Amplify Desmos Math CALIFORNIA

Grade 7

Intervention, Extension, and Investigation Resources

Contents

Mini-Lessons

Log in to the digital curriculum to review all program Mini-Lessons, Extensions, and Investigations.

Mini-Lessons (continued)

Contents Extensions

Extensions (continued)

Investigations

Mini-Lessons

Unit 1

Mini-Lessons

Name _____

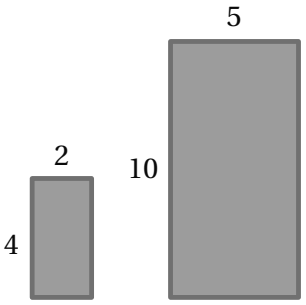
Connecting Scale Factors to Scaled Copies

ML 1.04

Modeled Review

Name: Shawn

What scale factor takes the original rectangle to its scaled copy? Show or explain your reasoning.

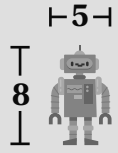
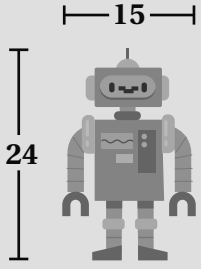
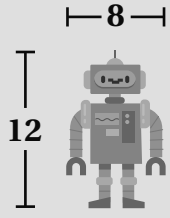
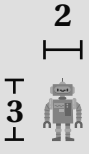


$\frac{\text{new length}}{\text{original length}}$ $\frac{5}{2}$ or $\frac{10}{4}$

$\frac{5}{2}$ is the scale factor.

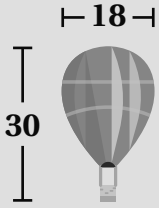
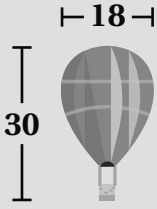
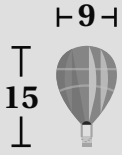
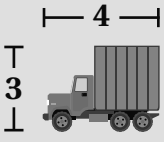
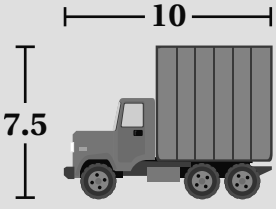
Guided Practice

1. Each original robot has a scaled copy. Complete the table by using the corresponding side lengths to determine the scale factor.

Original	Scaled copy	*Ratio* = $\frac{\textit{New Length}}{\textit{Original Length}}$	Scale factor
		$\frac{15}{5}$ or $\frac{24}{8}$	
		$\frac{\square}{\square}$ or $\frac{\square}{\square}$	

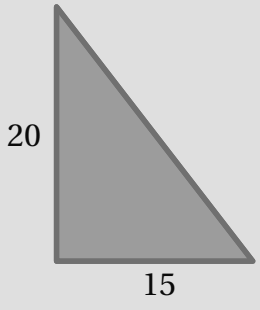
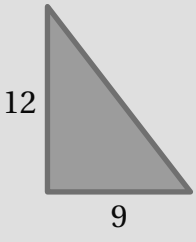
Guided Practice

2. For each pair of objects, determine the scale factor that takes the original to the scaled copy.

Original	Scaled copy	Scale factor
		
		
		

Check

What scale factor takes the original to the scaled copy?

Original	Scaled copy	Scale factor
		

Name _____

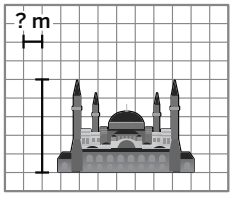
Determining Unknowns in Scale Drawings

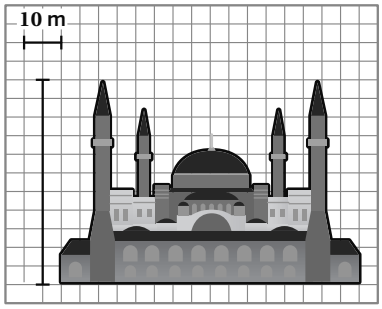
ML 1.10

Modeled Review

Name: Han

Here are two scale drawings of the same building. How many meters does the unknown scale represent? Show your thinking.





1 unit = 5 meters

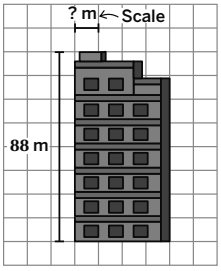
Height	Scale
$11 \cdot 5 = 55$	$\frac{55}{5} = 11$
55 meters	11 meters

The building in both the small and large drawing is 55 meters tall. The large drawing's scale is 2 units represents 10 meters, so the small drawing's scale is 1 unit represents 5 meters.

Guided Practice

Complete the scale for each building by determining the unknown measurement. Show your thinking.

1.



building height: _____ meters

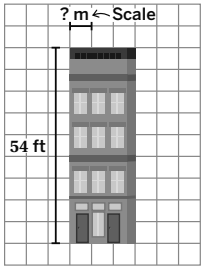
scaled drawing height: 8 units

_____ meters = _____

8 units

Scale = _____ meters

2.



building height: _____ feet

scaled drawing height: _____ units

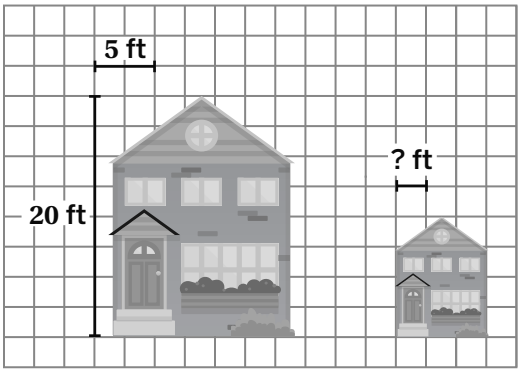
_____ = _____

Scale = _____

Guided Practice

For Problems 3-4, use the two scale drawings of the same building. How many feet does the unknown scale represent?

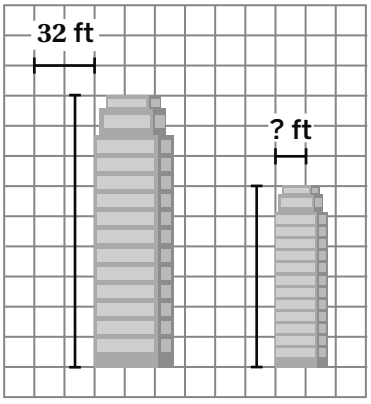
3.



unknown scale = _____ feet per unit

Height Scale

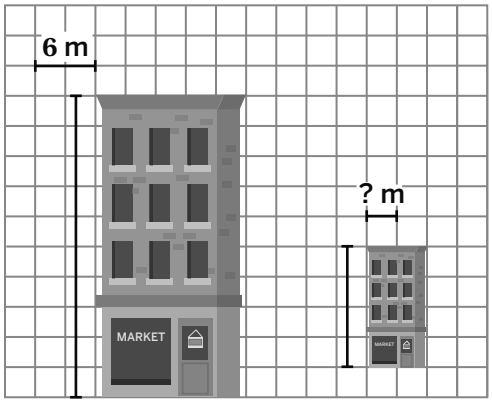
4.



unknown scale = _____

Check

Here are two scale drawings of an apartment building. How many meters does the unknown scale represent? Show your thinking.



unknown scale = _____

Unit 2

Mini-Lessons

Name _____

Determining the Constant of Proportionality

ML 2.03

Modeled Review

Name: Clare

Determine the constant of proportionality for the relationship and complete the table. What does the constant of proportionality tell you about the situation?

The constant of proportionality is 5.

The paint is made with 5 cups of yellow paint for every 1 cup of blue paint.

$2 \cdot 5 = 10$

$6 \cdot 5 = 30$

Blue paint (cups)	Yellow paint (cups)
2	10
1	5
6	30
52	260

Guided Practice

Determine the constant of proportionality for the relationship and complete the table.

1. $8 \cdot 3 = 24$

$12 \cdot \_\_\_\_ = \_\_\_\_$

$128 \cdot \_\_\_\_ = \_\_\_\_$

The constant of proportionality is _____.

Lemonade

	Volume (oz)	Sugar (g)
glass	8	24
bottle	12	
carton	32	96
jug	128	

2. $8 \cdot \_\_\_\_ = \_\_\_\_$

$12 \cdot \_\_\_\_ = \_\_\_\_$

$128 \cdot \_\_\_\_ = \_\_\_\_$

The constant of proportionality _____.

Apple juice

	Volume (oz)	Sugar (g)
glass	8	
bottle	12	
carton	32	80
jug	128	

Guided Practice

Determine the constant of proportionality for the relationship and complete the table. What does the constant of proportionality tell you about the situation?

3. The constant of proportionality is_____.

The paint is made with _____ cups of blue paint for every _____ cup of red paint.

Red paint (cups)	Blue paint (cups)
2	3
1	
6	
40	60

4. The constant of proportionality is_____.

Red paint (cups)	Yellow paint (cups)
2	8
1	
	32
50	

Check

Determine the constant of proportionality for the relationship and complete the table. What does the constant of proportionality tell you about the situation?

White paint (cups)	Red paint (cups)
2	4
1	
7	
	80

Name _____

Finding More Than One Equation to Represent Proportional Relationships

ML 2.06

Modeled Review

Name: Jason

A car traveled 150 miles in 2 hours at a constant speed.

1. What are two constants of proportionality for the relationship between distance in miles, d, and number of hours, t, that the car traveled?

$\frac{150}{2}$ and $\frac{2}{150}$

2. Write two equations that relate d and t in this situation.

$d = \frac{150}{2}t$ and $t = \frac{2}{150}d$

Guided Practice

Complete the table and determine the constant of proportionality. Write an equation to represent the situation.

1.

x	y
1	
2	10
4	
t	$5t$

constant of proportionality: _____

equation: _____

2.

x	y
1	
4	
8	
w	$\frac{1}{4}w$

constant of proportionality: _____

equation: _____

Guided Practice

There are 400 centimeters in every 4 meters.

3. Determine two constants of proportionality for the relationship between meters, x, and centimeters, y.

4. Write two equations that relate x and y in this situation.

It took 6 minutes, t, to fill a cooler with 8 gallons of water, w, at a steady rate.

5. Determine two constants of proportionality for the relationship between time, t, and gallons of water, w.

6. Write two equations that relate t and w in this situation.

Check

A polar bear swam 426 miles across the Beaufort Sea, north of Alaska, in 9 days.

1. Determine two constants of proportionality for the relationship between distance in miles, d, and number of days, t, that the polar bear swam.

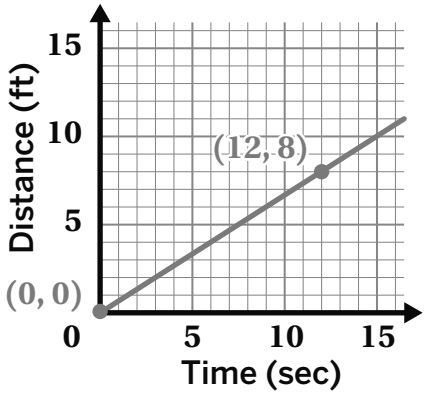
2. Write two equations that relate d and t in this situation.

Name _____

Comparing Proportional Relationships With Graphs

ML 2.10

Modeled Review



Name: Gabriela

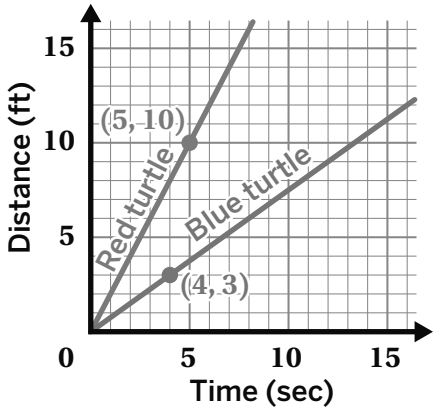
This line represents a proportional relationship because it begins at the origin. Its constant of proportionality is $\frac{8}{12}$ or $\frac{2}{3}$. The equation for the line is $y = \frac{2}{3}x$.

Guided Practice

1. A red turtle moves 10 feet in 5 seconds.
Its speed is 2 ft/sec. Equation: $y = 2x$
A blue turtle moves _____ feet in _____ seconds.
Its speed is $\frac{3}{4}$ ft/sec. Equation: $y = \frac{3}{4}x$
Which turtle moves *faster* ? How do you know?

_____

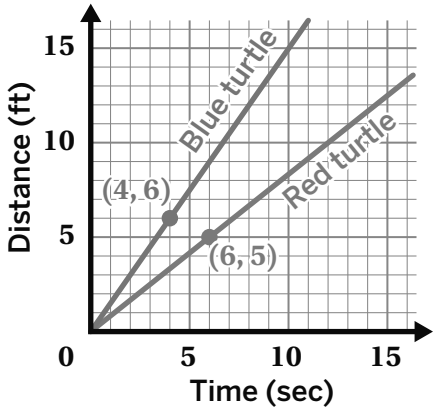
_____



2. A red turtle moves _____ feet in _____ seconds.
Its speed is _____ ft/sec. Equation: $y = \frac{5}{6}x$
A blue turtle moves _____ feet in _____ seconds.
Its speed is _____ ft/sec. Equation: $y = \frac{3}{2}x$
Which turtle moves slower ? How do you know?

_____

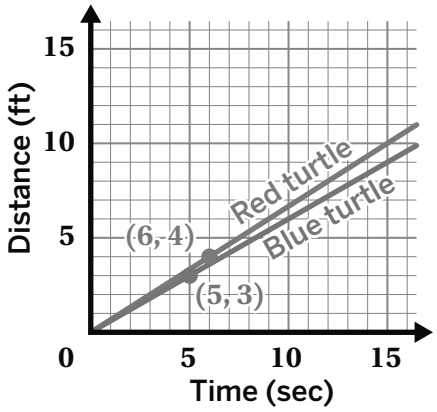
_____



Guided Practice

3. A red turtle moves _____ feet in _____ seconds.
Its speed is _____ ft/sec. Equation: _____
A blue turtle moves _____ feet in _____ seconds.
Its speed is _____ ft/sec. Equation: _____
Which turtle moves *slower* ? How do you know?

_____
_____

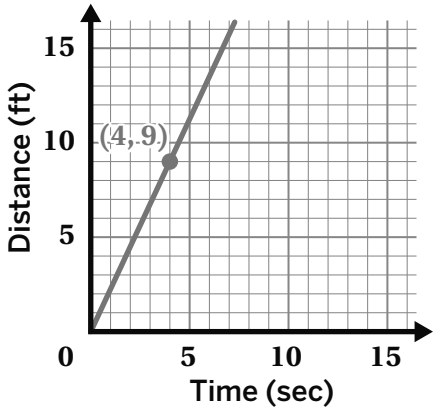


4. A red turtle moves according to the equation $y = 2x$, where y represents the distance in feet and x represents the time in seconds.

Here is a graph that shows the distance a blue turtle moves over time.

Which turtle moves *faster*? How do you know?

_____
_____
_____



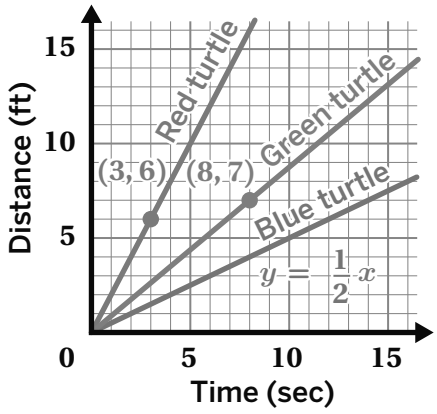
Check

Here is a graph that shows the distance three turtles move over time.

1. Which turtle moves the *fastest*? How do you know?

_____
_____

2. Write an equation for the turtle represented by the green line.



Name _____

Representing Proportional Relationships

ML 2.11

Modeled Review

Name: Priya

Here are four representations of a proportional relationship. Circle or show anywhere you can see the constant of proportionality.

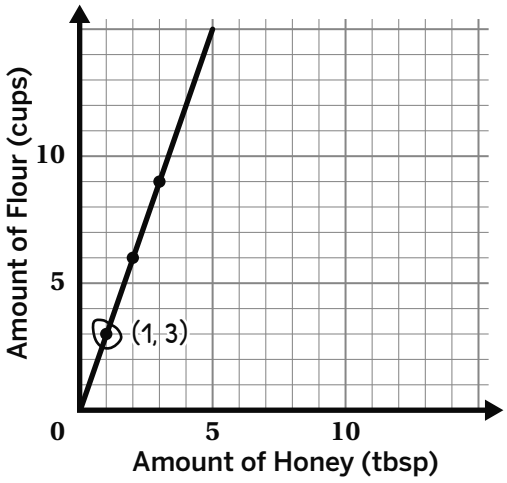
A baker uses 2 tablespoons of honey for every 6 cups of flour to make bread.

$k = \frac{y}{x}$ $k = \frac{6}{2}$

$f = 3h$

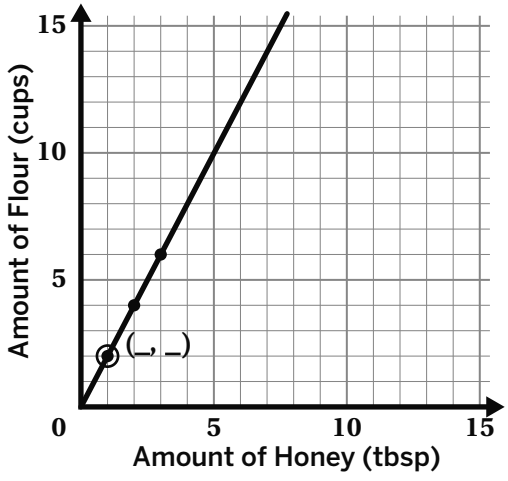
honey (tbsp), h	flour (cups), f
0	0
1	3
2	6

The honey represents the x-values and the flour represents the y-values.



Guided Practice

1. Analyze the graph and description of a proportional relationship to identify the constant of proportionality for each relationship.



A bakery uses 1 tablespoon of honey for every 2 cups of flour to make scones.

$k = \frac{2}{\square}$

$k =$

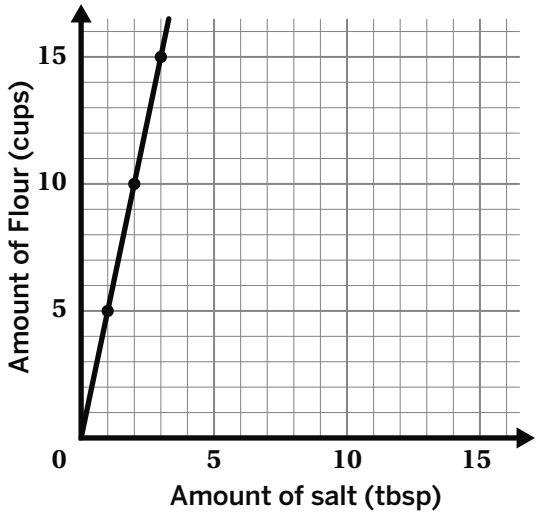
Guided Practice

2. Here are four representations of a proportional relationship. Circle or show where you can see the constant of proportionality in each representation.

A baker uses 3 tablespoons of salt for every 15 cups of flour to make pancakes.

$f = 5\ s$

salt (tbsp), s	flour (cups), f
0	0
1	5
2	10
3	15



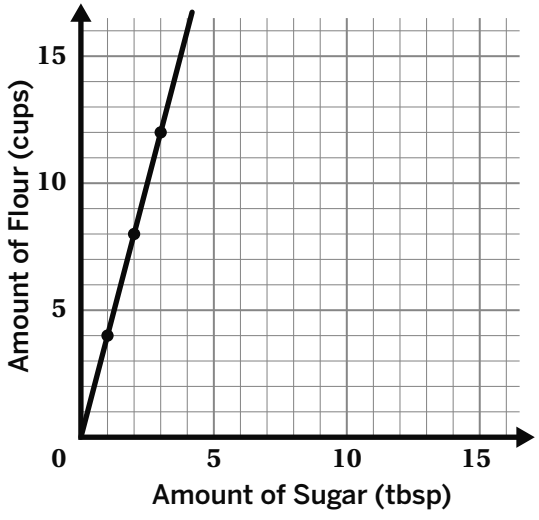
Check

Here are four representations of a proportional relationship. Circle or show anywhere you can see the constant of proportionality.

A baker uses 3 tablespoons of sugar for every 12 cups of flour to make muffins.

$f = 4\ s$

sugar (tbsp), s	flour (cups), f
0	0
1	4
2	8
3	12



Unit 3

Mini-Lessons

Name _____

Determining the Circumference of a Circle

ML 3.03

Modeled Review

Name: Shawn

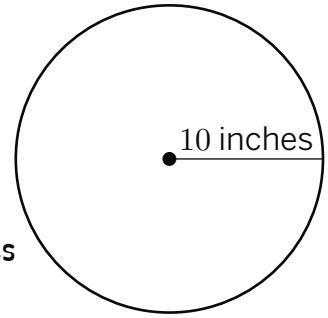
What is the approximate circumference of this circle? Show your thinking.

$C = 2\pi r$

$C = 2\pi 10$

$C = 20\pi$

Circumference: 20π inches



The radius is given because it is the distance from the center to the edge.

Guided Practice

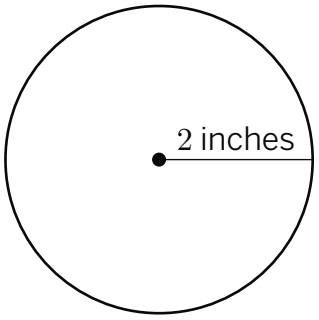
Determine the circumference using $C = 2\pi r$ or $C = \pi d$. Show your thinking.

1. $C = 2\pi r$ radius = 2

$C = 2\pi$ _____

$C =$ _____

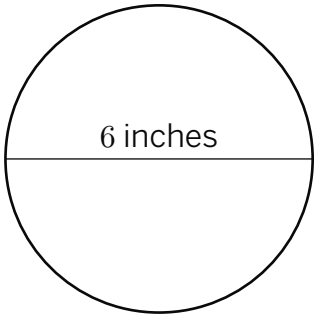
$C =$ _____ inches



2. $C = \pi d$ diameter = _____

$C =$ _____

$C =$ _____ inches



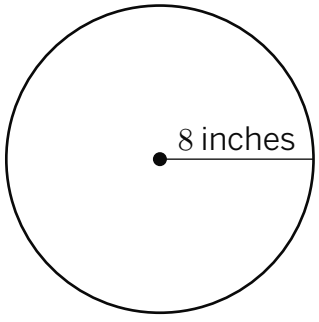
Guided Practice

3. Use the given information to determine the circumference of each object.

Object	Radius (cm)	Diameter (cm)	Circumference (cm)
Magnifying glass	5	10	
Lid	4		
Salad Plate		14	
Hand Mirror	6		
Clock		18	

Check

What is the approximate circumference of this circle?



Name _____

Calculating Areas of Complex Shapes

ML 3.09

Modeled Review

Name: Kai

What is the area of this shape?

Area of circle = πr^2

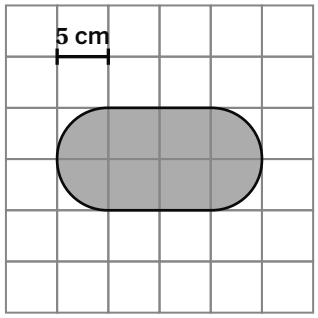
$\pi 5^2$

25π

Area of square = lw

10 × 10

100

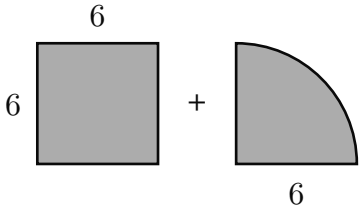


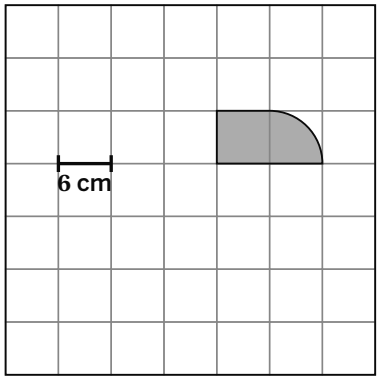
The shape is made up of a square and two semi-circles. Two semi-circles make one circle.

Area of the shape = 100 + 25π square centimeters

Guided Practice

1. Calculate the area of the shape.





Area of square = _____ square centimeters

Area of a quarter circle = $\frac{\pi r^2}{4}$

= _____ square centimeters

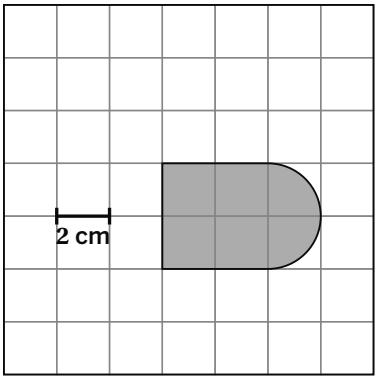
Area of shape = ___+___ square centimeters

Guided Practice

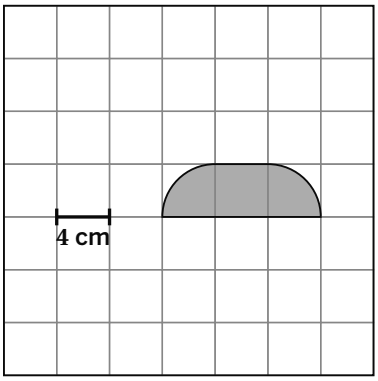
Calculate the area of the given shape.

Area of square = lw	Area of semi-circle = $\frac{\pi r^2}{2}$
Area of circle = πr^2	Area of quarter-circle = $\frac{\pi r^2}{4}$

2.



3.



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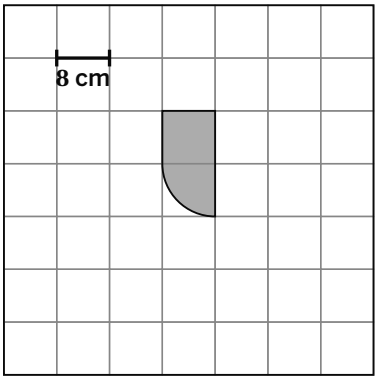

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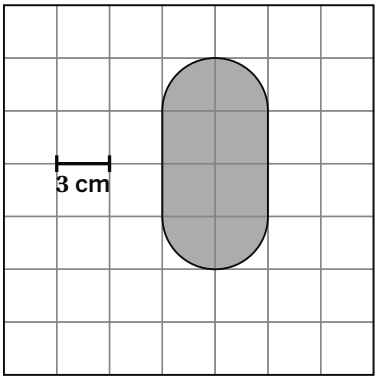
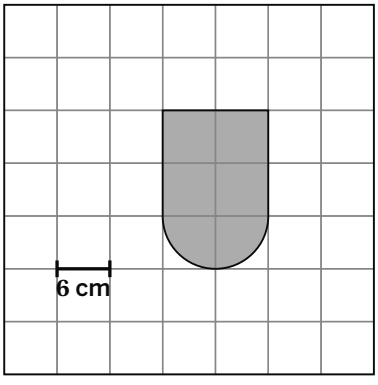


Check

Calculate the area of the shape.



Unit 4

Mini-Lessons

Name _____

Calculating Percentages

ML 4.01

Modeled Review

Name: Dylan

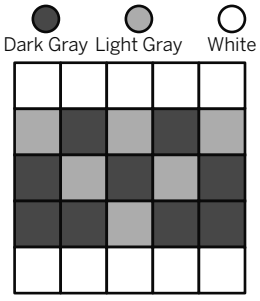
What percentage of the larger grid is white? Explain your thinking.

I counted 25 squares. Each square has a value of 4% because 100 divided by 25 is 4. There are 10 white squares. I can multiply 4 by 10 to get 40%.

$$\frac{100}{25} = 4\%$$

$$4 \cdot 10 = 40$$

$$40\%$$



Guided Practice

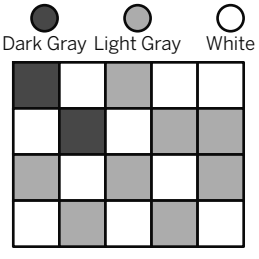
1. What percentage of the grid is light gray or white? Show your thinking.

Light Gray:

$\frac{100}{20} =$ _____

White:

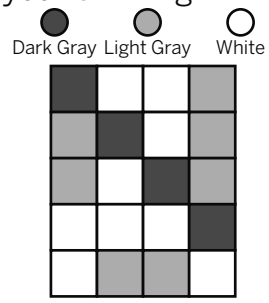
Color	Percentage
Dark Gray	10%
Light Gray	
White	



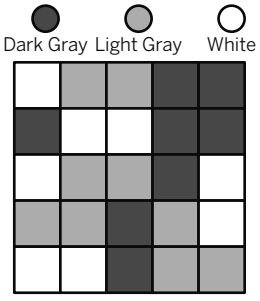
Guided Practice

Determine the percentages of the grids that are shaded each color.

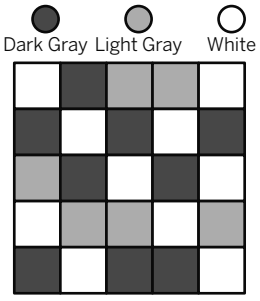
2. What percentage of the grid is dark gray? Show your thinking.



3. What percentage of the grid is white? Show your thinking.

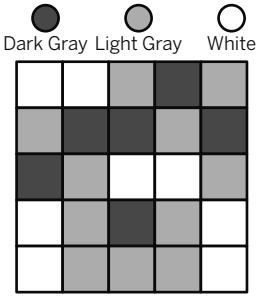


4. What percentage of the grid is light gray? Show your thinking.



Check

What percentage of the grid is white? Show your thinking.



Name _____

Finding Sales Tax and Tip

ML 4.07

Modeled Review

Name: Eva

Determine the missing values for the receipt.

Clare's meal costs $30.00 before tax.
There is a 7% sales tax.
What is the total after tax?

Original Cost	$30.00
7% Tax:	$2.10
Total:	$32.10

7% is $\frac{7}{100}$ or 0.07.

30 • 0.07 = 2.10
30 + 2.10 = 32.10

Guided Practice

Determine the missing values for these receipts.

1.

Smoothie:	$5.00
10% Tax:	$
Total:	$

5.00 • 0.10 = _____

5.00 + _____ = _____

2.

Meal:	$20.00
20% Tip:	$
Total:	$

20.00 • 0.20 = _____

20.00 + _____ = _____

Guided Practice

Determine the missing values for these receipts.

3. Han's meal costs $20.00 before tax.
There is a 7% sales tax.
What is the total after tax?

Original Cost	$20.00
7% Tax:	$
Total:	$

4. Tristan's meal costs $30.00 before tip.
Tristan tips 20%.
What is the total after tip?

Original Cost	$30.00
20% Tip:	$
Total:	$

5. Diego's meal cost $50.00 before tax.
There is a 5% sales tax.
What is the total after tax?

Original Cost	$50.00
5% Tax:	$
Total:	$

Check

Determine the missing values for the receipt.

Priya's meal costs $40.00 before tax.
There is an 8% sales tax.
What is the total after tax?

Original Cost	$40.00
8% Tax:	$
Total:	$

Name _____

Applying Fractions in Proportional Relationships

ML 4.12

Modeled Review

Name: Maya

Aba and Esteban each make peach cobbler. Whose peach cobbler has more peaches per serving? Show your thinking.

Aba	Esteban
To make 6 servings, use $\frac{3}{4}$ cups of peaches	To make 4 servings, use $\frac{1}{3}$ cups of peaches

Aba	Esteban
$\frac{3}{4} \div 6 = \frac{1}{8}$	$\frac{1}{3} \div 4 = \frac{1}{12}$
$\frac{1}{8}$ peaches per serving	$\frac{1}{12}$ peaches per serving

$\frac{1}{8}$ is greater than $\frac{1}{12}$ so Aba's recipe has more peaches per serving.

Aba's recipe has more peaches per serving.

Guided Practice

1. Han is making Recipe A for his family. The recipe makes 2 servings but he only wants to make 1 serving. Determine how much of each ingredient he needs for 1 serving. Show your thinking.

Strawberries: $3 \div 2 =$ ____

Sugar: $\frac{1}{2} \div$ ____ $=$ ____

Flour: _____

Recipe A Number of servings: 2	Adjusted Recipe A Number of servings: 1
3 cups of strawberries	____ cup of strawberries
$\frac{1}{2}$ cup of sugar	____ cup of flour
$\frac{1}{4}$ cup of flour	____ cup of flour

Guided Practice

Use the constant of proportionality to compare two proportional relationships. Show your thinking.

2. Diego and Dylan each make blackberry cobbler. Whose cobbler has more blackberries per serving?

Diego	Dylan
To make 6 servings, use 2 cups of blackberries	To make 4 servings, use $\frac{1}{2}$ cups of blackberries

3. Santiago and Tristan each make peach cobbler. Whose cobbler has more peaches per serving?

Santiago	Tristan
To make 2 servings, use $\frac{1}{4}$ cups of peaches	To make 3 servings, use $\frac{3}{4}$ cups of peaches

Check

Eva and Maya each make raspberry cobbler. Whose cobbler has more raspberries per serving? Show your thinking.

Eva	Maya
To make 3 servings, use $\frac{3}{4}$ cups of raspberries	To make 5 servings, use $\frac{1}{3}$ cups of raspberries

Unit 5

Mini-Lessons

Name _____

Adding and Subtracting Positive and Negative Numbers

ML 5.02

Modeled Review

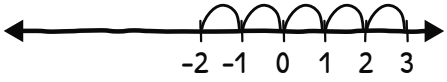
Name: Tristan

Evaluate each expression.

1. -5 + (-2) -7

I need to start at -5, and move left 2 times to represent adding -2.

2. -2 – (-5) 3

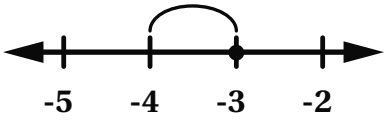


I need to start at -2, and move right 5 times.

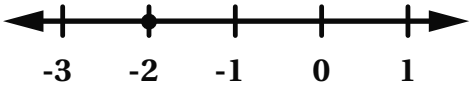
Guided Practice

Evaluate each expression using a number line.

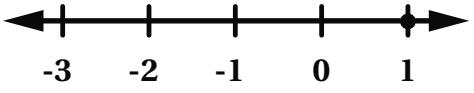
1. -3 – 2 ____



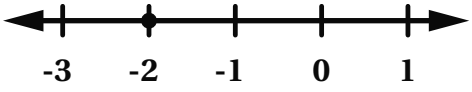
2. -2 + 3 ____



3. 1 – 3 ____



4. -2 – (-1) ____



Guided Practice

5. Evaluate each expression.

Addition Expression	Subtraction Expression
$-4 + 1$ ____	$4 - 1$ ____
$2 + (-3)$ ____	$3 - (-2)$ ____
$-1 + 5$ ____	$-5 - 1$ ____
$-3 + -3$ ____	$-3 - (-3)$ ____
$-2 + 5$ ____	$5 - (-2)$ ____

Check

Evaluate each expression.

Addition Expression	Subtraction Expression
$-4 + (-2)$ ____	$-2 - (-4)$ ____

Name _____

Dividing Integers

ML 5.08

Modeled Review

Name: Han

Select the two expressions that have the same value.

A. $\frac{-20}{4}$ = -5

B. $\frac{-20}{-4}$ = 5

C. $\frac{20}{-4}$ = -5

When dividing integers, like signs result in a positive quotient. Unlike signs result in a negative quotient.

Guided Practice

1. Evaluate each expression. Circle *all* the expressions with a negative quotient.

A. $\frac{-6}{2}$ **B.** $\frac{6}{-2}$ **C.** $\frac{-6}{-2}$

2. Evaluate each expression. Circle *all* the expressions with a negative quotient.

A. $\frac{-12}{3}$ **B.** $\frac{-12}{-3}$ **C.** $\frac{12}{-3}$

3. Evaluate each expression. Circle *all* the expressions with a negative quotient.

A. $\frac{-15}{5}$ **B.** $\frac{15}{-5}$ **C.** $\frac{-15}{-5}$

Guided Practice

Calculate the quotients of the expressions.

4. $\frac{10}{-2}$

5. $\frac{-12}{-6}$

6. $\frac{-20}{10}$

7. $\frac{36}{-6}$

8. $\frac{-18}{-9}$

9. $\frac{-16}{4}$

10. $\frac{-24}{-6}$

11. $\frac{-9}{-3}$

Check

Select the two expressions that have the same value.

A. $\frac{30}{-5}$

B. $\frac{-30}{-5}$

C. $\frac{-30}{5}$

Name _____

Solving Real-World Problems Involving Positive and Negative Numbers

ML 5.11

Modeled Review

Name: Santiago

Use the table to determine the change.

Date	Temperature in Kazan, Russia (°F)
December 20, 2019	-11
July 27, 2020	60

Positive change

What is the change in Kazan's temperature from December 20th to July 27th?

60 - (-11) = 60 + 11 = 71
71 degrees Fahrenheit

Since I am calculating the change, I need to find the difference between the final temperature and the initial temperature.

Guided Practice

Use the tables to determine if each temperature change in Fitzgerald, Canada is positive or negative.

1.

Date	Temperature (°F)
January 29, 2020	10
May 30, 2020	50

Positive Change or Negative Change

2.

Date	Temperature (°F)
June 24, 2019	60
February 9, 2020	-5

Positive Change or Negative Change

3.

Date	Temperature (°F)
December 19, 2019	-4
January 28, 2020	-8

Positive Change or Negative Change

4.

Date	Temperature (°F)
December 14, 2019	-2
July 8, 2020	70

Positive Change or Negative Change

Guided Practice

Use the tables to determine the change in temperature.

5.

Date	Temperature in Siberia, Russia (°F)
July 24, 2019	50
February 11, 2020	-10

What is the change in Siberia's temperature from July 24th to February 11th?

6.

Date	Temperature in Fairbanks, Alaska (°F)
January 2, 2020	-5
September 25, 2020	45

What is the change in Fairbanks's temperature from January 2nd to September 25th?

7.

Date	Temperature in Harbin, China (°F)
December 13, 2019	-8
August 20, 2020	68

What is the change in Harbin's temperature from December 13th to August 20th?

Check

Use the table to determine the temperature change.

Date	Temperature in Prospect Creek, Alaska (°F)
February 26, 2020	-7
August 9, 2020	63

What is the change in Prospect Creek's temperature from February 26th to August 9th?

Unit 6

Mini-Lessons

Name _____

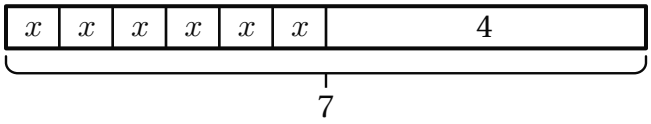
Writing Equations From Descriptions and Tape Diagrams

ML 6.03

Modeled Review

Name: Priya

Ella ran 6 times around her school building. Then she ran 4 miles home. Her phone told her that she ran 7 miles total.



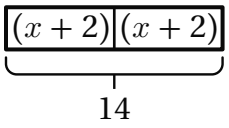
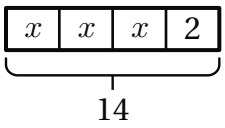
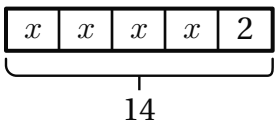
x appeared six times while 4 appeared only once. The sum is represented by 7, so the equation can be written as 6x + 4 = 7.

Write an equation that represents this situation.

$6x + 4 = 7$

Guided Practice

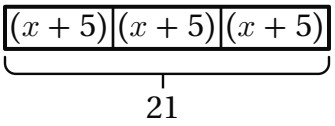
1. Match each equation to the correct tape diagram on the left and description on the right.

Tape diagram	Equation	Description
	$3x + 2 = 14$	Caleb collected 14 seashells at the beach, saving 2 for his collection and sharing the rest equally among his 3 teammates.
	$2(x + 2) = 14$	Fatima gathered 14 marbles and set aside 2 for herself before splitting the remainder equally among her 4 cousins.
	$4x + 2 = 14$	Felipe bought 2 books, one for each of his siblings to read. They each spent $2 on bookmarks. The total bill was $14.

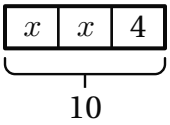
Guided Practice

Write an equation that matches the description and tape diagram.

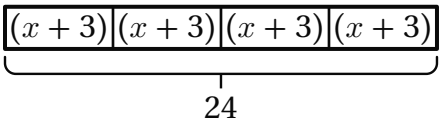
2. Ava ordered 3 slices of pizza, one for each friend at the sleepover. Each guest contributed \$5 for dessert. The total bill was \$21.



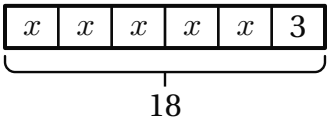
3. Mia baked 10 cupcakes and reserved 4 for herself before sharing the remaining treats equally among her 2 siblings.



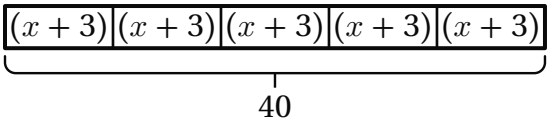
4. Maya purchased 4 tickets for a movie night, one for each family member. Afterward, each person bought a snack for \$3. The total bill was \$24.



5. Emma bought 18 stickers and decided to keep 3 for her collection before distributing the remaining stickers equally among her 5 cousins.



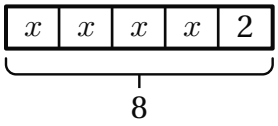
6. Alexis bought 5 servings of ice cream, one for each friend at the game. Each person chipped in \$3 for toppings. The total bill was \$40.



Check

Write an equation that matches the description and tape diagram.

Esteban walked 4 times around the park. Then he walked 2 miles to the school. His watch told him that he walked 8 miles total.



Name _____

Solving Equations With Positive and Negative Numbers

ML 6.07

Modeled Review

Name: Jack

Solve the equation. Show your thinking.

$-4(x-2)=12$

$$\frac{-4(x-2)}{-4}=\frac{12}{-4}$$

$$x-2=-3$$

$$+2 \quad +2$$

$$x=-1$$

Check:

$$-4(x-2)=12$$

$$-4(-1-2)=12$$

$$-4(-3)=12$$

$$12=12$$

Guided Practice

1. Solve the equations by completing the blanks in the equations and descriptions.

Equation	Moves
$-2x+3=7$ $-2x=$ ____ $x=$ ____	**Step 1:** Subtract ____ from each side. **Step 2:** Divide each side by ____.
$2(x+1)=6$ $x+1=$ ____ $x=$ ____	**Step 1:** _____ each side by ____. **Step 2:** _____ from each side.

Guided Practice

For Problems 2–5, solve the equation. Show your thinking.

2. $-4x - 2 = 10$

3. $3(x - 2) = 9$

4. $-3x + 5 = -1$

5. $-4(x + 3) = 20$

Check

Solve each equation. Show your thinking.

1. $-4x - 3 = 13$

2. $-3(x - 4) = 27$

Name _____

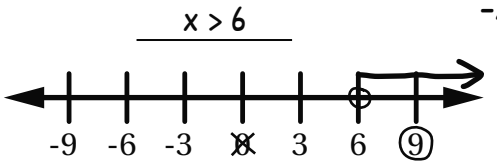
Solving Inequalities

ML 6.16

Modeled Review

Name: Priya

Solve and graph the solution to $-2x + 9 < -3$.



$-2x + 9 = -3$

$-9 \quad -9$

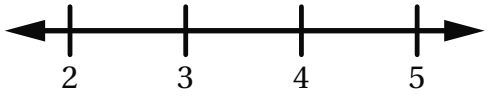
$\frac{-2x}{-2} = \frac{-12}{-2}$

$x = 6$

$x < 6$	$x > 6$
$-2(0) + 9 < -3$	$-2(9) + 9 < -3$
$0 + 9 < -3$	$-18 + 9 < -3$
$9 < -3$	$-9 < -3$
False	True

Guided Practice

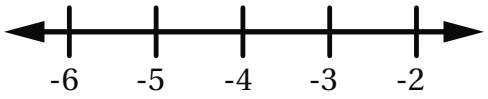
1. Graph the solution to the inequality $3x \geq 9$ by finding the boundary point and testing values on both sides of the boundary point.

Moves	Work
Move 1: Find the boundary point.	$3x = 9$ $x =$ ____
Move 2: Test the values on both sides of the point and determine whether the statement is true or false.	**Less than or equal to** $3(\_\_\_\_) \geq 9$ $\_\_\_\_ \geq 9$ _____ **Greater than or equal to** $3(\_\_\_\_) \geq 9$ $\_\_\_\_ \geq 9$ _____
Move 3: Graph the solution.	x _____ 3  2 3 4 5

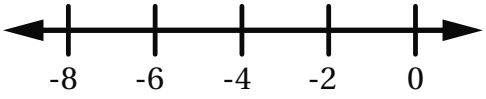
Guided Practice

Solve and graph the solution to each of the inequalities.

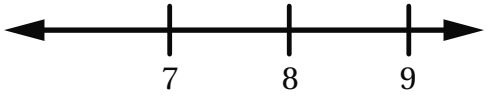
2. $2x + 7 \leq -1$ _____



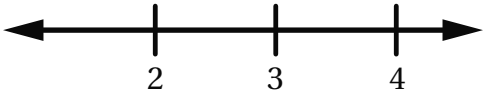
3. $-3x - 3 < 15$ _____



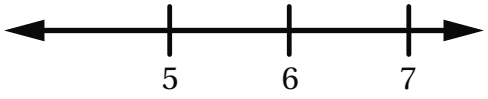
4. $2x - 4 \geq 10$ _____



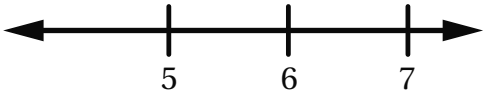
5. $-6x + 6 > -12$ _____



6. $4x - 5 \geq 15$ _____



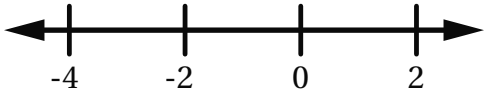
7. $-5x + 6 > -24$ _____



Check

Solve and graph the solution to $-4x - 3 < 5$.

_____



Unit 7

Mini-Lessons

Name _____

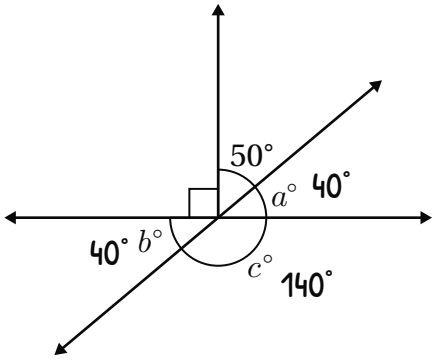
Determining Unknown Angle Measures

ML 7.04

Modeled Review

Name: Gabriel

Determine the values of a, b, and c.



$a + 50 = 90$
$-50 \quad -50$
$a = 40$

$a = b$
$b = 40$

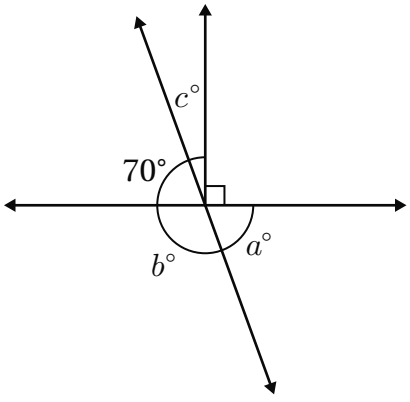
$c + a = 180$
$c + 40 = 180$
$-40 \quad -40$
$c = 140$

$a = 40, b = 40, c = 140$

Guided Practice

1. Fill in the missing values and solve the equations to determine the values of a, b, and c.

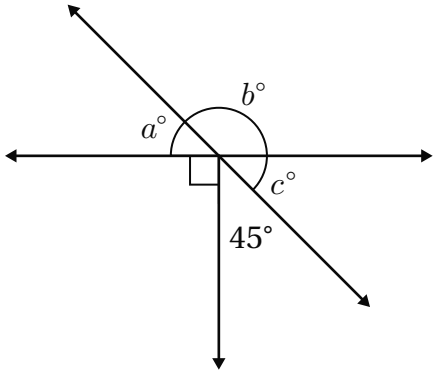
Equation	Relationship
$a = 70$	vertical angles
$c +$ _____ $= 90$	complementary angles
$b +$ _____ $= 180$	supplementary angles



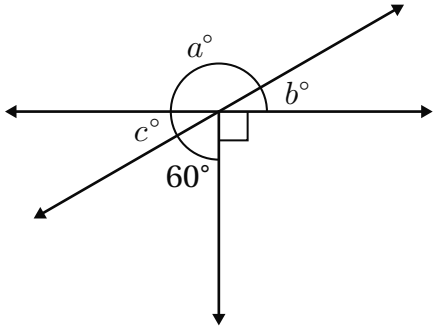
Guided Practice

Determine the values of a, b, and c for each diagram.

2.

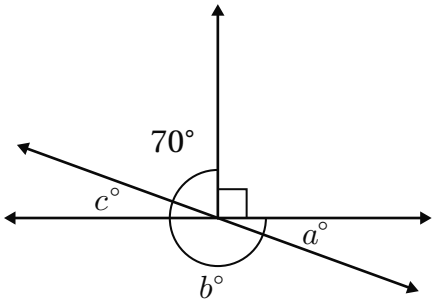


3.



Check

Determine the values of a, b, and c.



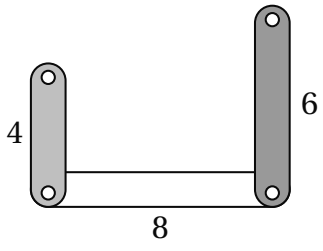
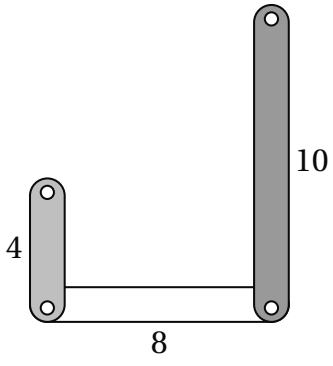
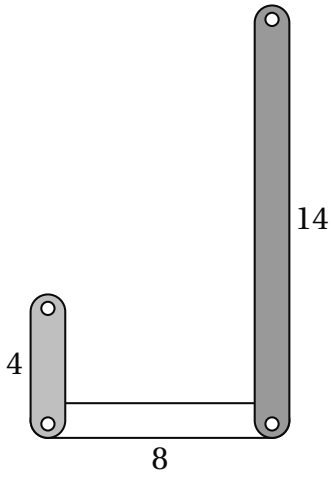
Name _____

Determining If Three Segments Form a Triangle

ML 7.05

Modeled Review

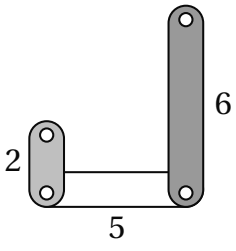
A triangle can be formed if the two shorter segments added together are longer than the third segment.

Triangle	Not a triangle
 	

Guided Practice

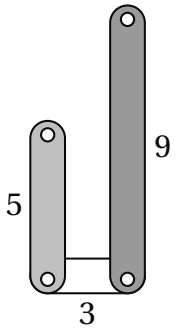
Determine whether or not the three segments will form a triangle.

1.



Yes No

2.



Yes No

Guided Practice

3. Select *all* the groups of segments that will form a triangle.

- ☐ 2, 3, 7
- ☐ 4, 2, 10
- ☐ 7, 10, 14
- ☐ 13, 8, 8
- ☐ 3, 4, 8

4. Select *all* the groups of segments that will form a triangle.

- ☐ 6, 5, 4
- ☐ 4, 7, 4
- ☐ 12, 5, 1
- ☐ 1, 8 ,6
- ☐ 6, 3, 2

5. Select *all* the groups of segments that will form a triangle.

- ☐ 9, 10, 19
- ☐ 14, 9, 10
- ☐ 12, 4, 6
- ☐ 7, 10, 16
- ☐ 8, 5, 14

6. Select *all* the groups of segments that will form a triangle.

- ☐ 11, 5, 4
- ☐ 4, 14, 6
- ☐ 16, 8, 19
- ☐ 3, 6, 3
- ☐ 9, 5, 12

Check

Select *all* the groups of segments that will form a triangle.

- ☐ 10, 5, 8
- ☐ 7, 4, 6
- ☐ 12, 5, 6
- ☐ 1, 6, 8
- ☐ 6, 2, 3

Name _____

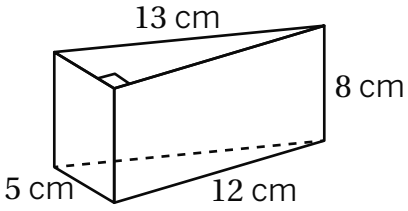
Calculating the Surface Area of Prisms

ML 7.12

Modeled Review

Name: Fatima

Calculate the surface area of this prism.



$$A = \frac{l \cdot w}{2} = \frac{12 \cdot 5}{2} = \frac{60}{2} = 30$$

There are two triangle faces and three rectangular faces.

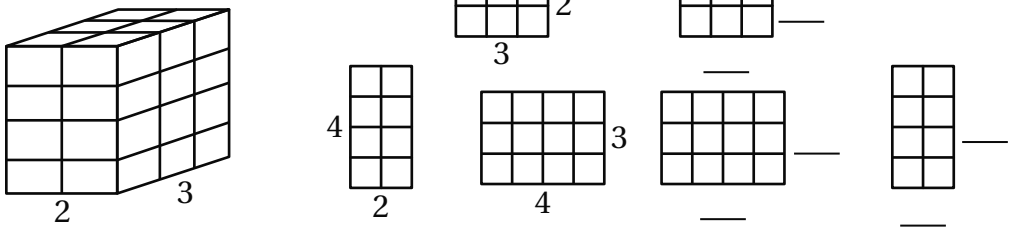
$A = 5 \cdot 8 = 40$
$A = 12 \cdot 8 = 96$
$A = 13 \cdot 8 = 104$

$30 + 30 + 40 + 96 + 104 = 300$

Surface area: 300 square centimeters

Guided Practice

1. Calculate the surface area of the prism.



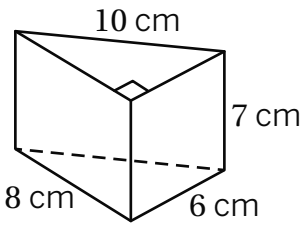
_____ + _____ + _____ + _____ + _____ + _____ = _____

Surface area: _____ square units

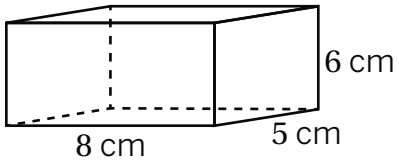
Guided Practice

Calculate the surface area of each prism.

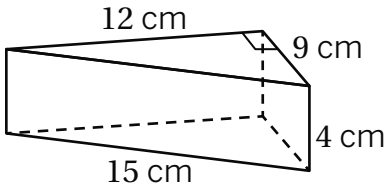
2.



3.

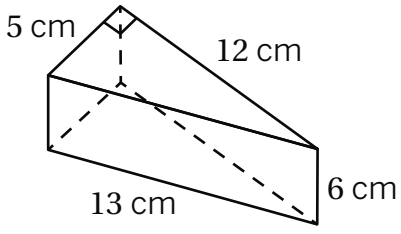


4.



Check

Calculate the surface area of the prism.



Unit 8

Mini-Lessons

Name _____

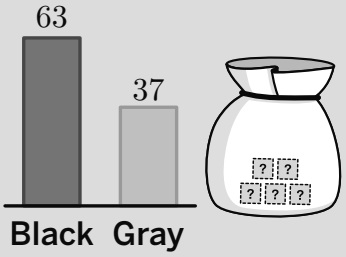
Predicting Sample Spaces Using Proportional Relationships

ML 8.03

Modeled Review

100 blocks were picked one at a time, and the results are shown in the histogram.

A new bag has 5 blocks. Some blocks are black and the others are gray. Based on the results of the histogram, how many blocks do you think are black?



Amir's work

$\frac{63}{100} = \frac{x}{5}$

$5 \cdot 0.63 = \frac{x}{5} \cdot 5$

$3.15 = x$

$x \cong 3$

3 blocks

Axel's work

$\frac{63}{100} \cong 60\%$

$x \cong 0.60 \cdot 5$

$x \cong 3$

3 blocks

Guided Practice

10 blocks were picked one at a time, and the results are shown in the histogram.

1. A new bag has 4 blocks. Some are black and some are gray. Based on the results of the histogram, how many blocks do you think are gray?

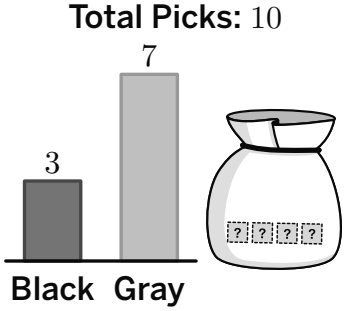
$\frac{\_\_\_\_}{10} = \_\_\_\_\ \%$

$x = \_\_\_\_ \cdot 4$

$x = \_\_\_\_$

$x \cong \_\_\_\_$

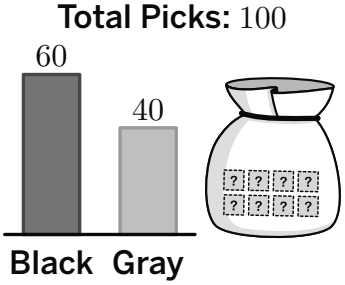
_____ blocks



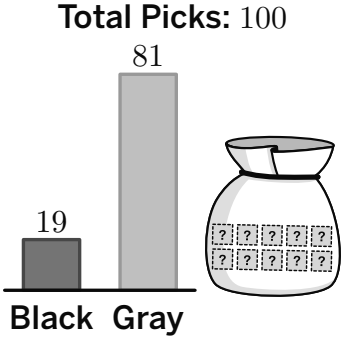
Guided Practice

100 blocks were picked one at a time, and the results are shown in the histogram.

2. A new bag has 8 blocks. Some are black and some are gray. Based on the results of the histogram, how many blocks do you think are black?



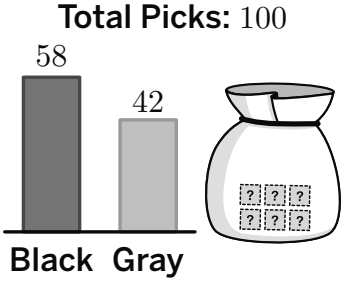
3. The bag has 10 blocks. Some are black and some are gray. Based on the results of the histogram, how many blocks do you think are gray?



Check

100 blocks were picked one at a time, and the results are shown in the histogram.

A new bag has 6 blocks. Some blocks are black and the others are gray. Based on the results of the histogram, how many blocks do you think are black?



Name _____

Predicting Population Using Sample Data

ML 8.12

Modeled Review

Name: Caleb

20 random students from Median Middle School were asked what superpower they wanted. The results are shown in the table. Median Middle School has 500 students. Estimate the total number of students who prefer teleportation.

$$\frac{4}{20} = \frac{x}{500}$$

$$500 \cdot 0.2 = \frac{x}{500} \cdot 500$$

$$100 = x$$

100 students

Superpower	Number of students
Teleportation	4
Flight	3
Super strength	3
Time travel	5
Invisibility	5

Guided Practice

1. 10 random 7th grade students from Birchwood Middle School were asked about their favorite type of pet. The results are shown in the table. Birchwood Middle School has 50 students in 7th grade. Estimate the total number of students who prefer fish.

$$\frac{\quad}{10} = \frac{x}{\quad}$$

$$\_\_\_\_ \cdot \frac{\quad}{\quad} = \frac{x}{\quad} \cdot \_\_\_\_$$

$$\frac{\quad}{\quad} = x$$

____ students

Pet	Number of students
Dogs	6
Cats	3
Fish	1

Guided Practice

2. 15 random students from Pinecrest Academy were asked what their favorite type of music was. The results are shown in the table. Pinecrest Academy has 100 students. Estimate the total number of students who prefer country music.

____ students

$$\frac{\quad}{\quad} = \frac{x}{\quad}$$

$$\frac{\quad}{\quad} \cdot \frac{\quad}{\quad} = \frac{x}{\quad} \cdot \frac{\quad}{\quad}$$

$$\frac{\quad}{\quad} = x$$

Music	Number of students
Pop	1
Rock	3
Hip-Hop	2
Country	9

3. 20 random students from Oakwood Elementary were asked what their favorite fruit was. The results are shown in the table. Oakwood Elementary has 400 students. Estimate the total number of students who prefer pineapple.

____ students

Fruit	Number of students
Apples	5
Bananas	3
Oranges	3
Strawberries	5
Pineapple	4

Check

25 random students from Valley High School were asked what their favorite sport is. The results are shown in the table. Valley High School has 400 students. Estimate the total number of students who prefer basketball.

____ students

Sport	Number of students
Soccer	8
Basketball	10
Baseball	3
Tennis	4

Prerequisite Skills and Concepts

Mini-Lessons

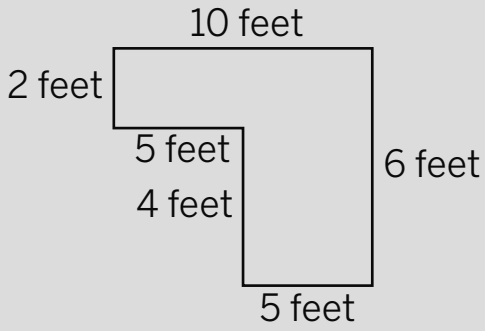
Name _____

Finding Area of Rectilinear Figures Without Grids

ML 2.11

Modeled Review

Students were asked to determine the area of the figure.

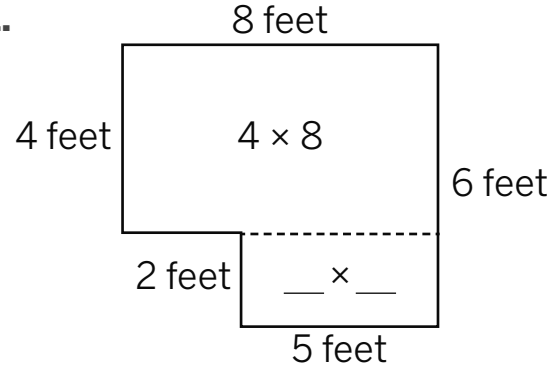


- Maya says you can determine the area by adding 2 × 10 and 4 × 5.
- Jack says you can determine the area by adding 2 × 5 and 6 × 5.

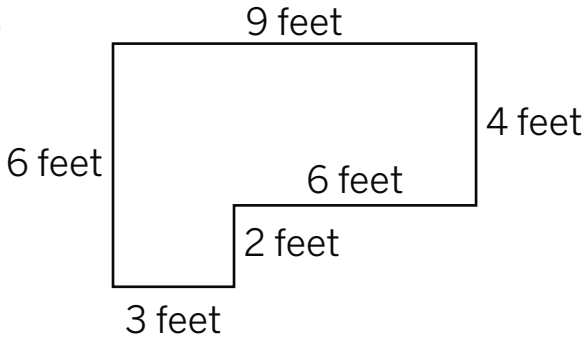
Guided Practice

Draw a line to decompose each figure into rectangles. Then write expressions that represent the area of each rectangle.

1.



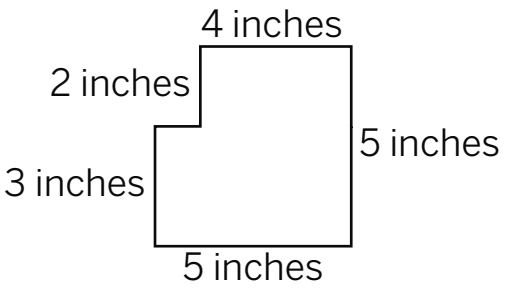
2.



Guided Practice

Determine the area of each figure. Show or explain your thinking.

3.



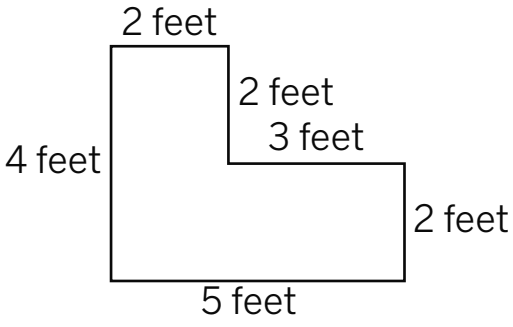
____ × ____ = ____

____ × ____ = ____

total area: ____ + ____ = ____

answer: ____ square inches

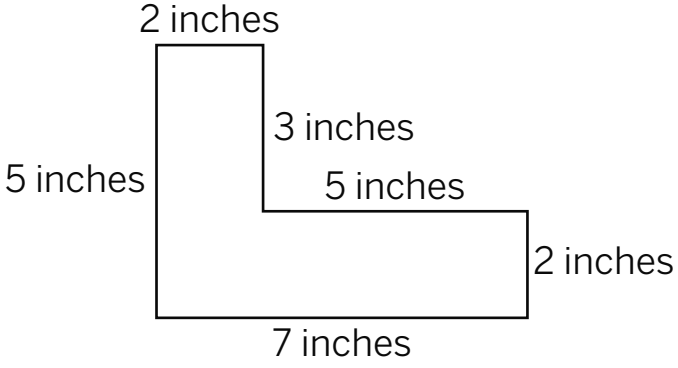
4.



answer: _____

Check

Determine the area of the figure. Show or explain your thinking.



answer: _____

Name _____

Solving Problems Involving Area and Perimeter

ML 6.21

Modeled Review

Name: Tristan

A square has a perimeter of 252 inches. What is the length of each side?

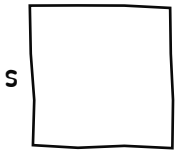
252 ÷ 4 = s

240 + 12

240 ÷ 4 = 60

12 ÷ 4 = 3

60 + 3 = 63



answer: 63 inches

Guided Practice

1. A rectangle has an area of 446 square inches. The length is 2 inches. What is the width?

answer: _____

2. A square has a perimeter of 136 inches. What is the length of each side?

answer: _____

Guided Practice

3. A rectangle measures 3 feet wide and 257 feet long. What is the area of the rectangle?

answer: _____

4. A rectangle measures 32 feet wide and 26 feet long. What is the area of the rectangle?

answer: _____

Check

A rectangle measures 17 inches wide and 48 inches long. What is the area of the rectangle?

answer: _____

Name _____

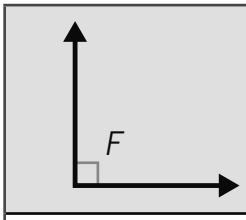
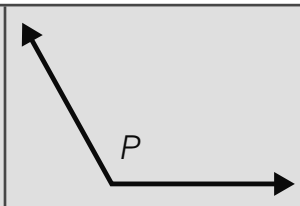
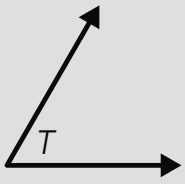
Identifying Acute, Obtuse, Right, and Straight Angles

ML 7.09

Modeled Review

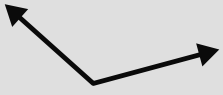
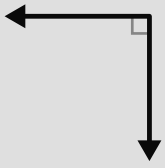
Name: Avery

Identify each angle as acute, right, obtuse, or straight. Then record the angle measure.

			
right 90°	obtuse 120°	straight 180°	acute 60°

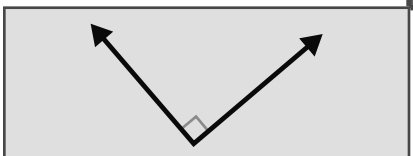
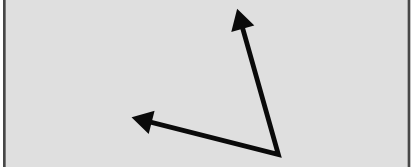
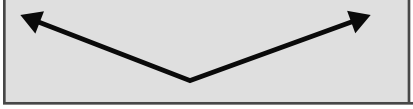
Guided Practice

1. Complete the table by labeling each angle. Use a protractor to measure if it is helpful.

	Acute, right, or obtuse	Greater than, less than, or equal to 90°
		greater than 90°
	acute	
		equal to 90°

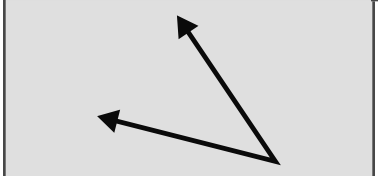
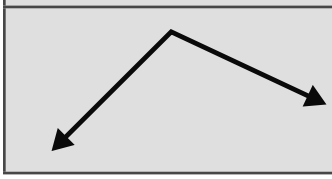
Guided Practice

2. Identify each angle as acute, right, obtuse, or straight. Then record the angle measure.

	Acute, right, obtuse, or straight	Measurement in degrees
		
		
		
		

Check

Identify each angle as acute, right, obtuse, or straight. Then record the angle measure.

	Acute, right, obtuse, or straight	Measurement in degrees
		
		

Name _____

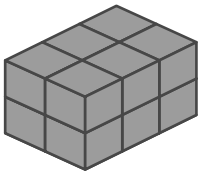
Using the Structure of Rectangular Prisms to Determine Volume

ML 1.04

Modeled Review

Name: Gabriela

Determine the volume of the prism. Show or explain your thinking.

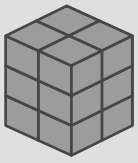
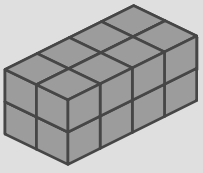
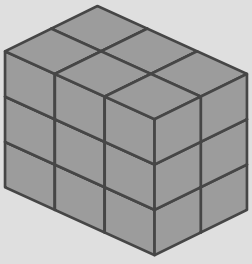


The bottom layer has 6 cubes.
There are 2 layers. 2 layers of
6 cubes is 12 cubes.

answer: 12 cubic units

Guided Practice

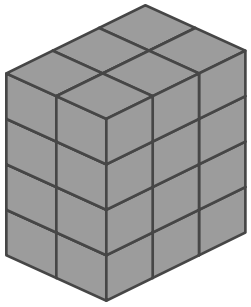
1. Complete the table.

Prism	Number of cubes in the bottom layer	Number of layers	Volume (cubic units)
	4	3	
	8		
			

Guided Practice

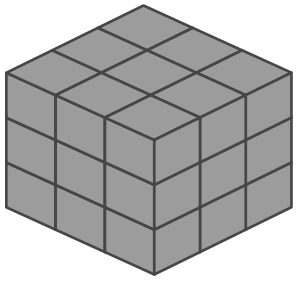
Determine the volume of each prism. Show or explain your thinking.

2.



answer: _____

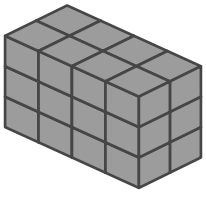
3.



answer: _____

Check

Determine the volume of the prism. Show or explain your thinking.



answer: _____

Name _____

Using Clues to Determine the Volumes of Figures

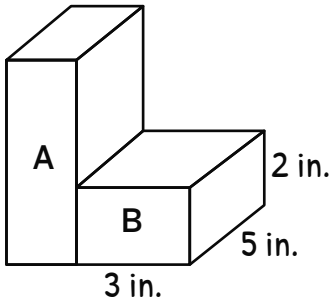
ML 1.14

Modeled Review

Name: Clare

Use the clues to determine the volume of the figure. Show or explain your thinking.

Clues
• Prism B has a length of 3 inches, a width of 5 inches, and a height of 2 inches. • The volume of Prism A is 2 times the volume of Prism B.



Prism B: 3 × 5 × 2 = 30

Prism A: 2 × 30 = 60

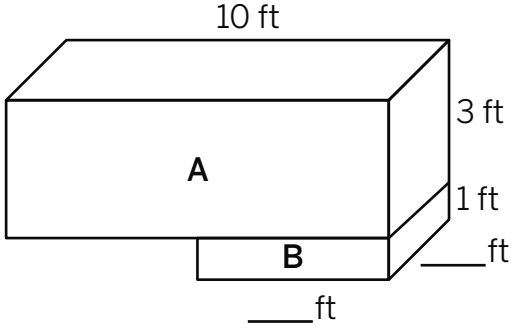
total volume: 30 + 60 = 90

answer: 90 cubic inches

Guided Practice

1. Use the clues to complete the dimensions of Prism A and Prism B.

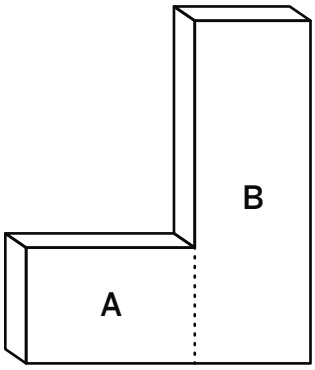
Clues
• The length of Prism A is twice the length of Prism B. • The height of Prism A is half the width of Prism B.



Guided Practice

2. Use the clues to determine the volume of the figure. Show or explain your thinking.

Clues
• Prism B has a length of 3 inches, a width of 2 inches, and a height of 9 inches. • Prism A has a height of 3 inches. • The width of each prism is the same. • The length of Prism A and B is 8 inches combined.

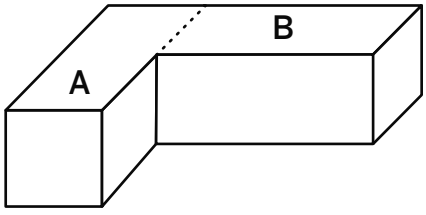


answer: _____

Check

Use the clues to determine the volume of the figure. Show or explain your thinking.

Clues
• Prism A has a length of 6 inches, a width of 2 inches, and height of 2 inches. • The volume of each prism is the same.



answer: _____

Name _____

Comparing Products Without Multiplying

ML 3.09

Modeled Review

Name: Avery

Compare the size of the product using <, >, or =.

1. $1,300 \times \frac{2}{3}$ ≤ $1,300$

2. $1,300 \times \frac{11}{9}$ ≥ $1,300$

If I multiply by a factor less than or equal to 1, my product will be less than or equal to the other factor.

Guided Practice

1. Fill in the blanks in the table using <, >, or =.

Expression	The product is (<, >, or =)
$100 \times \frac{2}{2}$	$100 \times \frac{2}{2}$ is _____ 100 because $\frac{2}{2}$ is _____ 1.
$100 \times \frac{1}{2}$	$100 \times \frac{1}{2}$ is _____ 100 because $\frac{1}{2}$ is _____ 1.
$100 \times \frac{3}{2}$	$100 \times \frac{3}{2}$ is _____ 100 because $\frac{3}{2}$ is _____ 1.

Guided Practice

Compare the size of the product using <, >, or =.

2. $545 \times \frac{3}{4}$ _____ 545

3. $545 \times \frac{7}{4}$ _____ 545

4. $650 \times \frac{3}{2}$ _____ 650

5. $650 \times \frac{7}{7}$ _____ 650

6. $800 \times \frac{3}{3}$ _____ 800

7. $800 \times \frac{2}{5}$ _____ 800

8. $1{,}050 \times \frac{10}{3}$ _____ 1,050

9. $1{,}050 \times \frac{8}{8}$ _____ 1,050

10. $1{,}200 \times \frac{6}{5}$ _____ 1,200

11. $1{,}200 \times \frac{1}{3}$ _____ 1,200

Check

Compare the size of the product using <, >, or =.

1. $1{,}100 \times \frac{3}{5}$ _____ 1,100

2. $1{,}100 \times \frac{10}{9}$ _____ 1,100

Name _____

Converting Inches, Feet, Yards, and Miles

ML 6.09

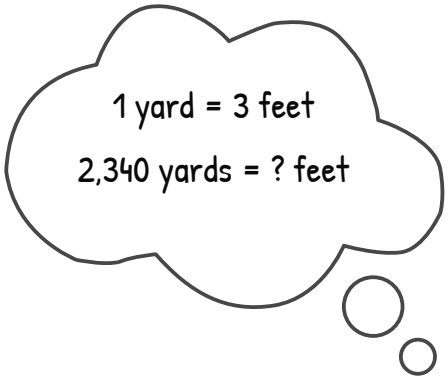
Modeled Review

Name: Han

A walking path is 2,340 yards long. How long is the walking path in feet?

2,340 × 3 = 7,020

```
  1 1
  2, 3 4 0
×       3
---------
  7, 0 2 0
```



answer: 7,020 feet

Guided Practice

Complete the conversions. Show your thinking.

1. 1 foot = 12 inches

3 feet = _____ inches

12 × 3 = _____

2. 1 yard = 3 feet

5 yards = _____ feet

3 × 5 = _____

3. 1 foot = _____ inches

$\frac{1}{2}$ foot = _____ inches

4. 1 mile = 5,280 feet

10 miles = _____ feet

Guided Practice

Solve each story problem. Show your thinking.

5. A football field is 100 yards. How long is a football field in feet?

answer: _____

6. The soccer team ran half a mile before practice. How many yards did they run?
1 mile = 1,760 yards

answer: _____

7. A soccer ball was kicked 120 inches. How far is that in feet?

answer: _____

8. A football goal post has a height of 30 feet. How tall is the goal post in inches?

answer: _____

Check

Solve the story problem. Show your thinking.

A football player kicked a 66 yard field goal. How far is that in feet?

answer: _____

Name _____

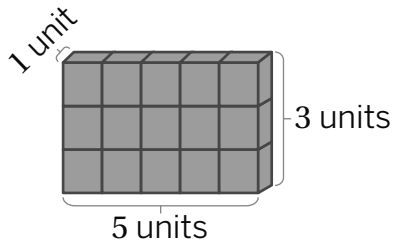
Determining Surface Areas of Rectangular Prisms

ML 1.10

Modeled Review

Name: Jaleel

Determine the surface area of the rectangular prism. Show or explain your thinking.



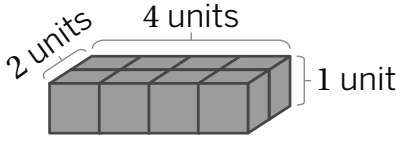
(5 · 3) · 2 = 30
(5 · 1) · 2 = 10
(3 · 1) · 2 = 6

30 + 10 + 6 = 46 square units

Guided Practice

Determine the surface area of each rectangular prism. Show or explain your thinking.

1.



(4 • 1) • 2 = 8

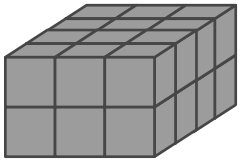
(1 • 2) • 2 = 4

(2 • 4) • 2 = 16

____ + ____ + ____ = ____

surface area: _____ square units

2.



(3 • 2) • 2 = ____

(2 • 4) • 2 = ____

(4 • 3) • 2 = ____

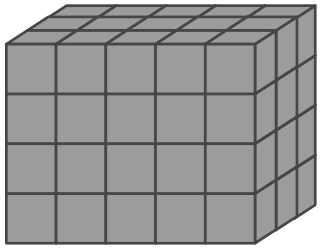
____ + ____ + ____ = ____

surface area: _____ square units

Guided Practice

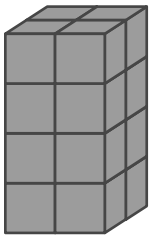
Determine the surface area of each rectangular prism. Show or explain your thinking.

3.



surface area: _____ square units

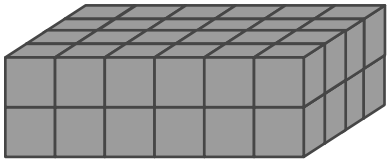
4.



surface area: _____ square units

Check

Determine the surface area of the rectangular prism. Show or explain your thinking.



surface area: _____ square units

Name _____

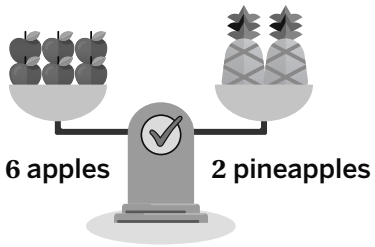
Generating Equivalent Ratios

ML 2.04

Modeled Review

Name: Tristan

The scale balances with 6 apples to 2 pineapples. Fill in the table of values that will balance the scale. Show your thinking.



Number of apples	Number of pineapples
3	1
6	2
12	4
24	8

× 2, × 2, × 2 (down the apples column)

2 ×, 2 ×, 2 × (down the pineapples column)

Guided Practice

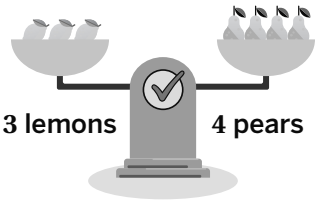
Use the tables below to record ratios that are equivalent to those shown on each balance scale.

1.

Number of lemons	Number of pears
3	4
6	
	16

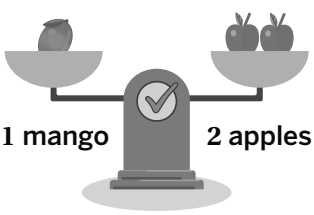
× 2, × 2 (down the lemons column)

× 2, × 2 (down the pears column)



2.

Number of mangos	Number of apples
1	2
10	



Guided Practice

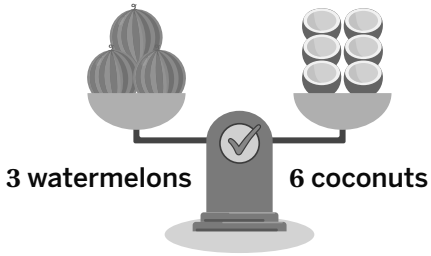
3. The scale balances with 3 watermelons to 6 coconuts. Select all of the equivalent ratios.

A. 1 watermelon and 2 coconuts

B. 6 watermelons and 12 coconuts

C. 9 watermelons and 15 coconuts

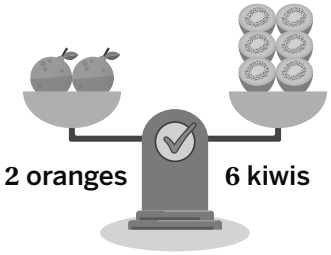
D. 30 watermelons and 60 coconuts



For Problems 4-5, use the tables below to record ratios that are equivalent to those shown on each balance scale.

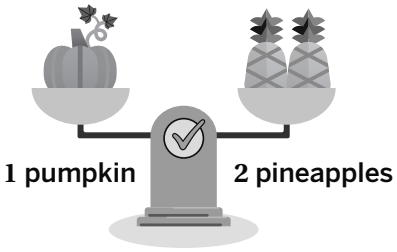
4.

Number of oranges	Number of kiwis



5.

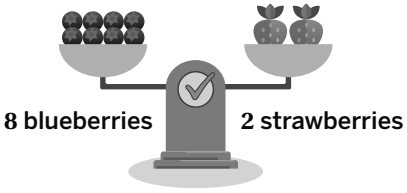
Number of pumpkins	Number of pineapples



Check

Use the table below to record ratios that are equivalent to those shown on the balance scale.

Number of blueberries	Number of strawberries
4	
	8



Name _____

Calculating Unit Rates

ML 3.04

Modeled Review

Name: Jason

During a sale, a clothing store is selling 5 shirts for $30. What is the unit rate? What does it represent?

30 ÷ 5 = 6

The unit rate is $6/shirt. This represents the price of one shirt.

Guided Practice

1. A commuter train travels 60 miles in 30 minutes without stopping. A subway train travels 5 miles in 2 minutes without stopping. Which train is traveling faster? How do you know?

Commuter train

Ratio: 60 miles/ 30 minutes

Unit rate: 2 miles/minute

Subway train

Ratio: _____ miles/ _____ minutes

Unit rate: _____ miles/minute

Faster train: The _____ train. It has a greater unit rate.

2. A local grocery store sells three gallons of milk for $12.00. A supermarket sells two gallons of milk for $8.50. Which is the better deal? How do you know?

Grocery store

Ratio: _____ dollars/ _____ gallons

Unit rate: _____ dollars/gallon

Supermarket

Ratio: _____ dollars/ _____ gallons

Unit rate: _____ dollars/gallon

Better deal: _____

Guided Practice

3. A cheetah has been observed traveling a distance of 315 meters in 10 seconds. How fast was the cheetah?

Cheetah

Ratio: _____ meters/ _____ seconds

Unit rate: _____ meters/second

A light aircraft needs to travel 30 meters per second to take off. Was the cheetah moving faster than this? How do you know?

_____

4. You are setting up a lemonade stand. Your goal is to sell 25 cups of lemonade. The recipe you use says that 12 lemons make 4 cups. You purchase 100 lemons to make 25 cups. Will your lemonade have more or less lemon flavor than the recipe? How do you know?

Recipe

Ratio: _____

Unit rate: _____

Lemonade stand

Ratio: _____

Unit rate: _____

_____

_____

Check

A red model train moves 180 centimeters in 30 seconds. A green model train moves 320 centimeters in 40 seconds. Which train is moving faster? How do you know?

_____

_____

Name _____

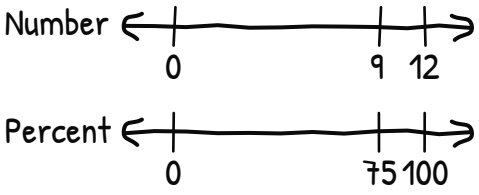
Calculating Unknown Percentages

ML 3.13

Modeled Review

Name: Tristan

What is 9 out of 12 as a percent?



75%

Name: Santiago

What is 9 out of 12 as a percent?

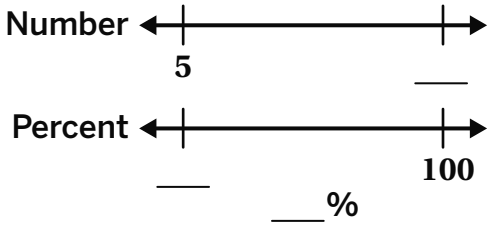
$\frac{9}{12} \cdot 100 =$ 75

75%

Guided Practice

For Problems 1 and 2, calculate the unknown percentage. Show your thinking.

1. What is 5 out of 10 as a percent?



2. What is 5 out of 50 as a percent?

$\frac{5}{50} \cdot 100 =$ _____

_____ %

Guided Practice

Calculate the unknown percentage. Show your thinking.

3. What is 18 out of 24 as a percent?

$\frac{18}{24} \cdot 100 =$ _____

4. What is 4 out of 16 as a percent?

_____ $\cdot 100 =$ _____

5. What is 13 out of 26 as a percent?

6. What is 3 out of 15 as a percent?

7. What is 7 out of 70 as a percent?

8. What is 19 out of 38 as a percent?

Check

What is 16 out of 64 as a percent? Show your thinking.

Name _____

Writing and Solving Equations

ML 6.05

Modeled Review

Name: Han

For Problems 1–3, use the equation $x + 2 = 10$.

1. Write a situation that represents this equation. Explain what x represents in your situation.

   Tristan spent $10 on a box of tissues and medicine. The box of tissues cost $2. The medicine cost x dollars.

2. Determine the solution to the equation.

   $x = 8$

   $x + 2 = 10$
   $-2 \quad -2$
   $x = 8$

3. Explain what the solution means in your situation.

   The medicine cost $8.

Guided Practice

For each situation, write the equation from the bank that best represents it.

$x + 6 = 13$	$6x = 13$	$x + 2 = 7$
$2x = 7$	$x - 2 = 7$	$2x = 10$

1. Santiago spent $\$7$ on a cup of coffee and a croissant. The cup of coffee cost $\$2$. The croissant cost x dollars.

   _____

2. Jada spent $\$10$ on two secondhand books. Each book cost x dollars.

   _____

3. Diego spent $\$13$ on an herb plant and a pot. The plant cost $\$6$. The pot cost x dollars.

   _____

Guided Practice

4. Complete the table for each equation shown.

Equation	Situation	Solution	Solution's meaning
$x + 6 = 15$	Rebecca spent \$15 on a deck of cards and a phone charger. The deck of cards cost \$6. The phone charger cost x dollars.	$x = 9$	
$4x = 20$	Esteban spent \$20 on four greeting cards. Each greeting card cost x dollars.		
$x + 3 = 14$			

Check

For Problems 1–3, use the equation $x + 5 = 12$.

1. Write a situation that represents the equation. Explain what x represents in your situation.

2. Determine the solution to the equation.

3. Explain what the solution means in your situation.

Name _____

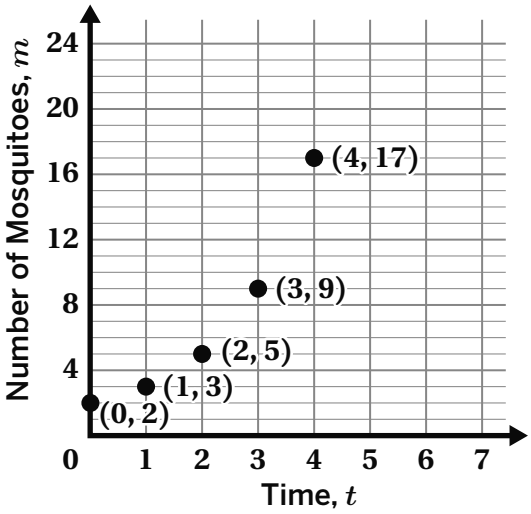
Interpreting Graphs

ML 6.14

Modeled Review

Name: Shawn

The graph represents the relationship between time, t, and the number of mosquitoes, m. Complete the table so it reflects the values in the graph.

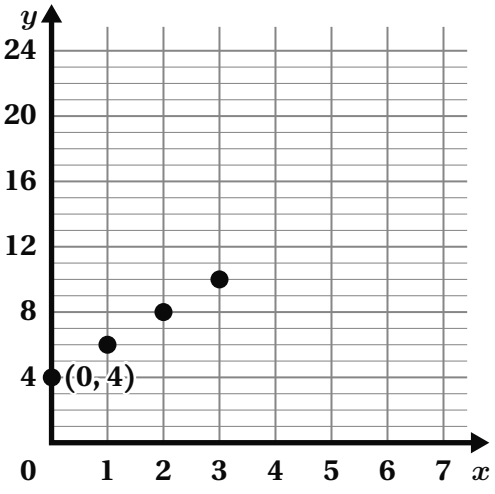


t	m
0	2
1	3
2	5
3	9
4	17

Time is the independent variable, so it represents the x-values. The number of mosquitos is the dependent variable, so it represents the y-values.

Guided Practice

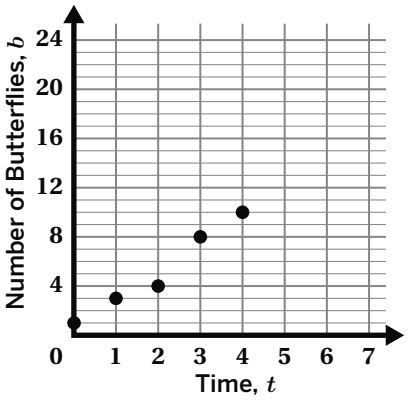
1. The graph represents the relationship between the independent variable, x, and the dependent variable, y. Complete the table so it reflects the values in the graph.



x	y
0	4

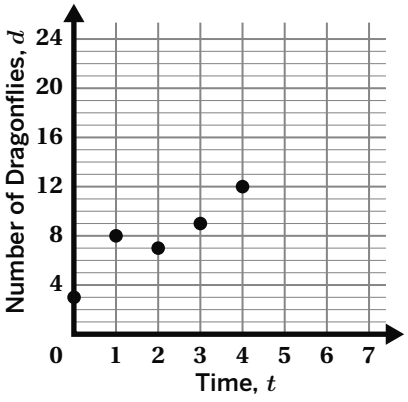
Guided Practice

2. This graph represents the relationship between time, t, and the number of butterflies, b. Complete the table so it reflects the values in the graph.



t	b

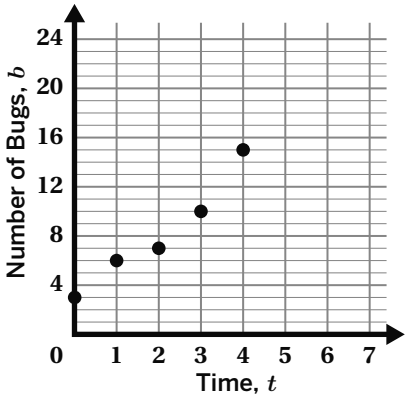
3. This graph represents the relationship between time, t, and the number of dragonflies, d. Complete the table so it reflects the values in the graph.



t	d

Check

This graph represents the relationship between time, t, and the number of bugs, b. Complete the table so it reflects the values in the graph.



t	b

Name _____

Ordering and Comparing Positive and Negative Numbers

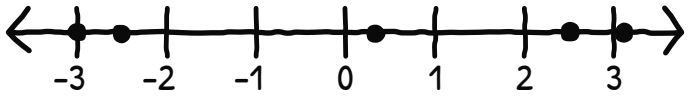
ML 7.04

Modeled Review

Name: Adam

Order the numbers from *least* to *greatest*.

−3	3.1	−2.5	2.5	$\frac{1}{4}$



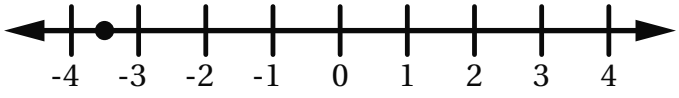
−3	−2.5	$\frac{1}{4}$	2.5	3.1
least				greatest

Guided Practice

Order each set of numbers from least to *greatest*. Use the number lines if they are helpful.

1.

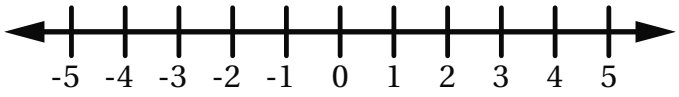
0	-3.5	3	-1



-3.5			
least			greatest

2.

4.5	-2	-4	$\frac{1}{4}$



least			greatest

Guided Practice

Order each set of numbers from *least* to *greatest*.

3.

0.2	-2	3.5	3	1.25
least				greatest

4.

0.5	3	$1\frac{1}{2}$	-2	1
least				greatest

5.

2.5	2	-2.5	$-\frac{1}{2}$	-3
least				greatest

6.

-3.5	$-\frac{1}{5}$	2.5	-5	3
least				greatest

Check

Order the numbers from *least* to *greatest*.

$-\frac{1}{2}$	1	-3	4	-4.5
least				greatest

Name _____

Calculating Mean Absolute Deviation

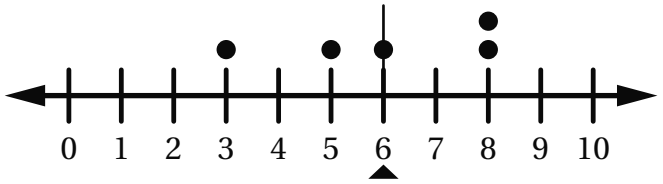
ML 8.09

Modeled Review

Name: Tristan

The dot plot shows the number of text messages Deven sent every day for 5 days. Calculate the MAD of this data.

Number of text messages Deven sent



Mean: 6 books

3 + 1 + 0 + 2 + 2 = 8

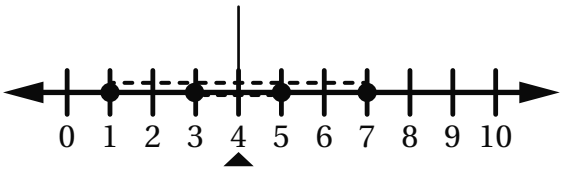
8 ÷ 5 = 1.6

MAD: 1.6

Guided Practice

1. The dot plot shows the number of books Jin read each month for 4 months. Calculate the MAD of this data.

Number of books Jin read



Mean: 4 books

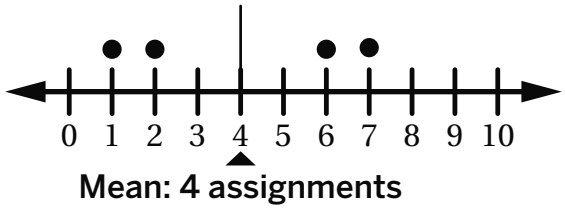
MAD: ____

Moves	Work
Step 1: Find the distance between each data point and the mean.	3 ___ ___ ___
Step 2: Add all the differences, then divide by the number of data points.	3 + ___ + ___ + ___ = ___ ___ ÷ 4 = ___

Guided Practice

2. The dot plot shows the number of homework assignments Neel completed each day for 4 days. Calculate the MAD of this data.

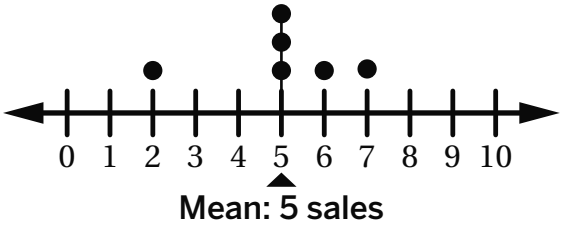
Number of assignments Neel completed



MAD: ____

3. The dot plot shows the number of sales Peter made each day for 6 days. Calculate the MAD of this data.

Number of sales Peter made

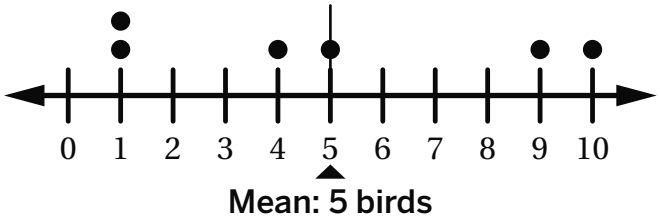


MAD: ____

Check

The dot plot shows the number of birds Tyler saw each day for 6 days. Calculate the MAD of this data.

Number of birds Tyler saw



MAD: ____

Name _____

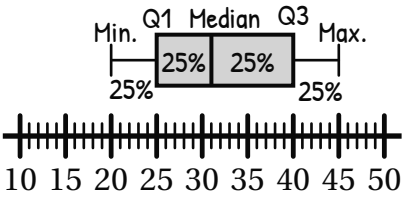
Interpreting Box Plots

ML 8.14

Modeled Review

Name: Santiago

Inola took the bus to school most days in January. She wrote down how many minutes her journey took each day and made this box plot.



1. Determine the median, IQR, and range for this data.

median: 30 **IQR:** 15 **range:** 25

40 – 25 = 15 45 – 20 = 25

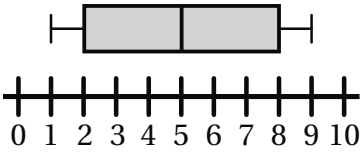
2. What percent of Inola's journey to school took 40 minutes or less?

A. 25% **B.** 50% **C.** 75% **D.** 100%

Guided Practice

For Problems 1–2, refer to the box plots to identify the statistics of the data set.

1. Mia measured the number of books she read each day for a week in June.



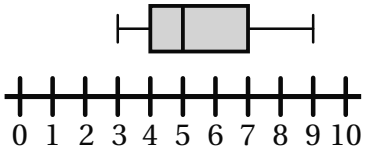
Number of books

Min.	Q1	Median	Q3	Max.
1	2			

What percent of the data was more than **5** books?

A. 25% **B.** 50%
C. 75% **D.** 100%

2. Zoe tracked the number of apples she ate each day for a week in January.



Number of apples

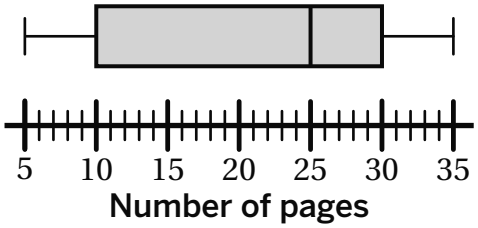
Min.	Q1	Median	Q3	Max.

What percent of the data was between **4** and **5** apples?

A. 25% **B.** 50%
C. 75% **D.** 100%

Guided Practice

Eliza tracked the number of pages she read each day for a week. She wrote down the amounts and created this box plot.



3. Determine the median, IQR, and range for this data.

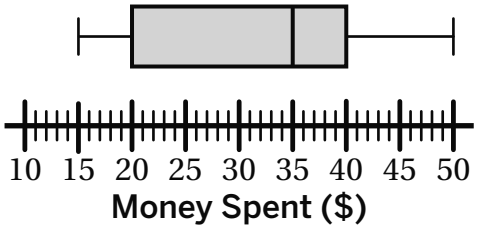
median: _____ **IQR:** _____ **range:** _____

4. On what percent of days did Eliza read less than 25 pages?

A. 25% **B.** 50% **C.** 75% **D.** 100%

Check

Evan tracked the amount of money he spent on clothes in January. He wrote down the amounts and created this box plot.



1. Determine the median, IQR, and range for this data.

median: _____ **IQR:** _____ **range:** _____

2. What percent of Evan's clothes cost at least $15?

A. 25% **B.** 50% **C.** 75% **D.** 100%

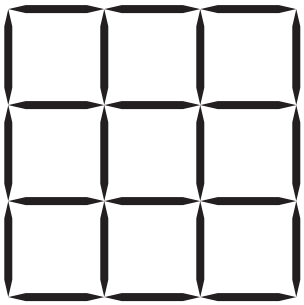
Extensions

Scaled Copies

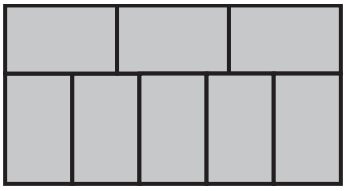
Name: Date: Period:

Student Choice Start with any problem. Remember to show or explain your thinking.

1 Remove eight toothpicks from the given design to create two squares.



2 Eight identical rectangles form a larger rectangle, as shown in the figure. Suppose you know that the length and width of the smaller rectangles are whole numbers, and the area of the larger rectangle is 480 square feet. Determine the perimeter of the larger rectangle.



Scaled Copies (continued)

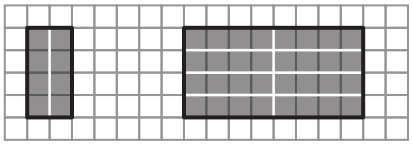
Name: ..... Date: ..... Period: .....

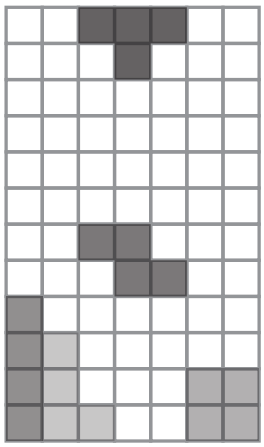
3

Tetrominoes are shapes made up of four squares. These are the tiles of the famous game Tetris.

All tetrominoes are examples of replicating tiles (*rep-tiles*), shapes that can tile an enlargement of themselves.

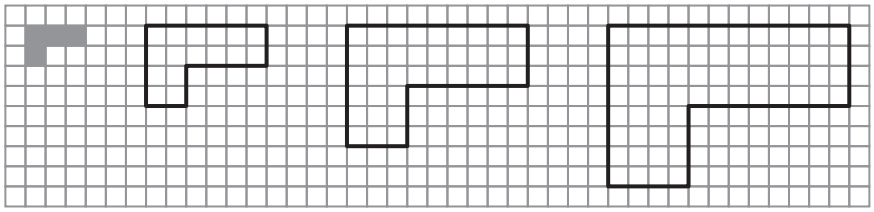
For example, the I tetromino can tile differently scaled copies of itself.





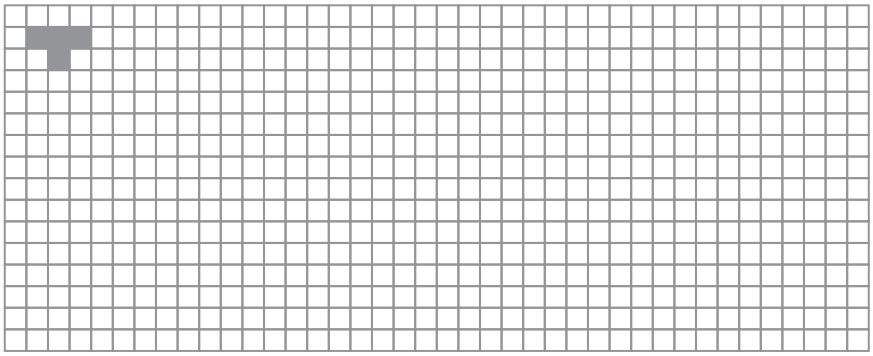
a Use the L shape tetromino to tile each of these scaled copies of itself and determine the scale factors.

× 1 × _____ × _____ × _____



b Do you think any enlargement of the L tetromino can be tiled using the L tetromino?

c Is it possible to use the T tetromino to fill any scaled copies of itself?



Scale Drawings

Name: ..... Date: ..... Period: .....

Student Choice Start with any problem. Remember to show or explain your thinking.

1

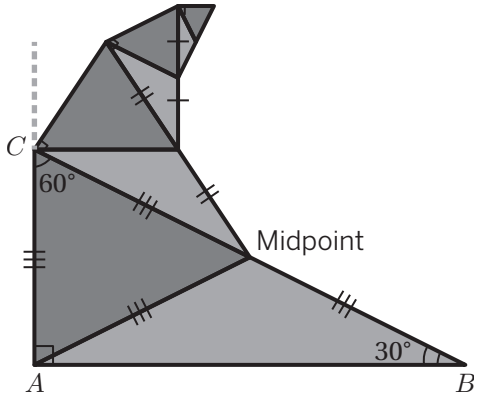
According to legend, the city of Delos in ancient Greece was once faced with a terrible plague.

An oracle told them that this was a punishment from the gods, and the plague would go away if they built a new temple that was exactly twice the volume of the existing one.

Delians doubled the lengths of the edges of the temple, but the plague did not stop. What do you think went wrong?

2

Here is a *spidron*. It's a continuous geometric figure made entirely from scaled copies of itself.



This spidron is a replica of the first spidron created by Dániel Erdély in 1979. It consists of equilateral triangles and isosceles triangles pasted together to form right triangles.

Use the grid to create your own spidron design by choosing a shape and a scale factor.

Scale Drawings (continued)

Name: Date: Period:

3 These tables show the distance between the Sun and the eight planets in our solar system.

Planet	Average Distance From Sun (millions of miles)
Mercury	35
Venus	67
Earth	93
Mars	142

Planet	Average Distance From Sun (millions of miles)
Jupiter	484
Saturn	887
Uranus	1,784
Neptune	2,795

a If you wanted to create a scale model of the solar system that could fit somewhere in your school, what scale would you use?

b The diameter of Earth is approximately $8{,}000$ miles. What would the diameter of Earth be in your scale model?

Proportional Relationships in Tables

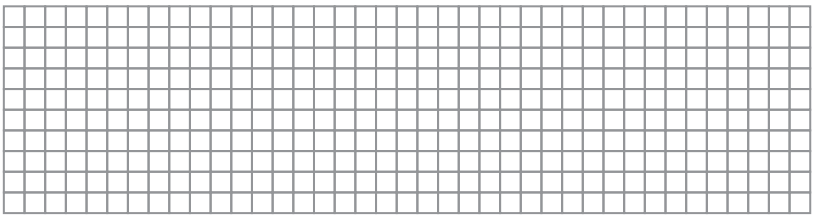
Name: Date: Period:

Student Choice Start with any problem. Remember to show or explain your thinking.

1

Farah used a blue marker to divide a plank of wood into into 5 equal parts. Raven then used a red marker to divide the same plank of wood into 7 equal parts.

a Name the colors on the plank from left to right. Draw a picture if it helps with your thinking.



b If the plank is cut at each marking, determine the lengths of the shortest and longest pieces.

2

If $a : b = 9 : 4$ and $b : c = 5 : 3$, determine the proportion $(a - b) : (b - c)$.

Proportional Relationships in Tables (continued)

Name: Date: Period:

3 Two thirds of Pablo's coins are nickels, one fourth are dimes, and the rest are quarters. If the total value of the coins is $\$6.65$, how many of Pablo's coins are nickels? Dimes? Quarters?

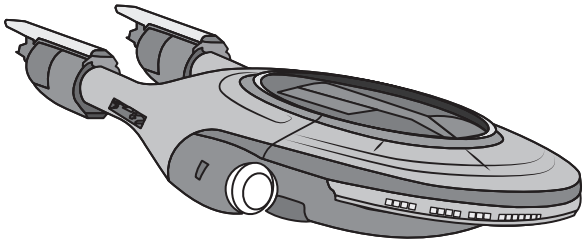
Proportional Relationships in Equations

Name: Date: Period:

Student Choice Start with any problem. Remember to show or explain your thinking.

1

The closest planet outside of our solar system orbits the star Proxima Centauri, which is 1.3 parsecs from Earth.



A *parsec* is about 3.26 light years.

A *light year* is the distance light travels in one year.

a How long does it take light from Proxima Centauri to reach Earth?

b How long would a voyage to Proxima Centauri take if a spaceship traveled at 90% the speed of light?

2

The International Space Station (ISS) is orbiting the Earth. The ISS is traveling in its orbit at a speed of 8 kilometers per second.

a What information do you need to determine how long it takes the ISS to orbit Earth?

b About how many hours does it take for the ISS to orbit Earth?

Proportional Relationships in Equations (continued)

Name: Date: Period:

3 A train traveling at 30 miles per hour reaches a tunnel that is 9 times as long as the train. The train takes 2 minutes to completely clear the tunnel. How long is the train?

Proportional Relationships in Graphs

Name: ..... Date: ..... Period: .....

Student Choice Start with any problem. Remember to show or explain your thinking.

1

Dhruv, who is right-handed, and Maneli, who is left-handed, are in the same class.

Dhruv has twice as many right-handed classmates as left-handed classmates, not including himself.

Maneli has three times as many right-handed classmates as left-handed classmates, not including herself.

How many left-handed students are in the class?

Proportional Relationships in Graphs (continued)

Name: Date: Period:

2 An old man wanted to leave his 17 camels to his 3 sons, to be divided such that:

- The eldest son should inherit $\frac{1}{2}$ of the old man's property.
- The middle son should inherit $\frac{1}{3}$ of the old man's property.
- The youngest son should inherit $\frac{1}{9}$ of the old man's property.

Because 17 is not evenly divisible by 2, 3, and 9, he asked for help from an oracle about how to do this. The oracle told him that she can give him a camel, for a total of 18.

She said: *Your eldest son should get* $18 \cdot \frac{1}{2} = 9$ *camels*, *your middle son should get* $18 \cdot \frac{1}{3} = 6$ *camels, and your youngest should get* $18 \cdot \frac{1}{9} = 2$ *camels. That's a total of* 17 *camels, and you can give my camel back afterward*.

How do you think the oracle thought of this trick? What advice could she have given the old man at the beginning?

Using Proportional Relationships

Name: Date: Period:

Student Choice Start with any problem. Remember to show or explain your thinking.

1 Here is a different version of the famous chicken-and-the-egg problem, which is said to be written by Fibonacci.

If a chicken and a half can lay an egg and a half in a day and a half, how many eggs can three chickens lay in three days?

2 30 people planted $1{,}000$ trees in 9 days. How many days would it take 36 people to plant $4{,}400$ trees?

3 The ratio of 7th graders to 8th graders in a school band was $3 : 4$ in the first quarter of a football game.

In the second quarter, three 7th graders joined the band and five 8th graders left.

As a result, the ratio of 7th graders to 8th graders became $6 : 5$.

How many members did the band have originally?

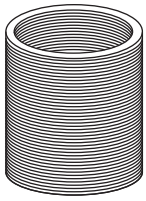
Circumference of a Circle

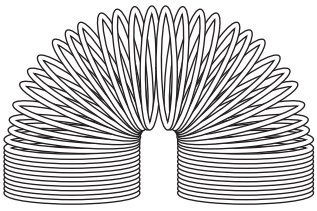
Name: ..... Date: ..... Period: .....

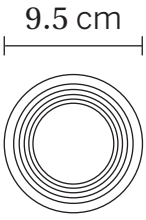
Student Choice Start with any problem. Remember to show or explain your thinking.

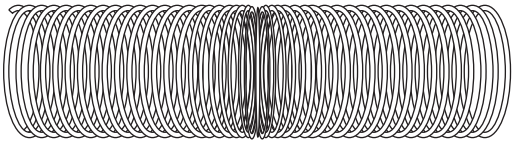
1 Explain how to determine the center of a paper circle without a ruler or a compass.

2 Consider the photos of a spring toy. If you could stretch out the spring completely straight, how long would it be?





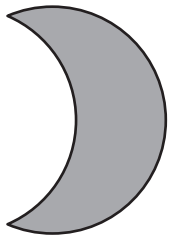




Circumference of a Circle (continued)

Name: Date: Period:

3 The shaded region is bound by two arcs, or parts of a circle, with the same radius of 7 centimeters. Determine the perimeter of the shaded region.



4 The circumference of Earth is approximately $40{,}000$ kilometers. Imagine you wrapped a circle of wire around the planet that is only 10 meters longer than its circumference. Could a flea, a mouse, or even a person fit under it?

Area of a Circle

Name: ..... Date: ..... Period: .....

Student Choice Start with any problem. Remember to show or explain your thinking

1

a What fraction of the whole circle is unshaded?

Figure A

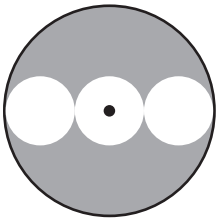


Figure B

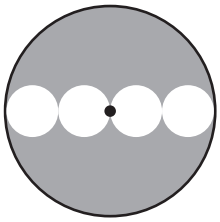


Figure C

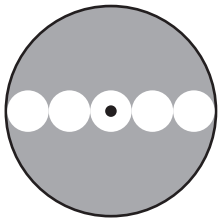
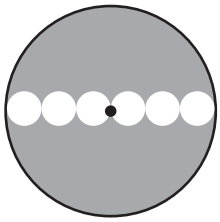


Figure D



b What fraction of the next figure circle will be unshaded?

Area of a Circle (continued)

Name: Date: Period:

2

a Determine the area of each figure. What do you notice?

Figure A

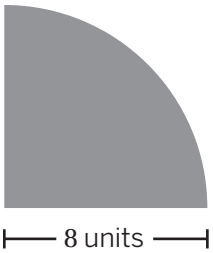


Figure B

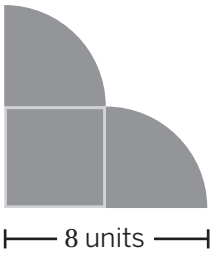


Figure C

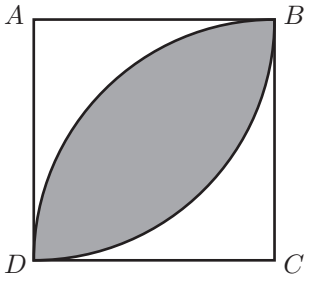
Figure D

b What would be the area of the next figure of this sequence?

c Could there be a figure with an area of 30 square units?

3

The square has side lengths of 10 units. Two arcs, parts of a circle, are shown with the center of each circle at A and C. Determine the area of the shaded region.



Percentages as Proportional Relationships

Name: ..... Date: ..... Period: .....

Student Choice Remember to show or explain your thinking.

1

A shop is experimenting with increasing and decreasing the prices of their items. What is the final price of a sweatshirt that had a price of $\$32.50$ if...

a The price was increased by 10%, then that price was decreased by 10%?

b The price was decreased by 10% first and then increased by 10%?

c The price was increased by 20%, then decreased by 20%. Will the price be more or less than the original price?

d The price was decreased by 20%, then decreased by 30%. How would this compare to a decrease of 50%?

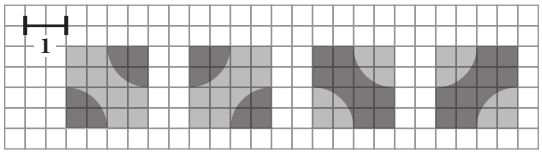
2

Egypt's population is about 100 million, and it grows by approximately 2% every year. If it continues growing at this same rate, determine how many years it will take to reach a population of 120 million.

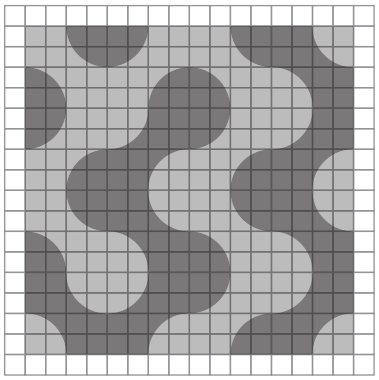
Percentages as Proportional Relationships (continued)

Name: ..... Date: ..... Period: .....

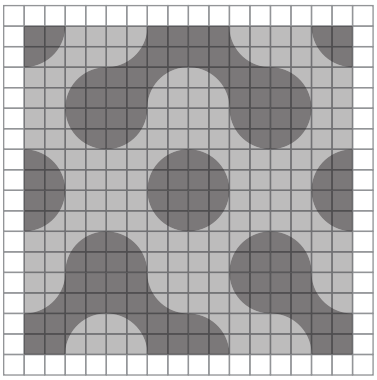
3 *Truchet tiles* are tiles that are designed so that when placed in a grid they generate interconnecting patterns. A common form of the Truchet tiles decorates each tile with two quarter-circles connecting the midpoints of adjacent sides



a What percent of each 16 by 16 square is shaded darker gray?

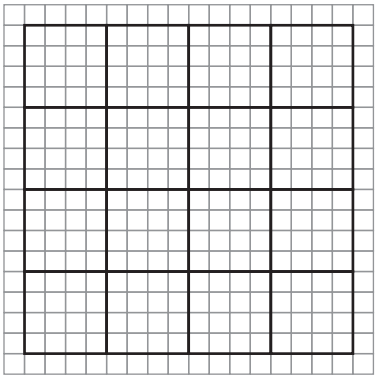


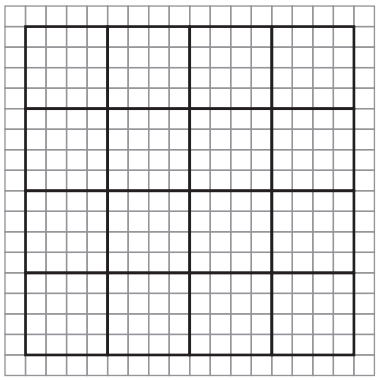
..... %



..... %

b Decorate each square so that the darker gray region is approximately 45% and 60% of each square respectively.





Applying Percentages

Name: Date: Period:

Student Choice Remember to show or explain your thinking.

1 Amy sold two sofas at a price of $\$1{,}200$ a piece. She made a 25% profit on the first sofa and a 20% loss on the other. She assumed that she still made a profit on the combined sale. Is she correct?

2

a Dinner at a restaurant costs $\$23.74$. The sales tax is 10% and the customer plans to tip 20%. How much more expensive is it to pay the tip with the tax already included, rather than pay the tip first without the tax included?

b The cost of a meal at the restaurant is c dollars, the sales tax is 10% and the customer plans to tip 20%. Write an expression to represent how much more expensive it would be for the customer to pay the tip on the price the tax, instead of the cost of the meal without the tax.

Applying Percentages (continued)

Name: ..... Date: ..... Period: .....

3 From 22 kg of fresh mushrooms, 2.5 kilogram of dried mushrooms are obtained, containing 12% water. What is the percentage of water in the fresh mushrooms?

Proportional Relationships with Fractions and Decimals

Name: Date: Period:

Student Choice Remember to show or explain your thinking.

1

A *repeating decimal* is a value where one or more of its digits eventually form a string of numbers that repeats forever, such as $0.181818\ldots$, which we represent as $0.\overline{18}$. A *terminating decimal* has digits that do not repeat forever, such as 0.625.

a Take a look at the fractions below. Which ones do you think can be written as a terminating decimal?

$\frac{2}{3}$	$\frac{3}{5}$	$\frac{3}{12}$	$\frac{5}{12}$	$\frac{7}{8}$	$\frac{5}{6}$	$\frac{13}{25}$	$\frac{9}{16}$	$\frac{3}{125}$	$\frac{35}{32}$	$\frac{9}{7}$

b What do you notice about the ones that repeat vs. terminate?

2

Here are some calculations for the fraction to decimal conversions.

$\frac{1}{9} = 0.11111\ldots$ $\frac{1}{99} = 0.010101010\ldots$ $\frac{1}{999} = 0.00100100100\ldots$

The decimal representations of $\frac{1}{9}$, $\frac{1}{99}$, and $\frac{1}{999}$ can be used to help you work out the decimal representations of other families of fractions.

Can you use what you now know to make predictions about the decimal representations of these and other fractions?

a $\frac{1}{3}, \frac{1}{33}, \frac{1}{333}$

b $\frac{1}{11}, \frac{1}{111}, \frac{1}{1111}$

c $\frac{13}{99}, \frac{28}{99}, \frac{54}{99}$

Proportional Relationships with Fractions and Decimals (continued)

Name: Date: Period:

3

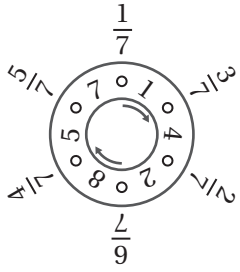
a Use a calculator to convert sevenths to decimals.

Fraction	Decimal
$\frac{1}{7}$	$0.1428571428571428\ldots$ $0.\overline{142857}$
$\frac{2}{7}$	
$\frac{3}{7}$	

Fraction	Decimal
$\frac{4}{7}$	
$\frac{5}{7}$	
$\frac{6}{7}$	

b What did you notice?

c Diego used sevenths circles to make the conversions. Can you explain how it works?



Adding and Subtracting

Name: ..... Date: ..... Period: .....

Student Choice Remember to show or explain your thinking.

1

Here are magic squares with positive and negative integers. Determine the unknown numbers.

a

		-1
2	0	
1	-4	

b

-4		
	-1	
	-3	2

c

0	-7	
-5		
-4		

2

Use the numbers -2, -1, 0, 1, 2 to complete the Kenken puzzle.

- Every row and column must use one of -2, -1, 0, 1, 2 only once.
- Each region must have the indicated result at the top left corner.
- For example, the *difference* (operation after the number) of the top left L-shape made up of three squares must be 0.

0-		-3+		0÷
0x		1+	3-	
	0x			3+
0-		1	0x	
		0		-1

3

Determine the sums. What patterns do you notice?

a $-1 + 2 - 3 + 4 - 5 + 6 - 7 + 8 \ldots - 97 + 98 - 99 + 100$

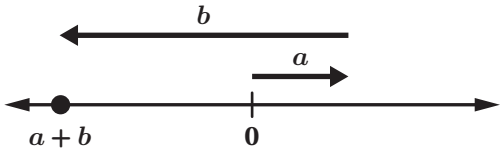
b $1 - 2 + 3 - 4 + 5 - 6 + 7 - 8 \ldots + 97 - 98 + 99 - 100$

Adding and Subtracting (continued)

Name: ..... Date: ..... Period: .....

4

Consider the following figure. For the numbers a and b, which expression is equivalent to $|a + b|$?



a $|a| + |b|$

b $|a| - |b|$

c $|b| - |a|$

5

Determine the following sums.

a $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16}$

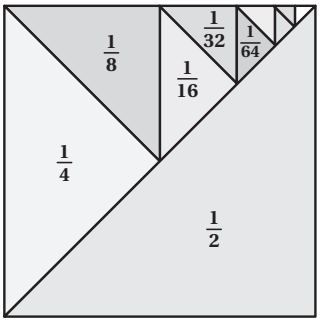
b $\frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \frac{1}{81}$

c $\frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \frac{1}{256}$

d Following the pattern in parts a–c, determine the value of $\frac{1}{5} + \frac{1}{25} + \frac{1}{125} + \frac{1}{625}$.

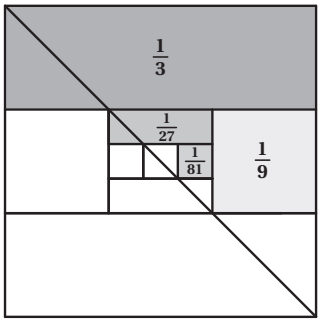
Continue the patterns to estimate each sum if it continues infinitely.

e



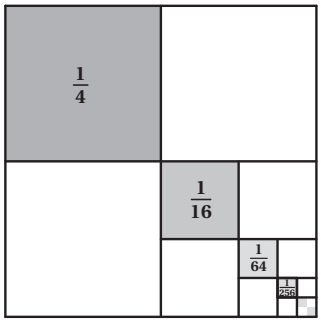
$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \ldots$

f



$\frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \frac{1}{81} + \frac{1}{243} + \ldots$

g



$\frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \frac{1}{256} + \frac{1}{1024} + \ldots$

h What do you notice about the infinite sums in parts e–g? Can you estimate the sum of $\frac{1}{n} + \frac{1}{n^2} + \frac{1}{n^3} + \frac{1}{n^4} + \ldots$

Multiplying and Dividing

Name: ..... Date: ..... Period: .....

Student Choice Start with any problem. Remember to show or explain your thinking.

1

Here is a group of six integers. Try to determine all the different ways to make the product equal -1 and 1.

$\square \cdot \square \cdot \square \cdot \square \cdot \square \cdot \square$

a For a group of six integers that multiply to equal 1, which of the following cannot be their sum?

A. -6 **B.** -2 **C.** 0 **D.** 2 **E.** 6

b For a group of six integers that multiply to equal -1, which of the following cannot be their sum?

A. -6 **B.** -4 **C.** 0 **D.** 4

2

Determine the sums.

a $(-1)^0 + (-1)^1$

b $(-1)^0 + (-1)^1 + (-1)^2 + (-1)^3$

c $(-1)^0 + (-1)^1 + (-1)^2 + (-1)^3 + \ldots + (-1)^{98} + (-1)^{99}$

3

a, b, and c are integers. If $-7 < a < 3$, $0 < b < 8$, and $-4 < c < 2$, determine the least possible value of $\frac{a \cdot b}{c}$.

Multiplying and Dividing (continued)

Name: Date: Period:

4

The following sequence of rational numbers is created by starting with 1. From then on, each term follows this pattern:

$$1 \quad 1+\frac{1}{1} \quad 1+\frac{1}{1+1} \quad 1+\frac{1}{1+\frac{1}{1+1}} \quad 1+\frac{1}{1+\frac{1}{1+\frac{1}{1+1}}} \ldots$$

a Evaluate the first five expressions in the sequence. Describe any patterns you notice.

b Predict the value of the next expression in the sequence.

c Determine the 10th term in this sequence.

5

Fill the blank with any whole number.

$$a = \frac{1}{2 \times \square + 1}$$

Simplify your fraction for a, then substitute it in the expression:

$$\frac{1+a}{1-a}$$

Try different whole numbers to determine a rule for any whole number.

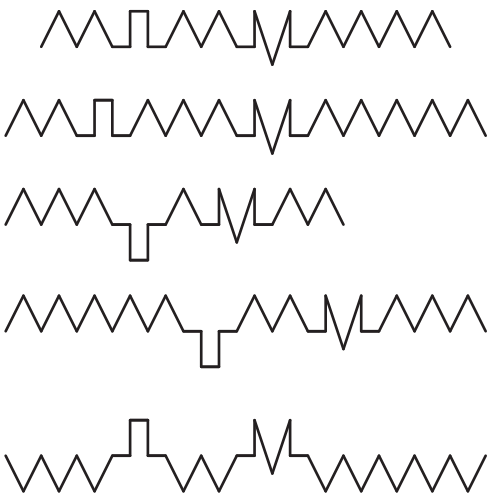
Applying Operations

Name: ..... Date: ..... Period: .....

Student Choice Start with any problem. Remember to show or explain your thinking.

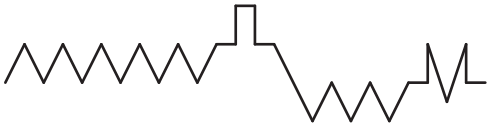
1

Astronomers have sent messages like this into outer space in order to establish communication with intelligent life on other planets. Researchers hope that even if such alien life forms cannot understand our written language, they might use radio for communication and be adept at mathematics.



a Can you decipher this mathematical message?

b Complete the message below



2

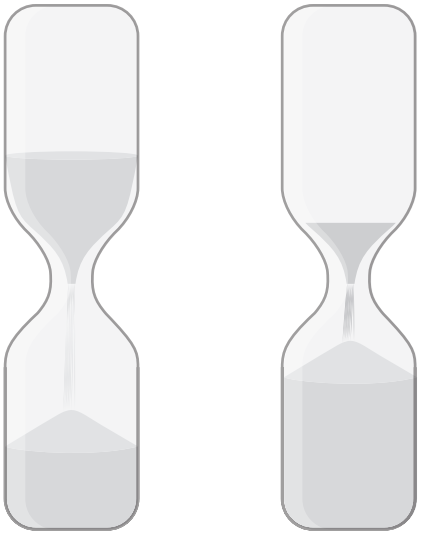
Complete the magic square using only the negative and positive integers from -8 to 8, so that the sum of every row and column, and the two diagonals equal zero.

	3		-4
5			
-8			
4	-2		

Applying Operations (continued)

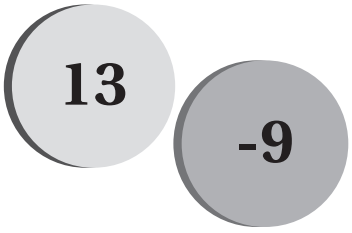
Name: ..... Date: ..... Period: .....

3 Can you measure exactly 15 minutes using nothing but an 11-minute hourglass and a 7-minute hourglass?



11-minute 7-minute

4 A faraway country uses a currency system with just two coins: worth 13 cents and -9 cents respectively. Is it possible to purchase an item that costs 19 cents?



Equations and Tape Diagrams

Name: ..... Date: ..... Period: .....

Student Choice Remember to show or explain your thinking.

1

Think of a number.

Step 1: Multiply the number by 5.

Step 2: Subtract 2.

Step 3: Add 7.

Step 4: Divide by 5.

Step 5: Subtract 1 more than your original number.

a How will your final answer be related to your original number?

b Try several numbers. What did you notice?

c Do you get the same result if you do step 3 before step 2?

2

Change the last step in the initial magic trick in Problem 1 so that you end up with your original number.

Equations and Tape Diagrams (continued)

Name: ..... Date: ..... Period: .....

3

a Lizzie skip counts by a number, y, starting from -4. The fourth number she says is 35, what is the value of y?

b Lizzie skip counts by 5 starting from x. The sum of the first five numbers she says is 65, what is the value of x?

4

In the diagram shown, each individual rectangle has dimensions 10 centimeters by 5 centimeters. If Figure n has a perimeter of 500 centimeters, determine the value of n.

Figure 1

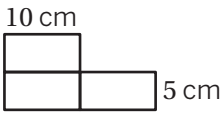


Figure 2

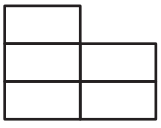
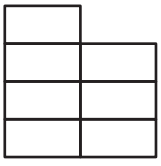


Figure 3



Solving Equations

Name: ..... Date: ..... Period: .....

Student Choice Remember to show or explain your thinking.

1

Consider the calendar and the date highlighted, May 8.

Sun	Mon	Tue	Wed	Thu	Fri	Sat
		1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30	31		

a Look at the numbers above, below, and to either side of the 8th. Calculate the mean of these numbers. What do you notice?

b Repeat part a by choosing a different date. Explain why the same thing will happen for any date in a location where it has a date above, below, and to either side.

c Assume you cover four dates next to May 8 with a square. Determine the difference of product of the corner numbers, i.e., $8 \cdot 2 - 1 \cdot 9$. Repeat by choosing different dates. What do you notice?

Sun	Mon	Tue	Wed	Thu	Fri	Sat
		1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30	31		

d Explain why the same thing will happen for any date.

2

You have an unknown number, x. In the first step, you add 1 then halve the sum. In the second step, you add 1 to that and halve the sum again.

a Write an expression to represent the first two steps of this situation.

b Continue adding 1 and halving the previous sum for the steps below. What did you notice?

3rd step 4th step 5th step

Solving Equations (continued)

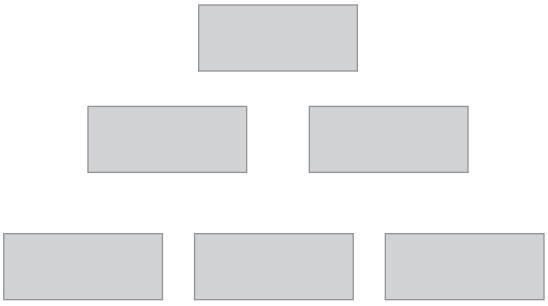
Name: Date: Period:

3

In an expression pyramid, each box is the sum of the two boxes under it. Complete the pyramids using the given expressions.

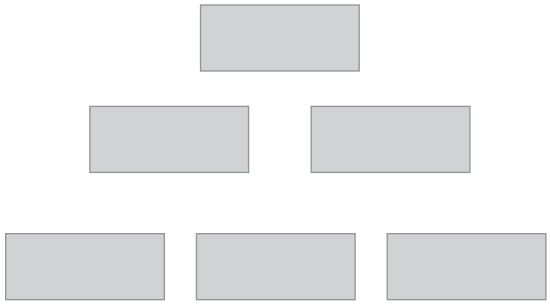
a

$2x + 1$	$2x$	$1 - x$
$3x - 1$	$3 - 2x$	$x - 2$



b

$-(y + 1)$	$2y$	$2(1 - x)$
$3(y - 1)$	$\frac{6 - 2y}{2}$	2



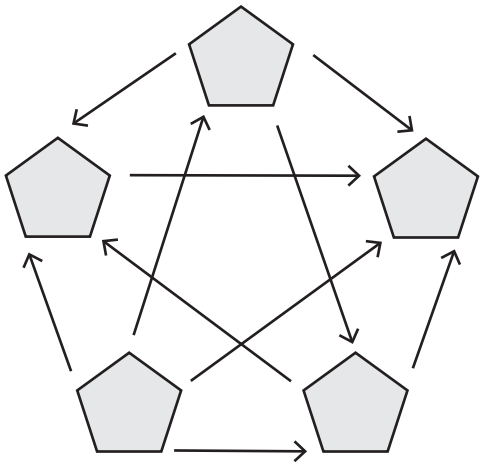
Inequalities

Name: Date: Period:

Student Choice Pick any problem to start with. Remember to explain your thinking.

1

Write an integer in each pentagon so that all arrows go from a larger number to a smaller number.



2

Using the numbers from 1–9 at most once, fill in the boxes so that the solution to each inequality is $x \leq 2$.

a Create an inequality.

$\square x - \square \leq \square$

b Create two inequalities.

$\square x - \square \leq \square$

$\square x - \square \leq \square$

Inequalities (continued)

Name: Date: Period:

3 Given $a > b$ and $c > d$, identify whether each of the following statements is always *true*, *sometimes* true or *never true*.

Inequalities	Always True	Sometimes True	Never True	Explanation
$a + c > b + d$	☐	☐	☐	
$a - c > b - d$	☐	☐	☐	
$a \cdot c > b \cdot d$	☐	☐	☐	
$\frac{a}{c} > \frac{b}{d}$ $(c \neq 0, d \neq 0)$	☐	☐	☐	

Angle Relationships

Name: ..... Date: ..... Period: .....

Student Choice Remember to show or explain your thinking.

1

Start with a rectangular piece of paper. Fold one corner at any angle, and then fold the other corner so that the edges of the paper meet, as shown.

a Unfold the paper to determine the measure of the angle at the bottom.

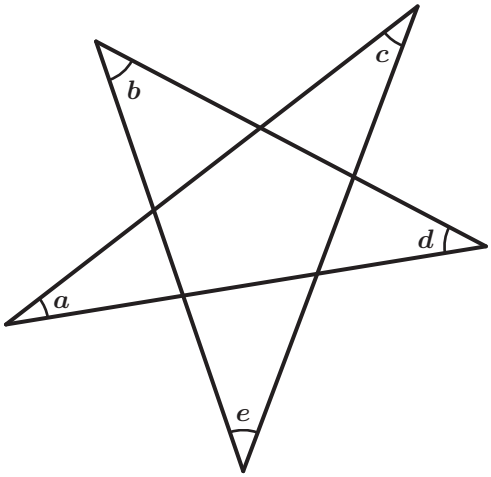
b Explain why the bottom angle always has the same measure.

2

Draw and cut out a pentagonal star on a piece of paper.

a Cut out each vertex angle to align next to each other to determine the sum.

b Do you think no matter how a pentagonal star is drawn, and no matter how it is distorted, its five angles will always add up to the same number?



Angle Relationships (continued)

Name: Date: Period:

3

The angle system we use today originates from ancient Babylonian mathematics. They divided a circle into 360 parts (degrees), and further subdivided each part into 60 smaller units (minutes and seconds).

$$60\ seconds\ (60'') = 1\ minute\ (1')$$

$$60\ minutes\ (60') = 1\ degree\ (1°)$$

For example we can represent 143.27 degrees in degree-minute-second (DMS) format by $0.27 \cdot 60 = 16.2$ minutes and $0.2 \cdot 60 = 12$ seconds so $143.27° = 143°16'12''$.

Convert these degree measurements to DMS format.

a $21.85° =$

b $60.73° =$

c Why do you think degrees are broken down into minutes and seconds?

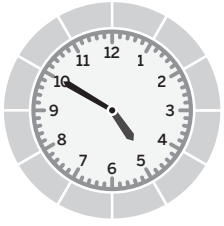
4

Determine the smaller angle between the hour and minute hands of the clock.

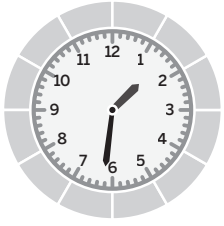
a at 2:30

b at 5:45

c at 4:50



d at 1:36



Drawing Triangles

Name: ..... Date: ..... Period: .....

Student Choice Remember to show or explain your thinking.

1

In geometry, drawing refers to sketching a geometric figure using any tools available, while *constructing* means creating a geometric figure with precise accuracy.

Complete the constructions and answer the following questions.

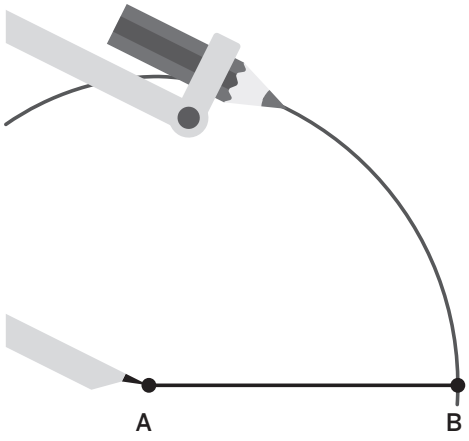
a There are several ways to construct an equilateral triangle. Here is one of them:

(The first step is done for you.)

Step 1: Draw a line segment AB. Place the compass on A and stretch it to point B. From point A, swing an arc above (or below) the line.

Step 2: Without changing the compass, place it on point B and swing another arc to intersect with the first.

Step 3: Label the intersection as C. Connect all three points to create the equilateral triangle.



b Constructing a triangle given all three sides, 4 centimeters, 6 centimeters, and 7 centimeters.

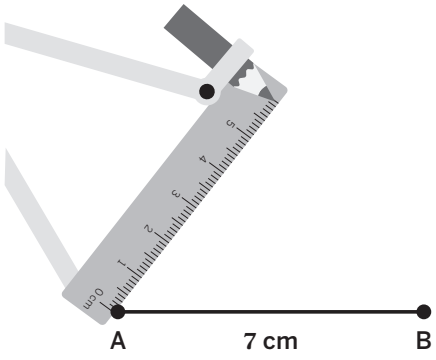
(The first step is done for you.)

Step 1: Use a ruler to draw the base, 7 centimeters.

Step 2: Set the compass for the second side, 6 centimeters, and draw an arc.

Step 3: Set the compass for the third side, 4 centimeters, and draw an arc.

Step 4: Mark the intersection point of the arcs as the third vertex and connect all three points to create the triangle.



Drawing Triangles (continued)

Name: ..... Date: ..... Period: .....

c Constructing a triangle given two angles and the included side, $50°$, 8 centimeters, and $30°$.

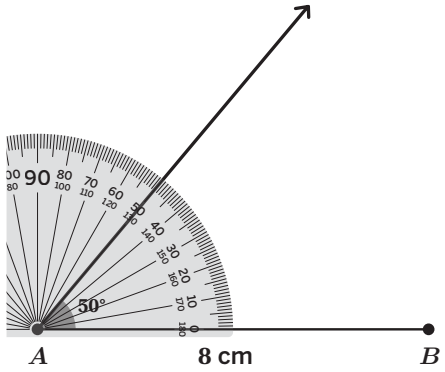
(The first two steps are done for you.)

Step 1: Use a ruler to draw the base, 8 centimeters.

Step 2: At point A, use a protractor to measure the angle $50°$, make a mark and then draw a straight line from point A through the mark.

Step 3: At point B use a protractor to measure the angle $30°$, make a mark and then draw a straight line from point B though the mark.

Step 4: Mark the intersection point of the lines as the third vertex of the triangle.



d Constructing a triangle given two sides and the included angle, 7 centimeters, $40°$, and 5 centimeters.

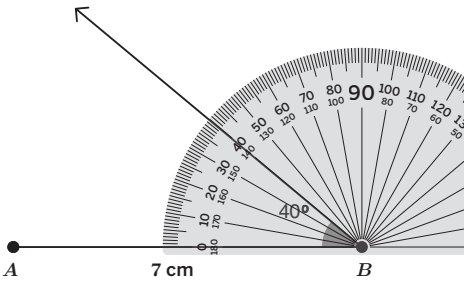
(The first two steps are done for you.)

Step 1: Use a ruler to draw the base, 7 centimeters.

Step 2: At point B, use a protractor to measure the angle $40°$, make a mark and then draw a straight line from point B through the mark.

Step 3: Open the compass to 5 centimeters. Place the point on point B and draw an arc. This second arc should cross the line drawn in step 2.

Step 4: Mark the intersection point of the lines as the third vertex of the triangle.



e How many unique triangles can be made with these measurements in parts b–d?

f What do all construction steps have in common?

g Construct a triangle given all three angles $30°$, $50°$, and $100°$. What do you notice?

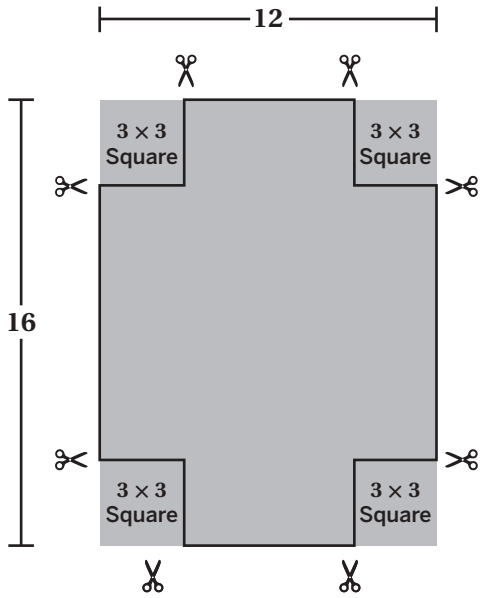
Solid Geometry

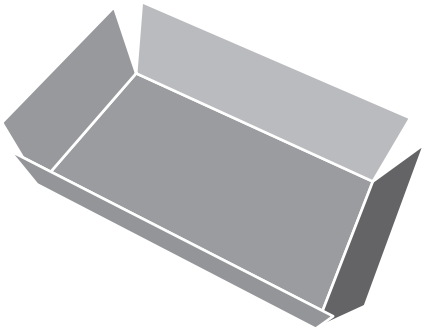
Name: ..... Date: ..... Period: .....

Student Choice Start with any problem. Remember to show or explain your thinking.

1

A 16×12 centimeter rectangular piece of card is used to create an open box by cutting four identical 3 centimeter by 3 centimeter squares from each of its corners. The remaining card is then folded to form an open box.





a What is the volume of the open box in cubic centimeters?

b Start by cutting out a 1 centimeter square from each corner – what would the volume be? Then cut a 2 centimeter square, a 3 centimeter square, and so on.

c What different volumes can you make by changing the size of the squares?

d What is the maximum size of the square you can cut out? Why?

e How did the volume change with the increasing size of the squares?

f What is the cut-off square size of the open box with the largest volume?

g The side length of the square doesn't necessarily have to be a whole number. Consider non-integer cutoff sizes to determine the dimensions of the box with the maximum volume.

Solid Geometry (continued)

Name: ..... Date: ..... Period: .....

2

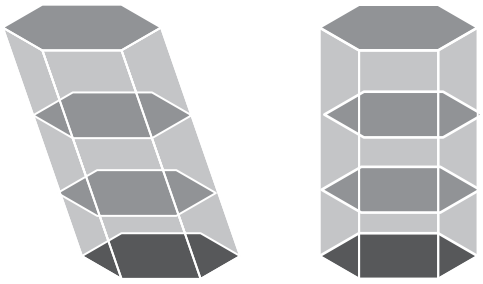
Here are three stacks of coins. Think about the base area, height, surface area and volume of each stack.

a Which one of those measures are the same? Which ones are different?

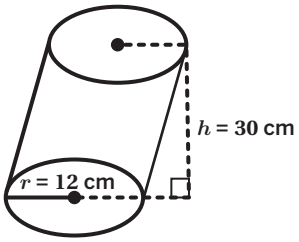
b An *oblique prism* is a slanted prism in which the lateral faces are not perpendicular to the bases. Here are some examples of right and oblique solids. Draw the heights for each pair.

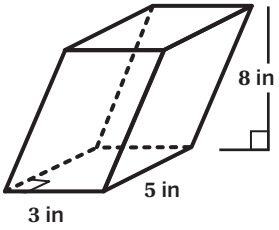
c Italian mathematician, Cavalieri, stated that if two solids have the same height and the same cross-sectional area at every level, then the two solids have the same volume.

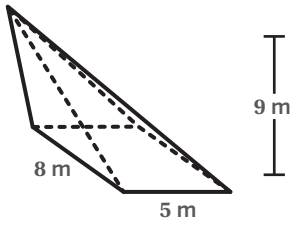
Use the coin stack example in part a to explain this principle.



d Determine the volume of the oblique solids.







Probability

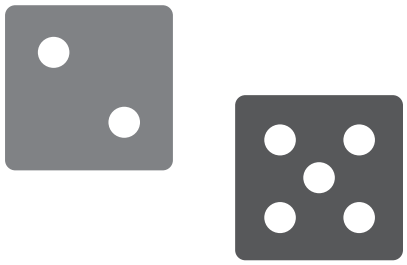
Name: ..... Date: ..... Period: .....

Student Choice Start with any problem. Remember to show or explain your thinking.

1

Two standard dice are rolled. What is the probability that the product of the numbers is

a odd? **b** even? **c** prime?



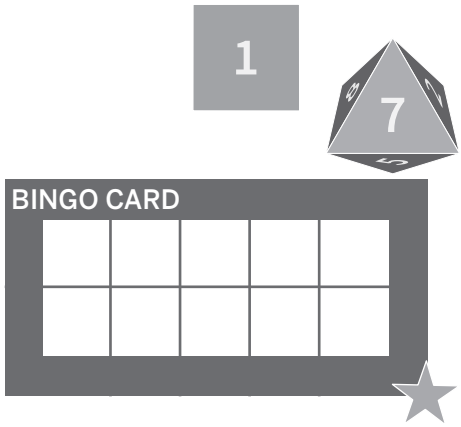
2

In a bingo game, each player will have a bingo card of 10 different numbers.

The caller rolls a cubic die (labeled 1–6) and an octahedron die (labeled 1–8), then finds the product to call out.

Players will cover the numbers on their bingo card as they are being called out. The one who covers all of the numbers first, wins.

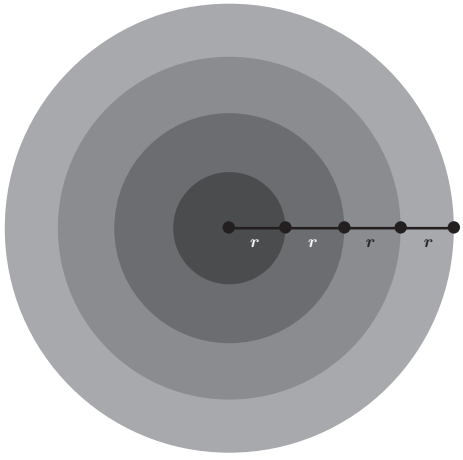
Pick the numbers on your bingo card to increase your chance of winning the game!



3

Here is a dartboard with a circle enclosed by three rings. Noa wants to divide the rings into sections so that every section is equally likely to be hit. (The innermost circle cannot be divided into sections.)

How many divisions should each ring have?

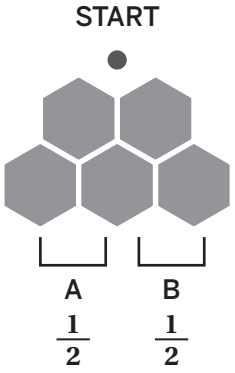


Probability (continued)

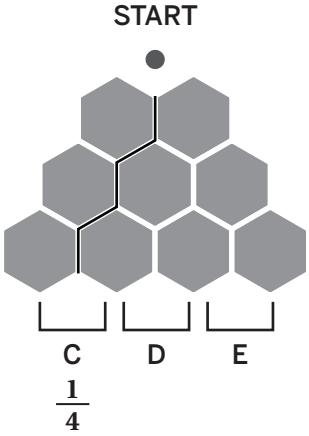
Name: ..... Date: ..... Period: .....

4

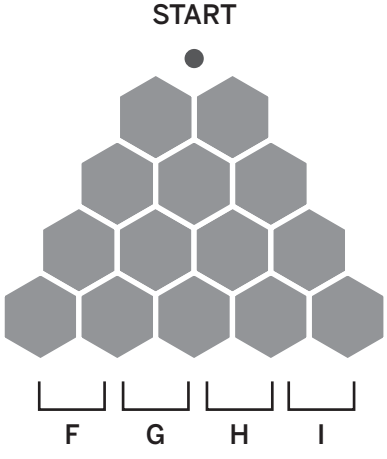
Here is a one layer example of a Galton board. A marble is rolled down from the start. It has a 50% chance of landing in bucket A and 50% chance of landing in bucket B.



a Another layer is added to the Galton Board. The probability of the marble ending in bucket C is $\frac{1}{4}$. What is the probability that the marble will land in bucket D? Bucket E?



b One more layer is added to the board. Determine the probabilities for marble to land in each bucket.



c How do the outcomes change as a new layer is added to the board? Can you estimate the outcomes in the next layer?

Sampling

Name: Date: Period:

Student Choice Start with any problem. Remember to show or explain your thinking.

1

Political parties often use samples to poll people about important issues. One common method is to call people and ask their opinions. In most places, though, they are not allowed to call cell phones. Explain how this restriction might lead to inaccurate samples of the population.

2

Here is 6 by 6 grid showing the ages of kids in a youth camp.

12	10	15	9	16	21
10	17	12	13	16	16
14	15	10	12	18	19
17	20	15	8	15	12
12	12	16	21	15	14
10	12	12	16	13	10

a Roll two dice showing the row and column number to pick 8 random numbers to calculate their mean.

b Compare each of these measures of center and spread for the population and the sample. What do you notice?

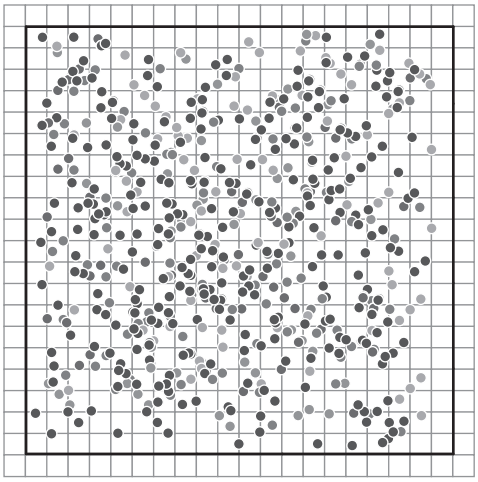
- Mean:
- Median:
- Mode:
- Range:
- IQR:

Sampling (continued)

Name: ..... Date: ..... Period: .....

3

How can you make an educational guess to determine the number of marbles in a 20 by 20 frame.



4

Here are the readings of a speed sensor in front of a school.

Sample data on the speeds of vehicles traveling

East (mph)	West (mph)
26, 30, 39, 34, 40, 21 25, 20, 24, 26, 22, 32	25, 35, 27, 34, 40, 22 20, 19, 18, 20, 19, 30

a Make a double box-and-whiskers diagram to represent the east and west end speeds (mph).

b Which measure would you use to convince the city council to add a stop sign to the east end? West end? Both?

Investigations

Investigation 1
Interior Design Room

Scale Drawings | 2-D and 3-D Connections | Proportional Relationships | Shapes in the World

7.RP.1, 7.RP.2, 7.RP.3, 7.G.1, 7.G.4, 7.G.6, SMP.1, SMP.2, SMP.4, SMP.5, SMP.6

Task 1

Name: ..... Date: ..... Period: .....

Dream Room Blueprint

Create a blueprint for your dream bedroom choosing from the room sizes provided.

An interior designer specializes in preparing homes and commercial spaces by enhancing their visual appeal and overall presentation. You will assume the role of an interior designer and design your dream bedroom by selecting a room type and various items to place in the room.

1. 💬 **Discuss:**

- What do you imagine your dream space to look like?
- What objects would it include?

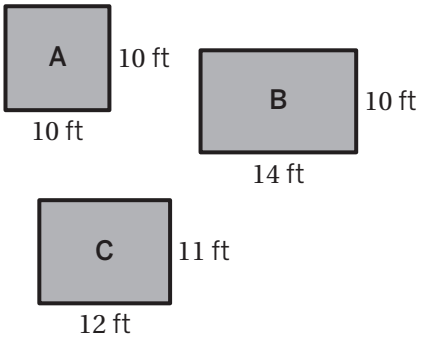
New Africa/Shutterstock.com

2. Select a room size.

A. 10 feet by 10 feet

B. 14 feet by 10 feet

C. 12 feet by 11 feet



Task 1

Name: ..... Date: ..... Period: .....

Dream Room Blueprint (continued)

3. Use the *Floorplan Shopping List* Sheet to select 10 objects that you would like to place in your room.

- **a** Record the objects and their dimensions in the table.
- **b** Use the scale 3 units to 2 feet to determine the dimensions of each object when placed in the blueprint.
- **c** Sometimes it's helpful to calculate the scale using a different measurement such as inches. What would the scale of 3 units to 2 feet be in units to inches? 3 units to ..... inches.

Object	Dimensions of Object	Dimensions of Scaled Object

4. Use the scaled dimensions to create a blueprint of your room on the *Floorplan Blueprint* Recording Sheet. If your items don't fit, consider repositioning them in your blueprint or selecting other items.

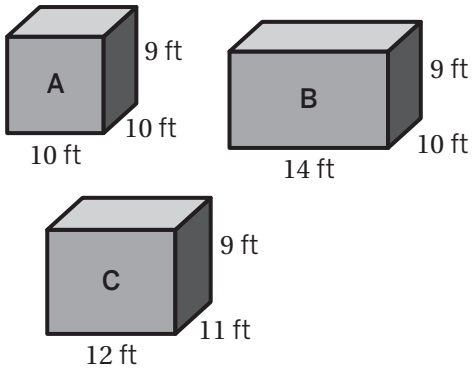
Task 2

Name: ..... Date: ..... Period: .....

Paint Time

Now that you have your blueprint planned, the next step is to plan what the walls, ceiling, and floor will look like.

Recall which room dimensions you chose for your blueprint. **Note:** All bedrooms have a ceiling height of 9 feet.

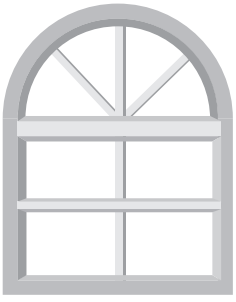


1. Each surface of the room can be represented by a rectangle. Use the Recording Sheets and the scale from Task 1 (3 units to 2 feet) to draw a rectangle to represent each wall and the ceiling of your room (you already drew the floor in your floorplan).

2. The next step is to decorate your walls. Your room will need one door and one large window.

a. Complete the table.

b. Add the items to your scale drawings on the *Wall* Recording Sheets.

Object	Dimensions of Real-Life Object	Dimensions of Scaled Object
Hallway Door	7 feet by 3 feet	
Large Window 	4 feet by 2 feet Diameter: 4 feet	

Task 2

Name: ..... Date: ..... Period: .....

Paint Time (continued)

3. Your walls need a new coat of paint.

a What color would you like to paint your walls? You may choose to use more than one color.

b Calculate the square footage that needs to be painted. **Note:** The door and window will not be painted.

4. Now consider the ceiling.

a What color would you like to paint your ceiling?

b Choose one lighting option from the Ceiling Lighting Shopping List and record it in the table.

Object	Dimensions of Object	Dimensions of Scaled Object

c Draw the lighting option on the scaled drawing of your ceiling.

d Calculate the square footage of the ceiling that needs to be painted. **Note:** The lighting will be installed prior to the painting so that area will not need to be painted.

5. The paint is sold in the size options shown.

a How much paint will you need for the walls? For the ceiling?

b What is the most cost effective way of buying the paint given the size options and your color preferences?

Size	1 Quart	1 Gallon	5 Gallon
Paint Coverage	100 sq. ft	400 sq. ft	2000 sq. ft
Container			
Cost	$24	$56	$245

6. 💬 **Discuss:**

- What was your favorite part of designing your own room?
- What are you most proud of?

Task 3

Name: ..... Date: ..... Period: .....

Gallery Tour

You will communicate the results of your Investigation by creating a poster.

1. As a class, decide on 5 class awards that will be given.

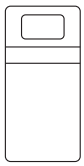
2. You will now create a poster. On your poster, be sure to include:

- Your blueprint
- A summary of your design choices
- A convincing statement of which award you should win

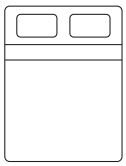
3. Take a look at your classmates' blueprints. Use the sticky notes to make comments and ask questions about your classmates' blueprints. Then, place a sticky note on blueprints for the different awards.

Floorplan Shopping List

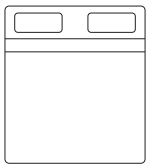
Twin Bed
40 in. by 76 in.



Queen Bed
60 in. by 80 in.



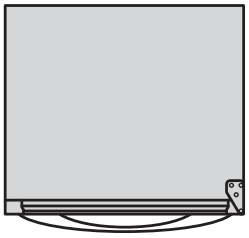
King Bed
76 in. by 80 in.



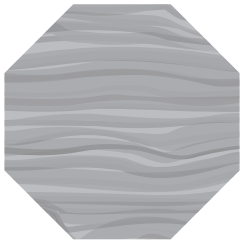
Desk
2 ft by 3 ft

Bookshelf
1 foot by 3 ft

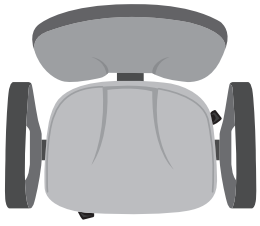
Mini fridge
20 in. by 20 in.



Accent table (octagonal)
Top to bottom: 2 ft
Left to right: 2 ft



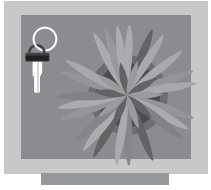
Standard Desk Chair
16 in. by 20 in.



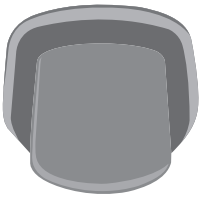
Dresser
16 in. by 44 in.

Storage Bench
16 in. by 40 in.

Nightstand
1.5 ft by 2 ft



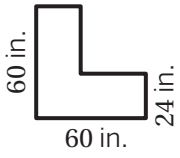
Accent Chair
32 in. by 40 in.



Sofa
84 in. by 38 in.

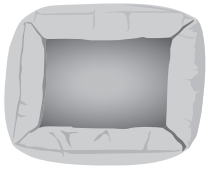
Laundry Basket (trapezoid shape)
Top: 12 in. Sides: 16 in.
Bottom: 20 in.

L-Shaped Desk
60 in. by 60 in. by 24 in.

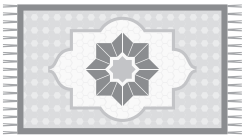


Fish Tank
4 ft by 1 ft

Dog Bed
36 in. by 24 in.

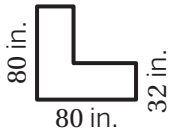


Rug
5 ft by 7 ft

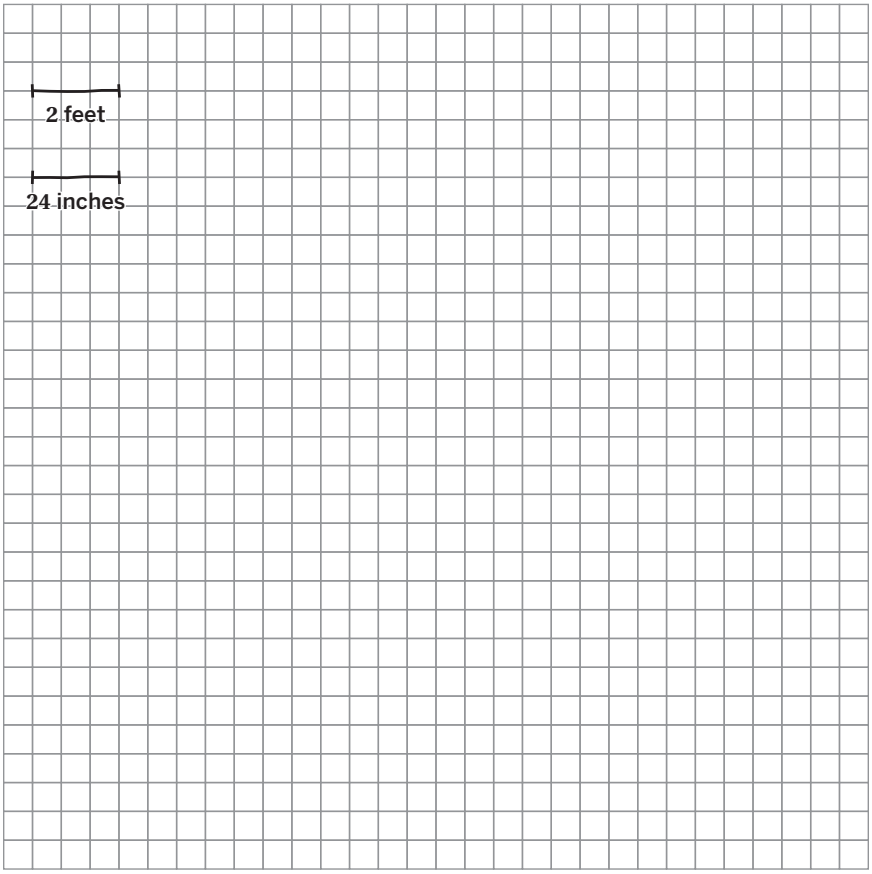


TV stand
12 in. by 48 in.

L-Shaped Couch
80 in. by 80 in. by 32 in.



Floorplan Blueprint Recording Sheet



Name: ..... Date: ..... Period: .....

Ceiling Lighting Shopping List

Recessed Lighting Radius: 4 in.	**Flush mounted lighting** Radius: 8 in.	**Chandelier** Radius (mount): 2 in.
Ceiling Fan Radius: 4 in.	**Pendant Lighting** Radius (mount): 2 in.	

Investigation 1
Information Sheet

Name: ..... Date: ..... Period: .....

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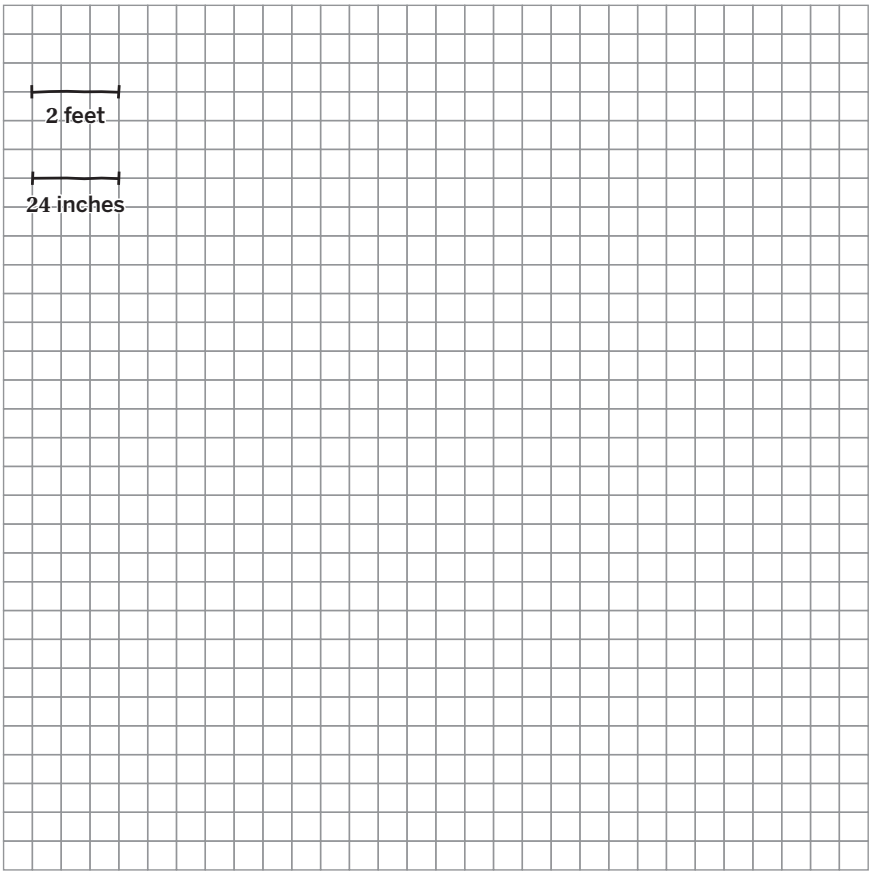
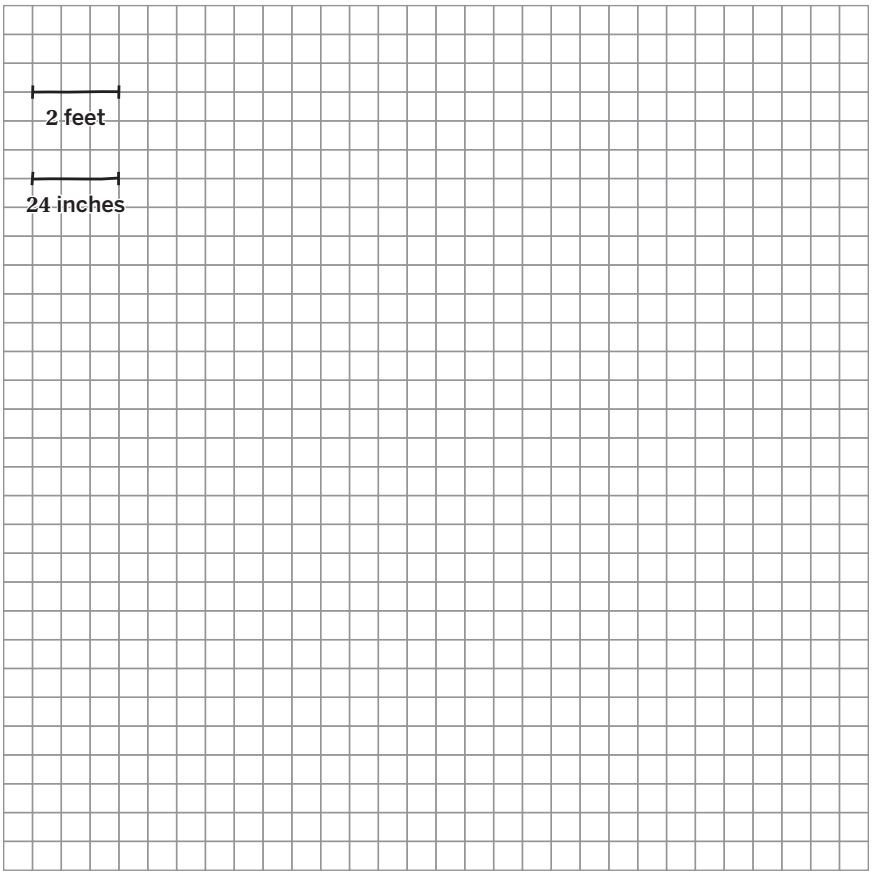
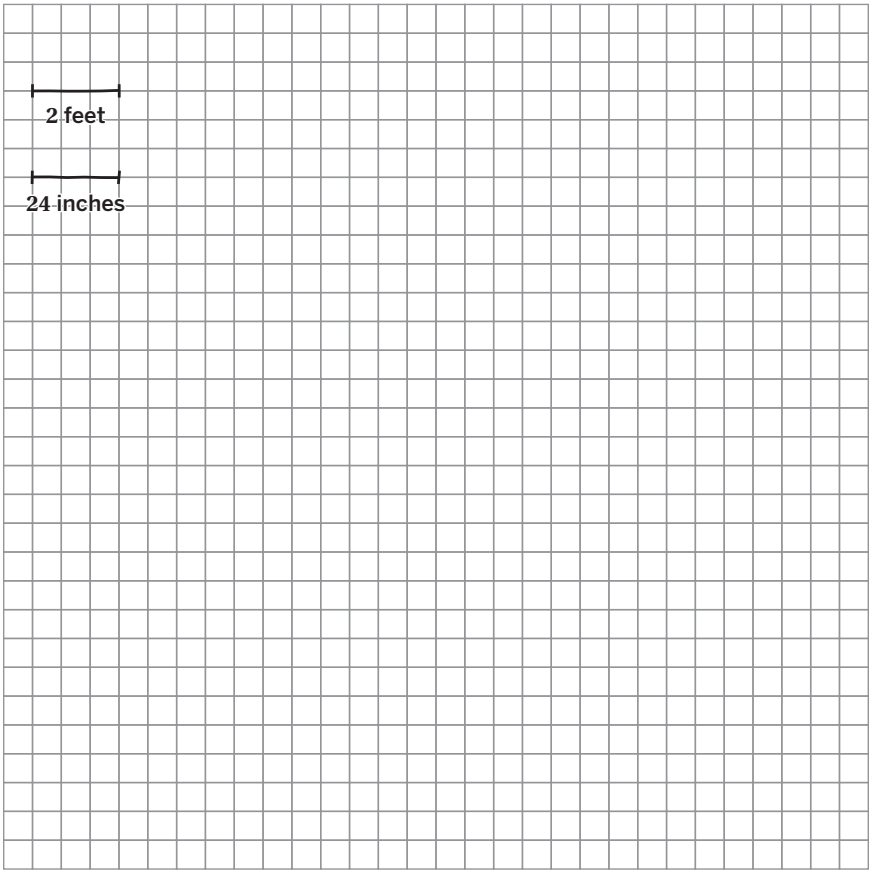
Ceiling Recording Sheet



Wall Recording Sheet 1



Wall Recording Sheet 2



Investigation 2

The California Spotted Owl

Populations and Samples | Visualize Populations | Proportional Relationships | Unit Rates in the World | Generalized Numbers Leading to Algebra | 7.SP.3, 7.SP.4, 7.EE.3, 7.RP.2.b, 7.RP.2.c, SMP.1, SMP.3, SMP.4, SMP.6, SMP.7

Name: ..... Date: ..... Period: .....

In early 2023, it was announced that the California Spotted Owl will be protected under the Endangered Species Act.

The owl lives in the Sierra Nevada forests and in the coastal and Southern California mountains.

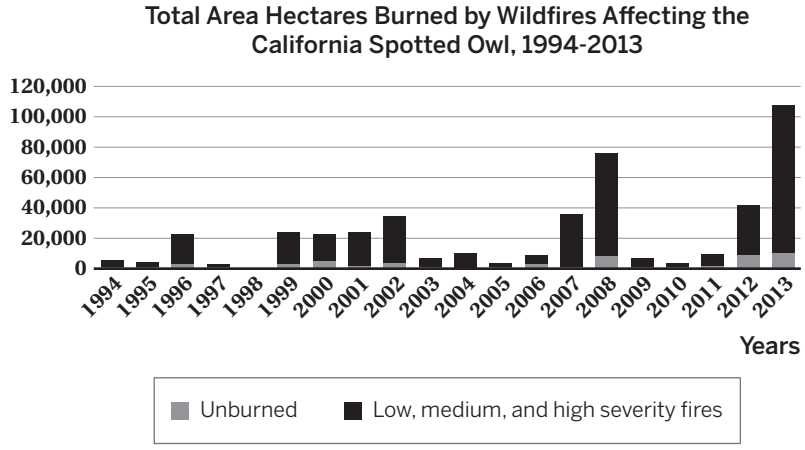
Its population has decreased by almost 50% since 1997 in the Sierra Nevada, according to the U.S. Department of Agriculture.

The owl needs older trees for nesting. Its primary threat is loss of habitat due to large-scale, high-severity wildfires.

tahirsphotography/Shutterstock.com

Sources: U.S. Fish and Wildlife Service; Forest Service. U.S. Department of Agriculture.

The graph shows the area burned by wildfires across the entire range of California Spotted Owls (CASPO) from 1994–2013.



Source: Forest Service. U.S. Department of Agriculture. Threats to the Viability of California Spotted Owls

Note: A hectare is a measure of area, equal to 10,000 square meters, or about 2.47 acres.

Data Talk! Discuss what you notice and wonder about the data.

- What do you notice about the units chosen? How can you make sense of them?
- Select a bar and describe what it means.
- Do you find any information surprising? Why?

Task 1

Name: ..... Date: ..... Period: .....

Wildfires and the California Spotted Owl (continued)

1. What are three questions you have about the California Spotted Owl that can be answered by looking for more information or data?

2. Your class will decide on 1–2 questions to answer during this Investigation. Record them here.

Question 1:

Question 2:

3. You will now investigate your questions by collecting data. Your teacher will provide you with these information and data sheets. Use the *Investigation* Recording Sheet if you find it helpful.

Data Sets	Information Sheet
California Wildfires, 1987–2023	Wildfire Causes
Causes of California Wildfires, 2001–2020	Protected Activity Centers (PACs)

Data Talk! Look at the data you collected.

- What do you notice about the data? What do you wonder?
- Are you able to answer any of your questions?
- Do you have any new questions?
- Do you need further information? What next steps can you take?

Task 2

Name: ..... Date: ..... Period: .....

Comparing Populations

1. With your group, choose one of the data sets to further explore and analyze. Circle the data set you choose.

California Wildfires, 1987–2023	Causes of California Wildfires, 2001–2020

2. Create at least two data displays to show part or all of the data your group chose. At least one of your displays should compare two populations. Use a separate sheet of paper to show more than two displays.

Data Display 1:

Data Display 2:

Task 2

Name: ..... Date: ..... Period: .....

Comparing Populations (continued)

3. For each display, answer these questions.

	Data Display 1	Data Display 2
What quantities are compared in the display? How are they measured?		
Why is the display you chose an appropriate way to summarize the data?		
Calculate any appropriate measure(s) of center. What do they mean in context?		
Calculate any appropriate measure(s) of variability. What do they mean in context?		
Do your displays show any potential outliers? If so, what do they mean?		
For the two populations you compared, how do they compare? Use the center, spread, and visual overlap to help you.		

Data Talk! Look at your data displays.

- Do your displays help you answer any of your questions? Why or why not?
- What other questions do you want to explore or data you might want to collect? Why?

Task 3

Name: ..... Date: ..... Period: .....

Gallery Tour

You will now communicate the results of your Investigation.

Your group will prepare a poster about the results of your Investigation and participate in a Gallery Tour.

Use the Investigation Presentation Checklist if you find it helpful.

Connecting to Your Community Think about your community or its surrounding area.

Investigate your area. Here are some questions that may inspire your investigation.
Or think of your own questions!

- What are some threatened or endangered species in your area?
- What is one species that most interests you? Why?
- Why might the species you selected be threatened or endangered?
- What has been done to help the species you chose?
- Are there other things you think may be able to help? If so, what are those things?

California Wildfires, 1987–2023

Year	Number of Fires	Acres Burned	Year	Number of Fires	Acres Burned
1987	13,476	873,000	2006	7,855	863,345
1988	13,290	345,000	2007	6,043	1,520,362
1989	10,024	173,400	2008	6,255	1,593,690
1990	10,548	365,200	2009	7,010	451,969
1991	9,609	44,200	2010	6,394	134,462
1992	12,047	282,745	2011	7,732	228,599
1993	8,689	309,779	2012	7,041	829,224
1994	10,269	526,219	2013	8,889	601,635
1995	8,492	209,815	2014	7,233	625,540
1996	10,610	752,372	2015	8,283	880,899
1997	9,502	283,885	2016	6,959	669,534
1998	7,572	215,412	2017	9,270	1,548,429
1999	11,125	1,172,850	2018	7,948	1,975,086
2000	7,622	295,026	2019	7,148	277,285
2001	9,317	377,340	2020	8,648	4,304,379
2002	8,171	538,216	2021	7,396	2,569,386
2003	8,287	965,770	2022	7,477	331,358
2004	7,898	311,024	2023	7,127	319,956
2005	7,237	279,214			

Causes of California Wildfires, 2001–2020

The table shows the number of lightning-caused wildfires and human-caused wildfires in California from 2001 to 2020, along with the total area burned each year.

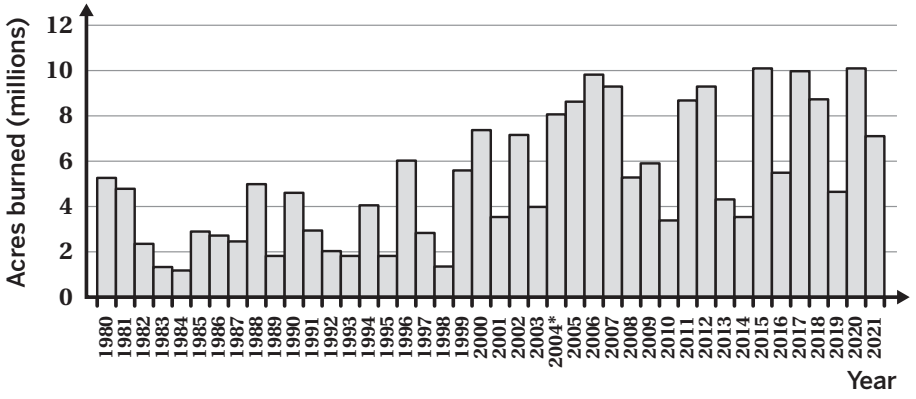
	Lightning-Caused				Human-Caused			
	Northern California		Southern California		Northern California		Southern California	
Year	Fires	Acres	Fires	Acres	Fires	Acres	Fires	Acres
2001	430	185,212	124	135,689	1,060	101,240	4,099	101,240
2002	420	42,688	76	5,661	3,789	39,560	4,060	412,447
2003	174	45,624	131	4,812	3,795	96,415	3,929	653,016
2004	728	3,689	188	8,333	3,613	146,720	3,845	84,075
2005	97	25,417	96	79,450	3,010	37,658	3,781	61,728
2006	785	174,654	397	24,232	3,676	146,999	3,166	342,864
2007	477	55,394	259	43,614	3,093	153,154	5,140	855,978
2008	596	852,133	274	26,140	3,407	91,022	5,208	454,249
2009	180	49,414	266	9,545	3,677	57,997	4,412	296,429
2010	301	12,973	258	16,660	2,502	22,701	3,394	67,236
2011	441	4,149	216	24,402	2,791	20,051	4,633	80,427
2012	890	592,668	179	19,375	3,356	178,818	4,146	80,539
2013	1,400	13,517	174	32,091	4,703	151,677	4,334	380,390
2014	574	292,861	291	6,136	3,605	181,965	3,527	74,082
2015	948	285,286	409	207,935	3,802	308,762	3,778	96,990
2016	186	3,007	272	14,489	3,266	93,699	3,900	464,718
2017	670	329,253	323	63,233	3,445	343,195	5,201	532,640
2018	966	92,487	428	24,621	3,428	1,404,463	4,322	324,102
2019	301	24,316	179	18,686	3,284	190,426	4,556	36,416
2020	2,238	1,549,012	832	216,492	4,248	1,549,012	5,295	216,492

Wildfire Causes

According to the Congressional Research Service, most wildfires in the United States are caused by human activity, about 89% of the average number of wildfires from 2018 to 2022. Natural causes of wildfires are typically the result of lightning. Wildfires that are caused by human activity typically result from . . .

- Unattended campfires
- Debris and open burning
- Improper disposal of smoking materials
- Equipment or vehicle use and malfunction near areas of dry grass
- Fireworks
- Arson
- Power generation, transmission, and/or distribution

The number of wildfires and acres burned have been increasing in the United States since 1980. The graph shows the number of millions of acres burned in the U.S. from wildfires from 1980–2021.



Source: National Interagency Fire Center, *2004 fires do not include state lands for North Carolina

In the western U.S., the largest fires occur in years with warm spring and summer temperatures and early spring dates for when the snow begins to melt. The table shows the average large wildfire burn time during different periods of time. The average number of days during the fire season each year was 84 days longer from 2003–2014 than in 1973–1982.

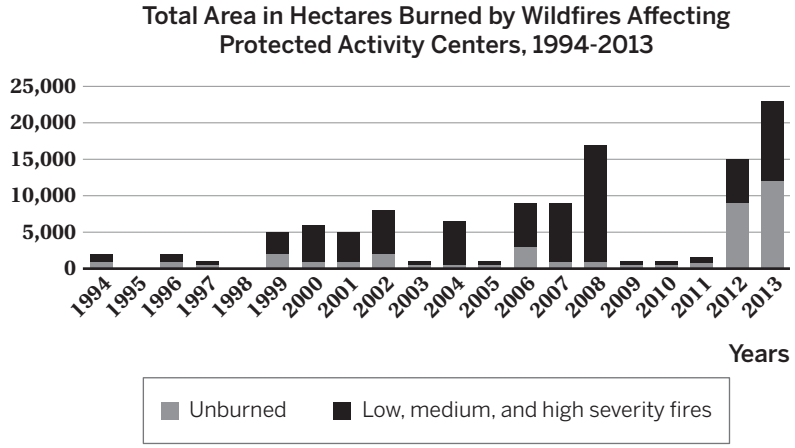
Years	Average large wildfire burn time, days
1973–1982	6
1983–1992	20
1993–2002	37
2003–2012	52

According to the Lawrence Livermore National Laboratory (LLNL), located about 50 miles from San Francisco, summer wildfires in California continue to break their own records. From 1996 to 2021, the average area burned by summer wildfires was five times greater than the years from 1971 to 1995.

Protected Activity Centers

Protected Activity Centers (PACs) were established by biologists to help protect the owl. Each center has a minimum area of 121 hectares, or about 300 acres. The PACs are designed to have the best quality habitat for the owls, which include large, mature trees near an existing nesting site.

The graph shows the area burned by wildfires in protected activity centers from 1994–2013.



Source: Forest Service. U.S. Department of Agriculture. Threats to the Viability of California Spotted Owls

Note: A hectare is a measure of area, equal to 10,000 square meters, or about 2.47 acres.

The California Spotted Owl

Record any data you find helpful on this sheet.

Source	Data that might be helpful	What question might the data help answer?

Display Checklist

Use this checklist to help you plan and organize your poster. You should include these items. Check them off as you complete them.

Your group's answers to the questions you decided on as a class.

An explanation of how the data you chose supports your answers.

At least 2 statistical displays of the data. For each display . . .

- ☐ Show the distribution of the data. Why are the displays you chose appropriate ways to summarize the data?
- ☐ Calculate appropriate measures of center and variability and interpret them in context.
- ☐ Indicate any potential outliers and what they mean in context.

At least one of your displays should compare two populations.

- ☐ Which populations did you compare? Why did you compare them?
- ☐ Calculate appropriate measures of center and variability and interpret them in context.
- ☐ Describe how the populations compare using the centers, spreads, and visual overlap of the distributions.
- ☐ Indicate any potential outliers and what they mean in context.

Your group's reflection on your experience during this Investigation. Consider these questions, if they are helpful.

- ☐ Did you find any of the data surprising? Why?
- ☐ What other questions might you want to explore?
- ☐ What other data might you want to collect?