Amplify Desmos Math CALIFORNIA

Grade 8

Volume 2: Units 5–8

Student Edition

About Amplify

Amplify is dedicated to collaborating with educators to create learning experiences that are rigorous and riveting for all students. Amplify creates K–12 core and supplemental curriculum, assessment, and intervention programs for today's students.

A pioneer in K–12 education since 2000, Amplify is leading the way in next-generation curriculum and assessment. All of our programs provide teachers with powerful tools that help them understand and respond to the needs of every student.

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Amplify gratefully acknowledges the work of distinguished program advisors from English Learners Success Forum (ELSF), who have been integral in the development of Amplify Desmos Math. ELSF is a 501(c)(3) nonprofit organization whose mission is to expand educational equity for multilingual learners by increasing the supply of high-quality instructional materials that center their cultural and linguistic assets.

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Dear Student,

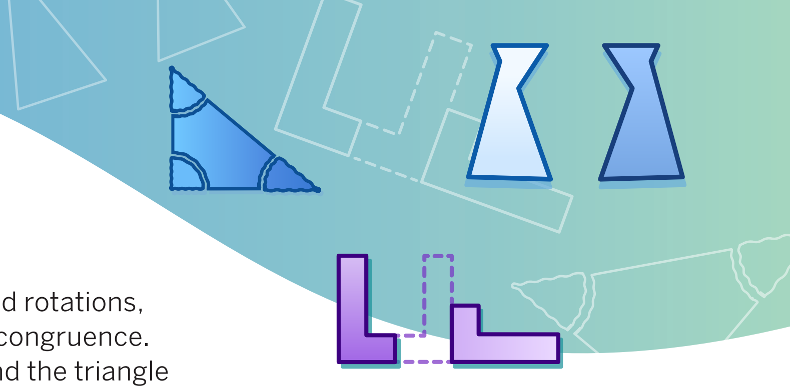
Welcome to Amplify Desmos Math California! We are excited to be partnering with you this year. You play an essential role in math class, so we wanted to reach out to introduce ourselves and tell you a bit about who we are.

Amplify Desmos Math California is a team of math educators on a mission to support you and your classmates in learning math. We hope each lesson inspires you to use your creativity, ask questions, and discover connections between math concepts and the world around us.

Here is what you can expect this year:

- A blend of learning on both paper and devices.
- Interactive lessons that encourage you to ask questions, explore, settle disputes, create challenges for your classmates, and more!
- Digital activities that show what your ideas mean with plenty of chances for you to revise.
- Opportunities for you to engage with interesting and important big ideas in mathematics.

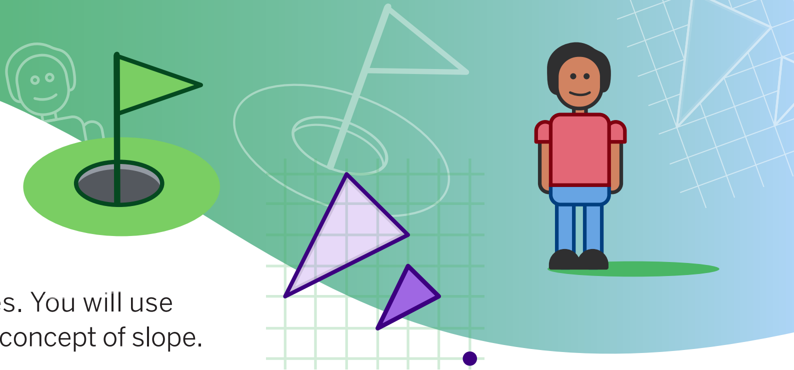
–The Amplify Desmos Math California Team



Unit 1 Rigid Transformations and Congruence

In this unit, you will investigate translations, reflections, and rotations, and use these transformations to make arguments about congruence. You will also explore angle relationships on parallel lines and the triangle sum theorem.

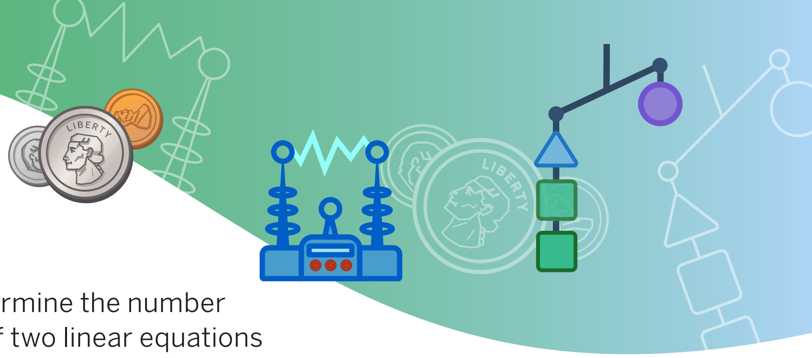
Unit 2 Dilations, Similarity, and Slope



In this unit, you will study dilations and similar figures. You will use similar triangles to explain, understand, and apply the concept of slope.

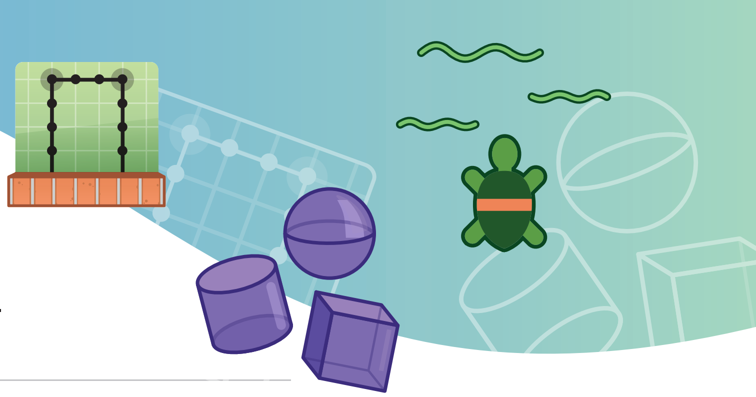
Unit 3 Proportional and Linear Relationships

In this unit, you will make connections between proportional relationships and non-proportional linear relationships. You will compare representations of linear relationships and interpret them in context. You will also determine solutions to two-variable linear equations of different forms.



Unit 4 Linear Equations and Linear Systems

In this unit, you will solve linear equations and determine the number of possible solutions. You will also solve systems of two linear equations algebraically and graphically.



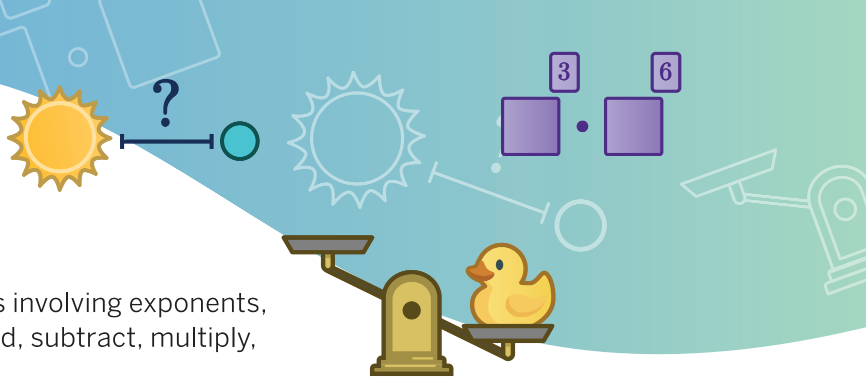
Unit 5 Functions and Volume

In this unit, you will learn about functions, analyze representations of functions, and examine functions in the context of the volume of cylinders, cones, and spheres.

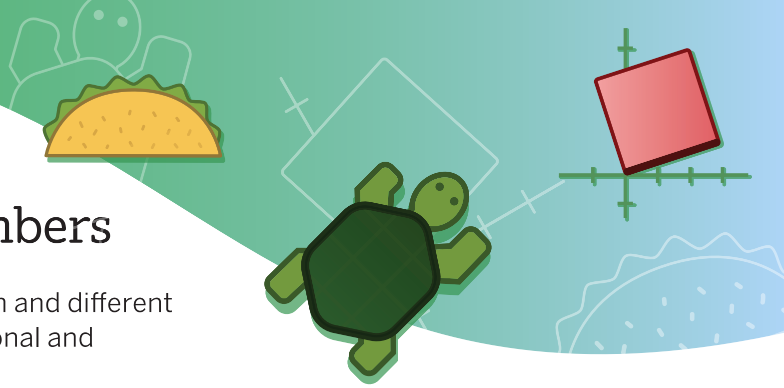
Unit 6 Associations in Data

In this unit, you will analyze data with two variables. You will use scatter plots and fitted lines to analyze numerical data and use two-way tables, bar graphs, and segmented bar graphs to analyze categorical data.

Unit 7 Exponents and Scientific Notation



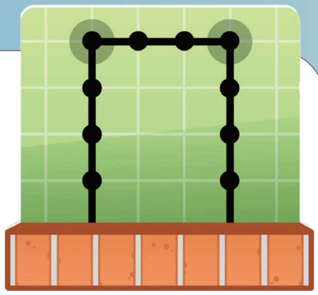
In this unit, you will develop fluency with expressions involving exponents, powers of 10, and scientific notation. You will also add, subtract, multiply, and divide numbers written in scientific notation.



Unit 8 The Pythagorean Theorem and Irrational Numbers

In this unit, you will explore the Pythagorean theorem and different types of numbers (square roots and cube roots, rational and irrational numbers).

Unit 5



Functions and Volume

Big Ideas in This Unit

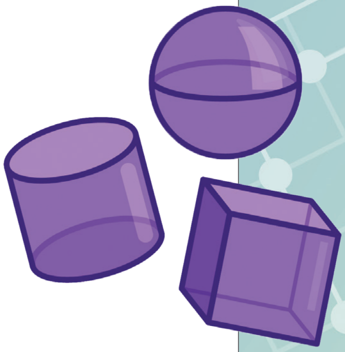
CC1 Data, Graphs, and Tables | Data Explorations | CC2 Linear Equations

Multiple Representations of Functions

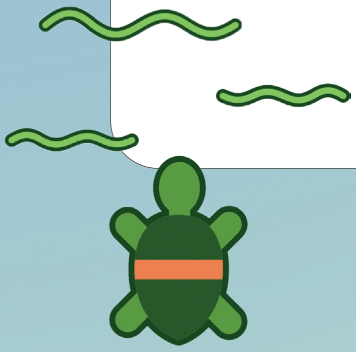
CC3 Cylindrical Investigations | CC4 Shape, Number, and Expressions

Questions for Investigation

- What makes a relationship a function?
- How are functions useful in representing situations?
- What are some relationships between a cylinder, a cone, and a sphere with common dimensions?



Explore: Graphs of Sounds
What could a graph of sound look like?



Watch Your Knowledge Grow

This is the math you'll explore in this unit. Rate your understanding to see how your knowledge grows!

○ Not yet — ○ Almost — ○ I got it!

I can . . .	Before	After
Describe what a function is.	○—○—○	○—○—○
Recognize that the independent variable is the input and the dependent variable is the output.	○—○—○	○—○—○
Justify whether a table or a graph represents a function.	○—○—○	○—○—○
Compare properties of functions represented in different ways.	○—○—○	○—○—○
Interpret the graph of a function in a situation in context.	○—○—○	○—○—○
Describe the relationship between two variables by analyzing a graph and using the terms increasing, decreasing, linear, or non-linear.	○—○—○	○—○—○
Sketch a graph representing a situation that is increasing, decreasing, linear, or non-linear.	○—○—○	○—○—○
Determine whether a function is linear or non-linear.	○—○—○	○—○—○
Determine the rate of change of a linear function.	○—○—○	○—○—○
Interpret the rate of change of a linear function in terms of the situation.	○—○—○	○—○—○

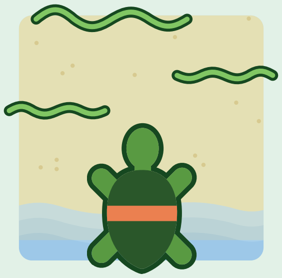
I can . . .	Before	After
Compare two linear functions and determine which function has the greater rate of change.	○—○—○	○—○—○
Determine the initial value of a linear function.	○—○—○	○—○—○
Interpret the initial value of a linear function in terms of the situation.	○—○—○	○—○—○
Determine whether a situation can be represented by a linear function.	○—○—○	○—○—○
Create a function to model a linear relationship.	○—○—○	○—○—○
Describe how the volumes of cylinders, cones, and spheres are related.	○—○—○	○—○—○
Determine the volume of a cylinder.	○—○—○	○—○—○
Analyze the relationship between the height or radius of a cylinder and its volume.	○—○—○	○—○—○
Develop a formula and use it to determine the volume of a cone.	○—○—○	○—○—○
Develop a formula and use it to determine the volume of a sphere.	○—○—○	○—○—○
Determine the unknown dimension of a cylinder, cone, or sphere given its volume or another dimension.	○—○—○	○—○—○

Sub-Unit

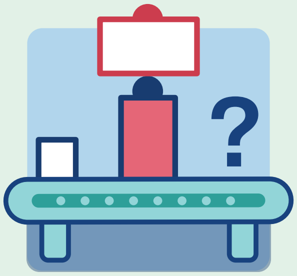
1

Introduction to Functions

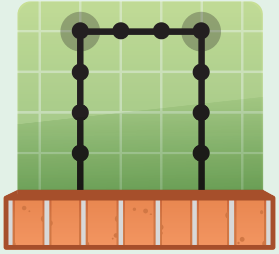
Explore
Graphs of Sounds



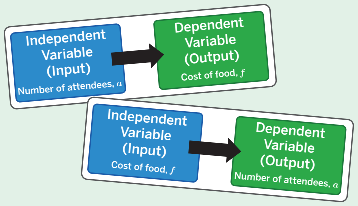
Lesson 1
Turtle Crossing



Lesson 2
Guess My Rule



Lesson 3
Function or Not?



Lesson 4
Dependence Day

Name: .... Date: .... Period: ....

Data, Graphs, and Tables Multiple Representations of Functions

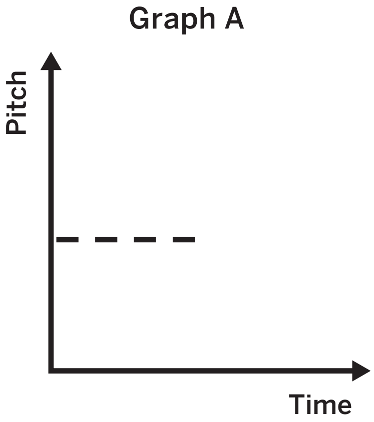
Building Toward 8.F.1, 8.F.5, SMP.2, SMP.4

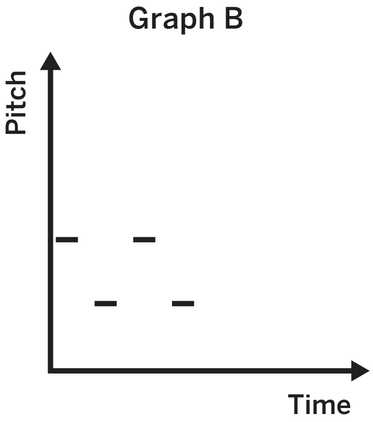
Explore: Graphs of Sounds

What could a graph of sound look like?

Warm-Up

1. You will listen to some sounds.Which graph could represent the sounds?
Hint: Pitch refers to how high or low a note sounds. ELD.PI.8.6.Em, Ex, Br





Explain your thinking.

Graph A represents Sound 2 and Graph B represents sound B. *Explanations vary.* Sound 1 had four sounds that had the same pitch, while Sound 2 alternated a sound with a high pitch and then a sound with a low pitch.

Name: .... Date: .... Period: ....

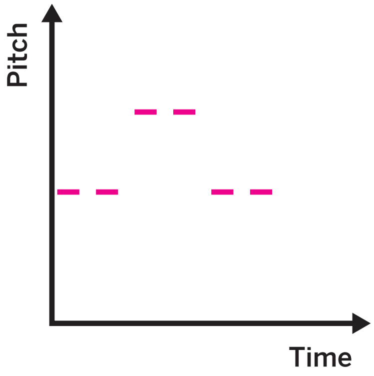
What's That Sound?

2. You will use a set of cards to complete this activity. Each card contains a sound you will make. Take turns with your partner as either the sound maker or the listener.

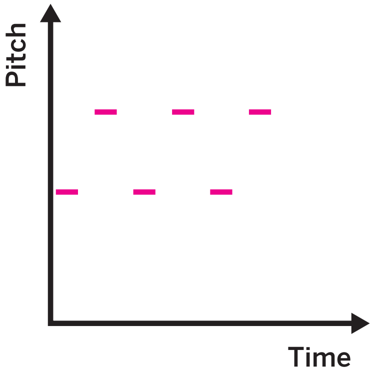
- Decide who will be the sound maker and who will be the listener. Start with Card 1.
- Sound Maker: Create the sound described on the card.
- Listener: Actively listen to your partner and the sound they are making.
- Together: Use the space below to sketch a graph that represents how the sound changes over time.
- Switch roles for Card 2 and repeat. Then do the same for Cards 3–8.

Responses vary.

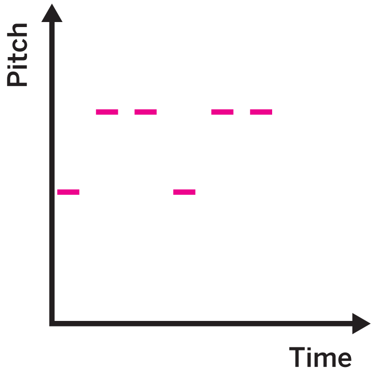
Card 1



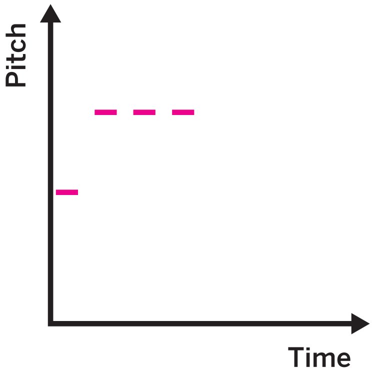
Card 2



Card 3



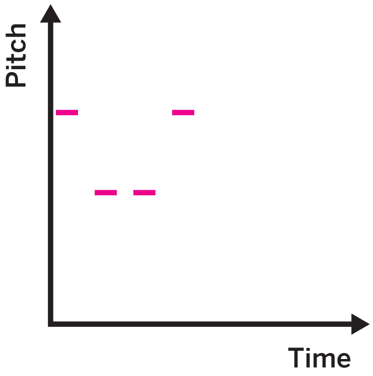
Card 4



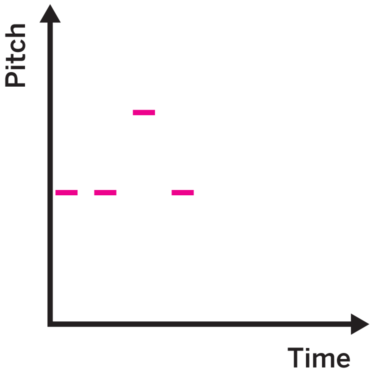
Name: ______________________ Date: __________ Period: __________

What's That Sound? (continued)

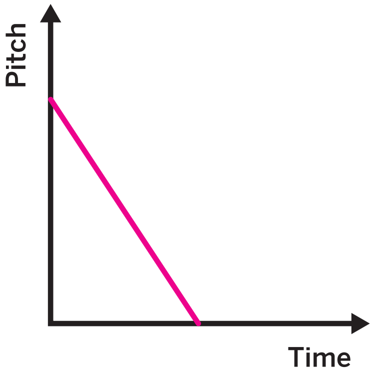
Card 5



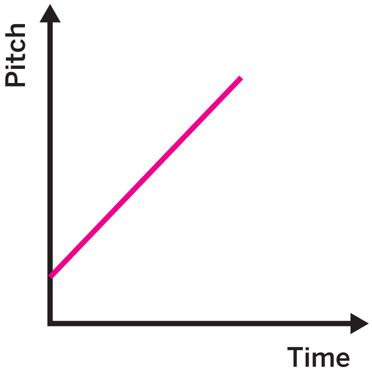
Card 6



Card 7

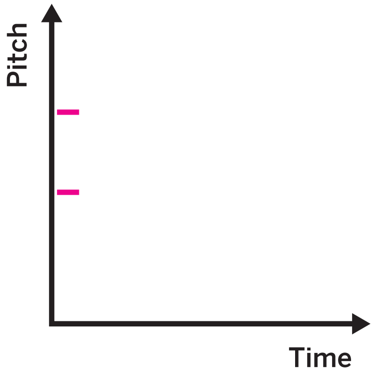


Card 8



3. Suppose you say "Fa" and your partner says "La" in a higher voice at the same time. Sketch a graph that could represent this scenario.

Responses vary. Sample response shown on graph.



Discuss: How is this graph similar to the ones from the ones you created in the activity? How is it different?

ELD.PI.8.1.Em, Ex, Br

Responses vary. This graph is similar because it shows horizontal lines. It is different because the graph has two lines graphed for the same time.

Name: .... Date: .... Period: ....

Building Math Habits of Mind

Discuss:

- Which of these habits of mind did you strengthen during this activity?
- How did you use the one(s) you selected?

I can slow down and first make sense of a challenging problem before trying to solve it.

○ Not yet ○ Almost ○ I got it!

I can represent real-world problems using equations and inequalities and interpret their solutions within the context of the problem.

○ Not yet ○ Almost ○ I got it!

I can justify my thinking and ask questions to help me understand the thinking of others.

○ Not yet ○ Almost ○ I got it!

I can apply the math that I know to solve real-world problems, make assumptions and revise my thinking as needed.

○ Not yet ○ Almost ○ I got it!

I can select an appropriate tool to help me solve problems.

○ Not yet ○ Almost ○ I got it!

I can communicate my thinking and solutions clearly to others.

○ Not yet ○ Almost ○ I got it!

I can look for structure or patterns to help me solve problems.

○ Not yet ○ Almost ○ I got it!

I can look for repeated calculations and other repeated steps to make generalizations.

○ Not yet ○ Almost ○ I got it!

Unit 5

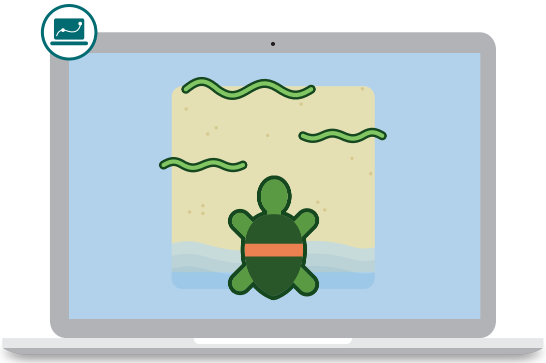
Lesson

1

Name: .... Date: .... Period: ....

Data, Graphs, and Tables | Multiple Representations of Functions

Building Toward 8.F.1, 8.F.5, SMP.2

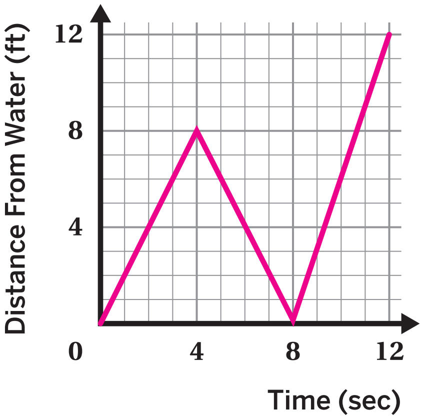


Turtle Crossing

Let's make sense of graphs.

Warm-Up

1 **a** Draw a distance vs. time graph to represent a turtle's journey across the sand.



b What story does your graph tell about the turtle's journey?

***Responses vary*. The turtle was headed toward the grass at 2 feet per second. It got scared so it headed back to the water. But the turtle suddenly felt brave and ran to the grass in the last 4 seconds.**

Activity 1

Name: .... Date: .... Period: ....

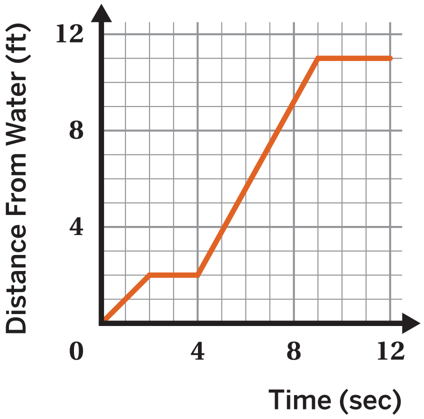
Turtle Graphs

2 Luca drew this graph to represent a turtle's journey.

What story does the graph tell about the turtle's journey?

Responses vary.

- **The turtle walks, stops, then walks again. Then it stops to rest.**
- **The turtle begins with its nose at the edge of the water and walks toward the grass for 2 seconds at a speed of 1 foot per second. Then the turtle stops for 2 seconds. Next, the turtle walks toward the grass for 4.5 seconds at a speed of 2 feet per second. After a total of 8.5 seconds, it stops on the grass, 11 feet away from the water.**



3 Let's watch an animation to see what Luca's turtle did.

💬 **Discuss:** How does this animation compare to your story?

Responses vary. **My story was similar to the animation because the turtle moves, stops for 2 seconds, and then moves faster.**

4 Use Luca's graph to answer each question.

a At 8 seconds, how far is the turtle from the water?

10 feet

b When is the turtle 4 feet from the water?

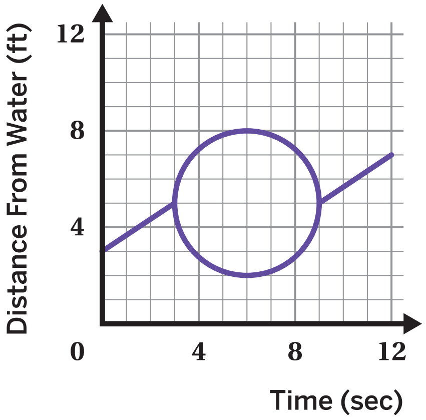
5 seconds

Name: .... Date: .... Period: ....

Turtle Graphs (continued)

5 Arnav drew this graph to represent a new turtle.

What story does the graph tell about the turtle's journey?



Responses vary.

- **The turtle walks forward then splits into two turtles, with one walking forward and one walking backward. The turtles then merge back into one turtle that continues walking forward.**
- **The turtle begins 3 feet away from the water and walks forward 2 feet over 3 seconds. Then it splits into two turtles that move in opposite directions for 3 seconds before they move toward each other for 3 seconds. The turtles then merge back into a single turtle that walks from 5 feet to 7 feet over 3 seconds.**

6 Let's watch an animation to see what Arnav's turtle did.

Discuss: How does this animation compare to your story?

Responses vary. **In my story, I predicted the turtle would move in a circle. I was surprised to see that the turtle was in two places at once for part of the journey.**

Name: .... Date: .... Period: ....

Dangerous Crossings

7 Let's watch the animation of another turtle crossing.

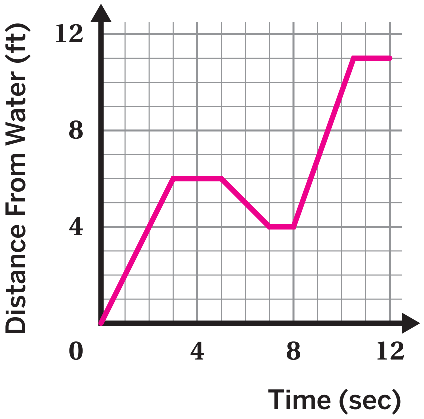
Discuss: What do you notice? What do you wonder?

Responses vary.

- I notice that the turtle was avoiding the green snakes.
- I notice the turtle had to walk backward at one point.
- I wonder what the graph will look like for this turtle crossing.
- I wonder if there are other ways the turtle can cross and still get to the other side.

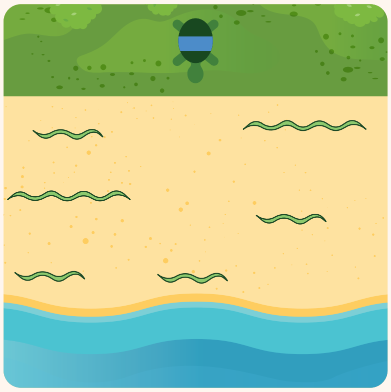
8 Draw a graph of the turtle's distance from the water over time based on the animation from the previous problem.

Responses vary.



You're invited to explore more.

9 You will use the Activity 2 Sheet to draw a new graph to help the turtle go from the grass to the water while avoiding the snakes.

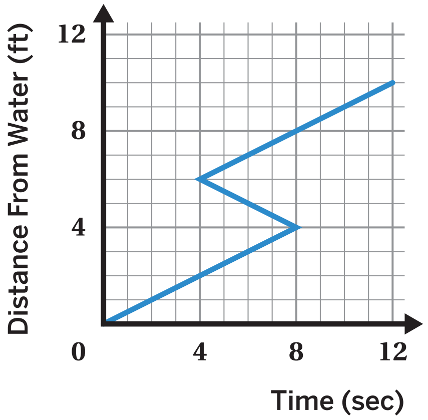


10 Synthesis

Carlos drew this graph to represent a new turtle.

Explain what story the graph is telling at 6 seconds. Does the turtle's journey seem realistic?

Responses vary. **At 6 seconds, the graph shows that the turtle is 3 feet, 5 feet, and 7 feet from the water. This doesn't seem realistic because the turtle cannot be in three locations at the same time.**

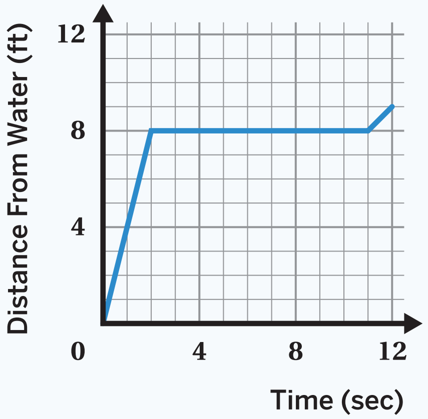


13 Summary 5.01

You can use a graph to represent a situation. Analyzing a point on a graph or pieces of a graph can help you interpret part of the situation.

For example, this graph represents a turtle's journey across sand. A turtle walks for 2 seconds until it is 8 feet from the water. It stops for 9 seconds and then continues walking forward.

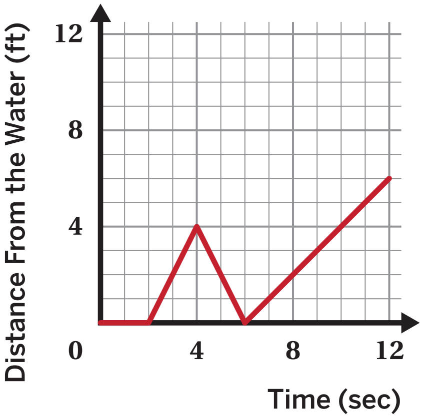
The point $(6, 8)$ represents the turtle's distance of 8 feet from the water after 6 seconds.



Practice
5.01

Name: .... Date: .... Period: ....

Problems 1–3: This graph represents a turtle walking across the sand.



1. What story does the graph tell about the turtle's journey?

***Responses vary*. The turtle stays still at the edge of the water, then walks away from the water, then walks back to the water, and then walks away from the water again.**

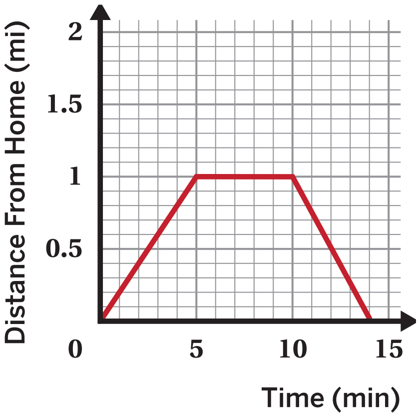
2. How far was the turtle from the water after 8 seconds?

2 feet

3. After how many seconds is the turtle's distance 2 feet from the water?

3 seconds, 5 seconds, and 8 seconds

Problems 4–6: This graph shows Takeshi's distance from home as time passes. Determine whether each statement is true or false.



4. Takeshi was 1 mile from home at 5 minutes.

True

5. Takeshi was 10 miles from home at 1 minute.

False

6. Takeshi's distance from home didn't change from 5 to 10 minutes.

True

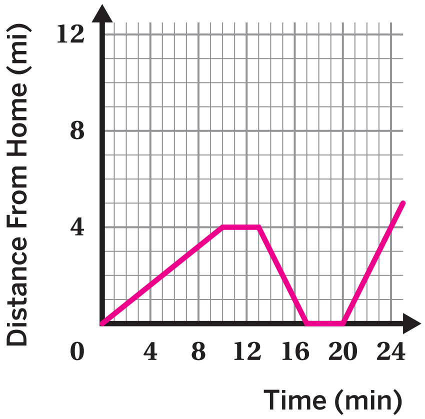
Practice
5.01

Name: ____________ Date: ________ Period: ________

7. Diamond's family went on vacation. First, they drove away from their house. After several minutes, they stopped to get gas. When Diamond's family left the gas station, they realized they forgot something and drove back to their house. After a few minutes, they drove away from their house again.

Sketch a graph that could represent Diamond's family's distance from home vs. time.

Responses vary.



Spiral Review

8. Solve this system of linear equations. Show or explain your thinking.

$$\begin{cases} y = x - 4 \\ y = 6x - 10 \end{cases}$$

$\left(\frac{6}{5}, -\frac{14}{5}\right)$. ***Explanations vary.***

$x - 4 = 6x - 10$

$6 = 5x$

$x = \frac{6}{5}$

$y = \left(\frac{6}{5}\right) - 4$

$y = -\frac{14}{5}$

9. Determine whether this system of equations has *one solution*, *no solution*, or *infinitely many solutions*.

$$\begin{cases} y = 3x + 5 \\ y = 3x + 7 \end{cases}$$

Show or explain your thinking.

No solution. *Explanations vary.* The slopes of the equations are the same, but the y-intercepts are different so I know the lines will never intersect.

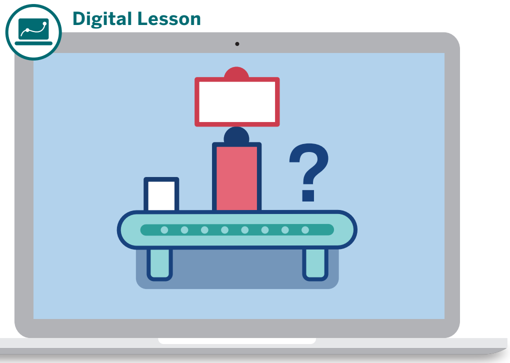
Unit 5

Lesson

2

Name: ______________________ Date: __________ Period: __________

Multiple Representations of Functions 8.F.1, SMP.3, SMP.6



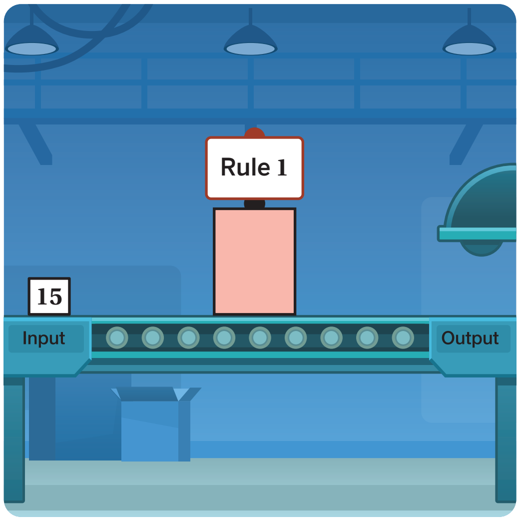
Guess My Rule

Let's explore rules to develop the concept of a function.

Warm-Up

1 This machine uses a secret rule, Rule #1, to turn inputs into outputs.

Rule #1 allows *all integers* as inputs.



a Let's watch this machine at work.

b What could Rule #1 be? Select *all* that apply.

- ☐ **A.** Divide by 2, then add 5.
- ☑ **B.** Divide by 3.
- ☑ **C.** Take the ones digit.
- ☐ **D.** Multiply by 3.
- ☑ **E.** Subtract 10.

2 Let's enter a different integer to help you decide what Rule #1 is.

What could Rule #1 be now? Select *all* that apply.

- ☐ **A.** Divide by 2, then add 5.
- ☐ **B.** Divide by 3.
- ☐ **C.** Take the ones digit.
- ☐ **D.** Multiply by 3.
- ☑ **E.** Subtract 10.

Note: If a student chooses a value greater than or equal to 10 but less than 20, both "subtract 10" and "take the ones digit" are possible.

Activity
1

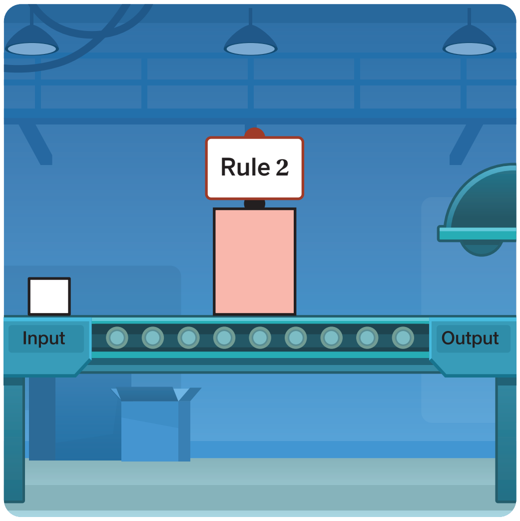
Name: .... Date: .... Period: ....

Guess My Rule

3 This machine uses a new rule called Rule #2.

Rule #2 allows *all numbers* as inputs.

Let's test several inputs to see how Rule #2 works.



4 Here are some inputs and outputs from other students.

If you input 6 into Rule #2, what do you think the output will be?

7

Explain your thinking.

***Explanations vary*. This rule outputs the number 7 no matter what the input is.**

Rule #2

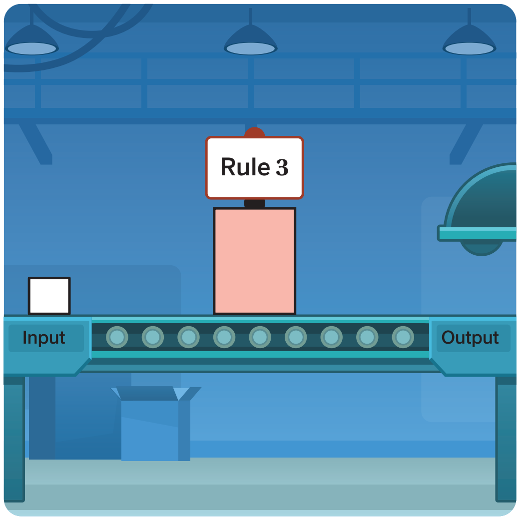
Input	Output
-13	7
-61	7
81	7
-100	7
60	7

5 Rule #3 allows *single words* as inputs.

a Let's test several inputs to see how Rule #3 works.

b **Discuss:** What do you think the rule might be?

***Responses vary*. Rule #3 takes the last letter of the word and outputs the next letter in the alphabet.**



Name: .... Date: .... Period: ....

Guess My Rule (continued)

6 Here are some inputs and outputs from other students.

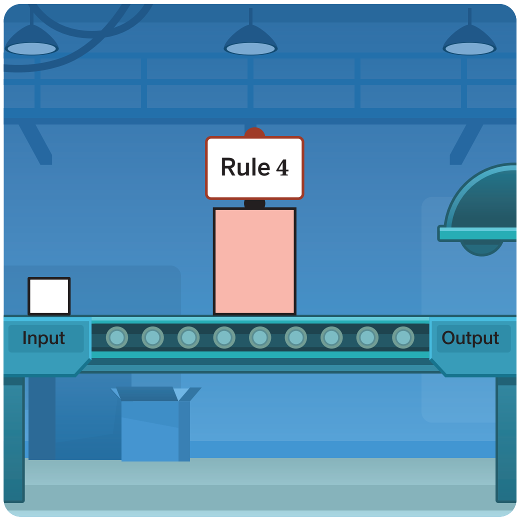
Test your understanding of Rule #3 by completing the table.

Rule #3

Input	Output
mint	u
hen	o
clear	s
friend	**e**
wallet	**u**
friend	**e**
party	**z**

7 Rule #4 allows *single letters* (like "A") as inputs.

Let's test several inputs to see how Rule #4 works.



8 Here are some inputs and outputs from other students.

Input	Output
W	William
A	Anand
A	Adam

What do you think the output will be if you input "A" into Rule #4? Explain your thinking.

Then compare your response with your classmates' responses.

***Responses vary.* The output could be "Alan" or "Adrian" if the input is "A" because I noticed that the first letter of the name is always the input.**

Activity
2

Name: .... Date: .... Period: ....

What Is a Function?

9 Rules #1, #2, and #3 all represent **functions**.

Rule #4 does *not* represent a function.

a What do you think makes a rule a function?

***Responses vary.* A rule is a function if an input produces the same output every time.**

b Compare your response with a classmate's. Then revise your response to make it stronger and clearer.

***Responses vary.* A rule is a function if it gives exactly one output for each input.**

Rule #1: Function

Input	Output
35	25
723	713
-4	-14
53	43
723	713

Rule #2: Function

Input	Output
15	7
18	7
262	7
-3	7
82.3	7

Rule #3: Function

Input	Output
hi	J
my	Z
name	F
is	T
Arturo	P

Rule #4: Not a Function

Input	Output
H	Hailey
J	Jada
M	Mai
H	Hamza
M	Madison

10 Determine whether each table represents a function.

a

Button Selected (Input)	Drink Received (Output)
A	Water
B	Seltzer
C	Juice
D	Water

Function

b

Money Spent (Input)	Number of Items (Output)
\$1	2
\$8	12
\$7	1
\$1	3

Not a function

c

Height in Feet (Input)	Height in Inches (Output)
5	60
4.5	54
6	72
5.4	64.8

Function

d

Time in Seconds (Input)	Musical Note (Output)
1.5	D
1.25	D
1.5	E
2	D

Not a function

11 This table does *not* represent a function.

Input	Output
1	5
2	10
3	15
2	20
1	24

a Change at least one number in the table so that it could represent a function.

Responses vary.

b Explain your thinking.

Responses vary.

- **I changed the input in the fourth row from 2 to 4 and the input in the fifth row from 1 to 5 so that each number on the input side is different.**
- **I changed the output in the fourth row from 20 to 10 and the output in the fifth row from 24 to 5 so that each input always goes to the same output.**

12 Synthesis

💬 **Discuss:** How can you determine whether a table could represent a function?

Use the examples if they help with your thinking.

Responses vary. **A table could represent a function if there is one output for each allowable input.**

Rule #1: Function

Input	Output
35	25
723	713
-4	-14
53	43
723	713

Rule #2: Function

Input	Output
15	7
18	7
262	7
-3	7
82.3	7

Rule #3: Function

Input	Output
hi	J
my	Z
name	F
is	T
Arturo	P

Rule #4: Not a Function

Input	Output
H	Hailey
J	Jada
M	Mai
H	Hamza
M	Madison

15 Summary 5.02

To determine if a table represents a **function**, you can look to see if any inputs are assigned to multiple outputs. You can also analyze the relationship to determine a rule.

This table represents a function because there is one output for exactly one input

This table does not represent a function because the input 10 has multiple outputs, 6 and 7.

Function

Input	Output
15	7
10	7
20	8
5	9

Not a function

Input	Output
10	6
10	7
20	8
5	9

function A relationship that assigns exactly one output to each possible input.

Practice
5.02

Name: ______ Date: ______ Period: ______

Problems 1–2: Complete each table based on its rule.

1. Rule: Divide by 4 and then add 2.

Input	Output
0	**2**
2	**2.5**
4	**3**
6	**3.5**
8	**4**

2. Rule: If odd, write 1. If even, write 0.

Input	Output
1	**1**
2	**0**
3	**1**
7	**1**
12	**0**

Problems 3–4: Determine whether each table could represent a function. Explain your thinking.

3.

Input	Output
4	-2
1	-1
0	0
1	1
4	2

Not a function. *Explanations vary.* There are two different output values for the input values 4 and 1.

4.

Input	Output
-2	4
-1	1
0	0
1	1
2	4

Function. *Explanations vary.* There is one output for each given input.

5. Which of the following tables could represent a function?

[✓ **A.**]

Input	Output
1	0
2	5
3	2.5
4	5
5	8

B.

Input	Output
0	1
5	2
2.5	3
5	4
8	5

C.

Input	Output
3	-8
3	-2
3	-1
3	6
3	12

D.

Input	Output
-1	0
-2	8
2	2
-1	-1
-2	9

Name: .... Date: .... Period: ....

6. Complete the table so that it could represent a function.

Responses vary.

Input	Output
-2	0
0	**5**
2	10
4	**15**
6	**20**

7. Many people consider mathematician Ada Lovelace to be the world's first computer programmer. Lovelace used functions to write programs for an early computing machine. Certain inputs (such as pressing a certain key on a keyboard) caused certain outputs.

Here's a table that shows how pressing different keys causes different movements in a video game character. Could this table represent a function? Explain your thinking.

Key Pressed	Movement
Right arrow	Walk right
Left arrow	Walk left
Up arrow	Walk right
Down arrow	Jump

Yes. *Explanations vary*. The table could represent a function because there is one output movement for each input key that's pressed.

Spiral Review

Problems 8–10: Determine whether each ordered pair is a solution for the equation $2x + 4y = 16$.

8. $(1, 3)$

No

9. $(6, 1)$

Yes

10. $(0, 4)$

Yes

11. Solve the equation $4z + 5 = -3z - 8$. Show your thinking.

$z = -\frac{13}{7}$. *Work varies.*

$\mathbf{4z + 5 = -3z - 8}$

$\mathbf{7z + 5 = -8}$

$\mathbf{7z = -13}$

$\mathbf{z = -\frac{13}{7}}$

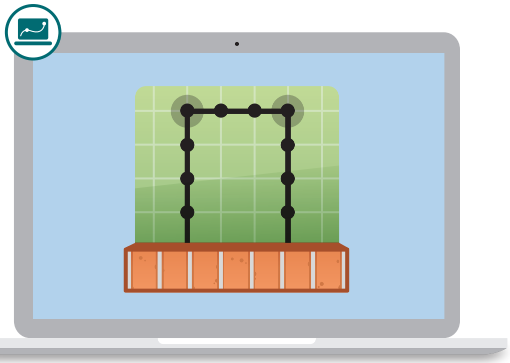
Unit 5

Lesson

3

Name: .... Date: .... Period: ....

Data, Graphs, and Tables | Multiple Representations of Functions | 8.F.1, SMP.3, SMP.8



Function or Not?

Let's determine whether a graph represents a function.

Warm-up

1 We learned that a function is a rule that assigns exactly one output to each possible input.

That means each input determines a single output.

Complete the table so that y is *not* a function of x.

Responses vary.

x	y
3	**1**
5	4
3	6

Activity
1

Name: .... Date: .... Period: ....

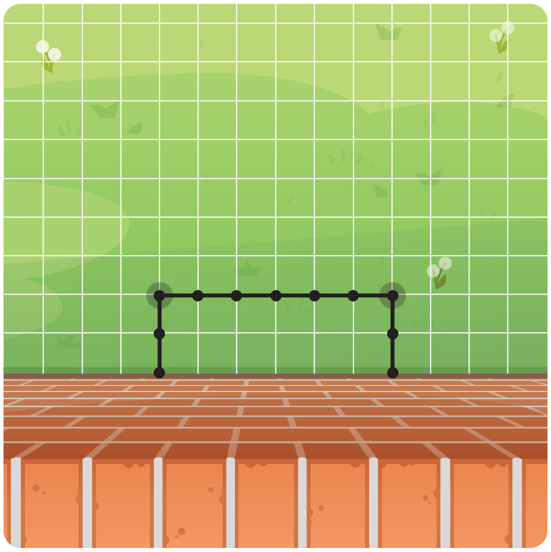
Rectangular Pen

2 Emma is hiring builders to create a rectangular pen, a closed space where animals are kept. She wants to build three sides of fencing against a brick wall.

Let's observe the relationship between the area and the amount of fencing.

Enter an amount of fencing and the area for a pen that you've observed. ***Responses vary.***

Amount of Fencing (m)	Area (sq. m)
10	**12**



3 Here is a table that shows the amount of fencing and the area for several pens.

Emma gave her builders 12 meters of fencing to build her pen.

Is it possible for the builders to determine the area of Emma's pen?

Yes [✓ No] I'm not sure

Explain your thinking.

Explanations vary. **It's not possible to determine the area of Emma's pen. This is because there are two rows in the table with an input (fencing amount) of 12 meters, each with a different output (area): 10 square meters and 16 square meters.**

Amount of Fencing (m)	Area (sq. m)
8	8
12	10
14	24
12	16
16	24
20	32

Name: Date: Period:

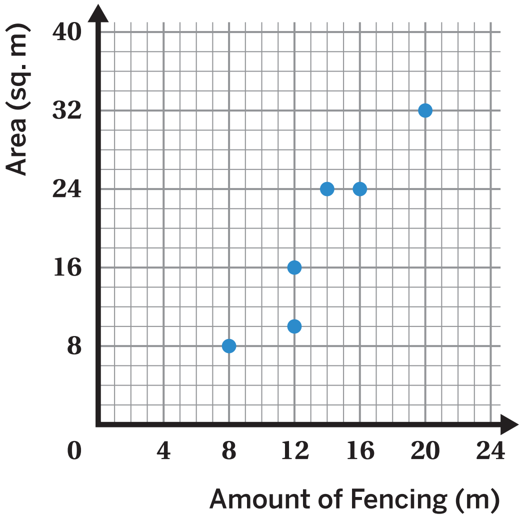
Rectangular Pen (continued)

4 Here is a graph that shows the relationship between area and amount of fencing for several pens.

How can you use the graph to quickly determine that area is not a function of the amount of fencing?

Area is not a function of the amount of fencing because . . .

Responses vary. **The points (12, 10) and (12, 16) tell me that for 12 meters of fencing, there is more than one possible area (10 square meters and 16 square meters). That means area is not a function of the amount of fencing.**



Name: .... Date: .... Period: ....

Turtle Crossing

5 Let's watch an animation of a turtle's journey across the sand.

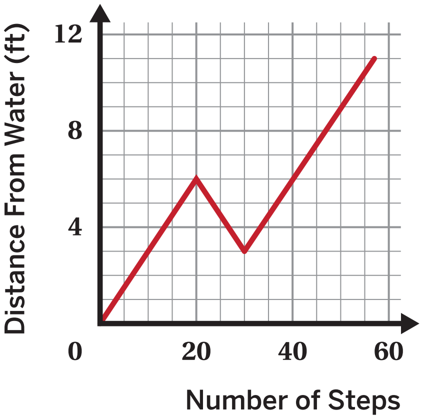
Write the turtle's number of steps and distance from the water for one point in time.

Responses vary.

Number of Steps	Distance from Water (ft)
20	6

6 How can you use this graph to decide whether distance from the water is a function of the number of steps?

Responses vary. Look along the horizontal axis at all of the possible input values. For each input value, look at how many corresponding output values there are. If every possible input value corresponds to exactly one output value, then the relationship is a function.



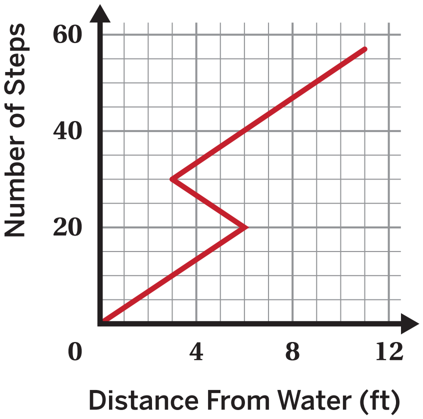
7 Here is the relationship in reverse.

Is the number of steps a function of distance? Circle one.

Yes [✓ No]

Explain your thinking.

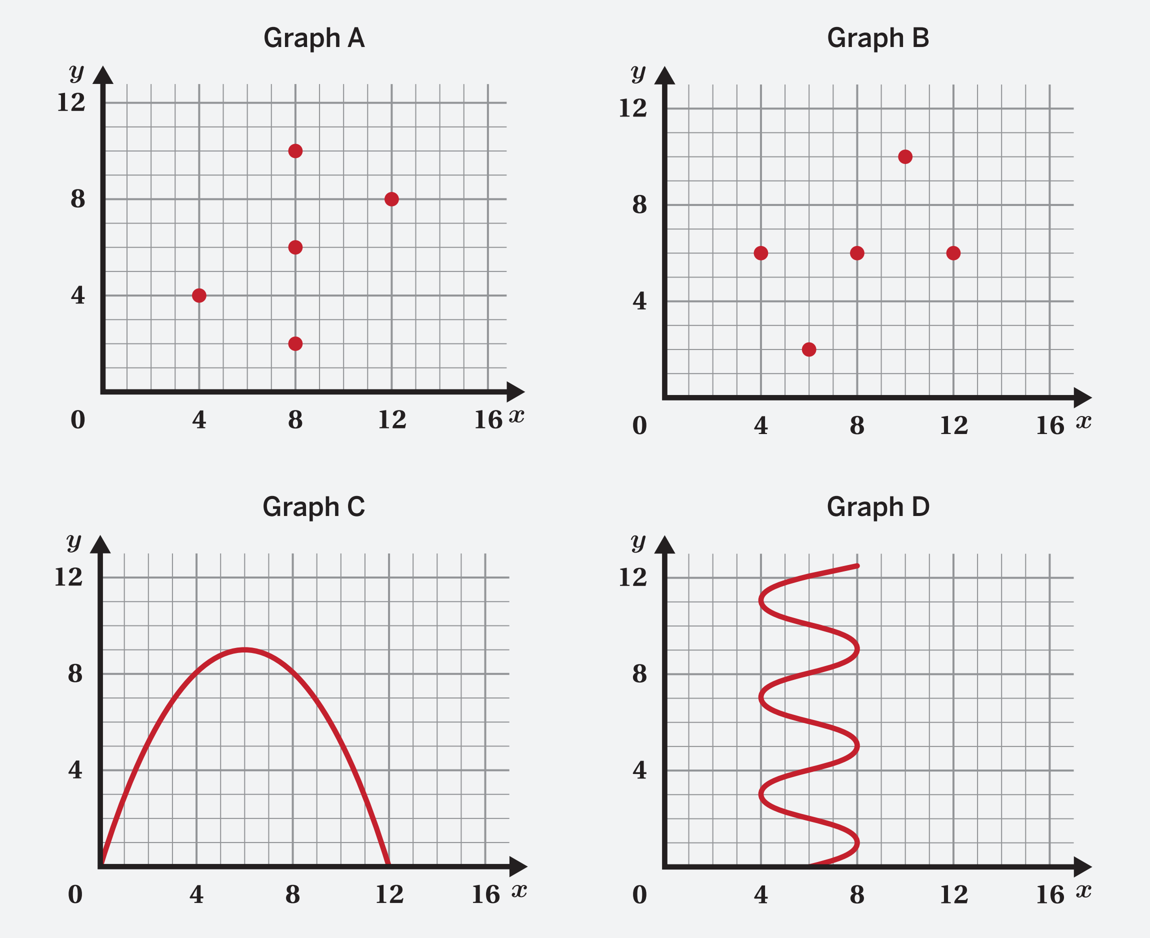
Explanations vary. There are some distances that correspond with multiple numbers of steps. For example, (6, 20) and (6, 40) represent the turtle at 6 feet from the water at both 20 steps and 40 steps.



Name: .... Date: .... Period: ....

Turtle Crossing (continued)

8 Determine whether each graph represents a function or not.



Function	Not a function
B, C	A, D

You're invited to explore more.

9 **a** You will use the Activity 2 Sheet to analyze some graphs that are surprising in some way.

b **Discuss:**

- Does this graph represent a function?
- What does the surprising part of this graph represent in context?

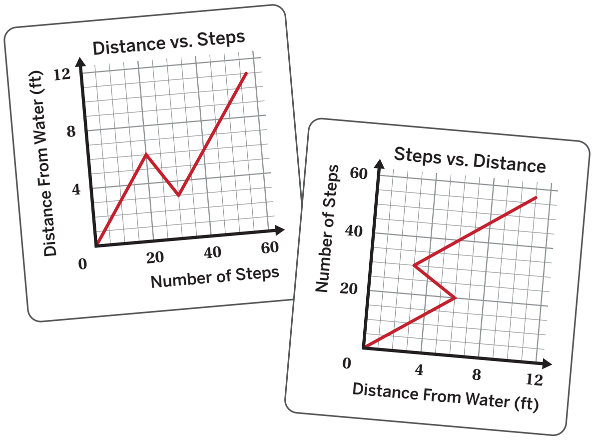
Responses vary.

10 Synthesis

Discuss: How can you determine whether a graph represents a function?

Use the examples if it helps with your thinking.

***Responses vary.* A graph represents a function when each possible input (x-value) has exactly one output (y-value).**

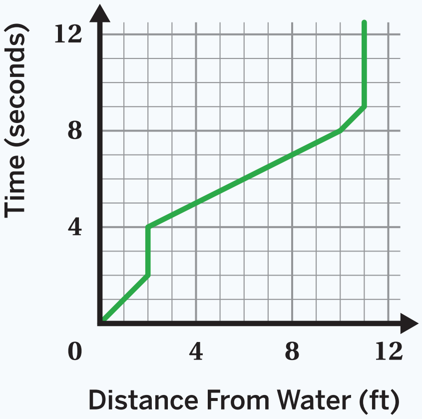


13 Summary 5.03

The graph of a relationship consists of the set of ordered pairs of inputs and outputs. A graph represents a function when each x-value, or input, only has one corresponding y-value, or output. If a graph has multiple y-values for the same x-value, it does not represent a function.

Here are two graphs of the same turtle's journey.

This graph represents a function because for every second, x, the turtle is at only one corresponding distance, y.



This graph does not represent a function because at both 2 feet and 11 feet, the turtle has multiple corresponding times.

Practice
5.03

Name: ______ Date: ______ Period: ______

Problems 1–3: A group of students are timed while sprinting 100 meters.

Time (seconds)	Speed (meters per second)
13.8	7.246
15.9	6.289
16.3	6.135
17.1	5.848
18.2	5.495

1. Is speed a function of time? Explain your thinking.

Yes. *Explanations vary*. For each time, there is only one possible speed.

2. Is distance a function of time? Explain your thinking.

Yes. *Explanations vary*. For each time, there is only one possible distance of 100 meters.

Time (seconds)	Distance (meters)
13.8	100
15.9	100
16.3	100
17.1	100
18.2	100

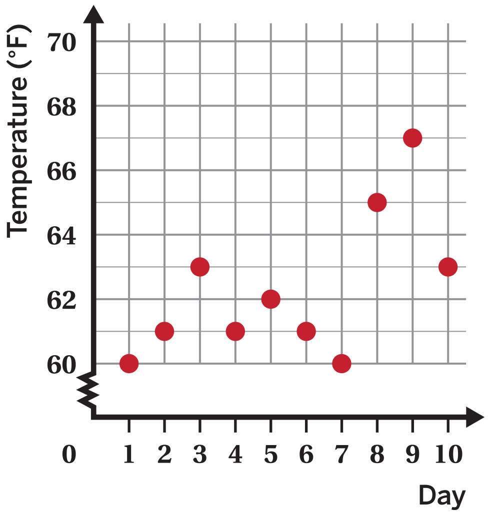
3. Is time a function of distance? Explain your thinking.

No. *Explanations vary*. For the distance of 100 meters, there are many possible times.

Distance (meters)	Time (seconds)
100	13.8
100	15.9
100	16.3
100	17.1
100	18.2

4. This graph represents a city's high temperatures over a 10-day period. Determine whether this statement is true or false: *The high temperature is a function of the day*. Explain your thinking.

True. *Explanations vary*. For each day, there is exactly one corresponding high temperature.



Practice
5.03

Name: Date: Period:

Problems 5–6: Determine whether each table could represent a function. Explain your thinking.

5.

Input	Output
1	0
2	0
3	0
4	0
5	0

Function. ***Explanations vary.*** **There is one output for each input.**

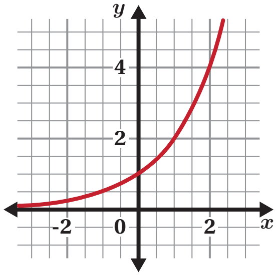
6.

Input	Output
0	1
0	2
0	3
0	4
0	5

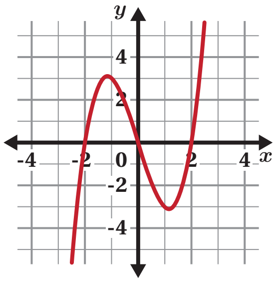
Not a function. ***Explanations vary.*** **The input, 0, has multiple outputs.**

7. Which graph does *not* represent y as a function of x?

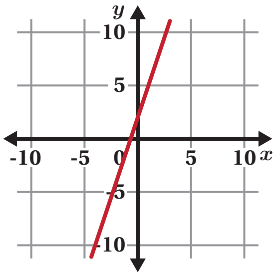
A.



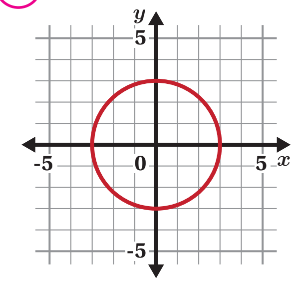
B.



C.



[✓ **D.**]



Spiral Review

Problems 8–10: Determine whether each system of equations has one solution, no solution, or infinitely many solutions.

8. $\begin{cases} y = x + 4 \\ y = x + 4 \end{cases}$

Infinitely many solutions

9. $\begin{cases} y = -\frac{4}{5}x + 7 \\ y = \frac{4}{5}x - 2 \end{cases}$

One solution

10. $\begin{cases} y = 2x + \frac{1}{5} \\ y = 2x + 42 \end{cases}$

No solution

11. $\begin{cases} y = -4x + 9 \\ y = 9x - 4 \end{cases}$

One solution

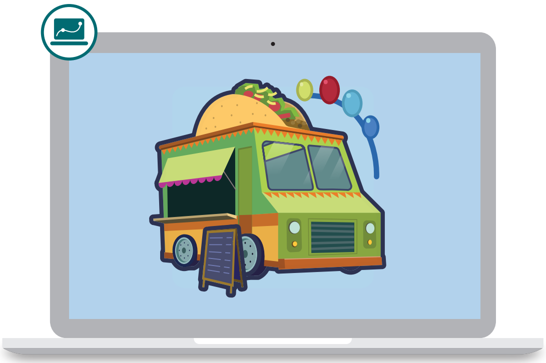
Unit 5

Lesson

4

Name: ______ Date: ______ Period: ______

Data, Graphs, and Tables | Multiple Representations of Functions | 8.F.1, SMP.3, SMP.4



Dependence Day

Let's explore the relationships between variables of functions.

Warm-up

The Metropolis Events Committee is in charge of planning events for the city.

1 What might they need to consider as part of the planning process?

***Responses vary.* They might need to decide what day and where to hold the event.**

2 The Events Committee wants to hire a taco truck to provide food for an event. The taco truck charges $150 for set-up and $10 for each person. How much would the taco truck cost if 80 people attend the event? Show or explain your thinking.

$950. *Explanations vary.*

$150 + (80 \cdot 10) = 950$

Activity
1

Name: Date: Period:

Party Planning

3 **Situation A:** The Events Committee is thinking about hiring the taco truck again. The taco truck always charges $150 for set-up and $10 for each person.

a Complete the table to show the cost of the taco truck for different numbers of attendees.

Number of Attendees	Food Cost ($)
80	**950**
120	**1350**
180	**1950**
200	**2150**

b Write an equation to calculate the food cost, f, of any number of attendees, a.

$f = 10 \cdot a + 150$ **(or equivalent)**

c Ari thinks the relationship between food cost and attendees represents a function because each input has only one possible output.

Do you agree with Ari's claim? Explain your thinking. **ELD.PI.8.11.Em, Ex, Br**

Yes. *Explanations vary*. Each input, the number of attendees, has exactly one possible output, the cost of food.

Name: .... Date: .... Period: ....

Party Planning (continued)

4 **Situation B:** The Events Committee has a food budget, which is the maximum food cost, and is planning on hiring the same taco truck.

a If the committee wants a food cost of $\$1{,}000$. What is the maximum number of people they can feed?

Approximately 85 people

b Complete the table to show the number of possible attendees for other food cost budgets.

Food Cost ($)	Number of Attendees
1,000	85
1,200	105
1,500	135
2,000	185

c Describe in words how to determine the number of attendees for any food cost.

***Responses vary.* Subtract 150 from the food cost to remove the set-up fee, then divide by 10 because each person's food cost is $10.**

d Write an equation to calculate the number of attendees, a, based on the budget for the food cost, f.

$a = \frac{f - 150}{10}$ **(or equivalent)**

e Does the relationship between food cost and attendees represent a function? Explain your thinking.

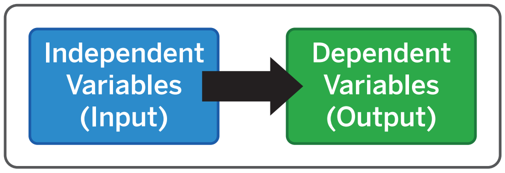
Yes. *Explanations vary.* Each amount of food cost will have only one number of attendees that can be fed for that cost.

Name: .... Date: .... Period: ....

Independent and Dependent Variables

In some relationships between two variables, one is called the **independent variable** and the other is called the **dependent variable**.

The independent variable is an input. The dependent variable, or output, depends on the input.



5 **Situation A:** The Events Committee is using the equation $f = 10 \cdot a + 150$ to calculate possible food costs for an event with a attendees. Explain why it would make sense to call the food cost, f, the dependent variable. **ELD.PI.8.10.Em, Ex, Br**

***Responses vary.* They could be thinking of the food cost as the dependent variable because it depends on how many people come to the event.**

6 **Situation B:** The Events Committee has raised money for their next event and wants to know how many attendees they can host with their budget for food costs.

a In this situation, what do you think they should use as the independent and dependent variable? **ELD.PI.8.10.Em, Ex, Br**

Independent variable: **Food cost, f**

Dependent variable: **Number of attendees, a**

Explain your thinking.

***Explanations vary*. The number of attendees will depend on the food cost.**

b What equation might they use? Circle one.

[✓ $a = \frac{f - 150}{10}$] $f = 10 \cdot a + 150$

c **Discuss:** How did you decide which equation they might use? **ELD.PI.8.3.Em, Ex, Br**

I chose the equation where the input is the number they have and the output is what they are trying to find.

Activity
3

Name: .... Date: .... Period: ....

Winter Carnival

The Events Committee is planning a Winter Carnival. At the carnival, attendees will get tickets that they can exchange to play games. Each game uses 3 tickets.

7 Match each variable or equation to the correct description. Some variables and equations will not have a match.

Number of Tickets, t	Ticket Cost, c	Number of Games, g	
$c = 3 + t$	$t = 3g$	$g = \frac{t}{3}$	$t = c - 3$

	How many games can you play with 28 tickets?	How many tickets should you purchase if you want to play 9 games?
Independent Variable	Number of Tickets, t	Number of Games, g
Dependent Variable	Number of Games, g	Number of Tickets, t
Equation	$g = \frac{t}{3}$	$t = 3g$

8 **Discuss:** How did you choose which variables were independent and dependent?
ELD.PI.8.3.Em, Ex, Br

The independent variable was the number given, or the input, and the dependent variable was the number I needed to find, or the output.

Explore More

9 Create your own situation where the dependent variable is the amount of space needed to hold an event.

Responses vary. **A school is renting a tent to host their graduation ceremony. The tent should have 4 square feet for each person, so the amount of space in the tent is dependent on the number of people who come to graduation.**

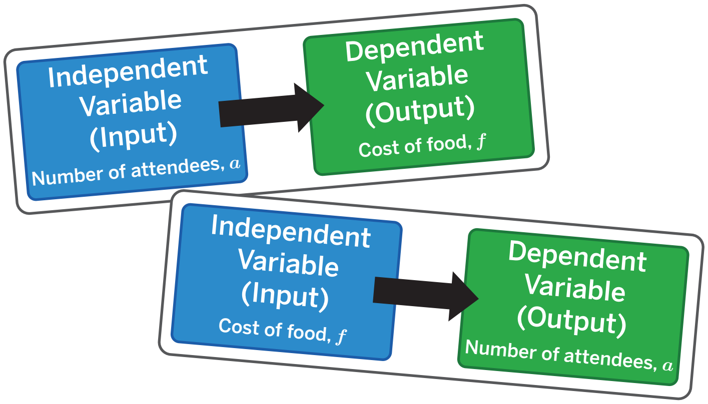
Synthesis

10 **Discuss:** How can you determine which variable is independent in a situation? Which is dependent?

ELD.PI.8.1.Em, Ex, Br

Use the example if it helps with your thinking.

Responses vary. **The independent variable represents the input of a function and the dependent variable represents the output of a function.**



Summary

In a situation represented by a function, the **independent variable** and **dependent variable** sometimes can switch depending on the problem you are trying to solve.

For example, in this situation, m represents the total number of miles walked and d represents the number of days of walking for someone who walks 2 miles a day.

Question	Independent and Dependent Variable	Equation	Explanation
How many miles have I walked, m, after d days?	Independent: Days, d Dependent: Miles, m	$m = 2d$	The number of miles walked depends on the number of days of walking.
How many days, d, will it take me to walk m miles?	Independent: Miles, m Dependent: Days, d	$d = \frac{m}{2}$	The number of days depends on the number of miles.

dependent variable (output) A variable whose value is based on the value of another variable or set of variables. The dependent variable is typically on the vertical axis of a graph and in the right-hand column of a table.

In a function, the dependent variable is sometimes called the output.

independent variable (input) A variable whose value is not based on the value of any other variable. The independent variable is typically on the horizontal axis of a graph and in the left-hand column of a table.

In a function, the independent variable is sometimes called the input.

Practice
5.04

Name: .... Date: .... Period: ....

Problems 1–2: Write an equation that expresses the output as a function of the input. Then determine the independent and dependent variables.

1. The perimeter, p, of a square with side length s:

Equation: $p = 4s$ (or equivalent)

Independent variable: side length of the square, s

Dependent variable: perimeter of the square, p

2. The total cost, c, after a sales tax of 7% is applied to the cost of a purchase, p.

Equation: $c = 1.07p$ (or equivalent)

Independent variable: cost of the purchase, p

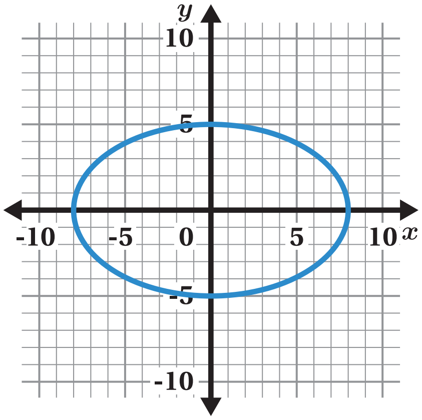
Dependent variable: total cost, c

3. Which of these could represent a function? Select *all* that apply.

☑ **A.** $y = \frac{2}{3}x - 5$

☐ **B.** $x = 4$

☐ **C.**



☑ **D.**

x	y
−1	7
−3	7
−2	7
−1	7

Practice
5.04

Name: ______ Date: ______ Period: ______

Problems 4–7: Rafael earns $\$10.50$ per hour helping his neighbor with chores.

4. Is the amount he earns a function of the number of hours he works? Explain your thinking.

Yes. *Explanations vary.* The amount he earns is a function of the number of hours worked because there is only one possible dollar amount for each number of hours worked.

5. Is the number of hours he works a function of the amount he earns? Explain your thinking.

Yes. *Explanations vary.* The number of hours worked is a function of the amount earned because every dollar amount he earns corresponds with one number of hours worked.

6. Write an equation that describes the situation. Use x to represent the independent variable and y to represent the dependent variable.

$y = 10.5x$ (or equivalent), where x is the number of hours worked and y is the amount earned in dollars.

7. How much will Rafael earn if he works 3 hours each weekday next week? Show or explain your thinking.

$\$157.50$. *Explanations vary.* $10.50 \cdot 3 \cdot 5 = 157.50$

Spiral Review

Problems 8–9: Solve each system of equations. Show your thinking.

8. $\begin{cases} y = 7x + 10 \\ y = \text{-}4x - 23 \end{cases}$

Solution: $(\text{-}3, \text{-}11)$. *Work varies.*

$7x + 10 = \text{-}4x - 23$

$7x + 4x = \text{-}23 - 10$

$11x = \text{-}33$

$x = \text{-}3$

$y = 7(\text{-}3) + 10$

$y = \text{-}11$

9. $\begin{cases} y = 3x - 6 \\ y = \text{-}2x - 1 \end{cases}$

Solution: $(1, \text{-}3)$. *Work varies.*

$3x - 6 = \text{-}2x - 1$

$3x + 2x = \text{-}1 + 6$

$5x = 5$

$x = 1$

$y = 3(1) - 6$

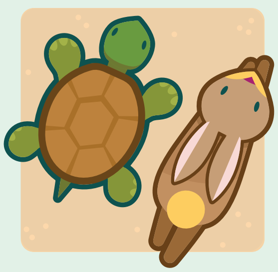
$y = \text{-}3$

Notes:

Sub-Unit

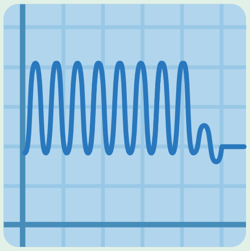
2

Representing and Interpreting Functions



Lesson 5

The Tortoise and the Hare

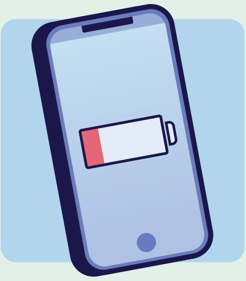


Lesson 6

Graphing Stories

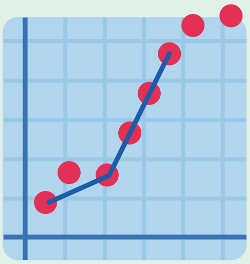
Lesson 7

Comparing Linear Functions



Lesson 8

Charge!



Lesson 9

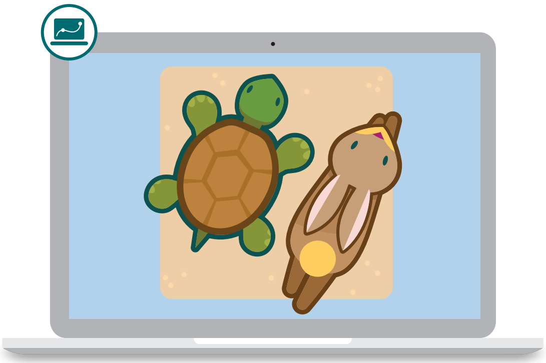
Piecing It Together

Unit 5
Lesson
5

Name: .... Date: .... Period: ....

Data, Graphs, and Tables | Linear Equations | Multiple Representations of Functions

8.F.5, SMP.2, SMP.3, SMP.6

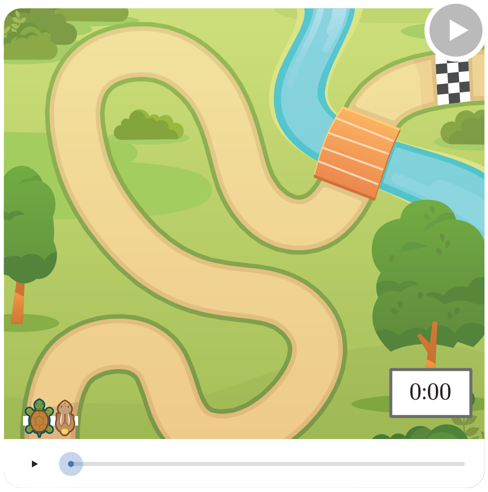


The Tortoise and the Hare

Let's interpret the graph of a function in context.

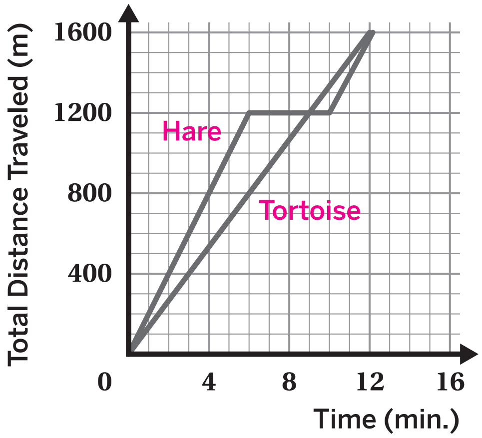
Warm-up

1 Let's watch an animation of the tortoise and the hare. Then tell a story based on what you see.



Responses vary. **Once upon a time, there was a race between a tortoise and a hare. The hare was faster than the tortoise but lost the race because it took a nap after 6 minutes. By the time the hare woke up from its nap (4 minutes later), the tortoise was too far ahead, and the hare could not catch up.**

2 This graph shows two functions representing the relationship between distance and time: one for the tortoise and one for the hare.



a Which animal does each function represent? Label the graph with "Tortoise" and "Hare".

b Explain your thinking.

Responses vary.

- **The linear graph represents the tortoise because the tortoise's distance changed at a constant rate.**
- **The graph with a horizontal section from 6 to 10 minutes represents the hare because the hare took a 4-minute nap during the race.**

Activity 1

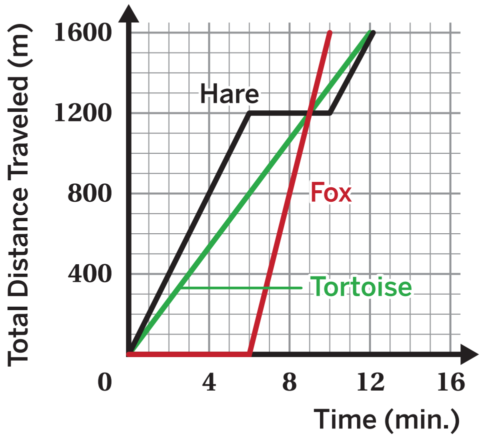
Name: .... Date: .... Period: ....

The Tortoise, the Hare, and the Fox

3 This graph shows the relationship between distance and time for a third animal, the fox.

Tell a story about the fox's journey during the race.

Include specific details about time and distance.



***Responses vary*. The fox began at the starting line and stayed there (possibly napping) for the first 6 minutes of the race. The fox then raced ahead, faster than the tortoise and the hare, and finished the race first after a total of 10 minutes (including only 4 minutes of running).**

4 Let's watch an animation of the race.

Discuss: How does your story compare to the actual race?

Responses vary.

5 Here are five statements about the race. Select *all* the true statements.

- ☐ **A.** The fox's distance was always increasing.
- ☑ **B.** Between 6 and 10 minutes, the fox was traveling faster than the other animals.
- ☑ **C.** When the hare reached 800 meters, the fox was still at the starting line.
- ☐ **D.** The graph of the tortoise represents a function, but the other two graphs do not represent functions.
- ☑ **E.** All three graphs represent functions.

Name: .... Date: .... Period: ....

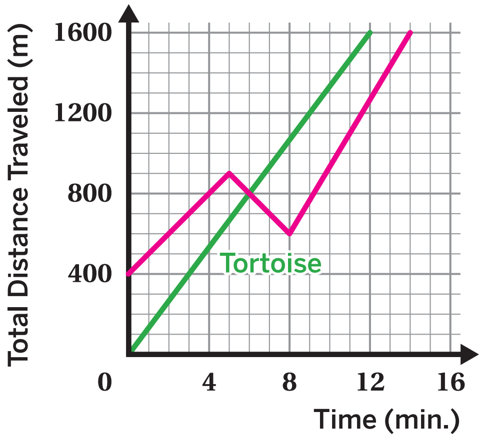
The Tortoise and the Dog

6 Next, the tortoise raced a dog.

Draw a graph representing the relationship between distance and time for a dog that makes *all* of these statements true:

- The dog got a head start but lost the race.
- The dog and tortoise were tied at 800 meters.
- The dog's distance was decreasing for 3 minutes.

Responses vary.



7 Let's watch an animation of the race described in Problem 6 and a graph that was made to represent it.

Revise your graph for Problem 6, as needed.

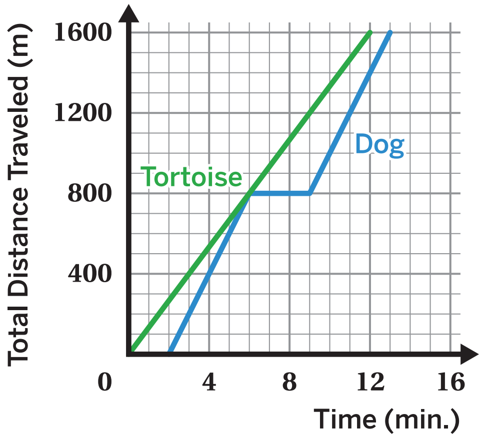
8 The blue graph shows what Brielle drew to represent the dog's race.

At least one of the following statements is false. Circle a false statement.

[✓ A.] The dog got a head start but lost the race.

B. The dog and tortoise were tied at 800 meters.

[✓ C.] The dog's distance was decreasing for 3 minutes.



Explain your thinking.

Explanations vary.

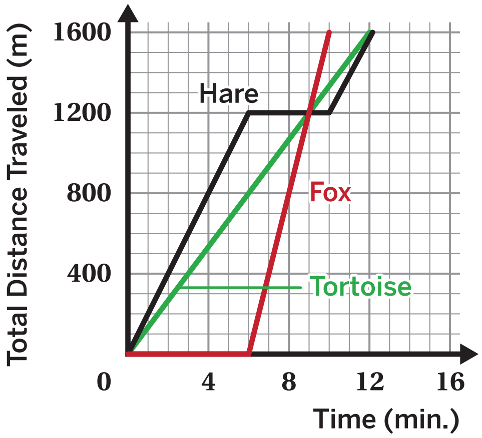
- **Statement A is false. The dog did not get a head start. Instead, it started the race 2 minutes late.**
- **Statement C is false. The dog's distance stopped increasing for 3 minutes, but it was never decreasing. During those 3 minutes, the dog was standing (or sitting) still.**

9 Synthesis

What can a function's graph tell us about a situation?

Use the example if it helps with your thinking.

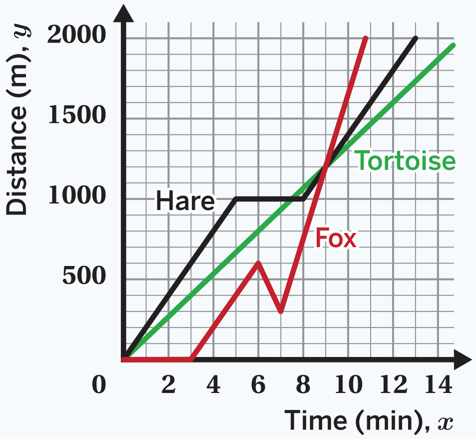
Responses vary. **Graphs of functions can provide a lot of information about a situation. For example, in this graph, I can tell the location of the tortoise, hare, and fox at any time in the race. I can use the slopes to determine which animal's speed is the fastest and when. I can also identify intersections, which represent that the animals are at the same location at the same time.**



12 Summary 5.05

A graph can be helpful when comparing multiple functions in a situation, such as by comparing the initial value, slope, and points of intersection.

For example, this graph represents a race between a hare, a tortoise, and a fox. From 0 to 5 minutes, the hare is moving at a steady pace of 200 meters per minute and is in first place. At 9 minutes, the race is tied. The fox does not begin the race until three minutes have passed, but it speeds up at 7 minutes to a pace of 450 meters per minute. The fox wins the race at about 11 minutes.

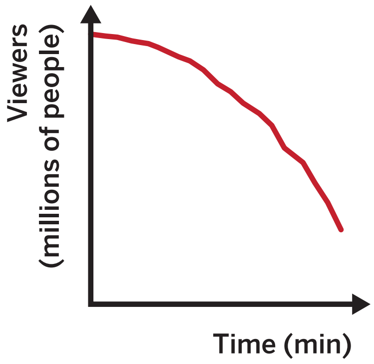


Practice
5.05

Name: .... Date: .... Period: ....

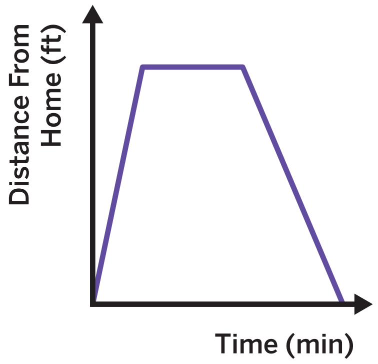
Problems 1–2: For each graph, write a story that the graph tells about each situation.

1. The relationship between number of viewers of a short video and time.



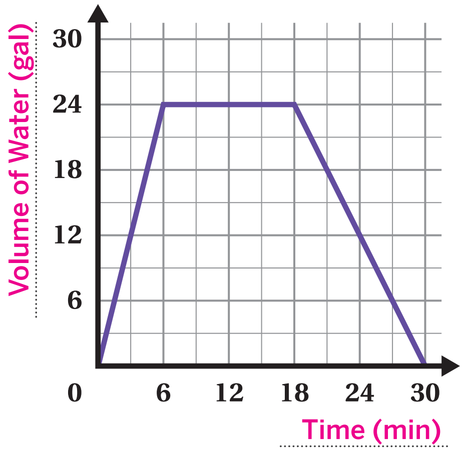
Responses vary. When the video begins it has a large number of viewers, but as the video continues playing over a number of minutes, the number of viewers declines.

2. The distance of a cat from its home as a function of time.



Responses vary. The cat walks away from its home and then comes to a complete stop for a number of minutes before returning home at a slower speed than when it was walking away from home.

Problems 3–8: Elena filled up the tub and gave her dog a bath. Then she let the water in the tub drain. The graph shows the amount of water in the tub, in gallons, as a function of time, in minutes.



3. Label the axes.

4. When did Elena turn off the water faucet?
   After 6 minutes

5. How much water was in the tub when she bathed her dog?
   24 gallons

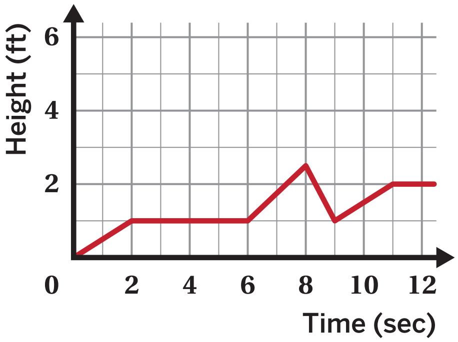
6. How long did it take the tub to drain completely?
   12 minutes

7. At what rate did the faucet fill the tub?
   4 gallons per minute

8. At what rate did the water drain from the tub?
   2 gallons per minute

Name: .... Date: .... Period: ....

9. This graph shows the height, in feet, that a butterfly is flying above the ground over time. At what interval is the butterfly's rate of change 0 feet per second? Select *all* that apply.



☐ **A.** Between 0 and 2 seconds

☑ **B.** Between 2 and 6 seconds

☐ **C.** Between 6 and 8 seconds

☐ **D.** Between 9 and 11 seconds

☑ **E.** Between 11 and 12 seconds

Spiral Review

10. A car is traveling at a speed of either 55 miles per hour or 35 miles per hour, depending on the speed limits, until it reaches its destination 200 miles away. Let x represent the amount of time, in hours, that the car is traveling at 55 miles per hour. Let y represent the amount of time, in hours, that the car is traveling at 35 miles per hour. The equation $55x + 35y = 200$ represents this situation.

If the car spends 2.5 hours traveling at 35 miles per hour on the trip, how long does it spend traveling at 55 miles per hour? Show or explain your thinking.

About 2 hours. *Explanations vary.* $55x + 35 \cdot 2.5 = 200$, so x is approximately 2.

11. The solution to a system of equations is $(1, 5)$. Select two equations that might make up the system.

☐ **A.** $y = -3x + 6$

☑ **B.** $y = 2x + 3$

☐ **C.** $y = -7x + 1$

☑ **D.** $y = x + 4$

☐ **E.** $y = -2x + 9$

Unit 5

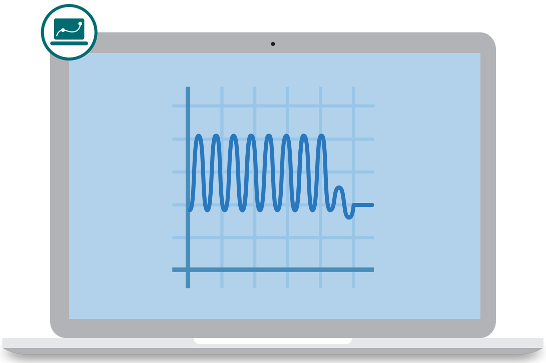
Lesson

6

Name: .... Date: .... Period: ....

Data, Graphs, and Tables | Linear Equations | Multiple Representations of Functions

8.F.5, SMP.1, SMP.2, SMP.4



Graphing Stories

Let's make connections between scenarios and the graphs that represent them.

Warm-up

1 Clem loves to play on the playground. Let's watch a short video of him on the swings. What different quantities are changing in this video?

Responses vary.

- Time
- Clem's waist height above the ground
- Clem's shoe height above the ground
- The number of times Clem swings back and forth
- The horizontal distance between Clem and the edge of the video screen

2 Let's look at two possible graphs to represent this situation.

Discuss: How are these graphs alike? How are they different? *Responses vary.*

Similarities:

- The initial height is the same in each graph (2 feet).
- The final height is the same in each graph (2 feet).
- Each graph has the same basic shape.
- Each graph represents Clem jumping off at the same time (around 12 seconds).

Differences:

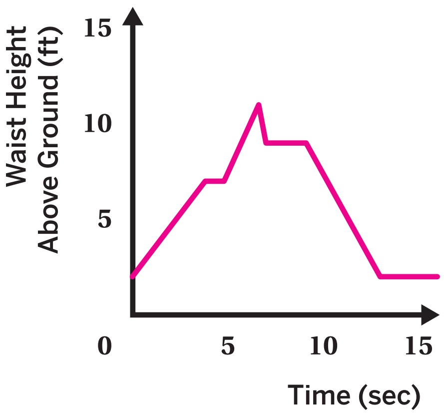
- The graph on the left shows a greater number of swings (8 compared to 4).
- The graph on the right shows a smaller distance between the highest and lowest waist heights.
- The graph on the right shows a smaller maximum waist height above the ground (3 feet compared to 4 feet).

Name: Date: Period:

Tyler on the Slide

3 Let's watch an animation of Tyler on the slide.

Sketch a graph representing the relationship between Tyler's waist height and time.



4 Let's watch an animation of Neel's graph representing the relationship of Tyler's waist height above the ground and time.

💬 **Discuss:** How does Neel's graph accurately represent the situation? How might he revise his graph?

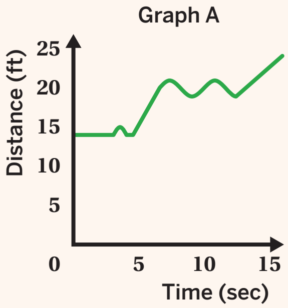
Responses vary.

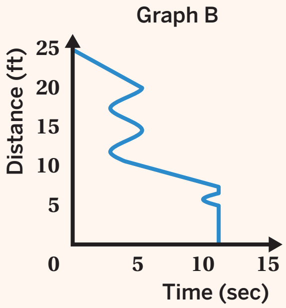
- The part of Neel's graph that is accurate is that it has an initial height of about 2 feet because it represents Tyler's waist height from the ground.
- I would revise the part of the graph from 0 to 4 seconds when Tyler is climbing the ladder. It is currently represented by a straight line, but it seems like he repeatedly steps and then pauses briefly before taking the next step.

You're invited to explore more.

5 Use the digital activity to watch the video of Tyler again. Which graph could represent the relationship between Tyler's distance from the right edge of the screen and time?

Graph A





Explain your thinking.

Explanations vary. Tyler begins about 14 feet away from the right edge of the screen and his distance remains mostly constant for the first 5 seconds of the video. His distance then increases while he climbs the stairs, remains constant while he sits at the top, and then goes back and forth between about 19 and 21 feet as he goes down the slide. At the end of the video, Tyler runs off the screen to the left, so his distance from the right edge continues to grow.

Activity
2

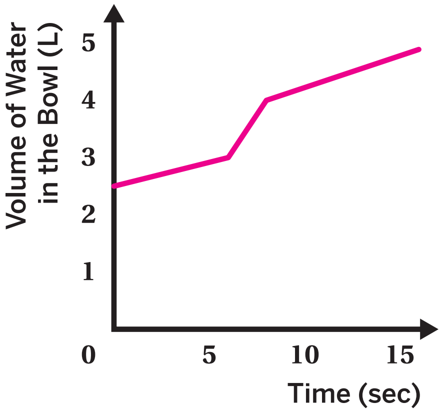
Name: .... Date: .... Period: ....

Water in a Bowl

6 Let's watch an animation of water in a bowl.

Sketch a graph representing the relationship between the volume of water in the 5-liter bowl and time.

Responses vary.



7 Let's watch an animation of Alisha's graph representing the relationship between volume of water in the bowl and time.

Discuss: How does Alisha's graph accurately represent the situation? How might she revise her graph?

Responses vary.

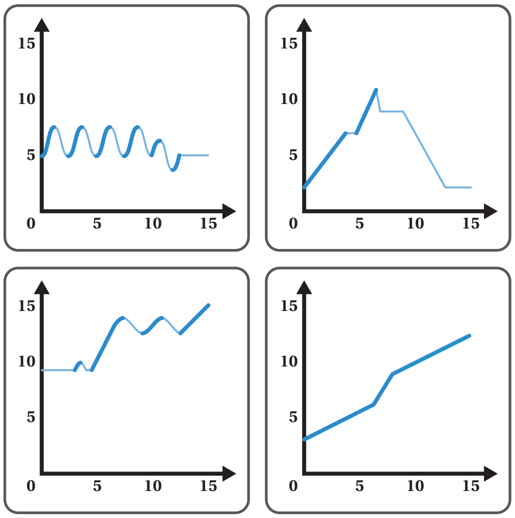
- **Alisha's graph is accurate in that the graph has a greater rate of change for the middle section of the graph.**
- **I would revise the middle section because it looks like a small amount of water is poured in initially and then it becomes more. I would use a curve instead of a straight line for that part of the graph.**

Name: ____________________ Date: __________ Period: __________

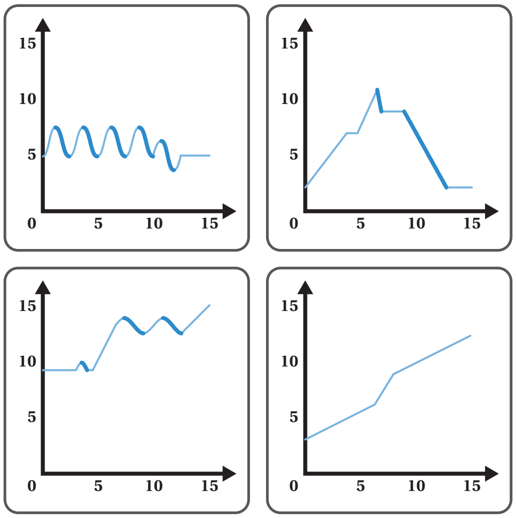
Describing Graphs

8 Here are some graphs from this lesson. Parts of the graphs are bolded to show where they are either **increasing**, **decreasing**, *linear*, or **non-linear**.

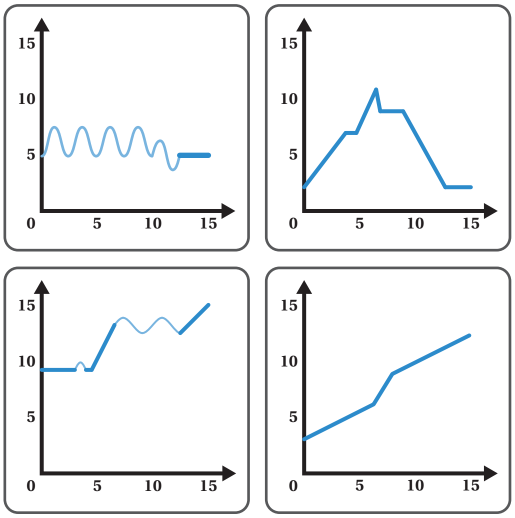
Increasing



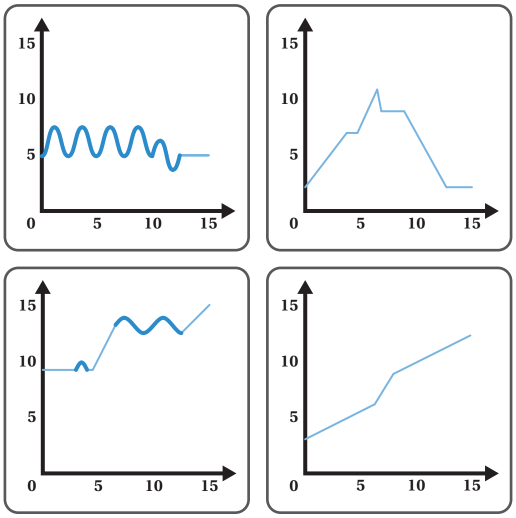
Decreasing



Linear



Non-Linear



Discuss: What do you think each term means?

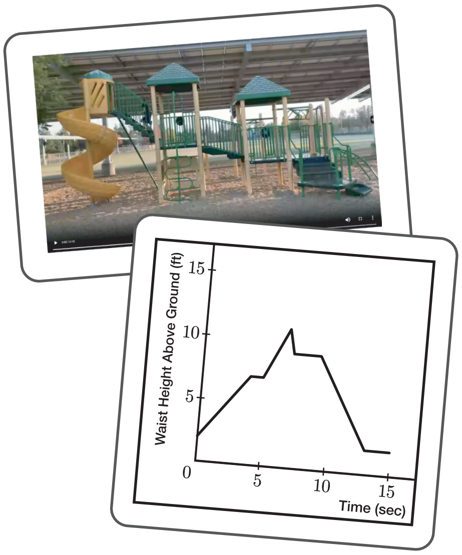
Responses vary.

- Increasing: Where the function is going up
- Decreasing: Where the function is going down
- Linear: Where the function's graph forms a straight line
- Non-linear: Where the function's graph does not form a straight line

9 Synthesis

Discuss: What can be helpful to consider when graphing a function that represents a real-world situation?

Use the example if it helps with your thinking.



Responses vary.

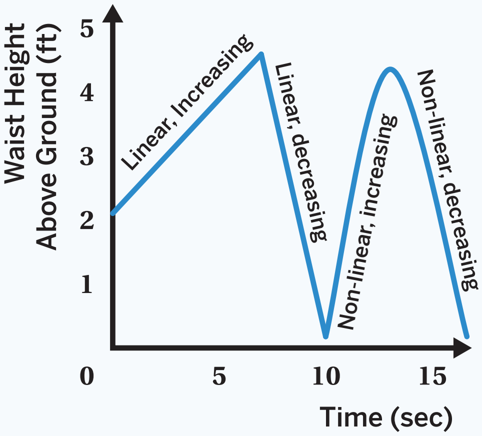
- **When graphing a function that represents a real-world situation, it may be helpful to identify specific points in the story and interpret the different x- and y-values that represent those points. For example, in a graph of waist height above the ground versus time, I might identify key moments in the story that have identified heights and seconds.**
- **It's important to consider when the function is increasing, decreasing, or neither, as well as when the function is linear or non-linear.**

12 Summary 5.06

We can use graphs to represent a story. When drawing a graph, it can be helpful to identify the variables involved to label the axes. Depending on the independent and dependent variables, different graphs can represent distinct details of the same story. It may also be helpful to identify key points in the story according to these chosen variables to help sketch these features.

For example, when part of the graph is:

- Going up from left to right, this part of the function is **increasing**.
- Going down from left to right, this part of the function is **decreasing**.
- A straight non-vertical line, this part of the function is *linear*.
- Not a straight line, this part of the function is **non-linear**.



decreasing A function, or interval of a function, is decreasing if the y-values go down when the x-values go up.

increasing A function, or interval of a function, is increasing if the y-values go up when the x-values go up.

non-linear A function whose graph is not a straight line.

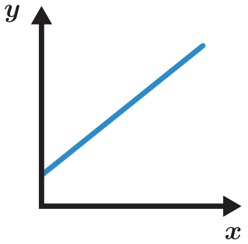
Practice
5.06

Name: .... Date: .... Period: ....

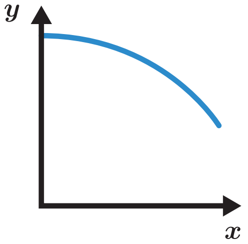
1. Determine which graph best represents the description.

Description	Graph
a. Linear and decreasing	d
b. Non-linear and increasing	c
c. Linear and increasing	a
d. Non-linear and decreasing	b

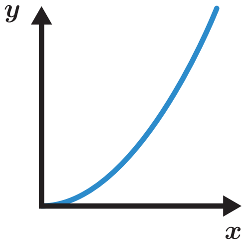
Graph A



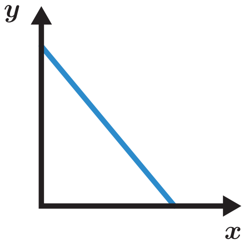
Graph B



Graph C

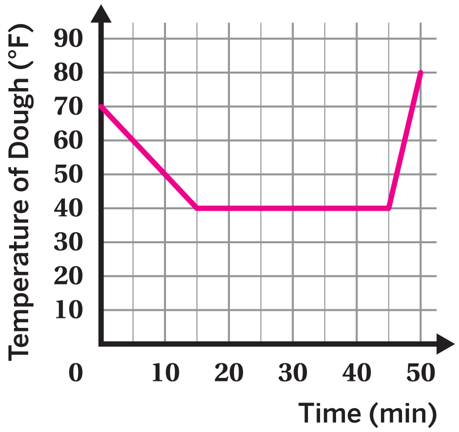


Graph D

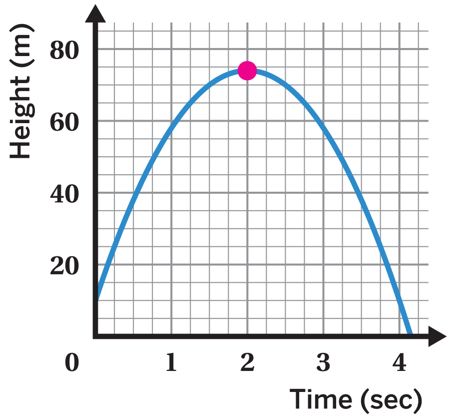


2. David places a batch of homemade pretzels in the refrigerator. The dough takes 15 minutes to cool from 70°F to 40°F. Once it is cool, the dough stays in the refrigerator for another 30 minutes. David then places the pretzels in the oven to bake. After 5 minutes in the oven, the temperature of the pretzel dough is 80°F.

   Sketch a graph that represents this situation.



Problems 3–6: This graph represents the height of an object that is launched upwards from a tower and then falls to the ground.



3. How tall is the tower from which the object was launched? **10 meters**

4. Plot the point that represents the greatest height of the object and the time it took the object to reach that height.

5. Determine one time interval when the height of the object was increasing.

   Responses between 0 and 2 seconds are considered correct.

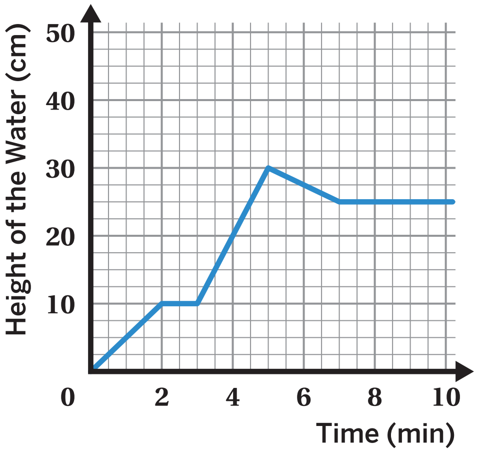
6. Determine one time interval when the height of the object was decreasing.

   Responses between 2 and 4.25 seconds are considered correct.

Name: .... Date: .... Period: ....

7. Kimaya fills her aquarium with water. This graph shows the height of the water in the aquarium vs. time.

Tell a story about how Kimaya fills the aquarium based on what you see. Include specific heights and times.



***Responses vary*. Kimaya turns on the water faucet, and the water in the aquarium increases at a constant rate for the first 2 minutes to a height of 10 centimeters. Then Kimaya's mom calls her to take out the trash, so she turns off the faucet for 1 minute. When she comes back, she turns on the water higher than before, and the water increases to a height of 30 centimeters in the next 2 minutes. This is high enough, and she turns off the water. Unfortunately, there is a slow leak in the aquarium, and the water height decreases to 25 centimeters. After 2 minutes, Kimaya notices the leak. She stops it, and the water height stays constant after that.**

Spiral Review

Problems 8–9: Solve each equation. Show your thinking.

8. $-(-2x + 1) = 9 - 14x$

$x = \frac{5}{8}$. ***Work varies.***

$2x - 1 = 9 - 14x$

$16x - 1 = 9$

$16x = 10$

$x = \frac{10}{16}$

$x = \frac{5}{8}$

9. $3x + \frac{3}{5} = \frac{1}{3}(5x + 5)$

$x = \frac{4}{5}$. ***Work varies.***

$9x + \frac{9}{5} = 5x + 5$

$4x + \frac{9}{5} = 5$

$4x = \frac{16}{5}$

$x = \frac{4}{5}$

10. Decide whether the equation $4(x + 3) = 7x + 12 - 3x$ has *one solution*, *no solution*, or *infinitely many solutions*. Show or explain your thinking.

Infinitely many solutions. *Explanations vary*. This equation is always true for any value of x because the expressions on either side of the equal sign are both equivalent to $4x + 12$.

Unit 5

Lesson

7

Name: ________ Date: ________ Period: ________

Data, Graphs, and Tables | Linear Equations | Multiple Representations of Functions

8.F.2, 8.F.3, 8.F.4, SMP.2, SMP.6

Comparing Linear Functions

Let's compare linear functions represented in different ways.

Warm-up

Jada has $50 in her savings account and saves $7 per week.

1. What could the independent variable and dependent variable be in this situation? Explain your thinking.

Responses vary.

- **The number of weeks Jada has spent saving is the independent variable, and the total amount of money in her savings account is the dependent variable. The amount of money in Jada's savings account depends on how many weeks she has spent saving.**
- **The total amount of money in Jada's savings account is the independent variable, and the number of weeks she has spent saving is the dependent variable. The amount of money in Jada's savings account tells us how long Jada has been saving.**

2. Write an equation representing this situation. Be sure to define the variables that you use.

Responses vary.

- $a = 7w + 50$**, where** a **is the total amount of money in Jada's savings account and** w **is the number of weeks she has spent saving.**
- $w = \frac{1}{7}a - \frac{50}{7}$**, where** a **is the total amount of money in Jada's savings account and** w **is the number of weeks she has spent saving.**

3. Is your relationship from Problem 2 a function? Explain your thinking.

Yes. *Explanations vary.* Each input value for the number of weeks spent saving has only one output value for the amount of money in the savings account.

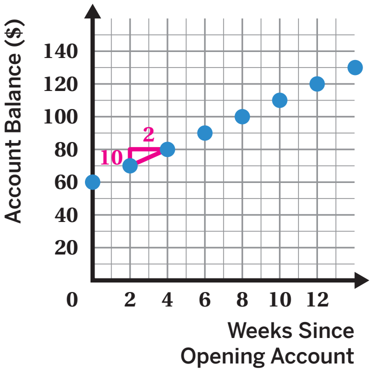
Activity
1

Name: ________ Date: ________ Period: ________

Which Is Growing Faster?

Noah and Elena both opened a savings account on the same day. Each person's account balance is a function of the number of weeks since the account was opened. Here is some information about each account. **ELD.PI.8.6.Em, Ex, Br, ELD.PI.8.11.Em, Ex, Br**

Noah's Account



Elena's Account

$a = 8w + 60$, where w is the number of weeks since the account was opened, and a is the account balance

4. Who started with more money in their account? Circle one.

Noah Elena [✓ They started with the same amount.]

Explain your thinking.

***Explanations vary.* On the graph representing Noah's account, the y-intercept is (0, 60). In the equation representing Elena's account, the constant term is 60. That means both Noah and Elena started with $60.**

5. Who is saving money at a faster rate? Circle one.

Noah [✓ Elena] They are saving at the same rate.

Explain your thinking.

***Explanations vary.* The slope of Elena's equation is 8, meaning she is saving $8 per week. The slope triangle drawn on the graph shows that the amount in Noah's account increases by $10 in two weeks, which is $5 per week.**

6. How much will Noah save over the course of a year if he does not make any withdrawals?

(Note: There are 52 weeks in a year.) Show or explain your thinking.

$320. *Explanations vary.* Noah's situation can be represented by the equation $a = 60 + 5w$, where w is the number of weeks since the account was opened and a is the total amount in the account. Noah can earn $320 in a year because $60 + 5(52) = 320$.

7. How long will it take Elena to save the same amount? Show or explain your thinking.

32.5 weeks. *Work varies.*

$320 = 8w + 60$

$260 = 8w$

$32.5 = w$

Activity
2

Name: ________ Date: ________ Period: ________

Making Deposits vs. Withdrawals

Take a look at the accounts of four customers. They each have an account balance, a, measured over w weeks.

Account A

The account balance, a, is represented by the function $a = 65 + 10w$, where w represents the number of weeks since the account was opened.

Account B

The account balance, a, starts at $\$40$ and decreases by $\$8.50$ per week.

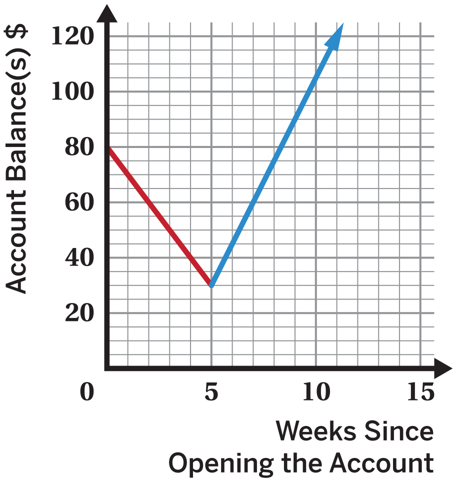
Account C

The account balance is represented by this table:

Number of Weeks	Account Balance ($)
1	71
3	48
7	2

Account D

The account balance is represented by this graph:



8. **Discuss:** **ELD.PI.8.1.Em, Ex, Br, ELD.PI.8.6.Em, Ex, Br**

a. Which account(s) show customers making deposits? Explain your thinking.

Accounts A and D. *Explanations vary*. Account A shows that deposits are being made because the coefficient w, which is the slope, is positive. Account D shows that deposits are being made after week 5 because the line is increasing at that point.

b. Which account(s) show customers making withdrawals? Explain your thinking.

Accounts B, C, and D. *Explanations vary*. Account B shows that withdrawals are being made because the description states that the account balance is decreasing. Account C shows that withdrawals are being made because the account balance is decreasing over time. Account D also shows withdrawals because the line is decreasing from $w = 0$ to $w = 5$.

Name: Date: Period:

Making Deposits vs. Withdrawals (continued)

9. Kiana says that all four relationships are **linear functions** and that each situation can be represented with one equation in the form $y = mx + b$. Is Kiana's claim correct? Explain your thinking. **ELD.PI.8.11.Em, Ex, Br**

No. *Explanations vary*. Account D is not a linear relationship because the graph is not a single line. It's a function made up of two different linear relationships. However, Accounts A, B, and C each have a starting balance ($\$b$) and a constant rate of change (m), so they can be represented in the form $y = mx + b$.

10. Write an equation for each linear function where a represents the account balance and w represents the number of weeks.

Account A: $a = 65 + 10w$
Account B: $a = 40 - 8.50w$
Account C: $a = 82.50 - 11.5w$

11. Which account will have the most money at the end of a year? Explain your thinking.

Account D. *Explanations vary*. Account D will have \$735 after 52 weeks and Account A will have \$585.

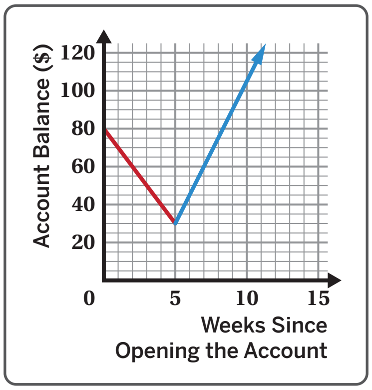
Synthesis

12. **Discuss:** How can you compare linear functions shown using different representations? **ELD.PI.8.1.Em, Ex, Br**

Responses vary. **Calculating the slope and the y-intercept can help you compare linear functions that are represented in different ways.**

Number of weeks	Account balance ($)
1	71
3	48
7	2

$a = 65 + 10w$



Summary 5.07

Linear relationships that have exactly one output for every possible input are called **linear functions**. All linear functions can be represented with an equation in the form $y = mx + b$, where m is the rate of change and b is the initial value.

You can compare properties of functions when they are represented in different ways.

- In a graph, you can find the rate of change using slope triangles to calculate the vertical and horizontal change between two points on the coordinate plane. You can find the initial value by locating the y-intercept.
- In a table, you can find the rate of change by determining the ratio of the difference between the y-values and the difference between the x-values. You can find the initial value by determining the dependent value when the independent value is 0.
- In an equation, you can find the rate of change by determining the coefficient of the independent variable. You can find the initial value by determining the constant, b, in the equation $y = mx + b$.
- In a verbal description, you can find the rate of change by determining how much the dependent variable changes for each unit of increase in the independent variable. You can find the initial value by determining the starting value.

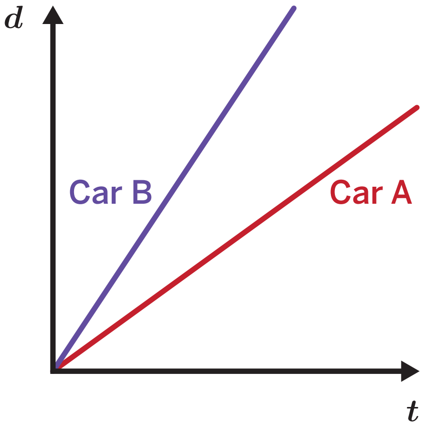
linear function A function that can be defined by an equation in the form $y = mx + b$, where m represents the slope and b represents the vertical intercept. A vertical line is not a linear function because it has an input with different outputs.

Practice
5.07

Name: .... Date: .... Period: ....

1. Two cars are traveling on the same highway in the same direction. The graphs show the distance, d, of each car from the starting point as a function of time, t. Which car is traveling faster? Explain your thinking.

Car B. *Explanations vary.* Car B is traveling faster because its line is steeper than Car A's line. This means that Car B's distance from the starting point is increasing at a faster rate.



Problems 2–4: Kiri and Chloe race each other home from school. They run at the same speed, but Kiri's house is slightly closer to the school than Chloe's house. Suppose there is a graph that shows their distances from home, in meters, as a function of the time, in seconds, from when they began the race.

2. If you were to read the graphs from left to right, would you expect the lines to increase or decrease? Explain your thinking.

Decrease. *Explanations vary.* As the time (the independent variable) increases, their distance from home (the dependent variable) decreases.

3. How would you expect the lines representing Kiri's run and Chloe's run to be different? Explain your thinking.

***Responses vary.* I would expect the y-intercepts to be different. Kiri's house is closer to the school than Chloe's, so her starting distance from home (the y-intercept) would be less than Chloe's.**

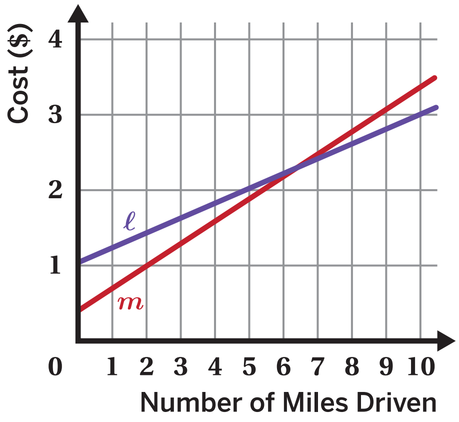
4. How would you expect the lines representing Kiri's run and Chloe's run to be alike? Explain your thinking.

***Responses vary.* I would expect the slopes to be the same because they run at the same speed.**

Practice
5.07

Name: .... Date: .... Period: ....

Problems 5–6: Two car services offer to pick up a customer and take them to their destination. Service A charges a flat fee of $\$0.40$ plus $\$0.30$ for each mile of the trip. Service B charges a flat fee of $\$1.10$ plus c dollars for each mile of their trip.



5. Match the services with the lines l and m.

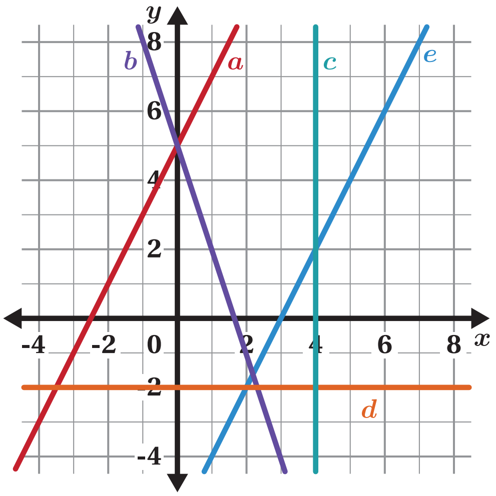
***Responses vary*. Line l represents Service B. Line m represents Service A.**

6. For Service B, is the additional charge per mile greater than or less than $\$0.30$ for each mile of the trip? Explain your thinking.

Less than. *Explanations vary*. The charge for each mile is less than $\$0.30$ because line l is not as steep as line m.

Spiral Review

Problems 7–8: Here are five lines.



7. Write an equation for each line shown on the graph.

Line a: $y = 5 + 2x$

Line b: $y = 5 - 3x$

Line c: $x = 4$

Line d: $y = -2$

Line e: $y = 2x - 6$

8. Select *all* of the lines where $(4, 2)$ is a solution.

- ☐ **A.** Line a
- ☑ **B.** Line b
- ☐ **C.** Line c
- ☐ **D.** Line d
- ☑ **E.** Line e

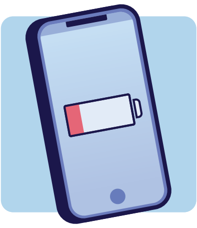
Unit 5
Lesson
8

Name: .... Date: .... Period: ....

Data Explorations Linear Equations 8.F.4, 8.F.5, SMP.3, SMP.4, SMP.5

Charge!

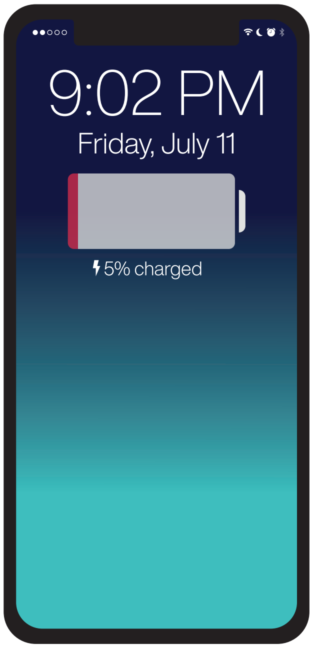
Let's use linear functions to model a real-world situation.



Warm-up

1. Tell a story about this image. **ELD.PI.8.10.Em, Ex, Br**

Responses vary. My dad has a smartphone. He saw that his phone was low on battery, so he set his alarm, plugged in his phone, and went to bed.



Name: Date: Period:

Charge!

2. Estimate the time when the phone will be fully charged. Explain your thinking.
ELD.PI.8.10.Em, Ex, Br

Responses vary. I think the phone will take two hours to charge, so it will be fully charged at 11:02 PM.

3. **a** What relevant information do you know that would help to answer this question?

Responses vary.
- The phone currently has a 5% charge.
- The current time is 9:02 PM.
- The phone is connected to Wi-Fi.

b What additional information would be helpful?

Responses vary.
- How fast does the phone charge?
- How old is the phone?
- What kind of phone is it?

4. Let's look at some screens with additional information. Record any information that you think is relevant.

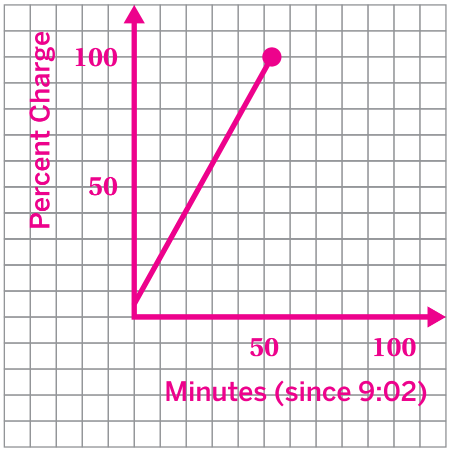
Responses vary.

5. **Discuss:** What mathematical representations might help you make a more precise prediction for when the phone will be fully charged? Explain your thinking.

Responses vary. **ELD.PI.8.3.Em, Ex, Br**

6. Use the additional information to revise your estimate for when the phone will be fully charged. Determine the initial value and rate of change then show or explain your thinking using *at least two* mathematical representations. **ELD.PI.8.10.Em, Ex, Br**

Responses vary.
- Equation: The phone will be fully charged about 84 minutes after 9:02 PM, or at 10:26 PM. I came up with an equation based on the first two data points, $y = \frac{9}{8}x + 5$. I set $y = 100$ and then solved for x.
- Table: I made a table to record the time since 9:02 PM and the percent charged. Then I determined an approximate "charging rate" by repeatedly picking two points from the table and finding the slope of a line through them. For example, $\frac{28-5}{20-0} = \frac{23}{20}$, which means the charging rate is an increase of $\frac{23}{20}$, or 1.15 percent per minute. Next, I used an equation to determine how long it would take at that rate to go from 28% to 100% charge, which I found was about 63 more minutes. Adding that to 9:22 PM, I got an estimate of 10:24 PM.
- Graph



Activity

2

Name: .... Date: .... Period: ....

Is it Linear?

Let's look at a graph that reveals when the phone is fully charged.

7. Is the relationship between the percent charge and time a function? Explain your thinking. **ELD.PI.8.10.Em, Ex, Br**

Yes. ***Explanations vary.*** **It's a function because at any given time, the phone is charged to a specific percentage.**

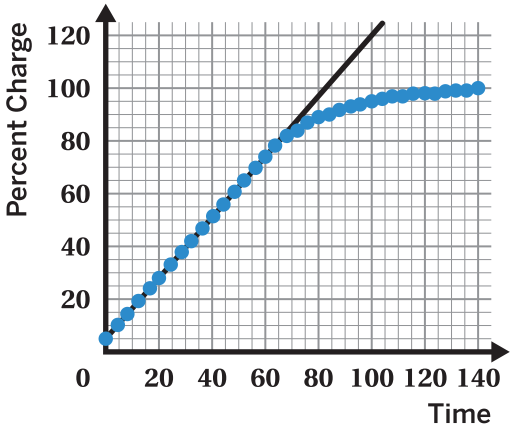
8. **Data Talk!** When might it be appropriate to use a linear function to model the data that describes the percent charge and the time? When might it *not* be appropriate? **ELD.PI.8.10.Em, Ex, Br**

Responses vary. **The relationship between percent charge and time creates a line until the phone reaches about 80% charge, so it makes sense to use a linear function for that interval. After that, the charging rate begins to decrease, so a single line is no longer appropriate. It might be appropriate to model the relationship using two separate lines: one before the phone is 80% charged and one after.**

Synthesis

9. **Discuss:** How might a linear function be helpful to model a situation? What are some limitations of using linear models?
ELD.PI.8.1.Em, Ex, Br

Responses vary. **An advantage of using a line to model a function is that it helps you make predictions and analyze the rate of change. But some situations are not linear, which can lead to inaccurate predictions.**



Summary 5.08

Sometimes a linear function can be used to model a situation. Although a function might be non-linear, parts of the data might be modeled by a linear function which can be used to help make predictions.

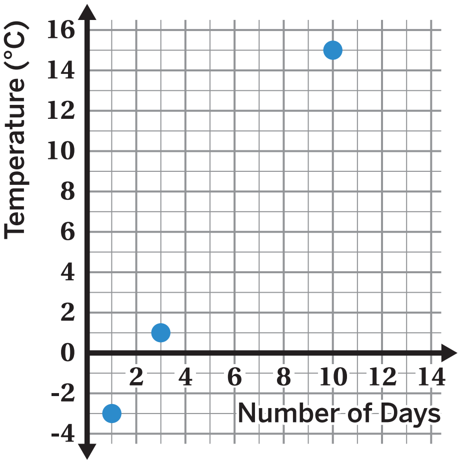
Practice
5.08

Name: .... Date: .... Period: ....

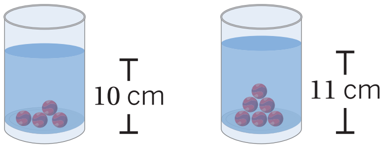
1. This graph shows the relationship between a city's high temperature, y, in degrees Celsius, and the number of days after the new year, x. Is the high temperature a linear function of the number of days after the new year? Explain your thinking.

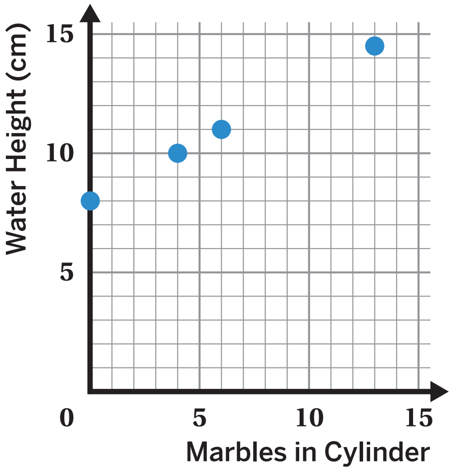
Responses vary.

- **Yes. The temperature increases at close to a constant rate for each day shown.**
- **No. Although this data could be modeled by a linear function, it doesn't make sense to use a linear model. The linear model would continue to increase and predict a temperature that's too hot 2 months after the new year.**



Problems 2–5: Use this information. Farah has a graduated cylinder filled partially with water. She is exploring the change in the water's height as she adds marbles to the cylinder. After dropping in 4 marbles, the height of the water is 10 centimeters. After dropping in 6 marbles, the height of the water is 11 centimeters.





2. How much does the height increase for each marble?

0.5 centimeters

3. What was the height of the water in the cylinder before any marbles were dropped in?

8 centimeters

4. What is the height of the water after 13 marbles are dropped in?

14.5 centimeters

5. Is the relationship between the height of the water and number of marbles a linear function? If so, what does the slope of the line represent? If not, explain why not.

Yes. ***Explanations vary.*** **The slope of the line means there is a constant increase in water height per marble added to the cylinder.**

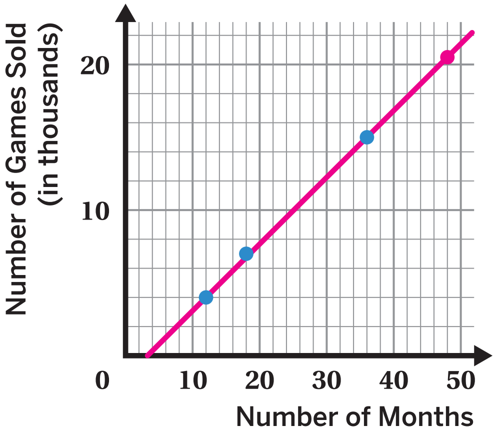
Practice
5.08

Name: .... Date: .... Period: ....

Problems 6–7: A board game company needs to know how many board games to produce. After the first 12 months, they sell 4,000 games. After 18 months, they sell 7,000 games. And after 36 months, they sell 15,000 games.

6. Do you think a single linear function could be used to estimate the number of games that will be sold after a certain number of months? If yes, draw a linear function to model the data. If no, explain your thinking.

Yes. Sample shown on graph.



7. Estimate the number of games sold after 48 months.

***Responses vary.* Approximately 20,500 games.**

Spiral Review

Problems 8–11: Solve each equation. Show your thinking.

8. $5y + 14 = -43 - 3y$

$y = -\frac{57}{8}$. *Work varies.*

$8y + 14 = -43$

$8y = -57$

$y = -\frac{57}{8}$

9. $4(2a + 2) = 8(2 - 3a)$

$a = \frac{1}{4}$. *Work varies.*

$8a + 8 = 16 - 24a$

$32a + 8 = 16$

$32a = 8$

$a = \frac{8}{32}$

$a = \frac{1}{4}$

10. $23n - 6 + 8n = -13n + 5$

$n = \frac{1}{4}$ (or equivalent)

Work varies.

$31n - 6 = -13n + 5$

$44n - 6 = 5$

$44n = 11$

$n = \frac{1}{4}$

11. $\frac{2}{3}c + 12 = \frac{8}{3}c - 4$

$c = 8$. *Work varies.*

$12 = \frac{6}{3}c - 4$

$16 = 2c$

$c = 8$

Unit 5

Lesson

9

Name: ______ Date: ______ Period: ______

Data Explorations   Linear Equations   8.F.4, 8.F.5, SMP.1, SMP.2, SMP.4

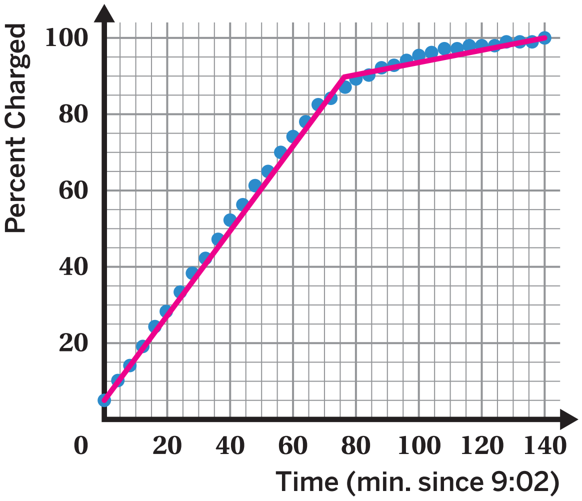
Piecing It Together

Let's create functions to model data sets.

Warm-up

1 **Data Talk!** Here is a data set for a phone charging over time.

A single linear function does not model this relationship very well.



a Sketch two connected line segments to model the relationship better.

Responses vary.

b When was the phone charging the slowest? Explain your thinking.

Responses vary.

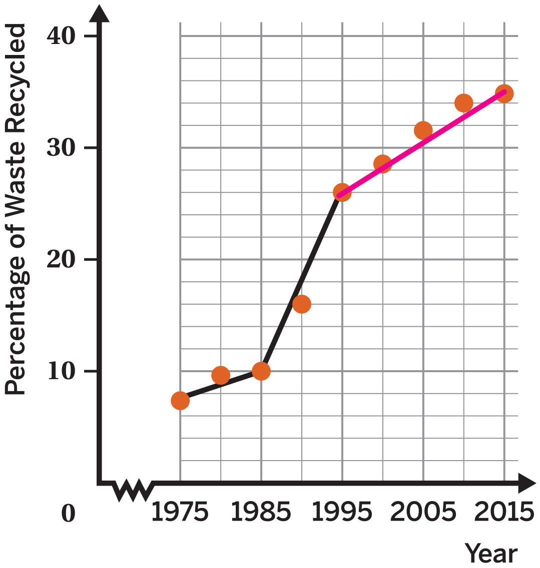
- **Once the phone reached about 85% charge, it began to charge more slowly until it was fully charged. You can see this in the graph because the points start increasing at a slower rate.**
- **The phone was charging the slowest between about 10:20 and 11:22, when it was fully charged. This aligns with my function, where my segment from about 10:20 to 11:22 has a very small slope.**

Name: .... Date: .... Period: ....

Recycling

2 **Data Talk!** This data set shows the percentage of waste produced in the United States that gets recycled over time.

A student started sketching a function to model this data.



a Sketch one more linear segment to complete the function.

Responses vary.

b Approximate the slope of the segment you created.

About 0.5 percent per year

Explain what this number means about the percentage of waste recycled.

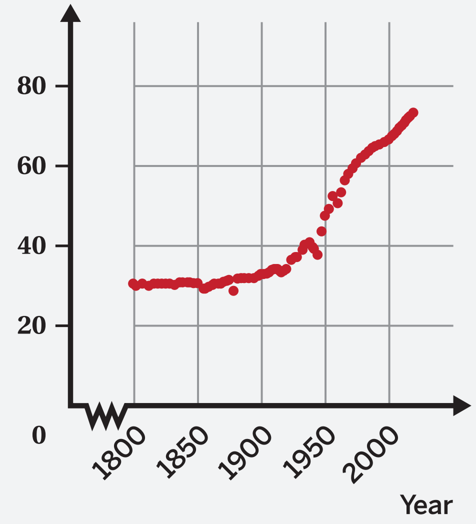
Explanations vary. The slope is the rate of change. It means that between 1995 and 2015, the percentage of waste recycled grew by 0.5 percent per year.

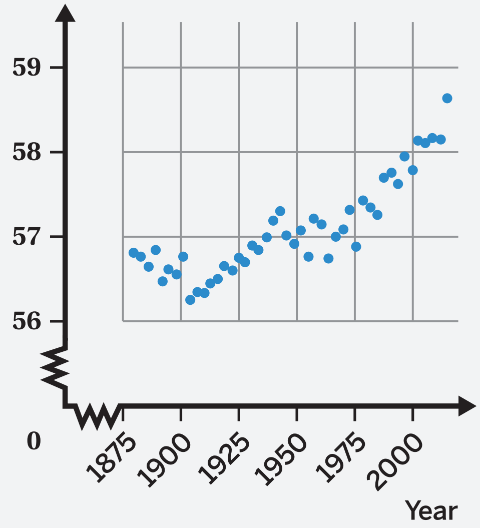
Activity
2

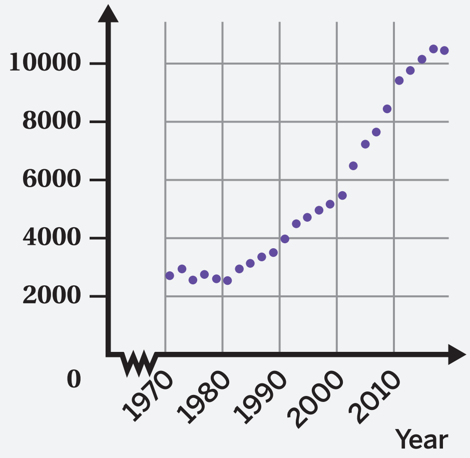
Name: ______________________ Date: __________ Period: __________

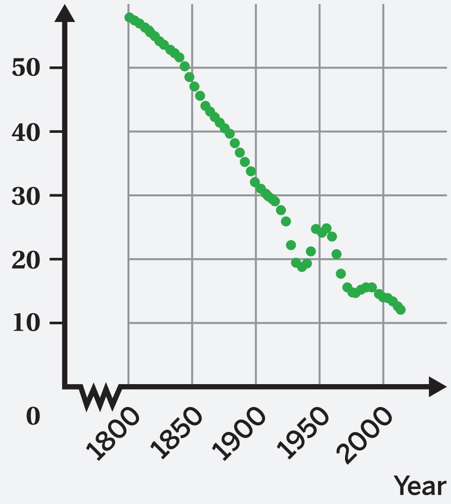
Four Data Sets

3 Here are four new data sets. Match each graph to the description you think it represents.


Graph A


Graph B


Graph C


Graph D

U.S. College Cost ($)	U.S. Births (per 1000 people)	Global Life Expectancy (age in years)	Global Temperature (°F)
C	D	A	B

Activity
3

Name: .... Date: .... Period: ....

Analyzing Data Sets

4 **Data Talk!** Look at the four data sets on the Activity 3 Sheet. Pick one that interests you.

Which data set are you choosing?

***Responses vary.* Sample responses are provided for the "U.S. College Cost" data set.**

What do you notice? What do you wonder?

Responses vary.

I notice:

- **I notice that the last data point for the cost of college shows a slight decrease in cost.**

I wonder:

- **I wonder what the cost of college will be when I'm old enough to enroll.**

5 **a** Using at least two line segments, sketch a function on the graph of the data set you chose on the Activity 3 Sheet to model the data.

Responses vary.

Word Bank

linear	increasing
non-linear	decreasing

b Describe your function using vocabulary from this unit.

***Responses vary.* Although the cost of Data Set D is non-linear, I can model it with a function made of four linear segments. From 1970 to 1980, the cost of college slightly decreased. Then it increased from 1980 to 2000, and then from 2000 on it increased at an even faster rate until it leveled off around 2016.**

6 **Data Talk!** During which interval of time did the data seem to change the most?

***Responses vary.* The cost of college seems to have changed the most in the decade between 2000 and 2010, from about $5,500 to about $9,000.**

Name: .... Date: .... Period: ....

Analyzing Data Sets (continued)

7 **a** Use your function to make a prediction for the year 2030.

***Responses vary.* In 2030, the cost of college will be $11,500**

b Do you think your function can be used to make an accurate prediction for the year 2050? Explain your thinking.

***Responses vary.* The trend in the cost of college has changed many times over the years. I don't think I can make a prediction for 2050 that's very precise or accurate.**

8 **Data Talk!** Share your findings with a student or a group that examined a different data set.

Discuss the following:

- What choices did you make as you sketched your function?
- What is your prediction for 2030, and how did you arrive at your prediction?
- Do you think your function can be used to make an accurate prediction for 2050?

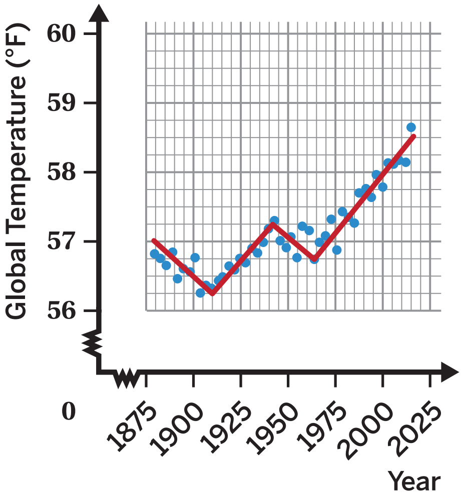
Responses vary.

9 Synthesis

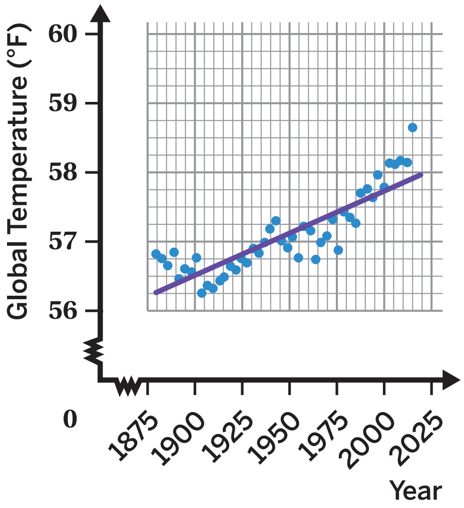
Data Talk! When is it helpful to model a data set with multiple linear segments instead of a single linear segment?

Use the examples if they help with your thinking.

Function A: Multiple Segments



Function B: Single Segment



Responses vary. **It can be helpful to use multiple linear segments, like in Function A, because it's very accurate. It's more likely than the other functions to produce a good prediction for the near future. One disadvantage of using Function A is that it doesn't represent long-term trends very well compared to a single linear segment in Function B.**

12 Summary 5.09

We can use either a single linear segment or multiple linear segments to represent a data set. Using multiple linear segments can help us analyze data more precisely, for example, by interpreting the slope of a specific segment. Using a single segment to represent a function can help us identify long-term trends.

Practice
5.09

Name: .... Date: .... Period: ....

Problems 1–2: On the first day after the new moon, 2% of the Moon's surface is illuminated. On the second day, 6% of the Moon's surface is illuminated.

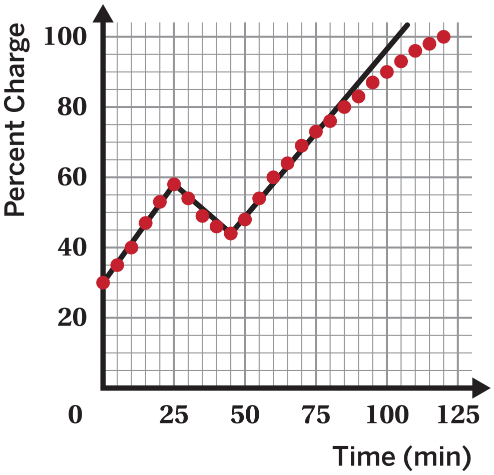
1. Assuming this data can be modeled with a linear function, complete the table. Round to the nearest day if necessary.

Day Number	Illumination (%)
1	2
2	6
. . .	. . .
13	50
26	100

2. The Moon's surface is actually 100% illuminated on day 14. How appropriate is it to use a linear function for this data?

***Responses vary.* A linear function is not appropriate for this data because the illumination of the Moon doesn't increase by the same percentage each day.**

Problems 3–4: Elena is charging her laptop. After 25 minutes, she unplugs her laptop to complete her homework. After she completes her homework, she plugs in her laptop again until it is fully charged. This graph shows the percent charge of Elena's laptop over time.



3. Describe the function used to model the data.

***Responses vary.* The data is modeled with three different linear functions, one for each interval: 0 to 25 minutes, 25 to 45 minutes, and 45 to 120 minutes.**

4. Which time interval is not modeled appropriately? Explain your thinking.

85 to 120 minutes. *Explanations vary.* After about 85 minutes, the charging rate begins to slow down, so the single line drawn for this interval is not appropriate. Two separate lines might be appropriate, one for 45 to 85 minutes and one for after 85 minutes.

5. The graph shows y as a function of x. For which intervals is the function increasing? Select *all* that apply.

☑ **A.** From -7 to -3

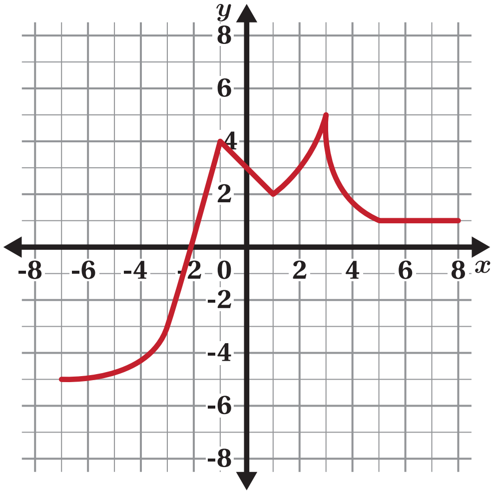
☑ **B.** From -3 to -1

☐ **C.** From -1 to 1

☑ **D.** From 1 to 3

☐ **E.** From 3 to 5

☐ **F.** From 5 to 7



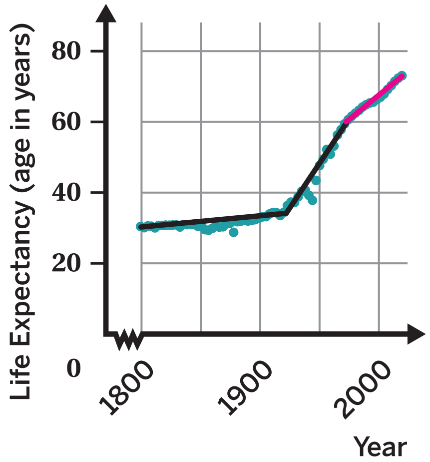
Practice
5.09

Name: ______________ Date: ______________ Period: ______________

Spiral Review

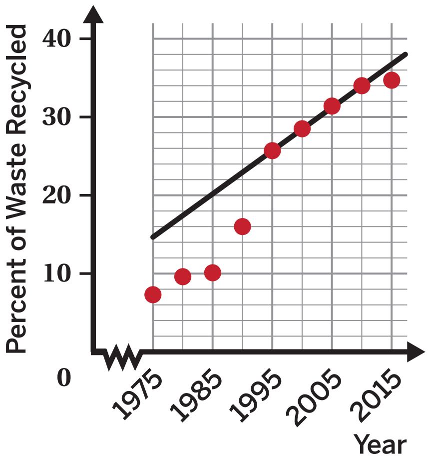
6. This graph shows global life expectancy over time. A student started to model the data with two linear functions. Sketch one more linear segment to complete the student's work.

***Responses vary.* Sample shown on graph.**



7. This graph shows the percentage of waste produced in the United States that gets recycled over time. A student draws a linear function that models the change from 1975 to 2015. For what years does the model make accurate predictions? For which years is it not as accurate?

***Responses vary.* The model is good at predicting the percentage of waste recycled from 1995 to 2015, but not as good from 1975 to 1990.**



8. Which linear function has a greater rate of change? Explain your thinking.

Function A

$$y = \frac{1}{6}x + \frac{2}{5}$$

Function B. *Explanations vary.* Function B has a rate of change of $\frac{1}{5}$. Function A has a rate of change of $\frac{1}{6}$.

Function B

x	y
-9	3
1	5
21	9

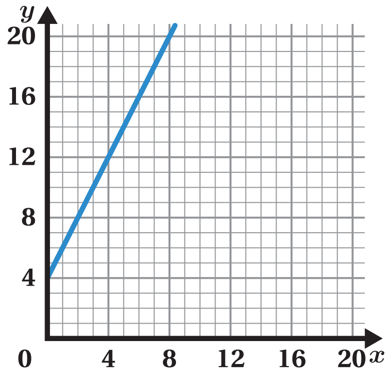
9. Which linear function has a smaller rate of change? Explain your thinking.

Function J

x	y
0.5	2
1	4
2.5	10

Function K. *Explanations vary.* Function J has a rate of change of 4. Function K has a rate of change of 2.

Function K



Name: .... Date: .... Period: ....

Data, Graphs, and Tables | Linear Equations | Multiple Representations of Functions

8.F.1, 8.F.2, 8.F.3, 8.F.4, 8.F.5

Practice Day 1

Let's practice what you've learned so far in this unit!

You will use task cards for this Practice Day. Record all of your responses here.

Task A: Two Truths and a Lie

1. Circle one: A [✓ B] C

Explanation:

Explanations vary. Takeshi and Mateo began watching 10 minutes into the movie. This is represented by the point (0, 10) on the graph.

2. Circle one: A [✓ B] C

Explanation:

Explanations vary. The volume of water is neither increasing nor decreasing between 35 and 45 minutes.

3. Circle one: A B [✓ C]

Explanation:

Explanations vary. A single linear model is not appropriate for this data. The rate of change between 30 and 40 therms is $1.50 per therm of gas. The rate of change between 40 and 60 therms is $6.25 per therm of gas.

You're invited to explore more.

Situation 1 false statement:

Responses vary. Takeshi and Mateo fast-forwarded the movie after 10 minutes of viewing.

Situation 2 false statement:

Responses vary. The volume of water in the cooler decreased by the same amount in the first 20 minutes of the soccer match as it did between 45 and 50 minutes.

Situation 3 false statement:

Responses vary. The relationship between the amount of gas used and the cost of gas for the month can be represented by a linear function.

Name: ______ Date: ______ Period: ______

Practice Day 1 (continued)

Task B: Function or Not?

1. Circle one:

Function [✓ Not a function]

Explain if not a function:

Explanations vary. An input of 0 has two outputs: -1 and 1.

2. Circle one:

Function [✓ Not a function]

Explain if not a function:

Explanations vary. Inputs like c and d have multiple outputs.

3. Circle one:

[✓ Function] Not a function

Explain if not a function:

4. Circle one:

Function [✓ Not a function]

5. Circle one:

[✓ Function] Not a function

6. Circle one:

[✓ Function] Not a function

7. Circle one: Function [✓ Not a function]

Explanation: *Explanations vary.* The total listening time is not a function of the number of songs she listens to because Eliza could listen to 5 songs, but some combination of songs might take 12 minutes and a different 5 songs could take 14 minutes to listen to.

You're invited to explore more.

Responses vary.

Function: A situation that represents a function could be if the input is someone's first name and the output is their last name.

Not a function: A situation that does not represent a function could be if the input is a household chore (e.g., vacuuming) and the output is the time it takes for someone to complete the task.

Task C: Three Restaurants

1. Circle one: [✓ Yes] or No

Explanation: *Explanations vary.* For every number of burgers ordered, there is only one possible cost.

2. Circle one:

McDougal's [✓ Sandy's] Burger Royalty

Explanation: *Explanations vary.* At McDougal's, 5 burgers cost $4.75(5) = $23.75. At Sandy's, 5 burgers cost $20. At Burger Royalty, 5 burgers cost $24.

3. McDougal's: $4.75 Sandy's: $5 Burger Royalty: $6

4. Advantage: *Responses vary.* One advantage of the equation over the graph is that you can get more precise values with an equation. These values may be hard to read in a graph.

You're invited to explore more.

Segment 1: $y = 6x$ Segment 2: $y = 18$

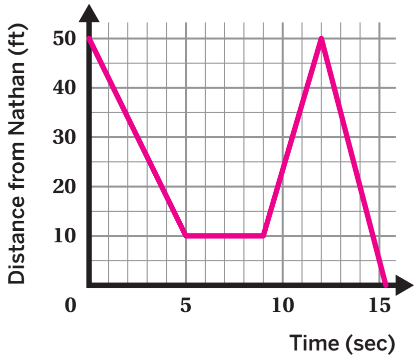
Segment 1: $y = 6x - 6$ Segment 1: $y = 36$

Name: .... Date: .... Period: ....

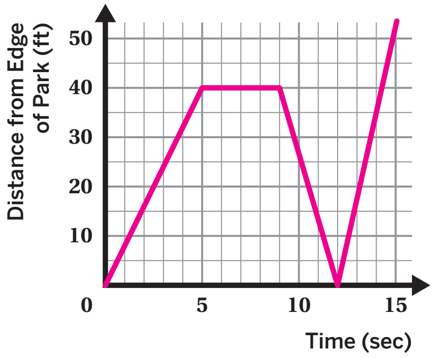
Practice Day 1 (continued)

Task D: Graphing Stories

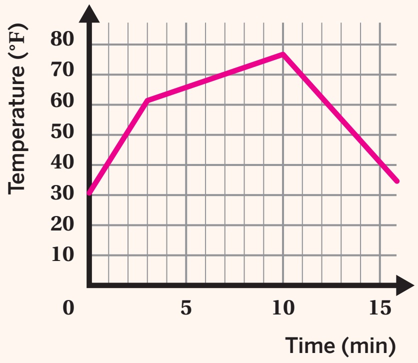
1.



2.



You're invited to explore more.



Responses vary.

Task E: Linear or Not?

1. Circle one: Linear or [✓ Not Linear]

2. Circle one: [✓ Linear] or Not Linear

3. Circle one: Linear or [✓ Not Linear]

4. Circle one: [✓ Linear] or Not Linear

You're invited to explore more.

Situation 1: **Not linear**

Situation 2: ***Responses vary.*** **$y = 5x + 40$, where x represents the number of weeks since opening the account and y represents the amount in the account.**

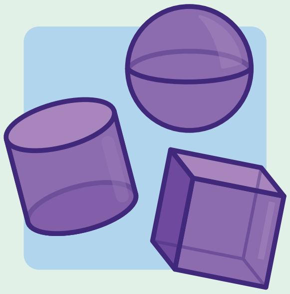
Situation 3: **Not linear**

Situation 4: ***Responses vary.*** **$y = -15x + 355$, where x represents the number of nights Tyrone has read and y represents the number of pages left in his book.**

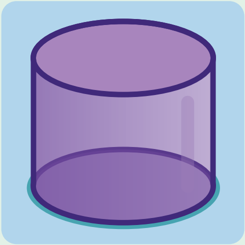
Sub-Unit

3

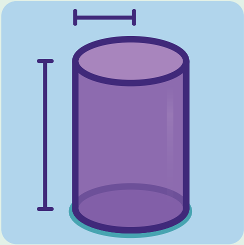
Volume



Lesson 10
Volume Lab

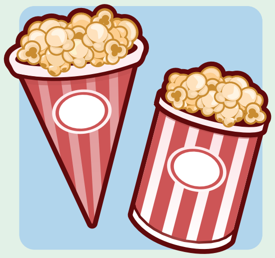


Lesson 11
Cylinders

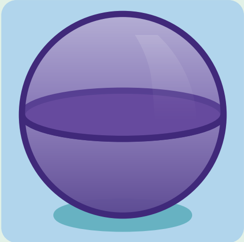


Lesson 12
Scaling Cylinders

Lesson 13
Cones



Lesson 14
Unknown Dimensions

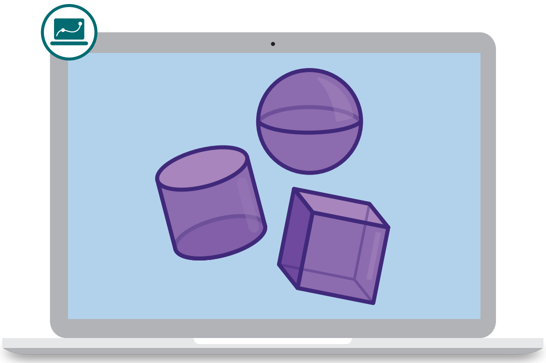


Lesson 15
Spheres

Unit 5
Lesson
10

Name: ________ Date: ________ Period: ________

Cylindrical Investigations | Shape, Number, and Expressions | Building Toward 8.G.9, SMP.1, SMP.7



Volume Lab

Let's estimate the volume of three-dimensional solids.

Warm-up

1 Which one doesn't belong? Explain your thinking.

Figure A

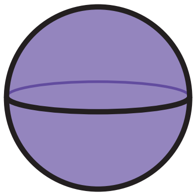


Figure B

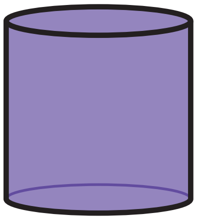


Figure C

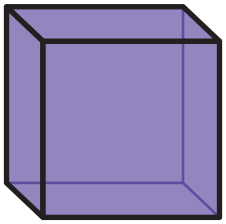
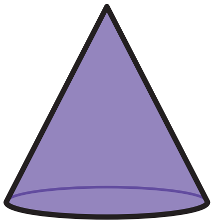


Figure D



Responses vary.

- Figure A: The sphere is the only object with no flat surfaces.
- Figure B: The cylinder is the only object with two faces that are circles. The cylinder is the only object that would work as a wheel for a machine that smooths roadways.
- Figure C: The cube is the only object without a radius.
- Figure D: The cone is the only object that would look different if you flipped it upside down.

Activity
1

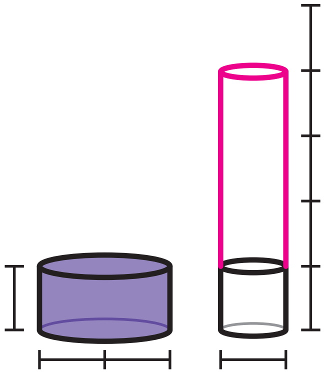
Name: .... Date: .... Period: ....

Comparing Volumes

2 Here are two **cylinders**.

Draw a new height for the cylinder on the right so that both cylinders have the same *volume*.

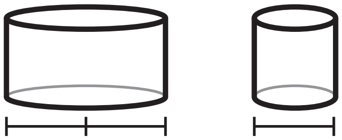
Response shown on diagram. The height of the cylinder on the right should be 4 units.



3 Here are two cylinders with the same height.

How many small cylinders do you think it would take to fill the large cylinder?

4 small cylinders

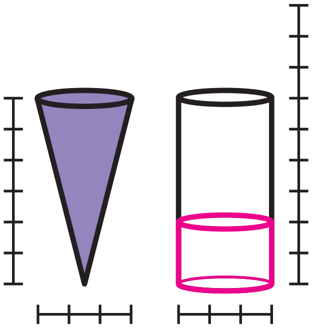


Name: .... Date: .... Period: ....

Comparing Volumes (continued)

4 Draw a new height for the cylinder so that both objects have the same volume.

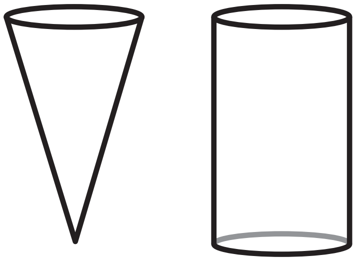
Response shown on diagram. The height of the cylinder should be 2 units.



5 Here is a **cone** and a cylinder with the same height and *diameter*.

How many cones would it take to fill the cylinder?

3 cones



Activity
2

Name: .... Date: .... Period: ....

Volume Lab

6 Let's use Screen 6 of the digital activity to explore some volume relationships!

a Select any *two* objects and adjust their dimensions. Then press "Compare." Repeat this with several pairs of objects. Draw or record something that you found interesting or surprising.

Responses vary.

- **I notice that a cylinder's volume is 3 times bigger than a cone with the same diameter and height.**
- **I notice that a cube always has a greater volume than a sphere when the side length and diameter are equal.**
- **I notice that when you double the diameter of a sphere, the volume of the sphere is 8 times larger.**

b Two cones have equal diameters. The height of one cone is 2 times as large as the height of the other cone. How are the volumes of the cones related?

Responses vary. **The volume of the large cone is 2 times that of the small cone.**

c Two cylinders have the same height. The diameter of one cylinder is 3 times as large as the diameter of the other cylinder. How are the volumes of the cylinders related?

Responses vary. **The volume of the large cylinder is 9 times that of the small cylinder.**

d Describe the relationship between the volume of two spheres, where the diameter of one sphere is twice the length of the other sphere.

Responses vary.
The volume of the larger sphere is 8 times the volume of the smaller sphere.

e Describe the relationship between two different objects, where one has twice the volume of the other.

Responses vary.

- **A sphere and cone where the cone's height and diameter are equal to the sphere's diameter. The sphere has twice the volume of the cone.**
- **A pair of cylinders where one cylinder's height is half of the other cylinder's height, and both cylinders have the same diameter.**

f Describe another interesting relationship between the volumes of two different objects.

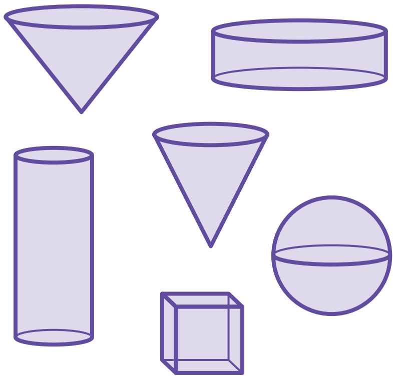
Responses vary. **(See sample responses for part a.)**

7 Synthesis

Describe one volume relationship you discovered during this lesson.

Responses vary.

- When a cylinder and a cone have the same dimensions, the cylinder has exactly 3 times as much volume.
- When the diameter of a sphere is doubled, its volume is multiplied by 8.
- When the height and diameter of a cylinder are equal, and a sphere has the same diameter, the volume of the cylinder is exactly 1.5 times the volume of the sphere.

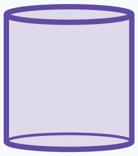
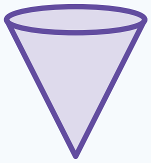
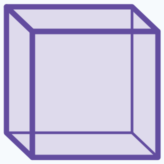
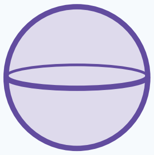


10 Summary 5.10

The *volume* of an object is the number of cubic units that fill its three-dimensional region without any gaps or overlaps.

You can often determine relationships between the volumes of different figures with similar measurements. For example, if the base of a **cone** and a **cylinder** have the same *diameter* and height, then the cylinder will have a volume that is three times greater than the cone.

There are also relationships between the volumes of the same figure with different measurements. For example, if the diameter of a **sphere** is doubled, or if the side length of a cube is doubled, the original volume of these figures will be multiplied by 8.

cone A 3-D figure that tapers from a circular base to a point.

cylinder A 3-D figure that has two parallel circular bases connected by a curved surface.

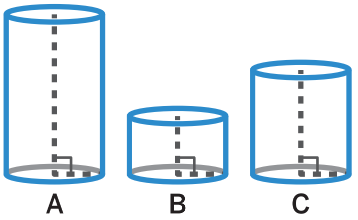
sphere A 3-D figure in which all cross-sections in every direction are circles.

Practice
5.10

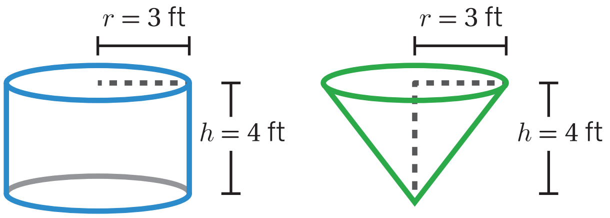
Name: ______________________ Date: __________ Period: __________

1. Cylinders A, B, and C have the same radius but different heights. Order the cylinders from *least* volume to *greatest* volume.

Cylinder B, Cylinder C, Cylinder A



2. Here is a cylinder and a cone with the same base and height. How much more water would you need to fill the cylinder than the cone?



[✓ **A.**] 3 times as much

B. 2 times as much

C. 5 times as much

D. 4 times as much

Spiral Review

3. The area of a circle is approximately 201.06 square inches. What is its radius in inches?

8 inches

4. Match each circle with its area.

- Circle A has a radius of 4 units.
- Circle B has a radius of 10 units.
- Circle C has a radius of 8 units.

Area of the Circle	Circle
About 314 square units	**B**
64π square units	**C**
16π square units	**A**

5. Here are two expressions that represent the volume of liquid in two different containers after t seconds:

- $1250 - 25t$ represents the volume of liquid in Container A.
- $50t + 250$ represents the volume of liquid in Container B.

What does the equation $1250 - 25t = 50t + 250$ represent in this situation?

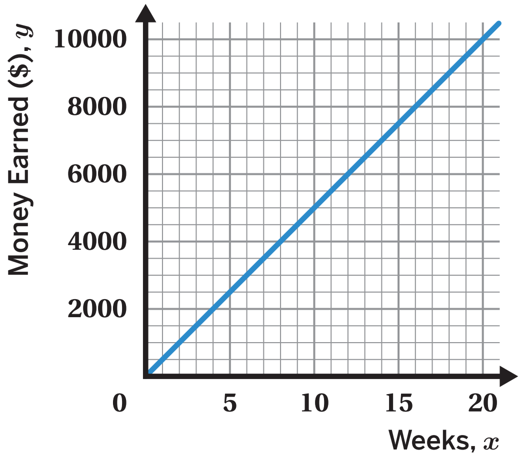
***Responses vary*. The equation says that the volume in one container is equal to the volume in the other container. This equation can be solved for t to determine the time at which both containers have the same volume of liquid inside them.**

Practice
5.10

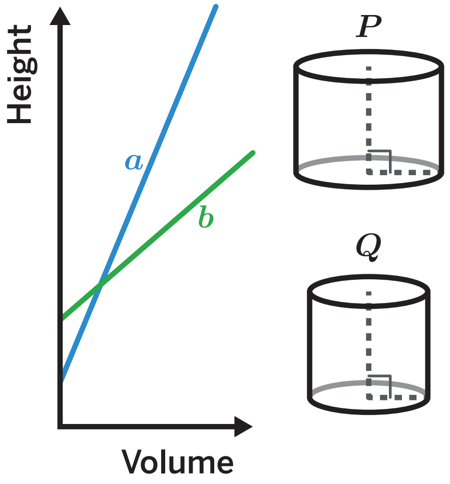
Name: ______ Date: ______ Period: ______

6. Mai earns \$1,710 every 3 weeks by working as a freelance photographer. Jayla is also a freelance photographer whose earnings are represented by the graph. Who earns more per week, and how much more? Show or explain your thinking.

Mai earns \$70 more per week. Explanations vary. Mai earns $\frac{1710}{3} = 570$, or \$570 per week. Jayla earns $\frac{1500}{3} = 500$, or \$500 per week.



Problems 7–9: Cylinders P and Q have the same height. Each starts off filled with different amounts of water. The graph shows the height of the water in each cylinder as the volume of water increases.



7. Match lines a and b to cylinders P and Q.

Cylinder	Line
P	**b**
Q	**a**

8. Describe what the slopes of lines a and b represent in this situation.

***Responses vary.* The slopes of lines a and b represent the rate the height of the water is increasing as the volume of the water increases in each cylinder.**

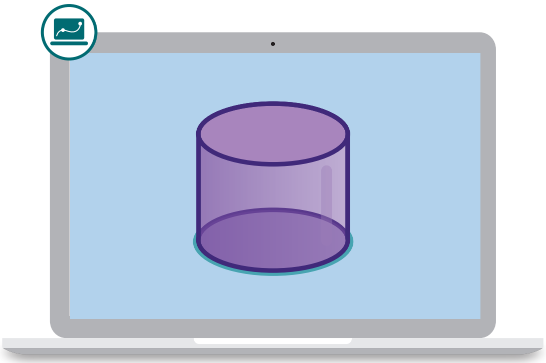
9. Which line has a greater rate of change? Explain your thinking.

***Responses vary.* Line a is steeper than line b. The rate at which the height of the water and volume are increasing for line a is greater than line b.**

Unit 5
Lesson
11

Name: ____________ Date: ____________ Period: ____________

Cylindrical Investigations | Shape, Number, and Expressions | 8.G.9, SMP.6, SMP.7, SMP.8



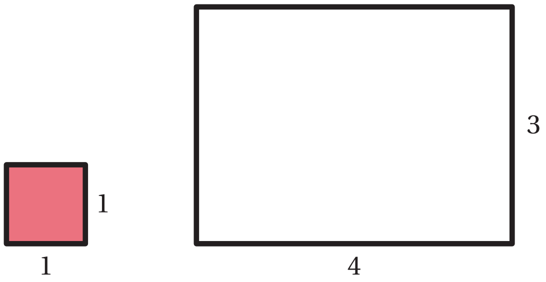
Cylinders

Let's calculate the volume of cylinders.

Warm-up

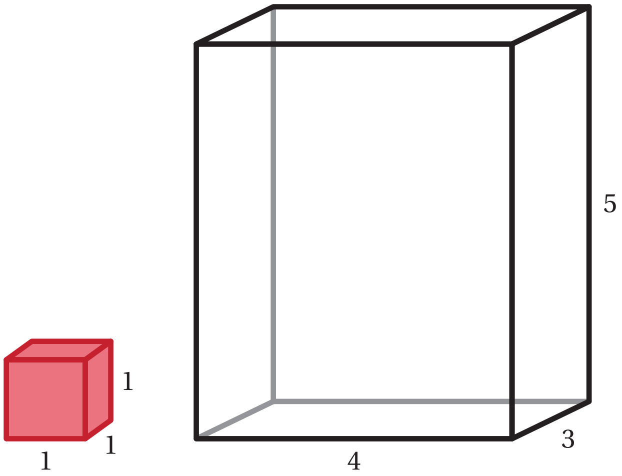
1 How many unit squares are needed to cover this rectangle?

12 unit squares



2 How many unit cubes are needed to fill this rectangular prism?

60 unit cubes



Name: .... Date: .... Period: ....

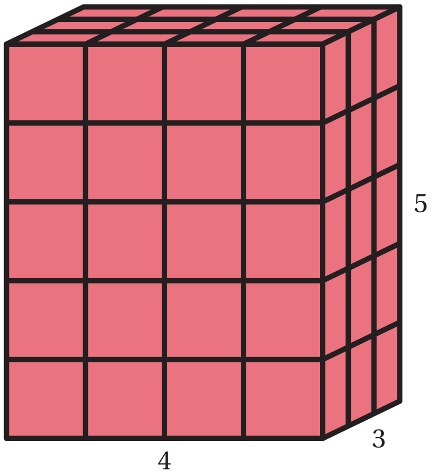
Volume Strategies

3 DeAndre calculated the volume of this rectangular prism using the equation

$V = 12 \bullet 5$

Explain what 12 and 5 represent in the diagram.

Responses vary. **12 represents the area of the bottom face (3 by 4). 5 represents the height.**

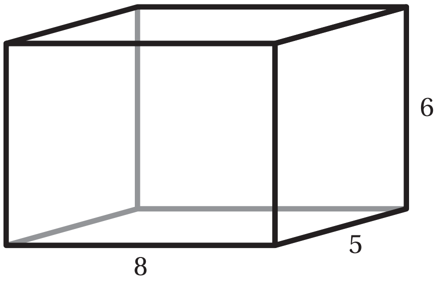


4 Here is a new rectangular prism.

Write an expression for its volume.

Responses vary.

- **$40 \bullet 6$ cubic units**
- **$30 \bullet 8$ cubic units**
- **$48 \bullet 5$ cubic units**

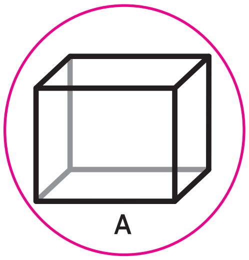


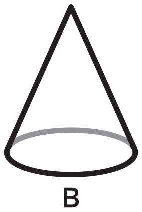
Name: .... Date: .... Period: ....

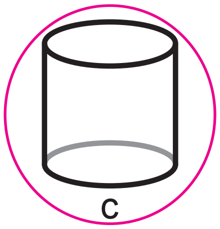
Volume Strategies (continued)

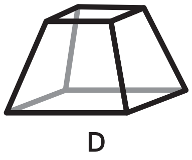
5 DeAndre described his strategy for calculating the volume of a solid.

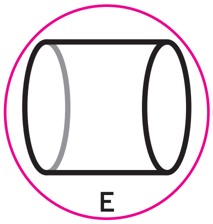
- First, find the area of the base.
- Then multiply that area by the height of the object.

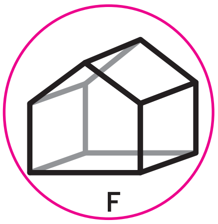












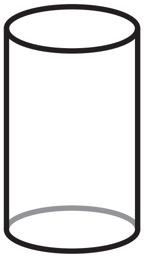
a Circle *all* of the objects for which DeAndre's strategy will work.

b **Discuss:** How did you decide which objects to choose?

***Responses vary.* I chose all the objects that had two congruent bases connected by perpendicular sides.**

6 DeAndre's strategy for calculating volume works for cylinders.

What information would you need to calculate the volume of this cylinder?



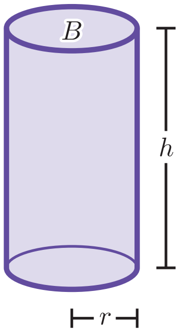
Responses vary.

- **I would need to know the area of the base and the height.**
- **I would need the length of the diameter or radius, and also the height.**
- **I would need to know how to calculate the area of a circle, $A = \pi r^2$, to determine the area of the base.**

7 Here are two formulas for the volume of a cylinder:

$V = \pi r^2 \bullet h$

$V = B \bullet h$



Describe how the formulas are related.

***Responses vary.* In the first formula, πr^2 represents the area of the base of the cylinder, while h represents the height, so the formulas are essentially the same. They both say "find the area of the base, then multiply by the height." The first formula can only be used for cylinders. The second formula can be used for both cylinders and prisms.**

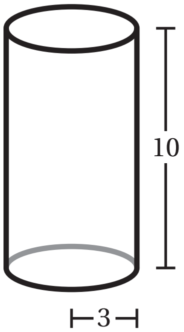
Activity

2

Name: .... Date: .... Period: ....

Cylinder Volumes

8 This cylinder has a height of 10 units and a *radius* of 3 units.



Calculate the volume of the cylinder.

90π cubic units (or equivalent)

Explain your thinking.

To determine the volume I can calculate the area of the base and multiply it to the height. The area of the base is $\pi \cdot 3^2$ or 9π square units. The base multiplied by the height is $9\pi \cdot 10$, or 90π cubic units.

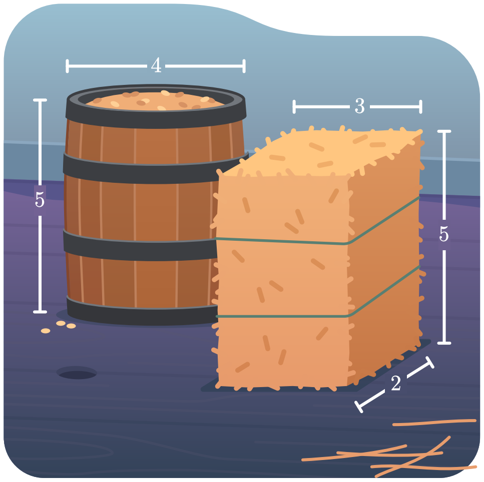
9 Caasi incorrectly determined the volume of the cylinder with this calculation:

$V = 3 \cdot 3 \cdot 10 = 90$

What did Caasi do well and what mistake did she make?

***Responses vary.* Caasi used the dimensions given. She made a mistake in that she didn't multiply by π. She might have been thinking she could find the area of the base by multiplying the radius by itself. This works when the base of a prism is a square, but not for a cylinder, where the base is a circle.**

10 At a farm, animals are fed bales of hay and buckets of grain. Each bale of hay is in the shape of a rectangular prism. The base has side lengths of 2 ft and 3 ft, and the height is 5 ft. Each bucket of grain is a cylinder with a diameter of 4 ft. The height of the bucket is 5 ft, which is the same height as the bale.



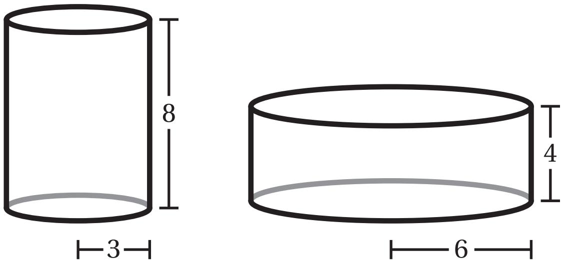
Which has the larger volume, the bale or the bucket? Explain your thinking.

The bucket. *Responses vary.* Both figures have the same height, so to determine the figure with the larger volume, I can determine which figure has a larger base. The cylindrical bucket has a base of $4\pi \approx 12.56$ square feet. The rectangular bale of hay has a base of 6 square feet. Therefore, the bucket has a greater volume.

Name: .... Date: .... Period: ....

Calculating Cylindrical Volume

11 Calculate the volume of each cylinder.



Tall cylinder:

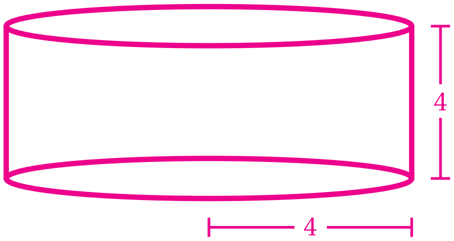
72π cubic units (or equivalent)

Short cylinder:

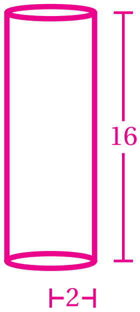
144π cubic units (or equivalent)

12 Sketch two different cylinders that have the same volume. What is the volume of each cylinder? ***Responses vary.***

Cylinder A



Cylinder B



The volume of each cylinder is 64π cubic units.

You're invited to explore more.

13 **a** Write dimensions for a rectangular prism and a cylinder with volumes that are close to equal. ***Responses vary.***

Rectangular prism: **1 by 2 by 3 units**

Cylinder: **$r = 1$ and $h = 2$ units**

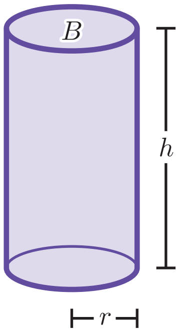
b Show that they have close to equal volumes. ***Responses vary.***

Rectangular prism volume: $1 \cdot 2 \cdot 3 = 6$ cubic units

Cylinder volume: $1^2 \cdot 2 \cdot \pi = 2\pi$ cubic units, which is about equal to 6.28 cubic units.

14 Synthesis

Describe a strategy for determining the volume of a cylinder given its radius and height.



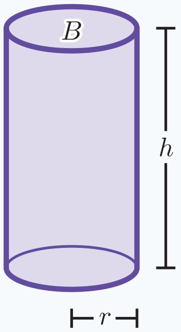
Responses vary.

- **I can multiply the area of the base of the cylinder by the height.**
- **I can determine the area of the base using $A = \pi r^2$ and then multiply that by the height.**

17 Summary 5.11

A prism has two congruent bases connected by perpendicular lines. Its volume can be determined by multiplying the area of its base by its height. A cylinder has two congruent circles for its base and the sides are perpendicular to the bases. This means you can also determine the volume of a cylinder by using the area of its base multiplied by its height.

If you know the radius and height of a cylinder, then you can determine the volume of the cylinder. The base area is determined using the expression $\pi \cdot r^2$. The volume, in cubic units, can be determined by multiplying the base area by the height, h. The formula for the volume of a cylinder is $V = \pi r^2 \cdot h$.

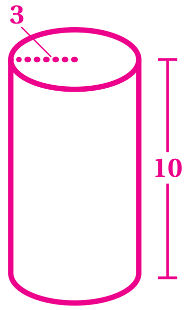


$V = \pi r^2 \cdot h$

Practice
5.11

Name: .... Date: .... Period: ....

Problems 1–3: Draw a cylinder.



1. Label the radius 3 units and the height 10 units.

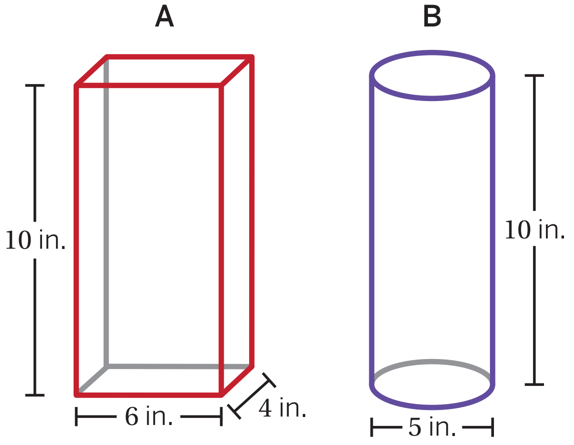
2. Determine the area of the base. Write your response in terms of π.

9π square units

3. Determine the volume of the cylinder. Write your response in terms of π.

90π cubic units

Problems 4–6: Containers A and B hold oatmeal. Container A is a rectangular prism and Container B is a cylinder.



4. The diameter of Container B is 5 inches. What is the radius of the container?

2.5 inches

5. Which container's base has a larger area? Explain your thinking.

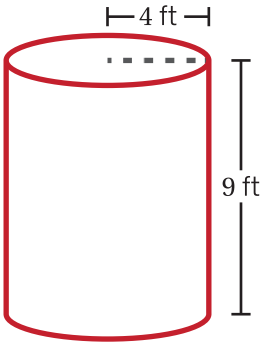
Container A. *Explanations vary.* The area of Container A's base is 24 square inches. The area of Container B's base is about 19.6 square inches.

6. Which has a larger volume: Container A or B? Explain your thinking.

Container A. *Explanations vary.* The two containers have the same height, but Container A's base area is larger, so its volume is larger.

7. Here is a cylinder with a radius of 4 feet and a height of 9 feet. What is the volume of the cylinder in cubic feet? Round your answer to the nearest hundredth.

Approximately 452.39 cubic feet

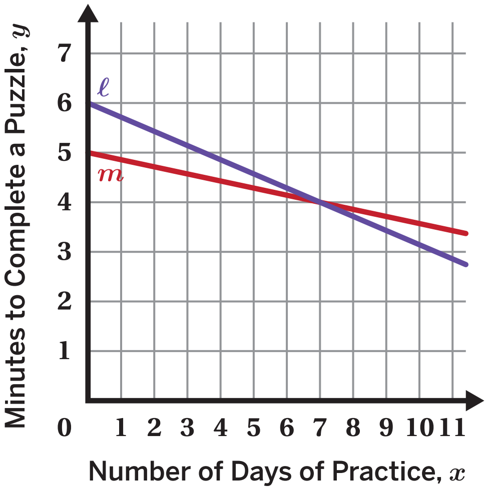


Practice
5.11

Name: Date: Period:

Spiral Review

Problems 8–9: Two students join a puzzle-solving club, and they each improve their completion time as they practice. Student A improves their completion time at a faster rate than Student B.



8. Match each student with the line that represents their time.

- **line ℓ represents Student A.**
- **line m represents Student B.**

9. Which student completed puzzles faster before practicing? Explain your thinking.

Student B. *Explanations vary*. The y-intercept of the line that represents Student B is less than the y-intercept of the line that represents Student A.

10. This table shows the radius, diameter, circumference, and area for two different circles. Complete the table.

Radius	Diameter	Circumference	Area
3 cm	**6 cm**	**6π cm**	**9π cm^2**
8 in	**16 in**	16π in	**64π in^2**

Unit 5

Lesson

12

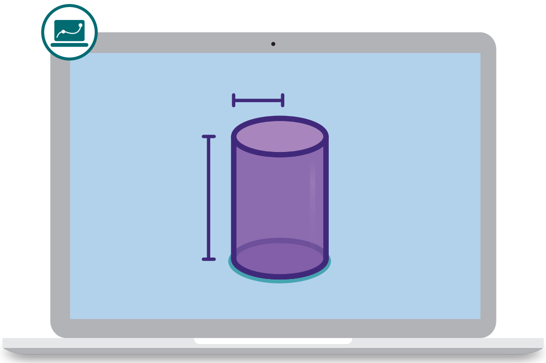
Name: .... Date: .... Period: ....

Multiple Representations of Functions | Cylindrical Investigations | Shape, Number, and Expressions

8.F.3, 8.G.9 SMP.1, SMP.2

Scaling Cylinders

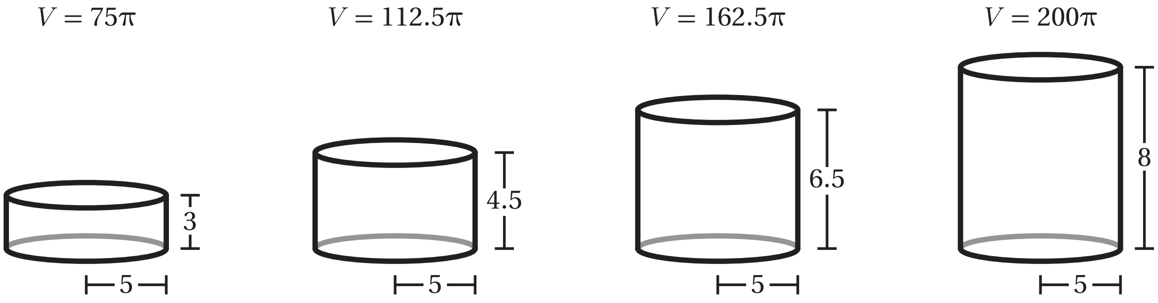
Let's see how changing a cylinder's radius or height impacts its volume.



Warm-up

1 We learned that in a function, the independent variable represents the input and the dependent variable represents the output.

Here are several cylinders:



In this situation, what could the independent and dependent variables be? ***Responses vary.***

Independent variable: **Height of the cylinder**

Dependent variable: **Volume of the cylinder**

Activity

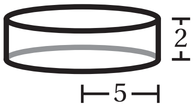
1

Name: .... Date: .... Period: ....

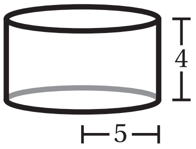
Changing the Height

2 Select the cylinder that represents the plotted point. Explain to a classmate how you chose. ***Explanations vary.* The point is located at $(4, 100\pi)$. Since the x-axis represents height, the height of the cylinder has to be 4 cm.**

A



[✓ B]



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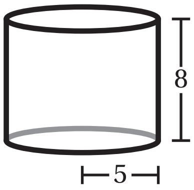

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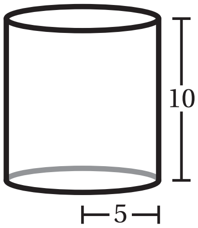
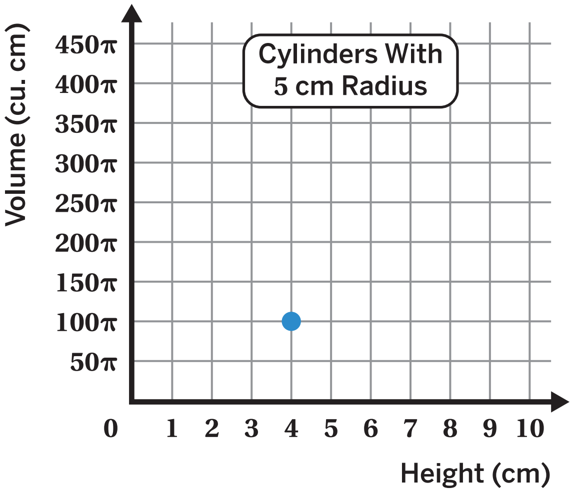




3 Let's watch an animation graphing the relationship between a cylinder's height and its volume.

Discuss: What do you notice? What do you wonder? ***Responses vary.***

- **I notice that as the height of the cylinder increases, the volume increases linearly.**
- **I notice that for every 2 centimeters of height, the cylinder adds 50π cubic centimeters of volume.**
- **I wonder if all cylinders have this linear pattern or just some.**

4 Let's look at a graph that represents the relationship between the height and the volume for cylinders with a radius of 5 centimeters.

Use the graph and the table to help you find the volume of each of the four cylinders.

Express each volume in terms of π.

Object	Height	Volume (cu. cm)
Cylinder A	2	**50π**
Cylinder B	4	100π
Cylinder C	8	**200π**
Cylinder D	16	**400π**

Activity
2

Name: Date: Period:

Changing the Radius

5 Let's see what happens if we keep the height of a cylinder constant but change the radius.

	Radius (cm)	Volume (cu. cm)
Cylinder 1	5	250π
Cylinder 2	7	490π

a Choose and record a radius for two different cylinders that each have a height of 10 centimeters.

Responses vary. Sample shown in table.

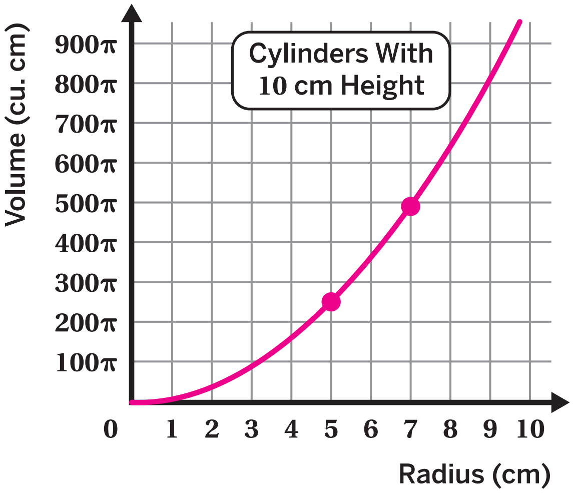
b Calculate the volume for each cylinder. Express each volume in terms of π.

Responses vary. Sample shown in table.

6 **a** Plot points to represent your two cylinders. *Responses vary.*

b Make a sketch of what you think the graph looks like for *all* cylinders with a height of 10 centimeters.

Responses vary.



7 Let's watch an animation graphing the relationship between a cylinder's radius and its volume.

💬 **Discuss:** Is this relationship a linear function? Explain your thinking.

Responses vary. The relationship between the radius and volume of the cylinder is a function but it is not linear. It is not linear because the graph is curved and not a straight line.

Name: ______ Date: ______ Period: ______

Changing the Radius (continued)

8 Let’s look at a graph that represents the relationship between the radius and the volume for cylinders with a height of 10 centimeters.

Use the graph and the table to help you find the volume of each of the four cylinders.

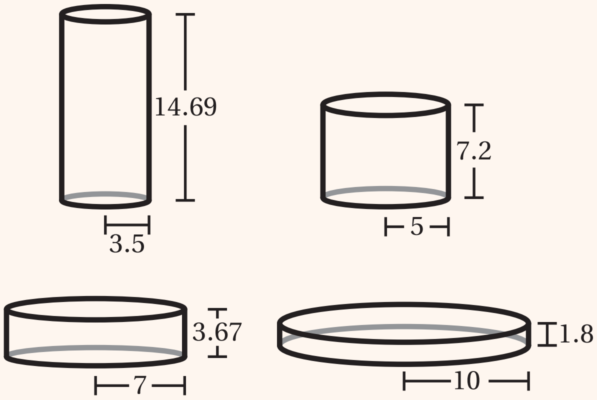
Express each volume in terms of π.

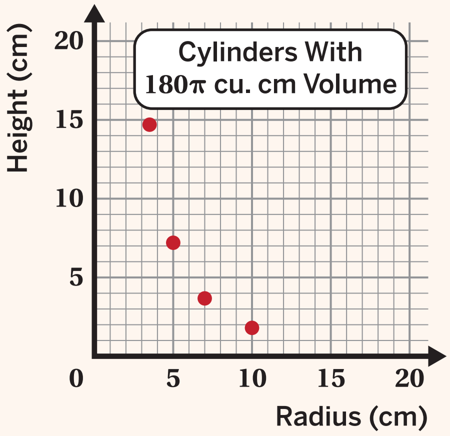
Object	Radius (cm)	Volume (cu. cm)
Cylinder E	2	40π
Cylinder F	4	160π
Cylinder G	8	640π
Cylinder H	16	2560π

You’re invited to explore more.

9 Explore the relationship between radius and height when the volume of a cylinder is fixed. Here are several cylinders that all have a volume of 180π.

Discuss: What do you notice? What do you wonder?



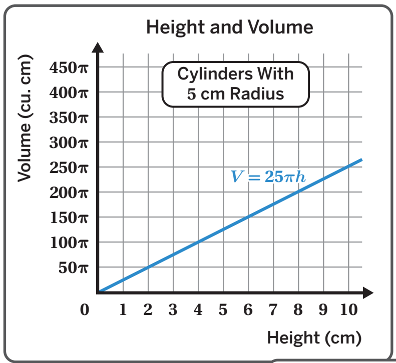


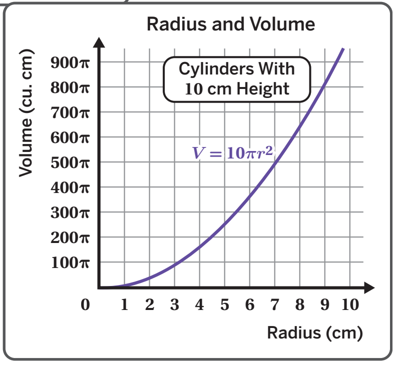
Responses vary.

- I notice that the relationship between the radius and height of a cylinder isn’t linear. I wonder why that is.
- I wonder if this graph will ever intersect the axes, or if not, how close it gets.
- I wonder if there is an equation that could create a graph of this relationship. I wonder how the graph would change if the fixed volume increased or decreased.

10 Synthesis

Here are the relationships we explored today.





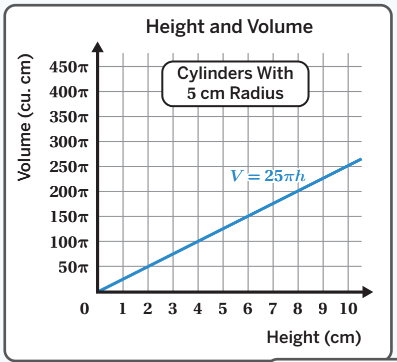
Discuss: How can you tell if the relationship between height and volume and the relationship between radius and volume are linear or non-linear?

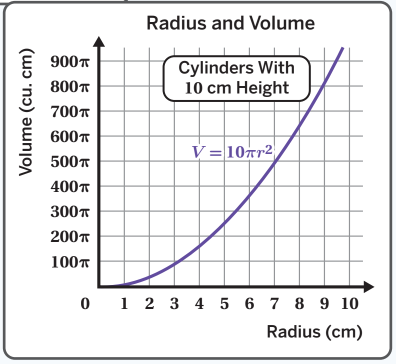
***Responses vary.* You can tell if a relationship is linear if there is a constant rate of change. Also the graph of a linear relationship is a straight line. So the relationship between height and volume is linear, but the relationship between radius and volume is not.**

13 Summary 5.12

The volume of a cylinder depends on the cylinder's radius and height. The formula for the volume of a cylinder is $V = \pi r^2h$, where r represents the radius and h represents the height.

When a cylinder's height, h, increases at a constant rate, the cylinder's volume, V, also increases at a constant rate. This means there is a proportional linear relationship between the height and volume. That's why we can represent the relationship between volume and height with a straight line.





On the other hand, we *cannot* represent the relationship between a cylinder's radius and volume with a line because the ratio of the volume to the radius changes as the radius increases. That's why the graph of the relationship between radius and volume is curved and non-linear.

Practice
5.12

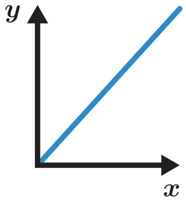
Name: ________________ Date: ________ Period: ________

1. Each row of this table lists information about a specific cylinder. Complete the table.

Diameter (units)	Area of Base (sq. units)	Height (units)	Volume (cu. units)
4	**4π**	10	**40π**
6	**9π**	**7**	63π
10	25π	6	**150π**

2. Which graph could represent the volume of water in a cylinder as a function of its height if the radius is held constant? Explain your thinking.

[✓ A.]



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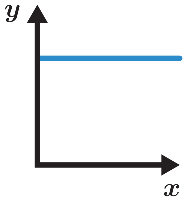
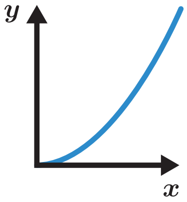
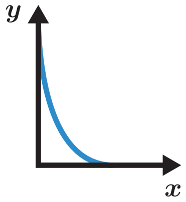


C.



D.



***Explanations vary*. As the height of water in a cylinder increases, the volume increases by a constant rate.**

Problems 3–6: Imagine several cylinders that all have a height of 18 meters. Let r represent the radiuses of the cylinders, in meters, and V represent the volume of the cylinders, in cubic meters.

3. Write an equation that represents the relationship between the volume, V, and the radius, r, for all cylinders with a height of 18 meters.

$V = \pi r^2 \cdot 18$

4. Complete this table:

r (m)	1	2	3
V (cu. m)	**18π**	**72π**	**162π**

5. If the radius of a cylinder is doubled, does the volume double? Explain your thinking.

No. *Explanations vary*.
- **From my table, I notice that 72π is not equal to $2 \cdot 18\pi$.**
- **If the radius is doubled, the volume will be multiplied by 4 because $(2r)^2 = 4r^2$.**

6. Is the graph representing the relationship between a cylinder's volume and its radius linear? Explain your thinking.

No. *Explanations vary*.
- **It's non-linear because the radius is squared.**
- **The values in my table aren't increasing by a constant amount.**

Practice
5.12

Name: ________________ Date: ________ Period: ________

7. A cylinder has a volume of 48π cubic centimeters and a height represented by h.

   Complete this table with the volumes of other cylinders that have the same radius but different heights.

Height (cm)	Volume (cu. cm)
h	**48π**
$2h$	**96π**
$5h$	**240π**
$\frac{h}{2}$	**24π**
$\frac{h}{5}$	**$\frac{48\pi}{5}$**

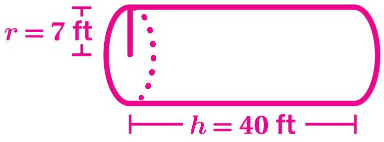
8. Which change do you think would increase the volume of a cylinder the most — doubling the radius or doubling the height? Explain your thinking.

   Doubling the radius. *Explanations vary*. When the radius is doubled, the volume is multiplied by 4 because the radius gets squared. When the height is doubled, the volume is multiplied by just 2.

Spiral Review

Problems 9–10: A gas company's delivery truck has a cylindrical tank with a diameter of 14 feet and a height of 40 feet.

9. Draw the tank, then label its radius and height.



10. How much gas can fit in the tank? Show or explain your thinking.

    $1960\,\pi \approx 6158$ cubic feet of gas. *Explanations vary.*

    $V = \pi r^2 h$

    If $r = \pi$ and $h = 40$, then

    $V = \pi \cdot 7^2 \cdot 40$

    $V = 1960 \cdot \pi$

    $V \approx 6157.52$

11. A cylinder has a volume of 63π *in*^{3} and a radius of 3 *in*. What is its height? Explain your thinking.

    7 inches

    Explanations vary.

    $V = \pi r^2 h$

    If $r = 3$ and $V = 63\pi$, then

    $63\pi = \pi \cdot 3^2 \cdot h$

    $63\pi = \pi \cdot 9 \cdot h$

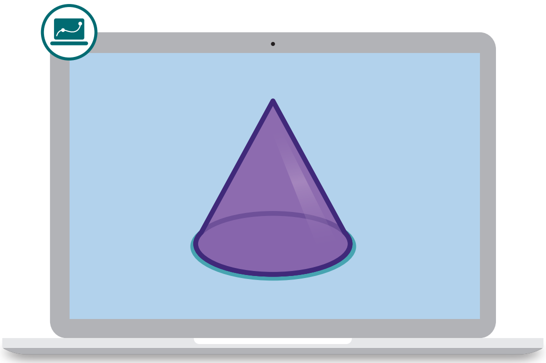
    $63\pi \div 9\pi = 9\pi \cdot h \div 9\pi$

    $7 = h$

Unit 5
Lesson
13

Name: .... Date: .... Period: ....

Cylindrical Investigations | Shape, Number, and Expressions | 8.G.9, SMP.1, SMP.8



Cones

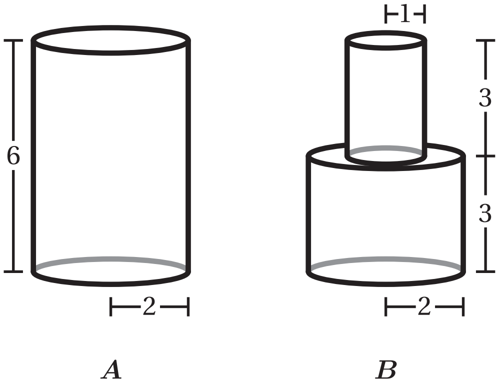
Let's explore cones and their volumes.

Warm-up

1 Determine the volume of figure A (a cylinder) and figure B (which is composed of two cylinders).

Write their volumes in terms of π.

Figure	Volume (cu. cm)
A	24π
B	15π

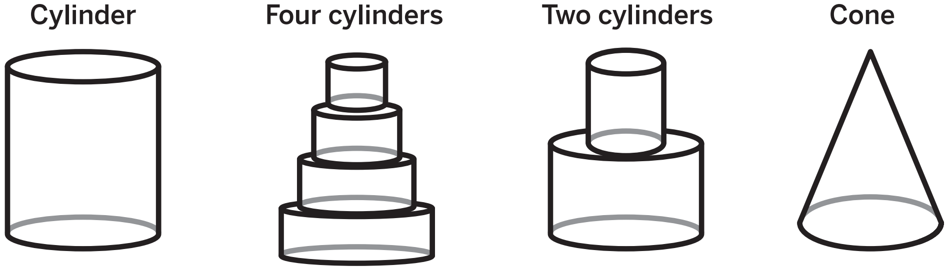


Activity
1

Name: ______________________ Date: __________ Period: __________

Estimating the Volume of a Cone

2 Here are four figures with the same height and the same radius for their largest bases.

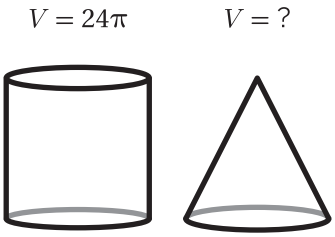


Order the figures by volume from *least* to *greatest*.

Cone	Four cylinders	Two cylinders	Cylinder

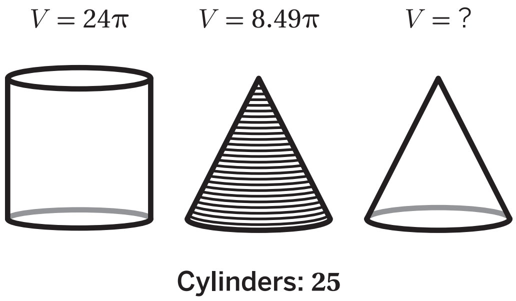
Least **Greatest**

3 This cylinder and cone have the same base and height. How does the volume of the cone compare to the volume of the cylinder? Explain your thinking.



***Responses vary*. Because the cone has sides that meet at a point at the top, the volume has to be less than the cylinder, which has vertical sides.**

4 Here is a stack of cylinders with the same base and height. Let's see what happens when we increase the number of cylinders.



a **Discuss:** What do you notice about the relationship between the volume of the cone and the volume of the stack of cylinders?

***Responses vary*. The more cylinders there are in the stack, the more it begins to look like the cone. I think that means the volume of the stack gets closer and closer to the volume of the cone, too.**

b What is the exact volume of the cone? Explain your thinking.

8π cubic units. *Responses vary*. When the stacked cylinders begin to look like the cone, their volume is about $\frac{1}{3}$ the volume of the original cylinder. To get the exact volume of the cone, I think I can divide the cylinder volume by 3.

Name: .... Date: .... Period: ....

Estimating the Volume of a Cone (continued)

5 Each row of the table shows the volumes of a cylinder and a cone with the same height and radius.

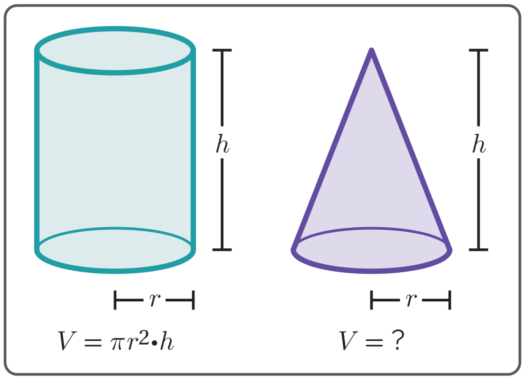
Fill in the unknown values.

Volume of Cylinder (cu. cm)	Volume of Cone (cu. cm)
24π	8π
30π	10π
120π	**40π**
60π	**20π**
45π	15π

6 One way of writing a formula for the volume of a cylinder with radius r and height h is $V = \pi r^2 \cdot h$.

Write a formula for the volume of a cone.

$V = \frac{1}{3}\pi r^2 h$ (or equivalent)



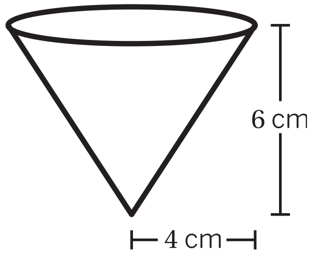
Name: ________ Date: ________ Period: ________

Comparing the Volume of a Cone

7 Let’s look at one way of writing a formula for the volume of a cone with radius r and height h.

Calculate the volume of this cone-shaped frozen yogurt cup that has a radius of 4 cm and a height of 6 cm.

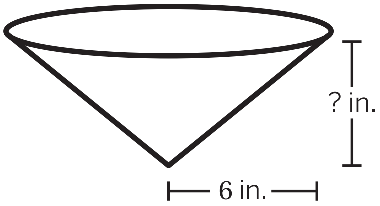
32π cubic inches (or equivalent)



8 The volume of this cone is 60π cubic inches.

What is the height of the cone?

5 inches



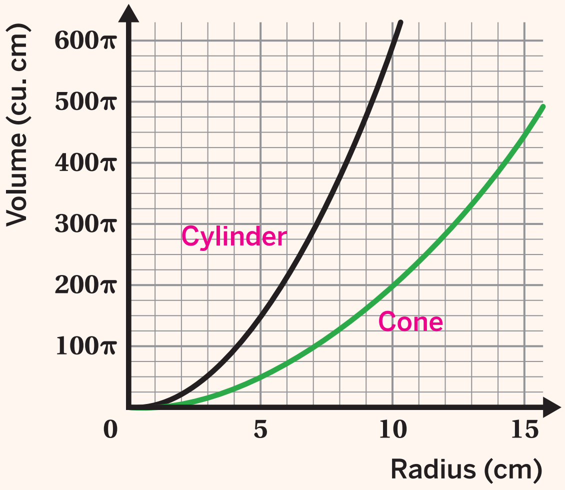
You’re invited to explore more.

9 This graph shows the relationship between radius and volume for cones and cylinders that have the same height.

a Label each curve to show which solid it represents.

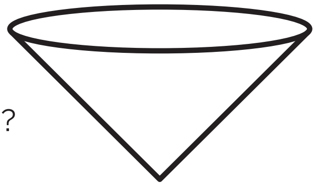
b What is the height of these cylinders and cones? Explain your thinking.

6 centimeters. *Explanations vary.* I noticed that a cylinder with a radius of 10 centimeters has a volume of 600π cubic centimeters. I plugged those values into the cylinder volume formula and solved for h, which gave me 6 centimeters.



10 Synthesis

One way of writing the formula for volume of a cone is $V = \frac{1}{3}\pi r^2 \cdot h$.



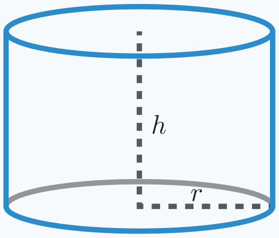
💬 **Discuss:** What does each part of the formula represent?

Responses vary. The πr^2 represents the area of the circle. This quantity is multiplied by h to get the volume of a cylinder with the same dimensions. That quantity is then multiplied by $\frac{1}{3}$ because the volume of a cone is $\frac{1}{3}$ of the volume of a cylinder that has the same radius and height.

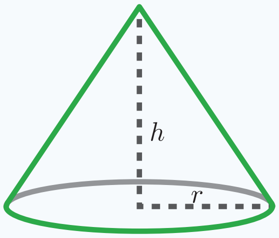
13 Summary 5.13

We learned that we can find the volume of a cylinder by calculating $V = \pi r^2 \cdot h$. If a cone and a cylinder have the same base and the same height, then the volume of the cone is one-third the volume of the cylinder.

If the radius and the height are known, we can determine the volume by using this formula for a cone: $V = \frac{1}{3}\pi r^2 \cdot h$.



Volume of a cylinder:
$V = \pi r^2 h$



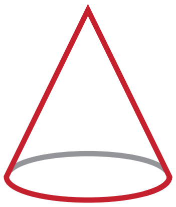
Volume of a cone:
$V = \frac{1}{3}\pi r^2 h$

Practice
5.13

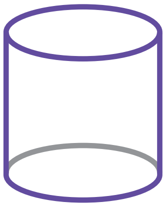
Name: .... Date: .... Period: ....

1. The volume of a cone is 36π cubic units. What is the volume of a cylinder with the same radius and the same height?

108π cubic units (or equivalent)



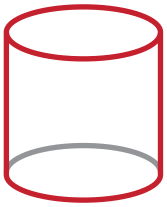
$V = 36\pi$



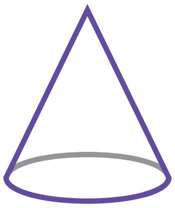
$V = ?$

2. The volume of a cylinder is 175π cubic units. What is the volume of a cone with the same radius and the same height?

$\frac{175}{3}\pi$ cubic units (or equivalent)



$V = 175\pi$



$V = ?$

3. A cylinder and a cone have the same height and radius. The height of each is 5 centimeters, and the radius is 2 centimeters. Calculate the volume of the cylinder and the cone (rounded to the nearest tenth). Use 3.14 as an approximation for π.

Cylinder: **62.8 cubic centimeters**

Cone: **20.9 cubic centimeters**

Problems 4–6: This table shows the radiuses of four cones with a height of 18 meters.

4. Complete the table with the volume of each cone.

Radius (m)	Volume (cu. m)
1	**6π**
2	**24π**
3	**54π**
4	**96π**

5. Based on your table, if the radius of a cone doubles, does the volume also double? Explain your thinking.

No. *Explanations vary.* The volume doesn't double. It is multiplied by four.

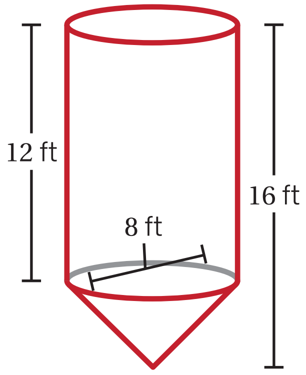
6. Based on your table, is the relationship between the radius of a cone and its volume linear? Explain your thinking.

No. *Explanations vary.* It's non-linear because the four points in the table do not lie on a straight line.

Practice
5.13

Name: .... Date: .... Period: ....

7. A silo is a large cylindrical container used on farms to hold grain. On Estaban's farm, a silo has a cone-shaped spout on the bottom to regulate the flow of grain going out. The diameter of the silo is 8 feet. The cylindrical part of the silo has a height of 12 feet, and the height of the entire silo is 16 feet.

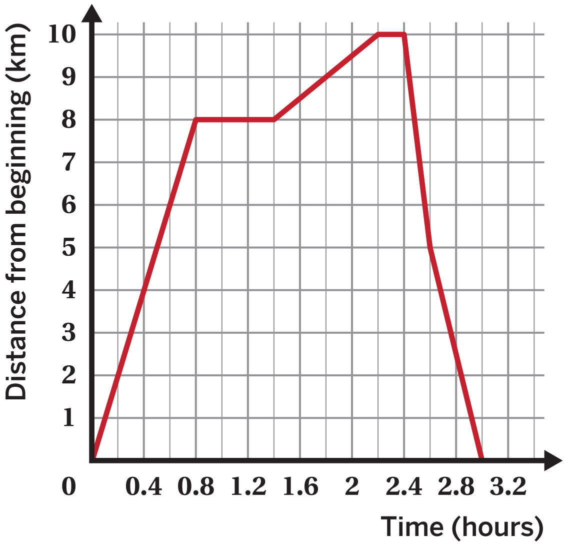


Approximately how many cubic feet of grain can the entire silo hold? Explain your thinking.

$213\pi \approx 669.87$ cubic feet. *Explanations vary*. The radius of the cone is 4 feet and the height is also 4 feet because $16 - 12 = 4$. The volume of the cone spout is $V = \frac{1}{3}\pi \cdot 4^2 \cdot 4 \approx 66.99$. The volume of the cylinder is $V = \pi \cdot 4^2 \cdot 12 \approx 602.88$. Adding these volumes together gives a total volume of approximately 669.87 cubic feet.

Spiral Review

Problems 8–11: This graph shows a trip on a bike trail.



8. When was the bike rider going the fastest?

Between 2.4 and 2.6 hours

9. During what times did the rider stop?

Between 0.8 and 1.4 hours and between 2.2 and 2.4 hours

10. During what times was the rider going back toward the beginning of the trail?

Between 2.4 and 3 hours

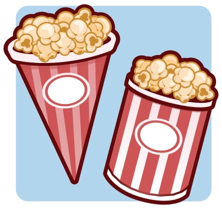
11. Approximately how many hours was the bike rider at least 8 miles away from the beginning of the trail?

***Responses vary*. Any value between 1.64 and 1.76 is considered correct.**

Name: Date: Period:

Cylindrical Investigations Shape, Number, and Expressions 8.G.9, SMP.3, SMP.7, SMP.8

Unknown Dimensions

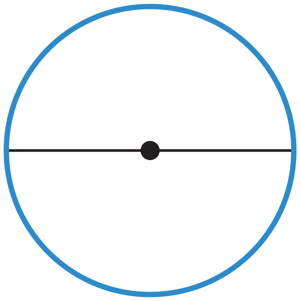


Let's determine unknown dimensions.

Warm-up ELD.PI.8.1.Em, Ex, Br

1. Here is a circle with an area of 81π square inches.

What is its diameter? Explain your thinking.



18 inches. *Explanations vary.* I determined the radius of the circle by calculating the square root of 81, which is 9. The diameter is 2 times the radius, so it's 18.

Activity 1

Name: .... Date: .... Period: ....

Cylinder Dimensions

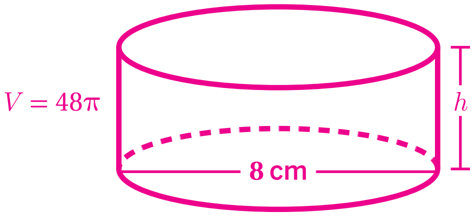
Each row of this table lists the dimensions of a different cylinder.

2. Complete the table.

Diameter (units)	Radius (units)	Base Area (sq. units)	Height (units)	Cylinder Volume (cu. units)
18	**9**	81π	2	**162π**
12	**6**	**36π**	**3**	108π
6	**3**	**9π**	11	99π
20	**10**	100π	**$\frac{1}{5}$**	20π
$2r$	**r**	**πr^2**	h	$\pi \cdot r^2 \cdot h$

3. A cylinder has a diameter of 8 centimeters and a volume of 48π cubic centimeters.

a Draw the cylinder.



b Determine its height. Show or explain your thinking.

3 centimeters. *Responses vary.* To get the height, I divided the volume (48π) by the area of the base (16π).

Activity
2

Name: ________ Date: ________ Period: ________

Cone Dimensions

Each row of this table lists the dimensions of a different cone.

4. Complete the table.

Diameter (units)	Radius (units)	Base Area (sq. units)	Height (units)	Cylinder Volume (cu. units)	Cone Volume (cu. units)
8	4	**16π**	3	**48π**	**16π**
12	**6**	36π	$\frac{1}{4}$	**9π**	**3π**
20	**10**	**100π**	**6**	**600π**	200π
8	**4**	**16π**	12	**192π**	64π

5. 💬 **Discuss:** How are determining the unknown dimensions of a cone and cylinder alike and different? **ELD.PI.8.1.Em, Ex, Br**

Alike:

- When we know two of the dimensions, we can use them to determine the quantities we don't know.
- We can use a formula we know (such as $V = \pi r^2 h$ for a cylinder) to identify what we know and don't know.
- We often need to find the base area in both a cylinder and a cone to identify unknown dimensions.

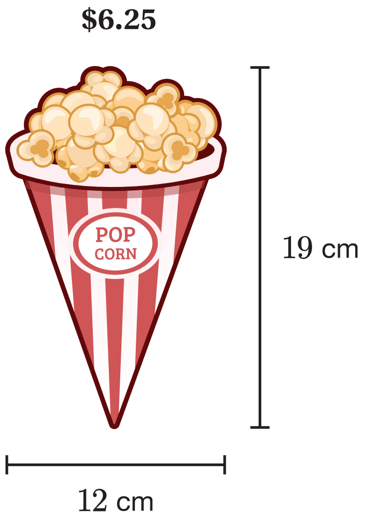
Different:

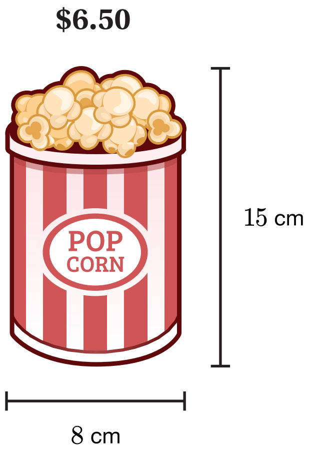
- We use different formulas to relate the dimensions for cylinders and cones.
- The volume of a cone is the volume of a cylinder with the same base divided by 3.

Name: ______________________ Date: __________ Period: __________

Which Is the Better Deal?

6. A movie theater offers two containers of popcorn for different prices:





Which container is the better deal? Show or explain your thinking.

The cylinder is a better deal. *Explanations vary.*

Cone:	Cylinder:
$V = \frac{1}{3}\pi r^2 h$	$V = \pi r^2 h$
The radius of the base is 6 cm and the height is 19 cm.	The radius of the base is 4 cm and the height is 15 cm.
$V = \frac{1}{3}\pi \cdot (6)^2 \cdot 19$	$V = \pi \cdot (4)^2 \cdot 15$
$V \approx 716.28$ cubic centimeters.	$V \approx 753.98$ cubic centimeters.
If you divide the volume by the cost of \$6.25, you get about 114.61. This means you get about 114.61 cubic centimeters of popcorn per dollar.	If you divide the volume by the cost of \$6.50, you get about 116. This means you get about 116 cubic centimeters of popcorn per dollar.

The cylinder is a better deal because you get slightly more popcorn per dollar than the cone.

You're invited to explore more.

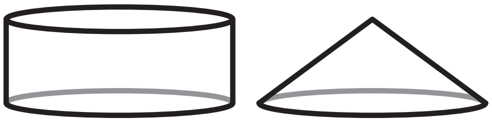
7. Change either the diameter or the height of one of the popcorn containers so that they're an equally good deal. **ELD.PI.8.11.Em, Ex, Br**

Responses vary. To make the cone just as good a deal as the cylinder, I would increase the height of the cone to 19.23 centimeters.

Synthesis

ELD.PI.8.10.Em, Ex, Br

8. Describe a strategy for determining an unknown dimension of a cylinder or cone.



Use the examples if they help with your thinking.

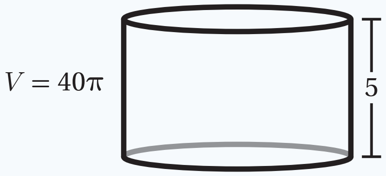
Responses vary. **We can use the dimensions we know to figure out the ones we don't. For example, if we know the area of the base and the volume, then we can determine the height by dividing the volume by the area of the base.**

Summary 5.14

The volume of a cylinder and a cone depend on their radius and height. In both cases, if you know the radius and the height, you can determine the volume using the formula $V = \pi r^2 h$ (for cylinders) and $V = \frac{1}{3}\pi r^2 h$ (for cones).

And if you happen to know the volume and either the radius or the height, you can determine the other dimensions, too.

For example, if a cylinder has a height of 5 inches and a volume of 40π, you can calculate the area of the base by dividing the volume by the height: 8π.



$$40\pi = B \cdot 5$$

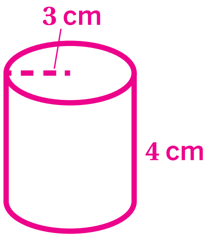
$$8\pi = B$$

Practice
5.14

Name: .... Date: .... Period: ....

1. Draw a cylinder with a diameter of 6 centimeters and a volume of 36π cubic centimeters.

***Responses vary*. Radius and height labeled as part of Problem 2 response.**



2. Determine the radius and the height of the cylinder from Problem 1. Show or explain your thinking. Then label your drawing with the cylinder's radius and height.

The radius is 3 centimeters. The height is 4 centimeters. *Explanations vary*. The radius of the cylinder is 3 centimeters because it's half the measure of the diameter. The height of the cylinder is 4 centimeters because $\frac{36\pi}{\pi(3)^2} = 4$.

3. A cylinder has a diameter of 14 centimeters and a volume of 1,000 cubic centimeters. What is its height? Express your answer to the nearest tenth of a centimeter.

Approximately 6.5 centimeters

4. Complete this table. The cylinder and cone in each row have the same dimensions.

Diameter (units)	Radius (units)	Base Area (sq. units)	Height (units)	Cylinder Volume (cu. units)	Cone Volume (cu. units)
10	5	**25π**	7	**175π**	**$\frac{175}{3}\pi$**
6	**3**	**9π**	**$\frac{40}{3}$**	**120π**	40π
12	**6**	36π	**4**	**144π**	48π

5. An ice cream shop offers two types of ice cream cones that hold the same volume. The waffle cone holds 12 ounces and is 5 inches tall. The sugar cone holds 12 ounces and is 8 inches tall. Which cone has a larger radius?

[✓ Waffle cone]   Sugar cone   They have the same radius

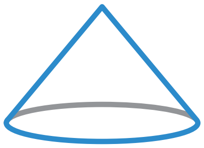
Explain your thinking.

***Explanations vary*. Because the waffle cone's height is smaller, its radius must be larger to have the same volume as the sugar cone.**

Practice
5.14

Name: ______________________ Date: __________ Period: __________

6. The volume of this cone is 33π cubic units. What is the volume of a cylinder that has the same base area and the same height? Explain your thinking.

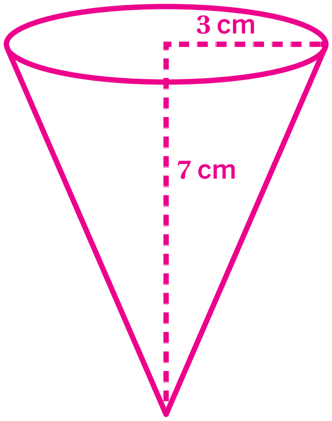


99π cubic units. *Explanations vary.* The volume of a cylinder is three times the volume of a cone with the same height and base area.

Spiral Review

Problems 7–8: A cone-shaped container is used to serve roasted almonds at a hockey game. The container has a diameter of 6 centimeters and a height of 7 centimeters.

7. Draw the cone. Label its height and radius.



8. If the container is filled completely with roasted almonds, how many cubic centimeters will it hold? Show or explain your thinking.

21π or about 65.94 cubic centimeters. *Explanations vary.*

$V = \frac{1}{3}\pi r^2 h$

If $r = 3$ and $h = 7$, then

$V = \frac{1}{3} \cdot \pi \cdot 3^2 \cdot 7$

$V = 21\pi$ **or about 65.94 cubic centimeters**

9. A cone-shaped frozen yogurt cup has a radius of 5 cm and a height of 9 cm. How many cubic centimeters of frozen yogurt can the cup hold? Approximate your answer to the nearest hundredth.

Approximately 235.62 cubic centimeters.
Work varies.

$V = \frac{1}{3}\pi r^2 h$

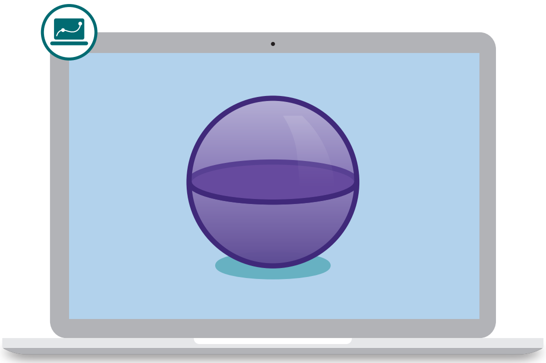
If $r = 5$ and $h = 9$, then

$V = \frac{1}{3}\pi \cdot 5^2 \cdot 9$

$V = 235.62$ **cubic centimeters**

Name: .... Date: .... Period: ....

Cylindrical Investigations Shape, Number, and Expressions 8.G.9, SMP.1, SMP.8



Spheres

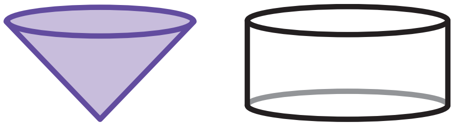
Let's develop a formula for the volume of a sphere.

Warm-up

1 A cone and a cylinder have the same height and radius.

What fraction of the cylinder will be filled by the cone?

$\frac{1}{3}$

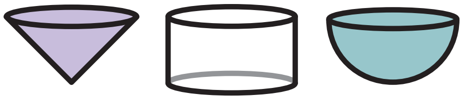


Activity 1

Name: Date: Period:

Hemispheres

2 What if we pour both a cone *and* a hemisphere into the cylinder?



Let's see what fraction of the cylinder will be filled.

Describe how the three volumes are related.

***Responses vary.* The volume of the cylinder is equal to the volumes of the cone and the hemisphere combined. We know that the volume of the cone is $\frac{1}{3}$ of the cylinder. The volume of the hemisphere must be $\frac{2}{3}$ the volume of the cylinder, and double the volume of the cone.**

3 The volume of the cylinder is 27π cubic units.

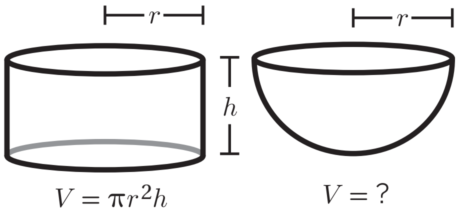
Write the volume of the cone and hemisphere.

Express the volumes in terms of π.

Object	Volume (cu. units)
Cone	**9π**
Cylinder	27π
Hemisphere	**18π**

4 One way of writing a formula for the volume of a cylinder is $V = \pi r^2 h$, where r is the radius and h is the height.

Write a formula for the volume of a hemisphere. Explain your thinking.



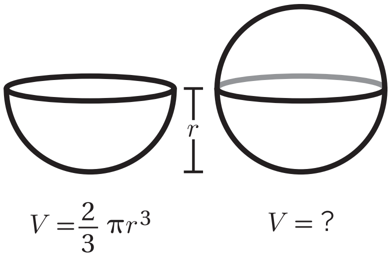
Responses vary.

- **$V = \frac{2}{3}\pi r^3$, because the radius and height of a hemisphere are the same.**
- **$V = \frac{2}{3}\pi r^2 h$, because the hemisphere fills up $\frac{2}{3}$ of the cylinder.**
- **$V = \pi r^2 h - \frac{1}{3}\pi r^2 h$, because the cylinder minus the cone equals the hemisphere.**

Name: .... Date: .... Period: ....

Finding Sphere Dimensions

5 One way of writing a formula for the volume of a hemisphere is $V = \frac{2}{3}\pi r^3$, where r is the radius.



Write a formula for the volume of a sphere. Explain your thinking.

$V = \frac{4}{3}\pi r^3$ (or equivalent)

***Explanations vary.* The sphere has twice the volume of the hemisphere, so I multiplied the hemisphere's volume formula by 2.**

6 Karima and Nasir are calculating the volume of a sphere with a radius of 2 units.

Karima

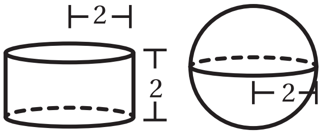
$V = \frac{4}{3}\pi r^3$ $\quad r = 2$

$V = \frac{4}{3}\pi(2)^3$

$V = \frac{4}{3}\pi \cdot 8$

$V = \frac{32}{3}\pi$

Nasir



$V = \pi r^2 h$

$V = \pi(2)^2 \cdot 2$

$V = 8\pi$

$8\pi \cdot \frac{4}{3}$

$V = \frac{32}{3}\pi$

Discuss: How did each student determine the volume of a sphere?

***Responses vary.* Karima used the formula for the volume of a sphere. Nasir used the formula for the volume of a cylinder with similar dimensions and multiplied it by $\frac{4}{3}$ to get the volume of the sphere.**

Name: .... Date: .... Period: ....

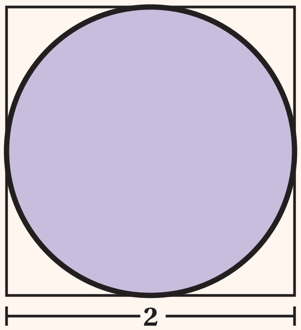
Finding Sphere Dimensions (continued)

7 Complete the table with the unknown dimensions of each sphere. Express your answers in terms of π.

Diameter (units)	Radius (units)	Sphere Volume (cu. units)
18	9	**972π**
8	**4**	**$\frac{256}{3}\pi$**
6	3	**36π**
12	6	**288π**
9	**4.5**	**121.5π**

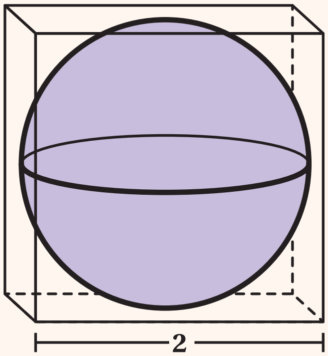
You're invited to explore more.

8 **a** What fraction of the square is filled by the circle?



$\frac{\pi}{4}$ (or equivalent)

b What fraction of the cube is filled by the sphere?



$\frac{\pi}{6}$ (or equivalent)

Activity
3

Name: .... Date: .... Period: ....

Melted Frozen Yogurt

A spherical scoop of frozen yogurt with a 3 inch diameter has melted.

9 How tall must a cone of the same diameter be to hold the melted frozen yogurt? Explain your thinking.

6 inches

Responses vary.

- **The height of the scoop (sphere) is 3 in. If the cone had a height of in., it could only fit half of the scoop because a sphere's volume is double the cone's volume with the same dimensions. The height of the cone will need to be double the height of the sphere.**
- $\frac{4}{3}\pi r^3 = \frac{1}{3}\pi r^2 h$

  If $r = 1.5$, then

  $\frac{4}{3}\pi(1.5)^3 = \frac{1}{3}\pi(1.5)^2 h$

  $4 \cdot 1.5 = h$

  $6 = h$

10 How tall must a cylinder of the same diameter be to completely be filled by the melted frozen yogurt? Explain your thinking.

2 inches

Responses vary.

- **The height of the scoop (sphere) is 3 in. If the cylinder had a height of 3 in., it would only be $\frac{2}{3}$ full because a sphere's volume is $\frac{2}{3}$ the cylinder's volume with the same dimensions. The height of the cylinder will need to be $\frac{2}{3}$ the height of the sphere.**
- $\frac{4}{3}\pi r^3 = \pi r^2 h$

  If $r = 1.5$, then

  $\frac{4}{3}\pi(1.5)^3 = \pi(1.5)^2 h$

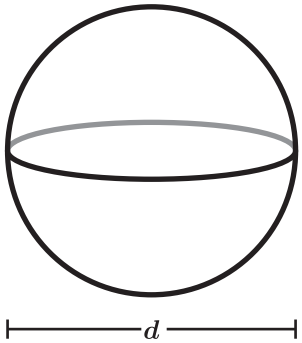
  $\frac{4}{3} \cdot 1.5 = h$

  $2 = h$

11 Synthesis

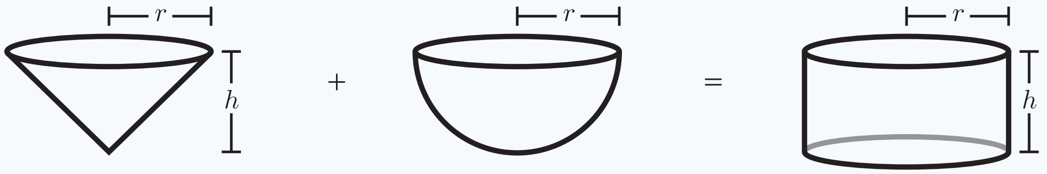
Describe a strategy for determining the volume of a sphere given its diameter.

Responses vary. Divide the diameter by 2 to calculate the radius. Then raise the radius to the third power. Finally, multiply that result by $\frac{4}{3}\pi$.



14 Summary 5.15

You can determine the volume of a sphere using the formula $V = \frac{4}{3}\pi r^3$. If the radius and height of a cone, hemisphere, and cylinder are all the same, you can make sense of the volume formulas for each solid by seeing how much of the others they fill.



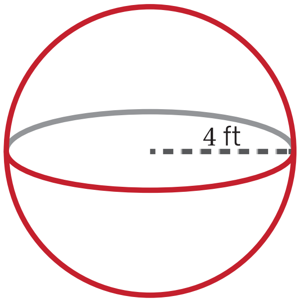
Volume of cone	**Volume of hemisphere**	**Volume of cylinder**
$\left(\frac{1}{3}V\text{ of cylinder}\right)$	$\left(\frac{2}{3}V\text{ of cylinder}\right)$	
$V = \frac{1}{3}\pi \cdot r^2h$	$V = \frac{2}{3}\pi \cdot r^2h$	$V = \pi \cdot r^2h$

The volume of the cone is $\frac{1}{3}$ of the volume of the cylinder. Since the volume of the cone and hemisphere together is equal to the cylinder, the volume of the hemisphere must be $1 - \frac{1}{3} = \frac{2}{3}$ of the volume of the cylinder. Then the volume of a sphere is twice the volume of a hemisphere, or $\frac{4}{3}$ the volume of the cylinder.

Practice
5.15

Name: .... Date: .... Period: ....

1. What is the volume of the sphere with a radius of 4 feet? Write your response in terms of π. Show or explain your thinking.



$\frac{256}{3}\pi$ cubic feet. *Explanations vary.*

$V = \frac{4}{3}\pi r^3$

If $r = 4$, then

$V = \frac{4}{3}\pi \cdot 4^3$

$V = \frac{256}{3}\pi$

2. Calculate the volume of a sphere with a diameter of 6 inches. Write your answer both in terms of π and by using 3.14 to approximate π.

In terms of π: 36π cubic inches

Using 3.14 as an approximation: 113.04 cubic inches

3. Calculate the volume of a cylinder with a height of 6 inches and a diameter of 6 inches. Write your answer both in terms of π and by using 3.14 to approximate π.

In terms of π: 54π cubic inches

Using 3.14 as an approximation: 169.56 cubic inches

4. Calculate the volume of a cone with a height of 6 inches and a diameter of 6 inches. Write your answer both in terms of π and by using 3.14 to approximate π.

In terms of π: 18π cubic inches

Using 3.14 as an approximation: 56.52 cubic inches

5. In the previous three problems, you found the volumes of three solids with the same height and diameter. How are these volumes related?

Responses vary. The volume of the cone plus the volume of the sphere equals the volume of the cylinder.

Practice
5.15

Name: ____________ Date: ________ Period: ________

6. A sphere has a radius of 5.1 centimeters. Determine the approximate volume of the sphere by using 3.14 for the value of π. Show or explain your thinking.

Approximately 555.37 cubic centimeters. *Explanations vary.*

$V = \frac{4}{3}\pi r^3$

$V \approx \frac{4}{3} \cdot 3.14 \cdot 5.1^3$

$V \approx 555.37$

7. A soccer ball has a diameter of 22 centimeters. Which equation can be used to find V, the volume of the soccer ball in cubic centimeters?

A. $V = \frac{4}{3}\pi(22)^2$ **B.** $V = \frac{4}{3}\pi(11)^2$ **C.** $V = \frac{4}{3}\pi(22)^3$ [✓ **D.**] $V = \frac{4}{3}\pi(11)^3$

Spiral Review

Problems 8–13: Evaluate each expression.

8. $\left(\frac{1}{2}\right)^2 \cdot 8 \quad = 2$

9. $3^2 + 2^3 \quad = 17$

10. $(2 \cdot 3)^2 = 36$

11. $\left(\frac{1}{3}\right)^2 \cdot 3^2 \quad = 1$

12. $\left(\frac{1}{5}\right)^2 \cdot 75 = 3$

13. $(6)^2 - (4)^2 \quad = 20$

Unit 5

Lessons 1–15

Name: .... Date: .... Period: ....

Data, Graphs, and Tables | Multiple Representations of Functions | Cylindrical Investigations

Shape, Number, and Expressions | 8.F.1, 8.F.2, 8.F.3, 8.F.4, 8.F.5, 8.G.9, SMP.3

Practice Day 2

Let’s practice what you’ve learned so far in this unit!

You will use task cards for this Practice Day. Record all of your responses here.

Task A: TV Snacks

1. Response:

- **Partner A: 2 pretzels were eaten per minute.**
- **Partner B: 10 pieces of popcorn were eaten per minute in the first 4 minutes.**

2. Circle One: Yes No

Explain:

- **Partner A: No. *Explanations vary*. From 0 to 4 minutes, the number of pieces of popcorn in the bag decreases. Between 4 and 6 minutes, the number of pieces of popcorn does not change.**
- **Partner B: Yes. *Explanations vary*. There is a constant decrease of 6 pretzels in the bag every 3 minutes.**

You’re invited to explore more.

Segment 1: $y = -10x + 90$

Segment 2: $y = 50$

Segment 3: $y = -20x + 170$

Segment 4: $y = 10$

Task B: Cylinders and Cones

1. Height:

- **Partner A: 9 feet**
- **Partner B: 27 feet**

2. Volume:

- **Partner A: 48π cubic feet**
- **Partner B: 432π cubic feet**

You’re invited to explore more.

Radius: **6 feet**

Height: **12 feet**

Responses vary.

Name: .... Date: .... Period: ....

Practice Day 2 (continued)

Task C: Basketball

1. Circle One: Yes No

Explain:

- Partner A: Yes. *Explanations vary*. For every point in time, there is only one possible height of the basketball.
- Partner B: No. *Explanations vary*. For example, a height of 8 feet occurs both at 0.25 seconds and 0.75 seconds.

2. Time:

- Partner A: The height of the basketball was increasing between 0 and 0.5 seconds and between 1.25 and 1.5 seconds.
- Partner B: The height of the basketball was decreasing between 0.5 and 1.25 seconds and between 1.5 and 1.75 seconds.

You're invited to explore more.

Statement 1: *Responses vary*. The maximum height of the basketball is 9 feet. (True)

Statement 2: *Responses vary*. The height of the basketball is 0 feet after 1.25 seconds. (True)

Statement 3: *Responses vary*. The basketball's height is always increasing. (Lie)

Task D: All the Spheres

1.

Radius (cm)	Volume of Sphere (cu. cm)
1	$\frac{4\pi}{3} \approx 4.2$
2	$\frac{32\pi}{3} \approx 33.5$
3	$36\pi \approx 113.04$
4	$\frac{256\pi}{3} \approx 267.9$

2. Circle One: Yes No

Explain:

- Partner A: No. *Explanations vary*. There is not a constant rate of change.
- Partner B: No. *Explanations vary*. There is not a constant rate of change.

You're invited to explore more.

Diameter: 12 meters

Name: ______ Date: ______ Period: ______

Practice Day 2 (continued)

Task E: Missing-Dimension Detective

1. Height: ______

- Partner A: 2.4 centimeters
- Partner B: 4.6 inches

2. Radius: ______

- Partner A: 3.8 centimeters
- Partner B: 1.5 inches

You're invited to explore more.

Circle One: [✓ Height] Radius

Explain: *Explanations vary*. The height of the cone is greater than the diameter of the sphere because when you set their volume formulas equal to each other and solve for the height, the result is $h = 4r$. The diameter of the sphere is twice the radius, or $2r$, which is not greater than $4r$.

Task F: Hydrate!

1. Milliliters of water: ______

- Partner A: 80 milliliters
- Partner B: 100 milliliters

2. Equation: ______

- Partner A: $y = \text{-}100x + 1200$
- Partner B: $y = \text{-}80x + 1080$

You're invited to explore more.

Miles: 6

Career Connection

Where in the universe can matter and light "fall in", but can't escape?

Maxal Tamor/Shutterstock.com

If you imagined a black hole, you're correct! A black hole is an approximate spherical region in space where the force of gravity is so strong that even light is not able to escape. They aren't really holes; they represent enormous amounts of matter that are compressed into very small spaces.

Our galaxy is home to Sagittarius A*, a supermassive black hole with a mass of about 4,000,000 Suns, but a diameter of only about 30 times that of our Sun.

Astrophysicists study the universe using concepts of matter, energy, and motion. They might calculate the approximate volume of stars, planets, and black holes using the formula for the volume of a sphere. These bodies in space closely resemble spheres because gravity pulls equally from all directions toward their centers.

Stephen Hawking' by 2°1°°. Public Domain.

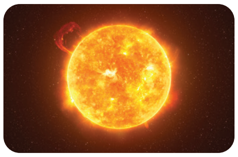
Meet Stephen Hawking

There is still much we do not know about black holes, but, thanks to Stephen Hawking, a British mathematician and theoretical physicist, we are much closer to understanding these extraordinary mysteries. As Stephen Hawking studied the masses and volumes of black holes, he theorized that black holes could gradually lose mass over time, by emitting small particles through a process now called Hawking radiation.

Are you interested in studying astrophysics? What can you do to learn more?

Math in the World

Our Sun is about 864,000 miles in diameter. Betelgeuse, one of the brightest stars visible, has a diameter about 724 times greater than our Sun. If Betelgeuse was placed at the center of our solar system, its size would reach past the orbit of Mars! How can you compare the volumes of these stars?



Our Sun

Lukasz Pawel Szczepanski/ Shutterstock.com.

Betelgeuse

Franco Tognarini/ Shutterstock.com.

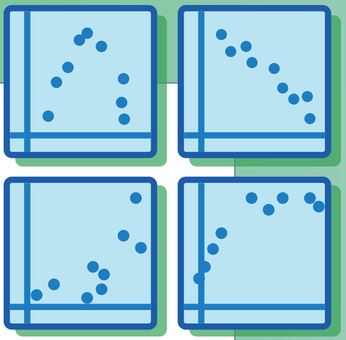
Math Mindset

How could you explain to a friend how the volume of a sphere is a function of its radius?

Responses vary for the Math in the World and Math Mindset questions.

Unit 6

Associations in Data



Big Ideas in This Unit

- CC1 Interpret Scatter Plots
- Data, Graphs, and Tables
- CC2 Slopes and Intercepts
- Linear Equations
- Multiple Representations of Functions

Questions for Investigation

- What is a scatter plot and what can it tell you?
- What are some ways you can describe trends in data?
- How can you analyze data with two variables that are categories instead of numbers?

Explore: Changes in the Ozone Layer
How has the ozone layer been changing?

Watch Your Knowledge Grow

This is the math you'll explore in this unit. Rate your understanding to see how your knowledge grows!

○—○—○

Not yet Almost I got it!

I can . . .	Before	After
Create a scatter plot from a table of data.	○—○—○	○—○—○
Explain whether data on a scatter plot has any clusters.	○—○—○	○—○—○
Explain whether data on a scatter plot has an outlier.	○—○—○	○—○—○
Explain whether the association of data on a scatter plot is positive, negative, or neither.	○—○—○	○—○—○
Explain whether data on a scatter plot has a linear or non-linear association.	○—○—○	○—○—○
Fit a line to data on a scatter plot.	○—○—○	○—○—○
Determine whether data on a scatter plot has a good line of fit.	○—○—○	○—○—○
Use the equation of a linear model to predict values that are not in the data.	○—○—○	○—○—○
Describe what the slope of a linear model in context represents.	○—○—○	○—○—○
Describe what the y-intercept of a linear model in context represents.	○—○—○	○—○—○

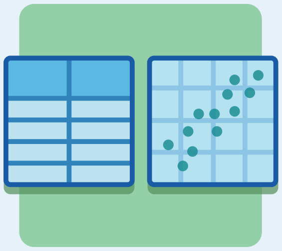
I can . . .	Before	After
Construct a two-way table.	○—○—○	○—○—○
Calculate relative frequencies given a two-way table.	○—○—○	○—○—○
Use relative frequencies in a two-way table to describe associations in data.	○—○—○	○—○—○

Sub-Unit

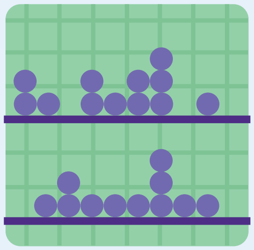
1

Organizing Numerical Data

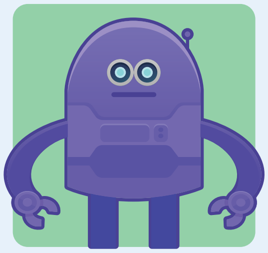
Explore
Changes in the Ozone Layer



Lesson 1
Click Battle



Lesson 2
Wingspan



Lesson 3
Robots

Name: ______ Date: ______ Period: ______

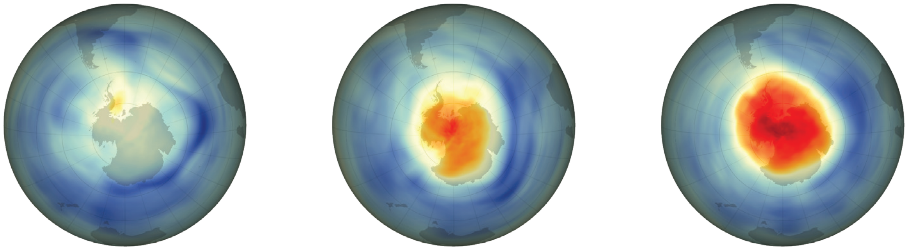
Interpret Scatter Plots Building Toward 8.SP.1, SMP.2, SMP.4

Explore: Changes in the Ozone Layer

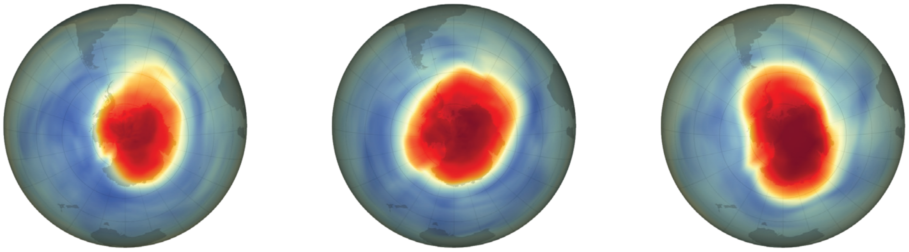
How has the ozone layer been changing?

Warm-Up

1. The images show the area of significant thinning in the ozone layer from 1980 to 2006, commonly referred to as the "ozone hole," over Antarctica. This "hole" forms each year during the Antarctic spring (August to October). The size and shape of the "hole" are monitored yearly to track changes and recovery progress.



October 16, 1980 **October 3, 1984** **October 7, 1989**



October 11, 1992 **October 6, 1998** **October 6, 2006**

"World of Change: Ozone Hole." NASA Earth Observatory

Discuss: What do you notice? What do you wonder?

Responses vary.

- I notice that the spot gets more red over time.
- I notice that the size and shape of the ozone "hole" in 2006 is very different from 1980.
- I wonder how the "hole" in the ozone layer is measured.
- I wonder if the red corresponds to a larger ozone "hole" size.

Name: .... Date: .... Period: ....

The Ozone Layer

2. The darker red shows a “hole” in the ozone layer. This is not a hole you can see with your eyes. Instead, it is the thinning of the ozone layer.

After examining the images from the Warm-Up, which statement do you agree with?

A. The size of the “hole” in the ozone layer seems to be decreasing.

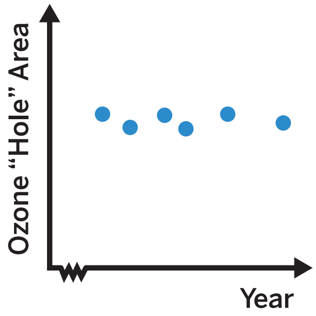
[✓ B.] The size of the “hole” in the ozone layer seems to be increasing.

C. The size of the “hole” in the ozone layer seems to remain the same each year.

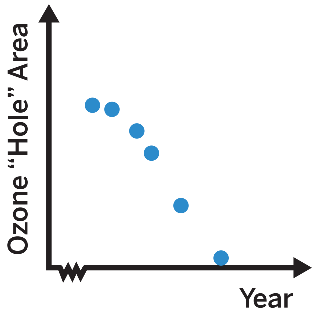
D. The size of the “hole” in the ozone layer does not seem to change in any pattern.

3. **Data Talk!** Which graph do you think could represent the changing size of the ozone “hole” from 1980 to 2006? Circle one.

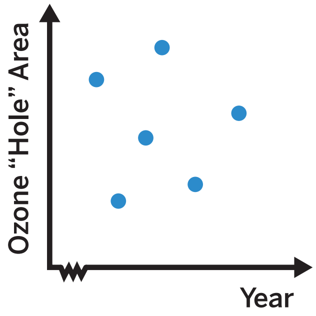
Graph A



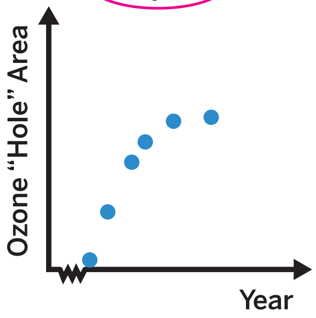
Graph B



Graph C



[✓ Graph D]



Explain your thinking. **ELD.PI.8.6.Em, Ex, Br, ELD.PI.8.11.Em, Ex, Br**

Responses vary. **The ozone “hole” increased in size through the 1980s and 1990s, but it began stabilizing after the late 1990s, so the points should be going up.**

Name: ______________________ Date: __________ Period: __________

The Ozone Layer (continued)

The ozone layer is a region in the stratosphere that acts like a shield, absorbing most of the Sun's harmful ultraviolet (UV) radiation. If the size of the ozone "hole" increases, regions under the "hole" experience less protection from harmful ultraviolet radiation, which can pose risks to human health and the environment.

4. Which do you think could have damaged the ozone layer? ***Responses vary.***

- people
- bicycles
- refrigerators
- electric cars
- water pollution
- air conditioners

Could damage the ozone layer	Could not damage the ozone layer	I'm unsure

5. Which do you think could be affected by the damaged ozone layer? ***Responses vary.***

- a person's lungs
- plants
- marine life
- a person's skin
- a person's health
- air quality

Could be affected by damaged ozone layer	Could not be affected by the damaged ozone layer	I'm unsure

6. There are changes to the ozone layer even until this day and data shows that our actions matter.

Discuss: What information could you collect to help determine whether your responses are correct?

Responses vary.

- **I could collect data on the production and use of ozone-depleting substances (e.g., CFCs) over time and compare that to the ozone hole size for the same years.**
- **I could collect information on the ozone layer and how it can affect a person's eyes.**

Activity

Name: ______________________ Date: __________ Period: __________

Building Math Habits of Mind

Discuss:

- Which of these habits of mind did you strengthen during this activity?
- How did you use the one(s) you selected?

I can slow down and first make sense of a challenging problem before trying to solve it.

○ Not yet ○ Almost ○ I got it!

I can represent real-world problems and interpret their solutions within the context of the problem.

○ Not yet ○ Almost ○ I got it!

I can justify my thinking and ask questions to help me understand the thinking of others.

○ Not yet ○ Almost ○ I got it!

I can apply the math that I know to solve real-world problems, making assumptions and revising my thinking as needed.

○ Not yet ○ Almost ○ I got it!

I can select an appropriate tool to help me solve problems.

○ Not yet ○ Almost ○ I got it!

I can communicate my thinking and solutions clearly to others.

○ Not yet ○ Almost ○ I got it!

I can look for structure or patterns to help me solve problems.

○ Not yet ○ Almost ○ I got it!

I can look for repeated calculations and other repeated steps to make generalizations.

○ Not yet ○ Almost ○ I got it!

Unit 6
Lesson
1

Name: Date: Period:

Interpret Scatter Plots Slopes and Intercepts 8.SP.1, SMP.7

Click Battle

Let's find ways to show patterns in data.

Warm-Up

1 Tap your pencil on your desk as many times as you can for 2 seconds. Record your number of taps.

Responses vary.

2 Tap your pencil on your desk as many times as you can for 6 seconds. Record your number of taps.

Responses vary.

Activity
1

Name: .... Date: .... Period: ....

Organizing Data

3 **Data Talk!** Let's look at some class data about button clicks, which are similar to pencil taps.

a Organize the data in a way that makes sense to you.

Responses vary.

b **Discuss:** What patterns do you see in the data?

Responses vary. **I organized the data in a two-column table with time (in seconds) in the left column and number of clicks in the right column. I noticed that, for the most part, students who had more time made more clicks.**

4 Let's look at one way to represent the data. What do you notice?

Responses vary.

- **I notice that the times are organized from least to greatest.**
- **I notice that some people had the same amount of time but made different numbers of clicks.**

5 **Data Talk!** Let's look at another way to represent the data.

Discuss the connections you see between the table and graph.

Responses vary.

- **Each row in the table appears as a point on the graph.**
- **The table headers (Time and Number of Clicks) appear as the axis labels.**
- **I see that values in the left column are between 2.5 and 8 and the points on the graph are between these same numbers when looking along the x-axis.**

Activity

2

Name: .... Date: .... Period: ....

Make a Prediction

Data Talk! Here is click data organized as a list, table, and scatter plot.

List

14 clicks in 2.5 seconds

17 clicks in 3 seconds

16 clicks in 4 seconds

42 clicks in 8 seconds

22 clicks in 5.5 seconds

24 clicks in 6.5 seconds

30 clicks in 5.5 seconds

24 clicks in 4.5 seconds

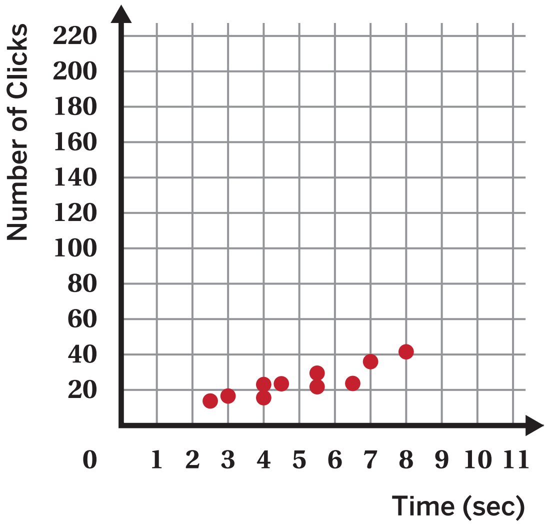
36 clicks in 7 seconds

23 clicks in 4 seconds

Table

Time (sec)	Number of Clicks
2.5	14
3	17
4	16
4	23
4.5	24
5.5	22
5.5	30
6.5	24
7	36
8	42

Scatter Plot



6 Select a representation and use it to answer this question:

How many clicks do you think a typical student in your class would make in 10 seconds?

***Responses vary.* 55 clicks**

Explain your thinking.

***Explanations vary.* I looked at the scatter plot. If I imagine a line that goes through roughly the middle of the points, there will be a point at approximately (10, 55).**

7 Test your prediction by counting the number of pencil taps you can make in 10 seconds.

Responses vary.

8 Synthesis

Data Talk! Discuss some advantages of using a list, a table, or a scatter plot to organize data.

Responses vary.

- **List: This is the easiest representation to create.**
- **Table: This makes it easier to see some numerical patterns (e.g., as students get more time, they tend to make more clicks).**
- **Scatter plot: This makes it easier to see some visual patterns (e.g., the data falls roughly on a line) and to use those patterns to make a prediction (e.g., the number of clicks in 10 seconds).**

11 Summary 6.01

You can organize and display data that includes numbers in different ways, including in a table and in a **scatter plot**.

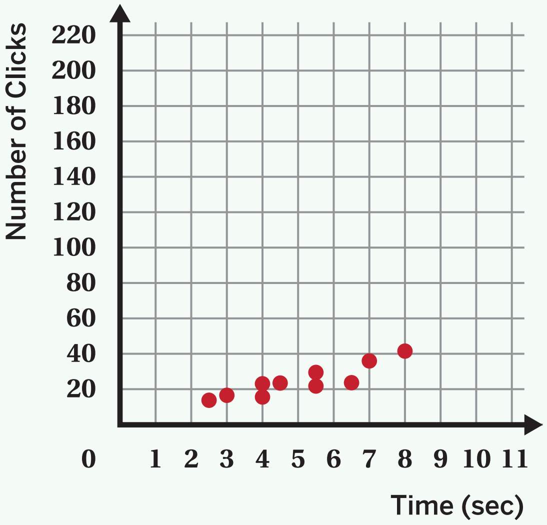
A table and a scatter plot both display the same data, but can be helpful in different ways. For example, you can use a scatter plot to investigate connections between two variables, while a table is helpful for looking for the exact values of specific data points.

Here is data showing the amount of time in seconds and the number of clicks of the button.

Table

Time (sec)	Number of Clicks
2.5	14
3	17
4	16
4	23
4.5	24
5.5	22
5.5	30
6.5	24
7	36
8	42

Scatter Plot



scatter plot A set of disconnected data points plotted on a coordinate plane. It allows us to investigate connections between two variables.

Practice
6.01

Name: .... Date: .... Period: ....

Problems 1–3: Here is data on the number of cases of whooping cough from 1944 to 1955.

Year	Number of Cases
1944	109,873
1945	133,792
1946	109,860
1947	156,517
1948	74,715
1949	64,479
1950	120,718
1951	68,687
1952	45,030
1953	37,129
1954	60,866
1955	62,786

1. Describe another way to sort this table. What is a question that can be answered when the table is sorted this way?

***Responses vary.* By the number of cases — in which year were there the fewest number of cases?**

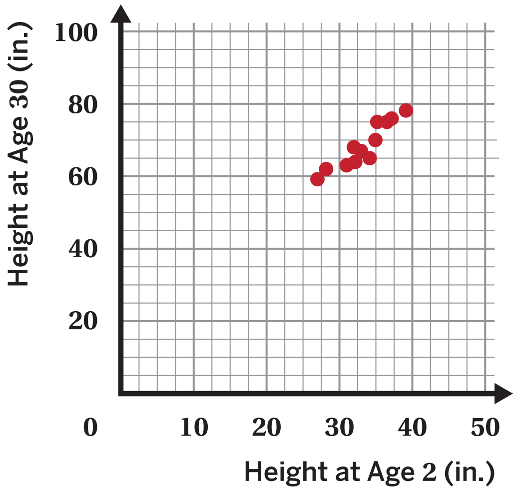
2. Which years in this period of time had more than 100,000 cases of whooping cough?

The years 1944, 1945, 1946, 1947, and 1950 had more than 100,000 cases of whooping cough.

3. Based on this data, would you expect 1956 to have closer to 50,000 cases or 100,000 cases? Explain your thinking.

This data seems to show the number of cases decreasing over time, so I would expect 1956 to have closer to 50,000 cases than 100,000.

Problems 4–5: A research study measured the heights of twelve people on their birthday at age 2 and at age 30.



4. What patterns do you notice in the data?

Responses vary.

- **I notice that the points have a general trend, as if arranged around a line that goes up and to the right.**
- **I notice that for many people, their height at age 30 was about double their height at age two.**

5. A two-year-old has a height of 38 inches. Based on this data, predict their height at age 30.

***Responses vary.* Answers between 70 and 85 are considered correct.**

Practice
6.01

Name: .... Date: .... Period: ....

6. Here is data a teacher collected after asking her students how many hours of sleep they had the night before a test.

	Hours of Sleep	Test Score
Ayaan	7	74
Emika	6	76
Inola	8	88
Kwasi	5	63
Zoe	7	90

How might you organize or display this data? Explain your thinking.

Responses vary.

- **Sort the table by hours of sleep or score because it is easier to see patterns.**
- **Create a scatter plot because it is easier to see the relationship between both variables.**

Spiral Review

7. A cylinder has a height of 6 feet and a diameter of 2 feet. Which measurement is closest to the volume of the cylinder in cubic feet?

A. 226.2 cubic feet

B. 75.4 cubic feet

[✓ **C.**] 18.8 cubic feet

D. 113.1 cubic feet

Problems 8–11: This cylinder has a radius of 4 centimeters and a height of 5 centimeters.

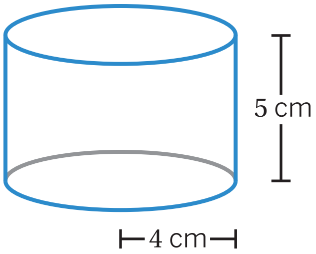


Figure may not be drawn to scale.

8. What is the volume of the cylinder?

80π cubic centimeters (or equivalent)

9. What is the volume of the cylinder when its radius is tripled?

720π cubic centimeters (or equivalent)

10. What is the volume of the cylinder when its radius is halved?

20π cubic centimeters (or equivalent)

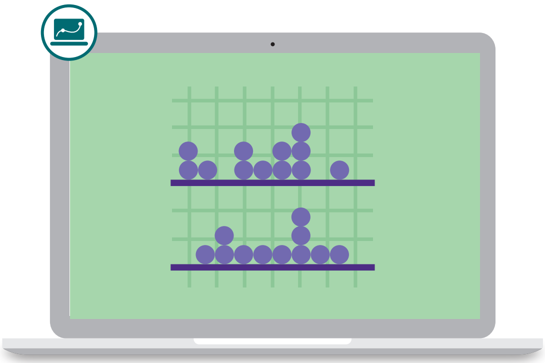
11. A cylinder has a volume of 120 cubic units. What is the volume of a cone with the same radius and height? Explain your thinking.

40 cubic units. *Explanations vary.* The volume of a cone is one-third the volume of a cylinder with the same radius and height.

Unit 6
Lesson
2

Name: ______ Date: ______ Period: ______

Interpret Scatter Plots 8.SP.1, SMP.6, SMP.7



Wingspan

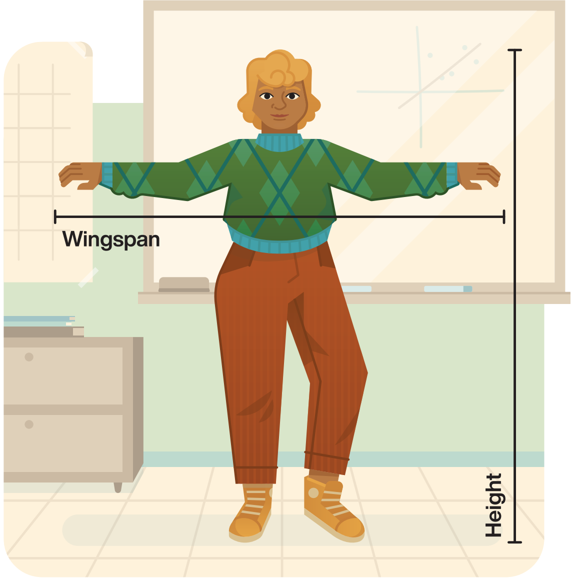
Let’s compare dot plots and scatter plots.

Warm-Up

1 With a partner, measure your height and wingspan to the nearest inch.

Responses vary.

Height (in.)	Wingspan (in.)
68	70



2 **Data Talk!** Let’s look at a table of height and wingspan data.

Discuss the following:

- How could we reorganize the data to make it more useful for analyzing?
- What are some questions this data could help answer?

Responses vary.

- **List the values for height in order from shortest to tallest.**
- **List the values for wingspan in order from shortest to longest.**
- **Do taller students always have longer wingspans?**
- **What is the difference in inches between the largest and smallest wingspans?**

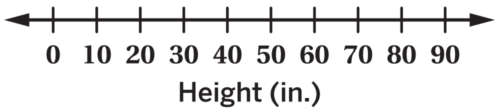
Activity
1

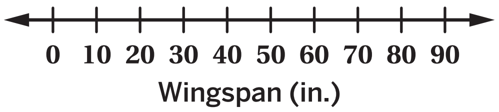
Name: .... Date: .... Period: ....

Visualizing Data

3 Plot points on the *dot plots* to represent your height and wingspan.

Responses vary based on students' heights and wingspans.





4 **Data Talk!** Let's look at dot plots of height and wingspan data.

a What is a question you *can* answer based on the dot plots?

Responses vary.

- **How many people are 62 inches tall?**
- **What is the range of heights in the class?**

b What is a question you *cannot* answer based on the dot plots?

Responses vary.

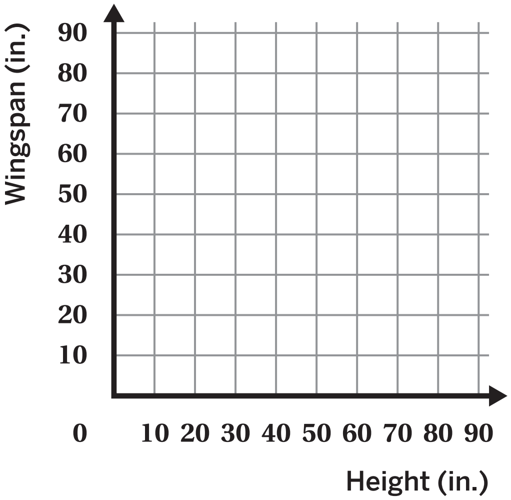
- **What is the wingspan of the person who is 60 inches tall?**
- **Is there a relationship between height and wingspan?**

Name: .... Date: .... Period: ....

Visualizing Data (continued)

5 Let's look at a table of some student's heights and wingspans. Plot the data shown in the table.

Responses vary based on students' heights and wingspans.



6 **Data Talk!** Let's look at a scatter plot that represents height and wingspan data.

a What is a question you *can* answer based on the scatter plot?

Responses vary.

- **Do people's heights and wingspans tend to be close?**
- **Is there a relationship between height and wingspan?**

b What is a question you *cannot* answer based on the scatter plot?

Responses vary.

- **Who is the tallest person?**
- **Are there other factors that influence a person's height?**

You're invited to explore more.

7 Use the Activity 1 Sheet to examine the heights, weights, wingspans, and hand lengths of professional basketball players.

What do you notice? What do you wonder?

Responses vary.

- **I notice that the graph of height on the x-axis and weight on the y-axis looks different than the graph of weight on the x-axis and height on the y-axis.**
- **I wonder how the graphs would be different if I added non-professional players to this data set.**

8 Synthesis

Data Talk! Discuss some advantages of using a dot plot or a scatter plot to represent data. Use the examples from the Summary if they help with your thinking.

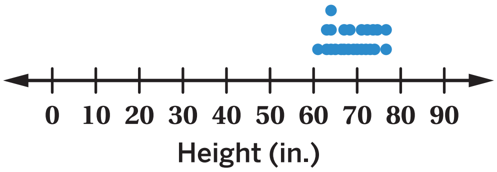
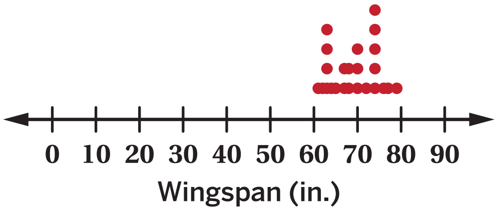
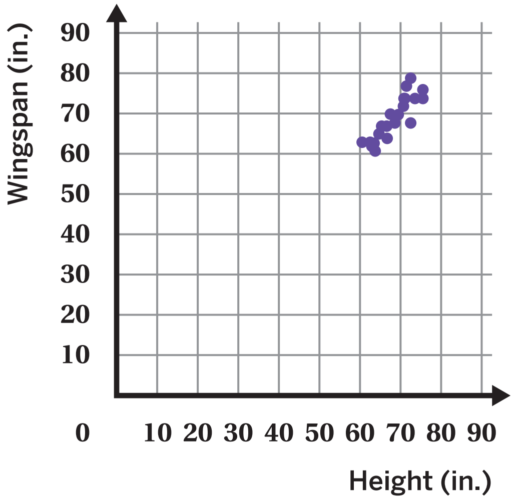
Responses vary.

- **Dot plot: This shows how many people share a value for one quantity. A dot plot would be helpful if you wanted to know how many people are the same height.**
- **Scatter plot: A point can represent two variables at the same time. Each point represents the height and the wingspan for one person, as opposed to showing the two variables separately.**

11 Summary 6.02

Data presented as numbers, quantities, or measurements that can be compared in a meaningful way is called *numerical data*, or *quantitative data*. You can investigate *univariate data*, which involves one variable, and *bivariate data*, which involves two variables.

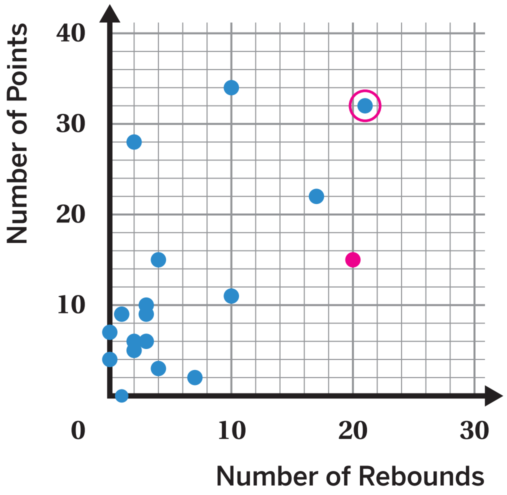
There are different ways to represent numerical data. A *dot plot* shows data for one variable and a scatter plot shows data for two variables at the same time. Seeing two numerical variables at the same time allows us to notice trends and connections.

Dot Plot	Scatter Plot
 	

Practice
6.02

Name: ____________ Date: ____________ Period: ____________

Problems 1–5: This scatter plot shows the number of rebounds and points for each player in a recent professional basketball game.



1. Circle the point on the graph that represents the player with the most rebounds. How many rebounds did that player have?

Response shown on graph. 21 rebounds

2. How many players had 0 rebounds?

2 players

3. What is another question you *can* answer based on this scatter plot?

***Responses vary.* Is there a relationship between rebounds and points?**

4. What is a question you *cannot* answer based on this scatter plot?

***Responses vary.* How many three-point and two-point shots did each player make?**

5. The table shows the data for another basketball player. Plot the point for the player on the graph. **Response shown on graph.**

Number of Rebounds	Number of Points
20	15

6. Which representation(s) are appropriate for comparing the heights of students on a volleyball team to the heights of students on a soccer team?

A. Scatter plot

[✓ B.] Dot plots

C. Both

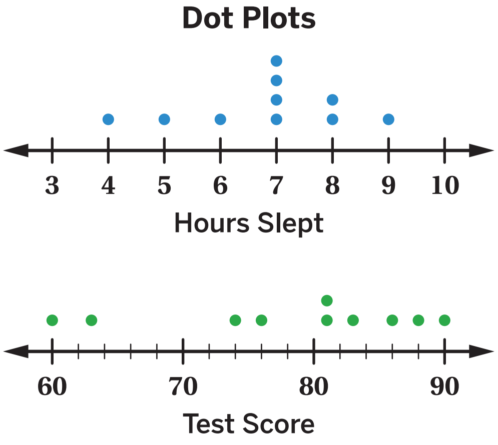
Explain your thinking.

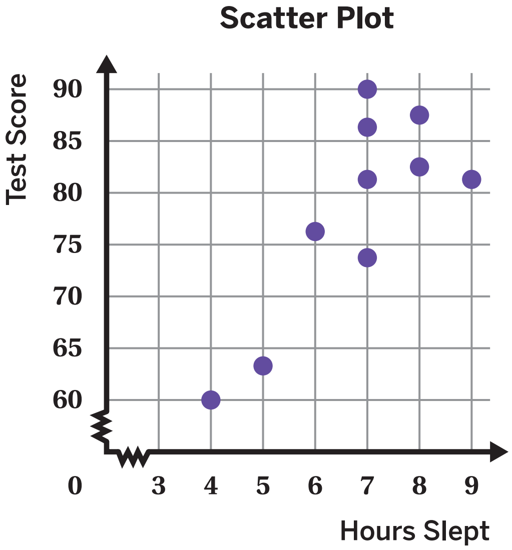
***Explanations vary.* Dot plots are appropriate because I am comparing two separate data sets to each other. A scatter plot would make sense if we were looking at two attributes for one set of humans, like heights and weights for players on the soccer team.**

Practice
6.02

Name: .... Date: .... Period: ....

7. A teacher collected data about her students' test scores and how many hours they slept the night before a test. She represented the data with dot plots and a scatter plot.


Dot Plots

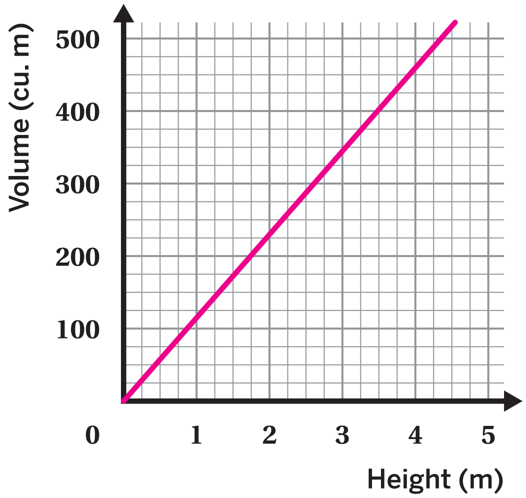

Scatter Plot

What is different about the two ways of representing the data?

***Responses vary.* Dot plots separate each variable and scatter plots show both variables at once.**

Spiral Review

Problems 8–11: There are many cylinders with a radius of 6 meters. Let h represent the height in meters and V represent the volume in cubic meters.



8. Write an equation that represents the volume, V, as a function of the height, h.

$V = 36\pi h$ (or equivalent)

9. Sketch the graph of the equation, using 3.14 as an approximation for π.

Response shown on graph.

10. If you double the height of a cylinder, what happens to the volume? Use the equation to help you explain your thinking.

***Responses vary.* If you double the height, the volume doubles. Replacing h with $2h$ in the equation gives $V = 36\pi \cdot 2h = 2(36\pi h)$, which is double the original volume.**

11. A cone has a radius of 4 feet and of height of 11 feet. What is the volume, in cubic feet, of the cone? Use 3.14 as an approximation for π. Round your answer to the nearest hundredth.

184.21 cubic feet

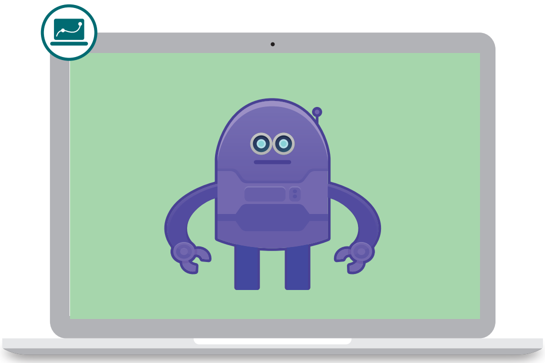
Unit 6

Lesson

3

Name: ______ Date: ______ Period: ______

Interpret Scatter Plots 8.SP.1, SMP.1, SMP.2, SMP.6



Robots

Let's investigate points on a scatter plot.

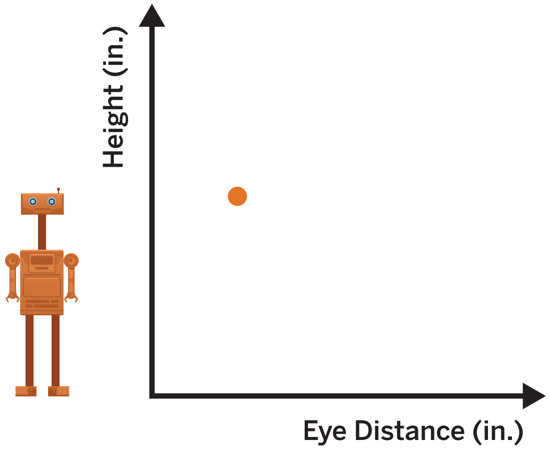
Warm-Up

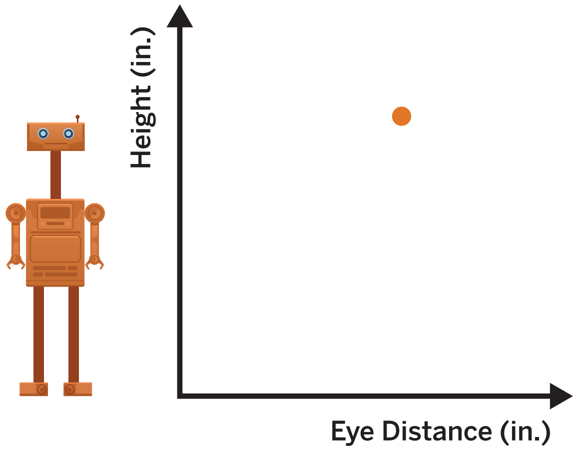
1 Here are two graphs and images of a robot.

Discuss: What do you notice?

Responses vary.

- **I notice that the robot's height is related to how high up the point is.**
- **I notice that the distance between the robot's eyes is related to where the point is horizontally.**





Activity 1

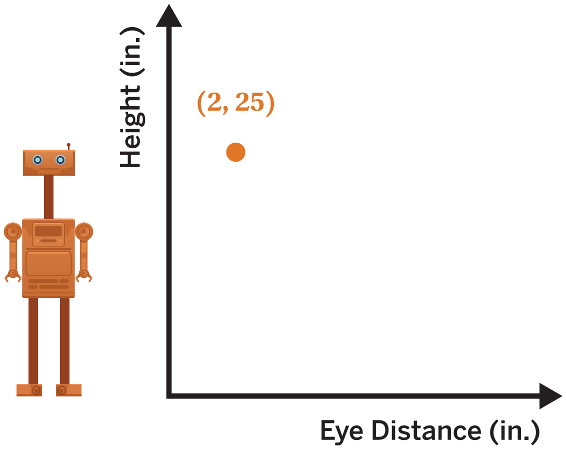
Name: ______________________ Date: __________ Period: __________

Robots

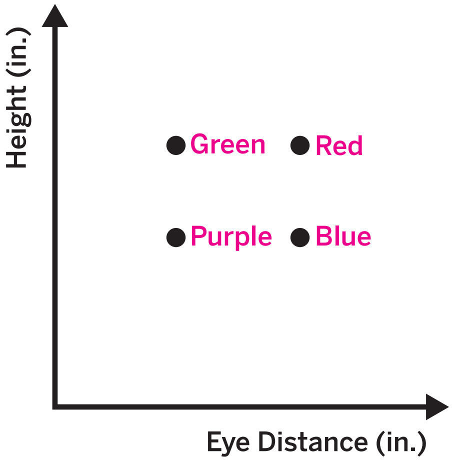
2 Describe something you know about the robot based on the graph.

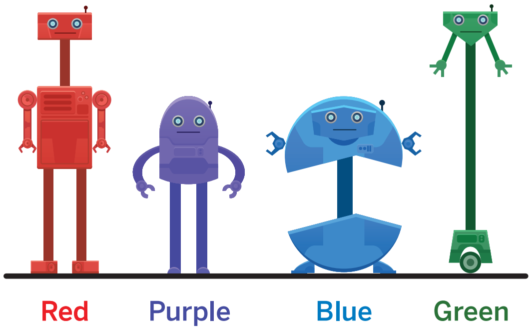
Responses vary.

- **The robot's height is 25 inches.**
- **The robot's eyes are 2 inches apart.**



3 Here are four different robots. Label each point on the graph with the color robot it represents.





Name: .... Date: .... Period: ....

Robots (continued)

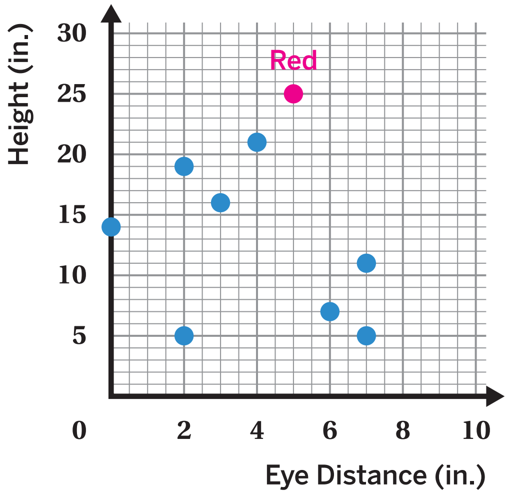
4 The table shows the heights and eye distances for five different robots.
Plot a point to represent the pink robot.

Robot Color	Eye Distance (in.)	Height (in.)
Teal	2	30
Black	4	10
Gray	8	10
Orange	6	20
Pink	8	20

5 The graph shows the heights and eye distances for eight blue robots.

Plot a point for a red robot to make this statement true: *The red robot is taller than all the blue robots, and its eye distance is 5 inches.*

***Responses vary.* Sample shown on graph. The point should be on the line $x = 5$ with a y-coordinate greater than 21.**



Activity
2

Name: ______________________ Date: __________ Period: __________

Challenge Creator

6 You will use a set of cards with scatter plots to create your own challenge.

a Choose one card that interests you and plot a point somewhere you think is interesting.

b On this page, write the point as an ordered pair. Then tell a story about this point.

Responses vary.

My Point	My Story

c Swap your challenge with one or more partners. Write the point they plotted as an ordered pair and tell a story about it.

	Point	Story
Partner 1		
Partner 2		
Partner 3		

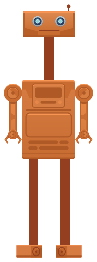
7 Synthesis

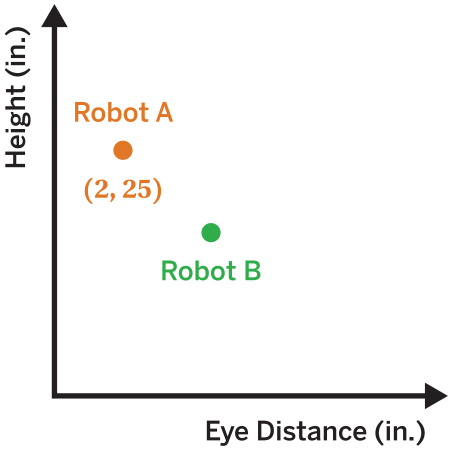
This graph shows the height and eye distance for two robots.

Describe some things you know about Robot B given the information about Robot A.

Responses vary.

- **Robot B's height is less than 25 inches.**
- **Robot B's eyes are more than 2 inches apart.**
- **Robot B is shorter than Robot A.**
- **Robot B's eyes are farther apart than Robot A's eyes.**

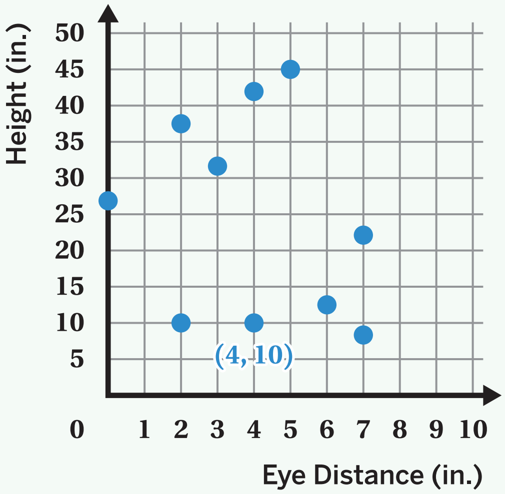




10 Summary 6.03

A point on a scatter plot represents two pieces of information. The axis labels tell you how to interpret the coordinates of each point.

In this example, the point $(4, 10)$ represents a robot with an eye distance of 4 inches and a height of 10 inches.

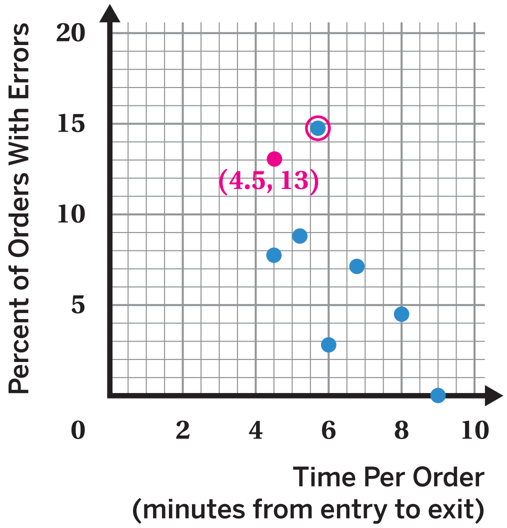


Practice
6.03

Name: .... Date: .... Period: ....

Problems 1–3: A study gathered data about different drive-thru restaurants. The table and scatter plot show the average time per order and the percent of orders with errors for each restaurant.

Restaurant	Time Per Order (min)	Percent of Orders With Errors
CraveBite	8	4.5
Taco Tango	6	2.8
Bite Master	9	0
Noodle Nest	4.5	7.7
Burger Whiz	5.2	8.8
Pajaro	6.8	7.1
NachoLoco	5.7	14.7



1. Circle the point on the scatter plot that represents the data for NachoLoco.

   Response shown on graph.

2. What does the point $(6, 2.8)$ represent?

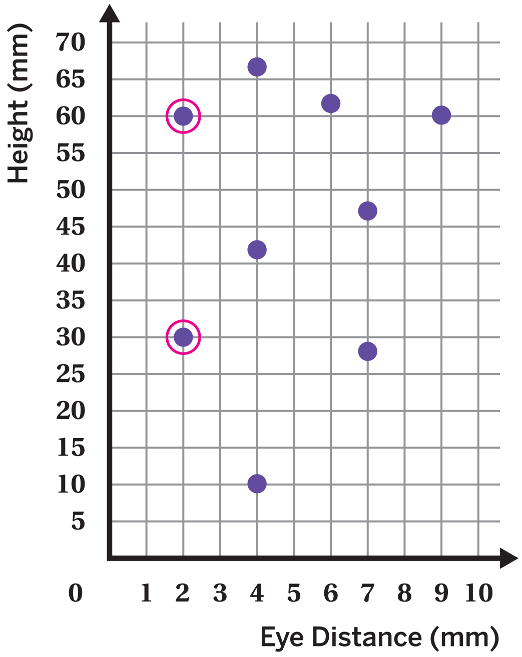
   Responses vary. It represents the time per order and the percent of orders with errors for Taco Tango.

3. In the same study, the data showed that Dumpling Delight takes 4.5 minutes per order and 13% of their orders had errors. Add a point to the scatter plot to represent Dumpling Delight.

   Response shown on graph.

4. Circle the point(s) for the robot(s) with the shortest eye distance. Write the height and eye distance of each point you circled.

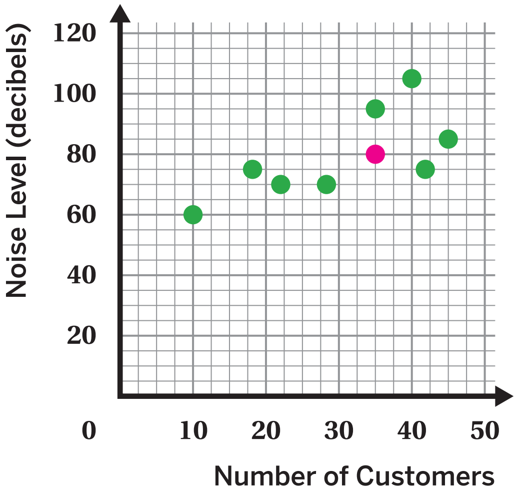
   Responses vary. The eye distances are both 2 millimeters, and their heights are 30 millimeters and 60 millimeters.



Practice
6.03

Name: ______ Date: ______ Period: ______

Problems 5–8: This scatter plot shows the noise level and the number of customers for eight restaurants.



5. What is the noise level at the loudest restaurant?

105 decibels

6. What is the noise level at the restaurant with the most customers?

85 decibels

7. What does the point (10, 60) tell you about the noise level and number of customers at that restaurant?

***Responses vary.* There are 10 customers and the noise level is 60 decibels.**

8. The noise level at a restaurant with 35 customers is 80 decibels. Plot a point on the graph that represents this restaurant.

Response shown on graph.

Spiral Review

9. Select *all* the representations that are appropriate for comparing exam score to hours of sleep the night before an exam.

- ☐ **A.** Histogram
- ☑ **B.** Scatter plot
- ☐ **C.** Dot plot
- ☑ **D.** Table
- ☐ **E.** Box plot

Problems 10–13: Evaluate each expression.

10. $-2 \cdot (-4)$ **= 8**

11. $-7 \cdot 2$ **= -14**

12. $9 \cdot (-10)$ **= -90**

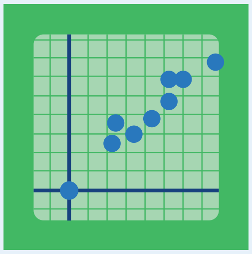
13. $-2 \cdot (-6) \cdot (5)$ **= 60**

Sub-Unit

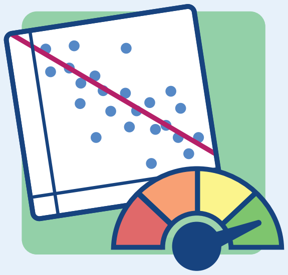
2

Analyzing Numerical Data

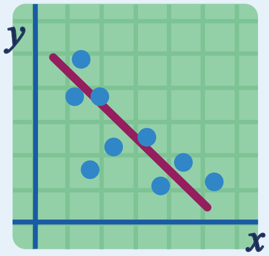
Lesson 4
Dapper Cats



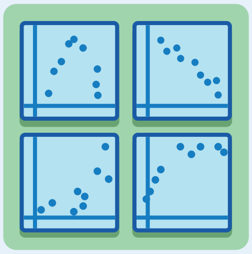
Lesson 5
Interpreting Scatter Plots



Lesson 6
Find the Fit



Lesson 7
Interpreting Slopes



Lesson 8
Scatter Plot City

Lesson 9
Animal Brains

Unit 6
Lesson
4

Name: .... Date: .... Period: ....

Interpret Scatter Plots | Multiple Representations of Functions | Slopes and Intercepts

8.SP.1, 8.SP.2, 8.SP.3, 8.EE.5, 8.EE.6, 8.F.4, SMP.2, SMP.7

Dapper Cats

Let's identify potential outliers and use a linear model to predict values.

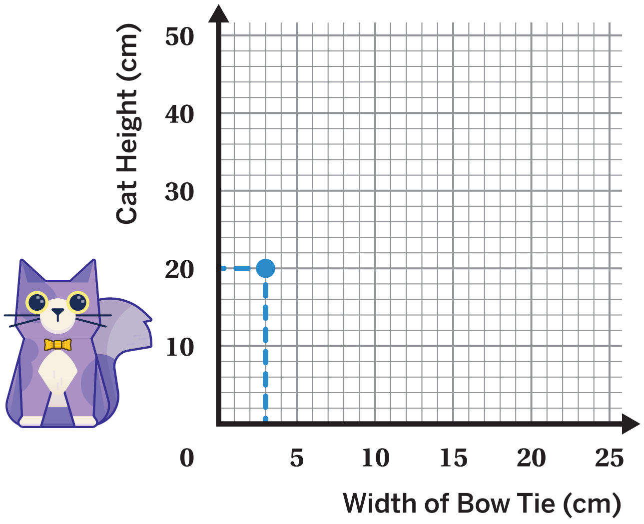
Warm-Up

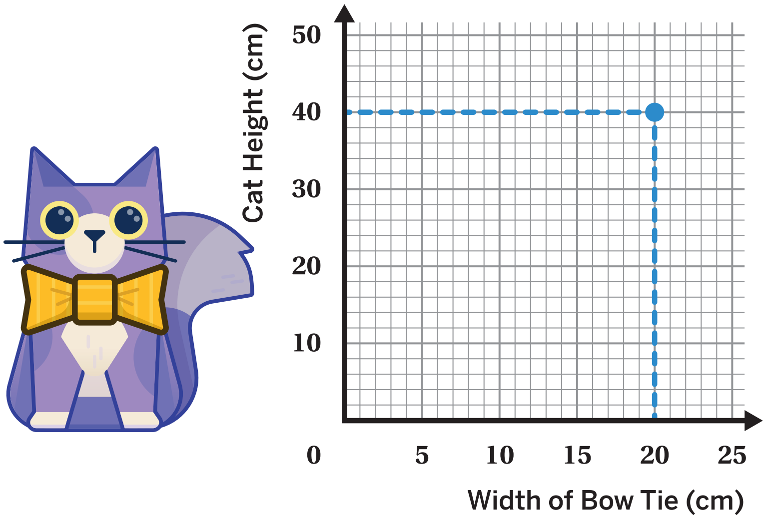
1 Here are two toy cats built at the Build-a-Cat workshop.

Discuss: What do you notice? What do you wonder?

Responses vary.

- I notice that the cats and their bow ties are different sizes depending on where the point is.
- I notice that the higher the point, the taller the cat is and the further the point is to the right, the wider the bow tie is.
- I notice that both of the points are on whole number values, not decimals.
- I wonder how big the biggest bow tie is.
- I wonder If there is a "correct" size bow tie.



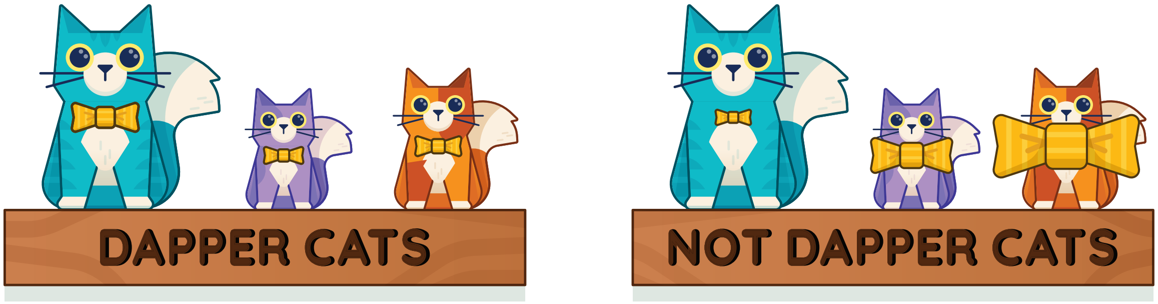


Activity 1

Name: .... Date: .... Period: ....

Dapper Cats

2 Some of these toy cats are dapper and some are not.



What do you think makes a toy cat "dapper"? What makes a toy cat "not dapper"?

Responses vary.

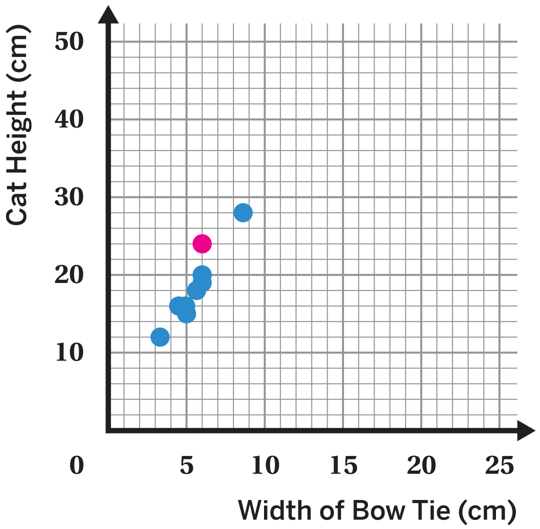
- The toy cats are "dapper" when their bow ties about as wide as the distance between the outside of their eyes. "Not dapper" toy cats have bow ties that are either wider or smaller than the distance between the outside of their eyes.
- A dapper cat has a perfect-sized bow tie. Cats are not dapper when they have a bow tie that is too big or too small.
- The size of the bow tie directly corresponds with the size of the cat.

3 **Data Talk!** This scatter plot shows many "dapper cat" orders.

Describe what you notice about the scatter plot.

Responses vary.

- I notice that there are different-sized dapper cats.
- I notice that the points fall roughly along a line.
- I notice that the points could lie on a line with a positive slope and a y-intercept of 0.
- I notice that the points are all clumped together.

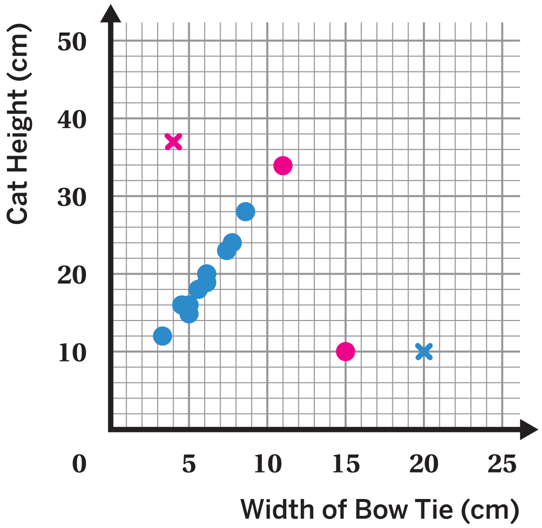


4 A customer just ordered a dapper cat that is 24 centimeters tall. Plot a point on the scatter plot to represent this cat.

Responses vary. Sample shown on graph.

Name: .... Date: .... Period: ....

Dapper Cats (continued)



5 Plot a point to represent a dapper cat that is taller than any of the other cats in the scatter plot.

Responses vary. **Sample shown on graph at $(11, 34)$.**

6 Plot a point to represent a not-dapper cat that is very short with a very large bow tie.

Responses vary. **Sample shown on graph at $(15, 10)$.**

7 **Data Talk!** A student plotted the point $(20, 10)$. It is shown on the scatter plot with an ×. This point is an **outlier** because it is far from the rest of the data.

a Plot another outlier on the scatter plot.

Responses vary. **Sample shown on graph at $(4, 37)$.**

b Describe the cat your point represents.

Responses vary.

- **The cat is dapper, but it's much taller than the other cats.**
- **The cat is not dapper. Its bow tie is too large for its height.**
- **The cat is not dapper. Its bow tie is too small for its height.**
- **The cat's bow tie is 5 centimeters wide and the cat is 40 centimeters tall.**

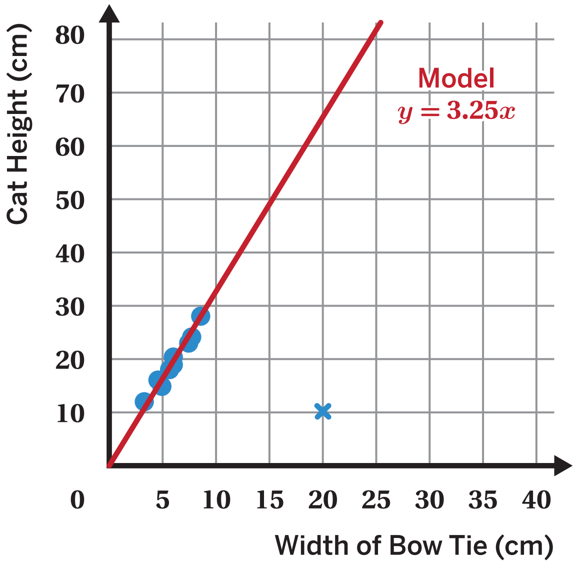
Name: ________________ Date: ________ Period: ________

Using a Linear Model to Predict Data

A **linear model** (also called a *line of fit*) can be used to help identify trends in data and to make predictions. The line and equation model the relationship between bow tie width, x, and cat height, y.

8-9 Use the linear model to predict the height of a dapper cat with a 12-centimeter bow tie.

Responses between 37 and 41 centimeters are considered correct.



10-11 Another dapper cat is 65 centimeters tall. Use the linear model to predict the width of its bow tie.

Responses between 18.5 and 21.5 centimeters are considered correct.

12 Lucy's cat has a 60-centimeter bow tie width. What does the linear model predict for its height?

195 centimeters

Explain your thinking.

***Explanations vary.* The linear model predicts that the height of Lucy's cat will be 195 centimeters. I know this because $3.25 \cdot 60 = 195$.**

13 Synthesis

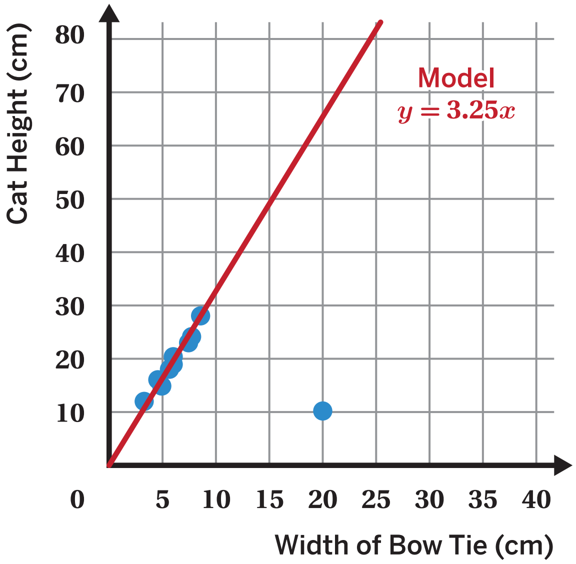
A line of fit and an equation are two ways to represent a linear model.

a How can a linear model be helpful?

***Responses vary.* A linear model is helpful because it helps us see the trend in the data more clearly so we can make predictions.**

b How can you identify an outlier on a scatter plot?

***Responses vary.* I can identify an outlier by looking for points that are far from the other points on the scatter plot.**



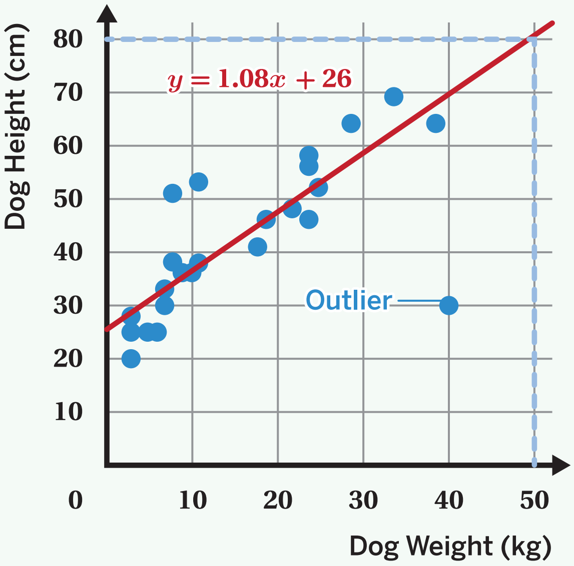
16 Summary 6.04

A **linear model** helps identify trends in data. You can use a linear model to make a prediction.

For example, there are two ways you can use this linear model to predict a dog's height when it weighs 50 kilograms.

- Use the graph to locate 50 on the x-axis and follow it up to meet the linear model, which shows a y-value of 80. This means when the dog's weight is 50 kilograms, its height is 80 centimeters.
- Use the equation for the linear model, $y = 1.08x + 26$, by replacing x with 50 and evaluating for y, which is approximately 80 centimeters.

You can identify an **outlier** by looking for points that are far away from the other points and from the predicted values. The point $(40, 30)$ is an outlier on the graph of dog weights and heights.



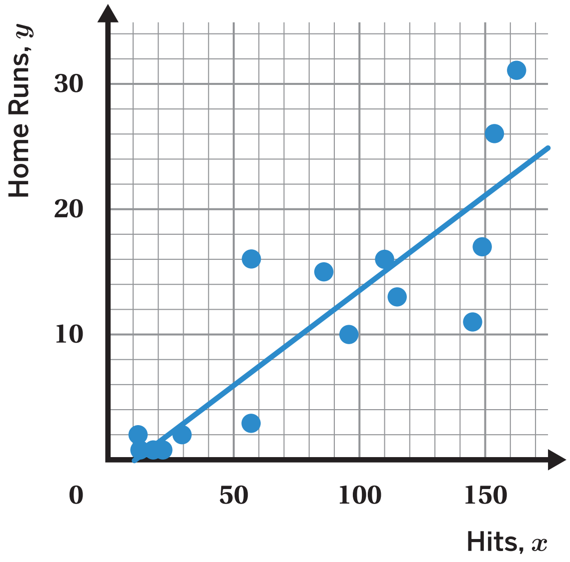
linear model (line of fit) A line that shows, or models, the general direction or trend of a group of points in a data set. We can use linear models to make predictions about values related to a given data set.

outlier A data value that is far from the other values in the data set.

Practice
6.04

Name: ______________________ Date: __________ Period: __________

Problems 1–3: This scatter plot shows the number of hits and home runs for 15 baseball players last season and the linear model $y = 0.15x - 1.5$.



1. How many home runs did the player with 154 hits have?

   26 home runs

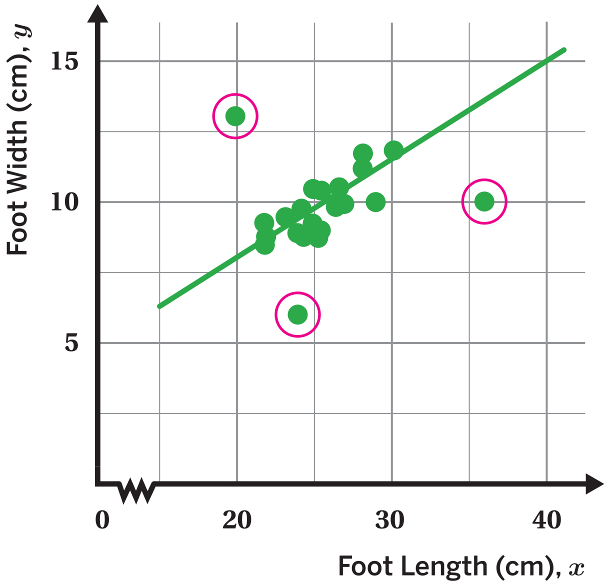
2. If a player has 20 home runs, how many hits does the linear model predict they will have?

   Responses between 141 and 145 are considered correct.

3. How many home runs does the linear model predict a player with 250 hits will have?

   36 home runs

Problems 4–5: This scatter plot shows several foot lengths and widths, along with a linear model represented by the equation $y = 0.35x + 1$.



4. Use the linear model's equation to predict the width of a foot that is 50 centimeters long.

   18.5 centimeters

5. Does the scatter plot appear to have any outliers? If so, circle them and describe what they represent about foot length and foot width.

   $(20, 13)$, $(24, 6)$, and $(36, 10)$ appear to be outliers. The point $(20, 13)$ shows a foot with a width much greater than predicted. The points $(24, 6)$ and $(36, 10)$ show feet with widths much smaller than predicted.

6. In your own words, what does an outlier represent?

   ***Responses vary.* An outlier represents a value that does not follow the data trend. It usually represents a measure that is much greater than or less than the rest of the data.**

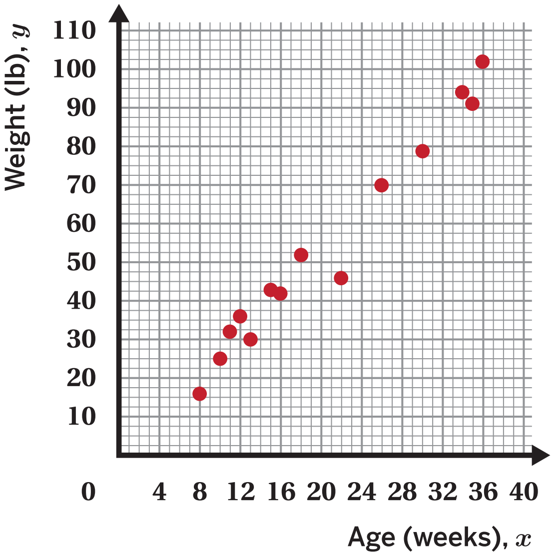
Practice
6.04

Name: .... Date: .... Period: ....

7. This scatter plot shows the weight and age for many dogs of a certain breed.

Which is the best prediction for the weight of a dog that is 24 weeks old?

- A. 10 pounds
- B. 45 pounds
- [✓ C.] 60 pounds
- D. 90 pounds



Spiral Review

Problems 8–9: Solve each system of equations. Write the solution as an ordered pair. Show your thinking.

8. $\begin{cases} y = -3x + 13 \\ y = -2x + 1 \end{cases}$

(12, -23) ***Work varies.***

$-3x + 13 = -2x + 1$ $\quad y = -2(12) + 1$

$13 = 1x + 1$ $\quad y = -24 + 1$

$12 = x$ $\quad y = -23$

9. $\begin{cases} y = x + 1 \\ y = -x + 5 \end{cases}$

(2, 3) ***Work varies.***

$x + 1 = -x + 5$ $\quad y = 2 + 1$

$2x + 1 = 5$ $\quad y = 3$

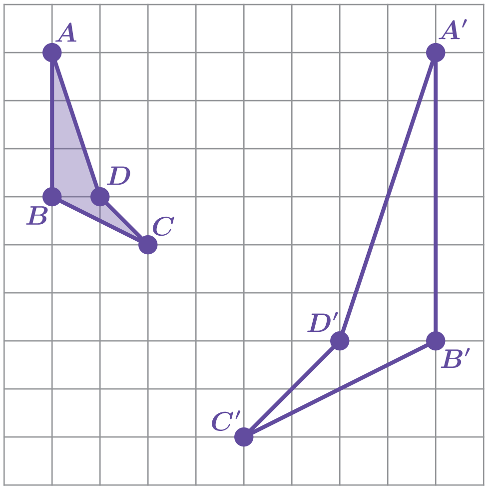
$2x = 4$

$x = 2$

Problems 10–11: Consider Polygon $ABCD$ and Polygon $A'B'C'D'$.

10. Describe a sequence of transformations that maps polygon $ABCD$ onto polygon $A'B'C'D'$.

Responses vary. **Reflect polygon $ABCD$ across a vertical line that passes through point C, then translate polygon $ABCD$ to the right 2 units and down 4 units. Then dilate the result using point C' as the center of dilation and a scale factor of 2.**



11. Which best describes polygon $ABCD$ and polygon $A'B'C'D'$?

- A. Congruent but not similar
- [✓ B.] Similar but not congruent
- C. Both similar and congruent
- D. Neither similar nor congruent

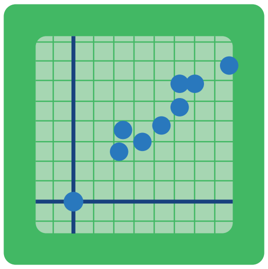
Unit 6
Lesson
5

Name: Date: Period:

Interpret Scatter Plots Slopes and Intercepts 8.SP.1, 8.F.4, SMP.2, SMP.3

Interpreting Scatter Plots

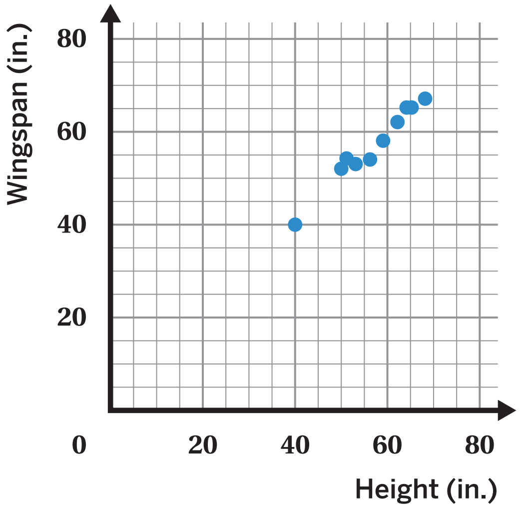
Let’s identify patterns in data.



Warm-Up

1. This table shows the height and wingspan data of 10 students.

Height (in.)	Wingspan (in.)
50	52
62	62
51	54
68	67
53	53
59	58
40	40
65	65
64	65
56	54



Sora and Saavani look for patterns in the data. They are trying to decide whether the point (40, 40) is an outlier.

- Sora claims that (40, 40) is an outlier.
- Saavani claims that (40, 40) is not an outlier

Discuss: Whose claim do you think is correct? Explain your thinking.

ELD.PI.8.3.Em, Ex, Br

Responses vary.

- I think Sora is correct. (40, 40) might be an outlier because the point is far from the data.
- I think Saavani is correct. (40, 40) might not be an outlier because the height and wingspan follows the trend of the data.

Activity
1

Name: ______________________ Date: __________ Period: __________

Scavenger Hunt

Start at any of the Scavenger Hunt Sheets.

- Record the sheet shape, solve the problem, and write your answer.
- Look for your answer at the top of another scavenger hunt sheet. Solve that problem.
- Repeat until you make it back to your starting sheet.

The sheet students starts with varies.

Sheet: Trapezoid

Work varies.

Answer 12,000 →

Sheet: Circle

Work varies.

↙ **Answer** C

Sheet: Pentagon

Work varies.

Answer B and D →

Sheet: Star

Work varies.

Answer 30 ↗

continued on next page...

Activity
1

Name: ____________ Date: ____________ Period: ____________

Scavenger Hunt (continued)

Sheet: Rectangle
Work varies.

Answer A →

Sheet: Triangle
Work varies.

↙ **Answer** 25

Sheet: Octagon
Work varies.

Answer D →

Sheet: Crescent
Work varies.

↙ **Answer** D and E

Sheet: Hexagon
Work varies.

Answer 9.16 →

Sheet: Oval
Work varies.

Answer B

Synthesis

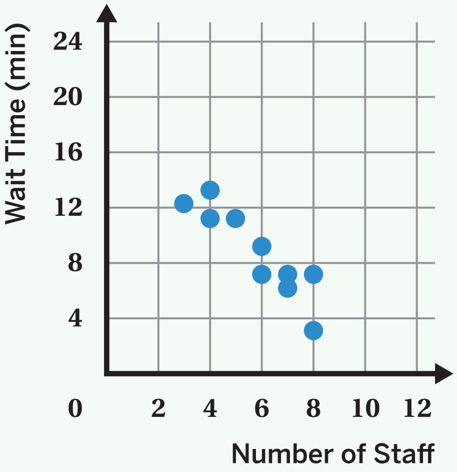
Discuss: How can a scatter plot help make sense of the relationship between two variables? **ELD.PI.8.1.Em, Ex, Br**

Responses vary. **Scatter plots help visualize how one variable generally affects the other.**

Summary 6.05

You can use a scatter plot to help identify patterns in data points and relationships between two variables.

For example, this scatter plot shows data about how long customers waited at a drive-thru restaurant and the number of staff working at that time.



The scatter plot shows both specific information and general trends, including:

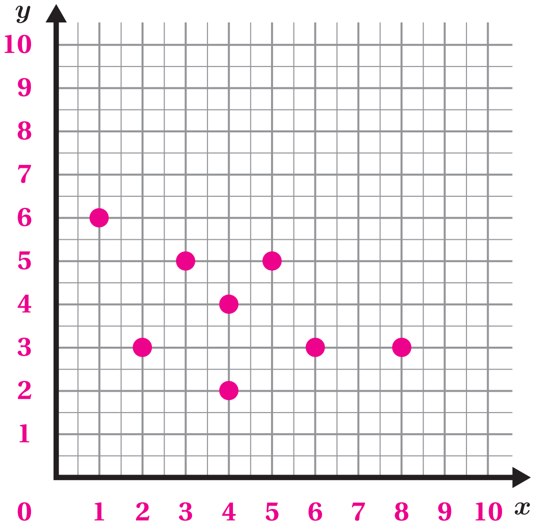
- When 3 staff were working, the wait time was about 12 minutes.
- The more staff there are, the shorter the wait time seems to be.

Name: ________________ Date: ________ Period: ________

Problems 1–2: Use the table.

x	y
2	3
5	5
4	2
8	3
6	3
3	5
1	6
4	4

1. Create a scale for the graph so it fits all the data. Then create a scatter plot of the data.

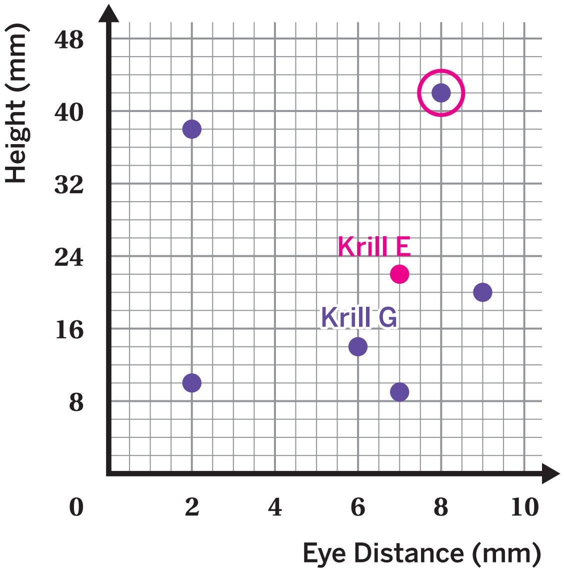


2. When do you think it is better to use a table to represent data? When do you think it is better to use a scatter plot?

Responses vary. **It is better to use a table when looking for precise values and details of the data. It is better to use a scatter plot when looking for an overall pattern (or the lack of an overall pattern).**

Problems 3–5: The table and scatter plot show the heights and eye distances of seven different krill (small shrimp-like crustaceans).

Krill	Eye Distance (mm)	Height (mm)
A	7	9
B	2	38
C	2	10
D	8	42
E	7	22
F	9	20
G	**6**	**14**



3. On the scatter plot, circle the point that represents the tallest krill.

Response shown on graph.

Practice
6.05

Name: ______________ Date: ______________ Period: ______________

4. On the scatter plot, plot a point that represents Krill E.

Response shown on graph.

5. Complete the table with the values that represent Krill G.

Response shown in table.

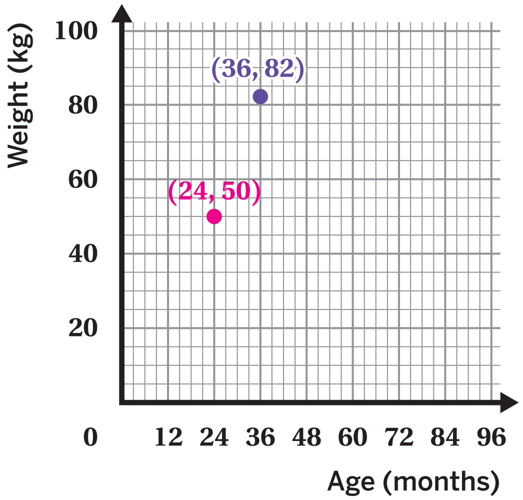
Problems 6–7: The graph shows the age and weight of a giant panda.

6. What does the point tell you about the panda?

***Responses vary.* The panda is 36 months old and weighs 82 kilograms.**

7. Plot a point to represent a different giant panda that is 24 months old and weighs 50 kilograms.

Response shown on graph.



Spiral Review

Problems 8–9: Solve each equation. Show your thinking.

8. $2(3 - 2c) = 30$

$c = -6$. *Work varies.*

$2(3 - 2c) \div 2 = 30 \div 2$

$3 - 2c = 15$

$3 - 2c - 3 = 15 - 3$

$-2c = 12$

$c = -6$

9. $3x - 2 = 7 - 6x$

$x = 1$. *Work varies.*

$3x - 2 + 6x = 7 - 6x + 6x$

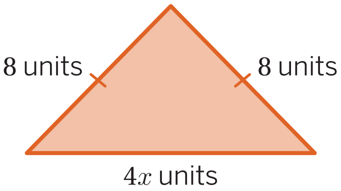
$9x - 2 = 7$

$9x - 2 + 2 = 7 + 2$

$9x = 9$

$x = 1$

10. The perimeter of the triangle is $10x$ units. Which equation represents the perimeter of the triangle?



A. $10x = 8 + 12x$

B. $10x = 4 + 16x$

C. $10x = 12 + 8x$

[✓ **D.**] $10x = 16 + 4x$

11. Determine whether the equation $4x + 9 = 8 + 1 - 4x$ has *one solution*, *no solutions*, or *infinitely many solutions*. Explain your thinking.

One solution. *Explanations vary.*

$4x + 9 = 8 + 1 - 4x$

$4x + 9 = 9 - 4x$

$8x + 9 = 9$

$8x = 0$

$x = 0$

Unit 6

Lesson

6

Name: ________ Date: ________ Period: ________

Interpret Scatter Plots Slopes and Intercepts 8.SP.1, 8.SP.2, 8.F.4, SMP.3, SMP.6

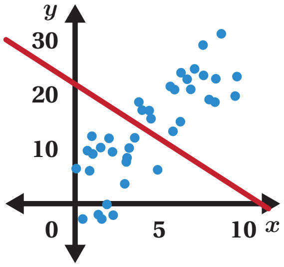
Find the Fit

Let's fit a line to data on a scatter plot.

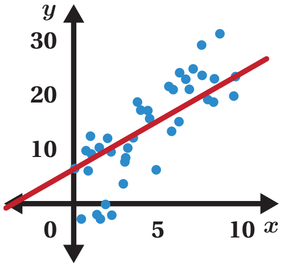
Warm-Up

1 **Data Talk!** Which one doesn't belong? Explain your thinking.

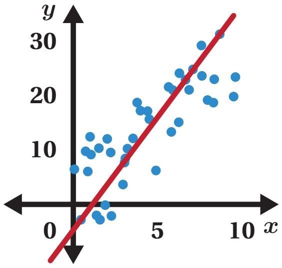
A



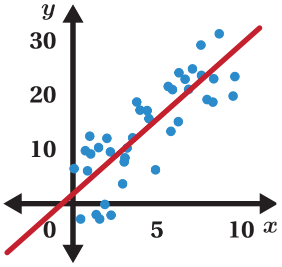
B



C



D



Responses vary.

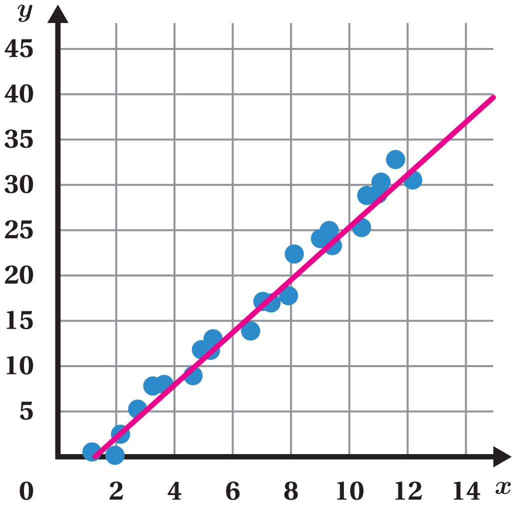
- Graph A doesn't belong because it is the only one where the line has a negative slope.
- Graph B doesn't belong because it is the only one where the line goes through the points that are furthest left and furthest right.
- Graph C doesn't belong because it is the only one where the line goes through the points that are lowest and highest.
- Graph D doesn't belong because it is the only one where the line goes right through the middle of the points and follows the trend of the data.

Activity
1

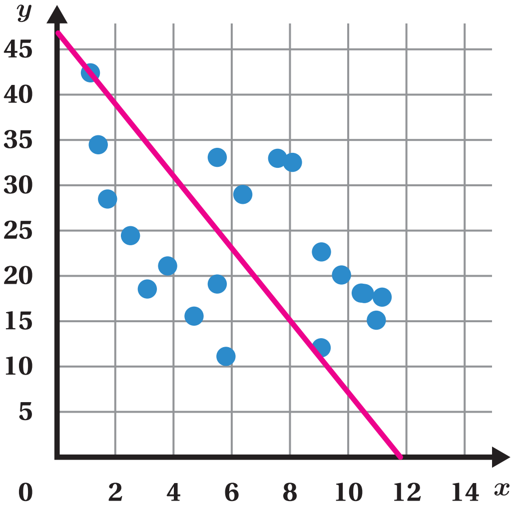
Name: .... Date: .... Period: ....

Lines of Fit

2 Create a line that is a good fit for each data set.



Responses vary. Sample shown on graph. An appropriate line created by students will have a positive slope and follow the trend of the data set.



Responses vary. Sample shown on graph. An appropriate line created by students will have a negative slope and follow the trend of the data set.

3 **Data Talk!** Let's look at some lines that other students created.

Discuss how you could decide if a line is a good fit for the data.

Responses vary.

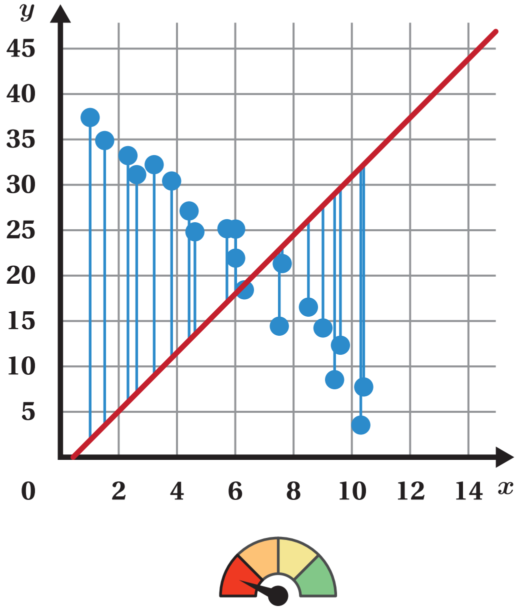
- A line is a good fit for the data if it's as close as possible to all the points.
- The slope should follow the trend of the data.

Name: .... Date: .... Period: ....

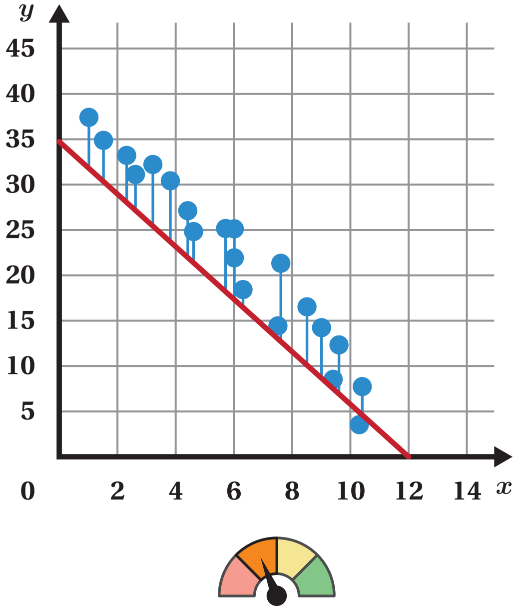
Lines of Fit (continued)

Here are four different lines for the same scatter plot. The meter shows a score for each line.

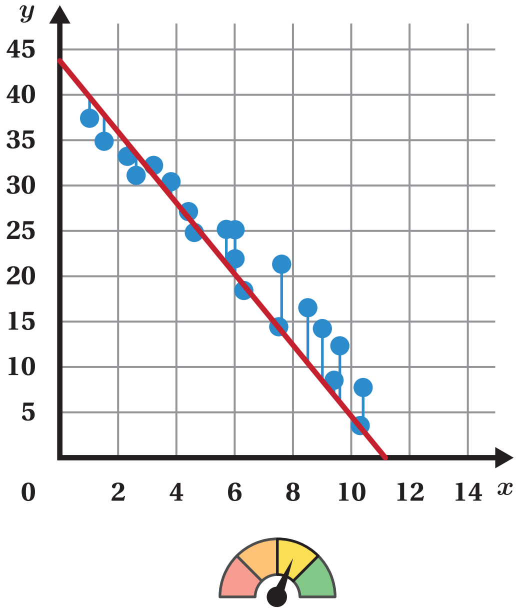
Graph A



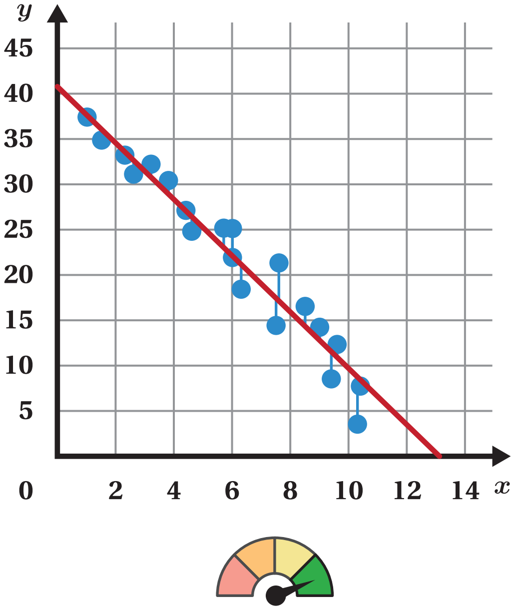
Graph B



Graph C



Graph D



4 Describe how to get a high score (green) on the meter.

Responses vary.

- Make a line that is as close to as many points as possible.
- The line should follow the trend of the data.
- The line should have a good balance of points that are above and below it.

Activity
2

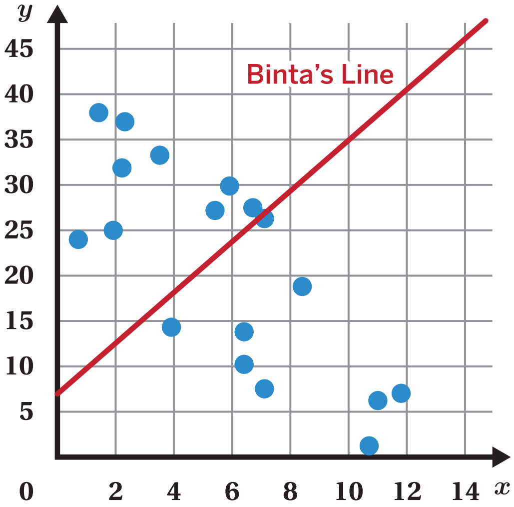
Name: ______________________ Date: __________ Period: __________

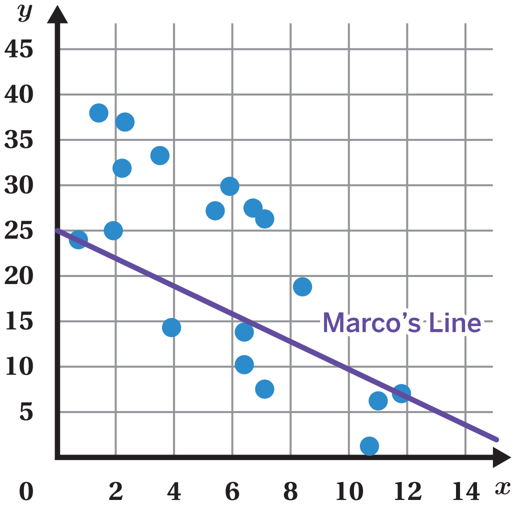
Find the Fit

5 **Data Talk!** Binta and Marco each sketched a *line of fit* on this scatter plot.

Binta says: *My line is a good fit because half of the points are on each side of the line*.

Marco says: *My line is a good fit because it passes through the leftmost and rightmost points*.





Whose line is a good fit for the data? Circle one.

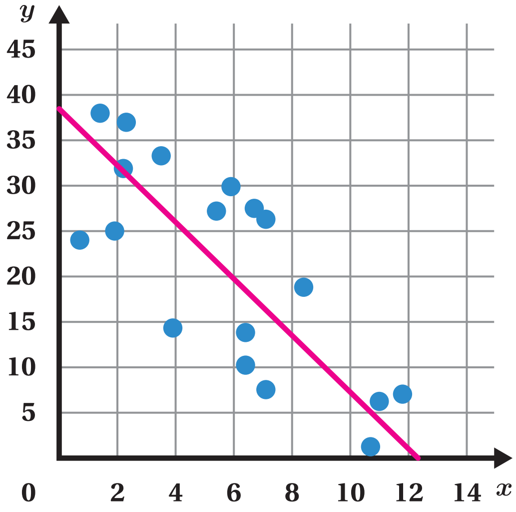
Binta's Marco's Both [✓ Neither]

Explain your thinking.

Explanations vary. **The data looks like it needs a line with a negative slope, but Binta's line has a positive slope. Marco's line is not a good fit either because most of the points are above it.**

6 Sketch a line that fits the data from the previous problem.

Responses vary. **Sample shown on graph.**



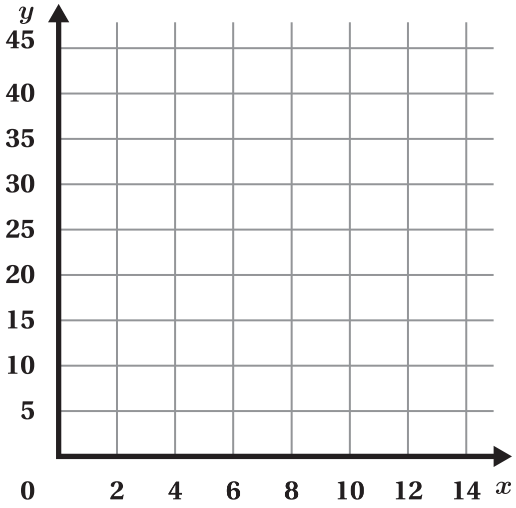
Name: .... Date: .... Period: ....

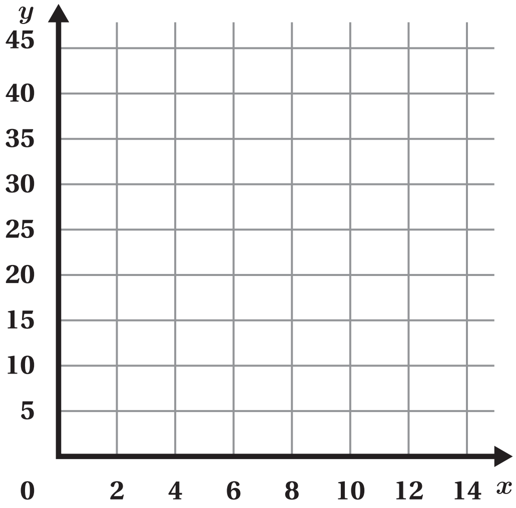
Challenge Creator

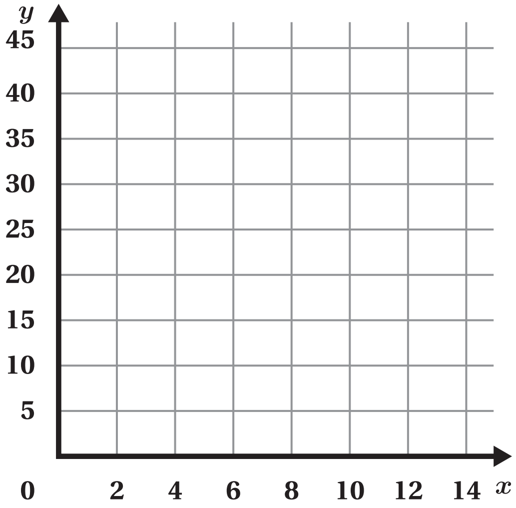
7 Follow these instructions to create scatter plots and solve your classmates' challenges.

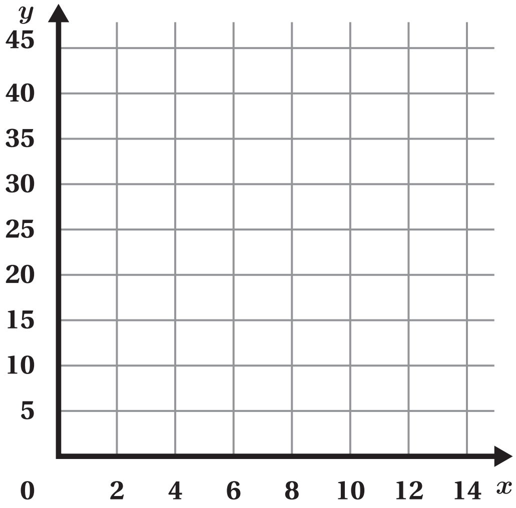
- **a** **Make It!** Create a scatter plot with at least ten points.
- **b** Pass your scatter plot to another student.
- **c** **Solve It!** Draw a line of fit for the data you received.
- **d** **Make It!** Create a new scatter plot in the next available space.
- **e** Repeat steps b–d until everyone has drawn lines for four scatter plots.

Responses vary.





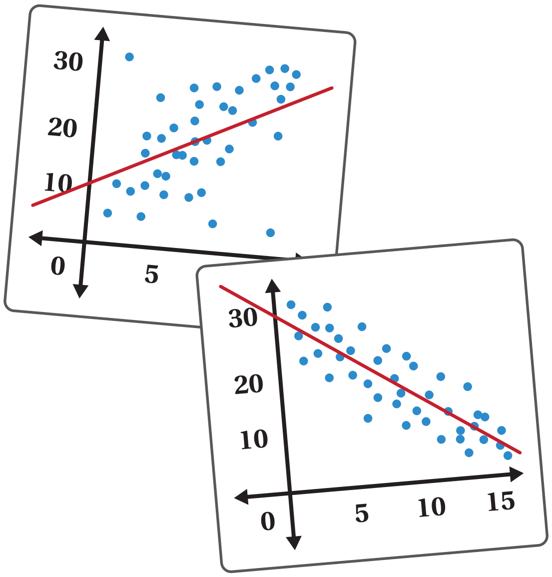




8 Synthesis

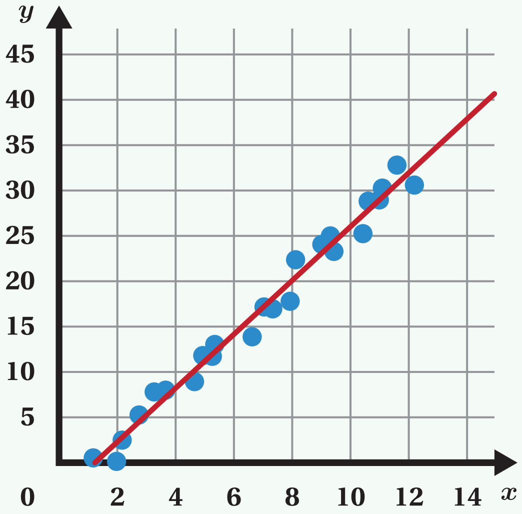
What are some things to consider when creating a line of fit? Use the examples if they help with your thinking.

***Responses vary.* Try to make the line go through the middle of the points and follow the trend of the data.**

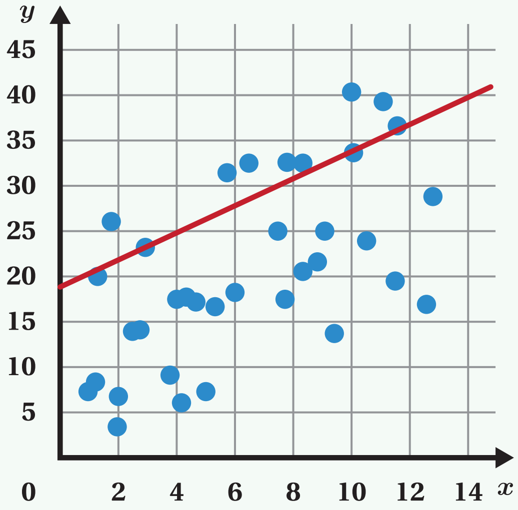


11 Summary 6.06

When creating a line of fit for a scatter plot, it's important to determine how well the line fits the data. A good line of fit follows the trend of the data, is as close to the plotted points as possible, and has about the same number of points above and below the line. The line may pass through some, all, or none of the points.



This line is a good fit for the data.

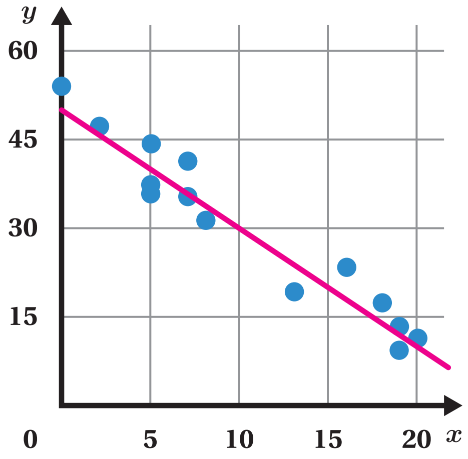


This line is not a good fit for the data.

Practice
6.06

Name: ______________________ Date: __________ Period: __________

Problems 1–2: Use this scatter plot.



1. Sketch a line of fit for the data.

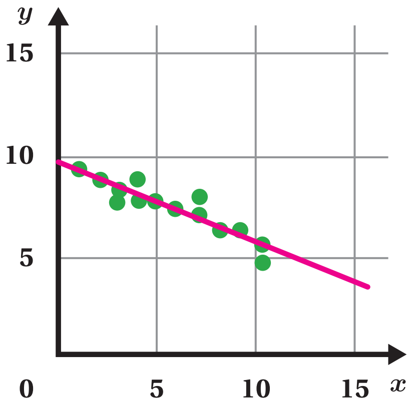
Responses vary. Sample shown on graph.

2. If a new data point has an x-value of 10, what does your line of fit predict for the value of y?

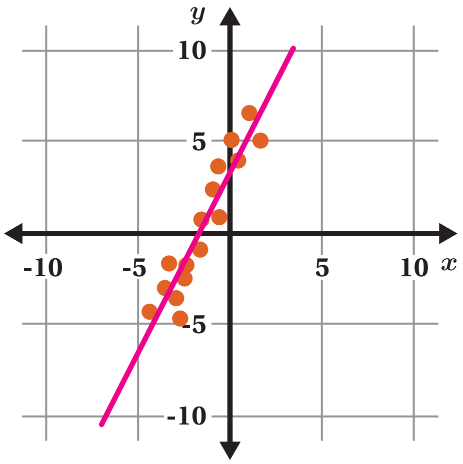
Responses between 25 and 35 are considered correct.

Problems 3–4: Sketch a line that fits the data. *Responses vary.* Samples shown on graphs.

3.

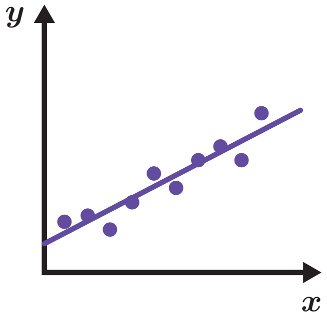


4.

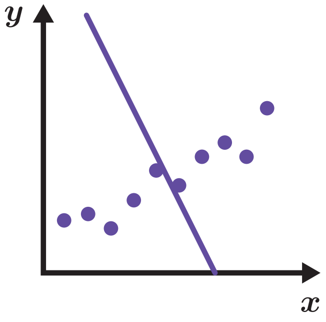


5. Which line best fits the data? Explain your thinking.

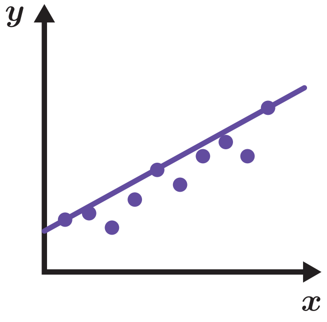
[✓ A.]



B.



C.

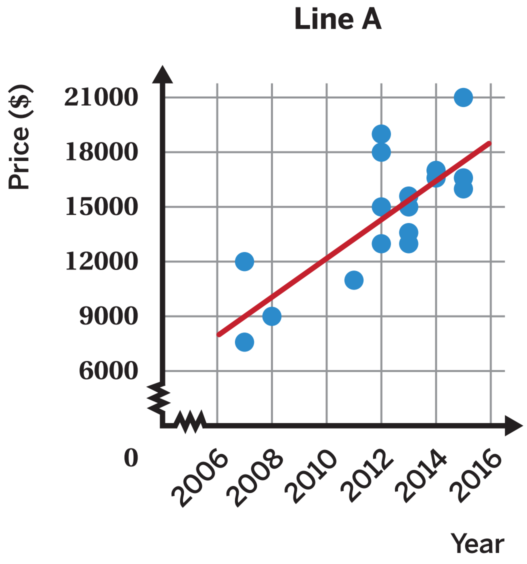


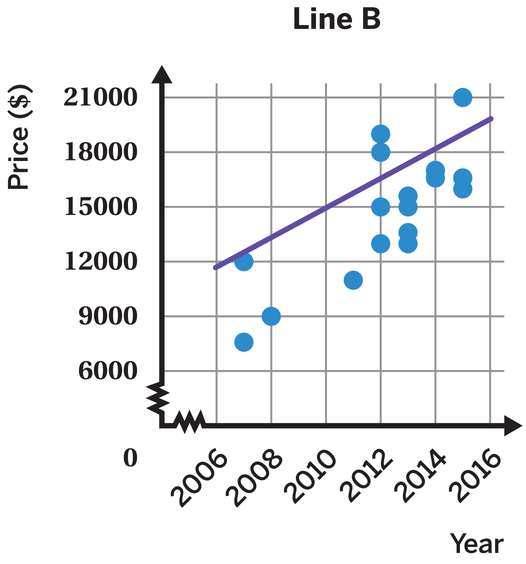
Explanations vary. The line in graph B balances points on either side of the line but doesn't follow the trend of the data. The line in graph C follows the positive trend of the data but doesn't balance the points on either side of the line. The line in graph A best fits the trend of the data and balances the data points on either side of the line.

Practice
6.06

Name: .... Date: .... Period: ....

6. Each line of fit applies to the same data.


Line A


Line B

Which line is a better fit for the data? **Line A**

Explain your thinking. ***Explanations vary.*** **Line A follows the trend of the data and has about half of the points above and below the line.**

Spiral Review

7. Match each equation with the scenario it represents.

Equation		Scenario
a $y = 3x$	c	A dump truck is hauling loads of dirt to a construction site. After 20 loads, there are 70 cubic feet of dirt.
b $\frac{1}{2}x = y$	a	You are making a water and salt mixture that has 2 cups of salt for every 6 cups of water.
c $y = 3.5x$	d	10 blueberries weigh 4 grams.
d $y = \frac{2}{5}x$	b	For every 48 cookies I bake, my students receive 24 cookies.

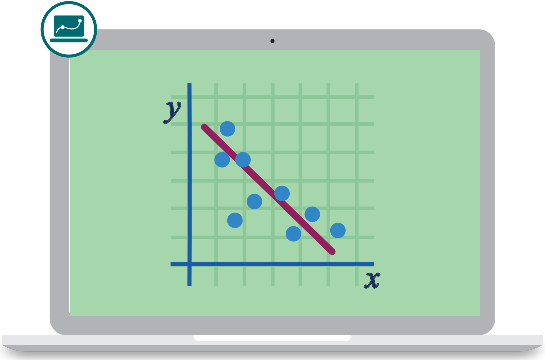
Name: .... Date: .... Period: ....

Interpret Scatter Plots | Linear Equations | Slopes and Intercepts

8.SP.1, 8.SP.2, 8.SP.3, 8.F.3, 8.F.4, SMP.2, SMP.6, SMP.7

Interpreting Slopes

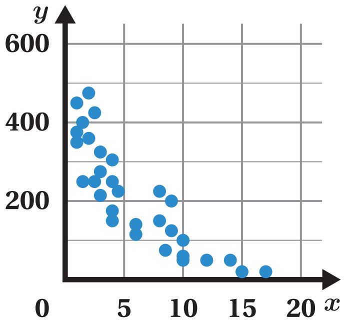
Let's identify different types of associations.



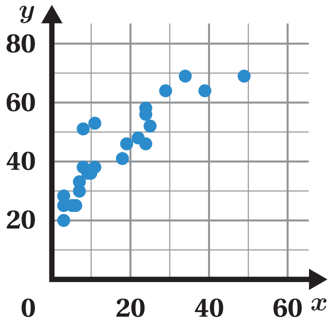
Warm-Up

1 **Data Talk!** Which one doesn't belong? Explain your thinking.

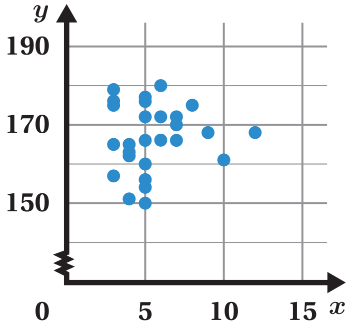
A.



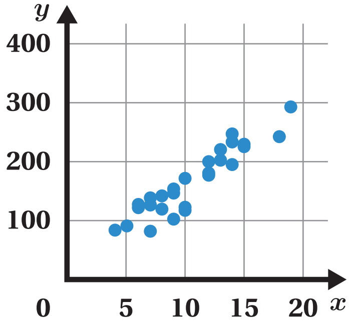
B.



C.



D.



Responses vary.

- Graph A doesn't belong because it is the only one where the points seem to decrease from left to right.
- Graph B doesn't belong because it is the only one where the x and y scales go by the same increment.
- Graph C doesn't belong because it is the only one where there is a break in the axis numbers, between 0 and 140 on the y-axis.
- Graph D doesn't belong because it is the only one where the points could be modeled by a line going through the origin.

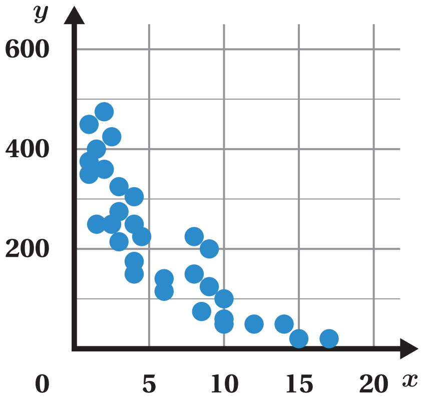
Activity
1

Name: .... Date: .... Period: ....

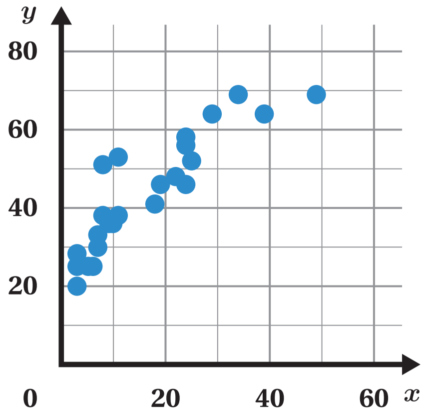
Associations

2 Match each scatter plot with the variables that it most likely represents.

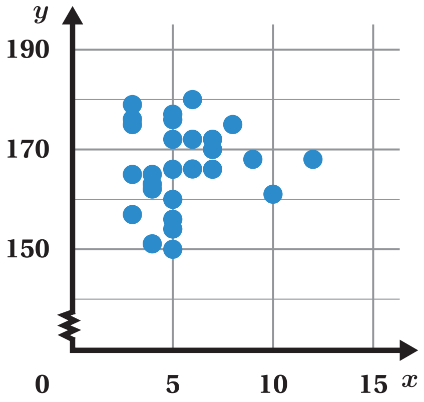
A.



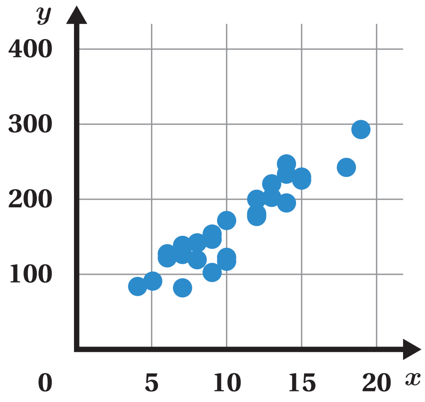
B.



C.



D.



x: Number of Floors
y: Building Height (ft)
D

x: Age of Bike (years)
y: Bike Price ($)
A

x: Dog Weight (kg)
y: Dog Height (cm)
B

x: Letters in Name
y: Height (cm)
C

3 **Data Talk!** How did you decide which scatter plot matches these variables?

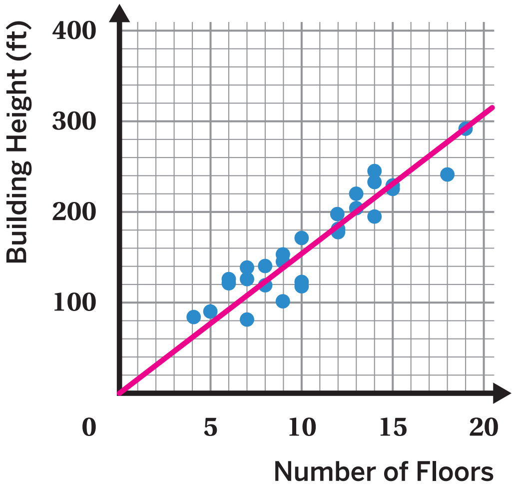
x: Number of Floors
y: Building Height (ft)

Responses vary. **I assumed that buildings with more floors are taller than those with fewer floors, so I chose a scatter plot where the points trend upward from left to right.**

Name: .... Date: .... Period: ....

Associations (continued)

4 **Data Talk!** Here is the scatter plot that shows data from some buildings.



a Sketch a linear model to fit the data.

Responses vary. Sample shown on graph.

b Esi sketched a linear model whose equation is $y = 13x + 30$.

Discuss:

- What is the *slope* of their line?
- What does the slope represent in this situation?

13. *Responses vary.* For every additional floor, the building's height increases by 13 feet.

5 An **association** is a relationship between two variables. There is a positive association if both variables increase together and a negative association if one variable decreases as the other increases.

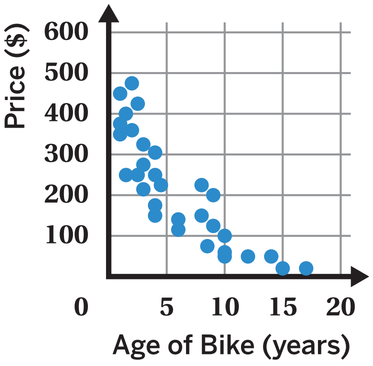
What type of association is there between building height and number of floors? Circle one.

[✓ Positive association] Negative association No association

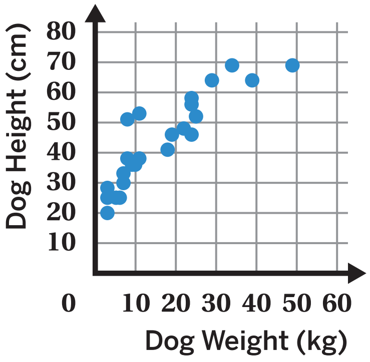
Explain your thinking.

Explanations vary. As the number of floors increases, the height of the building increases.

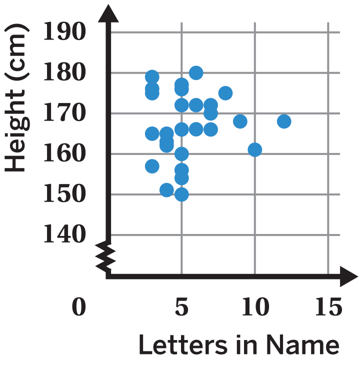
6 Determine what type of association each scatter plot shows. Discuss your thinking.



Negative association



Positive association

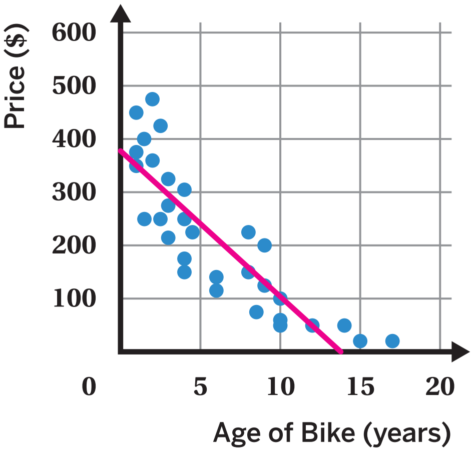


No association

Name: .... Date: .... Period: ....

Interpretations

7 **Data Talk!** Here is the scatter plot that shows the prices and ages of some used bikes.



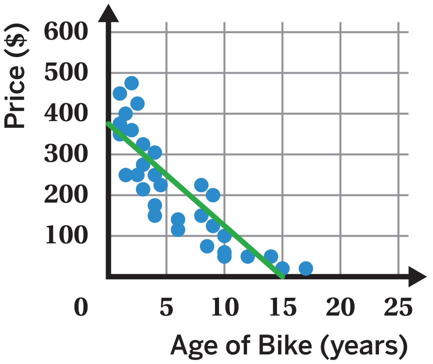
a Sketch a linear model that fits the data.

***Responses vary.* Sample shown on graph.**

b **Discuss:** How can you tell from the linear model that there is a negative association between bike age and price?

***Responses vary.* The linear model suggests a negative association because the line has a negative slope.**

8 Troy drew a linear model for the bike data.



a What is the equation of the line of fit that Troy drew?

$y = -25x + 375$

b Use Troy's model to finish this sentence:

The model predicts that as the age of a bike increases by 1 year:

A. The price will increase by $25.

[✓ B.] The price will decrease by $25.

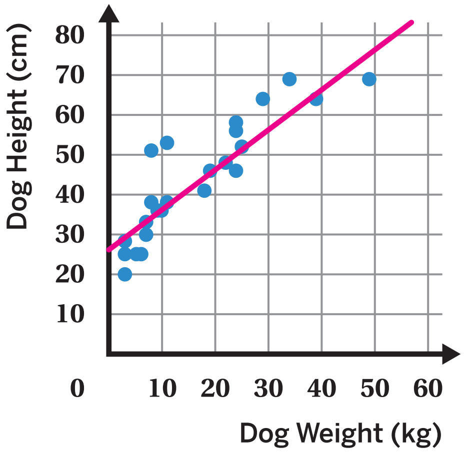
C. The price will increase by $375.

D. The price will decrease by $375.

Name: .... Date: .... Period: ....

Interpretations (continued)

9 Here is the scatter plot that shows the heights and weights of some dogs.



a Sketch a linear model that fits the data.

***Responses vary.* Sample shown on graph.**

b Nikolai sketched a linear model whose equation is $y = 1.2x + 28$.

Identify the slope of Nikolai's model and describe what it means in this situation.

1.2. *Responses vary.* This means that as a dog's weight increases by 1 kilogram, the dog's height is predicted to increase by 1.2 centimeters.

You're invited to explore more.

10 Fuel efficiency measures the number of miles a car can go using one gallon of gas (miles per gallon). This scatter plot shows the relationship between fuel efficiency and weight for 20 vehicles.

Write three statements about this scatter plot — two that are true and one that is a lie.

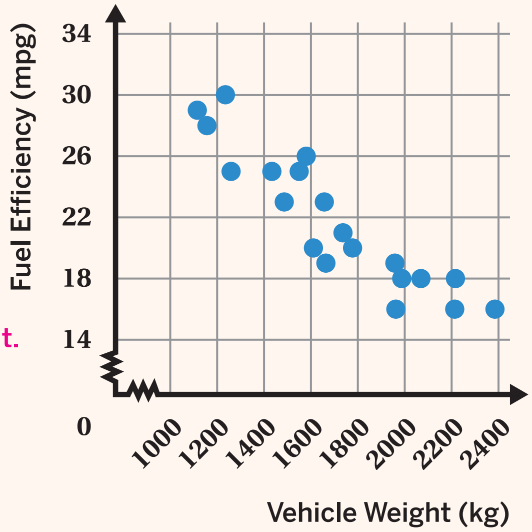
Sketch a line of fit if it helps with your thinking.

Responses vary.

Truth: As vehicle weight increases, fuel efficiency decreases.

Truth: There is a negative association between fuel efficiency and vehicle weight.

Lie: If the weight of the vehicle increases by 1 kilogram, the fuel efficiency decreases by 2 miles per gallon.

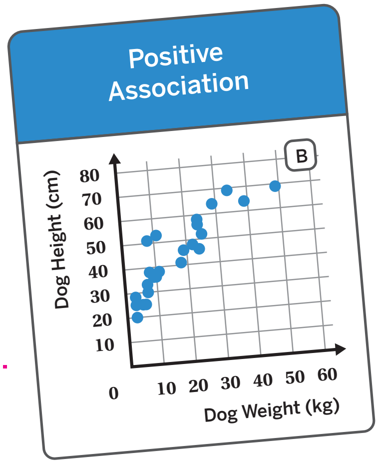


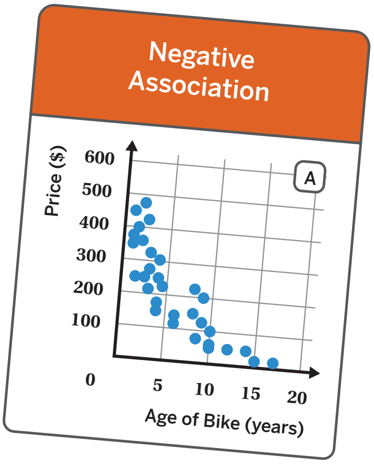
11 Synthesis

Data Talk! Discuss the following:

- What are some clues that a scatter plot might have a positive or negative association?

  Responses vary. **A scatter plot has a positive association if one quantity increases as the other increases. The data points will trend up and to the right, and a linear model for the data would have a positive slope. A scatter plot has a negative association if one quantity decreases as the other increases. The data points will trend down and to the right, and a linear model for the data would have a negative slope.**



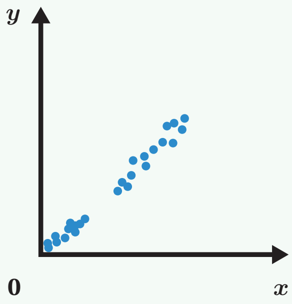


- What does the slope of a linear model tell you about the data?

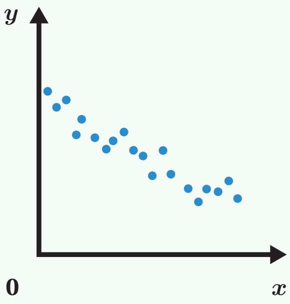
  Responses vary. **The slope of a linear model tells you what the model predicts as the quantity on the x-axis increases by 1.**

14 Summary 6.07

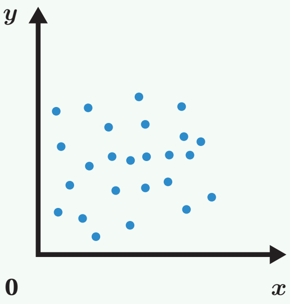
If two variables on a scatter plot are related, there is an **association**. The *slope* of a linear model can help determine the type of association. A positive association means that when one variable increases, the other also increases. A negative association means that when one variable increases, the other decreases. If the scatter plot shows no clear trend between the two variables, then the variables have no association.



Positive association



Negative association



No association

association When two variables are related to one another. Associations can be described as positive or negative, linear or non-linear.

Practice
6.07

Name: ________________ Date: ________ Period: ________

1. Which type of association does the scatter plot show?

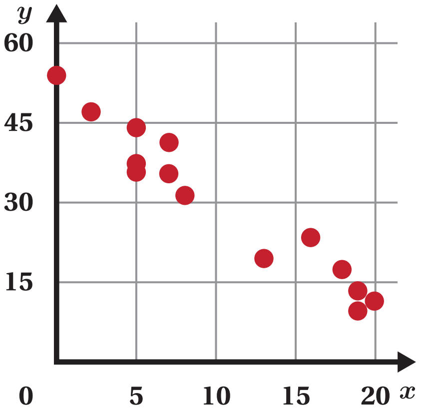
A. Positive association

[✓ B.] Negative association

C. No association

Explain your thinking.

***Explanations vary.* The scatter plot shows a negative association because as x increases, y decreases.**



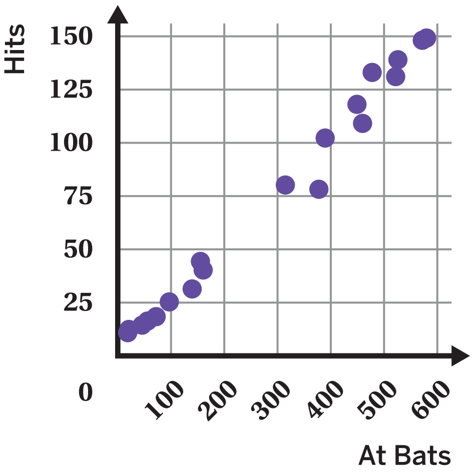
2. The scatter plot shows the number of hits and at bats for players on a baseball team. Which conclusion is best supported by the scatterplot?

[✓ A.] As the number of at bats increases, the number of hits also increases.

B. As the number of at bats increases, the number of hits decreases.

C. As the number of hits increases, the number of at bats remains the same.

D. There is no relationship between the number of at bats and the number of hits.



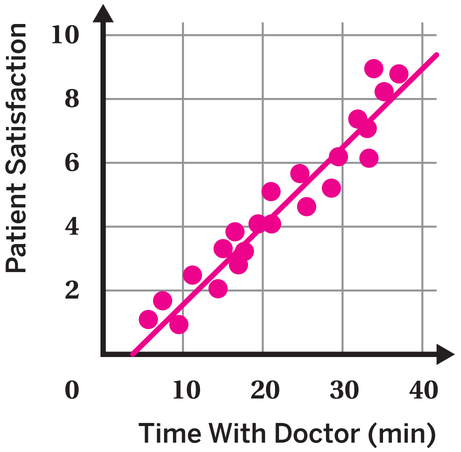
Problems 3–5: The doctors in a medical clinic looked at the relationship between patient satisfaction on a 0–10 scale and the number of minutes spent with a doctor. They found the variables had a positive association.

3. What does this positive association mean about the relationship between patient satisfaction and time with a doctor?

As time with the doctor increases, the patient's satisfaction also increases.

4. Create a scatter plot that represents this situation.

***Responses vary.* Sample shown on graph.**



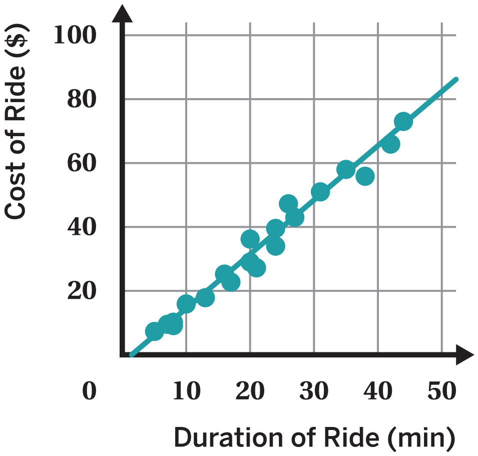
5. Sketch a line of fit for the data. How are the slope of the line and the association of the scatter plot similar?

***Responses vary.* The slope and the association of the scatter plot are both positive.**

Practice
6.07

Name: ______ Date: ______ Period: ______

Problems 6–8: The scatter plot shows the data from 20 taxi rides in Austin, Texas, along with a linear model whose equation is $y = 1.7x - 2.5$.



6. What is the slope of the linear model?

1.7

7. What does the slope represent in this situation?

For every minute added to the duration of the taxi ride, the cost of the ride increases by $1.70.

8. What type of association is there between duration and cost of a taxi ride?

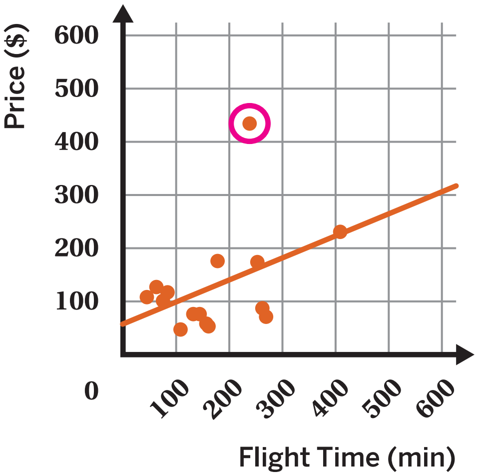
[✓ Positive association] Negative association No association

Explain your thinking.

***Explanations vary.* As the duration of the ride increases, the cost of the ride is expected to increase.**

Spiral Review

Problems 9–10: The scatter plot shows the flight time and price for several different flights from O'Hare Airport in Chicago.



9. Circle any data point(s) that appear to be outliers.

Response shown on graph.

10. Use the linear model to estimate the price of a 10-hour flight from O'Hare Airport.

Responses between $300 and $315 are considered correct.

11. The equation $y = 5280x$ gives the number of feet, y, in x miles. What does the number 5280 represent in this relationship?

***Responses vary.* There are 5280 feet in every mile. Each additional mile that someone travels is equivalent to traveling an additional 5280 feet.**

12. Solve this system of equations. Write your answer as an ordered pair (x, y).

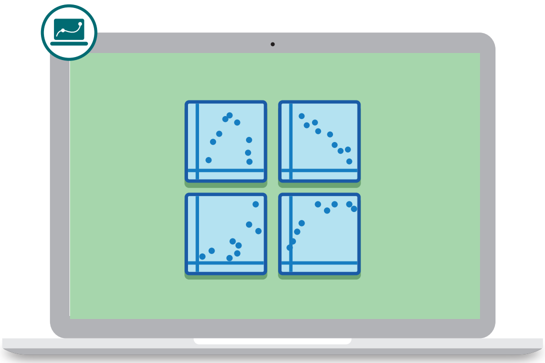
$$\begin{cases} y = -5x + 2 \\ y = -4x - 3 \end{cases}$$

(5, -23)

Name: .... Date: .... Period: ....

Interpret Scatter Plots | Linear Equations | Slopes and Intercepts

8.SP.1, 8.F.3, 8.F.4, 8.F.5, SMP.6



Scatter Plot City

Let's use precise language to describe the trends in a scatter plot.

Warm-Up

1 Play a few rounds of Polygraph with your classmates!

You will use a Warm-Up Sheet with scatter plots for four rounds. For each round:

- You and your partner will take turns being the Picker and the Guesser.
- Picker: Select a scatter plot from the Warm-Up Sheet. Keep it a secret!
- Guesser: Ask the Picker yes-or-no questions, eliminating scatter plots until you're ready to guess which scatter plot the Picker chose.

Record helpful questions from each round in the space below.

Responses vary.

Name: .... Date: .... Period: ....

Scatter Plot City

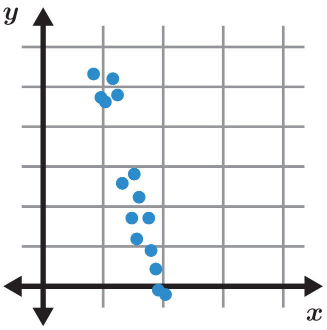
2 **Data Talk!** You will use the Activity 1 Sheet to see some scatter plots from the Polygraph and some terms that describe them.

Discuss what each term means. ***Responses vary.***

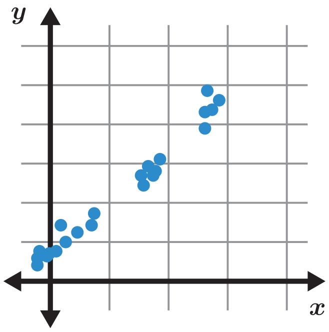
- *Linear* association
  When a straight line can model the data on a scatter plot.
- *Non-linear* association
  When a straight line cannot model the data on a scatter plot.
- With **clusters**
  When there is a grouping of data points around the same value.
- Without clusters
  When there is no grouping of data points around the same value.

Here are six new scatter plots.

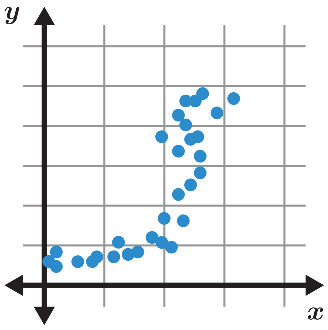
A.



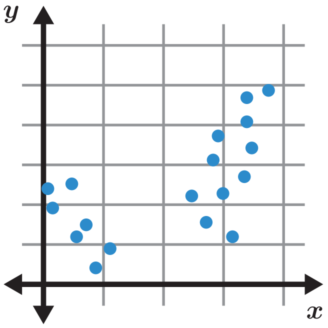
B.



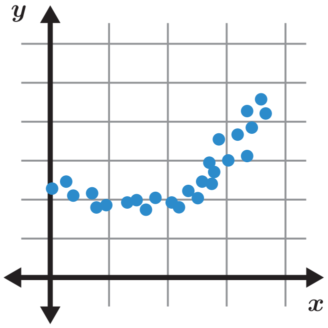
C.



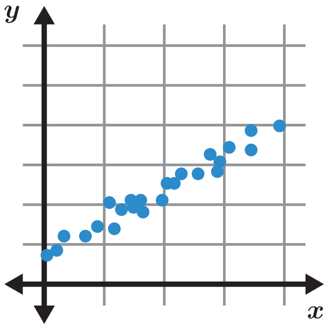
D.



E.



F.



3 Sort them according to their type of association.

Linear Association	Non-Linear Association
A, B, F	C, D, E

4 Sort them in a different way: those with clusters and those without.

With Clusters	Without Clusters
A, B, D	C, E, F

Activity

2

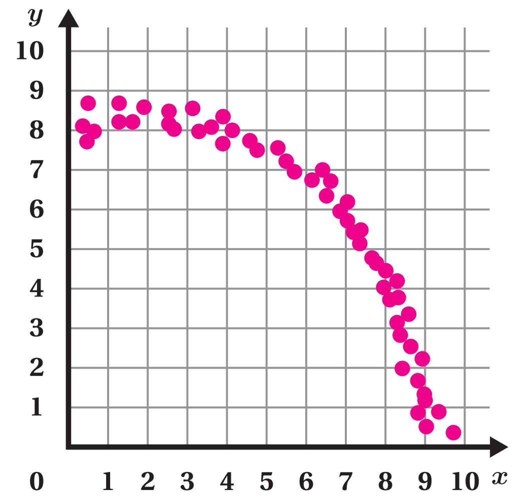
Name: Date: Period:

Putting It All Together

5 Data Talk!

a Create a scatter plot that has a negative non-linear association, without clusters.

Responses vary. Sample shown on graph.



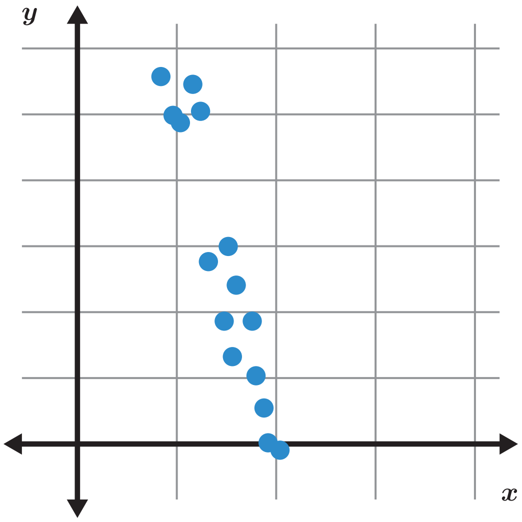
b Compare your scatter plot with your partner's.

6 Here is one of the scatter plots from before.

Describe the scatter plot using vocabulary from this unit.

positive association	negative association	clusters
linear association	non-linear association	outlier

Responses vary. This scatter plot has a negative linear association, with two clusters of points.

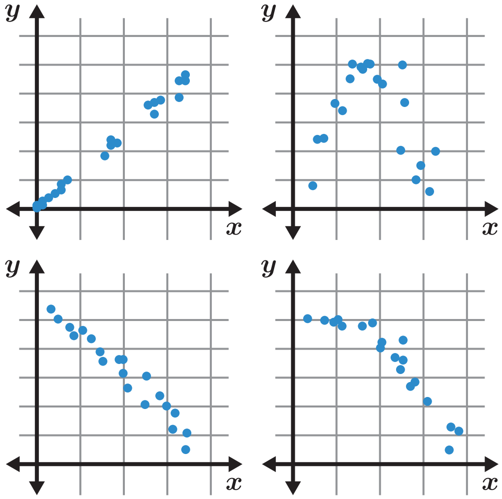


7 Synthesis

Data Talk! Discuss how you can identify a non-linear association or clusters in a scatter plot. Use the examples if they help with your thinking.

Responses vary.

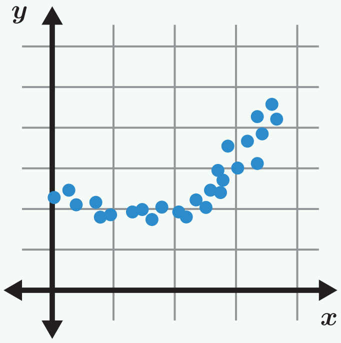
- **Non-linear association: A scatter plot has a non-linear association when there isn't a line that is a good fit for the data.**
- **Clusters: A scatter plot has clusters if the points are gathered together in groups.**



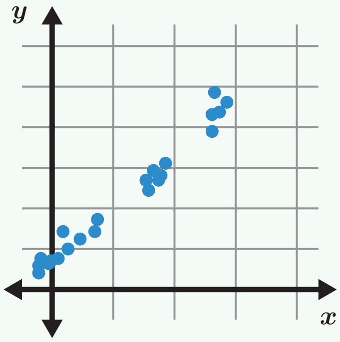
10 Summary 6.08

When you can model data on a scatter plot with a straight line, we say it has a *linear* association. Data showing a clear pattern that can't be modeled by a straight line has a *non-linear* association. Sometimes groups of data points appear close together, which are called **clusters**.

This scatter plot is an example of a non-linear association, without clusters.



This scatter plot is an example of a linear association, with clusters.

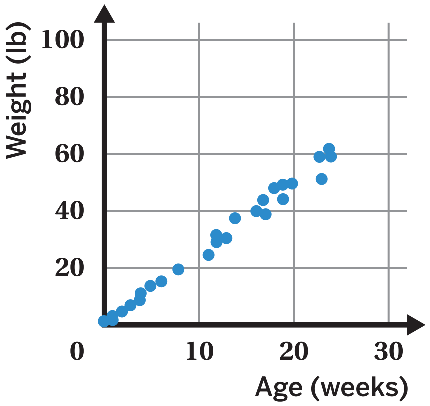


clusters Groups of data values that are close together.

Practice
6.08

Name: ____________________ Date: __________ Period: __________

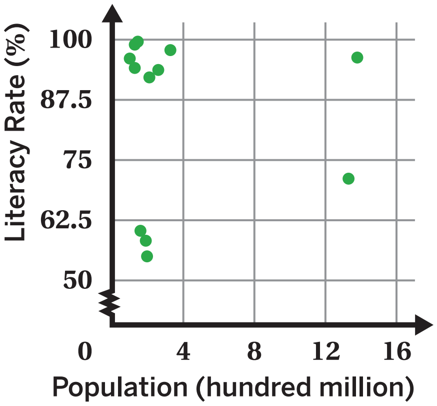
1. The graph shows the age and weight of babies in a nursery. Select *all* the terms that describe the association on the scatter plot.



- ☑ **A.** Linear association
- ☐ **B.** Non-linear association
- ☑ **C.** Positive association
- ☐ **D.** Negative association
- ☐ **E.** No association

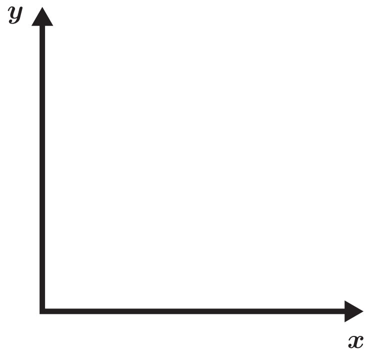
2. Decide whether there are clusters in the scatter plot showing the literacy rate and population for 12 countries. Explain your thinking.

***Responses vary*. There are clusters of points for countries with populations between 0 and 4 hundred million.**



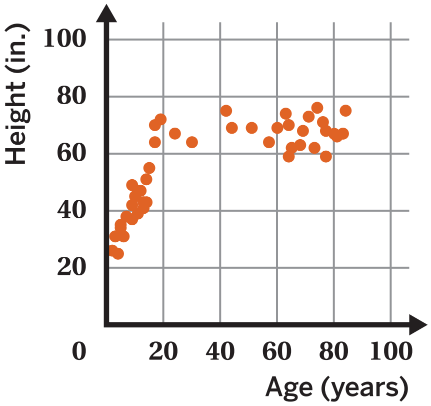
3. Create a scatter plot that has a positive linear association, with clusters.

***Responses vary*. Scatter plots should show points whose y-values generally increase as the x-values increase, and there should be at least one grouping of points.**



4. A doctor in a small town sees patients of all ages. Use the scatter plot of her patient's ages and heights to decide if each statement is true or false. Select True or False for each statement.

	True	False
There is a non-linear association between the ages and heights of the doctor's patients.	✓	
As patients' ages increase, their heights tend to decrease.		✓

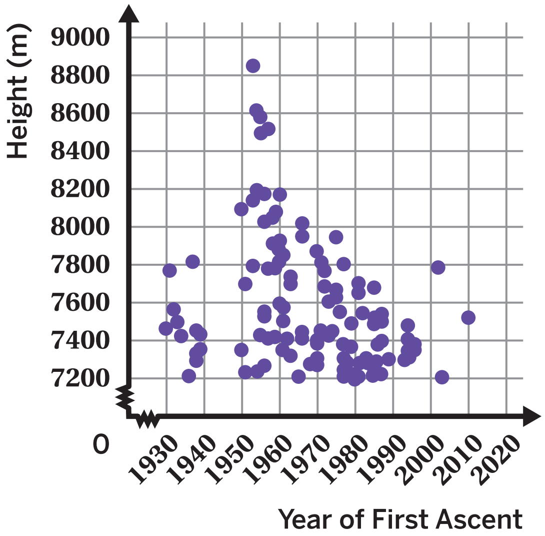


Practice
6.08

Name: ______________________ Date: __________ Period: __________

Spiral Review

5. The scatter plot shows data for some of the tallest mountains on Earth. Which of the following terms best describes the association between the heights of the mountains and years of first recorded ascent?



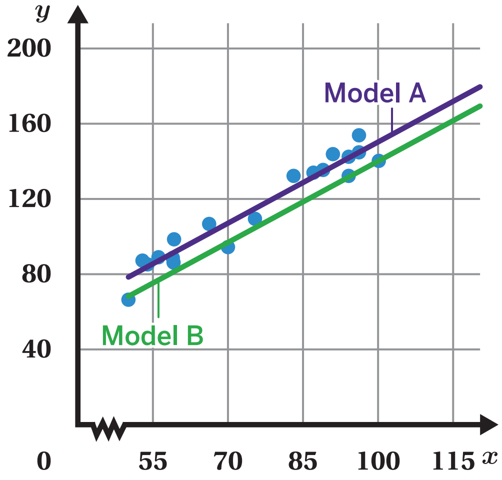
A. Linear association

B. Positive association

C. Negative association

[✓ **D.**] No association

Problems 6–7: Here are two different linear models for the same data.



6. Which model is a better fit for the data? Explain your thinking.

Model A. *Explanations vary.* In Model B, most of the points are above the line in the graph. In Model A, the points are more evenly arranged around the line.

7. Using the line of better fit, what is the approximate x-value for a point with a y-value of 120?

Responses should be between 78 and 83.

8. The points $(2, 4)$ and $(6, 7)$ fall on a line. What is the slope of the line?

A. 1 **B.** 2 **C.** $\frac{4}{3}$ [✓ **D.**] $\frac{3}{4}$

Problems 9–10: A cone has a volume of V cubic units.

9. Another cone has the same height and $\frac{1}{3}$ of the radius of the original cone. Write an expression for its volume.

$\frac{V}{9}$ cubic units (or equivalent)

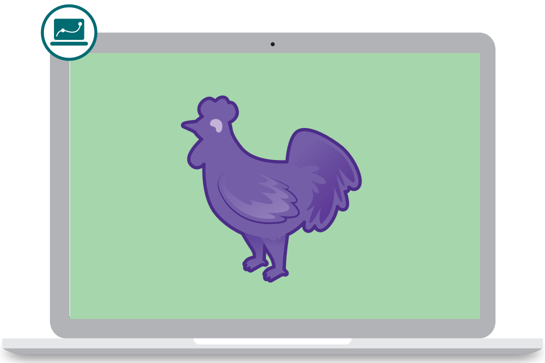
10. Another cone has the same height and 3 times the radius of the original cone. Write an expression for its volume.

$9V$ cubic units (or equivalent)

Name: .... Date: .... Period: ....

Interpret Scatter Plots | Data, Graphs, and Tables | Linear Equations | Slopes and Intercepts

8.SP.1, 8.SP.2, 8.SP.3, 8.F.3, 8.F.4, SMP.2, SMP.6



Animal Brains

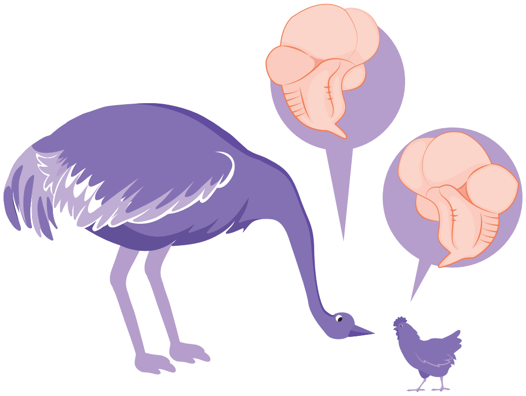
Let's analyze bivariate data.

Warm-Up

1 **Data Talk!** Do you think heavier animals have heavier brains? Explain your thinking.

Responses vary.

- **Yes, bigger animals will have bigger brains.**
- **No, sometimes smaller animals can have bigger brains than animals that weigh more.**



2 What information would help you determine whether heavier animals have heavier brains?

Responses vary. **Having data about the body weight and brain weight of several different animals would be helpful.**

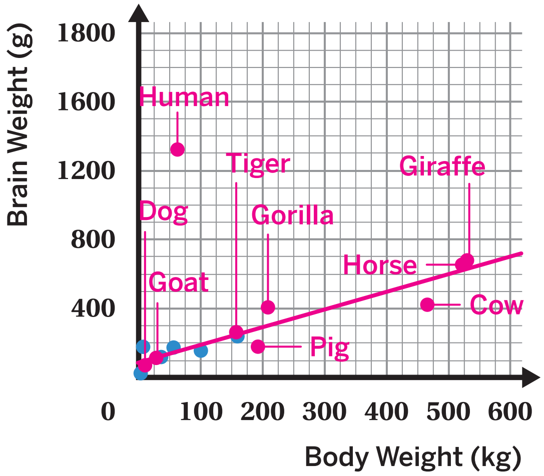
Activity
1

Name: ______________________ Date: __________ Period: __________

Creating a Scatter Plot

3 The scatter plot shows the body weight and brain weight of 6 different animals. The table shows the weights of 4 more animals. Plot and label a point on the graph to represent each animal's data from the table.

Animal	Body Weight (kg)	Brain Weight (g)
Giraffe	529	680
Tiger	157	264
Goat	28	115
Cow	465	423



Response shown on graph for Screens 3, 5, 6, and 8.

4 **Data Talk!** Look back at your prediction from the Warm-Up. Based on the scatter plot, what type of association does there appear to be between brain weight and body weight? Circle one.

[✓ Positive association]   Negative association   No association

Explain your thinking.

***Explanations vary*. There appears to be a linear association between brain and body weight, and a linear model with a positive slope would fit the data.**

5 The table shows the body weight of three more animals.

Plot and label points on the graph to predict the brain weight of each animal.

Complete the table with your predictions.

***Responses vary*. Actual values shown on graph and in table.**

Animal	Body Weight (kg)	Brain Weight (g)
Dog	10	**72**
Pig	192	**180**
Horse	521	**655**

Name: .... Date: .... Period: ....

Line of Fit

Fitting a line to data can help make predictions more accurate.

6 Draw a line that fits the data on the scatter plot on the previous page.

***Responses vary.* Sample shown on graph on the previous page.**

7 **Data Talk!** The equation for Inola's line of fit is $y = 0.9x + 79$.

a What is the *slope* of the line? What is the y-*intercept*?

Slope: **0.9**

y-intercept: **79**

b **Discuss:**

- What does each number mean in this situation?
- Do these values make sense in this situation?

Responses vary.

- **The slope of the line is 0.9, which means that for every additional kilogram of body weight, the predicted brain weight increases by 0.9 grams. This makes sense because larger animals tend to have larger brains.**
- **The y-intercept of the line is 79, which means that an animal with a body weight of 0 kilograms will have a predicted brain weight of 79 grams. This doesn't make sense because if the body weighs nothing, how could the brain weigh anything?**

8 Use your line of fit to predict the brain weight for a gorilla and a human. Plot and label points on the previous page to show your predictions.

***Responses vary.* Actual values shown in table.**

Animal	Body Weight (kg)	Brain Weight (g)
Gorilla	207	**406**
Human	62	**1320**

Name: .... Date: .... Period: ....

Line of Fit (continued)

9 **Data Talk!** Let's look at a scatter plot that shows body and brain weight data, including the data for the gorilla and human.

What do you notice? What do you wonder?

Responses vary.

I notice:	I wonder:
• **I notice that the point for the human is an outlier.** • **I notice there are two different clusters of points and one point that's really far away from the others.**	• **I wonder what animals' brains might be outliers in the direction opposite to humans.**

You're invited to explore more.

10 *Tyrannosaurus rex* (T. rex) is a dinosaur with an estimated body weight of 8,000 kilograms.

a Based on your line of fit from Activity 2, how much might a T. rex's brain weigh?

***Responses vary.* 7,735 grams**

b Do you think the point representing the actual brain weight of a T. rex will be above or below the line of fit?

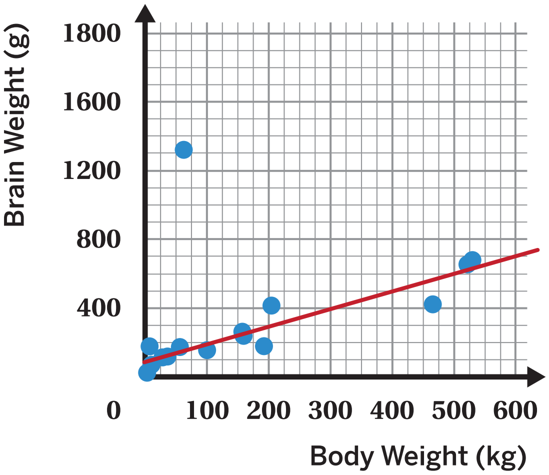
***Responses vary.* I think the T. rex's intelligence is below average, so the point would fall below the line.**

11 Synthesis

Data Talk! Describe an advantage and disadvantage of using a line of fit to make predictions.

Use the graph if it helps with your thinking.

***Responses vary*. An advantage of using a line of fit is that it helps me make reasonable predictions about where new points might be. A disadvantage is that it's not precise. Even if my prediction is reasonable, it might be wrong.**

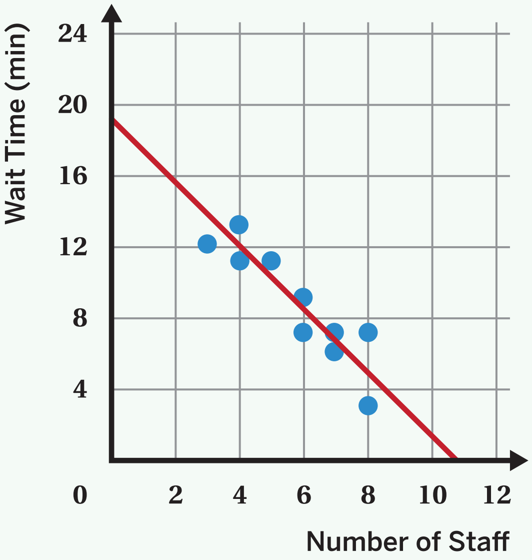


14 Summary 6.09

By understanding the association between two variables, you can make predictions about unknown values. When there's a linear association, using a linear model can often make predictions more accurate.

For example, this scatter plot shows data about how many minutes customers waited at a drive-through restaurant and the number of staff working at that time. This data can be modeled by the equation $y = -1.75x + 19$.

- The *slope* of the linear model is -1.75, which means that if the number of staff increases by 1 person, the wait time decreases by 1.75 minutes.
- The linear model predicts that if there are 2 staff working, the wait time will be approximately 15.5 minutes.
- But the linear model also predicts that when there are 0 staff working, the wait time will be 19 minutes, which is impossible!



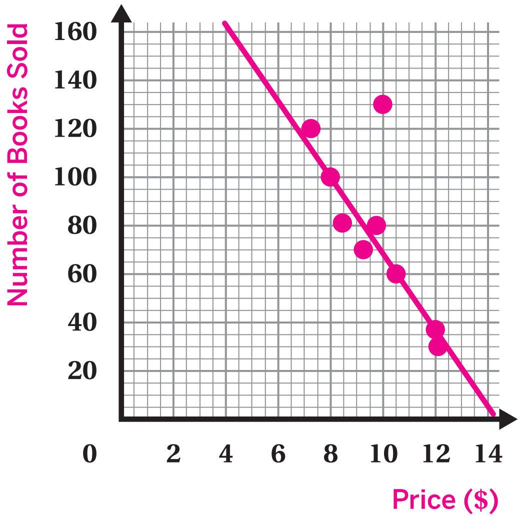
Name: .... Date: .... Period: ....

Problems 1–5: Stores across the country sell a particular book at different prices. The table shows the price of the book and the number of books sold at that price.

Price ($)	Number of Books Sold
10.50	60
12.10	30
8.45	81
9.25	70
9.75	80
7.25	120
12	37
9.99	130
7.99	100

1. Create a scatter plot for this data. Include labels for the horizontal and vertical axes.

Response shown on graph.



2. Are there any outliers? Explain your thinking.

Yes. *Explanations vary.* The point (9.99, 130) appears to be an outlier since the number of books sold is much higher compared to other books around the same price.

3. What type of association does there appear to be between the price of the book and the number of books sold? Explain your thinking.

Negative linear association. *Explanations vary.* There is a negative linear association between the variables. When the price increases, the number of books sold decreases.

4. Draw a line on the graph that you think is a good fit for the data.

***Responses vary.* Sample shown on graph.**

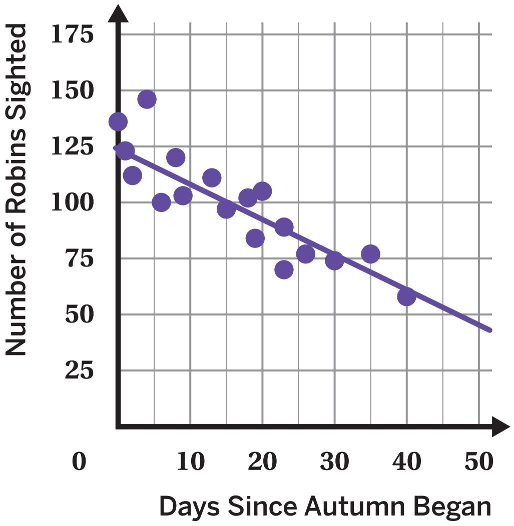
5. A bookstore plans to sell the book for $6. Use your line to predict the number of books the store will sell. Explain your thinking.

Responses between 125 and 145 books are considered correct. *Explanations vary.* The bookstore will sell 130 books. I chose this number because the point (6, 130) is on my line of fit.

Practice
6.09

Name: ______ Date: ______ Period: ______

Problems 6–7: This scatter plot shows data about the number of robins sighted at a local park and the number of days since autumn began.



6. Approximately how many robins were sighted 15 days since autumn began?

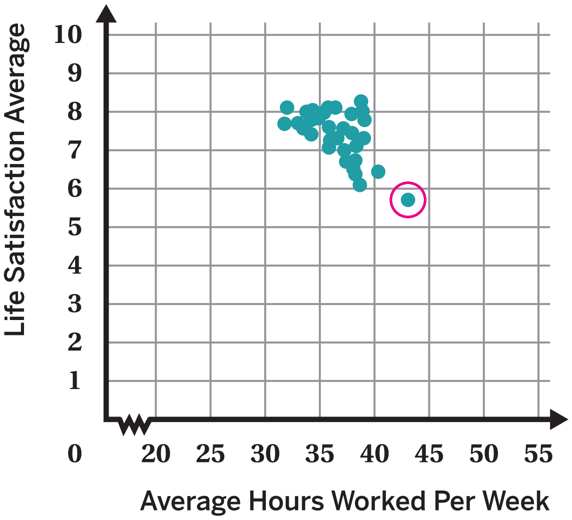
Responses between 90 and 100 robins are considered correct.

7. Use the line of fit to make a prediction for how many robins will be sighted 50 days since autumn began. Explain your thinking.

Responses between 40 and 50 robins are considered correct. *Explanations vary.*
47 robins, because the line of fit looks like it passes through the point (50, 47).

Spiral Review

Problems 8–10: Martina is interested in the quality of life in different countries. She created this scatter plot to show the average number of hours people worked in a week and their average life satisfaction (on a 0–10 scale). Each point on the graph represents a different country.



8. Select *all* the true statements about the data in the scatter plot.

☐ **A.** The variables have a positive association.

☑ **B.** The variables have a negative association.

☐ **C.** A linear model that fits the data would have a positive slope.

☑ **D.** A linear model that fits the data would have a negative slope.

☑ **E.** More than half of the countries have an average life satisfaction above 7.

9. Circle the point that represents the country where average life satisfaction is lowest. How many hours are in the average work week in that country?

43 hours per week

10. Write a sentence that describes what this point means in context to this situation.

***Responses vary.* The point (43, 5.8) indicates that people that work 43 hours per week are less satisfied in life than people that work fewer hours per week.**

Name: ________________ Date: ________ Period: ________

Interpret Scatter Plots Slopes and Intercepts

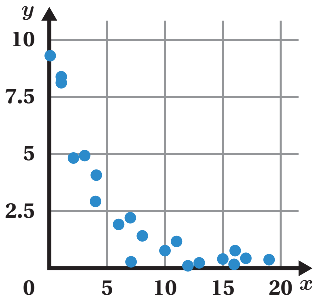
8.SP.1, 8.SP.2, 8.SP.3, SMP.1, SMP.2

Practice Day 1

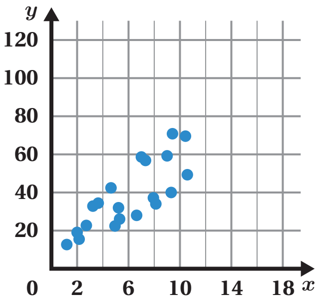
Let's practice what you've learned so far in this unit!

Here are eight scatter plots.

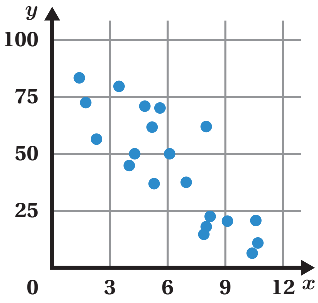
A



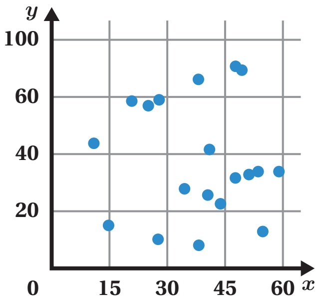
B



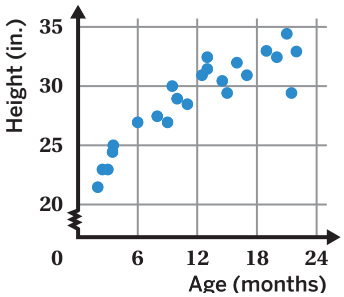
C



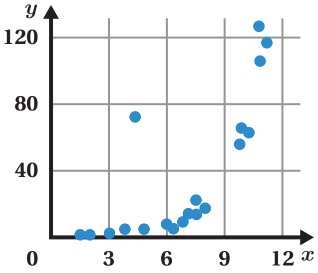
D



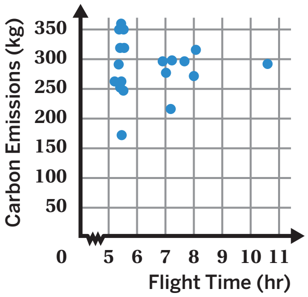
E



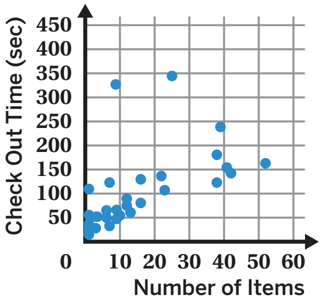
F



G



H



1. Sort each of the scatter plots based on these descriptions.

a

Positive Association	Negative Association	No Association
B, E, F, H	A, C	D, G

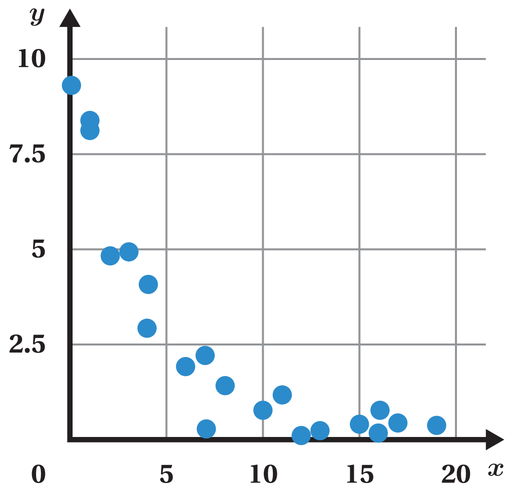
b

Linear Association	Non-Linear Association	No Association
B, C, E, H	A, F	D, G

Name: .... Date: .... Period: ....

Practice Day 1

2. Which pair of variables could this scatter plot represent?



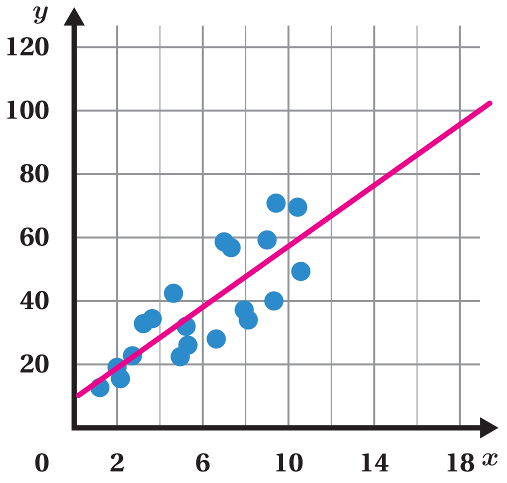
[✓ **A.**] x: Years of Experience
y: Time to Complete a Task (min)

B. x: Time Walking (hr)
y: Distance Traveled (mi)

C. x: Weight of Broccoli (lb)
y: Cost of Broccoli ($)

D. x: Number of Pets
y: Time Exercising (min)

3. Here is another scatter plot.



a Draw a line of fit to model the data.

***Responses vary.* Sample shown on graph.**

b Predict the y-value when $x = 14$.

Responses between 73 and 77 are considered correct.

c Which equation is the best fit for the data?

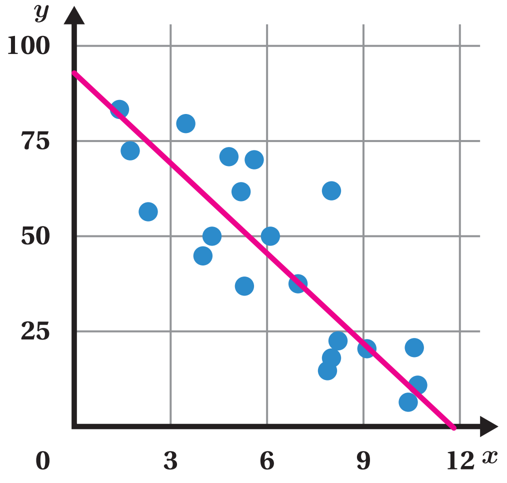
A. $y = -7x + 10$

B. $y = -7x - 10$

C. $y = 7x - 10$

[✓ **D.**] $y = 7x + 10$

4. Here is another scatter plot.



a Draw a line of fit to model the data.

***Responses vary.* Sample shown on graph.**

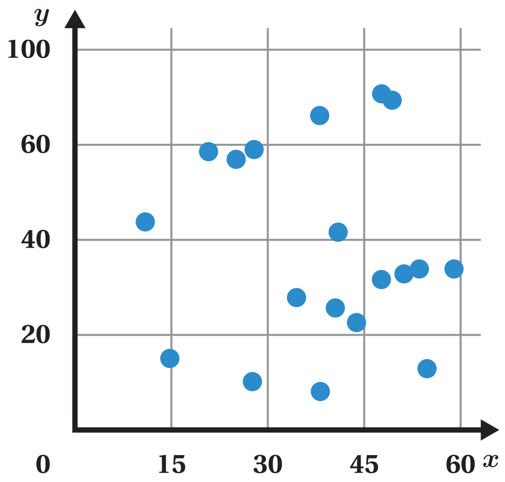
b A student found the equation $y = -7x + 88$ to be a good fit for the data. Use this model to predict y when x is 9.

25

Name: .... Date: .... Period: ....

Practice Day 1

5. Which pair of variables could this scatter plot represent?



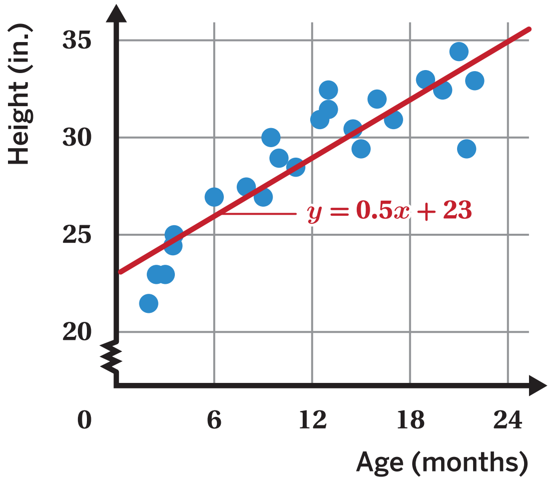
A. x: Number of Letters in First Name
y: Number of Letters in Last Name

B. x: Number of Cherries
y: Weight of Cherries (lb)

C. x: Outside Temperature (°F)
y: Heating Cost ($)

[✓ D.] x: Age (yr)
y: Number of Books Read

6. This scatter plot shows the ages and heights of children at a daycare center and a line of fit.



a What is the y-intercept of the line of fit and what does it represent in this context?

The y-intercept is 23. *Responses vary.* This means the model predicts a baby would be 23 inches at birth.

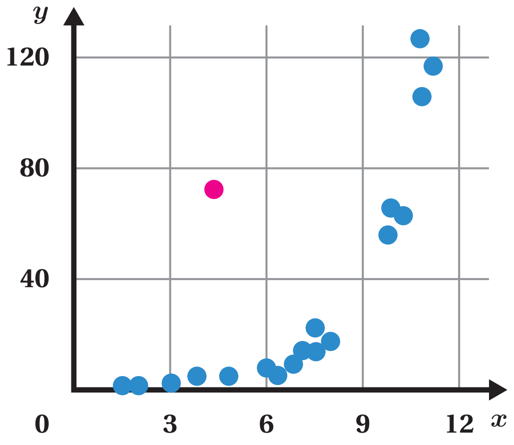
b What is the slope of the line of fit and what does it represent in this context?

The slope is 0.5. *Responses vary.* This means the model predicts babies grow 0.5 inches taller every month.

c What does the model predict for the height of a child on his first birthday?

***Responses vary.* 29 inches**

7. Here is another scatter plot.



a Select *all* the terms that describe this scatter plot.

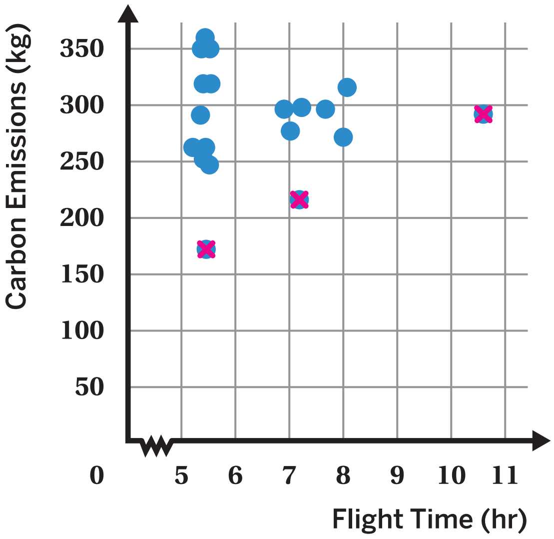
- ☑ A. Positive association
- ☐ B. Linear association
- ☑ C. Non-linear association
- ☑ D. With clusters
- ☐ E. Has outliers

b Add a point to the scatter plot that is an outlier. ***Responses vary.***

Name: Date: Period:

Practice Day 1

8. This scatter plot shows data for many possible flight options between Los Angeles and New York. Some options include a stop along the way.



a Put an × over any outlier(s).

Response shown on graph.

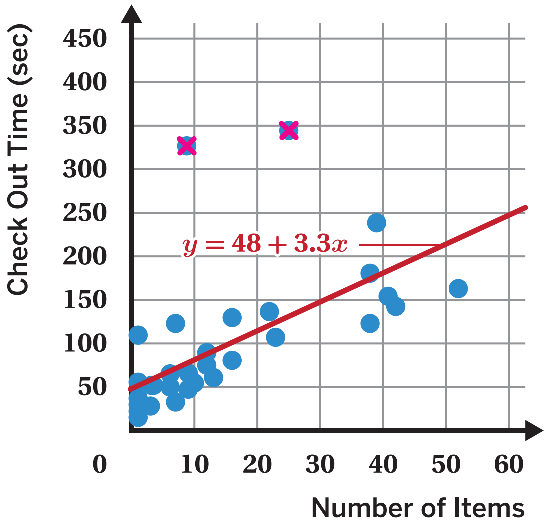
b Does the data have clusters? Explain your thinking.

Yes. *Explanations vary.* I see a cluster of points with x-values between 5 and 6 and another cluster with x-values between 6.8 and 8.1.

c What might explain why there is no association between the variables?

***Responses vary.* The type of plane might matter more than the flight duration. Some aircrafts are more fuel efficient than others.**

9. This scatter plot shows data from some recent transactions at a grocery store and a line of fit.



a Put an × over any outlier(s).

Response shown on graph.

b What is the slope of the line of fit and what does it represent in context?

The slope is 3.3. *Explanations vary.* This means the model predicts a transaction will take 3.3 more seconds for every additional item.

c Omar says: *The y-intercept for the line of fit should be 0 because a transaction with 0 items should take 0 seconds.*

Why do you think the y-intercept is 48?

***Responses vary.* A grocery store transaction includes time for things that aren't related to the number of items, like a cashier greeting the customer or a customer making a payment.**

Notes:

Sub-Unit

3

Categorical Data

Lesson 10
Tasty Fruit

Lesson 11
Finding Associations

Unit 6

Lesson

10

Name: ____________ Date: ________ Period: ________

Interpret Scatter Plots **Data, Graphs, and Tables**

8.SP.1, 8.SP.4, SMP.1

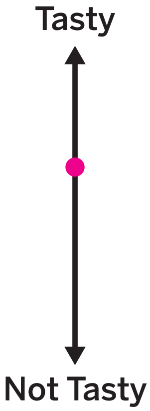
Tasty Fruit

Let’s explore two-way tables and bar graphs.

Warm-Up

1 Draw a point to show how tasty you think red apples are.

Responses vary.



2 Draw a point to show how easy you think red apples are to eat.

Responses vary.

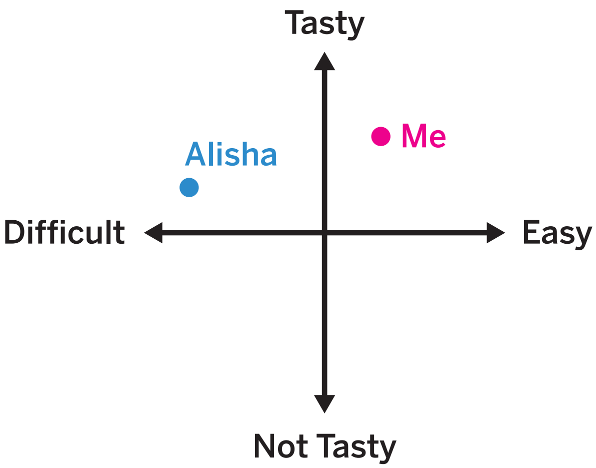
3 **Data Talk!** A scatter plot is one way to show how people feel about red apples.

Discuss how Alisha feels about red apples.

***Responses vary.* Alisha says that red apples are tasty but difficult to eat.**

Plot a point on the graph to represent *your* feelings about red apples.

Responses vary.



Activity
1

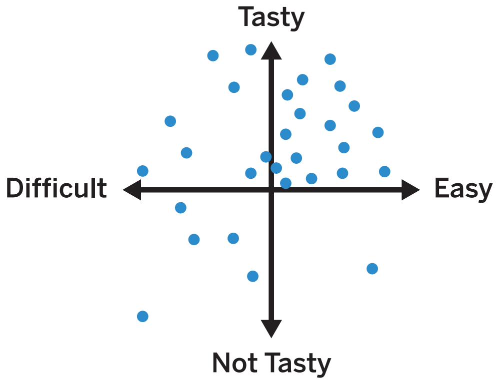
Name: .... Date: .... Period: ....

Displaying Categorical Data

Data Talk! Mr. Diaz's students are analyzing their class's opinions about red apples.

4 A student made a scatter plot and began to make a **two-way table**. The table shows **frequency**, which is the number of times each category appears in the data.

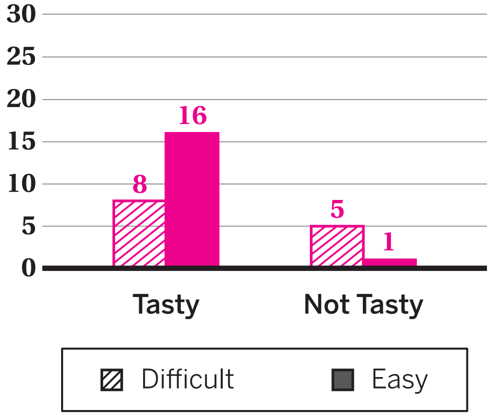
Complete the table to match the scatter plot.



	Difficult	Easy	Total
Tasty	8	16	24
Not Tasty	5	1	6
Total	13	17	30

5 Another student wanted to make a bar graph. Create a bar graph to match the table.

Response shown on graph.



6 The scatter plot, two-way table, and bar graph all represent *categorical* data about red apples.

a How many students said apples were easy to eat and tasty?

16 students

b How many students said apples are not tasty?

6 students

c How many students in total shared their opinions about red apples?

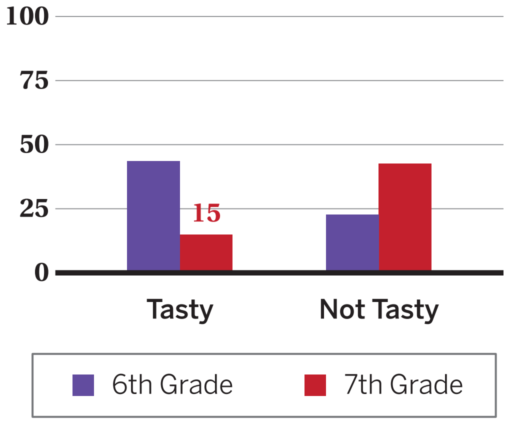
30 students

Activity
2

Name: .... Date: .... Period: ....

Analyzing Categorical Data

Data Talk! Abena surveyed the 6th and 7th graders at her school about grapes. The bar graph shows the results from Abena's survey.



7 The two-way table shows partial results from the survey. Complete the table.

	6th Grade	7th Grade	Total
Tasty	42	15	57
Not Tasty	22	41	63
Total	64	56	120

8 Based on the data, do 6th and 7th graders feel the same about grapes? Circle one.

Yes [✓ No] I'm not sure

Explain your thinking.

***Explanations vary.* Among 6th graders, there are more students who think grapes are tasty. In 7th grade, there are more students who think grapes are not tasty. It seems what grade you are in is related to whether or not you think grapes are tasty.**

You're invited to explore more.

9 150 students were asked what grade they are in and whether they play a sport.

The two-way table shows partial results from this survey.

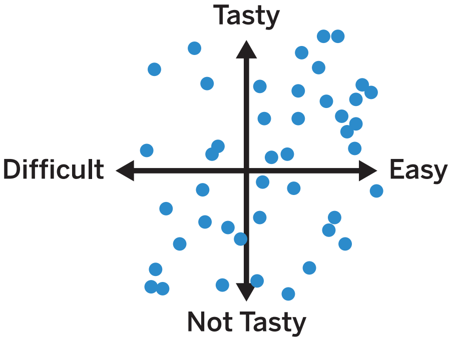
Complete the table.

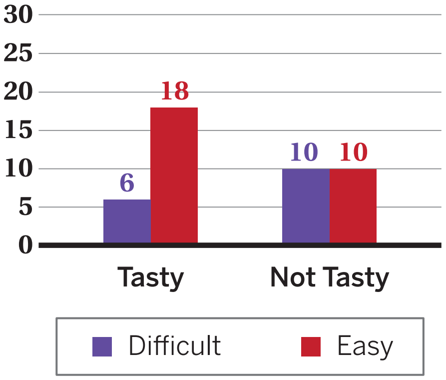
	Plays a Sport	Does Not Play a Sport	Total
6th Grade	46	11	57
7th Grade	19	1	20
8th Grade	16	5	21
9th Grade	29	23	52
Total	110	40	150

10 Synthesis

A school surveyed the 8th graders about how tasty bananas are, and how easy they are to eat.

	Difficult	Easy	Total
Tasty	6	18	24
Not Tasty	10	10	20
Total	16	28	44





Discuss: What are some advantages of using a scatter plot, a two-way table, or a bar graph to represent data? Use the examples if they help with your thinking.

Responses vary.

- **A scatter plot gives you more information about how tasty someone thinks bananas are.**
- **A two-way table organizes data so that it can be counted easily.**
- **A bar graph helps you visualize differences between the different groups.**

13 Summary 6.10

You can use a **two-way table** to compare two variables of *categorical data*, which is data that can be sorted into categories. Two-way tables show one of the variables across the top and the other down one side. Each entry in the table represents the **frequency**.

Two-way tables, scatter plots, and bar graphs can all be used to represent data and explore associations within data. Each representation has advantages and disadvantages. You can use these representations to investigate possible connections between variables.

For example, this two-way table shows data about whether students meditated on a certain day, and whether they felt calm or agitated that day. We can see there's a connection between meditating and feeling calm, since a majority of the people who felt calm also meditated.

	Meditated	Did Not Meditate	Total
Calm	45	8	53
Agitated	23	21	44
Total	68	29	97

frequency The number of times a value appears in a data set.

two-way table A way to compare two categorical variables. The table shows one of the variables across the top and the other variable down one side. Each entry is the frequency or relative frequency of the category in that column and row.

Practice
6.10

Name: .... Date: .... Period: ....

Problems 1–3: The table shows the results of a survey about TV watching habits.

	Watches TV Daily	Does Not Watch TV Daily	Total
Younger Than 18	30	80	110
18 or Older	60	35	95
Total	?	115	205

1. What do you notice?

***Responses vary.* I notice that out of the people who are 18 years or older, more people watch TV daily than don't.**

2. What do you wonder?

***Responses vary.* I wonder if people younger than 18 who don't watch TV daily use other types of electronic devices on a daily basis, such as laptops, phones, or tablets.**

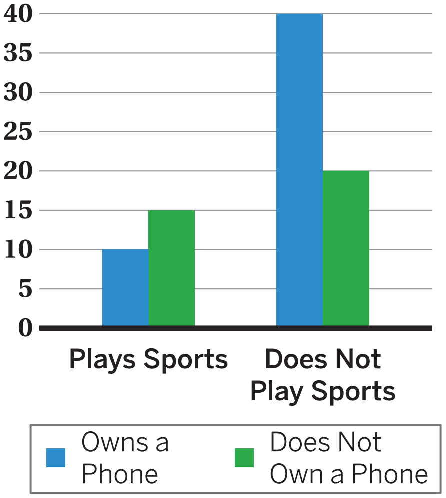
3. In total, how many people responded that they watch TV daily?

90 people

Problems 4–5: The bar graph shows data for a group of 8th grade students.

4. Complete the two-way table based on the information in the bar graph.

	Owns a Phone	Does Not Own a Phone	Total
Plays Sports	**10**	**15**	**25**
Does Not Play Sports	**40**	**20**	**60**
Total	**50**	**35**	**85**



5. Select *all* of the true statements.

☑ **A.** More students do not play sports than do.

☑ **B.** More students own a phone than don't.

☐ **C.** There are only 10 students who own a phone but don't play sports.

☐ **D.** There are no students who own a phone and play sports.

☐ **E.** There are 35 total students that own a phone.

6. Use the information in the two-way table to write two *true* statements about the data.

	Likes Riding a Bicycle	Does Not Like Riding a Bicycle
Kids	30	10
Adults	40	60

Responses vary.

- **The table shows that more adults like riding a bicycle than kids.**
- **More kids like riding a bicycle than don't.**

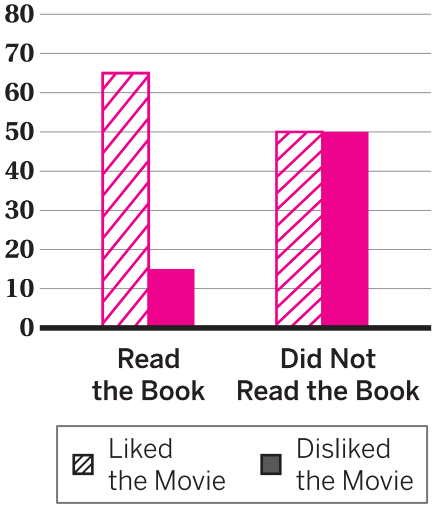
Practice
6.10

Name: .... Date: .... Period: ....

Problems 7–9: 180 people were surveyed about a movie they watched that was based on a book. Some people had already read the book and some had not.

7. Create a bar graph based on the information in the table.

	Liked the Movie	Disliked the Movie	Total
Read the Book	65	15	80
Did Not Read the Book	50	50	100
Total	115	65	180



8. What similarities and differences do you see in heights of the bars on the bar graph you created?

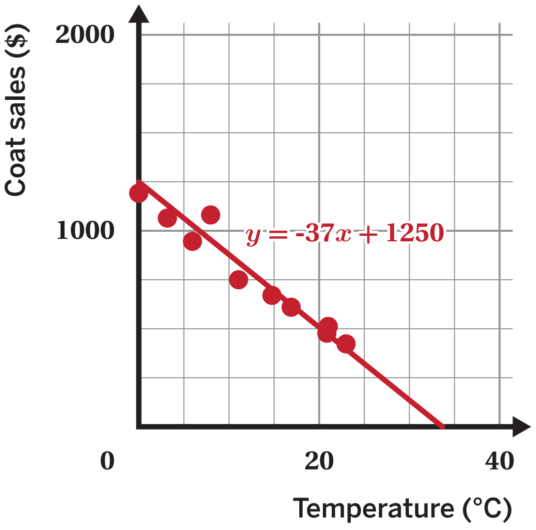
***Responses vary.* The bars that represent people who read the book have a big difference in their heights, while the bars that represent people who did not read the book are the same height.**

9. What claim might a person make based on this data?

***Responses vary.* People who read the book before watching the movie enjoy it more than those who have not read the book.**

Spiral Review

Problems 10–12: The scatter plot shows a store's daily coat sales and the outside temperature that day. The equation for the line of fit is $y = -37x + 1250$.



10. What is the slope and y-intercept of the line of fit?

The slope is -37. The y-intercept is 1,250.

11. What does the slope tell you about this situation?

***Responses vary.* The slope means that for every temperature increase of 1 degree, coat sales are predicted to decrease by $37.**

12. What does the y-intercept tell you about this situation?

***Responses vary.* The y-intercept means that if the temperature is 0°C, coat sales are predicted to be $1,250.**

Unit 6
Lesson
11

Name: ______________ Date: ______ Period: ______

Data, Graphs, and Tables 8.SP.4, SMP.3

Finding Associations

Let's use data displays to find associations.

Warm-Up

Data Talk! Shanice needs to book a flight. There are three airports near her home in Los Angeles. Shanice researched each airport to determine how many flights were delayed last month.

1 Of the 20,175 flights that left the three airports near Shanice last month, 4,465 were delayed.

What percent of flights were delayed?

Responses between 22% and 22.14% are considered correct.

DEPARTURES

Time	Destination	Status
11:05	BOSTON	DELAYED
11:25	MIAMI	DELAYED
12:05	LONDON	ON TIME
13:15	CHICAGO	DELAYED
13:30	NEW YORK	ON TIME
13:48	DUBAI	ON TIME
14:00	TOKYO	ON TIME
14:20	HOUSTON	DELAYED
15:05	TORONTO	ON TIME

2 This two-way table shows some data about flights that departed in the last month from the three airports near Shanice.

	On Time	Delayed	Total
Burbank	?	?	2,110
L.A.	?	?	13,765
Santa Ana	?	?	4,300
Total	15,710	4,465	20,175

Discuss: Do you think the data will show an association between airport and flight status?

Responses vary.

- **Yes, I think the airport will be associated with flight status. Los Angeles probably has a big airport, which could mean more congestion and delays.**
- **No, I think delays are more associated with things like weather or mechanical problems than with specific airports.**

Activity

1

Name: ______________________ Date: __________ Period: __________

Frequency and Relative Frequency

3 **Data Talk!** This table shows the frequencies of on-time and delayed flights from the three airports.

Based on the data, is there evidence of an association between airport and flight status? Circle one.

Yes No I'm not sure.

Explain your thinking.

Responses vary.

- **Yes. I calculated what percent of flights are on time for each airport. The percentages varied a lot between airports.**
- **I'm not sure. It's hard to say, because Los Angeles has more delays than the other airports but also more on-time flights.**

Frequencies

	On Time	Delayed	Total
Burbank	1,520	590	2,110
L.A.	11,530	2,235	13,765
Santa Ana	2,660	1,640	4,300
Total	15,710	4,465	20,175

4 One way to look for associations is to calculate **relative frequencies**. The relative frequency of a category is the percentage of data that is in that category.

The relative frequency of on-time flights from Burbank is about 72% because $\frac{1520}{2110} \approx 0.72$.

Relative Frequencies

	On Time	Delayed	Total
Burbank	72%	**28%**	100%
L.A.	**84%**	**16%**	100%
Santa Ana	**62%**	**38%**	100%

Complete the table. Round each percent to the nearest whole number.

5 Let's compare the two tables.

a Write a question you can answer using frequencies.

Responses vary. **Which airport has the most total flights?**

b Write a question you can answer using relative frequencies.

Responses vary. **Which airport has the highest percentage of on-time flights?**

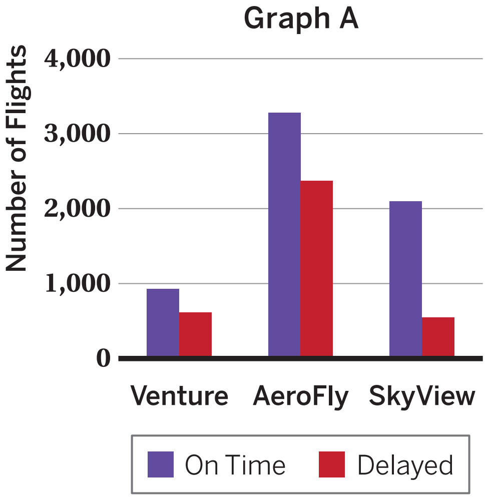
Activity
2

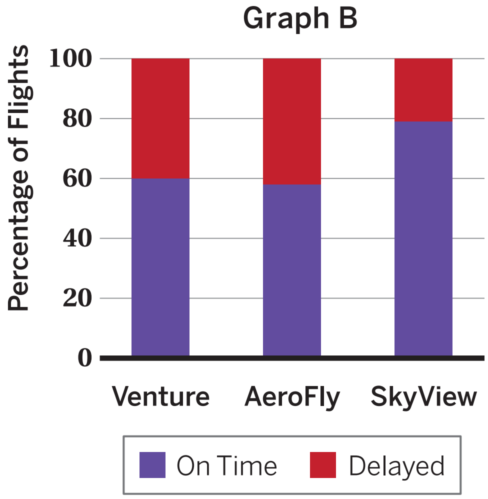
Name: .... Date: .... Period: ....

Analyzing Data Representations

6 **Data Talk!** Shanice also researched three airlines that she can fly with for her trip.

Discuss where you see the same information in each representation.





Frequencies

	On Time	Delayed	Total
Venture	914	605	1,519
AeroFly	3,288	2,369	5,657
SkyView	2,100	542	2,642
Total	6,308	3,516	9,818

Relative Frequencies

	On Time	Delayed	Total
Venture	60%	40%	100%
AeroFly	58%	42%	100%
SkyView	79%	21%	100%

***Responses vary.* The data is the same across the two tables, but the table on the left shows the number of flights and the table on the right shows the percent of flights, or frequencies versus relative frequencies.**

7 Graph B is called a **segmented bar graph**. Use the bar graph or segmented bar graph to help you answer: *Is there evidence of an association between airline and flight status?* **Yes**

Which graph was more helpful for determining if there is an association? Explain your thinking.

***Responses vary.* The bar graph shows how many on-time and delayed flights there are for each airline, but relative frequencies are more helpful for comparing airlines to one another.**

8 Consider the claim, “AeroFly Airlines has the highest rate of on-time flights because it has more on-time flights than Venture and SkyView combined.” Is this claim correct? Explain your thinking.

No. *Explanations vary.* AeroFly has more on-time flights than the other airlines because it has more flights overall. When making a claim about rate, it’s better to look at relative frequencies. AeroFly has a lower percentage of on-time flights than the other airlines.

Name: .... Date: .... Period: ....

Analyzing Data Representations (continued)

9 **Data Talk!** Based on the data, which airport and airline would you recommend for Shanice's flight? Explain your thinking.

Responses vary. I would recommend flying out of Los Angeles and using SkyView Airlines, if possible. Los Angeles has the highest percentage of on-time flights compared to other airports, and SkyView has the highest percentage of on-time flights compared to other airlines.

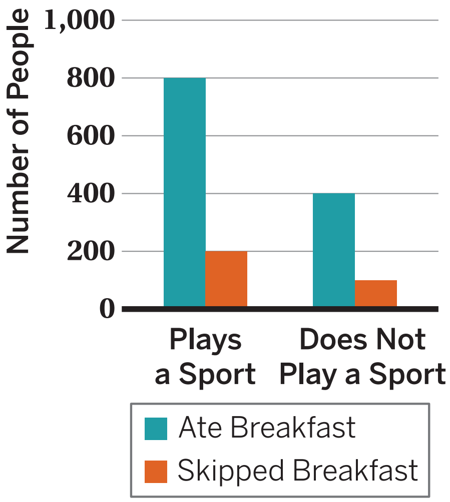
10 Here are four new data sets. For each data set, decide where there is evidence of an association between the variables.

	Left-Handed	Right-Handed
Has a Pet	83%	81%
No Pet	17%	19%
Total	100%	100%

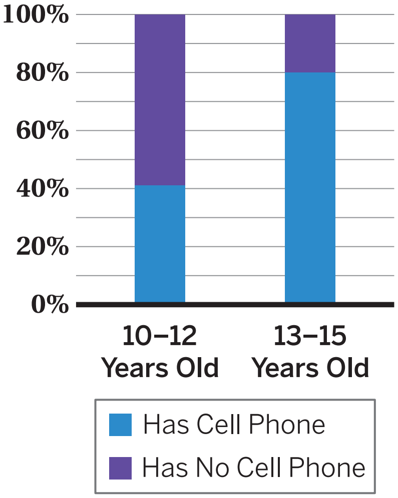
Association / [✓ No association]

	Completed Course	Did Not Complete	Total
Free Online Course	6%	94%	100%
In-Person Course	85%	15%	100%

[✓ Association] / No association



Association / [✓ No association]



[✓ Association] / No association

You're invited to explore more.

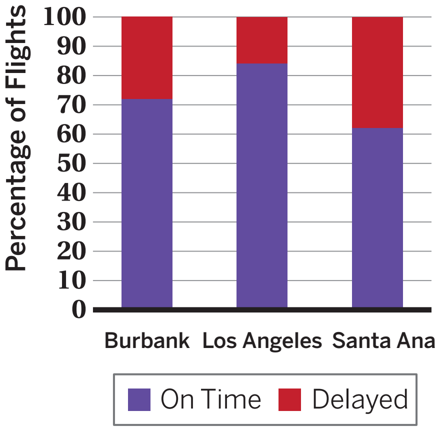
11 You will use the Activity 2 Sheet to look at representations of data collected by the National Household Travel Survey on whether people took public transit over a one-month period.

Responses vary.

12 Synthesis

Data Talk! Here are two representations of relative frequencies.

	On Time	Delayed	Total
Burbank	72%	28%	100%
L.A.	84%	16%	100%
Santa Ana	62%	38%	100%



Discuss how you can use relative frequencies to identify possible associations between variables.

Responses vary. **If variables aren't associated, then I'd expect their relative frequencies to be about the same. In this example, if there was no association, the percent of on-time flights would be the same for each airport. Since the relative frequencies vary by airport, that suggests there is an association.**

15 Summary 6.11

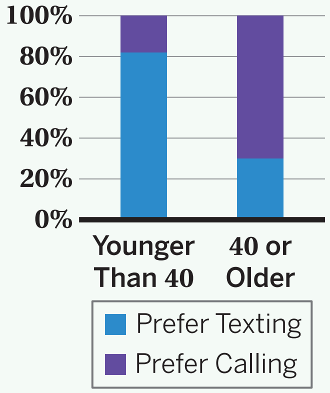
Specific types of two-way tables and bar graphs can be used to show frequencies and percentages within data sets, such as **relative frequency** tables and **segmented bar graphs**.

We can use these representations to identify associations between two categorical variables, which represent data that can be broken down into groups. For example, the table and graph show an association between categorical variables, age, and communication preference.

Relative Frequencies

	Prefer Texting to Communicate	Prefer Making a Phone Call	Total
Younger Than 40	82%	18%	100%
40 or Older	33%	67%	100%

Segmented Bar Graph



relative frequency The fraction or percent of the data that is in a category instead of the actual number of data points.

segmented bar graph A graph that compares two categories within a data set. The whole bar represents all the data within one category. Then each bar is separated into parts (segments) that show the percentage of each part in the second category.

Practice
6.11

Name: .... Date: .... Period: ....

Problems 1–4: A farmer brings produce to the farmer's market and records whether people bought lettuce, apples, both, or neither.

	Bought Apples	Did Not Buy Apples	Total
Bought Lettuce	14	58	72
Did Not Buy Lettuce	8	29	37
Total	22	87	109

1. How many people bought lettuce?

72

2. How many people bought lettuce *and* apples?

14

3. Complete the table to show the relative frequencies for each row.

Response shown in table.

	Bought Apples	Did Not Buy Apples	Total
Bought Lettuce	19%	81%	100%
Did Not Buy Lettuce	22%	78%	100%

4. Based on the data, is there an association between buying lettuce and buying apples? Explain your thinking.

No. *Explanations vary.* The relative frequency for buying apples is roughly the same whether lettuce was bought or not.

Problems 5–7: Researchers want to study news-reading habits among different age groups. They asked whether people primarily read news articles in print or on the internet.

	Internet Articles	Print Articles
18–25 Years Old	151	28
26–45 Years Old	132	72
46–65 Years Old	48	165

5. Calculate the relative frequencies for each age group.

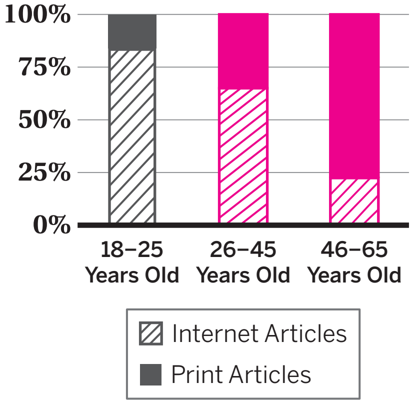
	Internet Articles	Print Articles	Total
18–25 Years Old	84%	16%	100%
26–45 Years Old	65%	35%	100%
46–65 Years Old	23%	77%	100%

Practice
6.11

Name: .... Date: .... Period: ....

6. Complete the segmented bar graph by drawing the missing bars. Create one segmented bar for each row of the table.

Response shown on graph.



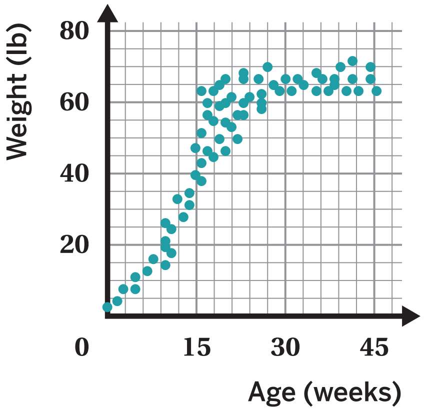
7. Is there an association between age groups and the method they use to read articles? Explain your thinking.

Yes. *Explanations vary.* Younger age groups are more likely to read internet articles than print articles, while the opposite is true for the oldest age group.

Spiral Review

8. Select *all* the phrases that describe the association on this scatter plot showing the age and weight of a group of male huskies.

- ☐ **A.** Linear association
- ☐ **B.** Negative association
- ☑ **C.** Non-linear association
- ☐ **D.** No association
- ☑ **E.** Positive association



Problems 9–10: In a class of 25 students, some students play a sport, some play a musical instrument, some do both, and some do neither.

9. Complete the table.

	Plays an Instrument	Does Not Play an Instrument	Total
Plays a Sport	**1**	11	**12**
Does Not Play a Sport	9	**4**	13
Total	10	**15**	25

10. How many students surveyed do not play a sport or an instrument?

4 students

Name: ________ Date: ________ Period: ________

Interpret Scatter Plots **Data, Graphs, and Tables**

8.SP.1, 8.SP.2, 8.SP.4, SMP.1, SMP.6, SMP.7

Practice Day 2

Let's practice what you've learned so far in this unit!

Start with any of the scavenger hunt sheets.

- Record the sheet shape, solve the problem, and write your answer.
- Look for your answer at the top of another scavenger hunt sheet. Solve that problem.
- Repeat until you make it back to your starting sheet.

The sheet students start with varies.

Sheet: Circle

Work varies.

Answer Negative →

Sheet: Star

Work varies.

↙ **Answer** A

Sheet: Triangle

Work varies.

Answer 12 →

Sheet: Crescent

Work varies.

Answer 9 ↗

continued on next page...

Unit 6

Lessons 1–11

Name: ______________________ Date: __________ Period: __________

Practice Day 2

Sheet: Oval

Work varies.

Answer 40%; 60% →

Sheet: Hexagon

Work varies.

↙ **Answer** C

Sheet: Octagon

Work varies.

Answer B →

Sheet: Rectangle

Work varies.

↙ **Answer** No association

Sheet: Pentagon

Work varies.

Answer 6 →

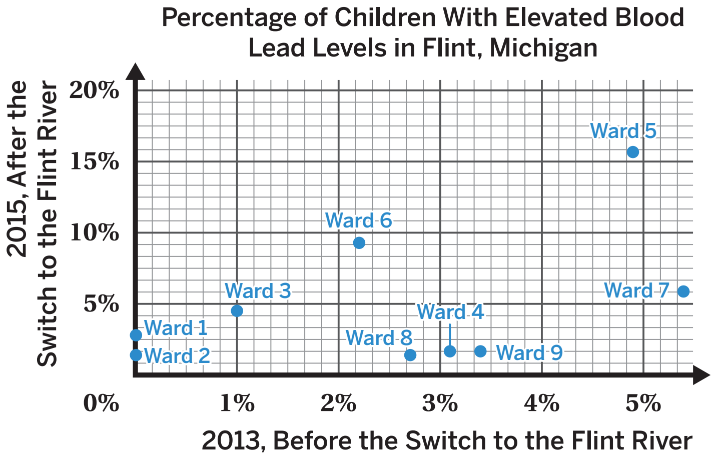
Sheet: Trapezoid

Work varies.

Answer 15 adults; 18 children

Career Connection

From over 1,200 miles away, 12-year-old Gitanjali Rao invented a device to detect water contamination.



In 2014, the city of Flint, Michigan switched the city's water supply source from Lake Huron to the Flint River. As a result, lead and other contaminants leaked into the city's drinking water supply.

12-year-old Gitanjali Rao, living in Colorado, wanted to help. She invented a device and an app to detect water contaminants and display the results. In 2020, she was named TIME magazine's first Kid of the Year.

Inventors identify areas in everyday life that can be improved and then design and create new products or processes. Many inventions today use STEM (Science, Technology, Engineering, and Math) to solve challenging problems in our world.

Meet Kimberly F. Sellers

Kimberly F. Sellers is the Head of the Department of Statistics at North Carolina State University. She is frequently requested to speak regarding her statistical research and her efforts to increase interest in STEM studies and careers. Dr. Sellers is also a principal researcher at the U.S. Census Bureau.

Are you interested in studying statistics or becoming an inventor like Gitanjali Rao? What can you do to learn more?

Community Connection

Share the scatter plot on this page — or a scatter plot from this unit — with at least 5 members of your community and ask them what they notice and wonder about the data.

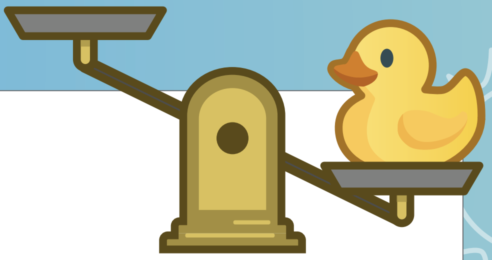
Responses vary.

Math Mindset

Describe a part of everyday life that might be able to be improved by an invention. How might STEM be involved in the invention?

Responses vary.

Unit 7



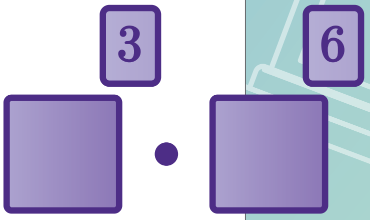
Exponents and Scientific Notation

Big Ideas in This Unit

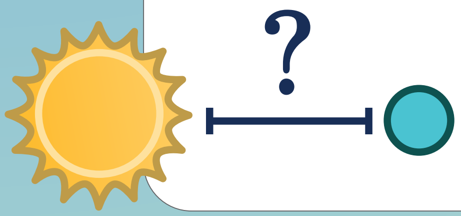
CC1 Data Explorations CC3 Big and Small Numbers

Questions for Investigation

- How can you use the properties of exponents to make connections between expressions?
- What is scientific notation and how can it be used to represent small and large numbers?



Explore: **How to Create a Sierpiński Triangle**
How can we create patterns using triangles?



Watch Your Knowledge Grow

This is the math you'll explore in this unit. Rate your understanding to see how your knowledge grows!

○	○	○
Not yet	Almost	I got it!

I can . . .	Before	After
Determine whether two expressions with exponents are equivalent.	○—○—○	○—○—○
Multiply powers with the same base.	○—○—○	○—○—○
Multiply powers with different bases and same exponents.	○—○—○	○—○—○
Divide powers with the same base.	○—○—○	○—○—○
Raise a power to another power.	○—○—○	○—○—○
Explain what it means for a number to be raised to a zero exponent.	○—○—○	○—○—○
Explain what it means for a number to be raised to a negative exponent.	○—○—○	○—○—○
Write rules for properties of exponents.	○—○—○	○—○—○
Use powers of 10 to estimate very large quantities.	○—○—○	○—○—○
Use powers of 10 to estimate very small quantities.	○—○—○	○—○—○

I can . . .	Before	After
Use powers of 10 to determine how many times larger one quantity is than the other.	○—○—○	○—○—○
Write numbers in scientific notation.	○—○—○	○—○—○
Multiply and divide numbers written in scientific notation.	○—○—○	○—○—○
Add and subtract numbers written in scientific notation.	○—○—○	○—○—○
Choose appropriate units of measurements for very large or very small quantities.	○—○—○	○—○—○
Interpret scientific notation shown on a calculator.	○—○—○	○—○—○

Sub-Unit

1

Exponent Properties

Explore
Creating a Sierpiński Triangle

Lesson 1
Circles

Lesson 2
Combining Exponents

Lesson 3
Power Pairs

Lesson 4
Rewriting Powers

Lesson 5
Negative and Zero Exponents

Lesson 6
Write a Rule

Name: .... Date: .... Period: ....

Big and Small Numbers Building Toward 8.EE.1, SMP.7, SMP.8

Explore: Creating a Sierpiński Triangle

How can we create patterns using triangles?

Warm-Up

1. Let's watch an animation. What do you notice? What do you wonder? **ELD.PI.8.2.Em, Ex, Br**

I notice:

Responses vary.

- **The pattern seems to continue forever because more and more triangles keep appearing.**
- **The larger triangle is filled with smaller triangles that vary in size, but there is a pattern where there are sets of 3 triangles that are the same size.**

I wonder. . .

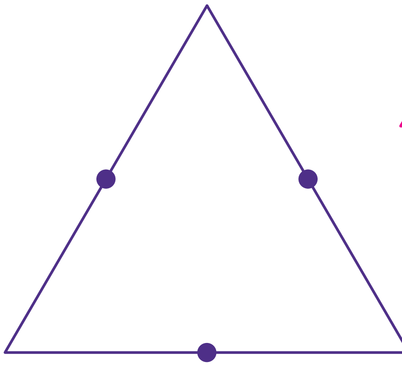
Responses vary.

- **Why does the pattern happen?**
- **Why is there one triangle in the middle that is larger but all of the other triangles are in sets of 3?**

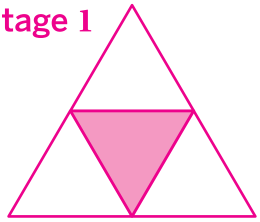
Name: .... Date: .... Period: ....

Drawing Triangles

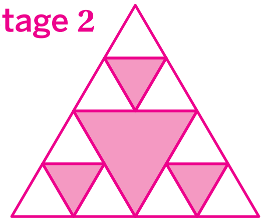
Let’s create a pattern similar to the one seen in the Warm-Up.



Stage 1



Stage 2



Stage	Number of Unshaded Triangles
1	3 or 3^1
2	9 or 3^2
3	27 or 3^3
4	81 or 3^4

2. The midpoints of the sides on the equilateral triangle are marked by dots. Connect the dots, and then shade the middle triangle formed by these dots. Write the number of unshaded triangles in the table for Stage 1.

3. For each unshaded triangle, find the midpoint of each side. Connect the dots, and then shade the middle triangles formed by these dots. Write the number of unshaded triangles in the table for Stage 2.

4. Continue the pattern and complete the table with the number of unshaded triangles for Stages 3 and 4.

5. What patterns do you notice in the number of unshaded triangles?

ELD.PI.8.6.Em, Ex, Br, ELD.PI.8.11.Em, Ex, Br

Responses vary.

- The number of unshaded triangles increases.
- The number of unshaded triangles triples at each stage.

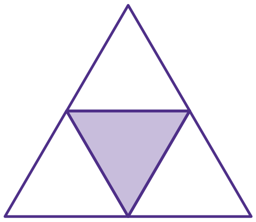
Name: .... Date: .... Period: ....

Drawing Triangles (continued)

The triangle pattern you created in the previous activity is called the Sierpiński triangle. In 1915, Waclaw Sierpiński discovered this geometric shape, which shows a repeating pattern at different scales.

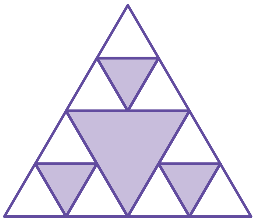
Study the Sierpiński triangles for the first four stages. The unshaded area of the triangle in each of the first three stages is written as a fraction of the total area, in square units.

Stage 1



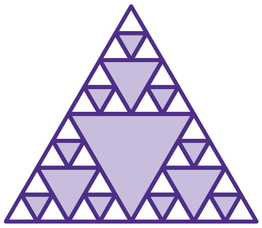
$\frac{3}{4}$

Stage 2



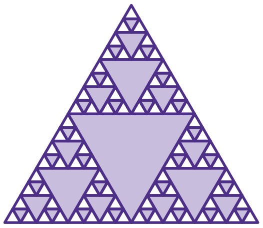
$\frac{9}{16}$

Stage 3



$\frac{27}{64}$

Stage 4



$\frac{81}{256}$ or $\frac{3^4}{4^4}$

6. How does the unshaded area change from one stage to the next?

ELD.PI.8.6.Em, Ex, Br, ELD.PI.8.11.Em, Ex, Br

Responses vary.

- **The area decreases for each successive stage.**
- **The numerator is multiplied by 3 and the denominator is multiplied by 4.**

7. Use the patterns to write the unshaded area for Stage 4.

Response shown under the triangle in Stage 4.

8. Do you think the area for the unshaded triangles in Stage 50 will be less than or greater than the surface area of a grain of salt? Be prepared to explain your thinking.

ELD.PI.8.11.Em, Ex, Br

Responses vary. **It will be less than the surface area of a grain of salt because the area decreases at each stage by a significant amount.**

Activity

Name: .... Date: .... Period: ....

Building Math Habits of Mind

Discuss:

- Which of these habits of mind did you strengthen during this activity?
- How did you use the one(s) you selected?

I can slow down and first make sense of a challenging problem before trying to solve it.

○ Not yet ○ Almost ○ I got it!

I can represent real-world problems using equations and inequalities and interpret their solutions within the context of the problem.

○ Not yet ○ Almost ○ I got it!

I can justify my thinking and ask questions to help me understand the thinking of others.

○ Not yet ○ Almost ○ I got it!

I can apply the math that I know to solve real-world problems, making assumptions, and revising my thinking as needed.

○ Not yet ○ Almost ○ I got it!

I can select an appropriate tool to help me solve problems.

○ Not yet ○ Almost ○ I got it!

I can communicate my thinking and solutions clearly to others.

○ Not yet ○ Almost ○ I got it!

I can look for structure or patterns to help me solve problems.

○ Not yet ○ Almost ○ I got it!

I can look for repeated calculations and other repeated steps to make generalizations.

○ Not yet ○ Almost ○ I got it!

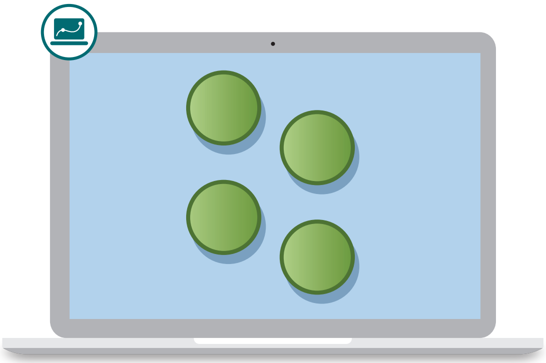
Unit 7

Lesson

1

Name: .... Date: .... Period: ....

Big and Small Numbers 8.EE.1, SMP.8



Circles

Let's revisit exponents.

Warm-Up

Evaluate each expression mentally. Try to think of more than one strategy.

1 $5 \cdot 2$

10

2 $5 \cdot 2 \cdot 2$

20

3 $5 \cdot 2 \cdot 2 \cdot 2$

40

4 $5 \cdot 2^4$

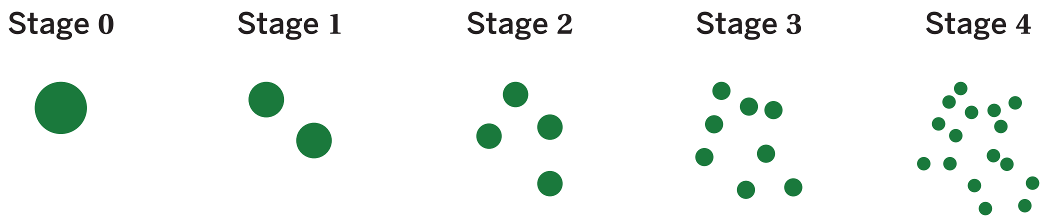
80

Activity
1

Name: .... Date: .... Period: ....

Lots of Circles

5 Let’s look at a pattern.



Discuss: What do you notice? What do you wonder?

Responses vary.

- I notice that the number of circles doubles with each stage.
- I wonder how many circles there will be at Stage 100.

6 How many circles will there be in Stage 5?

32 circles

Explain your thinking.

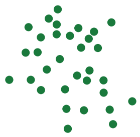
Explanations vary. I noticed that the number of circles double after each stage. Stage 4 has 16 circles, so if I double that I get 32 circles in Stage 5.

Name: ____________ Date: ____________ Period: ____________

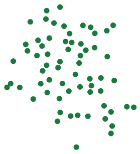
Lots of Circles (continued)

7 Here are Stages 5–12 of the same pattern.

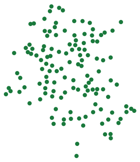
Stage 5



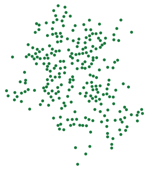
Stage 6



Stage 7

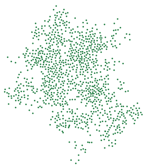


Stage 8

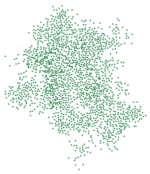


Stage 9

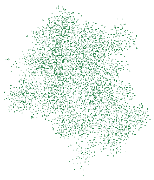
Stage 10



Stage 11



Stage 12



How many circles are there in Stage 12?

4,096 circles

8 Adah and Jamal were calculating the number of circles in Stage 12.

Adah wrote: $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$

Jamal wrote: 2^{12}

Whose expression is correct? Circle one.

Adah's Jamal's [✓ Both] Neither

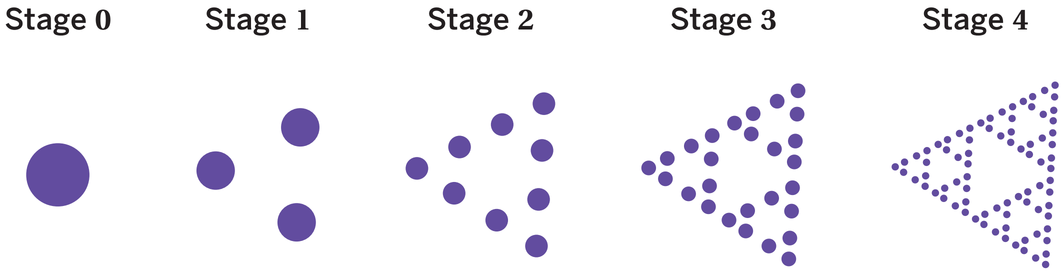
Explain your thinking.

***Explanations vary*. Both expressions are correct because $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$ has the same value as 2^{12}.**

Name: ______________ Date: ______ Period: ______

A New Pattern

9 Here is a new pattern.



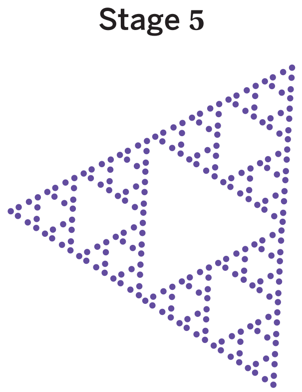
How many circles are there in Stage 4?

81 circles

Explain your thinking.

***Explanations vary*. Each stage has the number of circles in the previous stage multiplied by 3. Stage 1 has 3 circles, Stage 2 has 9 circles, Stage 3 has 27 circles, and Stage 4 has 81 circles.**

10 There are 243 circles in Stage 5.



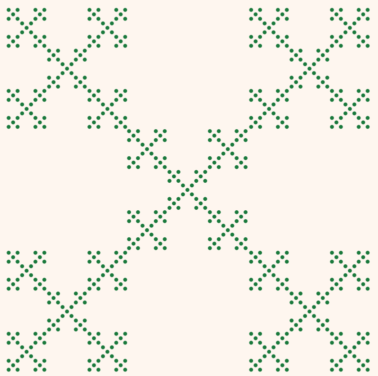
Select *all* the expressions that represent the number of circles in Stage 7.

- ☑ **A.** 3^7
- ☑ **B.** $243 \cdot 3^2$
- ☐ **C.** $243 \cdot (3 \cdot 2)$
- ☐ **D.** $243 + 243 + 243$
- ☑ **E.** $243 \cdot 3 \cdot 3$

You're invited to explore more.

11 How many circles are in this image?

625 circles



Explain your thinking.

***Explanations vary*. There are 25 circles in one X shape, and there are 5 of those in each of the bigger X shapes, so each of the 5 bigger X shapes has $25 \cdot 5 = 125$ circles. Because there are 5 of these in the image, there are $125 \cdot 5 = 625$ circles total.**

12 Synthesis

Discuss: When might it be helpful to write values or expressions using *exponents*?

Use the examples if they help with your thinking.

$7 \cdot 7 \cdot 7 \cdot 7 \cdot 7$

$6 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 5$

$10 + 10 + 10 + 10 + 10$

***Responses vary*. When an expression has repeated multiplication, it can be helpful to write the values using exponents. For example, $7 \cdot 7 \cdot 7 \cdot 7 \cdot 7$ can be written as 7^5 because 7 is multiplied 5 times. But I can't use exponents for $10 + 10 + 10 + 10 + 10$ because there isn't any repeated multiplication.**

15 Summary 7.01

Expressions with *exponents* are useful for representing repeated multiplication. In the expression 3^5, 5 is the exponent. When the exponent is a positive integer, it says how many times the number or expression is multiplied.

For example, $3^5 = \underbrace{3 \cdot 3 \cdot 3 \cdot 3 \cdot 3}_{\text{5 times}}$. Imagine writing 3^{100} using multiplication!

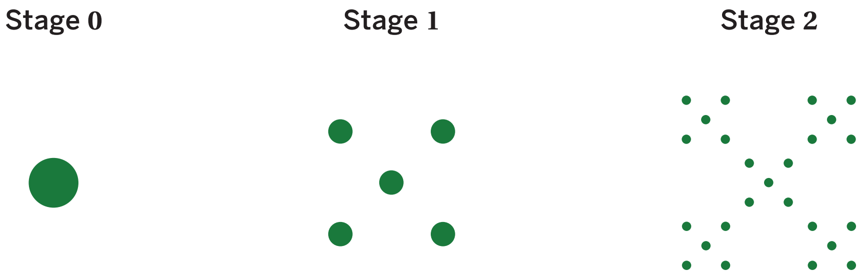
Here are a few more examples:

- $7 \cdot 7 \cdot 7 \cdot 7 \cdot 7 = 7^5$
- $5 \cdot 8 \cdot 5 \cdot 8 \cdot 5 \cdot 8 \cdot 5 = 5^4 \cdot 8^3$
- $10 \cdot 10 \cdot 10 + 10 \cdot 10 = 10^3 + 10^2$

Practice
7.01

Name: ______ Date: ______ Period: ______

1. Notice how the number of circles changes at each stage.



How many circles will there be in Stage 4?

5^4 (or equivalent)

2. Complete the table.

Expanded Expression	Exponent Expression	Value
$3 \cdot 3 \cdot 3 \cdot 3$	3^4	81
$7 \cdot 7 \cdot 7 \cdot 7 \cdot 7$	7^5	16,807
$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$	2^5	32
$4 \cdot 4 \cdot 4$	4^3	64

Problems 3–4: Write an equivalent expression that uses exponents.

3. $2 \cdot 9 \cdot 2 \cdot 9 \cdot 2 \cdot 2$

$2^4 \cdot 9^2$ (or equivalent)

4. $7 \cdot 7 \cdot 7 + 7 \cdot 7 \cdot 7 \cdot 7$

$7^3 + 7^4$ (or equivalent)

5. Each day, the number of grains of rice you have triples. On Day 1, you have 3 grains of rice. On Day 2, you have 9 grains of rice. On what day will you have 243 grains of rice?

Day 5

Show or explain your thinking.

Explanations vary. **$3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 = 243$, so I will have 243 grains of rice on Day 5.**

6. Adnan starts with two coins on Day 1. The number of coins doubles every day. How many coins will he have on Day 8? Write your answer as an expression with an exponent.

2^8

Practice
7.01

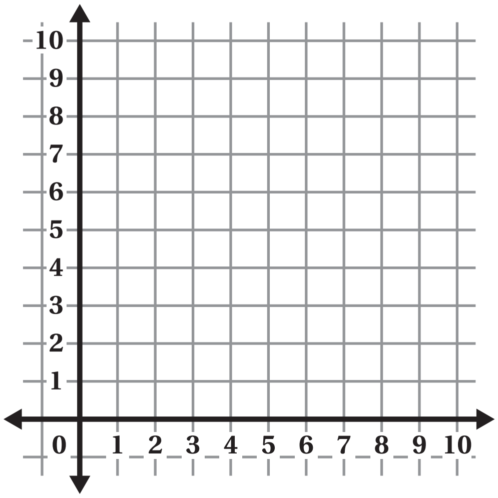
Name: ______ Date: ______ Period: ______

Spiral Review

7. Which expression is equivalent to $10000 + 225$?

- **A.** $10^3 + 9 \cdot 25$
- [✓ **B.**] $10^4 + 15^2$
- **C.** $100^3 + 9 \cdot 25$
- **D.** $1000^2 + 15^2$

Problems 8–9: The points $(2, 4)$ and $(6, 7)$ lie on a line. Sketch on the grid if it helps with your thinking.



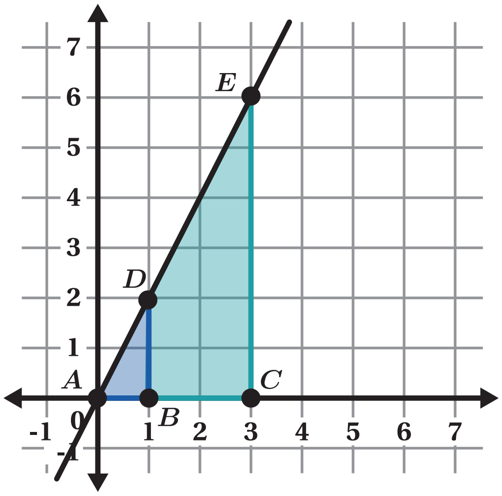
8. What is the slope of the line?

- **A.** 2
- **B.** 1
- **C.** $\frac{4}{3}$
- [✓ **D.**] $\frac{3}{4}$

9. What is the y-intercept of the line?

- **A.** 1.5
- **B.** 3.25
- [✓ **C.**] 2.5
- **D.** 2

Problems 10–11: Here is a diagram that shows a pair of similar figures.



10. What does the center of dilation need to be to dilate triangle ACE onto triangle ABD?

Point A or $(0, 0)$

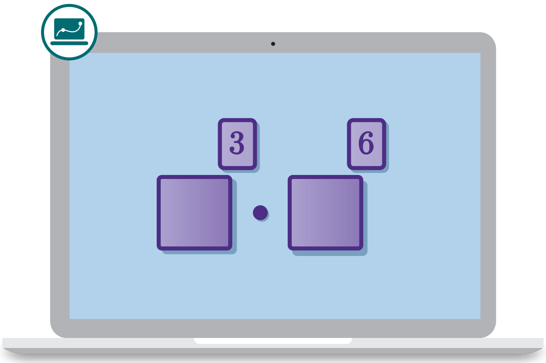
11. What does the scale factor need to be to dilate triangle ACE onto triangle ABD?

$\frac{1}{3}$

Unit 7
Lesson
2

Name: ________ Date: ________ Period: ________

Big and Small Numbers ☎ 8.EE.1, SMP.7



Combining Exponents

Let’s explore equivalent expressions with exponents.

Warm-Up

1 Which one doesn’t belong? Explain your thinking.

A. $(2^2)^3$

B. $2^3 \cdot 3 \cdot 2^2$

C. $2 \cdot 32$

D. $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$

Responses vary.

- Expression A doesn’t belong because it’s the only expression with parentheses.
- Expression B doesn’t belong because it’s the only expression that contains numbers with and without exponents.
- Expression C doesn’t belong because it’s the only expression that is the product of two numbers.
- Expression D doesn’t belong because it’s the longest expression.

Activity
1

Name: ______________________ Date: __________ Period: __________

Combining Exponents

2 Here are some different ways to build a billion, 10^9, by multiplying two **powers of ten**.

$(10 \cdot 10)\ (10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10)$

$10^2 \cdot 10^7 = 10^9$

$(10 \cdot 10 \cdot 10 \cdot 10)\ (10 \cdot 10 \cdot 10 \cdot 10 \cdot 10)$

$10^4 \cdot 10^5 = 10^9$

$(10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10)\ (10 \cdot 10 \cdot 10)$

$10^6 \cdot 10^3 = 10^9$

$(10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10)\ (10)$

$10^8 \cdot 10^1 = 10^9$

Discuss: What do you notice? What do you wonder?

Responses vary.

- **I notice that nine 10s are being grouped in different ways.**
- **I notice that the exponents of both powers of ten always add up to 9.**
- **I wonder if the pattern will continue for negative exponents, like $10^{11} \cdot 10^{-2}$.**
- **I wonder if this will work for more than two powers of ten.**
- **I wonder if there is a similar pattern for other powers that don't have a base of ten.**

3 Abdullah and Madison each rewrote the expression $(8^3)^2$ as a new expression.

Abdullah's Expression	Madison's Expression
Abdullah $8^3 \cdot 8^3$	Madison $(8 \cdot 8 \cdot 8) \cdot (8 \cdot 8 \cdot 8)$

a How are their strategies alike? How are they different?

Abdullah and Madison are both finding the product of a group of 8s and multiplying it to another group of 8s. The difference is that Abdullah is using exponents in his grouping, while Madison is using expanded notation in her grouping.

b **Discuss:** How do you know that the expressions are still equivalent, even though they're written differently?

Both strategies multiply two groups of three 8s, even though the notation is different.

4 Here is a new expression: $4^5 \cdot 2^5$. Is this expression equivalent to 8^5? Show or explain your thinking.

Yes. *Explanations vary.* $4^5 \cdot 2^5$ means there are five 4s multiplied by five 2s. If I group each 4 with a 2, then I get five groups of $4 \cdot 2$, which is the same as five 8s.

Activity
2

Name: .... Date: .... Period: ....

Odd One Out

5 **a** Two of these expressions are equivalent. Which expression is not equivalent to the others? Circle one.

[✓ $3 + 3 + 3 + 3 + 3$] $3^2 \cdot 3 \cdot 3 \cdot 3$ 3^5

b How could you change this expression so that it has the same value as the others?

Responses vary. **If I change the pluses (+) to multiplication dots (•), the expressions would all have the same value.**

6 **a** Two of these expressions are equivalent. Which expression is not equivalent to the others? Circle one.

$5 \cdot 5 \cdot 5 \cdot 4 \cdot 4 \cdot 4$ 20^3 [✓ $(5 \cdot 3) \cdot (4 \cdot 3)$]

b How could you change this expression so that it has the same value as the others?

Responses vary. **If I change the expression to $(5^3) \cdot (4^3)$, the expressions would all have the same value.**

7 **a** Two of these expressions are equivalent. Which expression is not equivalent to the others? Circle one.

$(2 \cdot 2 \cdot 2)\ (2 \cdot 2 \cdot 2)$ [✓ 6^2] $(2^3)^2$

b How could you change this expression so that it has the same value as the others?

Responses vary. **If I change the expression to 2^6, the expressions would all have the same value.**

Odd One Out (continued)

8 Here are some more pairs of *equivalent expressions*.

For each row, write one more equivalent expression. ***Responses vary.***

Expression 1	Expression 2	Expression 3
$4 \cdot 4 \cdot 4^3$	4^5	$4^2 \cdot 4^3$
6^4	$2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 3 \cdot 3$	$2^4 \cdot 3^4$
$(7 \cdot 7) \cdot (7 \cdot 7) \cdot (7 \cdot 7)$	$(7^2)^3$	7^6
5^{18}	$(5^3)^6$	$(5^2)^9$

You're invited to explore more.

9 Using whole numbers 0 through 9 without repeating, fill in the blanks so that all expressions have the same value.

$(5^{\square})^{\square}$

$5^{\square} \cdot 5^{\square}$

$5^{\square} \cdot 5^{\square} \cdot 5^{\square}$

$5^{\square}$

Responses vary.

- $(5^2)^4$
- $5^1 \cdot 5^7$
- $5^3 \cdot 5^5 \cdot 5^0$
- 5^8

10 Synthesis

Discuss: What are some strategies for writing equivalent expressions involving exponents?

$3^4 \cdot 3^2 = 3^6$

$3^5 \cdot 4^5 = 12^5$

$(3^2)^4 = 3^8$

Use the examples if they help with your thinking.

Responses vary. **I can expand and re-group expressions with exponents. For example, in 3^6 there are six 3s, so I can re-group the 3s into four 3s and two 3s, which is the same as $3^4 \cdot 3^2$. I could use this same strategy to expand $3^5 \cdot 4^5$ into a group of five 3s times a group of five 4s, then I can re-group to make five groups of $3 \cdot 4$, which is the same as five 12s or 12^5.**

13 Summary 7.02

Expanding is one strategy for determining if expressions with exponents are equivalent.

Here are two **powers of ten** that are equivalent to 10^8. Each expression can be expanded as $10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10$.

- $10^5 \cdot 10^3 = (10 \cdot 10 \cdot 10 \cdot 10 \cdot 10) \cdot (10 \cdot 10 \cdot 10) = 10^8$
- $(10^2)^4 = (10 \cdot 10) \cdot (10 \cdot 10) \cdot (10 \cdot 10) \cdot (10 \cdot 10) = 10^8$

power of ten A number written in the form 10^n, where n represents the number of times 10 is multiplied.

Practice
7.02

Name: .... Date: .... Period: ....

Problems 1–2: For each expression, write an equivalent expression with a single power. Two examples are shown.

1.

Expression	Single Power
$6^3 \cdot 6^9$	6^{12}
$2 \cdot 2^4$	2^5
$12^5 \cdot 12^{12}$	12^{17}
$7^6 \cdot 7^6 \cdot 7^6$	7^{18}

2.

Expression	Single Power
$(3^7)^2$	3^{14}
$(2^9)^3$	2^{27}
$(7^6)^3$	7^{18}
$(11^2)^3$	11^6

3. Which expression is equivalent to $6^4 \cdot 2^4$?

A. 8^4 **B.** 8^8 [✓ **C.**] 12^4 **D.** 12^{16}

Problems 4–6: Here is a large rectangular swimming pool. The pool is filled to the top with water.

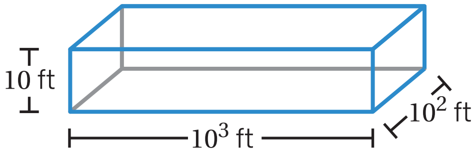


Figure may not be drawn to scale.

4. Write an expression with exponents to represent how much water the pool holds.

$10^3 \cdot 10^2 \cdot 10^1$ cubic feet
(or equivalent)

5. Write an equivalent expression with a single power.

10^6 cubic feet

6. You want to buy a cover to put on top of the pool. Write an expression with exponents to represent the area of the cover you need to buy.

$10^3 \cdot 10^2$ square feet
(or equivalent)

7. Select *all* the expressions that are equivalent to 6^8.

☑ **A.** $2^8 \cdot 3^8$
☐ **B.** $(6^4)^4$
☑ **C.** $(6^2)^4$
☐ **D.** $6^4 + 6^4$
☐ **E.** $6^4 \cdot 6^2$

Practice
7.02

Name: .... Date: .... Period: ....

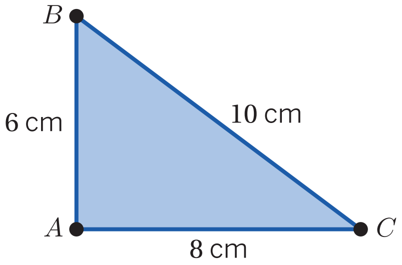
8. Fill in each blank using the whole numbers 1 to 9 only once to make a true statement.

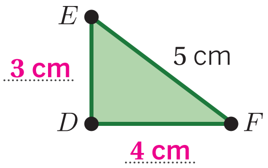
$6^{\square} \cdot 6^{\square} = (6^{\square})^{\square}$

Responses vary. $6^1 \cdot 6^5 = (6^2)^3$

Spiral Review

Problems 9–10: Triangle DEF is similar to triangle ABC.





9. Identify the transformation that maps triangle ABC to triangle DEF.

A dilation with a scale factor of $\frac{1}{2}$.

10. Label the side lengths for DE and DF.

11. Solve this system of equations:

$y = -\frac{3}{2}x + 4$

$y = 2x - 3$

$(2, 1)$

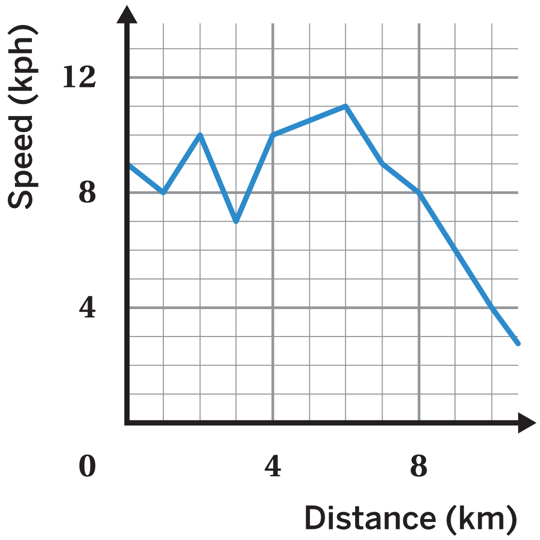
Problems 12–13: Diego runs a 10-kilometer race and keeps track of his speed.

12. Is speed a function of distance? Explain your thinking.

Yes. *Explanations vary.* For every value of distance, there is exactly one corresponding speed.

13. In 2–3 sentences, write a story that describes the changes in Diego's pace from the beginning of the race to the end.

Responses vary. At the beginning of the race, Diego started running at a pace of 9 kph. He slowed down then sped back up two times. Then at 4 km, his speed reached 11 kph. After that, he began to slowly decrease his speed and finished the race at a speed of 2 kph.

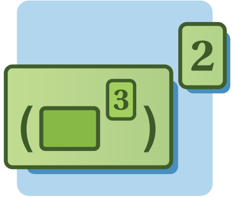


Unit 7
Lesson
3

Name: ________ Date: ________ Period: ________

Big and Small Numbers 8.EE.1, SMP.3, SMP.7

Power Pairs



Let's determine if expressions with exponents are equivalent.

Warm-Up

Fill in the blanks to create equivalent expressions.

1. $4^{(2+7)}$

$4^{\boxed{2}} \cdot 4^{\boxed{7}}$

$(4 \cdot 4) \cdot (4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4)$

$4^{\boxed{9}}$

2. $5^{(2 \cdot 3)}$

$\left(5^{\boxed{2}}\right)^{\boxed{3}}$

$(5 \cdot 5) \cdot (5 \cdot 5) \cdot (5 \cdot 5)$

$5^{\boxed{6}}$

Name: Date: Period:

Power Pairs

3. Here are two expressions.

$(3^4)^2$

$3^5 \cdot 3^3$

💬 **Discuss** Are these expressions equivalent? How do you know? **ELD.PI.8.3.Em, Ex, Br**

Yes. ***Explanations vary.*** **$(3^4)^2$ is the same as two groups of four 3s, which is equivalent to 3^8. $3^5 \cdot 3^3$ is the same as $(3 \cdot 3 \cdot 3 \cdot 3 \cdot 3) \cdot (3 \cdot 3 \cdot 3)$, which is also equivalent to 3^8.**

4. You will use a set of cards to complete this activity. Follow the instructions on the presentation screen to play the game *Power Pairs* with your group. Record the pairs of equivalent expressions in the table. ***Responses vary.***

Player's Name	Expression 1	Expression 2
	$(5 \cdot 5) \cdot (2 \cdot 2)$	10^2
	100	$5^2 \cdot 2^2$
	$16 \cdot 2$	2^5
	5^6	$5^4 \cdot 5^2$
	$2^4 \cdot 2^2$	$(2^3)^2$
	4^3	$2^3 \cdot 2 \cdot 2 \cdot 2$
	$(5^4)^2$	$5^3 \cdot 5^2 \cdot 5^3$
	25^4	$25 \cdot 25 \cdot 25 \cdot 25$
	10^4	$(10^1)^4$
	$10 \cdot 10 \cdot 10 \cdot 10$	10,000

Workspace:

Name: .... Date: .... Period: ....

Are They Equivalent?

5. Decide if the expressions in each pair are equivalent. Show your thinking. ***Work varies.***

	Expression 1	Expression 2	Equivalent? Circle one
a	$(5^5)^2$ $(5^5) \cdot (5^5)$	$5^4 \cdot 5^3$ $(5 \cdot 5 \cdot 5 \cdot 5) \cdot (5 \cdot 5 \cdot 5)$	Yes [✓ No]
b	$4^3 \cdot 2^5$ $(4 \cdot 4 \cdot 4) \cdot (2 \cdot 2 \cdot 2 \cdot 2 \cdot 2)$ $(4 \cdot 2)\ (4 \cdot 2)\ (4 \cdot 2) \cdot 2 \cdot 2$	8^8 $8 \cdot 8 \cdot 8 \cdot 8 \cdot 8 \cdot 8 \cdot 8 \cdot 8$	Yes [✓ No]
c	$15^3 \cdot 2^3$ 30^3	$(5 \cdot 2)^3 \cdot 3^3$ 30^3	[✓ Yes] No

6. Decide whether each expression is equivalent to 10^6. Rewrite any expressions that are *not* equivalent so that they are equivalent to 10^6.

a $\frac{10 \cdot 10^4 \cdot 10^2}{10}$

Equivalent

b 100^5

Not equivalent.
100^3

c $10^3 + 10^3$

Not equivalent.
$10^3 \cdot 10^3$

d $(10^2)^3$

Equivalent

You're invited to explore more.

7. Write at least four different expressions that are equivalent to $(3^4)^2$ using only numbers, multiplication, and exponents.

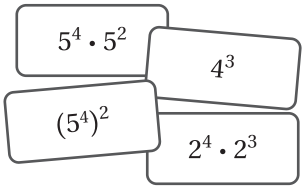
Responses vary.

- 3^8
- $3^4 \cdot 3^4$
- $(3 \cdot 3 \cdot 3 \cdot 3)^2$
- $3^5 \cdot 3^3$

Synthesis

8. **Discuss:** What are some important things to remember when determining whether expressions with exponents are equivalent? **ELD.PI.8.1.Em, Ex, Br**

Responses vary. **It's important to remember that I can rewrite a number or expression in different ways. Sometimes it's helpful to write out all the factors, for example: $4^3 = 4 \cdot 4 \cdot 4 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$. That is not equivalent to $2^4 \cdot 2^3$ because the expressions do not have the same amount of 2s.**



Summary 7.03

Rewriting powers can help you make sense of different bases with the same exponent. Here is one example.

If you expand $4^6 \cdot 3^6$, it is equal to $(4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4)\ (3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3)$. You can rearrange the factors to get $(4 \cdot 3)\ (4 \cdot 3)\ (4 \cdot 3)\ (4 \cdot 3)\ (4 \cdot 3)\ (4 \cdot 3)$, which is equal to 12^6. This means that $4^6 \cdot 3^6$ is equivalent to 12^6.

You may not need to expand an expression completely to determine whether it is equivalent to another expression. For example, $(12^4)^2$ is not equivalent to 12^6 because it is $(12^4) \cdot (12^4) = 12^8$.

Practice
7.03

Name: ____________ Date: ____________ Period: ____________

1. Write four different expressions that have the same value as $(5^2)^3$ using only numbers, multiplication, and exponents.

Responses vary.

$(25)^3$ $(5 \cdot 5)^3$ $5^2 \cdot 5^2 \cdot 5^2$ $25 \cdot 25 \cdot 25$

2. Show or explain why $6^5 \cdot 6^3$ is equivalent to $(6^4)^2$.

Responses vary.

$6^5 \cdot 6^3 = (6 \cdot 6 \cdot 6 \cdot 6 \cdot 6) \cdot (6 \cdot 6 \cdot 6) = (6 \cdot 6 \cdot 6 \cdot 6) \cdot (6 \cdot 6 \cdot 6 \cdot 6) = 6^4 \cdot 6^4 = (6^4)^2$

3. Write another expression that is equivalent to both $6^5 \cdot 6^3$ and $(6^4)^2$.

Responses vary. $(6^2)^4$

4. Select *all* the expressions that are equivalent to $2^3 \cdot 2^3$.

☑ **A.** $2^2 \cdot 2^4$
☐ **B.** 2^9
☐ **C.** 4^6
☑ **D.** $(2^3)^2$
☑ **E.** $2^5 \cdot 2$

Problems 5–8: For each pair, determine whether the expressions are equivalent.

5. Expression 1: $(12^2)^3$
Expression 2: $12^4 \cdot 12^2$

Circle one: [✓ Equivalent] Not equivalent

6. Expression 1: $(7 \cdot 2)^3$
Expression 2: $7^3 \cdot 2^3$

Circle one: [✓ Equivalent] Not equivalent

7. Expression 1: $10^2 + 10^4 + 10$
Expression 2: 10^7

Circle one: Equivalent [✓ Not equivalent]

8. Expression 1: 15^6
Expression 2: $(5 \cdot 3 \cdot 3 \cdot 5)^4$

Circle one: Equivalent [✓ Not equivalent]

Practice
7.03

Name: .... Date: .... Period: ....

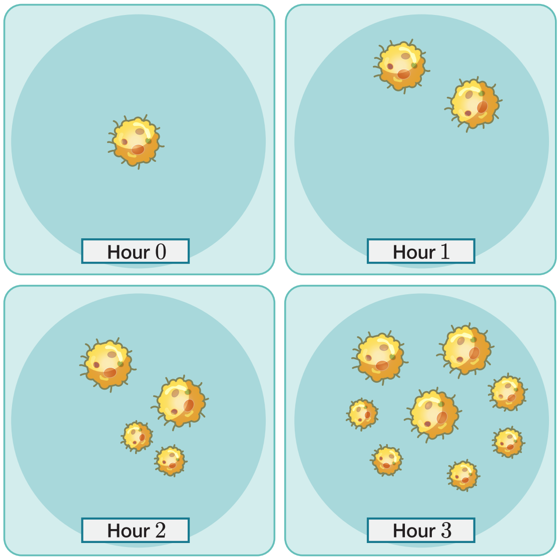
9. Fill in each blank using the digits 1 to 9 only once to make a true statement.

$$((2^{\square})^{\square})^{\square} = 2^{\square\square}$$

Responses vary. **$((2^1)^3)^9 = 2^{27}$**

Spiral Review

Problems 10–13: An amoeba (a single-celled animal) is being observed in a dish. Every hour, each amoeba divides to form two amoebas.



10. How many amoebas will there be after 5 hours?

32 amoebas

11. Write an expression for the number of amoebas after 8 hours.

2^8 (or equivalent)

12. Write an expression for the number of amoebas after 24 hours.

2^{24} (or equivalent)

13. Why might an expression with an exponent, like 2^6, be useful for determining the number of amoebas?

Responses vary. **Because the number of amoebas doubles every hour, it's more efficient to use an expression with exponents than expanded form.**

14. A delivery van can hold a total of 220 packages. The table shows how many minutes it took to load packages into a van.

Use the data in the table to determine whether each statement is True or False.

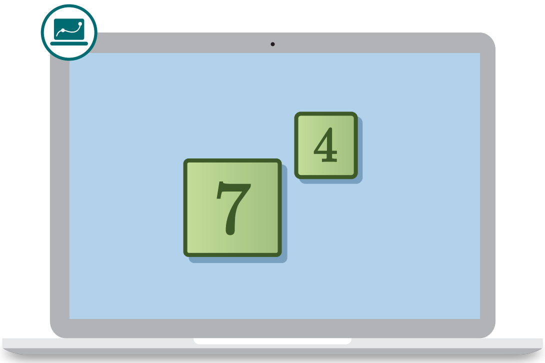
	True	False
The van starts with 220 packages.		✓
Every minute, 12 boxes are loaded.	✓	
The van is full in less than 20 minutes.	✓	

Packages Loaded	Time (min)
48	4
96	8
156	13

Unit 7
Lesson
4

Name: ______ Date: ______ Period: ______

Big and Small Numbers 8.EE.1, SMP.7



Rewriting Powers

Let's rewrite expressions with exponents as a single power.

Warm-Up

1 Order the expressions from what you think is *least complicated* to *most complicated*.

$$\frac{7^7}{7^3} \qquad \frac{7}{7}\cdot\frac{7}{7}\cdot\frac{7}{7}\cdot 7\cdot 7\cdot 7\cdot 7 \qquad \frac{7\cdot 7\cdot 7\cdot 7\cdot 7\cdot 7\cdot 7}{7\cdot 7\cdot 7}$$

$$\frac{7^3\cdot 7^3}{7^2} \qquad 7^4$$

Responses vary.

	Least Complicated
	Most Complicated

Activity

1

Name: ______________________ Date: __________ Period: __________

Single Powers

2 Circle one expression.

$\frac{7^3 \cdot 7^3}{7^2}$ $\frac{7}{7} \cdot \frac{7}{7} \cdot \frac{7}{7} \cdot 7 \cdot 7 \cdot 7 \cdot 7$ $\frac{7^7}{7^3}$ $\frac{7 \cdot 7 \cdot 7 \cdot 7 \cdot 7 \cdot 7 \cdot 7}{7 \cdot 7 \cdot 7}$

Show or explain how to rewrite this expression as the single power 7^4.

Responses vary. Using $\frac{7^3 \cdot 7^3}{7^2}$, I wrote out all the factors of 7 and looked for parts of the expression that could be rewritten as 1. I multiplied the remaining factors, then rewrote it as a single power: $\frac{7^3 \cdot 7^3}{7^2} = \frac{(7 \cdot 7 \cdot 7) \cdot (7 \cdot 7 \cdot 7)}{7 \cdot 7} = \frac{7}{7} \cdot \frac{7}{7} \cdot 7 \cdot 7 \cdot 7 \cdot 7 = 1 \cdot 1 \cdot 7 \cdot 7 \cdot 7 \cdot 7 = 7^4$.

3 Here is how Jayla rewrote $\frac{7^5 \cdot 7^2}{7^3}$ as a single power.

Discuss: How could you rewrite $\frac{4^9}{4^2 \cdot 4^4}$ as a single power?

Responses vary. I can expand the expression so there are nine 4s multiplied together in the numerator and six 4s multiplied together in the denominator. After dividing, I get $4 \cdot 4 \cdot 4$, which is the same as 4^3.

Jayla

$$\frac{7^5 \cdot 7^2}{7^3} = \frac{(7 \cdot 7 \cdot 7 \cdot 7 \cdot 7) \cdot (7 \cdot 7)}{7 \cdot 7 \cdot 7}$$

$$= \frac{7}{7} \cdot \frac{7}{7} \cdot \frac{7}{7} \cdot 7 \cdot 7 \cdot 7 \cdot 7$$

$$= 1 \cdot 1 \cdot 1 \cdot 7 \cdot 7 \cdot 7 \cdot 7$$

$$= 7^4$$

single power of 7

4 Rewrite each expression as a single power.

Expression	Single Power
$\frac{7^5 \cdot 7^2}{7^3}$	7^4
$\frac{4^9}{4^2 \cdot 4^4}$	**4^3**
$\frac{2^3 \cdot 2^3 \cdot 2^3}{2 \cdot 2 \cdot 2}$	**2^6**

Expression	Single Power
$2^4 \cdot 3^4$	**6^4**
$\frac{6^7}{2^7}$	**3^7**
$\frac{(8^4)^2}{8}$	**8^7**

Name: .... Date: .... Period: ....

Single Powers (continued)

5 Sort the expressions based on whether they are equivalent to 6^8.

$2^8 \cdot 3^8$ $2^3 \cdot 3^5$ $\frac{12^8}{2^8}$

$\frac{6^4 \cdot 6^4}{6^1}$ $\frac{6^7 \cdot 6^7}{(6^3)^2}$

Equivalent to 6^8	Not Equivalent to 6^8
$2^8 \cdot 3^8$ $\frac{6^7 \cdot 6^7}{(6^3)^2}$ $\frac{12^8}{2^8}$	$\frac{6^4 \cdot 6^4}{6^1}$ $2^3 \cdot 3^5$

6 Create one (or more) expressions that are equivalent to 4^5. Write something as unique and as complicated as you want!

Responses vary.

- $\frac{4^8}{4^3}$
- $4^2 \cdot 4^3$
- $\frac{(4^3)^2}{4^1}$
- $2^5 \cdot 2^5$

Name: ______________________ Date: __________ Period: __________

Challenge Creator

7 You will use a separate sheet of paper to create your own single power challenge.

- **a** **Make It!** On your sheet of paper, write down a single power, using a positive integer for the base and any integer for the exponent. (For example, 4^3 or 5^{16} or 2^{100}.)
- **b** **Solve It!** On this page, record your single power, then create an expression that is equivalent to it. (For example, $5^{10} \cdot 5^6$ is equivalent to 5^{16}.)

My Single Power	Equivalent Expression

- **c** **Swap It!** Swap your challenge with one or more partners. Record your partner's single power, then create an expression that is equivalent to it. *Responses vary.*

	Single Power	Equivalent Expression
Partner 1		
Partner 2		
Partner 3		
Partner 4		

8 Synthesis

Discuss: What is a strategy you could use for rewriting an expression as a single power?

Use these expressions if they help with your thinking.

$$\frac{6^7 \cdot 6^7}{(6^3)^2} \qquad 2^8 \cdot 3^8 \qquad \frac{12^8}{2^8}$$

Responses vary. First, write out all the exponents using repeated multiplication. Then look for things that can be combined (e.g., $2 \cdot 3 = 6$) or rewritten as 1 (e.g., $\frac{12}{12} = 1$). At the end, rewrite everything possible using exponents again.

11 Summary 7.04

You can rewrite expressions as a single power like 7^3 to help you make sense of more complex expressions, especially ones that involve division. Expanding is one strategy for rewriting expressions with exponents as single powers.

Here are two examples.

$$\begin{aligned}\frac{(3^3)^2}{3^4} &= \frac{(3 \cdot 3 \cdot 3) \cdot (3 \cdot 3 \cdot 3)}{3 \cdot 3 \cdot 3 \cdot 3}\\ &= \frac{3}{3} \cdot \frac{3}{3} \cdot \frac{3}{3} \cdot \frac{3}{3} \cdot 3 \cdot 3\\ &= 1 \cdot 1 \cdot 1 \cdot 1 \cdot 3 \cdot 3\\ &= 3^2\end{aligned}$$

$$\begin{aligned}\frac{9^2 \cdot 3^5}{3^3} &= \frac{(9 \cdot 9) \cdot (3 \cdot 3 \cdot 3 \cdot 3 \cdot 3)}{3 \cdot 3 \cdot 3}\\ &= \frac{(3 \cdot 3 \cdot 3 \cdot 3) \cdot (3 \cdot 3 \cdot 3 \cdot 3 \cdot 3)}{3 \cdot 3 \cdot 3}\\ &= \frac{3}{3} \cdot \frac{3}{3} \cdot \frac{3}{3} \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3\\ &= 1 \cdot 1 \cdot 1 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3\\ &= 3^6\end{aligned}$$

Knowing that $\frac{(3^3)^2}{3^4} = 3^2$ and $\frac{9^2 \cdot 3^5}{3^3} = 3^6$ can help you compare the two: $\frac{9^2 \cdot 3^5}{3^3}$ is greater than $\frac{(3^3)^2}{3^4}$ because 3^6 is greater than 3^2.

Practice
7.04

Name: ______ Date: ______ Period: ______

1. Select *all* the expressions that are equivalent to 10^6.

- ☑ **A.** $5^6 \cdot 2^6$
- ☐ **B.** $10^8 - 10^2$
- ☑ **C.** $10^3 \cdot 10 \cdot 10^2$
- ☑ **D.** $\frac{10^9}{10^3}$
- ☐ **E.** $(10^3)^3$

Problems 2–9: Rewrite each expression as a single power.

2. $4^4 \cdot 5^4 \; = \mathbf{20^4}$

3. $\frac{5^6}{5^3} \; = \mathbf{5^3}$

4. $(14^3)^6 \; = \mathbf{14^{18}}$

5. $\frac{21^3 \cdot 21^5}{21^2} \; = \mathbf{21^6}$

6. $8^3 \cdot 8^6 \; = \mathbf{8^9}$

7. $\frac{3^{10}}{3} \; = \mathbf{3^9}$

8. $(12^2)^7 \cdot 12 \; = \mathbf{12^{15}}$

9. $\frac{16^6}{2^6} \; = \mathbf{8^6}$

10. Fill in each blank using the whole numbers 1 to 9 only once to make a true statement.

$$(3 \cdot 5)^{\square} \cdot (2 \cdot 3)^{\square} \cdot (2 \cdot 5)^{\square} = 2^{\square} \cdot 3^{\square} \cdot 5^{\square}$$

Responses vary.

$\mathbf{(3 \cdot 5)^2 \cdot (2 \cdot 3)^4 \cdot (2 \cdot 5)^5 = 2^9 \cdot 3^6 \cdot 5^7}$

Practice
7.04

Name: .... Date: .... Period: ....

11. Deiondre says $3^6 \cdot 15^5 \cdot 5^6$ is equivalent to 15^{11}. Is Deiondre's claim correct? Show or explain your thinking.

Yes. *Explanations vary.* The factors of $3^6 \cdot 5^6$ can be regrouped to have six pairs of $3 \cdot 5$, which is the same as 15^6. Then the expression can be rewritten as $15^6 \cdot 15^5$, which is equivalent to 15^{11}.

12. What is the value of n in the equation $7^n = 7^{12} \cdot 7^4$?

16

13. Order the expressions from *least* to *greatest*.

$3^5 \cdot 4^5$	$2^7 \cdot 6^7$	$(12^2)^4$	$12^2 \cdot 12^2$	$12^3 \cdot (12^2)^3$

$12^2 \cdot 12^2$	$3^5 \cdot 4^5$	$2^7 \cdot 6^7$	$(12^2)^4$	$12^3 \cdot (12^2)^3$

Least **Greatest**

Spiral Review

Problems 14–16: Bananas cost $1.50 per pound and guavas cost $3.00 per pound. Demetrius spends $12 on fruit for a breakfast his family is hosting. He buys b pounds of bananas and g pounds of guavas.

14. Write an equation relating the two variables.

$1.5b + 3g = 12$ (or equivalent)

15. If Demetrius buys 4 pounds of bananas, how many pounds of guavas can he buy?

2 pounds

16. Demetrius remembers that his aunt loves papayas and wants to surprise her at the breakfast. If he buys one large papaya for $4.50 and one pound of guavas, how many pounds of bananas can he purchase?

3

Unit 7

Lesson

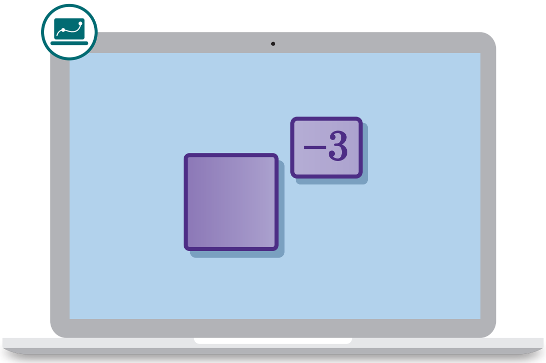
5

Name: ______ Date: ______ Period: ______

Big and Small Numbers 8.EE.1, SMP.1, SMP.7

Negative and Zero Exponents

Let's explore exponents that are not positive.



Warm-Up

1 Order the expressions by value from *least* to *greatest*.

$\frac{2^6}{2^3}$	$\frac{2^5}{2^5}$	2^6	0	$\frac{2^3}{2^4}$

Least

0

$\frac{2^3}{2^4}$

$\frac{2^5}{2^5}$

$\frac{2^6}{2^3}$

2^6

Greatest

Activity

1

Name: .... Date: .... Period: ....

Negative and Zero Exponents

2 Complete as much of the table as you can.

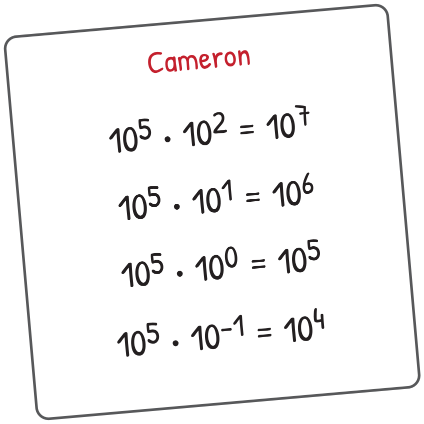
Exponent Form	Expanded Form	Value
10^4	$10 \bullet 10 \bullet 10 \bullet 10$	10,000
10^3	$10 \bullet 10 \bullet 10$	**1,000**
10^2	$\mathbf{10 \bullet 10}$	100
$\mathbf{10^1}$	10	10
10^0	**1**	**1**
10^{-1}	$\frac{1}{10}$	**0.1** or $\frac{1}{10}$
10^{-2}	$\frac{1}{10 \bullet 10}$ or $\frac{1}{10} \bullet \frac{1}{10}$	$\frac{1}{100}$

3 What patterns do you see in the table? Describe as many as you can.

Responses vary.

- As I move down the left column, the exponents decrease by 1 for each row.
- Positive exponents describe the number of factors of 10.
- Negative exponents describe the number of factors of 10 in the denominator.
- Negative exponents describe the number of factors of $\frac{1}{10}$.
- As I move down the table, the values get closer to 0.
- There is a form of mirror symmetry in each column of the table.

4 Cameron wanted to investigate more about negative and zero exponents. Cameron decided to write some expressions and apply exponent properties.


Cameron

$10^5 \cdot 10^2 = 10^7$

$10^5 \cdot 10^1 = 10^6$

$10^5 \cdot 10^0 = 10^5$

$10^5 \cdot 10^{-1} = 10^4$

Discuss:

- What patterns do you notice in Cameron's expressions?
- How could you use Cameron's work to determine the values of 10^0 and 10^{-1}?

  Responses vary.

  - Each line on Cameron's work decreases by a factor of 10.
  - The third line suggests that 10^0 must be 1 because 1 is the only number that Cameron can multiply by and still get 10^5. The fourth line suggests that 10^{-1} must be $\frac{1}{10}$ because to get from 10^5 to 10^4 Cameron must divide by 10 or multiply by $\frac{1}{10}$.

Name: ____________________ Date: __________ Period: __________

Negative and Zero Exponents (continued)

5 Jayla and Cameron wrote $\frac{10^4}{10^4}$ as equivalent expressions.

Discuss: How are their expressions equivalent to each other?

Responses vary. **Jayla's expression equals 1 because a fraction with the same value in the numerator and denominator is equal to 1. Cameron's expression also equals 1 because any number with a zero exponent is equal to 1.**

Jayla

$$\frac{10^4}{10^4} = \frac{10 \cdot 10 \cdot 10 \cdot 10}{10 \cdot 10 \cdot 10 \cdot 10}$$

$$= \frac{10}{10} \cdot \frac{10}{10} \cdot \frac{10}{10} \cdot \frac{10}{10}$$

$$= 1 \cdot 1 \cdot 1 \cdot 1$$

$$= 1$$

Cameron

$$\frac{10^4}{10^4} = 10^{4-4}$$

$$= 10^0$$

6 Jayla and Cameron wrote $\frac{10^2}{10^5}$ as equivalent expressions.

Discuss: How are their expressions equivalent to each other?

Responses vary. **Cameron's expression is equal to Jayla's because a number with a negative exponent will be less than 1 and can be rewritten as a fraction using 1 in the numerator and the same base with an exponent of the opposite sign in the denominator.**

Jayla

$$\frac{10^2}{10^5} = \frac{10 \cdot 10}{10 \cdot 10 \cdot 10 \cdot 10 \cdot 10}$$

$$= \frac{10}{10} \cdot \frac{10}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10}$$

$$= 1 \cdot 1 \cdot \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10}$$

$$= \frac{1}{10^3}$$

Cameron

$$\frac{10^2}{10^5} = 10^{2-5}$$

$$= 10^{-3}$$

Name: .... Date: .... Period: ....

Beyond 10

7 Complete this new table about powers of 3.

Exponent Form	Expanded Form	Value
3^3	$3 \cdot 3 \cdot 3$	27
3^2	$3 \cdot 3$	**9**
3^1	3	3
3^0	**1**	**1**
3^{-1}	$\frac{1}{3}$	$\frac{1}{3}$
3^{-2}	$\frac{1}{3 \cdot 3}$ **or** $\frac{1}{3} \cdot \frac{1}{3}$	$\frac{1}{9}$ **(or equivalent)**
3^{-3}	$\frac{1}{3 \cdot 3 \cdot 3}$ **or** $\frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{3}$	$\frac{1}{27}$ **(or equivalent)**

8 The value of $3^6 = 729$. Predict the value of 3^{-6}.

$\frac{1}{729}$ **(or equivalent)**

Name: .... Date: .... Period: ....

Beyond 10 (continued)

9 Group the equivalent expressions. Some expressions may have no match.

$\left(\frac{1}{3}\right)^5$

$\frac{1}{5} \bullet \frac{1}{5} \bullet \frac{1}{5}$

3^{-5}

$\frac{1}{5 \bullet 5 \bullet 5}$

5^{-3}

-15

$\frac{1}{3^5}$

$\frac{1}{15}$

Group 1	Group 2	Left over
3^{-5}	5^{-3}	-15
$\frac{1}{3^5}$	$\frac{1}{5} \bullet \frac{1}{5} \bullet \frac{1}{5}$	$\frac{1}{15}$
$\left(\frac{1}{3}\right)^5$	$\frac{1}{5 \bullet 5 \bullet 5}$	

10 Here are two expressions from earlier.

3^{-5}

-15

Are these equivalent? Circle one.

Yes [✓ No] I'm not sure.

Explain your thinking.

***Explanations vary.* These are not equivalent because the expression 3^{-5} is the same as $\frac{1}{3^5}$, which is a positive number, whereas -15 is a negative number.**

You're invited to explore more.

11 Write as many different expressions that are equivalent to $\left(\frac{2}{3}\right)^{-3}$ as you can.
Here is one example: $\left(\frac{3}{2}\right)^3$

Responses vary.

- $\frac{3}{2} \bullet \frac{3}{2} \bullet \frac{3}{2}$
- $\frac{27}{8}$
- $\left(\frac{2}{3}\right)^{-5} \bullet \left(\frac{2}{3}\right)^2$
- $\frac{2^{-3}}{3^{-3}}$
- $\frac{1}{\left(\frac{2}{3}\right)^3}$

12 Synthesis

Discuss: How could you use the table to convince someone that $6^0 = 1$ and $6^{-1} = \frac{1}{6}$?

Exponent Form	Value
6^3	216
6^2	36
6^1	6
6^0	
6^{-1}	

Responses vary. **Each time the exponent decreases by 1, the value gets divided by 6. Based on this pattern and the fact that $6^1 = 6$, I can determine that $6^0 = 1$. I can use the same logic for 6^{-1}. If I divide both sides of $6^0 = 1$ by 6, I get $6^{-1} = \frac{1}{6}$.**

15 Summary 7.05

Positive, negative, and zero exponents are all related.

For example, this table shows that each time the exponent decreases by 1, the value gets divided by 4. Based on this pattern we can determine that $4^0 = 1$. Similarly, if we divide both sides of $4^0 = 1$ by 4, we get $4^{-1} = \frac{1}{4}$.

We can use these patterns to make generalizations about powers with zero and negative exponents.

Any power with an exponent of 0 is equal to 1.

Examples:

- $\left(\frac{1}{5}\right)^0 = 1$
- $(-3)^0 = 1$

Powers with negative exponents are equal to 1 divided by the power with the corresponding positive exponent.

Examples:

- $5^{-2} = \left(\frac{1}{5}\right)^2 = \frac{1}{5} \bullet \frac{1}{5} = \frac{1}{25}$
- $10^{-3} = \frac{1}{10^3} \bullet \frac{1}{10 \bullet 10 \bullet 10} = \frac{1}{1000}$

Exponent Form	Expanded Form	Value
4^2	$4 \bullet 4$	16
4^1	4	4
4^0	1	1
4^{-1}	$\frac{1}{4}$	$\frac{1}{4}$
4^{-2}	$\frac{1}{4} \bullet \frac{1}{4}$	$\frac{1}{16}$

Practice
7.05

Name: .... Date: .... Period: ....

Problems 1–2: Prisha says: *I can determine the value of* 5^0 *by looking at other powers of* 5. She organizes her work in a table.

Exponent Form	Value
5^3	125
5^2	25
5^1	5
5^0	

1. What pattern do you notice in Prisha's table?

***Responses vary.* When the power of 5 decreases by 1, the value is divided by 5.**

2. If this pattern continues, what would the value of 5^0 be?

1

3. Select *all* the expressions that are equivalent to 4^{-3}.

☐ **A.** -12

☑ **B.** 2^{-6}

☑ **C.** $\frac{1}{4^3}$

☑ **D.** $\left(\frac{1}{4}\right) \cdot \left(\frac{1}{4}\right) \cdot \left(\frac{1}{4}\right)$

☐ **E.** $(-4) \cdot (-4) \cdot (-4)$

Problems 4–7: Determine if each equation is *true* or *false*. Then change one side of each false equation to make it true.

	Equation	True	False	Changed Equation
4.	$\frac{5^3}{5^3} = 5^0$	✓		
5.	$\frac{5^{-3}}{5^3} = 5^0$		✓	*Responses vary.* $\frac{5^{-3}}{5^3} = 5^{-6}$
6.	$5^0 \cdot 5^{-6} \cdot 5^5 = \frac{1}{5}$	✓		
7.	$(5^{-4})^0 = \frac{1}{5^4}$		✓	*Responses vary.* $(5^{-4})^0 = 1$

Practice
7.05

Name: .... Date: .... Period: ....

Problems 8–10: Rewrite each expression as a single power.

8. $\frac{3^7}{3^{11}}$ **3^{-4} or $\frac{1}{3^4}$**

9. $2^{-5} \cdot 3^{-5}$ **6^{-5} or $\frac{1}{6^5}$**

10. $\frac{7^0 \cdot 8^{-3}}{4^{-3}}$ **2^{-3} or $\frac{1}{2^3}$**

11. In the expressions, a and b represent integers. The value of Expression 2 is 100 times greater than the value of Expression 1. What is the value of b?

Expression 1: 10^{a+b} Expression 2: 10^a

A. 2
B. 0
[✓ **C.**] -2
D. -4

Spiral Review

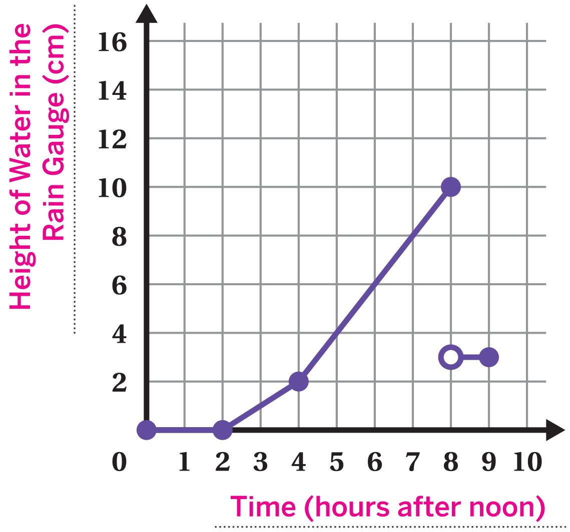
Problems 12–13: Adrian set up a rain gauge to measure rainfall in his backyard. The graph shows the total rainfall (in centimeters) in Adrian's rain gauge from noon to 9:00 PM.

12. Label the axes on the graph, including the units of measurement in parentheses.

Responses vary.

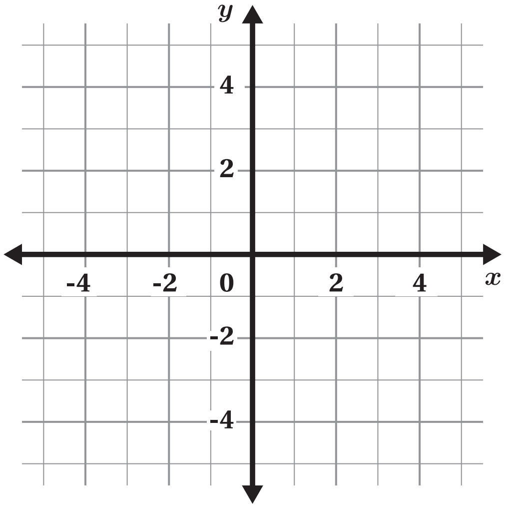
13. Use the graph to write a story that describes the rainfall that day.

***Responses vary.* At 2 PM, the gauge was empty, but it began to fill as it rained. Two hours later, the gauge had 2 centimeters of water in it. At 8 PM, Adrian found that the gauge had 10 centimeters of water in it. But then he accidentally knocked the gauge over, spilling all but 3 centimeters of water. At 9 PM, there were no further changes in the water level.**



14. Write the equation of the line passing through the points (-4, -2) and (4, 4). Use the graph it helps with your thinking

$y = \frac{3}{4}x + 1$ (or equivalent)



Unit 7

Lesson

6

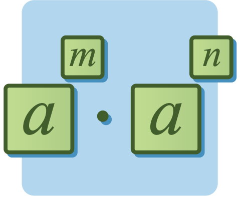
Name: ______ Date: ______ Period: ______

Big and Small Numbers

8.EE.1, SMP.3, SMP.7, SMP.8

Write a Rule

Let’s describe the rules for exponents.



Warm-Up

1. You and your partner will use a set of cards. Sort the expressions into groups in a way that makes sense to you.

Responses vary.
Group 1: Card E, Card L
Group 2: Card B, Card I
Group 3: Card C, Card D
Group 4: Card F, Card K
Group 5: Card G, Card H
Group 6: Card A, Card J

Activity
1

Name: .... Date: .... Period: ....

Write Your Own Rules

2. You and your partner will use the cards from the Warm-Up and the instructions on the screen to create your own rules for three exponent situations. **ELD.PI.8.10.Em, Ex, Br, ELD.PII.8.5.Em, Ex, Br**

Multiplying Powers With the Same Base

Examples	Rule (First Draft)	Rule (Second Draft)
$6^4 \cdot 6^{11} = 6^{15}$ Card E: $\left(\frac{3}{5}\right)^4 \cdot \left(\frac{3}{5}\right)^2 = \left(\frac{3}{5}\right)^6$ Card L: $10^{14} \cdot 10^1 \cdot 10^{-2} = 10^{13}$	*Responses vary*. When multiplying, we can add exponents.	*Responses vary*. When multiplying exponential terms with the same base, we can rewrite the expression by adding the exponents.

Dividing Powers With the Same Base

Examples	Rule (First Draft)	Rule (Second Draft)
$\frac{3^9}{3} = 3^8$ Card B: $\frac{(0.5)^8}{(0.5)^3} = (0.5)^5$ Card I: $\frac{10^2}{10^7} = 10^{-5}$	*Responses vary*. When dividing, we can subtract the exponents.	*Responses vary*. When dividing exponential terms with the same base, we can rewrite the expression by subtracting the exponents.

Powers of Powers

Examples	Rule (First Draft)	Rule (Second Draft)
$\left(\left(\frac{7}{8}\right)^5\right)^2 = \left(\frac{7}{8}\right)^{10}$ Card C: $(5^3)^4 = 5^{12}$ Card D: $(10^2)^{-3} = 10^{-6}$	*Responses vary*. We can multiply the exponents.	*Responses vary*. When raising an exponential term to a power, we can rewrite the expression by multiplying the exponents.

3. Choose any one of the three rules. Show or explain how you know the rule always works.

Responses vary. I chose the Dividing Powers With the Same Base rule. I know this rule always works because I get the same answer whether I expand out the expression and reduce the numerator and denominator or keep the common base and just subtract the exponents. For example: $\frac{10^2}{10^7} = \frac{10 \cdot 10}{10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10} = \frac{1}{10^5} = 10^{-5}$ and $\frac{10^2}{10^7} = 10^{2-7} = 10^{-5}$.

Activity
2

Name: ______________________ Date: __________ Period: __________

Three More Rules

4. You will use the cards from the Warm-Up and the instructions on the screen to create your own rules for three more exponent situations. **ELD.PI.8.10.Em, Ex, Br, ELD.PII.8.5.Em, Ex, Br**

Negative Exponents

Examples	Rule (First Draft)	Rule (Second Draft)
$3^{-6} = \frac{1}{3^6}$ Card F: $10^{-2} = \frac{1}{10^2}$ Card K: $\frac{1}{10^3} = 10^{-3}$	*Responses vary.* We can flip negative exponents to make them positive.	*Responses vary.* Exponential terms with negative exponents are equivalent to their reciprocal with a positive exponent.

Powers With Different Bases

Examples	Rule (First Draft)	Rule (Second Draft)
$7^3 \cdot 3^3 = 21^3$ Card G: $5^4 \cdot \left(\frac{1}{2}\right)^4 = \left(\frac{5}{2}\right)^4$ Card H: $10^{11} = 2^{11} \cdot 5^{11}$	*Responses vary.* Multiply the bases together and keep the exponent.	*Responses vary.* When multiplying exponential terms that have the same exponent but different bases, we can rewrite the expression by multiplying the bases and keeping the exponent as is.

Zero Exponents

Examples	Rule (First Draft)	Rule (Second Draft)
$10^0 = 1$ Card A: $(-7)^0 = 1$ Card J: $(2.98)^0 = 1$	*Responses vary.* Anything to the power of zero is 1.	*Responses vary.* Any exponential base raised to the power of zero is equal to 1.

5. Choose any one of the three rules. Show or explain how you know the rule always works.

Responses vary. I chose the Powers With Different Bases rule. I know this rule always works because expanding the factored form gives me the same answer as multiplying the bases and keeping the exponent as is. For example: $7^3 \cdot 3^3$ becomes $(7 \cdot 7 \cdot 7) \cdot (3 \cdot 3 \cdot 3)$. When I regroup the factors, I get $(7 \cdot 3) \cdot (7 \cdot 3) \cdot (7 \cdot 3)$, which is equivalent to $21 \cdot 21 \cdot 21 = 21^3$.

Synthesis

6. Describe how exponent rules can help you rewrite exponential expressions.

ELD.PI.8.10.Em, Ex, Br

Responses vary. **Exponent rules can help to rewrite exponential expressions more quickly. For example, if I'm dividing two terms with the same base, like $\frac{3^7}{3^2}$, I can quickly rewrite the expression by subtracting the exponents: $3^{7-2} = 3^5$.**

Summary 7.06

There are several rules about powers that can be helpful when rewriting or comparing expressions with exponents.

Types of Powers	Rule With Variables	Example
Multiplying Powers With the Same Base	$a^n \bullet a^m = a^{n+m}$	$6^2 \bullet 6^7 = 6^{2+7} = 6^9$
Dividing Powers With the Same Base	$\frac{a^n}{a^m} = a^{n-m}$	$\frac{3^{11}}{3^4} = 3^{11-4} = 3^7$
Powers of Powers	$(a^n)^m = a^{n \bullet m}$	$(1.7^5)^3 = 1.7^{(5 \cdot 3)} = 1.7^{15}$
Negative Exponents	$a^{-n} = \frac{1}{a^n} = \left(\frac{1}{a}\right)^n$	$14^{-2} = \frac{1}{14^2} = \left(\frac{1}{14}\right)^2$
Powers With Different Bases	$a^n \bullet b^n = (ab)^n$ $\frac{a^n}{b^n} = \left(\frac{a}{b}\right)^n$	$2^3 \bullet 5^3 = 10^3$ $\frac{2^3}{5^3} = \left(\frac{2}{5}\right)^3$
Zero Exponents	$a^0 = 1$	$\left(\frac{5}{9}\right)^0 = 1$

Note: The variables a and b are not equal to 0, and n and m are integers.

Practice
7.06

Name: ………………………………… Date: ………………… Period: …………………

Problems 1–3: Evaluate each expression.

1. $5^0 = 1$

2. $\frac{6^3}{6^3} = 1$

3. $2^2 + 2 + 2^0 = 7$

Problems 4–9: Rewrite each expression as a single power.

4. $\frac{5^3 \cdot 5^4}{5^5} = 5^2$

5. $(7^4) \cdot \frac{7^{12}}{7^7} = 7^9$

6. $\left(\frac{3^5}{3^3}\right)^4 = 3^8$

7. $\frac{2^4 \cdot 2^5 \cdot 2^6}{2^3 \cdot 2^7} = 2^5$

8. $\frac{(10^5)^2}{(10^2)^3} = 10^4$

9. $\left(\left(\frac{2}{3}\right)^4\right)^6 = \left(\frac{2}{3}\right)^{24}$

10. Determine the value of n for the equation $6^n = \frac{1}{6^4}$.

-4

Problems 11–13: Rewrite each expression as a single power with a negative exponent.

	Expression	Single Power With Negative Exponent
	$\frac{1}{6} \cdot \frac{1}{6}$	6^{-2}
11.	$\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2}$	2^{-3}
12.	$\frac{1}{5^7}$	5^{-7}
13.	$\left(\frac{1}{10^3}\right)^3$	10^{-9}

Practice
7.06

Name: Date: Period:

14. Circle the expression that is not equivalent to the others.

1 10^0 $\frac{8^2}{8^2}$ $8^3 \cdot 8^{-3}$ [✓ 0]

15. Kai tried to write $\left(\frac{5^8}{5^{10}}\right)^3$ as a single power of 5:

$$\left(\frac{5^8}{5^{10}}\right)^3 = (5^2)^3 = 5^{2 \cdot 3} = 5^6$$

Is Kai's work correct? **No**

Explain your thinking.

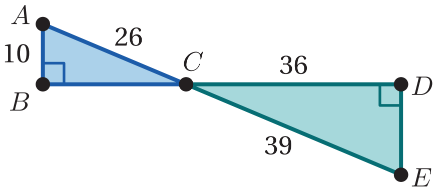
Explanations vary. **When dividing terms with the same base, we can rewrite the expression by keeping the base as is and subtracting the exponents. So** $\frac{5^8}{5^{10}} = 5^{8-10} = 5^{-2}$**, not** 5^2**.**

16. Fill in each blank using the digits 0 to 9 only once to create a result with the greatest exponent. ***Responses vary.*** $\frac{3^5}{3^1} \cdot (3^9)^8 = 3^{76}$

$$\frac{3^{\square}}{3^{\square}} \times \left(3^{\square}\right)^{\square} = 3^{\square\square}$$

Spiral Review

Problems 17–19: Here is a diagram.



17. Explain why triangle ABC is similar to triangle EDC.

Responses vary. **Both triangles contain a right angle, and angles** ACB **and** ECD **are vertical angles. The triangles are similar because two pairs of corresponding angles are congruent.**

18. Determine the length of side BC.

24

19. Determine the length of side DE.

15

Name: ______________________ Date: __________ Period: __________

Big and Small Numbers 8.EE.1, SMP.6, SMP.7

Practice Day 1

Let's practice what you have learned so far in this unit!

You will use task cards for this Practice Day. Record all of your responses here.

Task A: Write It!

1. Choose six problems to write using a single exponent.

A. $10^{-3} \cdot 10^{8}$
10^{5}

B. $\frac{3^{5}}{3^{28}}$
3^{-23} or $\frac{1}{3^{23}}$

C. $(7^{2})^{3}$
7^{6}

D. $\frac{2^{-5}}{2^{4}}$
2^{-9} or $\frac{1}{2^{9}}$

E. $3^{5} \cdot 3^{6}$
3^{11}

F. $(5^{3})^{-3}$
5^{-9} or $\frac{1}{5^{9}}$

G. $2^{-4} \cdot 2^{-3}$
2^{-7} or $\frac{1}{2^{7}}$

H. $(10^{-8})^{-4}$
10^{32}

I. $\frac{6^{5}}{6^{-8}}$
6^{13}

J. $(12^{-3})^{5}$
12^{-15} or $\frac{1}{12^{15}}$

K. $\frac{10^{0}}{10^{-20}}$
10^{20}

L. $\left(\frac{5}{6}\right)^{4} \cdot \left(\frac{5}{6}\right)^{5}$
$\left(\frac{5}{6}\right)^{9}$

2. Problem and Explanation: ***Responses vary.* I skipped Problem L because I wasn't sure how to use exponents and fractions together.**

Task B: Write It, Part 2!

1. Choose three problems to write using a single *positive* exponent.

A. $10^{-7} = \frac{1}{10^{7}}$

B. $\frac{5^{3}}{5^{7}} = \frac{1}{5^{4}}$

C. $\frac{9^{6}}{9^{11}} = \frac{1}{9^{5}}$

D. $\left(\frac{1}{2}\right)^{-32} = 2^{32}$

E. $7^{-8} = \frac{1}{7^{8}}$

F. $\left(\frac{8}{5}\right)^{-5} = \left(\frac{5}{8}\right)^{5}$

2. Problem and Explanation: ***Responses vary.* I would assign Problem F to my best friend because fractions are a nice challenge.**

Name: .... Date: .... Period: ....

Practice Day 1

Task C: Evaluate It!

1. Choose three problems to evaluate (write without any exponents).

A. $\frac{10^5}{10^5} = \mathbf{1}$

B. $\left(\frac{5}{4}\right)^2 = \mathbf{\frac{25}{16}}$

C. $\left(\frac{2}{3}\right)^3 = \mathbf{\frac{8}{27}}$

D. $(3^4)^0 = \mathbf{1}$

E. $2^8 \cdot 2^{-8} = \mathbf{1}$

F. $\left(\frac{7}{2}\right)^2 = \mathbf{\frac{49}{4}}$

2. Expression and Explanation: ***Responses vary.***

- **Problem A evaluates to 1 because any number divided by itself is 1.**
- **Problem D evaluates to 1 because any non-zero number to the power of 0 is 1.**

Task D: Single Exponents

1. Choose three problems to write using a single exponent. (Note: Not all problems can be written using a single exponent.)

A. $10^3 \cdot 10^3$
$\mathbf{10^6}$

B. $3^2 \cdot 2^3$
Not possible.

C. $5^6 \cdot 9^6$
$\mathbf{45^6}$

D. $2^3 \cdot 4^3 \cdot 6^3$
$\mathbf{48^3}$

E. $7^5 \cdot 8^5$
$\mathbf{56^5}$

F. $\left(\frac{2}{3}\right)^4 \cdot \left(\frac{2}{3}\right)^4$
$\mathbf{\left(\frac{2}{3}\right)^8}$ **or** $\mathbf{\left(\frac{4}{9}\right)^4}$

2. Problem and Explanation: **Problem B. *Explanations vary*. Problem B can't be written using a single exponent because both the bases and the exponents are different. I could combine some factors to get $6^2 \cdot 2$, but I can't combine using a single exponent.**

You're invited to explore more.

$x =$ **20**

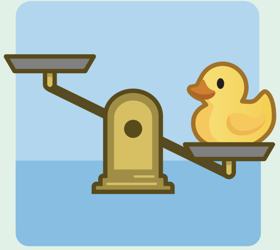
Explanation: ***Explanations vary*. Because $9 = 3^2$, I can write the expression $3^{(x+4)} = 3^{2(12)}$. This means $x + 4 = 2(12)$, so $x = 20$.**

Sub-Unit

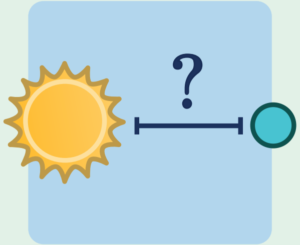
2

Scientific Notation

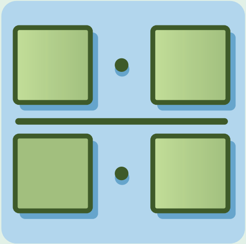
Lesson 7
Scales and Weights, Part 1



Lesson 8
Scales and Weights, Part 2



Lesson 9
Specific and Scientific



Lesson 10
Multiplying and Dividing

Lesson 11
Balance the Scale

Lesson 12
Use Your Powers

Lesson 13
City Lights

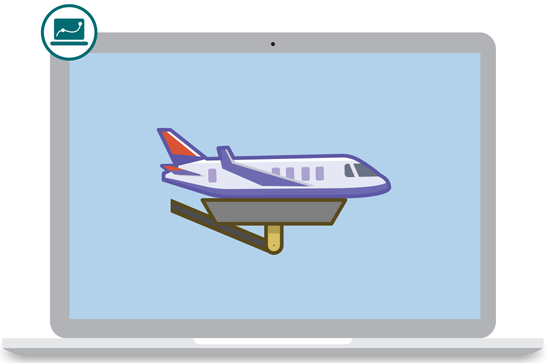
Lesson 14
Star Power

Name: ______________________ Date: __________ Period: __________

Big and Small Numbers 8.EE.3, SMP.3

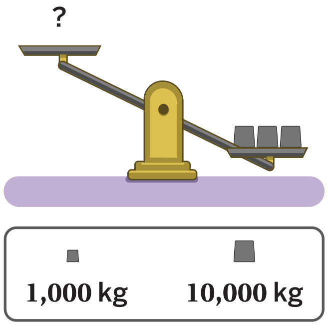
Scales and Weights, Part 1

Let's explore ways to represent large numbers.



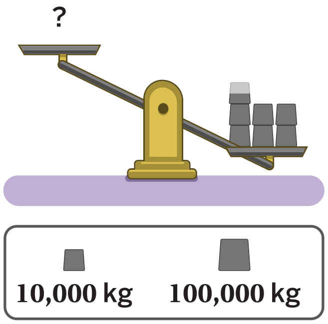
Warm-Up

1 **a**



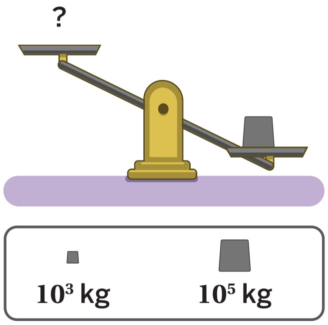
How many 1,000 kg weights are needed to balance with three 10,000 kg weights? **30**

b



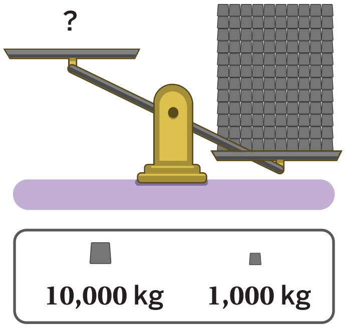
How many 10,000 kg weights are needed to balance with 6.5 100,000 kg weights? **65**

c



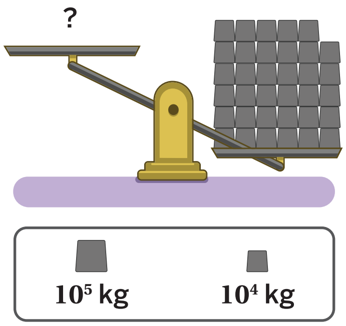
How many 10^3 kg weights are needed to balance with one 10^5 kg weight? **100**

d



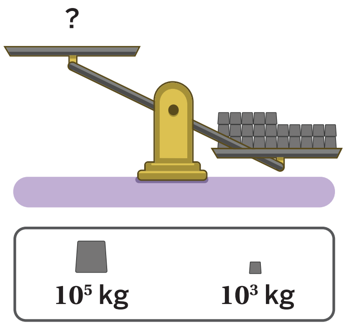
How many 10,000 kg weights are needed to balance with 120 1,000 kg weights? **12**

e



How many 10^5 kg weights are needed to balance with 35 10^4 kg weights? **3.5**

f



How many 10^5 kg weights are needed to balance with 25 10^3 kg weights? **0.25**

Activity
1

Name: .... Date: .... Period: ....

Scales and Weights

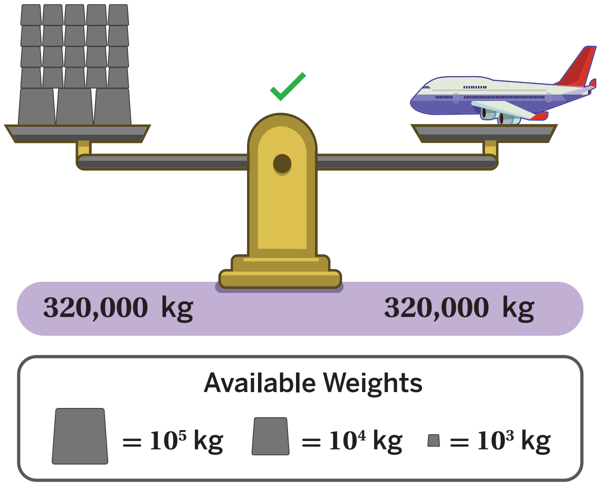
2-3 A plane weighs 320,000 kilograms.

The table shows how a student balanced the scale using the weights provided.

Write two other combinations of weights that will balance the scale.

Responses vary.

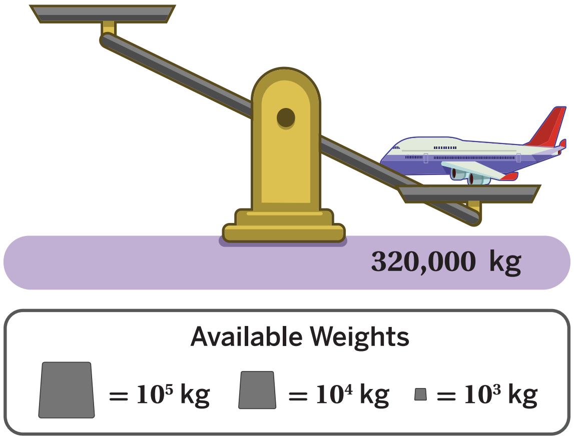
10^5 kg Weights	10^4 kg Weights	10^3 kg Weights
3	0	20
3	**2**	**0**
0	**30**	**20**



4 Another way to write 320,000 kilograms is to use a combination of powers of 10. For example: $3 \cdot 10^5 + 2 \cdot 10^4$ kilograms.

a **Discuss:**
What does each part of the expression represent in terms of weights?

Responses vary. **$3 \cdot 10^5$ represents three 100,000 kg weights, or 300,000 kg. The $2 \cdot 10^4$ represents two 10,000 kg weights, or 20,000 kg.**



b Write an expression to represent a *different* combination of available weights that will balance the scale.

Responses vary.

- **$3 \cdot 10^5 + 20 \cdot 10^3$ kilograms**
- **$32 \cdot 10^4$ kilograms**
- **$320 \cdot 10^3$ kilograms**

Name: .... Date: .... Period: ....

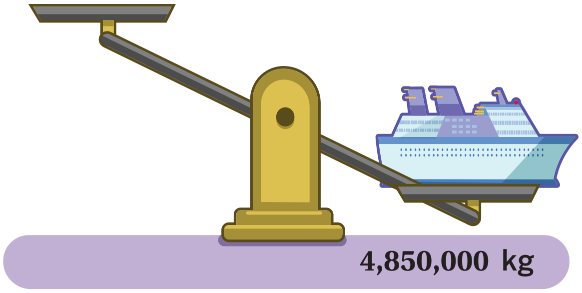
Ships and Shuttles

5 A ship weighs 4,850,000 kilograms.

Write an expression to represent a combination of available weights that will balance the scale.

Responses vary.

- **$4 \cdot 10^6 + 8 \cdot 10^5 + 5 \cdot 10^4$ kilograms**
- **$485 \cdot 10^4$ kilograms**
- **$48.5 \cdot 10^5$ kilograms**



Available Weights

■ = 10^6 kg ■ = 10^5 kg ▪ = 10^4 kg

6 Rishi and Parv tried to balance the scale with a 2,030,000-kilogram space shuttle.

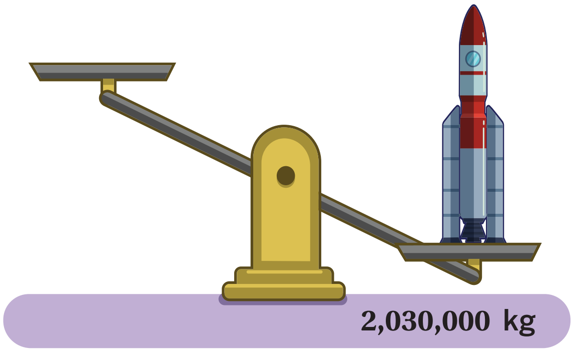
- Rishi wrote: $2.03 \cdot 10^6$ kilograms
- Parv wrote: $20 \cdot 10^5 + 3 \cdot 10^4$ kilograms

Whose expression represents a combination of weights that will balance the scale? Circle one.

Rishi's Parv's [✓ Both] Neither

Explain your thinking.

Responses vary. I know that Rishi's expression will balance the scale because $2.03 \cdot 10^6 = 203 \cdot 10^4 = 2030000$. I know that Parv's expression will also balance the scale because $20 \cdot 10^5 = 2000000$ and $3 \cdot 10^4 = 30000$, which makes 2,030,000 altogether.



Available Weights

■ = 10^6 kg ■ = 10^5 kg ▪ = 10^4 kg

7 Write an expression to represent a *different* combination of available weights that will balance the scale.

Responses vary.

- **$2 \cdot 10^6 + 3 \cdot 10^4$ kilograms**
- **$203 \cdot 10^4$ kilograms**
- **$1 \cdot 10^6 + 103 \cdot 10^4$ kilograms**

Activity
2

Name: .... Date: .... Period: ....

Ships and Shuttles (continued)

8 Match each expression with the value it is equivalent to. One expression will have no match.

$4.7 \cdot 10^5$ $4.7 \cdot 10^4$ $7 \cdot 10^4 + 400 \cdot 10^3$ $4 \cdot 10^4 + 7 \cdot 10^2$

$40 \cdot 10^3 + 70 \cdot 10^2$ $4 \cdot 10^5 + 7 \cdot 10^3$ $4 \cdot 10^5 + 5 \cdot 10^4 + 20 \cdot 10^3$

407,000	47,000	470,000
$4 \cdot 10^5 + 7 \cdot 10^3$	$4.7 \cdot 10^4$ $40 \cdot 10^3 + 70 \cdot 10^2$	$4.7 \cdot 10^5$ $7 \cdot 10^4 + 400 \cdot 10^3$ $4 \cdot 10^5 + 5 \cdot 10^4 + 20 \cdot 10^3$

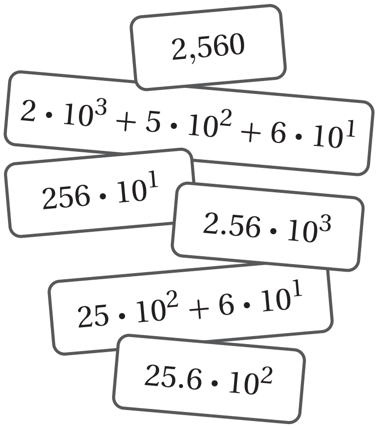
9 In the previous problem, how did you decide which value this expression is equivalent to?

$7 \cdot 10^4 + 400 \cdot 10^3$

Responses vary. I matched this expression with 470,000 because $7 \cdot 10^4 = 7 \cdot 10000 = 70000$ and $400 \cdot 10^3 = 400 \cdot 1000 = 400000$. When combined, $70000 + 400000 = 470000$.

10 Synthesis

Discuss: What are some strategies for writing a number as a combination of powers of 10? Use the examples if they help with your thinking.



Responses vary.

- **I think about the place value of each digit. For example, in 2,560 the 2 represents 2,000, so I could write it as $2 \cdot 10^3$. I can apply similar logic to the 5 and 6.**
- **I can write a number as a single multiple of a power of 10 using exponent rules. For example, $2560 = 2560 \cdot 10^0$, because $10^0 = 1$. Then I can borrow powers of 10 like this: $2560 \cdot 10^0 = 256 \cdot 10^1 = 25.6 \cdot 10^2 = 2.56 \cdot 10^3$.**

13 Summary 7.07

You can write large numbers as a combination of powers of 10 to prevent having to count 0s.

The number 90,700,000 can be written many different ways using powers of 10.

For example:

- $90700000 = 9 \cdot 10^7 + 7 \cdot 10^5$
- $90700000 = 90 \cdot 10^6 + 7 \cdot 10^5$
- $90700000 = 9.07 \cdot 10^7$

Practice

7.07

Name: .... Date: .... Period: ....

Problems 1–3: Write each value as a number times 10^3.

1. 42,300

$42.3 \cdot 10^3$

2. 2,000

$2 \cdot 10^3$

3. 301,000

$301 \cdot 10^3$

Problems 4–6: Rewrite each expression as a combination of powers of 10. ***Responses vary.***

4. 4,200,000

- **$4 \cdot 10^6 + 2 \cdot 10^5$**
- **$4.2 \cdot 10^6$**
- **$42 \cdot 10^5$**

5. 40,700

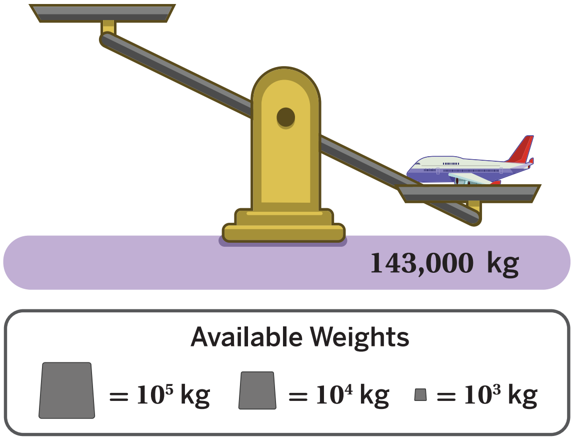
- **$4.07 \cdot 10^4$**
- **$4 \cdot 10^4 + 7 \cdot 10^2$**
- **$40700 \cdot 10^0$**

6. 999

- **$9.99 \cdot 10^2$**
- **$99.9 \cdot 10^1$**
- **$9 \cdot 10^2 + 9 \cdot 10^1 + 9 \cdot 10^0$**

Problems 7–8: Three students tried to balance the scale using weights measuring 10^5 kg, 10^4 kg, and 10^3 kg.

- Lucy wrote the weight of the plane as $14.3 \cdot 10^4$ kilograms.
- Parv wrote the weight of the plane as $143 \cdot 10^2$ kilograms.
- Kiri wrote the weight of the plane as $1 \cdot 10^5 + 4 \cdot 10^4 + 3 \cdot 10^3$ kilograms.



7. Which of these expressions are accurate?

Lucy's and Kiri's expressions

8. Write an expression to represent a different combination of available weights that will balance the scale.

***Responses vary.* $1.43 \cdot 10^5$ kilograms**

Practice
7.07

Name: .... Date: .... Period: ....

9. Which of the following numbers has the greatest value?

A. $48.34 \bullet 10^3$ [✓ B.] $4.834 \bullet 10^5$

C. $4.834 \bullet 10^4$ D. $4.83 \bullet 10^5$

10. Which expression is equivalent to 5,124,000?

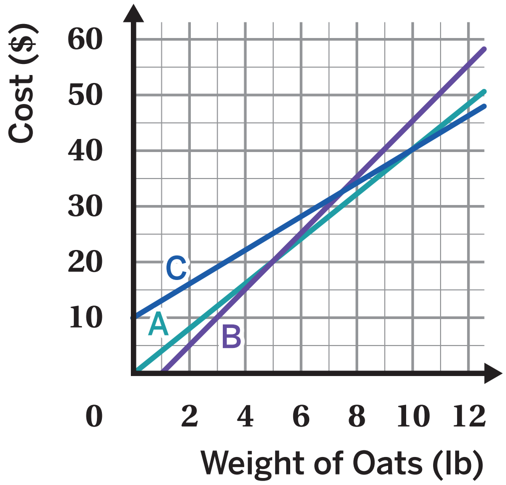
A. $5.12 \bullet 10^3$ B. $51.24 \bullet 10^6$

C. $5124 \bullet 10^3$ [✓ D.] $5.124 \bullet 10^6$

Spiral Review

Problems 11–13: Three stores are selling oats by the pound.

- Store A sells oats for a dollars per pound.
- Store B sells oats for b dollars per pound, with a coupon for $5 off the total purchase.
- Store C is an online store. They sell oats for c dollars per pound, with a $10 delivery fee.



11. Which store has the lowest price for two pounds of oats? **Store B**

12. If a customer wants to buy 6 pounds of oats to make granola, which store has the lowest price? **Store A**

13. How many pounds of oats would a customer need to buy to make Store C a good option? Explain your thinking.

Responses vary.

- **10 pounds. If a customer orders 10 pounds of oats, Store C and Store A have the lowest price.**
- **11 pounds, because Store C is the cheapest for *all* oat purchases above 10 pounds.**

14. Select *all* expressions that are equivalent to 10^{-3}.

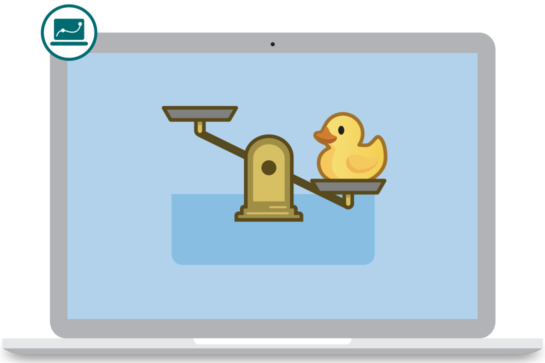
☑ A. $(5 \bullet 2)^{-3}$ ☐ B. $\frac{10^6}{10^3}$ ☑ C. $\frac{1}{10^3}$ ☐ D. $(10^{-3})^{-1}$ ☑ E. $\frac{1}{1000}$

Name: .... Date: .... Period: ....

Big and Small Numbers 8.EE.3, SMP.3

Scales and Weights, Part 2

Let's explore ways to represent small numbers with powers of 10.



Warm-Up

1 Match each number to a verbal description. One number will have no match.

1,000 10^{-3} 0.000001 10^{-6}

0.00001 10^{3} 0.001

One thousand	One thousandth	One millionth
1,000 10^{3}	0.001 10^{-3}	0.000001 10^{-6}

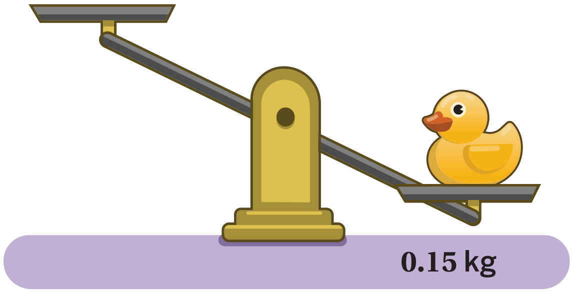
Activity
1

Name: ______________________ Date: __________ Period: __________

Light Weights

2-3 A rubber duck weighs 0.15 kilograms. A student balanced the scale using the weights shown in the top row of the table.

Write two other combinations of weights that will balance the scale. ***Responses vary.***



Available Weights

■ $= 10^{-1}$ kg ▪ $= 10^{-2}$ kg ▫ $= 10^{-3}$ kg

10^{-1} kg	10^{-2} kg	10^{-3} kg
1	5	0
0	**15**	**0**
0	**0**	**150**

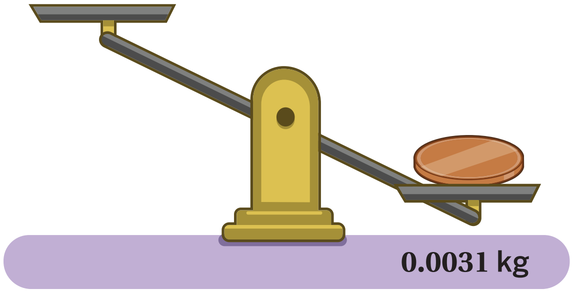
4 A penny weighs 0.0031 kilograms. Here's one way to write its weight using a combination of powers of 10:

$3 \cdot 10^{-3} + 1 \cdot 10^{-4}$

Write an expression to represent a *different* combination of available weights that will balance the scale.

Responses vary.

- **$3.1 \cdot 10^{-3}$ kilograms**
- **$30 \cdot 10^{-4} + 10 \cdot 10^{-5}$ kilograms**
- **$30 \cdot 10^{-4} + 0.1 \cdot 10^{-3}$ kilograms**



Available Weights

■ $= 10^{-3}$ kg ▪ $= 10^{-4}$ kg ▫ $= 10^{-5}$ kg

Activity
1

Name: .... Date: .... Period: ....

Light Weights (continued)

5 Two students made mistakes writing 0.0031 using combinations of powers of 10.

Arturo

$3.1 \cdot 10^{-2}$

Kimaya

$3 \cdot 10^{-4} + 1 \cdot 10^{-3}$

Circle your favorite mistake. ***Responses vary.***

Arturo's mistake Kimaya's mistake

a What is correct about this student's work?

Responses vary.

- **Arturo correctly identified that 0.0031 could be written as 3.1 times a negative power of 10.**
- **Kimaya correctly identified that 0.0031 could be made by adding 3 times a power of 10 and 1 times a power of 10.**

b What could you add or change to make *all* of their work correct?

Responses vary.

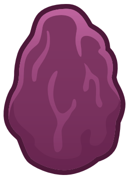
- **In Arturo's work, I could change the power of 10 from -2 to -3.**
- **In Kimaya's work, I could swap the -4 and -3 in the powers of 10.**

Name: .... Date: .... Period: ....

Getting Smaller and Smaller

6 The weight of a raisin is 0.000572 kilograms.
Here is how two students rewrote 0.000572.

Lukas	Alina
$57.2 \cdot 10^{-5}$	$5 \cdot 10^{-4} + 7 \cdot 10^{-5} + 2 \cdot 10^{-6}$



0.000572 kg

Discuss:

- What are some advantages of Alina's strategy?
- What are some advantages of Lukas's strategy?

Responses vary.

- **Alina's strategy multiplies each digit by a power of 10 that represents the digit's place value. This makes it clearer what value each digit represents.**
- **Lukas's strategy uses fewer numbers and symbols because there is only one power of 10.**

7 Lukas's strategy was to rewrite the raisin's weight, 0.000572 kilograms, as a number times a single power of 10:

$57.2 \cdot 10^{-5}$

Write the same weight as:

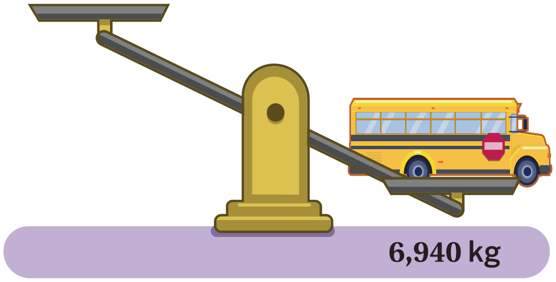
- A number times 10^{-6}: **$572 \cdot 10^{-6}$ kg**
- A number times 10^{-4}: **$5.72 \cdot 10^{-4}$ kg**

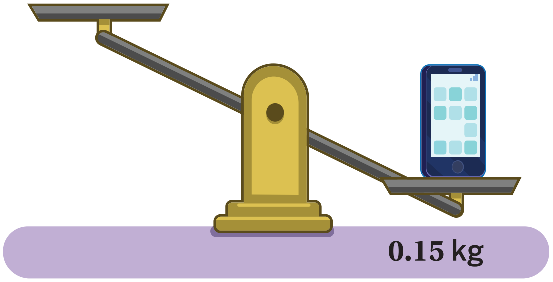
Activity
3

Name: ________ Date: ________ Period: ________

Repeated Challenges

8 Write the weight of each object using a number times a single power of 10.



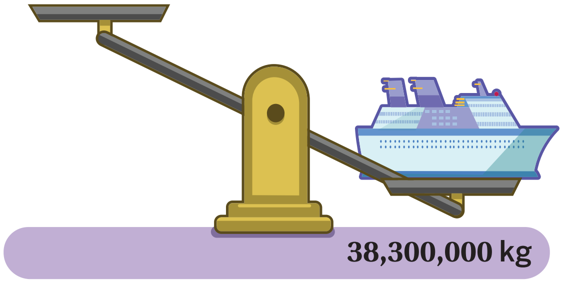


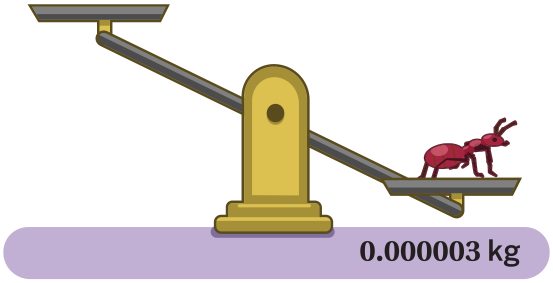
a Write the weight of the bus (6,940 kg) as a number times 10^3.

$6.94 \cdot 10^3$

b Write the weight of the cell phone (0.15 kg) as a number times 10^{-2}.

$15 \cdot 10^{-2}$





c Write the weight of the cruise ship (38,300,000 kg) as a number times 10^7.

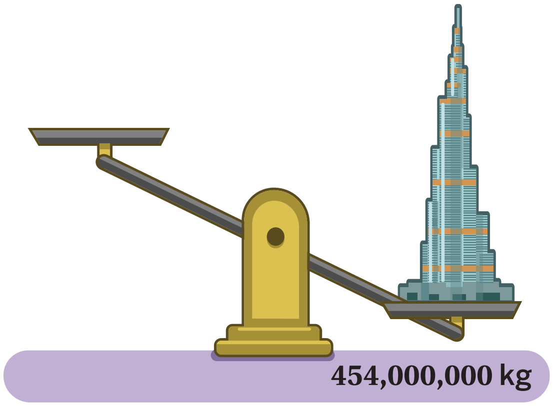
$3.83 \cdot 10^7$

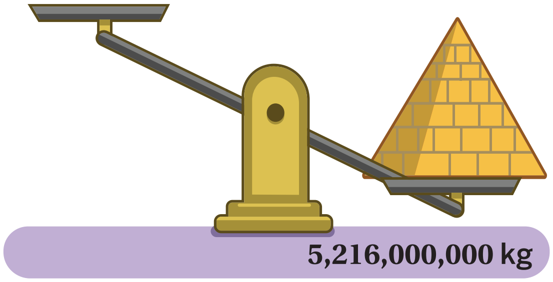
d Write the weight of the ant (0.000003 kg) as a number times 10^{-5}.

$0.3 \cdot 10^{-5}$

Name: .... Date: .... Period: ....

Repeated Challenges (continued)



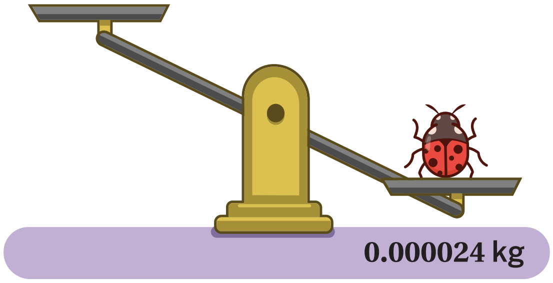


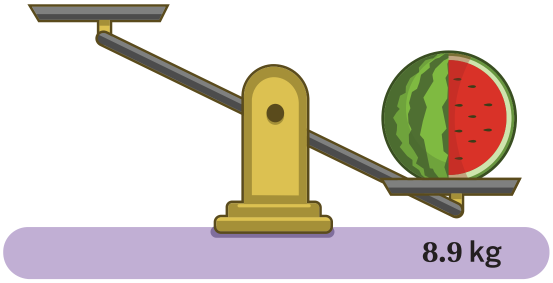
e Write the weight of the Burj Khalifa (454,000,000 kg) as a number times 10^7.

$45.4 \cdot 10^7$

f Write the weight of the pyramid (5,216,000,000 kg) as a number times 10^9.

$5.216 \cdot 10^9$





g Write the weight of the ladybug (0.000024 kg) as a number times 10^{-6}.

$24 \cdot 10^{-6}$

h Write the weight of the watermelon (8.9 kg) as a number times 10^1.

$0.89 \cdot 10^1$

9 Synthesis

Discuss: What are some strategies for writing very small values as a number times a single power of 10? Use the examples if they help with your thinking.

0.00083

$8.3 \cdot 10^{-4}$

$83 \cdot 10^{-5}$

Responses vary.

- **I can use the place value of each digit to rewrite the number using powers of 10. For example, in 0.00083, the 83 represents $\frac{83}{100000}$ or $\frac{83}{10^5}$, so I could write it as $83 \cdot 10^{-5}$. I can apply similar logic if I think of it as $\frac{8.3}{10000}$ to write it as $8.3 \cdot 10^{-4}$.**
- **I can use exponent rules to write the number as a combination of powers of 10. For example, $0.00083 = 0.00083 \cdot 10^0$ because $10^0 = 1$. Then I can borrow powers of 10 like this: $0.00083 \cdot 10^0 = 0.0083 \cdot 10^{-1} = 0.083 \cdot 10^{-2} = 0.83 \cdot 10^{-3} = 8.3 \cdot 10^{-4}$, and so on.**

12 Summary 7.08

Like large numbers, you can write small numbers using combinations of powers of 10. Numbers less than 1 will use negative powers of 10.

For example:

- $0.000000877 = 8 \cdot 10^{-7} + 7 \cdot 10^{-8} + 7 \cdot 10^{-9}$
- $0.00000000034 = 3 \cdot 10^{-10} + 4 \cdot 10^{-11}$
- $0.00000049 = 4 \cdot 10^{-7} + 9 \cdot 10^{-8}$

You can write large and small values as a number times a single power of 10 to help compare those values and get a sense of their scale.

For example:

- $42000000000 = 4.2 \cdot 10^{10}$
- $2500000000 = 25 \cdot 10^{8}$
- $0.00000000034 = 3.4 \cdot 10^{-10}$
- $0.00000049 = 49 \cdot 10^{-8}$

Practice
7.08

Name: .... Date: .... Period: ....

1. Select *all* expressions that are equivalent to $\frac{1}{1000}$.

☑ **A.** $1 \bullet 10^{-3}$ ☐ **B.** $-1 \bullet 10^{3}$ ☐ **C.** $1 \bullet 10^{\frac{1}{3}}$

☑ **D.** $10 \bullet 10^{-4}$ ☐ **E.** $10 \bullet 10^{-3}$

2. Order the expressions from *least* to *greatest*.

$2 \bullet 10^{-3}$	$3 \bullet 10^{-2}$	$-3 \bullet 10^{-2}$	$-2 \bullet 10^{-3}$	$3 \bullet 10^{2}$

$-3 \bullet 10^{-2}$	**$-2 \bullet 10^{-3}$**	**$2 \bullet 10^{-3}$**	**$3 \bullet 10^{-2}$**	**$3 \bullet 10^{2}$**

Least **Greatest**

Problems 3–4: Write each sum as a decimal.

3. $3 \bullet 10^{-4} + 2 \bullet 10^{-5} + 3 \bullet 10^{-6}$

0.000323

4. $2 \bullet 10^{-7} + 3 \bullet 10^{-5} + 5 \bullet 10^{-3}$

0.0050302

Problems 5–6: Write each value as a number times a single power of 10.

5. $\frac{7}{10000}$

$7 \bullet 10^{-4}$ (or equivalent)

6. 0.0013

$1.3 \bullet 10^{-3}$ (or equivalent)

7. Write 0.00573 in three different ways, using a single multiple of 10.

Responses vary.

$573 \bullet 10^{-5}$

$57.3 \bullet 10^{-4}$

$5.73 \bullet 10^{-3}$

Practice
7.08

Name: .... Date: .... Period: ....

8. Fill in each blank using the digits 0 to 9 to make a sum that's as close to 10 as possible.

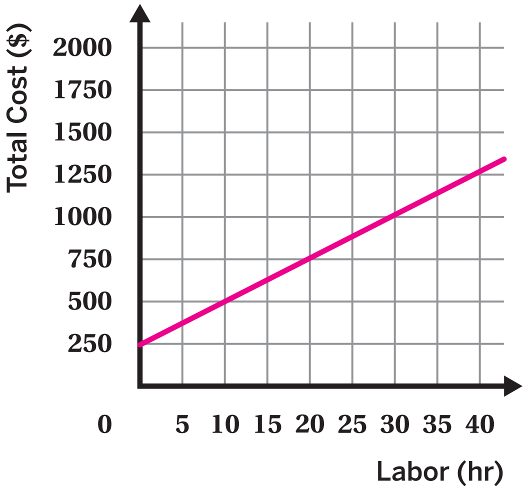
$\square\square \cdot 10^{-\square} + \square\square \cdot 10^{-\square}$

$98 \cdot 10^{-1} + 76 \cdot 10^{-3}$

Spiral Review

Problems 9–11: An electrician charges a flat rate of \$250, plus \$25 for each hour of labor.

9. Graph a line representing the relationship between the number of hours of labor and the total cost.



10. What is the total cost for 20 hours of labor?

\$750

11. What is the slope of this line?

25

Explain its meaning in context.

***Explanations vary*. The slope is the same as the price per hour, in dollars, that the electrician charges for labor.**

12. This table of values represents a linear function. Which equation could represent the linear function?

x	y
3	0
6	2
9	4

A. $y = \frac{2}{3}x + 3$

[✓ B.] $y = \frac{2}{3}x - 2$

C. $y = \frac{3}{2}x + 2$

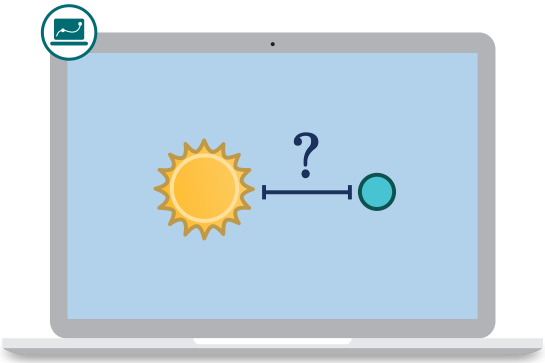
D. $y = \frac{3}{2}x - 3$

Name: ………… Date: ………… Period: …………

Big and Small Numbers 8.EE.4, SMP.7

Specific and Scientific

Let's explore scientific notation.



Warm-Up

1 Order these numbers from *least* to *greatest*.

$75 \cdot 10^5$	4,000,000	$0.6 \cdot 10^7$	$5 \cdot 10^5$

$5 \cdot 10^5$	4,000,000	$0.6 \cdot 10^7$	$75 \cdot 10^5$

Least **Greatest**

2 Order these numbers from *least* to *greatest*.

$4 \cdot 10^6$	$6 \cdot 10^6$	$5 \cdot 10^5$	$7.5 \cdot 10^6$

$5 \cdot 10^5$	$4 \cdot 10^6$	$6 \cdot 10^6$	$7.5 \cdot 10^6$

Least **Greatest**

3 Which list was easier to sort? Explain your thinking.

Responses vary.

- List 1: My strategy for ordering the lists was to do the multiplication for every number that was written as a product. List 1 only had three of these, compared to List 2 which had four.
- List 2: Powers of 10 helped guide my sorting of List 2. For instance, I could see that three numbers were in the millions (10^6) and one was not. To order the three numbers in the millions, I could use the leading digit, because I know 7 million is greater than 6 million is greater than 4 million.

Activity
1

Name: .... Date: .... Period: ....

Scientific Notation

Scientific notation is a specific way of writing very large or very small numbers that can help us compare numbers.

4 Some of these numbers are written in scientific notation and some are not.

What do you think it means for a number to be written in scientific notation?

Responses vary. A number is written in scientific notation if it's written as a product of two numbers, where the first part is a number between 1 and 10 and the second part is an integer power of 10.

In Scientific Notation	Not in Scientific Notation
$3 \cdot 10^{9}$	3,000,000,000
$1.257 \cdot 10^{5}$	$125.7 \cdot 10^{3}$
$2 \cdot 10^{-1}$	0.2
$5.1 \cdot 10^{-4}$	$0.51 \cdot 10^{-3}$

5 Sort the numbers based on whether they are written in scientific notation.

0.00099	48,200	$0.78 \cdot 10^{-3}$
$5.23 \cdot 10^{8}$	$8.7 \cdot 10^{-12}$	$36 \cdot 10^{5}$

In Scientific Notation	Not in Scientific Notation
$5.23 \cdot 10^{8}$ $8.7 \cdot 10^{-12}$	$0.78 \cdot 10^{-3}$ 48,200 $36 \cdot 10^{5}$ 0.00099

6 Some calculators use "E" notation to write scientific notation. For example, $6.02 \cdot 10^{23}$ is represented as 6.02E23. What do you think "E23" in the second part of the expression represents?

Responses vary. E23 represents 10^{23}

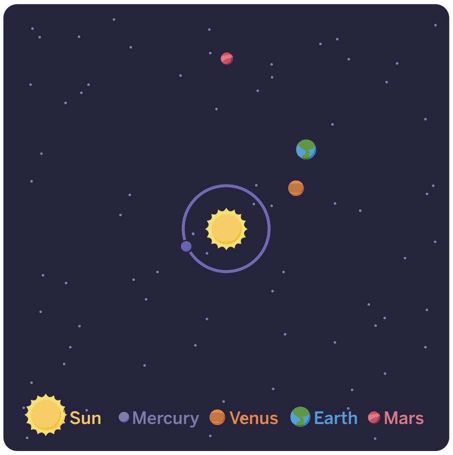
Activity
2

Name: .... Date: .... Period: ....

Solar System and Test Tubes

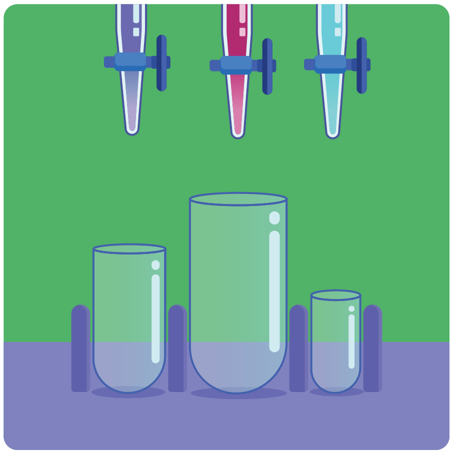
7 Here are the distances of four planets from the Sun. Write each distance in scientific notation. Mercury has been done for you.

Planet	Distance From Sun (mi)	Scientific Notation (mi)
Mercury	36,000,000	$3.6 \cdot 10^7$
Venus	67,000,000	$6.7 \cdot 10^7$
Earth	92,960,000	$9.296 \cdot 10^7$
Mars	$1417 \cdot 10^5$	$1.417 \cdot 10^8$



8 We can use scientific notation to represent small numbers, too! Write each volume in scientific notation.

Liquid Color	Volume (L)	Scientific Notation (L)
Purple	0.000125	$1.25 \cdot 10^{-4}$
Red	0.0002	$2 \cdot 10^{-4}$
Blue	$325 \cdot 10^{-8}$	$3.25 \cdot 10^{-6}$



9 Match each situation with its approximate rate or speed.

Situation	Rate or Speed
a Approximate average rate that tectonic plates move	c $1.7 \cdot 10^3$ miles per hour
b Approximate rate the Earth spins at the equator	a $6 \cdot 10^{-1}$ inches per year
c Approximate speed that the International Space Station travels in orbit around earth	d $3 \cdot 10^8$ meters per second
d Approximate speed of light in a vacuum	b $1.6 \cdot 10^3$ kilometers per hour

10 Synthesis

Discuss: What is one strategy for writing a number in scientific notation? Use the examples if they help with your thinking.

Responses vary. To write a number in scientific notation, I can use exponent rules. For example, $6700000 = 6700000 \cdot 10^0$, because $10^0 = 1$. Then I can borrow powers of 10 to shift the place values, like this: $6700000 \cdot 10^0 = 670000 \cdot 10^1$. I can continue this until the first number is between 1 and 10. Large numbers will end up with positive integer powers of 10 and small numbers less than 1 will have negative integer powers of 10.

Not in Scientific Notation	In Scientific Notation
36,000,000	$3.6 \cdot 10^7$
6,700,000	
0.00024	
$417 \cdot 10^3$	

13 Summary 7.09

There are many ways to express a number using a power of 10. One specific way is called scientific notation, which can be helpful for comparing very large or very small numbers.

For example:

- 425,000,000 is $4.25 \cdot 10^8$ in scientific notation
- 0.0000000000783 is $7.83 \cdot 10^{-11}$ in scientific notation

scientific notation A way to write very large or very small numbers. In scientific notation, a number between 1 and 10 is multiplied by a power of 10.

Practice
7.09

Name: .... Date: .... Period: ....

Problems 1–4: Write each value as a number times a power of 10. ***Responses vary.***

1. 0.04
- **$4 \cdot 10^{-2}$**
- **$0.4 \cdot 10^{-1}$**

2. 0.072
- **$7.2 \cdot 10^{-2}$**
- **$72 \cdot 10^{-3}$**

3. 0.0000325
- **$3.25 \cdot 10^{-5}$**
- **$325 \cdot 10^{-7}$**

4. 0.003
- **$3 \cdot 10^{-3}$**
- **$0.3 \cdot 10^{-2}$**

Problems 5–10: Write each value in scientific notation.

5. 0.00083

$8.3 \cdot 10^{-4}$

6. 760,000,000

$7.6 \cdot 10^{8}$

7. $147 \cdot 10^{6}$

$1.47 \cdot 10^{8}$

8. 0.038

$3.8 \cdot 10^{-2}$

9. 3.8

$3.8 \cdot 10^{0}$

10. $38 \cdot 10^{-4}$

$3.8 \cdot 10^{-3}$

11. There are a total of 367,400 books in a library. When the number of books is written in scientific notation, what is the power of 10?

5

12. Luis wrote 0.0000683 as $0.683 \cdot 10^{-4}$. Is this number written in scientific notation? Explain your thinking.

No. *Explanations vary.* The first part of a number written in scientific notation must be greater than or equal to 1. But the first part of Luis's number is less than 1.

13. Angela says: *A positive number multiplied by* 10^5 *will be greater than any other number multiplied by* 10^3.

Is this statement *always*, *sometimes*, or *never* true? Explain your thinking.

Sometimes. *Explanations vary.* For example, if 3.7 is multiplied by 10^5 and 8.1 is multiplied by 10^3, then $3.7 \cdot 10^5$ will be greater than $8.1 \cdot 10^3$, making the statement true. But, if 37 is multiplied by 10^5 and 8,100 is multiplied by 10^3, then $8100 \cdot 10^3$ will be greater than $37 \cdot 10^5$, making the statement not true.

Practice
7.09

Name: .... Date: .... Period: ....

14. What is 5,000,000 written in scientific notation?

A. $0.5 \cdot 10^6$

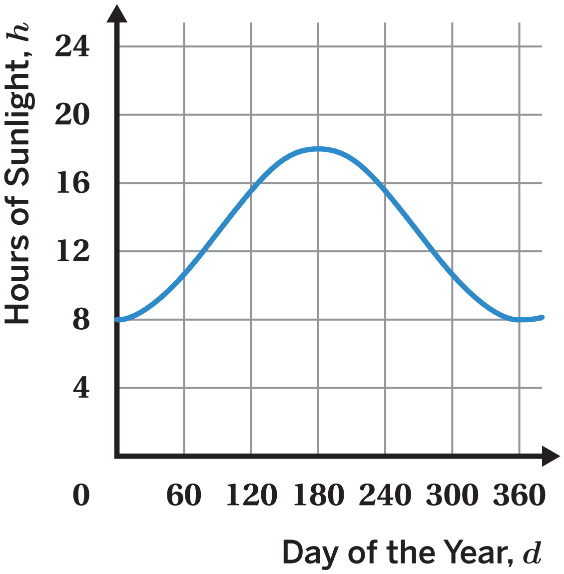
B. $0.5 \cdot 10^7$

[✓ C.] $5 \cdot 10^6$

D. $5 \cdot 10^7$

Spiral Review

Problems 15–18: Here is the graph representing the predicted number of hours of sunlight, h, on a given day of the year, d, in Metropolis.



15. Is hours of sunlight a function of days of the year? Explain your thinking.

Yes. *Explanations vary*. For every value of d, there is only one value of h.

16. For what days of the year do the hours of sunlight increase?

***Responses vary*. Around Day 0 to around Day 180**

17. For what days of the year do the hours of sunlight decrease?

***Responses vary*. Around Day 180 to Day 360**

18. Which day of the year has the most hours of sunlight?

Day 180

19. Select *all* the expressions that are equivalent to $4 \cdot 10^{-3}$.

☑ A. $4 \cdot \left(\frac{1}{10}\right) \cdot \left(\frac{1}{10}\right) \cdot \left(\frac{1}{10}\right)$

☐ B. $4 \cdot (-10) \cdot (-10) \cdot (-10)$

☑ C. $4 \cdot 0.001$

☐ D. $4 \cdot 0.0001$

☑ E. 0.004

☐ F. 0.0004

Unit 7

Lesson

10

Name: ________ Date: ________ Period: ________

Big and Small Numbers 8.EE.4, SMP.6

Multiplying and Dividing

Let's explore how to multiply and divide with scientific notation.

Warm-Up

1. What is the value of $(2 \cdot 10^3) \cdot (4 \cdot 10^6)$?

A. $6 \cdot 10^9$

B. $8 \cdot 10^{18}$

[✓ **C.**] $8 \cdot 10^9$

D. $6 \cdot 10^{18}$

Explain your thinking.

Explanations vary.

- Choice C is correct because

$$\begin{aligned}(2 \cdot 10^3) \cdot (4 \cdot 10^6) &= 2 \cdot 10 \cdot 10 \cdot 10 \cdot 4 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \\ &= 2 \cdot 4 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \\ &= 8 \cdot 10^9.\end{aligned}$$

- Choice C is correct because

$$\begin{aligned}(2 \cdot 10^3) \cdot (4 \cdot 10^6) &= 2000 \cdot 4000000 \\ &= 8000000000 \\ &= 8 \cdot 10^9.\end{aligned}$$

Activity
1

Name: ____________________ Date: __________ Period: __________

Multiplication Strategies

Anya and Rudra are trying to multiply $(4 \cdot 10^5) \cdot (3 \cdot 10^6)$.
Here is each student's work.

Anya	Rudra
$(4 \cdot 10^5) \cdot (3 \cdot 10^6)$	$(4 \cdot 10^5) \cdot (3 \cdot 10^6)$
$400{,}000 \cdot 3{,}000{,}000$	$4 \cdot 3 \cdot 10^5 \cdot 10^6$
$1{,}200{,}000{,}000{,}000$	$12 \cdot 10^{5+6}$
$1.2 \cdot 10^{12}$	$1.2 \cdot 10^{11}$
	$1.2 \cdot 10^{12}$

2. **Discuss:** How are Anya's and Rudra's strategies alike? How are they different?

ELD.PI.8.3.Em, Ex, Br, ELD.PI.8.6.Em, Ex, Br

Responses vary. **Anya's and Rudra's strategies are alike because they multiply two numbers written in scientific notation to get a new number written in scientific notation. They also both involve multiplying a 4 by a 3, resulting in a 12. Anya's strategy includes expanding out each number, multiplying them together, and then rewriting in scientific notation. Rudra's strategy is to multiply the first parts together and then use exponent properties to add the exponents.**

3. Here are four new expressions.

- For each expression, circle whether you would use Anya's strategy, Rudra's strategy, or another strategy. ***Responses vary.***
- Choose two expressions to multiply. Write your answers in scientific notation.
- Compare your strategies and responses with a partner. ***Responses vary.***

a $(3 \cdot 10^5) \cdot (6 \cdot 10^2)$

Anya Rudra Other

$\mathbf{1.8 \cdot 10^8}$

b $(4 \cdot 10^2) \cdot (2 \cdot 10^{-3})$

Anya Rudra Other

$\mathbf{8 \cdot 10^{-1}}$

c $(6 \cdot 10^3) \cdot 400$

Anya Rudra Other

$\mathbf{2.4 \cdot 10^6}$

d $(5 \cdot 10^{22}) \cdot (7 \cdot 10^{17})$

Anya Rudra Other

$\mathbf{3.5 \cdot 10^{40}}$

Activity
2

Name: ______________________ Date: __________ Period: __________

Division Strategies

Anya and Rudra are trying to divide $\frac{7 \cdot 10^6}{2 \cdot 10^4}$. Here is each student's work.

Anya

$$\frac{7 \cdot 10^6}{2 \cdot 10^4}$$

$$\frac{7000000}{20000}$$

$$350$$

$$3.5 \cdot 10^2$$

Rudra

$$\frac{7 \cdot 10^6}{2 \cdot 10^4}$$

$$\frac{7}{2} \cdot \frac{10^6}{10^4}$$

$$3.5 \cdot 10^{6-4}$$

$$3.5 \cdot 10^2$$

4. **Discuss:** How are Anya's and Rudra's strategies alike? How are they different?

ELD.PI.8.3.Em, Ex, Br, ELD.PI.8.6.Em, Ex, Br

Responses vary. **Anya's and Rudra's strategies are alike because they both involve doing division and they result in the same answer. They are also alike because they both divide 7 by 2, resulting in 3.5. Anya's strategy includes expanding out each number, dividing the two terms, and then rewriting in scientific notation. Rudra's strategy is to divide the first parts and then use exponent properties to subtract the exponents.**

5. Here are four new expressions.

- For each expression, circle whether you would use Anya's strategy, Rudra's strategy, or another strategy. ***Responses vary.***
- Choose two expressions to divide. Write your answers in scientific notation.
- Compare your strategies and responses with a partner. ***Responses vary.***

a $\frac{8 \cdot 10^3}{2 \cdot 10^2}$

Anya Rudra Other

$4 \cdot 10$

b $\frac{9 \cdot 10^{-4}}{3}$

Anya Rudra Other

$3 \cdot 10^{-4}$

c $\frac{3 \cdot 10^5}{6 \cdot 10^2}$

Anya Rudra Other

$5 \cdot 10^2$

d $\frac{8 \cdot 10^9}{4 \cdot 10^{-4}}$

Anya Rudra Other

$2 \cdot 10^{13}$

Activity

3

Name: ______________ Date: ______________ Period: ______________

Two Truths and a Lie

Problems 6–9: For each problem, two statements are true and one is false. Circle the false statement.

6. A. $1.5 \cdot 10^7 = (5 \cdot 10^2) \cdot (3 \cdot 10^4)$

[✓ B.] $1.5 \cdot 10^7 = (5 \cdot 10^2) \cdot (3 \cdot 10^5)$

C. $1.5 \cdot 10^7 = (3 \cdot 10^3) \cdot (5 \cdot 10^3)$

7. A. $(7 \cdot 10^{-4}) \cdot (4 \cdot 10^{15}) = (7 \cdot 4) \cdot 10^{(-4 + 15)}$

[✓ B.] $(7 \cdot 10^{-4}) \cdot (4 \cdot 10^{15}) = 2.8 \cdot 10^{10}$

C. $(7 \cdot 10^{-4}) \cdot (4 \cdot 10^{15}) = 2.8 \cdot 10^{12}$

8. A. $(6 \cdot 10^3) \cdot 200 = (6 \cdot 200) \cdot 10^3$

B. $(6 \cdot 10^3) \cdot 200 = 6 \cdot 10^3 \cdot 2 \cdot 10^2$

[✓ C.] $(6 \cdot 10^3) \cdot 200 = (6 \cdot 200) \cdot 10^{(3 + 2)}$

9. A. $\frac{9 \cdot 10^7}{2 \cdot 10^3} = (9 \div 2) \cdot 10^{(7 - 3)}$

[✓ B.] $\frac{9 \cdot 10^7}{2 \cdot 10^3} = 7 \cdot 10^4$

C. $\frac{9 \cdot 10^7}{2 \cdot 10^3} = 4.5 \cdot 10^4$

10. Write two multiplication or division statements that are true and one that is false. Write your statements in scientific notation. ***Responses vary.***

A. ______________ = ______________

B. ______________ = ______________

C. ______________ = ______________

Trade with a classmate.

- -

Name ______________ Which statement is false? ______________

Synthesis

11. 💬 **Discuss:** What is one strategy for multiplying or dividing numbers written in scientific notation? Use the examples if they help with your thinking.

$(4 \cdot 10^{-2}) \cdot (5 \cdot 10^{8})$

$\frac{5 \cdot 10^{9}}{2 \cdot 10^{4}}$

ELD.PI.8.10.Em, Ex, Br

Responses vary.

- **When multiplying two numbers written in scientific notation, multiply the first parts together and then use exponent properties to multiply the powers of 10.**
- **When dividing two numbers written in scientific notation, divide the first part in the numerator by the first part in the denominator, and then use exponent properties to divide the powers of 10.**
- **Sometimes you will need to rewrite your answer so that it is in scientific notation.**

Summary 7.10

Multiplying numbers written in scientific notation is an extension of multiplying decimals.

To multiply two numbers written in scientific notation:

- Multiply the first parts of each number.
- Multiply the powers of 10 using exponent properties.

For example: $(2 \cdot 10^{3}) \cdot (4 \cdot 10^{6}) = (2 \cdot 4) \cdot 10^{(3+6)} = 8 \cdot 10^{9}$.

To divide two numbers written in scientific notation, it can be helpful to rewrite the expression as a fraction.

- Divide the first part in the numerator by the first part in the denominator.
- Divide the powers of 10 using exponent properties.

For example: $\frac{8 \cdot 10^{7}}{4 \cdot 10^{2}} = \frac{8}{4} \cdot 10^{(7-2)} = 2 \cdot 10^{5}$.

Practice

7.10

Name: ____________ Date: ________ Period: ________

1. Select *all* the expressions that are equivalent to $4 \cdot 10^3$.

☑ **A.** $(2 \cdot 10^{-5}) \cdot (2 \cdot 10^{8})$

☐ **B.** $\frac{8 \cdot 10^5}{2 \cdot 10^8}$

☑ **C.** $\frac{8 \cdot 10^{-5}}{2 \cdot 10^{-8}}$

☐ **D.** $(4 \cdot 10^3) \cdot (1 \cdot 10^3)$

☑ **E.** $(4 \cdot 10) \cdot (100)$

Problems 2–5: Determine the value of a that makes each equation true.

2. $(5 \cdot 10^a) \cdot (3 \cdot 10^3) = 1.5 \cdot 10^8$

$a =$ 4

3. $(6 \cdot 10^{-5}) \cdot (2 \cdot 10^{-4}) = 1.2 \cdot 10^a$

$a =$ -8

4. $\frac{3.2 \cdot 10^7}{1.6 \cdot 10^a} = 2 \cdot 10^2$

$a =$ 5

5. $\frac{6.2 \cdot 10^a}{2 \cdot 10^{-3}} = 3.1 \cdot 10^{-4}$

$a =$ -7

6. What is the value of $\frac{4.6 \cdot 10^{-5}}{2.3 \cdot 10^{-2}}$? Write your answer in scientific notation.

$2 \cdot 10^{-3}$

7. Changing the sign of which part of the expression would make the value of $\frac{-3 \cdot 10^{-4}}{-2.4 \cdot 10^{-2}}$ greater than 1?

A. -3 [✓ **B.** -4] **C.** -2.4 **D.** -2

Practice

7.10

Name: .... Date: .... Period: ....

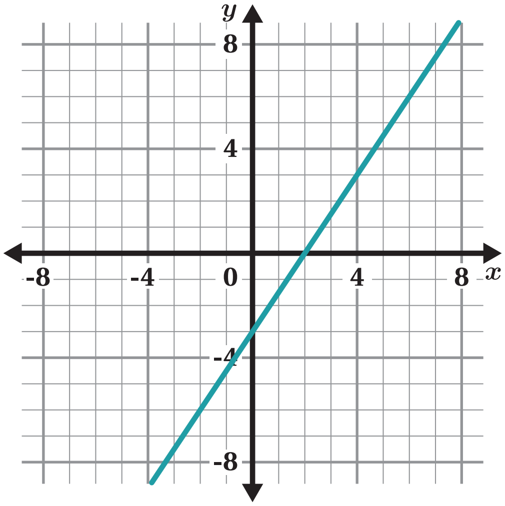
8. Fill in each blank using the digits 1 to 9 only once to create the largest product possible.

☐☐ • $10^{-☐}$ • ☐☐ • $10^{-☐}$

$(96 \cdot 10^{-1}) \cdot (87 \cdot 10^{-2})$

Spiral Review

Problems 9–11: Here is a graph of the equation $y = \frac{3}{2}x - 3$, which is part of a system of equations.



9. Write a second equation so that the system has infinitely many solutions.

Equation: **$y = \frac{3}{2}x - 3$ (or equivalent)**

10. Write a second equation so that the system has no solutions.

Equation: ***Responses vary.* $y = \frac{3}{2}x + 2$.**

Any line with the same slope and a different y-intercept.

11. Write a second equation so that the system has one solution at $(4, 3)$.

Equation: ***Responses vary.* $y = \frac{1}{2}x + 1$ (or equivalent)**

12. Determine whether this equation shown has *one solution*, *no solutions*, or *infinitely many solutions*. Explain your thinking.

$$\frac{1}{4}(12 - 4m) = 6 - m$$

No solutions. *Explanations vary.*

$$3 - m = 6 - m$$
$$3 = 6$$

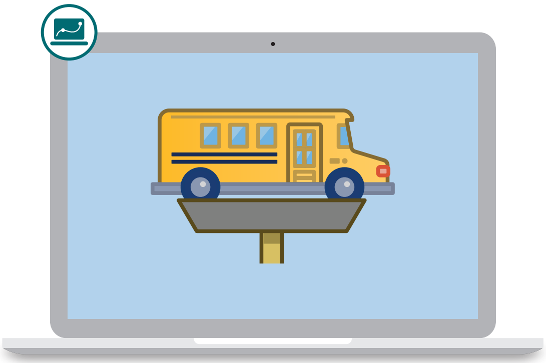
This equation is never true for any value of m.

Name: ______ Date: ______ Period: ______

Big and Small Numbers 8.EE.3, 8.EE.4, SMP.2, SMP.3, SMP.6

Balance the Scale

Let's use multiplication and division to compare large and small numbers in scientific notation.

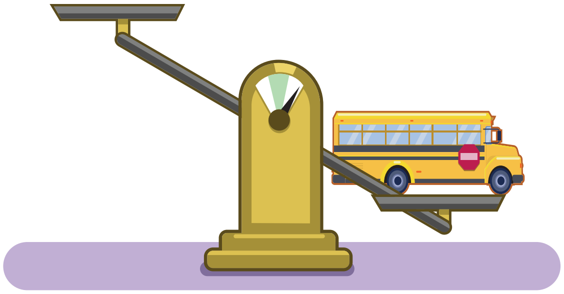


Warm-Up

1 A school bus sits on one side of the scale.

How many jelly beans do you think it would take to balance the scale? Use scientific notation.

Responses vary.

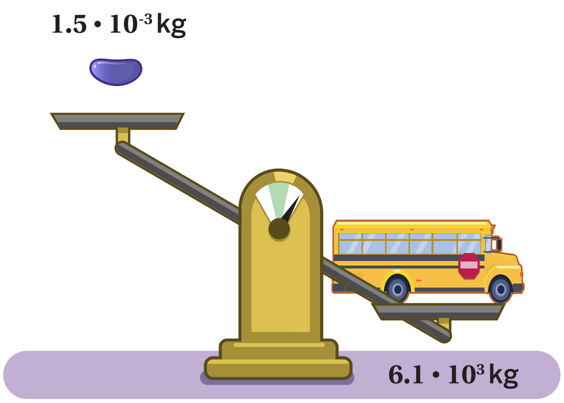


Activity
1

Name: .... Date: .... Period: ....

Balance the Scale, Part 1

2 One jelly bean weighs $1.5 \cdot 10^{-3}$ kilograms. A school bus weighs $6.1 \cdot 10^{3}$ kilograms.

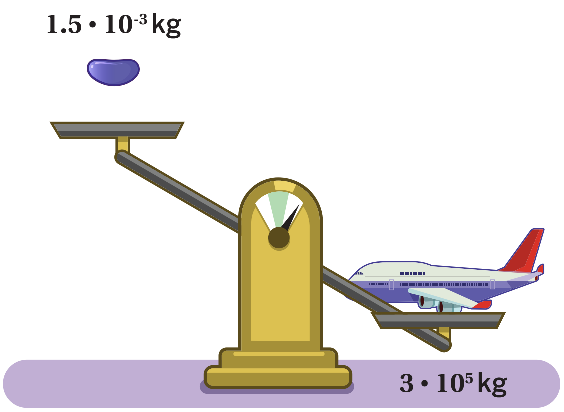


Describe a strategy for determining about how many jelly beans weigh as much as a bus.

Responses vary.

- **Start with the lighter weight. Multiply its first part so that it reaches the bigger weight's first part ($1.5 \cdot 4 \approx 6.1$). Then multiply by the power of 10 needed to reach the bigger weight's power of 10 ($10^{-3} \cdot 10^{6} = 10^{3}$).**
- **Begin by rounding the first parts to the nearest whole or half number. Then divide the larger quantity by the smaller quantity. This will result in dividing the first parts and subtracting the exponents.**
- **I expanded each number out, so the weight of a jelly bean is 0.0015 kilograms and the weight of the school bus is 6,100 kilograms. Then I divided 6,100 by 0.0015 to get about 4,066,667 jelly beans.**

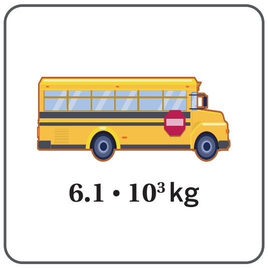
3 One jelly bean weighs $1.5 \cdot 10^{-3}$ kilograms. A jumbo jet weighs $3 \cdot 10^{5}$ kilograms.



How many jelly beans will it take to balance the scale?

$2 \cdot 10^{8}$ jelly beans

4 Basheera says the jumbo jet weighs about 200 times as much as the school bus. Elena says it weighs about 50 times as much as the school bus.



Whose claim is correct? Circle one.

Basheera's [✓ Elena's] Neither

Explain your thinking.

Explanations vary.

- **I rounded 6.1 to 6. When I multiplied $6 \cdot 10^{3}$ by 50, I got $300 \cdot 10^{3}$, which is the same as $3 \cdot 10^{5}$, the jumbo jet's weight.**
- **I rounded 6.1 to 6. If I divide $3 \cdot 10^{5}$ by $6 \cdot 10^{3}$, I get $0.5 \cdot 10^{2}$, which is the same as 50.**

Name: .... Date: .... Period: ....

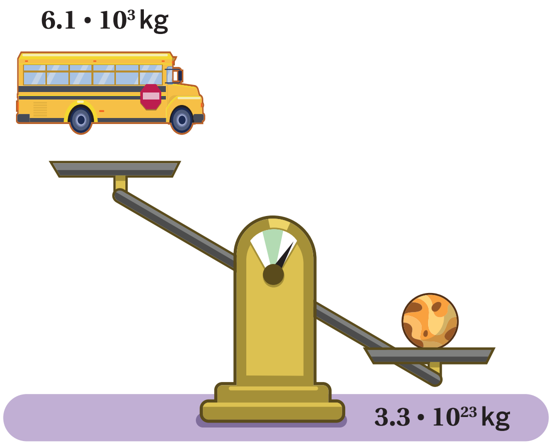
Balance the Scale, Part 2

5 A school bus weighs $6.1 \cdot 10^3$ kilograms. Mercury weighs $3.3 \cdot 10^{23}$ kilograms.

About how many school buses will it take to balance the scale? Write your answer in scientific notation.

Responses vary.

- $5 \cdot 10^{19}$ school buses
- $5.4 \cdot 10^{19}$ school buses



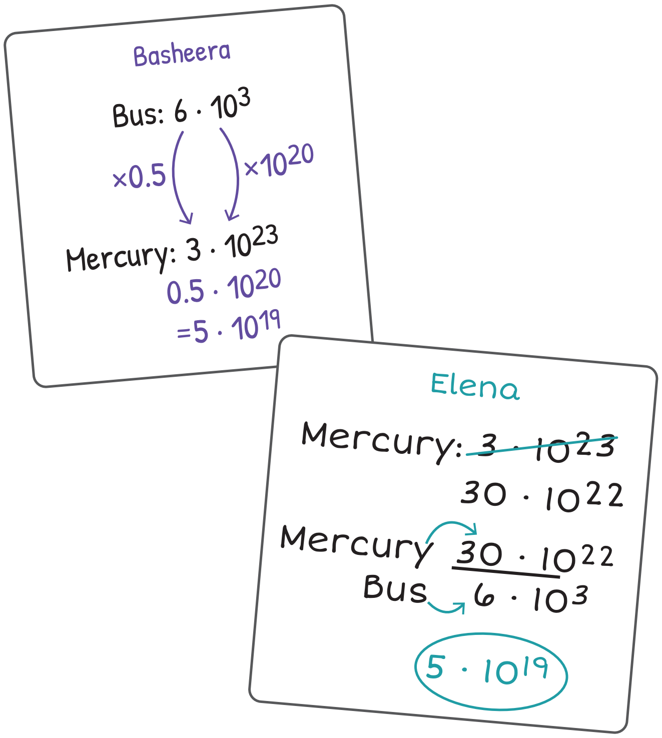
6 Here are two students' strategies for the previous problem. Examine their work.

Explain how Basheera and Elena each arrived at the answer $5 \cdot 10^{19}$ buses.

Responses vary.

Basheera: She rounded each number first. Then she analyzed the weights of the bus and Mercury in pieces to see how much to multiply. To get from 6 to 3, she multiplied by 0.5. To get from 10^3 to 10^{23}, she multiplied by 10^{20}. Then she rewrote $0.5 \cdot 10^{20}$ in scientific notation.

Elena: She also rounded each number first. Then she rewrote Mercury's weight by borrowing a power of 10 to make the division easier. Then she used division: $\frac{30}{6} = 5$ and $\frac{10^{22}}{10^3} = 10^{19}$.



Basheera

Bus: $6 \cdot 10^3$

$\times 0.5$ $\times 10^{20}$

Mercury: $3 \cdot 10^{23}$

$0.5 \cdot 10^{20}$

$= 5 \cdot 10^{19}$

Elena

Mercury: $\cancel{3 \cdot 10^{23}}$

$30 \cdot 10^{22}$

Mercury / Bus $\frac{30 \cdot 10^{22}}{6 \cdot 10^3}$

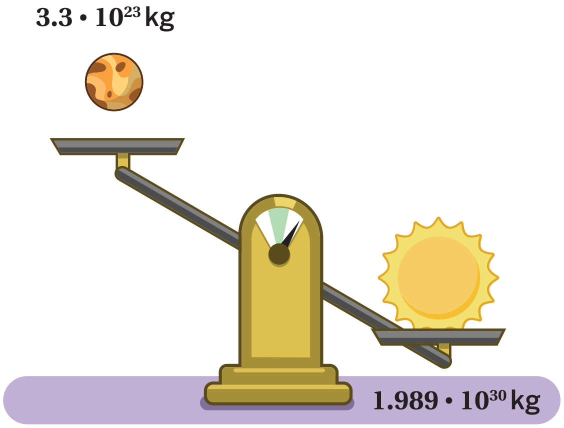
$5 \cdot 10^{19}$

7 Mercury weighs $3.3 \cdot 10^{23}$ kilograms. The Sun weighs $1.989 \cdot 10^{30}$ kilograms.

About how many Mercurys will it take to balance the scale? Write your answer in scientific notation.

Responses vary.

- $6.03 \cdot 10^6$ Mercurys
- $6.7 \cdot 10^6$ Mercurys



Name: .... Date: .... Period: ....

Balance the Scale, Part 2 (continued)

8

- Choose an object from each row and then compare them.
- Use these weights to determine how many of your first object weighs as much as your second object.
- Complete as many comparisons as you have time for.

Watermelon $8.9 \cdot 10^0$ kilograms	**Horse** $7.1 \cdot 10^2$ kilograms	**Ant** $3 \cdot 10^{-6}$ kilograms	**Cell Phone** $1.5 \cdot 10^{-1}$ kilograms	**Penny** $3.1 \cdot 10^{-3}$ kilograms
Bus $7.81 \cdot 10^3$ kilograms	**Moon** $7.348 \cdot 10^{22}$ kilograms	**Rocket** $2.03 \cdot 10^6$ kilograms	**Cruise Ship** $3.83 \cdot 10^7$ kilograms	**Pyramid** $5.216 \cdot 10^9$ kilograms

Responses vary.

Comparison 1

How many .... weigh about as much as the ....?

Write your answer in scientific notation.

Comparison 2

How many .... weigh about as much as the ....?

Write your answer in scientific notation.

Comparison 3

How many .... weigh about as much as the ....?

Write your answer in scientific notation.

Comparison 4

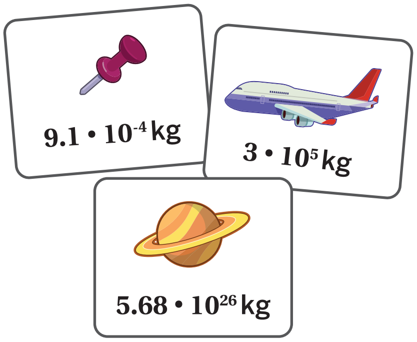
How many .... weigh about as much as the ....?

Write your answer in scientific notation.

9 Synthesis

Discuss: What is one strategy for determining how many times as large one number is compared to another?

Responses vary. **Divide the larger quantity by the smaller quantity. Sometimes it's helpful to round the numbers first. I can also make the division easier by borrowing powers of 10. After dividing, sometimes the result needs to borrow more powers of 10 so that it's in scientific notation.**



12 Summary 7.11

You can use scientific notation when comparing quantities.

Here is one example. How many jelly beans weigh as much as one Egyptian pyramid?

- Jelly bean weight: $1.5 \cdot 10^{-3}$ kilograms
- Egyptian pyramid weight: $5.216 \cdot 10^9$ kilograms

It can be helpful to round the first parts of both numbers before calculating.

- $1.5 \cdot 10^{-3}$ is about $2 \cdot 10^{-3}$.
- $5.216 \cdot 10^9$ is about $5 \cdot 10^9$.

There are many strategies you can use when comparing quantities in scientific notation. Here are two:

Divide the larger number by the smaller number.

$$\frac{5 \cdot 10^9}{2 \cdot 10^{-3}} = 2.5 \cdot 10^{12}$$

$2.5 \cdot 10^{12}$ jelly beans weigh about the same as the pyramid.

Multiply the smaller number by the number needed to equal the larger number.

$$2 \cdot 10^{-3} \cdot ? = 5 \cdot 10^9$$

$2.5 \cdot 10^{12}$ jelly beans weigh about the same as the pyramid.

Practice
7.11

Name: ______ Date: ______ Period: ______

1. Which number is greater? Circle one.

[✓ $17 \cdot 10^8$] $4 \cdot 10^8$

About how many times greater is one than the other?

$17 \cdot 10^8$ is about 4 times greater than $4 \cdot 10^8$.

2. Which number is greater? Circle one.

$2 \cdot 10^6$ [✓ $7.839 \cdot 10^6$]

About how many times greater is one than the other?

$7.839 \cdot 10^6$ is about 4 times greater than $2 \cdot 10^6$.

3. Which number is greater? Circle one.

$42 \cdot 10^7$ [✓ $8.5 \cdot 10^8$]

About how many times greater is one than the other?

$8.5 \cdot 10^8$ is about 2 times greater than $42 \cdot 10^7$.

Problems 4–5: Complete each sentence by writing a number in scientific notation.

Responses vary.

4. $10.3 \cdot 10^9$ is about **$2 \cdot 10^6$** times as large as $5.2 \cdot 10^3$.

5. $12.5 \cdot 10^{11}$ is about $4 \cdot 10^8$ times as large as **$3.1 \cdot 10^3$**.

6. The mass of a penny is $3.1 \cdot 10^{-3}$ kilograms and the mass of an Egyptian pyramid is $5.216 \cdot 10^9$ kilograms. Based on this information, how many pennies weigh about as much as one pyramid? Write your answer in scientific notation.

Responses between $1.667 \cdot 10^{12}$ and $1.7 \cdot 10^{12}$ are considered correct.

Practice
7.11

Name: ____________ Date: ____________ Period: ____________

7. How many times greater is 200 billion than 5 thousand?

A. $4 \cdot 10^9$ times greater

B. $4 \cdot 10^8$ times greater

[✓ C.] $4 \cdot 10^7$ times greater

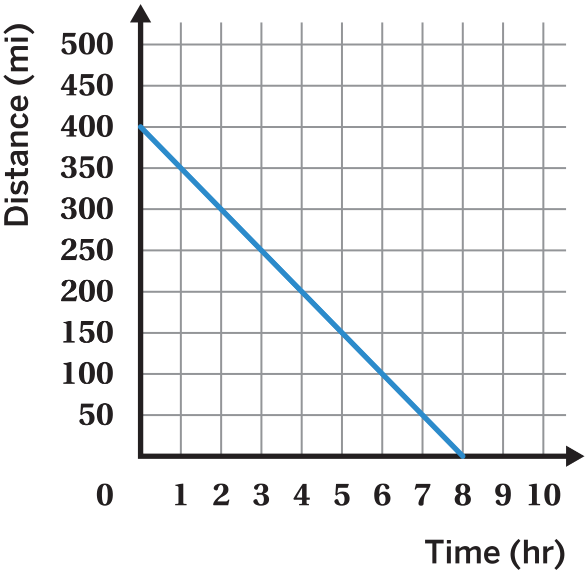
D. $4 \cdot 10^6$ times greater

8. A number is $3 \cdot 10^5$ times as large as another number. Determine two numbers that make this relationship true and complete the statement below. ***Responses vary.***

$8.1 \cdot 10^9$ is $3 \cdot 10^5$ times as large as $2.7 \cdot 10^4$.

Spiral Review

Problems 9–12: A family sets out on a road trip to visit their cousins. They travel at a steady rate. The graph shows the distance to their cousins' house each hour of the trip.



9. How far is the trip?

400 miles

10. How long did the trip take?

8 hours

11. How fast are they traveling?

50 miles per hour.

Explain your thinking

***Explanations vary.* If they travel 400 miles in 8 hours, I can divide 400 by 8 to get how many miles they travel per hour. This gives me 50 miles per hour.**

12. Is the slope positive or negative? **Negative**

Explain how you know and why that fits the situation.

***Explanations vary.* The slope is negative because the line moves down toward the right. It shows the change in remaining miles for each hour. 50 fewer miles remain after each hour, which means the car is traveling at a steady rate of 50 miles per hour.**

Unit 7

Lesson

12

Name: .... Date: .... Period: ....

Data Explorations Big and Small Numbers 8.EE.3, 8.EE.4, SMP.1, SMP.2, SMP.4

Use Your Powers

Let's use scientific notation to help us make calculations with large numbers.

Warm-Up

Complete each sentence by writing a number in scientific notation.

Responses vary.

1. $6.1 \cdot 10^{13}$ is about $3 \cdot 10^{11}$ times as large as $2.1 \cdot 10^{2}$.

2. $2.9 \cdot 10^{7}$ is about $2 \cdot 10^{10}$ times as large as $1.5 \cdot 10^{-3}$.

3. $1.4 \cdot 10^{9}$ is about $2.5 \cdot 10^{3}$ times as large as $5.8 \cdot 10^{5}$.

Name: .... Date: .... Period: ....

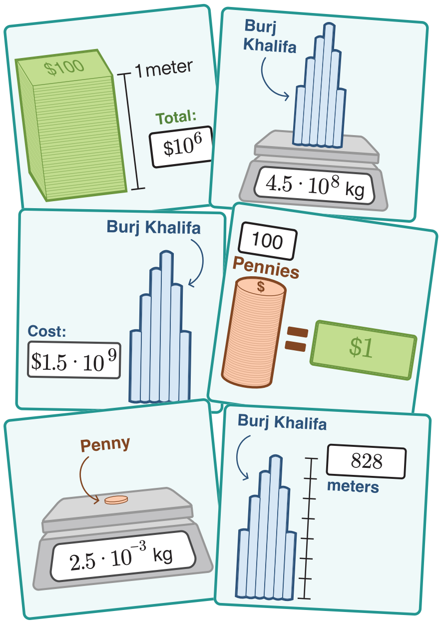
Make a Poster

4. Read each situation. Then circle one situation to explore. **ELD.PI.8.6.Em, Ex, Br**

Situation 1	Situation 2	Situation 3	Situation 4
The Burj Khalifa	Food waste	Student debt	Distance to the Moon

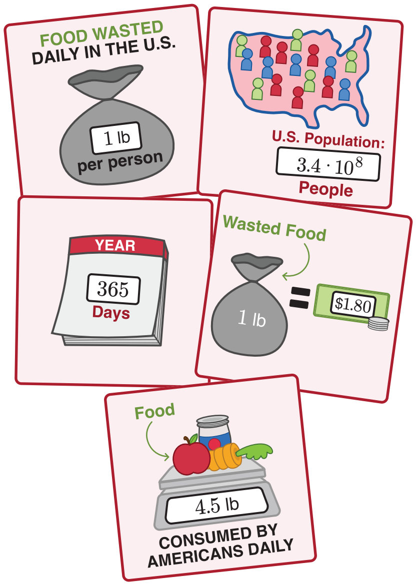
Situation 1

The Burj Khalifa



Situation 2

Food waste



Situation 1

a Which is taller: the Burj Khalifa or the stack of 50 dollar bills it cost to build the Burj Khalifa?

Sample shown in part b.

b How many times as tall?

***Responses vary.* The stack of cash is taller because $\frac{1.5 \cdot 10^9}{5 \cdot 10^5} = 3 \cdot 10^3$. So the stack of cash is about 3,000 meters tall, which is about 3.5 times as tall as the Burj Khalifa.**

Situation 2

a How many dollars worth of food is wasted in the U.S. each day?

Sample shown in part b.

b How many additional people could be fed with the food that is thrown away?

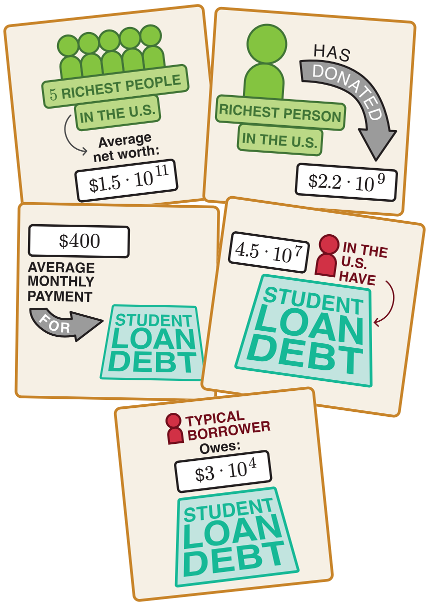
***Responses vary.* About $6 \cdot 10^8$ dollars worth of food is wasted in the U.S. each day. Eating 4.5 pounds per day, $7.6 \cdot 10^7$ additional people could be fed on the food that is thrown away because $\frac{3.4 \cdot 10^8}{4.5} = \frac{34 \cdot 10^7}{4.5}$, or about $7.6 \cdot 10^7$.**

Name: .... Date: .... Period: ....

Make a Poster (continued)

Situation 3

Student Debt



a Which is greater: the net worth of the five richest people in the U.S. or all student debt in the U.S.?

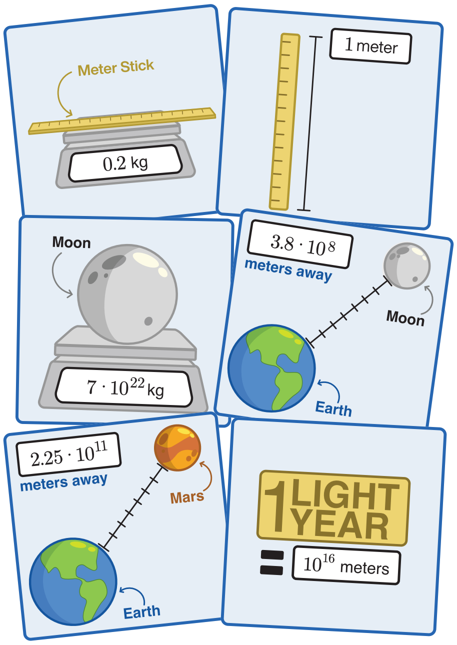
Sample shown in part b.

b How many times greater?

***Responses vary.* The net worth of the five richest people in the U.S. is $7.5 \cdot 10^{11}$ dollars. All of the student debt in the U.S. is $1.35 \cdot 10^{12}$ dollars. Student debt is approximately 1.8 times greater because $\frac{1.35 \cdot 10^{12}}{7.5 \cdot 10^{11}}$ is approximately 1.8.**

Situation 4

Distance to the Moon



a How many meter sticks does it take to equal the mass of the Moon?

Sample shown in part b.

b If you took all those meter sticks and lined them up end to end, how many times would they reach from Earth to the Moon?

***Responses vary.* It would take $3.5 \cdot 10^{23}$ meter sticks to equal the mass of the Moon. Placed end to end, the meter sticks would reach from Earth to the Moon about 10^{15} times because $4 \cdot 10^{23}$ is 10^{15} times as much as $4 \cdot 10^{8}$.**

5. Create a poster answering the questions for the situation you chose. Your poster should include: **ELD.PI.8.10.Em, Ex, Br**

- ☐ A summary of the situation.
- ☐ All of the measurements you used to answer each question. Include an explanation of how you chose units of appropriate size.
- ☐ Your calculations.
- ☐ Your answers (with units).

Synthesis

6. Describe something you learned about scientific notation while making your poster.

ELD.PI.8.10.Em, Ex, Br

Responses vary. **I learned that when working with scientific notation, it is helpful to compare the first parts of the expressions and the exponents for the powers of 10. Then I can decide what operations to use and if rounding can help make my calculations easier.**

Summary 7.12

You can use scientific notation and exponent rules to solve real-world problems that include very large or very small numbers. For example, you can use the rules to calculate how many dollars worth of food are wasted in the United States each year or the total amount of student debt in the United States.

When solving a real-world problem, it is important to look at the information you know, determine what information is needed to solve the problem, and think about appropriate units of measurement. You can also use rounding to make some quantities simpler to work with.

Practice
7.12

Name: ______________________ Date: __________ Period: __________

1. Select *all* the expressions that are equal to $9 \cdot 10^8$.

☑ A. $(9 \cdot 10^{10}) \cdot (1 \cdot 10^{-2})$

☐ B. $\frac{18 \cdot 10^4}{2 \cdot 10^4}$

☑ C. $3 \cdot 3 \cdot 10^6 \cdot 10^2$

☐ D. 90^8

☑ E. $\frac{18 \cdot 10^{12}}{2 \cdot 10^4}$

2. The Sun is roughly $1 \cdot 10^2$ times as wide as Earth. The star KW Sagittarii is roughly $1 \cdot 10^5$ times as wide as Earth. About how many times as wide is KW Sagittarii as the Sun?

$1 \cdot 10^3$ (or 1,000) times as wide

Show or explain your thinking.

Explanations vary. **I know how both the Sun's and KW Sagittarii's widths compare to Earth's width, so I can use these comparisons to help me. KW Sagittarii is about $\frac{1 \cdot 10^5}{1 \cdot 10^2} = 1 \cdot 10^3$ times as wide as the Sun.**

3. The mass of Saturn is about $5.68 \cdot 10^{26}$ kilograms. If the average mass of an apple is about 0.085 kilograms, how many apples does it take to equal the mass of Saturn?

About $6.7 \cdot 10^{27}$ apples

Show or explain your thinking.

Explanations vary. $\frac{5.68 \cdot 10^{26}}{8.5 \cdot 10^{-2}} \approx \frac{6 \cdot 10^{26}}{9 \cdot 10^{-2}} \approx 6.7 \cdot 10^{27}$

Problems 4–6: Here are some interesting facts.

- The average distance from Earth to the Sun is about $1.5 \cdot 10^{11}$ meters.
- The Helios 2 spacecraft traveled at a speed of $2.53 \cdot 10^8$ meters per hour.
- There are about 3,000 blades of grass in a square foot.
- A soccer field has an area of 81,000 square feet.

4. The rings of Saturn are about $2.82 \cdot 10^8$ meters across. How long would it take for Helios 2 to travel across the rings of Saturn? Show or explain your thinking.

About 1.2 hours or 1 hour and 12 minutes. ***Explanations vary.*** $\frac{2.82 \cdot 10^8}{2.53 \cdot 10^8} \approx \frac{3 \cdot 10^8}{2.5 \cdot 10^8} \approx 1.2$

5. How long would it take for Helios 2 to travel from Earth to the Sun?
Show or explain your thinking.

About $5.93 \cdot 10^2$ hours, or 24.7 days. ***Explanations vary.*** $\frac{1.5 \cdot 10^{11}}{2.53 \cdot 10^8} = \frac{15 \cdot 10^{10}}{2.53 \cdot 10^8} \approx 5.93 \cdot 10^2$

6. Which is greater: the time it would take for Helios 2 to travel from Earth to the Sun or the number of blades of grass on a soccer field? Show or explain your thinking.

The number of blades of grass on a soccer field. ***Explanations vary.***
Hours needed to travel from Earth to the Sun: $\frac{1.5 \cdot 10^{11}}{2.53 \cdot 10^8} = \frac{15 \cdot 10^{10}}{2.53 \cdot 10^8} \approx 5.93 \cdot 10^2$
Number of blades of grass on a soccer field:
$(3 \cdot 10^3) \cdot (8.1 \cdot 10^4) \approx (3 \cdot 10^3) \cdot (8 \cdot 10^4) \approx 2.4 \cdot 10^8$

Practice
7.12

Name: .... Date: .... Period: ....

7. The Burj Khalifa in Dubai is the tallest structure in the world at approximately $8.3 \cdot 10^2$ meters tall. The Big Ben clock tower in London is 96 meters tall. About how many times taller is the height of the Burj Khalifa than Big Ben? Show or explain your thinking.

The Burj Khalifa is about 8 times taller. ***Explanations vary.*** $\frac{8.3 \cdot 10^2}{96} = \frac{8.3 \cdot 10^2}{9.6 \cdot 10^1} \approx \frac{8 \cdot 10^2}{1 \cdot 10^2} \approx 8$

8. The body of a 154-pound person contains approximately $1.4 \cdot 10^2$ grams of potassium and $2 \cdot 10^{-3}$ grams of silver. Based on this information, how many times greater is the number of grams of potassium in the body than the number of grams of silver in the body?

70,000 (or equivalent)

Spiral Review

9. Select *all* expressions that are equivalent to 6^{-3}.

☐ **A.** -18

☑ **B.** $\frac{6}{6^4}$

☑ **C.** $\frac{1}{6^3}$

☑ **D.** $\left(\frac{1}{6}\right) \cdot \left(\frac{1}{6}\right) \cdot \left(\frac{1}{6}\right)$

☐ **E.** $\frac{12^6}{2^9}$

10. What is the value of m in the equation $4^m = 4^{12} \cdot 4^{-2}$?

$m = 10$

11. Here is a scatter plot that shows the number of points and assists by a set of hockey players.

Select *all* descriptions that apply to the association in the scatter plot.

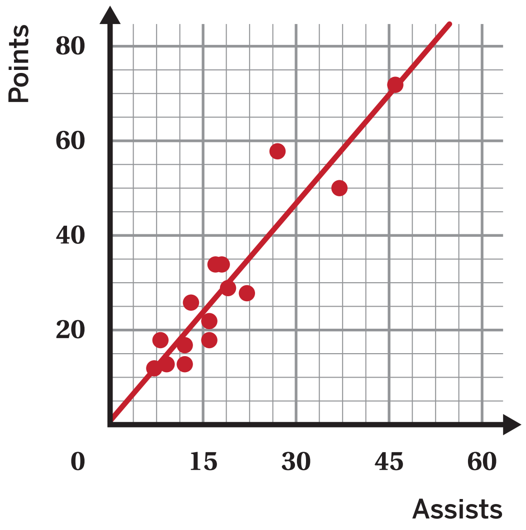
☑ **A.** Linear

☐ **B.** Non-linear

☑ **C.** Positive

☐ **D.** Negative

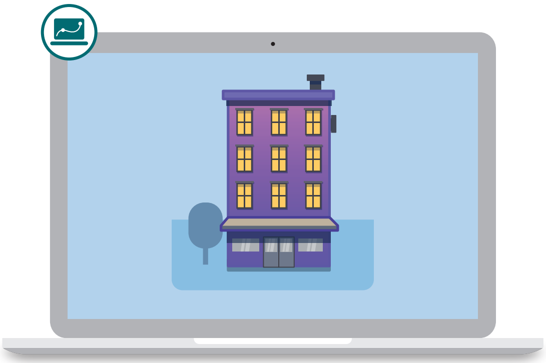
☐ **E.** No association



Unit 7
Lesson
13

Name: Date: Period:

Big and Small Numbers 8.EE.4, SMP.1, SMP.6



City Lights

Let's apply our understanding of place value to add and subtract with scientific notation.

Warm-Up

1 Ariel says: $2 \cdot 10^2 + 3 \cdot 10^3 = 5 \cdot 10^5$.

Is Ariel's claim correct? Circle one.

Yes [✓ No] I'm not sure

Explain your thinking.

Explanations vary. **Ariel's claim is not correct because $2 \cdot 10^2 = 200$ and $3 \cdot 10^3 = 3000$. 3,200 is not equal to $5 \cdot 10^5$.**

Name: ______ Date: ______ Period: ______

City Lights, Part 1

2 We use renewable and non-renewable energy every day. Renewable energy comes from sources that won't run out, like solar or wind power. Non-renewable energy comes from sources that can run out, like coal or oil.

Discuss:

- Why might someone use renewable energy to power their home?
- Why might someone use non-renewable energy to power their home?

Responses vary.

- **Someone might use renewable energy because it could be better for the environment.**
- **Someone might use non-renewable energy because it's readily available in their neighborhood.**

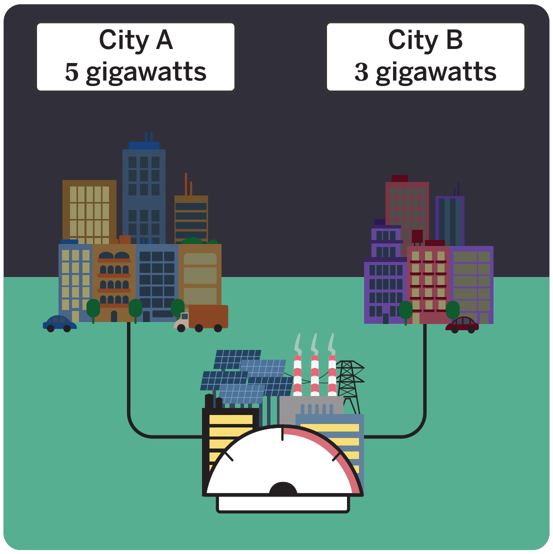
3 City A gets electricity from renewable energy sources and City B gets electricity from non-renewable energy sources.

City A needs 5 gigawatts of electricity.

City B needs 3 gigawatts.

How many gigawatts are needed to power both cities?

8 gigawatts



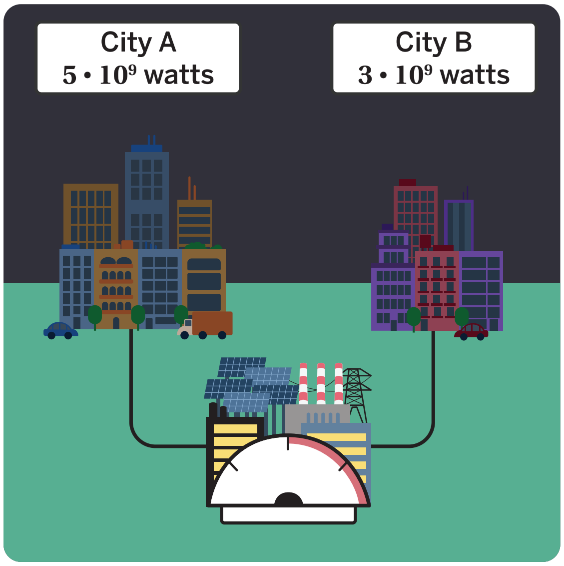
4 1 gigawatt is equal to 10^9 watts.

City A needs $5 \cdot 10^9$ watts of electricity.

City B needs $3 \cdot 10^9$ watts.

How many watts are needed to power both cities? Write your answer in scientific notation.

$8 \cdot 10^9$ watts.



Activity
1

Name: ______________________ Date: ____________ Period: ____________

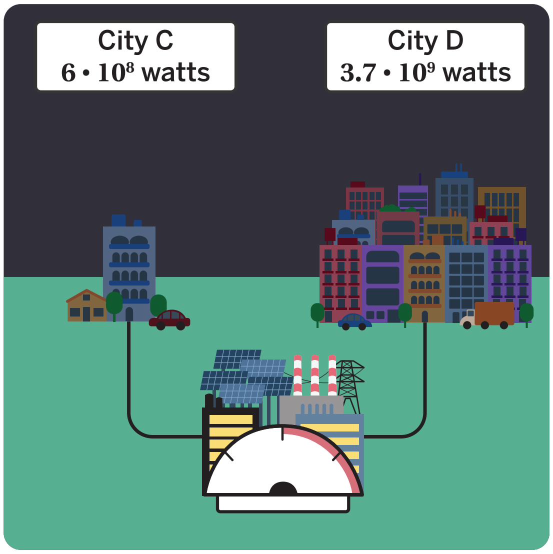
City Lights, Part 1 (continued)

5 Here are two new cities: City C and City D.

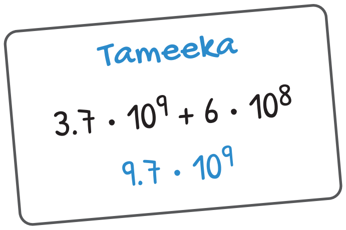
City C needs $6 \cdot 10^8$ watts of electricity.
City D needs $3.7 \cdot 10^9$ watts.

How many watts are needed to power both cities? Write your answer in scientific notation.

$4.3 \cdot 10^9$ watts.



6 Tameeka made a mistake on the previous problem.



Tameeka

$3.7 \cdot 10^9 + 6 \cdot 10^8$

$9.7 \cdot 10^9$

a What do you think Tameeka did well?

Responses vary.

- **I think Tameeka understood that the first part of each number needs to be combined and that the power of 10 will stay the same.**
- **Tameeka used the larger power of 10 as the common power for both numbers.**

b What would you recommend Tameeka change in her work?

***Responses vary.* The terms have different powers of 10. I would recommend that Tameeka rewrite $3.7 \cdot 10^9$ as $37 \cdot 10^8$. From there, she can add $37 \cdot 10^8$ and $6 \cdot 10^8$ because they both have the same power of 10: $37 \cdot 10^8 + 6 \cdot 10^8 = 43 \cdot 10^8$, or $4.3 \cdot 10^9$.**

Name: ______ Date: ______ Period: ______

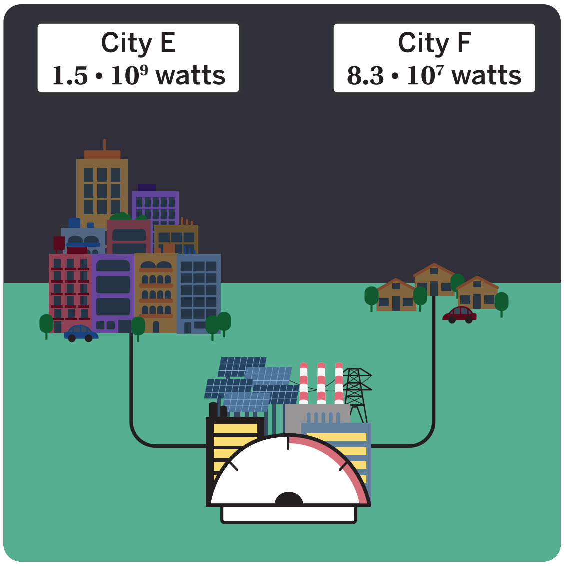
City Lights, Part 2

7 Here are two new cities: City E and City F.

City E needs $1.5 \cdot 10^9$ watts of electricity.

City F needs $8.3 \cdot 10^7$ watts.

How many watts are needed to power both cities? Write your answer in scientific notation.



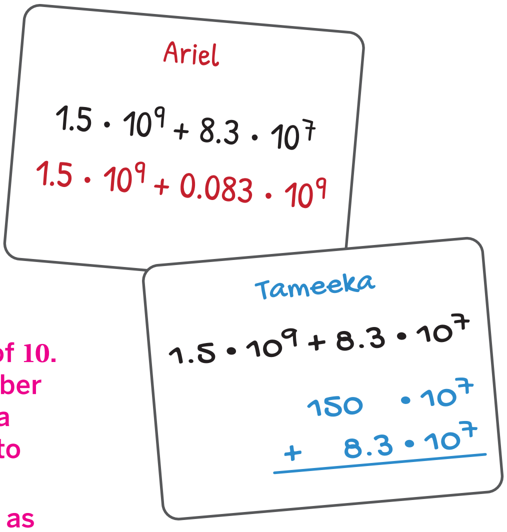
$1.583 \cdot 10^9$ watts. ***Work varies.***

$$1.5 \cdot 10^9 + 8.3 \cdot 10^7 = 1.5 \cdot 10^9 + 0.083 \cdot 10^9$$
$$= 1.583 \cdot 10^9$$

8 Here are Ariel's and Tameeka's strategies for the previous problem.

Discuss:

- How might each student finish the problem?
- After seeing both strategies, how would you add $3.6 \cdot 10^6 + 2.5 \cdot 10^5$?



Ariel

$1.5 \cdot 10^9 + 8.3 \cdot 10^7$

$1.5 \cdot 10^9 + 0.083 \cdot 10^9$

Tameeka

$1.5 \cdot 10^9 + 8.3 \cdot 10^7$

$$\begin{array}{r} 150 \cdot 10^7 \\ + \quad 8.3 \cdot 10^7 \\ \hline \end{array}$$

Responses vary.

- **Both students rewrote the problem so that the numbers they are adding have the same power of 10. Then they would add the first parts of each number together. Ariel would get $1.583 \cdot 10^9$ and Tameeka would get $158.3 \cdot 10^7$. Tameeka would then have to rewrite her answer in scientific notation.**
- **To add $3.6 \cdot 10^6 + 2.5 \cdot 10^5$, I would rewrite $2.5 \cdot 10^5$ as $0.25 \cdot 10^6$. Then I could add $3.6 \cdot 10^6$ and $0.25 \cdot 10^6$ to get $3.85 \cdot 10^6$.**

Name: .... Date: .... Period: ....

City Lights, Part 2 (continued)

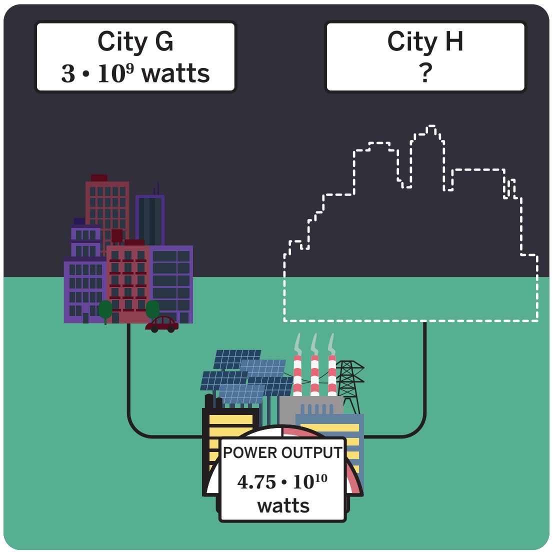
9 Here are two new cities: City G and City H.

The power plant provides exactly enough electricity for both cities: $4.75 \cdot 10^{10}$ watts.

City G uses $3 \cdot 10^{9}$ watts.

How many watts does City H use? Write your answer in scientific notation.

$4.45 \cdot 10^{10}$ watts.



10 Match each value with the correct situation. Note: All units are in watts.

$3.76 \cdot 10^{10}$	$1.03 \cdot 10^{12}$	$1.6 \cdot 10^{10}$	$1.03 \cdot 10^{11}$

Situation A	Situation B	Situation C	Situation D
City 1: $8 \cdot 10^{10}$	City 1: $4.5 \cdot 10^{10}$	City 1: $9.6 \cdot 10^{10}$	City 1: **$3.76 \cdot 10^{10}$**
City 2: $9.5 \cdot 10^{11}$	City 2: **$1.6 \cdot 10^{10}$**	City 2: $7 \cdot 10^{9}$	City 2: $2.4 \cdot 10^{9}$
Total: **$1.03 \cdot 10^{12}$**	Total: $6.1 \cdot 10^{10}$	Total: **$1.03 \cdot 10^{11}$**	Total: $4 \cdot 10^{10}$

11 Synthesis

What are some important things to remember when adding or subtracting numbers written in scientific notation?

$4.6 \cdot 10^7 + 3.2 \cdot 10^6$

$1.57 \cdot 10^8 - 4 \cdot 10^6$

Use the examples if they help with your thinking.

***Responses vary*. Make sure the powers of 10 are the same. To rewrite a number with a different power of 10, multiply the first part of the number by 10 to make the exponent smaller by 1, or divide the first part by 10 to make the exponent larger by 1.**

14 Summary 7.13

Scientific notation can be useful for adding or subtracting very large or very small numbers. It is important to pay attention to place value when adding and subtracting numbers written in scientific notation.

For example: Let's add $3.4 \cdot 10^5 + 2.1 \cdot 10^6$.

It may appear that you can add the first parts: 3.4 and 2.1. However, these numbers *do not* have the same place value because they are multiplied by different powers of 10.

If you rewrite one number so that both numbers have the same power of 10, then you can add their first parts. In this case, let's rewrite $2.1 \cdot 10^6$ as $21 \cdot 10^5$.

$$\begin{aligned} 3.4 \cdot 10^5 + 2.1 \cdot 10^6 &= 3.4 \cdot 10^5 + 21 \cdot 10^5 \\ &= 24.4 \cdot 10^5 \\ &= 2.44 \cdot 10^6 \end{aligned}$$

Now that the power of 10 is the same, you can add 3.4 and 21. The sum is $24.4 \cdot 10^5$, or $2.44 \cdot 10^6$ when rewritten in scientific notation.

Practice
7.13

Name: ____________ Date: ____________ Period: ____________

Problems 1–4: Determine the value of each expression. Write your answers in scientific notation.

1. $5.3 \cdot 10^4 + 4.7 \cdot 10^4$ **$= 1 \cdot 10^5$**

2. $3.7 \cdot 10^6 - 3.3 \cdot 10^6$ **$= 4 \cdot 10^5$**

3. $4.8 \cdot 10^{-3} + 6.3 \cdot 10^{-3}$ **$= 1.11 \cdot 10^{-2}$**

4. $6.6 \cdot 10^{-5} - 6.1 \cdot 10^{-5}$ **$= 5 \cdot 10^{-6}$**

5. Write the value of $2.3 \cdot 10^4 + 4.1 \cdot 10^5$ in scientific notation.

$4.33 \cdot 10^5$

Problems 6–8: Decide whether each statement is *true* or *false*. Show or explain your thinking.

6. $3 \cdot 10^2 + 4 \cdot 10^3 = 7 \cdot 10^5$

False. *Explanations vary.* The digits 3 and 4 do not have the same place value because the powers of 10 are not the same.

7. $8 \cdot 10^2 - 5.1 \cdot 10^3 = 2.9 \cdot 10^2$

False. *Explanations vary.* $5.1 \cdot 10^3$ is greater than $8 \cdot 10^2$, so the difference must be negative.

8. $7 \cdot 10^{-4} + 9 \cdot 10^{-3} = 9.7 \cdot 10^{-3}$

True. *Explanations vary.* $7 \cdot 10^{-4} = 0.7 \cdot 10^{-3}$, which will result in a sum of $9.7 \cdot 10^{-3}$.

Practice
7.13

Name: ______________ Date: ______ Period: ______

9. Fill in each blank. Try to make an expression none of your classmates will.

$\square \cdot 10^{\square} + \square \cdot 10^{\square} = 3.6 \cdot 10^4$

Responses vary. $3 \cdot 10^4 + 6 \cdot 10^3$

Spiral Review

Problems 10–13: Multiply the numbers in each expression. Write your answers in scientific notation.

10. $4.1 \cdot 10^7 \cdot 2$ $= 8.2 \cdot 10^7$

11. $3 \cdot (1.5 \cdot 10^{11})$ $= 4.5 \cdot 10^{11}$

12. $(3 \cdot 10^3)^2$ $= 9 \cdot 10^6$

13. $(9 \cdot 10^6) \cdot (3 \cdot 10^6)$ $= 2.7 \cdot 10^{13}$

Problems 14–15: Calculate each quotient. Write your answers in scientific notation.

14. $\dfrac{3.6 \cdot 10^8}{6 \cdot 10^4}$

$6 \cdot 10^3$

15. $(1.4 \cdot 10^{-3}) \div (7 \cdot 10^5)$

$2 \cdot 10^{-9}$

Problems 16–17: Diego was trying to solve an equation. But when he checked his answer, he saw his solution was incorrect.

Diego

$-4(7 - 2x) = 3(x + 4)$

$-28 - 8x = 3x + 12$

$-28 = 11x + 12$

$-40 = 11x$

$-\frac{40}{11} = x$

16. What would you recommend Diego change in his work?

Responses vary. It looks like Diego multiplied -4 by 7 to get -28, but he needs to also multiply -4 by $-2x$, which would give him $+8x$, not $-8x$.

17. What is the correct solution to the equation?

$x = 8$

Name: .... Date: .... Period: ....

Data Explorations Big and Small Numbers 8.EE.3, 8.EE.4, SMP.2, SMP.6

Star Power

Let's compare the net worths of different celebrities.

Warm-Up

You will use the Warm-Up Card to complete this activity.

1. Choose *four* of the celebrities listed on the card. Record their names and their net worths in the table.
2. Record each celebrity's net worth written in scientific notation. ***Responses vary.***

Name	Net Worth ($)	Net Worth Written in Scientific Notation ($)

3. Order the celebrities in your table from lowest to highest net worth. ***Responses vary.***

Lowest Net Worth **Highest Net Worth**

Activity
1

Name: .... Date: .... Period: ....

Star Power

Jack Mesos is the founder of BuyNSell.com. His net worth is about $1.5 \cdot 10^{11}$ dollars.

4. You will use the Warm-Up Card to help you answer: *Who has more money?*

[✓ **A.**] Jack Mesos
B. All 10 celebrities combined
C. I'm not sure.

Explain your thinking.

***Explanations vary.* The net worth of the 10 celebrities combined is about $6.1 \cdot 10^9$ dollars, which is less than Jack Mesos's net worth of $1.5 \cdot 10^{11}$ dollars.**

5. Which unit would you choose to represent Jack Mesos's net worth?

Responses vary.

A. Hundred dollars
B. Thousand dollars
C. Million dollars
D. Billion dollar

6. Determine Jack Mesos's net worth using the units that you selected in Problem 5.

***Responses vary.* 150 billion dollars.**

7. As of 2023, the median salary of a full-time worker in the U.S. was around $60,000 per year. How long would someone with this salary need to work to earn the equivalent of Jack Mesos's net worth? Write your answer in scientific notation.

$2.5 \cdot 10^6$ years

8. Which unit do you think is most appropriate to use for your response to Problem 7?

Responses vary.

A. Days
B. Years
C. Centuries
D. Millennia

Explain your thinking. **ELD.PI.8.11.Em, Ex, Br**

***Explanations vary.* I think using millennia as the unit emphasizes how long it would take for someone with a salary of $60,000 to earn the equivalent of Jack Mesos's net worth. It would take $2.5 \cdot 10^3$ (or 2,500) millennia, which is a very long time considering we're only living in the third millennium.**

Name: .... Date: .... Period: ....

What Would You Buy?

9. If you had $\$1.5 \bullet 10^{11}$ and you wanted to spend it all, what would you buy?

Responses vary.

10. You will use the Activity 2 Sheet that lists the costs of various items. Without going over, spend as close to $\$1.5 \bullet 10^{11}$ as you can by purchasing *at least four* different items. Record each item you chose in the table. Then calculate how much money you spent. ***Responses vary.***

Item	Price Per Item	Quantity	Total Cost
	Total Spent		

11. How many dollars away were you from spending exactly $\$1.5 \bullet 10^{11}$? **Responses vary.**

Explain your thinking.

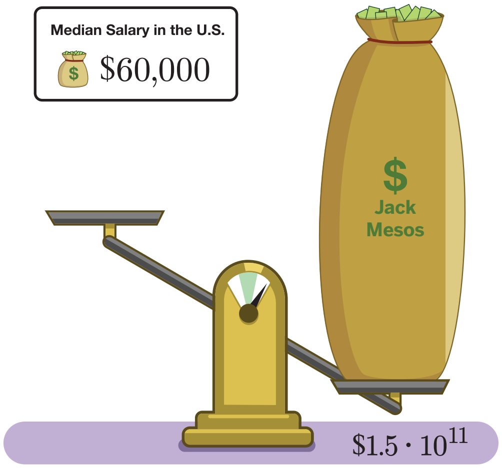
Explanations vary. **I took the total cost I spent and wrote it as a power of 10^{11}. Then I subtracted the total cost from $1.5 \bullet 10^{11}$ to figure out how close I was to spending all of the money.**

Synthesis

12. What are some important things to remember about adding, subtracting, multiplying, and dividing numbers written in scientific notation?

ELD.PI.8.10.Em, Ex, Br

***Responses vary*. When adding or subtracting numbers written in scientific notation, it's helpful to have the same power of 10 for both expressions. When multiplying or dividing numbers written in scientific notation, I can multiply or divide the first parts of the expressions and then multiply or divide the powers of 10.**



Summary 7.14

Scientific notation is a useful tool for adding, subtracting, multiplying, dividing, and comparing very small or very large numbers.

- You can rewrite the number 39,000,000,000,000 as $3.9 \cdot 10^{13}$ and still convey just how large the number is.
- To add or subtract numbers written in scientific notation, it is useful to rewrite the numbers so they have the same power of 10.
- To multiply or divide numbers written in scientific notation, it is useful to multiply or divide the numbers that come before the powers of 10. Then you can use exponent rules to multiply or divide the powers of 10.
- If the product or quotient is not written in scientific notation, you can always rewrite it to be in that form.
- Sometimes it can be helpful to round numbers written in scientific notation when exact values are less important.

Some situations that involve very large or very small numbers include salaries of wealthy people, talking about large groups like the total number of workers at a company, or the sizes of microscopic objects like cells and bacteria.

Practice

7.14

Name: .... Date: .... Period: ....

Problems 1–3: In 2022, the United States had an approximate population of $3.3 \cdot 10^8$ people. California, Texas, Florida, and New York had the greatest populations out of all the states.

State	Population (people)
California	$3.9 \cdot 10^7$
Texas	$3.0 \cdot 10^7$
Florida	$2.2 \cdot 10^7$
New York	$2.0 \cdot 10^7$

1. What was the total population of all four states? Write your answer in scientific notation.

   About $1.1 \cdot 10^8$ people

2. What was the total population for the other 46 states? Write your answer in scientific notation.

   About $2.2 \cdot 10^8$ people

3. About how many times greater was the population of California than the population of Florida?

   About 2 times greater

Problems 4–7: Here is a table about different life forms on our planet.

Creature	Population	Mass of One Individual (kg)
Humans	$8 \cdot 10^9$	$6.2 \cdot 10^1$
Sheep	$1.38 \cdot 10^9$	$6 \cdot 10^1$
Chickens	$3.44 \cdot 10^{10}$	$2.6 \cdot 10^0$
Antarctic krill	$8 \cdot 10^{14}$	$1 \cdot 10^{-3}$
Bacteria	$5 \cdot 10^{30}$	$1 \cdot 10^{-12}$

4. Which is greater: the total mass of all humans or the total mass of all Antarctic krill?

   The total mass of all Antarctic krill. *Work varies.*

   Humans:
   $(8 \cdot 10^9) \cdot (6.2 \cdot 10^1)$
   $\approx 5 \cdot 10^{11}$ kilograms

   Antarctic krill:
   $(8 \cdot 10^{14}) \cdot (1 \cdot 10^{-3})$
   $= 8 \cdot 10^{11}$ kilograms

5. How can you tell which creature has the greatest total mass?

   ***Responses vary.* Bacteria has the greatest total mass because the population of bacteria times the mass of one bacteria is 10^{18}, which is larger than the mass of any of the other creatures.**

6. About how many more chickens are there than sheep? Write your answer in scientific notation. Show or explain your thinking.

   Responses between $3 \cdot 10^{10}$ and $3.302 \cdot 10^{10}$ are considered correct. *Explanations vary.* $3.44 \cdot 10^{10} - 1.38 \cdot 10^9$ can be rewritten as $34.4 \cdot 10^9 - 1.38 \cdot 10^9$, which equals $33.02 \cdot 10^9$ or $3.302 \cdot 10^{10}$ (about 33 billion).

7. Do you think kilograms would be an appropriate unit for measuring the mass of each creature listed in the table? Explain your thinking.

   ***Responses vary.* I think it would be appropriate to use kilograms to measure all the different creatures, because it would make it easier to compare the different masses. But I also think it's usually better to use a smaller unit, such as nanograms, to measure the mass of bacteria.**

Practice
7.14

Name: ________ Date: ________ Period: ________

Problems 8–9: Here is a list of facts about space, the world, and the human body.

- The Milky Way is about 10^5 light years across.
- One light year is about 10^{16} meters long.
- There are about $3.7 \cdot 10^{13}$ cells in a human body.
- The world's population is about $8 \cdot 10^9$.

8. Which is greater: the number of meters across the Milky Way or the total number of cells in all the humans in the world? Show or explain your thinking.

Number of human cells. *Explanations vary*. The Milky Way is about $10^5 \cdot 10^{16}$, or 10^{21}, meters across. The total number of human cells is $(3.7 \cdot 10^{13}) \cdot (8 \cdot 10^9)$, or $2.96 \cdot 10^{23}$, which is greater than the approximate number of meters across the Milky Way.

9. Yona says that there are about 30 times as many cells in all humans as there are meters across the Milky Way. Is she correct? Show or explain your thinking.

No. *Explanations vary*. The number of cells is about $\frac{3 \cdot 10^{23}}{1 \cdot 10^{21}} = 3 \cdot 10^2$, or 300, times the number of meters across the Milky Way.

Spiral Review

10. Write a number and power of 10 to show the value of $(6.2 \cdot 10^5) \cdot (3.4 \cdot 10^2)$ in scientific notation.

Write a number and a power of 10 to show the value of the expression in scientific notation.

2.108 • **10^8**

11. Select *all* the expressions that are equivalent to 3^8.

☑ A. $(3^2)^3 \cdot 3^2$

☐ B. $\frac{1}{3^4 \cdot 3^4}$

☑ C. $\frac{(3^4)^3 \cdot 3^0 \cdot 3^3}{3^2 \cdot 3^5}$

☐ D. $(3^2 \cdot 3^2)^4$

☑ E. $\frac{3^3 \cdot 3^3 \cdot 3^4}{3^2}$

12. Order the expressions by value from *least* to *greatest*.

$2.3 \cdot 10^{-2}$	$2.3 \cdot 10^2$	$23 \cdot 10^{-4}$	$0.23 \cdot 10^5$	23

$23 \cdot 10^{-4}$	**$2.3 \cdot 10^{-2}$**	**23**	**$2.3 \cdot 10^2$**	**$0.23 \cdot 10^5$**

Least **Greatest**

Name: .... Date: .... Period: ....

Big and Small Numbers 8.EE.1, 8.EE.3, 8.EE.4, SMP.2, SMP.6, SMP.7

Practice Day 2

Let's practice what you've learned so far in this unit!

You will use problem cards for this Practice Day. Record all of your responses here.

Card 1

$3.4 \cdot 10^6$

Card 2

$5.4 \cdot 10^7$

Card 3

$3 \cdot 10^{12}$

Card 4

$38 \cdot \boxed{}^{8} = 12^{\boxed{}}$

Responses vary.

$3^8 \cdot \boxed{4}^{8} = 12^{\boxed{8}}$

$3^8 \cdot \boxed{48}^{8} = 12^{\boxed{16}}$

Card 5

$2.95 \cdot 10^6$

Card 6

$2.1 \cdot 10^5$

Card 7

$7.27 \cdot 10^6$

Card 8

$7.82 \cdot 10^{-6}$

Name: ________ Date: ________ Period: ________

Practice Day 2 (continued)

Card 9

Larger: $2.9 \cdot 10^9$

Times as large: About 500

Card 10

$(7^{\square})^{\square} = \frac{1}{7^{24}}$

Responses vary. Any values where the two values multiply to -24 is correct.

Card 11

$4.08 \cdot 10^5$

Card 12

$4 \cdot 10^7$

Card 13

Moon: Earth

Times as much: 82

Card 14

Mercury: Earth

Times as much: 18

Card 15

Country: United States

Times as many emissions: about 2

Card 16

Country: India

Times as many people: about 5

Notes:

Career Connection

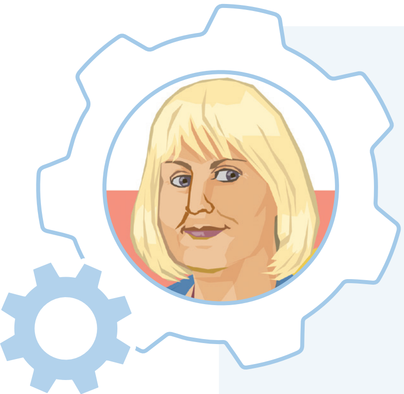
How do you measure a galaxy?


nednapa/Shutterstock.com

Meters may be fine for measuring distances on Earth, but to measure distances in outer space, we need much bigger units of measurement. Our galaxy, the Milky Way, is about 1.0×10^{21} meters across. This is the equivalent of 1 *zettameter*.

The closest major galaxy to the Milky Way is the Andromeda Galaxy, with a diameter of about 2.1 zettameters. The Andromeda galaxy is about 23.7 zettameters away (2.5 million light-years). That's probably not high on your list for a weekend getaway — yet.

Astrophysicists study the universe including how galaxies, stars, and planets form and change. Computational astrophysicists design and use computer models to simulate physical processes that occur in space and are difficult to observe on Earth.



Meet Sophie Wilson

Sophie Wilson is an English computer scientist. She designed her first microcomputer while studying at the University of Cambridge, and went on to lead the development of the BBC BASIC programming language. In the 1980s, Wilson helped design processors that could reach speeds of up to 10 MHz, or 10^7 operations per second. While today's devices can perform billions of operations every second, Wilson was among the pioneers who helped engineer these greater speeds.

Are you interested in studying computer science or astrophysics? What can you do to learn more?

Math in the World

Despite its name, a *light-year* measures distance (not time). It represents how far light travels in 1 year. Light travels incredibly fast, about 3×10^8 meters per second! That's about a *million times faster* than a commercial airplane!

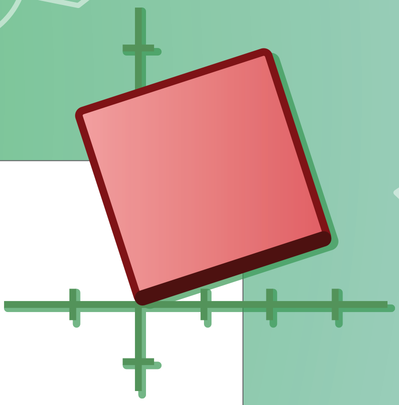
Light-years are used to indicate distances in space because of the vastness of space, making them a more understandable measure than meters.

How many meters does light travel in 1 year? **9.46×10^{15} meters**

Math Mindset

Suppose you estimated the diameter of the Andromeda Galaxy to be 2 zettameters. What is the difference between this estimate and 2.1 zettameters in terms of actual meters? Is this a big difference?

1×10^{20} meters, or
100,000,000,000,000,000,000 meters
Responses vary.



Unit 8

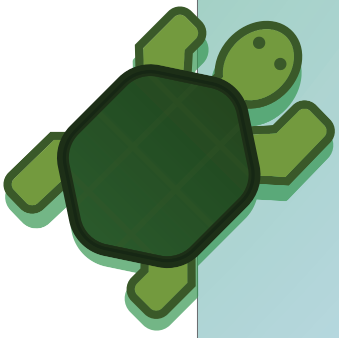
The Pythagorean Theorem and Irrational Numbers

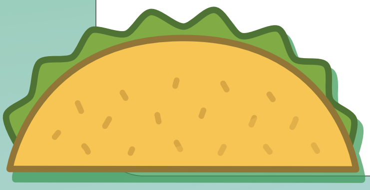
Big Ideas in This Unit

CC3 Pythagorean Explorations
CC4 Shape, Number, and Expressions
NS Number Line Understanding
Generalized Numbers Leading to Algebra

Questions for Investigation

- How can you estimate the square root of a number? What does the square root of a number represent?
- Is it true that $\text{leg}^2 + \text{leg}^2 = \text{hypotenuse}^2$ for all right triangles? If so, can you prove it?
- What is the difference between a rational number and an irrational number?





Explore: The Longest Cut
What's the longest length in a rectangle or rectangular prism?

Watch Your Knowledge Grow

This is the math you'll explore in this unit. Rate your understanding to see how your knowledge grows!

○	○	○
Not yet	Almost	I got it!

I can . . .	Before	After
Calculate the area of a square with vertices at the intersections of grid lines.	○—○—○	○—○—○
Use square root notation to represent the side length of a square given its area.	○—○—○	○—○—○
Use cube root notation to represent the edge length of a cube given its volume.	○—○—○	○—○—○
Approximate the value of a square root or cube root.	○—○—○	○—○—○
Represent a square root or cube root as a point on a number line.	○—○—○	○—○—○
Determine which two whole number values a square root or cube root is between.	○—○—○	○—○—○
Identify values as perfect squares or perfect cubes.	○—○—○	○—○—○
Recognize that the relationship $a^2 + b^2 = c^2$ is true for right triangles.	○—○—○	○—○—○
Explain a proof of the Pythagorean theorem.	○—○—○	○—○—○
Apply the Pythagorean theorem to determine unknown side lengths in right triangles.	○—○—○	○—○—○

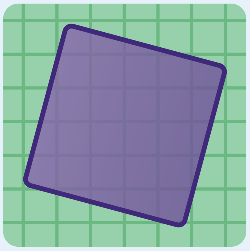
I can . . .	Before	After
Determine the side length of a square or the length of the diagonal of a rectangular prism using the Pythagorean theorem.	○—○—○	○—○—○
Use the converse of the Pythagorean theorem to determine if a triangle is a right triangle.	○—○—○	○—○—○
Solve real-world problems using the Pythagorean theorem.	○—○—○	○—○—○
Calculate the distance between two points on the coordinate plane using the Pythagorean theorem.	○—○—○	○—○—○
Understand whether a unit fraction will be a repeating or a terminating decimal.	○—○—○	○—○—○
Express a repeating or terminating decimal as a fraction.	○—○—○	○—○—○
Justify whether a number is rational or irrational.	○—○—○	○—○—○

Sub-Unit

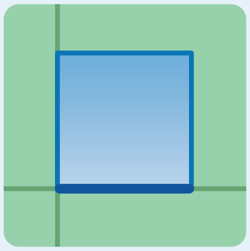
1

Square Roots and Cube Roots

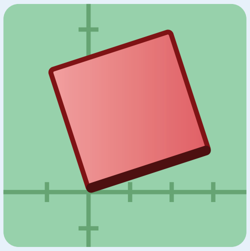
Explore
The Longest Cut



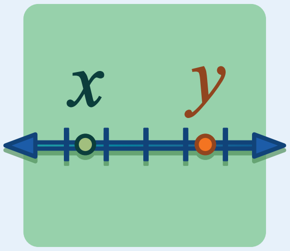
Lesson 1
Tilted Squares



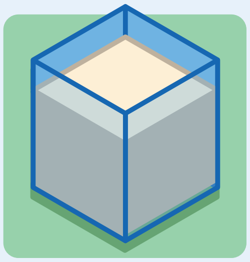
Lesson 2
From Squares to Roots



Lesson 3
Between Squares



Lesson 4
Root Down



Lesson 5
Filling Cubes

Name: .... Date: .... Period: ....

Pythagorean Explorations   Building Toward 8.G.7, SMP.6, SMP.7

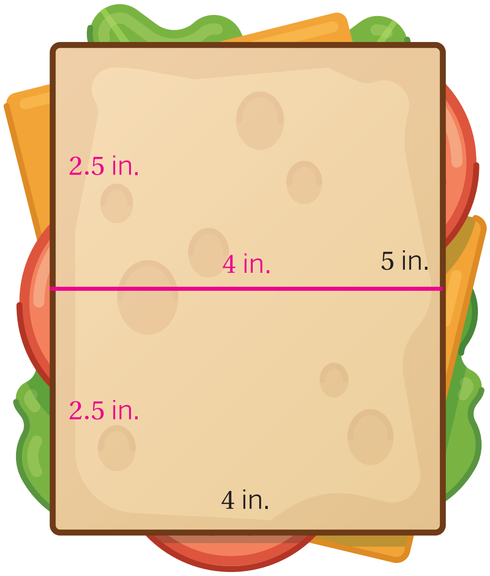
Explore: The Longest Cut

What's the longest length in a rectangle or rectangular prism?

Warm-Up

1. Draw a line to "cut" the sandwich into two pieces. Estimate the length of the cut and the sides of each new part.

   ***Drawings vary*. Sample response shown.**



Activity

Name: .... Date: .... Period: ....

The Longest Cut

2. You will use the Activity 1 Sheet to complete Problems 1–2. Measure each side of the sandwich. Then determine the longest, straight cut you can make on each sandwich. Record your results in the table. ***Responses vary.* Sample responses shown.**

	Side (in.)	Side (in.)	Length of cut (in.)
Sandwich A	4	5	6.5
Sandwich B	3	4	5
Sandwich C	4	4	5.7
Sandwich D	2	3	3.5

3. **Discuss:** What do you notice about the cuts? What do you wonder?
ELD.PI.8.1.Em, Ex, Br

Responses vary.

- **I notice that the cuts are all diagonal cuts.**
- **I notice the longest cut is longer than either side of the sandwich.**
- **I notice that most of the longest cuts are not whole number lengths.**
- **I wonder if the longest cut is always the same cut for any sandwich size.**
- **I wonder if I can determine the longest cut without using a ruler.**

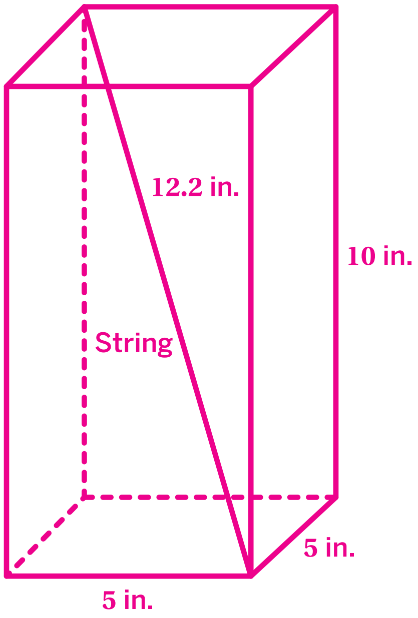
Name: .... Date: .... Period: ....

The Longest Cut (continued)

You will use a box and string for Problems 4–7.

4. Sketch your box. Then label the length, width, and height of the box.

Sketches vary. Sample response shown.



5. What is the longest straight length of string you can create that fits completely inside the box?

Responses vary. 12.2 inches.

6. Sketch the measure of the string length you found inside the box and label its length.

Sketches vary. Sample response shown in image.

7. Compare your results with your classmates.

💬 **Discuss:** What is alike? What is different? ELD.PI.8.1.Em, Ex, Br, ELD.PI.8.3.Em, Ex, Br

Responses vary. Each of the longest strings started from one corner and ended in the opposite corner of the box. The string lengths were different based on the box size.

Activity

Name: .... Date: .... Period: ....

Building Math Habits of Mind

Discuss:

- Which of these habits of mind did you strengthen during this activity?
- How did you use the one(s) you selected?

I can slow down and first make sense of a challenging problem before trying to solve it.

○ Not yet ○ Almost ○ I got it!

I can represent real-world problems using equations and inequalities and interpret their solutions within the context of the problem.

○ Not yet ○ Almost ○ I got it!

I can justify my thinking and ask questions to help me understand the thinking of others.

○ Not yet ○ Almost ○ I got it!

I can apply the math that I know to solve real-world problems, make assumptions and revise my thinking as needed.

○ Not yet ○ Almost ○ I got it!

I can select an appropriate tool to help me solve problems.

○ Not yet ○ Almost ○ I got it!

I can communicate my thinking and solutions clearly to others.

○ Not yet ○ Almost ○ I got it!

I can look for structure or patterns to help me solve problems.

○ Not yet ○ Almost ○ I got it!

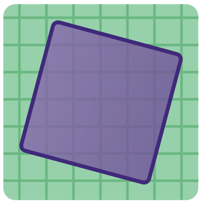
I can look for repeated calculations and other repeated steps to make generalizations.

○ Not yet ○ Almost ○ I got it!

Unit 8
Lesson
1

Name: .... Date: .... Period: ....

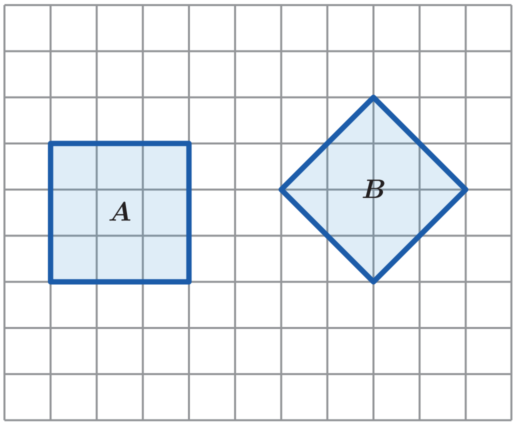
Pythagorean Explorations   Shape, Number, and Expressions   Building Toward 8.EE.2, 8.G.6, 8.NS.2, SMP.7



Tilted Squares

Let’s explore finding the areas of tilted squares.

Warm-Up



1. Which shaded region is larger?

Square A

Explain your thinking. **ELD.PI.8.10.Em, Ex, Br**

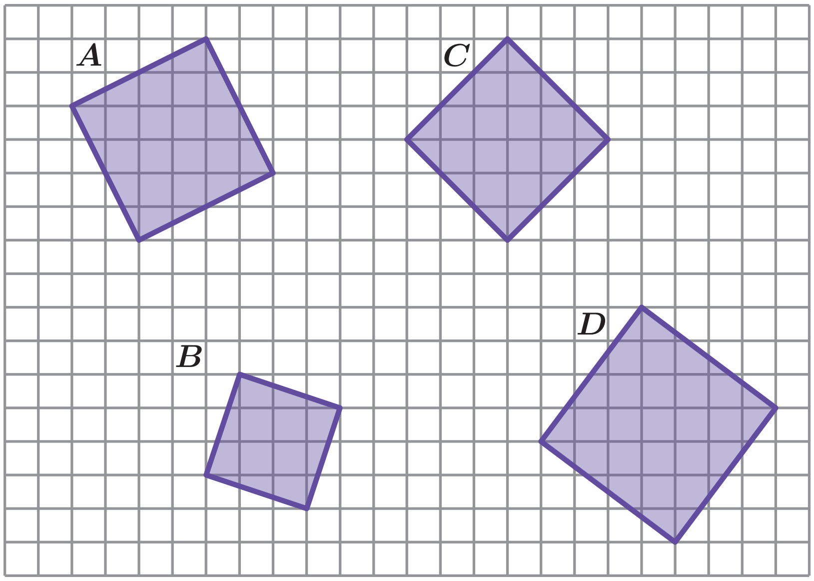
Explanations vary. Square A has an area of 9 square units, which I found by multiplying the base length by the height length: $3 \cdot 3$. To find the area of square B, I divided the figure into four congruent triangles and calculated the area of each triangle. $4 \cdot \frac{1}{2}(2 \cdot 2) = 8$ square units.

Activity
1

Name: .... Date: .... Period: ....

Area of Tilted Squares

2. Determine the area of each tilted square (in square units). Record the areas in the table.



Square	A	B	C	D
Area (sq. units)	20	10	18	25

3. What strategies did you use to determine the areas of the tilted squares?

ELD.PI.8.10.Em, Ex, Br

Responses vary.

- First, I drew a larger square around a tilted square. The area of the large square is equal to the area of the tilted square plus the area of the four congruent triangles. I found the area of the large square and then subtracted the area of each triangle.
- The area of a tilted square can be divided into four congruent triangles and a square. I calculated the area of one triangle, multiplied that by four, and then added the area of the square.
- I counted the number of unit squares within a tilted square. Since the square is tilted, there were a number of partial square units. I counted those partial square units and estimated how many full square units they would equal.

4. What is the side length of square D?

5 units

Explain your thinking.

Explanations vary. The area of square D is 25 square units. The side length of the square must be 5 units because 5^2 is equal to 25.

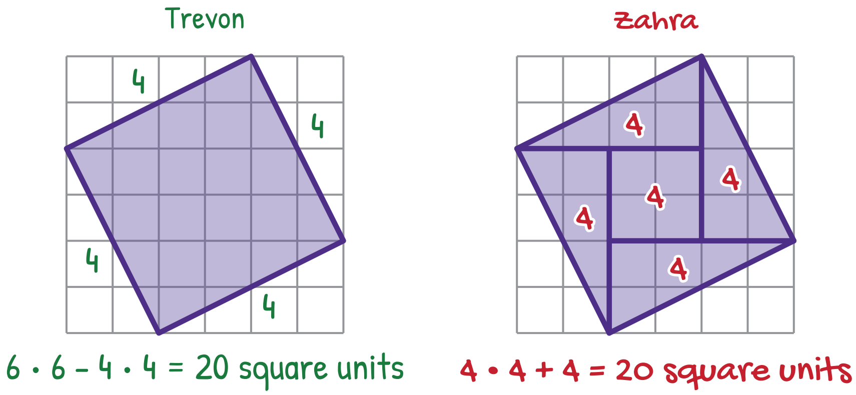
Activity
2

Name: .... Date: .... Period: ....

Different Strategies

Here are Trevon's and Zahra's strategies for finding the area of tilted square A.

ELD.PI.8.6.Em, Ex, Br, ELD.PI.8.11.Em, Ex, Br



5. How are Trevon's and Zahra's strategies alike? How are they different?

Responses vary. **Both strategies require finding the area of congruent triangles and squares. In Trevon's strategy, the original tilted square is surrounded by a larger square, the area of the larger square is calculated, and then the area of the four congruent triangles are subtracted. In Zahra's strategy, the tilted square is decomposed into four congruent triangles and one square. The area of one triangle is found, multiplied by four, and then added to the area of the center square.**

6. How does each strategy compare to your own?

Responses vary.

- **I used the same strategy as Trevon to find the area by surrounding and subtracting.**
- **I used the same strategy as Zahra to find the area by decomposing the tilted square.**
- **I used something different than Trevon and Zahra and found an approximate area by counting the unit squares and estimating.**

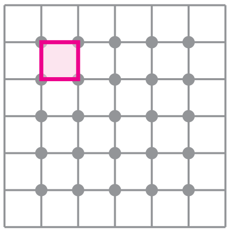
Activity
3

Name: ______________________ Date: ____________ Period: ____________

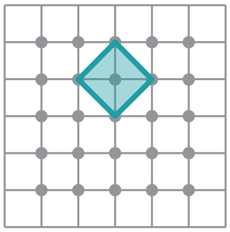
Building Squares With Different Areas

7. Here are squares with areas of 2 square units and 9 square units. On each dot grid, try to draw a square with the given area. Then circle “P” for any area that is possible to draw and “N” for any area that’s not possible to draw. **Samples shown on grids.**

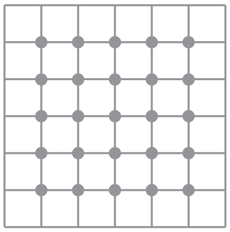
Area:
1 square unit
[✓ P] / N



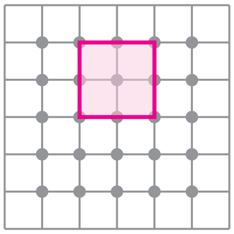
Area:
2 square units
[✓ P] / N



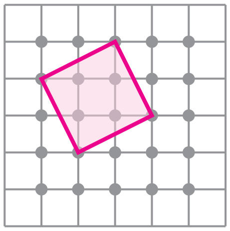
Area:
3 square units
P / [✓ N]



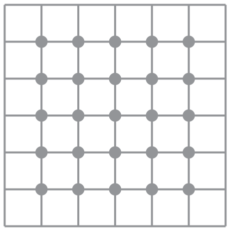
Area:
4 square units
[✓ P] / N



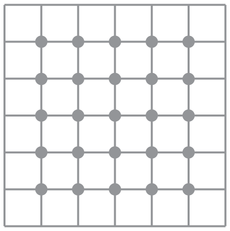
Area:
5 square units
[✓ P] / N



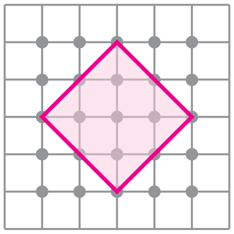
Area:
6 square units
P / [✓ N]



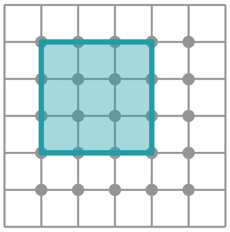
Area:
7 square units
P / [✓ N]



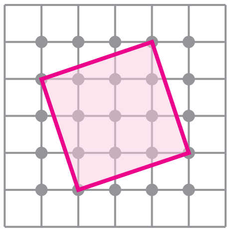
Area:
8 square units
[✓ P] / N



Area:
9 square units
[✓ P] / N



Area:
10 square units
[✓ P] / N



8. Choose one of the squares and determine its side length.

Responses vary.

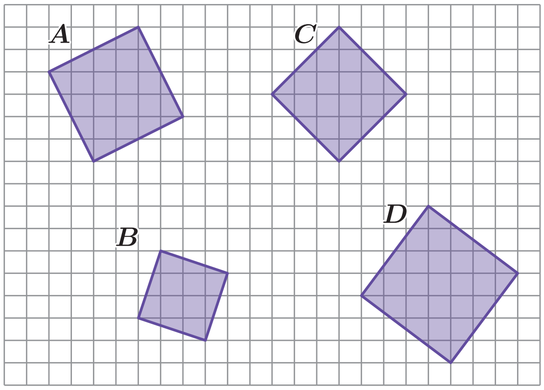
- **The square with 4 square units has a side length of 2 units.**
- **The square with 9 square units has a side length of 3 units.**

Synthesis

9. Describe a strategy for determining the area of a tilted square.

Use the examples if they help with your thinking. **ELD.PI.8.10.Em, Ex, Br**

***Responses vary.* Draw a larger square around the tilted square. Then find the areas of the triangles and subtract them from the area of the larger square.**

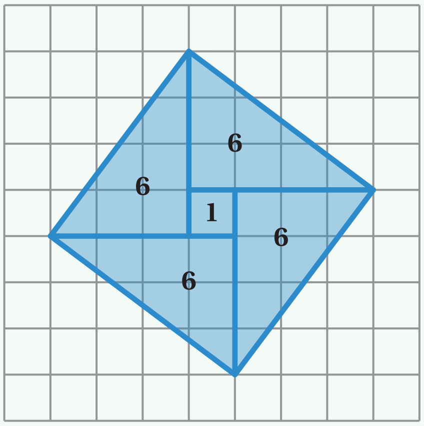


Summary 8.01

There are many strategies for determining the area of a tilted square. Here are two strategies called "decompose and rearrange" and "surround and subtract."

Decompose and Rearrange

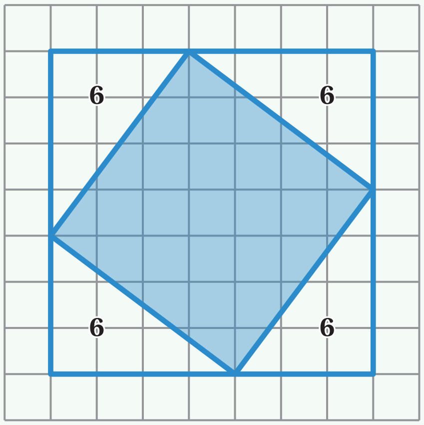
Area is calculated by adding the areas of the four triangles and one center square.



$4 \cdot 6 + 1 = 25$ square units

Surround and Subtract

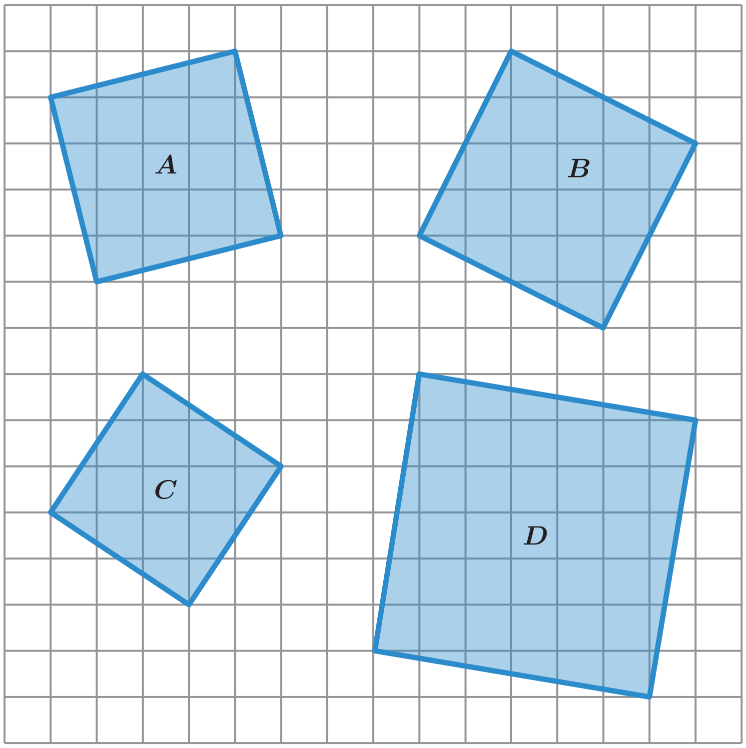
Area is calculated by finding the area of the large square minus the area of the four triangles.



$7 \cdot 7 - 4 \cdot 6 = 25$ square units

Practice
8.01

Name: ______ Date: ______ Period: ______



Problems 1–4: Determine the area of each tilted square. Each square grid represents 1 square unit.

1. Square A
17 square units

2. Square B
20 square units

3. Square C
13 square units

4. Square D
37 square units

Problems 5–7: Determine the area of each square given its side length.

5. Side length: 3 inches
9 square inches

6. Side length: 100 centimeters
10,000 square centimeters

7. Side length: x units
x^2 square units

Problems 8–10: Here are the areas of three squares. Determine the side length of each square.

8. Area: 81 square inches
9 inches

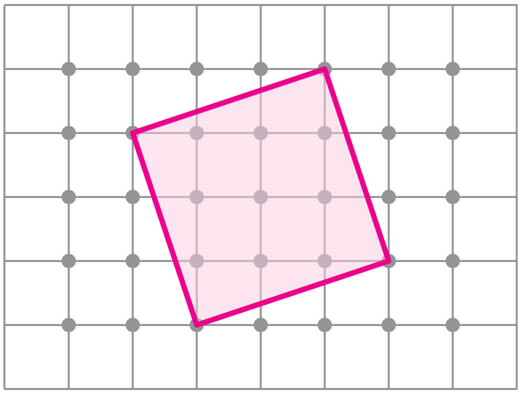
9. Area: $\frac{4}{25}$ square centimeters
$\frac{2}{5}$ centimeters

10. Area: m^2 units
m units

Practice
8.01

Name: .... Date: .... Period: ....

Problems 11–12: Use the rectangular grid.



11. Sketch the largest tilted square possible on the rectangular dot grid.
Sample response shown.

12. Determine the area of your square.
10 square units

Spiral Review

13. Select *all* the expressions that are equivalent to 3^8.

☐ **A.** $3^6 \cdot 10^2$ ☐ **B.** 8^3 ☑ **C.** $\frac{3^6}{3^{-2}}$ ☑ **D.** $(3^4)^2$ ☑ **E.** $(3^2)^4$

14. In July 2023, the population of Arlington, Texas, was 392,786. What is the value of the exponent when this number is written in scientific notation?
5

15. Which expression is equal to $(3.1 \cdot 10^4) \cdot (2 \cdot 10^6)$?

A. $5.1 \cdot 10^{10}$ **B.** $5.1 \cdot 10^{24}$ [✓ **C.** $6.2 \cdot 10^{10}$] **D.** $6.2 \cdot 10^{24}$

16. Here is Oliver's work for solving this problem:

Evaluate $5.4 \cdot 10^5 + 2.3 \cdot 10^4$ and write the answer in scientific notation.

Oliver

$5.4 \cdot 10^5$ can be rewritten as $54 \cdot 10^4$

$54 \cdot 10^4 + 2.3 \cdot 10^4 = 56.3 \cdot 10^4$

Is Oliver's solution correct?
No

Explain your thinking.
***Explanations vary.* Oliver's calculations are correct, but his final answer isn't in scientific notation. To finish the problem, he should convert his answer to be $5.63 \cdot 10^5$.**

Unit 8

Lesson

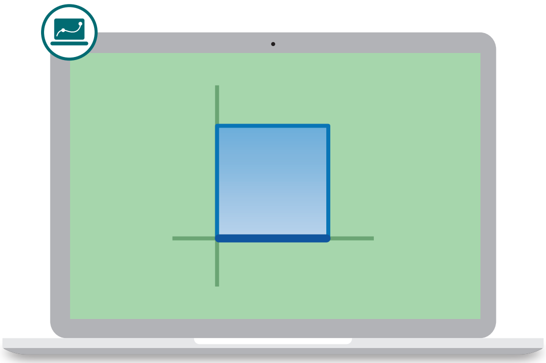
2

Name: .... Date: .... Period: ....

Shape, Number, and Expressions 8.NS.1, 8.NS.2, SMP.6, SMP.7

From Squares to Roots

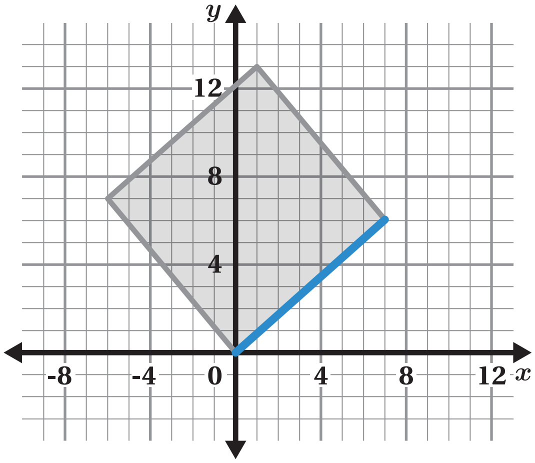
Let's explore the connection between the area and side length of a square.



Warm-Up

1 Estimate the side length of the square.

Responses vary. Approximately 9 units

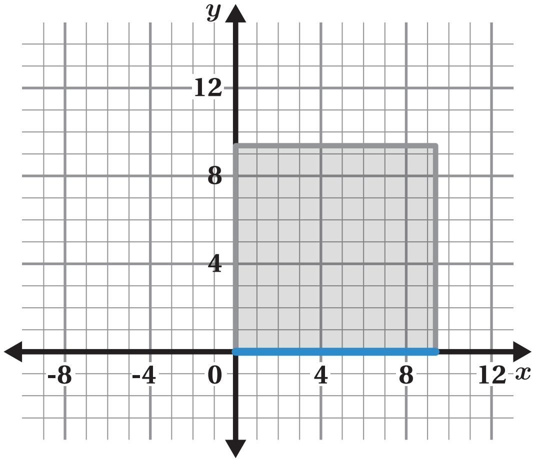


2 You can *approximate* the side length of a tilted square by rotating it onto an axis.

a Here is the square from the previous problem rotated so that the highlighted side length is along the x-axis.

b Write a new estimate for the side length of the square.

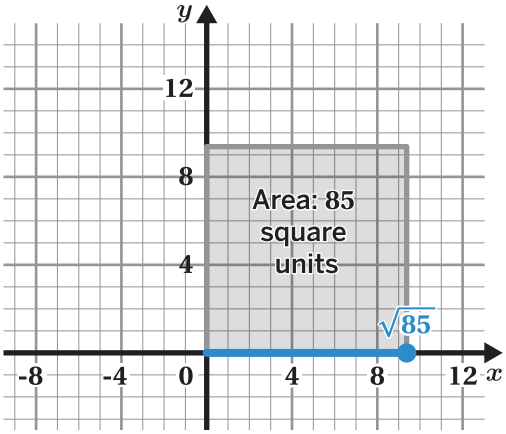
Responses vary. Approximately 9.2 units



Name: ________ Date: ________ Period: ________

Square Roots

3 The *exact* side length of this square is the **square root** of 85, written as $\sqrt{85}$.



a Take a look at the square and how the side length is written.

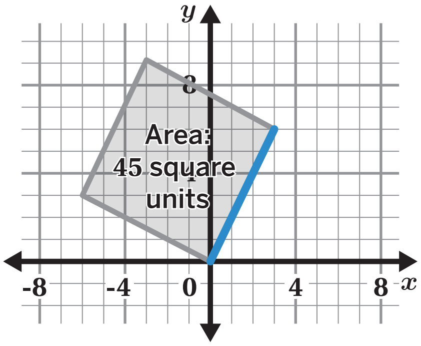
b **Discuss:** What do you think a square root is?

Responses vary. **A square root, $\sqrt{n}$, has the same value as a side length of a square with an area of n square units.**

4 **Rational numbers** are numbers that can be written as a fraction of two non-zero integers, such as $\frac{3}{1}$, or 3. $\sqrt{9}$ is a rational number because it's equivalent to 3.

A number that is not rational is called an **irrational number**, such as $\sqrt{85}$.

The area of this square is 45 square units.



a Write the exact value of the side length.

$\sqrt{45}$ units

b Is this number rational or irrational?

irrational

5 Determine the unknown side lengths and areas for each square.

Square	Side Length (units)	Area (sq. units)
A	$\sqrt{55}$	55
B	**$\sqrt{81}$ or 9**	81
C	2.5	**6.25**
D	**$\sqrt{14}$**	14
E	$\sqrt{44}$	**44**
F	**$\sqrt{32}$**	32

Name: .... Date: .... Period: ....

Square Roots (continued)

6 Order the squares from *smallest* to *largest* area.

A	B	C	D
Area of 50 square units	Side length of $\sqrt{81}$ units	Side length of $\sqrt{55}$ units	Side length of 8 units

A	C	D	B

Smallest **Largest**

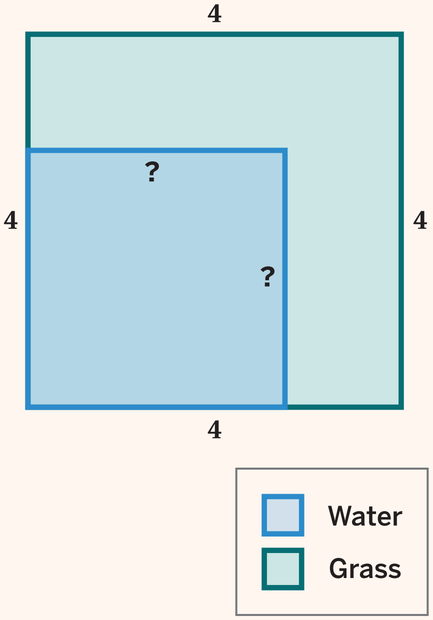
You’re invited to explore more.

7 Metropolis has a park surrounded by a square fence with 4-meter side lengths.

The city would like to build a square pool as shown in the figure.

What should the side length of the pool be so that half of the area is grass and half is water? Explain your thinking.

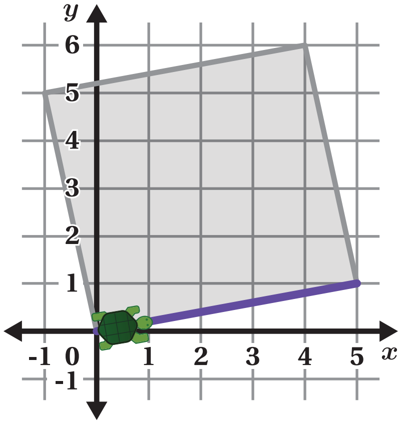
$\sqrt{8}$ meters. *Explanations vary*. The area surrounded by the fence is 16 square meters, so we want the area of both the grassy region and the water region to be 8 square meters. For the blue square in the figure to have an area of 8 square meters, the side length needs to be $\sqrt{8}$ meters.



Name: .... Date: .... Period: ....

Turtle Tracing

Tiam the turtle is walking on one side of a square.

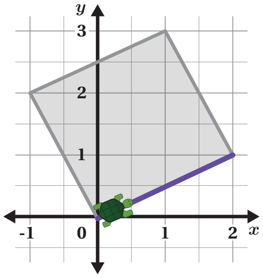
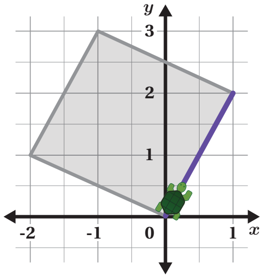
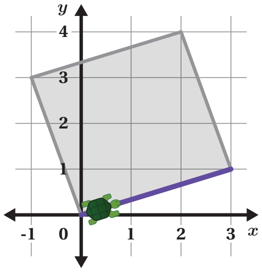
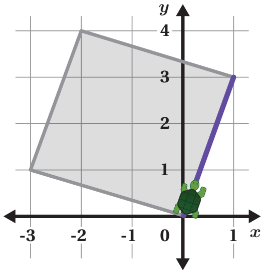
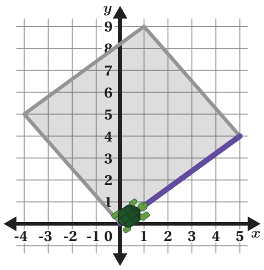
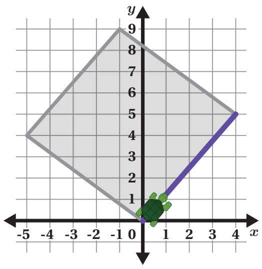


8 Exactly how far does Tiam need to travel?

$\sqrt{26}$ units

9 Complete this chart with a partner.

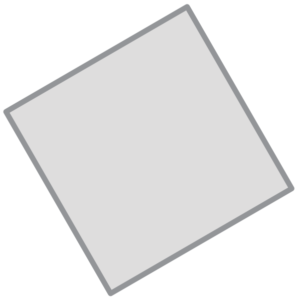
- Decide with your partner who will complete Column A and who will complete Column B.
- Determine how far Tiam the turtle needs to travel, and then compare your solutions. The solutions in each row should be the same. Discuss and resolve any differences.
- Determine the side lengths of as many squares as you have time for.

	Column A	Column B
a	$\sqrt{5}$ units 	$\sqrt{5}$ units 
b	$\sqrt{10}$ units 	$\sqrt{10}$ units 
c	$\sqrt{41}$ units 	$\sqrt{41}$ units 

10 Synthesis

Describe the relationship between the side length and the area of a square using the term *square root*.

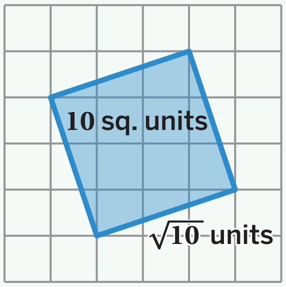
Responses vary. **The square root of a square's area will give you the exact value of the square's side length.**



13 Summary 8.02

There is a known relationship between the area of any square and its side length. The exact value of the side length of a square can be written as the **square root** of its area.

For example, $\sqrt{10}$ is the exact value for the side length of a square with an area of 10 square units.



Square roots can be rational or irrational. $\sqrt{9}$ is a **rational number** because 3 can be written as a fraction of integers, $\frac{3}{1}$.

Repeating and terminating decimals are also rational numbers. Some examples are 2.25, $1.333 \ldots$, and $3.000 \ldots$. If a decimal ends in repeating zeros, it is called terminating.

Irrational numbers include decimals that never repeat or terminate. $\sqrt{2}$ and $\sqrt{10}$ are irrational numbers because when written as a decimal it never repeats.

irrational number A number that is not rational; it cannot be written as a fraction of two non-zero integers.

rational number A number that can be written as a fraction of two non-zero integers.

square root A positive number that can be squared to get n. Written as $\sqrt{n}$, the square root is also the side length of a square with an area of n.

Practice
8.02

Name: .... Date: .... Period: ....

1. Square A has an area of 81 square feet. Select *all* the expressions that are equal to the side length of this square (in feet).

☐ **A.** 3
☐ **B.** $\frac{81}{2}$
☑ **C.** $\sqrt{81}$
☐ **D.** $\sqrt{9}$
☑ **E.** 9

Problems 2–5: Here are the areas of different squares. Determine the side length for each square.

2. Area: 37 square units

$\sqrt{37}$ units (or equivalent)

3. Area: $\frac{100}{9}$ square units

$\frac{10}{3}$ units (or equivalent)

4. Area: $\frac{2}{5}$ square units

$\sqrt{\frac{2}{5}}$ units (or equivalent)

5. Area: 0.0001 square units

0.01 units (or equivalent)

Problems 6–8: Here is some information about three squares. Determine which side length matches each square.

- Square A is smaller than square B.
- Square B is smaller than square C.
- The three squares' side lengths are $\sqrt{26}$, 4.2, and $\sqrt{11}$ units.

6. Square A

$\sqrt{11}$ units

7. Square B

4.2 units

8. Square C

$\sqrt{26}$ units

Problems 9–11: Here are the side lengths of different squares. Determine the area of each square.

9. Side length: $\frac{1}{5}$ cm

$\frac{1}{25}$ square centimeters (or equivalent)

10. Side length: $\frac{3}{7}$ units

$\frac{9}{49}$ square units (or equivalent)

11. Side length: 0.1 meter

0.01 square meters (or equivalent)

Practice
8.02

Name: ______________________ Date: __________ Period: __________

Spiral Review

12. Which expression is equivalent to 7^8.

A. $7^2 \cdot 7^4$ **B.** $7^7 + 7^1$ [✓ **C.** $\frac{7^{12}}{7^4}$] **D.** $7^{10} - 7^2$

13. Which expression is equivalent to $(12^3)(12^{-8})$?

A. 12^{-24} **B.** -12^5 [✓ **C.** $\frac{1}{12^5}$] **D.** $\frac{1}{12^{-5}}$

Problems 14–15: Here is a table showing the areas of six large countries.

Country	Area (sq. km)
Russia	$1.71 \cdot 10^7$
Canada	$9.98 \cdot 10^6$
China	$9.60 \cdot 10^6$
United States	$9.53 \cdot 10^6$
Brazil	$8.52 \cdot 10^6$
India	$3.29 \cdot 10^6$

14. About how many times greater is the area of Russia than the area of Canada?

About 1.7 times greater

15. The Eastern Hemisphere countries on this list are Russia, China, and India. The Western Hemisphere countries are Canada, the United States, and Brazil. Which has the greater total area?

[✓ **A.** The three Eastern Hemisphere countries]

B. The three Western Hemisphere countries

Explain your thinking.

***Explanations vary.* The Eastern Hemisphere countries' areas sum to 2.999×10^7 square kilometers whereas the Western Hemisphere countries' areas sum to 2.803×10^7 square kilometers.**

16. Select *all* the expressions that are equivalent to 10^{-6}.

☐ **A.** $-\frac{1}{1000000}$

☑ **B.** $\left(\frac{1}{10}\right)^6$

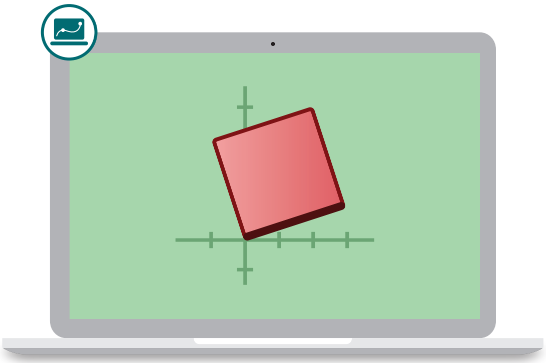
☐ **C.** $10^8 \cdot 10^{-2}$

☑ **D.** $\frac{1}{10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10}$

☑ **E.** $\frac{1}{10^6}$

Name: Date: Period:

Shape, Number, and Expressions 8.EE.2, 8.NS.2, SMP.1, SMP.7

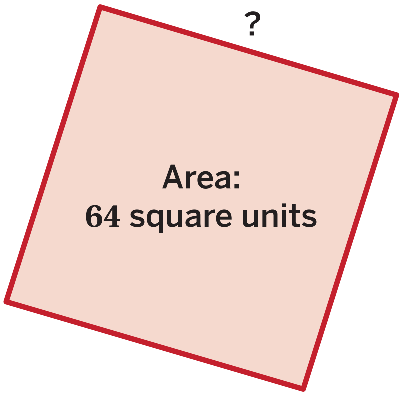


Between Squares

Let's approximate the value of square roots.

Warm-Up

1 Select *one* correct expression for the side length of this square.



☐ **A.** $\frac{64}{2}$

☑ **B.** $\sqrt{64}$

☑ **C.** 8

☐ **D.** $\sqrt{8}$

☐ **E.** 4

☐ **F.** $\frac{64}{4}$

Explain your thinking.

Explanations vary.

- **$\sqrt{64}$. The side length of a square is the square root of the area.**
- **8. You can determine the area of a square by squaring the side length, and 8^2 is 64 square units.**

Activity 1

Name: .... Date: .... Period: ....

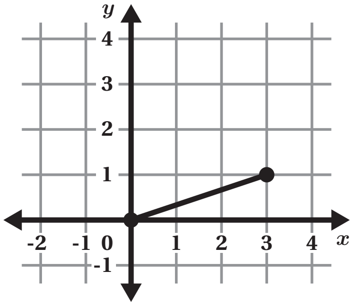
Squaring Lines

2 What do you think is the length of this segment?

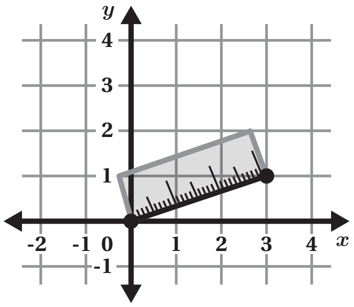
Use the ruler, circle, or square to help with your thinking.

Responses vary.

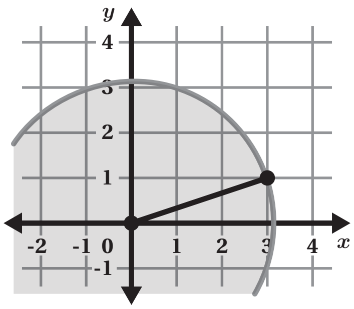
- **$\sqrt{10}$ units**
- **3.2 units**



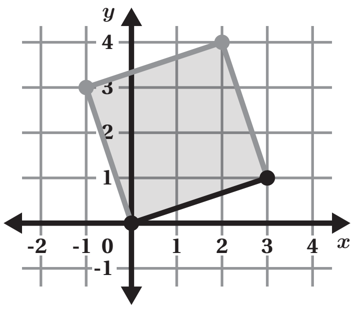
Ruler



Circle



Square



3 Ava says that the segment length is $\sqrt{10}$ units because the area of the square is 10 square units.

Rebecca says that the segment length is about 3.2 units because that's the approximate length of the circle's radius.

Discuss:

- How are Ava's and Rebecca's strategies alike? How are they different?
- What is helpful about each strategy?

Responses vary.

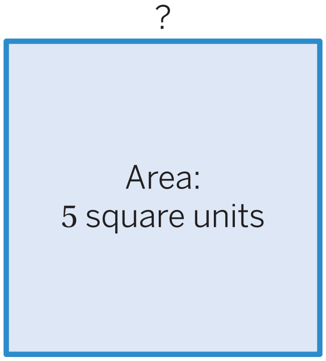
- **Ava's and Rebecca's strategies are alike in that they both use shapes to find the length of the line segment. Their strategies are different because Ava used the area of a square to determine the exact length of the segment, and Rebecca used the radius of a circle to approximate the length of the segment.**
- **Ava's strategy is helpful for determining the exact value of a line segment. Rebecca's strategy is helpful for determining the approximate value of a line segment.**

Activity
2

Name: .... Date: .... Period: ....

Using Squares to Estimate

4 Here is a square with an area of 5.



a What is the exact side length of the square?

$\sqrt{5}$ units

b What two numbers does side length fall between? Circle one.

Between 0 and 1 Between 1 and 2 [✓ Between 2 and 3]

Explain your thinking.

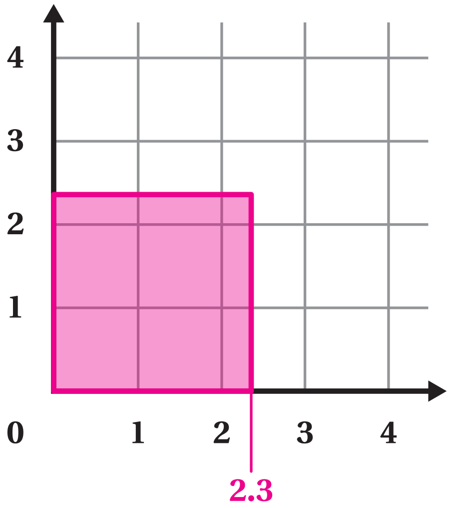
2^2 is 4 and 3^2 is 9. Because the area of the square is 5 and it falls between 4 and 9, then $\sqrt{5}$ is going to be between 2 and 3.

5 **a** Sketch a square to help you estimate $\sqrt{5}$.

Sample response shown on grid.

b Do you think $\sqrt{5}$ is greater than or less than 2.5? Explain your thinking.

***Responses vary.* When the side length of a square is 2.5 units, its area is 6.25 square units. For the area of a square to be 5 square units (or $\sqrt{5} \cdot \sqrt{5}$), its side length must be less than 2.5 units.**



Activity
3

Name: ______ Date: ______ Period: ______

Closest Decimal Approximation

6 $\sqrt{5}$ is a number that equals 5 when squared.

Use a calculator to approximate the value of $\sqrt{5}$ as closely as you can. Record each guess, n, and its square, n^2, in the table.

***Responses vary.* The exact value of n is between 2.236 and 2.237.**

n	n^2
2.0	$(2.0)^2 = 4.00$
2.2	**4.84**
2.3	**5.29**
2.25	**5.0625**
2.23	**4.9729**

7 Describe your strategy for finding a decimal approximation that is as close as possible to $\sqrt{5}$.

***Responses vary.* I started with 2.1, but its square was too low. So then I tried 2.4, but its square was too high. I kept using each approximation to make a better approximation the next time.**

8 Use a calculator to approximate the value of $\sqrt{30}$ as closely as you can. Record each guess, n, and its square, n^2, in the table.

***Responses vary.* The exact value of n is between 5.477 and 5.478.**

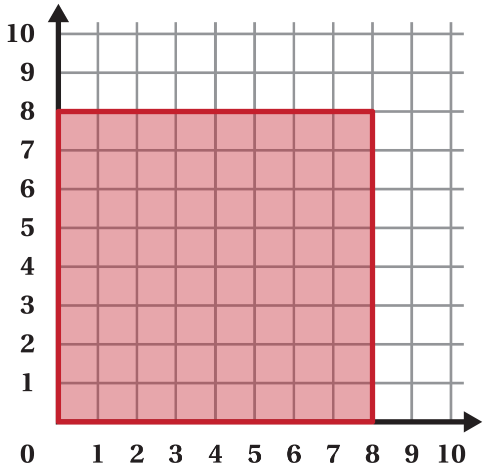
n	n^2
5.2	$(5.2)^2 = 27.04$
5.3	**28.09**
5.4	**29.16**
5.5	**30.25**
5.45	**29.7025**

9 Synthesis

Discuss: What are some strategies for approximating a square root, such as $\sqrt{75}$?

Responses vary.

- **I could make a square with one side along the x-axis and an area of approximately 75 square units. Then I could use the x-axis scale to estimate the square's side length, which would approximate $\sqrt{75}$.**
- **I could use a calculator to square different approximations of the square root, trying to get as close to the target as possible. With $\sqrt{75}$, I might start with 8.5. This is because $8^2 = 64$ and $9^2 = 81$, so $\sqrt{75}$ must be somewhere in between. Then depending on whether 8.5^2 is greater than or less than 75, I would revise my estimate and try again.**



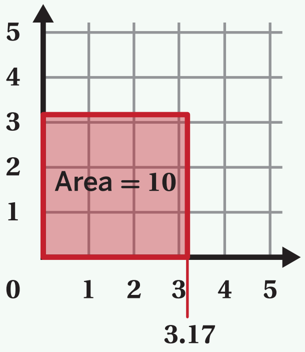
n	n^2
8.0	**64**

12 Summary 8.03

You can use several strategies to approximate the values of square roots. One strategy is to use the areas of squares. The side length of a square is equal to the square root of its area. Another strategy is to create a table of values for n and determine n^2. Remember that $(\sqrt{n})^2 = n$. Below is a description of how each strategy can be used to approximate $\sqrt{10}$.

Using Squares

- Create a square that has an area equal to about 10 square units.
- Approximate the side length of the square created.



Using a Table

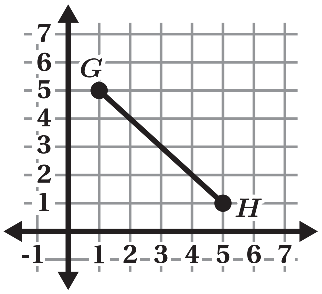
- Create a table of decimal value guesses for n.
- Calculate n^2 for each guess of n.
- The closer n^2 is to 10, the better that value of n is as an approximation for $\sqrt{10}$.

n	n^2
3.1	9.61
3.16	9.9856
3.17	10.0489
3.165	10.017225

Practice
8.03

Name: .... Date: .... Period: ....

Problems 1–2: Here is the line segment GH. Each grid square represents 1 square unit. Use the ruler, circle, or square if they help with your thinking.

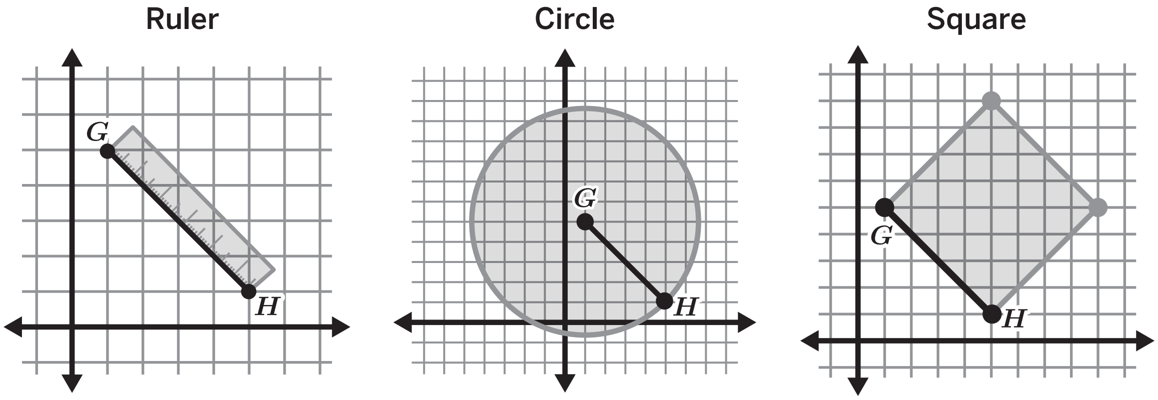


1. Determine the approximate length of GH.

Responses between 5 and 6 are considered correct.

2. Determine the exact length of GH.

$\sqrt{32}$ units



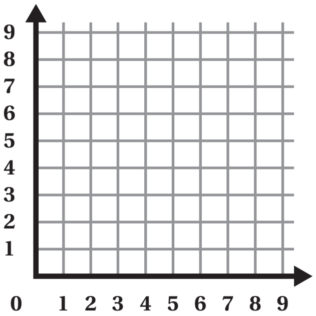
3. Determine the value of $\sqrt{16}$.

4

Problems 4–5: Estimate each square root. Draw a square if it helps with your thinking.

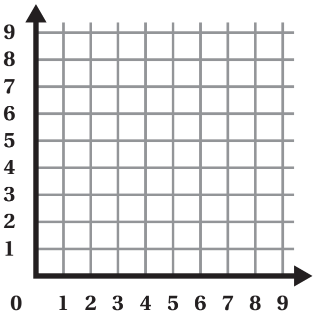
4. $\sqrt{35}$

Responses between 5 and 6 are considered correct.



5. $\sqrt{66}$

Responses between 8 and 9 are considered correct.



Problems 6–7: Determine which two whole numbers each square root is between.

6. $\sqrt{7}$

Between 2 and 3

7. $\sqrt{31}$

Between 5 and 6

Practice
8.03

Name: ______________________ Date: __________ Period: __________

8. Here is a list of values ordered from least to greatest. One value is unknown. Which could be the unknown value?

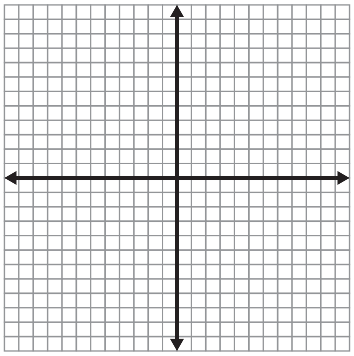
$2.5, \frac{19}{3}, \sqrt{51},$ ______

[✓ **A.**] $(3.1)^2$ **B.** $\frac{15}{4}$ **C.** 6.89 **D.** 2.1

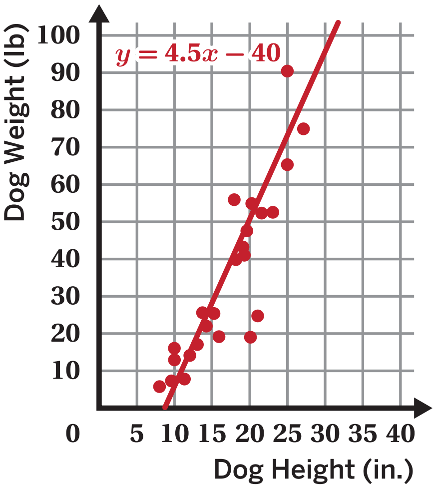
Spiral Review

9. Identify two points on a line that would create a slope of 6. Use the coordinate plane if it helps with your thinking.

Responses vary. **The points $(1, 1)$ and $(2, 7)$.**



Problems 10–12: Here is a scatter plot that shows the heights and weights of 25 dogs, as well as a linear model for the same situation and its equation.



10. What does the slope of the linear model mean in this situation?

For every 1 inch increase in height, the weight is predicted to increase by 4.5 pounds.

11. Based on the model, what will be the weight of a dog that is 25 inches tall?

72.5 pounds

12. Does the data show a positive or negative association?

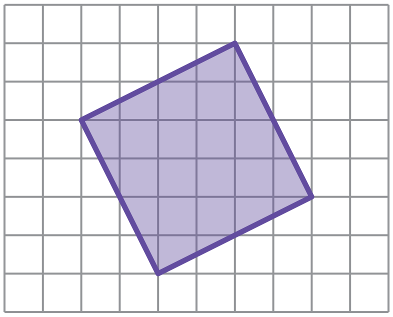
Positive association

Explain your thinking.

Explanations vary. **As the dog height increases, the weight also tends to increase.**

13. What is the area of this square?

20 square units



Unit 8
Lesson
4

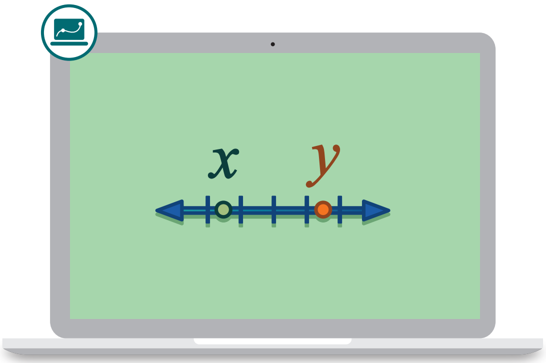
Name: ______ Date: ______ Period: ______

Shape, Number, and Expressions

8.EE.2, 8.NS.2, SMP.2, SMP.3, SMP.6

Root Down

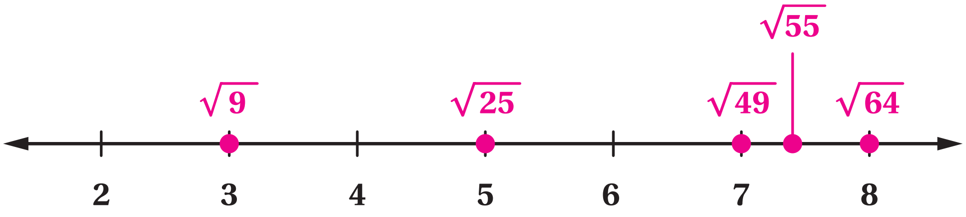
Let's estimate the value of square roots and represent them on a number line.



Warm-Up

1 Plot these values on the number line.

$\sqrt{49}$	$\sqrt{64}$	$\sqrt{9}$	$\sqrt{25}$	$\sqrt{55}$



2 9, 25, 49, and 64 are **perfect squares**. 55 is not.

What do you think a perfect square is?

***Responses vary.* A perfect square is a number that is the square of an integer. For example, 9 is a perfect square because $3 \cdot 3 = 9$, but 55 is not a perfect square because there is no square of an integer that equals 55.**

Activity
1

Name: .... Date: .... Period: ....

Between Whole Numbers

3 Match each value to the whole numbers it is between. Two values will not have a match. Note: The numbers x, y, and z are positive numbers.

The value of x when $x^2 = 50$	The value of y when $y^2 = 20$	The value of z when $z^2 = 80$	$\sqrt{24}$
$\sqrt{62}$	$\sqrt{60}$	$\sqrt{17}$	$\sqrt{15}$

Between 4 and 5	Between 7 and 8	No Match
The value of y when $y^2 = 20$ $\sqrt{24}$ $\sqrt{17}$	$\sqrt{62}$ $\sqrt{60}$ **The value of x when $x^2 = 50$**	**The value of z when $z^2 = 80$** $\sqrt{15}$

4 Esi thinks that the description *The value of z when $z^2 = 80$* doesn't belong in either category.

Between 7 and 8 **Between 4 and 5**

The value of z when $z^2 = 80$

a Which two whole numbers is the value of z between?

A. 4 and 5

B. 6 and 7

C. 7 and 8

[✓ **D.**] 8 and 9

b Of those two numbers, which would z be closer to?

9

Explain your thinking.

Explanations vary. **Since $9^2 = 81$, $\sqrt{80}$ is slightly less than 9.**

Name: .... Date: .... Period: ....

Between Whole Numbers (continued)

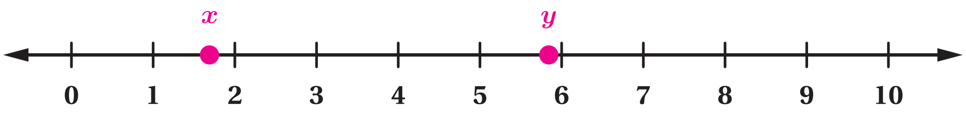
5 Order the numbers from *least* to *greatest*.

$\sqrt{99}$	$\sqrt{75}$	9	9.5	10
$\sqrt{75}$	9	9.5	$\sqrt{99}$	10

Least **Greatest**

6 The numbers x and y are positive. $x^2 = 3$ and $y^2 = 35$.

a Plot x and y on the number line.



b **Discuss:** How did you decide where to plot each point?

Responses vary. I know $\sqrt{4} = 2$, and because 3 is less than 4, x should be slightly less than 2. I know $\sqrt{36} = 6$, and because 35 is less than 36, y should be slightly less than 6.

Activity
2

Name: ____________________ Date: __________ Period: __________

Challenge Creator

7 Use blank paper to create your own challenge. ***Responses vary.***

a **Make It!** On the paper, write five numbers in a random order. Include at least one square root.

b **Solve It!** On this page, order the numbers from *least* to *greatest.*

My Challenge

Least **Greatest**

c **Swap It!** Swap your challenge on the blank paper with one or more partners. For each partner's challenge, order the numbers from *least* to *greatest*.

Partner 1's Challenge

Least **Greatest**

Partner 2's Challenge

Least **Greatest**

Partner 3's Challenge

Least **Greatest**

Partner 4's Challenge

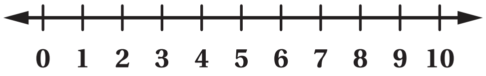
Least **Greatest**

8 Synthesis

Discuss: What are some strategies for plotting square roots on a number line?

$\sqrt{25}$ $\sqrt{36}$ $\sqrt{31}$ $\sqrt{40}$

Use the number line and examples if they help to show your thinking.



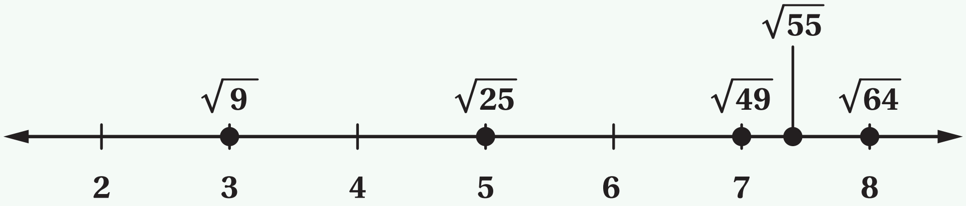
Responses vary. **To figure out where to plot a square root on a number line, determine the two whole numbers it is between. For example, $\sqrt{40}$ is between 6 and 7 because $\sqrt{36} = 6$, $\sqrt{49} = 7$, and 40 is between 36 and 49.**

11 Summary 8.04

You can represent a square root on a number line. We write a solution to an equation, such as $x^2 = 3$, using square root notation. The positive solution to this equation is $x = \sqrt{3}$.

You can approximate a square root on a number line by observing the whole numbers around it.

For example, you can determine that $\sqrt{55}$ is between 7 and 8 because $7^2 = 49$ and $8^2 = 64$, and 55 is between 49 and 64. More precisely, $\sqrt{55}$ should be plotted slightly left of 7.5 since it is closer to 7 than 8.



perfect square The square of an integer.

Practice
8.04

Name: .... Date: .... Period: ....

1. The number z is positive. Determine the exact value of z if $z^2 = 60$.

A. 30 **B.** $\sqrt{7.75}$ **C.** 7.75 [✓ **D.**] $\sqrt{60}$

2. Which statement *best* describes the value of $\sqrt{41}$?

[✓ **A.**] The value of $\sqrt{41}$ is between 6 and 6.5.

B. The value of $\sqrt{41}$ is between 6.5 and 7.

C. The value of $\sqrt{41}$ is between 7 and 7.5.

D. The value of $\sqrt{41}$ is between 7.5 and 8.

3. Write two square roots that are between 7 and 8.

***Responses vary.* $\sqrt{50}$ and $\sqrt{61}$**

4. Explain how you know that $\sqrt{30}$ is between 5 and 6.

***Responses vary.* $\sqrt{25} = 5$ and $\sqrt{36} = 6$, and $\sqrt{30}$ is between $\sqrt{25}$ and $\sqrt{36}$.**

5. Explain how you know that $\sqrt{37}$ is a little more than 6.

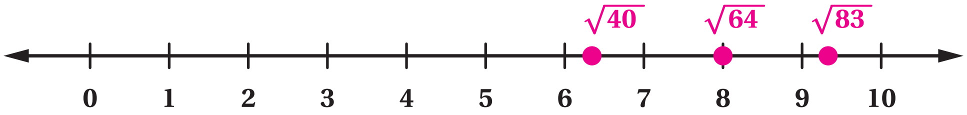
***Responses vary.* $\sqrt{36}$ is exactly 6, and $\sqrt{37}$ is a little more than that.**

6. Explain how you know that $\sqrt{95}$ is a little less than 10.

***Responses vary.* $\sqrt{100}$ is exactly 10, and $\sqrt{95}$ is a little less than that.**

7. Estimate each number and plot it on the number line.

$\sqrt{83}$ $\sqrt{40}$ $\sqrt{64}$



Practice
8.04

Name: .... Date: .... Period: ....

8. Select *all the* numbers that are greater than 10 and less than 11.

☑ **A.** $\sqrt{120}$ ☐ **B.** $\sqrt{122}$ ☐ **C.** $\sqrt{130}$

☐ **D.** $\sqrt{95}$ ☑ **E.** $\sqrt{105}$

9. Order the numbers from *least* to *greatest*.

$\sqrt{83}$	8.5	$\frac{75}{9}$	$\sqrt{67}$	9
$\sqrt{67}$	$\frac{75}{9}$	8.5	9	$\sqrt{83}$

Least **Greatest**

10. Fill in each blank using the digits 1 to 9 only once to make the inequality true.

$\square < \sqrt{\square\square} < \square$

Responses vary. $5 < \sqrt{32} < 6$

Spiral Review

Problems 11–12: Evaluate each expression. Write your answer in scientific notation.

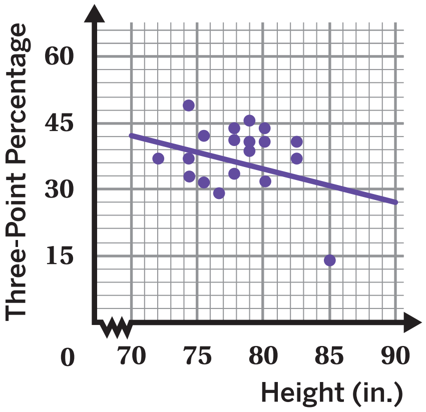
11. $(2 \cdot 10^3)(3.4 \cdot 10^{11})$

$6.8 \cdot 10^{14}$

12. $\frac{4.6 \cdot 10^3}{2 \cdot 10^5}$

$2.3 \cdot 10^{-2}$

Problems 13–14: This scatter plot shows the heights (in inches) and the three-point percentages for different basketball players last season.



13. Predict the three-point percentage for a player who is 70 inches tall.

Responses between 42% and 44% are considered correct.

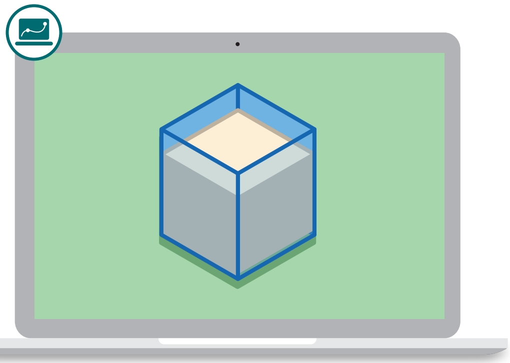
14. Identify a data point that appears to be an outlier.

$(85, 14)$

Unit 8
Lesson
5

Name: .... Date: .... Period: ....

Shape, Number, and Expressions 8.EE.2, 8.NS.2, SMP.2, SMP.3, SMP.6



Filling Cubes

Let's explore the relationship between the edge length and the volume of a cube.

Warm-Up

1 Order each value from *least* to *greatest*. Note: Let a, b, c, and d be positive numbers.

a when $a^2 = 9$	b when $b^3 = 8$	c when $c^2 = 8$	d when $d^3 = 9$

b when $b^3 = 8$	**d when $d^3 = 9$**	**c when $c^2 = 8$**	**a when $a^2 = 9$**

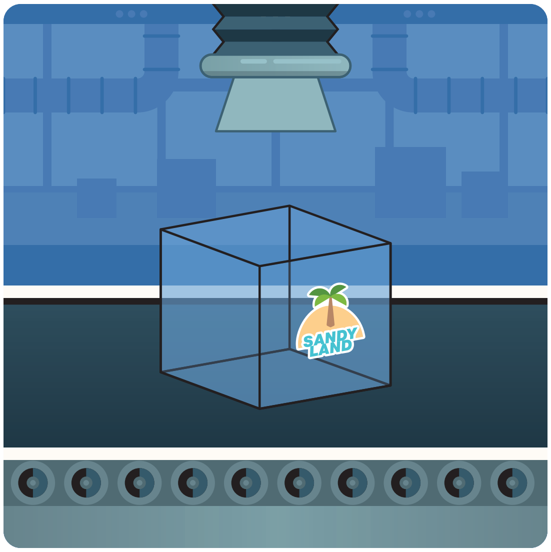
Least **Greatest**

Activity
1

Name: .... Date: .... Period: ....

Filling Cubes

Your job is to make sure the right amount of sand ends up in each cube. Use a calculator if it helps with your thinking.



2 This cube has an edge length of 6 inches.

How much sand is needed to fill it?

Edge Length (in.)	Amount of Sand (cu. in.)
6	216

3 Four new orders just came in. Complete the table for each order.

Edge Length (in.)	Amount of Sand (cu. in.)
3	27
2.1	9.261
4	64
5	125

4 Describe a strategy you used to find the unknown edge lengths.

Responses vary. To determine the edge lengths, I thought about what number raised to the third power would result in that amount of sand. For example, to determine the edge length of a box with 64 cubic inches, I know that $4^3 = 64$, so the edge length would be 4 inches.

Activity
1

Name: ______________________ Date: __________ Period: __________

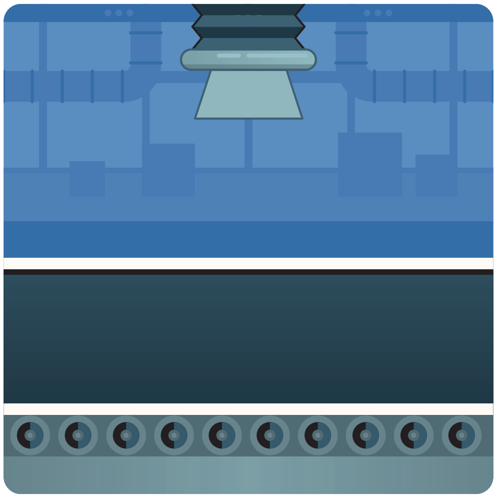
Filling Cubes (continued)

5 A customer wants a cube filled with 100 cubic inches of sand.

Let's try to find the *exact* edge length of this cube.

Choose an edge length and use your calculator to determine how much sand will fill that cube. Keep revising your estimate to get as close to the target as possible.

***Responses vary*. The exact edge length is between 4.641 and 4.642 inches.**



Edge Length (in.)	Amount of Sand (cu. in.)
4.5	$(4.5)^3 = 91.125$
4.7	**103.823**
4.65	**100.544625**
4.64	**99.897344**
4.642	**100.026577288**

6 The equation $x^3 = 100$ can help you determine the edge length of a cube that holds 100 cubic inches of sand. The exact solution to this equation is a **cube root**: $x = \sqrt[3]{100}$.

a Enter $\sqrt[3]{100}$ on your calculator to see its approximate value. $\sqrt[3]{100} \approx 4.64159$

b **Discuss:** What is the relationship between the edge length and the volume of a cube?

***Responses vary*. The edge length of a cube is equal to the cube root of its volume.**

7 Determine the *exact* unknown value for each cube or **perfect cube**.

Edge Length (in.)	Amount of Sand (cu. in.)
$\sqrt[3]{200}$	200
$\sqrt[3]{150}$	150
$\sqrt[3]{91.125}$ **or 4.5**	91.125
$\sqrt[3]{42}$	**42**

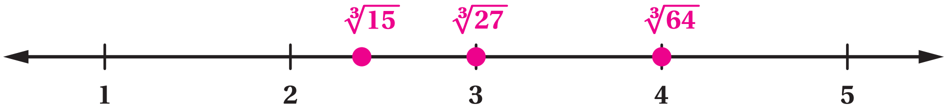
Name: ________ Date: ________ Period: ________

The Number Line

8 Here are three cube roots.

$\sqrt[3]{27}$	$\sqrt[3]{64}$	$\sqrt[3]{15}$

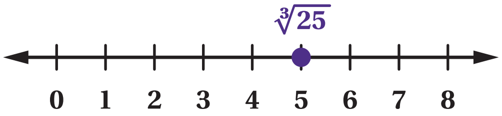
a Plot the cube roots on the number line.



b Describe your strategy for plotting $\sqrt[3]{15}$.

Responses vary. **15 is between the perfect cubes 8 and 27. Since $\sqrt[3]{8} = 2$ and $\sqrt[3]{27} = 3$, $\sqrt[3]{15}$ must be between 2 and 3.**

9 Nia incorrectly plotted $\sqrt[3]{25}$.



Discuss:

- What mistake could Nia have made?
- What question could you ask Nia to help her correct her work?

Responses vary.

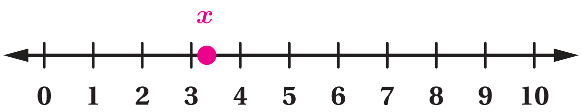
- **Nia might have been thinking about $\sqrt{25}$ instead of $\sqrt[3]{25}$.**
- **What does the cube root symbol mean?**
- **What two whole numbers is $\sqrt[3]{25}$ between?**

10 Here is an equation: $x^3 = 30$.

a Write the exact solution to the equation.

$x = \sqrt[3]{30}$

b Plot the solution on the number line.

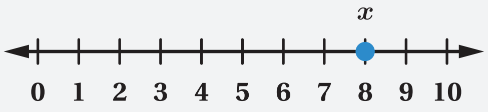
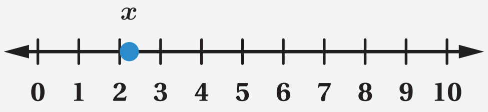
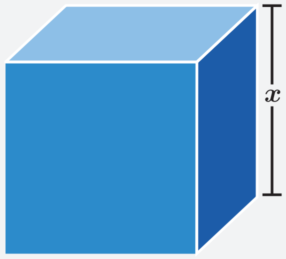


Name: .... Date: .... Period: ....

The Number Line (continued)

11 Match each equation to the visual that represents the same value of x.

$x = \sqrt{10}$	$x = \sqrt{64}$	$x = \sqrt[3]{10}$	$x = \sqrt[3]{64}$

Visual	Equation
	$x = \sqrt{64}$
	$x = \sqrt[3]{10}$
Volume: 64 cu. in. 	$x = \sqrt[3]{64}$
Area: 10 sq. in. 	$x = \sqrt{10}$

You're invited to explore more.

12 **a** If you double the edge length of a cube, what happens to the volume?

Responses vary. **If I double the edge length, the volume is multiplied by 2^3, or 8.**

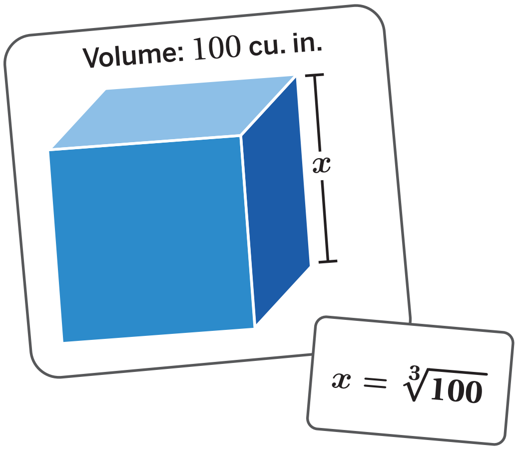
b If you double the volume of a cube, what happens to the edge length?

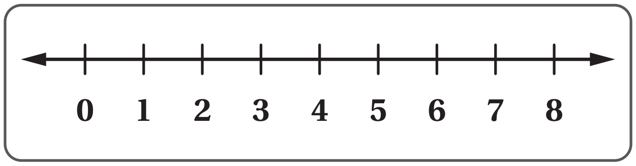
Responses vary. **If I double the volume, the edge length is multiplied by $\sqrt[3]{2}$.**

13 Synthesis

Explain a strategy for determining where to plot a cube root on the number line.

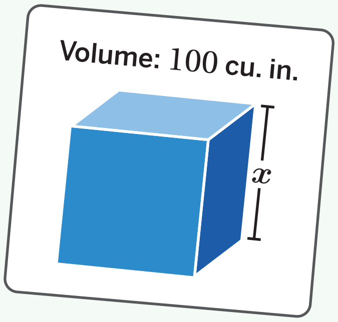
Responses vary. To determine where to plot a cube root on the number line, start by identifying the whole numbers around it. For example, $\sqrt[3]{100}$ is between 4 and 5 because $4^3 = 64$ and $5^3 = 125$. 100 is a little bit closer to 125 than 64, so $\sqrt[3]{100}$ should be just over halfway between 4 and 5.



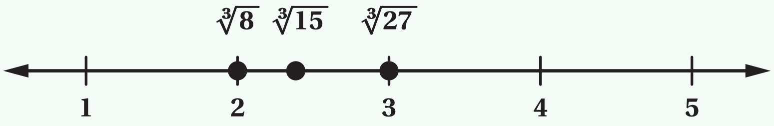


16 Summary 8.05

A **cube root** describes the edge length of a cube given its volume. For the cube shown with a volume of 100 cubic inches, the equation $x^3 = 100$ can help you find its edge length. Its exact solution would be represented as $x = \sqrt[3]{100}$.



We can approximate a cube root on a number line by observing the whole numbers around it. For example, you can determine that $\sqrt[3]{15}$ is between 2 and 3 because $2^3 = 8$ and $3^3 = 27$, and 15 is between 8 and 27. 8 and 27 are **perfect cubes** because they are both the cube of an integer.



cube root The cube root of a number n (written as $\sqrt[3]{n}$) is the number that can be cubed to get n. The cube root is also the edge length of a cube with a volume of n.

perfect cube The cube of an integer is called a perfect cube.

Practice
8.05

Name: ________ Date: ________ Period: ________

1. What is the volume of each cube based on its edge length?

	Edge Length	Volume
Cube A	4 cm	**64 cu. cm**
Cube B	$\sqrt[3]{11}$ ft	**11 cu. ft**
Cube C	s units	**s^3 cu. units**

2. What is the exact edge length of each cube based on its volume?

	Edge Length	Volume
Cube D	**10 cm**	1,000 cu. cm
Cube E	**$\sqrt[3]{23}$ in.**	23 cu. in.
Cube F	**$\sqrt[3]{v}$ units**	v cu. units

Problems 3–6: Write an equivalent expression that doesn't use a cube root symbol.

3. $\sqrt[3]{1}$ $= 1$

4. $\sqrt[3]{216}$ $= 6$

5. $\sqrt[3]{\frac{27}{125}}$ $= \frac{3}{5}$

6. $\sqrt[3]{\frac{1}{64}}$ $= \frac{1}{4}$

Problems 7–10: Write an exact solution to each equation.

7. $t^2 = 64$

$t = 8$ or $t = -8$

8. $f^3 = 181$

$f = \sqrt[3]{181}$

9. $m^2 = 8$

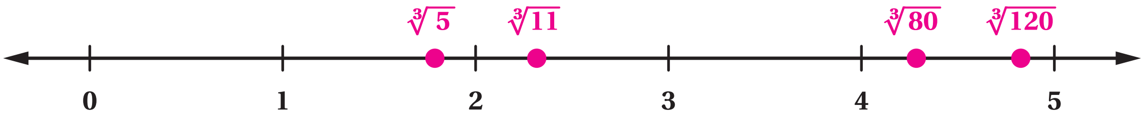
$m = \sqrt{8}$ or $m = -\sqrt{8}$

10. $c^3 = 343$

$c = 7$

11. Plot each cube root on the number line.

$\sqrt[3]{5}$ $\sqrt[3]{11}$ $\sqrt[3]{80}$ $\sqrt[3]{120}$



Practice
8.05

Name: .... Date: .... Period: ....

12. Order the following from *least* to *greatest*. Use the number line if it helps with your thinking.

$\sqrt[3]{27}$	$\sqrt[3]{530}$	$\sqrt{48}$	$\sqrt{121}$	π	$\frac{19}{2}$

$\sqrt[3]{27}$	π	$\sqrt{48}$	$\sqrt[3]{530}$	$\frac{19}{2}$	$\sqrt{121}$

Least **Greatest**

13. Cube B has a volume that is double the volume of cube A. What could be the edge lengths of each cube?

***Responses vary*. Cube A has an edge length of 2 and cube B has an edge length of $\sqrt[3]{16}$.**

Spiral Review

14. What two consecutive whole numbers does $\sqrt{40}$ lie between?

A. 4 and 5 **B.** 5 and 6 [✓ **C.**] 6 and 7 **D.** 7 and 8

15. What is the approximate value of $\sqrt{61}$ rounded to the nearest tenth?

7.8

16. Which set of ordered pairs represents a function?

A. $\{(-5, 10), (-20, 0), (-20, 25)\}$ **B.** $\{(-10, 0), (-10, 5), (-10, 25)\}$

C. $\{(-25, 0), (5, -10), (-25, 30)\}$ [✓ **D.**] $\{(-20, 0), (5, -10), (30, -25)\}$

Unit 8

Lessons 1–5

Set A

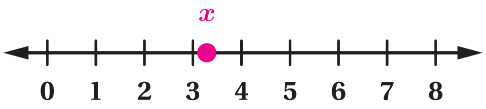
Name: ______________ Date: ______________ Period: ______________

Pythagorean Explorations | Shape, Number, and Expressions | 8.EE.2, 8.NS.2, SMP.6

Practice Day 1

Let's practice what you've learned so far in this unit!

1. $x^2 = 10$ and x is a positive number. Plot x on the number line.



2. Evaluate $\sqrt{64}$.

8

3. What is the exact side length of a cube that has a volume of 10 cubic units?

$\sqrt[3]{10}$ units

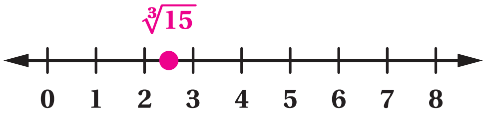
4. What is the exact side length of a square that has an area of 8 square inches?

$\sqrt{8}$ inches

5. Evaluate $\sqrt[3]{27}$.

3

6. Plot a point at the approximate location of $\sqrt[3]{15}$ on the number line.



7. The side length of a square is $\sqrt{27}$ inches. What is the area of the square?

27 square inches

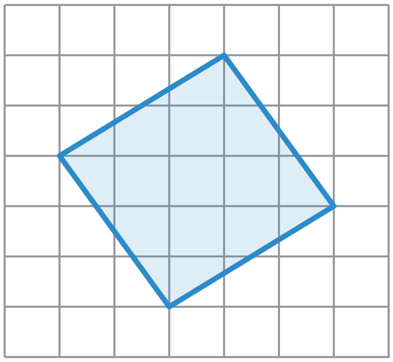
8. $\sqrt{18}$ is between which two consecutive integers?

4 and 5

Name: ____________ Date: ________ Period: ________

Practice Day 1

9. Determine the area of the square.



13 square units

10. $\sqrt[3]{50}$ is between which two consecutive integers?

3 and 4

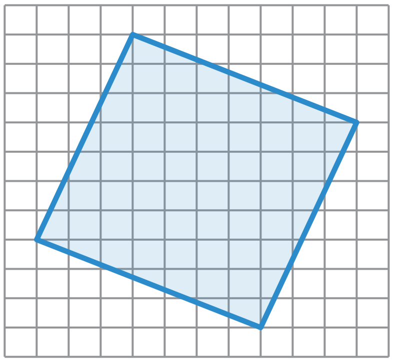
11. If $x^2 = \frac{1}{4}$ and x is a positive number, what is the value of x?

$x = \frac{1}{2}$

12. The volume of a cube is 125 cubic inches. What is the side length of the cube?

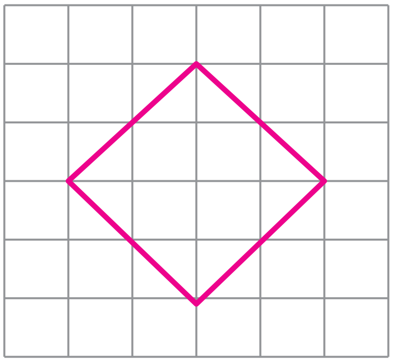
5 inches

13. Determine the exact side length of the shaded square.



$\sqrt{58}$ units

14. Draw a square that has an area of 8 square units.



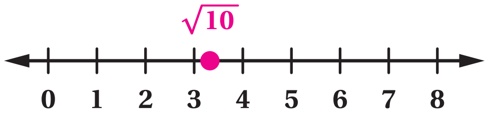
Responses vary. Sample shown on graph.

Name: ________ Date: ________ Period: ________

Practice Day 1

Let's practice what you've learned so far in this unit!

1. Plot $\sqrt{10}$ on the number line.



2. What is the side length of a square that has an area of 64 square inches?

$\sqrt{64}$ or 8 inches

3. What is the exact solution to the equation $x^3 = 10$?

$x = \sqrt[3]{10}$

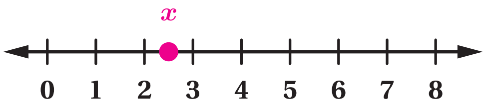
4. If x is positive and $x^2 = 8$, what is the exact value of x?

$x = \sqrt{8}$

5. Evaluate $\sqrt{9}$.

3

6. $x^3 = 15$. Plot x on the number line.



7. The side length of a cube is 3 inches. What is the volume of the cube?

27 cubic inches

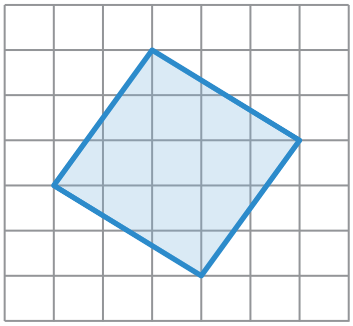
8. $\sqrt{24}$ is between which two consecutive integers?

4 and 5

Name: ______________________ Date: __________ Period: __________

Practice Day 1

9. Determine the area of the tilted square.



13 square units

10. $\sqrt[3]{60}$ is between which two consecutive integers?

3 and 4

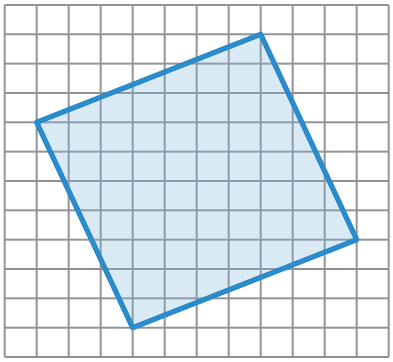
11. If $x^3 = \frac{1}{8}$, what is the value of x?

$x = \frac{1}{2}$

12. The side length of a cube is $\sqrt[3]{5}$ inches. What is the volume of the cube?

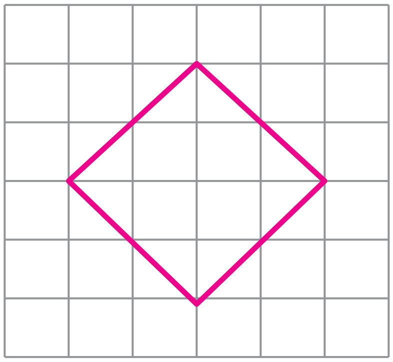
5 cubic inches

13. Determine the exact side length of the shaded square.



$\sqrt{58}$ units

14. Draw a square that has a side length of $\sqrt{8}$ units.



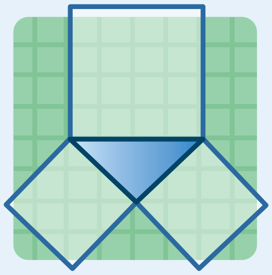
Responses vary. Sample shown on graph.

Notes:

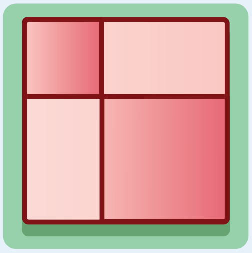
Sub-Unit

2

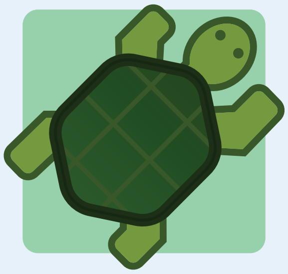
The Pythagorean Theorem



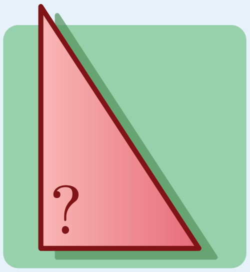
Lesson 6
The Pythagorean Theorem



Lesson 7
Pictures to Prove It

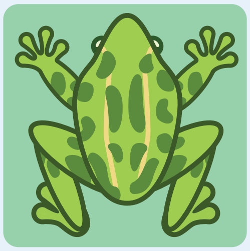


Lesson 8
Triangle-Tracing Turtle



Lesson 9
Make It Right

Lesson 10
Taco Truck



Lesson 11
Pond Hopper

Unit 8

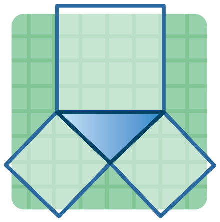
Lesson

6

Name: ________ Date: ________ Period: ________

Pythagorean Explorations Shape, Number, and Expressions 8.G.6, 8.G.7, SMP.1, SMP.3, SMP.7

The Pythagorean Theorem

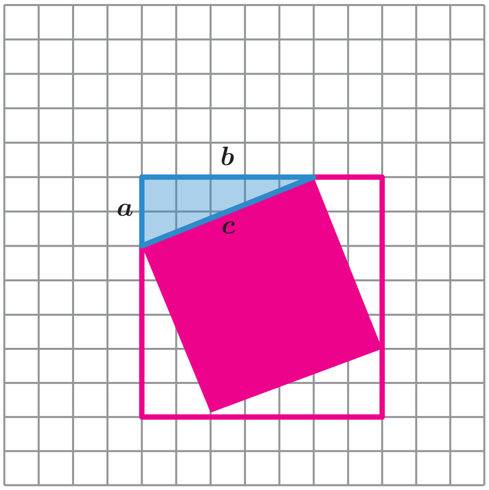


Let's explore the relationship between the squares of side lengths in triangles.

Warm-Up

1. Use any strategy to determine the value of c^2.

29 square units



Explain your thinking.

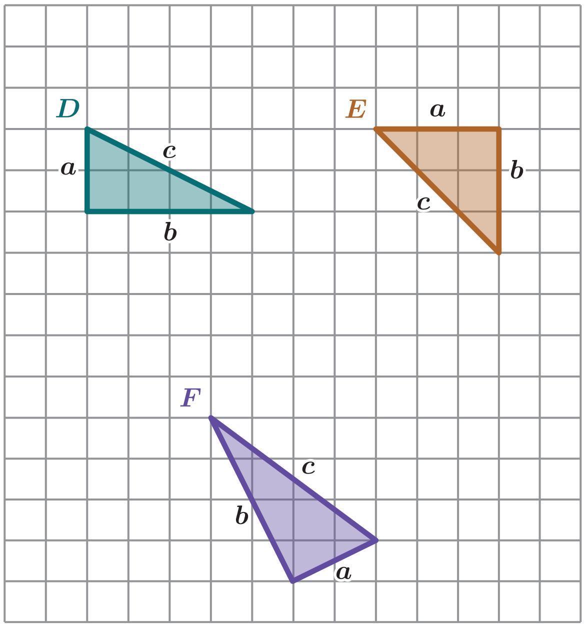
Explanations vary. I created a tilted square with side length c inside a larger square with an area of 49 square units. I then determined the area of each triangle outside the tilted square to be 5 square units. Then I subtracted the area of the four congruent triangles from the area of the large square to find the area of just the tilted square, which is the value of c^2.

Activity
1

Name: Date: Period:

Squares of Side Lengths

2. Use these triangles to complete the table.



Triangle	a^2	b^2	c^2
D	**4**	**16**	**20**
E	**9**	**9**	**18**
F	**5**	**20**	**25**

3. What do you notice? What do you wonder? ***Responses vary.*** **ELD.PI.8.6.Em, Ex, Br**

I notice . . .

- **For each of these triangles, c^2 is larger than a^2 or b^2.**
- **If you add a^2 and b^2 together, the result is equal to c^2.**

I wonder . . .

- **If $a^2 + b^2 = c^2$ is true for all triangles?**
- **What type of triangles are these?**

Name: .... Date: .... Period: ....

True for Every Triangle?

4. You will use a set of cards for this problem.

a Work with a partner to create groups where $a^2 + b^2 = c^2$ and $a^2 + b^2 \neq c^2$. Complete the table with the card numbers.

$a^2 + b^2 = c^2$	$a^2 + b^2 \neq c^2$
Cards 1, 4, and 6	Cards 2, 3, and 5

b Revisit your noticings and wonderings from Activity 1. What do you notice and wonder now?

I notice . . .

Responses vary. It looks like the triangles where $a^2 + b^2 = c^2$ are right triangles.

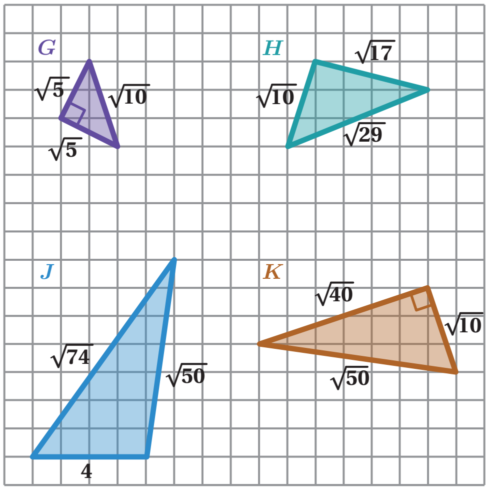
I wonder . . . **ELD.PI.8.6.Em, Ex, Br**

Responses vary. For what type of triangles is $a^2 + b^2 > c^2$?

For each triangle, let a and b represent the two shorter sides and c represent the longest side.

5. Circle one triangle where the equation $a^2 + b^2 = c^2$ is true.

[✓ Triangle G] Triangle H Triangle J [✓ Triangle K]



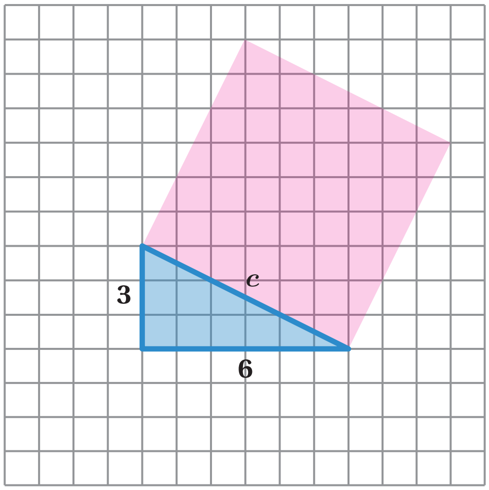
Show or explain your thinking.

Explanations vary. Triangle G. The sum of the squares of the two shorter sides is $(\sqrt{5})^2 + (\sqrt{5})^2 = 10$. This is the same measure as the square of the longest side, $(\sqrt{10})^2 = 10$, which shows that $a^2 + b^2 = c^2$ is true for this triangle. This triangle is also a right triangle, which matches what I saw in the card sort.

6. Sai says that the value of c is $\sqrt{45}$. Do you agree? Circle one.

ELD.PI.8.11.Em, Ex, Br

[✓ Agree] Disagree There's not enough information



Show or explain your thinking.

Explanations vary. I drew a square with c as one of its sides and determined its area to be 45 square units. To find the value of c, I took the square root of the area to determine $c = \sqrt{45}$.

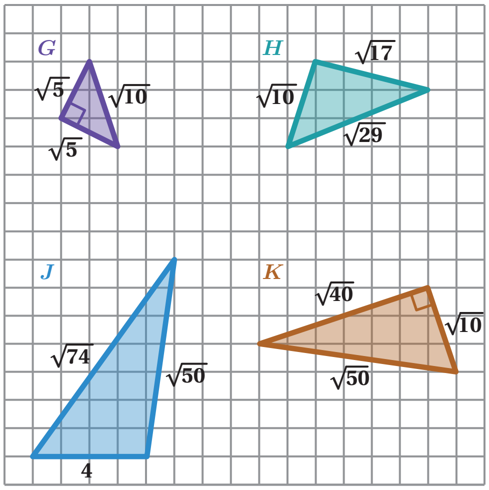
Synthesis

7. The **Pythagorean theorem** says that for right triangles, $a^2 + b^2 = c^2$. The date of the first discovery is unknown, but the Babylonians used the Pythagorean theorem over 3,500 years ago (1,000 years before Pythagoras was born).

ELD.PI.8.11.Em, Ex, Br

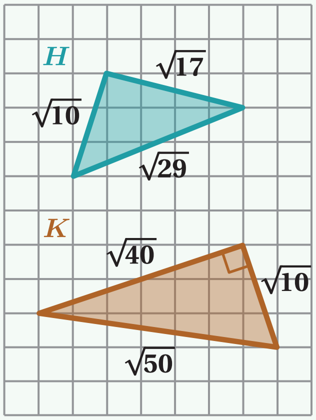
Explain the Pythagorean theorem in your own words. Use the triangles if they help with your thinking.

Responses vary. **The Pythagorean theorem tells us that for right triangles, the sum of the squares of the shorter side lengths is equal to the square of the longest side length.**



Summary 8.06

The **Pythagorean theorem** says that for right triangles, $a^2 + b^2 = c^2$, where a and b represent the lengths of the two shorter sides and c represents the length of the longest side.



For triangle H:

$(\sqrt{10})^2 + (\sqrt{17})^2 = 27$

$c^2 = (\sqrt{29})^2 = 29$

$27 \neq 29$ so $a^2 + b^2 = c^2$ is not true.

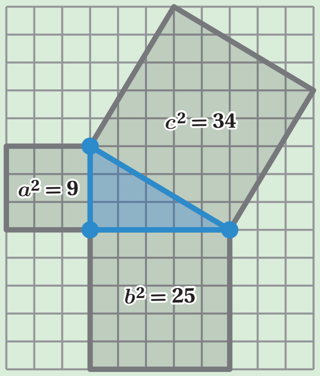
For triangle K:

$(\sqrt{10})^2 + (\sqrt{40})^2 = 50$

$c^2 = (\sqrt{50})^2 = 50$

$50 = 50$ so $a^2 + b^2 = c^2$ is true.

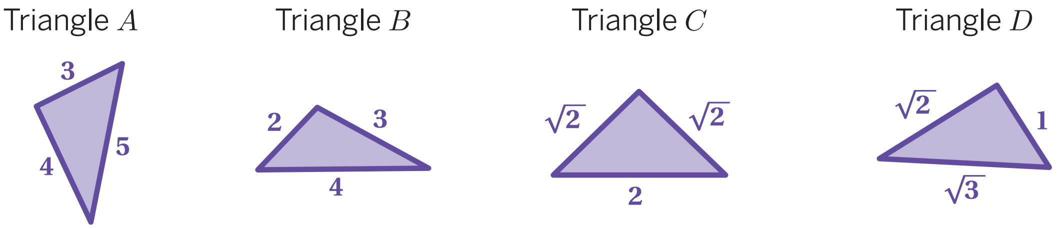
Pythagorean theorem The theorem that describes the relationship between the side lengths of a right triangle. The Pythagorean theorem says that the square of the hypotenuse is equal to the sum of the squares of the legs. We can write this as $a^2 + b^2 = c^2$.



Practice
8.06

Name: .... Date: .... Period: ....

1. For which of the following triangles is $a^2 + b^2 = c^2$ true? **Triangles A, C, and D**



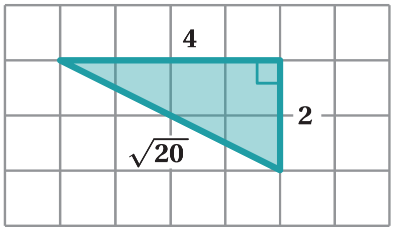
Show or explain your thinking. ***Work varies.***

Triangle A: $3^2 + 4^2 = 25$ and $5^2 = 25$

Triangle C: $(\sqrt{2})^2 + (\sqrt{2})^2 = 4$ and $2^2 = 4$

Triangle D: $1^2 + (\sqrt{2})^2 = 3$ and $(\sqrt{3})^2 = 3$

2. Select *all* the equations that represent the relationship between the side lengths.

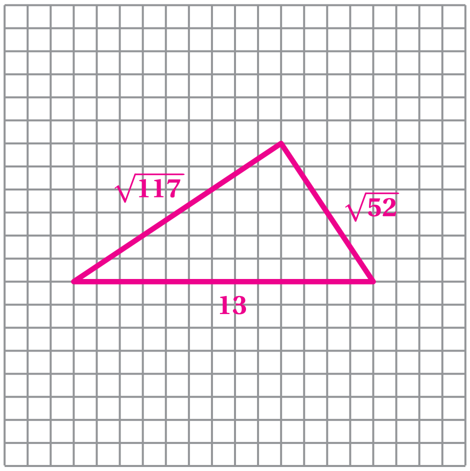


- ☐ A. $(\sqrt{20})^2 + 4^2 = 2^2$
- ☑ B. $(\sqrt{20})^2 = 4^2 + 2^2$
- ☑ C. $2^2 + 4^2 = (\sqrt{20})^2$
- ☐ D. $2^2 + (\sqrt{20})^2 = 4^2$
- ☑ E. $(\sqrt{20})^2 = 2^2 + 4^2$

3. Draw a triangle where $a^2 + b^2 = c^2$. What are its side lengths?

Responses vary.

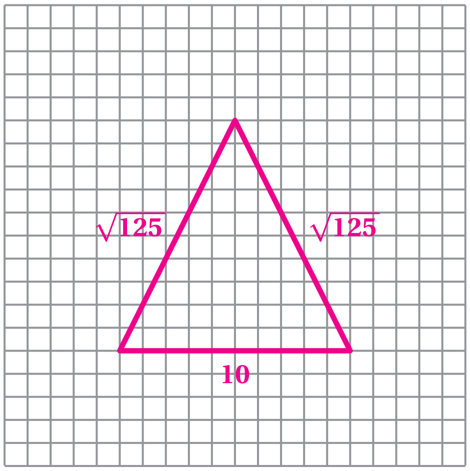
$a =$ **$\sqrt{117}$**, $b =$ **$\sqrt{52}$**, $c =$ **13**



4. Draw a triangle where $a^2 + b^2 \neq c^2$. What are its side lengths?

Responses vary.

$a =$ **$\sqrt{125}$**, $b =$ **$\sqrt{125}$**, $c =$ **10**



Practice
8.06

Name: .... Date: .... Period: ....

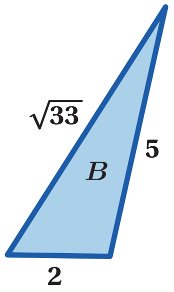
Problems 5–6: Here is a triangle.

5. Felipe claims the Pythagorean theorem is true for any triangle. Is his claim correct?

No

Explain your thinking using triangle B as an example.

Explanations vary. The Pythagorean theorem only applies to right triangles. Triangle B is an obtuse triangle, so the Pythagorean theorem does not apply.



6. Felipe wants to make triangle B a right triangle. If he keeps the same lengths for the shorter sides, 2 and 5, what would the length of the longest side be?

$\sqrt{29}$

Spiral Review

7. Order the following expressions from *least* to *greatest*.

$25 \div 10$	$250000 \div 1000$	$2.5 \div 1000$	$0.025 \div 1$

$2.5 \div 1000$	$0.025 \div 1$	$25 \div 10$	$250000 \div 1000$

Least **Greatest**

Problems 8–10: A teacher tells her students she is just over 1.5 billion seconds old.

8. Write her age in seconds using scientific notation.

1.5×10^9

9. What is a more reasonable unit of measurement for this situation?

Responses vary. Years

10. Convert the teacher's age to a new unit. Show or explain your thinking.

Responses vary. There are 31,536,000 seconds in a year. $1.5 \times 10^9 \div 31536000$ is about 47.6, so she is about 48 years old.

11. Kwasi borrows money from his older brother to buy a new game and begins to pay him back. The amount of money he owes his brother can be represented by the equation $y = -6x + 30$ where y is the amount of remaining money, in dollars, to be paid after x months. Based on the equation, what amount of money, in dollars, will Kwasi still owe after 3.5 months?

A. 6

[✓ **B.**] 9

C. 21

D. 30

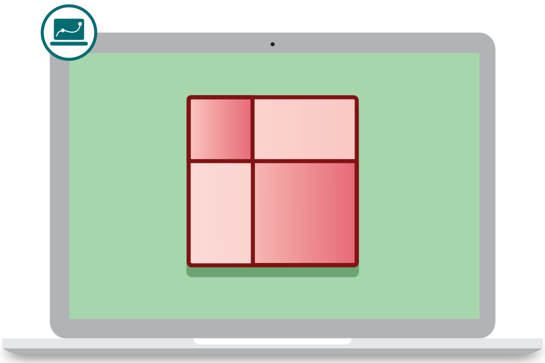
Unit 8

Lesson

7

Name: ______________________ Date: __________ Period: __________

Pythagorean Explorations   Shape, Number, and Expressions   8.G.6, 8.G.7, SMP.1, SMP.3



Pictures to Prove It

Let’s prove the Pythagorean theorem.

Warm-Up

1 Here are two figures.

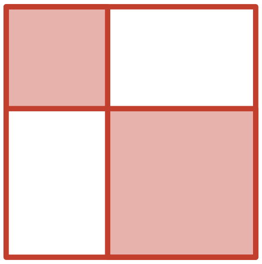


Figure A

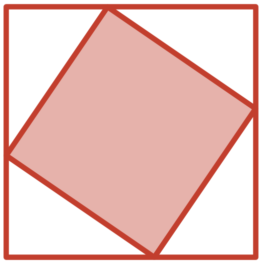


Figure B

Which one do you think has a larger *shaded* area? Circle one.

Figure A Figure B The areas are equal   I’m not sure

Explain your thinking.

Responses vary.

- **Figure A. There are two shaded squares, whereas figure B only has one.**
- **Figure B. There is one big tilted square.**
- **The areas are equal. The triangles in figure B have the same area as the non-shaded rectangles in figure A.**
- **I’m not sure. There are no measurements or units on the squares. I need more information.**

Name: .... Date: .... Period: ....

Pictures to Prove It

Note: For this lesson, assume figures that look like squares are squares.

2 Determine the total area of the shaded region in each figure. Mark the diagram if it helps with your thinking.

	Figure C	Figure D
Total Shaded Area (sq. units)	13	13

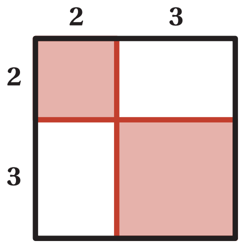


Figure C

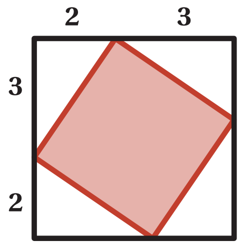


Figure D

3 Here are three different dimensions for a pair of new figures.

a Choose one of the options and circle your choice.

Option 1

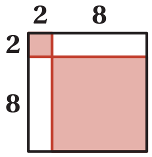


Figure E

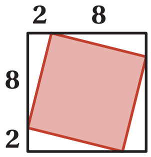


Figure F

Option 2

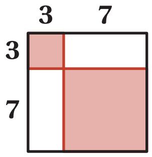


Figure E

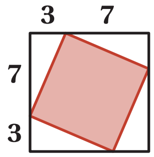


Figure F

Option 3

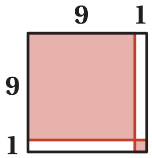


Figure E

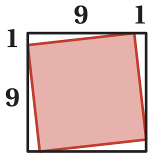


Figure F

b Determine the total area of the shaded region in each figure.

	Figure E	Figure F
Total Shaded Area (sq. units)	Option 1: 68 Option 2: 58 Option 3: 82	Option 1: 68 Option 2: 58 Option 3: 82

4 Do you think the total shaded area will *always* be equal between figures like E and F, even if their outer dimensions change?

Yes

Explain your thinking.

***Explanations vary*. The two figures have the same total area, and the two white rectangles have the same area as the four white triangles in the other figure, so the shaded areas must also be equal.**

Activity
2

Name: ______ Date: ______ Period: ______

Thinking More Generally

5-6 Let's generalize using variables instead of numbers.

Determine the area of each region in figure G and figure H using the variables a, b, or c.

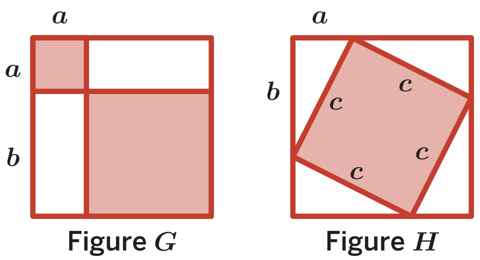


Figure G Figure H

	Figure G	Figure H
Total Area (sq. units)	$(a + b)(a + b)$	$(a + b)(a + b)$
Total Unshaded Area (sq. units)	$2ab$	$2ab$
Total Shaded Area (sq. units)	$a^2 + b^2$	c^2

7 The Pythagorean theorem states that for any *right* triangle, $a^2 + b^2 = c^2$.

How can we use these figures to prove that the Pythagorean theorem is true?

Mark the diagram if it helps to show your thinking.

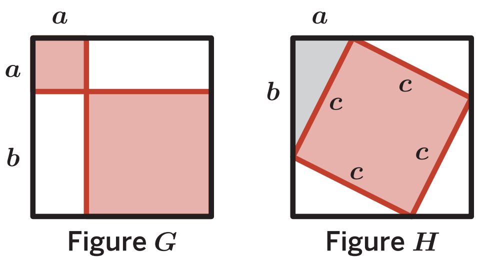


Figure G Figure H

Responses vary. **Figure H shows a shaded right triangle with legs a and b and a hypotenuse c. Figures G and H both have a total area of $(a + b)^2$, so they are equal. We also know that the unshaded areas are equal. In figure G, the unshaded area is $2ab$ and in figure H, the unshaded area is $(4)\left(\frac{1}{2}ab\right)$, which simplifies to $2ab$. This means the remaining shaded areas in each figure also have to be equal. The shaded area in figure G can be expressed as $a^2 + b^2$ and the shaded area in figure H can be expressed as c^2, so $a^2 + b^2 = c^2$.**

Name: ________________ Date: ________ Period: ________

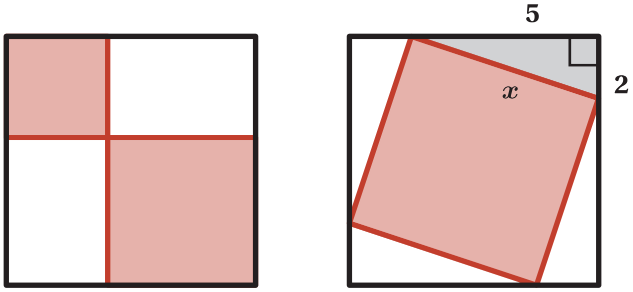
Let’s Put It to Work

Let’s put the Pythagorean theorem to work.

8 Calculate the value of x.

Draw on the diagram if it helps with your thinking.

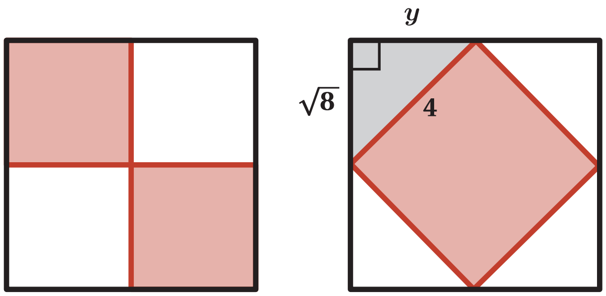
$\sqrt{29}$



9 Calculate the value of y.

Draw on the diagram if it helps with your thinking.

$\sqrt{8}$



You’re invited to explore more.

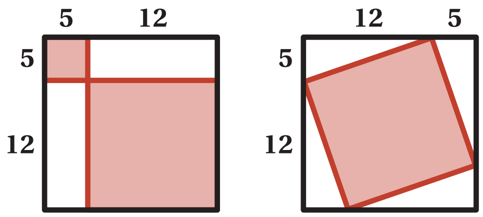
10 You will need the Activity 3 Sheet. Use scissors to cut along the dashed lines.

a Arrange the pieces in the smaller squares to fit in the large square.

b Describe what you notice about the relationship between the areas of the two smaller squares and the area of the large square.

11 Synthesis

Explain how the equation $5^2 + 12^2 = 13^2$ is related to the figures on the right and to the Pythagorean theorem.



Responses vary. In the left figure, I see $5^2 + 12^2$ as the total area of the two shaded squares. The Pythagorean theorem says that the area of the shaded square in the right figure will be equal to the sum of the shaded squares in the left figure, so its area is 169 square units, or 13^2 square units.

14 Summary 8.07

The Pythagorean theorem states that for any right triangle, $a^2 + b^2 = c^2$. There are many proofs for the Pythagorean theorem.

For example, you can draw squares on the sides of a right triangle, as shown in Figure 1.

In Figure 2 and Figure 3, each total area is equal to $(a + b)^2$.

Since the area of the right triangle is equal to $\frac{1}{2}ab$, the unshaded areas of Figures 2 and 3 are equal and each sum to $2ab$.

Since the total areas and the unshaded areas of Figures 2 and 3 are equal, then their shaded areas must be equal. This shows that $a^2 + b^2 = c^2$.

Figure 1

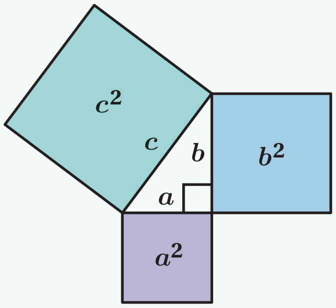


Figure 2

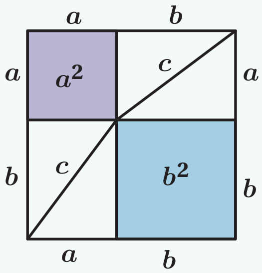
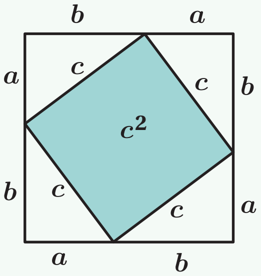


Figure 3

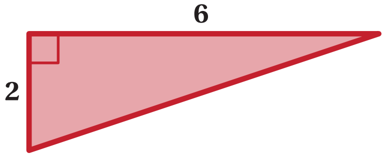


Practice
8.07

Name: ________________ Date: ________ Period: ________

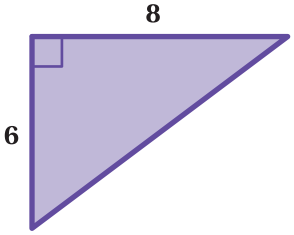
Problems 1–2: Determine the length of each unlabeled side.

1.



$\sqrt{40}$ units (or equivalent)

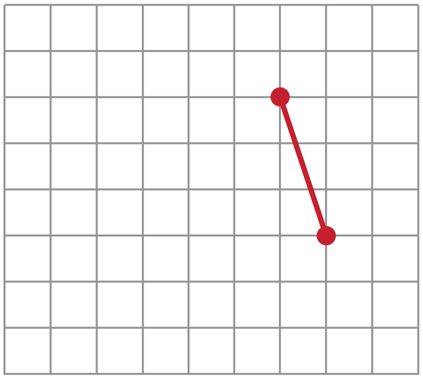
2.



10 units (or equivalent)

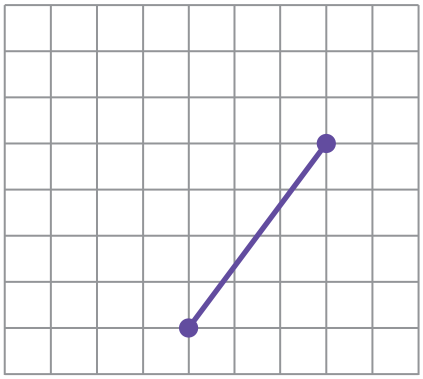
Problems 3–4: Determine the exact length of each segment. Each grid line represents one unit.

3.



$\sqrt{10}$ units

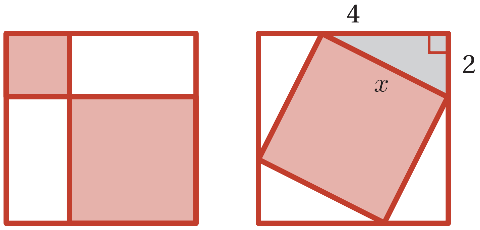
4.



5 units (or equivalent)

5. Determine the exact value of x.

$\sqrt{20}$ units



6. Determine a set of values for a, b, and c that meet the following criteria:

- Make the shaded areas of figure G and figure H equal.
- Make $a^2 + b^2 = c^2$ true.

Responses vary. $a = 5$, $b = 8$, and $c = \sqrt{89}$.

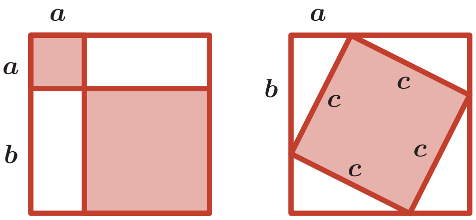


Figure G Figure H

Practice
8.07

Name: ____________________ Date: __________ Period: __________

Spiral Review

Problems 7–10: For each square root, write which two consecutive whole numbers the value is between. For example, $\sqrt{2}$ is between 1 and 2.

7. $\sqrt{10}$

3 and 4

8. $\sqrt{54}$

7 and 8

9. $\sqrt{18}$

4 and 5

10. $\sqrt{99}$

9 and 10

Problems 11–13: Rewrite each expression as a single power of 10. For example, $(10^5)^0$ can be rewritten as 10^0.

11. $10^5 \cdot 10^0$

$\mathbf{10^5}$

12. $\frac{10^9}{10^0}$

$\mathbf{10^9}$

13. Which expression is equivalent to $3^{-4} \cdot 3^9$?

A. $\frac{3^{-4}}{3^{-1}}$

B. $(3^5)^{-1}$

[✓ C.] $\frac{3^4}{3^{-1}}$

D. $(3^{-1})^5$

14. Select *all* possible values for x in the equation $x^3 = 64$.

[✓ A.] $\sqrt[3]{64}$

B. $\sqrt[3]{4}$

[✓ C.] 4

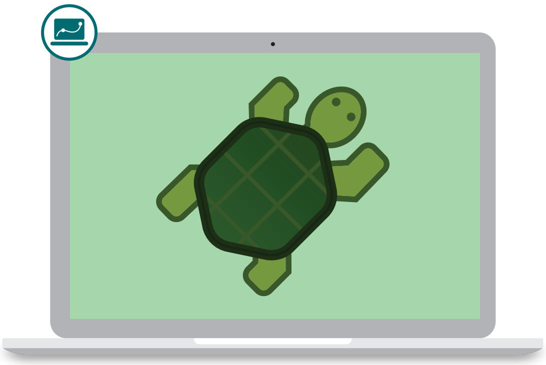
[✓ D.] $2 \cdot \sqrt[3]{8}$

E. 8

Unit 8
Lesson
8

Name: ________ Date: ________ Period: ________

Pythagorean Explorations 8.G.7, SMP.3, SMP.6

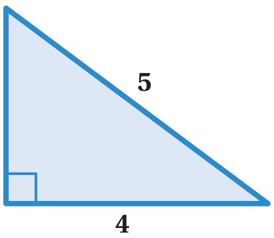


Triangle-Tracing Turtle

Let’s calculate unknown side lengths in right triangles.

Warm-Up

1 Select *all* the equations that could help you calculate the unknown side length of this triangle.



- ☑ A. $a^2 + 4^2 = 5^2$
- ☐ B. $a = \sqrt{4^2 + 5^2}$
- ☐ C. $b^2 = 5^2 + 4^2$
- ☑ D. $4^2 + b^2 = 5^2$
- ☑ E. $b = \sqrt{5^2 - 4^2}$

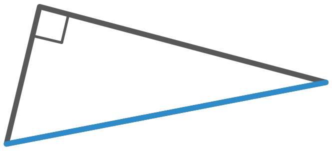
Name: .... Date: .... Period: ....

Hypotenuse

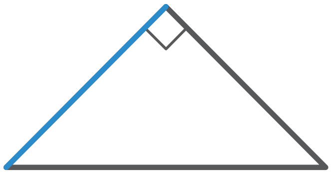
2 The **hypotenuse** is the side of a right triangle that is opposite the right angle. The **legs** of a right triangle are the sides that make the right angle.

Select *all* the triangles where a hypotenuse is highlighted.

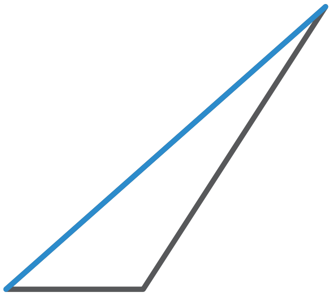
☑ **A.**



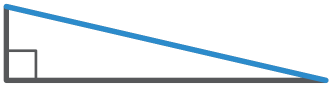
☐ **B.**



☐ **C.**



☑ **D.**



3 Melissa incorrectly thinks that triangles A, C, and D from the previous problem have a highlighted hypotenuse.

a What do you think Melissa did well?

***Responses vary.* She selected A and D, both of which are right triangles with the longest side highlighted.**

b What mistake might she have made?

***Responses vary*. She selected a triangle that is not a right triangle. Hypotenuses can only be part of right triangles.**

Activity
2

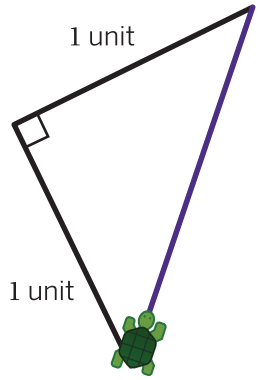
Name: ____________ Date: ________ Period: ________

Turtle Tracing

Tiam the turtle is walking on one side of a triangle.

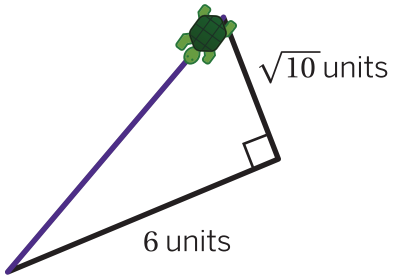
4 Exactly how far does Tiam need to travel?

$\sqrt{2}$ units



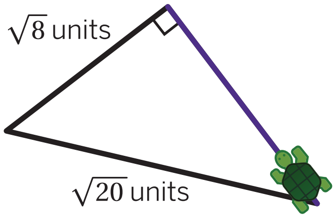
5 Exactly how far does Tiam need to travel?

$\sqrt{46}$ units



6 Exactly how far does Tiam need to travel?

$\sqrt{12}$ units



Name: .... Date: .... Period: ....

Turtle Tracing (continued)

7 You will use a separate sheet of paper to create your own triangle challenge.

a **Make It!** On the paper, sketch a right triangle. Label two of the sides with their approximate lengths.

b **Solve It!** On this page, write the two side lengths you labeled on your triangle. Then calculate the length of the third side.
Responses vary.

My Sides	My Lengths (units)
Leg 1	
Leg 2	
Hypotenuse	

c **Swap It!** Swap your challenge with one or more partners. Calculate the unknown side length for each partner's triangle.
Responses vary.

	Side	Length (units)
Partner 1	Leg 1	
	Leg 2	
	Hypotenuse	
Partner 2	Leg 1	
	Leg 2	
	Hypotenuse	
Partner 3	Leg 1	
	Leg 2	
	Hypotenuse	

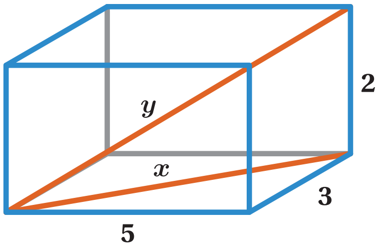
Name: .... Date: .... Period: ....

Three Dimensions

8 Here is a rectangular prism.

Segment y represents the diagonal of the prism.

Calculate the exact length x. Show your thinking.



$\sqrt{34}$ units

Steps vary.

$$3^2 + 5^2 = x^2$$
$$9 + 25 = x^2$$
$$34 = x^2$$
$$\sqrt{34} = x$$

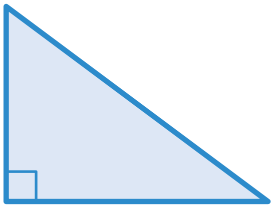
9 Calculate the exact length of segment y. Show your thinking.

$\sqrt{38}$ units

Steps vary.

$$(\sqrt{34})^2 + 2^2 = y^2$$
$$34 + 4 = y^2$$
$$38 = y^2$$
$$\sqrt{38} = y$$

10 Synthesis

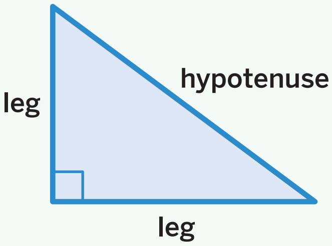


Discuss: If you know two side lengths of a right triangle, how can you calculate the third side length?

Use the image if it helps to show your thinking.

Responses vary. **I can calculate the third side of a right triangle by substituting the two known side lengths into the Pythagorean theorem and solving for the unknown side.**

13 Summary 8.08



The Pythagorean theorem says that in a right triangle, the sum of the squares of the lengths of the **legs** is equal to the square of the length of the **hypotenuse**. This can be represented by the equation $a^2 + b^2 = c^2$, where a and b represent the lengths of the legs and c represents the length of the hypotenuse.

When any two side lengths of a right triangle are known, the Pythagorean theorem can be used to calculate the length of the third side, whether it is the hypotenuse or a leg. You can substitute the lengths you know into the equation $\text{leg}^2 + \text{leg}^2 = \text{hypotenuse}^2$ or $a^2 + b^2 = c^2$, and then solve for the unknown value.

hypotenuse The side of a right triangle that is opposite the right angle. The hypotenuse is always the longest side of a right triangle.

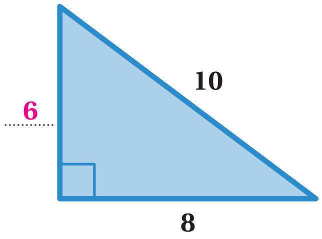
legs The two sides of a right triangle that are not the hypotenuse. The legs are the sides that form the right angle.

Practice
8.08

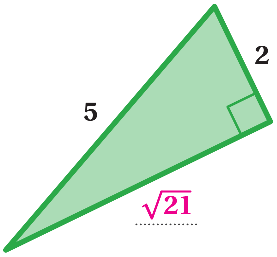
Name: .... Date: .... Period: ....

Problems 1–3: Calculate the exact value of the unknown side length in each right triangle.

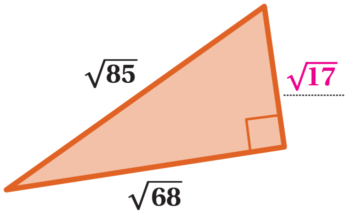
1.



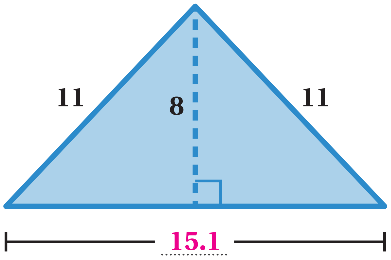
2.



3.



4. Calculate the value of the unknown side length to the nearest tenth.

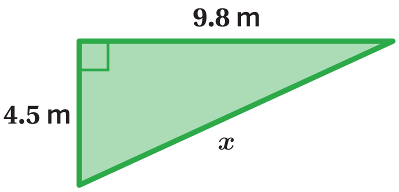


5. Ivory is walking home from a friend's house. Ivory start by walking 2 kilometers west and then 3 kilometers south. How many kilometers, to the nearest tenth, does Ivory live from the friend's house?

3.6 kilometers

6. This diagram shows a right triangle.

Which measurement is closest to the value of x in meters?



A. 3.8 meters

B. 8.7 meters

[✓ **C.**] 10.8 meters

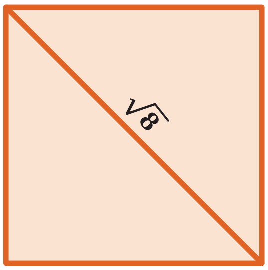
D. 11.2 meters

Practice
8.08

Name: ________ Date: ________ Period: ________

7. What is the area of this square?

4 square units



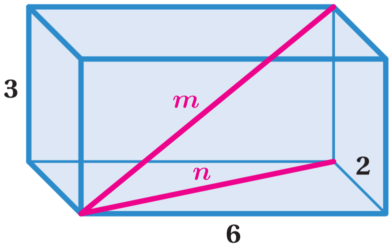
8. Calculate the length of the diagonal of the prism. Show your thinking.

7 units (or equivalent)

Steps vary.

$$2^2+6^2=n^2$$
$$4+36=n^2$$
$$40=n$$
$$\sqrt{40}=n$$

$$(\sqrt{40})^2+3^2=m^2$$
$$40+9=m^2$$
$$49=m^2$$
$$\sqrt{49}=m$$
$$7=m$$



Spiral Review

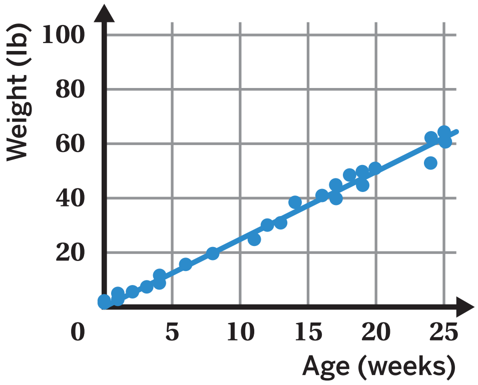
9. In 2015, there were roughly $1 \cdot 10^6$ high school football players and $2 \cdot 10^3$ professional football players in the United States. About how many times more high school football players were there?

There were approximately 500 times more high school football players.

Show or explain your thinking.

Explanations vary. $$\frac{1 \cdot 10^6}{2 \cdot 10^3}=0.5 \cdot 10^3$$
$$=5 \cdot 10^2$$
$$=500$$

Problems 10–11: The scatter plot shows some ages and weights for a large dog breed. The scatter plot shows the model of $y=2.45x+1.22$.



10. What does the slope mean in this situation?

Responses vary. **The slope means that this type of dog can be expected to gain 2.45 pounds per week.**

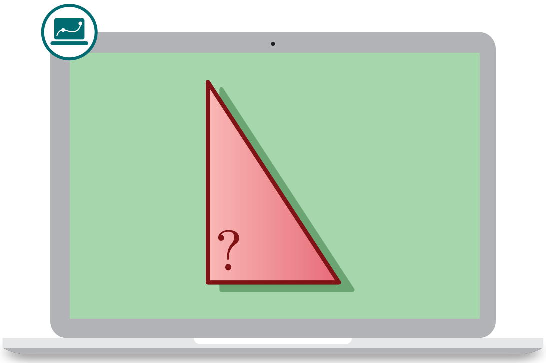
11. Based on this model, how heavy would you expect a newborn puppy to be?

1.22 pounds

Unit 8
Lesson
9

Name: ________ Date: ________ Period: ________

Pythagorean Explorations 8.G.6, 8.G.7, SMP.3, SMP.7



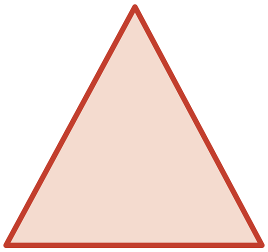
Make It Right

Let's determine if a triangle is a right triangle.

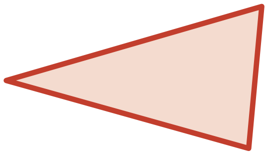
Warm-Up

1 Which one doesn't belong?

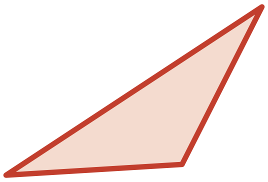
A.



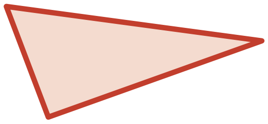
B.



C.



D.



Explain your thinking.

Responses vary.

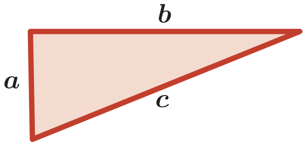
- **Choice A doesn't belong because it's the only one that looks like it has three equal sides and three equal angles.**
- **Choice B doesn't belong because it's the only one that has all acute angles, and no sides look like they are the same length.**
- **Choice C doesn't belong because it's the only one that has an obtuse angle.**
- **Choice D doesn't belong because it's the only one that looks like a right triangle.**

Activity
1

Name: .... Date: .... Period: ....

Is the Converse True?

2 Mathematicians sometimes think about a statement's *converse*, which is a statement in the opposite direction.



The converse of the Pythagorean theorem says: *If a triangle has side lengths such that* $a^2 + b^2 = c^2$*, it is a right triangle.*

Do you think this statement is always, sometimes, or never true? Circle one.

Responses vary.

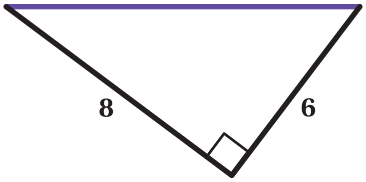
Always Sometimes Never

Explain your thinking.

Explanations vary.

- **I think this is always true because I know there are right triangles with side lengths that make $a^2 + b^2 = c^2$ true.**
- **I think this is sometimes true because you could have lots of different triangles with side lengths a, b, and c, but only one of them would be a right triangle.**

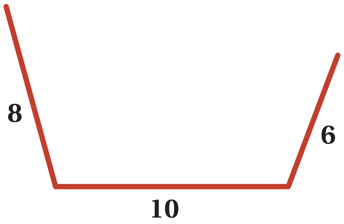
3 Let's explore the converse of the Pythagorean theorem by focusing on a specific example. Here is a right triangle with legs that are 6 and 8 units long.



What is the length of its hypotenuse?

10 units

4 You will use the cutouts from the Activity 1 Card to create several triangles.



a Experiment with making two other triangles with sides lengths measuring 6, 8, and 10 units.

Responses vary.

b **Discuss:** What do you notice? What do you wonder?

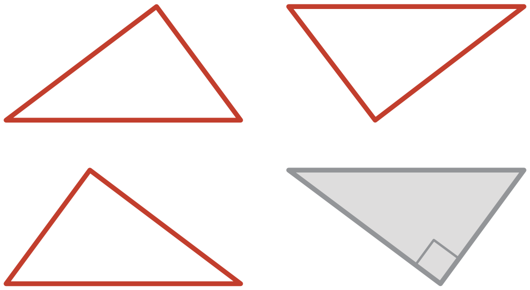
Responses vary.

- **I notice that all the triangles appear to have a right angle.**
- **I notice that the triangles are reflections or rotations of each other.**
- **I wonder if the triangles are all right triangles.**
- **I wonder if the triangles are all the same triangle.**

Name: .... Date: .... Period: ....

Is the Converse True? (continued)

5 Yosef says that every triangle with sides lengths measuring 6, 8, and 10 units *must* be a right triangle, because the triangles with these lengths are all *congruent*.



Do you agree? Circle one. Use tracing paper to help with your thinking.

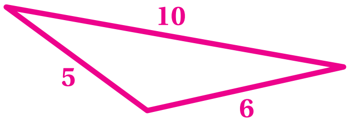
[✓ Yes]   No   I'm not sure

Explain your thinking.

***Explanations vary.* The Pythagorean theorem tells us that a right triangle with leg lengths 6 and 8 has a hypotenuse of length 10. We can use rigid transformations to show that any other triangle with those side lengths is congruent to the right triangle.**

6 Draw a non-right triangle and label the side lengths. Try to think of a triangle that no one else in the class will draw. Does your triangle satisfy the equation $a^2 + b^2 = \mathrm{c}^2$, where c is the longest length? Show or explain your thinking.

Responses vary.



Let's check.

$$5^2 + 6^2 = 10^2$$
$$25 + 36 = 100$$
$$61 = 100$$

My triangle does not satisfy the equation $a^2 + b^2 = c^2$.

7 Compare your work with a classmate's.

Discuss:

- Is the equation $a^2 + b^2 = c^2$ true for any of the non-right triangles?
- Do you think the equation will ever be true for any non-right triangle? Explain your thinking.
- How can you use the equation $a^2 + b^2 = c^2$ to prove whether a triangle is a right or non-right triangle?

Responses vary.

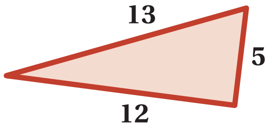
- **The equation $a^2 + b^2 = c^2$ is not true for any of the non-right triangles drawn.**
- **I think this equation will not be true for any non-right triangle because the Pythagorean theorem is only used for right triangles.**
- **If the side lengths of a triangle do not satisfy the equation $a^2 + b^2 = c^2$, then it is not a right triangle. If the side lengths of a triangle satisfy the equation $a^2 + b^2 = c^2$, then it is a right triangle.**

Activity
2

Name: ________ Date: ________ Period: ________

Make It Right

8 What type of triangle is this? Circle one.



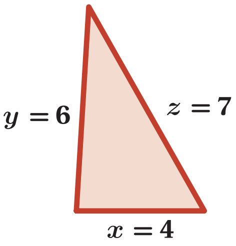
[✓ A right triangle] Not a right triangle

Explain your thinking.

***Explanations vary.* This is a right triangle because $5^2 + 12^2 = 13^2$.**

9 Change *one* of the values to make this triangle a right triangle.

There are many different solutions. Try to find *at least* four. ***Responses vary.***



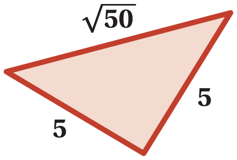
x	y	z
4	6	$\sqrt{52}$
4	6	$\sqrt{20}$
4	$\sqrt{65}$	7
4	$\sqrt{33}$	7
$\sqrt{13}$	6	7
$\sqrt{85}$	6	7

Name: .... Date: .... Period: ....

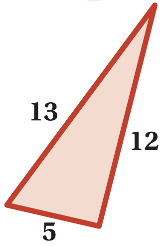
Make It Right (continued)

10 Circle whether each triangle is a right triangle.

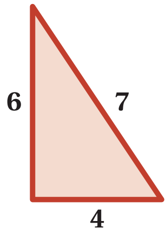
a [✓ Right Triangle] Not a Right Triangle



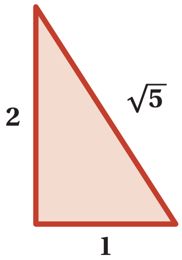
b [✓ Right Triangle] Not a Right Triangle



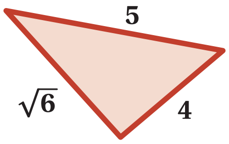
c Right Triangle [✓ Not a Right Triangle]



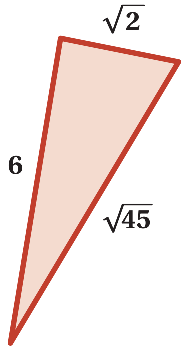
d [✓ Right Triangle] Not a Right Triangle



e Right Triangle [✓ Not a Right Triangle]



f Right Triangle [✓ Not a Right Triangle]



You're invited to explore more.

11 Here is an obtuse triangle, an acute triangle, and a right triangle. *All* triangles are one of these three types.

Decide whether triangles X, Y, and Z are acute, right, or obtuse based on their side lengths.

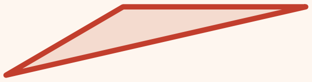
Triangle X, side lengths: 15, 20, 8
Triangle Y, side lengths: 8, 15, 13
Triangle Z, side lengths: 17, 8, 15

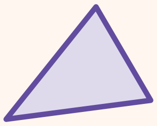
Triangle X: Obtuse. *Work varies.* $8^2 + 15^2 < 20^2$.

Triangle Y: Acute. *Work varies.* $8^2 + 13^2 > 15^2$.

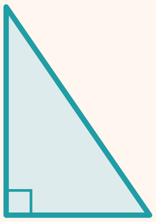
Triangle Z: Right. *Work varies.* $8^2 + 15^2 = 17^2$.

Obtuse





Acute

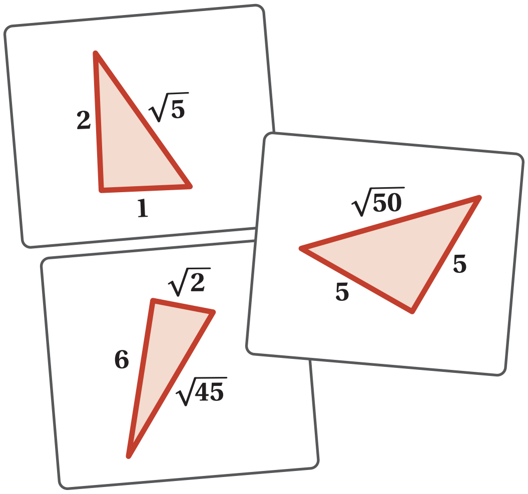


Right

12 Synthesis

How can you tell from just the side lengths if a triangle is a right triangle?

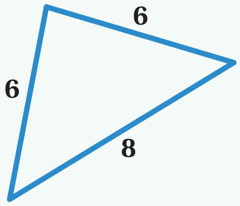
Responses vary. **Square the lengths of the two shorter sides and add them together. If the sum is equal to the square of the length of the longest side, then the triangle is a right triangle.**



15 Summary 8.09

If a triangle has side lengths a, b, and c, where c is the longest side and $a^2 + b^2 = c^2$, then the converse of the Pythagorean theorem says that you must have a right triangle. We can use this to determine whether any triangle is a right triangle. If the sides of a triangle *do not* make the equation $a^2 + b^2 = c^2$ true, then you know it is *not* a right triangle.

In the triangle shown, let $a = 6$, $b = 6$, and $c = 8$. You can use substitution to determine whether the triangle is a right triangle.



$$a^2 + b^2 = 36 + 36$$
$$= 72$$

Because $c^2 = 8^2$, or 64, the triangle cannot be a right triangle because $a^2 + b^2 \neq c^2$.

Practice
8.09

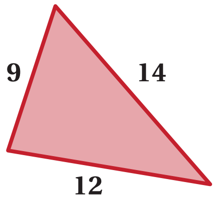
Name: .... Date: .... Period: ....

1. The lengths of two sides of a triangle are 5 and 6. Which side length would make the triangle a right triangle?

A. $\sqrt{8}$
[✓ B.] $\sqrt{11}$
C. 9
D. 10

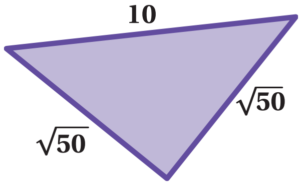
Problems 2–4: Determine whether each triangle is a right triangle.

2.



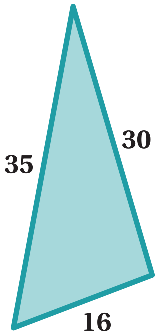
No

3.



Yes

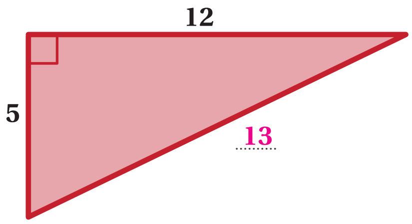
4.



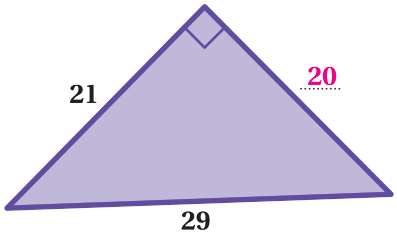
No

Problems 5–6: Calculate the value of the unknown side length so that the triangle is a right triangle.

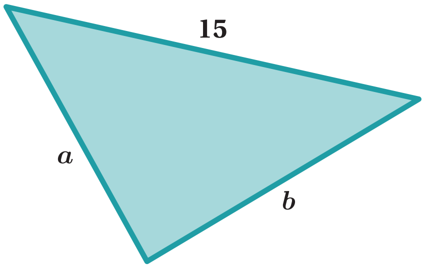
5.



6.



7. The longest side of this triangle has a length of 15 centimeters. What could the lengths of the other two sides be so that this is a right triangle?



Responses vary.

- Side a: $\sqrt{200}$ centimeters
  Side b: 5 centimeters
- Side a: $\sqrt{125}$ centimeters
  Side b: 10 centimeters

Practice
8.09

Name: .... Date: .... Period: ....

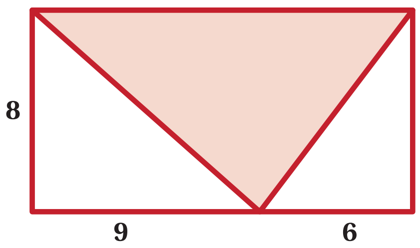
8. The side lengths of a triangle are 5, 7, and 9 units. Does the length of the longest side need to *increase*, *decrease*, or *stay the same* to make the triangle a right triangle? Circle one.

Increase [✓ Decrease] Stay the same

Explain your thinking.

***Explanations vary.* $5^2 + 7^2 < 9^2$. The length of the longest side needs to decrease so that the square of its side length is equal to $5^2 + 7^2$, or 74.**

9. Here is a 15-by-8 rectangle divided into triangles. Is the shaded triangle a right triangle? Circle one.



Yes [✓ No]

Explain your thinking.

***Explanations vary.* I used the Pythagorean theorem to calculate the lengths of the legs of the shaded triangle, which are $\sqrt{145}$ and 10. The length of the longest side of the shaded triangle is 15 (the same as the length of the rectangle). For the triangle to be a right triangle, the side lengths must make the equation $a^2 + b^2 = c^2$ true. Because $145 + 100 = 245$, not 225, I know that the shaded triangle is not a right triangle.**

Spiral Review

10. Determine which two whole numbers $\sqrt{53}$ is between.

Between 7 and 8

11. Write an expression equivalent to $\sqrt[3]{\frac{64}{125}}$ that does not use a cube root symbol.

$\frac{4}{5}$ ***Responses vary.***

12. What is the approximate value of $\sqrt{24}$?

A. 4

[✓ B.] 4.9

C. 5

D. 4.4

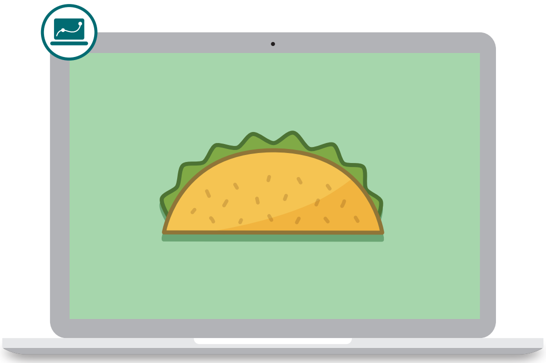
Unit 8

Lesson

10

Name: ____________________ Date: __________ Period: __________

Pythagorean Explorations 8.G.7, 8.EE.2, SMP.1, SMP.2



Taco Truck

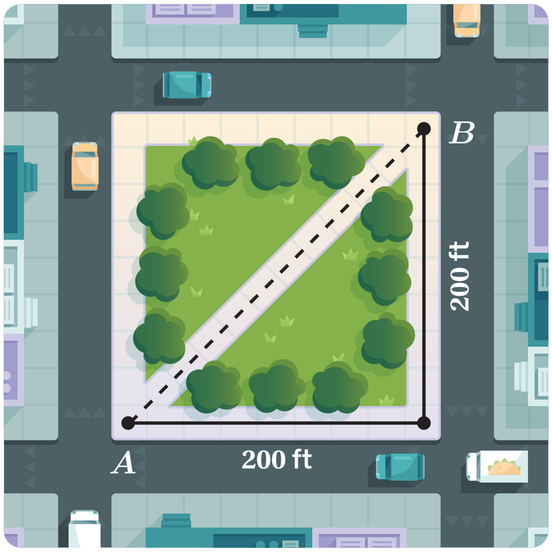
Let's solve problems with the Pythagorean theorem.

Warm-Up

1 Alma is going to walk through the park from point A to point B.

What distance will she walk?

$\sqrt{80000} \approx 282.8$ feet (or equivalent)



2 If Alma walks at a speed of 4 feet per second, how long will it take for her to walk across the park?

About 70.7 seconds (or equivalent)

Name: .... Date: .... Period: ....

Taco Truck

3 Imagine you're on the beach, and you're getting hungry.

Sketch the route you would take to the taco truck.

***Responses vary.* Sample shown in image.**

Explain your thinking.

Explanations vary.

- **The quickest way to get from one point to another is to follow a straight line between them.**
- **I hate walking on sand, so I would walk to the boardwalk as quickly as possible and walk the rest of the way on the boardwalk.**

4 Bao and Eva choose different routes to get to the taco truck.

a Who do you think will reach the taco truck first? ***Responses vary.***

Bao

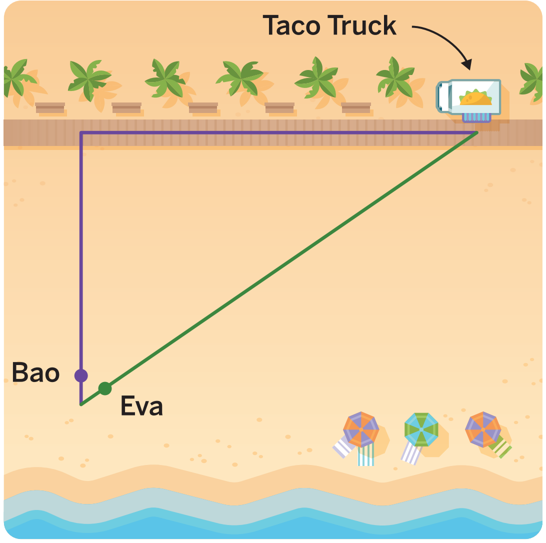
Eva

They'll arrive at the same time

b What information would help you know for sure?

Responses vary.

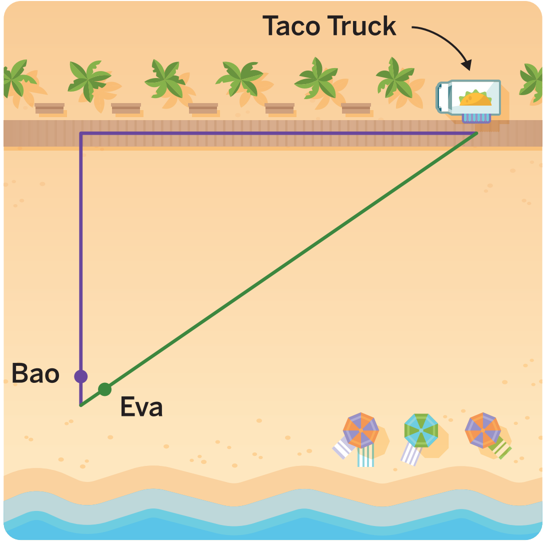
- **Both Bao's and Eva's speeds.**
- **The distance to the taco truck.**
- **The distance to the boardwalk from the starting point and the distance along the boardwalk to the taco truck.**



Name: .... Date: .... Period: ....

Let's Eat

5 Let's look at some additional information about Bao's and Eva's routes on the screen.



Use the information to calculate how long it will take for each person to get to the taco truck. Show your thinking.

- **Bao's time: 207 seconds (or equivalent).** ***Work varies.***

  $$\frac{327.6}{3} + \frac{489}{5} = 207$$

- **Eva's time: About 196.2 seconds (or equivalent).** ***Work varies.***

  $$\frac{\sqrt{327.6^2 + 489^2}}{3} \approx 196.2$$

You're invited to explore more.

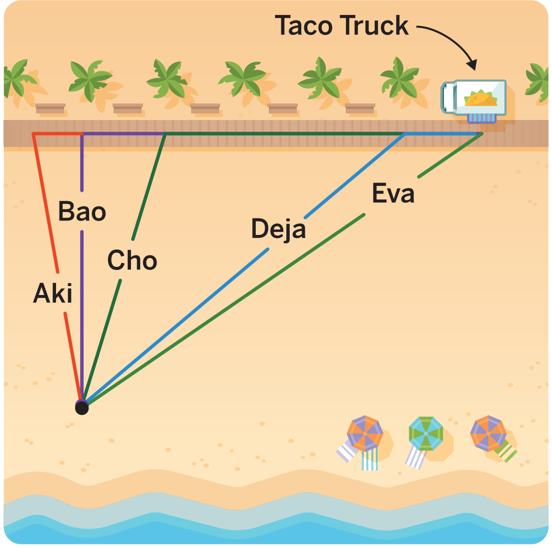
6 Determine the speed on the boardwalk that would make Eva and Bao arrive at the same time.

Speed on Sand	Speed on Boardwalk
3 feet per second	**About 5.6 feet per second**

Name: ______________ Date: ______ Period: ______

Three More Paths

7 Here are three more possible paths.



Who do you think will reach the taco truck first? ***Responses vary.***

☐ Aki
☐ Bao
☐ Cho
☐ Deja
☐ Eva

Explain your thinking.

Explanations vary.

- **Cho will get there first because Cho spent more time walking faster on the boardwalk.**
- **Deja will arrive first because she has almost a direct path to the truck, but she saves some time on the boardwalk.**
- **Eva will get to the truck first because she walks in a straight line.**

8 You will watch a race between Aki, Bao, Cho, Deja, and Eva.

Discuss: What do you notice? What do you wonder?

Responses vary.

- **Bao is the first person to the boardwalk, but he doesn't get to the truck first.**
- **Eva is the last person to the boardwalk, but she isn't the last person to the truck.**
- **Why would Aki take a route that moves away from the truck at first?**
- **Did Deja find the fastest route to the truck?**

9 Let's look at some information about the winning path.

Remember that:

- The speed on the *boardwalk* is 5 feet per second.
- The speed on the *sand* is 3 feet per second.

Use this information to calculate the amount of time it took the winner to get to the taco truck. Show your thinking.

About 189.5 seconds (or equivalent). ***Work varies.***

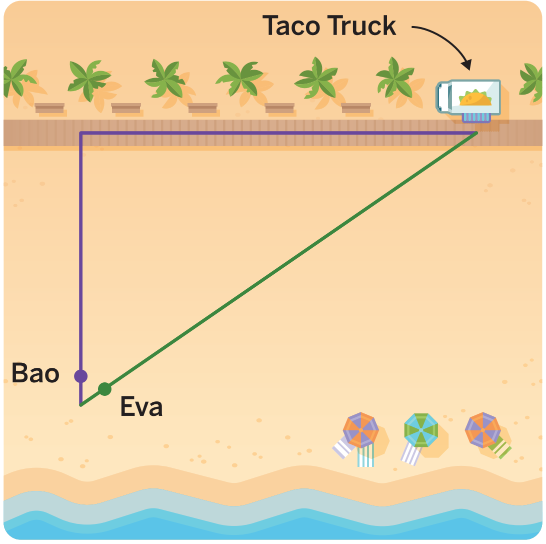
$$\frac{\sqrt{327.6^2 + 389^2}}{3} + \frac{100}{5} \approx 189.52$$

10 Synthesis

Discuss: What are some important things to remember when using the Pythagorean theorem to solve problems?

Responses vary.

- **The Pythagorean theorem can only be applied to right triangles.**
- **I need to use the squares of the side lengths rather than the lengths themselves.**
- **I need to identify which sides are the legs (vs. the hypotenuse) of the right triangle.**

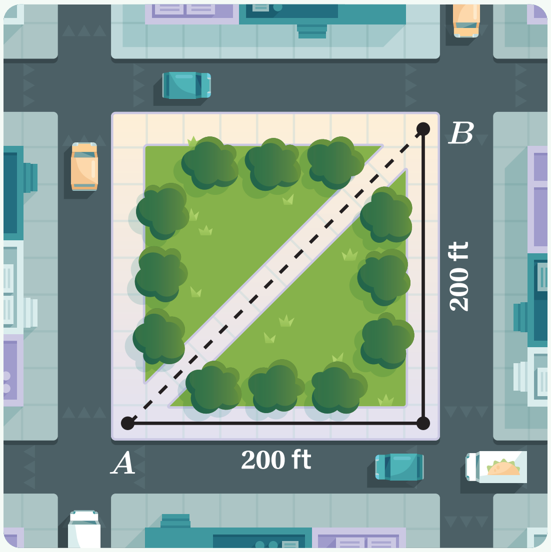


10 Summary 8.10

The Pythagorean theorem can be used to solve problems that can be modeled with right triangles. The sides of a triangle might represent units such as the length of an object or the distance between two objects.

To apply the Pythagorean theorem, the lengths of two sides must be known so the length of the third side can be determined.

For example, you can use the Pythagorean theorem to calculate the distance to walk through the park from point A to point B.

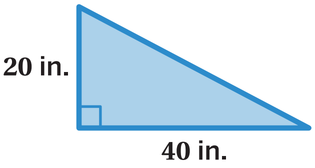


Practice
8.10

Name: .... Date: .... Period: ....

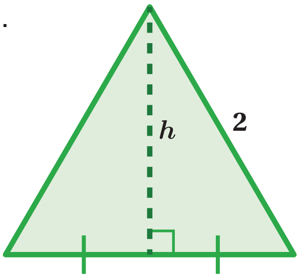
1. Here is a skateboarding ramp. The length of the base is 40 inches, and the height of the ramp is 20 inches. What is the approximate length of the ramp in inches?

   About 44.7 inches



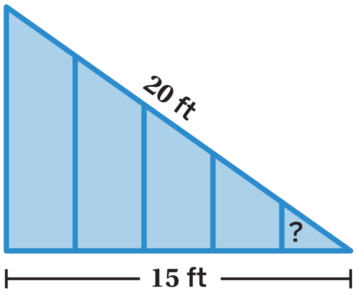
2. Here is an equilateral triangle. The length of each side is 2 units. A height, h, is drawn. Determine the exact height.

   $\sqrt{3}$ units

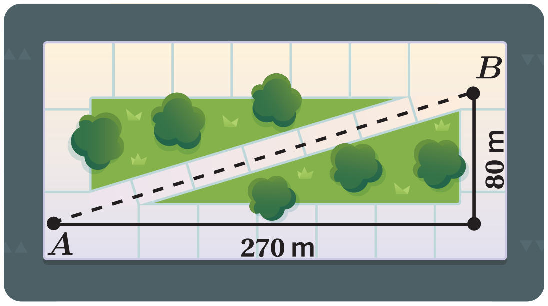


3. A 20-foot roof needs 5 support beams placed equidistant along the floor. Each support is a different length. Determine the exact length of the shortest support beam.

   $\sqrt{7}$ feet



Problems 4–5: A standard city block in Manhattan is a rectangle measuring 80 meters by 270 meters. Sol is thinking about cutting diagonally through the park to get from point A to point B.



4. Determine the distance Sol would walk if Sol cut through the park.

   $\sqrt{79300} \approx 282$ meters

5. If Sol walks an average of 1.42 meters per second, how much time will Sol save by cutting through the park instead of walking around it? Round your answer to the nearest second.

   About 48 seconds

Practice
8.10

Name: .... Date: .... Period: ....

Spiral Review

6. Select *all* the sets of side lengths that form a right triangle.

☐ **A.** $7, 8, 13$

☑ **B.** $4, 10, \sqrt{84}$

☑ **C.** $\sqrt{8}, 11, \sqrt{129}$

☑ **D.** $\sqrt{1}, 2, \sqrt{3}$

☐ **E.** $\sqrt{2}, 3, \sqrt{13}$

Problems 7–9: Circle the number that is greater.

7. [✓ $12 \cdot 10^9$] or $4 \cdot 10^9$

8. $1.5 \cdot 10^{-12}$ or [✓ $3 \cdot 10^{-12}$]

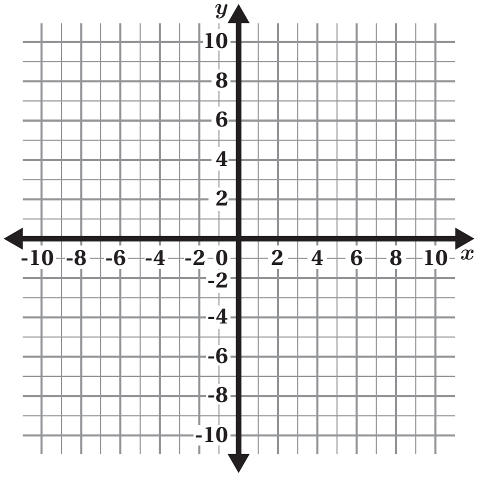
9. $20 \cdot 10^4$ or [✓ $6 \cdot 10^5$]

10. A line contains the point $(3, 5)$. If the line has a negative slope, which of these points could also be on the line?

Use the graph if it helps with your thinking.

A. $(6, 5)$ **B.** $(4, 7)$

[✓ **C.**] $(5, 4)$ **D.** $(2, 0)$



11. Which expression is equivalent to $(5^{-2})^5 \cdot 5^4$?

A. 5^{12}

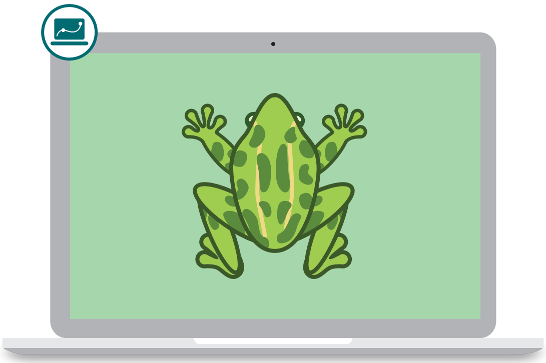
B. 5^7

[✓ **C.**] $\frac{1}{5^6}$

D. $\frac{1}{5^{40}}$

Name: .... Date: .... Period: ....

Pythagorean Explorations 8.G.8, SMP.3, SMP.7



Pond Hopper

Let's calculate distances between points on the coordinate plane.

Warm-Up

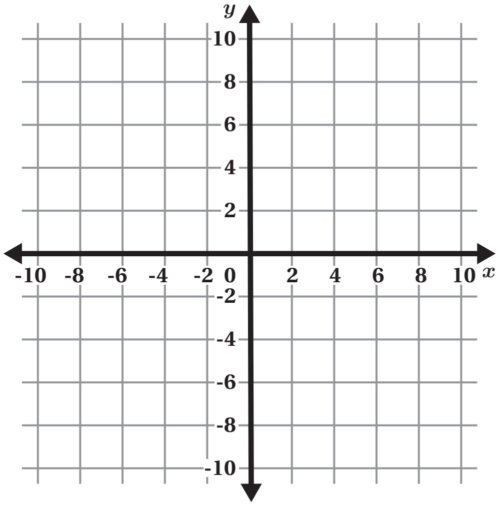
1 Order the pairs of points from closest together to farthest apart.

Use the graph if it helps with your thinking.

Pair A:	$(-8, 1)$ and $(-8, 8)$
Pair B:	$(7, 0)$ and $(7, -9)$
Pair C:	$(2, 3)$ and $(2, 9)$
Pair D:	$(-3, 6)$ and $(5, 6)$

C	A	D	B

Closest Together **Farthest Apart**



Activity
1

Name: ________ Date: ________ Period: ________

Pond Hopper

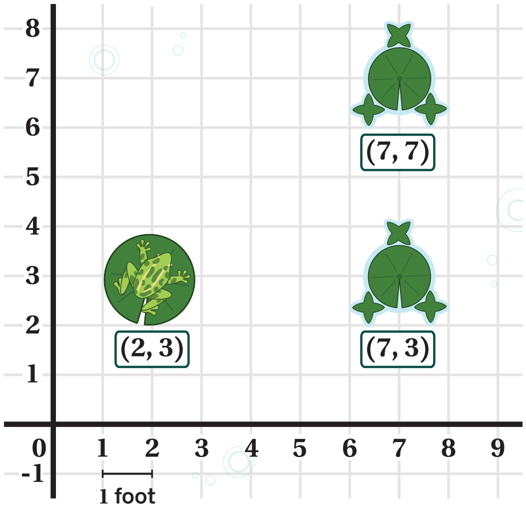
2 Let's help the frog hop to all the lily pads.

Pick a lily pad. Write its coordinates in the table.

Then write the distance between the frog and the lily pad.

Repeat until you have hopped to all the lily pads.

Responses vary.



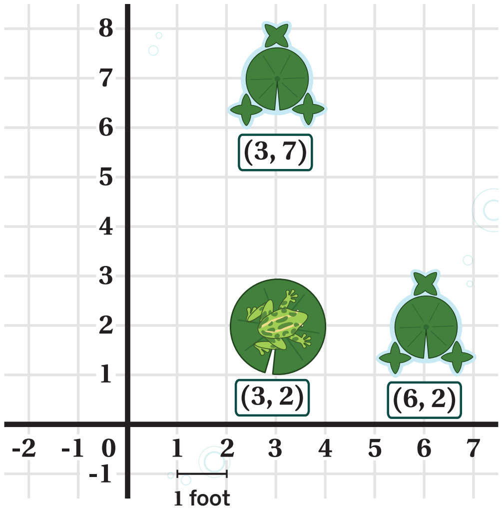
	Lily Pad Coordinates	Distance (ft)
Hop 1	(7, 3)	5
Hop 2	(7, 7)	4
Hop 3		
Hop 4		

Another route:

- Hop $\sqrt{41}$ feet to (7, 7)
- Hop 4 feet to (7, 3)

3 Help the frog hop to all the lily pads in *as few hops as possible*.

Responses vary.



	Lily Pad Coordinates	Distance (ft)
Hop 1	(6, 2)	3
Hop 2	(3, 7)	$\sqrt{34}$
Hop 3		
Hop 4		

Another route:

- Hop 5 feet to (3, 7)
- Hop $\sqrt{34}$ feet to (6, 2)

Name: ________ Date: ________ Period: ________

Pond Hopper (continued)

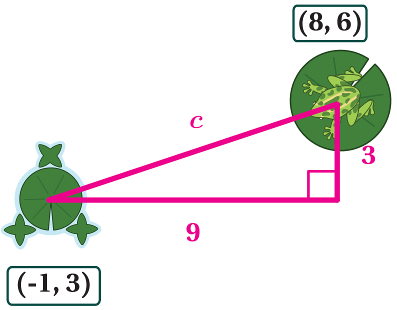
4 Which expression represents the distance between the frog and the lily pad?

A. $\sqrt{7^2 - 3^2}$

B. $\sqrt{7^2 + 3^2}$

C. $\sqrt{9^2 - 3^2}$

[✓ D.] $\sqrt{9^2 + 3^2}$



All measurements in feet

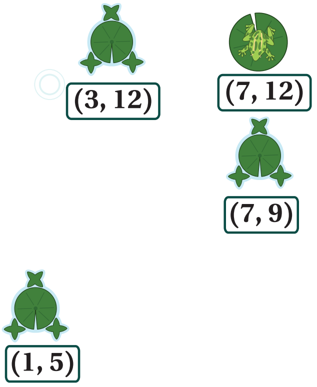
Show or explain your thinking.

Explanations vary. I can create a right triangle by drawing a line segment straight down from (8, 6) and another line segment straight left to (-1, 3). Then I can use the Pythagorean theorem to calculate the hypotenuse, which is the same as the distance between the frog and the lily pad. The legs of the triangle are 9 feet and 3 feet, so $9^2 + 3^2 = c^2$ and $c = \sqrt{9^2 + 3^2}$.

5 Help the frog hop to all the lily pads in *as few hops as possible*.

Responses vary. Samples shown below.

	Lily Pad Coordinates	Distance (ft)
Hop 1	(3, 12)	4
Hop 2	(7, 9)	5
Hop 3	(1, 5)	$\sqrt{52}$
Hop 4		
Hop 5		



All measurements in feet

Another route:

- Hop 3 feet to (7, 9)
- Hop $\sqrt{52}$ feet to (1, 5)
- Hop $\sqrt{53}$ feet to (3, 12)

Activity
2

Name: .... Date: .... Period: ....

Challenge Creator

6 You will use the Activity 2 Sheet to complete this activity.

a **Make It!** On the Activity 2 Sheet, create a lily pad challenge by sketching lily pads and a rock.

b **Solve It!** On this page, help the frog hop to all the lily pads in your challenge in *as few hops as possible*.

Responses vary.

My Challenge

	Hop 1	Hop 2	Hop 3	Hop 4	Hop 5	Hop 6	Hop 7	Hop 8
Lily Pad Coordinates								
Distance (ft)								

c **Swap It!** Swap your challenge with one or more partners. Help the frog hop to all the lily pads in each of your partners' challenges in *as few hops as possible*.

Responses vary.

Partner 1

	Hop 1	Hop 2	Hop 3	Hop 4	Hop 5	Hop 6	Hop 7	Hop 8
Lily Pad Coordinates								
Distance (ft)								

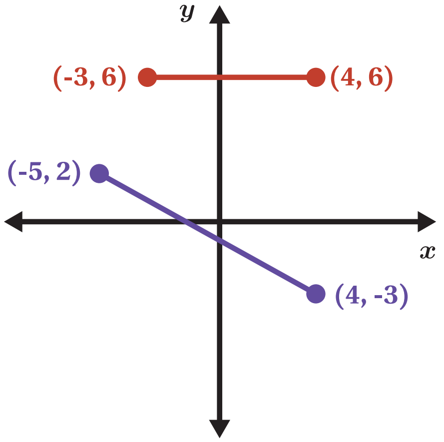
Partner 2

	Hop 1	Hop 2	Hop 3	Hop 4	Hop 5	Hop 6	Hop 7	Hop 8
Lily Pad Coordinates								
Distance (ft)								

7 Synthesis

💬 **Discuss:** What are some strategies to calculate the distance between two points on the coordinate plane?

Use the examples in the graph if they help to show your thinking.



Responses vary.

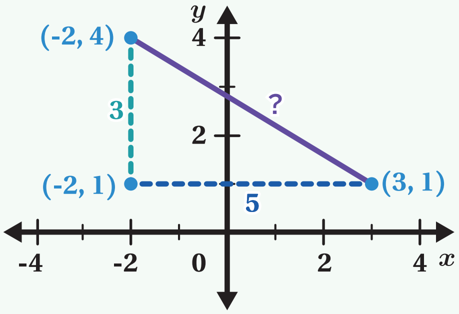
- **When points have the same x- or y-coordinate, they will be perfectly horizontal or vertical. I can determine the distance between them by just subtracting the coordinates that are different. If the value is negative, I will take the opposite because distances are always positive.**
- **When points are not aligned horizontally or vertically, I can make a right triangle and use the Pythagorean theorem to calculate the distance between the two points. I can determine the length of one leg by subtracting the points' x-coordinates, and the other leg by subtracting the y-coordinates. If either value is negative, I will take the opposite because lengths are positive.**

10 Summary 8.11

You can use the Pythagorean theorem to calculate the distance between two points that are on a diagonal line segment. To do this, start by drawing horizontal and vertical legs to form a right triangle. Then use the Pythagorean theorem to calculate the length of the hypotenuse, which will be the distance between the two points.

When two points are on a horizontal line segment, you can calculate the distance between them by determining the absolute value of the difference between their x-coordinates. For the points $(-2, 1)$ and $(3, 1)$ the distance is $|-2 - 3| = 5$ units.

Similarly, when two points are on a vertical line segment, you can calculate the distance between them by determining the absolute value of the distance between their y-coordinates. For the points $(-2, 4)$ and $(-2, 1)$ the distance is $|4 - 1| = 3$ units.



$$\text{leg}^2 + \text{leg}^2 = \text{hypotenuse}^2$$

$$3^2 + 5^2 = x^2$$

$$9 + 25 = x^2$$

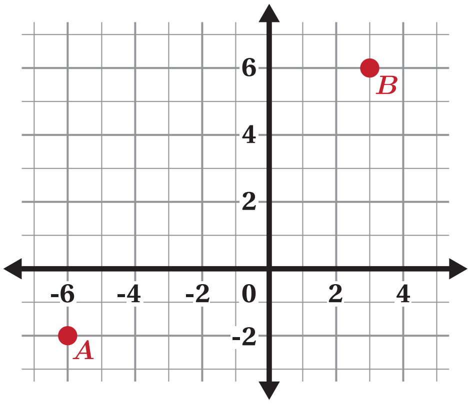
$$34 = x^2$$

$$\sqrt{34} = x$$

Practice
8.11

Name: ______ Date: ______ Period: ______

1. Here is a coordinate plane with labeled points.



What is the distance between point A and point B?

A. $\sqrt{17}$

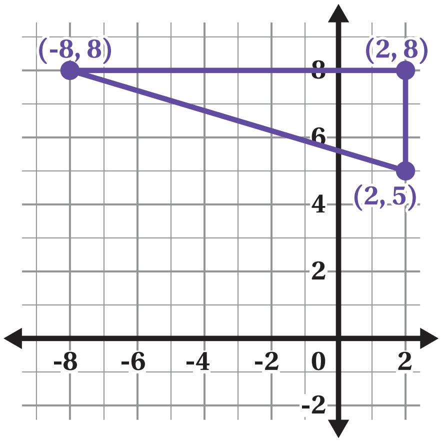
B. $\sqrt{128}$

[✓ C.] $\sqrt{145}$

D. 17

Problems 2–3: Calculate the length of each side of these right triangles.

2.

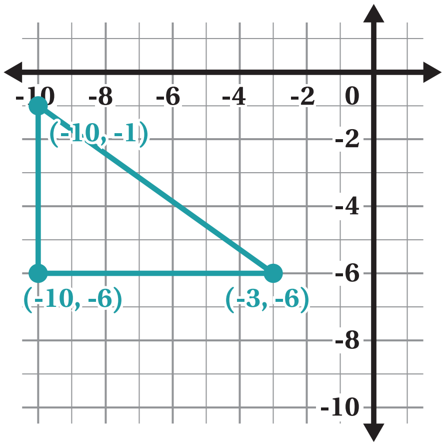


Shorter leg: 3 units

Longer leg: 10 units

Hypotenuse: $\sqrt{109}$ units

3.

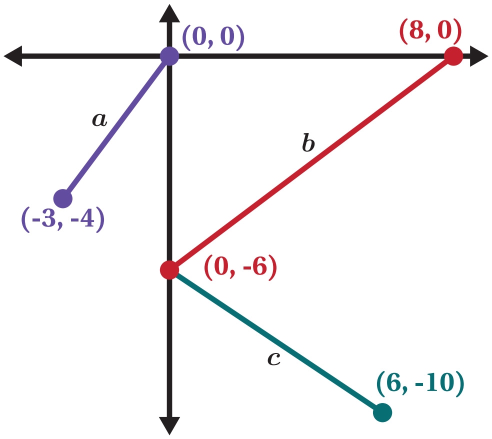


Shorter leg: 5 units

Longer leg: 7 units

Hypotenuse: $\sqrt{74}$ units

Problems 4–6: Calculate the length of each segment.



4. Segment a

5 units

5. Segment b

10 units

6. Segment c

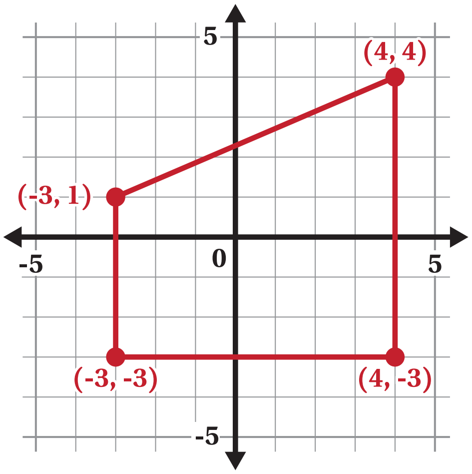
$\sqrt{52}$ units

Practice
8.11

Name: ______ Date: ______ Period: ______

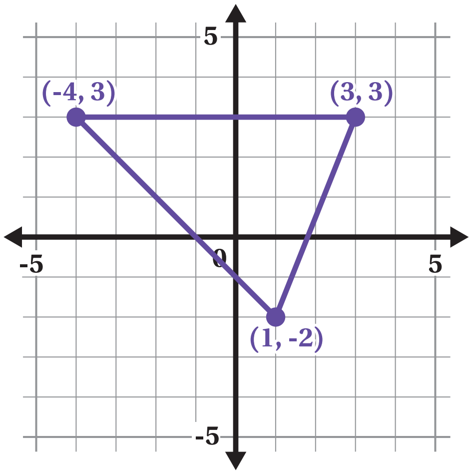
Problems 7–8: Calculate the perimeter of each polygon.

7.



$18 + \sqrt{58}$, or about 25.6 units

8.



$7 + \sqrt{50} + \sqrt{29}$, or about 19.5 units

9. The distance between $(0, -8)$ and another point is 5 units. What are possible coordinates for the second point if the two points do not lie on a horizontal or vertical line?

Responses vary.

- $(-4, -5)$
- $(4, -5)$
- $(-3, -12)$
- $(3, -12)$

Spiral Review

Problems 10–11: Determine the value of the expression. Write your answer in scientific notation.

10. $(5.6 \cdot 10^8) + (7.3 \cdot 10^8)$

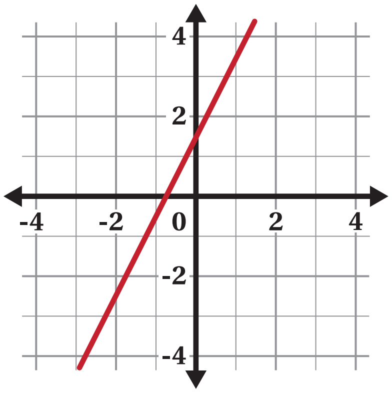
$1.29 \cdot 10^9$

11. $(8.7 \cdot 10^3) - (2.2 \cdot 10^2)$

$8.48 \cdot 10^3$

12. Write an equation for the line.

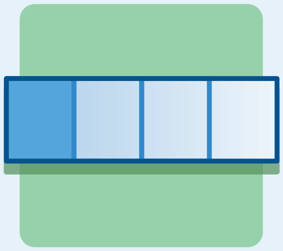
$y = 2x + 1.5$



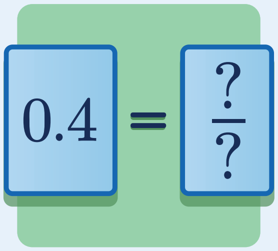
Sub-Unit

3

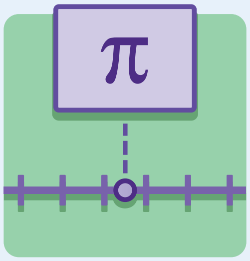
Rational and Irrational Numbers



Lesson 12
Fractions to Decimals



Lesson 13
Decimals to Fractions



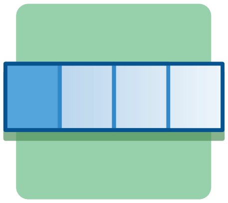
Lesson 14
Hit the Target

Unit 8
Lesson
12

Name: ________ Date: ________ Period: ________

Number Line Understanding 8.NS.1, SMP.1, SMP.6, SMP.7

Fractions to Decimals

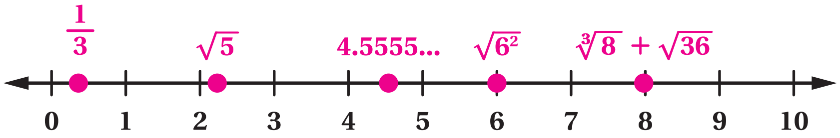


Let's explore connections between fractions and their decimal representations.

Warm-Up

1. Plot these values on the number line.

$\sqrt{5}$	$\frac{1}{3}$	$\sqrt[3]{8}+\sqrt{36}$	4.5555...	$\sqrt{6^2}$



Activity
1

Name: ______ Date: ______ Period: ______

Terminating or Repeating

2. Every fraction can be written as a decimal. Here are some fractions written as decimals.

Fraction	Decimal	Terminating	Repeating	Neither
$\frac{1}{8}$	0.125	✓		
$\frac{3}{5}$	0.6	✓		
$\frac{341}{100}$	3.41	✓		
$\frac{1}{3}$	$0.333...$		✓	
$\frac{243}{99}$	$2.454545...$		✓	
$\frac{121}{15}$	$8.0666...$		✓	

What do you notice? What do you wonder? ***Responses vary.*** **ELD.PI.8.6.Em, Ex, Br**

I notice:

- **All of the examples either terminate or repeat.**
- **For repeating decimals, the denominator is a multiple of 3.**

I wonder:

- **Are there any decimal representations that would be in the neither category?**
- **What is the easiest way to convert a fraction to its decimal representation?**

3. We can also use **bar notation** to write repeating decimals.

For example, $0.333... = 0.\overline{3}$ and $2.454545... = 2.\overline{45}$.

Order these numbers from least to greatest: 8.06, $8.0\overline{63}$, $8.0\overline{6}$, 8.063.

$\mathbf{8.06}$	$\mathbf{8.063}$	$\mathbf{8.0\overline{63}}$	$\mathbf{8.0\overline{6}}$

Least **Greatest**

4. Write $\frac{11}{50}$ as a decimal and decide whether it is *terminating*, *repeating*, or *neither*.

Fraction	Decimal	Terminating	Repeating	Neither
$\frac{11}{50}$	$\mathbf{0.22}$	✓		

Activity

2

Name: .... Date: .... Period: ....

Converting Unit Fractions

5. Use long division to write each *unit fraction* as a decimal. Use the workspace or blank paper if it helps with your thinking.

Unit Fraction	Decimal	Terminating	Repeating	Neither
$\frac{1}{2}$	0.5	✓		
$\frac{1}{3}$	$0.\overline{3}$		✓	
$\frac{1}{4}$	**0.25**	✓		
$\frac{1}{5}$	**0.2**	✓		
$\frac{1}{6}$	$0.1\overline{6}$		✓	
$\frac{1}{7}$	$0.\overline{142857}$		✓	
$\frac{1}{8}$	**0.125**	✓		
$\frac{1}{9}$	$0.\overline{1}$		✓	
$\frac{1}{10}$	**0.1**	✓		
$\frac{1}{11}$	$0.\overline{09}$		✓	
$\frac{1}{12}$	$0.08\overline{3}$		✓	

Workspace:

6. Write another unit fraction that terminates when written as a decimal.

Responses vary.

- $\frac{1}{20}$
- $\frac{1}{40}$
- $\frac{1}{100}$
- $\frac{1}{125}$
- $\frac{1}{800}$

7. Write another unit fraction that repeats when written as a decimal.

Responses vary.

- $\frac{1}{13}$
- $\frac{1}{17}$
- $\frac{1}{30}$
- $\frac{1}{41}$
- $\frac{1}{101}$

Activity

2

Name: ______ Date: ______ Period: ______

Converting Unit Fractions (continued)

8. How can you predict whether a unit fraction will terminate, repeat, or neither when written as a decimal? **ELD.PI.8.10.Em, Ex, Br**

Responses vary.

- **Write the denominator in factored form. If the factors consist only of 2s and 5s, then the decimal representation will terminate. Otherwise, it will repeat.**
- **If you can write an equivalent fraction with a power of 10 as the denominator, the decimal representation will terminate. If not, it will repeat.**

You're invited to explore more.

9. Complete the table. Then answer these questions:

a How are the decimal representations in the table similar to each other?

Each decimal contains the same six repeating digits ($1, 4, 2, 8, 5, 7$) in the same order.

b How are the decimal representations in the table similar to each other?

Each decimal starts at a different digit in the cycle.

c Add the decimal representations of $\frac{3}{7}$ and $\frac{4}{7}$. What is the result? How does this compare to when you add the fractions $\frac{3}{7}$ and $\frac{4}{7}$?

$\mathbf{0.\overline{428571} + 0.\overline{571428} = 0.\overline{999999} = 0.\overline{9}}$**;**
$\mathbf{\frac{3}{7} + \frac{4}{7} = \frac{7}{7}}$**; This suggests that $\mathbf{0.\overline{9} = 1}$.**

Fraction	Decimal
$\frac{1}{7}$	$\mathbf{0.\overline{142857}}$
$\frac{2}{7}$	$\mathbf{0.\overline{285714}}$
$\frac{3}{7}$	$\mathbf{0.\overline{428571}}$
$\frac{4}{7}$	$\mathbf{0.\overline{571428}}$
$\frac{5}{7}$	$\mathbf{0.\overline{714285}}$
$\frac{6}{7}$	$\mathbf{0.\overline{857142}}$

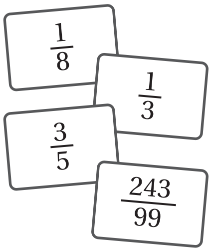
Synthesis

10. Explain a strategy for writing fractions as decimals.

ELD.PI.8.10.Em, Ex, Br

Responses vary.

- **To write a fraction as a decimal, use long division.**
- **If the denominator is a factor of 100, you can multiply the numerator and denominator to make the fraction out of 100, then write the number of hundredths as a decimal.**



Summary 8.12

You can write every number as a decimal. Some fractions can be written as terminating decimals, while others can be written as repeating decimals. To write a fraction as a decimal, you can use long division.

For example, here is how you can use long division to rewrite $\frac{1}{15}$ as a decimal.

$$\begin{array}{r} 0.06666...\\ 15\overline{)1.00}\\ -90\\ \hline 100\\ -90\\ \hline 100\\ -90\\ \hline 10 \end{array}$$

To avoid writing the repeating part of a decimal over and over, you can use **bar notation**. For example, when writing $\frac{1}{15}$ as a decimal, you would write $0.06666...$ as $0.0\overline{6}$.

bar notation A way to represent the repeating digits of a decimal number where a small line is written over the digits that repeat.

Practice
8.12

Name: ______________________ Date: __________ Period: __________

Problems 1–4: Write each fraction as a decimal.

1. $\frac{99}{100}$

0.99

2. $\frac{7}{9}$

$0.\overline{7}$

3. $\frac{1}{15}$

$0.0\overline{6}$

4. $\frac{3}{7}$

$0.\overline{428571}$

5. Which decimal is the equivalent of $\frac{5}{18}$?

A. $0.\overline{27}$

B. 0.27

[✓ **C.**] $0.2\overline{7}$

D. $\overline{0.27}$

Problems 6–7: Determine whether the decimal representation of each fraction will repeat or terminate. Explain your thinking.

6. $\frac{1}{16}$

Terminate. *Explanations vary.* $\frac{1}{16} = \frac{1}{2 \cdot 2 \cdot 2 \cdot 2}$. The denominator only includes factors of 2, so the decimal will terminate.

7. $\frac{1}{12}$

Repeat. *Explanations vary.* $\frac{1}{12} = \frac{1}{2 \cdot 2 \cdot 3}$. The denominator includes factors other than 2 or 5, so it will repeat.

Practice
8.12

Name: .... Date: .... Period: ....

8. Order these numbers from *least* to *greatest*:

$1.04,\ 1.0\overline{47},\ 1.0\overline{4},\ 1.047,\ 1.04\overline{7}$

1.04	**$1.0\overline{4}$**	**1.047**	**$1.0\overline{47}$**	**$1.04\overline{7}$**

Least **Greatest**

9. What is the smallest whole number n that would make $\frac{n}{36}$ a terminating decimal?

9

Explain your thinking. ***Explanations vary.* Because $\frac{n}{36} = \frac{n}{2 \cdot 2 \cdot 3 \cdot 3}$, I need to eliminate factors other than 2 and 5. $n = 3 \cdot 3 = 9$ would result in the smallest denominator equivalent to a multiple of 2.**

Spiral Review

Problems 10–11: The numbers x and w are positive. Determine the exact value of each variable.

10. $x^2 = 90$

$x = \sqrt{90}$

11. $w^2 = 36$

$w = 6$

12. A restaurant serves food on 11-by-15 inch trays.

The trash can at the restaurant has a rectangular opening that measures 7-by-9 inches.

Jin claims that it is impossible to accidentally drop a tray in the trash can, because the shortest side of the tray is longer than either edge of the opening.

Is Jin's claim correct? Explain your thinking.

No. *Explanations vary.* I used the Pythagorean theorem to calculate the length of the diagonal of the trash can, $\sqrt{7^2 + 9^2} \approx 11.4$. The short side of the tray can fit diagonally through the opening of the trash can which measures approximately 11.4 inches.

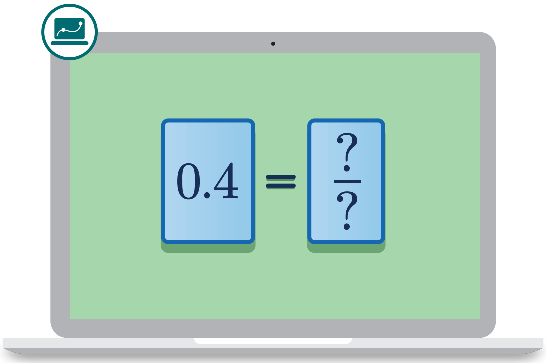
Unit 8
Lesson
13

Name: ____ Date: ____ Period: ____

Number Line Understanding 8.NS.1, SMP.2, SMP.7

Decimals to Fractions

Let's develop a strategy for rewriting repeating decimals as fractions.



Warm-Up

Determine the value of each expression mentally. Try to think of more than one strategy.

1 $234 - 34$

200

2 $9.7 - 0.7$

9

3 $100.\overline{25} - 99.\overline{25}$

1

4 $18.8\overline{3} - 1.4\overline{3}$

17.4

Activity
1

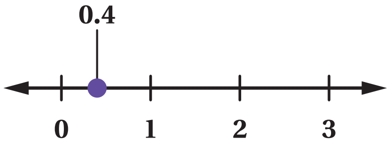
Name: ________ Date: ________ Period: ________

Terminating Decimals to Fractions

5 Write a fraction as close to 0.4 as you can.

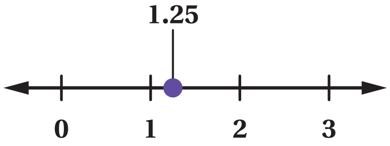
Use a calculator to check how close you are and revise as needed.

$\frac{2}{5}$ (or equivalent)



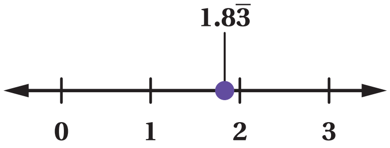
6 Write a fraction as close to 1.25 as you can.

$\frac{5}{4}$ (or equivalent)



7 Write a fraction as close to $1.8\overline{3}$ as you can.

$\frac{11}{6}$ (or equivalent)



Activity
2

Name: ________________ Date: ________ Period: ________

Repeating Decimals to Fractions

8 Here is some of the work Mai did to write $1.8\overline{3}$ as a fraction.

Mai

$$\begin{aligned} x &= 1.8\overline{3} \\ 10x &= 18.\overline{3} \\ 100x &= 183.\overline{3} \end{aligned}$$

$$\begin{aligned} 100x &= 183.\overline{3} \\ -(10x &= 18.\overline{3}) \\ \hline 90x &= 165 \\ x &= ? \end{aligned}$$

a **Discuss:** What did Mai do? Why do you think she chose these steps?

Responses vary.

- Mai set x equal to $1.8\overline{3}$ and created two equivalent equations by multiplying each side by 10 and 100. Mai then subtracted the two equations.
- I think Mai chose to multiply by 10 and 100 to create equations that were easy to subtract from each other, so she could get rid of the repeating decimal.

b Write $1.8\overline{3}$ as a fraction.

$\frac{165}{90}$ (or equivalent). *Work varies.*

$$\frac{90x}{90} = \frac{165}{90}$$

$$x = \frac{165}{90}$$

9 Use Mai's strategy to write $2.7\overline{4}$ as a fraction. Show your thinking.

$\frac{247}{90}$ (or equivalent). *Work varies.*

$$\begin{aligned} x &= 2.7\overline{4} \\ 10x &= 27.\overline{4} \\ 100x &= 274.\overline{4} \end{aligned}$$

$$\begin{aligned} 100x &= 274.\overline{4} \\ -(10x &= 27.\overline{4}) \\ \hline 90x &= 247 \\ x &= \frac{247}{90} \end{aligned}$$

Activity
3

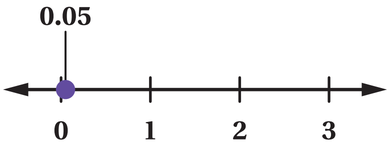
Name: .... Date: .... Period: ....

Repeated Challenges

10 Write each decimal as a fraction. Complete as many problems as you have time for.

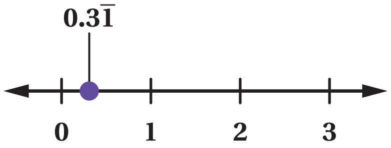
a 0.05

$\frac{1}{20}$ (or equivalent)



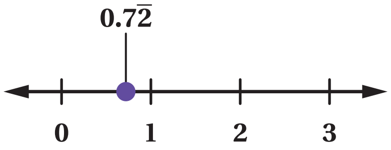
b $0.3\overline{1}$

$\frac{14}{45}$ (or equivalent)



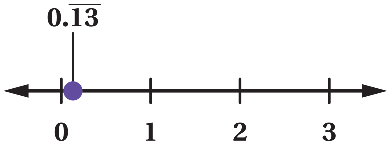
c $0.7\overline{2}$

$\frac{13}{18}$ (or equivalent)

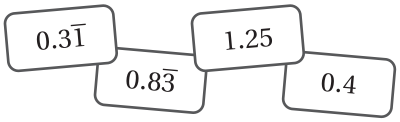


d $0.\overline{13}$

$\frac{13}{99}$ (or equivalent)



11 Synthesis



Discuss: How is writing a repeating decimal as a fraction like writing a terminating decimal as a fraction? How is it different?

Responses vary.

- **Both repeating and terminating decimals can be written as fractions by creating an equation where x equals the decimal. For repeating decimals, you can multiply the equation by 10 and again by 100 to create two new equations, then subtract the two equations to get rid of the repetition. Then you can solve for x to determine how to write the decimal in fraction form. For terminating decimals, you only need one equation because there is no repetition.**
- **A terminating decimal can be expressed as a fraction using place value, while a repeating decimal can't. For example, 0.4 can be read as "four tenths" and can be written as the fraction $\frac{4}{10}$.**

14 Summary 8.13

You can express all repeating decimals as fractions. One way to do this is to multiply equations by factors of 10 until the repeating decimals can subtract to 0. Once the repetition is removed, the resulting equation can be solved and left in fraction form.

For example, see these steps to represent $0.\overline{57} = 0.575757575...$ as a fraction.

If a decimal expansion of a number is a repeating or terminating decimal, the number can be written as a fraction. If the digits in the decimal expansion do not repeat (non-repeating) and do not terminate (non-terminating), then the number cannot be written as a fraction.

$$\begin{aligned} x &= 0.\overline{57} \\ 10x &= 5.\overline{75} \\ 100x &= 57.\overline{57} \end{aligned}$$

$$\begin{aligned} 100x &= 57.\overline{57} \\ -(x &= 0.\overline{57}) \\ \hline 99x &= 57 \\ x &= \frac{57}{99} \\ 0.\overline{57} &= \frac{57}{99} \end{aligned}$$

Practice
8.13

Name: ______________________ Date: __________ Period: __________

Problems 1–2: Here are the numbers 0.444 and $0.\overline{4}$.

1. How are the numbers alike?

***Responses vary.* They're both decimals between 0.4 and 0.5, and the first three digits in their decimal expansions are the same.**

2. How are the numbers different?

Responses vary.

- **$0.\overline{4}$ is greater than 0.444 because it has a greater digit in the ten-thousandths place.**
- **0.444 is a terminating decimal, while $0.\overline{4}$ is an infinitely repeating decimal.**

Problems 3–4: Match each fraction with its decimal representation.

3.

Decimal		**Fraction**
a.	$0.\overline{6}$	a $\frac{2}{3}$
b.	0.66	b $\frac{33}{50}$

4.

Decimal		**Fraction**
a.	$0.4\overline{8}$	d $\frac{7}{90}$
b.	$0.\overline{48}$	c $\frac{7}{100}$
c.	0.07	b $\frac{48}{99}$
d.	$0.0\overline{7}$	a $\frac{44}{90}$

Problems 5–10: Write each decimal as a fraction.

5. $3.\overline{45}$

$\frac{38}{11}$ **(or equivalent)**

6. 3.45

$\frac{69}{20}$ **(or equivalent)**

7. $0.\overline{7}$

$\frac{7}{9}$ **(or equivalent)**

8. $0.1\overline{3}$

$\frac{12}{90}$ **(or equivalent)**

9. $0.6\overline{38}$

$\frac{632}{990}$ **(or equivalent)**

10. $0.\overline{03}$

$\frac{3}{99}$ **(or equivalent)**

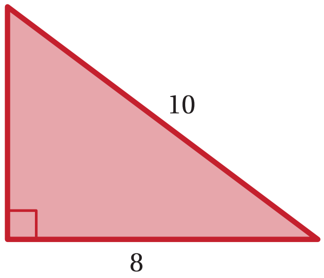
Practice
8.13

Name: ________________ Date: ________ Period: ________

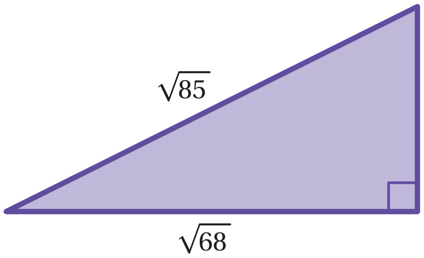
Spiral Review

Problems 11–12: Fill in the blank with the unknown side length of each right triangle.

11. **6**



12. **$\sqrt{17}$**



13. Determine whether each comparison is true or false.

Comparison	True	False
$2 > \sqrt{2}$	✓	
$\sqrt{15} < 3$		✓

14. Mohamed and Jalen are comparing $\sqrt{38}$ to 6.9. Mohamed says that $\sqrt{38}$ is greater than 6.9 and Jalen says that $\sqrt{38}$ is less than 6.9. Whose claim is correct? **Jalen's is correct.**

Explain your thinking.

Explanations vary. **I agree with Jalen's response because $\sqrt{38}$ is closer to $\sqrt{36}$ (which is equal to 6) than to $\sqrt{49}$ (which is equal to 7).**

15. Order the expressions from *least* to *greatest*.

$\sqrt[3]{125}$	$\sqrt{125}$	$\sqrt[3]{15}$	π	$\frac{11}{3}$

$\sqrt[3]{15}$	π	$\frac{11}{3}$	$\sqrt[3]{125}$	$\sqrt{125}$

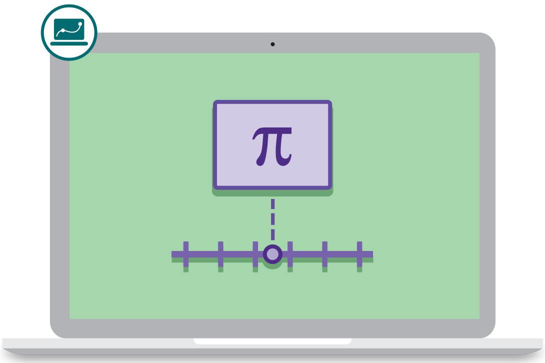
Least **Greatest**

Unit 8
Lesson
14

Name: ________ Date: ________ Period: ________

Number Line Understanding **Generalized Numbers Leading to Algebra**

8.NS.1, 8.NS.2, 8.EE.2, SMP.2, SMP.3, SMP.6



Hit the Target

Let's build an understanding of two new types of numbers.

Warm-Up

1 Which number is greater? Circle one.

[✓ $\sqrt{13}$] π I'm not sure

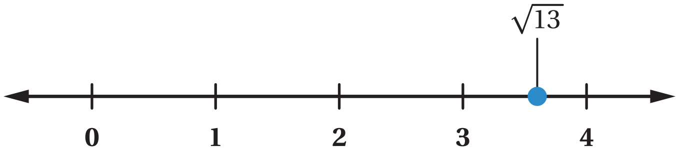
Explain your thinking.

Explanations vary. π is approximately equal to 3.14. $\sqrt{13}$ is greater than that because it's closer to $\sqrt{16} = 4$ than $\sqrt{9} = 3$.

Name: .... Date: .... Period: ....

Hit the Target

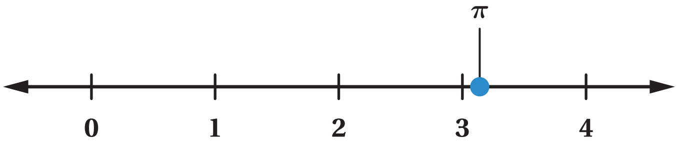
2 Write a fraction as close to $\sqrt{13}$ as you can (without using the square root symbol). Use a calculator to check how close you are, then revise your fraction to get as close to the target as possible.



Responses vary.

- $\frac{18}{5}$
- $\frac{361}{100}$
- $\frac{360555}{100000}$

3 Write a fraction as close to π as you can (without using the π symbol). Use a calculator to check how close you are, then revise your fraction to get as close to the target as possible.



Responses vary.

- $\frac{314}{100}$
- $\frac{22}{7}$
- $\frac{31415926}{10000000}$

Activity
2

Name: ………………………… Date: ………… Period: …………

Irrational Numbers

4 π and $\sqrt{13}$ are irrational numbers. How does your previous work build on what you know about irrational numbers?

Responses vary. **Irrational numbers are non-terminating, non-repeating decimals and cannot be written as a fraction.**

5 Is $\sqrt{\frac{9}{4}}$ rational or irrational? Circle one.

[✓ Rational] Irrational I'm not sure

Explain your thinking.

Explanations vary. **$\sqrt{\frac{9}{4}} = \frac{3}{2}$ because $\left(\frac{3}{2}\right)^2 = \frac{9}{4}$. Since $\sqrt{\frac{9}{4}}$ can be written as a fraction with non-zero integers for the numerator and denominator, it must be rational.**

6 Sort the numbers into groups based on whether they are rational or irrational.

$\frac{8}{4}$	$2\frac{3}{20}$	$1.\overline{73}$	$\sqrt{2}$
$2 \cdot \sqrt{13}$	$\sqrt{10}$	2π	$\sqrt{\frac{1}{4}}$
$\sqrt[3]{9}$	$\sqrt[3]{8}$	1.73205080757...	1.73

Rational	Irrational	I'm Not Sure
$\sqrt{\frac{1}{4}}$ 1.73 $1.\overline{73}$ $\frac{8}{4}$ $\sqrt[3]{8}$ $2\frac{3}{20}$	$\sqrt{2}$ $2 \cdot \sqrt{13}$ 2π $\sqrt[3]{9}$ $\sqrt{10}$ 1.73205080757...	**There is no expectation yet that students can prove that the numbers in the irrational category are irrational, so they may place them here instead.**

Activity
2

Name: ______________________ Date: __________ Period: __________

Irrational Numbers (continued)

7 Jada claims that any number written with a square root or a cube root is irrational. Is Jada correct? Circle one.

Yes [✓ No]

Explain your thinking.

***Explanations vary.* Many numbers written with a square root or a cube root are rational. For example, $\sqrt{16}$ is rational, as it equals $\frac{4}{1}$. So is $\sqrt[3]{\frac{8}{125}}$, as it equals $\frac{2}{5}$.**

You're invited to explore more.

8 Here are some problems to explore why $\sqrt{2}$ is irrational. Use a calculator if it helps with your thinking.

a $\left(\frac{577}{408}\right)^2$ is very close to 2, but is it exactly equal to 2?

***No.* Note: While some simpler calculators display 2 as the result for $\left(\frac{577}{408}\right)^2$, more accurate tools show that $\left(\frac{577}{408}\right)^2 > 2$.**

b If $\left(\frac{577}{408}\right)^2 = 2$, then $408^2 \cdot 2 = 577^2$. Diya says, "I know it's not true even though I haven't computed any of these numbers." How can Diya know this?

***Responses vary.* $408^2 \cdot 2$ is even. 577^2 is odd. Therefore, $408^2 \cdot 2$ cannot be equal to 577^2.**

c How does this show that $\frac{577}{408} \neq \sqrt{2}$?

***Responses vary.* $\sqrt{2}$ is the solution to the equation $x^2 = 2$. Since $\frac{577}{408}$ is not a solution to that equation, $\frac{577}{408} \neq \sqrt{2}$.**

d Is $\frac{1414213562375}{1000000000000} = \sqrt{2}$? Explain your thinking.

***No. Explanations vary.* $\sqrt{2}$ is the solution to the equation $x^2 = 2$. Since $\frac{1414213562375}{1000000000000}$ is not a solution to that equation, $\frac{1414213562375}{1000000000000} \neq \sqrt{2}$.**

9 Synthesis

Discuss: What is an irrational number? Give at least one example.

Responses vary. **An irrational number is a number that is not rational. In other words, an irrational number *cannot* be written as a fraction using non-zero integers for the numerator and denominator. $\sqrt{13}$ is an example of an irrational number.**

12 Summary 8.14

A rational number is a number that can be written as a fraction of two non-zero integers. An irrational number is a number that cannot be written as a fraction of two non-zero integers.

Here are some examples of rational and irrational numbers.

Examples of Rational Numbers

- Fractions: $\frac{10}{5}$, $3\,\frac{11}{20} = \frac{71}{20}$
- Terminating decimals: $1.5 = \frac{3}{2}$, $1.73 = \frac{173}{100}$
- Repeating decimals: $1.\overline{73} = \frac{172}{99}$, $0.1212... = \frac{12}{99}$
- Square roots of perfect squares and cube roots of perfect cubes: $\sqrt[3]{8} = \frac{2}{1}$, $\sqrt{64} = \frac{8}{1}$, $\sqrt{\frac{1}{9}} = \frac{1}{3}$

Examples of Irrational Numbers

- Non-terminating, non-repeating decimals: π, $0.743...$, $2.742050...$
- Square roots of non-perfect squares and cube roots of non-perfect cubes: $\sqrt{2}$, $3 \cdot \sqrt{5}$, $\sqrt[3]{9}$

Practice
8.14

Name: ________________ Date: ________ Period: ________

1. Sort the numbers based on whether they are rational or irrational.

0.1234	$-\sqrt{12}$	$\frac{-13}{3}$
$-\sqrt{100}$	$\sqrt{37}$	-77

Rational	Irrational
0.1234 **$\frac{-13}{3}$** **$-\sqrt{100}$** **-77**	**$-\sqrt{12}$** **$\sqrt{37}$**

2. Select *all* the rational numbers.

☐ **A.** π^2

☐ **B.** $\sqrt{14}$

☐ **C.** $-\sqrt{99}$

☑ **D.** $-\sqrt{25}$

☑ **E.** $-\frac{123}{45}$

☑ **F.** $\sqrt{64}$

3. Determine whether each number is rational or irrational.

Number	Rational	Irrational
$\frac{1}{\sqrt{16}}$	✓	
$\sqrt{10}$		✓
$-2\frac{1}{3}$	✓	
0.532	✓	

4. Eliza claims that $\sqrt{\frac{7}{9}}$ is rational. Is her claim correct?

No

Explain your thinking.

Explanations vary. **$\sqrt{\frac{7}{9}} = \frac{\sqrt{7}}{3}$. This is an irrational number because it cannot be written as a fraction where both the numerator and denominator are non-zero integers.**

Practice
8.14

Name: .... Date: .... Period: ....

5. Zoe claims that the solution to equation $x^3 = 30$ is irrational. Is her claim correct?

Yes

Explain your thinking.

Explanations vary. **The solution is $x = \sqrt[3]{30}$. $\sqrt[3]{30}$ is irrational because it is not a perfect cube, so it cannot be written as a terminating or repeating decimal.**

Spiral Review

Problems 6–7: Rewrite each expression as a single power.

6. $3^{-5} \bullet 4^{-5}$ $= 12^{-5}$ **(or equivalent)**

7. $(3^{-3})^2$ $= 3^{-6}$ **(or equivalent)**

Problems 8–9: Write an equation that expresses the relationship between a right triangle's side lengths if they measure:

8. 10, 6, and 8 units

Responses vary. $6^2 + 8^2 = 10^2$

9. $\sqrt{5}$, $\sqrt{3}$, and $\sqrt{8}$ units

Responses vary. $(\sqrt{3})^2 + (\sqrt{5})^2 = (\sqrt{8})^2$

10. A square has vertices $(0, 0)$, $(5, 2)$, $(3, 7)$, and $(-2, 5)$. Which of these statements is true?

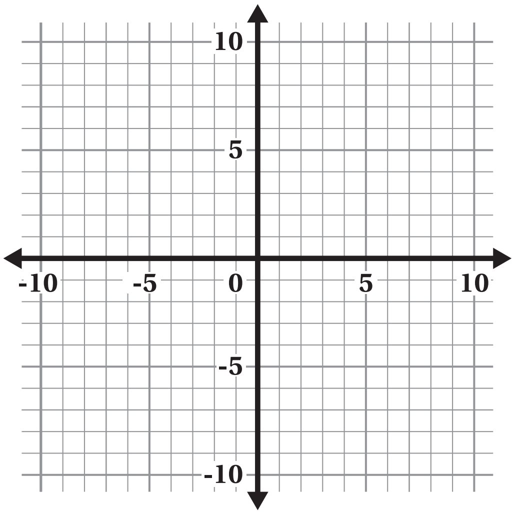
Use the graph if it helps with your thinking.

A. The square's side length is 5.

B. The square's side length is between 6 and 7.

[✓ **C.**] The square's side length is between 5 and 6.

D. The square's side length is 7.



11. Fill in each blank using the digits 0 to 9 only once to make the equation true.

$$\sqrt{\square\square} + \sqrt{\square\square} = \sqrt{\square\square}$$

Responses vary. $\sqrt{09} + \sqrt{25} = \sqrt{64}$

Name: .... Date: .... Period: ....

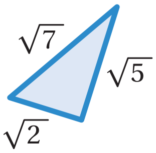
Pythagorean Explorations 8.EE.2, 8.G.7, 8.G.8, 8.NS.1, 8.NS.2, SMP.6

Practice Day 2

Let's practice what you've learned so far in this unit!

1. Which triangles are right triangles? Show your thinking. *Work varies.*

a.

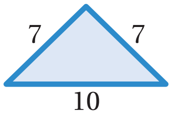


Circle one: [✓ Yes] No

$(\sqrt{2})^2 + (\sqrt{5})^2 \stackrel{?}{=} (\sqrt{7})^2$

$2 + 5 = 7$

b.

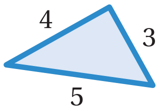


Circle one: Yes [✓ No]

$(7)^2 + (7)^2 \stackrel{?}{=} (10)^2$

$49 + 49 \neq 100$

c.

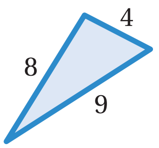


Circle one: [✓ Yes] No

$(3)^2 + (4)^2 \stackrel{?}{=} (5)^2$

$9 + 16 = 25$

d.



Circle one: Yes [✓ No]

$(8)^2 + (4)^2 \stackrel{?}{=} (9)^2$

$64 + 16 \neq 81$

2. Determine the exact length of the line segment between (-4, 5) and (6, -1).

$\sqrt{136}$ units

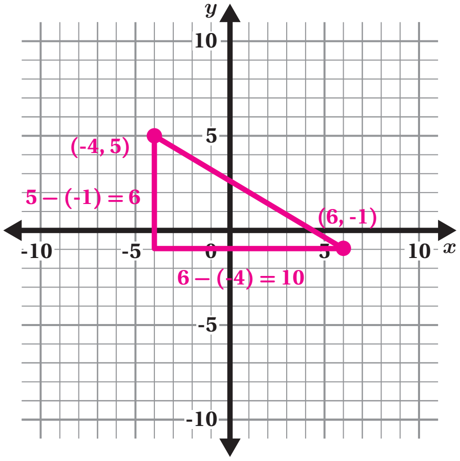
Show your thinking.

Work varies.

Vertical leg: $5 - (-1) = 6$

Horizontal leg: $6 - (-4) = 10$

Hypotenuse: $\sqrt{6^2 + 10^2} = \sqrt{136}$



3. Which of the following numbers is not a solution to the equation $x^2 = 16$?

A. 4

B. $\sqrt{16}$

[✓ C.] 8

D. -4

Explain your thinking.

Explanations vary. $8^2 = 64$, so $x = 8$ can't be a solution to this equation.

Name: .... Date: .... Period: ....

Practice Day 2

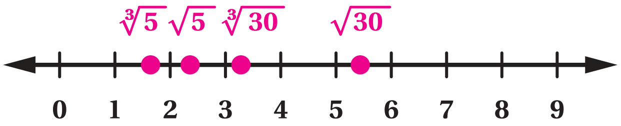
4. **a** Complete the table. Workspace:

Fraction	Decimal
$\frac{3}{10}$	**0.3**
$\frac{5}{8}$	**0.625**
$\frac{7}{12}$	$\mathbf{0.58\overline{3}}$
$\frac{23}{100}$ **(or equivalent)**	0.23
$\frac{23}{99}$ **(or equivalent)**	$0.\overline{23}$

b Are any of these numbers irrational? Explain your thinking.

No. *Explanations vary.* All of these numbers can be expressed as a fraction made up of non-zero integers, so they're all rational numbers.

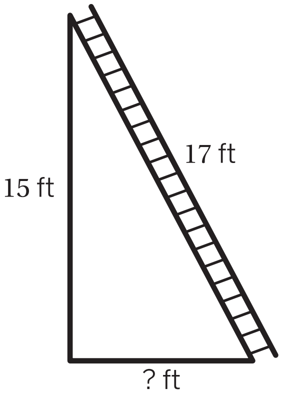
5. Plot the irrational numbers $\sqrt{5}$, $\sqrt{30}$, $\sqrt[3]{5}$, and $\sqrt[3]{30}$ on the number line. Show your thinking.



Work varies.

- $2^2 = 4$ **and** $3^2 = 9$**, so** $\sqrt{5}$ **is closer to 2 than 3.**
- $5^2 = 25$ **and** $6^2 = 36$**, so** $\sqrt{30}$ **is between 5 and 6.**
- $1^3 = 1$ **and** $2^3 = 8$**, so** $\sqrt[3]{5}$ **is between 1 and 2.**
- $3^3 = 27$ **and** $4^3 = 64$**, so** $\sqrt[3]{30}$ **is closer to 3 than 4.**

6. A 17-foot ladder leans against a wall. The ladder reaches a window 15 feet up the wall. How far from the wall is the base of the ladder? Show your thinking.



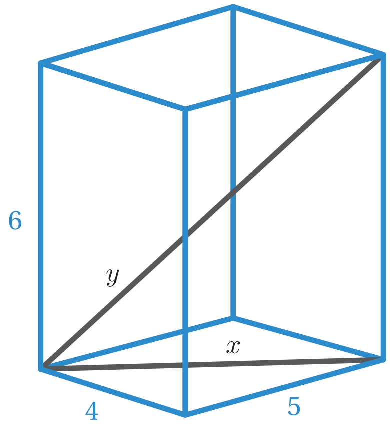
8 feet. *Work varies.*

$$\begin{aligned} 15^2 + x^2 &= 17^2 \\ 225 + x^2 &= 289 \\ x^2 &= 64 \\ x &= 8 \end{aligned}$$

Name: .... Date: .... Period: ....

Practice Day 2

7. Here is a rectangular prism.



a Calculate the exact length of segment x.

$\sqrt{41}$ units

Show your thinking.

Work varies.

$$(4)^2 + 5^2 = x^2$$
$$16 + 25 = x^2$$
$$41 = x^2$$
$$\sqrt{41} = x$$

b Calculate the exact length of segment y.

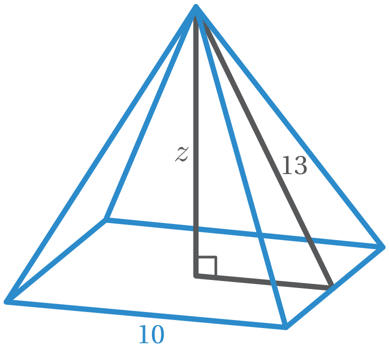
$\sqrt{77}$ units

Show your thinking.

Work varies.

$$(\sqrt{41})^2 + (6)^2 = y^2$$
$$41 + 36 = y^2$$
$$77 = y^2$$
$$\sqrt{77} = y$$

8. Calculate the exact height, z, of the triangular pyramid. $\sqrt{144} = 12$ units



Show your thinking.

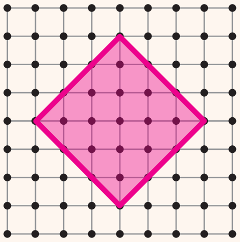
Work varies.

$$10 \div 2 = 5$$
$$(5)^2 + (z)^2 = 13^2$$
$$25 + z^2 = 169$$
$$z^2 = 169 - 25$$
$$z^2 = 144$$
$$z = \sqrt{144}$$

You're invited to explore more.

9. **a.** On the dot grid, draw a square with an area greater than 10 and less than 20 square units.

Responses vary.



b. Calculate the exact side length of the square you drew.

Responses vary.

- $\sqrt{18}$ units
- $\sqrt{17}$ units
- $\sqrt{13}$ units

Notes:

Career Connection

How does that dot on a digital map know your *exact* location?

PintoArt/Shutterstock.com.

While it has its roots in the Pythagorean Theorem, the technology behind our use of digital maps today – the Global Positioning System (GPS) – is built on the structures found in triangles, circles, and spheres. GPS is a network of at least 24 satellites that work together using math and technology to determine precise locations on Earth with great accuracy.

Geographic Information Systems (GIS) specialists design and maintain digital ways to collect, analyze, and display geographic data. They work in a variety of areas, including city planning, health, transportation, and environmental science.

“Gladys West” by Adrian Cadiz, Courtesy United States Air Force

Meet Gladys West

The next time you use a digital map on your phone or other device, think of pioneering mathematician Gladys West, one of GPS’ founders. Advances in GPS technology were made possible in large part because of her work. In the 1960s and 1970s, West worked for the Naval Proving Ground, where she collected critical data about Earth’s precise shape which was then used as a basis for the newly developed GPS system. You may be surprised to learn that we do *not* live on a perfect sphere!

Are you interested in studying geographic data? What can you do to learn more?

Math in the World

Two points determine a location in two dimensions and 3 points determine a location in three dimensions. However, to accurately determine a location on Earth, a GPS device must receive a signal from 4 *satellites!* The GPS device uses the 4 satellites to calculate its own latitude, longitude, altitude, and *time*. Why do you think time might be important when using GPS?

Responses vary.

Math Mindset

Describe a moment during this unit or in your everyday life where you used math to help solve a problem that you were unsure of solving at first. What steps did you take?

Responses vary.