Amplify Desmos Math CALIFORNIA

Algebra 1

Intervention, Extension, and Investigation Resources

Contents

Mini-Lessons

Log in to the digital curriculum to review all program Mini-Lessons, Extensions, and Investigations.

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 Extensions

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Investigations

Mini-Lessons

Prerequisite Skills and Concepts

Mini-Lessons

Name _____

Determining the Constant of Proportionality

ML 2.03

Modeled Review

Name: Clare

Determine the constant of proportionality for the relationship and complete the table. What does the constant of proportionality tell you about the situation?

The constant of proportionality is 5.

The paint is made with 5 cups of yellow paint for every 1 cup of blue paint.

$2 \cdot 5 = 10$

$6 \cdot 5 = 30$

Blue paint (cups)	Yellow paint (cups)
2 ×5→	10
1	5
6 ×5→	30
52	260

Guided Practice

Determine the constant of proportionality for the relationship and complete the table.

1. $8 \cdot 3 = 24$

$12 \cdot \_\_\_\_ = \_\_\_\_$

$128 \cdot \_\_\_\_ = \_\_\_\_$

The constant of proportionality is _____.

Lemonade

	Volume (oz)	Sugar (g)
glass	8 ×3→	24
bottle	12	
carton	32	96
jug	128	

2. $8 \cdot \_\_\_\_ = \_\_\_\_$

$12 \cdot \_\_\_\_ = \_\_\_\_$

$128 \cdot \_\_\_\_ = \_\_\_\_$

The constant of proportionality _____.

Apple juice

	Volume (oz)	Sugar (g)
glass	8	
bottle	12	
carton	32	80
jug	128	

Guided Practice

Determine the constant of proportionality for the relationship and complete the table. What does the constant of proportionality tell you about the situation?

3. The constant of proportionality is_____.

The paint is made with _____ cups of blue paint for every _____ cup of red paint.

Red paint (cups)	Blue paint (cups)
2	3
1	
6	
40	60

4. The constant of proportionality is_____.

Red paint (cups)	Yellow paint (cups)
2	8
1	
	32
50	

Check

Determine the constant of proportionality for the relationship and complete the table. What does the constant of proportionality tell you about the situation?

White paint (cups)	Red paint (cups)
2	4
1	
7	
	80

Name _____

Writing Equivalent Expressions Using the Area Model

ML 6.08

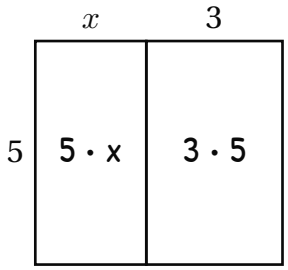
Modeled Review

Name: Jada

Write *two* equivalent expressions that represent the area of the rectangle.

Expression 1: 5(*x* + 3)

Expression 2: 5*x* + 15



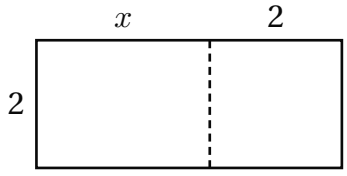
5 times *x* + 3 or 5(*x* + 3)

5 · *x* + 5 · 3 or 5 · (*x* + 3)
5*x* + 15

Guided Practice

Write *two* equivalent expressions using the area of a rectangle.

1.



$2 \cdot (x + \_\_\_)$

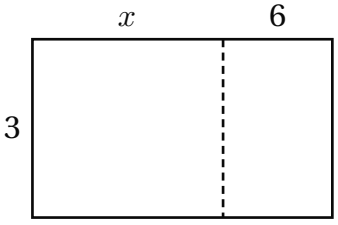
$2 \cdot x + \_\_\_\_$

$\_\_\_\_ + \_\_\_\_$

Expression 1: _____

Expression 2: _____

2.



$3 \cdot (\_\_\_ + \_\_\_)$

$\_\_\_\_ + \_\_\_\_$

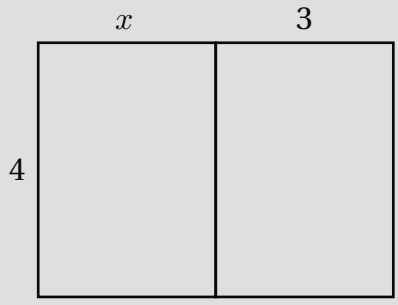
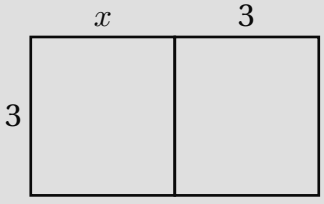
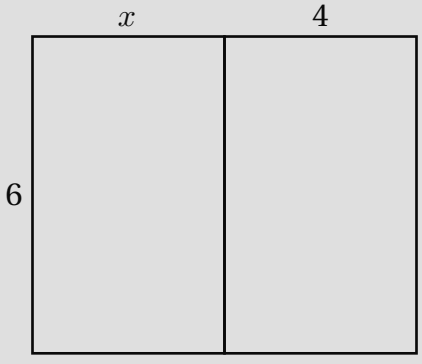
$\_\_\_\_ + \_\_\_\_$

Expression 1: _____

Expression 2: _____

Guided Practice

3. Write *two* equivalent expressions that represent the area of each rectangle.

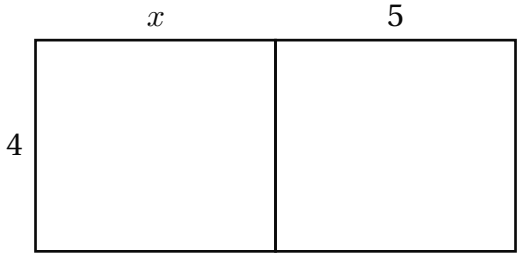
Area Model	Expression 1	Expression 2
	$4(x+3)$	
		
		

Check

Write *two* equivalent expressions that represent the area of the rectangle.

Expression 1: _____

Expression 2: _____



Name _____

Evaluating Expressions With Exponents

ML 6.11

Modeled Review

Name: Kai

Determine the value of each expression.

1. $2 \cdot 3^2$ = 18

$2 \cdot (3 \cdot 3)$
$2 \cdot (9)$
18

I need to evaluate the part of the expression with the exponent first and then multiply.

2. $5 + (4 - 3)^2$ = 6

$5 + (1)^2$
$5 + (1 \cdot 1)$
$5 + 1$
6

I need to evaluate the grouped part of the expression first, then the exponent part. Then I can add.

Guided Practice

Determine the value of each expression.

1. 2^2

$2 \cdot$ ___

_____

2. $2 + 3^2$

$2 + ($___ $\cdot$ ___$)$

___ $+$ ___

_____

3. $(6 - 2)^2$

(___)^2

(_____)

_____

4. $1 + (3 - 2)^2$

$1 + ($___$)^2$

_____

_____

_____

Guided Practice

5. Determine the value of each expression.

Expression	Value
$3^2 + 6$	
$(4 + 3)^2$	
$3 + (4 + 1)^2$	
$2 \cdot 4^2$	
$(7 - 1)^2 + 2$	
$6 + 7^2$	

Check

Determine the value of each expression.

1. $3 \cdot 5^2$ _____

2. $7 + (5 - 2)^2$ _____

Name _____

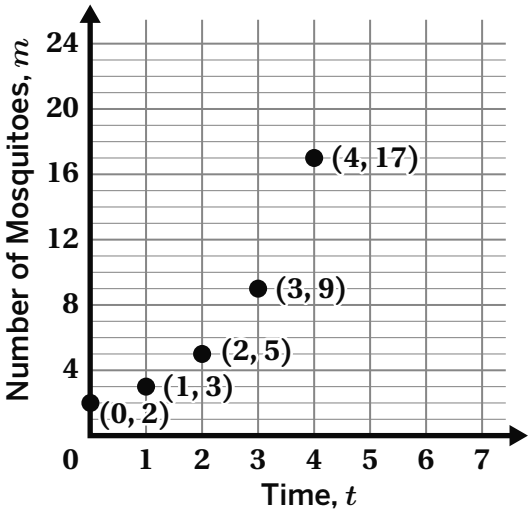
Interpreting Graphs

ML 6.14

Modeled Review

Name: Shawn

The graph represents the relationship between time, t, and the number of mosquitoes, m. Complete the table so it reflects the values in the graph.

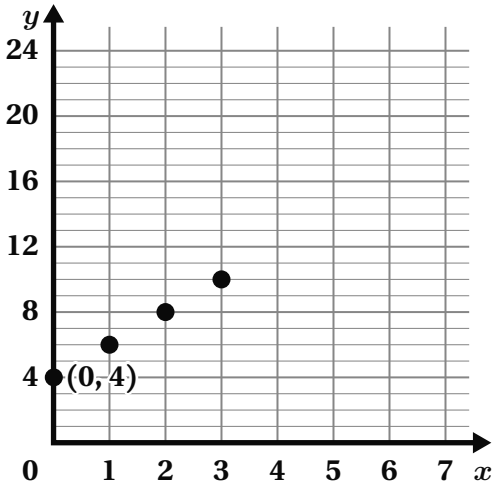


t	m
0	2
1	3
2	5
3	9
4	17

Time is the independent variable, so it represents the x-values. The number of mosquitos is the dependent variable, so it represents the y-values.

Guided Practice

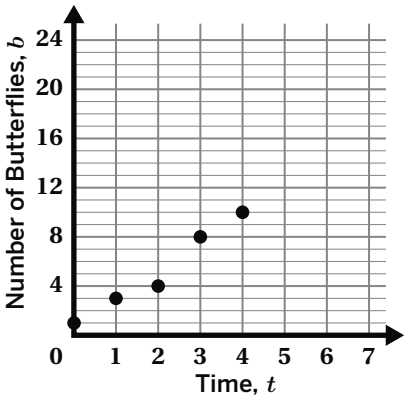
1. The graph represents the relationship between the independent variable, x, and the dependent variable, y. Complete the table so it reflects the values in the graph.



x	y
0	4

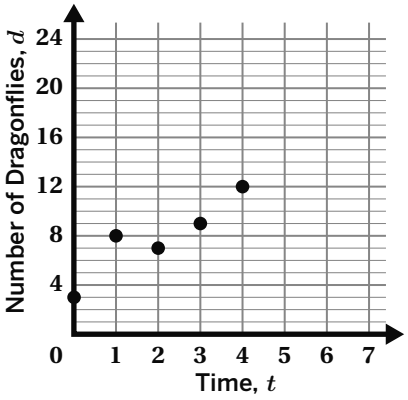
Guided Practice

2. This graph represents the relationship between time, t, and the number of butterflies, b. Complete the table so it reflects the values in the graph.



t	b

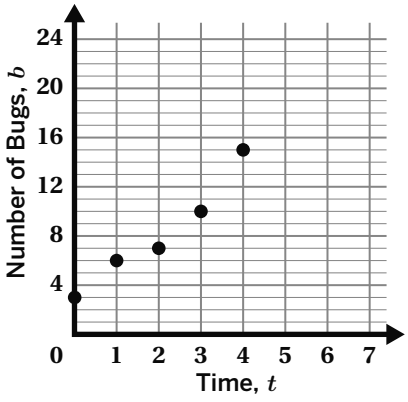
3. This graph represents the relationship between time, t, and the number of dragonflies, d. Complete the table so it reflects the values in the graph.



t	d

Check

This graph represents the relationship between time, t, and the number of bugs, b. Complete the table so it reflects the values in the graph.



t	b

Name _____

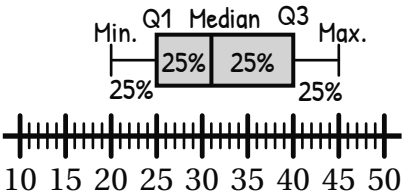
Interpreting Box Plots

ML 8.14

Modeled Review

Name: Santiago

Inola took the bus to school most days in January. She wrote down how many minutes her journey took each day and made this box plot.



1. Determine the median, IQR, and range for this data.

median: 30 **IQR:** 15 **range:** 25

40 – 25 = 15 45 – 20 = 25

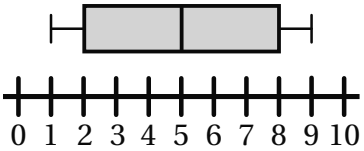
2. What percent of Inola's journey to school took 40 minutes or less?

A. 25% **B.** 50% **C.** 75% **D.** 100%

Guided Practice

For Problems 1–2, refer to the box plots to identify the statistics of the data set.

1. Mia measured the number of books she read each day for a week in June.



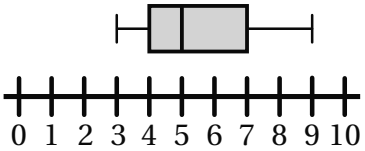
Number of books

Min.	Q1	Median	Q3	Max.
1	2			

What percent of the data was more than **5** books?

A. 25% **B.** 50%
C. 75% **D.** 100%

2. Zoe tracked the number of apples she ate each day for a week in January.



Number of apples

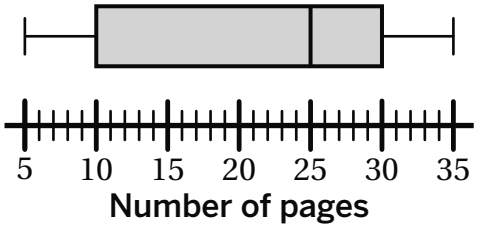
Min.	Q1	Median	Q3	Max.

What percent of the data was between **4** and **5** apples?

A. 25% **B.** 50%
C. 75% **D.** 100%

Guided Practice

Eliza tracked the number of pages she read each day for a week. She wrote down the amounts and created this box plot.



3. Determine the median, IQR, and range for this data.

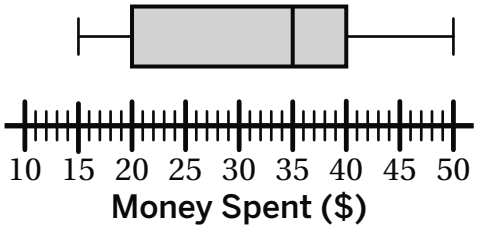
median: _____ **IQR:** _____ **range:** _____

4. On what percent of days did Eliza read less than 25 pages?

A. 25% **B.** 50% **C.** 75% **D.** 100%

Check

Evan tracked the amount of money he spent on clothes in January. He wrote down the amounts and created this box plot.



1. Determine the median, IQR, and range for this data.

median: _____ **IQR:** _____ **range:** _____

2. What percent of Evan's clothes cost at least $15?

A. 25% **B.** 50% **C.** 75% **D.** 100%

Name _____

Calculating Percentages

ML 4.01

Modeled Review

Name: Dylan

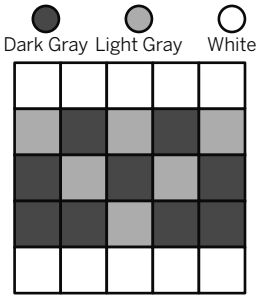
What percentage of the larger grid is white? Explain your thinking.

I counted 25 squares. Each square has a value of 4% because 100 divided by 25 is 4. There are 10 white squares. I can multiply 4 by 10 to get 40%.

$$\frac{100}{25} = 4\%$$

$$4 \cdot 10 = 40$$

$$40\%$$



Guided Practice

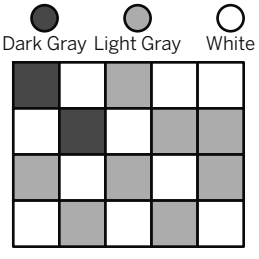
1. What percentage of the grid is light gray or white? Show your thinking.

Light Gray:

$\frac{100}{20} =$ _____

White:

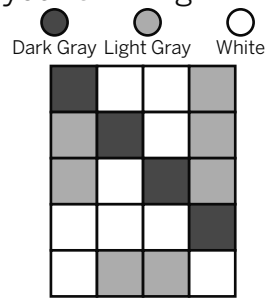
Color	Percentage
Dark Gray	10%
Light Gray	
White	



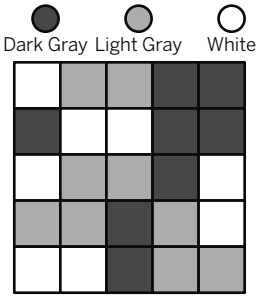
Guided Practice

Determine the percentages of the grids that are shaded each color.

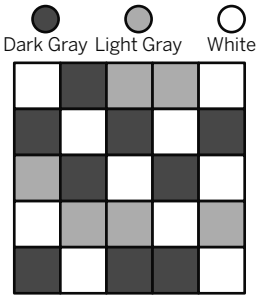
2. What percentage of the grid is dark gray? Show your thinking.



3. What percentage of the grid is white? Show your thinking.

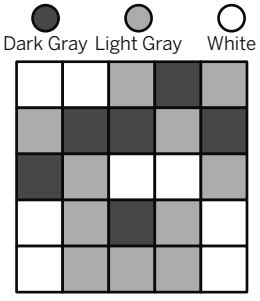


4. What percentage of the grid is light gray? Show your thinking.



Check

What percentage of the grid is white? Show your thinking.



Name _____

Solving Equations With Positive and Negative Numbers

ML 6.07

Modeled Review

Name: Jack

Solve the equation. Show your thinking.

$-4(x-2)=12$

$\frac{-4(x-2)}{-4} = \frac{12}{-4}$

$x - 2 = -3$

$+2 \quad +2$

$x = -1$

Check:

$-4(x-2) = 12$

$-4(-1-2) = 12$

$-4(-3) = 12$

$12 = 12$

Guided Practice

1. Solve the equations by completing the blanks in the equations and descriptions.

Equation	Moves
$-2x+3=7$ $-2x=\_\_\_\_$ $x=\_\_\_\_$	**Step 1:** Subtract ____ from each side. **Step 2:** Divide each side by ____.
$2(x+1)=6$ $x+1=\_\_\_\_$ $x=\_\_\_\_$	**Step 1:** _____ each side by ____. **Step 2:** _____ from each side.

Guided Practice

For Problems 2–5, solve the equation. Show your thinking.

2. $-4x - 2 = 10$

3. $3(x - 2) = 9$

4. $-3x + 5 = -1$

5. $-4(x + 3) = 20$

Check

Solve each equation. Show your thinking.

1. $-4x - 3 = 13$

2. $-3(x - 4) = 27$

Name _____

Solving Inequalities

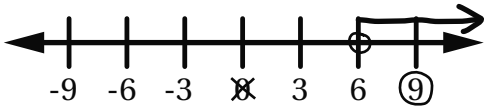
ML 6.16

Modeled Review

Name: Priya

Solve and graph the solution to $-2x + 9 < -3$.

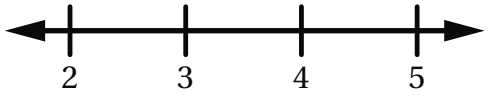
x > 6



	x < 6	x > 6
-2x + 9 = -3	-2(0) + 9 < -3	-2(9) + 9 < -3
-9 -9	0 + 9 < -3	-18 + 9 < -3
-2x = -12	9 < -3	-9 < -3
-2 -2	False	True
x = 6		

Guided Practice

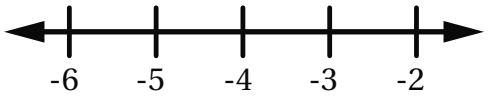
1. Graph the solution to the inequality $3x \geq 9$ by finding the boundary point and testing values on both sides of the boundary point.

Moves	Work
Move 1: Find the boundary point.	$3x = 9$ $x =$ ____
Move 2: Test the values on both sides of the point and determine whether the statement is true or false.	**Less than or equal to** $3(\_\_\_\_) \geq 9$ $\_\_\_\_ \geq 9$ _____ **Greater than or equal to** $3(\_\_\_\_) \geq 9$ $\_\_\_\_ \geq 9$ _____
Move 3: Graph the solution.	x ____ 3 

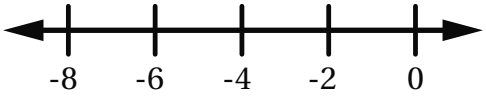
Guided Practice

Solve and graph the solution to each of the inequalities.

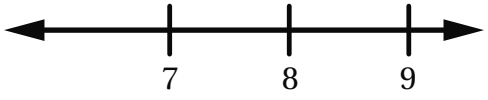
2. $2x + 7 \leq -1$ _____



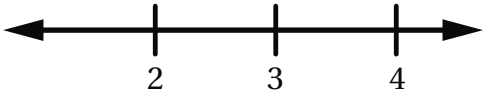
3. $-3x - 3 < 15$ _____



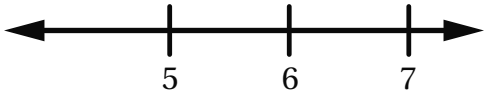
4. $2x - 4 \geq 10$ _____



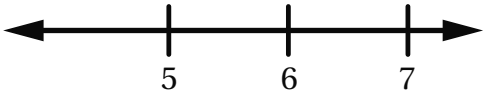
5. $-6x + 6 > -12$ _____



6. $4x - 5 \geq 15$ _____



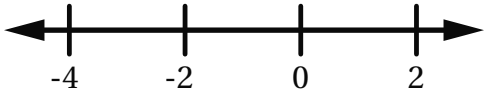
7. $-5x + 6 > -24$ _____



Check

Solve and graph the solution to $-4x - 3 < 5$.

_____



Name _____

Calculating Slope By Drawing Triangles on a Coordinate Plane

ML 2.10

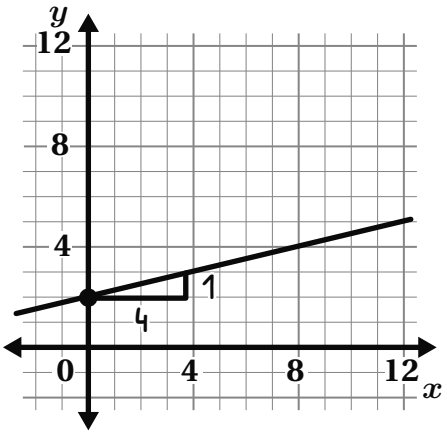
Modeled Review

Name: Priya

Determine the slope of the line. Show your thinking.

The slope of the line is the ratio between the height of the triangle to its base.

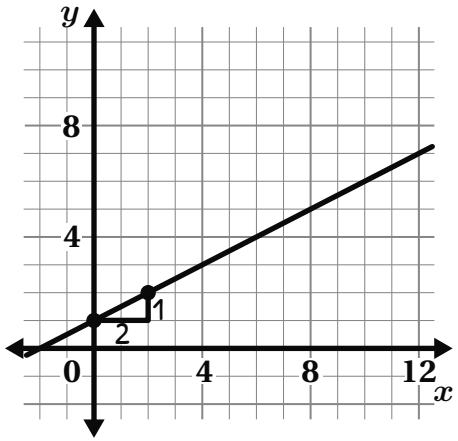
Slope is $\frac{1}{4}$.



Guided Practice

Determine the slope of the line using a slope triangle.

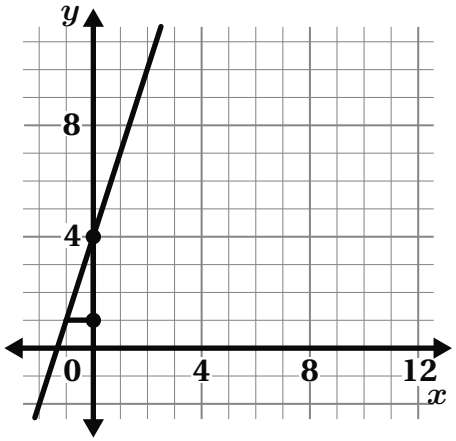
1.



slope = $\frac{\textit{height of slope triangle}}{\textit{base of slope triangle}} = \frac{\square}{\square}$

_____

2.



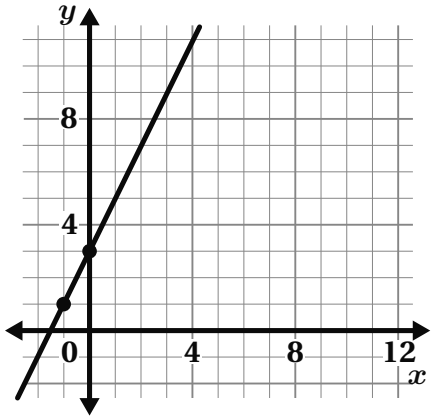
slope = $\frac{\textit{height of slope triangle}}{\textit{base of slope triangle}} = \frac{\square}{\square} = \square$

_____

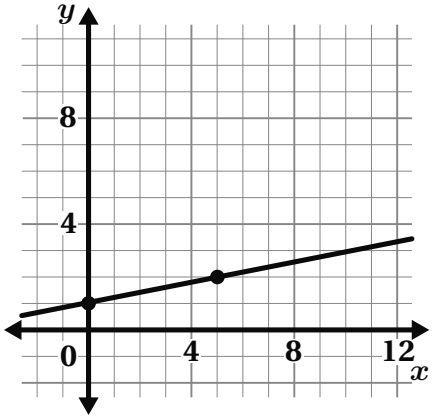
Guided Practice

Determine the slope of the line. Show your thinking.

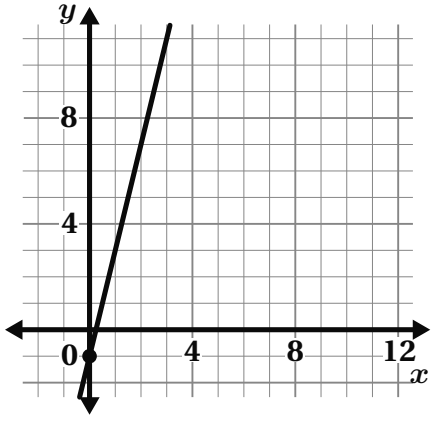
3.



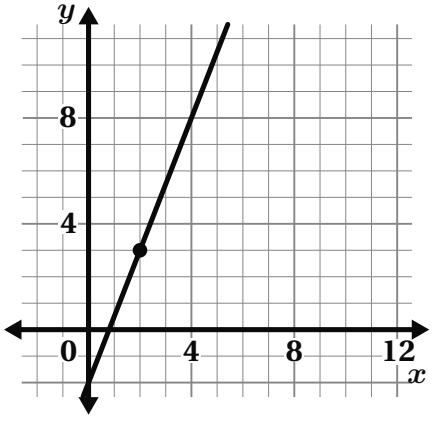
4.



5.

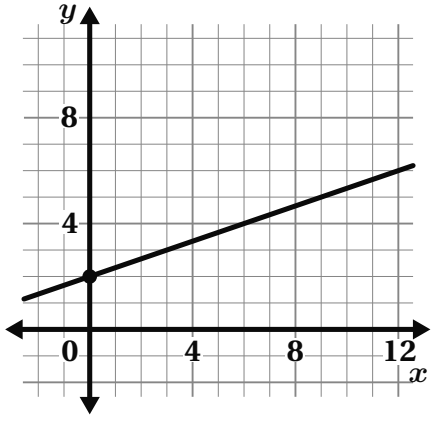


6.



Check

Determine the slope of the line. Show your thinking.



Name _____

Interpreting Slope and Intercepts of Linear Relationships

ML 3.05

Modeled Review

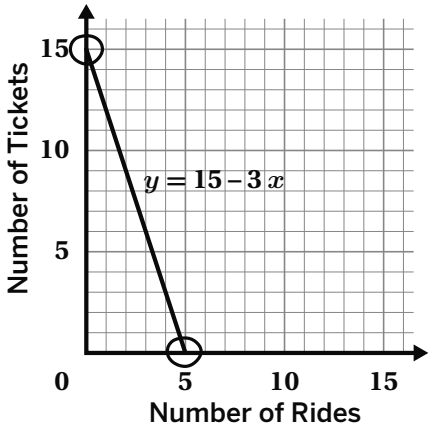
Name: Jack

The line $y = 15 - 3x$ represents Marc purchasing tickets at a carnival to go on rides.

1. Write the vertical intercept as a coordinate pair. What does it represent in this situation?
   (0,15); Marc started with 15 tickets.

2. Write the horizontal intercept as a coordinate pair. What does it represent in this situation?
   (5, 0); Marc went on 5 rides and then ran out of tickets.

3. What is the slope and what does it represent?
   -3; Marc uses 3 tickets for each ride.



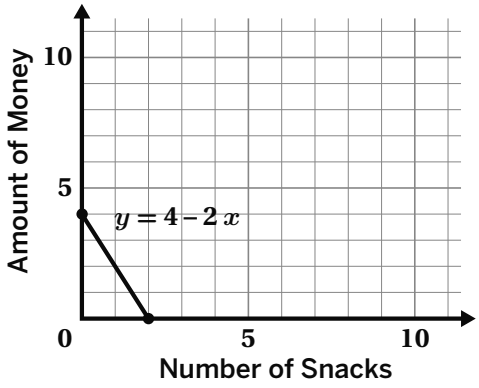
Guided Practice

The line $y = 4 - 2x$ represents Mia purchasing snacks at the theater.

1. Write the vertical intercept as a coordinate pair. What does it represent in this situation?
   (0,__);

2. Write the horizontal intercept as a coordinate pair. What does it represent in this situation?
   (0);

3. What is the slope, and what does it represent?
   __; This means that Mia's _____ _____at a constant rate of _____



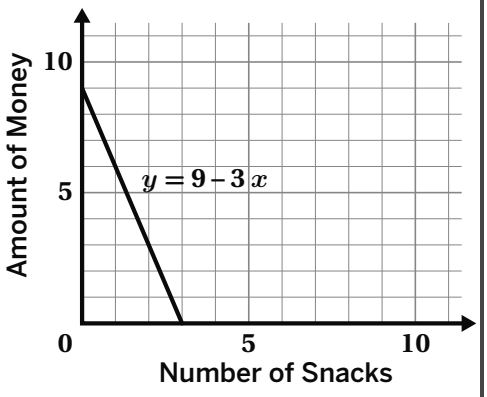
Guided Practice

The line $y = 9 - 3x$ represents Kai purchasing snacks at a soccer game.

4. Write the vertical intercept as a coordinate pair. What does it represent in this situation?

5. Write the horizontal intercept as a coordinate pair. What does it represent in this situation?

6. What is the slope, and what does it represent?



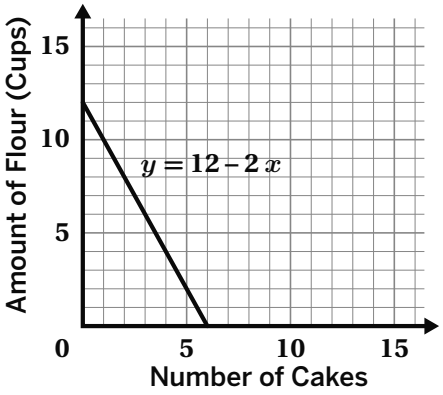
Check

The line $y = 12 - 2x$ represents Amy using flour to make cakes.

1. Write the vertical intercept as a coordinate pair. What does it represent in this situation?

2. Write the horizontal intercept as a coordinate pair. What does it represent in this situation?

3. What is the slope, and what does it represent?



Name _____

Calculating Slope Given Two Points

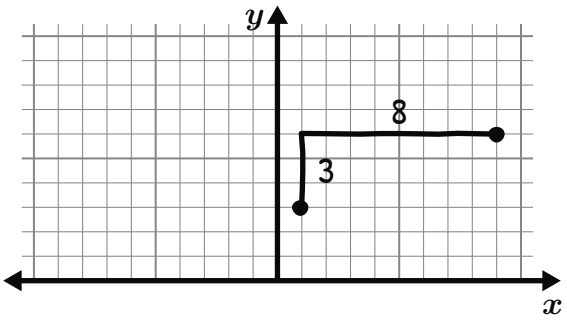
ML 3.09

Modeled Review

Name: Han

Calculate the slope of the line that passes through $(1, 3)$ and $(9, 6)$. Use the graph if it is helpful.

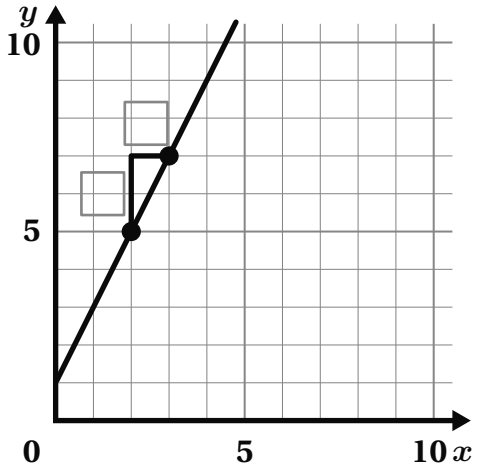
slope $= \frac{3}{8}$



Guided Practice

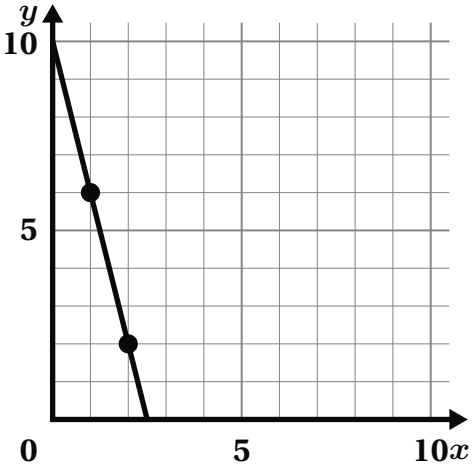
Calculate the slope of the line that passes through the given points.

1.



slope: _____

2.



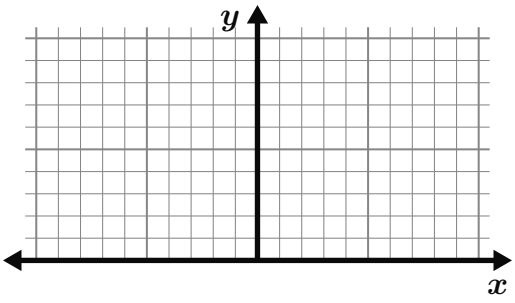
slope: _____

Guided Practice

Calculate the slope of the line that passes through the given points. Use the graph if it is helpful.

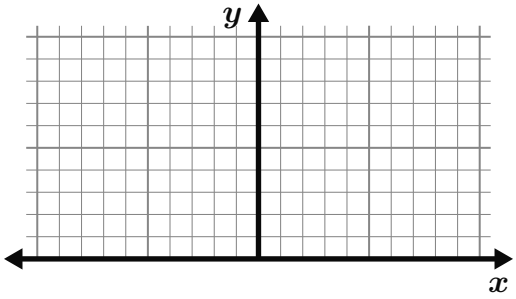
3. $(2, 4)$ and $(3, 1)$

slope: _____



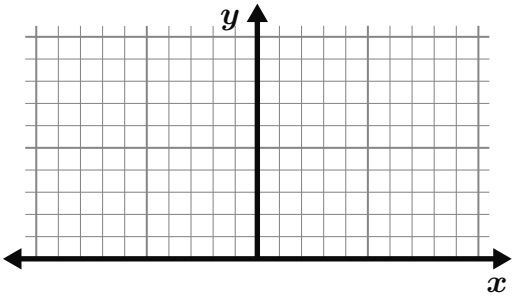
4. $(-1, 2)$ and $(4, 4)$

slope: _____



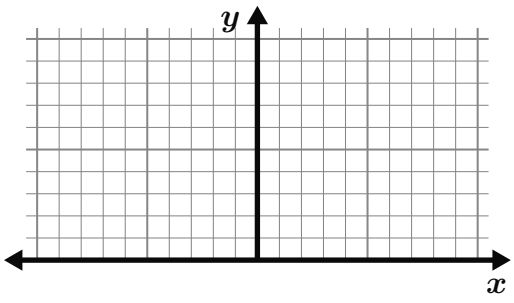
5. $(4, 2)$ and $(8, 5)$

slope: _____



6. $(6, 3)$ and $(9, 1)$

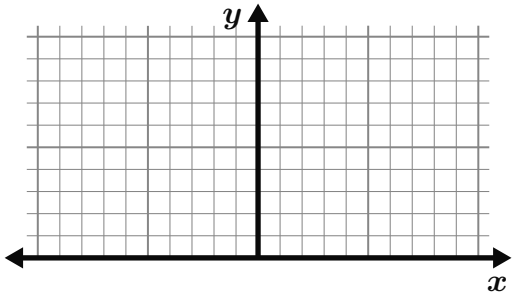
slope: _____



Check

Calculate the slope of the line that goes through $(2, 1)$ and $(4, 6)$. Use the graph if it is helpful.

slope: _____



Name _____

Solving Linear Equations With Parentheses

ML 4.04

Modeled Review

Name: Kayla

Solve the equation.

$-3(2x-4)=18$

$-3(2x-4) = 18$
$-6x + 12 = 18$
$-12 \quad -12$
$-6x = 6$
$x = -1$

Name: Isaiah

Solve the equation.

$-3(2x-4)=18$

$\underline{-3(2x-4)} = \underline{18}$
$-3 \qquad -3$
$2x - 4 = -6$
$+4 \quad = +4$
$2x = -2$
$x = -1$

Guided Practice

Solve each equation. Complete the missing steps.

1. $4x + 9 = 5x + 7$

$-4x \qquad -4x$

$9 = \_\_\_ + 7$

$-7 \qquad -7$

$\_\_\_ = x$

solution: $x =$ ___

2. $x - 8 = 10 - 5x$

$+\_\_\_ \qquad +\_\_\_$

$6x - 8 = 10$

$+\_\_\_ + \_\_\_$

$6x = 18$

$x =$ ___

solution: $x =$ ___

Solve the equation $\frac{1}{2}(4x-2)=11$ using two different methods.

3. $\frac{1}{2}(4x-2)=11$

$\times 2 \qquad \times 2$

$4x - 2 = \_\_\_$

$\_\_\_ \quad \_\_\_$

$4x = 24$

$x =$ ___

solution: $x =$ ___

4. $\frac{1}{2}(4x-2)=11$

$2x - \_\_\_ = 11$

$+ \ 1 \quad +\_\_\_$

$2x =$ ___

$x =$ ___

solution: $x =$ ___

Guided Practice

Solve each equation.

5. $4(3x + 5) = -2(-8x + 6)$

$\_\_\_\,x + 20 = \_\_\_\,x - \_\_\_$

$\_\_\_\_\_ \quad \_\_\_\_\_$

$20 = \_\_\_\,x - 12$

$\_\_\_\_\_ \quad \_\_\_\_\_$

$32 = \_\_\_\,x$

$\_\_\_ = x$

solution: $x =$ ___

6. $-2(2x + 3) = 3(x + 5)$

solution: $x =$ ___

7. $\frac{1}{3}(x - 5) = 2(x - 1)$

solution: $x =$ ___

8. $0.5(x - 4) = -3(2x + 5)$

solution: $x =$ ___

Check

Solve the equation.

$$-2(5x - 3) = 3(-4x + 8)$$

solution: $x =$ ___

Name _____

Solving Systems of Linear Equations by Graphing

ML 4.11

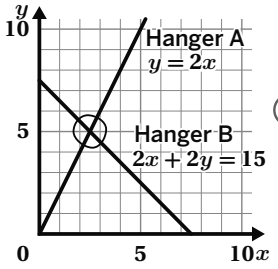
Modeled Review

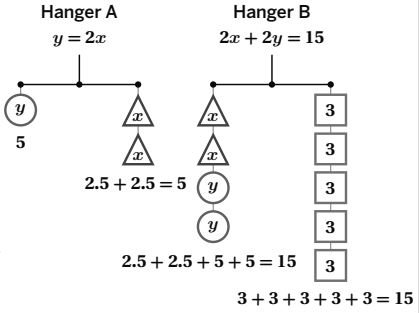
Name: Clare

Use these hanger diagrams and the related graph to complete the problems.

1. Determine the solution to the system of equations. (2.5, 5)

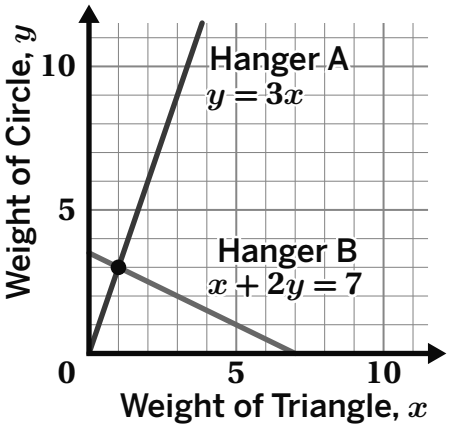
2. What does the solution tell you about the weight of the circle and the triangle to balance both hangers? Each triangle weighs 2.5 and each circle weighs 5.

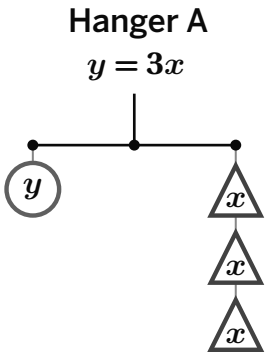


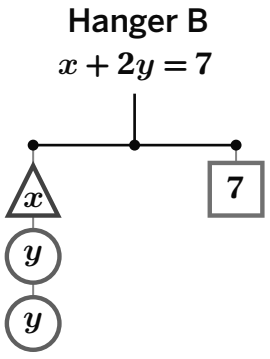


Guided Practice

Use these hanger diagrams and the related graph to complete the problems.







1. Determine the solution to the system of equations.

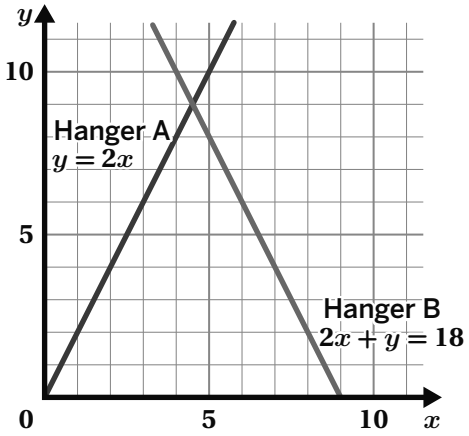
   _____

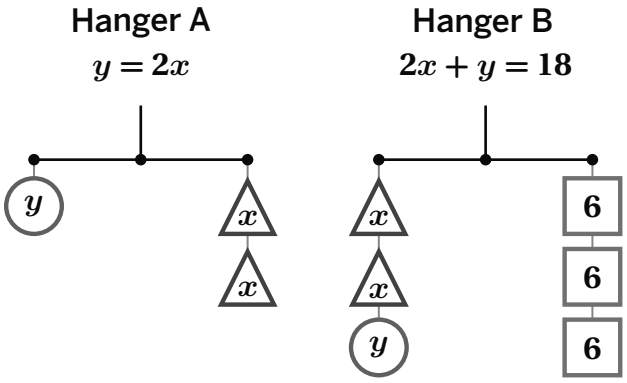
2. What does the solution tell you about the weight of the circle and the triangle to balance both hangers?

   Each triangle weighs 1 and each circle weighs ___.

Guided Practice

Use these hanger diagrams and the related graph to complete the problems.





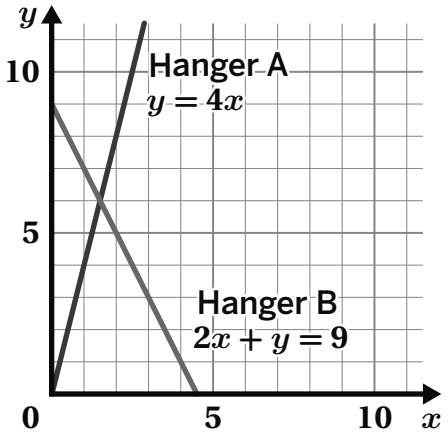
3. Determine the solution to the system of equations.

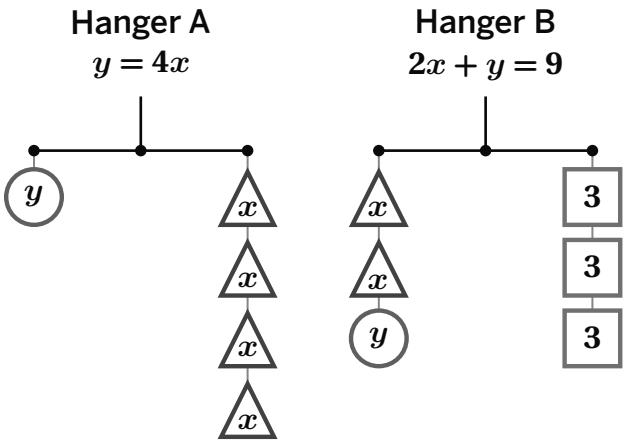
_____

4. What does the solution tell you about the weight of the circle and the triangle to balance both hangers?

Check

Use these hanger diagrams and the related graph to complete the problems.





3. Determine the solution to the system of equations.

_____

4. What does the solution tell you about the weight of the circle and the triangle to balance both hangers?

Name _____

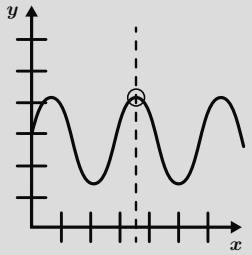
Justifying Whether a Graph Represents a Function

ML 5.03

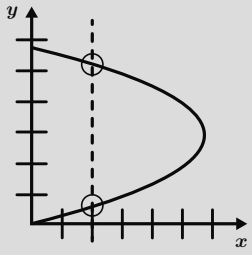
Modeled Review

Function: a rule that assigns exactly one output to each possible input.

Function



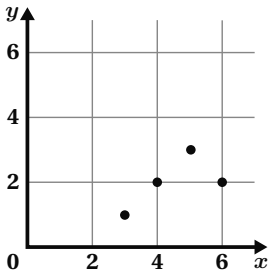
Not a function



Guided Practice

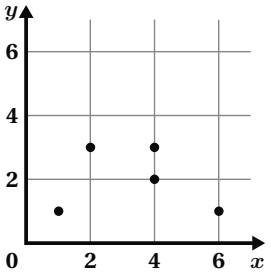
Determine whether the graph is a function. If the graph is not a function, draw a vertical line where the graph shows more than one output for the same input.

1.



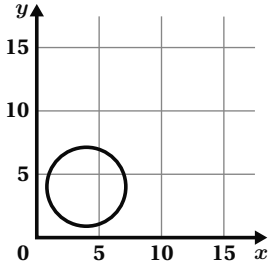
Function Not a function

2.



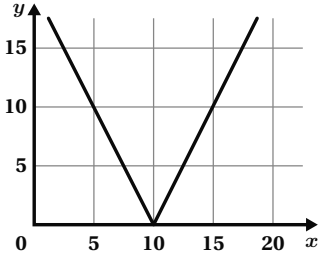
Function Not a function

3.



Function Not a function

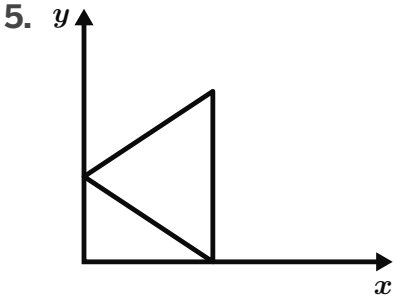
4.

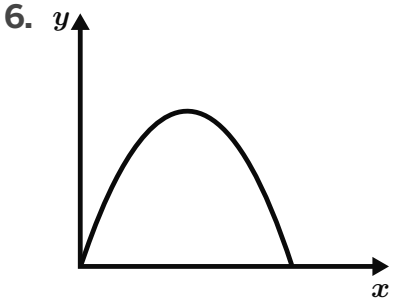


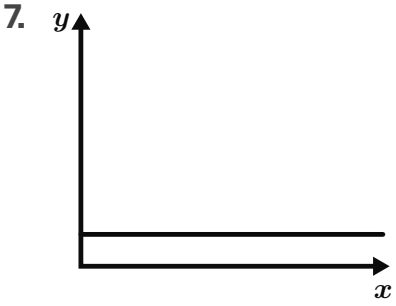
Function Not a function

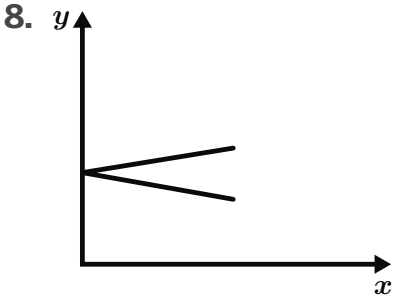
Guided Practice

Determine whether y is a function of x and write "function" or "not a function." If the graph is not a function, draw a vertical line where the graph shows more than one output for the same input.

5. 

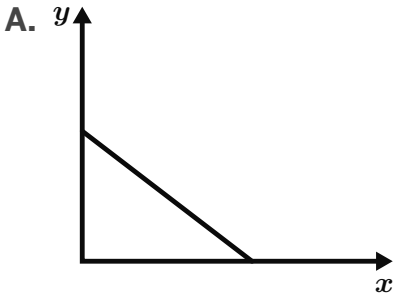
6. 

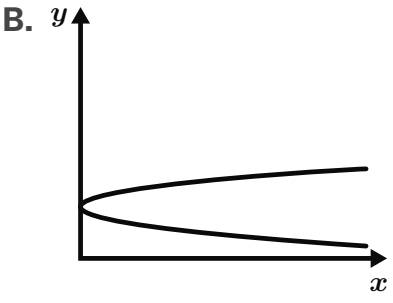
7. 

8. 

Check

Select the graph in which y is a function of x. On the graph that is not a function, draw a vertical line where the graph shows more than one output for the same input.

A. 

B. 

Name _____

Comparing Properties of Linear Functions

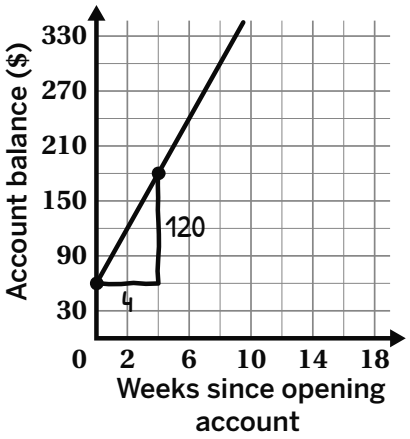
ML 5.07

Modeled Review

Name: Maya

The graph and table each show a different savings account with changes occurring at a constant rate.

Kai



Slope = $\frac{120}{4}$ = 30 y- intercept = 60

Jada

Number of weeks	Account balance ($)
1	75
2	60
3	45

+1 -15

Slope = $\frac{-15}{1}$ = -15

y = mx + b (1,75)
y = -15x + b
75 = -15(1) + b
75 = -15 + b
90 = b

1. Whose balance changes at a faster rate? Explain your thinking.
   Kai; Kai's account is increasing by $30 every week, while Jada's account is decreasing by $15 every week.
2. Who started with the larger amount in their account? Explain your thinking.
   Jada; At 0 weeks she had $90 in her account, while Kai at 0 weeks had $60.

Guided Practice

1. Which representation has the fastest rate of change?

   A. $y = -3x + 1$ **B.** $y = 2x + 1$ **C.** $y = -\frac{1}{2}x + 1$

2. Which representation has the least y-intercept?

   A. The account balance, *a*, starts at $30 and increases by $5 per week.

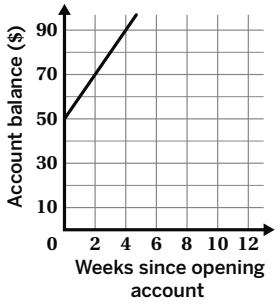
   B. The account balance, *a*, starts at $50 and increases by $8 per week.

   C. The account balance, *a*, starts at $20 and increases by $9 per week.

Guided Practice

3. The savings accounts of three customers are being compared. Circle the representation with the fastest rate of change. Explain your reasoning.

A.



B.

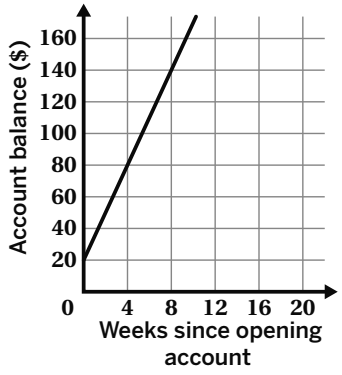
Number of weeks	Account balance ($)
1	90
2	70
3	50

C. The account balance, a, starts at $30 and increases $18 per week.

Check

The graph and table each show a different person's savings account.

Jack



Han

Number of weeks	Account balance ($)
1	100
2	80
3	60

1. Whose account balance changes at a faster rate? Explain your thinking.

2. Who started with the larger initial amount saved? Explain your thinking.

Name _____

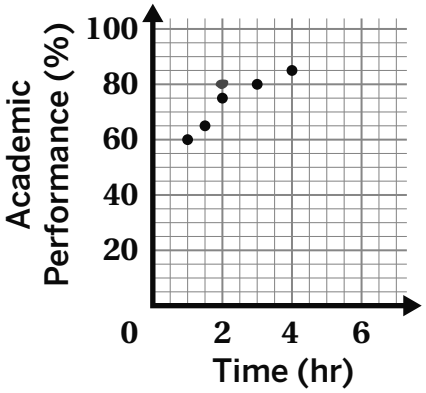
Interpreting Points on a Scatter Plot

ML 6.03

Modeled Review

Name: Diego

This scatter plot shows the academic performance and the number of hours students studied for a math test.



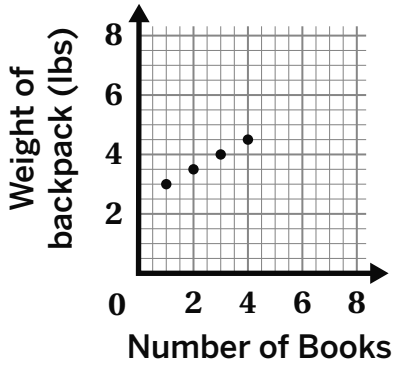
1. What is the highest academic performance a student received?
   85%
2. What is the academic performance of the student who studied the least?
   60%
3. Another student studied 2 hours and earned an academic performance of 80%. Plot a point on the graph that represents this student.

Guided Practice

The table and scatter plot show the number of books each student carried in their backpack and the corresponding weight of each backpack.

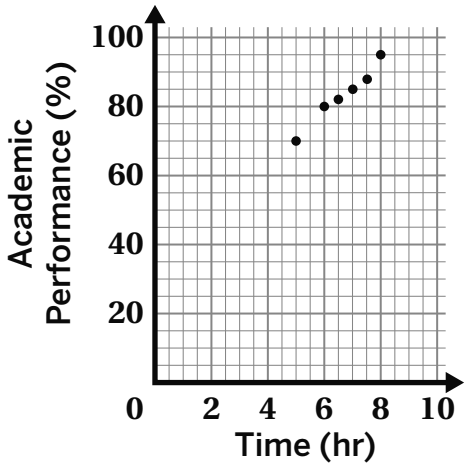
Student	Number of books	Backpack weight (lbs)
Isaiah	1	3
Kayla	3	4
Mia	2	3.5
Felipe	4	4.5

1. Circle the point on the scatter plot that represents the data for Mia.
2. What does the point $(3, 4)$ represent?
3. In the same study, the data showed that Arjun had a backpack weighing 5 pounds with 6 books. Add a point to the scatter plot to represent Arjun.



Guided Practice

This scatter plot illustrates the connection between the amount of sleep students get each night and their academic performance.



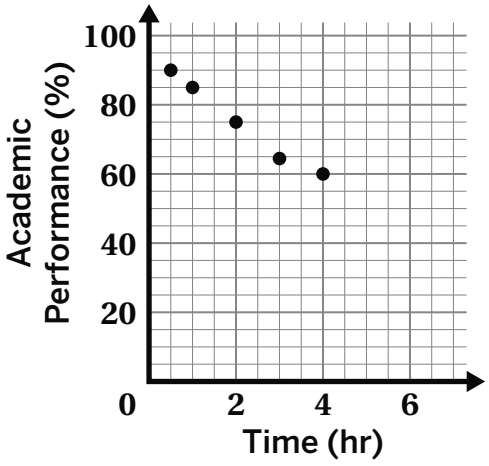
4. What is the highest academic performance a student received?

5. How many hours did the student who received the highest academic performance sleep?

6. The academic performance for a student who slept 5 hours is 50%. Plot a point on the graph that represents this student.

Check

This scatter plot shows the academic performance and the number of hours of screen time students have.



1. What is the highest academic performance observed in a student?

2. What is the lowest academic performance observed in a student?

3. The academic performance for a student who had 4 hours of screen time is 75%. Plot a point on the graph that represents this student.

Name _____

Using Lines of Fit to Make Predictions

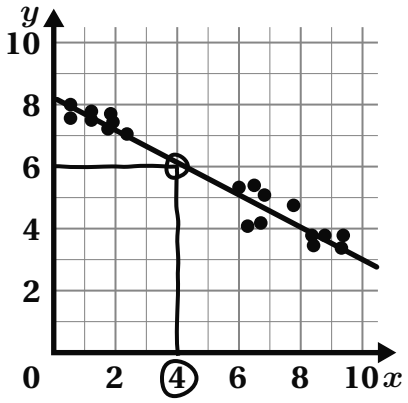
ML 6.09

Modeled Review

Name: Tristan

Use the line of fit to predict the y-value of a new data point whose x-value is 4.

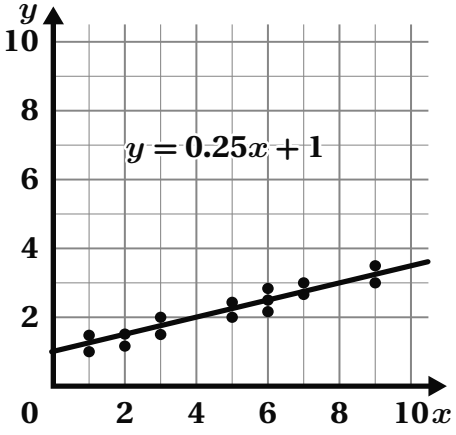
6



Guided Practice

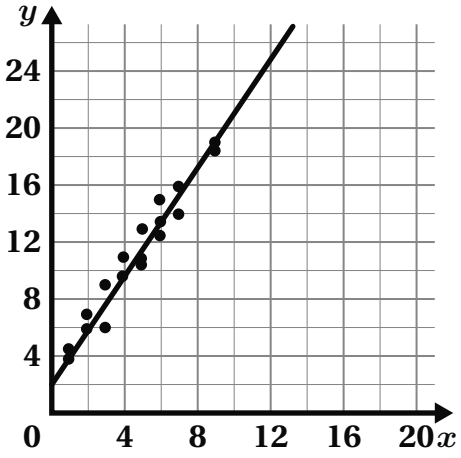
1. Use the line of fit to make predictions and complete the table.

x	y
4	
8	

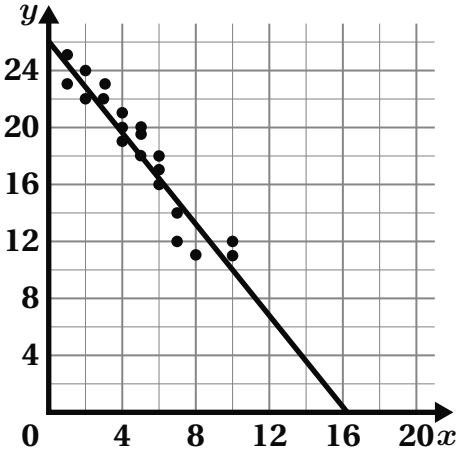


Guided Practice

2. Use the line of fit to predict the y-value of a new data point whose x-value is 8.

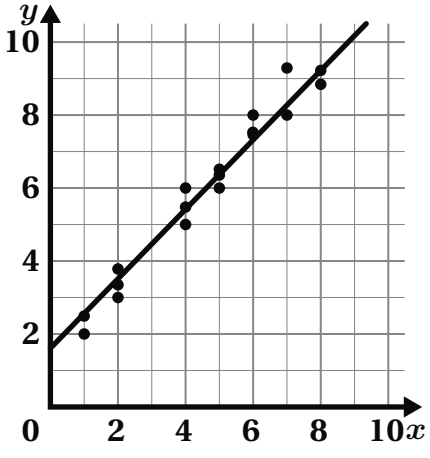


3. Use the line of fit to predict the y-value of a new data point whose x-value is 12.



Check

Use the line of fit to predict the y-value of a new data point whose x-value is 3.



Name _____

Calculating Missing Values in Two-Way Tables

ML 6.10

Modeled Review

Name: Avery

A group of people were asked whether or not they liked their food and if they were satisfied with the service at a restaurant. Some of the responses are recorded in the two-way table. Calculate the missing values.

81 − 72 = 9
29 − 21 = 8
72 + 8 = 80
81 + 29 = 110

	Satisfied	Dissatisfied	Total
Liked food	72	8	80
Did not like food	9	21	30
Total	81	29	110

Guided Practice

For Problems 1–2, use the number bank to fill in the missing values in each two-way table.

60	33	10	12	5	46

1. Customers were surveyed about their preference for outdoor or indoor seating and whether they found the restaurant's ambiance satisfactory. Some of the responses are recorded in the two-way table.

	Satisfied	Unsatisfactory	Total
Outdoor	20		25
Indoor		35	45
Total	30	40	70

2. Participants in a wellness retreat were surveyed on their interest in meditation or yoga and their perception of mental health benefits. Some of the responses are recorded in the two-way table.

	Beneficial	Not beneficial	Total
Meditation		13	25
Yoga	27		
Total	39		85

Guided Practice

3. Visitors to a museum were surveyed on their enjoyment of the interactive exhibits and if they found the staff's assistance helpful. Some of the responses are recorded in the two-way table. Calculate the missing values.

	Helpful	Not helpful	Total
Enjoyed	45	25	
Did not enjoy		15	
Total	65		105

4. Patients in a healthcare facility were surveyed on their comfort with medical staff and satisfaction with care quality. Some of the responses are recorded in the two-way table. Calculate the missing values.

	Satisfied	Not satisfied	Total
Comfortable	33	21	
Not comfortable		27	
Total	52		100

5. Students were polled about their preference for group projects or individual assignments and their satisfaction with the level of challenge in the coursework. Some of the responses are recorded in the two-way table. Calculate the missing values.

	Satisfied	Not satisfied	Total
Individual	56		65
Group	37	13	
Total		22	

Check

Participants in a fitness program were asked about their preference for cardio or strength training and how they rated the effectiveness of the program. Some of the responses are recorded in the two-way table. Calculate the missing values.

	Effective	Not effective	Total
Cardio		20	
Strength	12		20
Total	92	28	

Name _____

Multiplying Powers

ML 7.03

Modeled Review

Name: Jack

Decide if the expressions in each pair are equivalent. Show your thinking.

Expression 1	Expression 2	Equivalent?
$(5^5)^2$ $(5^5) \cdot (5^5)$ $(5 \cdot 5 \cdot 5 \cdot 5 \cdot 5) \cdot (5 \cdot 5 \cdot 5 \cdot 5 \cdot 5)$	$5^4 \cdot 5^3$ $(5 \cdot 5 \cdot 5 \cdot 5) \cdot (5 \cdot 5 \cdot 5)$	Yes **No**
$4^3 \cdot 2^5$ $(4 \cdot 4 \cdot 4) \cdot (2 \cdot 2 \cdot 2 \cdot 2 \cdot 2)$ $(4 \cdot 2)(4 \cdot 2)(4 \cdot 2) \cdot 2 \cdot 2$	8^8 $(8 \cdot 8 \cdot 8 \cdot 8 \cdot 8 \cdot 8 \cdot 8 \cdot 8)$	Yes **No**
$15^3 \cdot 2^3$ $(15 \cdot 15 \cdot 15) \cdot (2 \cdot 2 \cdot 2)$ $(15 \cdot 2)(15 \cdot 2)(15 \cdot 2)$	$(5 \cdot 2)^3 \cdot 3^3$ $(10 \cdot 10 \cdot 10) \cdot (3 \cdot 3 \cdot 3)$ $(10 \cdot 3)(10 \cdot 3)(10 \cdot 3)$	**Yes** No

Guided Practice

1. Match each expression to the correct expanded form on the right.

Expression	Expanded Form
$(3^4)^2$	$3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$
3^9	$(3 \cdot 3 \cdot 3) \cdot (3 \cdot 3 \cdot 3)$
$(3^3) \cdot (3^3)$	$(3 \cdot 3 \cdot 3 \cdot 3) \cdot (3 \cdot 3 \cdot 3 \cdot 3)$

Guided Practice

2. Decide if the expressions in each pair are equivalent. Show your thinking.

Expression 1	Expression 2	Equivalent?
$(7^4)^2$	49^4	Yes No
$6^3 \cdot 2^4$	$(2 \cdot 2)^3 \cdot 3^3$	Yes No
$5^3 \cdot 2^3$	10	Yes No
$(6^4)^2$	$6^4 \cdot 6^2$	Yes No
$8^4 \cdot 3^3$	$(3 \cdot 2)^4 \cdot 4^3$	Yes No

Check

Decide if the expressions in each pair are equivalent. Show your thinking.

Expression 1	Expression 2	Equivalent?
$(4^3)^2$	$4^3 \cdot 4^3$	Yes No
$3^2 \cdot 2^3$	6^5	Yes No

Name _____

Approximating Square Roots

ML 8.03

Modeled Review

Name: Evan

Approximate $\sqrt{18}$.

$\sqrt{18}$ is between 4.2 and 4.3

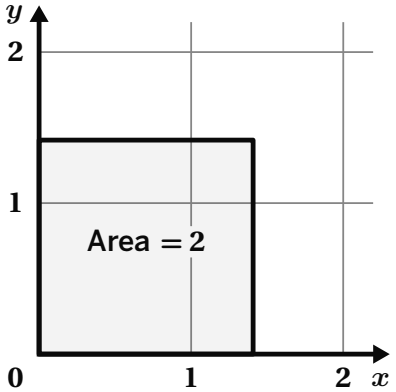
I multiplied 4.1, 4.2, and 4.3 by themselves to find their squares.

n	n^2
4.1	16.81
4.2	17.64
4.3	18.49

Guided Practice

Approximate the square root using the side lengths of the squares and the tables.

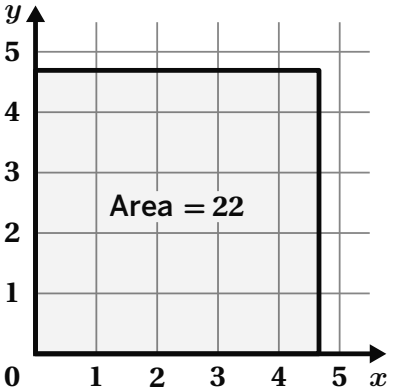
1. $\sqrt{2}$



n	n^2
1.4	1.96
1.5	

$\sqrt{2}$ is between _____ and _____

2. $\sqrt{22}$



n	n^2
4.6	

$\sqrt{22}$ is between _____ and _____

Guided Practice

For Problems 3–6, complete the table to approximate the square root.

3. $\sqrt{6}$ is between _____ and _____

n	n^2
2.3	5.29
2.4	

4. $\sqrt{10}$ is between _____ and _____

n	n^2
3.1	

5. $\sqrt{15}$ is between _____ and _____

n	n^2

6. $\sqrt{40}$ is between _____ and _____

n	n^2

Check

Approximate $\sqrt{28}$.

$\sqrt{28}$ is between _____ and _____

n	n^2

Name _____

Identifying Rational and Irrational Numbers

ML 8.14

Modeled Review

Rational	Irrational
all positive and negative numbers that can be written as fractions, including whole numbers	a number that cannot be written as a fraction of two non-zero integers
Examples	**Examples**
$\frac{3}{4}$ | $2.5 = \frac{5}{2}$ | $0.\overline{66}$ | $\sqrt{16}$	π | $\sqrt{7}$ | $\sqrt{3} = 1.732050...$

Guided Practice

1. Determine if the numbers in the bank are rational or irrational. Then add them to the correct column.

1.75	$\sqrt{3}$	9π	$\frac{1}{2}$
$-\frac{1}{4}$	$0.\overline{33}$	$\sqrt{10}$	$\sqrt{15} = 3.872983...$

Rational	Irrational
$\frac{1}{2}$	$\sqrt{15} = 3.872983...$

Guided Practice

2. Is the number rational or irrational? Add a check mark to the correct column.

Rational	Rational	Irrational
$\sqrt{36}$		
$0.\overline{72}$		
$1 + \sqrt{2} = 2.414213...$		
$-\frac{7}{8}$		
$\sqrt{11}$		
$\pi^2 = 9.869604...$		
8π		
$0.\overline{11}$		
4		

Check

Is the number rational or irrational? Add a check mark to the correct column.

Number	Rational	Irrational
$\sqrt{14}$		
5π		
$-\frac{11}{5}$		

Extensions

Sequences

Name: ..... Date: ..... Period: .....

Student Choice Pick any problem to start with. Remember to show or explain your thinking.

1

Let's assume the initial thickness of a paper is 0.001 centimeters and you can fold this very very thin paper infinitely many times.

a How many folds would result in the same thickness of a 500-page book made from the same paper?

b How many folds would result in the same thickness of 500 of those books?

c How many folds would result in the same height as the Empire State Building?

d How many times do you need to fold the paper to get to the Moon?

e How many times do you need to fold the paper to get to the Moon and come back?

Sequences (continued)

Name: Date: Period:

2 The table shows a sequence resulting from consecutive sums.

Consecutive sums	Sequence
1	1
$1+2$	3
$1+2+3$	6
$1+2+3+4$	
$1+2+3+4+5$	
$1+2+3+4+5+6$	
$1+2+3+\ldots+10$	

a Complete the table using the consecutive sums.

b Create a visual pattern of this sequence.

c This sequence is called the *triangular number* sequence.

Why do you think these numbers are called triangular? Is there a constant rate or difference between the terms of the sequence?

d Determine a way to find the n^{th} term of a sequence.

Variables and Relationships

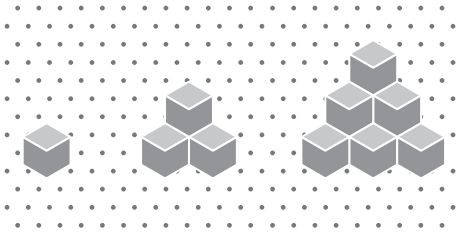
Name: ..... Date: ..... Period: .....

Student Choice Start with any problem. Remember to show or explain your thinking.

1

Here are Stages 1–3 of four visual patterns. Complete the table.

- For each pattern, determine the number of tiles or cubes that will be in Stage 4 and Stage.
- For each pattern, write an explicit expression for the number of tiles or cubes in Stage n.

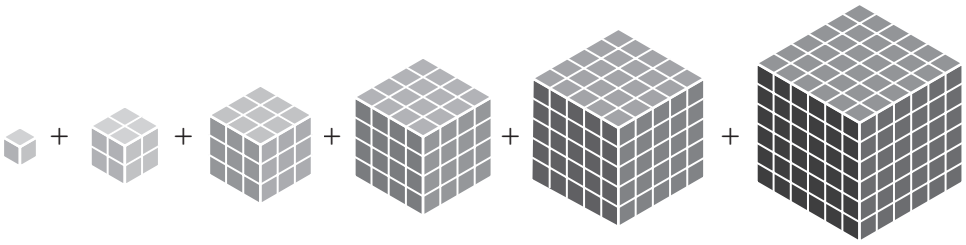
Stages 1–3	Stage 4	Stage 5	n^{th} term
 1 tile 3 tiles 9 tiles			
 1 cube 7 cubes 19 cubes			
 3 tiles 10 tiles 21 tiles			
 1 cube 4 cubes 10 cubes			

Variables and Relationships (continued)

Name: ..... Date: ..... Period: .....

2

The result of multiplying the same number three times is called a *cube number*. Here is a visual of the sum of the first six cube numbers.



a What is the sum of the first three cube numbers? The first four? The first six?

b What is the sum of the first n cube numbers?

3

The Fibonacci series is a series of numbers in which each number is the sum of the two preceding numbers: $1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89 \ldots$

a Complete the table to extend the Fibonacci series backward from 0.

F_{-6}	F_{-5}	F_{-4}	F_{-3}	F_{-2}	F_{-1}	F_0	F_1	F_2	F_3	F_4	F_5	F_6	F_7
						0	1	1	2	3	5	8	13

b What do you notice?

One-Variable Equations

Name: ..... Date: ..... Period: .....

Remember to show or explain your thinking.

1

A magician asked everyone in the audience to think of a number, and then follow these steps:

- Step 1: Add 4.
- Step 2: Multiply by 4.
- Step 3: Subtract 4.
- Step 4: Divide by 4.
- Step 5: Subtract your original number

a Think about a number between 1 and 10 and follow the steps. What is your final number?

b Emma's initial number was 7. What is the final number she ended up with?

c Mayra thought of an integer and followed the same steps. What answer did she end up with?

d Will this work for numbers that are not whole numbers between 1 and 10? What numbers will it work for?

e Repeat the same trick with 2s, 3s, . . ., n instead of 4s. What are the final answers?

One-Variable Equations (continued)

Name: Date: Period:

2 A magician asked everyone in the audience to think of a number, and then follow these steps:

- Step 1: Triple the number.
- Step 2: Add 45 to the result.
- Step 3: Double the result.
- Step 4: Divide the result by 6.
- Step 5: Subtract the number with the first number you started with.

Then she added, "When you are finished, you should all have the same number."

What is the number? How did the magician know it would be the same for everyone?

3 Design your own math magic trick that ends with the number 8. Your trick should have multiple steps.

Multi-Variable Equations

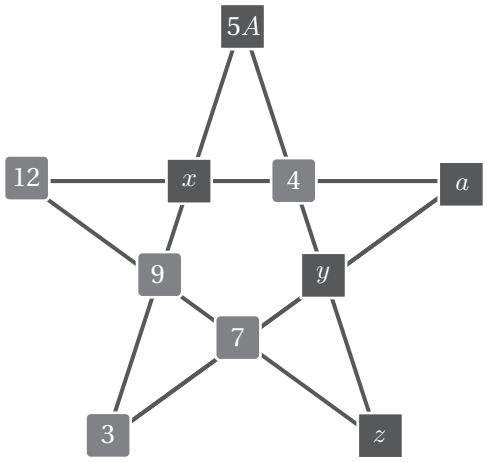
Name: ..... Date: ..... Period: .....

Remember to show or explain your thinking.

1

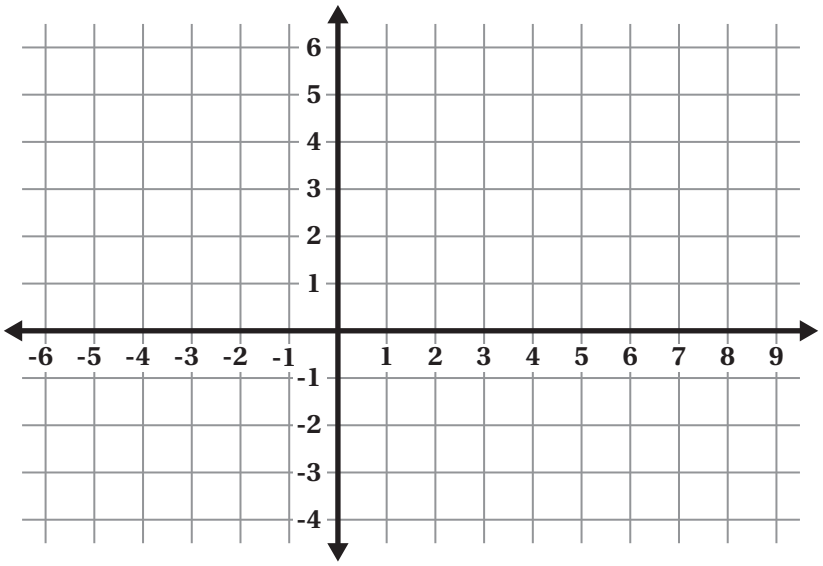
Here is a figure where the four values along each line have a sum of 30.

Determine the values of x, y, z, and a.



2

a A triangle is formed by the lines $y = -2$, $y = 2x + 8$, and $2x + 3y = 12$. What is the area of the triangle?



b John added a line that divides the triangle into two equal areas. What could be an equation of John's line?

Multi-Variable Equations (continued)

Name: Date: Period:

3

Let x and y be the first two terms of a Fibonacci-like sequence, where each term is the sum of the two preceding terms

a Write the next five terms. What do you notice?

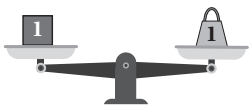
b The fifth term of the sequence is 19 and the eighth term is 81. What are the first two terms, x and y, of this sequence?

4

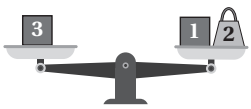
A set of five weights can balance any object with an integer weight from 1 to 120 kilograms on a two-sided scale.

Here are examples of two of the five weights.

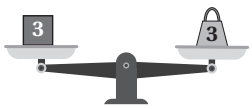
To weigh a 1 kg object, you can use the 1 kg weight.



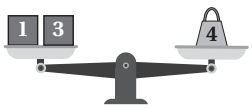
To weigh a 2 kg object, you can use the 1 kg and 3 kg weights.



To weigh a 3 kg object, you can use the 3 kg weights.



To weigh a 4 kg object, you can again use the 1 kg and 3 kg weights.



a What one additional weight can you add to the set to weigh any object up to 13 kilograms?

b Complete the set of five weights so that it can balance any object with a weight up to 120 kilograms.

One-Variable and Two-Variable Inequalities

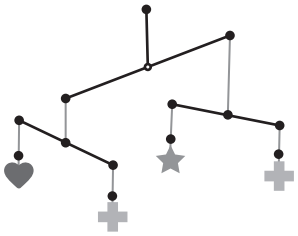
Name: ..... Date: ..... Period: .....

Student Choice Remember to show or explain your thinking.

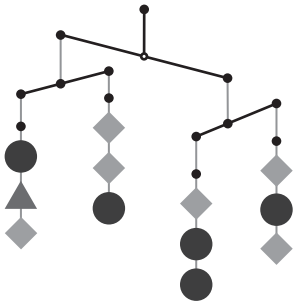
1 Here are three imbalanced hangers.

For each hanger, write an inequality statement that orders the shapes from lightest to heaviest.

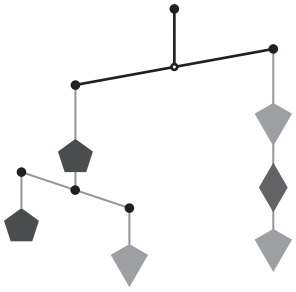
Hanger A



Hanger B



Hanger C



2 Create your own imbalanced hanger where there is only one way to order the shapes from lightest to heaviest.

One-Variable and Two-Variable Inequalities (continued)

Name: Date: Period:

3

Determine a pair of values x and y for which each statement is true and false.

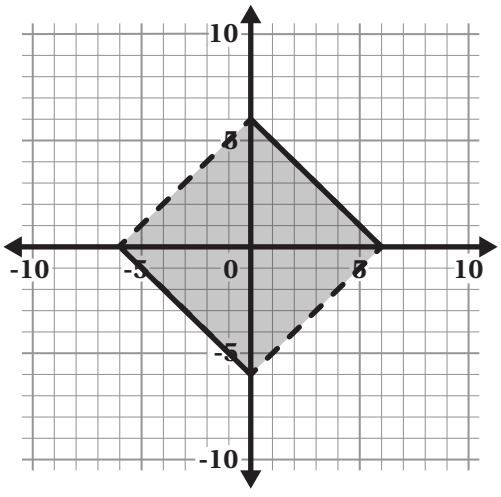
	Statement	True	False
a	If $x \geq y$, then $\lvert x \rvert \leq \lvert y \rvert$		
b	If $x \leq 0$, and $y > 0$, then $x \cdot y \geq 0$		
c	If $x > y$, then $x \cdot y \geq y^2$		
d	$\lvert x + y \rvert > x + y$		

4

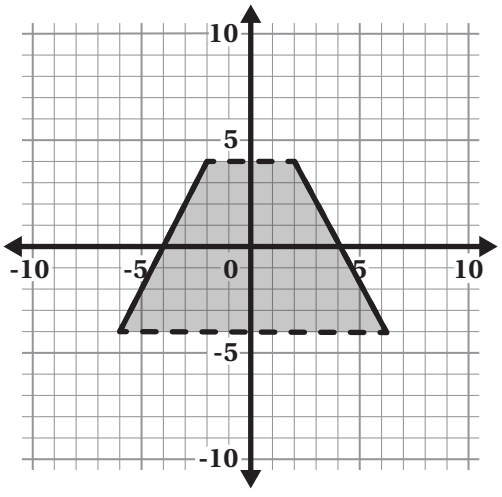
Anna draws polygons on a coordinate plane using inequalities.

Determine the set of inequalities she used to create each polygon.

a



b



Representing Categorical Data

Name: ..... Date: ..... Period: .....

Student Choice Start with any problem. Remember to show or explain your thinking.

1

On April 15, 1912, the Titanic struck an iceberg and rapidly sank with only 710 of the 2,204 passengers and crew surviving.

Data on the survival of first and second-class passengers are summarized in the following two-way table.

	Survived	Did not survive
First-class passengers	201	123
Second-class passengers	118	166

a Create a total relative frequency table to represent the data.

b Interpret the data. Is there evidence to support any associations in this data? Could you generalize these results? Be sure to include which data displays and statistical calculations you used when explaining your thinking.

c Data on the third-class passenger is added with 181 surviving and 528 not surviving. Create a relative frequency table including the new data and interpret your findings.

Representing Categorical Data (continued)

Name: Date: Period:

2 In 1973, UC Berkeley was sued for gender bias against women applying to graduate school. The following facts were true:

- The overall acceptance rate in the four departments was approximately 30% for female applicants and approximately 44% for male applicants.
- The following data on the four departments show that the acceptance rate for female applicants was actually higher than the acceptance rate for male applicants.

Department	Number of male applicants	Number of female applicants	Percentage of males admitted	Percentage of females admitted
1	825	108	62	82
2	560	25	63	68
3	373	341	6	7
4	417	375	33	35

Explain how it is possible for both of these facts to be true. Use data displays and statistical evidence to support your thinking.

Summarizing One-Variable Data

Name: ..... Date: ..... Period: .....

Remember to show or explain your thinking.

1

a If a fixed number n is added to all measurements in a data set, how does the mean of the new data set change?

b If all measurements in a data set are multiplied by a fixed number n, how does the mean of the new data set change?

c Are these properties of mean true for median? Give an example.

d If a fixed number n is added to all measurements in a data set, how do the deviations $(x - \overline{x})$ of the new data set change?

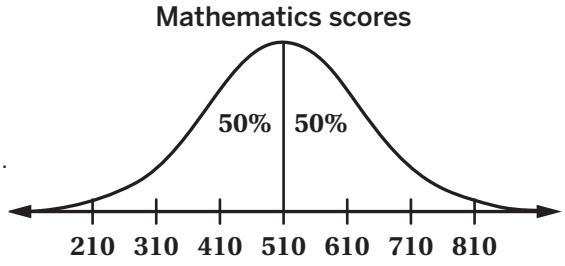
e If all measurements in a data set are multiplied by a fixed number n, how do the deviations $(x - \overline{x})$ of the new data set change?

Summarizing One-Variable Data (continued)

Name: ..... Date: ..... Period: .....

2 When working with normally distributed data:

- The mean is equivalent to the median.
- 50% of the data lies to the left of the mean.
- 50% of the data lies to the right of the mean.
- 68% of the data are within 1 standard deviation of the mean.
- 95% of the data are within 2 standard deviations of the mean.
- 99.7% of the data are within 3 standard deviations of the mean.



The scores shown are for a standardized mathematics test in a particular year.

a Based on the given data, identify the mean and the standard deviation for the mathematics test.

b Based on the given bell curve, if 150 students are in a given sample, how many would you expect to score between 410 and 710?

Summarizing Two-Variable Data

Name: ..... Date: ..... Period: .....

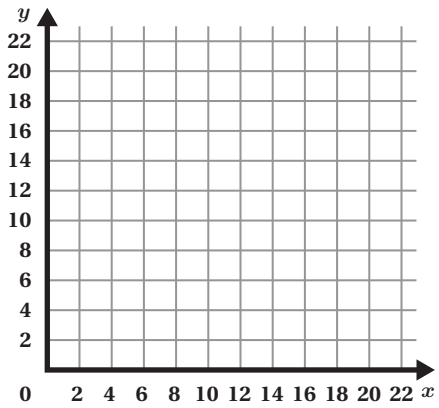
Remember to show or explain your thinking.

1

Here's a set of data.

Days since Team Trees' campaign launch	Total donations (in millions of dollars)
1	1
2	4
4	6
6	10
7	11
8	12
10	14
12	14
14	16
19	20

a Plot a scatter plot diagram and add what you think is the line of best fit.



b Another strategy for fitting a line to data is called the *three-median method*. Divide the data set into thirds. (If they do not divide evenly and there is one extra data value, put it in the middle group, If there are two extra values, put an extra value in each of the outer groups.)

- **Step 1:** Determine the medians of each group.

- **Step 2:** Draw an initial line to connect the medians of the lower and upper groups. Calculate its slope and y-intercept.

- **Step 3:** Move this line $\frac{1}{3}$ of the way towards the median of the central group by maintaining the slope of your initial line.

Summarizing Two-Variable Data (continued)

Name: ..... Date: ..... Period: .....

c Plot the line of the three-median method on the same scatter plot diagram in part a.

d Compare the lines you drew in part a and part c. Why do you think the *three-median method* works? Can there be examples where this method does not work well?

e Use the three-median regression line to predict the value in donations Team Trees can expect to receive in 30 days.

f Use the three-median regression line to predict how many days it will take for Team Trees to reach their goal of 25 million dollars.

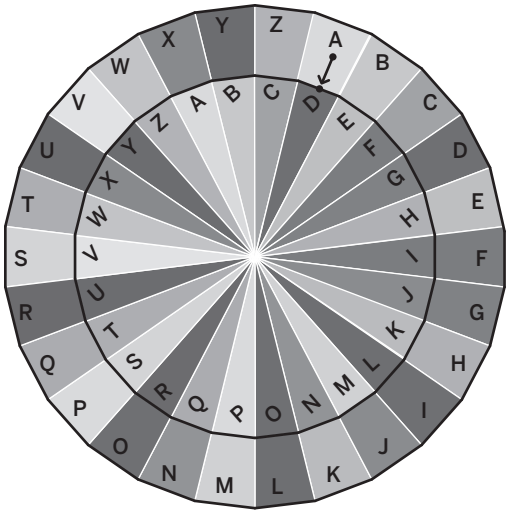
Function Notation

Name: ..... Date: ..... Period: .....

Student Choice Complete Problem 1 before starting Problem 2. Remember to show or explain your thinking.

1

The *Caesar Cipher* is simply a type of substitution cipher, i.e., each letter of a given text is replaced by a letter with a fixed number of positions down the alphabet.



a The smaller disk is rotated to shift each letter 3 steps forward: $s(x) = x + 3$

Encode the word "Amplify" using the Caesar Cipher with the shift $s(x) = x + 3$.

b Encode a word you like and exchange with a classmate to decode.

c Decode "Ohw'v ohduq wrjhwkhu" using $s(x) = x + 3$

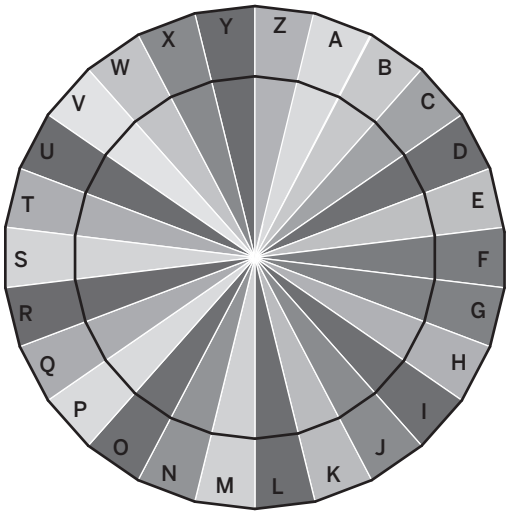
d What other way can you write the same function, $s(x) = x + 3$, on the Caesar wheel?

Function Notation (continued)

Name: ..... Date: ..... Period: .....

2

Here is a blank cipher where the coding rule is not known.



a Decode the text.

X jhts id utta vjxain iwpi X hetci paa spn eapnxcv vpbth, lwxat X lph hjeedhts id qt sdxcv bpiwtbpixrh. iwtc, lwtc X sxhrdktgts hjggtpa cjbqtgh, X gtpaxots iwpi eapnxcv vpbth xh bpiwtbpixrh.Ydwc Rdclpn

b Write the rule of the cipher using function notation.

3

The mathematician Hedy Lamarr's patented "frequency hopping" allowed for secret codes to be passed over the radio. It was not possible for the code to be cracked without the key. A coded message was created by assigning each letter of the alphabet a number from 1 to 26 in chronological order, A is 1, B is 2, etc. Then it was multiplied by 4 and added with 3.

a Write a function that represents how the message was encoded.

b Decode the message $79\ 87\ 67\ 23\ 75$.

"Hedy Lamarr in 'The Heavenly Body'. Movie of MGM (1944)" by MGM Public Domain via Wikimedia Commons.

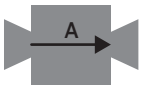
Key Features of Functions

Name: ..... Date: ..... Period: .....

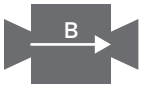
Student Choice Start with any problem. Remember to show or explain your thinking.

1

Here are two function machines with their input/output table.



Input	Output
2	5
3	7
4	9



Input	Output
5	11
7	17
9	23

a Fabiana wants to reverse the way that function machine A works. What is the rule she needs to write?

b Diego wants to reverse the way that function machine B works. What is the rule he needs to write?

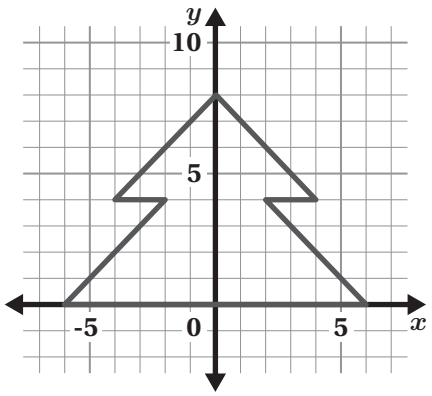
c Ada wants to combine two function machines by writing a single rule for a new machine. What is the new function Ada needs to use to get the same outputs of machine B using the inputs of machine A?

Key Features of Functions (continued)

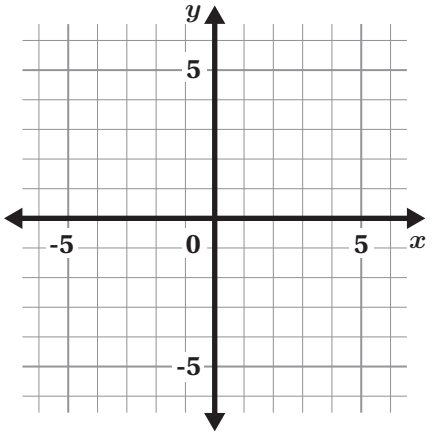
Name: Date: Period:

2 Here is the logo of a holiday store company designed in the Desmos Graphing Calculator using the graphs of several functions.

a Write the functions and their domains needed to create this logo.



b Design your own logo using graphs of functions and their domains.



Special Types of Functions

Name: ..... Date: ..... Period: .....

Student Choice Start with any problem. Remember to show or explain your thinking.

1

The Lucas Number sequence, like the Fibonacci sequence, is a sequence in which each term is the sum of the two terms that come before it.

a Take a look at the Lucas sequence below. Complete the recursive definition for it.
$2, 1, 3, 4, 7, 11, 18, 29, 47, 76, 123, 199, 322, 521$

$$L(n) = \begin{cases} 2 & n = 0 \\ \text{.....} & n = 1 \\ \text{.....} & \text{.....} \end{cases}$$

b As in Fibonacci sequence, one can extend the Lucas numbers to negative integers such as..., -11, 7, -4, 3, -1, 2, 1, 3, 4, 7, 11,...

Write the function $L(\text{-}n)$ in terms of $L(n)$.

c The Lucas numbers are related to the Fibonacci numbers in so many ways. For example:

$L(n) = F(n-1) + F(n+1)$

n:	0,	1,	2,	3,	4,	5,	6,	7,	8,	...
F(n):	0,	1,	1,	2,	3,	5,	8,	13,	21,	...
L(n):	2,	1,	3,	4,	7,	11,	18,	29,	47,	...

Write at least two more relationships using the function notation.

2

a The *floor function* $\lfloor x \rfloor$ is defined to be the greatest integer less than or equal to the real number x.

For example:

$\lfloor 3 \rfloor = 3$

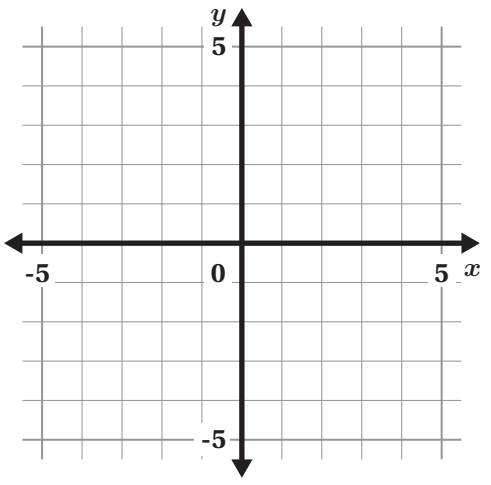
$\lfloor \text{-}3 \rfloor = \text{-}3$

$\lfloor 4.26 \rfloor = 4$

$\lfloor \text{-}4.26 \rfloor = \text{-}5$

$\lfloor \pi \rfloor = 3$

Plot the graph of the floor function.



Special Types of Functions (continued)

Name: ..... Date: ..... Period: .....

b The *fractional part function* $\{x\}$ is defined to be the difference $x - \lfloor x \rfloor$.

For example:

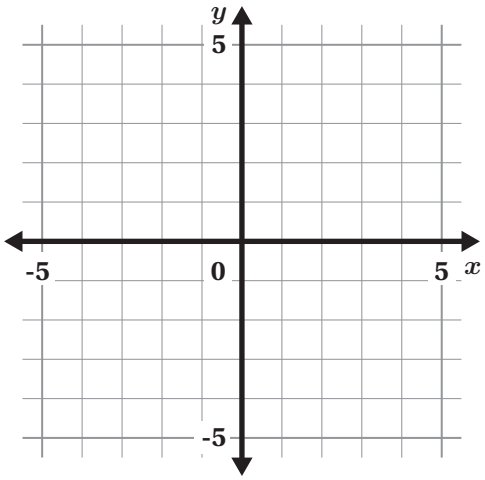
$\{3\} = 0$

$\{\text{-}3\} = 0$

$\{4.26\} = 4.26 - \lfloor 4.26 \rfloor = 4.26 - 4 = 0.26.$

$\{\text{-}4.26\} = \text{-}4.26 - \lfloor \text{-}4.26 \rfloor = \text{-}4.26 - (\text{-}5) = 0.74.$

Plot the graph of the fractional part function.



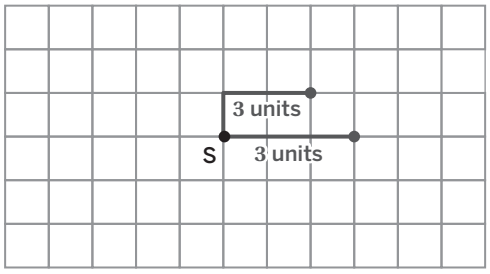
c What is the range of the fractional part function $\{x\}$?

d What is the sum of $\{x\}$ and $\{\text{-}x\}$?

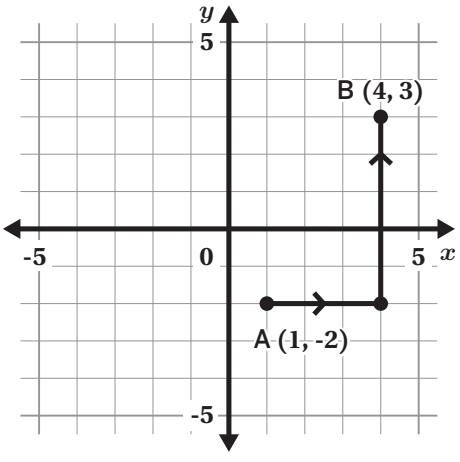
3

Taxicab geometry is a form of geometry in which you can only move along the lines of a grid. It gets its name from taxis since they can only drive along streets rather than moving as the bird flies.

a Determine all the points which are 3 units away to the given point S.



b What is the taxicab distance function between the points (x_1, y_1) and (x_2, y_2).



Systems of Equations

Name: ..... Date: ..... Period: .....

Student Choice Start with any problem. Remember to show or explain your thinking.

1

Here is a 3×3 square and a 5×5 square. Except for the last row and column of each square, the sums of each column and row is given. Determine the value of each letter.

a

X	Z	Y	10
Y	X	X	14
Y	Z	Z	?
13	7	?	

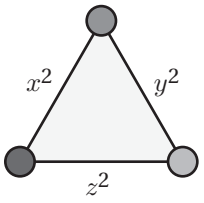
b

C	A	C	C	A	16
D	A	A	D	B	19
C	A	D	A	B	17
B	A	A	C	E	16
E	C	C	E	B	?
12	22	18	16	?	

2

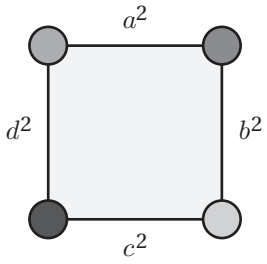
Determine the whole number value of each circle so that the sum of the two numbers along any given side is a square of another number.

a



○ + ○ = y^2
○ + ○ = x^2
○ + ○ = z^2

b



○ + ○ = a^2
○ + ○ = b^2
○ + ○ = c^2
○ + ○ = d^2

Systems of Equations (continued)

Name: Date: Period:

3 A rectangle has a perimeter of 29 centimeters. If Trevon adds 3 centimeters to the width and subtracts 4 centimeters from the length, the area does not change. Determine the length and width of the rectangle.

4 The point $(8, -3)$ is the intersection of the graphs of $ax + by = 25$ and $3ax - 5by = 3$. Determine the values of a and b.

5 Prisha has 30 coins, consisting of nickels, dimes, and quarters, worth $\$4.60$. There are two more dimes than quarters. How many of each kind of coin does she have?

Systems of Inequalities

Name: ..... Date: ..... Period: .....

Remember to show or explain your thinking.

1

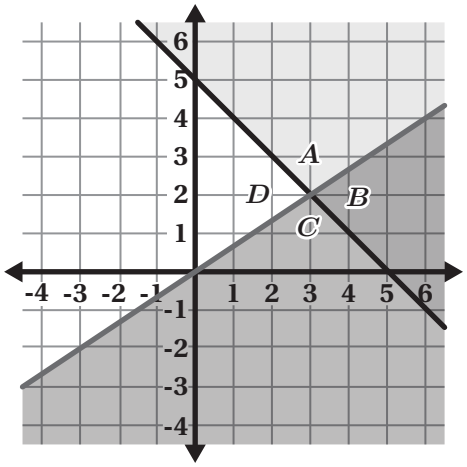
Determine the system of inequalities. . .

a whose solution is Region A.

b whose solution is Region B.

c whose solution is Region C.

d whose solution is Region D.



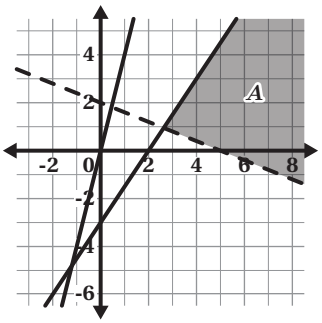
2

Here is a system of inequalities with missing inequality symbols.

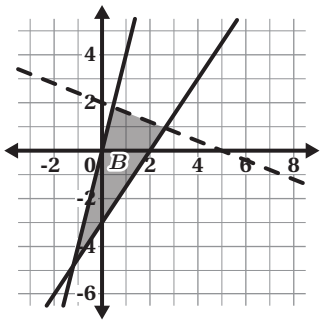
$$y \square 4x$$
$$2x + 5y \square 10$$
$$3x - 2y \square 6$$

a Determine the symbols if there is no solution.

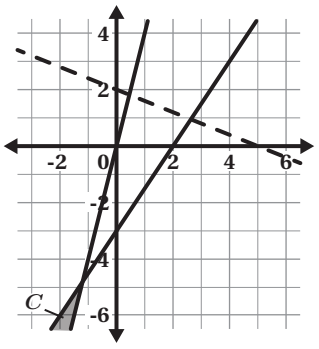
b Determine the symbols if the solution is Region A.



c Determine the symbols if the solution is Region B.



d Determine the symbols if the solution is Region C.

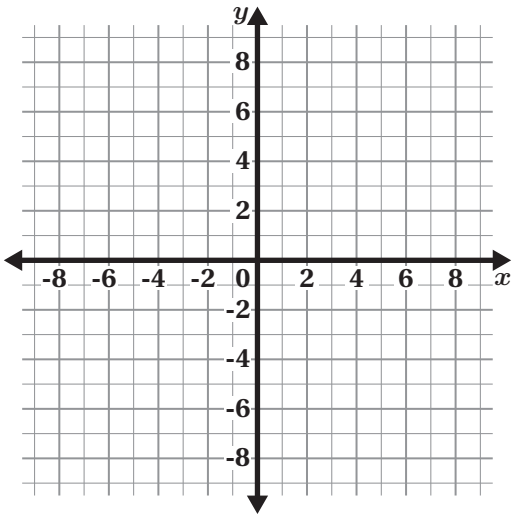


Systems of Inequalities (continued)

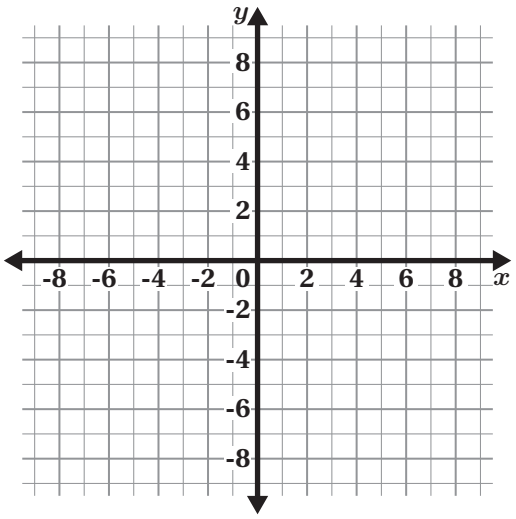
Name: Date: Period:

3

a Create your own system of inequalities whose solution is a single point. How do you know the solution is a single point?



b Create your own system of inequalities whose solution is a square. How do you know the solution is a square?



Comparing Linear and Exponential Functions

Name: ..... Date: ..... Period: .....

Student Choice Start with any problem. Remember to show or explain your thinking.

1

Use these assumptions to determine if vampires really exist?

- One vampire does exist.
- That vampire must bite someone each week to survive.
- Once a person is bitten by a vampire, they become a vampire.

2

Here is a pattern called the *Hilbert Curve*.

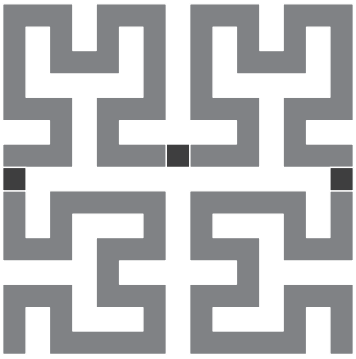
Figure 1

7 units long

Figure 2

$4 \cdot 7 + 3$ units long

Figure 3

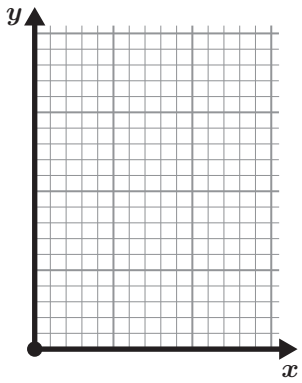


..... units long

a Complete the table for the first five figures of the Hilbert Curve.

Figure	Length
1	7
2	31
3	
4	
5	

b Plot the points on a graph.



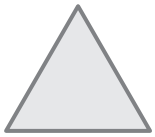
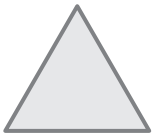
c Write a recursive and/or explicit expression to represent the growth.

Comparing Linear and Exponential Functions (continued)

Name: ..... Date: ..... Period: .....

3 Here is a fractal called the *Sierpinski triangle*.

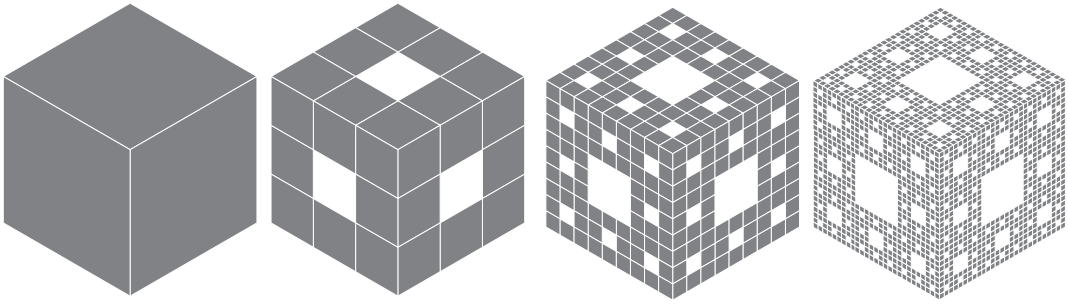
a Complete the table.

	Figure 0	Figure 1	Figure 2	Figure 3	Figure 4	Figure n
						
Number of shaded triangles:	1	3				
Shaded Area:	1	$\frac{3}{4}$	$\frac{9}{16}$			

b Which measure(s) of the Sierpinski triangle demonstrate exponential growth, and which one(s) show decay?

c What about the total perimeter of the Sierpinski triangle? What do you notice about the perimeter and the area of the Sierpinski triangle?

d Here is another fractal called the *Menger Sponge*. You start with a solid cube, and repeatedly drill smaller and smaller holes into its faces. Every new iteration of holes has $\frac{1}{3}$ of the width of the previous iteration of holes.



Which measure(s) of the Menger Sponge do you think will demonstrate exponential growth, and which one(s) will show decay?

Exponential Growth and Decay

Name: ..... Date: ..... Period: .....

Remember to show or explain your thinking.

1

A certain bacteria population doubles in size every 12 hours.

a By how much will it grow in 2 days?

b By how much will it grow in 10 days?

c Complete the *doubling-time growth formula* for a population of size N_0, that doubles every d years (where d is measured in years, not hours, days, or any other unit of time) to determine the number N of the population at a time t.

$$N = \square \cdot \square^{\frac{t}{d}}$$

2

All radioactive elements decay at a very predictable rate — this is determined by their *half-life*. A half-life is the time required for something's quantity to decrease by half. For example, the half-life of radioactive radium is 1600 years. (That's why Marie Curie's notebooks are still stored in lead-lined boxes!)

a Complete the table to determine by how much radium will remain after each year.

Time in years	0	1600	3200	4800	5400	t
Amount left	N_0	$N_0 \cdot \frac{1}{2}$				

b Compare the *half-life decay* and *doubling-time growth* formulas. How are they similar? How are they different?

c Carbon-14 has a half-life of approximately $6{,}000$ years. Scientists use C-14 in a process called *Carbon dating* to determine the ages of organic materials such as ancient fossils or mummies. If the amount N of C-14 is $\frac{1}{8}$ of the estimated amount N_0 that was in the sample when it was alive. What is the approximate age of the sample?

Exponential Growth and Decay (continued)

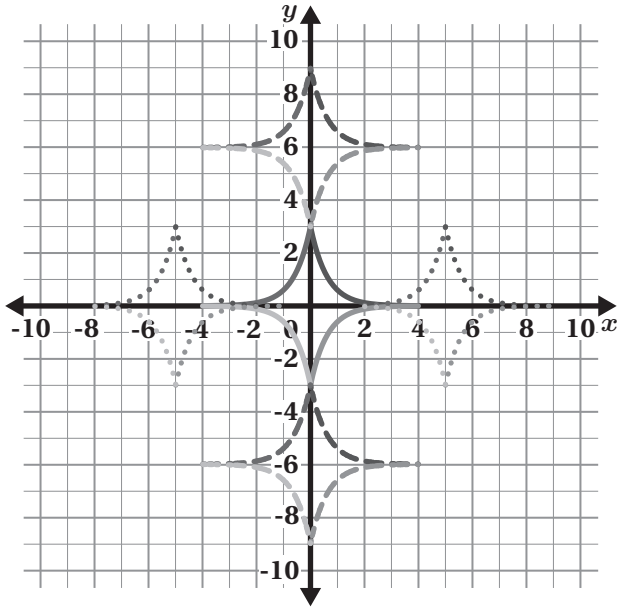
Name: ..... Date: ..... Period: .....

3

The central diamond-like shape made up of the exponential graphs:

- $y = 3 \cdot 5^{x}\ \{-4 < x < 4\}\ \{-3 < y < 3\}$
- $y = (-3) \cdot 5^{-x}\ \{-4 < x < 4\}\ \{-3 < y < 3\}$
- $y = 3 \cdot 5^{-x}\ \{-4 < x < 4\}\ \{-3 < y < 3\}$
- $y = (-3) \cdot 5^{x}\ \{-4 < x < 4\}\ \{-3 < y < 3\}$

The set of graphs is translated in four directions to create an image. Determine as many functions as you can to create the given image. Focus only on the functions rather than the range and domain restrictions.



Upper four graphs	Right four graphs	Lower four graphs	Left four graphs

Modeling With Exponentials

Name: Date: Period:

Student Choice Start with any problem. Remember to show or explain your thinking.

1

You are given $\$50{,}000$ that will double every year for the next 5 years.

a Write an equation that represents this situation where y represents the value of the investment after 5 years.

b Your equation has three numbers: starting value, growth factor, and the exponent. You are allowed to double one of these numbers. Which one do you think will result in the highest outcome?

2

Kiran wants to invest $\$1{,}000$ in an account with a 5% annual interest rate. He knows if the money is compounded semi-annually, he will earn more interest than if it is compounded annually. He wonders how much interest can be earned by compounding more and more often.

a Complete the table to investigate what happens to the end-of-year balance as the interest is compounded more and more frequently. Let n represent the number of times the interest is compounded in one year.

n	Year-end balance
1	
2	
3	
5	
10	
20	
50	

b What do you notice?

Modeling With Exponentials (continued)

Name: Date: Period:

3 Consider the following approximate data on the human population over time.

Year	1804	1927	1960	1974	1987	1999	2011	2024
World population in billions	1	2	3	4	5	6	7	8

a Generate an approximate model to represent the scenario.

b Use your model to determine when the human population will exceed 10 billion, 50 billion, and 100 billion.

c Do you think your model is accurate for these years?

Introduction to Quadratic Functions

Name: ..... Date: ..... Period: .....

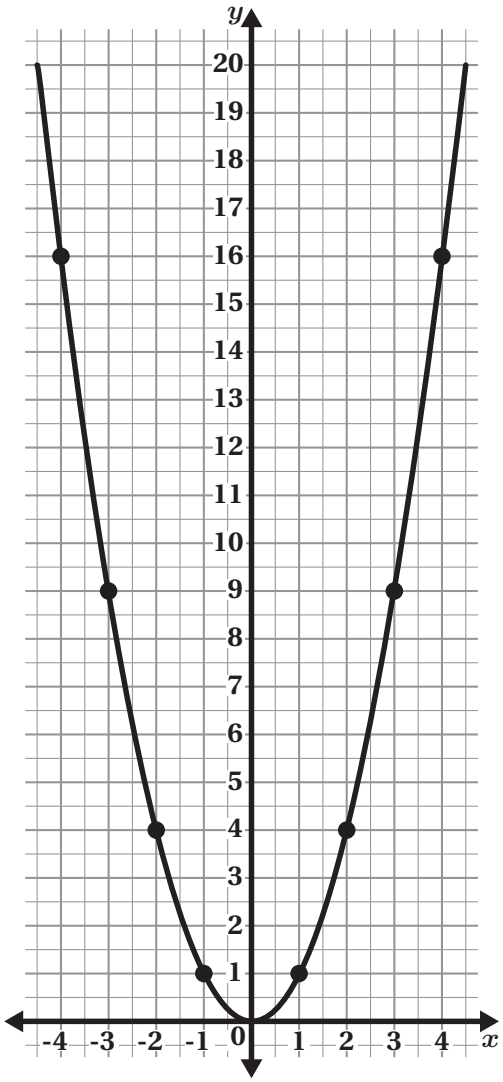
Remember to show or explain your thinking.

1

How many rectangles with whole-number side lengths have numerical perimeters equal to their numerical areas? What are the side lengths?

2

Here is the graph of $y = x^2$.



a Select any two points on the graph, one with a negative x-value and another with a positive x-value. Connect these two points with a line, then determine the y-intercept of the line.

b Repeat the steps in Problem 2a several times. What do you notice?

c Predict the y-intercept of the line connecting the points $(-6, 36)$ and $(8, 64)$?

d Investigate the slope of each line. What do you notice?

Introduction to Quadratic Functions (continued)

Name: ..... Date: ..... Period: .....

3

Here are three quadratic number sequences given as tables. Determine the second difference for each. Is there a connection between the value of the second difference and the coefficient of the n^2 term?

a

Term number	Term
1	3
2	6
3	11
4	18
5	27
n	$n^2 + 2$

b

Term number	Term
1	6
2	8
3	14
4	24
5	38
n	$2n^2 - 4n + 8$

c

Term number	Term
1	2
2	5
3	14
4	29
5	50
n	$3n^2 - 6n + 5$

4

Here are new sequences. Determine the first, second, third, . . . differences if needed. What do you notice? How do you think the coefficient of the largest power of the rule is connected to the value of the difference?

a

Term number	Term
1	1
2	8
3	27
4	64
5	125
6	216
n	n^3

b

Term number	Term
1	3
2	17
3	55
4	129
5	251
6	433
n	$2n^3 + 1$

c

Term number	Term
1	0
2	14
3	78
4	252
5	620
6	1290
n	$n^4 - n$

Standard Form and Factored Form

Name: ..... Date: ..... Period: .....

Remember to show or explain your thinking.

1

Han created a table to match the quadratic functions that are in *standard form* and *factored form*. Here is Han's table. He wants to determine the axis of symmetry of each parabola.

a Complete the table.

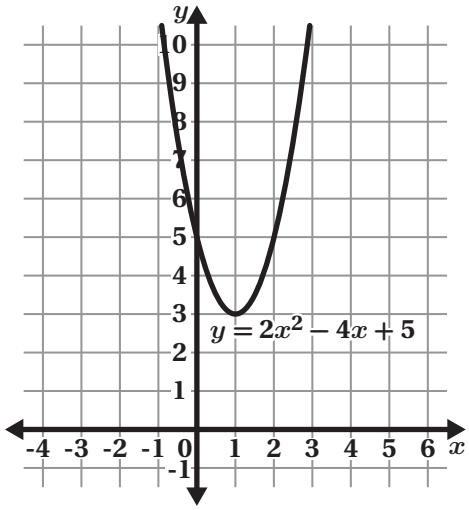
Standard form	Factored form	x-intercepts (m and n)	Axis of symmetry
$y = x^2 - 2x - 15$	$y = (x - 5)(x + 3)$		
$y = x^2 + 6x + 5$	$y = (x + 1)(x + 5)$		
$y = 2x^2 + 6x + 4$	$y = (2x + 4)(x + 1)$		
$y = 2x^2 + 6x - 36$	$y = (2x - 6)(x + 6)$		
$y = 3x^2 + 9x - 30$	$y = (x + 5)(3x - 6)$		

b Han thinks that there is a relationship between the axis of symmetry of each graph and the coefficients of x^2 and x in standard form. He determined the equation of the axis of symmetry as $x = \frac{-b}{a}$ for any quadratic function $f(x) = ax^2 + bx + c$. Is he correct? Explain why or why not.

c Here is a new quadratic function given in the standard form.

$y = 2x^2 - 4x + 5$

Use its graph to verify your conclusion in part b for the axis of symmetry.



Standard Form and Factored Form (continued)

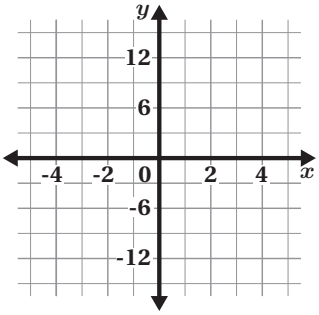
Name: Date: Period:

2

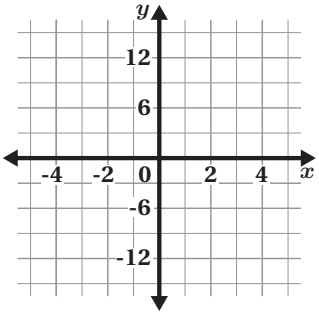
Here are three more functions.

a Graph each one on the given coordinate planes.

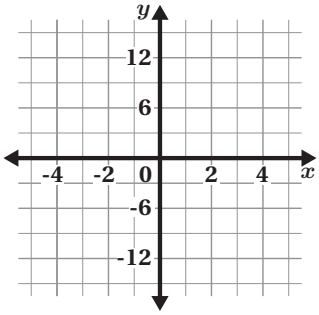
$y = (x - 3)^2$



$y = (x + 1)^2$



$y = (2x - 1)^2$

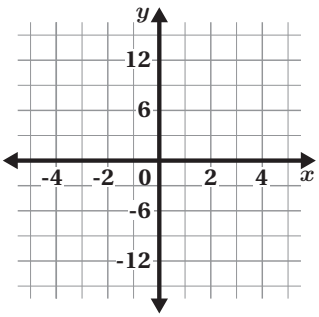


b What do you notice about the x- and y-intercepts, vertex, and axis of symmetry for each?

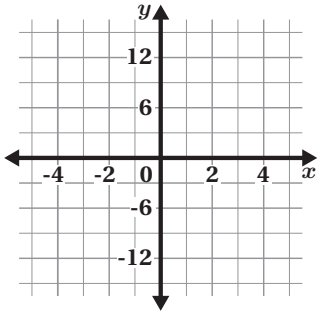
3

a Determine the x- and y-intercepts and sketch the graphs of the functions.

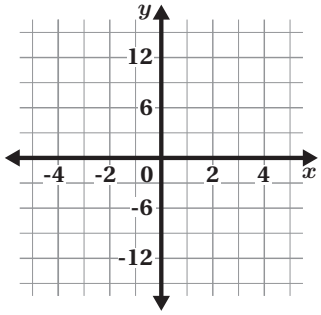
$(x + 2)(x - 1)$



$(x + 2)(x - 1)(x - 3)$



$(x + 2)(x - 1)(x - 3)\ (x + 1)$



b Use Desmos Graphing Calculator to graph the equations in part a. Which parts of your sketches match the actual graph?

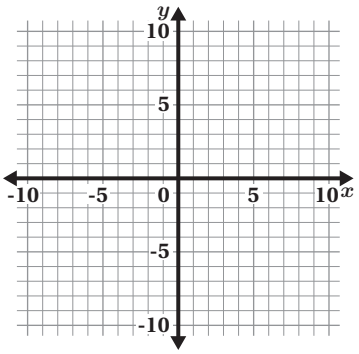
Vertex Form

Name: Date: Period:

Remember to show or explain your thinking.

1 Consider the equation $x = (y - 2)^2 + 1$.

a Determine the vertex and the line of symmetry.



b Graph the equation.

c Is y a function of x?

2

a Describe the graph of a parabola with the equation $y = x(b - x)$.

b Write expressions for the coordinates of its intercepts and vertex in terms of b.

c Do these expressions work for the negative values of b?

Vertex Form (continued)

Name: Date: Period:

3 The graph of $y = x^2$ has one x-intercept. Using the equation $y = a(x - h)^2 + k$, describe how to translate the graph of $y = x^2$ by changing the values of a, h, and k in order to create equations with the specified number of x-intercepts.

a No x-intercepts

b Only one x—intercept

c Two x-intercepts

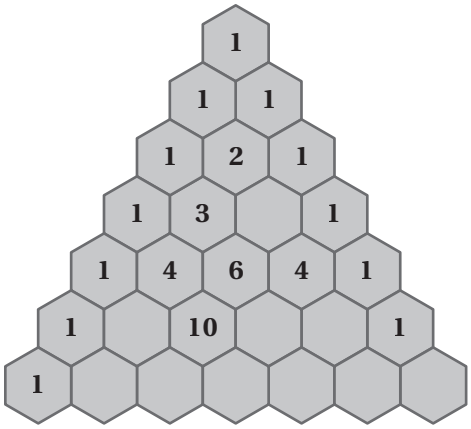
Multiplying and Factoring

Name: ..... Date: ..... Period: .....

Student Choice Start with any problem. Remember to show or explain your thinking.

1

Pascal's triangle is an arrangement of numbers in a triangular array. It starts with a single 1 at the top, and every following cell is the sum of the two cells directly above.



a Complete the missing cells of Pascal's triangle.

b Multiply the given expressions.

$(a + b)^2 = (a + b)\ (a + b)$

$(a + b)^3 = (a + b)(a + b)(a + b)$

$(a + b)^4 = (a + b)(a + b)(a + b)(a + b)$

c What do you notice? Can you connect the numbers in each row with the product expressions you wrote in part b?

d Amy is working on the standard form of $(a + b)^5$. Help her to complete the equation using Pascal's triangle.

$$[...]a^5 + [...]a^4b + [...]a^3b^2 + [...]a^2b^3 + [...]ab^4 + [...]b^5$$

e Can you predict the $(a + b)^6$ based on Amy's expansion of the fifth power and Pascal's triangle?

Multiplying and Factoring (continued)

Name: ..... Date: ..... Period: .....

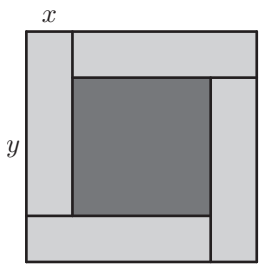
2

Here are commonly used identities to help writing the factored form and standard form of a given quadratic expression.

Square of the sum of two numbers $(a+b)^2$	Square of the difference of two numbers $(a-b)^2$	Product of the sum and difference of two numbers $(a+b)(a-b)$
$(a+b)(a+b) = a^2 + 2ab + b^2$	$(a-b)(a-b) = a^2 - 2ab + b^2$	$(a+b)(a-b) = a^2 - b^2$

These identities help solve many puzzle-like problems in mathematics. Here are some examples.

a Four identical rectangles with dimensions x and y enclosed the shaded region. Select the expression that represents the area of the shaded region.



A. $(x-y)^2$

B. $y^2 - x^2$

C. $x^2 - y^2$

D. $(x+y)^2$

b The perimeter of a rectangle is 46 cm and the length of one of its diagonals is 17 cm. Determine the area of the rectangle.

c How can you use the identities above to calculate the quotient of $\frac{105^2 - 2 \cdot 105 \cdot 64 + 64^2}{21^2 - 20^2}$ without a calculator?

d $9^x - 18 \cdot 3^x + 81 = 0$. Determine the value of x.

e Use $\frac{1}{a} - a = 7$ to determine the value of $\frac{1}{a} + a$.

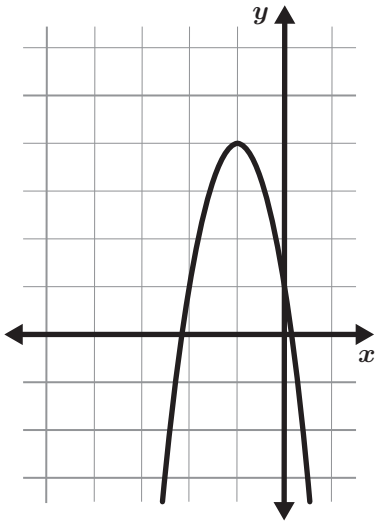
Solving Equations and Completing the Square

Name: ..... Date: ..... Period: .....

Student Choice Start with any problem. Remember to show or explain your thinking.

1

Here is the graph of the function $y = ax^2 + bx + c$.



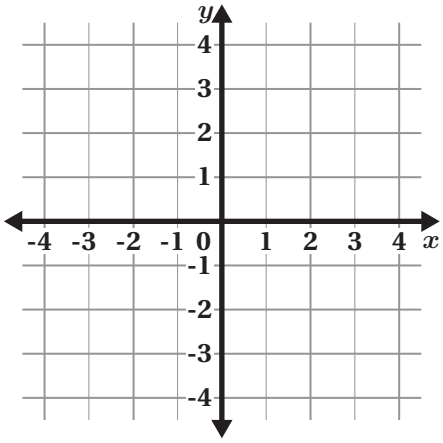
a Determine the signs of a, b, and c.

b Determine its axis of symmetry.

c Determine the coordinates of the vertex.

d Determine the equation of the graph.

e Draw a graph of the function $y = cx^2 + bx + a$.



2

$5x^2 + 20x + 25$ is not a perfect square, so you cannot rearrange into a single square. However, it can be arranged into the sum of squares. How can you rewrite this equation as the sum of two squares?

Solving Equations and Completing the Square (continued)

Name: Date: Period:

3 Determine a value of n if $x^2 + 16x = n$ has

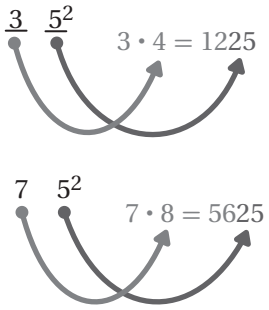
a no real solution

b one solution

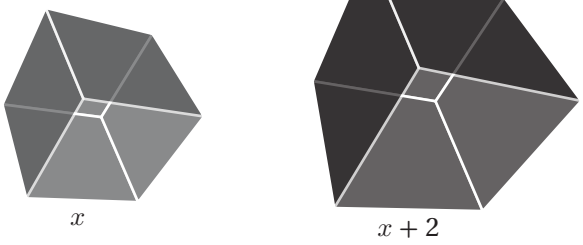
c two solutions

4 The square of a two-digit number $a5$ can easily be calculated using an algebraic trick of finding the product $a(a + 1)$, then placing the result as digits in front of 25.

Can you explain why this trick works?



5 Each edge of a cube is 2 inches longer than each edge of another cube. If the difference between the volumes is 98 cubic inches, find the edge lengths of the cubes.



The Quadratic Formula and More

Name: ..... Date: ..... Period: .....

Student Choice Start with any problem. Remember to show or explain your thinking.

1

In a chess tournament where each player competes against every other player, 36 games were played. How many people participated in this tournament?

2

Here is the Fibonacci sequence again: $1, 1, 2, 3, 5, 8, 13, 21, 34, 55, \ldots$
Let a, b, c be the three consecutive numbers in this sequence. As one of the many properties of the Fibonacci sequence, it is known that the ratio between the consecutive terms converges to the same number, i.e., $\frac{a}{b} = \frac{b}{a}$ which can also be written as $\frac{a+b}{4} = \frac{b}{a}$ because $c = b + a$.

Eva uses the following steps to write a quadratic equation to determine the value of this ratio.

Eva's Work

$\frac{a+b}{b} = \frac{b}{a}$

$\frac{a}{b} + \frac{b}{b} = \frac{b}{a}$ substitute $x = \frac{a}{b}$

$x + 1 = \frac{1}{x}$

$x^2 + x - 1 = 0$

a Help Eva to solve $x^2 + x - 1 = 0$ to determine the ratio between the consecutive Fibonacci numbers.

b What is so special about the solutions

3

Solve each equation.

a $x^4 - 7x^2 + 12 = 0$

b $x^4 - x^2 - 30 = 0$

The Quadratic Formula and More (continued)

Name: Date: Period:

4 Here is a table showing the standard form and factored form of the equations.

a Complete the table.

Standard form	Factored form	x-intercepts (m and n)
$x^2 + x - 6$	$(x - 2)(x + 3)$	2, -3
$x^2 - 2x - 3$		
	$(x - 6)(x + 6)$	
$2x^2 - 11x + 12$	$(2x - 3)(x - 4)$	
$3x^2 - 9x + 30$		

a Where do you see $m + n$ in standard form?

c Where do you see $m \cdot n$ in standard form?

d Determine two numbers whose sum is -7 and product is 12. How can the numbers you determined help you write the factored form of $x^2 + 7x + 12$?

e Suppose m and n are the solutions of the equation $x^2 - 7x + 5 = 0$. Determine the value of $m^2 + n^2$.

f Suppose m and n are the solutions of the equation $2x^2 + 15x + 16 = 0$.

Determine the value of $\frac{1}{m} + \frac{1}{n}$.

Investigations

Investigation 1
Exploring Global Warming

Investigate Data | Model with Functions | Function Investigations | Growth and Decay

A–CED.2, S–ID.6.a, S–ID.6.b, S–ID.6.c, S–ID.7, S–ID.8

Task 1

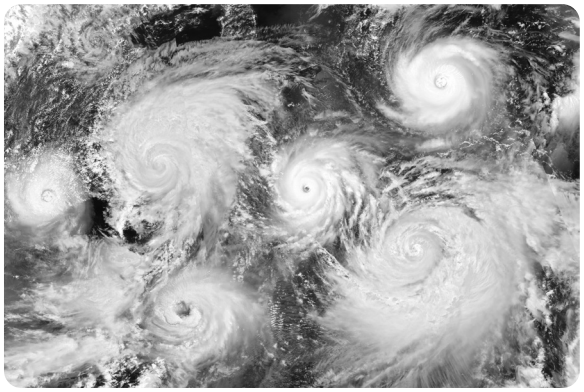
Name: ..... Date: ..... Period: .....

Global Connections

Here are some images of occurrences around the world.

Alexey Seafarer/Shutterstock.com

Toa55/Shutterstock.com



Elena11/Shutterstock.com

kapichka/Shutterstock.com

1. What do you notice? What do you wonder?

2. What associations come to mind when you hear the term *global warming*?

Task 1

Name: ..... Date: ..... Period: .....

Global Connections (continued)

Global warming refers to the rise in global temperatures which scientists say is mainly due to the increase of Greenhouse gases (GHG's) in the atmosphere. Let's explore some changes in global temperature. Use Screen 2 of the digital activity to view the scatterplot for the Global Temperature data set.

3. Create a scatter plot of the data and draw a line of best fit.

4. Use rows 3 and 5 to generate a line of best fit and the correlation coefficient for this data set.

5. Predict the average change in global temperature for the year 2030.

Let's explore a global topic further. Your group will choose a topic and be given the data set to analyze. You will use the digital activity to answer the following questions based on your topic choice.

6. Create a scatter plot of the data and draw a line of best fit.

7. **Data Talk!**

- What kind of trend does the data show?
- Describe the association between the variables.

8. Use row 3 to generate the correlation coefficient for your data.

9. What does the correlation coefficient tell you about the association between the variables in your topic?

10. Use row 5 to generate the line of best fit and write the equation model for your data.

11. Can your line of best fit be used to make reasonable predictions? Explain your thinking.

12. Use the line of best fit to predict an output value for your data set for the year 2030.

13. **Data Talk!** Discuss the similarities and differences between the Global Temperature Change data and the data for your topic.

Task 2

Name: ..... Date: ..... Period: .....

Gallery Tour

1. Your group will prepare a poster about the results of the data set you explored in Problems 6–12 and participate in a Gallery Tour.

   Use the **Display Checklist Sheet** if you find it helpful.

2. 💬 **Discuss:** Do you think there could be a possible association between global warming and all the topics presented during the Gallery Tour?

Task 3

Name: ..... Date: ..... Period: .....

Preventing Global Warming

Greenhouse gases (GHG's) trap heat in the atmosphere. The primary GHG emitted by human activity is carbon dioxide (CO_2). GHG's contribute to global warming by absorbing energy and creating an insulating layer in the Earth's atmosphere thus trapping heat.

Your carbon footprint is the total amount of GHG's that are released into the atmosphere through your daily activities. The average carbon footprint for a person in the United States per year is 16 tons of CO_2 while globally the average carbon footprint for a person is closer to 4 tons of CO_2.

1. Have you ever thought about your carbon footprint?

2. **Discuss:** What do you think should be considered when investigating an individual's carbon footprint?

Let's calculate your carbon footprint. Use the **Carbon Footprint Calculation Sheet** to find your carbon footprint value.

3. Your carbon footprint value: .....

4. Was the value for your carbon footprint surprising?

Knowing your carbon footprint is important because it allows for areas to be identified where CO_2 emissions can be reduced and empowers you to take action to a more sustainable future.

Review the **Actions to Reduce Your Carbon Footprint Sheet.**

5. **Discuss:**

a. What did you find interesting about the list of carbon footprint reducing actions?

b. Why do you think it is important for everyone to try to reduce their carbon footprint?

Task 3

Name: ..... Date: ..... Period: .....

Preventing Global Warming (continued)

Lowering an individual carbon footprint is not a task that happens overnight! Small actions can have a big impact when everyone contributes.

6. Decide on three actions from the **Actions to Reduce Your Carbon Footprint Sheet** that you can take to reduce your carbon footprint.

Action 1: .....

Action 2: .....

Action 3: .....

Connecting to Your Community!

Think about your community or its surrounding area.

Investigate your area. Here are some questions that may inspire your investigation.

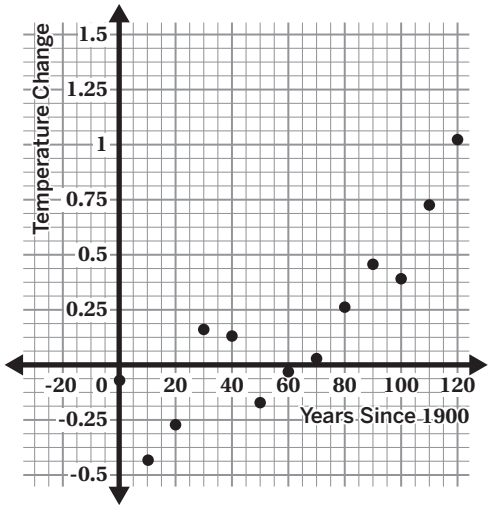
Or think of your own questions!

- What ways can you hold yourself accountable and be consistent with implementing your identified actions to reduce your carbon footprint?
- What can you do in your community to help reduce CO_2 emissions?
- What has been done in your state to reduce CO_2 emissions?

Global Average Temperature Change

Year	Years Since 1900	Temperature Change ($°F$) (compared to 1951–1980 average)
1900	0	-0.07
1910	10	-0.43
1920	20	-0.27
1930	30	0.16
1940	40	0.13
1950	50	-0.17
1960	60	-0.03
1970	70	0.03
1980	80	0.26
1990	90	0.45
2000	100	0.39
2010	110	0.72
2020	120	1.02

Source: *National Oceanic and Atmospheric Administration*



Line of Best Fit: .....

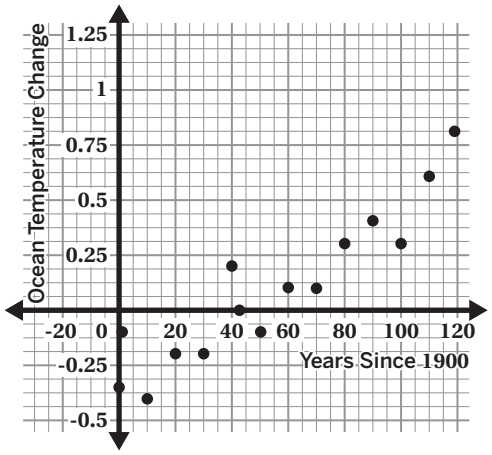
Correlation Coefficient: .....

Topic Choice: Global Ocean Temperatures Changes

Global ocean temperatures have changed over the years. This data set shows the change in global ocean temperatures relative to the average temperature in the 20th century. For example, a value of 0.3 for 2000 means that the average global temperature in 2000 was 0.3°C higher than the 20th century average.

Year	Years Since 1900	Temperature Change (°C)
1901	1	-0.1
1910	10	-0.4
1920	20	-0.2
1930	30	-0.2
1940	40	0.2
1950	50	-0.1
1960	60	0.1
1970	70	0.1
1980	80	0.3
1990	90	0.4
2000	100	0.3
2010	110	0.6
2019	119	0.8

Source: *National Oceanic and Atmospheric Administration*



Line of Best Fit: .....

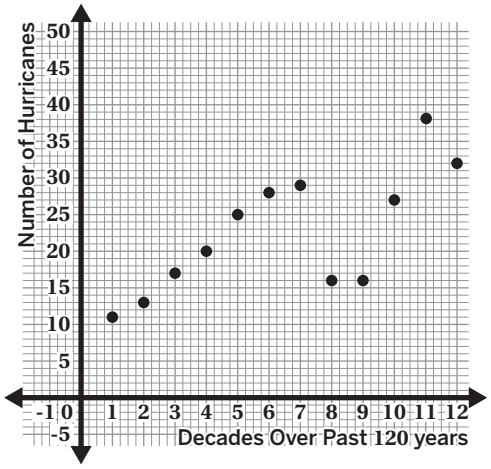
Correlation Coefficient: .....

Topic Choice: Number of Major Hurricanes in the Atlantic

Major hurricanes (category 3, 4, or 5) are characterized by high wind speeds and significant storm surges can cause devastating effects. This dataset shows the number of major hurricanes in the Atlantic Basin over the past 12 decades.

Years	Decade	Number of Major Hurricanes (per decade)
1901–1910	1	11
1911–1920	2	13
1921–1930	3	17
1931–1940	4	20
1941–1950	5	25
1951–1960	6	28
1961–1970	7	29
1971–1980	8	16
1981–1990	9	16
1991–2000	10	27
2001–2010	11	38
2011–2020	12	32

Source: *National Oceanic and Atmospheric Administration*



Line of Best Fit: .....

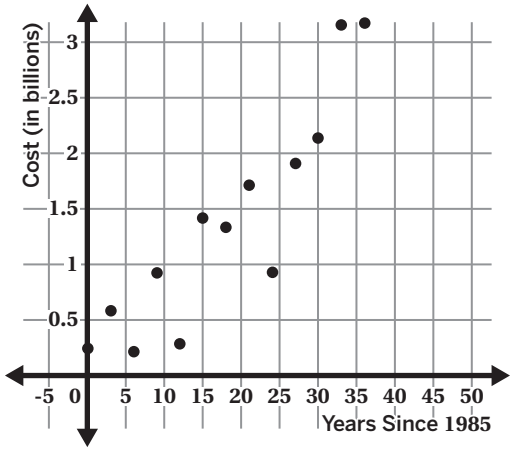
Correlation Coefficient: .....

Topic Choice: Cost of Fire Suppression

The federal government spends money on suppression, disaster assistance, and other indirect costs when it comes to fire suppression like those due to forest fires. This data set shows the cost in billions of dollars for firefighting suppression costs by the federal government.

Year	Years Since 1985	Cost in Billions of Dollars
1985	0	0.240
1988	3	0.579
1991	6	0.206
1994	9	0.918
1997	12	0.284
2000	15	1.411
2003	18	1.327
2006	21	1.704
2009	24	0.921
2012	27	1.902
2015	30	2.131
2018	33	3.143
2021	36	3.166

Source: *National Oceanic and Atmospheric Administration*



Line of Best Fit: .....

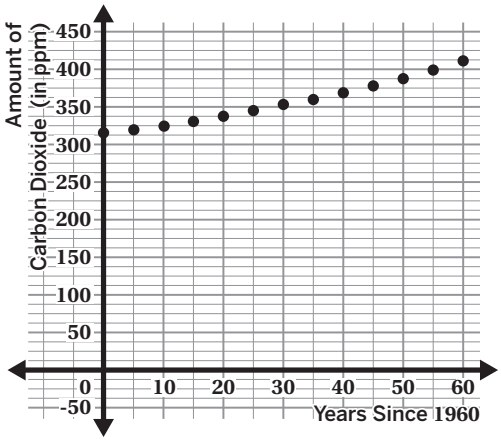
Correlation Coefficient: .....

Topic Choice: Atmospheric Carbon Dioxide

Atmospheric carbon dioxide refers to the concentration of carbon dioxide gas present in the Earth's atmosphere. Carbon dioxide plays a major role in trapping heat and influencing the planet's temperature. This data set shows the amount of atmospheric carbon dioxide in parts per million (ppm) since 1960.

Year	Years Since 1960	Amount of Carbon Dioxide (parts per million)
1960	0	316
1965	5	320
1970	10	325
1975	15	331
1980	20	338
1985	25	345
1990	30	353
1995	35	360
2000	40	369
2005	45	378
2010	50	388
2015	55	400
2020	60	412

Source: *National Oceanic and Atmospheric Administration*



Line of Best Fit: .....

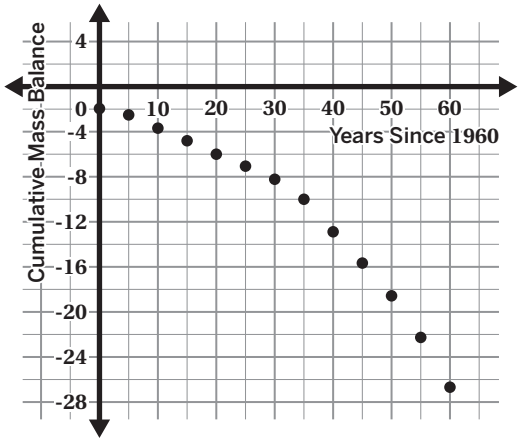
Correlation Coefficient: .....

Topic Choice: Mass Balance of Glaciers

Cumulative mass balance is the balance between snow accumulation and melting in glaciers. This is measured in meters of water. This data set shows the average cumulative mass balance of glaciers worldwide.

Year	Years Since 1960	Cumulative Mass Balance (meters of water equivalent)
1960	0	-2.01
1965	5	-2.52
1970	10	-3.72
1975	15	-4.82
1980	20	-5.98
1985	25	-7.10
1990	30	-8.27
1995	35	-10.02
2000	40	-12.91
2005	45	-15.68
2010	50	-18.61
2015	55	-22.29
2020	60	-26.74

Source: *Environmental Protection Agency*



Line of Best Fit: .....

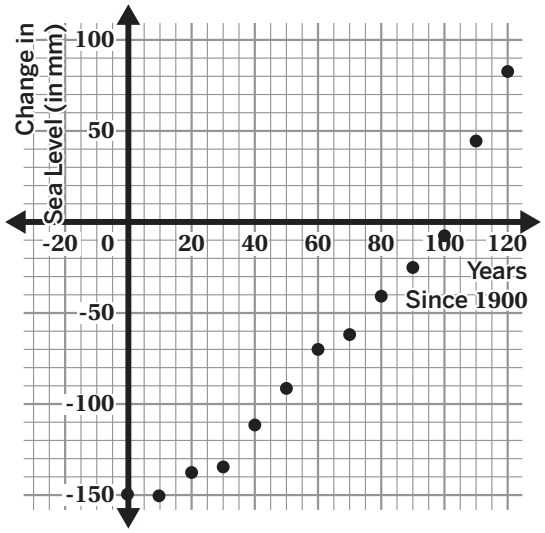
Correlation Coefficient: .....

Topic Choice: Global Sea Level Change

Global sea levels have changed over time This data set shows the change in global sea levels relative to the average sea level from 1993–2008. For example, a value of -135 for 1930 means that the sea level in 1930 was 135 mm lower than the 1993–2008 sea level average.

Year	Years Since 1900	Change in Sea Level (mm)
1900	0	-150
1910	10	-151
1920	20	-138
1930	30	-135
1940	40	-112
1950	50	-92
1960	60	-70
1970	70	-62
1980	80	-41
1990	90	-25
2000	100	-8
2010	110	45
2020	120	83

Source: *National Oceanic and Atmospheric Administration*



Line of Best Fit: .....

Correlation Coefficient: .....

Carbon Footprint Calculation

Let's get an idea of your current carbon footprint by answering a few questions below that are based on the Environmental Protection Agency's (EPA) Carbon Footprint Calculator. Please note that this list is not exhaustive.

Each answer choice is connected to an annual value of CO_2 output in pounds per year. That value is placed within the parenthesis. Place the sum of all these values on the column on the right for each choice selected per question. Then sum up all the values at the end to get a total.

Questions **Value**

1. Select all the energy sources that you use at home? Select *all* that apply. 1. .....

- ☐ **A.** Natural Gas (3090)
- ☐ **B.** Electricity (2725)
- ☐ **C.** Fuel Oil (4860)
- ☐ **D.** Propane (2234)

2. Do you turn off all the lights when you leave your home? 2. .....

A. Yes (266) **B.** No (536)

3. Are your clothes washed in cold water? 3. .....

A. Yes (-153) **B.** No (0)

4. How are your clothes dried? 4. .....

A. Hang to dry (0) **B.** Dryer (800) **C.** Both (400)

5. What type of transportation does your household mostly use? 5. .....

- ☐ **A.** Drive your own car (10000)
- ☐ **B.** Carpool (3000)
- ☐ **C.** Bus or Train (1100)
- ☐ **D.** Walk or Bike (0)

6. Does your household recycle? 6. .....

A. Yes (0) **B.** No (692)

7. What does your household recycle? Select *all* that apply. 7. .....

- ☐ **A.** Aluminum and Steel Cans (-89)
- ☐ **B.** Plastic (-36)
- ☐ **C.** Newspapers and Magazines (-141)
- ☐ **D.** Glass (-25)

8. Do you enable the power saving features on your computer? 8. .....

A. Yes (-800) **B.** No (0)

9. Does your household have an energy efficient refrigerator? 9. .....

A. Yes (-65) **B.** No (0)

Total value: .....

Display Checklist

Use this checklist to help you plan and organize your poster. You should include these items. Check them off as you complete them.

Table of Data

Graphs

- ☐ Shows scatterplot with line of fit drawn.

Calculations

- ☐ Determines the correlation coefficient.
- ☐ Determines equation for line of best fit.
- ☐ Determines a prediction for the year 2030.

Reflection Question Answers

- ☐ Were any of your findings surprising? Why or why not?
- ☐ What connections do you think there are between your data, topic, and global warming?
- ☐ What other questions do you have regarding your analysis?

Actions to Reduce Your Carbon Footprint

Here is a list of actions and the amount of pounds of CO_2 reduced over a year.

Action	Pounds of CO_2 Saved Per Year
Use an electric vehicle	8500
Use a hybrid vehicle	6400
Carpool to school	5000
Ride your bike instead of driving	8100
Take public transportation instead of driving	8100
Turn off lights	400
Reduce the winter household thermostat	1600
Increase the summer household thermostat	600
Use rechargeable batteries	400
Manage computer energy use	800
Line dry clothing	800
Reduce waste	800
Print double sided	600
Eat a low carbon diet	3400

Investigation 2
Tuition Costs

Model with Functions | Function Investigations | Growth and Decay | Financial LIteracy

F–IF.2, F–LE.1, N–Q.3, S–ID.6.a

Task 1

Name: ..... Date: ..... Period: .....

Types of Post-Secondary Education

After high school, many people further their education to better prepare themselves for a career. There are many options for post-secondary education, and tuition costs can vary between these different options. Here are some examples of post-secondary options.

Four-Year Colleges and Universities: Students attend these colleges and universities to earn bachelor's degrees. These vary in location, size, admission criteria, and the types of courses offered. Some are *public*, offering lower tuition for students who are residents of the state where the college is located. Others are *private*, which typically have higher tuition rates than public colleges or universities. A student can choose to attend a public school in any state, but the cost of tuition will be lower at schools in the state in which a student resides.

Two-Year Colleges: Students can attend these colleges to earn associate's degrees. Like four-year institutions, community colleges and junior colleges can be public or private. Typically, tuition rates are lower than at four-year institutions, but they are not typically residential. Graduates of two-year colleges often go on afterwards to earn a bachelor's degree from a four-year college.

Vocational Schools: These technical and trade schools offer specialized training to students for various lengths of time depending on the specialization. Some career choices may require this type of training before securing a job. Program examples include culinary, cosmetology, computer technician, medical assistant, and plumbing. Some two-year colleges offer vocational programs as well.

1. What type of career are you interested in?

2. **Discuss:** Based on your career choice, what type of post-secondary school interests you? What might you want to study while there?

Task 1

Name: ..... Date: ..... Period: .....

Types of Post-Secondary Education (continued)

3. You will now further explore post-secondary education options by looking more closely at two schools. Choose any two post-secondary schools that may be of interest to you. These can be California-based schools or schools located in other states. Be sure to pick institutions from two different categories (public 4-year, private 4-year, 2-year, vocational school).

School 1	School 2

4. Now that you have chosen two schools of interest, let's find out some more information about them.

School	Location	Enrollment	Two interesting facts about the school

5. What else are you interested in learning about for each of the schools you chose?

6. How might school costs impact a student's choice for the type of school to attend? Explain your thinking.

Task 1

Name: ..... Date: ..... Period: .....

Analyzing Tuition Costs

There are many costs associated with post secondary education, and one of these costs is *tuition*. You will use Screens 2–5 of the digital activity to explore tuition trends for two types of post secondary institutions (Screen 2: Public 4-Year, Screen 3: Private 4-Year, Screen 4: Public 2-Year, Screen 5: Private 2-Year):

Choice 1: (circle one)

4-Year Public 4-Year Private 2-Year Public 2-Year Private

1. **Data Talk!** Look closely at the data from 1983–2022 (1983 represents the 1983–1984 school year) for your selection. How would you describe tuition costs during that time period?

2. Generate a linear *and* an exponential model for the data. Hint: For a linear model, type $y_1 \sim ax_1 + b$ in Row 3. For an exponential model, type $y_1 \sim a \cdot b^{x_1}$ in Row 5.

3. Record the slope of the linear model and the growth factor of the exponential model. Then explain what each value says about how the tuition is changing each year.

Linear Model	Exponential Model
Slope: This means . . .	Growth Factor: This means . . .

4. Which model do you think better fits the data for the tuition costs from 1983–2022? Explain your thinking.

5. Use each model to predict the tuition costs for the year you graduate high school.

Linear model prediction: Exponential model prediction:

6. Do you think either model should be used to predict the tuition costs in 2075? Explain your thinking.

Task 2

Name: ..... Date: ..... Period: .....

Analyzing Tuition Costs (continued)

Choice 2: (circle one)

4-Year Public 4-Year Private 2-Year Public 2-Year Private

7. **Data Talk!** Look closely at the data from 1983–2022 for your selection. How would you describe tuition costs during that time period?

8. Generate a linear and an exponential model for the data. Hint: For a linear model, type $y_1 \sim ax_1 + b$ in Row 3. For an exponential model, type $y_1 \sim a \cdot b^{x_1}$ in Row 5.

9. Record the slope of the linear model and the growth factor of the exponential model. Then explain what each value says about how the tuition is changing each year.

Linear Model	Exponential Model
Slope: This means . . .	Growth Factor: This means . . .

10. Which model do you think better fits the data for the tuition costs from 1983–2022? Explain your thinking.

11. Use each model to predict the tuition costs for the year you graduate high school in.

Linear model prediction: Exponential model prediction:

12. Do you think either model should be used to predict the tuition costs in 2075? Explain your thinking.

Task 3

Name: ..... Date: ..... Period: .....

Comparing Costs

1. **Discuss:** Compare your models and findings for tuition costs from each school choice that you analyzed in Task 2. What do you notice? What do you wonder?

2. Was the same type of model (linear or exponential) the best fit for both sets of data? Why might this be the case?

3. Let's look at one of the specific schools that you chose in Task 1. Determine the tuition costs for this school year. How does this amount compare to the average tuition costs given to you in Task 2?

4. Tuition costs are one component of the total cost for attending a post-secondary school. What are some of the other costs associated with post-secondary education, and how might they differ depending on the type of school attended?

5. **Discuss:** There are many resources that exist to help make the costs of post-secondary education more manageable. Use the *Types of Financial Aid* Sheet to learn more about the types of aid that may be available to you. What do you notice? What do you wonder?

6. How might this impact your current thoughts about the type of school that you would like to attend after high school?

Types of Financial Aid

Financial aid is money that can be used to help pay for college or career school. Financial aid can include grants, work-study, loans, and scholarships, and it can help make college or career school more affordable. Here is some more information about each of these types of aid:

Grants: A grant is a type of financial aid that does not have to be repaid.

Scholarships: Scholarships are also a type of financial aid that do not have to be repaid. This type of aid can be based on academic merit, talent, or a particular area of study. Many organizations (both non-profit and private) offer scholarships to help students pay for school, and many schools offer their own scholarships as well.

Work-Study Jobs: The Federal Work-Study Program allows you to work part-time to earn money to pay for education expenses. The program encourages employment in civic education and work related to the student's course of study when possible. These can be on-campus or off-campus jobs.

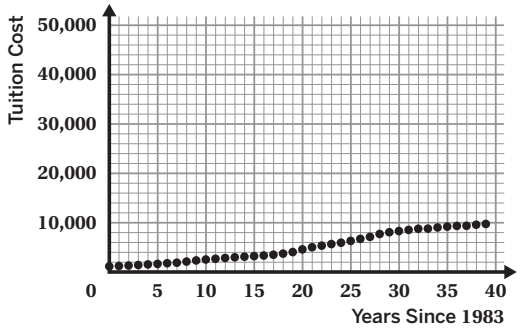
Loans: A loan is money that is borrowed and must be paid back with interest. Student loans allow you to borrow money to attend a college or career school. After you complete your program, you will begin to repay the loan, as well as the interest that accrues. Loans can be through the federal government or private banks, but federal loans usually have the most benefits.

Source: *Federal Student Aid, An Office of the U.S. Department of Education*

Name _____ Date _____

Average Tuition Costs: Public 4-Year Universities

School Year, Fall	Years Since 1983	Average Tuition Costs	School Year, Fall	Years Since 1983	Average Tuition Costs
1983	0	1148	2003	20	4587
1984	1	1228	2004	21	5027
1985	2	1318	2005	22	5351
1986	3	1414	2006	23	5666
1987	4	1537	2007	24	5943
1988	5	1646	2008	25	6312
1989	6	1780	2009	26	6717
1990	7	1888	2010	27	7132
1991	8	2117	2011	28	7713
1992	9	2349	2012	29	8070
1993	10	2537	2013	30	8312
1994	11	2681	2014	31	8543
1995	12	2848	2015	32	8778
1996	13	2987	2016	33	8804
1997	14	3110	2017	34	9036
1998	15	3229	2018	35	9212
1999	16	3349	2019	36	9349
2000	17	3501	2020	37	9374
2001	18	3735	2021	38	9596
2002	19	4046	2022	39	9750

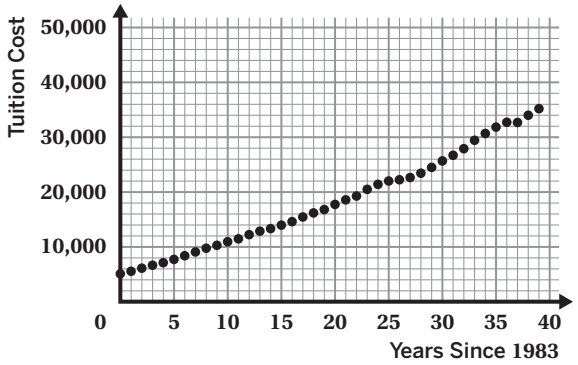


Source: *National Center for Education Statistics*

Name _____ Date _____

Average Tuition Costs: Private 4-Year Universities

School Year, Fall	Years Since 1983	Average Tuition Costs	School Year, Fall	Years Since 1983	Average Tuition Costs
1983	0	5093	2003	20	17763
1984	1	5556	2004	21	18604
1985	2	6121	2005	22	19292
1986	3	6658	2006	23	20517
1987	4	7116	2007	24	21427
1988	5	7722	2008	25	22040
1989	6	8396	2009	26	22269
1990	7	9083	2010	27	22677
1991	8	9759	2011	28	23464
1992	9	10294	2012	29	24523
1993	10	10952	2013	30	25707
1994	11	11481	2014	31	26739
1995	12	12243	2015	32	27942
1996	13	12881	2016	33	29476
1997	14	13344	2017	34	30723
1998	15	13973	2018	35	31883
1999	16	14616	2019	36	32764
2000	17	15470	2020	37	32728
2001	18	16211	2021	38	34051
2002	19	16826	2022	39	35248

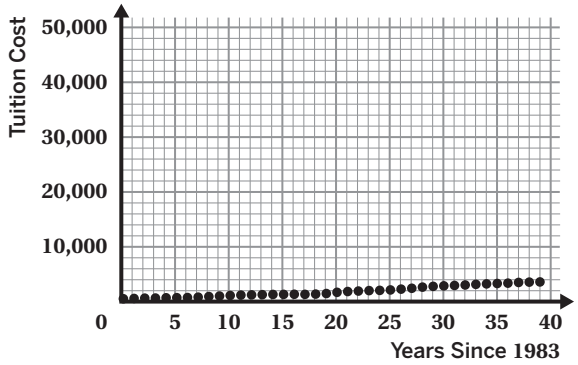


Source: *National Center for Education Statistics*

Name _____ Date _____

Average Tuition Costs: Public 2-Year Institutions

School Year, Fall	Years Since 1983	Average Tuition Costs	School Year, Fall	Years Since 1983	Average Tuition Costs
1983	0	528	2003	20	1702
1984	1	584	2004	21	1849
1985	2	641	2005	22	1935
1986	3	660	2006	23	2017
1987	4	706	2007	24	2058
1988	5	730	2008	25	2136
1989	6	756	2009	26	2283
1990	7	824	2010	27	2441
1991	8	936	2011	28	2651
1992	9	1025	2012	29	2792
1993	10	1125	2013	30	2881
1994	11	1192	2014	31	2955
1995	12	1239	2015	32	3038
1996	13	1276	2016	33	3156
1997	14	1314	2017	34	3242
1998	15	1327	2018	35	3312
1999	16	1348	2019	36	3377
2000	17	1333	2020	37	3503
2001	18	1380	2021	38	3563
2002	19	1483	2022	39	3598

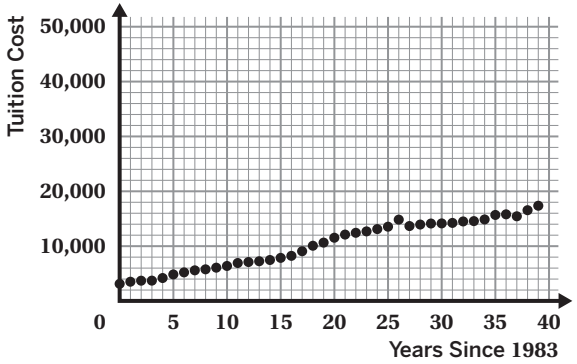


Source: *National Center for Education Statistics*

Name _____ Date _____

Average Tuition Costs: Private 2-Year Institutions

School Year, Fall	Years Since 1983	Average Tuition Costs	School Year, Fall	Years Since 1983	Average Tuition Costs
1983	0	3099	2003	20	11545
1984	1	3485	2004	21	12122
1985	2	3672	2005	22	12450
1986	3	3684	2006	23	12708
1987	4	4161	2007	24	13128
1988	5	4817	2008	25	13567
1989	6	5196	2009	26	14862
1990	7	5570	2010	27	13687
1991	8	5754	2011	28	13961
1992	9	6059	2012	29	14149
1993	10	6370	2013	30	14170
1994	11	6914	2014	31	14261
1995	12	7094	2015	32	14528
1996	13	7236	2016	33	14589
1997	14	7464	2017	34	14894
1998	15	7854	2018	35	15720
1999	16	8225	2019	36	15831
2000	17	9067	2020	37	15473
2001	18	10076	2021	38	16588
2002	19	10651	2022	39	17408



Source: *National Center for Education Statistics*