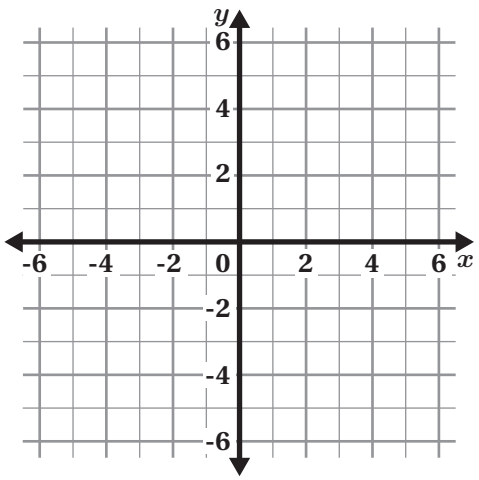
9. Here's a square. Write up to four different inequalities to create a design.

Inequality 1: .....

Inequality 2: .....

Inequality 3: .....

Inequality 4: .....



Spiral Review

10. Select *all* the equations that are equivalent to $3x + 5 = 20 - x$.

- ☐ **A.** $4x = 15$
- ☐ **B.** $2x = 25$
- ☐ **C.** $-4x + 20 = -5$
- ☐ **D.** $4x - 15 = 0$
- ☐ **E.** $x - 20 = 5 + 3x$

Problems 11–12: Solve for y.

11. $-6x + 3y = 24$

12. $2 - 6x = 4 + 4y$

Name: ..... Date: ..... Period: .....

Comparing Models | Modeling with Functions | Systems of Equations | Financial Literacy

A–CED.3, A–CED.4, A–REI.3, A–REI.10, A–REI.12, SMP.2

Practice Day 2

Let's practice what you've learned so far in this unit!

Problems 1–5: Solve each equation or inequality. Show your thinking for each problem.

1. $3x - 1 = 4(x + 2)$

2. $3 = 6x + 11 + 2x$

3. $-4(3m - 1) \leq 2m - 24$

4. Solve for y:
$y - 18x = 12$

5. Solve for h:
$9y = 6x - 3h$

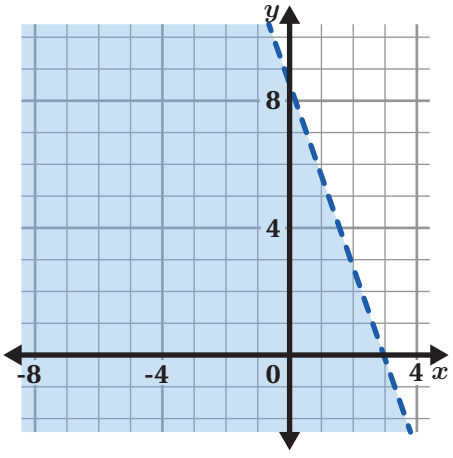
6. Which inequality matches the graph?

A. $3x + y > 9$

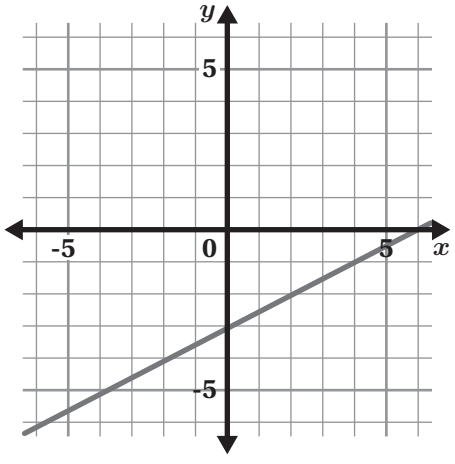
B. $3x + y < 9$

C. $x - 9y > 3$

D. $x + 9y < 3$



7. Graph all the solutions to the inequality:
$-2x + 4y \leq -12$.



Name: ..... Date: ..... Period: .....

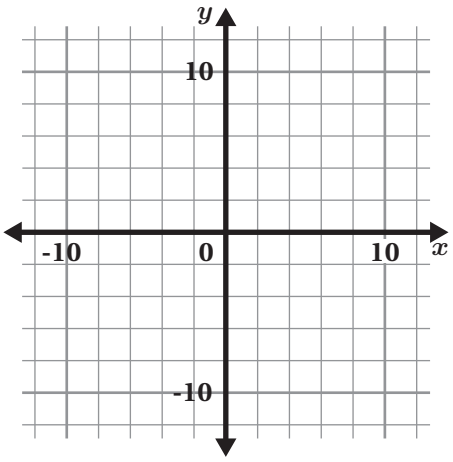
Practice Day 2

8. Here is an inequality: $2x + y < 8$.

a Select *all* the coordinate pairs that are solutions to the inequality.

- ☐ **A.** $(0, 0)$
- ☐ **B.** $(5, -2)$
- ☐ **C.** $(-1, 4)$
- ☐ **D.** $(-3, -5)$
- ☐ **E.** $(7, 7)$

b Graph all the solutions to the inequality.



9. The city of Metropolis holds an outdoor concert every summer. They sell two kinds of tickets: lawn tickets and seat tickets. This summer, they sold fewer than 400 tickets in total.

a Write an inequality that describes this situation. Describe what each variable represents.

b Write the coordinates of one solution to the inequality and explain what it represents in this situation.

10. At a concert in Metropolis, lawn tickets cost $\$30$ and seat tickets cost $\$50$. The concert organizers want to make at least $\$8{,}000$ from ticket sales.

a Write an inequality that describes this situation.

b Write the coordinates of one solution to the inequality and explain what it represents in this situation.

11. Are there any coordinates that are solutions to *both* inequalities in Problems 9 and 10? Explain your thinking.

Name: ..... Date: ..... Period: .....

Comparing Models | Modeling with Functions | Systems of Equations | Financial Literacy

A–CED.3, A–CED.4, A–REI.3, A–REI.10, A–REI.12, SMP.2

Practice Day 2

Let's practice what you've learned so far in this unit!

Problems 1–5: Solve each equation or inequality. Show your thinking for each problem.

1. $2(4 - x) = -5x - 19$

2. $6x - 4 + 3x = -13$

3. $10m + 15 \leq -7(-2m - 1)$

4. Solve for y:
$0 = 12 - y + 18x$

5. Solve for h:
$2h - 4x = -6y$

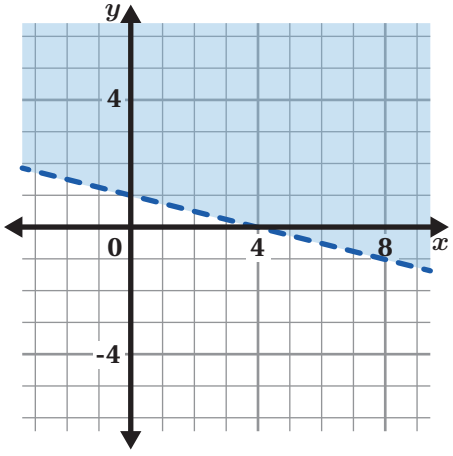
6. Which inequality matches the graph?

A. $x - 4y > 4$

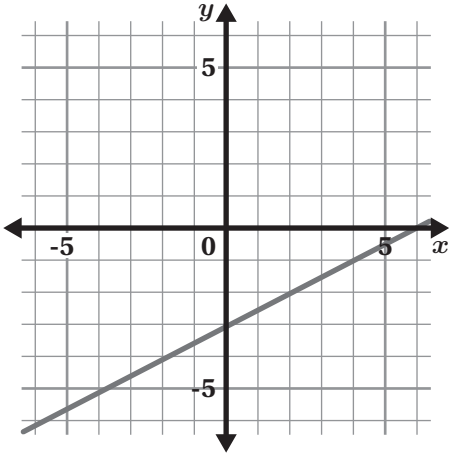
B. $x + 4y > 4$

C. $4x - y < 4$

D. $4x + y < 4$



7. Graph all the solutions to the inequality:
$-x + 2y \leq -6$.



continued on next page...

Name: Date: Period:

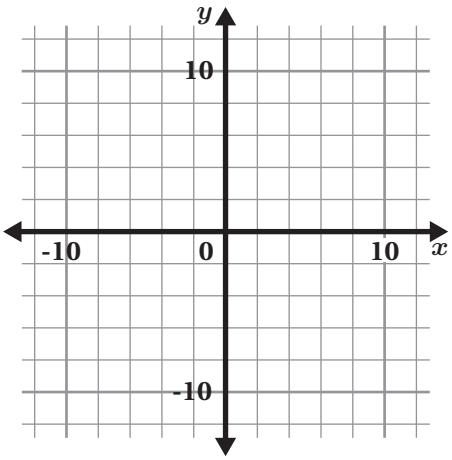
Practice Day 2

8. Here is an inequality: $6x + 3y < 24$.

a Select *all* the coordinate pairs that are solutions to the inequality.

- ☐ **A.** $(0, 0)$
- ☐ **B.** $(5, -2)$
- ☐ **C.** $(-1, 4)$
- ☐ **D.** $(-3, -5)$
- ☐ **E.** $(7, 7)$

b Graph all the solutions to the inequality.



9. At Treetop Adventure Park, you can buy tickets for the zip line adventure or the high ropes obstacle course. This week, the park sold fewer than 400 tickets in total.

a Write an inequality that describes this situation. Describe what each variable represents.

b Write the coordinates of one solution to the inequality and explain what it represents in this situation.

10. At Treetop, zip line tickets cost $30 and high ropes tickets cost $50. Treetop wants to make at least $8,000 from ticket sales.

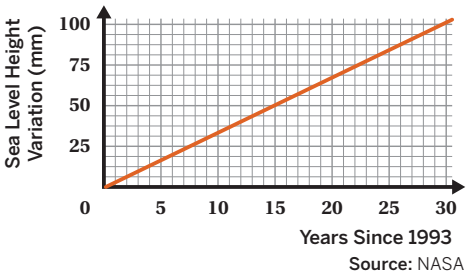
a Write an inequality that describes this situation.

b Write the coordinates of one solution to the inequality and explain what it represents in this situation.

11. Are there any coordinates that are solutions to both inequalities in Problems 9 and 10? Explain your thinking.

Career Connection

Who was able to measure to an accuracy of 4.2 cm from a distance of 832.6 miles?



Source: NASA

The satellite TOPEX/Poseidon, that's who! Launched in 1992, it was the first major satellite to collect data on Earth's oceans. The mission ended in 2006, but it started a 30+ year endeavor to track sea levels by satellite. Since 1993, sea levels have risen by about 3.4 mm per year, $\pm$ 0.4 mm per year. The linear equation $y = 3.4x - 0.9$ models this data. What does the y-intercept tell you?

Satellite engineers or aerospace engineers design, build, and test satellites for many purposes, including for scientific research. They use equations and formulas to ensure satellites stay in orbit.


Nicole Hernandez Hammer

Meet Nicole Hernandez Hammer

Nicole Hernandez Hammer, born in Guatemala, is an American scientist who studies the change in global temperatures and the accompanying changes in sea level. Her research focuses on how these changes have affected communities of color and low-income communities.

Are you interested in studying the changes in Earth's sea levels?

Math in the World

The deepest part of Earth's oceans is the Challenger Deep located at -10,935 m, relative to sea level. The average ocean depth relative to sea level is about -3,682 m. What does x represent in the equation $|-10{,}935| - |-3{,}682| = x$? What is its value?

Math Mindset

When using linear equations to model real-world situations, how do you identify the important quantities?

Unit 3

Systems of Linear Equations and Inequalities

Big Ideas in This Unit

CC2 Systems of Equations | Modeling with Functions | Comparing Models

CC4 Building with Triangles | Transformations and Congruence | NS Financial Literacy

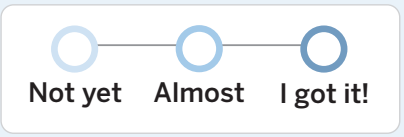
Questions for Investigation

- How can you solve systems of equations and inequalities symbolically and graphically?
- How can real situations be represented using systems of equations or inequalities?
- How can you use coordinates to prove geometric theorems and solve problems?

Explore:
A Community Fundraiser
How do different constraints affect profit?

Watch Your Knowledge Grow

This is the math you'll explore in this unit. Rate your understanding to see how your knowledge grows!



I can . . .	Before	After
Solve a system of linear equations using an elimination strategy.		
Explain how replacing an equation in a system with one that is equivalent creates a new system that has the same solution.		
Solve a system of linear equations using substitution.		
Solve a system of linear equations using a graph.		
Analyze equations to determine whether the system has zero, one, or infinitely many solutions.		
Interpret what the solution to a system of equations means in real-world situations.		
Represent a situation with a system of equations.		
Describe how ordered pairs from the graph of a system of inequalities relate to constraints in a situation.		
Determine if a point is a solution to a system of inequalities using a graph.		
Determine the solution region of a system of inequalities given the boundary lines.		

I can . . .	Before	After
Describe how to determine the solution region of a system of inequalities on a graph.		
Graph a system of inequalities to determine the solution region.		
Write a system of inequalities to represent a graph.		
Write a system of inequalities to represent constraints in a real situation.		
Use the solutions to a system of inequalities to make decisions.		
Prove the slope criteria for parallel and perpendicular lines.		
Write the equation for a line that goes through a given point and is either parallel or perpendicular to a given line.		
Precisely define perpendicular line, parallel line, and line segment.		
Use coordinates and the Pythagorean theorem to derive the distance formula.		
Use the distance formula to determine the length of the segment that connects two points.		
Determine the perimeter and area of polygons using the distance formula.		

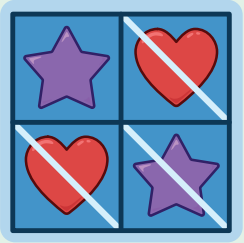
Sub-Unit

1

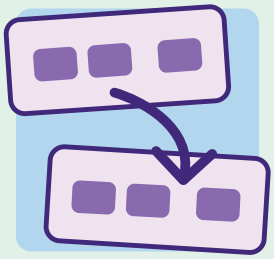
Systems of Equations

Explore
A Community Fundraiser

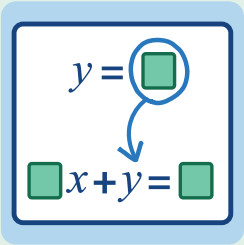
Lesson 1
Shape It Up



Lesson 2
Eliminating Shapes



Lesson 3
Process of Elimination



Lesson 4
Solution by Substitution

Lesson 5
Lizard Lines

Lesson 6
City Development

Lesson 7
Bus Systems

Lesson 8
Electric Line Zapper

Name: ..... Date: ..... Period: .....

Systems of Equations | Financial Literacy

Building Toward: A–CED.3, A–REI.6, A–REI.11, SMP.2, SMP.6

Explore: A Community Fundraiser

How do different *constraints* affect *profit*?

Warm-Up

Nonprofit organizations are operated for the benefit of the public.

This is different from a business, where the goal is to generate income for the business's owners.

Examples of nonprofit organizations include animal shelters and food pantries.

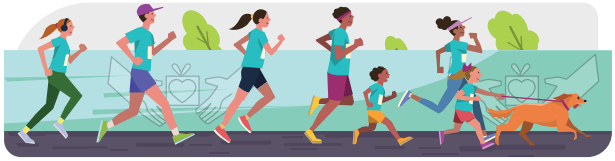
1. What other community needs could nonprofits provide?

2. How might nonprofit organizations earn money so they can keep running?

Name: ..... Date: ..... Period: .....

Breaking Even and Making a Profit

One way nonprofit organizations raise money is through fundraising events.



Each year, the local animal shelter hosts a 5 kilometer road race in the community.

The animal shelter pays 2,000 dollars in upfront costs to have everything they need for the day of the race such as a timing company and entertainment. In addition to the upfront costs, each participant receives a T-shirt that costs 5 dollars per person to make.

3. How can the animal shelter ensure they leave the event with money for their organization?

The point when the upfront costs are paid and money can start to be earned is called the break-even point.

4. The animal shelter decides to charge each participant 30 dollars to enter the race. How many participants would need to participate to break-even?

Explain or show your thinking.

5. If this was your event, how much would you charge each participant? Explain or show your thinking.

Name: ..... Date: ..... Period: .....

Breaking Even and Making a Profit (continued)

A *profit* is the difference between the amount of money earned and the amount of money spent. Most organizations create a fundraising goal that shares what they hope to earn at that particular event.

6. What might be a good fundraising goal for the animal shelter? Explain your choice.

7. With the 30 dollars participant cost, how many participants is needed to reach this goal?

Explain or show your thinking.

8. You explored different costs of entry and fundraising goals and how that impacts how the animal shelter profits. Is there anything else you would consider or change if this was your fundraiser? Explain your thinking.

Name: ..... Date: ..... Period: .....

Building Math Habits of Mind

Discuss:

- Which of these habits of mind did you strengthen during this activity?
- How did you use the one(s) you selected?

I can slow down and first make sense of a challenging problem before trying to solve it.

Not yet Almost I got it!

I can represent real-world problems using equations and inequalities and interpret their solutions within the context of the problem.

Not yet Almost I got it!

I can justify my thinking and ask questions to help me understand the thinking of others.

Not yet Almost I got it!

I can apply the math that I know to solve real-world problems, make assumptions, and revise my thinking as needed.

Not yet Almost I got it!

I can select an appropriate tool to help me solve problems.

Not yet Almost I got it!

I can communicate my thinking and solutions clearly to others.

Not yet Almost I got it!

I can look for structure or patterns to help me solve problems.

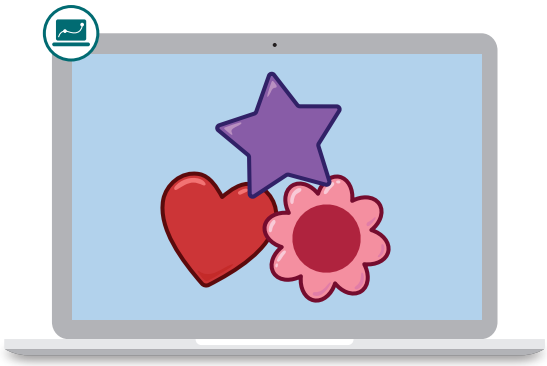
Not yet Almost I got it!

I can look for repeated calculations and other repeated steps to make generalizations.

Not yet Almost I got it!

Name: ..... Date: ..... Period: .....

Systems of Equations | Building Toward A–REI.6, SMP.1



Shape It Up

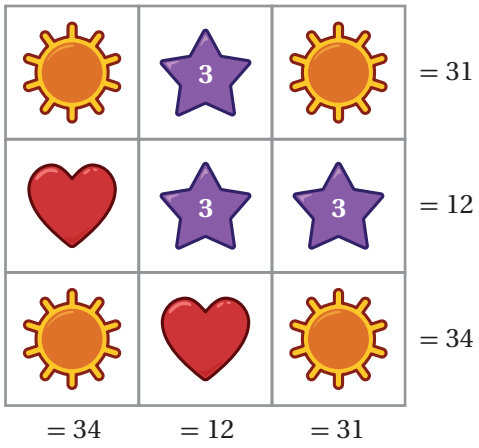
Let's use reasoning to solve shape puzzles.

Warm-Up

1 Here is a shape puzzle. The *sum* of each row and column is shown.

Determine the value of each heart and sun.

Shape	Value
Heart	
Star	3
Sun	



Activity 1

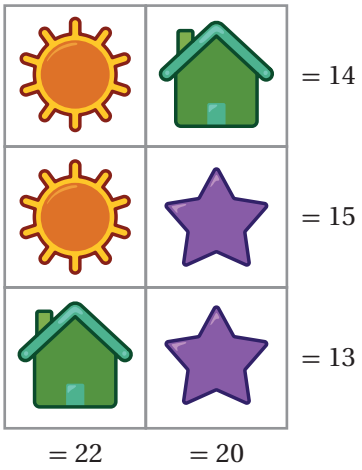
Name: ..... Date: ..... Period: .....

Shape Puzzle Strategies

2 Here is a different shape puzzle.

Jayden thinks that each sun has a value of 10.

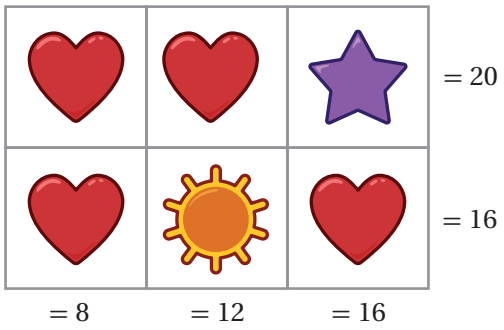
Show or explain why that is not possible.



3 Here is a shape puzzle.

Determine the *solution* for this puzzle.

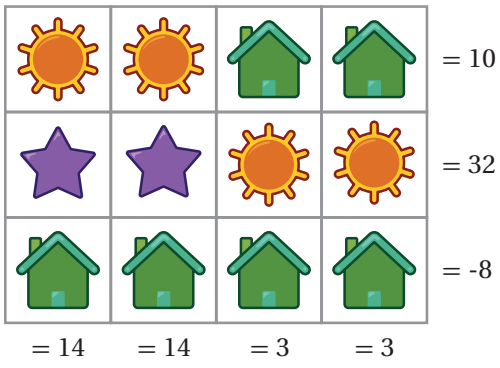
Shape	Value
Heart	
Star	
Sun	



4 Here is a shape puzzle.

Determine the solution for this puzzle.

Shape	Value
Star	
Sun	
House	



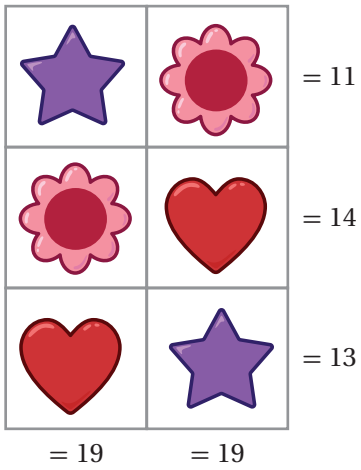
Name: ..... Date: ..... Period: .....

Shape Puzzle Strategies (continued)

5 Here is a shape puzzle.

Determine the solution for this puzzle.

Shape	Value
Heart	
Star	
Flower	



6 Let's take a look at Jayden's first step for solving the puzzle on the previous problem.

How is this helpful in solving the puzzle?

Activity 2

Name: ..... Date: ..... Period: .....

Make Your Own Puzzle

7 In the digital activity, create your shape puzzle. Use this page to support your thinking.

a **Make It!** Fill your puzzle with shapes! You can use as many as four different shapes.

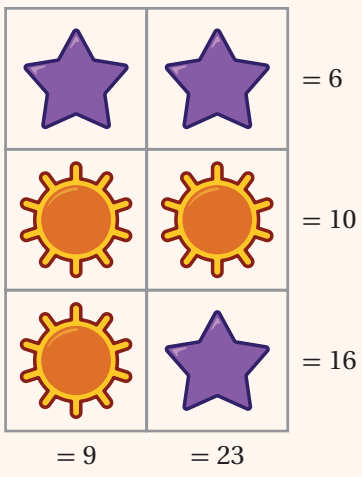
b **Solve It!** Determine the value of each shape in your puzzle.

You're invited to explore more.

8 Here is a different shape puzzle.

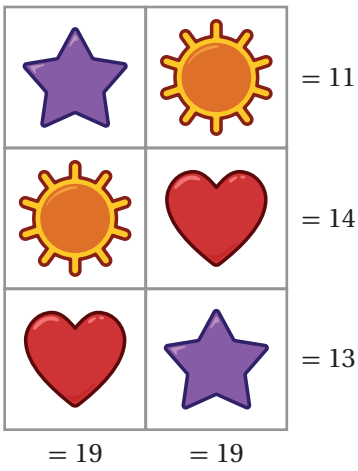
a Try to determine the solution for this puzzle.

b Describe what you notice about this puzzle.



9 Synthesis

Describe some strategies for solving shape puzzles.

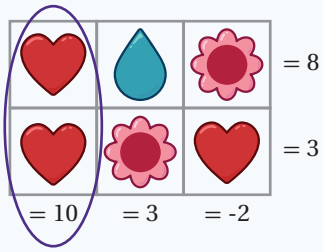


12 Summary 3.01

There are many different ways to solve problems and puzzles using math. Let's look at a few strategies for determining the values of shapes in a puzzle.

Strategy 1: Look for single-shape rows/columns

If we know the value of two hearts, we can determine the value of one heart and substitute that value in the other parts of the puzzle.

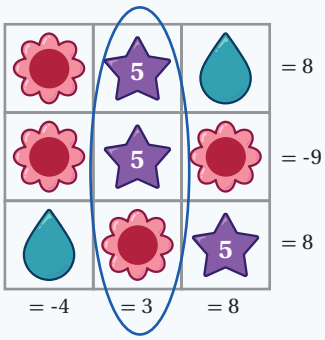


2(♥) = 10

♥ = 5

Strategy 2: Substitute known shape values to solve for missing values

If we know the value of one star, then we can substitute that value in other parts of the puzzle.



2(★) + ✿ = 3

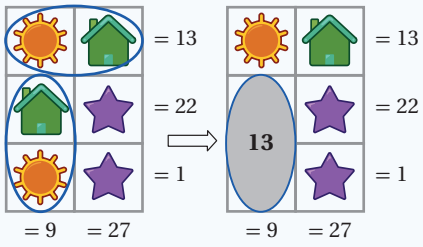
2(5) + ✿ = 3

10 + ✿ = 3

✿ = -7

Strategy 3: Look for repeating shape patterns

If we know the value of a shape combination and we see it repeat in the puzzle, then we can substitute the value for the shape combination.



☀ + ⌂ = 13

☀ + (⌂ + ☀) = 9

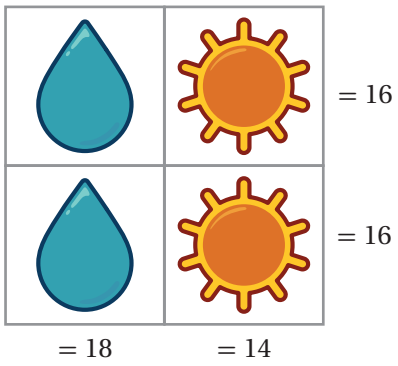
☀ + 13 = 9

☀ = -4

Practice
3.01

Name: ..... Date: ..... Period: .....

Problems 1–2: Here is a shape puzzle. The sum of each row and column is shown.



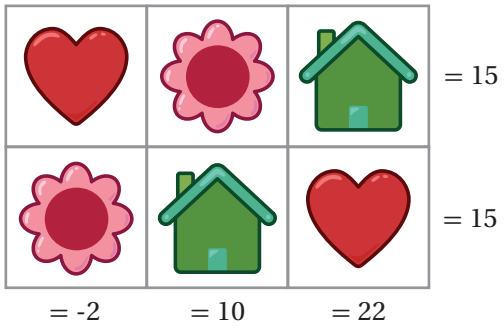
1. Select *all* the true statements.

- ☐ **A.**  +  = 18
- ☐ **B.**  +  = 14
- ☐ **C.**  = 14
- ☐ **D.**  +  = 18
- ☐ **E.**  +  = 16

2. Show or explain why this statement is *false*:  = 8

3. Determine the solution for this puzzle.

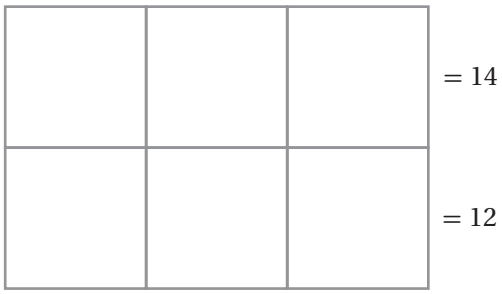
Shape	Value
Heart	
Flower	
House	



Problems 4–5: Use these two equations:

$$x + x + y = 14$$
$$y + y + y = 12$$

4. Draw a shape puzzle to represent these equations.



5. Determine the values of x and y.

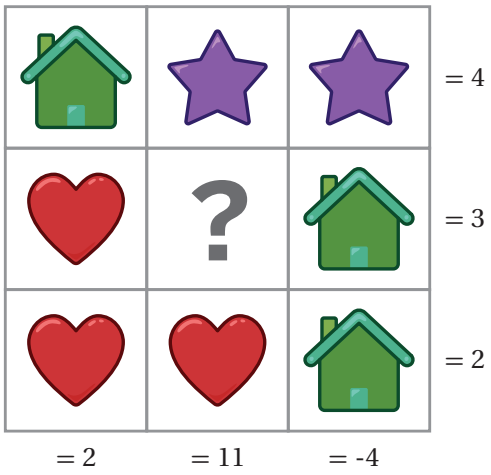
$x =$ ..... $y =$ .....

Name: _____ Date: _____ Period: _____

6. Determine the missing shape in the center of this puzzle. Circle your choice.

Heart House Star

Show or explain your thinking.



Spiral Review

Problems 7–10: **Test Practice** Determine the value of the variable that makes each equation true.

7. $2.5 + (-3) = a$

8. $12 + b = -9$

9. $2c + 3 = 15$

10. $3d + 2 = 35$

Problems 11–12: Determine the solutions to each inequality.

11. $4x + 4 < 3(x + 5)$

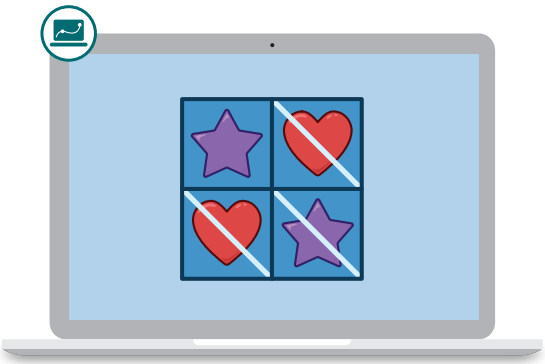
12. $6 - \frac{1}{3}x < x - 8$

Name: ..... Date: ..... Period: .....

Systems of Equations A–REI.6, A–SSE.1.a, SMP.3, SMP.7

Eliminating Shapes

Let's solve systems of equations by adding or subtracting the equations to eliminate a variable.



Warm-Up

Determine an expression that makes each equation true for any value of x and y.

1. $3x + \_\_\_\_\_ = 0$

2. $3x - \_\_\_\_\_ = 0$

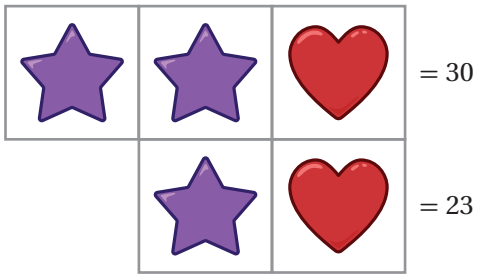
3. $(3x + y) - (\_\_\_\_\_) = 0$

4. $(3x + y) + (\_\_\_\_\_) = 0$

Name: ..... Date: ..... Period: .....

Adding and Subtracting Equations

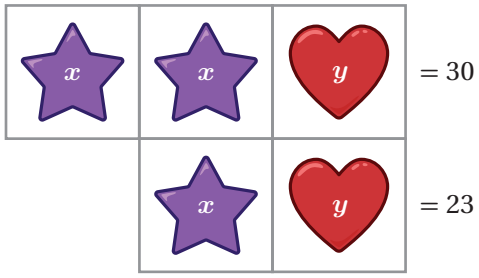
5 Here is a shape puzzle. The sum of each row is shown.



Determine the solution for this puzzle.

Shape	Value
Heart	
Star	

6 This shape puzzle could be written as a **system of equations**, where x is the value of each star and y is the value of each heart.



$$2x + y = 30$$

$$x + y = 23$$

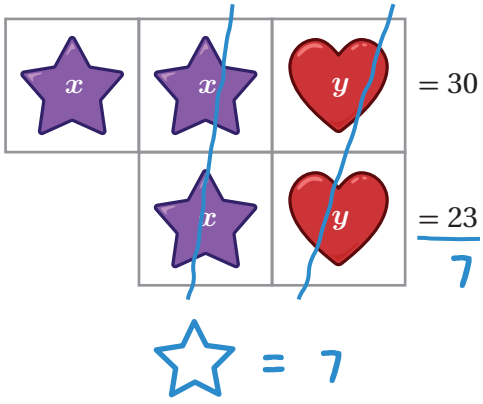
Explain how this system of equations is like the puzzle.

Name: ..... Date: ..... Period: .....

Adding and Subtracting Equations (continued)

7 Here is how Ebony and Diego each determined the value of a star.

Ebony



Diego

$$2x + y = 30$$
$$-(x + y = 23)$$

x + 0 = 7

x = 7

Discuss how you see subtraction in each strategy.

Activity 2

Name: ..... Date: ..... Period: .....

Elimination

8 Here is a new system of equations.

$$x + 2y = 10$$
$$x + y = 7$$

Determine the values of x and y that make both equations true (the *solution to the system*).

Draw a puzzle if it helps with your thinking.

$x =$ ....., $y =$ .....

9 Ebony and Diego want to eliminate the y's in this system of equations.

$$-2x + y = 9$$
$$8x - y = 3$$

- Ebony says to *add* the equations.
- Diego says to *subtract* the equations.

Whose strategy will eliminate the y's ?

A. Ebony's **B.** Diego's **C.** Both **D.** Neither

Name: ..... Date: ..... Period: .....

Elimination (continued)

10 Determine the **solution to the system of equations** from the previous problem:

$-2x + y = 9$

$8x - y = 3$

$x =$ ....., $y =$ .....

11 The strategy of adding or subtracting equations to eliminate a variable is called **elimination**.

Diego says elimination works because it's like adding or subtracting the same value from each side of an equation.

Explain what Diego is saying in your own words.

Diego

$$2x + y = 30$$
$$-(x + y = 23)$$

$$x + 0 = 7$$
$$x = 7$$

12 Determine the solution to this system of equations:

$-7x - 5y = 15$

$7x + 3y = 12$

$x =$ ....., $y =$ .....

Name: ..... Date: ..... Period: .....

Elimination Repeated Challenges

13 Choose four of the systems of equations below and solve them using elimination.

A. $5x + 3y = 21$
$2x + 3y = 12$

B. $8x + 5y = 12$
$8x + 3y = 4$

C. $2x + 3y = 14$
$-2x + 7y = 6$

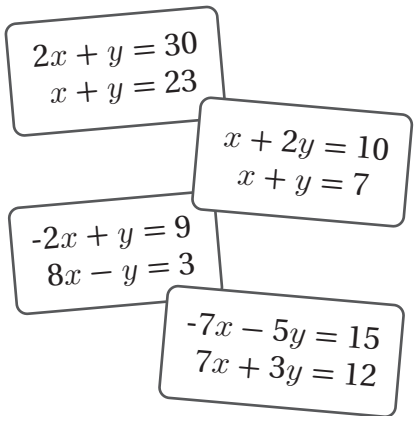
D. $9x + 3y = -3$
$4x - 3y = -23$

E. $2x + 3y = 4$
$2x + 7y = -12$

F. $y = 4x - 1$
$y = 6x - 7$

14 Synthesis

Discuss: How can you determine whether to add or subtract equations in order to eliminate a variable?



17 Summary 3.02

There are many solutions to a linear equation in two variables, but there might be only one (or no) **solution to a system of linear equations** in two variables. One strategy used to determine the ordered pair that makes both equations in a system true is called **elimination**. Let's look at some examples.

If the equations in the system share the same *coefficient* with opposite signs on the same variable, you can eliminate a variable by adding. You can solve this system by adding to eliminate the y-variable.

$$\begin{aligned} -2x + y &= 9 \\ +(8x - y &= 3) \\ \hline 6x + 0 &= 12 \\ x &= 2 \end{aligned}$$

$$\begin{aligned} -2(2) + y &= 9 \\ y &= 13 \end{aligned}$$

If the equations in the system share the same *coefficient* with the same signs on the same variable, you can eliminate a variable by subtracting. You can solve this system by subtracting to eliminate the y-variable.

$$\begin{aligned} x + 2y &= 30 \\ -(x + y &= 23) \\ \hline y &= 7 \end{aligned}$$

$$\begin{aligned} x + (7) &= 23 \\ x &= 16 \end{aligned}$$

elimination A method of solving systems of equations where you add or subtract the equations to produce a new equation with fewer variables.

system of equations Two or more equations that represent the constraints on a shared set of variables.

solution to a system of equations A solution to a system of equations is a set of values that makes all equations in that system true.

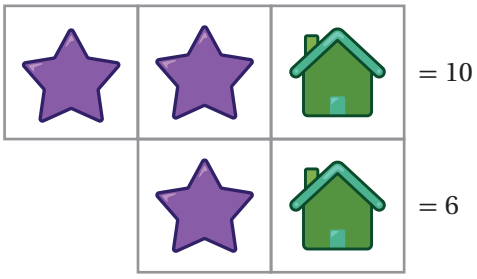
Practice
3.02

Name: _____ Date: _____ Period: _____

1. Solve this system of equations. Use the shape puzzle if it helps with your thinking.

$2x + y = 10$

$x + y = 6$



$x =$ _____ $y =$ _____

Problems 2–3: Mateo made a mistake as he started to solve this system of equations.

$$2x + y = 19$$
$$-(x - y = 11)$$

$x + 0 = 8$

$x = 8$

2. Describe one thing Mateo did *correctly*.

3. Describe one thing Mateo did incorrectly.

Problems 4–5: **Test Practice** Determine the solution for each system of equations.

4. $3x + 4y = 6$
$3x + 2y = 18$

$x =$ _____ $y =$ _____

5. $5x + 6y = 26$
$-5x + 2y = -18$

$x =$ _____ $y =$ _____

Practice
3.02

Name: ..... Date: ..... Period: .....

Spiral Review

Problems 6–8: Write an equivalent expression by combining like terms.

6. $5a + 3b - 2a$

7. $3(c - 2) + 2c$

8. $5d - 2(7d + 3g)$

9. Consider the equation of the line $y - 3 = -6(x + 4)$. Determine the slope of the line and a point that the line passes through.

Problems 10–11: Use the coordinate grid which contains lines a, b, and c.

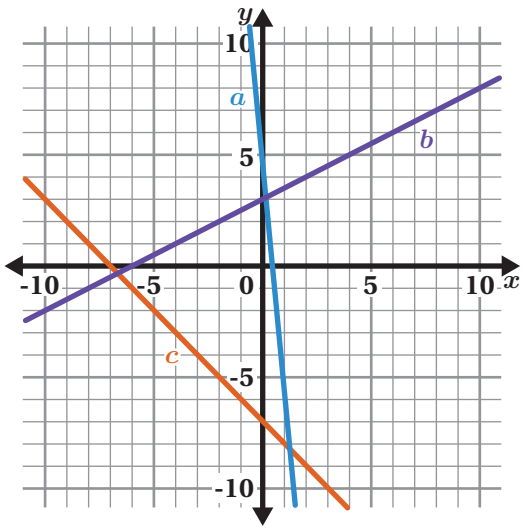
10. Which line represents $x - 2y = -6$?

A. Line a

B. Line b

C. Line c

Explain your thinking.

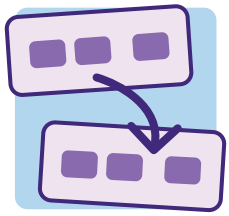


11. Write the equation for line c in slope-intercept form.

Name: ..... Date: ..... Period: .....

Systems of Equations A–REI.5, A–REI.6, A–SSE.1.a, SMP.3, SMP.6, SMP.7

Process of Elimination



Let's create equivalent equations to eliminate a variable.

Warm-Up

Here are some linear equations.

Equation A	Equation B	Equation C
$x + 2y = 11$	$4x + y = 2$	$5x + 10y = 55$
Equation D	**Equation E**	**Equation F**
$y = 2 - 4x$	$2x + \frac{1}{2}y = 1$	$x + 2y - 11 = 0$

1. Sort the equivalent linear equations into two groups. Record your groupings.

Group 1	Group 2

2. Choose one equation. Write a new *equivalent equation* that would belong in that group.

..... *is equivalent to* ..... *because . . .*

Activity 1

Name: ..... Date: ..... Period: .....

First Steps of Elimination

Caasi is solving this system of equations, but she got stuck.

Here's how Caasi started.

Caasi

$$x + 2y = 11$$
$$-(4x + y = 2)$$
$$-3x + y = 9$$

3. **Discuss:**

- What was Caasi's first step?
- Why do you think she got stuck?

Diego is trying to solve this system of equations.
Here's how Diego started.

4. What was Diego's first step?

Diego

$x + 2y = 11$
$4x + y = 2$ → $x + 2y = 11$
$8x + 2y = 4$

$x + 2y = 11$
$-(8x + 2y = 4)$
$-7x = 7$
$x = -1$

5. Diego got stuck using his method after solving for $x = -1$. What do you think he should do next?

6. Ariel suggests that Diego could solve the following system:

$$x = -1$$
$$x + 2y = 11$$

Discuss:

- How did Ariel create the first equation $x = -1$ in this system?
- How can you prove whether both systems will have the same solution?

7. Finish Diego's work to solve the system.

$x =$ ..... and $y =$ .....

Name: Date: Period:

More Than One Way?

8. Caasi and Kwabena started solving this system in different ways.

$$\begin{aligned} 4x - y &= 5 \\ x + 2y &= 8 \end{aligned}$$

With a partner, solve the system both ways. Compare your solutions.

Caasi: *Multiply the first equation by* 2. Kwabena: *Multiply the second equation by* -4.

9. 💬 **Discuss:**

- What is similar about Caasi's and Kwabena's methods for solving the linear system?
- What is different about their methods?

Name: _____ Date: _____ Period: _____

Prepare to Be Eliminated

You will use a set of cards for this activity.

10. Here are the instructions for each round.

Select a card from A–F.

- Discuss two possible first steps you could take to solve the system.
- Choose a different first step from your partner. Solve your system individually.
- Compare your solutions and support each other to make adjustments as needed.

Round 1, Card _____

Equation 1: _____

Equation 2: _____

Solution: $x =$ _____ and $y =$ _____

Round 2, Card _____

Equation 1: _____

Equation 2: _____

Solution: $x =$ _____ and $y =$ _____

Name: _____ Date: _____ Period: _____

Prepare to Be Eliminated (continued)

Round 3, Card _____

Equation 1: _____

Equation 2: _____

Solution: $x =$ _____ and $y =$ _____

Round 4, Card _____

Equation 1: _____

Equation 2: _____

Solution: $x =$ _____ and $y =$ _____

You're invited to explore more.

11. The solution to this system of equations is $x = 5$ and $y = 2$.

$$Ax - By = 24$$
$$Ax + By = 16$$

What are possible values for A and B?

$A =$ _____ and $B =$ _____

Synthesis

12. Describe how writing equivalent equations can help you solve systems of equations.

Use this system if it helps you explain your thinking.

$$x + 3y = 6$$
$$2x + y = 7$$

Summary 3.03

It can be helpful to write *equivalent equations* when using elimination to solve systems of equations. You can create equivalent equations by multiplying each term of the first or second equation by a number. Your goal is to end up with a system of equations where one variable has the same or *opposite* coefficients so you can add or subtract them to eliminate a variable.

Here is a system of equations:

$$9x - 4y = 2$$
$$3x + y = 10$$

You can multiply the second equation by -3 to eliminate the x-variables.

$$9x - 4y = 2$$
$$\text{-3}\,(3x + y = 10)$$

$$9x - 4y = 2$$
$$+\quad \text{-}9x - 3y = \text{-}30$$
$$\overline{\qquad\qquad}$$
$$0 - 7y = \text{-}28$$
$$\boxed{y = 4}$$

$$3x + (4) = 10$$
$$3x = 6$$
$$\boxed{x = 2}$$

Or you can multiply the second equation by 4 to eliminate the y-variables.

$$9x - 4y = 2$$
$$4\,(3x + y = 10)$$

$$9x - 4y = 2$$
$$+\ 12x + 4y = 40$$
$$\overline{\qquad\qquad}$$
$$21x + 0 = 42$$
$$\boxed{x = 2}$$

$$9(2) - 4y = 2$$
$$18 - 4y = 2$$
$$\text{-}4y = \text{-}16$$
$$\boxed{y = 4}$$

Practice
3.03

Name: ..... Date: ..... Period: .....

1. Select *all* equations that are equivalent to $6x + 8y = 10$.

- ☐ **A.** $12x + 16y = 20$
- ☐ **B.** $2x + 6y = 10$
- ☐ **C.** $3x + 4y = 5$
- ☐ **D.** $1.5x + 2y = 2.5$
- ☐ **E.** $4x + 5y = 4$

2. Arnav and Omari are solving this system of equations. They disagree about what the first step should be to eliminate a variable.

$$4x + 2y = 62$$
$$-8x - y = 59$$

Arnav's strategy: Multiply $4x + 2y = 62$ by 2

Omari's strategy: Multiply $-8x - y = 59$ by 2

Whose strategy will eliminate a variable once the equations are added? Circle your choice.

A. Arnav's **B.** Omari's **C.** Both **D.** Neither

Explain your thinking.

Problems 3–4: **Test Practice** Determine the solution to each system of equations.

3.
$$2x - 4y = 10$$
$$x + 5y = 40$$

$x =$ ..... $y =$ .....

4.
$$5x + 2y = 20$$
$$2x - 3y = -11$$

$x =$ ..... $y =$ .....

Name: ..... Date: ..... Period: .....

5. Using the digits 0–9, without repeating, fill in each blank to create two equivalent equations.

$\square x + \square y = \square$ $\square x + \square y = \square$

Spiral Review

Problems 6–9: Solve each equation.

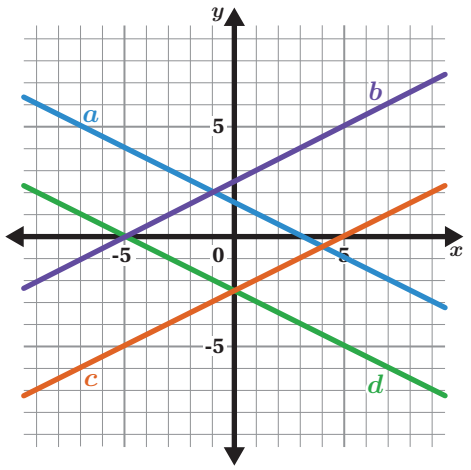
6. $x - 2 = 5$

7. $3(x - 2) = 3 \cdot 5$

8. $8x - 16 = 40$

9. Solve the equation $3x - 9y = 72$ for y.

Problems 10–11: Use the graph.



10. Which line represents $5x + 10y = 15$?

A. Line a

B. Line b

C. Line c

D. Line d

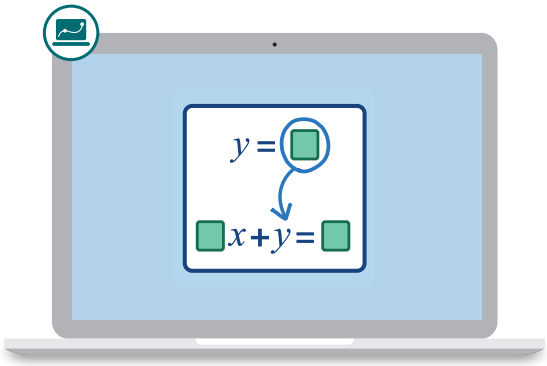
11. Write the equation for line c in slope-intercept form.

Name: ..... Date: ..... Period: .....

Systems of Equations A–REI.6, SMP.1, SMP.5, SMP.6, SMP.7

Solution by Substitution

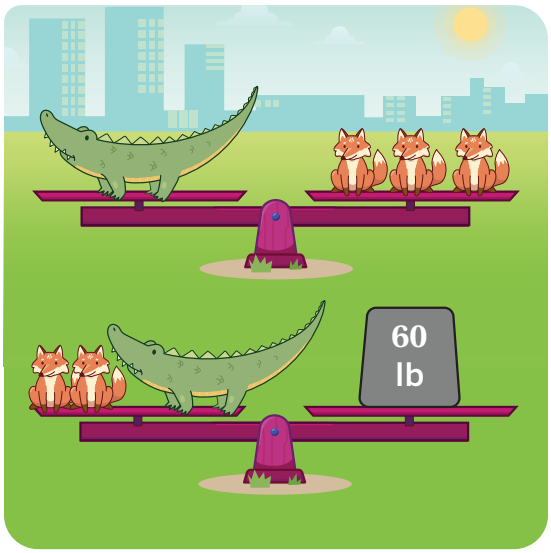
Let's use substitution to solve systems of equations.



Warm-Up

1 Here are two scales showing the weights of foxes and alligators.

What do you notice? What do you wonder?



2 Let's watch an animation.

a **Discuss:** What happened in the animation?

b Determine the weight of each animal.

Fox: ..... pounds Alligator: ..... pounds

Activity 1

Name: ..... Date: ..... Period: .....

Introducing Substitution

3 Riya determined the weight of a fox by writing a system of equations and doing these steps.

Then she needed to solve a new system of equations.

Show or explain what Riya's first step might be as she solves the new system.

Riya

$y = 3x$ $\quad 2x + y = 60$

$2x + (3x) = 60$

$5x = 60$

$x = 12$

New $\quad y = 2x + 3 \quad 4x + y = 15$

4 Here is the system from the previous problem:

$y = 2x + 3$

$4x + y = 15$

Determine the solution.

$x =$ ....., $y =$ .....

5 Riya's strategy is called solving by **substitution**.

Substitution is when a variable is replaced with an expression that is equal to it.

Show or explain the first step to solving the new system of equations with substitution.

$y = 3x$ $\quad 2x + y = 60$

$2x + (3x) = 60$

$y = 2x + 3$ $\quad 4x + y = 15$

$4x + (2x + 3) = 15$

New $\quad 5x - 2y = 15 \quad y = 3x - 12$

Name: ..... Date: ..... Period: .....

Introducing Substitution (continued)

6 Natalia made a mistake as she solved the system of equations from the previous problem.

What did Natalia do well? What should she fix?

Natalia

$5x - 2y = 15$ $\quad y = 3x - 12$

$5x - 2(3x - 12) = 15$

$5x - 6x - 24 = 15$

$-x = 39$

$x = -39$

7 Determine the solution to the previous problem.

$5x - 2y = 15$

$y = 3x - 12$

$x =$ ..... , $y =$ .....

Activity 2

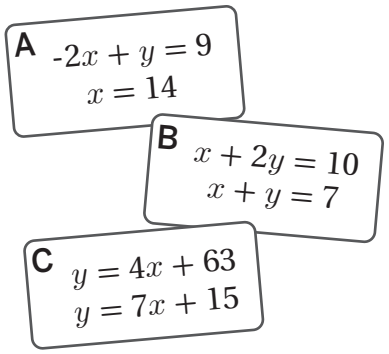
Name: ..... Date: ..... Period: .....

Practicing Substitution

8 Here are three systems of equations.

Discuss:

- What would be your first step in solving each of these systems using substitution?
- Would you prefer to solve each system using substitution or elimination? Why?



9 Determine the solution to each system of equations.

$y = 7x + 12$
$y = -3x + 2$

$x =$ ..... , $y =$ .....

$2x + 2y = 8$
$x = 4 + 3y$

$x =$ ..... , $y =$ .....

$-2x + 4y = 9$
$y = x - 1$

$x =$ ..... , $y =$ .....

$3x - 2y = 14$
$x + 3y = 1$

$x =$ ..... , $y =$ .....

You're invited to explore more.

10 Solve this system of *four* equations. All values in the solution are integers.

$3x + 2y - z + 5w = 20$

$y = 2z - 3w$

$z = w + 1$

$2w = 8$

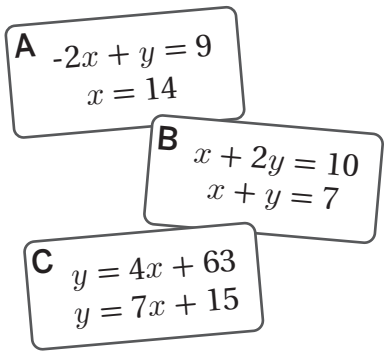
$w =$ ..... , $x =$ ..... , $y =$ ..... , $z =$ .....

11 Synthesis

Substitution and *elimination* are two strategies for solving systems of equations.

Discuss: How are these strategies alike? How are they different?

Use the examples if they help with your thinking.



14 Summary 3.04

Substitution can be used to solve a system of equations, and it is a useful strategy when one variable is already isolated in an equation.

Here are two examples of systems of equations where substitution may be a useful strategy.

In this system, both y-variables are already isolated. We can substitute the expression $-4x + 6$ in for y in the second equation.

$$y = -4x + 6$$
$$y = 3x - 15$$

$$\begin{aligned} y &= -4x + 6 \\ -4x + 6 &= 3x - 15 \\ -7x &= -21 \\ x &= 3 \\ y &= 3(3) - 15 \\ y &= -6 \end{aligned}$$

In this system of equations, y is already isolated, so we can substitute the expression $2x - 5$ in for y in the first equation.

$$-3x - 2y = 3$$
$$y = 2x - 5$$

$$\begin{aligned} -3x - 2y &= 3 \\ -3x - 2(2x - 5) &= 3 \\ -3x - 4x + 10 &= 3 \\ -7x + 10 &= 3 \\ -7x &= -7 \\ x &= 1 \\ y &= 2(1) - 5 \\ y &= -3 \end{aligned}$$

substitution A method for solving systems of equations where a variable is replaced with an equivalent expression in order to produce a new equation with fewer variables.

Name: ..... Date: ..... Period: .....

Problems 1–2: Show or explain what your first step would be for solving each system of equations.

1. $4x - y = 20$
$x + y = 5$

2. $6x - 12y = 24$
$y = 2x - 1$

3. Determine the solution to this system of equations:

$7x - y = -3$

$y = x - 3$

Problems 4–5: Alma made a mistake as she started to solve this system of equations.

$y = \frac{1}{2}x - 1$

$4x - 2y = 11$

Alma

$$4x - 2\left(\frac{1}{2}x - 1\right) = 11$$
$$4x - x - 2 = 11$$
$$3x - 2 = 11$$
$$3x = 13$$
$$x = \frac{13}{3}$$

4. What is the error in Alma's work?

5. Determine the solution to this system of equations.

Practice
3.04

Name: ..... Date: ..... Period: .....

Spiral Review

Problems 6–7: Kadeem made a mistake as he started to solve this system of equations.

$$5x - 4y = 6$$
$$5(x + y = 25)$$

Kadeem

$5x - 4y = 6$

$5x + 5y = 125$

$-1y = 131$

$y = -131$

6. Show or explain one thing Kadeem did correctly.

7. Show or explain Kadeem's mistake.

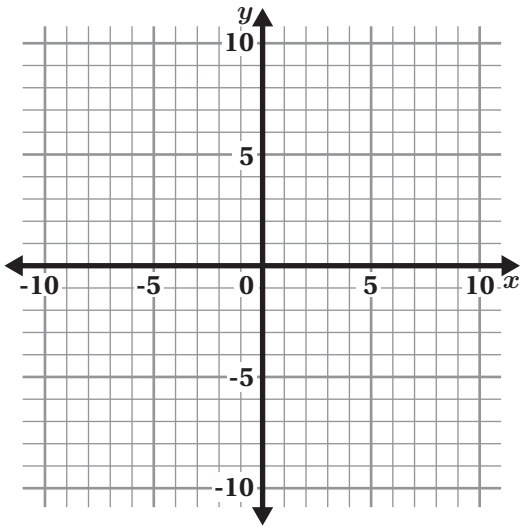
Problems 8–10: Solve each equation for the given variable.

8. Solve for k.
$2t + k = 6$

9. Solve for x.
$4x + 3y = 12$

10. Solve for y.
$4x + 3y = 12$

11. **Test Practice** Graph the equation $4x - 6y = 24$.



Name: ..... Date: ..... Period: .....

Systems of Equations A–REI.6, A–REI.11, SMP.2, SMP.3, SMP.7

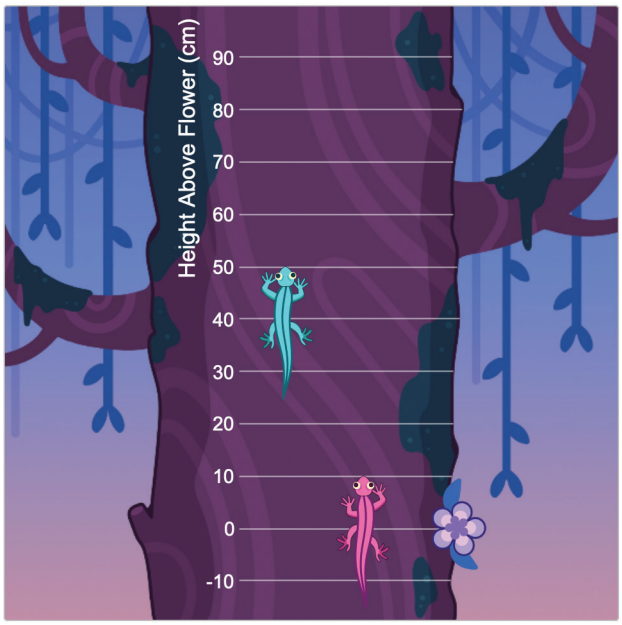
Lizard Lines

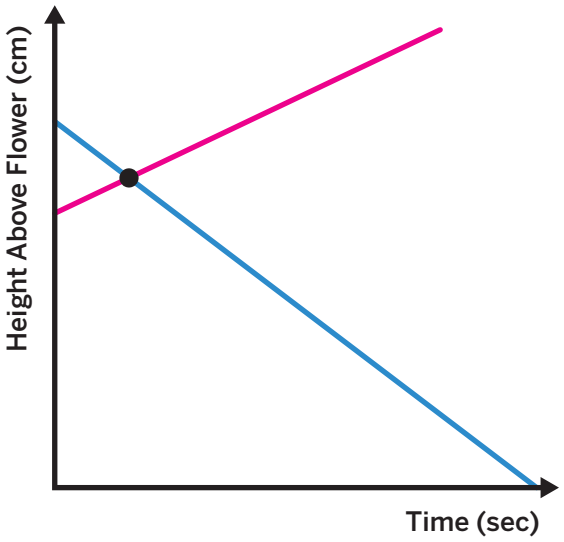
Let's explore systems of equations using graphs.

Warm-Up

1 Let's watch different pairs of lizards walk along a tree trunk together.

2 Here are the starting positions for the blue and the pink lizards.





a **Discuss:** How might you use the graph to estimate when the lizards will meet and at what height they will meet?

b Use your strategy to estimate the time and height at which the lizards will meet.

Name: ..... Date: ..... Period: .....

Making Connections

3 Here are equations for each lizard's height above the flower, y, as a function of time, x, in seconds:

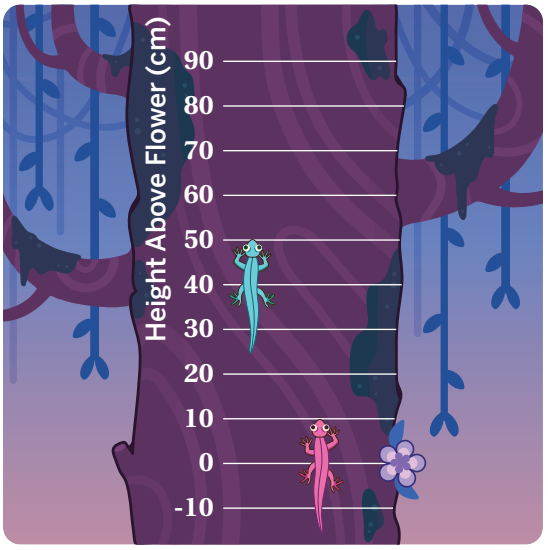
$$y = -5x + 50$$

$$y = 5x + 10$$

When and where will the lizards have the same position?

Time (sec), x: .....

Height (cm), y: .....



4 Here are Jin's and Nasir's strategies for figuring out when the lizards will be in the same position.

Jin

$y = -5x + 50$

$y = 5x + 10$

$5x + 10 = -5x + 50$

$10x = 40$

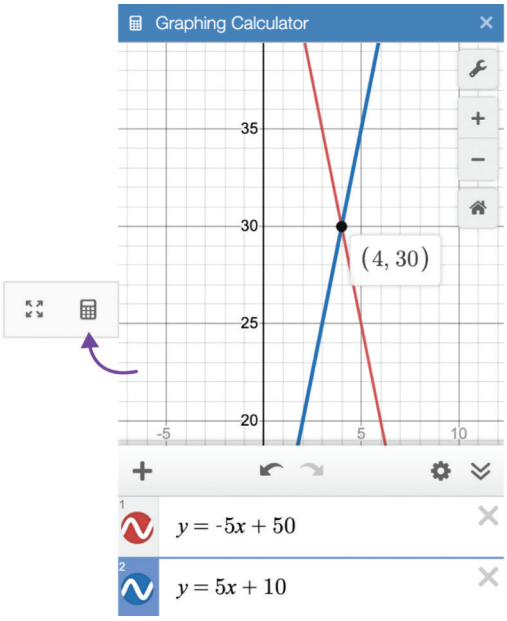
$x = 4$

$y = 5x + 10$

$y = 5(4) + 10$

$y = 30$

Nasir



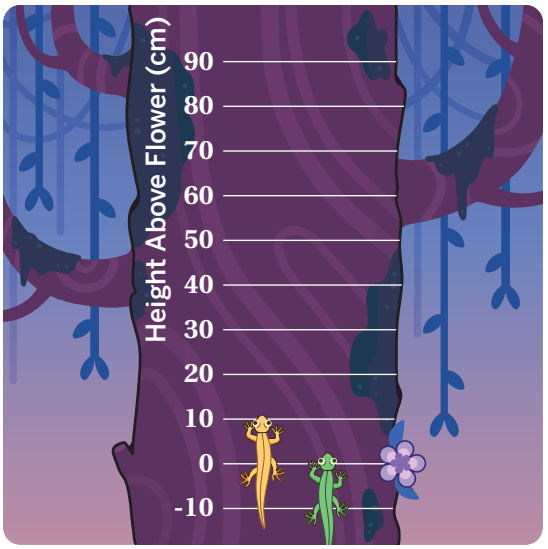
Discuss: Where do you see the solution in each strategy?

Activity 2

Name: ..... Date: ..... Period: .....

Will They Meet?

You will use a graphing calculator for this activity.



5 Here are equations for each lizard's height above the flower, y, as a function of time, x, in seconds:

$$y = -2x + 11$$

$$y = 4x + 2$$

When and where will the lizards have the same position?

Use a graphing calculator if it helps with your thinking.

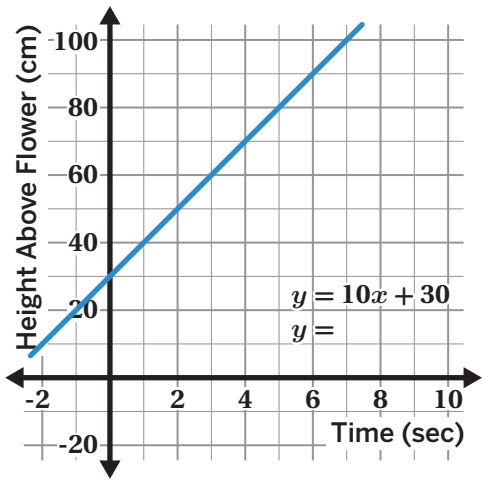
Time (sec), x: .....

Height (cm), y: .....

6 The blue line represents the graph of a blue lizard.

Create a line for a green lizard so that the lizards meet at exactly 5 seconds.

Try to make a line that none of your classmates will make.



7 Here are equations for two lizards' heights above the flower, y, as a function of time, x, in seconds.

$$y = 8x + 60$$

$$y = 8x + 35$$

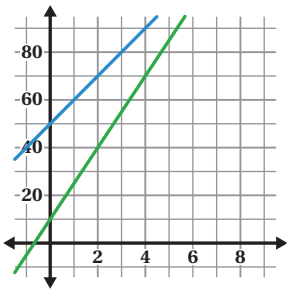
Will these lizards meet? Explain your thinking.

Name: ..... Date: ..... Period: .....

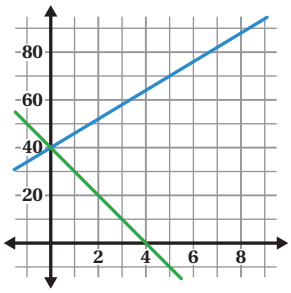
Graphing Systems

8 Here are some systems of equations.

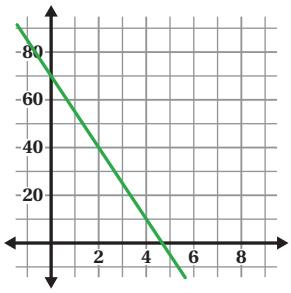
$y = 10x + 50$
$y = 15x + 10$



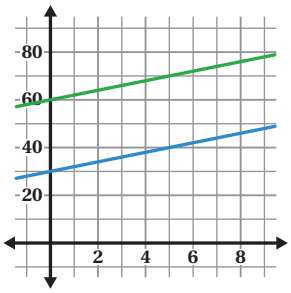
$y = 6x + 40$
$y = -10x + 40$



$y = -15x + 70$
$y = -15x + 70$



$y = 2x + 60$
$y = 2x + 30$



Select each type of system that is possible to make.

- ☐ **A.** No solutions
- ☐ **B.** Exactly one solution
- ☐ **C.** Exactly two solutions
- ☐ **D.** Infinitely many solutions

9 For each system of equations, circle the number of solutions that it has. If there is one solution, what is the solution?

a $y = \frac{1}{2}x - 1$
$y = \frac{1}{2}x + 2$

No solutions | One solution (....., ...) | Infinitely many solutions

b $y = x + 2$
$y = -3x - 2$

No solutions | One solution (....., ...) | Infinitely many solutions

c $y = 2x + 6$
$y = 2(x + 3)$

No solutions | One solution (....., ...) | Infinitely many solutions

d $y - 5x = -7$
$y = 5x$

No solutions | One solution (....., ...) | Infinitely many solutions

e $y = 20x$
$20y = x$

No solutions | One solution (....., ...) | Infinitely many solutions

Name: ..... Date: ..... Period: .....

Graphing Systems (continued)

10 Group each system of equations based on the number of solutions it has.

A

$2x + 4y = 16$

$y = \frac{1}{2}x + 2$

B

$y = \frac{2}{3}x + 10$

$y = \frac{2}{3}x - 7$

C

$y = 2x + \frac{1}{4}$

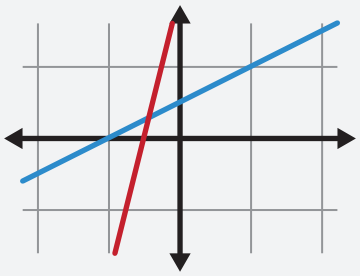
$y = 4x + \frac{1}{4}$

D

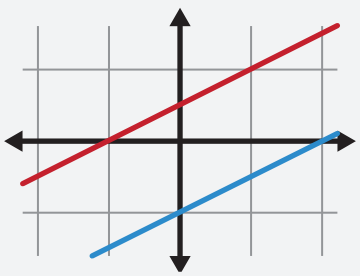
$y = \frac{1}{2}x + 3$

$2y = x + 6$

E



F



No Solutions	One Solution	Infinitely Many Solutions

11 Jaleel and Irene are trying to decide when a system of equations has *no solutions*.

A system of equations has no solutions when . . .

Jaleel: . . . the *slopes* are the same.

Irene: . . . the *y-intercepts* are the same.

Whose claim is correct? Explain your thinking.

No Solutions

$y = \frac{2}{3}x + 10$

$y = \frac{2}{3}x - 7$

$y = 2x + \frac{1}{4}$

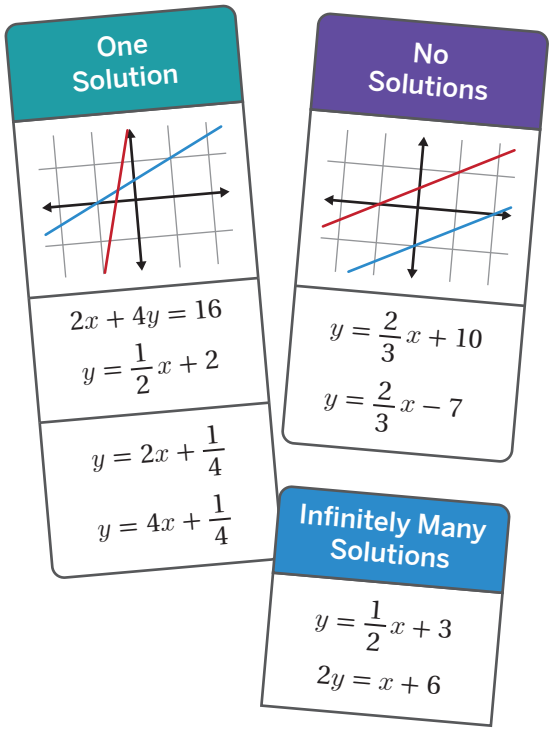
$y = 4x + \frac{1}{4}$

12 Synthesis

Select one and explain.

How can you tell if a system of equations has:

- ☐ **A.** No solutions?
- ☐ **B.** One solution?
- ☐ **C.** Infinitely many solutions?



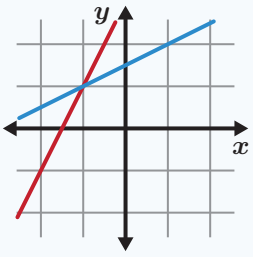
15 Summary 3.05

You can solve systems of equations using strategies like elimination, substitution, or graphing.

On a coordinate plane, you can see the solution of a system of equations at the point(s) where the two lines intersect. A system of linear equations can have:

One Solution

The lines intersect at (-2, 2).

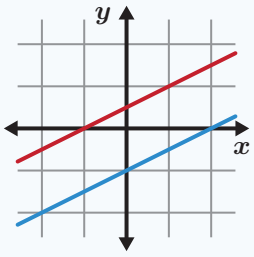


$y = 2x + 6$
$y = \frac{1}{2}x + 3$

The equations have different *slopes* and *y-intercepts*.

No Solutions

The lines are *parallel*.

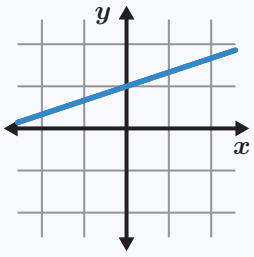


$y = \frac{1}{2}x + 1$
$y = \frac{1}{2}x - 2$

The equations have the same slope and different *y*-intercepts.

Infinitely Many Solutions

The lines are the same.



$y = \frac{1}{3}x + 2$
$3y - x = 6$

The equations are equivalent.

Practice
3.05

Name: ..... Date: ..... Period: .....

1. The point (-2, 2) is on the line $y = x + 4$. Explain how you can determine if this point is the solution to this system of equations:

$y = x + 4$
$y = 2x - 1$

2. Solve this system of equations. Write the solution as a coordinate pair.

$y = -\frac{1}{2}x - 8$

$y = 3x + 6$

3. Match each system of equations to the number of solutions it has.

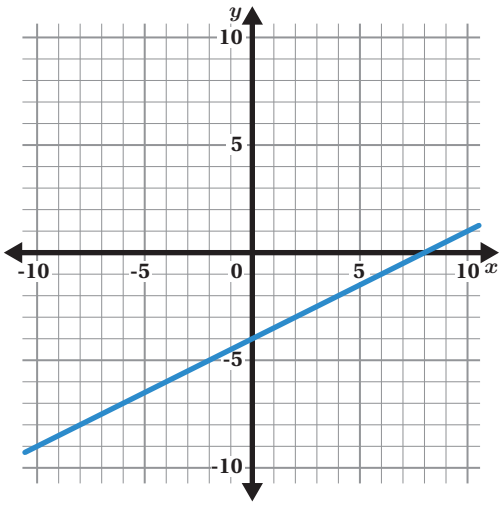
a $y = -2x + 1$
$2y = -4x + 2$ ..... No solutions

b $y = -2x + 1$
$y = -2x + 4$ ..... One solution

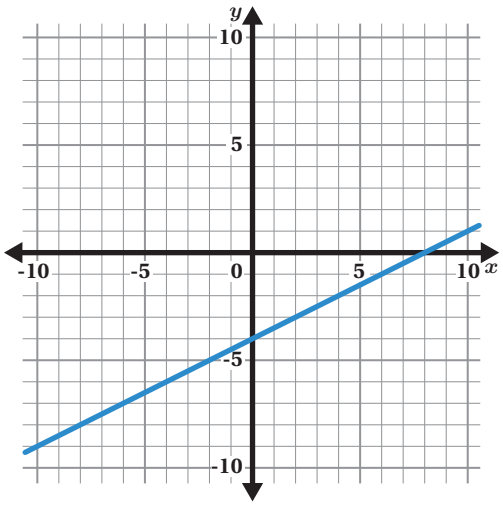
c $y = -2x + 1$
$y = 2x + 1$ ..... Infinitely many solutions

Problems 4–5: Here is a graph of $y = \frac{1}{2}x - 4$. Graph a second line to make this system of equations have:

4. No solutions.



5. One solution at (2, -3).



Name: ..... Date: ..... Period: .....

6. Here is one equation in a system of equations: $y = 3x - 2$. Write a different second equation so that the system of equations has infinite solutions.

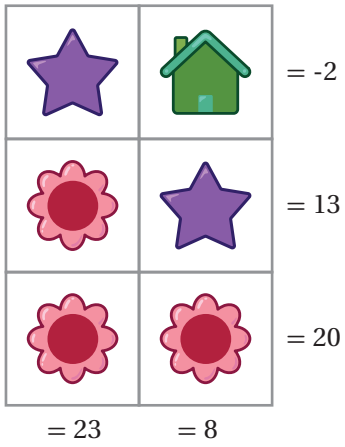
7. **Test Practice** A system of equations has no solutions. Select *all* of the statements that must be true about the equations in this system.

- ☐ **A.** The equations have different slopes
- ☐ **B.** The equations have the same slope
- ☐ **C.** The equations have different y-intercepts
- ☐ **D.** The equations have the same y-intercepts

Spiral Review

8. Here is a shape puzzle. What is the value of each shape?

Shape	Value
Star	
House	
Flower	

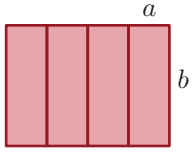


9. Solve this inequality: $3(x - 3) > 2x - 6$

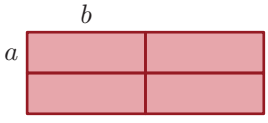
10. Here are two arrangements of identical rectangles. Determine the dimensions a and b.

$a =$ ..... units

$b =$ ..... units



Perimeter: 56 units



Perimeter: 64 units

Name: ..... Date: ..... Period: .....

Systems of Equations A–REI.6, F–IF.7, F–IF.7.a, N–Q.1, SMP.1, SMP.2

City Development

Let's solve systems of equations using multiple methods and interpret the solution in context.

Warm-Up

1. Cities often develop plots of land for specific purposes.

What might a city develop on unused plots of land?

2. What community needs could they meet?

3. What environmental effects should a city consider before developing unused plots of land?

Name: Date: Period:

Apartments and Green Space

Metropolis won a grant of 4 million dollars to develop 14 plots of unused land. They want to use all of the money and build on every plot.

Proposal 1: They can build an apartment building on two plots of land for $\$800{,}000$ and green space on one plot of land for $\$200{,}000$.

Here are two equations about this situation.

$$2a + g = 14$$
$$0.8a + 0.2g = 4$$

- a is the number of apartments.
- g is the number of green spaces.

4. Show or explain how each part of these equations connect to the situation.

Name: ..... Date: ..... Period: .....

Apartments and Green Space (continued)

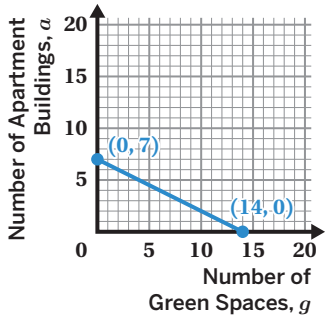
5. Here is incomplete work from three students who were solving the system. With your group, identify the strategy each student is using to solve the system and then complete the strategy.

Diamond

$g + 2a = 14$

a-intercept	g-intercept
$(0) + 2a = 14$	$g + 2(0) = 14$
$a = 7$	$g = 14$
$(0, 7)$	$(14, 0)$

$0.2g + 0.8a = 4$



Jamal

$-0.2 \cdot (g + 2a = 14)$

$0.2g + 0.8a = 4$

$-0.2g + -0.4a = -2.8$

$+ \quad 0.2g + 0.8a = 4$

Angela

$g + 2a = 14$

$-2a \quad -2a$

$g = 14 - 2a$

$0.2g + 0.8a = 4$

$0.2(14 - 2a) + 0.8a = 4$

$2.8 - 0.4a + 0.8a = 4$

6. In your group, compare your solutions and work together to revise any mistakes.

Discuss:

- What strategy did the student start with?
- How did you complete the strategy?

7. What does the solution represent about the Metropolis building project?

Activity 2

Name: Date: Period:

Houses and Green Space

Metropolis won a grant of $4 million to develop 14 plots of unused land. They want to use all of the money and build on every plot.

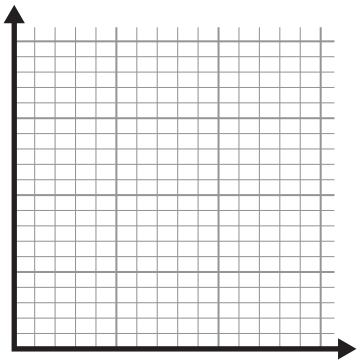
Proposal 2: They can build a house on a plot of land for $500,000 and green space on a plot of land for $200,000.

Here are two equations about this situation.

$$h + g = 14$$
$$0.5h + 0.2g = 4$$

- h is the number of houses.
- g is the number of green spaces.

8. Solve this system graphically and symbolically.

Graphically	Symbolically
	

9. What does your solution tell you about the city's plan to develop the land?

10. Which solving strategy did you prefer? Explain your thinking.

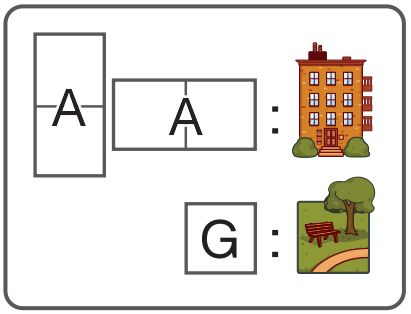
Name: ..... Date: ..... Period: .....

Comparing Proposals

11. Metropolis City Council is deciding between Proposal 1 and Proposal 2. Use the solutions to the linear systems from Activities 1 and 2 to design maps of neighborhoods that meet the *constraints* for Proposal 1 and Proposal 2.

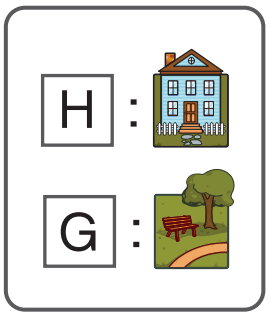
Proposal 1: Apartments and Green Space

Mark the location of each apartment building, A, and each plot of green space, G.



Proposal 2: Houses and Green Space

Mark the location of each house, H, and each plot of green space, G.



12. Compare your Metropolis proposals.

Discuss: What are some advantages and disadvantages of building houses versus apartments? Consider the needs of community members and environmental effects.

Synthesis

13. **Discuss:** When would you choose to solve a system of equations graphically? Symbolically?

$$2a + g = 14$$
$$0.8a + 0.2g = 4$$

Summary 3.06

Systems of equations can represent *constraints* in a situation. There are different ways to solve systems of equations to determine the values that satisfy these constraints. Graphing a system of equations by hand can help to estimate an intersection point, while symbolic strategies like substitution or elimination can be used to find an exact solution.

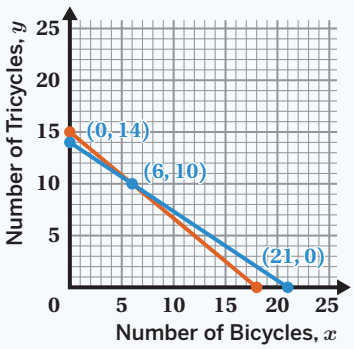
For example, a bike shop makes 2-wheel bicycles and 3-wheel tricycles. This week, they have 42 wheels and enough materials to make 16 bikes total.

Here is a system of equations about this situation:

$$x + y = 16$$
$$2x + 3y = 42$$

- x is the number of bicycles
- y is the number of tricycles

Graphing



The point of intersection $(6, 10)$ is the solution.

Substitution

$x + y = 16$
$-y \quad -y$
$x = (16 - y)$

$2x + 3y = 42$
$2(16 - y) + 3y = 42$
$32 - 2y + 3y = 42$
$\boxed{y = 10}$

$x = 16 - 10$
$\boxed{x = 6}$

Elimination

$-2 \cdot (x + y = 16)$
$2x + 3y = 42$

$-2x - 2y = -32$
$+ \ 2x + 3y = 42$
$0 + y = 10$
$\boxed{y = 10}$

$x + (10) = 16$
$\boxed{x = 6}$

In this situation, the solution $x = 6$ and $y = 10$ means that the bike shop will use all of their wheels and materials if they make 6 bicycles and 10 tricycles.

Practice
3.06

Name: ..... Date: ..... Period: .....

1. Match each system of equations to its solution.

a $y = 3x + 1$
$y = -3x + 25$

..... $(1, 4)$

b $y = -2x$
$y = x - 12$

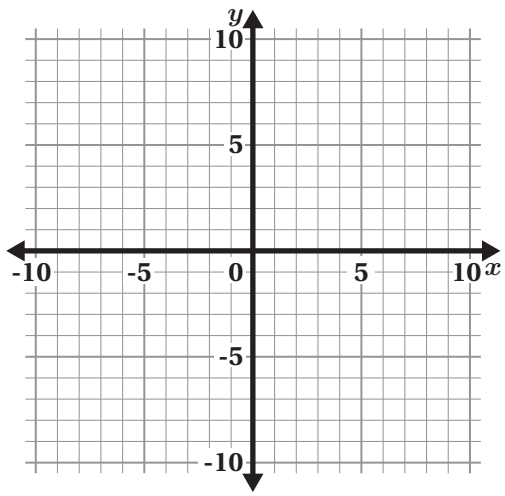
..... $(4, -8)$

c $4x + 2y = 12$
$y = 3 + x$

..... $(4, 13)$

Problems 2–3: **Test Practice** Here is a system of equations: $y = 8x - 6$, $8x + 2y = 12$

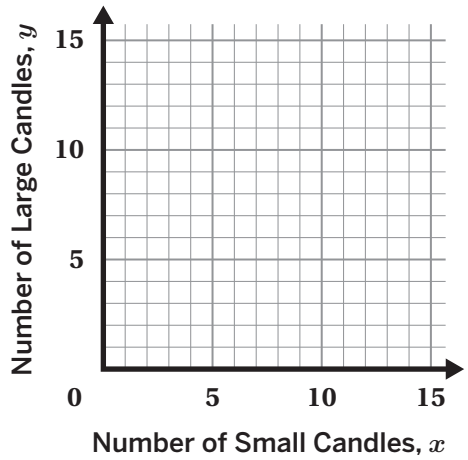
2. Graph the system of equations.



3. Determine the solution to the system of equations.

Problems 4–6: Dylan needs to use 40 ounces of wax to make 7 candles. A small candle requires 4 ounces of wax and a large candle requires 8 ounces of wax. Dylan wrote this system of equations: $x + y = 7$, $4x + 8y = 40$

4. Graph the system of equations.



5. Determine the solution to the system.

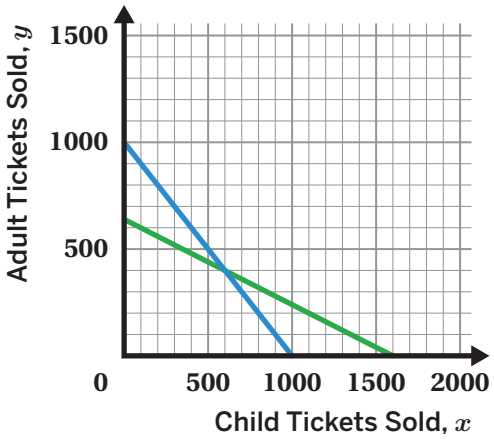
6. What does the solution to the system tell you?

Name: ..... Date: ..... Period: .....

Problems 7–8: A child ticket to a state fair costs \$10. An adult ticket costs \$25. The fair sold 1,000 tickets and made \$16,000 total. Here is a system of equations and a graph that represent the situation.

$$10x + 25y = 16000$$
$$x + y = 1000$$

7. Label each line with the equation it represents.



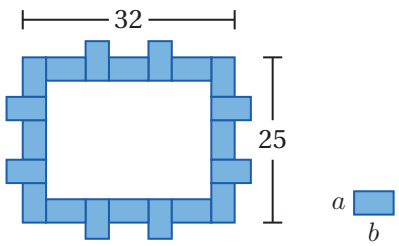
8. Determine the solution to the system and explain what it means.

Spiral Review

9. All the small rectangles are identical. Determine the dimensions, a and b, of one rectangle.

$a =$ .....

$b =$ .....



Problems 10–11: Using only the number 4 exactly four times and the four basic operations, write an expression to create each number.

10. 1

11. 15

Name: ..... Date: ..... Period: .....

Systems of Equations | Modeling with Functions | Comparing Models

A–CED.2, A–CED.3, A–REI.6, F–IF. 7, F–IF.7.a, SMP.1, SMP.2, SMP.5

Bus Systems

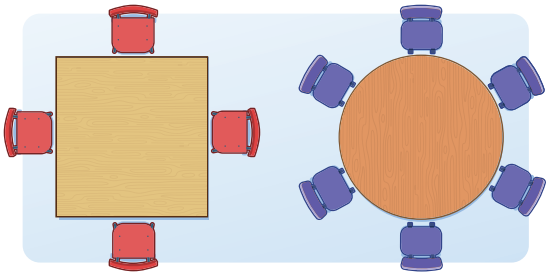
Let's model real-world situations using systems of equations.

Warm-Up

1. Sol is a party planner and uses square and round tables for each party. For each situation, write an equation to represent the number of tables Sol will use.

x: number of round tables

y: number of square tables



Situation 1: A party will have 240 people. Round tables seat 6 people and square tables seat 4 people.

Situation 2: There will be 3 round tables for every square table.

Situation 3: The number of square tables will be 10 more than double the number of round tables.

Situation 4: An event space will have 18 total tables.

Activity 1

Name: ..... Date: ..... Period: .....

Seats and Handholds

The Metropolis Transit Association (MTA) is deciding how many seats and handholds will be on their new buses. There are four configurations with different ratios of seats and handholds.

2. With your group, define the variables in this situation.

x represents . . .

y represents . . .

3. Choose *two* configurations and write a system of equations for them.

	Configuration A	Configuration B	Configuration C	Configuration D
Ratio	4 handholds for every seat	1 handhold for every seat	4 handholds for every 5 seats	1 handhold for every 7 seats
Space	• A bus has 240 sq. ft of space to fill with seats and handholds. • Each seat uses 4 sq. ft of space and each handhold uses 2 sq. ft of space.			
System of Equations				

4. Solve each of the two systems of equations that you wrote using any strategy. Use the next page for your thinking.

5. With your group, come to an agreement on the solutions for each configuration.

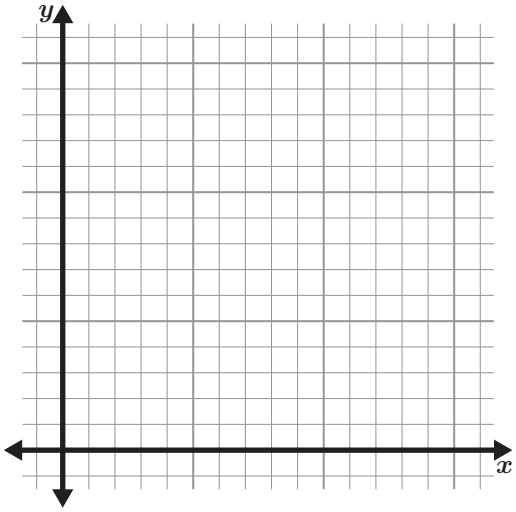
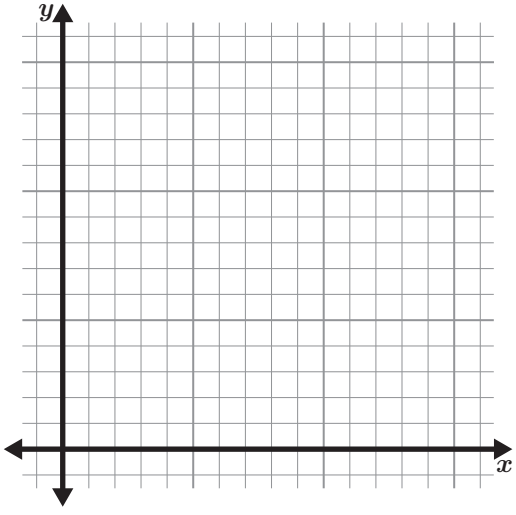
	Configuration A	Configuration B	Configuration C	Configuration D
Solution	**seats, handholds**	**seats, handholds**	**seats, handholds**	**seats, handholds**

6. **Discuss:** What does each solution mean about the number of seats and handholds?

Name: ..... Date: ..... Period: .....

Seats and Handholds (continued)

Use this space to solve the systems of equations you wrote.

Configuration		
Configuration		

Activity 2

Name: ..... Date: ..... Period: .....

Making Recommendations

The MTA is considering using the new buses from Activity 1 for two existing bus lines.

Bus Line 1	Bus Line 2
• **Route:** Between the airport and parking garage. • **Average time per person:** 5 minutes • Buses often have a large number of people and their bags.	• **Route:** Along the main street of a neighborhood. • **Average time per person:** 15 minutes • Popular stops include: the grocery store, the laundromat, and the post office.

7. Which configuration would you recommend for Bus Line 1? Circle one.

Configuration A Configuration B Configuration C Configuration D

Explain your thinking.

8. Which configuration would you recommend for Bus Line 2? Circle one.

Configuration A Configuration B Configuration C Configuration D

Explain your thinking.

9. **Discuss:** What else should the MTA consider when choosing a configuration for a bus line?

Synthesis

10. Describe how to write a system of equations to represent a situation.

Use the example if it helps with your explanation.

Bus Configuration A

- The bus has 240 sq. ft of space for seats and handholds.
- Each seat uses 4 sq. ft of space and each handhold uses 2 sq. ft of space.
- There are 4 handholds for every seat on the bus.

System of Equations

$$4x + 2y = 240$$

$$y = 4x$$

x: number of seats

y: number of handholds

Summary 3.07

You can write systems of equations to help represent constraints in real-world situations.

Here are some things to consider when you write a system of equations:

- Identify the constraints of the situation.
- Identify what each variable will represent.
- Determine whether you want to use *standard form* ($Ax + By = C$) or *slope-intercept form* ($y = mx + b$) to represent each constraint.

Let's look at an example. The architect of an apartment building has enough space to fit 50 apartments and 80 parking spaces. The city requires 2 parking spaces for every big apartment, and 1 parking space for every small apartment.

Constraints

- There is space to fit 50 apartments and 80 parking spaces.
- Every big apartment has 2 parking spaces and every small apartment has 1 parking space.

Variables

x represents the number of big apartments.

y represents the number of small apartments.

System of Equations

$$x + y = 50$$

$$2x + y = 80$$

Practice
3.07

Name: ..... Date: ..... Period: .....

1. Match the system of equations to the situation.

a $2x + 2y = 17$
$y = 5 + x$

..... The sum of two numbers is 17. The difference between two numbers is 5.

b $x + y = 17$
$x - y = 5$

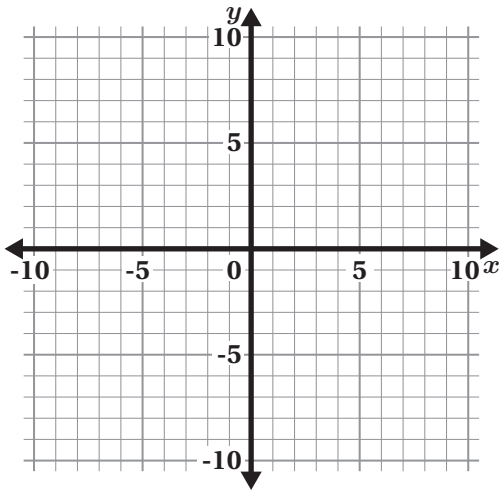
..... A rectangle has a perimeter of 17 units. The length of the rectangle is 5 more units than the width.

c $x + y = 17$
$x = 5$

..... The sum of two numbers is 17. One of the numbers is 5.

2. Solve this system of equations. Sketch a graph if it helps with your thinking.

$3x + 4y = 24$
$y = 2x - 5$



Problems 3–5: A school choir has a concert coming up. They are figuring out what prices to set for student tickets, s, and adult tickets, a.

- They estimate that 150 students and 50 adults will buy tickets.
- They want to make $1,000 in ticket sales.
- They would like the adult ticket price to be double the student ticket price.

3. **Test Practice** Write a system of equations that represents this situation.

4. Solve the system.

$s =$ ..... $a =$ .....

5. Explain the solution in context.

Name: ..... Date: ..... Period: .....

6. Determine the solution to the system of equations.

$y = 2x - 8$

$3x + y = 2$

7. Determine values for A and B so that the system has infinitely many solutions.

$12x + Ay = 8$

$Ax + By = 4$

A: .....

B: .....

8. Determine the solution to the system of equations.

$x - 3y = 17$

$3x - y = 3$

Spiral Review

Problems 9–11: For each equation, identify the slope of its graph.

9. $y = 5 + 2x$

10. $2x + 4y = 4$

11. $y + 1 = 9x$

Name: ..... Date: ..... Period: .....

Systems of Equations A–REI.6, SMP.5, SMP.7

Electric Line Zapper

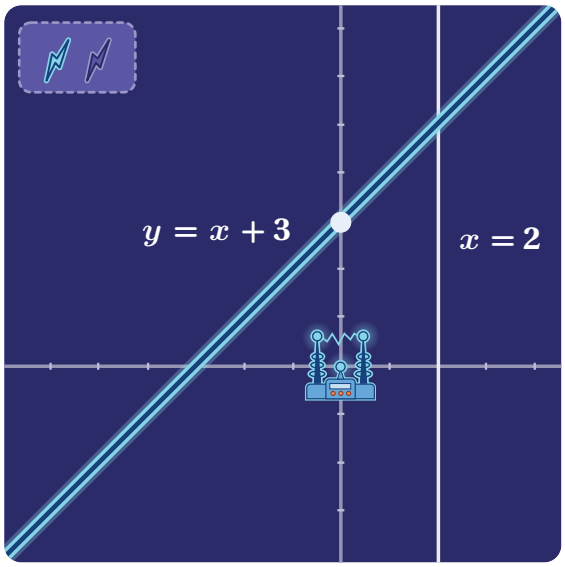
Let's solve systems of equations strategically.

Warm-Up

1 Use the digital activity to zap a point on each of these lines to light it up.

$y = x + 3$
$x = 2$

Zap	Ordered Pair
Zap 1	$(0, 3)$
Zap 2	



Activity 1

Name: ..... Date: ..... Period: .....

Zapping Two Lines

You'll use the digital activity for Problems 2–4.

2 These two lines are hidden in the graph:

$3x + 4y = 3$

$-3x + 3y = 18$

Zap a point on each line to light it up.

3 These two lines are hidden in the graph:

$y = 2x - 4$
$y = 0.5x + 5$

Adah zapped the point $(-4, 5)$ but did not light up either line.

Help her light up both lines with *one* zap.

4 Let's look at two ways to start solving the system of equations from the previous problem.

Discuss: Which strategy would you use? Why?

Name: Date: Period:

Zapping Many Lines

5 These three lines are hidden in a graph:

$y = 3x + 6$

$2x + 2y = 20$

$x - y = 10$

Use the digital activity to zap *two* points to light up all three lines.

6 Adah wants to light up two lines with one zap by using *elimination* to solve a system of equations.

Which two lines from the previous problem might she choose to zap? Circle two.

Line A: $y = 3x + 6$

Line B: $2x + 2y = 20$

Line C: $x - y = 10$

Explain your thinking.

Activity 3

Name: Date: Period:

Repeated Challenges

7 You'll use the digital activity to play a few rounds of Line Zapper. Use this page to show your thinking.

a $y = 2x + 10$
$y = 7 + 3x$

b $2x - y = 12$
$x + 4y = 15$

c $y = 4x + 5$
$4x + 2y = 16$
$x - y = 7$

8 Adah tried to light up these lines with one zap:

$y = 3x + 4$
$y = 3x - 2$

What does her work say about this system of equations?

Adah

$y = 3x + 4$ $y = 3x - 2$

$3x + 4 = 3x - 2$

$3x + 6 = 3x$

$6 = 0$

9 Synthesis

Discuss: What are some ways you can decide what strategy to use when solving a system of equations?

Use the examples if they help with your thinking.

$$\begin{aligned} x &= 2 \\ y &= x + 3 \end{aligned}$$

$$\begin{aligned} 3x + 4y &= 3 \\ -3x + 3y &= 18 \end{aligned}$$

$$\begin{aligned} y &= 3x + 6 \\ 2x + 2y &= 20 \end{aligned}$$

$$\begin{aligned} y &= 2x - 4 \\ y &= 0.5x + 5 \end{aligned}$$

12 Summary 3.08

You can solve systems of equations symbolically using either *substitution* or *elimination*. Looking for specific structures in the equations can help you decide which strategy to use.

- It may be helpful to use *substitution* when at least one of the equations has an isolated variable or at least one is in *slope-intercept form*.
- It may be helpful to use *elimination* when both equations are in the same form or if the equations have a pair of same or opposite terms.

When solving a system of equations symbolically, sometimes all of the variables are eliminated.

No Solutions

When the result is a false statement there are *no solutions* to the system of equations. This means the lines are parallel and will never intersect.

$$\begin{aligned} y &= 3x + 6 \qquad y = 3x - 6 \\ 3x + 6 &= 3x - 6 \\ 3x + 12 &= 3x \\ 12 &= 0 \end{aligned}$$

Infinitely Many Solutions

When the result is a true statement there are *infinitely many solutions* to the system of equations. The equations are equivalent and represent the same line.

$$\begin{aligned} 2 \cdot (2x + 4y &= 6) \\ -4x - 8y &= -12 \end{aligned}$$

$$\begin{aligned} 4x + 8y &= 12 \\ +\ -4x - 8y &= -12 \\ \hline 0 + 0 &= 0 \\ 0 &= 0 \end{aligned}$$

Practice
3.08

Name: ..... Date: ..... Period: .....

Problems 1–3: Show or explain what your *first step* would be to solve each system of equations.

1. $6x + 21y = 103$
$-6x + 23y = 51$

2. $2x + y = 10$
$y = 6$

3. $y = \frac{2}{3}x + 7$
$y = \frac{2}{3}x - 3$

Problems 4–5: Solve each system of equations. Write the solution as a coordinate pair.

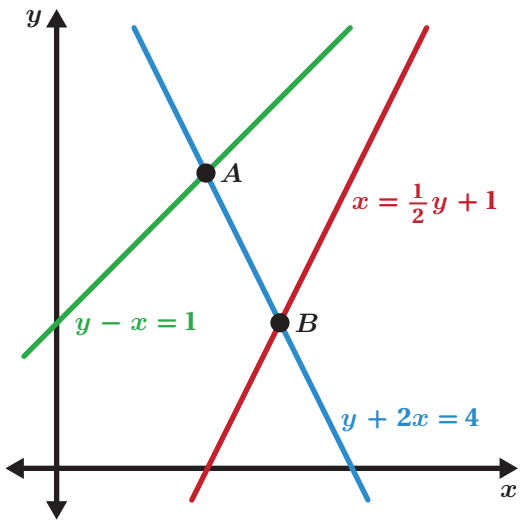
4. $5x + 2y = 29$
$5x - 2y = 41$

5. $2x + 3y = 2$
$x = 4y + 12$

6. Determine the coordinates of points A and B, the intersections of the lines on the graph.

A: .....

B: .....



Name: _____ Date: _____ Period: _____

7. Here is a system of equations.

$$y = 6x - 7$$
$$3y = 6x + 15$$

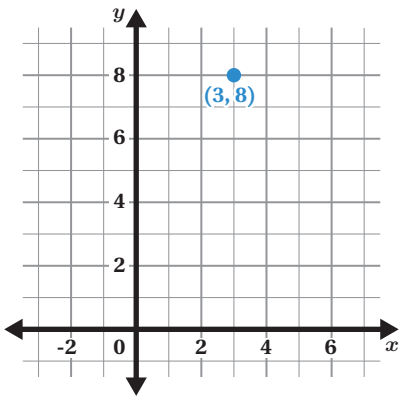
Juliana claims that this system has no solutions. Do you agree with Juliana's claim? Explain your thinking.

8. Write a system of equations where $(3, 8)$ is the solution.

Use the graph if it helps your thinking.

Equation 1: _____

Equation 2: _____



9. **Test Practice** Three systems are shown below. Choose the letter that describes the number of solutions of each system.

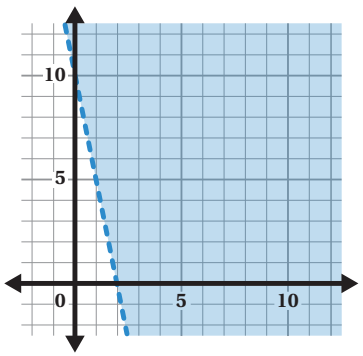
	System	Number of Solutions
_____	$3x + 3y = 24$ $5x + 4y = 25$	**A.** Infinitely many solutions
_____	$3x + 3y = 12$ $4x + 4y = 16$	**B.** No solution
_____	$3x + 3y = 24$ $3x + 3y = 12$	**C.** One solution

Spiral Review

10. Select *all* the ordered pairs that are solutions to the inequality $6y < 30 - 5x$.

☐ **A.** $(0, 0)$ ☐ **B.** $(6, 3)$ ☐ **C.** $(0, 5)$ ☐ **D.** $(4, 1)$ ☐ **E.** $(-5, 0)$

11. Which inequality represents this graph?



A. $5x + y < 10$

B. $5x + y \leq 10$

C. $5x + y > 10$

D. $5x + y \geq 10$

Name: ..... Date: ..... Period: .....

Systems of Equations | Modeling with Functions | A–CED.3, A–REI.6, F–IF.7, SMP.2, SMP.5, SMP.7

Practice Day 1

Let's practice what you've learned so far in this unit!

Start with any one of the scavenger hunt sheets.

- Record the sheet shape, solve the problem, and write your answer.
- Look for your answer at the top of another scavenger hunt sheet. Solve that problem.
- Repeat until you make it back to your starting sheet.

Sheet: ..... **Answer** →	**Sheet:** ..... ↙ **Answer**
Sheet: ..... **Answer** →	**Sheet:** ..... ↙ **Answer**

continued on next page...

Name: _____ Date: _____ Period: _____

Practice Day 1 (continued)

Sheet: _____

Answer

Sheet: _____

Answer

Sheet: _____

Answer

Sheet: _____

Answer

Sheet: _____

Answer

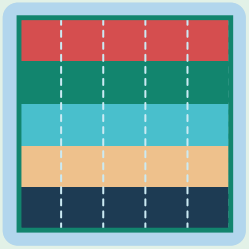
Sheet: _____

Answer

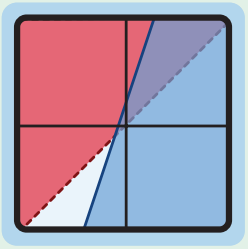
Sub-Unit

2

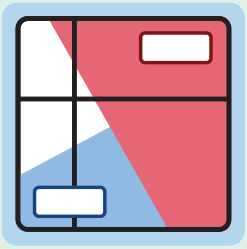
Systems of Inequalities



Lesson 9
Quilts



Lesson 10
Seeking Solutions

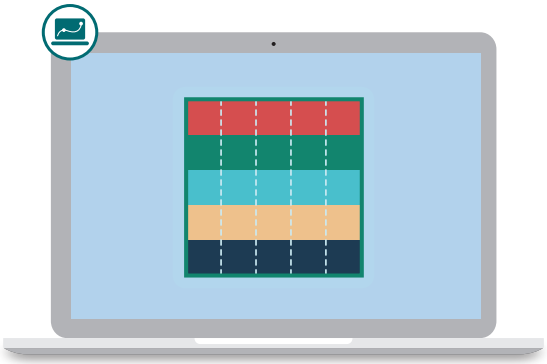


Lesson 11
Boundaries and Shading

Lesson 12
Community Meals

Name: ..... Date: ..... Period: .....

Modeling with Functions A–CED.3, SMP.2, SMP.3



Quilts

Let's explore what solutions to systems of inequalities mean.

Warm-Up

1 People across many different cultures make quilts. They can be used for warmth, storytelling, political involvement, income, and more.

a Let's look at a variety of different quilts.

b What are some decisions people might make when designing a quilt?

2 There is a longstanding patchwork quilt tradition in Gee's Bend, Alabama. Spend a few minutes researching and learning about Gee's Bend and quilters like Annie Mae Young.

Discuss:

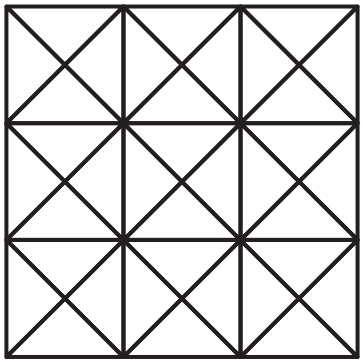
- What *constraints* might quilters like Annie Mae Young have experienced when creating quilts?
- How has quilting been a source of income and political involvement in this community?

Name: ..... Date: ..... Period: .....

Sai's Quilt

3 **a** Let's make a quilt square together.

b **Discuss:** Why did you choose to design your quilt the way you did?



Solid Fabric Patterned Fabric

4 Sai is making a quilt using solid and patterned fabric.

Sai needs *at least* 35 sq. ft of fabric to cover the bed.

Fabric Constraint

$$x + y \geq 35$$

What are some combinations of fabric Sai could use?

Solid Fabric (sq. ft), x	Patterned Fabric (sq. ft), y

5 Sai wants to spend no more than $30 on fabric.

- Solid fabric costs $0.50 per sq. ft.
- Patterned fabric costs $1 per sq. ft.

Cost Constraint

$$0.50x + y \leq 30$$

What are some combinations of fabric Sai could use?

Solid Fabric (sq. ft), x	Patterned Fabric (sq. ft), y

Name: ..... Date: ..... Period: .....

Sai's Quilt (continued)

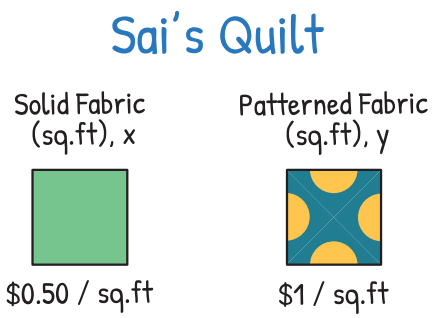
6 Sai wants the quilt to have at least 35 sq. ft of fabric and cost no more than \$30. Sai wrote a **system of inequalities** to represent these two constraints.

$$x + y \geq 35$$

$$0.50x + y \leq 30$$

Sai designed a quilt using 10 sq. ft of solid fabric and 28 sq. ft of patterned fabric.

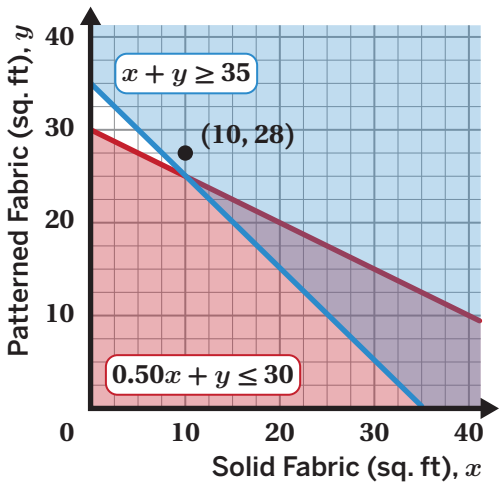
Does Sai's design meet both constraints?



7 This graph represents Sai's system of inequalities.

The fabric used in Sai's design is represented by the point $(10, 28)$.

How can the graph help Sai decide whether the design meets both constraints?



8 Let's test several fabric combinations.

Discuss: How can you see which constraints a point meets by looking at the graph?

Activity 2

Name: ..... Date: ..... Period: .....

Evan's Quilt

9 Evan is making a quilt using different fabrics.

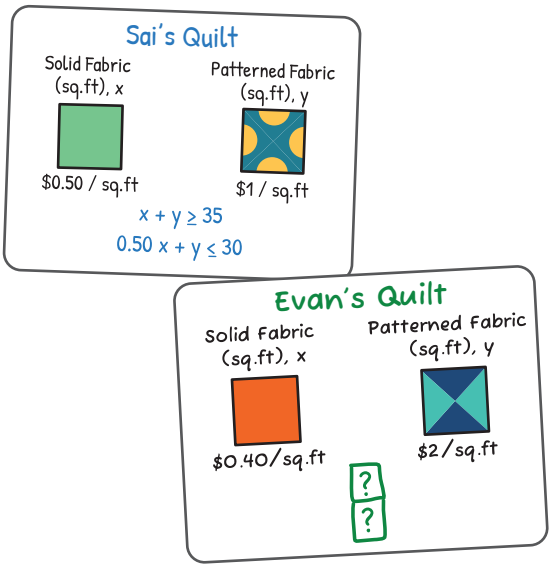
- Solid fabric costs $0.40 per sq. ft.
- Patterned fabric costs $2 per sq. ft.

Evan wants his quilt to have *at least* 35 sq. ft of fabric and cost *no more than* $30.

Write a system of inequalities to represent Evan's quilt.

Fabric inequality: .....

Cost inequality: .....



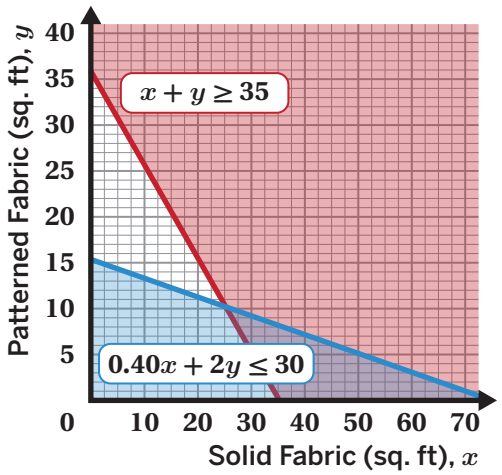
10 This graph represents Evan's system of inequalities:

$$x + y \geq 35$$
$$0.40x + 2y \leq 30$$

Determine a combination of solid and patterned fabric that meets both constraints.

Solid Fabric (sq. ft), x: .....

Patterned Fabric (sq. ft), y: .....



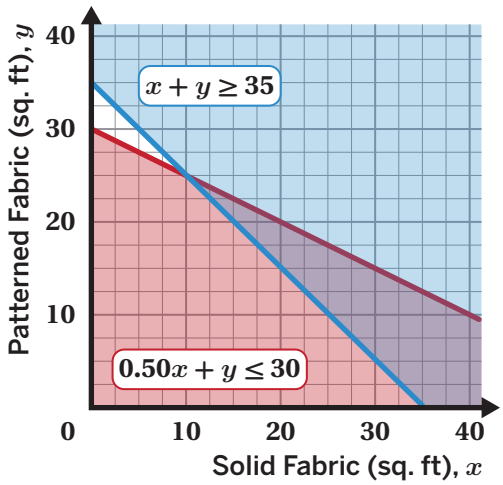
11 Titus says $(10, 10)$ *is* a solution to Evan's system of inequalities. Alma says it *is not* a solution.

Whose thinking is correct? Explain your thinking.

12 Synthesis

How can you determine if a point is a solution to a system of inequalities?

Draw on the graph if it helps to show your thinking.



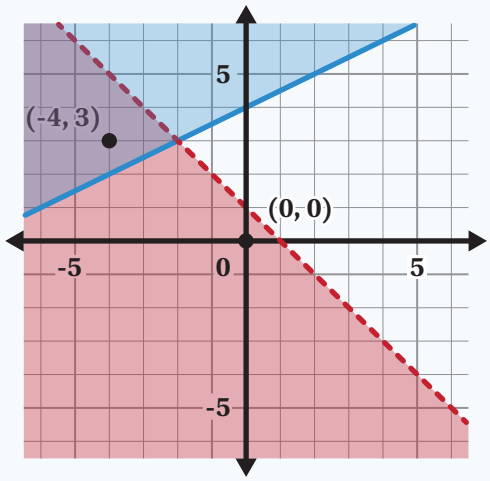
15 Summary 3.09

You can use different strategies to determine if a point is a solution to a **system of inequalities**.

- If the point is in the shaded region for both inequalities, then it is a solution to the system.
- If the x- and y-values of the point are substituted into both inequalities and the inequalities are true, then the point is a solution to the system.

Here is a graph for this system of inequalities.

$$x + y < 1$$
$$y \geq \frac{1}{2}x + 4$$



You can see that the point $(-4, 3)$ is a solution because it is in the shaded region for both inequalities.

You can also substitute points into both inequalities to determine if they are solutions. $(0, 0)$ is not a solution and $(-4, 3)$ is a solution.

system of inequalities A system with two or more inequalities that represent the constraints on a shared set of variables.

Practice
3.09

Name: ..... Date: ..... Period: .....

1. **Test Practice** Kwasi needs at least 3 new school uniforms (3 shirts and 3 pants). Each pair of pants costs \$26 and each shirt costs \$18. Kwasi has a \$150 budget. Which inequality could be part of the system that represents Kwasi's situation?

A. $18s + 26p \leq 150$

B. $18s + 26p \geq 3$

C. $3s + 3p \geq 6$

D. $3s + 3p \leq 150$

Problems 2–4: It costs Lukas \$5.00 to mail a package. Lukas has *postcard stamps*, p, that are worth \$0.34 each and *first-class stamps*, f, that are worth \$0.49 each.

2. Lukas wrote the inequality $0.34p + 0.49f \geq 5$. What does this inequality represent?

3. Lukas wrote another inequality: $p + f \leq 12$. What does this inequality represent?

4. If Lukas uses 1 postcard stamp and 9 first-class stamps, will this satisfy both constraints? Explain your thinking.

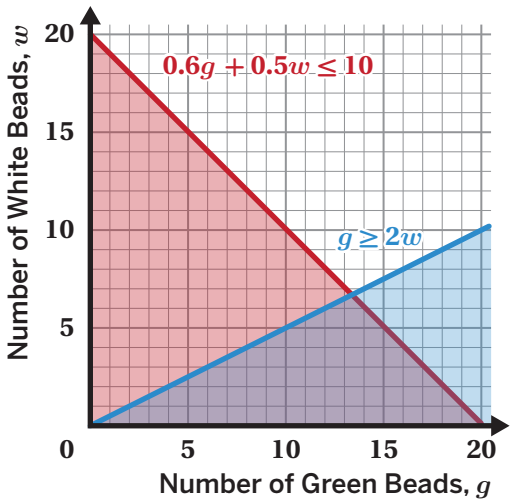
Problems 5–6: Arjun is making a bracelet.

He has \$10 to spend on beads.

Green beads cost \$0.60 each and white beads cost \$0.50 each.

His bracelet design needs at least twice as many green beads as white beads.

The graph shows the system of inequalities that represents this situation.



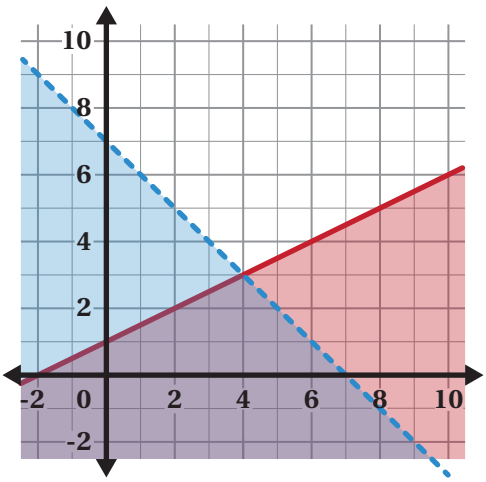
5. What is a combination of green and white beads that meets both constraints?

6. What is a combination of green and white beads that meets *only one* constraint?

Name: _____ Date: _____ Period: _____

7. Using the digits 0–9 without repeating, fill in each blank such that each statement is true:

- A is a solution to both inequalities
- B is a solution to only one inequality
- C is a solution to only the other inequality
- D is not a solution to either inequality



$A = (\square, \square)$ $B = (\square, \square)$

$C = (\square, \square)$ $D = (\square, \square)$

Spiral Review

8. Solve this system of equations. Write your solution as a coordinate pair.

$2x + y = 8$
$y = 2x + 4$

9. Jayla is at the market with \$14 to buy fruit. She decides to buy apples and grapes. Apples, a, cost \$1.67 per pound and grapes, g, cost \$1.87 per pound. Write an inequality to represent this situation.

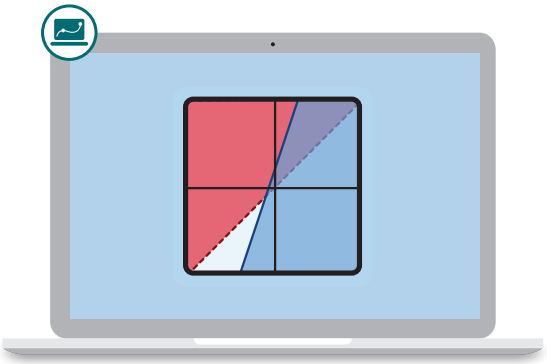
10. Esteban claims that $(5, 10)$ is *not* a solution to the inequality $y < 3x - 5$. Do you agree with Esteban's claim? Explain your thinking.

Name: ..... Date: ..... Period: .....

Systems of Equations A–REI.12, F–IF.7, SMP.3, SMP.5, SMP.6

Seeking Solutions

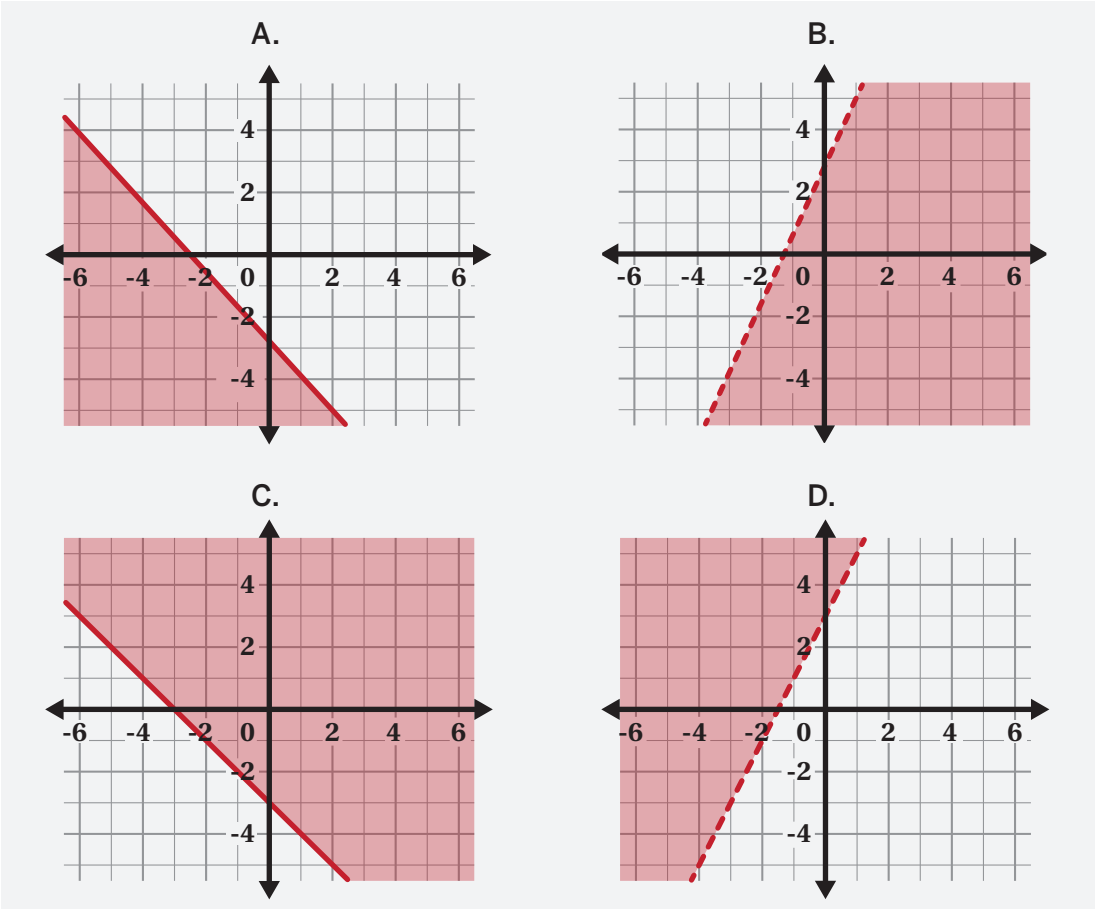
Let's explore strategies for determining the solution region for a system of inequalities.



Warm-Up

1 Match each inequality to its graph. There will be two graphs without a match.

$x + y \geq -3$	$y > 2x + 3$



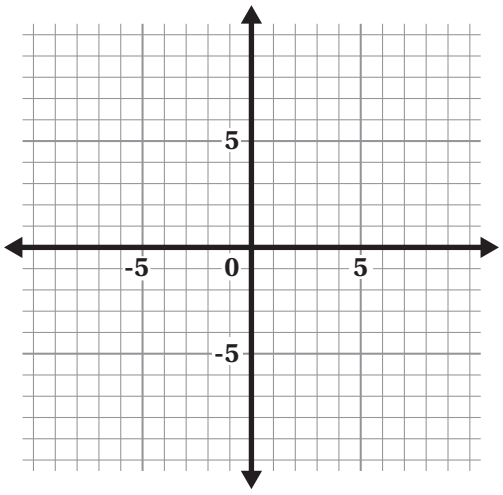
Activity 1

Name: ..... Date: ..... Period: .....

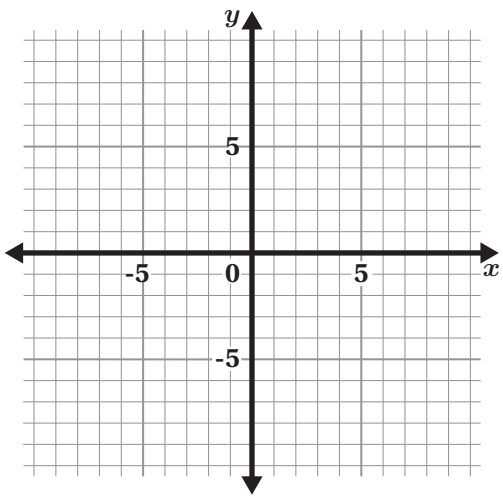
The Overlap

2 Graph the solution to each inequality.

$y > x$



$-3x + y \leq 2$



3 Let's look at a graph that shows the system of inequalities from the previous question.

a Watch as the point is moved to different regions of the graph.

b **Discuss:**

- How many regions do you see?
- When is each inequality highlighted? When are both highlighted?
- What happens when the point is on the dashed line? On the solid line?

Name: ..... Date: ..... Period: .....

The Overlap (continued)

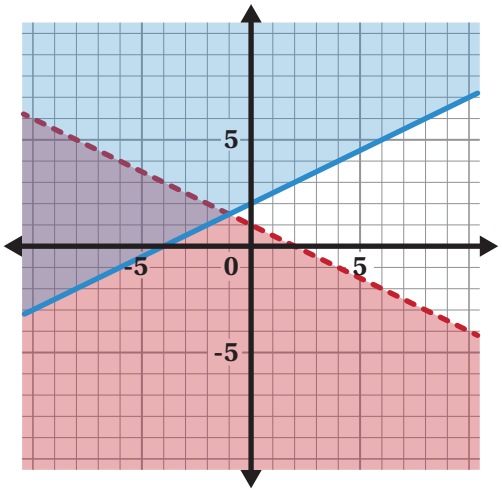
4 The **solutions to a system of inequalities** are all the points that make both inequalities true.

On a graph, the solutions are located in the same region.

Draw a point in the *solution region* of this system of inequalities:

$$\frac{1}{2}x + y < 1$$

$$-x + 2y \geq 4$$



Activity 2

Name: ..... Date: ..... Period: .....

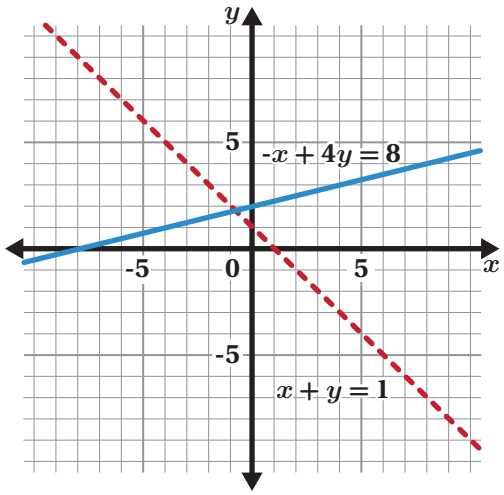
Where Is the Solution Region?

5 This graph shows the *boundary lines* and their equations for this system of inequalities:

$$x + y > 1$$

$$-x + 4y \leq 8$$

How can you determine where the solution region is?



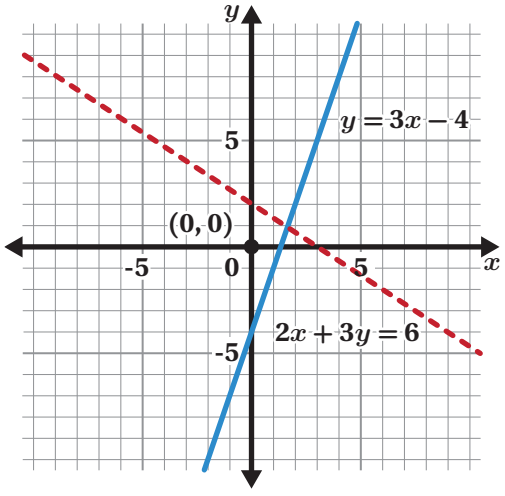
6 Plot a point on the solution region for the system of inequalities in the previous problem.

7 Terrance is trying to graph the solutions to this system of inequalities. First, he tests the point $(0, 0)$.

$2x + 3y > 6$

$y \geq 3x - 4$

Dashed Line	Solid Line
$2(0) + 3(0) > 6$	$0 \geq 3(0) - 4$
$0 + 0 > 6$	$0 \geq 0 - 4$
$0 > 6$	$0 \geq -4$
False!	True!



Discuss: What can Terrence do next to determine the solution region?

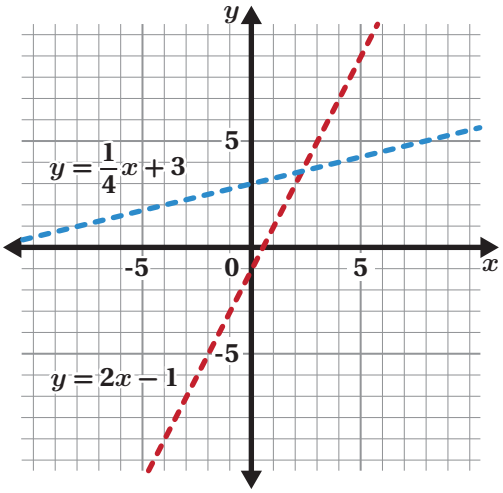
Name: ..... Date: ..... Period: .....

Solution Region Practice

8 Plot a point in the solution region for each system of inequalities.

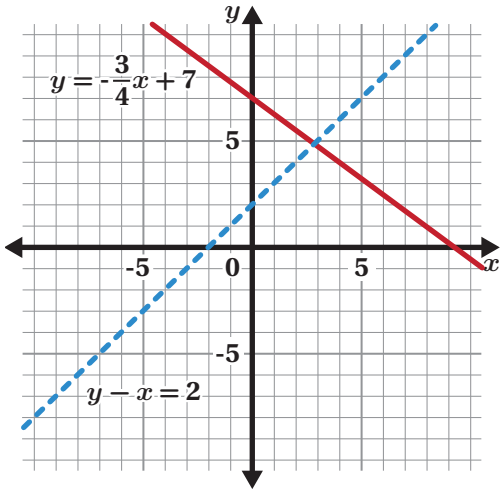
a $y > 2x - 1$

$y < \frac{1}{4}x + 3$



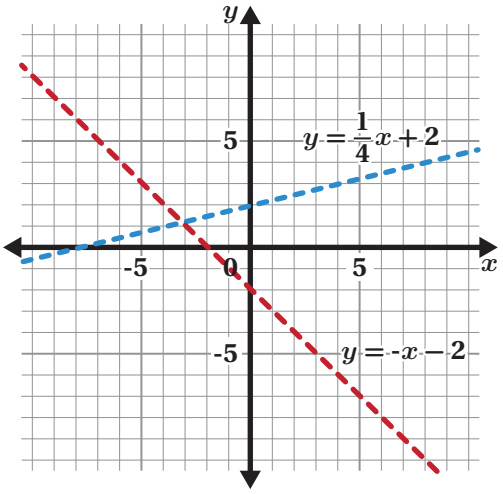
b $y \leq -\frac{3}{4}x + 7$

$y - x < 2$



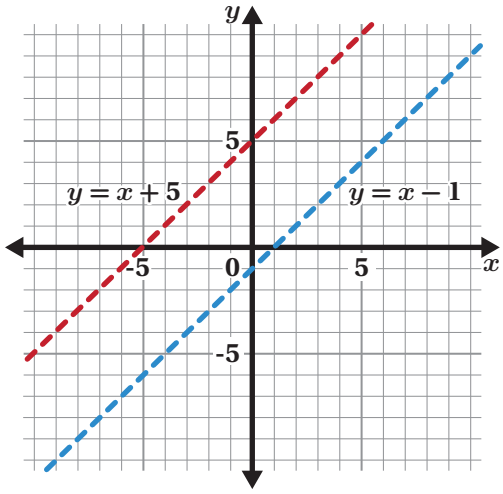
c $y < -x - 2$

$y < \frac{1}{4}x + 2$



d $y > x + 5$

$y > x - 1$

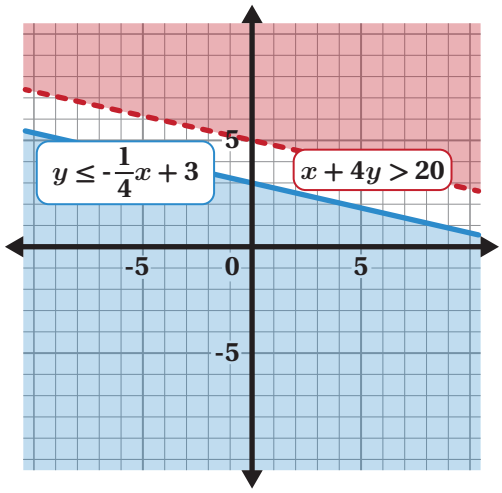


Name: ..... Date: ..... Period: .....

Solution Region Practice (continued)

9 This system of inequalities has *no solutions*.

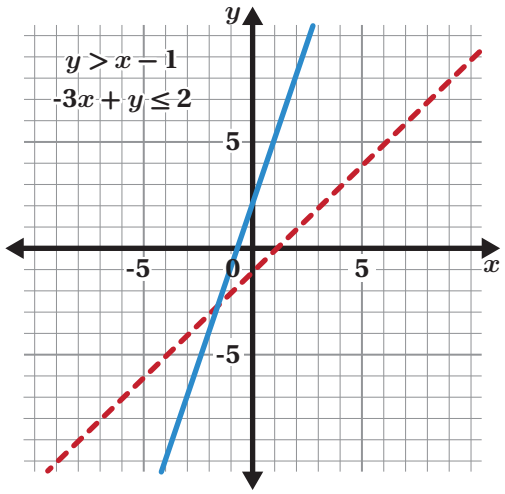
How would you convince a classmate that there are no solutions to this system?



10 Synthesis

Discuss: What are some things you should keep in mind when determining the solution region of a system of inequalities?

Use the graph if it helps with your thinking.



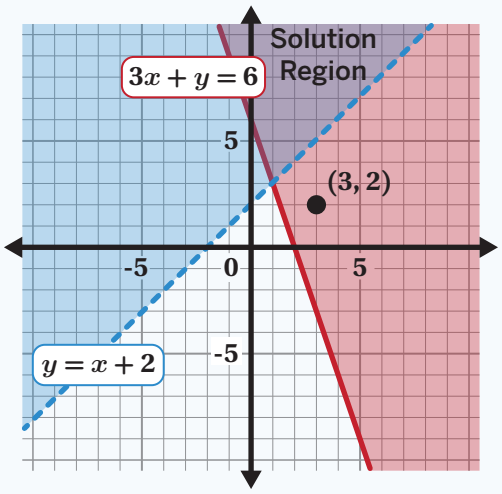
13 Summary 3.10

The **solution to a system of inequalities** can be seen in the region where the graphs overlap. One strategy for determining the location of the *solution region* is to test a point.

Let's look an example for this system of inequalities:

$$3x + y \geq 6$$
$$y > x + 2$$

You can test the point (3, 2) to help determine the solution region.



Solid Line	Dashed Line
$3x + y \geq 6$	$y > x + 2$
$3(3) + 2 \geq 6$	$0 > 3 + 2$
$11 \geq 6$	$0 > 5$
True ✓	False ✗
Shade the side of the solid line that includes (3, 2)	Shade the side of the dashed line that does not include (3, 2)

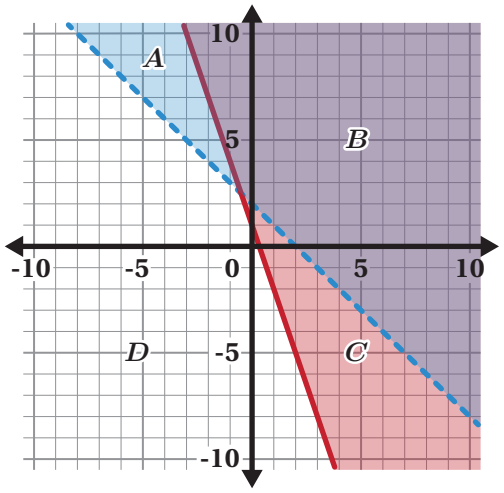
solution (to a system of inequalities) An ordered pair that makes each inequality in a system true. Every ordered pair that is a solution to a system is located in the *solution region* where the graphs overlap.

Practice
3.10

Name: ..... Date: ..... Period: .....

Problems 1–2: Here is the graph of a system of inequalities.

$$y > -x + 2$$
$$3x + y \geq 1$$



1. **Test Practice** Which letter represents the solution region to the system of inequalities? Circle one:

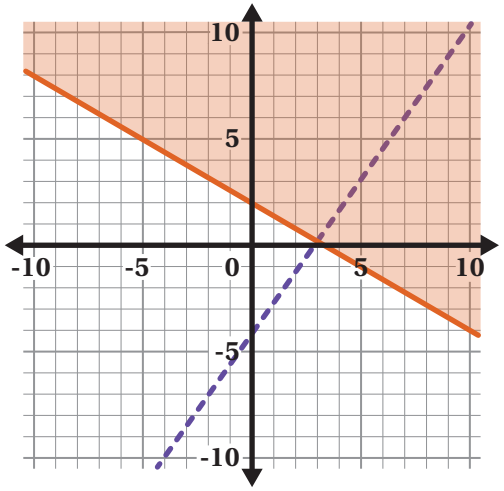
A B C D

2. Is the point (5, -4) a solution to the system? Circle one:

Yes No

Problems 3–4: Javier graphed the first inequality and the boundary line of the second inequality.

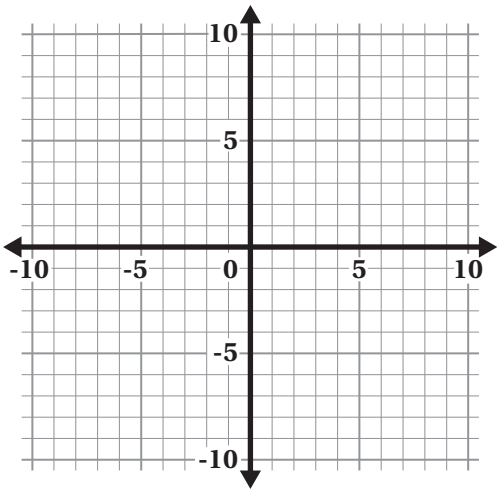
$$3x + 5y \geq 10$$
$$4x - 3y < 9$$



3. Complete the graph of the second inequality.

4. Explain how you knew where to shade the second inequality.

5. Make a graph of a system of inequalities that has no solutions.



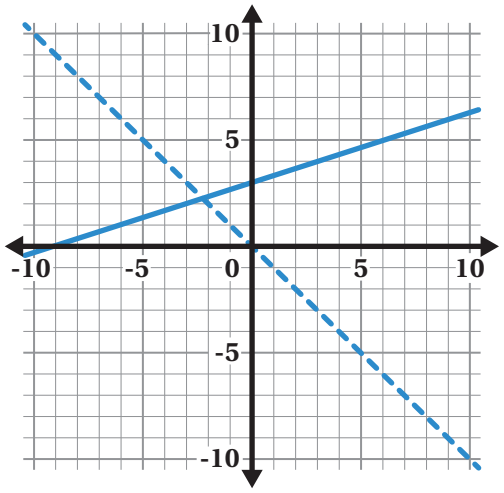
6. Explain how you know it has no solutions.

Name: _____ Date: _____ Period: _____

Problems 7–8: Nyanna graphed the boundary lines of this system of inequalities:

$$x + y > 0$$
$$-x + 3y \leq 9$$

7. Complete the graph of the system of inequalities.



8. Identify a coordinate pair that is in the solution region.

Problems 9–10: Fill in each blank with an inequality symbol such that:

9. The system has no solutions.

$x - y$ ☐ 0

$x - y$ ☐ 0

10. Only points with matching x- and y-coordinates are a solution.

$x - y$ ☐ 0

$x - y$ ☐ 0

Spiral Review

Problems 11–13: Determine the value of each expression.

11. $\frac{4}{6} \cdot 8$

12. $\frac{3}{5} \cdot 2 \cdot \frac{10}{9}$

13. $\frac{3}{6} \cdot 3 \cdot \frac{6}{9} \cdot 2$

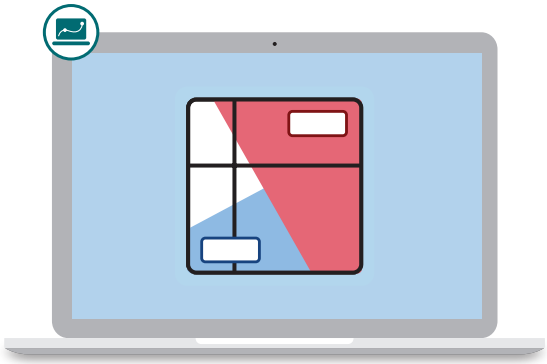
14. Solve $-2x + 3y = 12$ for y to show that it is equivalent to $y = \frac{2}{3}x + 4$. Show or explain your thinking.

Name: ..... Date: ..... Period: .....

Systems of Equations A–REI.12, F–IF.7, F–IF.7.a, SMP.6, SMP.7

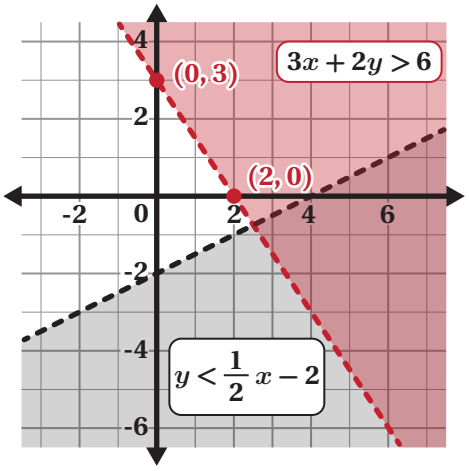
Boundaries and Shading

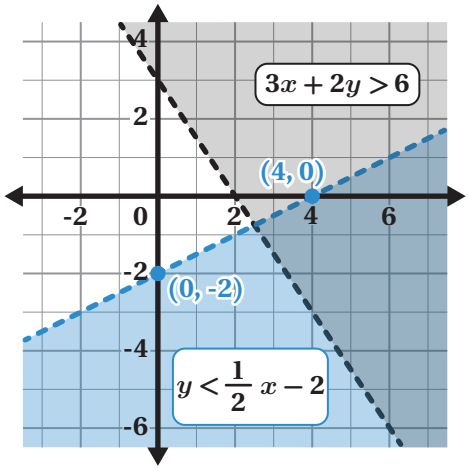
Let's write and graph systems of inequalities.



Warm-Up

The graphs show a system of inequalities. The x- and y-*intercepts* are shown for the boundary lines of two inequalities. Choose one inequality to talk about with a partner.





1. **Discuss:** For the inequality you chose, how can you determine the intercepts of a boundary line without a graph?

2. **Discuss:** How can you use the intercepts of the boundary line to graph this inequality?

Activity 1

Name: ..... Date: ..... Period: .....

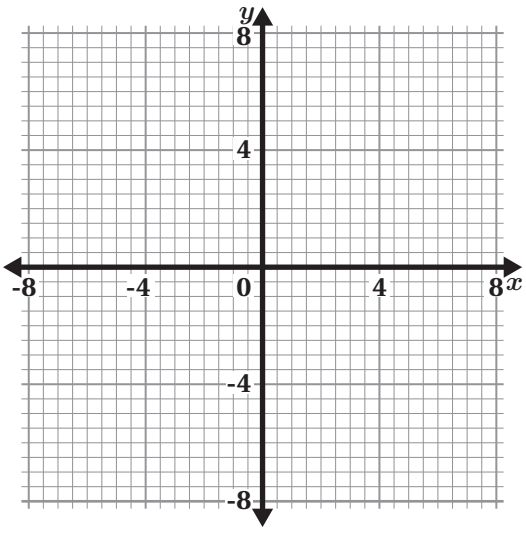
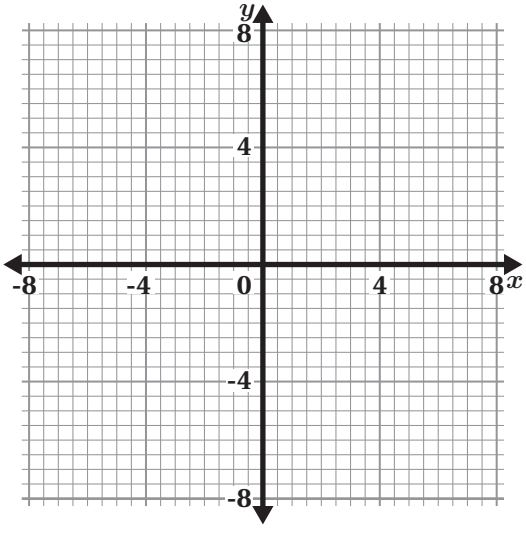
Rounds of Systems

You and your partner will use a set of cards for this activity.

3. Graph a system of inequalities for each round.

Instructions for Each Round

Step 1: Select a card. Graph the inequality on your card.
Step 2: Trade cards with your partner. This will be the second inequality in your system.
Step 3: Graph the second inequality on the same set of axes as the first.
Step 4: Determine one solution to the system. Explain or show how you know it is a solution.
Step 5: Highlight the solution region on the graph.

Round 1	Round 2
My inequality:	My inequality: .....
My partner's inequality:	My partner's inequality: .....
	
What is one solution to the system? Show or explain your thinking.	What is one solution to the system? Show or explain your thinking.

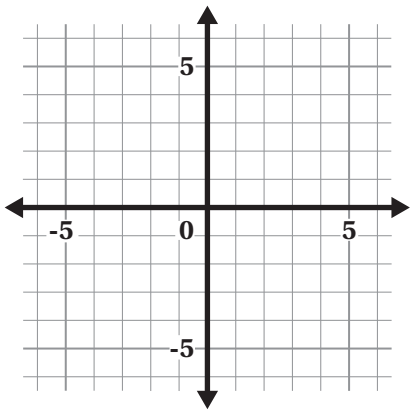
Name: ..... Date: ..... Period: .....

Graphing Systems

4. Select four of these six problems to complete with your partner. Use the graphing calculator to check your work.

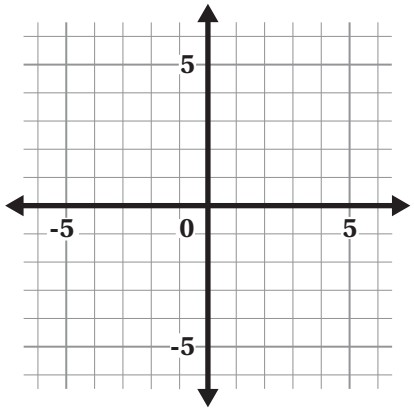
a Graph the system of inequalities and mark the solution region.

$$2x - y < 4$$
$$y \geq \frac{1}{4}x + 3$$

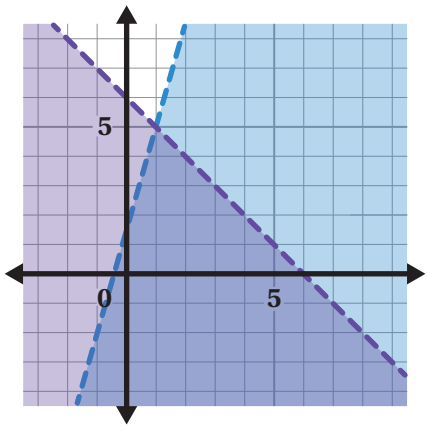


b Graph the system of inequalities and mark the solution region.

$$3x - 6y \geq 12$$
$$y \leq 1$$



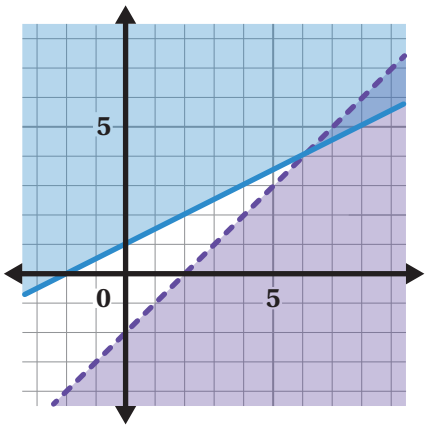
c Write a system of inequalities that this graph could represent.



Inequality 1: .....

Inequality 2: .....

d Write a system of inequalities that this graph could represent.



Inequality 1: .....

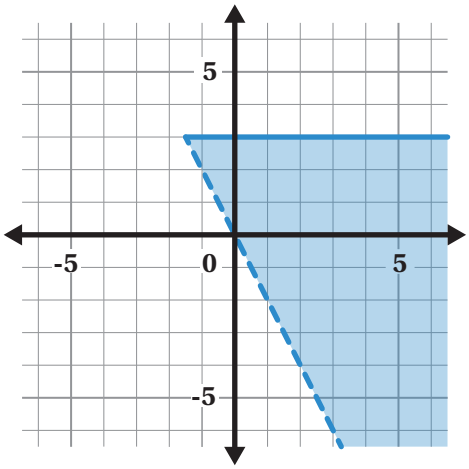
Inequality 2: .....

Name: ..... Date: ..... Period: .....

Graphing Systems (continued)

e Here is the graph of a solution region.

Write a system of inequalities with this solution region.

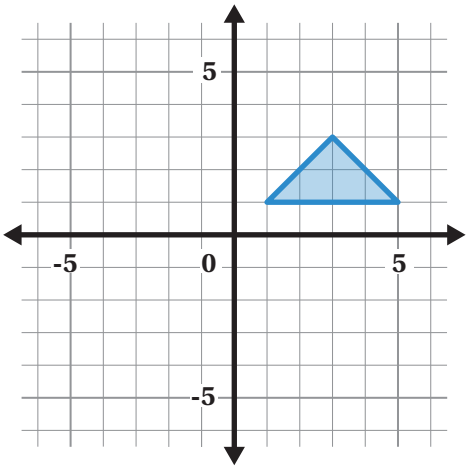


Inequality 1: .....

Inequality 2: .....

f Here is the graph of a solution region.

Write a system of inequalities with this solution region.



Inequality 1: .....

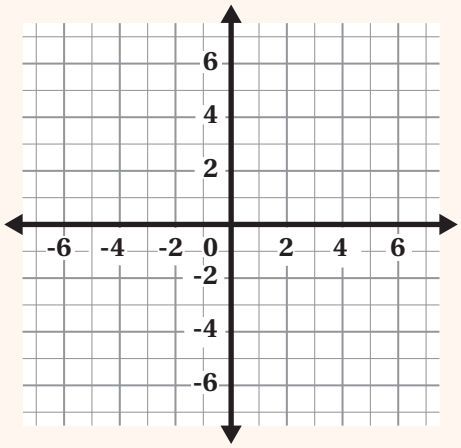
Inequality 2: .....

Inequality 3: .....

You're invited to explore more.

5. Draw and shade a shape on the graph that uses only straight lines.

Write a system of inequalities so that your shape is the solution region.

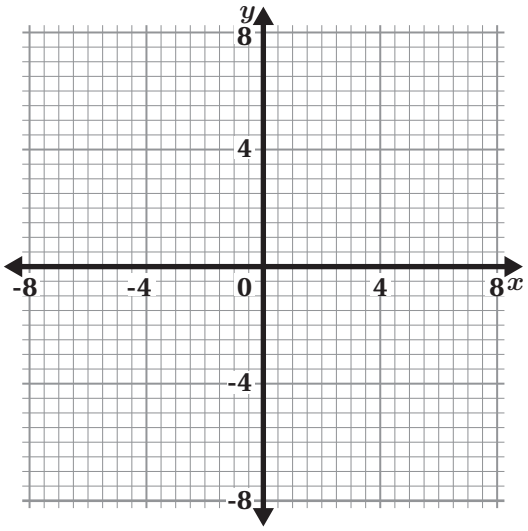


Synthesis

6. Describe a strategy for graphing the solution region for a system of inequalities.

Use this example if it helps with your thinking.

$$x + y < 3$$
$$y \geq 2x$$



Summary 3.11

You can graph a system of linear inequalities by graphing the boundary line of each inequality and testing a point to determine which side of the boundary lines to shade.

You can use different strategies to help you graph boundary lines of inequalities.

- A strategy to graph boundary lines written in *slope-intercept form* ($y = mx + b$) is to plot the y-intercept and use the slope to determine other points.
- A strategy to graph boundary lines written in *standard form* ($Ax + By = C$) is to plot and connect the x- and y-*intercepts*.

If an inequality uses a $\leq$ or $\geq$ symbol, then the boundary line is solid and included in the solution region. If an inequality uses a $<$ or $>$ symbol, then the boundary line is dashed and not part of the *solution region*.

You can also write a system of linear inequalities from a graph. Use the boundary lines and test points to help you determine the inequality symbols. You can also use test points to check the accuracy of your system of inequalities.

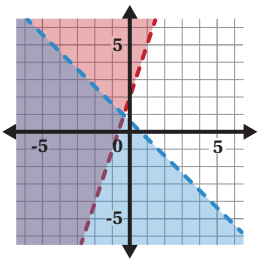
Practice
3.11

Name: ..... Date: ..... Period: .....

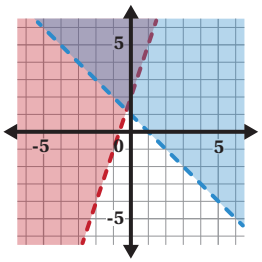
1. **Test Practice** Which graph matches this system of inequalities?

$$-3x + y \geq 2$$
$$2x + 2y \geq 2$$

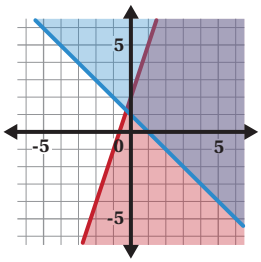
A.



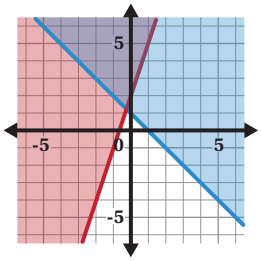
B.



C.



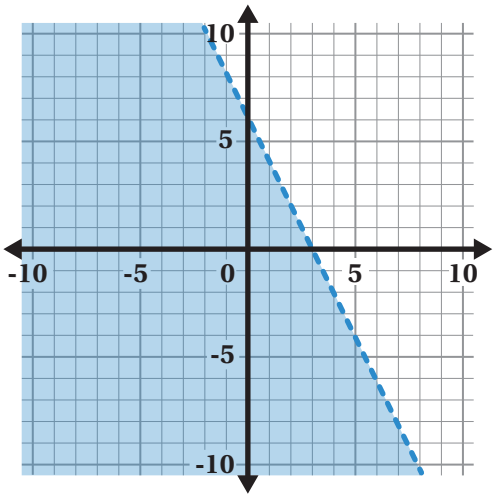
D.



Problems 2–4: The first inequality of this system of inequalities is graphed below.

$$5y < 30 - 10x$$
$$2x + 3y > 12$$

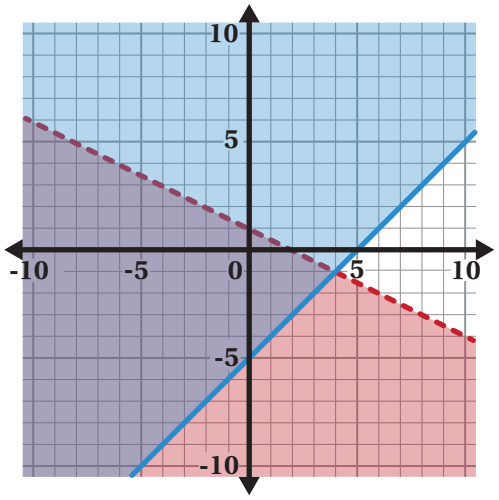
2. Graph the second inequality.



3. Write a point that is *not* a solution to this system.

4. Write a point that *is* a solution to this system.

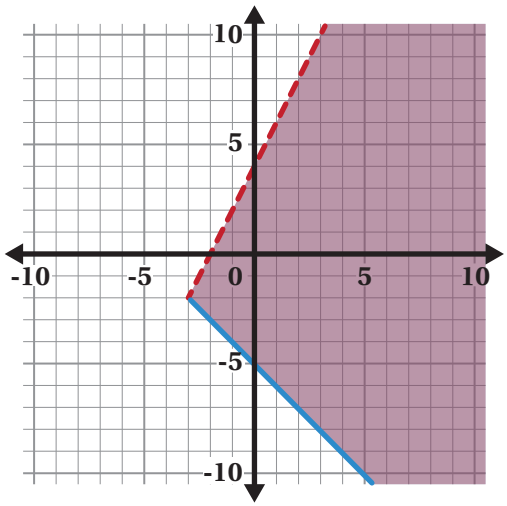
5. Write a system of inequalities to represent this graph.



Practice
3.11

Name: ..... Date: ..... Period: .....

6. Here is the graph of a solution region. Write a system of inequalities that has this solution region.

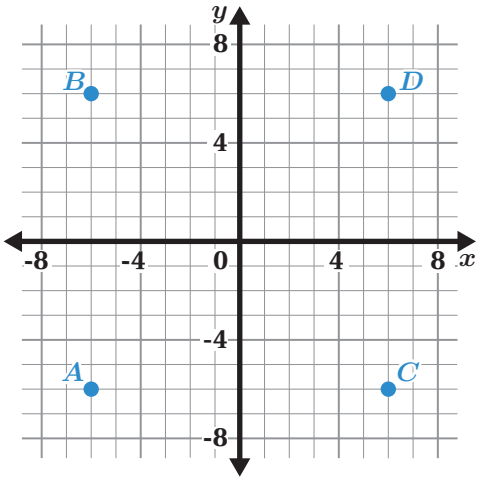


7. Using the digits 0–9 without repeating, fill in each blank such that all of the following statements are true.

- A is a solution to both inequalities.
- B is a solution to only one inequality.
- C is a solution to only the other inequality.
- D is not a solution to either inequality.

$\square x + \square y < \square$

$\square x + \square y < \square$



Spiral Review

Problems 8–10: Consider the line $f(x) = 3x - 4$ and the point $(5, c)$. Write a value for c so that the point is:

8. On the line.

$c =$ .....

9. Above the line.

$c =$ .....

10. Below the line.

$c =$ .....

Problems 11–12: Solve each equation for y.

11. $8x - 3y = -57$

12. $13x - 5y = 35$

Name: _____ Date: _____ Period: _____

Modeling with Functions | Systems of Equations | A–CED.3, A–REI.12, F–IF.7, N–Q.1, N–Q.2, SMP.1, SMP.4

Community Meals

Let's use systems of inequalities to model real-world situations.

Warm-Up

Here is a situation with hidden information.

Double Trellis Food Initiative is a community organization in Philadelphia that prepares and serves free meals to community members. Each day, Chef Malik plans to make at least _____ meals, some vegetarian and some with meat. Preparing a vegetarian meal costs _____, while a non-vegetarian meal costs _____ each. Chef Malik's total daily budget is _____.

1. Let's make sense of this situation together as a class.

Activity 1

Name: ..... Date: ..... Period: .....

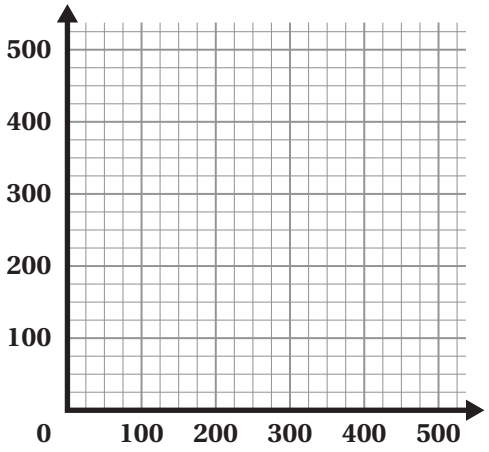
Meal Prep

Double Trellis Food Initiative is a community organization in Philadelphia that prepares and serves free meals to community members.

Each day, Chef Malik plans to make at least 325 meals, some vegetarian and some with meat. Preparing a vegetarian meal costs $1.50, while a non-vegetarian meal costs $2. Chef Malik's total daily budget is $600.

2. Define the variables in this situation.

- x represents . . .
- y represents . . .



3. Write a system of inequalities to represent these *constraints*.

4. Graph the system of inequalities you wrote. Mark the solution region. Be sure to label each axis.

5. Liam thinks that Chef Malik can make 400 vegetarian and 100 non-vegetarian meals each day and meet his constraints.

Is Liam's thinking correct? Show or explain your thinking.

6. How many vegetarian meals and non-vegetarian meals do you recommend Chef Malik plan to make? Explain your thinking.

Activity 2

Name: ..... Date: ..... Period: .....

Make It To-Go

Chef Malik is considering packaging and delivering his meals in order to reach more community members. Each to-go vegetarian meal would cost $\$2.25$ to make and each to-go non-vegetarian meal would cost $\$2.75$.

Chef Malik wants to serve at least 325 meals and spend no more than $\$600$.

7. Write a system of inequalities to represent these constraints. Be sure to define your variables.

- x represents . . .
- y represents . . .

8. Use Screen 6 in the digital companion to graph the system you created.

9. What does the solution region for this system tell us about Chef Malik's plan?

10. Chef Malik was able to get an *additional* $\$200$ in funding to help with the added cost of the to-go meals.

Will that additional money allow him to meet the constraints for his meals? Explain your thinking.

11. Select one question that interests you.

Discuss:

- ☐ **A.** What are some advantages and disadvantages of offering to-go meals versus sit-down meals?
- ☐ **B.** What would you want to know to help Chef Malik decide which option to offer?
- ☐ **C.** If you were in Chef Malik's situation, what would you do?

Name: ..... Date: ..... Period: .....

Planning Your Own System

12. Imagine you just received a $\$1{,}500$ grant to fund work in your local community.

- What group of community members would you like to serve?
- What services would you want to provide?
- What would you want to know or learn about as you create a plan?

13. Create a system of inequalities to model what you plan to do with the grant money. Be sure to define your variables.

14. Use Screen 8 in the digital companion to graph the system you created.

15. Describe the choices you made while creating your plan and what that means about how you will serve the community.

Synthesis

16. **Discuss:** How can systems of inequalities help solve real-world problems in your community?

Summary 3.12

Systems of inequalities can help you solve problems involving real-world *constraints*.

Here is an example about a juice shop.

You can write and graph a system of inequalities to represent these constraints.

Let x represent the number of 12-ounce jars and y can represent the number of 16-ounce jars.

$$12x + 16y \leq 144$$

$$2.50x + 4.50y > 33.50$$

A juice shop has 144 ounces of orange juice to fit in jars of 12 and 16 ounces. They earn \$2.50 for each 12-ounce jar and \$4.50 for each 16-ounce jar. They need to earn more than \$33.50 from the juice.

You can graph each boundary line and use the test point $(2, 7)$ to help us determine the solution region.

Solid Line

$12x + 16y \leq 144$

$12(2) + 16(7) \leq 144$

$136 \leq 144$

True ✓

Shade the side of the solid line that includes $(2, 7)$

Dashed Line

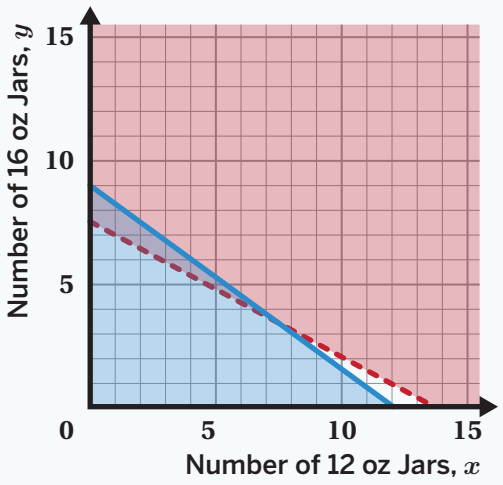
$2.50x + 4.50y > 33.50$

$2.50(2) + 4.50(7) > 33.50$

$36.50 > 33.50$

True ✓

Shade the side of the dashed line that includes $(2, 7)$



You can use the graph to help you determine some possible combinations of 12- and 16-ounce jars of juice that meet the constraints. Such as:

- Zero 12 oz jars of juice and eight 16 oz jars of juice
- Zero 12 oz jars of juice and nine 16 oz jars of juice
- One 12 oz jars of juice and eight 16 oz jars of juice

Practice
3.12

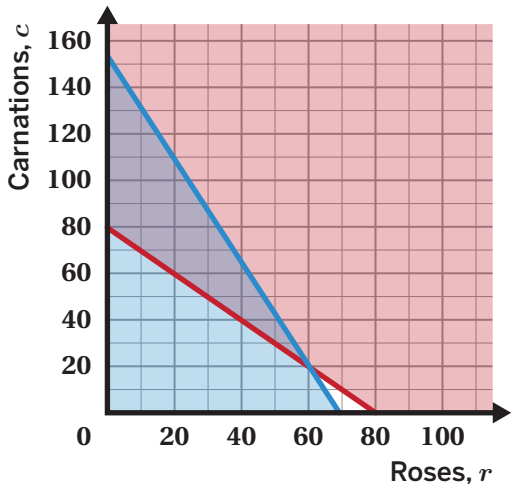
Name: ..... Date: ..... Period: .....

Problems 1–4: Victor has $100 to spend on flowers for a school celebration. Roses, r, cost $1.45 each and carnations, c, cost $0.65 each. Victor wants to buy enough flowers so that all 80 people can take home at least one flower.

1. **Test Practice** Select *all* inequalities that represent the constraints in this situation.

- ☐ **A.** $r + c \leq 80$
- ☐ **B.** $r + c \geq 80$
- ☐ **C.** $1.45r + 0.65c \leq 80$
- ☐ **D.** $1.45r + 0.65c \geq 100$
- ☐ **E.** $1.45r + 0.65c \leq 100$

2. Victor created a graph using inequalities. What is a combination of carnations and roses that meets the constraints?



3. What is a combination of carnations and roses that *does not* meet the constraints?

4. Victor wants to get *the most roses possible* while still meeting the constraints. How many roses and carnations should he get?

Explain your thinking.

Problems 5–6: Cho needs at least 3 pounds of fruit to make a fruit salad for a family gathering. She decides to buy blueberries and apples. She has $12 to spend. Blueberries cost $4.85 per pound and apples cost $1.31 per pound.

- b represents pounds of blueberries.
- a represents pounds of apples.

5. Write a system of inequalities to represent Cho's constraints.

6. Can Cho buy 1.5 pounds of blueberries and 2.5 pounds of apples? Circle your choice.

Yes No

Show or explain your thinking.

Practice
3.12

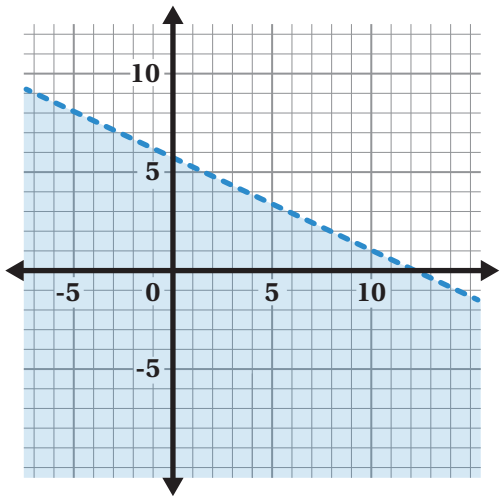
Name: ..... Date: ..... Period: .....

Spiral Review

7. The first inequality of this system of inequalities is graphed. Graph the second inequality.

$x + 2y < 12$

$y \leq x - 4$



Problems 8–9: Solve each system. Write the solution as a coordinate pair.

8. $2x - 3y = 14$
$2x + y = 6$

9. $y = 9x + 17$
$y = 6 - 2x$

Problems 10–12: Evaluate each expression.

10. 25% of 200

11. 12% of 200

12. 8% of 200

Problems 13–14: Solve each formula for the indicated variable.

13. Solve for l.
$2l + 2w = P$

14. Solve for h.
$V = \frac{1}{3}Bh$

Name: ..... Date: ..... Period: .....

Systems of Equations | Modeling with Functions

A–CED.3, A–REI.6, A–REI.12, F–IF.7, SMP.2, SMP.5, SMP.7

Practice Day 2

Let's practice what you've learned so far in this unit!

You will use problem cards for this Practice Day. Record all of your responses here.

Card 1

a

b

Card 2

a

b

Card 3

Card 4

Card 5

a

b

Card 6

a

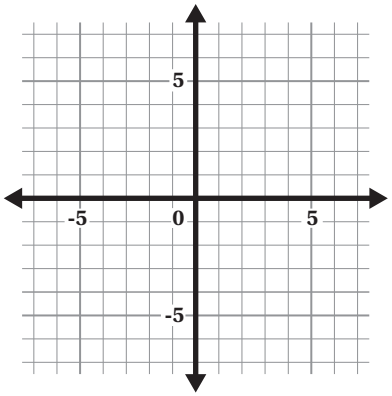
b

Name: ..... Date: ..... Period: .....

Practice Day 2 (continued)

Card 7

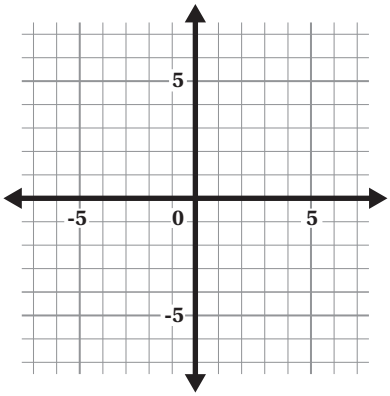
a



b

Card 8

a



b

Card 9

a No solutions:

b Infinitely many solutions:

c One solution:

Card 10

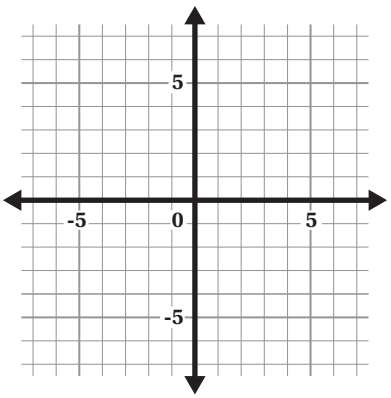
a No solutions:

b Infinitely many solutions:

c One solution:

Card 11

Card 12

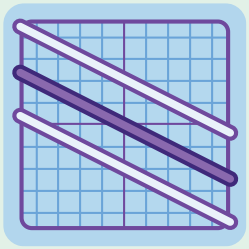


Notes:

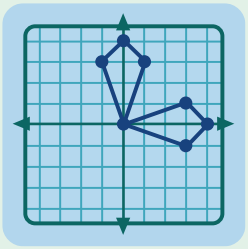
Sub-Unit

3

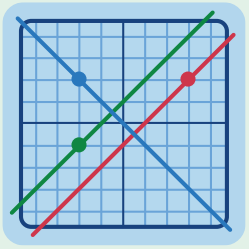
Proving Geometric Theorems Algebraically



Lesson 13
Parallel Lines on the Coordinate Grid



Lesson 14
Perpendicular Lines on the Coordinate Grid



Lesson 15
It's All on the Line

Lesson 16
Going the Distance

Lesson 17
Restaurant Math

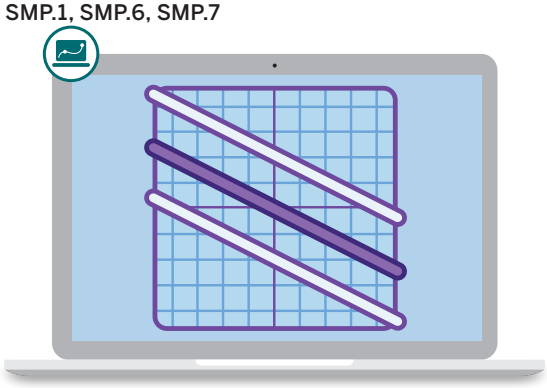
Name: ..... Date: ..... Period: .....

Modeling with Functions | Building with Triangles | Transformation and Congruence

CA CCSSM: G–CO.1, G–GPE.4, G–GPE.5, N–Q.1, SMP.1, SMP.6, SMP.7

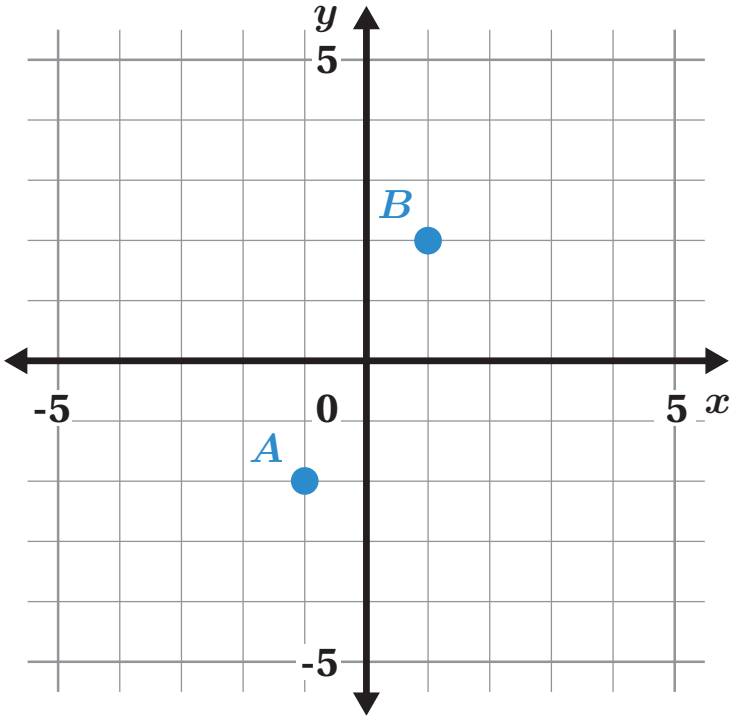
Parallel Lines on the Coordinate Grid

Let's use coordinates to investigate parallel lines.



Warm-Up

1 Translate each point to the right 2 units and up 1 unit. Plot these new points on the coordinate grid, labeling them A' and B'.

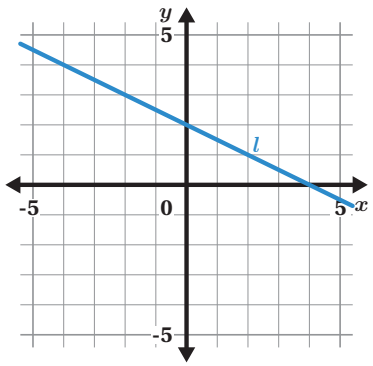


Activity 1

Name: ..... Date: ..... Period: .....

Translated Lines

2 Here is line l represented by the equation $y = -\frac{1}{2}x + 2$.



a Translate the line three times using any translation rule. Graph each of the translated lines then label them as a, b and c.

b **Discuss:** What do you notice? What do you wonder?

I notice . . .

I wonder . . .

3 Eva says that translated lines are always parallel to the original line. Do you agree or disagree with Eva?

Agree Disagree I'm not sure

Explain your thinking.

4 When a line is translated, **parallel lines** are created.

a Find the slope of each of your translated lines. Show your thinking.

b **Discuss:** What do you notice? Do you think this will always be true? Why or why not?

Line	Slope
l	$-\frac{1}{2}$
a	
b	
c	

Activity 2

Name: ..... Date: ..... Period: .....

Build a Parallelogram

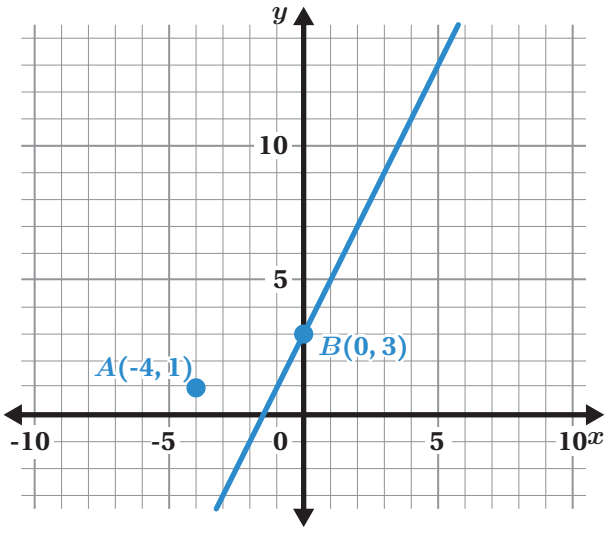
Equations and points can be used to build a parallelogram.

The line $y = 2x + 3$ contains one side of parallelogram $ABCD$. $A(-4, 1)$ and $B(0, 3)$ are two vertices of parallelogram $ABCD$.

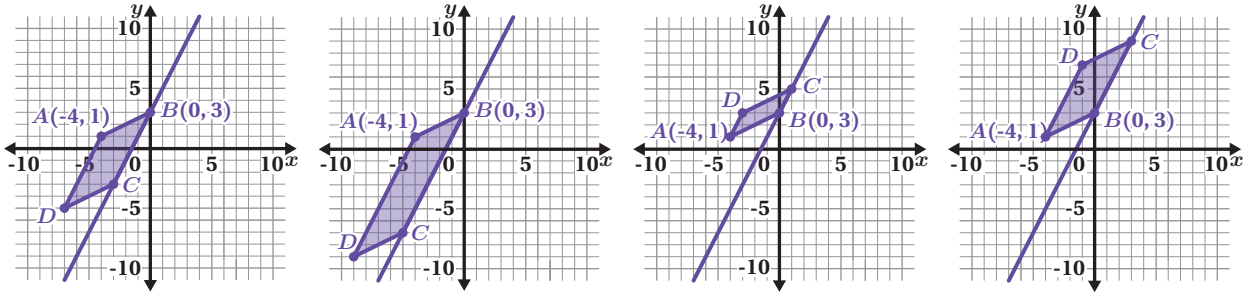
5 Write the equations for the three other sides of parallelogram $ABCD$.

Then graph these lines on the coordinate grid.

Side	Equation containing the side
BC	$y = 2x + 3$
AB	
CD	
DA	



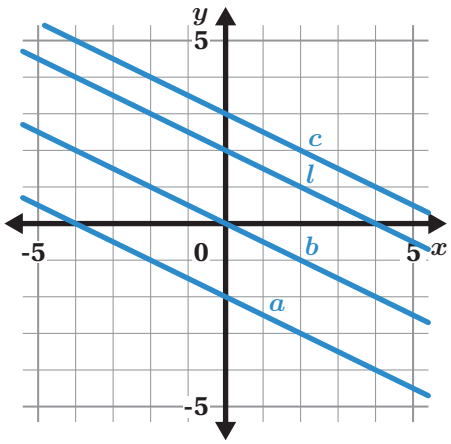
6 You can create many different parallelograms based on the location of the vertex C.



Explain how you know each of these quadrilaterals is a parallelogram.

7 Synthesis

Discuss: Explain what is true about the slope of a parallel line and how we know. Use the visual if it helps explain your thinking.



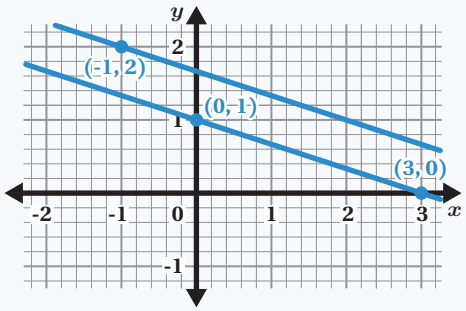
10 Summary 3.13

When a line is translated, each and every point on the line moves the same distance and direction from the original position. A translated line will always be *parallel* to the original line.

In this lesson, you used translations to prove the slope criteria for **parallel lines**. This theorem states that two lines with the same slope are parallel, and if two lines are parallel, then they have the same slope. This information can be used to solve problems.

For example: Write an equation of a line that passes through $(-1, 2)$ and is parallel to a line with x-intercept $(3, 0)$ and y-intercept $(0, 1)$.

- Find the slope of the line: $m = \frac{\Delta y}{\Delta x} = \frac{1-0}{0-3} = -\frac{1}{3}$
- Parallel lines have the same slope, so $m = -\frac{1}{3}$
- Write the equation: $y - 2 = -\frac{1}{3}(x + 1)$
- This can be put in slope-intercept form by distributing $-\frac{1}{3}$, then isolating y: $y = -\frac{1}{3}x + \frac{5}{3}$



parallel lines Lines that never cross or intersect. The distance between corresponding points on two parallel lines is always the same. On a graph, these are two lines with the same slope and different y-intercepts.

Practice
3.13

Name: ..... Date: ..... Period: .....

1. **Test Practice** What equation represents a line that is parallel to the line $y = -3x + 4$?

A. $y = -3x + 3$ **B.** $y = -\frac{1}{3}x + 4$ **C.** $y = \frac{1}{3}x + 3$ **D.** $y = 3x + 4$

2. Write an equation of the line that passes through the point $(-1, 2)$ and is parallel to the line with the equation $y = \frac{1}{5}x + 8$.

3. Write an equation of the line that passes through the point $(2, 4)$ and is parallel to the line that contains points $(0, 9)$ and $(1, -2)$.

4. Wey Wey says that the slope of a line parallel to $-4x + y = 1$ is 4. Is Wey Wey correct?

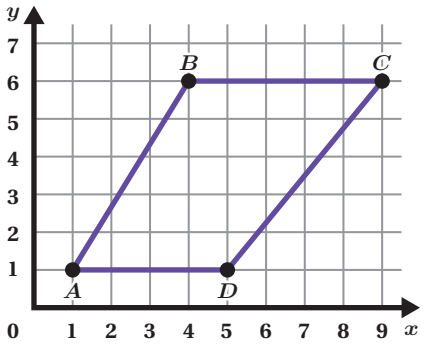
Yes No

Explain your thinking.

5. Is figure $ABCD$ a parallelogram?

Yes No

Show or explain your thinking.



Name: ..... Date: ..... Period: .....

6. Select *all* the equations that are parallel to $2x + 5y = 8$.

- ☐ **A.** $y = \frac{2}{5}x + 4$
- ☐ **B.** $y = -\frac{2}{5}x + 4$
- ☐ **C.** $y - 2 = \frac{5}{2}(x + 1)$
- ☐ **D.** $y - 2 = -\frac{2}{5}(x + 1)$
- ☐ **E.** $10x + 5y = 40$

7. Fill in each blank using the digits 1 to 9 only once so that the lines through each pair of points are parallel.

$(\square, \square)$ and $(\square, \square)$

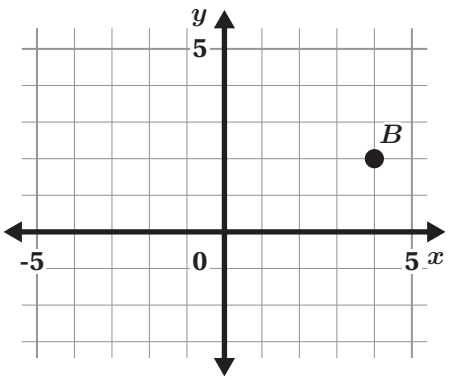
$(\square, \square)$ and $(\square, \square)$

Spiral Review

8. Write an equation of the line with slope $\frac{2}{3}$ that goes through the point $(-2, 5)$.

For Problems 9–11: Imagine that point B is rotated $90°$ counterclockwise about the origin $(0, 0)$.

9. Graph the image of point B.



10. Calculate the slope of the line from the origin to point B.

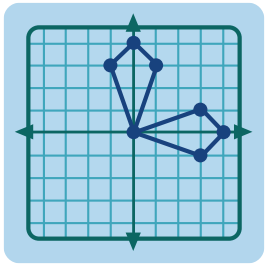
11. Calculate the slope from the origin to point B'.

Name: ..... Date: ..... Period: .....

Modeling with Functions | Building with Triangles | Transformations and Congruence

G–CO.1, G–CO.2, G–GPE.4, G–GPE.5, N–Q.1, SMP.1, SMP.3, SMP.7

Perpendicular Lines on the Coordinate Grid



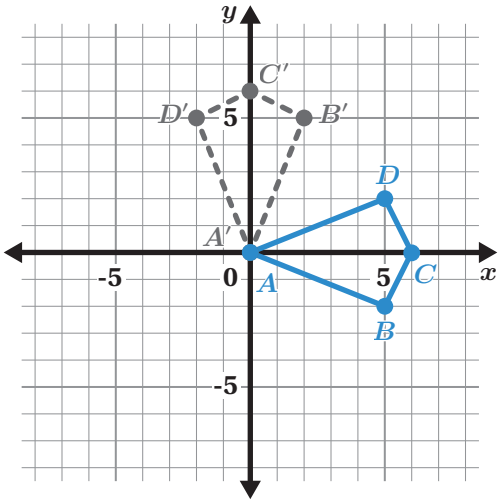
Let's investigate the slopes of perpendicular lines.

Warm-Up

Quadrilateral $A'B'C'D'$ is the image of quadrilateral $ABCD$ after a rigid transformation.

1. Consider the table and the graph. What do you notice? What do you wonder?

Original Point	Pre-Image Coordinates	Image Coordinates
A	(0, 0)	(0, 0)
B	(5, -2)	(2, 5)
C	(6, 0)	(0, 6)
D	(5, 2)	(-2, 5)



I notice . . .

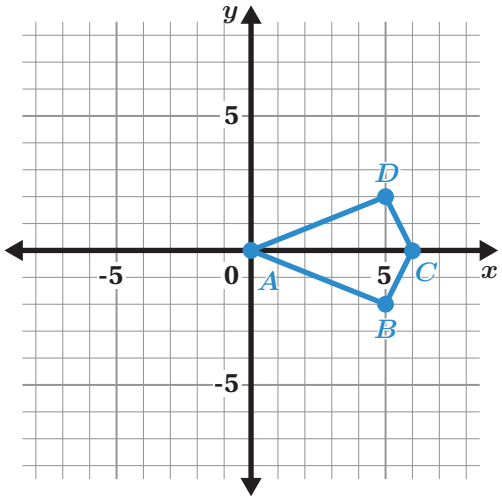
I wonder . . .

Name: ..... Date: ..... Period: .....

Make a Conjecture

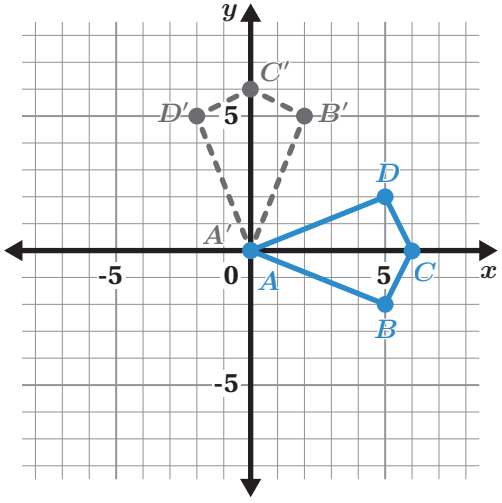
2. Consider Quadrilateral $ABCD$ from the Warm-Up. Complete the table with the slope of each segment in Quadrilateral $ABCD$.

Segment	Slope
AB	
BC	
CD	
DA	



3. Consider the image of Quadrilateral $ABCD$ after a 90° counterclockwise rotation around center $(0, 0)$. Complete the table with the slope of each segment in Quadrilateral $A'B'C'D'$.

Segment	Slope
$A'B'$	
$B'C'$	
$C'D'$	
$D'A'$	



4. What pattern do you notice in your responses to Problems 2–3?

Name: ..... Date: ..... Period: .....

Make a Conjecture (continued)

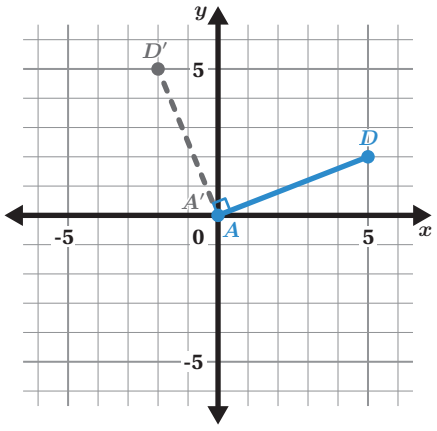
5. Complete the table by calculating the product of the slopes of each pair of segments.

Segments	Product of Slopes
AB and $A'B'$	
BC and $B'C'$	
CD and $C'D'$	
DA and $D'A'$	

6. What pattern do you notice in your responses to Problem 5?

When you identify a pattern, like the one in Problem 5, you can use that pattern to make a **conjecture**, which is an idea that you think is true based on evidence but without proof.

7. A 90° counterclockwise rotation of a pre-image results in the image and pre-image having corresponding lines that are **perpendicular**, meaning they intersect to form a right angle.



Considering this information, Amir says: *"If two lines are perpendicular, then they have opposite reciprocal slopes. If two lines have opposite reciprocal slopes, then they are perpendicular."*

Do you agree or disagree with Amir's conjecture? Circle one.

Agree Disagree I'm not sure

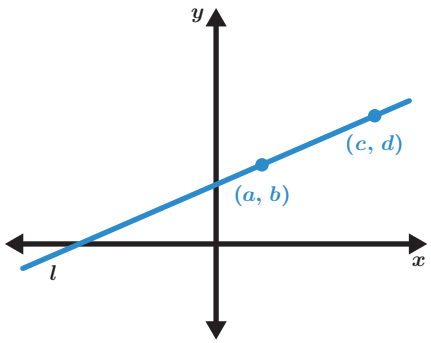
Explain your thinking.

Activity 2

Name: Date: Period:

Prove It

Let's prove part of Amir's conjecture from Activity 1 by looking at a line l that goes through the points (a, b) and (c, d). Assume that the line is not horizontal or vertical.



8. Determine the slope of line l.

9. How can you create a line m that is perpendicular to line l?

10. Determine the coordinates of two points that lie on line m. Use the graph and your responses from the Warm-up if it helps with your thinking.

11. Determine the slope of line m.

12. Use lines l and m to prove that perpendicular lines have opposite reciprocal slopes.

13. Consider the line given by the equation $y = -\frac{2}{3}x + 4$. Write an equation of the line perpendicular to this line that passes through the point $(4, 3)$.

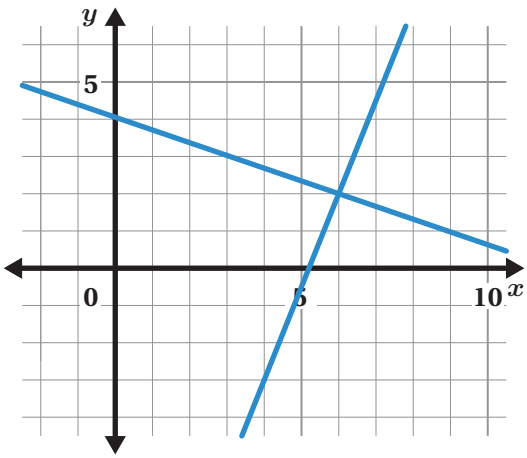
Explain your thinking.

Synthesis

14. Are these lines perpendicular? Circle one.

Yes No

Justify your response.

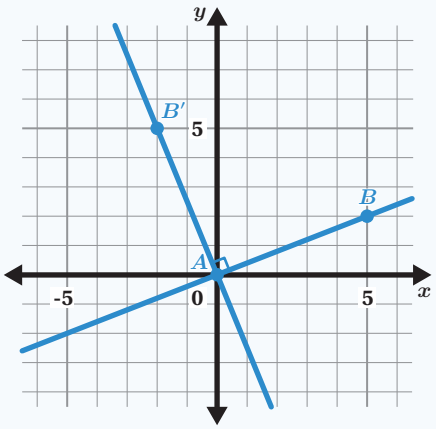


Summary 3.14

Perpendicular lines can be created on the coordinate plane by rotating any line 90° or 270° around the origin.

You can use this fact to formulate a **conjecture** about the slopes of perpendicular lines. Once proved, this theorem states that:

- If two lines have **opposite reciprocal** slopes, then the lines are perpendicular.
- If two lines are perpendicular and neither line is vertical, then their slopes must be opposite reciprocals.



This information can be used to solve problems. For example, if you have a point and a line l, you can use what you know about the slopes of perpendicular lines to write an equation for the line that passes through the point and is perpendicular to line l.

conjecture A statement that is believed to be true based on evidence.

opposite reciprocals Two numbers that have a product of -1. For example, 3 and $-\frac{1}{3}$ are opposite reciprocals.

perpendicular Intersecting at a right angle.

Practice
3.14

Name: ..... Date: ..... Period: .....

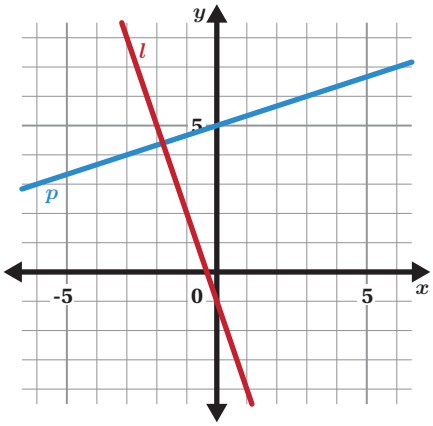
Problems 1–4: Are lines l and p perpendicular? Explain your thinking.

1. Line l has a slope of 4.
Line p has a slope of -4.

2. The equation of line l is $y = \frac{2}{3}x + 7$.
The equation of line p is $y = -\frac{3}{2}x - 5$.

3. The equation of line l is $y - 2.25 = -2(x - 2)$.
The equation of line p is $-2x - 4y = 10$.

4.



Problems 5–6: Write an equation of the line that satisfies each description.

5. Passes through the point $(-6, -4)$ and is perpendicular to $y = \frac{1}{2}x - 9$.

6. Passes through the point $(-8, 11)$ and is perpendicular to $y = -5(x + 4)$.

7. **Test Practice** Determine the equation of the line that is perpendicular to $3x + y = 5$ and passes through $(2, -4)$.

A. $y = \frac{1}{3}x + \frac{10}{3}$ **B.** $y = 3x - 10$ **C.** $y = -3x + 2$ **D.** $y = \frac{1}{3}x - \frac{14}{3}$

Name: ..... Date: ..... Period: .....

8. Determine the point of intersection of these two lines:

- $y = \frac{1}{4}x - 4$
- The line perpendicular to $y = \frac{1}{4}x - 4$ that passes through $(2, 5)$.

9. Four lines contain the sides of a rectangle. None of the lines are vertical or horizontal. Write the equations of four lines that would make this situation true.

First equation	
Second equation	
Third equation	
Four equation	

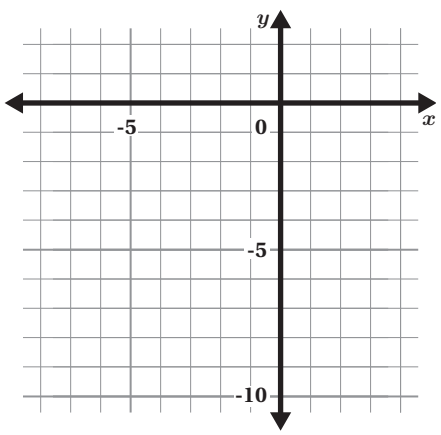
Spiral Review

10. Solve this system of equations. Write the solution as a coordinate pair.

$5x - 2y = 15$

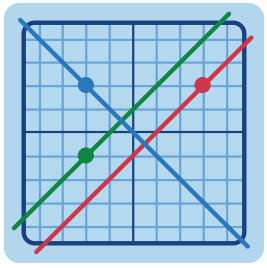
$3x - 3y = 18$

11. Calculate the exact distance between the points (-7, -9) and (2, -6). Use the coordinate plane if it helps with your thinking.



Name: ..... Date: ..... Period: .....

Modeling with Functions | Transformations and Congruence | A–CED.2, G–GPE.5, SMP.6, SMP.7

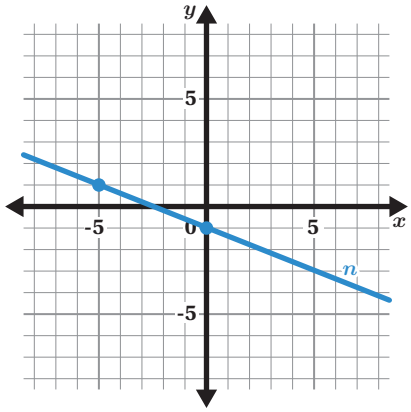


It's All on the Line

Let's work with both parallel and perpendicular lines.

Warm-Up

The image shows line n.



1. Write an equation for line p that is *perpendicular* to line n and whose y-intercept is $(0, 5)$. Graph and label line p.

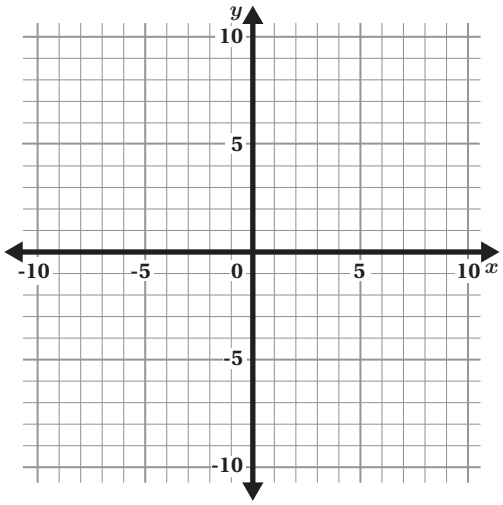
2. Write an equation for line t that is *parallel* to line n and that passes through the point $(3, 1)$. Graph and label line t.

Activity 1

Name: ..... Date: ..... Period: .....

Three Lines

3. Line l is represented by the equation $y = \frac{2}{3}x + 2$. Graph and label line l.



4. Write an equation of the line perpendicular to l that passes through the point $(-6, 4)$. Graph the line and label it line p.

5. Write an equation of the line perpendicular to the line p that passes through the point $(3, -2)$. Graph the line and label it line n.

6. Examine your graphs for lines l, p, and n.

a What do you notice about lines l and n?

b Does this always happen? Explain your thinking.

You're invited to explore more.

7. Prove that the line $Ax + By = C$ is always perpendicular to the line that passes through (A, B) and the origin.

Activity 2

Name: ..... Date: ..... Period: .....

Partner Problems

With your partner, decide who will complete Column A and who will complete Column B. After each pair of problems, share your solution with your partner. Compare your solutions, and discuss and resolve any differences.

	Column A	Column B
8.	Write an equation of the line that passes through the point $(0, 8)$ and is parallel to the line $y = -2x + 5$.	Write an equation of the line that passes through the point $(1, 6)$ and is perpendicular to the line $y = \frac{1}{2}x + 1$.
9.	Write an equation for a line that passes through the point $(3, -4)$ and is perpendicular to the line $y = \frac{2}{5}x + 1$.	Write an equation for a line that is a translation of the line $y = -\frac{5}{2}x + 5$ and passes through the point $(3, -4)$.
10.	Write an equation of the line that passes through the point $(2, 1)$ and is perpendicular to the line $y = -\frac{1}{3}x$.	Write an equation of the line that passes through the point $(0, -5)$ and is parallel to the line $y = 3x + 4$.
11.	Write an equation of the line that is a translation of the line $y = 2x - 1$ and passes through the point $(-2, 2)$.	Write an equation of the line that passes through the point $(-2, 2)$ and is perpendicular to the line $y = -\frac{1}{2}x + 3$.

Synthesis

Discuss:

12. What are the different ways that you can determine the equation of a line that is parallel to $y = 2x + 1$?

13. How can you use slope to help determine the equation of a line that is perpendicular to $y = 2x + 1$?

Summary 3.15

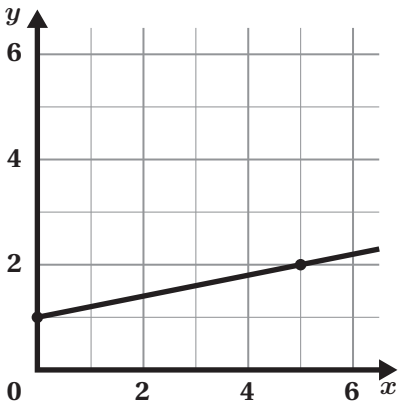
There are different ways to determine the equation of a parallel or perpendicular line. Depending on the information given, one method might be more efficient than another.

Parallel	Perpendicular
• Determining the slope of the given line and writing an equation with the same slope. For example, to find the equation of the line parallel to $2x + 4y = 12$ through the point (-1, 2), we would first find the slope of the given line: $2x + 4y = 12y \Rightarrow y = -\frac{1}{2}x + 3$ The slope of the given line is $-\frac{1}{2}$. The slope of a line parallel would also be $-\frac{1}{2}$. The equation of the parallel line can be found using point-slope form: $y - 2 = -\frac{1}{2}(x + 1)$	• Determining the slope of the given line and writing an equation with the opposite reciprocal slope. For example, to find the equation of the line perpendicular to $6x + 2y = 12$ through the point (-1, 2), we would first find the slope of the given line: $6x + 2y = 12 \Rightarrow y = -3x + 6$ The slope of the given line is -3. The slope of a line perpendicular would be $\frac{1}{3}$. The equation of the perpendicular line can be found using point-slope form: $y - 2 = \frac{1}{3}(x + 1)$
• Translating or rotating the given line $180°$ using any point not on the line as the center of rotation.	• Rotating the line $90°$ using any point as the center.

Practice
3.15

Name: ..... Date: ..... Period: .....

Problems 1–6: Here is the graph of a line. Determine whether each equation is parallel to this line, perpendicular to this line, or neither.



1. $y = 0.2x$

2. $y = -2x + 1$

3. $y = 5x - 3$

4. $y - 3 = -5(x - 4)$

5. $y - 1 = 2(x - 3)$

6. $5x + y = 3$

7. Write the equation of a line that passes through the point $(4, -2)$ and is parallel to the line $2x - y = 9$.

8. Write the equation of a line that passes through the point $(0, 12)$ and is perpendicular to the line $6x + 2y = 10$.

9. Cedar Street is parallel to Ash Street. Ash Street is parallel to Beech Street. Beech Street is perpendicular to Front Street. What is the relationship between Cedar Street and Front Street? Explain your thinking.

10. **Test Practice** A line with equation $y = mx + b$ passes through the point $(-5, 1)$ and is parallel to $y = 2x - 8$. What is the value of b?

A. 9 **B.** 1 **C.** 11 **D.** -11

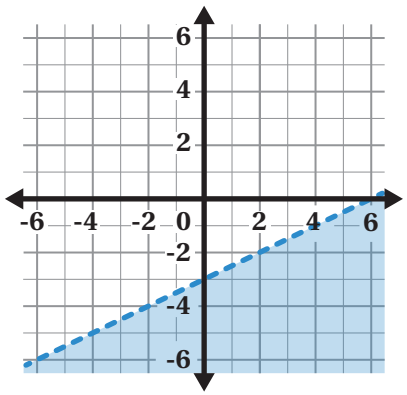
Name: ..... Date: ..... Period: .....

Spiral Review

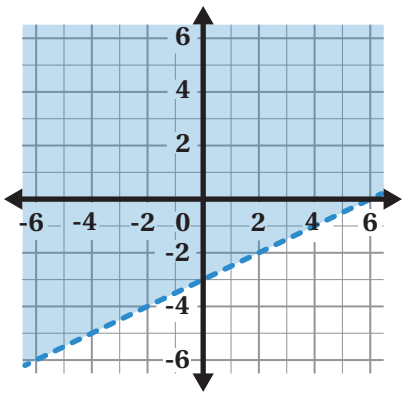
11. In which graph does the shaded region represent the solution set for the inequality shown below?

$$2x - 4y < 12$$

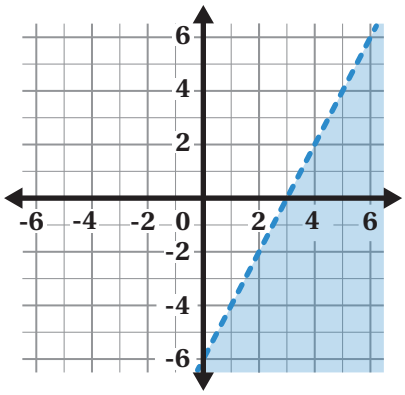
A.



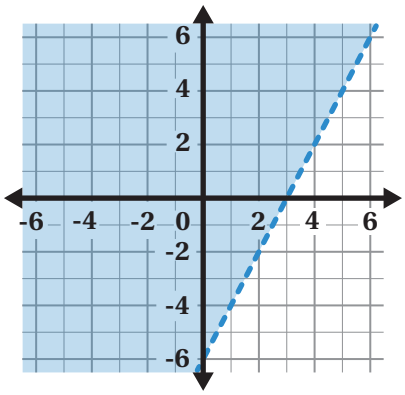
B.



C.



D.



12. Show or explain how you can determine that the point $(2, 1)$ is a solution to this system of equations.

$2x - y = 3$

$x + 3y = 5$

Name: ..... Date: ..... Period: .....

Building with Triangles | Transformations and Congruence

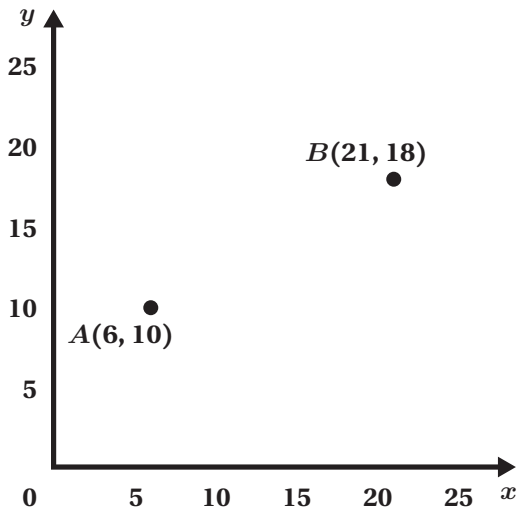
G–CO.1, G–GPE.4, SMP.3, SMP.6, SMP.8

Going the Distance

Let's explore another way to find the length of the segment that connects two points.

Warm-Up

1. Determine the distance between point A and point B.



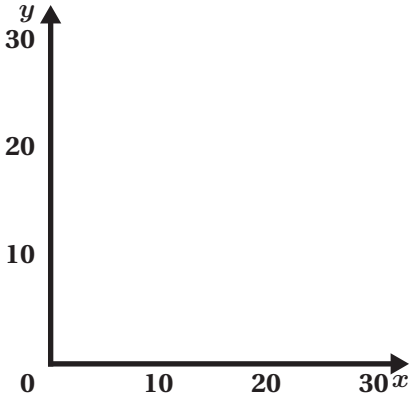
2. When point A and point B are connected they form **line segment** AB. What do you think this term means?

Name: ..... Date: ..... Period: .....

Segment Length

3. Here are three points: $A(6, 10)$, $B(6, 27)$, and $C(2, 10)$.

a **Discuss:** What do you notice? What do you wonder?

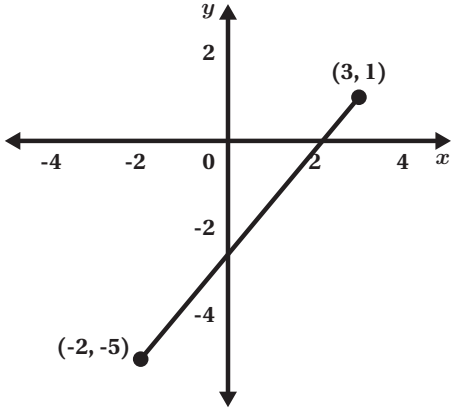


b Determine the length of the following segments.

Use the coordinate grid if it helps with your thinking.

Segment	Length of Segment
AB	
AC	
BC	

4. Determine the length of the segment that connects the point $(3, 1)$ to point $(-2, -5)$.



Name: ..... Date: ..... Period: .....

Segment Length (continued)

5. Write an equation to determine the length of the segment connecting point (-2, -5) to point (x, y).

6. Pilar wrote an equation to determine the length of the segment connecting point (-2, -5) to point (x, y). Some of her work is correct and some of her work is incorrect.

$$\sqrt{(x-2)^2+(y-5)^2} = \text{hypotenuse}$$

a Explain what Pilar did well.

b Explain what you think is incorrect about Pilar's work.

c What question could you ask Pilar to help her correct her work?

7. Suppose you have a line segment with endpoints $A(x_1, y_1)$ and $B(x_2, y_2)$.

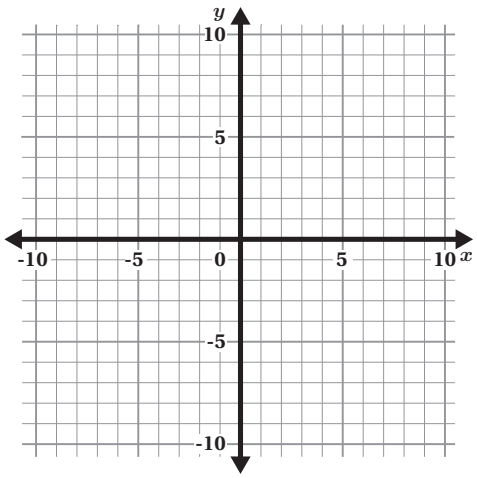
Determine the length of segment AB.

Activity 2

Name: ..... Date: ..... Period: .....

Determining Distance

8. Determine the length of the segment that connects point A at $(0, 9)$ to point B at $(3, 3)$. Show your thinking. Use the grid if it helps.



9. Melanie found the incorrect length of the segment that connects point D at (-7, 2) to point E at $(1, 4)$.

Melanie's work:

Length of segment DE $= \sqrt{(1 - 2)^2 + (-7 - 4)^2}$

$= \sqrt{(-1)^2 + (-11)^2}$

$= \sqrt{1 + 121}$

$= \sqrt{122}$

a. **Discuss:** What did Melanie do well? What is correct about her work?

b. **Discuss:** Describe the mistake in Melanie's work.

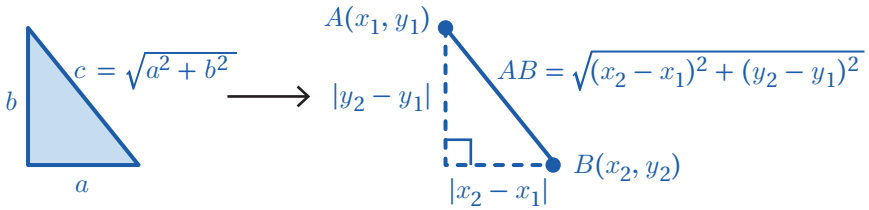
c. Determine the correct length of segment DE.

d. What advice would you give Melanie to help her prevent mistakes like this one?

10. Find the length of the segment that connects point $W(-3.5, -2)$ to point $Z(-6, -4)$. Show your thinking.

Synthesis

11. Explain how the Pythagorean theorem relates to the equation $AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

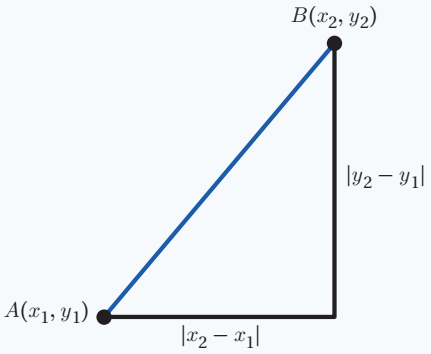


Summary 3.16

There are multiple strategies for determining the length of the segment between two points.

Line segment AB has endpoints $A(x_1, y_1)$ and $B(x_2, y_2)$. We can think of AB as the hypotenuse of a right triangle and determine its length using the Pythagorean theorem.

- The horizontal segment length (leg of right triangle) is the change in the x-values: $|x_2 - x_1|$.
- The vertical segment length (other leg of right triangle) is the change in the y-values: $|y_2 - y_1|$.
- Next, substitute each side length (legs of right triangle) into the Pythagorean theorem: $(x_2 - x_1)^2 + (y_2 - y_1)^2 = AB^2$
- Finally, solve for AB: $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$. This is called the **distance formula**.



line segment The part of a line that connects two points, called endpoints.

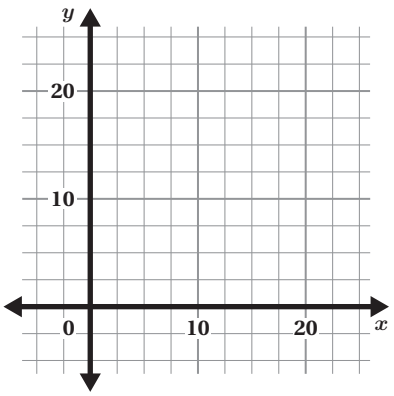
Distance Formula The equation used to determine the length of the segment with endpoints at (x_1, y_1) and (x_2, y_2). The distance can be found by evaluating the expression $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

Practice
3.16

Name: ..... Date: ..... Period: .....

Problems 1–4: Determine the exact length of the segment that connects the two points.

1. $A(0, 0)$ and $B(7, 0)$



2. $C(2, 9)$ and $D(2, 15)$

3. $A(0, 0)$ and $C(2, 9)$

4. $B(7, 0)$ and $D(2, 15)$

Problems 5–6: What is the exact length of the line segment that connects the two points? Show your thinking.

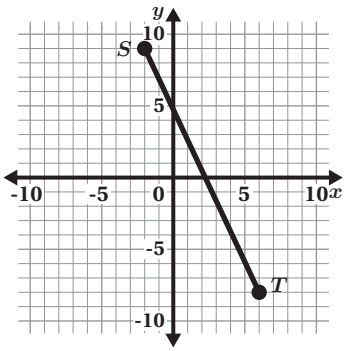
5. $D(-3, 6)$ and $E(4, -1)$.

6. $E(8.2, -6)$ and $F(3.7, -4.1)$.

Name: ..... Date: ..... Period: .....

7. **Test Practice** Given $S(-2, 9)$ and $T(6, -8)$.

What is the length of $\overline{ST}$?



A. $\sqrt{23}$

B. 17

C. $5\sqrt{13}$

D. $\sqrt{353}$

8. Determine the two possible locations of the x-coordinate for a segment with:

- A length of $\sqrt{34}$.
- One endpoint at $(-2, 6)$ and the other endpoint at $(x, 1)$.

Spiral Review

9. Match each line with a line perpendicular to it.

a $y = \frac{1}{3}x + 4$ ..... $y = 5x - 1$

b $x + 5y = 17$ ..... The line through $(1, -1)$ and $(-1, 5)$

c The line through $(-2, 6)$ and $(2, 8)$ ..... $2x + y = 13$

10. Solve this system of equations. Write the solution as a coordinate pair.

$$\begin{cases} 2x - 3y = -4 \\ y = x + 5 \end{cases}$$

Name: ..... Date: ..... Period: .....

Building with Triangles G–GPE.7, SMP.1, SMP.4, SMP.6

Restaurant Math

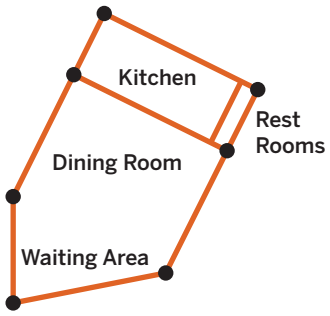
Let's explore the perimeter and area using coordinates.

Warm-Up

A chef plans to open a new farm-to-table fine dining restaurant in an existing building. The floor plan for the new restaurant is shown.

I notice . . .

I wonder . . .

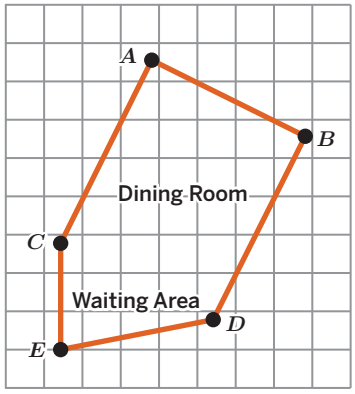


Activity 1

Name: ..... Date: ..... Period: .....

Finding Dimensions

The floor plan for a portion of the restaurant has been superimposed on a grid. Each unit represents 1 meter. The coordinates of each point are as follows: $A(4, 18)$, $B(24, 8)$, $C(-8, -6)$, $D(12, -16)$, and $E(-8, -20)$.



1. **Discuss:** How can the coordinates be used to determine the length of each wall?

The restaurant plans to put LED lighting all along the perimeter of the waiting area and dining room.

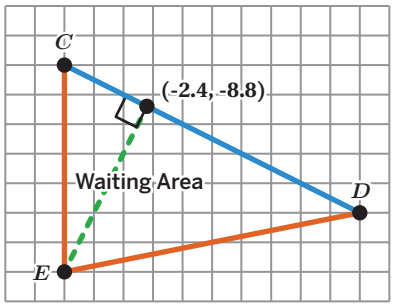
2. Determine the number of meters of LED lighting needed. Round your answer to the nearest tenth of a meter. Show your thinking.

Activity 2

Name: ..... Date: ..... Period: .....

Capacity

The owner of the restaurant needs to know if there is enough space to set up a 52.8 square meter pick-up station for takeout orders in the waiting area.



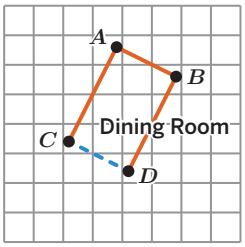
3. What is the floor space of the entire waiting area? Round your answer to the nearest square meter. Note: The perpendicular line drawn from point E to the point (-2.4, -8.8) on the opposite side is called the **altitude**.

The recommended floor space is 18 square feet or approximately 1.67 square meters per guest in the waiting area.

4. How many guests would fit in the remaining waiting area if the pick-up station is installed?

Health and Safety regulations require 20 square feet or approximately 1.85 square meters per guest in the dining room for fine dining restaurants.

5. What is the capacity of this restaurant? Show or explain your thinking.



You're invited to explore more.

6. The owner must determine how many of each type of table to place in the dining room. What would you recommend and why?

Number of seats	2	4	5
Square meters needed	7.5	9.7	11.2

Synthesis

Discuss:

7. What information do you need to find the perimeter and area of a polygon?

8. What tools do you have to obtain that information if the coordinates of the vertices are known?

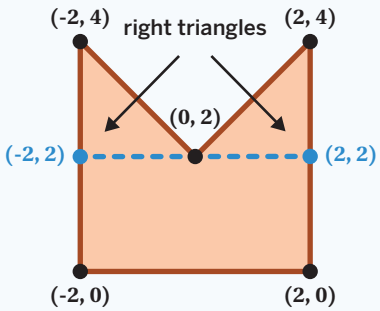
Summary 3.17

Often geometric shapes can be decomposed into familiar polygons.

Coordinates of the vertices can be used to determine the lengths of the sides with the distance formula.

In right triangles with two known lengths, the Pythagorean theorem can be used to find the length of a missing side.

Once the dimensions of the polygon are known, calculate the perimeter and area.

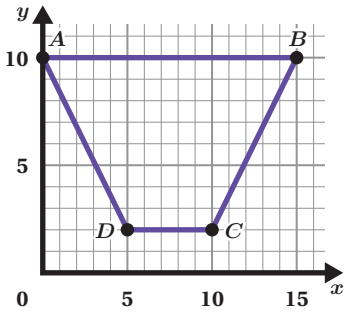


altitude The perpendicular line segment from a vertex of a triangle to the opposite side.

Practice
3.17

Name: ..... Date: ..... Period: .....

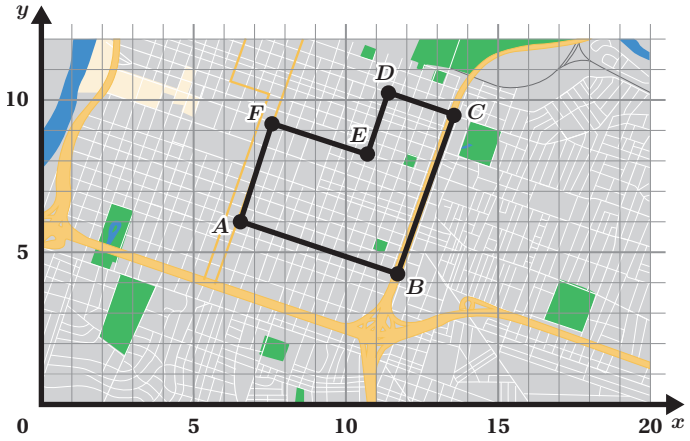
Problems 1–2: Here is polygon $ABCD$.



1. Determine the perimeter of polygon $ABCD$. Round to the nearest tenth.

2. Determine the area of Polygon $ABCD$.

Problems 3–4: The points $A(6.5, 6)$, $B(11.7, 4.3)$, $C(13.5, 9.5)$, $D(11.4, 10.2)$, $E(10.7, 8.2)$, and $F(7.5, 9.2)$ form the vertices of a polygon that approximates the area of Midtown Sacramento.



3. Use the coordinate points to approximate the perimeter of Midtown. Round to the nearest hundredth.

4. Use the coordinate points to approximate the area of Midtown. Round to the nearest hundredth.

5. **Test Practice** Triangle ABC has vertices at (-4, 0), (-1, 6), and (3, -1). What is the perimeter of triangle ABC, rounded to the nearest tenth?

Name: ..... Date: ..... Period: .....

6. Nicolas was calculating the distance between $C(-3, 8)$ and $D(5, -1)$ but made a mistake in his calculations. Identify Nicolas's mistake and correct it to find the length of CD.

$$CD = \sqrt{(-3 + 5)^2 + (8 - 1)^2}$$

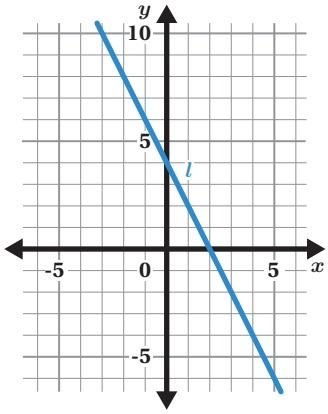
$$CD = \sqrt{(2)^2 + (7)^2}$$

$$CD = \sqrt{4 + 49}$$

$$CD = \sqrt{53} \approx 7.3$$

Spiral Review

Problems 7–9: Here is the graph of line l.



7. Write an equation to represent line l.

8. Line m contains the point $(4, 4)$ and is parallel to line l. Write an equation to represent line m.

9. Line n has a y-intercept at $(0, 8)$ and is perpendicular to line l. Write an equation to represent line n.

10. What is the solution to this system of equations?

Equation 1: $y = -2x + 3$

Equation 2: $12x + 6y = 24$

A. All real numbers **B.** No solution **C.** $(0, 3)$ **D.** $(2, 0)$

Name: ..... Date: ..... Period: .....

Systems of Equations | Modeling with Functions | Building with Triangles | Transformations and Congruence | A–CED.3, A–REI.6, A–REI.12, G–GPE.5, G–GPE.7, SMP.2, SMP.6, SMP.7

Practice Day 3

Let's practice what you've learned so far in this unit!

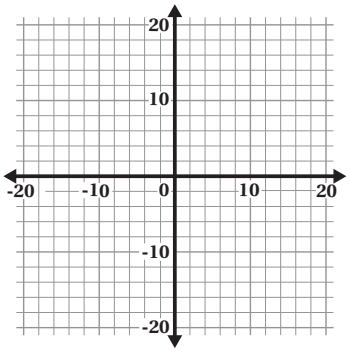
You will use problem cards for this Practice Day. Record all of your responses here.

Card 1

Card 2

Card 3

a



b

Card 4

a

b

Card 5

Card 6

Name: Date: Period:

Practice Day 3 (continued)

Card 7

a

b

Card 8

Card 9

a

b

Card 10

Card 11

Notes:

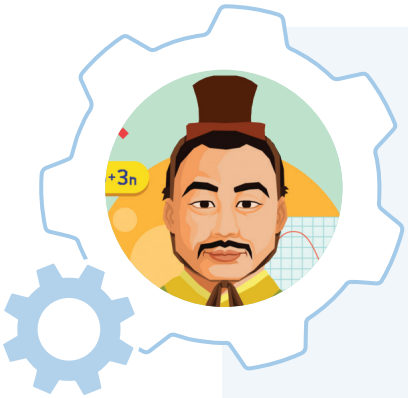
Career Connection

How is city planning related to solving a puzzle?

KieferPix/Shutterstock.com

Solving a puzzle can involve thinking about different strategies, trying out ideas, and thinking about constraints. One famous historical puzzle, the "Hundred Fowls Problem," was created by 5th century Chinese mathematician Zhang Quijian.

City planners think about constraints as they design cities to be attractive and functional. Some constraints might be geographic location, access to natural water sources, or available budget to spend. City planners might create systems of equations to help decide where to place buildings, public transit systems, and parks.



Meet Zhang Qiujian

While little is known about Chinese mathematician Zhang Quijian's biography, he wrote a famous puzzle called the "Hundred Fowls" problem in his 5th century book, *Zhang Qiujian Suanjing*. Can you solve it? How can a system of equations help you get started?

"Roosters cost 5 qian each, hens cost 3 qian each, and three chicks cost 1 qian. If 100 fowls are bought for 100 qian, how many roosters, hens, and chicks are there?"

Are you interested in solving more puzzles like this one or studying city planning? What can you do to learn more?

Math in the World

For more than 80 years, the city of Copenhagen, Denmark, has been designed around the concept of a hand with 5 fingers. Public transit systems and major roads lie along each of the "5 fingers" and areas of green space are preserved between them. What are some constraints that might have been involved in designing Copenhagen?

trabantos/Shutterstock.com

Math Mindset

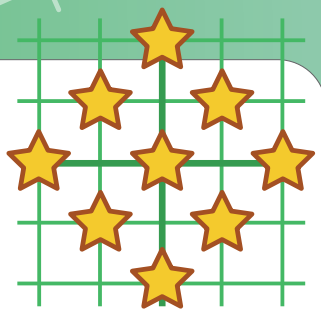
Which strategy would you choose to solve this system of equations? Why would you choose this strategy?

$8x - 5y = -12$

$2x + y = 6$

Unit 4

Describing Functions



Big Ideas in This Unit

CC2 Modeling with Functions | Systems of Equations | Comparing Models

CC3 Composing Functions

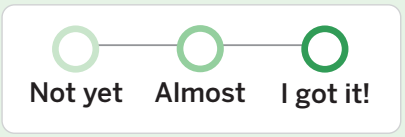
Questions for Investigation

- What are the characteristics of a function and how can a function represent a situation?
- What are key features of functions and how can we use them to describe and interpret functions?
- How can we use function notation as a tool to communicate precisely?

Explore: School Commutes
How can a graph represent a situation?

Watch Your Knowledge Grow

This is the math you'll explore in this unit. Rate your understanding to see how your knowledge grows!



I can . . .	Before	After
Determine whether or not a rule is a function.		
Interpret what each part of an equation written in function notation means in a situation.		
Evaluate a function that is written in function notation for a specific input value.		
Write an equation of a function using function notation.		
Describe the key features of functions in graphs and tables using terms like *minimum*, *maximum*, *positive*, and *negative*.		
Create the graph of a function given a description of its key features.		
Calculate the average rate of change of a function over a specified interval.		
Interpret what the average rate of change means for a specific context.		
Compare key features of graphs.		
Define the function using terms domain and range.		
Determine and describe the domain and range of a function within a context.		
Represent the domain and range of a function using inequalities.		
Determine the domain and range from a graph.		

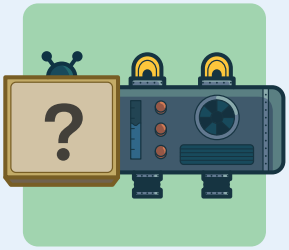
I can . . .	Before	After
Explain why sequences are functions.		
Write a recursive definition for a sequence using function notation.		
Write an explicit definition for a sequence using function notation.		
Evaluate outputs for an absolute value function.		
Graph absolute value functions in the form $f(x) = \|x - h\| + k$.		
Identify how the minimum, a vertical stretch, and a vertical compression affect the graph of an absolute value function.		

Sub-Unit

1

Function Notation

Explore
School Commutes



Lesson 1
Mystery Rule

Lesson 2
Pricing Pizzas

Lesson 3
Toy Factory

Name: ..... Date: ..... Period: .....

Modeling with Functions | Comparing Models | Building Toward F–IF.1, F–IF.4, N–Q.1, SMP.1, SMP.2, SMP.4

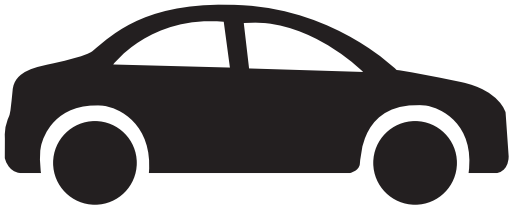
Explore:
School Commutes

How can a graph represent a situation?

Warm-Up

1. How do you get to school in the mornings? Be sure to include all parts of your commute.

2. How long does it usually take you to get to school?



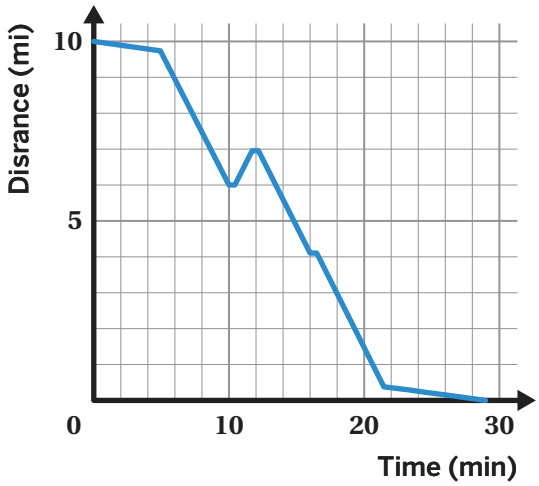
3. About how many miles from school do you live?

4. If you could choose any type of transportation to get to school, what would you choose? Explain your thinking.

Name: ..... Date: ..... Period: .....

School Commute Descriptions and Graphs

Irelle made a graph to represent the commute to school.



5. What do you notice? What do you wonder?

6. What do you think the point $(0, 10)$ represents in Irelle's commute?

7. What do you think the point $(29, 0)$ represents in Irelle's commute?

8. **Discuss:** What might the horizontal segments represent in Irelle's commute?

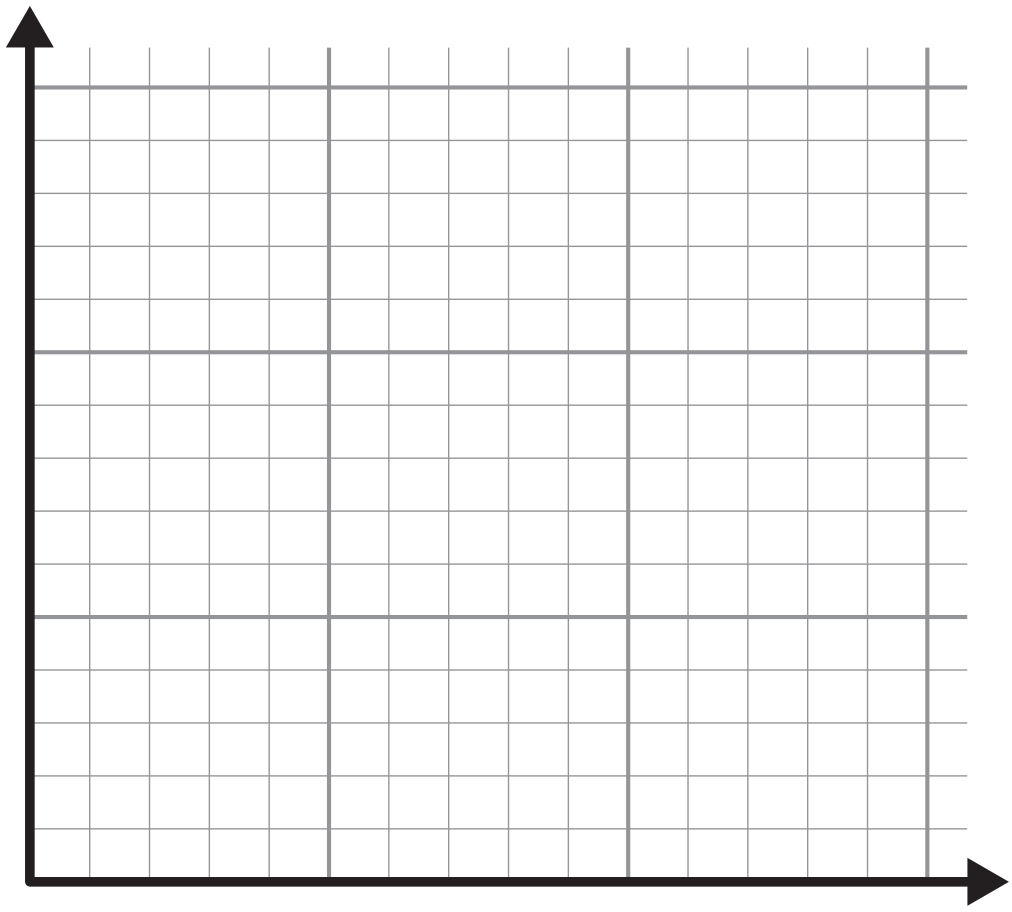
9. Describe how you think Irelle got to school.

Name: ..... Date: ..... Period: .....

School Commute Descriptions and Graphs (continued)

10. Using your information from the Warm-Up, write a brief description of your commute to school.

11. Sketch a graph that represents your description in Problem 10. As you create your graph, consider including axes labels and appropriate scales for both axes.



Name: ..... Date: ..... Period: .....

Building Math habits of Mind

Discuss:

- Which of these habits of mind did you strengthen during this activity?
- How did you use the one(s) you selected?

I can slow down and first make sense of a challenging problem before trying to solve it.

Not yet Almost I got it!

I can represent real-world problems and interpret their solutions within the context of the problem.

Not yet Almost I got it!

I can justify my thinking and ask questions to help me understand the thinking of others.

Not yet Almost I got it!

I can apply the math that I know to solve real-world problems, make assumptions and revise my thinking as needed.

Not yet Almost I got it!

I can select an appropriate tool to help me solve problems.

Not yet Almost I got it!

I can communicate my thinking and solutions clearly to others.

Not yet Almost I got it!

I can look for structure or patterns to help me solve problems.

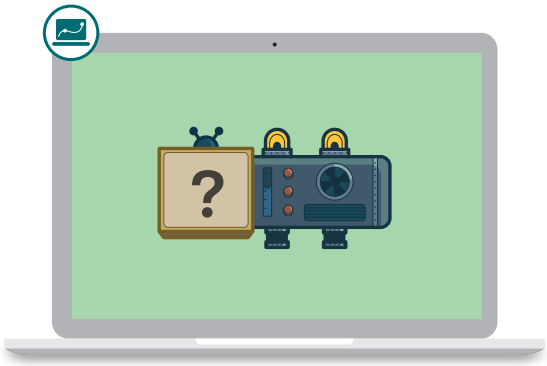
Not yet Almost I got it!

I can look for repeated calculations and other repeated steps to make generalizations.

Not yet Almost I got it!

Name: ..... Date: ..... Period: .....

Modeling with Functions | Building Toward F–IF.1, SMP.3, SMP.6, SMP.8



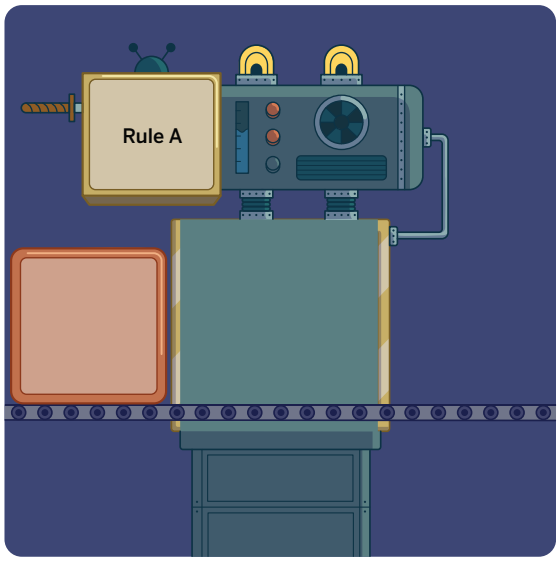
Mystery Rule

Let's consider whether or not rules are functions.

Warm-Up

1 This machine uses Rule A to turn *inputs* into *outputs*.

Let's test several inputs to see how Rule A works. Record the results in the table.



Input	Output
5	16

2 Predict the output for 101. Explain your reasoning.

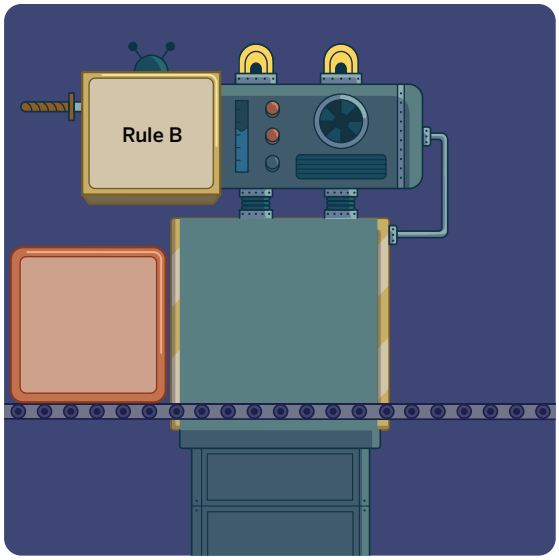
Name: ..... Date: ..... Period: .....

What Is a Function?

3 Rule B's inputs are words.

Let's test several inputs to see how Rule B works. Record the results in the table.

Input	Output
howdy	8



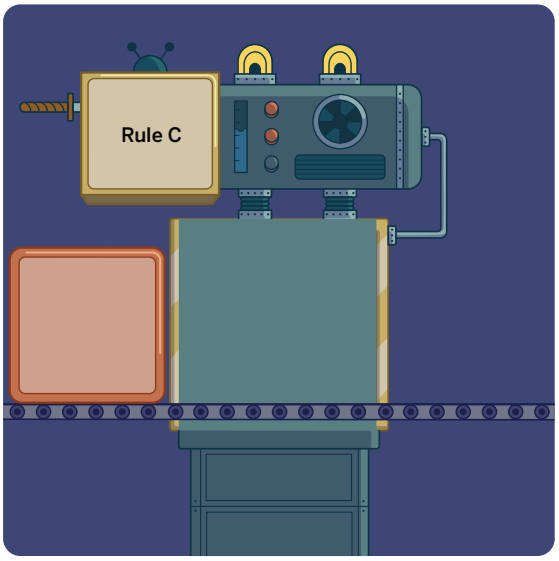
4 Predict the output for "give". Explain your reasoning.

Name: ..... Date: ..... Period: .....

What Is a Function? (continued)

5 Rule C's inputs are whole numbers from 1 to 15.

Let's test several inputs to see how Rule C works. Record the results below.



Input	Output

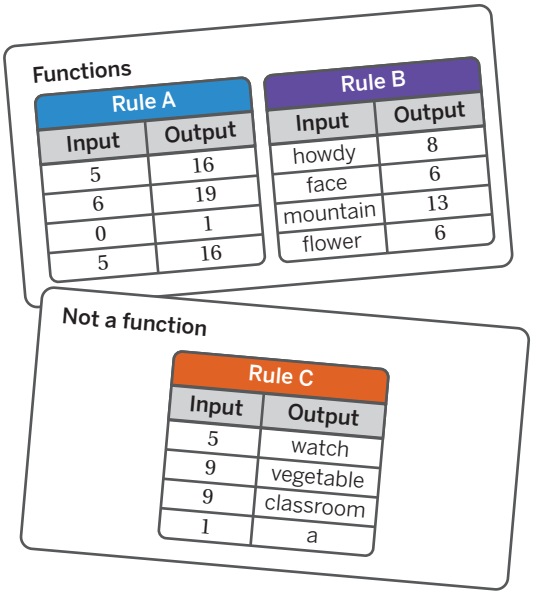
6 **a** Predict the output for 6.

b Compare your response with a partner's. How are your responses alike and different?

7 Rules A and B are examples of a **function**.

Rule C is *not* a function.

What do you think makes Rule C not a function?



Name: ..... Date: ..... Period: .....

Which Rules Are Functions?

8 Here are four rules.

Discuss: Is each rule a function? Why or why not?

Rule D takes temperatures in Fahrenheit and outputs temperatures in Celsius.

Input	Output
50	10
77	25
100	37.8
212	100

Rule E takes any integer and outputs one of its factors.

Input	Output
24	6
13	1
13	13
9	3

Rule F takes any number and rounds it to a whole number.

Input	Output
32.5	33
$\frac{4}{3}$	1
0.1	0
23	23

Rule G takes any word and shifts each letter one place in the alphabet.

Input	Output
bird	cjse
house	ipvtf
hello	ifmmp
world	xpsme

Name: ..... Date: ..... Period: .....

Which Rules Are Functions? (continued)

9 Rule E is *not* a function.

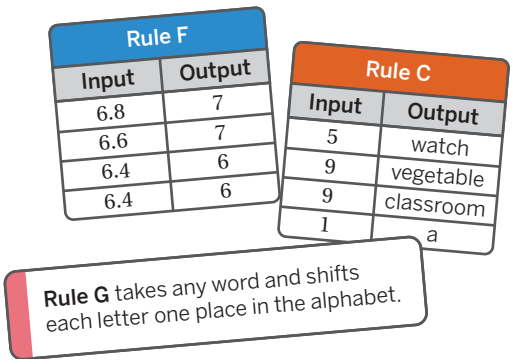
How can you tell by looking at the table?

Rule E takes any integer and outputs one of its factors.

Rule E	
Input	Output
1200	30
1200	10
20	10

10 Synthesis

How can you decide whether a rule is a function?



13 Summary 4.01

We can represent rules with verbal descriptions or as a table of *inputs* and *outputs*. All sets of inputs and outputs are called *relations*. A **function** is a special kind of relation that assigns exactly one output to each possible input.

You can determine whether a rule is a function by organizing the inputs and outputs into a table. If one input has multiple possible outputs, then the rule is not a function.

Here are two examples.

Rule A takes an integer and outputs an integer that is one less.

Input	Output
1	0
2	1
2	1
4	3

In this relationship, Rule A is a function because each input has exactly one output.

Rule B takes a number and outputs a random number that is greater.

Input	Output
0	2
0	10
-2	0
-1.6	-1.2

In this relationship, Rule B is *not* a function because the input 0 has multiple outputs.

Practice
4.01

Name: ..... Date: ..... Period: .....

1. Rule A takes any word as an input and writes the word backwards as an output.

Is Rule A a function? Explain your thinking.

Rule A

Input	hat	sock	racecar
Output	tah	kcos	racecar

Problems 2–3: Here is Rule B.

2. Is Rule B a function? Explain your thinking.

Rule B

Input	4	6	6	5
Output	blue	purple	yellow	white

3. Predict what the output could be when the input is 3.

Problems 4–5: Here is Rule C.

4. Is Rule C a function? Explain your thinking.

Rule C

Input	6	8	12	14
Output	4	6	10	?

5. Predict the missing output for Rule C.

6. **Test Practice** This table shows the total number of days in each month of a given year.

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
28		✕										
29		✕										
30				✕		✕			✕		✕	
31	✕		✕		✕		✕	✕		✕		✕

Imagine a rule where the input is a month and the output is the number of days in that month. Does this rule represent a function? Explain your thinking.

Name: _____ Date: _____ Period: _____

7. A machine uses Rule D to turn inputs into outputs. The table shows two inputs and their outputs.

Hoang tried the input 4 again and the output was not 27. He claims that this is enough information to determine whether or not Rule D is a function.

Is Hoang correct? Explain how you know.

Rule D

Input	Output
3	21
4	27

8. Create a table for Rule E that is *not* a function.

Explain your thinking.

Input	Output

Spiral Review

9. Complete the table using this rule:
Add 2 to the input, then multiply by 3 to get the output.

Input	-5	0	4
Output	-9		

Problems 10–11: A school sells two types of tickets for a play: adult tickets and student tickets. Adult tickets are $5 each and student tickets are $2 each. The school collects $400 total.

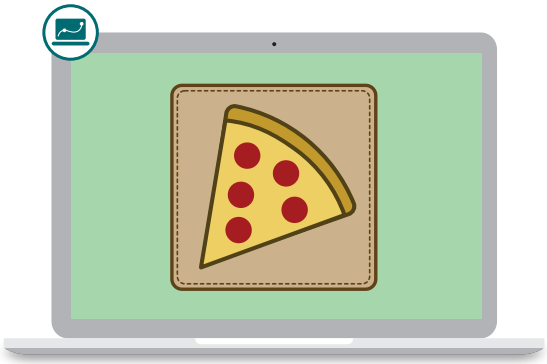
10. Write an equation where x represents the number of adult tickets sold and y represents the number of student tickets sold.

11. How many of each ticket type could the school sell to collect $400?

Adult tickets: _____ Student tickets: _____

Name: ..... Date: ..... Period: .....

Modeling with Functions F–IF.1, F–IF.2, SMP.2, SMP.3, SMP.6



Pricing Pizzas

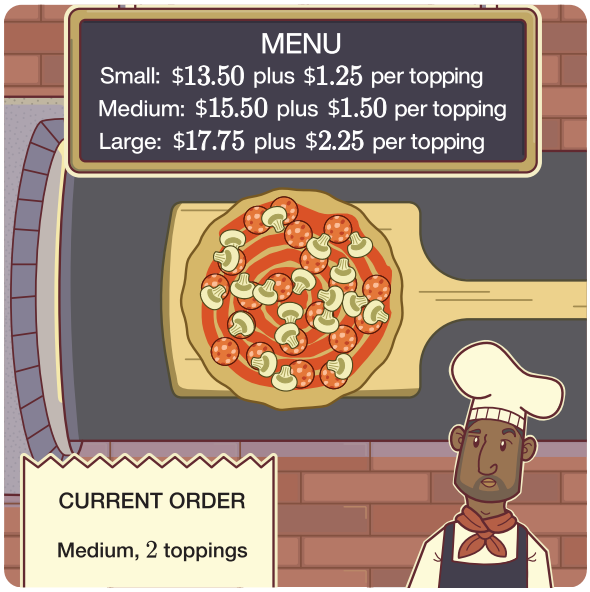
Let's learn what function notation is and interpret function notation statements in context.

Warm-Up

1 Desmos Pizza offers small, medium, and large pizzas.

Use the menu to determine the price of each pizza order.

Pizza Order	Price ($)
Medium, 2 toppings	
Large, 4 toppings	
Small, 3 toppings	
Large, 6 toppings	



Activity 1

Name: ..... Date: ..... Period: .....

Pricing Pizzas

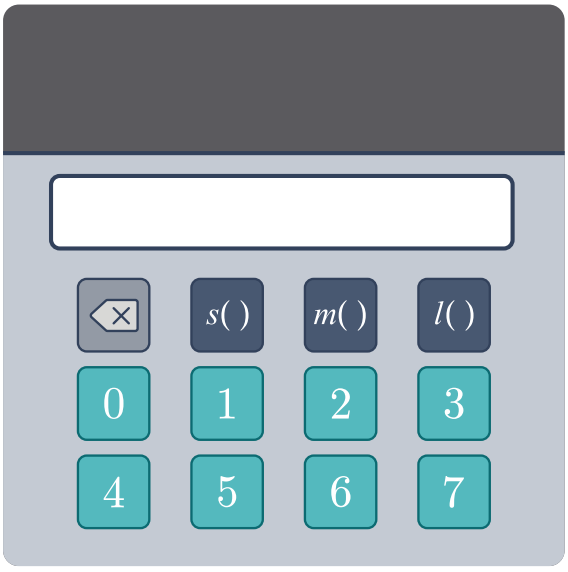
2 A worker at Desmos Pizza made a list of all the large pizza orders one night.

Discuss: How is this list like a function?

3 Desmos Pizza uses a cash register with three functions — one for each size pizza.

In the digital activity, use the functions to determine the price of each pizza order.

Pizza Order	Price ($)
Small, 3 toppings	$s(3) = 17.25$
Large, 2 toppings	
Medium, 1 topping	



Name: ..... Date: ..... Period: .....

Pricing Pizzas (continued)

4 $s(3)$ is an example of a statement in **function notation**.

We read $s(3)$ as "s of three."

a Say $s(3) = 17.25$ aloud to a classmate.

b Select *all* the ideas that $s(3)$ represents.

- ☐ **A.** The price of a small pizza with 3 toppings.
- ☐ **B.** The price of a small pizza multiplied by 3.
- ☐ **C.** The function s with an input of 3.
- ☐ **D.** The function s with an output of 3.
- ☐ **E.** The price of 3 small pizzas.

5 On the previous problem, Luca said:

s times 3 is 17.25, so a small pizza with 3 toppings will cost $5.75.

What would you say to help him understand his mistake?

Luca

$$\frac{s(3)}{3} = \frac{17.25}{3}$$

$$s = 5.75$$

Activity 2

Name: ..... Date: ..... Period: .....

Interpreting Function Notation

6 Match each function notation statement with its correct interpretation(s). Three cards will have no match.

Card A	Card B	Card C	Card D
The function s with the input 0.	A small pizza that costs $\$0$.	The price of a medium pizza with x toppings.	5 large pizzas

Card E	Card F	Card G
The output of l when the input is 5.	The price of a small pizza with 0 toppings.	The price of a medium pizza multiplied by x.

$l(5)$	$s(0)$	$m(x)$

7 Emma and her friends are texting about their pizza order.

Emma writes: $m(7) < l(5)$.

What do you think this means?

8 Select *all* the true statements.

- ☐ **A.** $s(2) > s(1)$
- ☐ **B.** $m(4) < s(4)$
- ☐ **C.** $l(4) > s(3)$
- ☐ **D.** $l(1) = m(3)$
- ☐ **E.** $l(0) < m(2)$

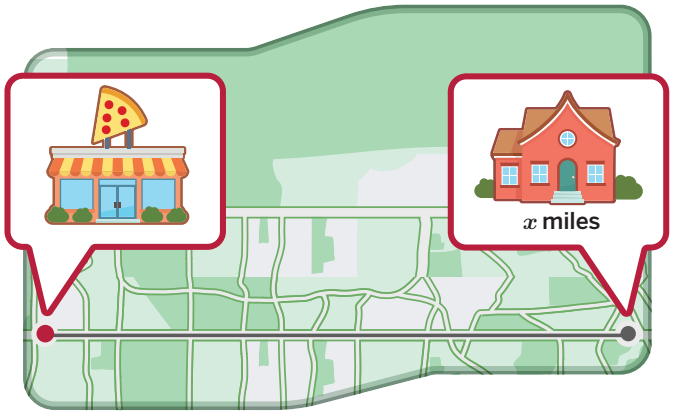
Activity 3

Name: ..... Date: ..... Period: .....

Delivering Pizzas

9 Desmos Pizza uses the function $d(x)$ to estimate the number of minutes it takes to deliver pizza x miles.

What would $d(2) = 30$ mean in this situation?



10 $d(x)$ estimates the number of minutes it takes to deliver pizza x miles.

Match each function notation card to a description. One card will have no match.

Card A

It takes longer to deliver 5 miles away than 1 mile away.

Card B

The number of minutes to make a delivery 5 miles away.

Card C

A delivery 1 mile away will take more than 5 minutes.

Card D

A delivery 5 miles away will take more than 1 minute.

Card E

Delivering 5 pizzas takes longer than delivering 1 pizza.

$d(5) > 1$	$d(5)$	$d(5) > d(1)$	$d(1) > 5$

11 Synthesis

This lesson introduced function notation.

a. Say the equation $m(5) = 23$ aloud to a classmate.

b. **Discuss:** Describe what each part of the equation means.

14 Summary 4.02

Function notation replaces the dependent variable, y, with the function notation $f(x)$ which is read as "f of x". For example, $f(4) = 9$ is a statement written in function notation. It says that when the input of the function f is 4, the output is 9. In other words, when the value of the *independent variable* is 4, the value of the *dependent variable* is 9.

We can also interpret the meaning of function notation in a specific context, such as determining the price of a slice of pizza based on the number of toppings. Here is an example of a menu and a table showing some input-output pairs for the function $s(t)$

Menu

Slice of Pizza $1.75 plus $0.50 per topping

Number of Toppings	Price ($)
0	1.75
1	2.25
2	2.75

$s(2) = 2.75$ is a statement written in function notation.

- $s(2)$ can be read as "s of two".
- For this situation, the number of toppings is the *independent variable*, t, and the price of a slice of pizza is the *dependent variable*, $s(t)$.
- $s(2) = 2.75$ means the price of a slice of pizza with 2 toppings is $2.75.

function notation A way of expressing the relationship between the inputs and outputs of a function using an equation that is often denoted by a name such as $f(x)$.

Practice
4.02

Name: ..... Date: ..... Period: .....

Problems 1–2: The function $f(t)$ models the temperature, in degrees Celsius, t hours after midnight.

1. **Test Practice** Select the equation that represents the statement: *At 1 AM, the temperature was* 20 °C.

A. $f(100) = 20$ **B.** $f(20) = 100$ **C.** $f(1) = 20$ **D.** $f(20) = 1$

2. Use function notation to represent each statement.

The temperature at 2 AM. .....

The temperature was the same at 9 AM and at 11 AM. .....

The temperature was higher at 9 AM than at 2 AM. .....

t hours after midnight, the temperature was 24 °C. .....

3. A restaurant sells three different salads. They use the functions $c(x)$, $g(x)$, and $p(x)$ to represent the cost of their caesar, garden, and pasta salads in dollars, with x additional ingredients added. Explain the meaning of each statement.

$g(0) = 10$	
$g(3) > c(1)$	
$p(2) < g(3)$	

Problems 4–6: Use the table to determine the missing values in the function statements.

4. $f($.....$) = 23$

5. $f(-5) =$ .....

6. $f($.....$) = -5$

x	$f(x)$
-5	17
-2	-5
5	23

Practice
4.02

Name: _____ Date: _____ Period: _____

Problems 7–9: Desmos Pizza's online menu offers small, medium, and large pizzas. Fill in each blank to make each equation true.

7. $s(7) = m$ (_____)

8. m (_____) $= l(5)$

9. l(_____) $= s$ (_____)

Spiral Review

10. Here are Rules A and B.

Which rule is a function? Circle one.

Rule A Rule B Both Neither

Rule A

Input	Output
4	2
9	-3
9	3

Rule B

Input	Output
1	5
2	9
2	9

Problems 11–12: Here are two data sets.

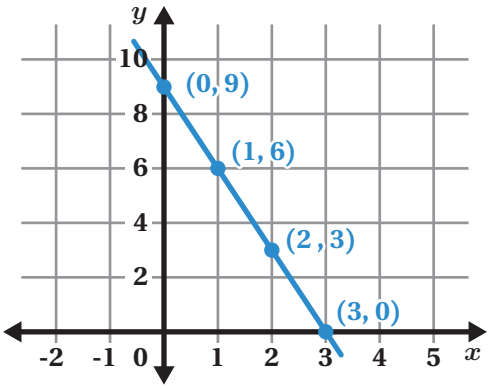
Data Set A: 6, 7, 8, 8, 8, 8.5, 9

Data Set B: 4, 7, 8, 8, 9, 12, 12

11. Which one has the larger standard deviation?

12. Which one has a mean that is greater than the median?

13. Here's a graph of a relationship and its table of values.



x	y
0	9
1	6
2	3
3	0

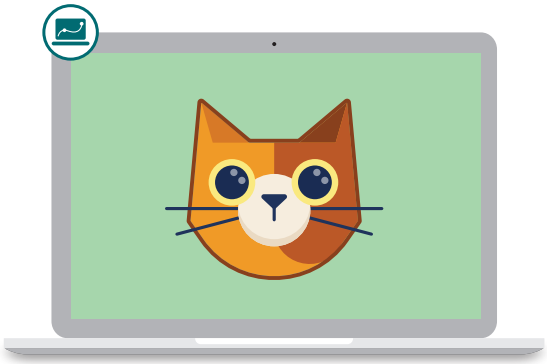
Write an equation to represent this relationship.

Name: ..... Date: ..... Period: .....

Modeling with Functions | Comparing Models | F–IF.1, F–IF.2, F–BF.1, SMP.7, SMP.8

Toy Factory

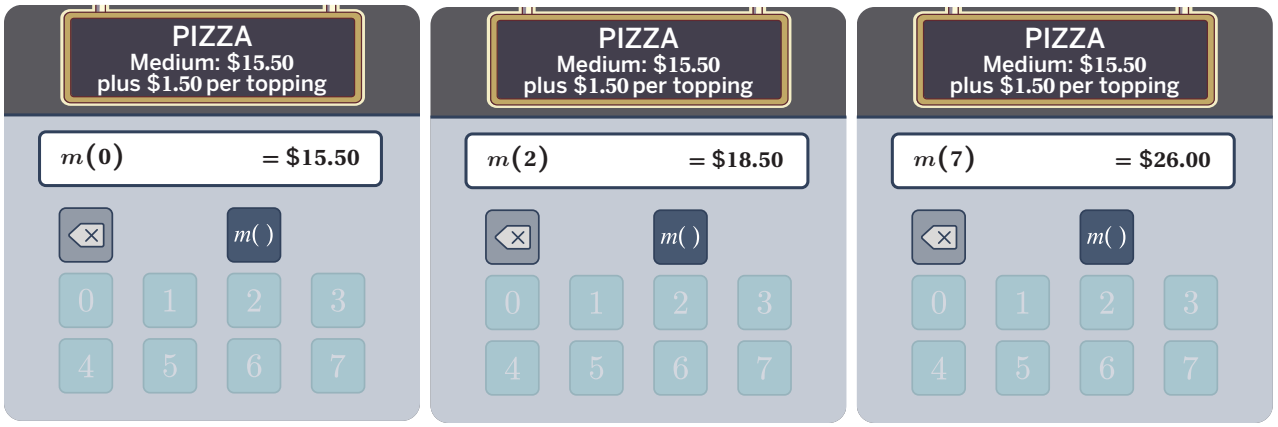
Let's explore functions represented as equations written in function notation.



Warm-Up

1 The cash register uses $m(x)$ to determine the price of a medium pizza with x toppings.

a Here are three pizzas and their prices.



b Describe how the cash register calculates prices.

2 Which equation represents $m(x)$?

A. $m(x) = 15.5 + 1.5$ **B.** $m(x) = 15.5x + 1.5$ **C.** $m(x) = 15.5 + 1.5x$

Explain your thinking.

Name: ..... Date: ..... Period: .....

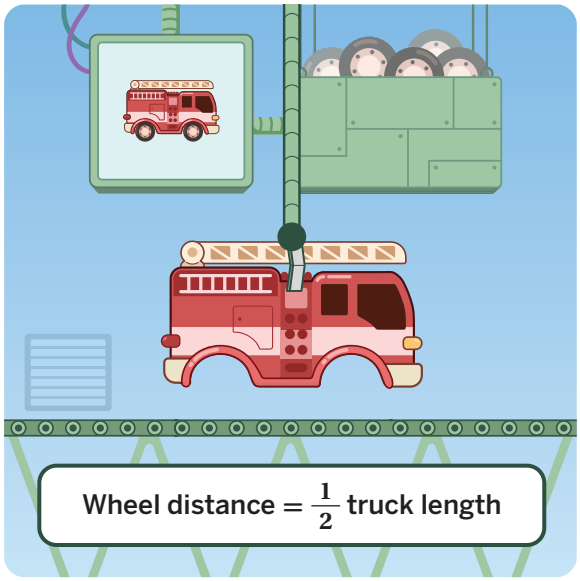
Exploring Equations of Functions

3 A toy factory makes fire trucks in a variety of sizes.

The distance between the truck's wheels is always half the length of the truck.

Complete the table to put wheels on the toy truck.

Truck Length (in.)	Wheel Distance (in.)
10	
6	
3	



4 Kanna wrote the function $d(x) = \frac{1}{2}x$ to determine the wheel distance for a truck length of x.

a **Discuss:** What does $d(x) = \frac{1}{2}x$ mean?

b What is the value of $d(7)$?

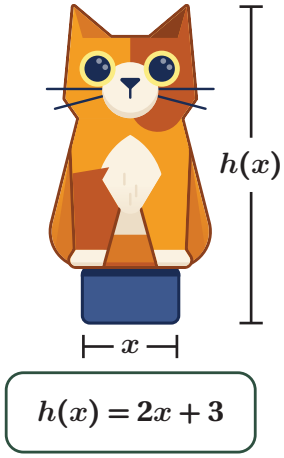
Name: Date: Period:

Exploring Equations of Functions (continued)

5 The factory also makes toy cats.

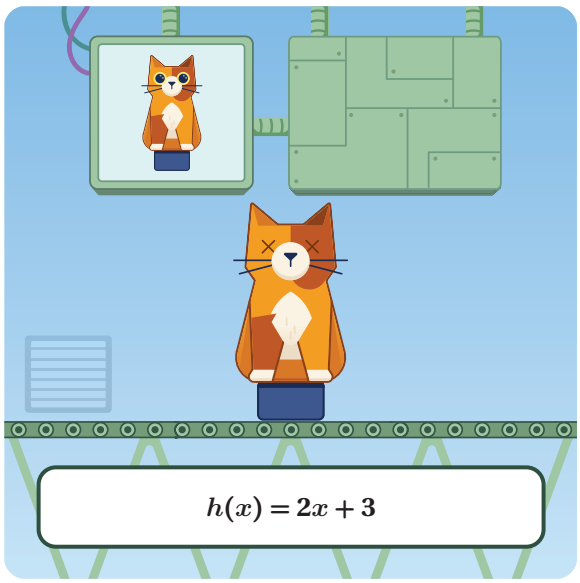
They use this diagram and function to determine where to place the cat's eyes. All units are in inches, and x represents the distance between the eyes of the cat.

What does $h(x) = 2x + 3$ mean in this situation?



6 Calculate the value of each function notation expression.

Expression	Eye Height (in.)
$h(5)$	
$h(3)$	
$h(7.5)$	



Discuss: Why is it useful to write a function as an equation?

Activity 2

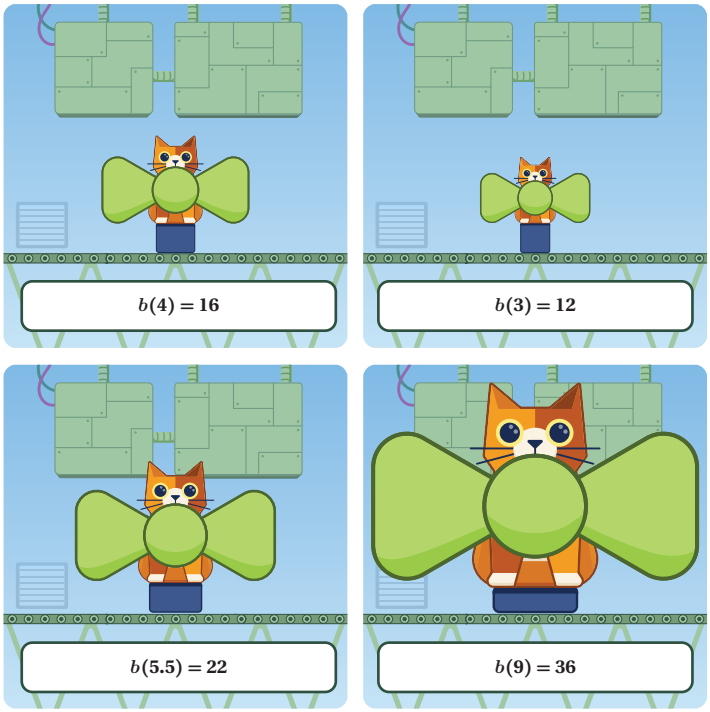
Name: ..... Date: ..... Period: .....

Writing Equations of Functions

7 Each toy cat needs a bow tie. The function $b(x)$ determines the width of the bow tie, where x is the width of the base. All units are in inches.

a Let's see how the function $b(x) = 4x$ works.

b Change the equation $b(x) = 4x$ to make bow ties that fit better.



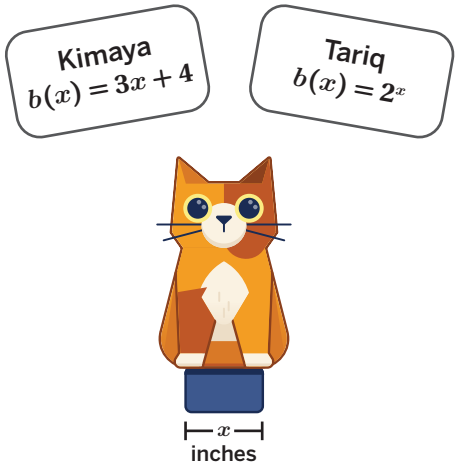
8 $b(x)$ determines the width of the bow tie, where x is the width of the base.

Kimaya says that her function will produce a wider bow tie than Tariq's function for any base width.

Is she correct? Circle one.

Yes No

Explain your thinking.



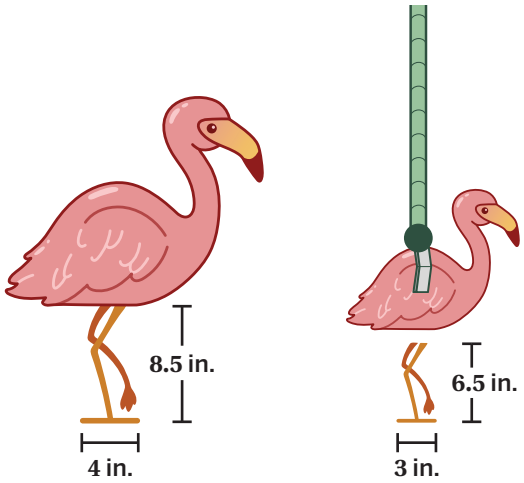
Name: ..... Date: ..... Period: .....

Writing Equations of Functions (continued)

9 Flamingo Frank is made by attaching a flamingo to legs at a specific height.

This height is determined by the width of the base.

Base Width (in.)	Height (in.)
4	8.5
3	6.5
1	2.5
5.5	11.5



How can you determine the height for any base width?

10 This machine assembles Flamingo Frank by attaching it to legs at a specific height. That height, $f(x)$, depends on the width of the base, x. Write an equation for $f(x)$.

You're invited to explore more.

11 Here are four figures in a visual pattern. The number of tiles is a function of the figure number.

Figure, n	Number of Tiles, $t(n)$
1	3
2	6
4	18
6	38

Figure 1

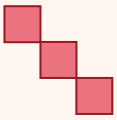


Figure 2

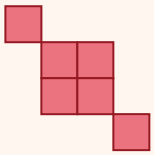


Figure 4

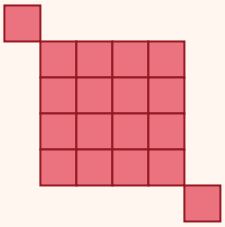
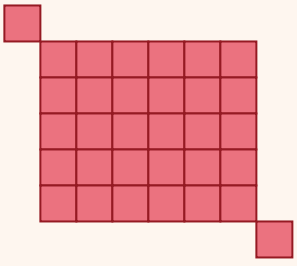


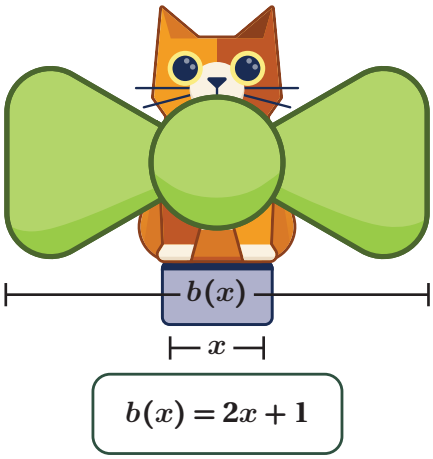
Figure 6



Write an equation for $t(n)$.

12 Synthesis

A toy factory uses this diagram and function to determine the width of the bow tie. All units are in inches.



What does $b(x) = 2x + 1$ mean in this situation?

What does $b(4) = 9$ mean?

15 Summary 4.03

You can represent function rules with equations, verbal descriptions, and tables.

For example, the function $s(t)$ describes the relationship between the cost of a slice of pizza and the number of toppings, t.

Let's represent this function rule with a table and an equation:

Description

Menu
Slice of Pizza
$1.75 plus $0.50 per topping

Table

Number of Toppings	Price ($)
0	1.75
1	2.25
2	2.75

Equation

$s(t) = 1.75 + 0.50t$

You can use the equation to determine different values of the function.

Let's determine the value of $s(4)$:

$s(4) = 1.75 + 0.50(4)$

$s(4) = 3.75$

This means the price of a slice of pizza with 4 toppings is $3.75.

Practice
4.03

Name: ..... Date: ..... Period: .....

1. Let $f(x) = 2x + 5$.

Calculate the value of each function notation expression.

The first value is already completed.

Expression	Value
$f(0)$	5
$f(4)$	
$f(6)$	
$f(-3)$	

Problems 2–3: A toy factory makes toy bunnies. Each toy bunny holds a carrot. A bunny's height, $h(x)$, is three times the length of the carrot, x.

2. Complete the table.

x	1	2	3	4	5	6
$h(x)$						

3. Write an equation for the function $h(x)$.

4. The functions $f(x)$ and $g(x)$ are defined by these equations:

- $f(x) = -15x + 80$
- $g(x) = 10x + 25$

Circle which is greater: $f(2)$ or $g(2)$. Explain your thinking.

Problems 5–6: The function $p(s)$ models the perimeter of a square of side length s. The perimeter is represented by the equation $p(s) = 4s$.

5. What is the value of $p(20)$?

6. What does your answer mean in this situation?

Name: ..... Date: ..... Period: .....

7. **Test Practice** Model rockets are created in various sizes. The height of a rocket in inches, $h(x)$, depends on the radius of the base of the rocket in inches, x.

Use the table to write an equation for $h(x)$ that outputs the height of the rocket with a base radius of x.

Radius (in.), x	Height (in.), $h(x)$
1	5
3	13
5	21
10	41

Problems 8–9: The function $w(t)$ models the weight of a pumpkin, in pounds, as a function of how many months, t, it has been growing. Explain the meaning of each statement.

8. $w(2) = 5$

9. $w(6) > w(4)$

Spiral Review

Problems 10–12: Here are the first four figures in a pattern.

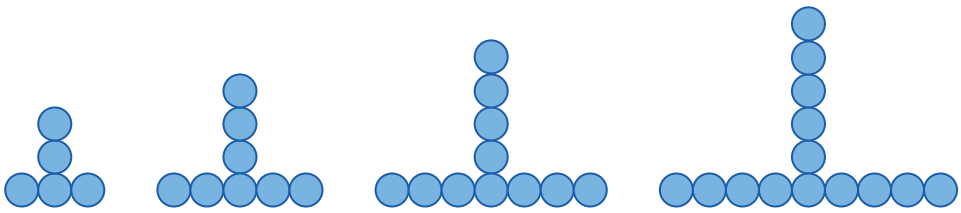


Figure 1 **Figure 2** **Figure 3** **Figure 4**

10. How many dots will be in Figure 5?

11. Write an equation for the number of dots, $d(n)$, in Figure n.

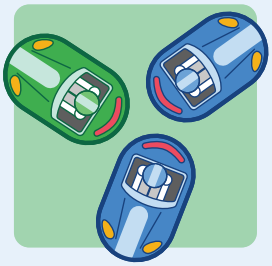
12. Is this pattern arithmetic or geometric?

Notes:

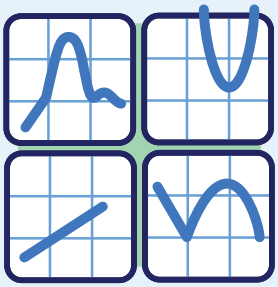
Sub-Unit

2

Key Features of Functions



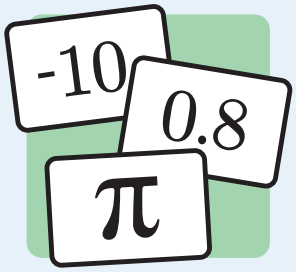
Lesson 4
Function Carnival



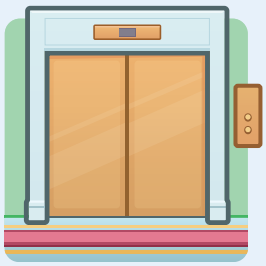
Lesson 5
Craft-a-Graph

Lesson 6
Plane, Train, and Automobile

Lesson 7
Space Race



Lesson 8
Ins and Outs

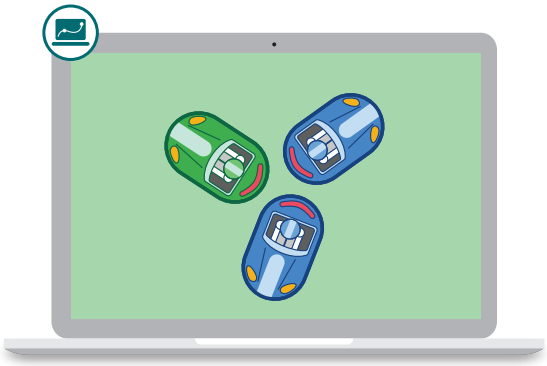


Lesson 9
Elevator Stories

Lesson 10
Marbleslides

Name: ..... Date: ..... Period: .....

Modeling with Functions | Comparing Models | F–IF.1, F–IF.4, N–Q.2, SMP.2, SMP.4, SMP.6



Function Carnival

Let's create and analyze graphs that represent stories.

Warm-Up

1 Let's watch a video.

Write a story about what you see.

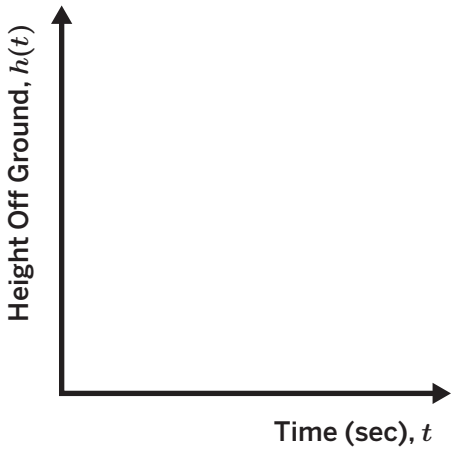
Activity 1

Name: Date: Period:

Cannon Person

2 $h(t)$ represents the person's height off the ground at time t.

Sketch a graph of $h(t)$.



3 The sketch of $h(t)$ is the graph of a function that represents all the y-values that follow the rule, $h(t)$.

Here is a precise graph of $h(t)$. The x-values are values of the variable t. The y-values are values of the function $h(t)$. This means that $y = h(t)$.

Select a true statement.

- ☐ **A.** $h(3) > h(5)$
- ☐ **B.** $h(3) = h(5)$
- ☐ **C.** $h(0) < h(10)$
- ☐ **D.** $h(0) = h(10)$

Explain what the statement says about the Cannon Person.

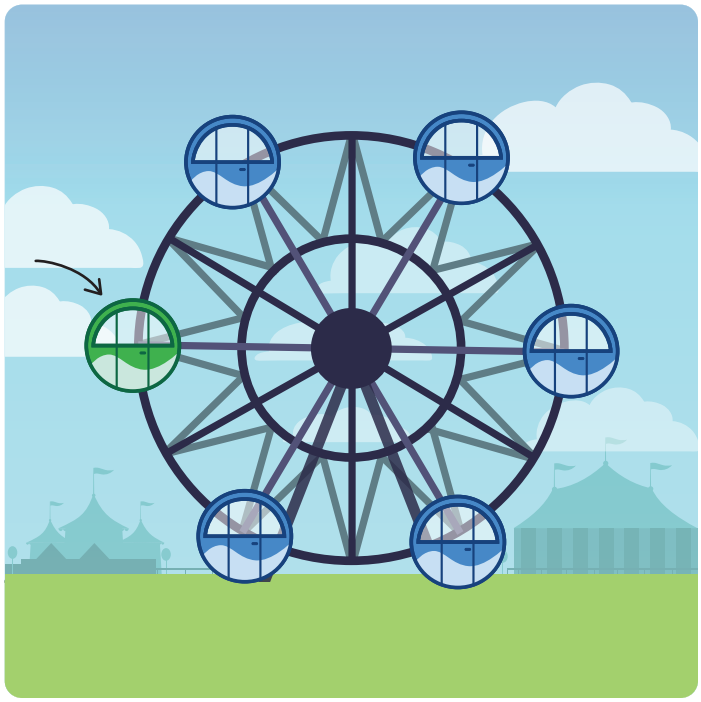
Activity 2

Name: ..... Date: ..... Period: .....

Ferris Wheel

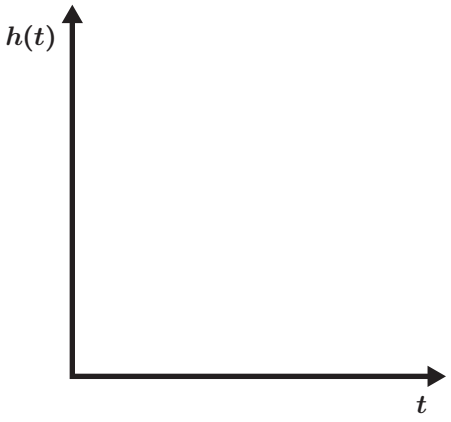
4 Let's watch a video of a Ferris wheel.

Describe what happens to the height of the green cart.



5 $h(t)$ represents the height of the Ferris wheel at time t.

Sketch a graph of $h(t)$.



6 Let's look at the graph that Liam drew.

What do you think happened to the green cart when he pressed play?

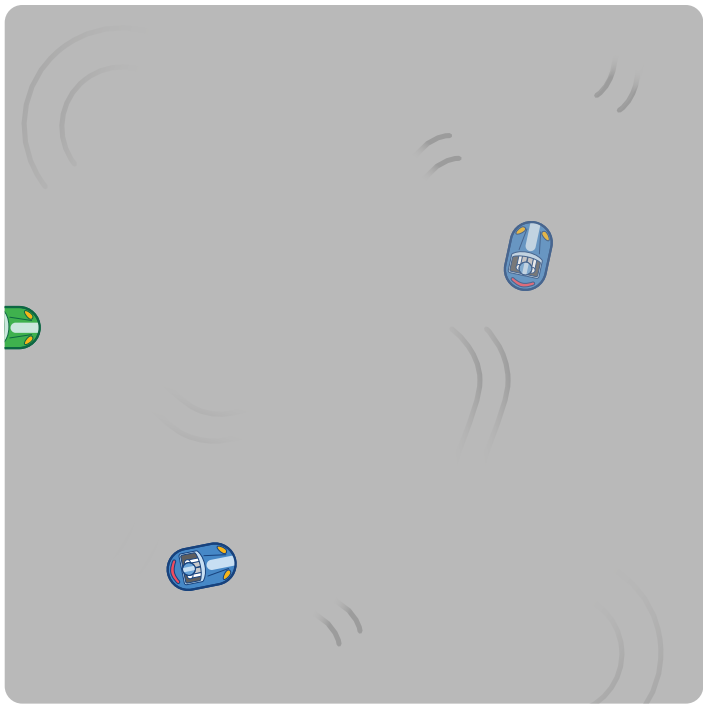
Activity 3

Name: ..... Date: ..... Period: .....

Bumper Cars

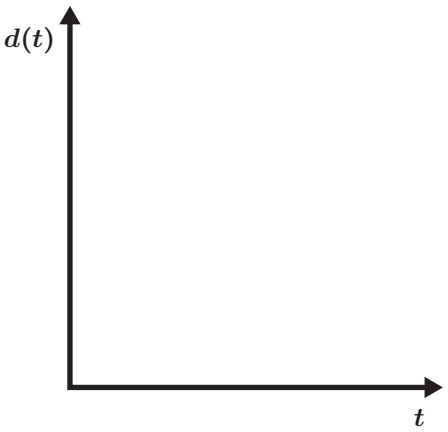
7 Let's watch a video of some bumper cars.

What are some different things you could measure and display in a graph?



8 $d(t)$ represents the distance traveled by the green car at time t.

Sketch a graph of $d(t)$.



Name: ..... Date: ..... Period: .....

Bumper Cars (continued)

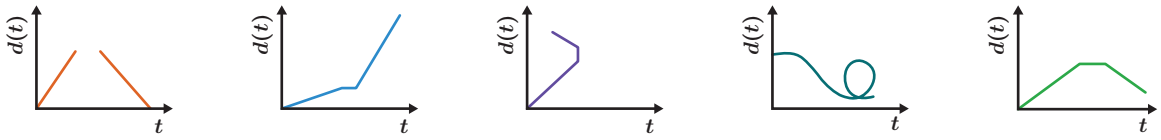
9 $d(t)$ represents a roller coaster's distance from the start at time t.

Match each description to its graph. Two graphs will have no match.

a The roller coaster moves slowly, stops for a moment, then goes very fast.

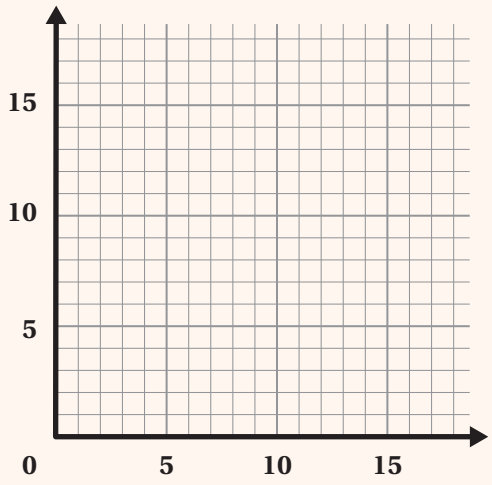
b The roller coaster moves forward, stops, and then backs up.

c The roller coaster starts in the middle, backs up, and then splits into multiple roller coasters.



You're invited to explore more.

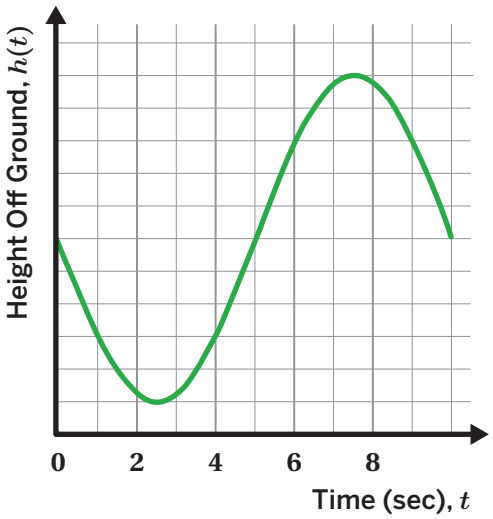
10 Draw your own graph and label the x-axis and y-axis. Then write a story about what it describes.



11 Synthesis

Discuss: How can a graph help tell a story about a situation?

Use this example of a Ferris wheel graph if it supports your explanation.



14 Summary 4.04

A graph can reveal in more detail what is happening during a situation. Here is an example:

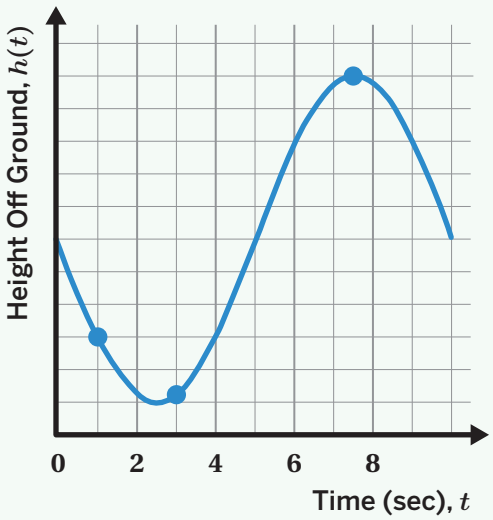
The function $h(t)$ represents the height of the cart on the Ferris wheel at time t.

We can use the graph to describe many parts of the situation. For example:

- At around 7.5 seconds, the Ferris wheel cart is at its maximum height.
- $h(1)$ is greater than $h(3)$. This means the Ferris wheel cart was higher off the ground at 1 second than at 3 seconds.

While we can use the graph to describe many things, there are lots of things the graph cannot describe. For example:

- How much fun the people are having
- How many people are riding the Ferris wheel

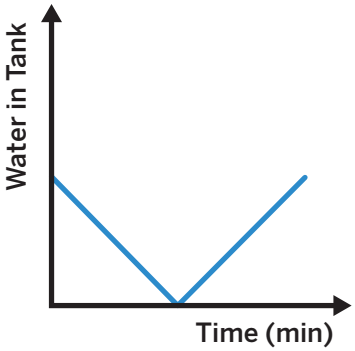


Practice
4.04

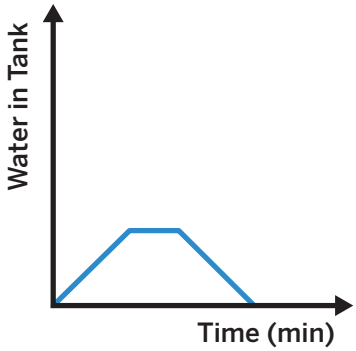
Name: ..... Date: ..... Period: .....

1. An empty water tank is filled until it is half full. Two minutes later, it drains until it is empty again. Which graph could represent this situation? Circle your choice.

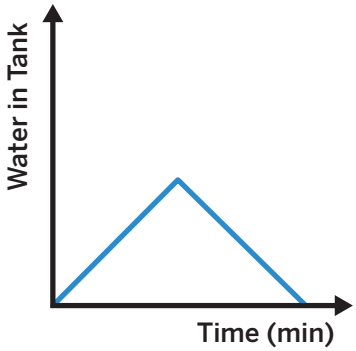
Graph A



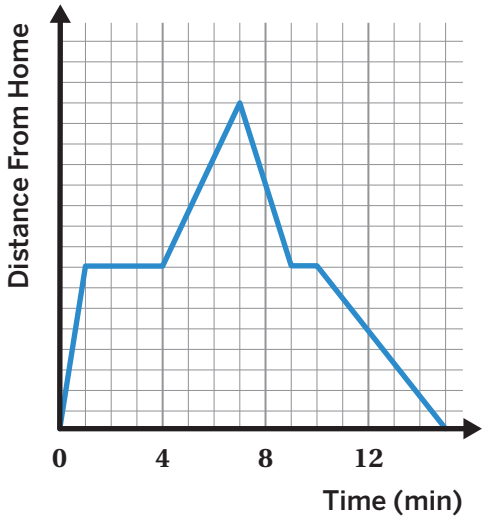
Graph B



Graph C



Problems 2–4: Prisha rode her bike around town. Her fitness tracker made a graph to represent the distance she was from her home at any given time during her ride.



2. How many minutes was Prisha's bike ride?

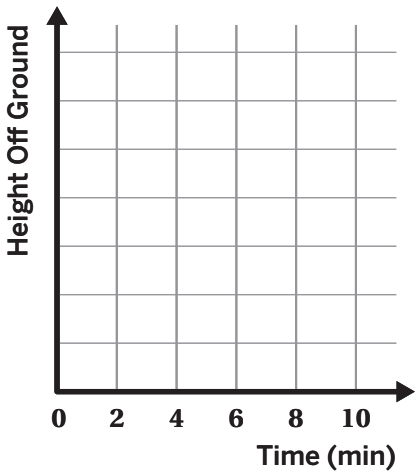
3. At what time was Prisha the farthest distance from her home?

4. How long did Prisha rest during her ride?

5. Here is some information about a hot air balloon ride.

- Ascends (goes up) quickly for 2 minutes.
- Ascends slowly for another minute until it reaches its maximum height.
- Maintains its maximum height for 3 minutes.
- Descends (goes down) for the next 4 minutes until it lands on the ground.

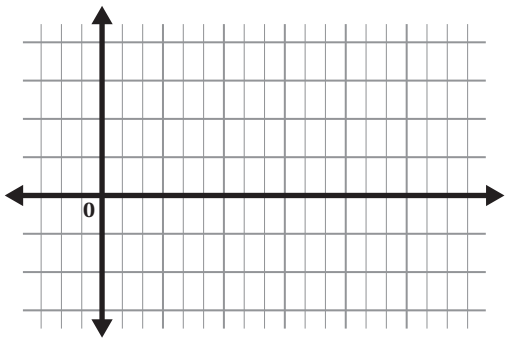
Make a graph that could represent the height of a hot air balloon over time.



Name: ..... Date: ..... Period: .....

Problems 6–7: Aba describes her morning at school yesterday like this:

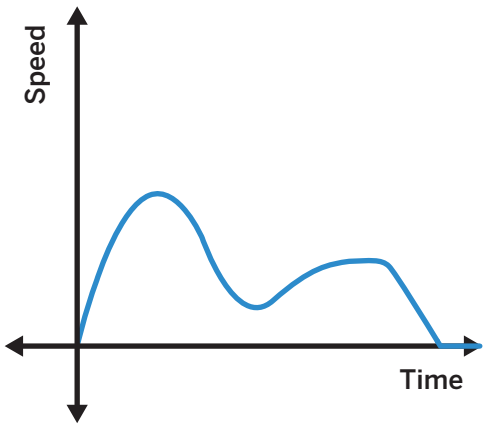
I entered the school on the ground floor, then walked up the stairs to the third floor to attend an hour-long class. Afterward, I had an hour-long class in the basement, then I went up to the ground floor and sat outside to eat my lunch.



6. Label both axes.

7. Sketch a possible graph of Aba's height from the ground floor as a function of time.

Test Practice Problems 8–9: Here is a graph of speed and time.



8. Which sport could this graph represent?

A. Fishing
B. Skydiving
C. 100-yard sprint
D. Golf
E. Soccer

9. Describe how you think that sport fits the graph.

Spiral Review

10. $p(t)$ represents the height of water in a bathtub, in inches, after t minutes. Match each sentence to its equation.

a. After 20 minutes, the bathtub is empty. ..... $p(10) = 4$

b. The bathtub starts out with no water. ..... $p(t) = w$

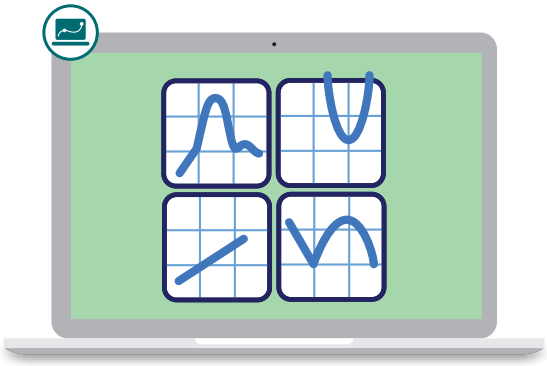
c. After 10 minutes, the height of the water is 4 inches. ..... $p(20) = 0$

d. The height of the water is 10 inches after 4 minutes. ..... $p(0) = 0$

e. The height of the water is w inches after t minutes. ..... $p(4) = 10$

Name: ..... Date: ..... Period: .....

Modeling with Functions | Comparing Models | F–IF.4, SMP.6



Craft-a-Graph

Let's describe and create graphs of functions using key features.

Warm-Up

1 Play a few rounds of Polygraph with your classmates!

You will use a Warm-Up Sheet with functions. In each round:

- You and your partner will take turns being the Picker and the Guesser.
- Picker: Select a function from the Warm-Up Sheet. Keep it a secret!
- Guesser: Ask the Picker yes-or-no questions, eliminating functions until you're ready to guess which function the Picker chose.

Record helpful questions from each round in the space below.

Name: ..... Date: ..... Period: .....

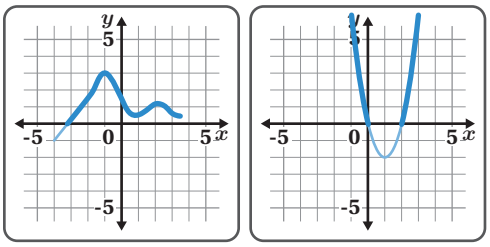
Describe It

2 Here are some functions from Polygraph, along with some terms that describe parts of their graphs.

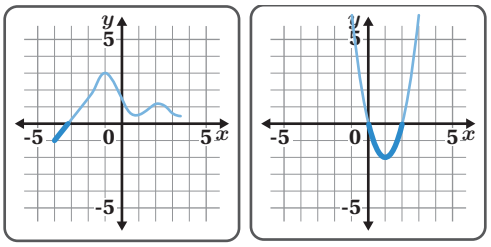
a Take a look at each term and where it appears on the graph.

b **Discuss:** What does each term mean?

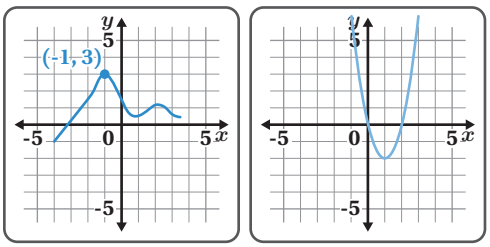
Positive



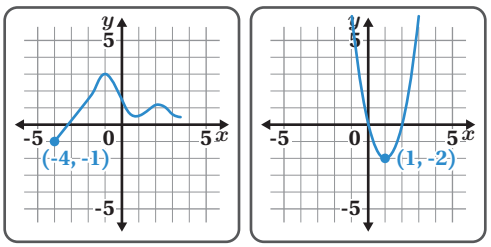
Negative



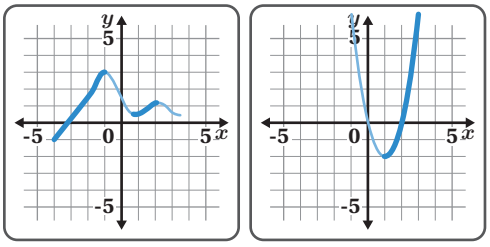
Maximum



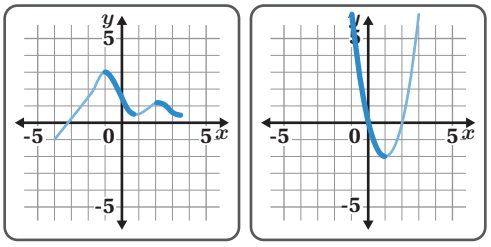
Minimum



Increasing



Decreasing



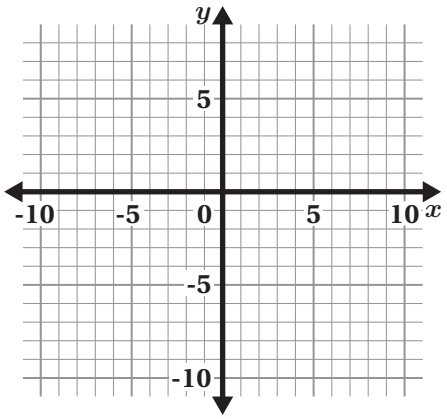
Activity 2

Name: ..... Date: ..... Period: .....

Build It

3 Now it's your turn to make a function! Draw any function.

Describe your function using some of these terms: **positive**, **negative**, **maximum**, **minimum**, *increasing, decreasing*

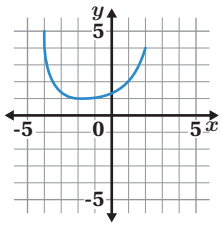


4 Latifah described her function this way:

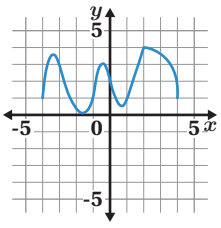
- My function is always positive.
- The maximum is at $(2, 4)$.

Select *all* the functions that could be Latifah's.

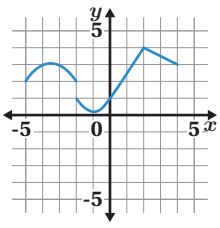
☐ A.



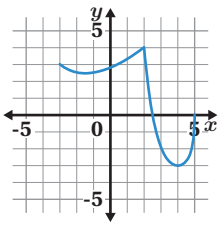
☐ B.



☐ C.



☐ D.



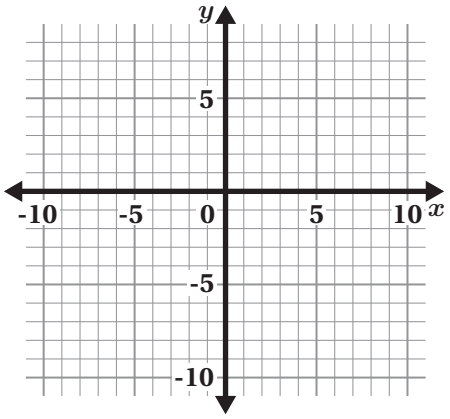
Name: ..... Date: ..... Period: .....

Build It (continued)

5 Haruto described his function this way:

- My function is increasing when $x > 0$.
- It has a minimum at (-3, -4).

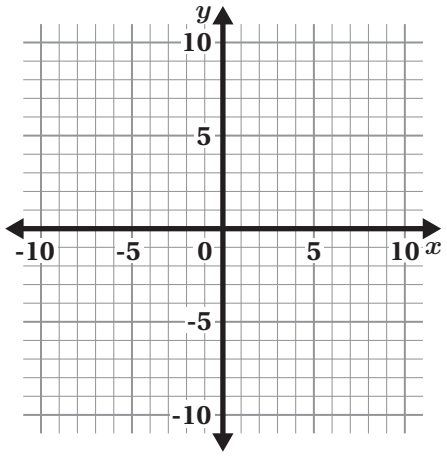
Create a function that could be Haruto's.



6 Andrea described her function this way:

- Positive when $x > -2$
- Decreasing when $x > 1$

Is it possible for a function to have both features? Explain your thinking.



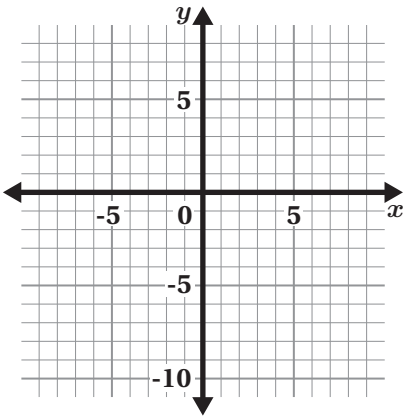
Activity 3

Name: Date: Period:

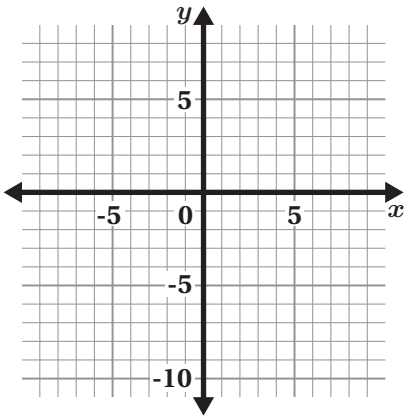
Repeated Challenges

7 Build a function that meets these criteria.

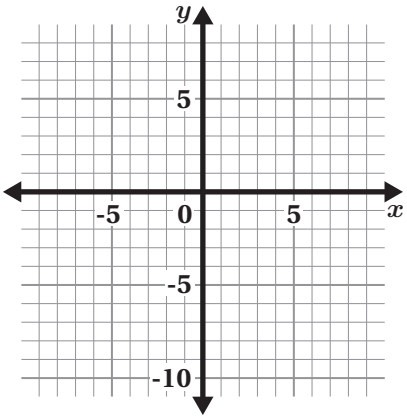
Positive when $x < 4$
Decreasing when $x > -1$



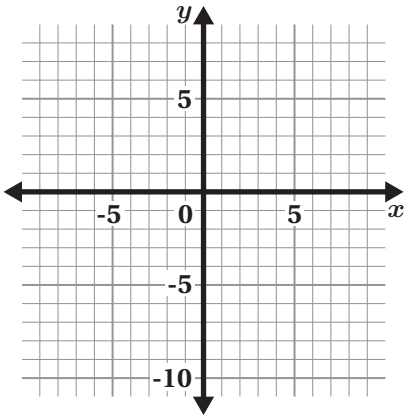
Increasing when $x > 1$
Maximum at $(-3, 5)$



Negative when $x < 2$
Decreasing when $x > -5$



Positive when $x > -2$
Minimum at $(-3, -1)$

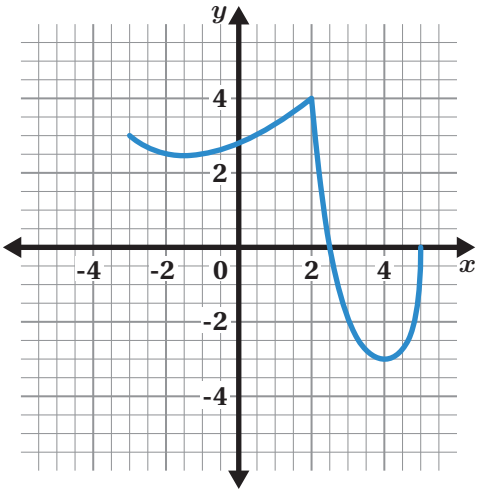


8 Synthesis

Here are some of the terms to describe functions that we learned about today.

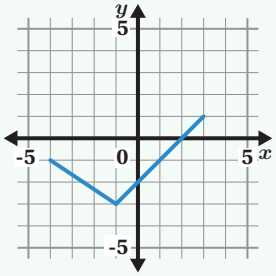
Positive	Negative
Maximum	Minimum
Increasing	Decreasing

Select three terms. Write the meaning of each term you selected.

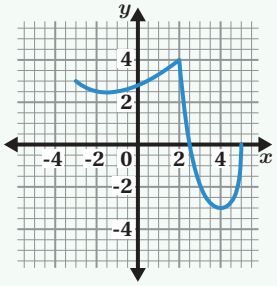


11 Summary 4.05

We can use the key features of a graph to help describe a function or sketch a possible graph of a function. Some terms that describe parts of graphs are **maximum**, **minimum**, **positive**, **negative**, *increasing*, *and decreasing*.



- The function has a minimum at (-1, -3) and a maximum at (3, 1).
- It's positive when $x > 2$ and negative when $x < 2$.
- It increases when $x > -1$ and decreases when $x < -1$.



- The function has a minimum at (4, -3) and a maximum at (2, 4).
- It's positive when $x < 2.5$ and negative when $2.5 < x < 5$
- It increases when $-1.5 < x < 2$ and $x > 4$. It decreases when $x < -1.5$ and $2 < x < 4$.

maximum (of a function) The highest point on a graph.

minimum (of a function) The lowest point on a graph.

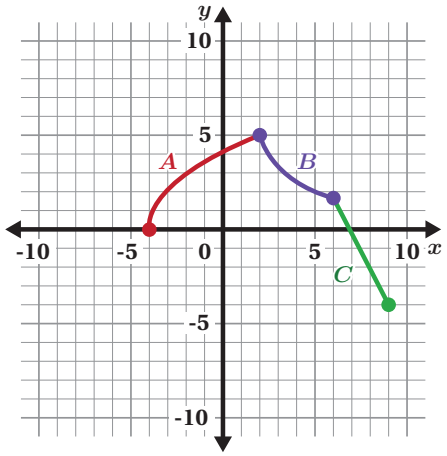
negative (interval or function) When a function's outputs are negative and its graph is below the x–axis.

positive (interval or function) When a function's outputs are positive and its graph is above the x–axis.

Practice
4.05

Name: ..... Date: ..... Period: .....

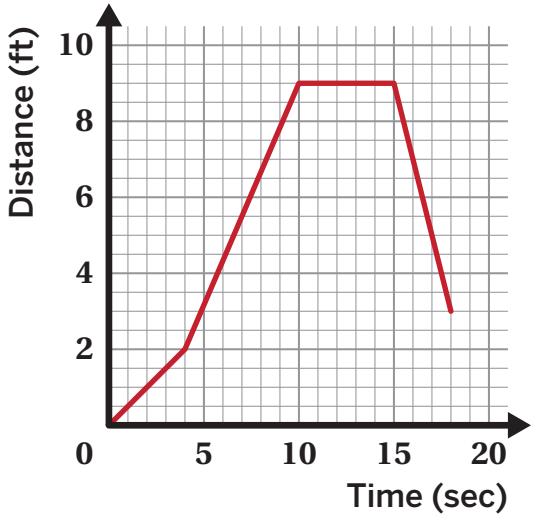
1. **Test Practice** Select *all* the true statements about this graph.



- ☐ **A.** This graph is a function.
- ☐ **B.** Part A is decreasing.
- ☐ **C.** Part B is decreasing.
- ☐ **D.** The maximum is at $(2, 5)$.
- ☐ **E.** The minimum is at $(-4, 0)$.

2. Manuel is watching his little brother at the park. The graph represents the distance Manuel is from his brother as a function of time.

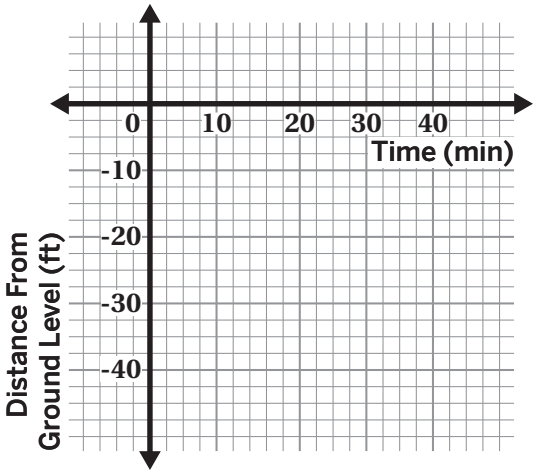
Describe Manuel's distance from his brother over time. Use terms that you learned in this lesson.



3. Ivory goes on a tour of a cave. The tour starts at ground level.

- The tour stays at ground level for 15 minutes.
- Then the tour descends (goes down) for 15 minutes to a depth of 20 feet below ground level.
- The tour stays at this level for 10 minutes.
- The tour spends the last 5 minutes ascending (going up) to ground level.

Sketch a graph describing Ivory's elevation as a function of time.



Practice
4.05

Name: ..... Date: ..... Period: .....

Problems 4–6: Here is a table that lists Seattle's temperatures for one day.

Hours After Midnight	Temperature (°F)
1	32
2	33
3	35
4	36
5	35

4. How many hours after midnight is the minimum temperature?

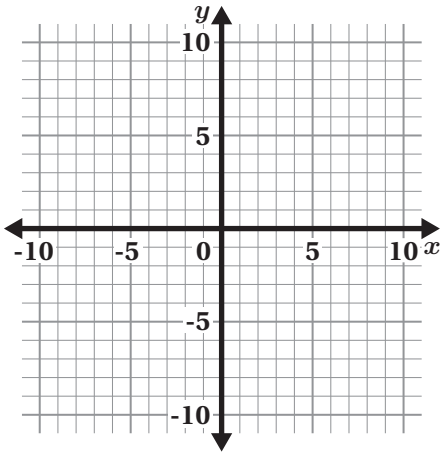
5. How many hours after midnight is the maximum temperature?

6. Between what hours is the temperature decreasing?

Problems 7–8: Here are three statements:

- The function is always positive.
- The function is always increasing.
- The function is always decreasing.

7. Sketch a graph of a function so that two of the statements are true and one is false.



8. Explain how you know that your graph makes two of the statements true and one of the statements false.

Spiral Review

Problems 9–11: Determine each quotient.

9. $\frac{5}{6} \div \frac{2}{3}$

10. $\frac{8}{6} \div \frac{4}{3}$

11. $\frac{12}{20} \div \frac{18}{16}$

Name: ..... Date: ..... Period: .....

Modeling with Functions F–IF.6, N–Q.1, SMP.2, SMP.3

Plane, Train, and Automobile

Let's calculate the average rate of change over a specified interval.

Warm-Up

1 A wedding is happening in New York City! Many relatives are coming from out of town.

a Here are three wedding guests and how they traveled to the wedding.

Arjun

Troy

Mayra

b What do you notice? What do you wonder?

Activity 1

Name: ..... Date: ..... Period: .....

Arjun's Automobile Trip

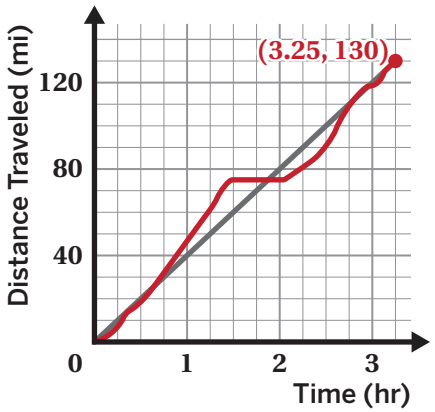
2 Arjun's car trip was 130 miles and took 3.25 hours.

Imagine Arjun traveled at a constant speed the whole trip.

What would be Arjun's speed in miles per hour?

3 **a** Take a look at the map and the graph of Arjun's actual trip from Hartford, Connecticut to New York City.

b Tell a story about Arjun's trip.

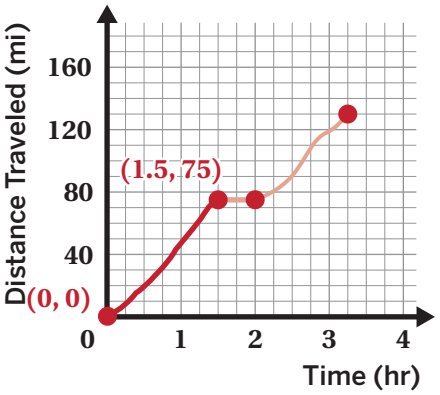


4 An **interval** is a set of numbers that has a starting value and ending value, such as 0 to 1.5 hours (highlighted in the graph).

The **average rate of change** is the *slope* of the line that connects the point where the interval begins and the point where it ends.

For Arjun's trip, the average rate of change was 40 miles per hour.

How would you calculate the average rate of change for the interval 0 to 1.5?

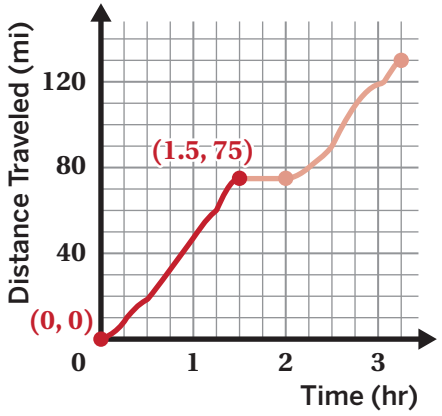


Name: ..... Date: ..... Period: .....

Arjun's Automobile Trip (continued)

5 Let's examine some intervals of Arjun's trip.

Time (hr)	Distance Traveled (mi)
0	0
1.5	75
2	75
3.25	130



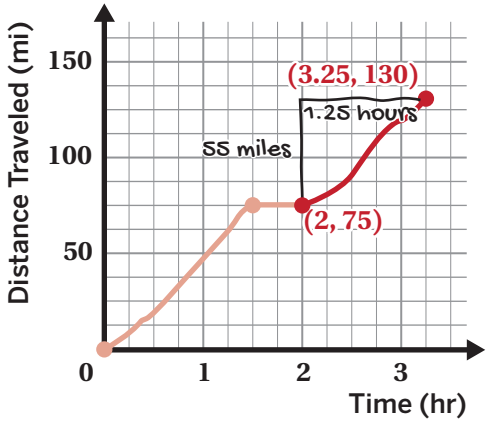
Determine the average rate of change of each interval.

Interval	0 to 1.5 hours	1.5 to 2 hours	2 to 3.25 hours
Average Rate of Change (mph)			

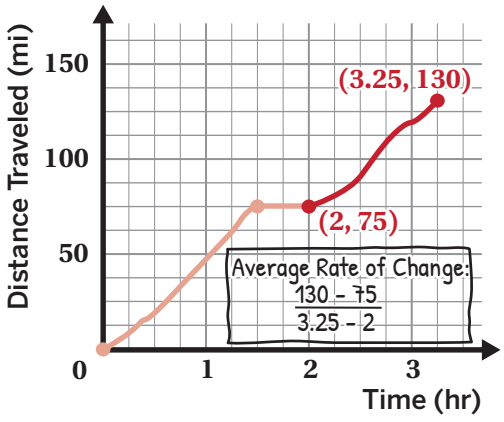
6 Two students calculated the average rate of change between 2 and 3.25 hours.

a Take a look at Naoki's and LaShawn's work.

Naoki



LaShawn



b How are their methods similar?

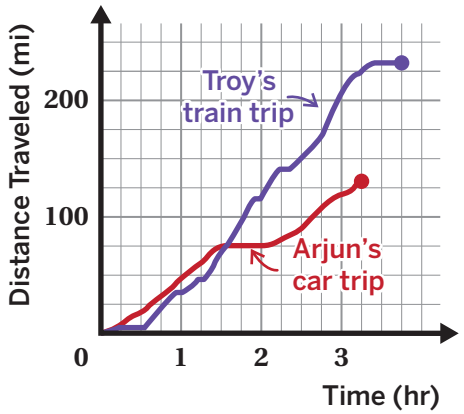
Name: Date: Period:

Troy's Train Trip

7 Let's watch an animation of Troy's trip.

Here is the graph for Arjun's and Troy's trips.

What are some ways Troy's trip is different from Arjun's?



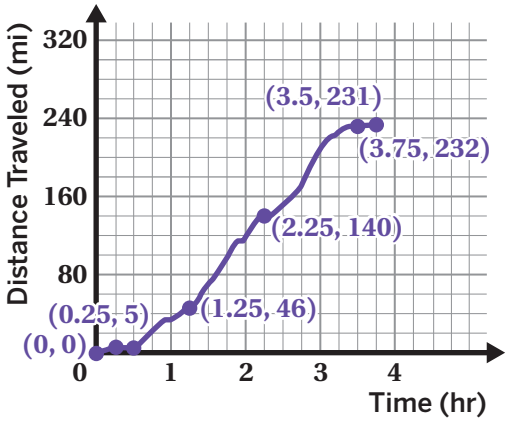
8 Let's examine some intervals of Troy's trip.

Choose two points on the graph. Then calculate the average rate of change for the interval you selected.

Can you find:

- Troy's average rate of change for the full trip?
- An interval where Troy moved fast? Slow?

Interval	Average Rate of Change (mph)

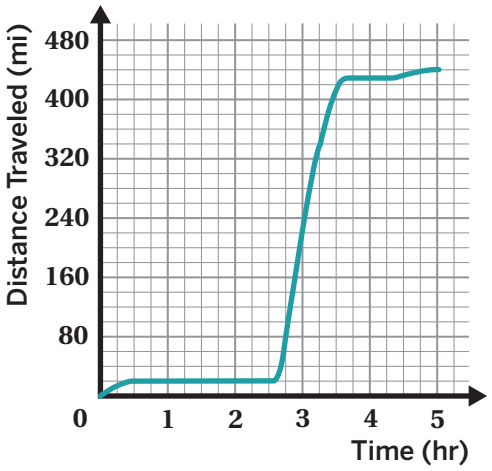


Activity 3

Name: ..... Date: ..... Period: .....

Mayra's Flight

9 Mayra's plane trip from Pittsburgh was 440 miles and took 5 hours.

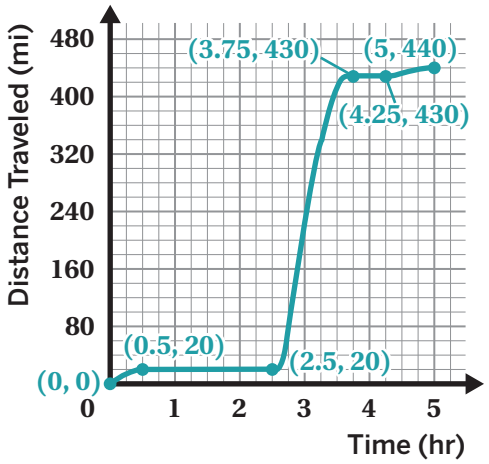


a **Discuss:** What questions do you have about Mayra's trip?

b Label three or more intervals of the graph with what you think was happening at that time.

10 Mayra said: *The flight felt fast, but the trip felt slow.*

Choose two points on the graph. Calculate the average rate of change for the interval you selected.



Can you find:

- Mayra's average rate of change for the full trip?
- Mayra's average rate of change *during the flight*?

Interval	Average Rate of Change (mph)

Name: ..... Date: ..... Period: .....

Mayra's Flight (continued)

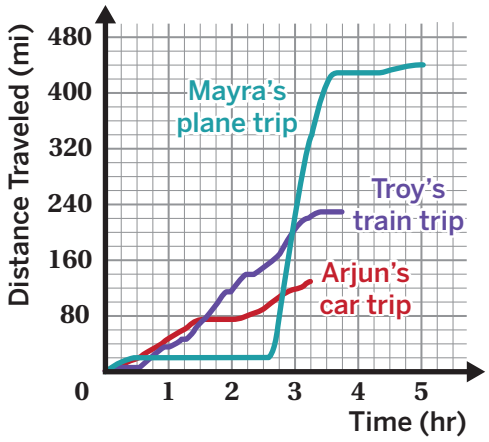
11 There are many reasons why someone may choose to travel by car, train, or plane.

Here are the graphs for all three people's trips.

Circle the trip you think is best.

Arjun (car) Troy (train) Mayra (plane)

Explain why you chose that trip.

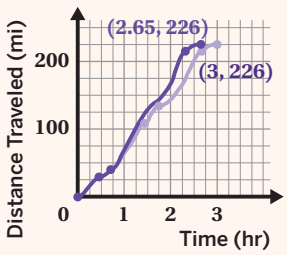


You're invited to explore more.

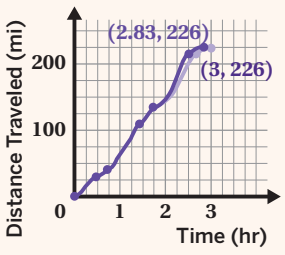
12 Public transit agencies often look at average speed (rate of change) when they look for ways to improve service.

a Explore these three proposals to improve the train from Washington to New York:

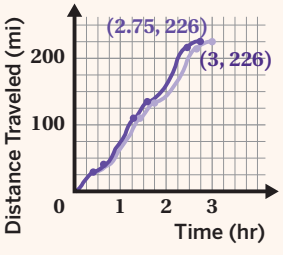
Proposal A: Skip stops along the way.



Proposal B: Upgrade one section of the tracks.



Proposal C: Buy trains with a higher top speed.



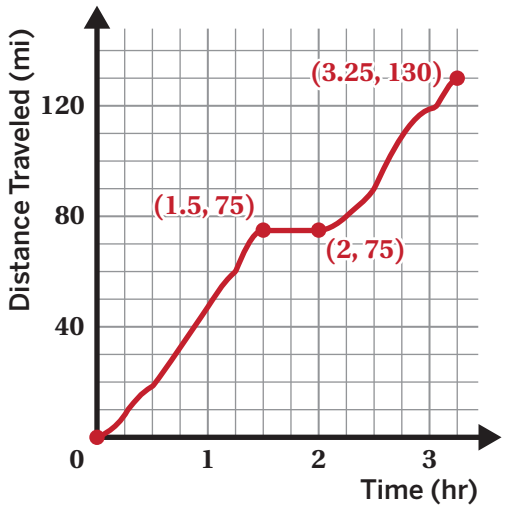
b **Discuss:**

- What is the train's average speed?
- Who might benefit and who might be harmed?

13 Synthesis

Discuss: How can you calculate the average rate of change for an interval of a function?

Use this graph if it helps with your thinking.



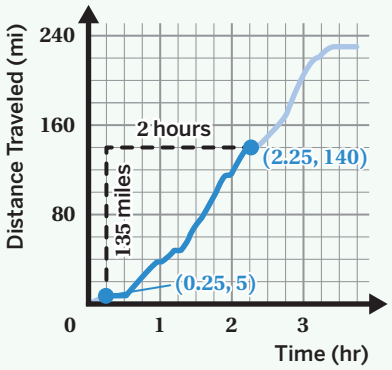
16 Summary 4.06

Functions can have different rates of change over different **intervals**.

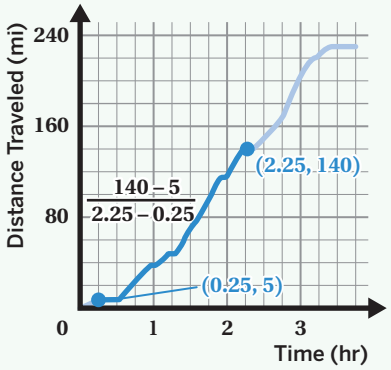
We can calculate the **average rate of change** over an interval, like the interval from 0.25 to 2.25 hours.

This graph represents Troy's train trip. Here are two different strategies for calculating the average rate of change.

Strategy 1



Strategy 2



The average rate of change for the interval 0.25 to 2.25 hours is 67.5.
That means that Troy's average speed was 67.5 miles per hour in that interval.

average rate of change A measure of how much a function changes, on average, over an interval. To calculate the average rate of change over an interval, find the slope between the point on the graph of the function where the interval begins and the point where it ends.

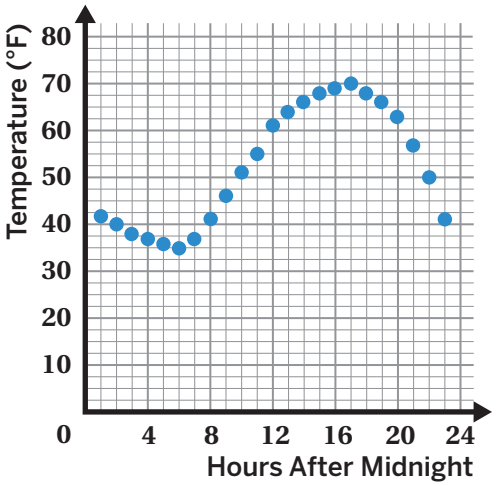
interval A set of numbers that has a starting value and ending value.

Practice
4.06

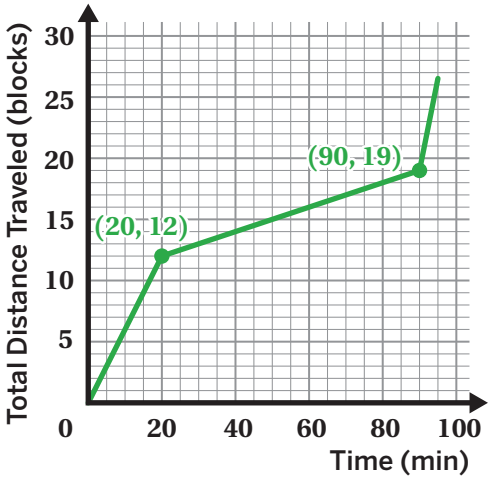
Name: ..... Date: ..... Period: .....

1. The temperature was recorded at several times in a 24-hour period. The function $t(n)$ gives the temperature in degrees Fahrenheit n hours after midnight. Use the graph to determine if the average rate of change for each interval is *positive* (+), *negative* (–), *or zero*. Place a check in the appropriate column.

	+	–	0
$n = 1$ to $n = 5$			
$n = 5$ to $n = 7$			
$n = 10$ to $n = 20$			



Problems 2–3: This graph shows the total distance in city blocks, $d(t)$, that Pilar walked as a function of time in minutes, t.

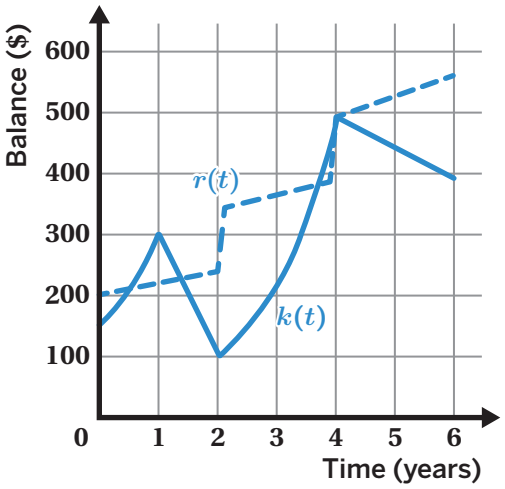


2. Determine the average rate of change between $t = 20$ and $t = 90$.

3. What do you think the average rate of change you calculated means in this situation?

4. **Test Practice** $r(t)$ and $k(t)$ model the savings account balances of Rafael and Katie after t years. Select *all* the statements that are true.

- ☐ **A.** Katie has a lower average rate of change in the last two years.
- ☐ **B.** Katie's balance is always less than Rafael's.
- ☐ **C.** $r(2) = 100$
- ☐ **D.** Rafael's balance is increasing from year 0 to year 6.
- ☐ **E.** Rafael has a higher average rate of change in the first four years.



Practice
4.06

Name: ..... Date: ..... Period: .....

Problems 5–7: This table shows the population of a city from 1988 to 2016.

Year, t	Population, $p(t)$
1988	35,700
1992	42,700
1996	33,100
2000	33,700
2004	45,000
2008	48,400
2012	40,900
2016	43,000

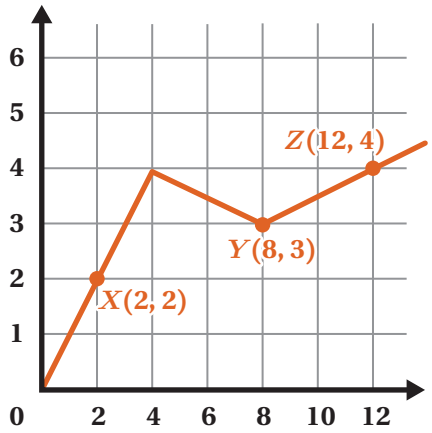
5. Determine the average rate of change for $p(t)$ between 1992 and 2000.

6. State two values of t that create an interval with a *negative* average rate of change.

7. State two values of t that create an interval with a *positive* average rate of change.

8. Match each interval to its average rate of change.

Interval	Average Rate of Change
A. X to Y	..... $\frac{1}{5}$
B. Y to Z	..... $\frac{1}{4}$
C. X to Z	..... $\frac{1}{6}$



Spiral Review

9. Jada is walking to school. The function $d(t)$ gives her distance from school, in meters, t minutes since she left home. Which equation represents this statement?
Jada is 600 *meters from school after* 5 *minutes.*

A. $d(5) = 600$ B. $d(600) = 5$ C. $t(5) = 600$ D. $t(600) = 5$

10. Complete the arithmetic sequence with the missing terms: ....., 6, ..., 22, 30

11. Complete the geometric sequence with the missing terms:

30, -10, ..., ..., $\frac{10}{27}$

Name: ..... Date: ..... Period: .....

Systems of Equations | Modeling with Functions | F–IF.2, F–IF.4, F–IF.6, A–REI.11, SMP.6

Space Race

Let's make connections between function notation and key features of graphs.

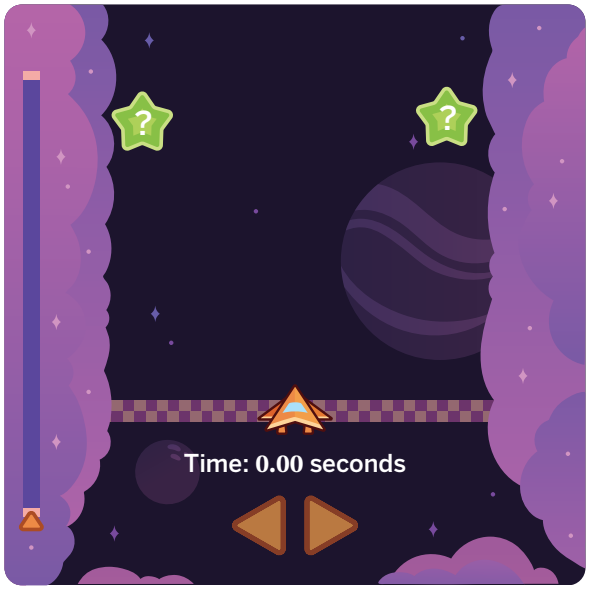
Warm-Up

1 It's time to play a game called Space Race!

Your mission: Get your spaceship to the finish line as quickly as possible.

a. Use the digital activity to play Round 1.

b. **Discuss:** How does this game work?



2 Use the digital activity to race your spaceship several more times. Then compare your graphs with a partner's graphs.

Name: ..... Date: ..... Period: .....

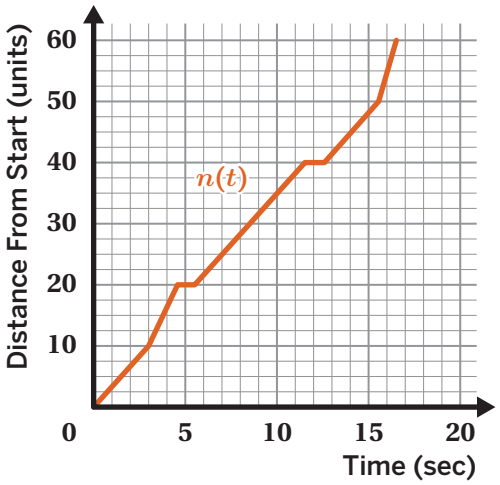
Comparing Graphs By Key Features

3 Nekeisha also played a round of Space Race.

$n(t)$ represents the distance of Nekeisha's spaceship after t seconds.

What is a value of t for which $n(t) = 10$?

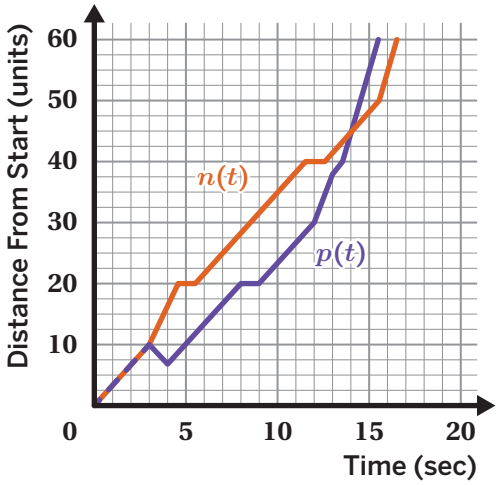
What does that tell you about the situation?



4 Let's watch Nekeisha and Polina race their spaceships.

Discuss:

- What were some interesting moments during this race?
- Where do you see those moments on the graph?



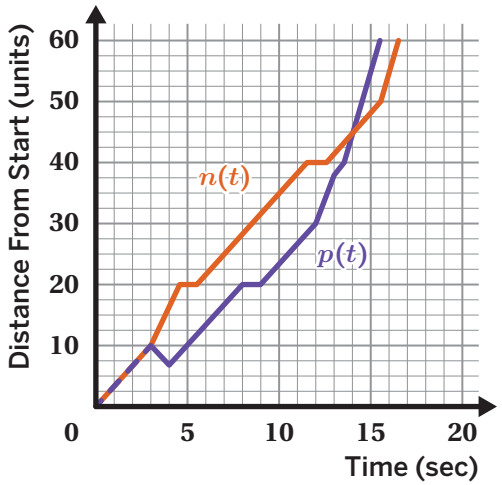
Name: ..... Date: ..... Period: .....

Comparing Graphs By Key Features (continued)

5 $n(t)$ and $p(t)$ represent the distances of Nekeisha's and Polina's spaceships after t seconds.

One student described part of the graph like this: *Polina's function was decreasing, and Nekeisha's function was increasing.*

Write a value of t that is in this interval of time.



Describe what was happening in the race at that moment.

6 Who had a greater average rate of change from 12 to 14 seconds?

A. Polina **B.** Nekeisha **C.** Their average rates of change was the same

Explain your thinking.

Activity 2

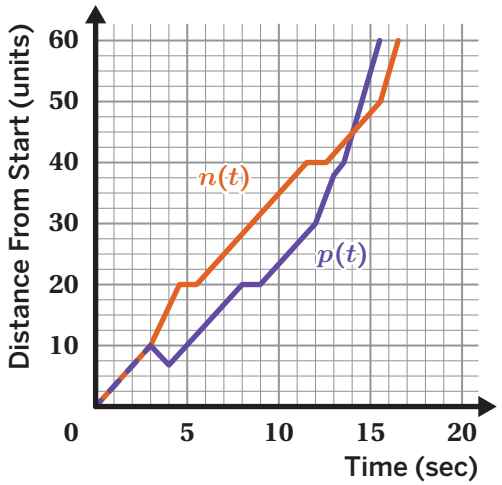
Name: ..... Date: ..... Period: .....

Comparing Using Function Notation

7 Nekeisha traveled farther than Polina in the first 11 seconds.

You can express that by writing $n(11) > p(11)$.

a **Discuss:** How can you use the graph to tell that $n(11) > p(11)$?



b Which statement could you use to show who traveled farther after 15 seconds?

A. $n(15) > p(15)$

B. $n(15) = p(15)$

C. $p(15) \geq n(15)$

D. $p(15) > n(15)$

8 We know that $n(11) > p(11)$.

What is a different value of t for which $n(t) > p(t)$?

Name: ..... Date: ..... Period: .....

Comparing Using Function Notation (continued)

9 What is a value of t for which $n(t) = p(t)$?

Describe what was happening in the race at that moment.

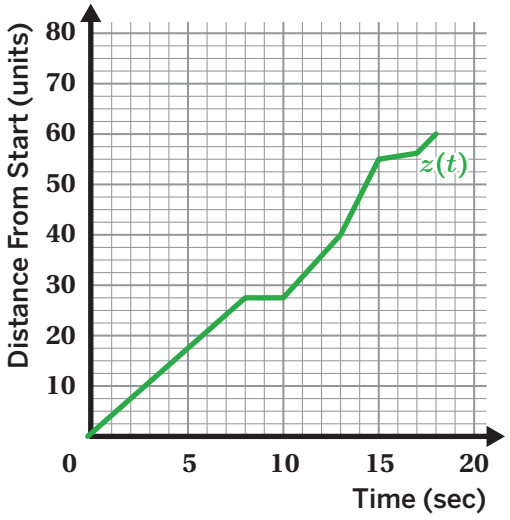
10 Valeria and Zion also raced their spaceships.

$v(t)$ and $z(t)$ represents the distances of Valeria's and Zion's spaceships after t seconds.

Here is some information about their race:

- $v(0) = z(0)$
- $v(4) < z(4)$
- $v(10) = 20$
- $v(13) = z(13)$
- $v(15) > z(15)$

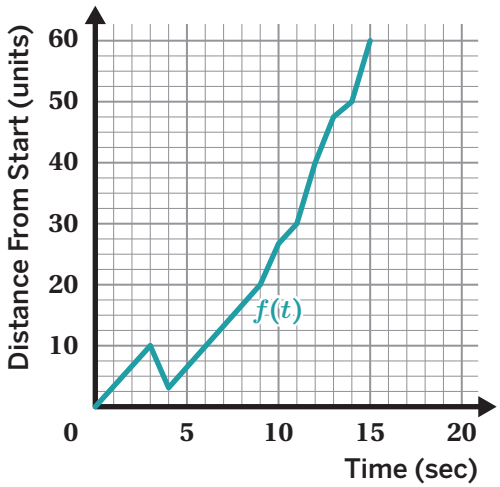
Make a graph that could represent Valeria's distance traveled after t seconds.



11 Synthesis

How can statements in function notation and terms like *maximum*, *increasing*, and *average rate of change* help us compare graphs of functions?

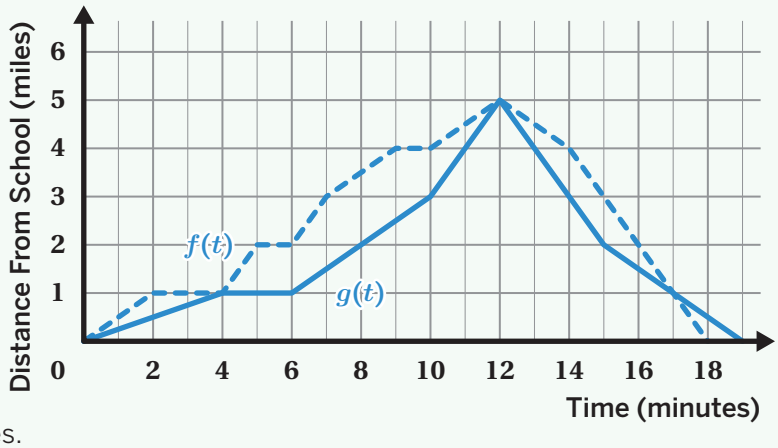
Use the example if it helps to show your thinking.



14 Summary 4.07

We can analyze functions by comparing the key features of different intervals on a graph, then using function notation to describe them.

For instance, here are some true statements about these two graphs:



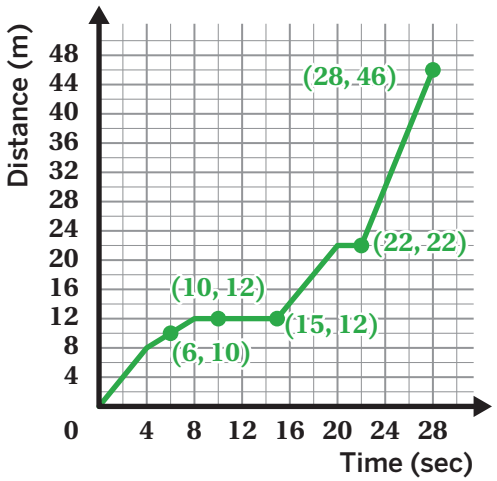
- When $t = 4$, $f(t) = g(t)$.
- $f(8) > g(8)$
- $f(12) = g(12)$
- $f(15) > g(15)$
- $f(t)$ and $g(t)$ have the same maximum.
- $f(t)$ and $g(t)$ are both decreasing from 12 to 15 minutes.
- $f(t)$ and $g(t)$ have the same average rate of change from 5 to 6 minutes.

Practice
4.07

Name: ..... Date: ..... Period: .....

Problems 1–2: Mai built a model race car for a school competition.

$m(t)$ represents the distance of Mai's car, in meters, after t seconds.



1. Use the graph to determine the missing value in each function statement.

$m($.....$) = 10$

$m(10) =$ .....

$m(22) =$ .....

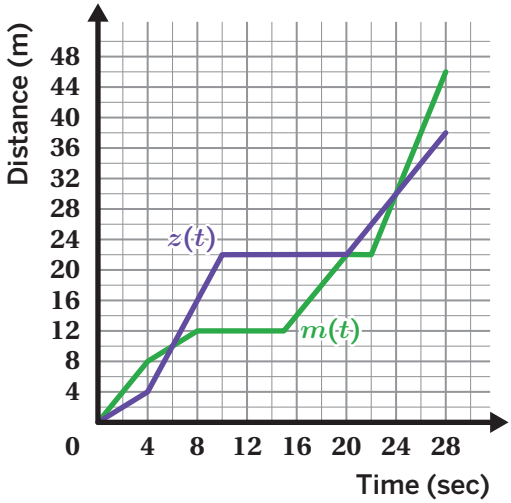
$m($.....$) = 46$

2. Over what interval did Mai's car travel the slowest?

A. 0 to 4 sec **B.** 4 to 8 sec **C.** 8 to 15 sec **D.** 15 to 20 sec

Problems 3–6: Zion also built a model race car for the school competition.

$z(t)$ represents the distance of Zion's car, in meters, after t seconds.



Did Zion or Mai have the greater average rate of change over the following intervals? Explain your thinking.

3. $t = 4$ to $t = 8$

4. $t = 10$ to $t = 20$

5. Name a time when Zion's and Mai's cars had traveled the same distance.

6. **Test Practice** Select *all* the true statements.

☐ **A.** $m(t)$ has a greater maximum than $z(t)$.

☐ **B.** $z(t)$ and $m(t)$ have the same minimum at $(6, 10)$.

☐ **C.** $z(20) = m(20)$

☐ **D.** $m(15) > z(15)$

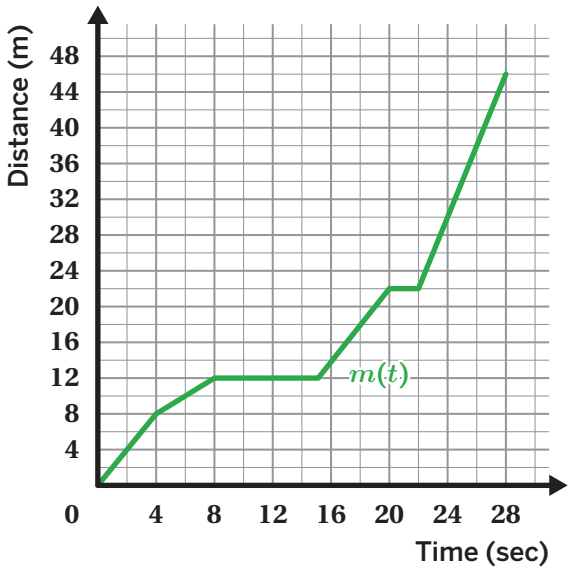
☐ **E.** $m(t)$ and $z(t)$ both increase from 22 to 28 seconds.

Name: _____ Date: _____ Period: _____

7. Parv built a race car to race against Mai.

Use this information to make a graph that could represent the distance of Parv's race car, $p(t)$, after t seconds:

- $p(8) < m(8)$
- $p(12) = m(12)$
- The average rate of change of $p(t)$ and $m(t)$ is the same from $t = 22$ to $t = 28$.
- $m(t)$ has a greater maximum than $p(t)$.



Spiral Review

Problems 8–11: Nekeisha goes for a bike ride.

$d(t)$ represents Nekeisha's distance from home, in miles, t minutes after she leaves.

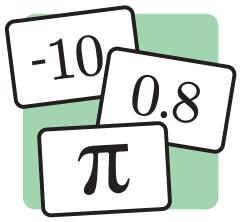
Explain the meaning of each statement in context:

8.	$d(0) = 0$	
9.	$d(30) = d(60)$	
10.	$d(90) = 0$	
11.	$d(15) = 2$	

Name: ..... Date: ..... Period: .....

Modeling with Functions F–IF.1, F–IF.5, N–Q.2, SMP.3, SMP.6

Ins and Outs



Let's explore the possible inputs and outputs of functions.

Warm-Up

1. Here are four sets of numbers.

Which set doesn't belong? Explain your thinking.

A	B
1, 2, 3, 4	0, 1, 2, 3, 4, . . .
C	**D**
5, 10, 15, 20, . . .	-1, -2, -3, -4, . . .

Activity 1

Name: _____ Date: _____ Period: _____

Possible or Impossible?

You will need a set of cards for this activity. For each **function**, determine which numbers are possible inputs and which ones are impossible. Use the same set of cards for each function.

2. Brielle's Bike Rentals charges a $10 rental fee plus an additional $2 for every hour you rent a bike. $b(h) = 2h + 10$ represents the cost of a bike rental for h hours.

Possible inputs: _____

Impossible inputs: _____

What do the possible inputs have in common?

3. The price of a medium pizza is $10 plus an additional $2 for every topping (up to 4 toppings). $p(t) = 2t + 10$ represents the cost of a pizza with t toppings.

Possible inputs: _____

Impossible inputs: _____

What do the possible inputs have in common?

4. A student is graphing the function $f(x) = 2x + 10$.

Possible inputs: _____

Impossible inputs: _____

What do the possible inputs have in common?

Name: Date: Period:

Possible or Impossible? (continued)

5. The set of all possible inputs of a function is called the **domain**.

Match each domain description with $p(t)$, $b(h)$, or $f(x)$. Be prepared to share your thinking.

Function	Domain
	All Numbers
	All Numbers Greater Than 0
	Whole Numbers From 0 to 4

6. Each of the scenarios $p(t)$, $b(h)$, and $f(x)$ can be modeled by the same linear relationship.

Discuss: Why are the domains of these three functions different from each other?

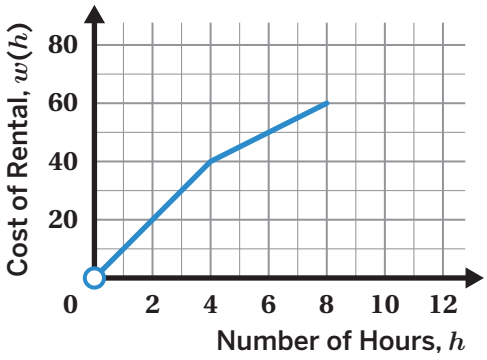
Activity 2

Name: Date: Period:

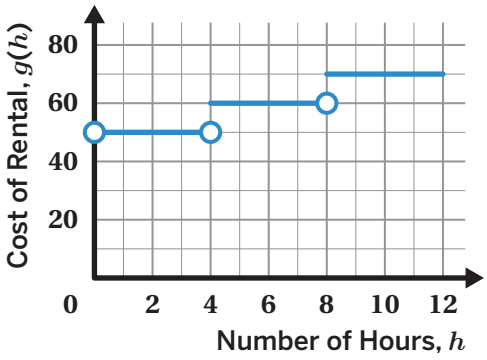
What About the Outputs?

Two bike rental companies decided to graph the cost of a bike as a function of time. The functions $w(h)$ and $g(h)$ represent the cost of a bike rental for h hours.

Wheels-R-Us



Gears-2-Go



7. Deja paid $\$40$ to rent a bike.

Discuss: Is this enough information for you to know which company she rented from? Why or why not?

8. Explain why Deja would never pay $\$55$ to rent her bike from Gears-2-Go.

9. The set of all possible outputs of a function is called the **range**. How would you describe the range of each function?

$w(h)$: $g(h)$:

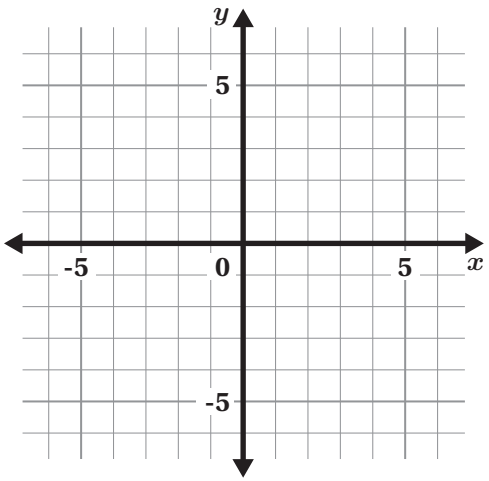
Name: ..... Date: ..... Period: .....

What About the Outputs? (continued)

Here is the domain and range of a function:

- Domain: All numbers from -3 to 4.
- Range: All numbers from 0 to 5.

10. Sketch a graph that matches the domain and range.



11. Compare your graph with a partner.

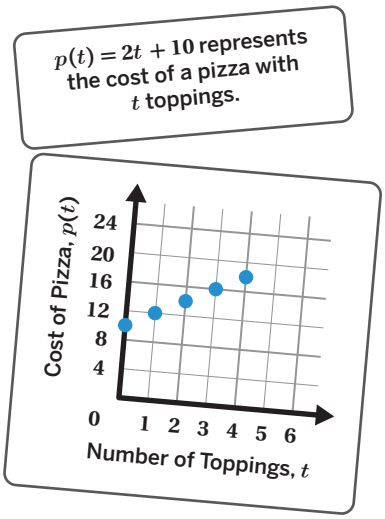
Discuss:

- What's the same about your graphs?
- What's different?

Synthesis

12. Discuss:

- How can you determine if a number is in the domain of a function?
- How can you determine if a number is in the range of a function?



Summary 4.08

The **domain** and **range** of a relation can be *discrete* or *continuous*. *Discrete* means that the possible values are separate and countable, like whole numbers. *Continuous* means that the possible values are all connected, like the values along the graph of a line. You can determine the domain by looking at the x-values as you read the graph from left to right and the range by looking at the y-values as you read the graph from bottom to top.

When a relation assigns each element of the domain to exactly one element of the range, the relation is considered a **function**. Having a context for what the function is describing helps to make sense of the domain and range.

domain The set of all possible input values for a function or relation. The domain can be described in words or as an inequality.

range The set of all possible output values for a function or relation. The range can be described in words or as an inequality.

function A relation from one set (called the domain) to another set (called the range) that assigns to each element of the domain exactly one element of the range.

Practice
4.08

Name: ..... Date: ..... Period: .....

Problems 1–2: Aniyah is working at the mall wrapping presents during the holidays. She earns $2 for each present she wraps. The function $f(x) = 2x$ represents Aniyah's daily earnings for x number of presents wrapped.

1. Are the values 2.3, -10, and 0.5 possible inputs for this situation? Explain your thinking.

2. Why is $24.50 an impossible daily earning amount for Aniyah?

Problems 3–6: A concert is being held at a local venue. Tickets are $25 per person. The concert venue has a maximum capacity of 500 people. The function $f(t) = 25t$ represents the amount of money raised for t tickets sold.

3. Select *all* the possible values in the domain of $f(t)$.

☐ **A.** -0.1 ☐ **B.** 25 ☐ **C.** 3.7 ☐ **D.** 525 ☐ **E.** 173

4. Describe the domain of $f(t)$.

5. Select *all* the possible values in the range of $f(t)$.

☐ **A.** -25 ☐ **B.** 0 ☐ **C.** 25 ☐ **D.** 860 ☐ **E.** $12{,}500$

6. Describe the range of $f(t)$.

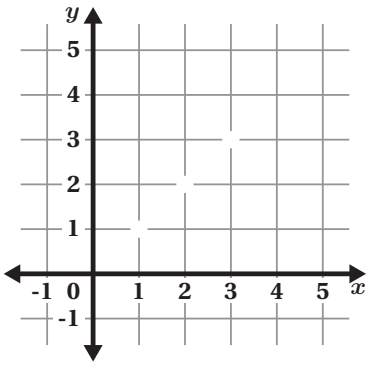
7. **Test Practice** Match each scenario with its domain description.

Scenario	All Numbers	Whole Numbers From 0 to 5	All Numbers From 0 to 24
The graph of a linear function: $p(t) = 10t + 1$			
The function $f(t)$ describes the volume of water in a pool over the course of t hours in 1 day.			
The price of a medium frozen yogurt, $m(t)$: $6 plus $1 per topping, t, with a maximum of 5 toppings.			

Name: ..... Date: ..... Period: .....

8. Sketch a graph of $f(x)$ that meets the following conditions:

- Domain: All numbers from 0 to 4
- Range: All whole numbers from 0 to 3



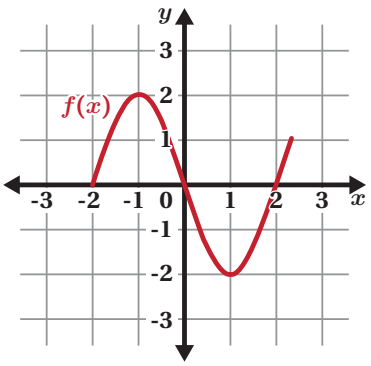
Spiral Review

9. Use the equations for $f(x)$ and $g(x)$ to determine the outputs for each input.

	$x = -5$	$x = 0$	$x = 7$
$f(x) = 2x + 5$			
$g(x) = -x + 2$			

10. Emmanuel is describing the graph of $f(x)$. Select *all* the true statements.

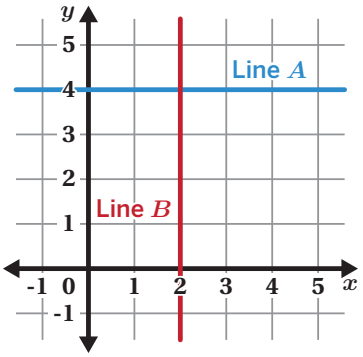
- ☐ **A.** The minimum of $f(x)$ is at $(1, -2)$.
- ☐ **B.** $f(0) = -2$
- ☐ **C.** The graph is decreasing for x, where $x \geq 1$.
- ☐ **D.** $f(0) > f(1)$
- ☐ **E.** The maximum of $f(x)$ is at $(-1, 2)$.



11. Write the equation for each line.

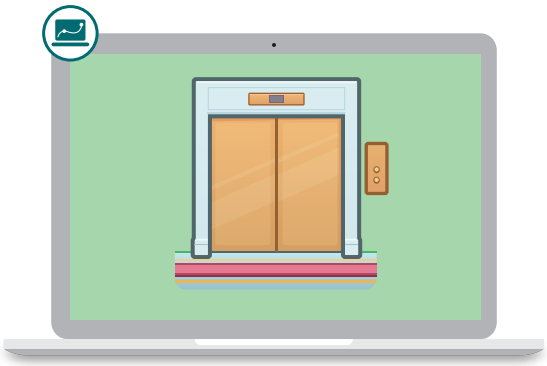
Line A: .....

Line B: .....



Name: ..... Date: ..... Period: .....

Modeling with Functions F–IF.5, SMP.1, SMP.2, SMP.3, SMP.6



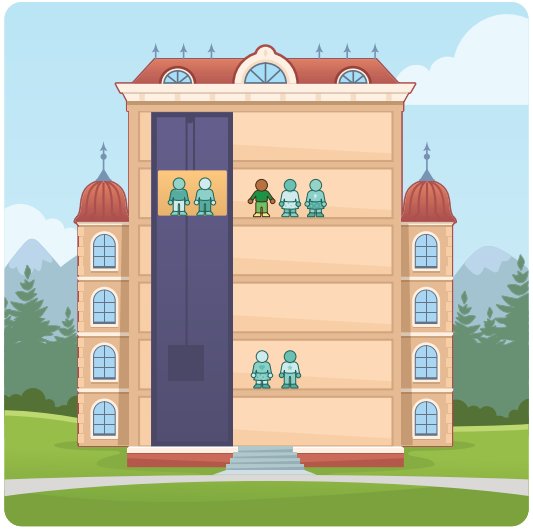
Elevator Stories

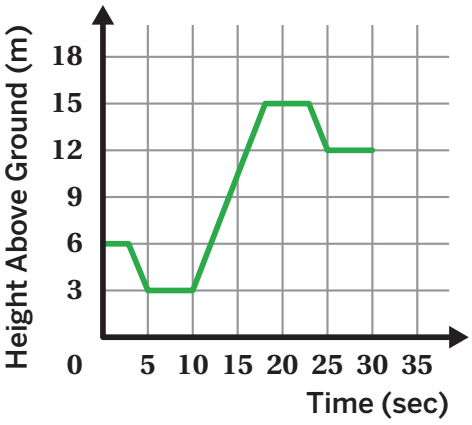
Let's use compound inequalities to describe the domain and range of functions from their graphs.

Warm-Up

1 Let's watch an animation of Amari's elevator ride.

Tell a story about what you see.





Activity 1

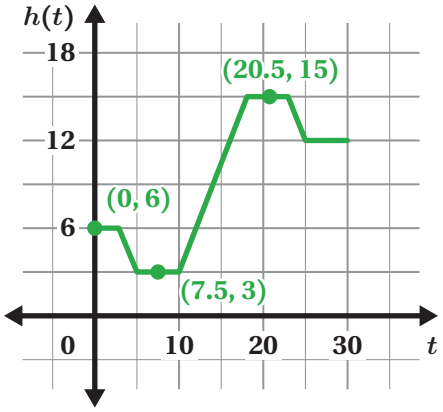
Name: ..... Date: ..... Period: .....

Describing the Domain

2 Here is a graph of $h(t)$, which represents the height of the elevator at a certain time, t, during Amari's ride.

Complete the input and output table for $h(t)$.

t	$h(t)$
5	
20	
30	



3 **a** Select *all* the numbers that are in the domain of $h(t)$.

- ☐ **A.** -5
- ☐ **B.** $\frac{1}{2}$
- ☐ **C.** 2
- ☐ **D.** 12
- ☐ **E.** 18
- ☐ **F.** 23.5
- ☐ **G.** 32
- ☐ **H.** 60

b Describe the domain of $h(t)$.

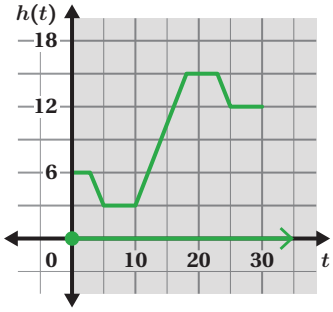
Name: ..... Date: ..... Period: .....

Describing the Domain (continued)

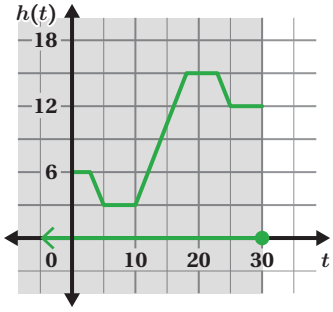
4 The domain of $h(t)$ is all the numbers from 0 to 30.

a Think about these inequalities and their relationship with the graph of $h(t)$.

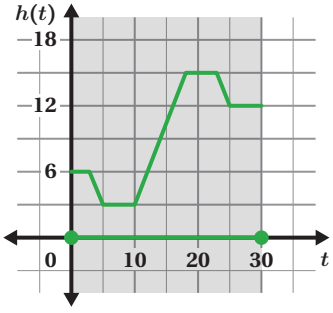
$t \geq 0$



$t \leq 30$



$t \geq 0$ and $t \leq 30$



b **Discuss:**

- Which inequality describes the domain of $h(t)$?
- Why don't the other inequalities describe the domain?

5 Each of these *compound inequalities* accurately describes the domain of $h(t)$.

a **Discuss:** How are they alike? How are they different?

$t \geq 0$ and $t \leq 30$

$0 \leq t$ and $t \leq 30$

$0 \leq t \leq 30$

b Explain how a compound inequality can help you describe the domain of $h(t)$.

Activity 2

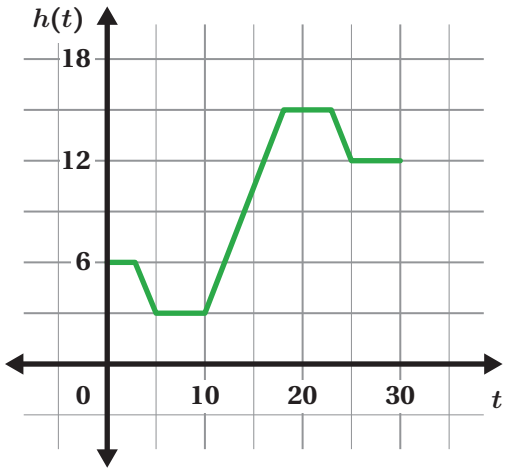
Name: ..... Date: ..... Period: .....

Distinguishing Domain and Range

6 **a** Write a compound inequality that describes the range of $h(t)$.

..... $\leq h(t) \leq$ .....

b Describe in words what the compound inequality says about the range.



7 Here is the graph of another elevator ride.

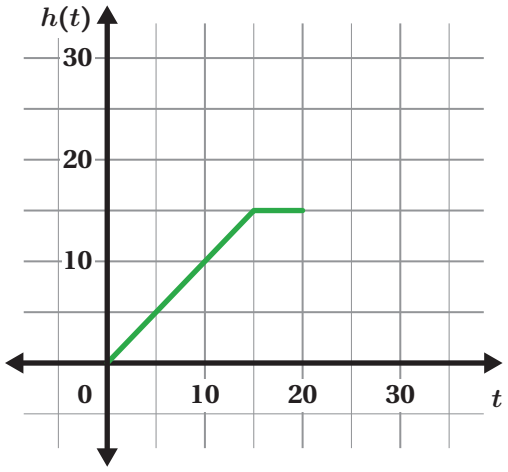
Two students described the range of this function.

- Ali says the range is $0 \leq t \leq 15$.
- Shanice says the range is $0 \leq h(t) \leq 15$.

Whose thinking is correct? Circle one.

Ali's Shanice's Both Neither

Explain your thinking.

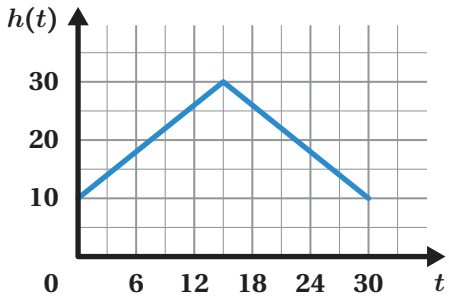


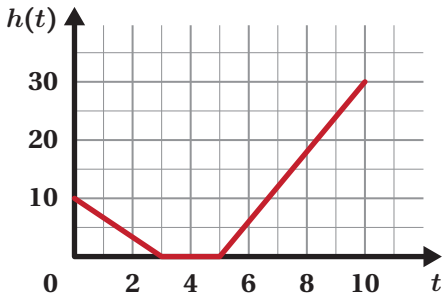
Name: Date: Period:

Distinguishing Domain and Range (continued)

8 Match each graph of a function with its domain and range. Two inequalities will have no match.

$0 \leq t \leq 10$	$0 \leq h(t) \leq 30$	$0 \leq t \leq 30$
$10 \leq h(t) \leq 30$	$0 \leq h(t) \leq 10$	$10 \leq t \leq 30$





9 The function $h(t)$ represents the height of the elevator at a certain time, t.

The range of $h(t)$ is $-30 \leq h(t) \leq 0$.

What could you say about this elevator ride?

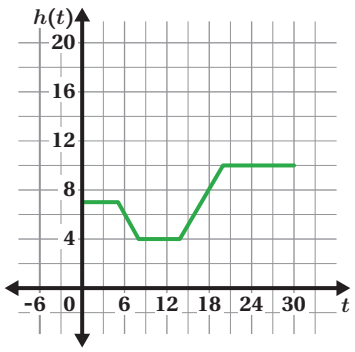
Activity 3

Name: ..... Date: ..... Period: .....

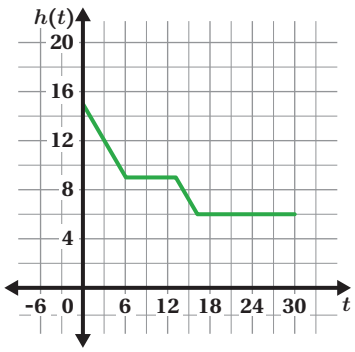
Writing Compound Inequalities

10 Complete the compound inequality to describe the range of $h(t)$.

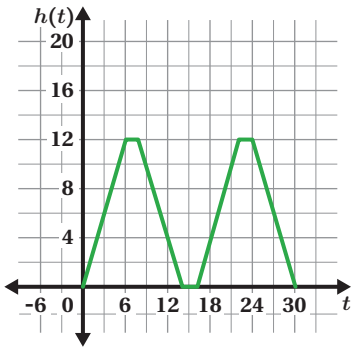
..... $\leq h(t) \leq$ .....



..... $\leq h(t) \leq$ .....



..... $\leq h(t) \leq$ .....



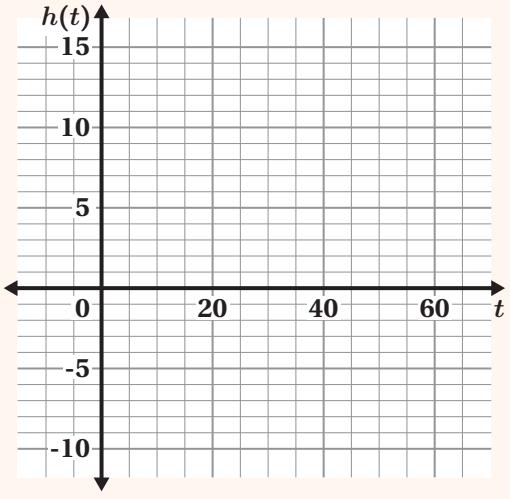
You're invited to explore more.

11 $h(t)$ represents the height of an elevator at a certain time, t, during an elevator ride.

- The domain is $0 \leq t \leq 60$.
- The range is $-6 \leq h(t) \leq 12$.

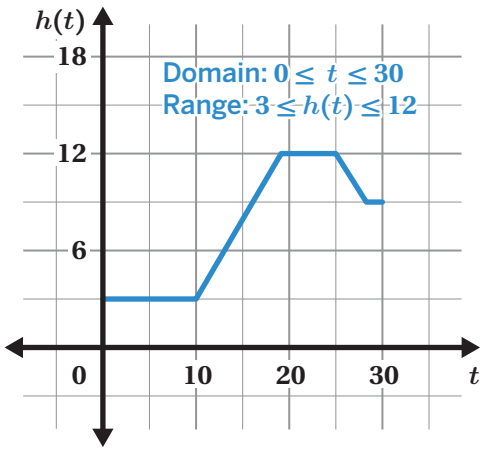
a Create a graph that matches the domain and range.

b Tell a story about your graph.



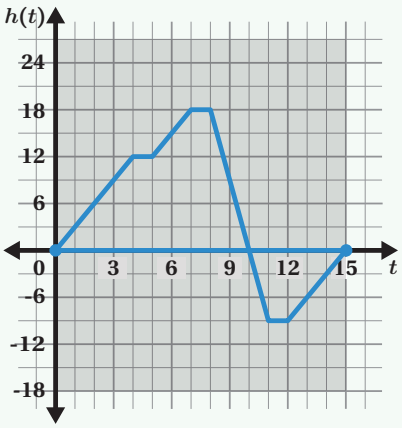
12 Synthesis

How can you determine the domain and range of a function from its graph? Draw on the graph if it helps with your thinking.



15 Summary 4.09

The *domain* and *range* of a function can each be described using a *compound inequality.* You can write a compound inequality using symbols or using the words "and" or "or". A graph can help you visualize the domain and range of a function, making it easier to describe them using compound inequalities.



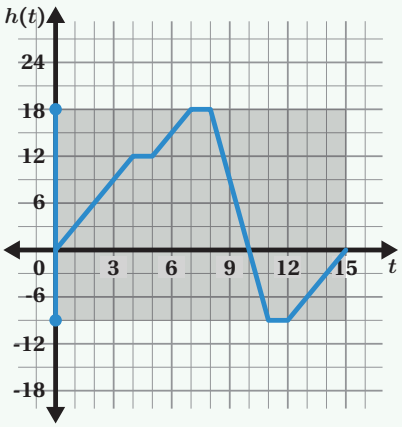
Domain: The domain is the set of all inputs to the function, or all values of the independent variable. You can see the domain as the number of included values in the graph from left to right. The domain of this function is all the t values from 0 to 15.

$$t \geq 0 \text{ and } t \leq 15$$

$$0 \leq t \text{ and } 15 \geq t$$

$$0 \leq t \leq 15$$

Range: The range is the set of all outputs of the function, or all values of the dependent variable. You can see the range as the number of included values in the graph from bottom to top. The range of this function is all the $h(t)$ values from -9 to 18.



$$h(t) \geq -9 \text{ and } h(t) \leq 18$$

$$-9 \leq h(t) \text{ and } 18 \geq h(t)$$

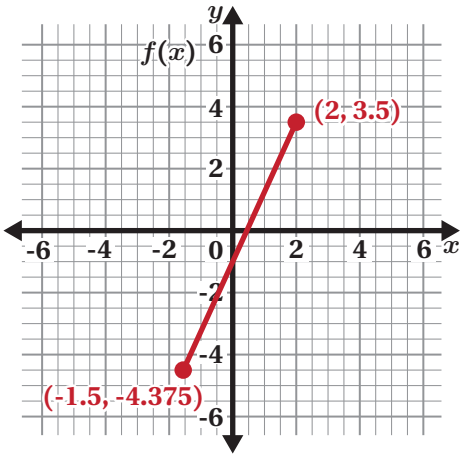
$$-9 \leq h(t) \leq 18$$

Practice
4.09

Name: ..... Date: ..... Period: .....

Problems 1–2: Valeria and Thiago disagree about the domain of $f(x)$.

- Valeria says the domain is $-1.5 \leq x \leq 2$.
- Thiago says the domain is $-4.375 \leq x \leq 3.5$.

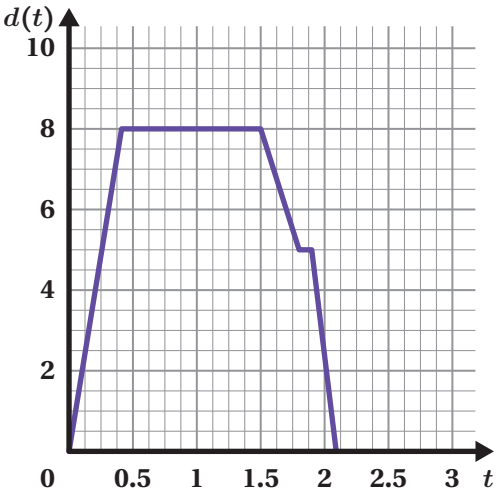


1. Whose answer is correct? Circle one.

Valeria's Thiago's

2. Explain why the other person's answer is incorrect.

Test Practice Problems 3–5: Haru bikes to his friend's house. After a while, he heads home. On the way, he stops at the store to buy a bottle of water. $d(t)$ represents Haru's distance from his house, in kilometers, after t hours. This graph shows Haru's distance over time.



3. Which inequality describes the domain of $d(t)$?

A. $0 \leq d(t) \leq 2.1$ **B.** $0 \leq d(t) \leq 8$

C. $0 \leq t \leq 2.1$ **D.** $0 \leq t \leq 8$

4. Which inequality describes the range of $d(t)$?

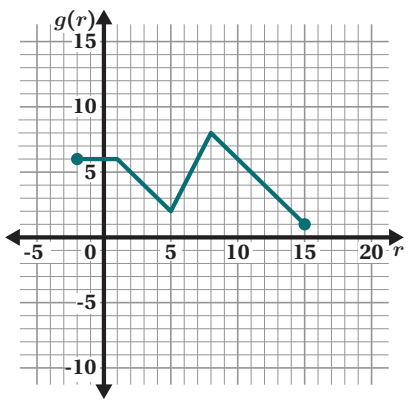
A. $0 \leq d(t) \leq 2.1$ **B.** $0 \leq d(t) \leq 8$

C. $0 \leq t \leq 2.1$ **D.** $0 \leq t \leq 8$

5. If Haru had not stopped at the store, would that change the domain or the range? Circle one.

Domain Range Both Neither

Problems 6–8: Here is the graph of $g(r)$.



6. Write a compound inequality to describe the domain.

7. Write a compound inequality to describe the range.

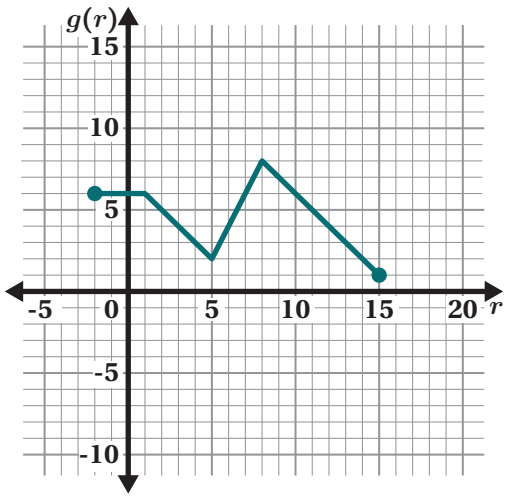
Practice
4.09

Name: ..... Date: ..... Period: .....

Spiral Review

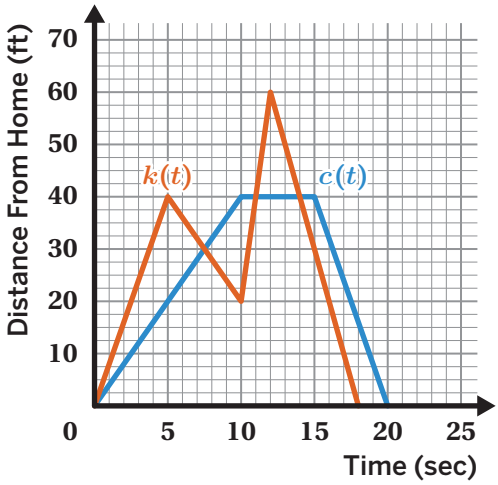
8. Determine the average rate of change for each interval in the graph of $g(r)$.

Interval	Average Rate of Change
$r = -2$ to $r = 1$	
$r = -2$ to $r = 5$	
$r = -2$ to $r = 8$	



9. Functions $c(t)$ and $k(t)$ represent the distance of two cats from home after t seconds. Select *all* of the true statements.

- ☐ **A.** $k(5) > c(5)$
- ☐ **B.** $k(t)$ and $c(t)$ have the same domain and range.
- ☐ **C.** $k(t)$ keeps increasing from 0 to 13 seconds.
- ☐ **D.** $k(11) = c(11)$
- ☐ **E.** Both cats return home.



Problems 10–11: Tickets to the state fair cost \$10 each. The function $c(t) = 10t$ gives the total cost in dollars, $c(t)$, for the number of tickets purchased, t.

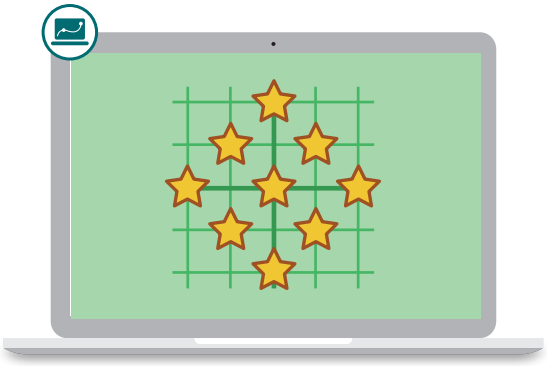
10. Select all the values that are possible outputs for $c(t)$.

☐ **A.** 0 ☐ **B.** 70 ☐ **C.** 105 ☐ **D.** 880 ☐ **E.** 963

11. Explain why -5 would be an impossible input for this situation.

Name: ..... Date: ..... Period: .....

Modeling with Functions F–IF.5, SMP.1, SMP.6



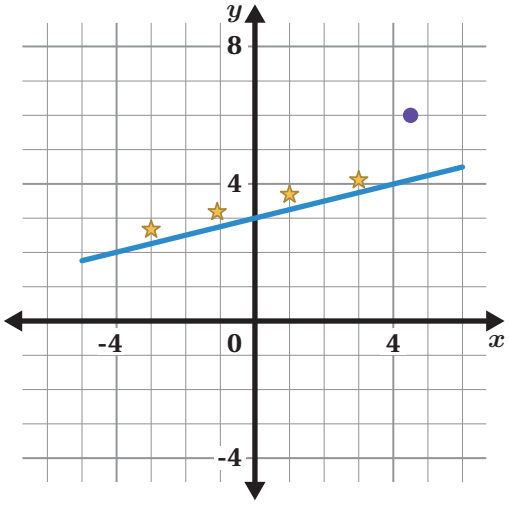
Marbleslides

Let's practice restricting the domain and range of a graph.

Warm-Up

1 Your goal is to capture all the stars.

Use the digital activity to see what happens when we press "Launch."



Activity 1

Name: ..... Date: ..... Period: .....

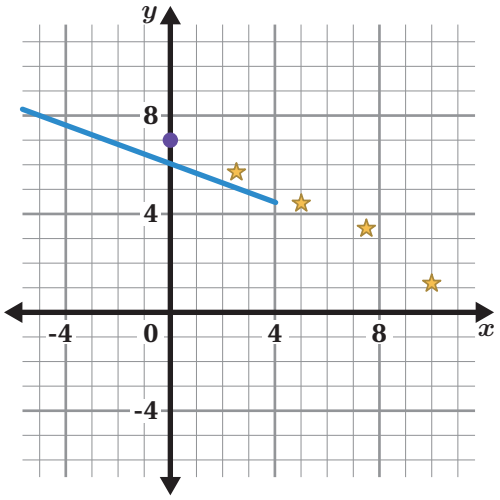
Restrict the Domain and Range

You'll use the digital activity for Problems 2–3.

2 Change one number to capture all the stars.

Original equation:
$y = -0.4x + 6\ \{x < 4\}$

Your equation:

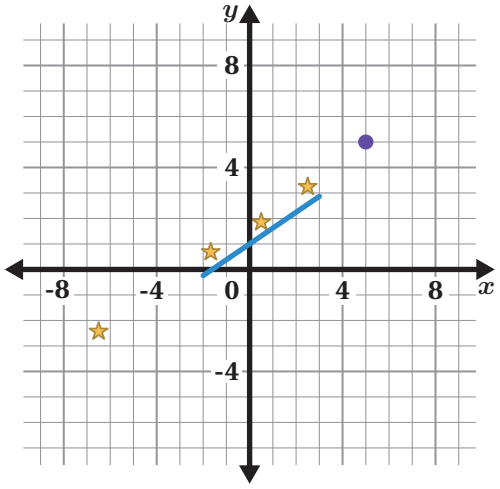


3 Change the domain to fix the Marbleslide.

Original equation:
$y = 0.6x + 1\ \{-2 < x < 3\}$

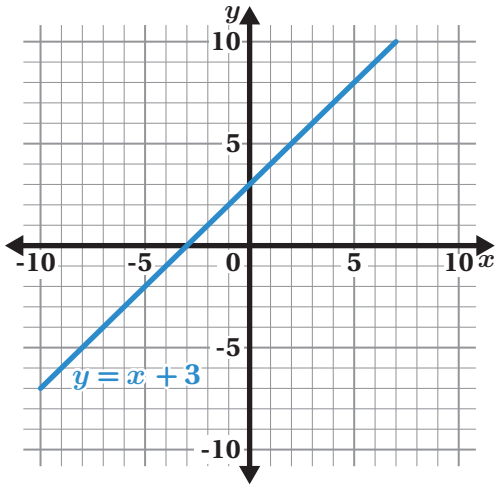
Your equation:

$y = 0.6x + 1\ \{\qquad\qquad\}$



4 If we included the range restriction $\{-2 < y < 4\}$, what would happen to the graph?

Draw on the graph if it helps with your thinking.



Name: ..... Date: ..... Period: .....

Restrict the Domain and Range (continued)

You'll use the digital activity for Problems 5–6.

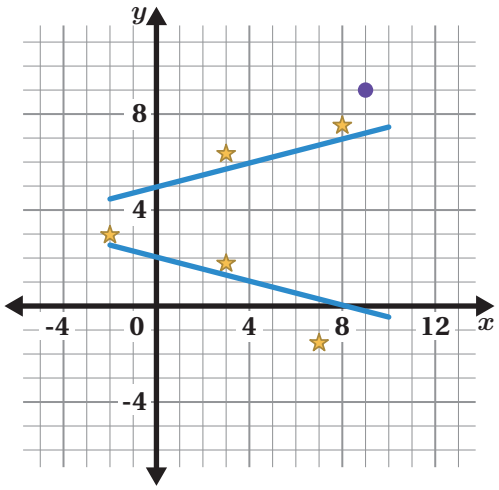
5 Change the domain, range, or equations to collect all the stars.

Original equations:

$y = \frac{1}{4}x + 5 \{-2 < x < 10\}$

$y = -\frac{1}{4}x + 2 \{-0.5 < y < 2.5\}$

Your equations:



6 **a** Change the domain or range so that the line only appears from point M to point P.

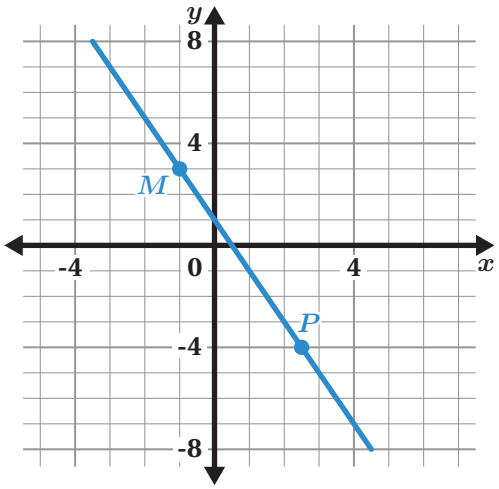
Original equation:

$y = -2x + 1$

Your equation:

$y = -2x + 1 \{$.....$\}$

b **Discuss:** What was your strategy?



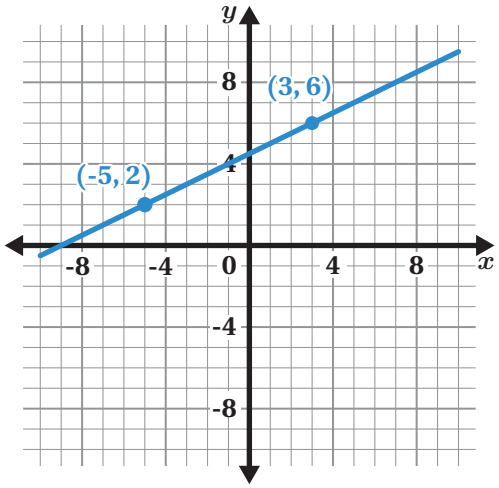
7 Two students are discussing how to restrict the graph of $y = \frac{1}{2}x + 4.5$ between (-5, 2) and (3, 6).

Hailey says to include $\{-5 < x < 3\}$.
Ricardo says to include $\{2 < y < 6\}$.

Whose thinking is correct? Circle one.

Hailey's Ricardo's Both Neither

Explain your thinking.



Activity 2

Name: ..... Date: ..... Period: .....

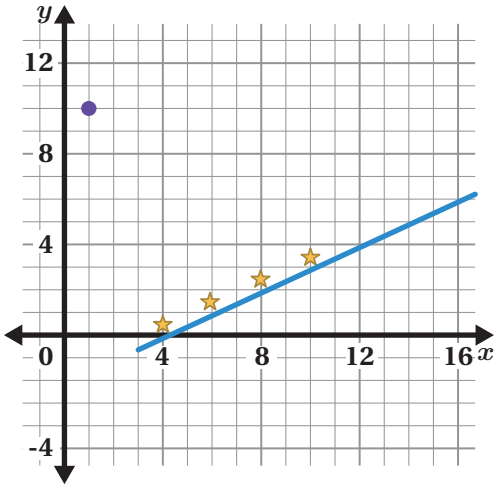
More Challenges!

You'll use the digital activity for Problems 8–12.

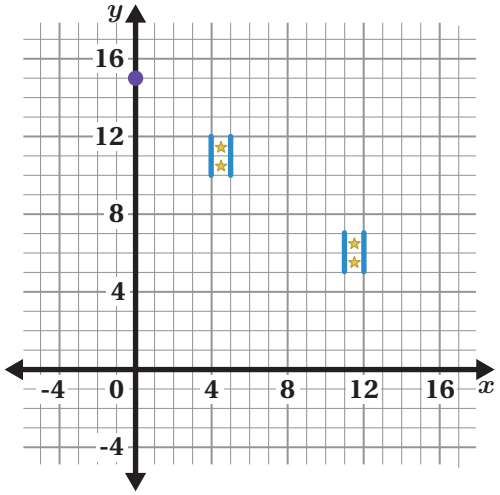
8 Create as many equations as you need to collect all the stars.

We have included the equation of a line that might help you start.

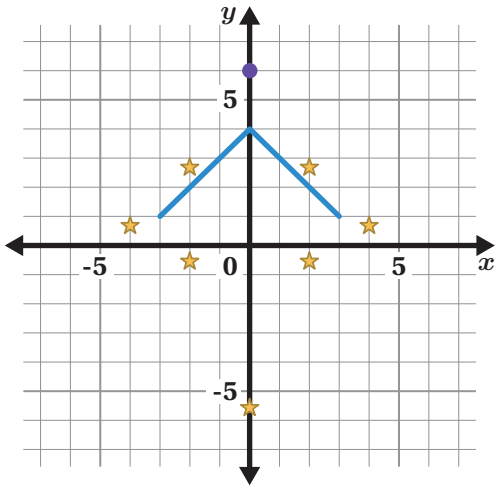
$$y = \frac{1}{2}x - 2.1 \ \{x > 3\}$$



9 Create as many equations as you need to collect all the stars.



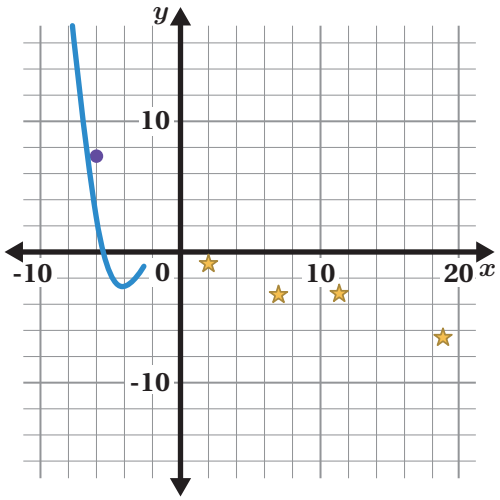
10 Create as many equations as you need to collect all the stars.



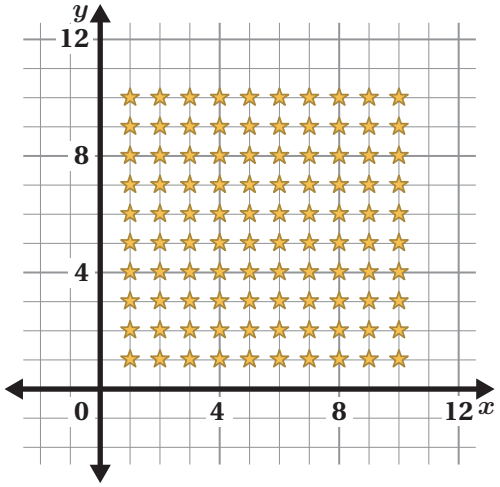
Name: ..... Date: ..... Period: .....

More Challenges! (continued)

11 Create as many equations as you need to collect all the stars.



12 Challenge yourself to collect as many stars as you can!

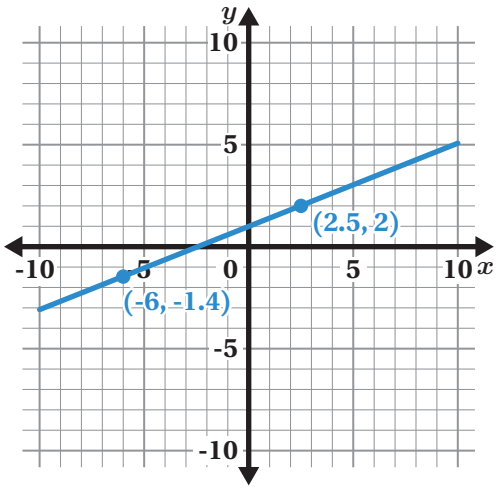


13 Synthesis

Discuss:

a. How can you use the domain to restrict the graph from $(-6, -1.4)$ to $(2.5, 2)$?

b. How can you use the range?



16 Summary 4.10

You can restrict a function's domain or range to highlight specific portions of a graph. Inequalities are one way to represent these restrictions symbolically. One strategy you can use to create a precise domain and range restriction is to use the ordered pairs at the boundaries of the interval.

Here is an example: Let's restrict the domain and range of $h(x)$ to highlight the interval from $(-3, 7)$ to $(6, 1)$.

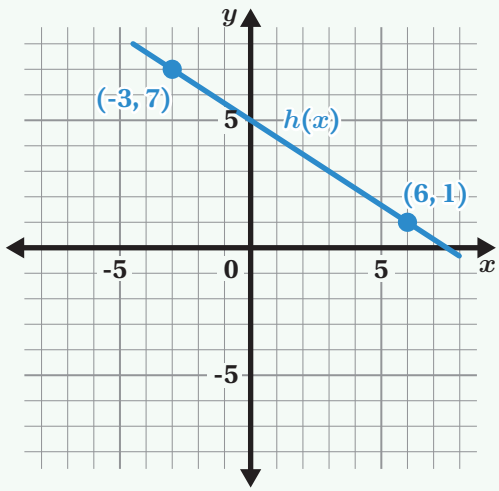
To restrict the domain, use the x-values of each ordered pair:

Domain: $-3 \leq x \leq 6$

To restrict the range, use the y-values of each ordered pair:

Range: $1 \leq h(x) \leq 7$

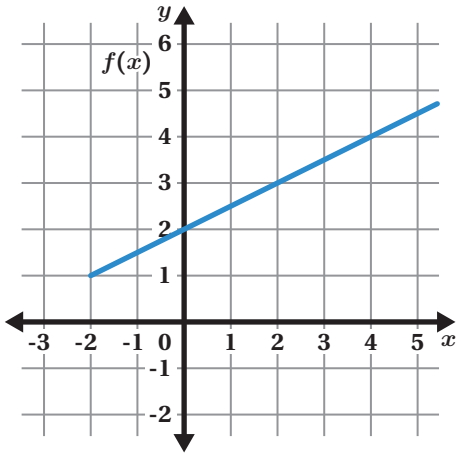
When you restrict the domain, you are restricting the x-values, or the input values. When you restrict the range, you are restricting the y-values, or the output values of $h(x)$.



Practice
4.10

Name: ..... Date: ..... Period: .....

Problems 1–2: See the graph of $f(x)$.



1. What is the domain of $f(x)$?

A. $x \geq 0$ **B.** $x \geq -2$

C. $x \leq -2$ **D.** $x \geq 1$

2. What is the range of $f(x)$?

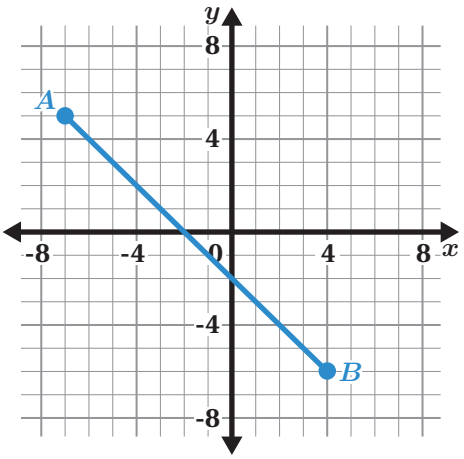
A. $f(x) \geq 0$ **B.** $f(x) \geq -2$

C. $f(x) \leq 1$ **D.** $f(x) \geq 1$

3. Fill in the blanks for the domain and range of $y = -x - 2$ from point A to point B.

..... $\leq x \leq$ .....

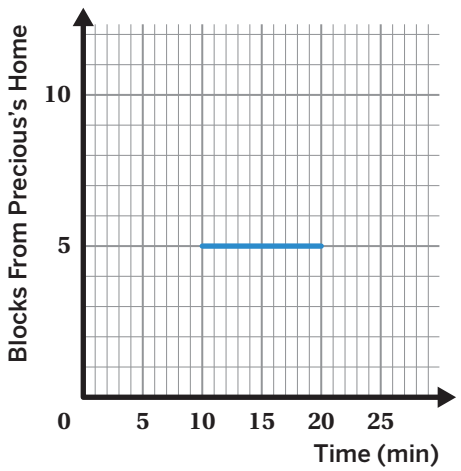
..... $\leq y \leq$ .....



Test Practice Problems 4–5: Precious leaves her home to go to the grocery store. This is her path:

- She walks to the store, which is 5 blocks away, at a speed of half a block per minute.
- She is in the store for 10 minutes.
- She runs back home at a speed of 1 block per minute.

The graph shows part of her path.



4. Sketch the missing pieces of Precious's path on the graph.

5. Which equation represents the beginning piece of Precious's path? Circle your choice.

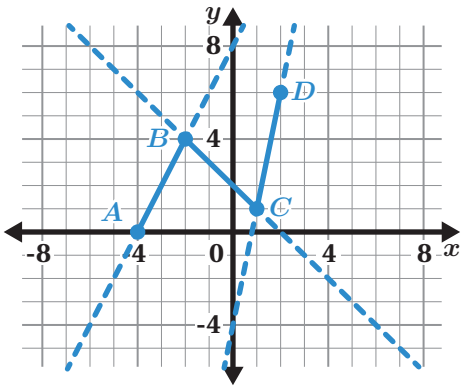
A. $y = 0.5x$ $\{0 < x < 5\}$ **B.** $y = 0.5x$ $\{0 < x < 10\}$ **C.** $y = -x$ $\{20 < x < 25\}$ **D.** $y = -x + 15$ $\{10 < x < 20\}$

Explain your thinking.

Practice
4.10

Name: ..... Date: ..... Period: .....

Problems 6–8: Sneha tries to connect points A to B to C to D with three segments. She graphs three lines but needs help restricting each line's domain. Determine the domain restriction for each of the lines.

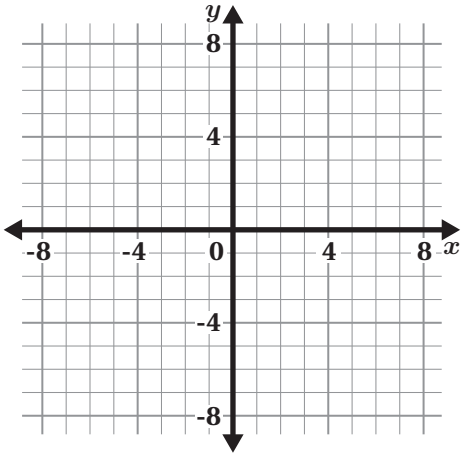


6. Domain restriction for A to B:

7. Domain restriction for B to C:

8. Domain restriction for C to D:

9. Create a design on the graph using at least six lines. Then write the equations with their domain or range restriction to represent each line segment in your design.



Spiral Review

10. Elena is deciding between two cafeteria meal plans. She estimates that she will buy 15 meals from the cafeteria each month.

Which meal plan would cost Elena less? Circle one.

Plan A Plan B

Explain your choice.

Meal Plans

Plan A: $2.50 per meal
Plan B: $30 per month

Name: ..... Date: ..... Period: .....

Modeling with Functions | Comparing Models | Systems of Equations

F–IF.4, A–REI.11, F–IF.2, F–IF.5, F–IF.6, F–BF.1, SMP.2, SMP.6

Practice Day 1

Let's practice what you've learned so far in this unit!

You will use task cards for this Practice Day. Record all of your responses here.

Task A: Milwaukee Temperatures

1. Statement: .....

2. Explanation:

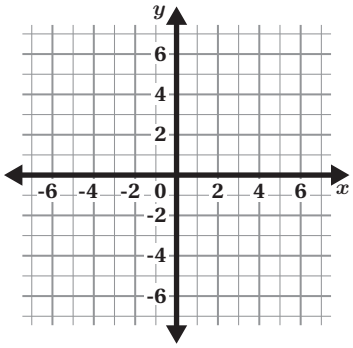
3. The temperature was decreasing: .....
The temperature was negative: .....
The temperature was increasing: .....

4. Inequality: ..... $\leq$ $h(t) \leq$ .....

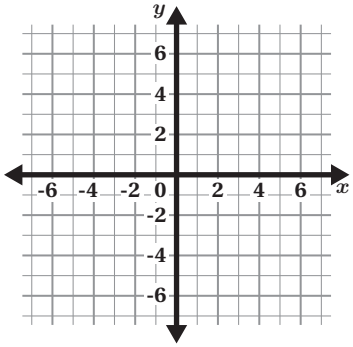
5. Explanation:

Task B: Creating Functions

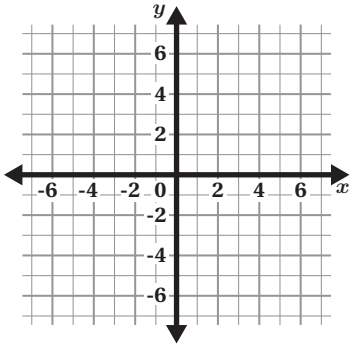
1.



2.



3.



Task C: Savings

1. Story:

2. Solution: .....

3. Inequality: .....

4. $t =$ .....

5. Solution: .....
Explanation:

Name: ..... Date: ..... Period: .....

Practice Day 1 (continued)

Task D: Daylight Hours

1.

Function Notation	Words
$f(1) = 8.6$	On the first day of the year, Fargo had 8.6 hours of daylight.
	On the 154th day of the year, El Paso had 14.1 hours of daylight.
$p(313) > f(313)$	
	On the 78th day of the year, El Paso and Fargo had the same amount of daylight.

2. Select *all* that apply: A B C D E

3. Statement 1: Statement 2:

Task E: Carbon Emissions

1. **a** Circle one: Positive Negative Zero

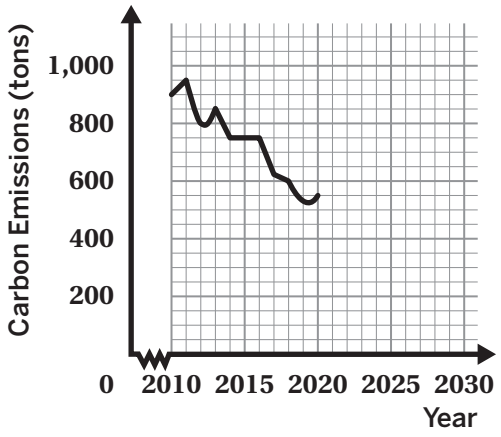
b Circle one: Positive Negative Zero

c Circle one: Positive Negative Zero

2. ..... tons of carbon per year

Explanation:

3. Graph:



4. ..... tons of carbon per year

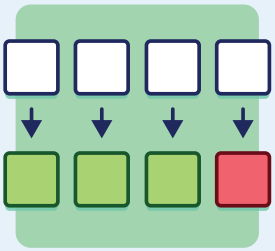
Explanation:

Notes:

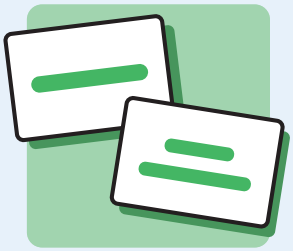
Sub-Unit

3

Special Types of Functions



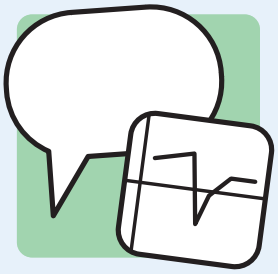
Lesson 11
Recursion Excursion



Lesson 12
Functions and Sequences

Lesson 13
What's Your Score?

Lesson 14
Absolute Value Machines

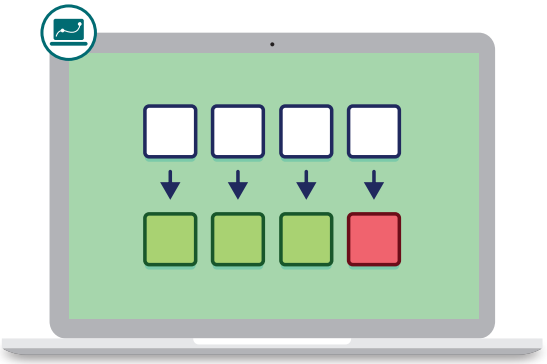


Lesson 15
Our Math Stories

Name: ..... Date: ..... Period: .....

Composing Functions F–IF.3, F–BF.2, SMP.1, SMP.2, SMP.8

Recursion Excursion

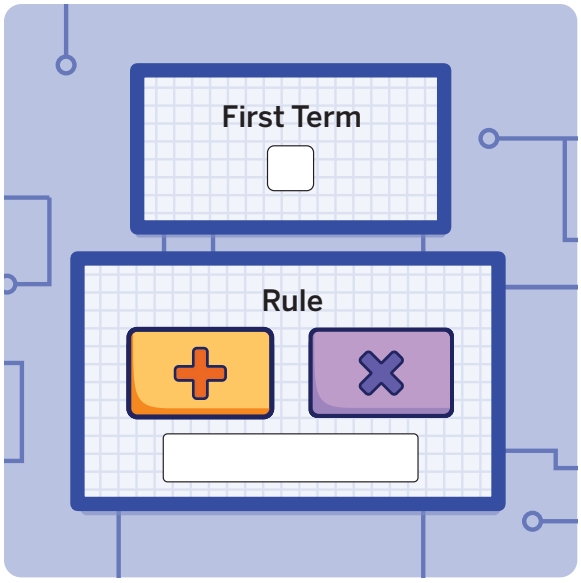


Let's write a recursive definition for a sequence using function notation.

Warm-Up

1 Let's play with the first term and rule, and notice what happens to the *sequence*, graph, and table.

Discuss: What do you remember about sequences?



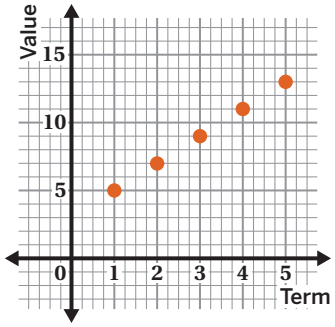
Activity 1

Name: ..... Date: ..... Period: .....

Sequences as Functions

2 Ariel made this sequence during the Warm-Up.

First Term: 5
Rule: Constant Difference of 2



Term	Value
1	5
2	7
3	9
4	11
5	13

She said: *If the domain is the term number, then all sequences are functions.*

Explain what Ariel might have been thinking.

3 Let $f(n)$ represent the value of term n in this sequence.

a **Discuss:** What does $f(4) = 11$ mean?

b What is the value of $f(7 - 1)$?

4 A sequence's *recursive definition* is made up of its first term and rule.

Here are some recursive definitions.

5, 7, 9, 11, 13	10, 6, 2, -2, -6	80, 40, 20, 10, 5
First Term: 5 Rule: Constant difference of 2	First Term: 10 Rule: Constant difference of -4	First Term: 80 Rule: Constant ratio of 0.5

a Here is how we write these rules using function notation.

$$f(n) = \begin{cases} 5 & n = 1 \\ f(n-1) + 2 & n \geq 2 \end{cases} \qquad f(n) = \begin{cases} 10 & n = 1 \\ f(n-1) + (-4) & n \geq 2 \end{cases} \qquad f(n) = \begin{cases} 80 & n = 1 \\ f(n-1) \cdot 0.5 & n \geq 2 \end{cases}$$

b **Discuss:** What does $f(n - 1)$ mean?

Name: ..... Date: ..... Period: .....

Sequences as Functions (continued)

5 These two recursive definitions will make the same sequence.

How are the definitions alike? How are they different?

$$f(n) = \begin{cases} 7 & n = 1 \\ f(n-1) + 2 & n \geq 2 \end{cases}$$

$$f(1) = 7$$
$$f(n) = f(\boldsymbol{n} - 1) + 2$$

Alike:

Different:

6 Match each recursive definition to the sequence that it makes. One sequence will have two matches.

a. $d(n) = \begin{cases} 3 & n = 1 \\ d(n-1) + n & n \geq 2 \end{cases}$ ..... 3, 5, 8, 12, . . .

b. $h(1) = 5$
$h(n) = h(n-1) \cdot 3$ ..... 3, 8, 13, 18, . . .

c. $k(n) = \begin{cases} 3 & n = 1 \\ k(n-1) + 5 & n \geq 2 \end{cases}$ ..... 5, 8, 11, 14, . . .

d. $m(1) = 5$
$m(n) = m(n-1) + 3$ ..... 5, 15, 45, 135, . . .

e. $v(1) = 3$
$v(n) = v(n-1) + 5$

Activity 2

Name: ..... Date: ..... Period: .....

Sequence Challenges

7 Here is the recursive definition for a sequence:

$h(1) = 5$

$h(n) = h(n-1) \cdot 2$

Write the first five terms of the sequence.

Term, n	Value
1	
2	
3	
4	
5	

8 Complete the recursive definition so that it makes the sequence 7, 12, 17, 22, 27.

$g(1) =$ .....

$g(n) = g(n-1)$ .....

Term, n	Value
1	7
2	12
3	17
4	22
5	27

Name: ..... Date: ..... Period: .....

Sequence Challenges (continued)

9 Here's a recursive definition Nyanna wrote for the sequence $7, 12, 17, 22, 27$.

a What did Nyanna do well?

Nyanna's Recursive Definition

$g(1) = \boxed{7}$

$g(n) = \boxed{5(n-1)}$

b What would you recommend Nyanna change to get a recursive definition that creates the sequence?

10 Write a recursive definition that will make the sequence $448, 224, 112, 56, 28$.

$f(1) =$ .....

$f(n) =$ .....

Term, n	Value
1	448
2	224
3	112
4	56
5	28

Activity 3

Name: ..... Date: ..... Period: .....

Choose Your Own Sequence

11 The Fibonacci sequence is a sequence in which each term is the sum of the two terms that come before it.

A common Fibonacci sequence is $1, 1, 2, 3, 5, 8, 13$.

Write a recursive definition for it.

$f(1) =$ .....

$f(2) =$ .....

$f(n) =$ .....

12 You will be designing a challenge for your classmates to solve.

a Make your sequence.

- Write the first five terms of your sequence.
- Write a recursive definition that will make your sequence. Your definition can include $f(n-1)$, $f(1)$, and/or n.

My Challenge	Recursive Definition
....., ..., ..., ..., ...	$f(1) =$ $f(n) =$

b Solve your classmates' sequence challenges.

- Share your sequence with a classmate. Keep your recursive definition a secret!
- Write a recursive definition that will make their sequence.

Challenges	Recursive Definition
.....'s Sequence ....., ..., ..., ..., ...	$f(1) =$ $f(n) =$
.....'s Sequence ....., ..., ..., ..., ...	$f(1) =$ $f(n) =$
.....'s Sequence ....., ..., ..., ..., ...	$f(1) =$ $f(n) =$

13 Synthesis

Explain the different parts of a recursive definition and what they tell you about a function.

Use one or both examples if they help with your thinking.

$$f(n) = \begin{cases} 25 & n = 1 \\ f(n-1) + 5 & n \geq 2 \end{cases}$$

$$f(1) = 25$$

$$f(n) = f(n-1) + 5$$

16 Summary 4.11

There are several ways we can define, or describe, a *sequence*. When you define a sequence recursively, you are determining each term using the previous term.

You can define a sequence recursively by identifying the first term of the sequence and writing a rule for how the sequence changes between terms by either a *constant ratio* or a *constant difference.* When writing the rule in function notation, you can write the recursive definition by referencing the previous term, which can be written as $f(n-1)$.

Here is an example of a recursive definition for the sequence $32, 16, 8, 4, 2, 1$ written in function notation:

$$f(n) = \begin{cases} 32 & n = 1 \\ 0.5\, f(n-1) & n \geq 2 \end{cases}$$

The first term is 32, and the sequence changes by a constant ratio of 0.5. The rule is to multiply the previous term $f(n-1)$ by 0.5.

The domain of any sequence includes the term numbers. Since each input (term number) has exactly one output (term value), that means all sequences are functions with the same domain.

Practice
4.11

Name: _____ Date: _____ Period: _____

Problems 1–3: Let $h(n)$ represent the value of term n in this sequence: $2, 6, 10, 14, 18, \ldots$. Write the number that makes each equation true.

1. $h(1) =$ _____

2. $h(8 - 1) =$ _____

3. $h(n) = h(n - 1) +$ _____

4. **Test Practice** Match each sequence with one of the definitions.

	$2, 8, 14, 20, 26$	$2, 14, 98, 686$	$2, 1, 0.5, 0.25, 0.125$
$f(1) = 2$ $f(n) = 7 \cdot f(n - 1)$			
$g(1) = 2$ $g(n) = \frac{1}{2} \cdot g(n - 1)$			
$h(1) = 2$ $h(n) = h(n - 1) + 6$			

Problems 5–6: Write the first four terms of each sequence.

5. $a(1) = 1$
$a(n) = 10 \cdot a(n - 1)$
_____, _____, _____, _____

6. $b(1) = 1$
$b(n) = b(n - 1) + 10$
_____, _____, _____, _____

Problems 7–10: Here are the first five terms of some sequences. Write a recursive definition for each one.

7. $2, 8, 32, 128, 512$
$g(1) =$ _____
$g(n) =$ _____

8. $240, 180, 120, 60, 0$
$h(1) =$ _____
$h(n) =$ _____

9. $768, 192, 48, 12, 3$
$b(1) =$ _____
$b(n) =$ _____

10. $4.5, 17, 29.5, 42, 54.5$
$f(1) =$ _____
$f(n) =$ _____

Name: ..... Date: ..... Period: .....

11. An arithmetic sequence $a(n)$ and geometric sequence $g(n)$ both have the same first and fourth term. Determine a recursive definition for each.

Arithmetic Sequence	Geometric Sequence
$a(1) =$	$g(1) =$ .....
$a(n) =$	$g(n) =$ .....

Spiral Review

Problems 10–12: Determine whether each sequence is arithmetic, geometric, or neither.

12. $162, 54, 18, 6 \ldots$ Arithmetic Geometric Neither

13. $2, 4, 8, 14 \ldots$ Arithmetic Geometric Neither

14. $6, 12, 18, 24 \ldots$ Arithmetic Geometric Neither

15. A linear function $f(x)$, is represented by the equation $f(x) = 3x + 2$

Another linear function, $g(x)$, is represented in the table:

x	$g(x)$
0	-3
2	1

Compare the parameters of these two functions:

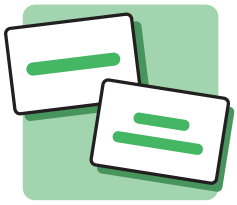
a Which function has the greater y-intercept?

b Which function has the greater rate of change?

Name: _____ Date: _____ Period: _____

Modeling with Functions | Composing Functions | F–BF.1.a, F–BF.2, F–IF.3, F–LE.2, SMP.5, SMP.7

Functions and Sequences



Let's look at different ways to write sequences.

Warm-Up

Here are two definitions of the same sequence.

$f(n) = 20 + 5n$

$f(1) = 25$
$f(n) = f(n-1) + 5$

1. Which definition is *recursive* and which is *explicit*? How can you tell?

2. How could each definition be used to determine the value of $f(3)$?

3. How would you explain to someone else why $f(n)$ is a function?

Name: ..... Date: ..... Period: .....

Recursive and Explicit

4. Complete the table below.

Sequence	Recursive Definition	Explicit Definition
Term, n / $a(n)$: 1 → 10; 2 → 20; 3 → 40; 4 → 80		$a(n) = 5 \cdot 2^n$
Term, n / $b(n)$: 1 → 32; 2 → 16; 3 → 8; 4 → 4	$b(1) = 32$ $b(n) = b(n-1) \cdot \frac{1}{2}$	
Term, n / $c(n)$: 1 → ; 2 → 35; 3 → 30; 4 → 25		

5. **Discuss:**

- What strategy did you use to write *recursive definitions*? *Explicit definitions*?
- What are some errors someone might make when writing definitions of sequences?

Name: Date: Period:

Recursive and Explicit (continued)

6. Two students wrote an explicit definition for this sequence: 12, 15, 18, 21, . . .

Madison

$f(n) = 9 + 3n$

Anya

$f(n) = 12 + 3(n-1)$

Whose explicit definition is correct?

Madison's Anya's Both Neither

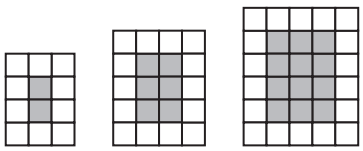
Explain your thinking.

Activity 2

Name: ..... Date: ..... Period: .....

Situations

In this visual pattern, $w(n)$ represents the number of white tiles in Figure n.

Sequence	Recursive Definition	Explicit Definition
Figure 1 Figure 2 Figure 3 	$w(1) = 10$ $w(n) = w(n-1) + 4$	$w(n) = 4n + 6$

7. Would you rather use the recursive or explicit definition to answer each question below?

a	How many tiles will there be in Figure 10?	Recursive	Explicit
b	How many tiles will there be in Figure 100?	Recursive	Explicit
c	How is the pattern growing?	Recursive	Explicit

8. You will use a set of cards. Read each card and write your responses in the space below. Use Screens 3–4 of the digital activity to help with your thinking.

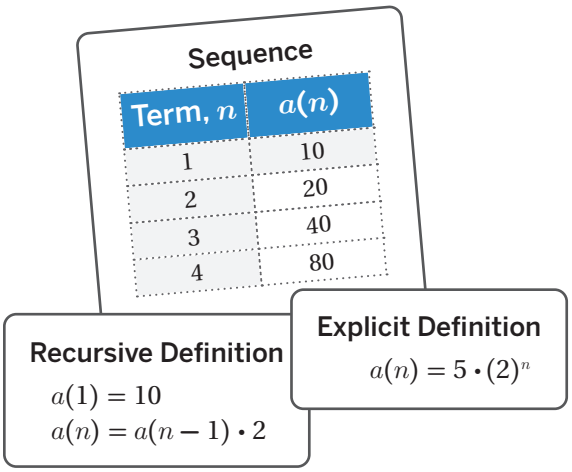
Stacking Cups	Visual Patterns
a	**a**
b	**b**
Bank Account	**Going Viral**
a	**a**
b	**b**

Synthesis

9. Discuss:

a How are recursive definitions and explicit definitions similar?

b How are they different?



Summary 4.12

You can define a sequence explicitly by writing a function where the input is the term number and the output is the value of that term in the sequence. It can be helpful to look for the pattern between the terms (for example, if they have a *constant difference* or a *constant ratio*). Then determine the first term in the sequence and/or the sequence value when term n is 0.

Here are two sequences defined recursively and explicitly:

Sequence	Recursive Definition	Explicit Definition
15, 12, 9, 6, 3, . . .	$a(1) = 15$ $a(n) = a(n-1) - 3$	$a(n) = 18 - 3n$ or $a(n) = 15 - 3(n-1)$
Term, n / $b(n)$ 1 / 20 2 / 10 3 / 5 4 / 2.5	$b(1) = 20$ $b(n) = \frac{1}{2} \cdot b(n-1)$	$b(n) = 40 \cdot \left(\frac{1}{2}\right)^n$ or $b(n) = 20 \cdot \left(\frac{1}{2}\right)^{n-1}$

Practice
4.12

Name: ..... Date: ..... Period: .....

1. **Test Practice** Match each sequence with one of the definitions.

	0, 4, 8, 12, 16	6, 18, 54, 162	5, 7, 9, 11, 13
$f(n) = 2n + 3$			
$g(n) = 2 \cdot 3^n$			
$h(n) = 4n - 4$			

Problems 2–4: Fill in the blank and write an explicit definition to make each sequence arithmetic or geometric.

2. $q =$ ....., 2, 4, . . .

$q(n) =$ .....

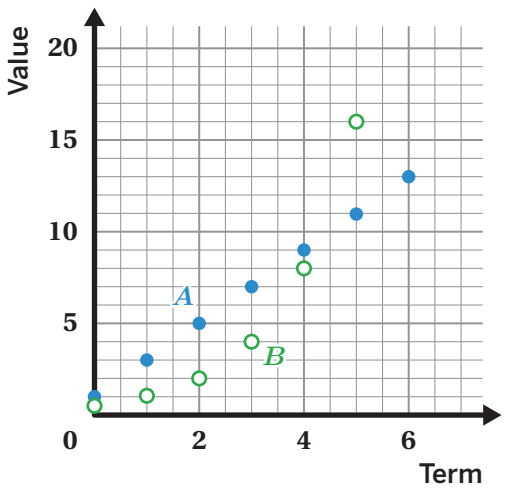
3. $r = 16,$ ....., 8, . . .

$r(n) =$ .....

4. $s = 3, 9,$ ....., . . .

$s(n) =$ .....

Problems 5–6: Write an explicit formula for each graphed sequence.



5. **Sequence** A

$A(n) =$

6. **Sequence** B

$B(n) =$

Name: ..... Date: ..... Period: .....

7. Write an explicit definition for a sequence that is neither arithmetic nor geometric.

$f(n) =$

Explain your thinking.

Spiral Review

Problems 8–10: Determine the slope for each problem.

8. $y = \frac{20}{2}x$

Slope:

9. $2x + y = 0$

Slope:

10.

x	y
0	0
2	1
4	2
6	3

Slope:

11. Here are the first five terms of a sequence. Write a recursive definition for the sequence.

$4, 12, 36, 108, 324$

$b(1) =$

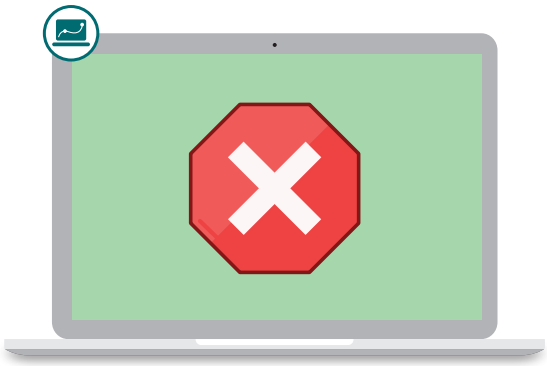
$b(n) =$

Name: ..... Date: ..... Period: .....

Modeling with Functions F–IF.2, F–IF.7, SMP.2, SMP.7

What's Your Score?

Let's make sense of absolute value functions.



Warm-Up

1 Which one doesn't belong?

A. $x = |\text{-}3|$

B. $|x| = 3$

C. $x = |9| - |12|$

D. $|9 - 12| = x$

Explain your thinking.

Activity 1

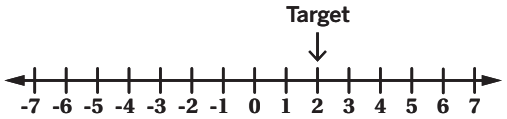
Name: ..... Date: ..... Period: .....

Target Numbers

2 Let's play a game. In the digital activity, press "Stop" to stop the arrow and get a score.

a Play up to five times.

b **Discuss:** How are the scores determined?



3 Here are Adriana's scores.

Adriana got a score of 4 on her next try.

What number do you think she stopped on? Why?

Number	Score
5	3
1	1
2	0
-4	6

4 Now there is a mystery target in this game!

a Play several rounds of the game on the digital activity. Record your number and score in the table.

b **Discuss:** What do you think the target is? Why?

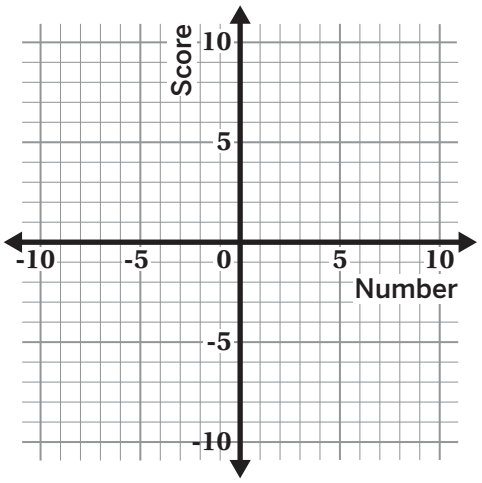
Number	Score

Name: ..... Date: ..... Period: .....

Target Numbers (continued)

5 **a** Plot the scores on the graph.

Number	Score
5	1
-1	5
3	1
0	4
-3	7



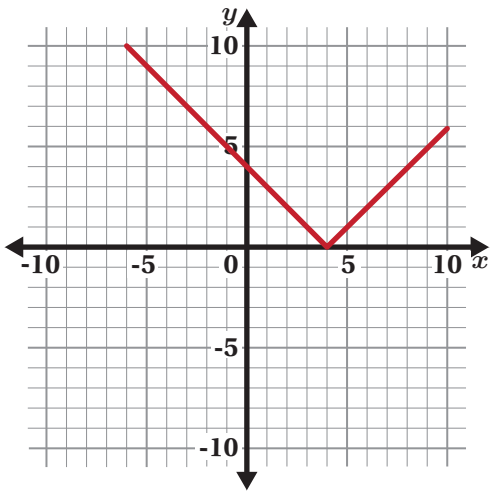
b **Discuss:**

- What do you think the graph of all possible scores looks like?
- Where can you see the mystery target?

6 The function $f(x) = |x - 4|$ is an example of an **absolute value function**.

This particular function tells you how far away you are from a target value of 4.

What is the value of $f(-2)$?



Name: ..... Date: ..... Period: .....

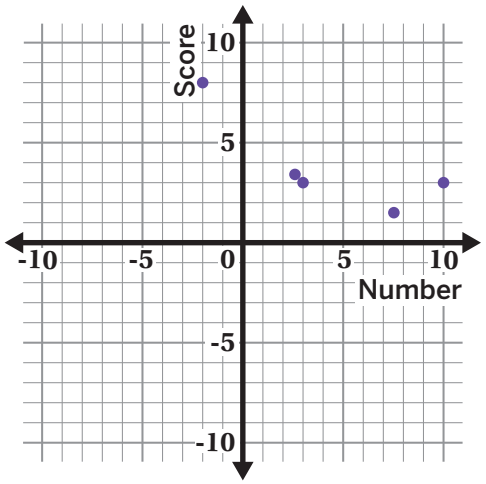
Absolute Value Functions

7 Use the digital activity to play another game. In this game, your score is how far away your guess is from a mystery number.

a Use the digital activity to enter up to five guesses.

b Tell a partner what the mystery number is and why.

8 Here are some guesses and scores.
Which function gives the score for each guess in this game?



A. $a(x) = |x| + 6$

B. $b(x) = |x + 6|$

C. $c(x) = |x| - 6$

D. $d(x) = |x - 6|$

Explain your thinking.

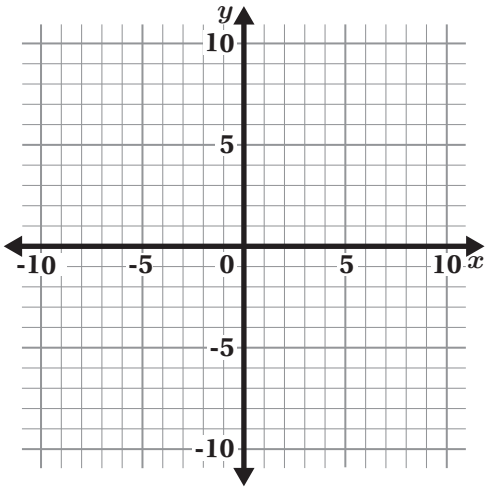
Name: ..... Date: ..... Period: .....

Absolute Value Functions (continued)

9 There is a new mystery number. The function $f(x) = |x + 3|$ gives the score for each guess, x.

Complete the table and plot the ordered pairs.

x	$f(x)$
5	
-1	
-5	
-3	
3	



You're invited to explore more.

10 Here are some guesses and scores for a new mystery number. Can these be scores for the same mystery number? Circle one.

Yes No

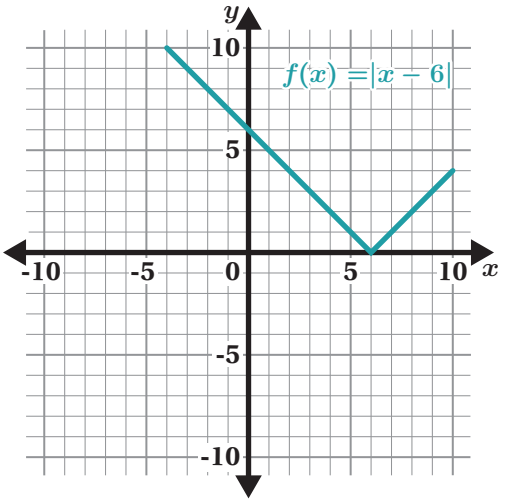
Explain your thinking.

Guess	Score
1	4
6	2

11 Synthesis

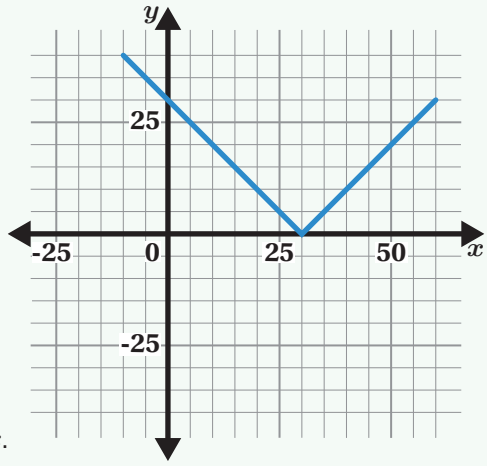
How is an **absolute value function** related to the distance from a number?

Use the graph and equation if they help with your thinking.



14 Summary 4.13

The equation of an **absolute value function** is defined using absolute value symbols, and its graph forms the shape of a V. We can write absolute value functions in the form $f(x) = |x - h|$, where $f(x)$ gives the distance of any input, x, from h. Let's look at an example.



Mr. DeAndre asked his students to guess a mystery number and gave each student a score. Their score was how far away their guess was from his mystery number, 30. Here is the graph of the function $f(x) = |x - 30|$, which gives the score for each guess, x.

We can use the equation to determine the value of $f(25)$ and interpret its meaning.

$$f(25) = |25 - 30|$$
$$= |-5|$$
$$= 5$$

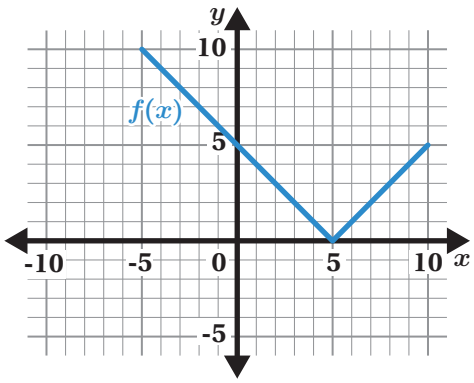
This means a student who guessed 25 was 5 away from the mystery number.

absolute value function A function that is defined using absolute value symbols. When written in the form $f(x) = |x - h|$, its output is the distance of its input from a given value, h.

Practice
4.13

Name: ..... Date: ..... Period: .....

Problems 1–3: Use the graph of $f(x)$ to determine each value.

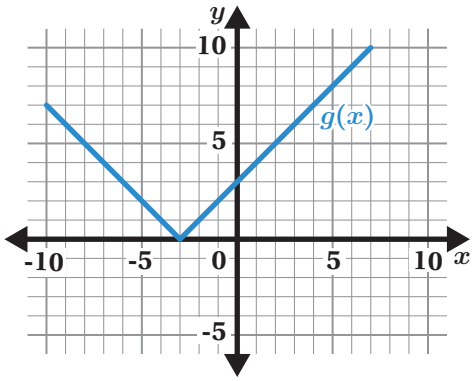


1. $f(0) =$ .....

2. $f(8) =$ .....

3. $f(5) =$ .....

4. **Test Practice** Which equation represents the graph of $g(x)$? Circle your choice.



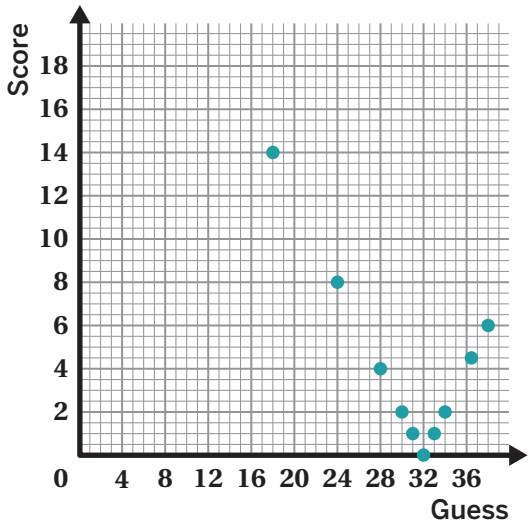
A. $g(x) = |x| - 3$

B. $g(x) = |x - 3|$

C. $g(x) = |x| + 3$

D. $g(x) = |x + 3|$

Problems 5–7: Ricardo's teacher challenged his class to guess how many marbles were in a jar. Each student received a score equal to how far away their guess was from the actual number of marbles in the jar. The graph shows each student's guess, x, and score, $m(x)$.



5. How many marbles are in the jar?

6. Circle the point that represents the furthest guess from the actual number of marbles.

7. Ricardo writes the equation $m(25) = 7$.

What does his equation mean?

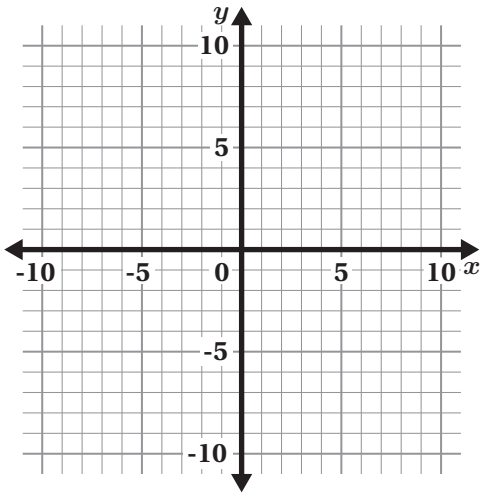
Practice
4.13

Name: ..... Date: ..... Period: .....

Problems 8–9: Here is a function: $h(x) = |x - 2|$

8. Complete the table for $h(x)$.

x	$h(x)$
-1	
0	
2	
3	
5	



9. Use the table from Problem 8 to help you sketch the function.

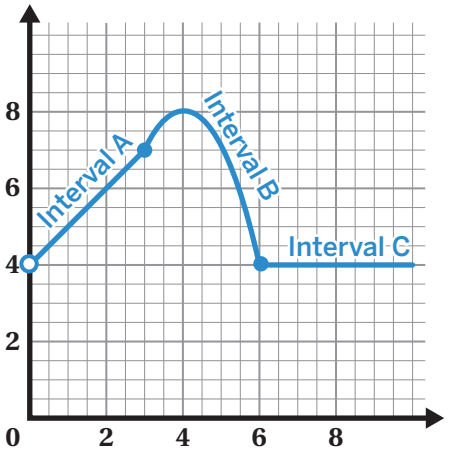
Spiral Review

Problems 10–12: Match each domain to an interval of the function on the graph.

10. $3 \leq x \leq 6$ .....

11. $x \geq 6$ .....

12. $0 < x \leq 3$ .....



13. The California Department of Fish and Wildlife estimated there were 460,420 deer in the state in 2021. They estimated that the deer population in 2018 was 470,000.

Calculate the average rate of change during this time interval and explain what it tells us about the deer population.

Unit 4
Lesson
14

Name: ..... Date: ..... Period: .....

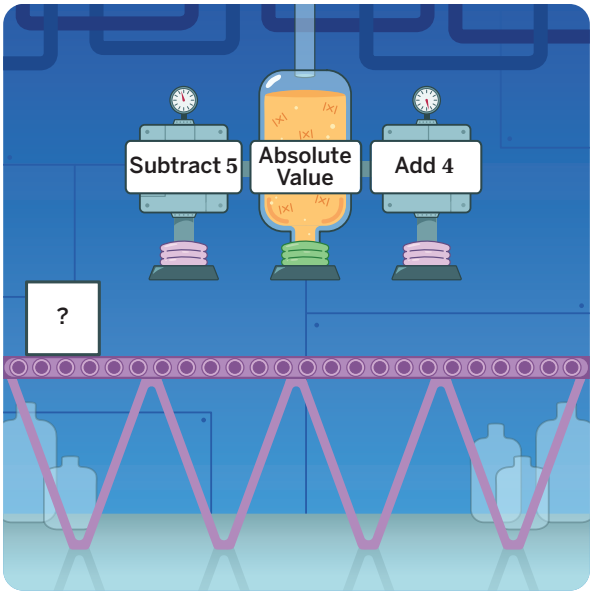
Modeling with Functions | Comparing Models | Composing Functions | A–CED.1, F–BF.3, F–IF.7, SMP.3, SMP.6, SMP.7

Absolute Value Machines

Let's graph absolute value functions.

Warm-Up

1 Here is a machine for $f(x) = |x - 5| + 4$.



a Let's watch how the machine works. Write down what happens to each input value at each solving step.

x	$x - 5$	$\lvert x - 5 \rvert$	$\lvert x - 5 \rvert + 4$

b **Discuss:** What do you notice and wonder?

Activity 1

Name: ..... Date: ..... Period: .....

Features of Absolute Value Functions

2 Kiri tried the numbers in the table.

She says: *The minimum value the machine can make is* 4.

Do you agree? Circle one.

Yes No Not enough information

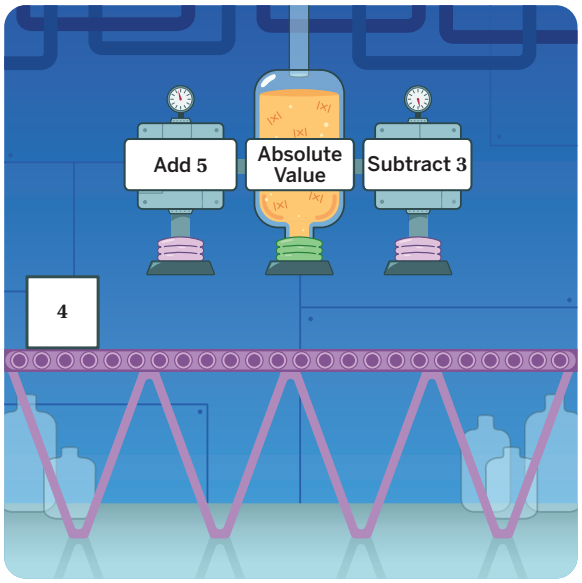
Explain your thinking.

x	$x-5$	$\lvert x-5\rvert$	$\lvert x+5\rvert+4$
-1	-6	6	10
2	-3	3	7
5	0	0	4
6	1	1	5

3 Here is a machine for $g(x) = |x + 5| - 3$.

What number will come out of the machine if we enter 4, -1, and -6?

Use the table if it helps with your thinking.



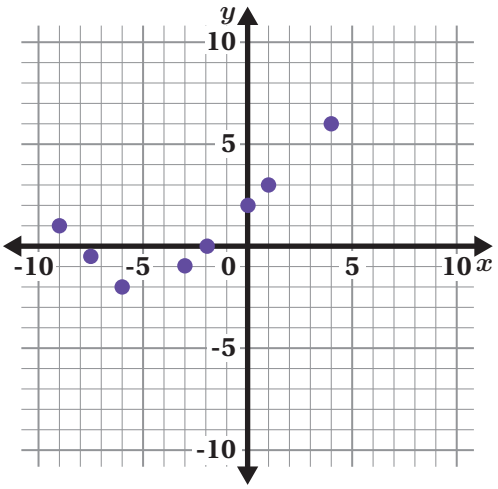
	x	$x+5$	$\lvert x+5\rvert$	$\lvert x+5\rvert-3$
a	4			
b	-1			
c	-6			

Name: ..... Date: ..... Period: .....

Features of Absolute Value Functions (continued)

4 Here are some points on the graph of $g(x) = |x + 5| - 3$.

a Draw a sketch that shows what all the points look like.



b Describe your sketch using some of these terms:

positive	maximum	increasing	domain
negative	minimum	decreasing	range
symmetry			

5 Here are descriptions of $g(x) = |x + 5| - 3$ from other students.

Select *all* the descriptions that are true.

- ☐ **A.** The domain is all numbers.
- ☐ **B.** The minimum is at (-6, -2).
- ☐ **C.** The range is $g(x) \geq -3$.
- ☐ **D.** $g(x)$ is increasing when $x > -6$.
- ☐ **E.** The minimum is at (-5, -3).

Activity 2

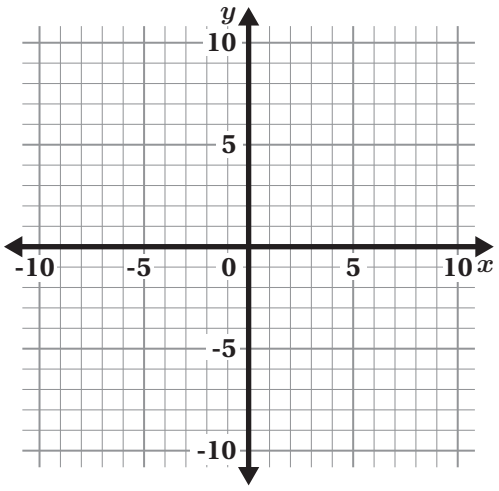
Name: ..... Date: ..... Period: .....

Graphing Absolute Value Functions

6 Here is a new function: $f(x) = |x - 6| + 2$.

Complete the table and then plot the graph of $f(x)$.

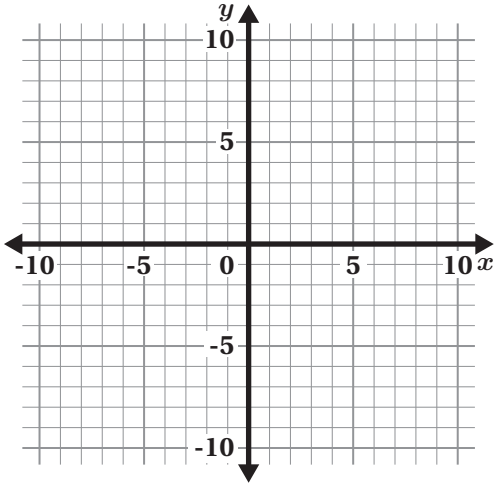
x	$f(x)$
9	
2	
0	
6	
-1	



7 Draw a graph of $j(x) = |x + 3| + 1$.

Use the table if it helps with your thinking.

x	$j(x)$



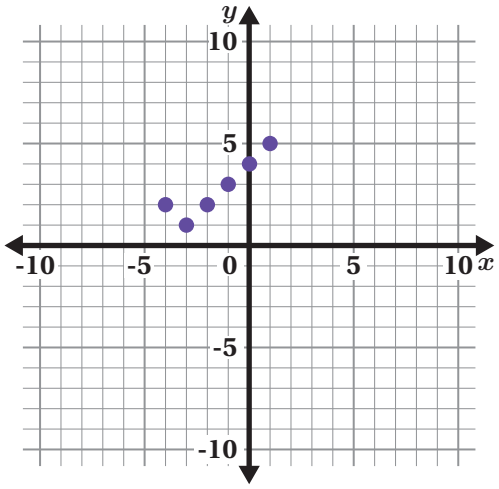
Name: ..... Date: ..... Period: .....

Graphing Absolute Value Functions (continued)

8 Here are the points that Tiana plotted for $j(x) = |x + 3| + 1$.

Tiana says: *I can use symmetry to plot more points on the graph.*

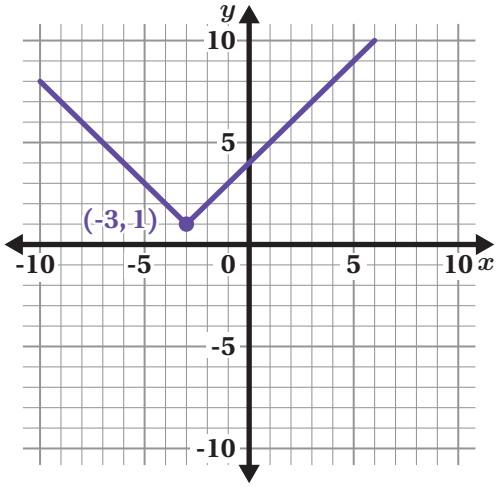
Show or describe what you think this means.



9 Here is the graph of $j(x) = |x + 3| + 1$.

The minimum value is shown.

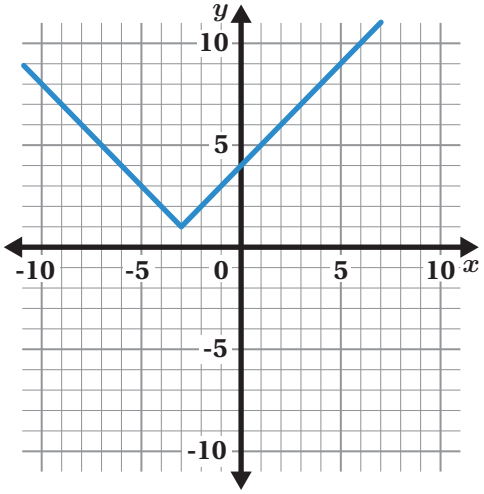
How can you see the minimum value in the equation?



10 Here is the graph of $j(x) = |x + 3| + 1$.

How would the graph of $m(x) = |x + 3|$ compare to the graph of $j(x)$?

Sketch the graph of $m(x)$.



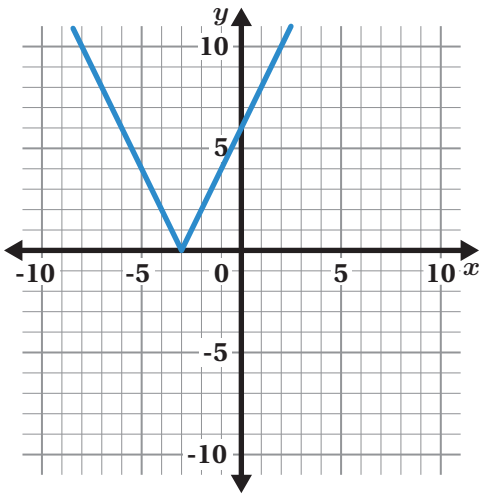
Activity 3

Name: Date: Period:

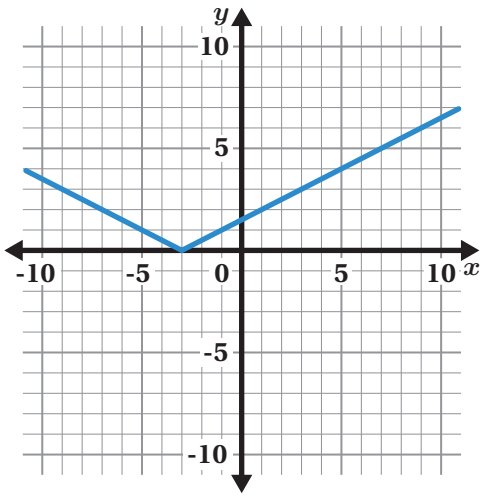
More Graphs of Absolute Value Functions

Laila and Bao graphed two different absolute value functions.

Laila graphed $h(x) = 2|x + 3|$.



Bao graphed $k(x) = \frac{1}{2}|x + 3|$.



11 **Discuss:** How are the two absolute value functions and their graphs alike? How are they different?

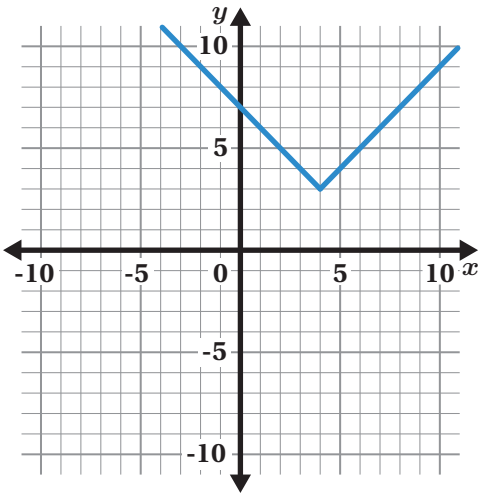
12 How do Laila's and Bao's functions and graphs compare to the function and graph on for $m(x)$ on Screen 10?

13 How does the coefficient in front of the absolute value affect the graph of the function?

Name: ..... Date: ..... Period: .....

More Graphs of Absolute Value Functions (continued)

14 Here is the graph of an absolute value function.



Laila says the function is $f(x) = |x + 4| + 3$.

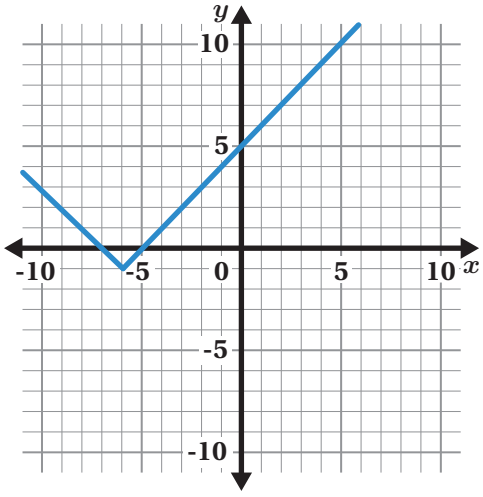
Bao says the function is $f(x) = |x - 4| + 3$.

Whose function represents the graph?

Laila Bao Neither Both

Explain your thinking.

15 Here is the graph of another absolute value function.



Write an equation for $g(x)$.

$g(x) =$ .....

16 Synthesis

Discuss: What can you know about the graph of an absolute value function by looking at its table or equation?

Use the example if it helps with your thinking.

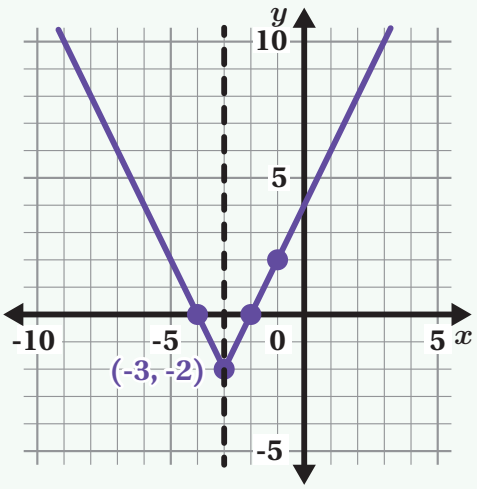
$f(x) = |x - 4| + 3$

x	$f(x)$
-2	9
0	7
2	5
4	3
6	5

19 Summary 4.14

You can determine key features of the graph of an *absolute value function* by analyzing its table or equation, which are both helpful in sketching its graph.

Here are the graph and table for the absolute value function $f(x) = 2|x + 3| - 2$.



x	$f(x) = 2\|x + 3\| - 2$
-4	0
-3	-2
-2	0
-1	2

Evaluating $f(x)$ at $x = -3$ makes the equation equal to -2. The values in the table show that there is symmetry around the point (-3, -2). This tells us that (-3, -2) is the minimum value of the function, which we can see by looking at the graph. The coefficient of 2 in front of the absolute value bars tells us that the graph is **vertically stretched**, which we can also see by looking at the graph.

vertical compression The sides of a function's graph appear to move away from the line of symmetry. This occurs when the coefficient in front of the absolute value is between 0 and 1.

vertical stretch The sides of a function's graph appear to move towards the line of symmetry. This occurs when the coefficient in front of the absolute value is greater than 1.

Practice
4.14

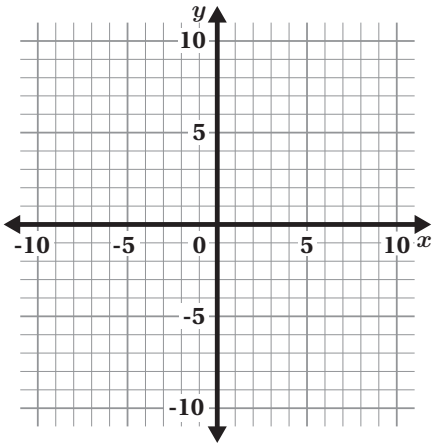
Name: ..... Date: ..... Period: .....

1. **Test Practice** Determine the minimum value of the function $k(x) = 3|x + 1|$.

A. $(3, -1)$ **B.** $(-1, 3)$ **C.** $(-1, 0)$ **D.** $(1, 0)$

2. Graph $g(x) = |x + 1| + 4$. Use the table if it helps with your thinking.

x	$g(x)$
-3	
-1	
0	



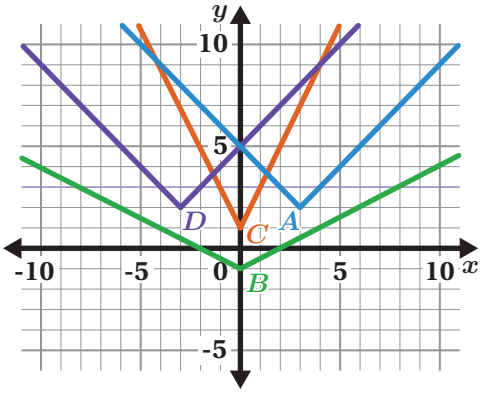
3. Match each function with its graph.

$f(x) = |x + 3| + 2$ .....

$g(x) = |x - 3| + 2$ .....

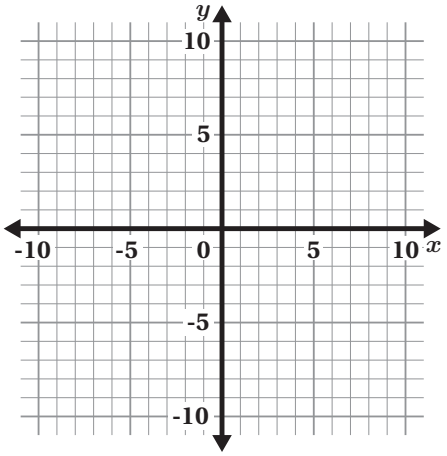
$h(x) = \frac{1}{2}|x| - 1$ .....

$j(x) = 2|x| + 1$ .....



4. Graph $h(x) = 3|x - 2| + 1$. Use the table if it helps with your thinking.

x	$g(x)$
0	
1	
2	

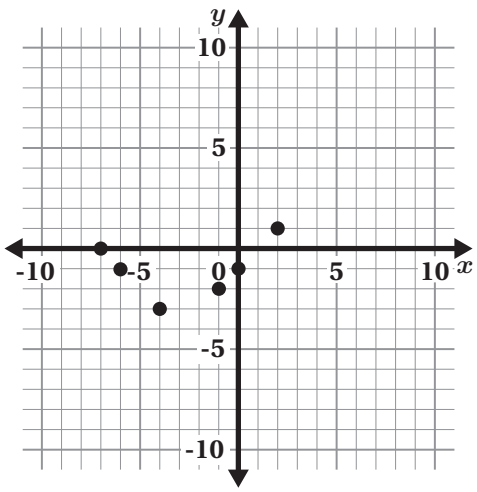


Name: ..... Date: ..... Period: .....

5. Here are some points on the graph of $h(x) = |x + 3| - 4$.

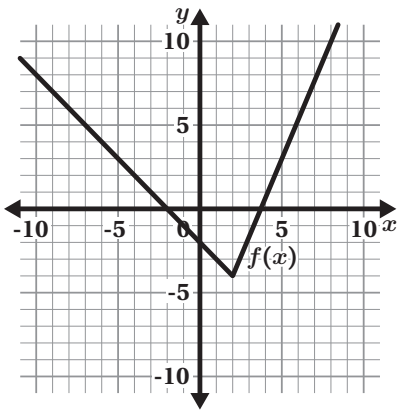
- Sketch a graph of $h(x)$.
- Describe the graph using these terms:

positive	maximum	increasing	domain
negative	minimum	decreasing	range
symmetry			



6. Here is the graph of $f(x)$.
Sketch the graph of $g(x)$ so that two of the statements below are true and one is false.

- $f(5) = g(\text{-}5)$
- $f(\text{-}5) = g(5)$
- $g(2) < f(2)$



Spiral Review

Problems 7–9: For each square root, write which two consecutive whole numbers the value is between.

7. $\sqrt{14}$

8. $\sqrt{60}$

9. $\sqrt{88}$

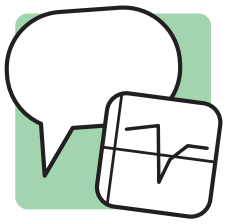
10. Write an explicit definition for the sequence, $f(n)$, given its recursive definition:
$f(1) = 10$ and $f(n) = f(n - 1) \cdot 2.5$

Name: ..... Date: ..... Period: .....

Modeling with Functions | Comparing Models | F–IF.2, F–IF.4, F–IF.5, N–Q.1, SMP.2, SMP.4, SMP.6

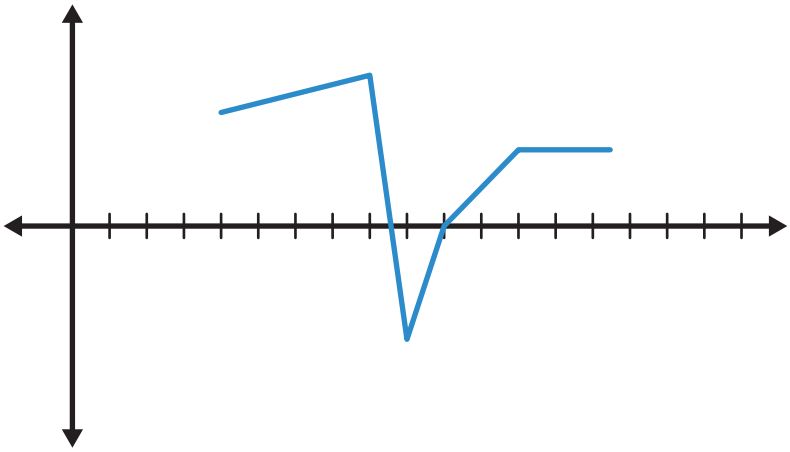
Our Math Stories

Let's use functions and graphs to model math stories.



Warm-Up

Let's look at the graph of a function. New information will be revealed each round.



1. **Discuss:** For each round, what do you notice? What do you wonder?

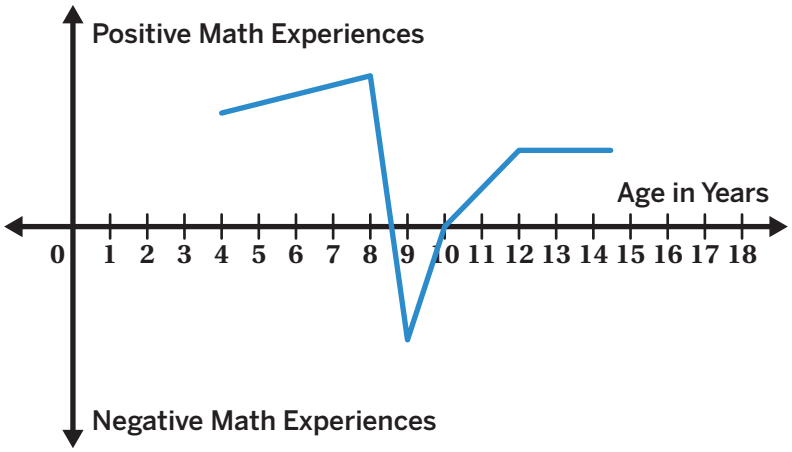
Round 1: **Round 2:** **Round 3:**

Activity 1

Name: ..... Date: ..... Period: .....

Storytime

Adrian is a 9th grade student. He graphed his math experiences, $f(x)$, as a function of his age, x.



What does each statement say about Adrian's math experiences?

2. $f(x)$ increases from $x = 9$ to $x = 12$.

3. $f(5) > f(11)$

4. The domain of $f(x)$ is $4 \leq x \leq 14.5$.

5. Circle the part of Adrian's graph that you are most curious about. Why are you most curious about that part?

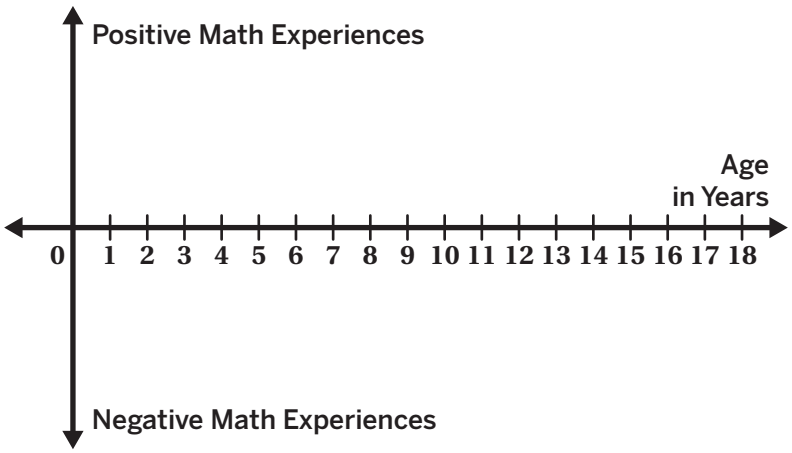
Name: ..... Date: ..... Period: .....

Storytime (continued)

Faaria is another student in Adrian's class. She shared these moments from her math story:

- My earliest math experience was when I was a toddler in daycare. We counted with our fingers.
- In 2nd grade, I was obsessed with playdough. My favorite thing to do was make smaller shapes and put them together to make more complicated pieces.
- In middle school, my experiences changed the most because my classes got harder. I had no idea what was going on. Sometimes, even showing up and taking notes was a struggle.
- In high school, math was difficult for me, but my experiences were generally positive.

6. Sketch a graph that could represent Faaria's story.



7. Use the vocabulary you've learned in this unit to compare your graph with a classmate's.

Discuss: How are your graphs the same? How are they different?

Activity 2

Name: ..... Date: ..... Period: .....

Our Math Stories

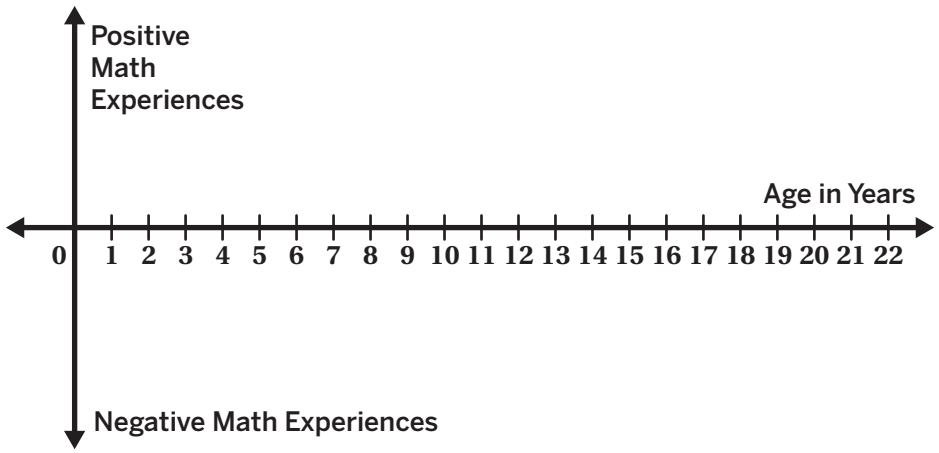
In this activity, you will create a graph of a math story and share it with a partner. You will use the Activity 2 Sheet.

8. Decide if you want to share your story (option 1) or a fictional student's story (option 2).

9. Option 1: Respond to the reflection questions on the Activity 2 Sheet.

Option 2: Read the story on the Activity 2 Sheet.

10. Create a graph of a math story.



11. Share the graph you made with a partner. Tell them whose math story the graph represents (yours, Makayla's, or Neel's).

Identify three parts on your partner's graph that are interesting to you. Ask your partner about the parts you identified. Switch roles so that each person has the opportunity to share.

12. What can you learn from listening to other people's stories?

Synthesis

13. Select *one* question and answer it below.

A. What are some ways that graphs can help you understand your math story?

B. What are some ways that graphs can help you understand someone else's math story?

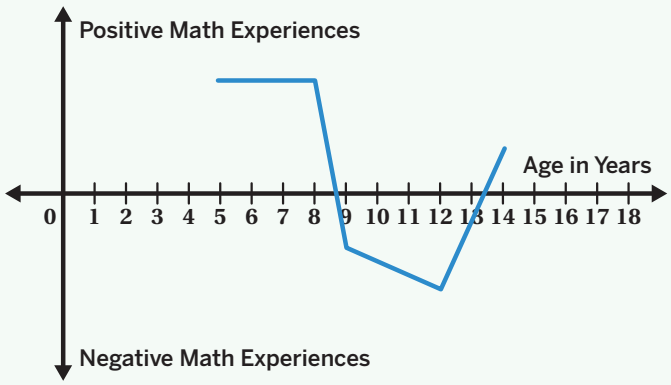
Summary 4.15

Storytelling is a powerful way to learn more about others and to reflect on your own journey. Graphing a story can help us see interesting self-discoveries and have deeper discussions.

When you use equations, tables, words, graphs, and their key features to represent real-world relationships, pay close attention to the scale and units. You can look for the maximums or minimums; intercepts; intervals where the graph increases, decreases, or remains constant; and domain and range to make sense of the situation or someone's story. Let's look at an example.

Here's a graph of someone's math experiences over time. From the graph, we can learn that:

- Most of their positive math experiences are from ages 5 to 8, and their most negative math experience was at age 12.
- Their math experiences decreased from age 8 to 12 and increased after age 12.
- This person graphed their experiences from age 5 to age 14. It's possible they drew this graph at age 14.



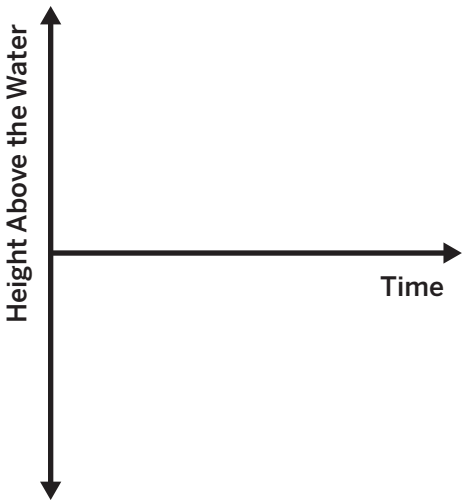
There is also a lot we can't tell from the graph of someone's story. For example, we can't tell what the positive or negative math experiences were, or what emotions they were feeling at the time. The graph gives us only a window into someone else's story, not the full image.

Practice
4.15

Name: ..... Date: ..... Period: .....

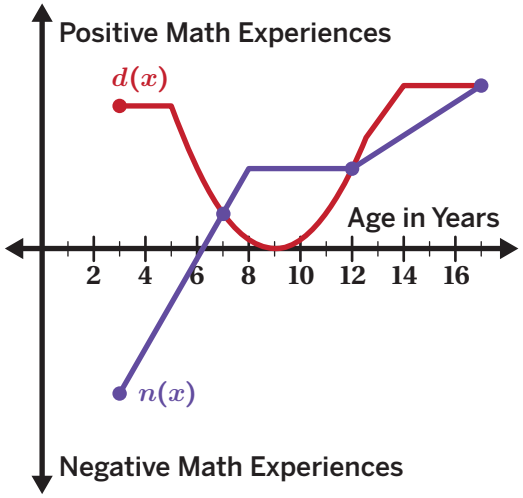
1. Sketch a graph to represent this scenario:

Pablo went swimming at the city pool. When he first arrived, he climbed to the top of the high-diving board where he dove into the deep end of the pool. He swam to the shallow end where he stayed for several minutes.



Problems 2–6: Adhira and Anand are twins. This graph represents their math stories.

$d(x)$ represents Adhira's math experience as a function of age and $n(x)$ represents Anand's math experience as a function of age.



2. What does $n(17) > n(12)$ say about Anand's math experience?

3. What does $d(3) = d(5)$ say about Adhira's math experience?

4. **Test Practice** True or false: The domain for Adhira's graph is the same as the domain for Anand's graph.

5. **Test Practice** True or false: The range for Adhira's graph is the same as the range for Anand's graph.

6. Who had a greater average rate of change between ages 3 and 17: Adhira or Anand? Explain how you know.

Name: ..... Date: ..... Period: .....

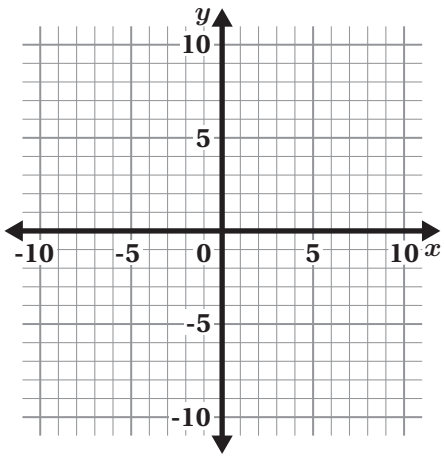
7. Create a story that could be modeled by this function.

Create a graph if it helps with your thinking.

$$f(x) = \begin{cases} 4 & 0 \le x < 8 \\ -4x + 36 & 8 \le x < 9 \\ 2x - 18 & 9 \le x \le 12 \\ 6 & 12 < x \le 15 \end{cases}$$

Spiral Review

8. This is the graph of the function $g(x) = |x - 2| + t$. Use the graph to determine the value of t.



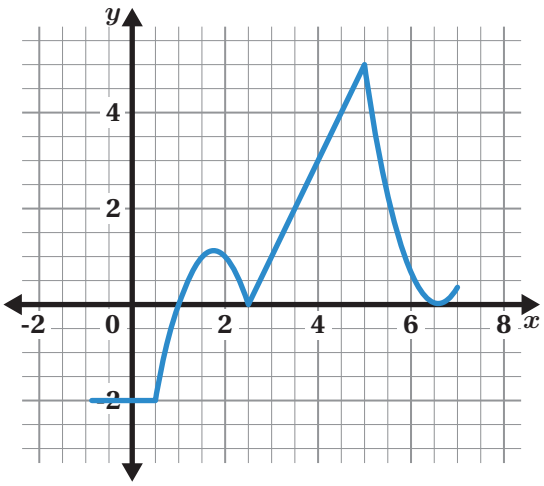
Problems 9–11: Evaluate each expression.

9. $\left(\frac{1}{2}\right)^3$

10. $\left(\frac{1}{2}\right)^{-3}$

11. $\left(\left(\frac{1}{2}\right)^3\right)^{-2}$

12. Select all the statements that are true about the function that is graphed.



A. The graph is continuously increasing.

B. The graph is increasing at a constant rate when $2.5 \le x \le 5$.

C. The range of the graph is $-2 \le f(x) \le 7$.

D. The range of the graph is $-2 \le f(x) \le 5$.

E. The maximum of the graph is at $(5, 5)$.

Name: Date: Period:

Modeling with Functions | Comparing Models | Systems of Equations | Composing Functions

F–IF.2, F–IF.4, F–IF.5, F–IF.6, F–BF.1.a, F–BF.2, F–BF.3 SMP.2, SMP.3, SMP.6

Practice Day 2

Let's practice what you've learned so far in this unit!

You will use problem cards for this Practice Day. Record all of your responses here.

Card 1

Circle the false statement: A B C

Explanation:

Card 2

Circle the false statement: A B C

Explanation:

Card 3

Circle the false statement: A B C

Explanation:

Card 4

Circle the false statement: A B C

Explanation:

Card 5

Circle the false statement: A B C

Explanation:

Name: _____ Date: _____ Period: _____

Practice Day 2 (continued)

Card 6

Circle the false statement: A B C

Explanation:

Card 7

Circle the false statement: A B C

Explanation:

Card 8

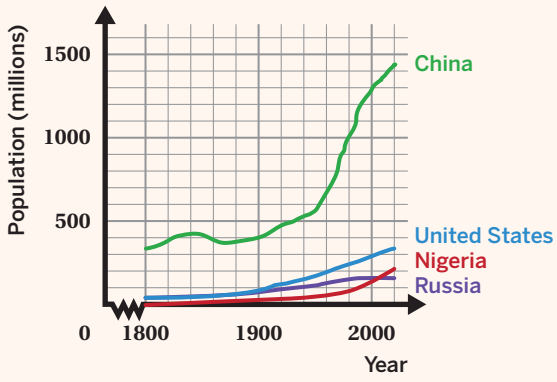
Circle the false statement: A B C

Explanation:

You're invited to explore more.

Here is a graph of the populations of China, Nigeria, Russia, and the United States.

a What do you notice about this graph?



b What are some questions you could explore based on this graph?

Career Connection

How is it possible that a friend can receive your text message from thousands of miles away?

gezzeg/Shutterstock.com

Radio waves to the rescue! A radio wave is one type of electromagnetic wave. It can travel through the air and many kinds of walls. While your cell phone might only be able to send a signal several miles away, cell towers receive and then resend signals so that your signal reaches your friend's cell phone. This happens *incredibly* fast because these waves travel at the speed of light!

Electrical engineers use math functions to design and manage electrical equipment, such as cell phones, computers, and electrical systems of automobiles and aircraft. Electrical engineers can work in a variety of areas, such as telecommunications, energy, signal processing, and more!

Walter P. Reuther Library, Archives of Labor and Urban Affairs, Wayne State University.

Meet Mary Golda Ross

Mary Golda Ross was a Cherokee mathematician and engineer of the mid 1900s. She was the first Native American woman to work in the U.S. space program as an engineer. She was also one of 40 founding engineers and the only woman working on the renowned "Skunks Works" project at the aerospace company Lockheed. Part of her work included satellite projects that set the foundation for computers and cell phones to access signals all over the world.

Are you interested in electrical engineering or cell phone technology? What can you do to learn more?

Math in the World

Cell towers can pinpoint a cell phone's location by a process called *trilateration*. Part of this process involves the function $d = rt$ that relates distance, rate, and time. Because cell phone signals travel at the speed of light, the rate r is 300,000,000 meters per second. Write this equation in function notation where distance is a function of time t.

Steve Heap/Shutterstock.com

Math Mindset

Describe a time during this unit where you used a function to model a real-world problem.