

 **Amplify** Desmos Math **CALIFORNIA**

Grade 7

Volume 1: Units 1–4

Student Edition

About Amplify

Amplify is dedicated to collaborating with educators to create learning experiences that are rigorous and riveting for all students. Amplify creates K–12 core and supplemental curriculum, assessment, and intervention programs for today’s students.

A pioneer in K–12 education since 2000, Amplify is leading the way in next-generation curriculum and assessment. All of our programs provide teachers with powerful tools that help them understand and respond to the needs of every student.

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Amplify gratefully acknowledges the work of distinguished program advisors from English Learners Success Forum (ELSF), who have been integral in the development of Amplify Desmos Math. ELSF is a 501(c)(3) nonprofit organization whose mission is to expand educational equity for multilingual learners by increasing the supply of high-quality instructional materials that center their cultural and linguistic assets.

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Dear Student,

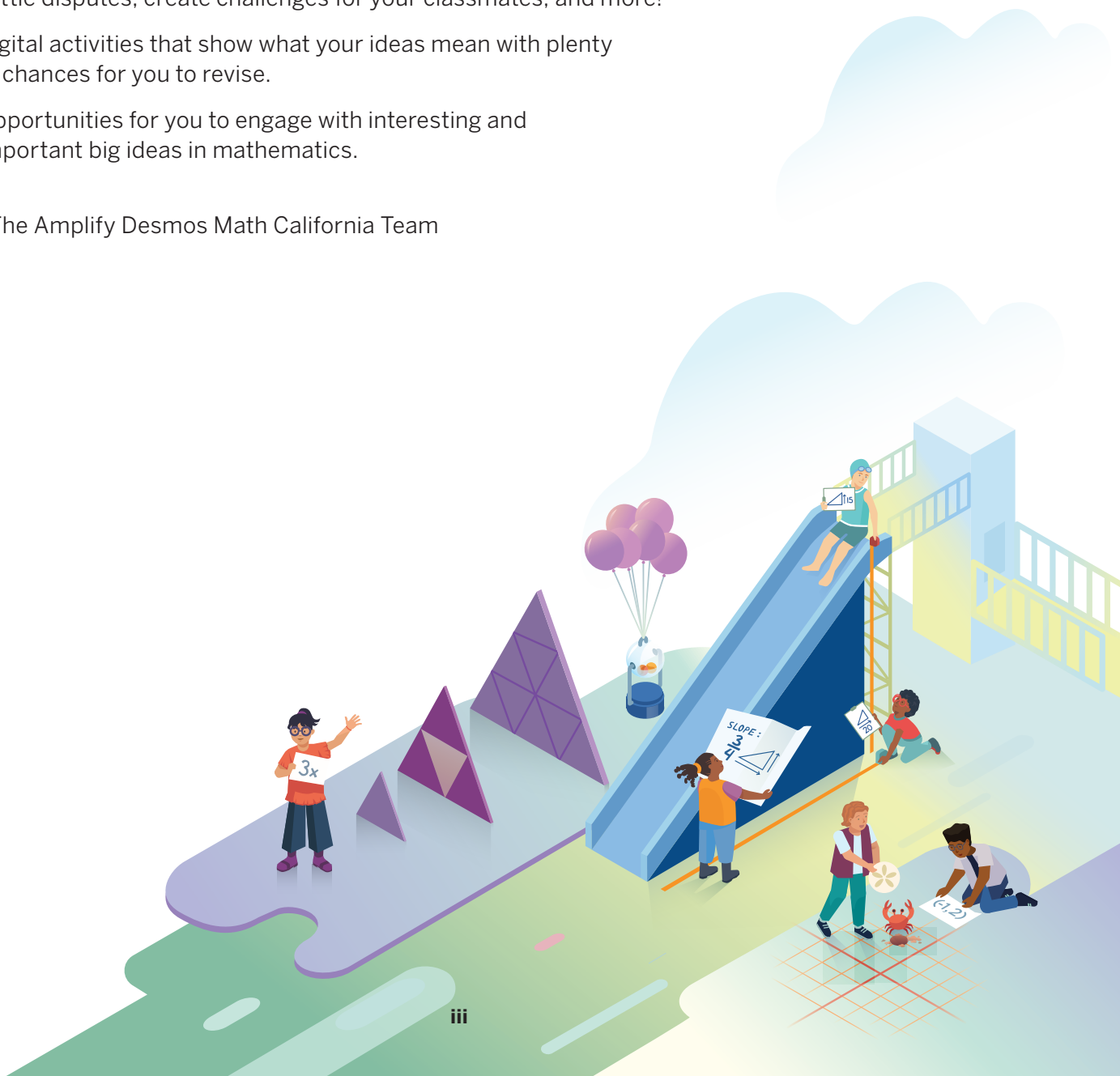
Welcome to Amplify Desmos Math California! We are excited to be partnering with you this year. You play an essential role in math class, so we wanted to reach out to introduce ourselves and tell you a bit about who we are.

Amplify Desmos Math California is a team of math educators on a mission to support you and your classmates in learning math. We hope each lesson inspires you to use your creativity, ask questions, and discover connections between math concepts and the world around us.

Here is what you can expect this year:

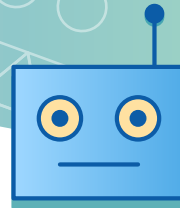
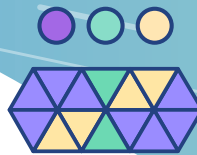
- A blend of learning on both paper and devices.
- Interactive lessons that encourage you to ask questions, explore, settle disputes, create challenges for your classmates, and more!
- Digital activities that show what your ideas mean with plenty of chances for you to revise.
- Opportunities for you to engage with interesting and important big ideas in mathematics.



–The Amplify Desmos Math California Team



Unit 1 Scale Drawings

In this unit, you will examine and create scaled copies of figures, as well as explore scale drawings of real-life objects.



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In this unit, you will explore proportional relationships in a variety of contexts and make comparisons using tables, equations, and graphs.



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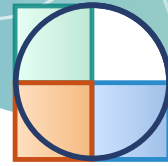
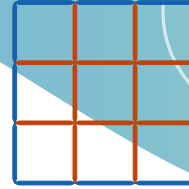
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Unit 3 Measuring Circles

In this unit, you will explore the relationships between the radius, diameter, circumference, and area of a circle. You will also practice writing and using equations to calculate missing measurements.

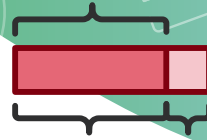


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In this unit, you will apply what you have learned about proportional relationships to solve problems involving percent change and fractional quantities.



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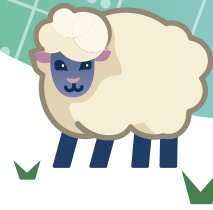
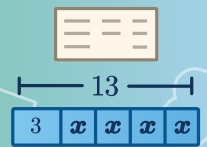
In this unit, you will develop and use a variety of strategies for adding, subtracting, multiplying, and dividing positive and negative numbers.



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In this unit, you will solve equations of the form $px + q = r$ and $p(x + q) = r$, and equations that include expanding, factoring, or combining like terms. You will also solve inequalities and graph their solutions on a number line.



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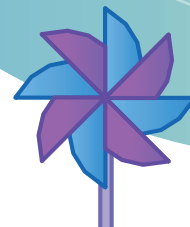
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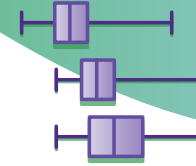
Unit 7 Angles, Triangles, and Prisms





In this unit, you will solve real-life and mathematical problems involving angle measures, volume, and surface area. You will also explore whether it is possible to draw no triangles, one triangle, or more than one triangle given three measures of sides or angles.

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Unit 8 Probability and Sampling



In this unit, you will learn about probability as a way to describe the likelihood of unknown events and use simulations to estimate the probability of real-world situations. You will also use samples to draw conclusions about and compare populations.

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Unit 1

Scale Drawings

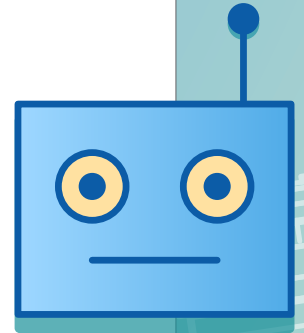
Big Ideas in This Unit


CC2 Proportional Relationships Unit Rates in the World

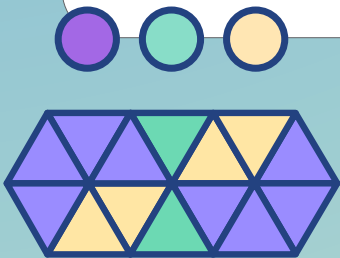
CC4 Scale Drawings Shapes in the World

Questions for Investigation

- How can you tell whether a figure is a scaled copy of another figure using unit rates and proportional reasoning?
- How are lengths, angles, and areas affected when creating scaled copies?
- How do scales and scale drawings allow us to represent figures in the real world?

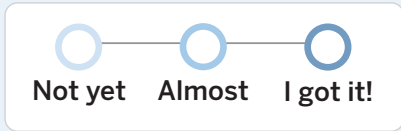


 **Explore: Scale-y Shapes**
Which shapes can be used to build larger copies of themselves?







Watch Your Knowledge Grow

This is the math you'll explore in this unit. Rate your understanding to see how your knowledge grows!



I can . . .	Before	After
Recognize proportional relationships between scaled copies.	<input type="radio"/> — <input type="radio"/> — <input type="radio"/>	<input type="radio"/> — <input type="radio"/> — <input type="radio"/>
Calculate actual lengths of a scale drawing.	<input type="radio"/> — <input type="radio"/> — <input type="radio"/>	<input type="radio"/> — <input type="radio"/> — <input type="radio"/>
Create a scale drawing using a different scale.	<input type="radio"/> — <input type="radio"/> — <input type="radio"/>	<input type="radio"/> — <input type="radio"/> — <input type="radio"/>
Solve problems involving scale drawings.	<input type="radio"/> — <input type="radio"/> — <input type="radio"/>	<input type="radio"/> — <input type="radio"/> — <input type="radio"/>
Decide if corresponding side lengths are in a proportional relationship.	<input type="radio"/> — <input type="radio"/> — <input type="radio"/>	<input type="radio"/> — <input type="radio"/> — <input type="radio"/>
Check whether side lengths are in equivalent ratios.	<input type="radio"/> — <input type="radio"/> — <input type="radio"/>	<input type="radio"/> — <input type="radio"/> — <input type="radio"/>
Determine the scale used by calculating a unit rate.	<input type="radio"/> — <input type="radio"/> — <input type="radio"/>	<input type="radio"/> — <input type="radio"/> — <input type="radio"/>
Recognize and solve problems involving a unit rate.	<input type="radio"/> — <input type="radio"/> — <input type="radio"/>	<input type="radio"/> — <input type="radio"/> — <input type="radio"/>
Calculate the actual area of a scaled drawing.	<input type="radio"/> — <input type="radio"/> — <input type="radio"/>	<input type="radio"/> — <input type="radio"/> — <input type="radio"/>
Solve real-world problems involving scale drawings.	<input type="radio"/> — <input type="radio"/> — <input type="radio"/>	<input type="radio"/> — <input type="radio"/> — <input type="radio"/>

I can . . .	Before	After
Solve problems involving scale drawings measured in like or different units.		
Solve problems involving scale drawings using fractions and decimals.		

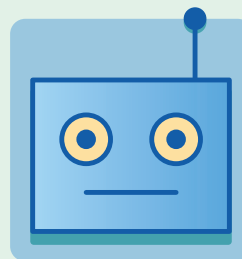
Scaled Copies



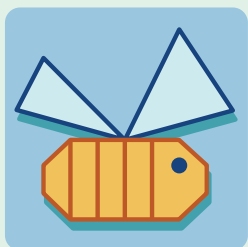
Explore
Scale-y Shapes



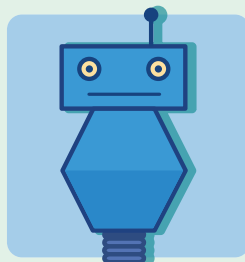
Lesson 1
Scaling Machines



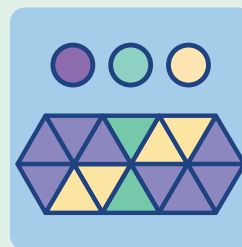
Lesson 2
Scaling Robots



Lesson 3
Make It Scale



Lesson 4
Scale Factor Challenges



Lesson 5
Tiles



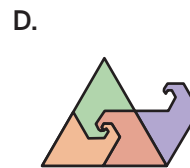
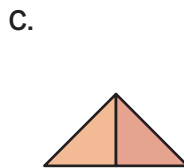
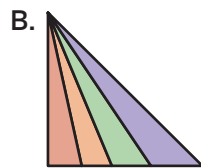
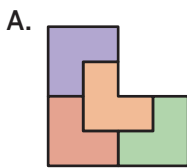
Explore: Scale-y Shapes

Which shapes can be used to build larger copies of themselves?



Warm-up

1. Study the images. Which image doesn't belong? Explain your thinking.





Rep-tiles

You will be given pattern blocks.

2. Follow the directions to build each shape. Then use the space provided on this page to trace both the original shape and the shape that you built.

- a** Using only triangles, build another triangle.



- b** Using only rhombuses, build another rhombus.



- c** Using only trapezoids, build two trapezoids.





Rep-tiles (continued)

3. Compare each original shape with the shapes you and your classmates built.

a What is the same about all of the triangles? What is different?

b What is the same about all of the rhombuses? What is different?

c What is the same about all of the trapezoids? What is different?

4. How many rhombuses are needed to build a rhombus that has side lengths twice as long as the original rhombus? Three times as long?

5. Consider the figures below. What do you notice? What do you wonder?



Figure A



Figure B

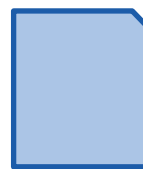


Figure C

a I notice . . .

b I wonder . . .



Building Math Habits of Mind



Discuss:

- Which of these habits of mind did you strengthen during this activity?
- How did you use the one(s) you selected?

I can slow down and first make sense of a challenging problem before trying to solve it.

— —
 Not yet Almost I got it!

I can represent real-world problems and interpret their solutions within the context of the problem.

— —
 Not yet Almost I got it!

I can justify my thinking and ask questions to help me understand the thinking of others.

— —
 Not yet Almost I got it!

I can apply the math that I know to solve real-world problems, make assumptions and revise my thinking as needed.

— —
 Not yet Almost I got it!

I can select an appropriate tool to help me solve problems.

— —
 Not yet Almost I got it!

I can communicate my thinking and solutions clearly to others.

— —
 Not yet Almost I got it!

I can look for structure or patterns to help me solve problems.

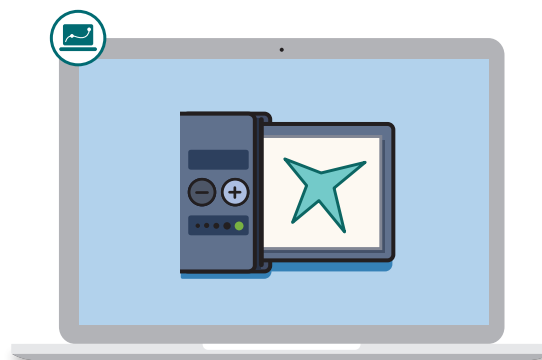
— —
 Not yet Almost I got it!

I can look for repeated calculations and other repeated steps to make generalizations.

— —
 Not yet Almost I got it!

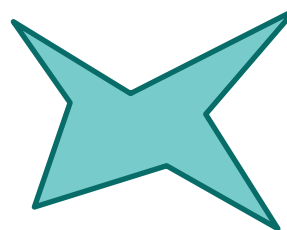
Scaling Machines

Let's describe the characteristics of scaled copies.



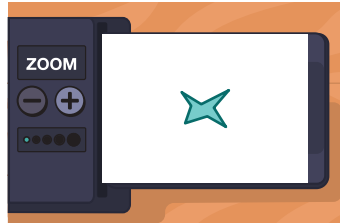
Warm-Up

- 1 What does this shape remind you of? Explain your thinking or tell a story about it.



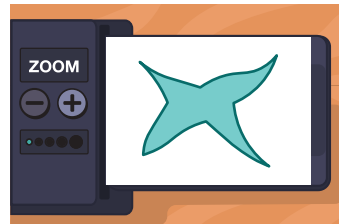
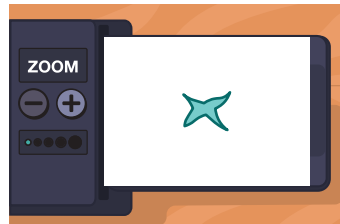
Printer Problems

2 Here is a working printer and printouts from two different zoom levels.



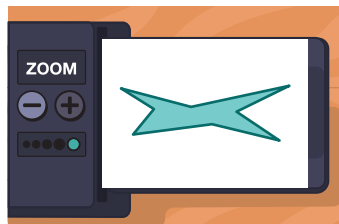
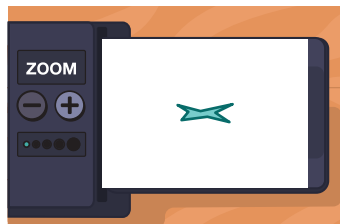
What stays the same no matter the zoom level? What changes?

3 Here is a *broken* printer and printouts from two different zoom levels.



Describe how the printer is broken.

4 Here is *another* broken printer that is broken in a *different* way.



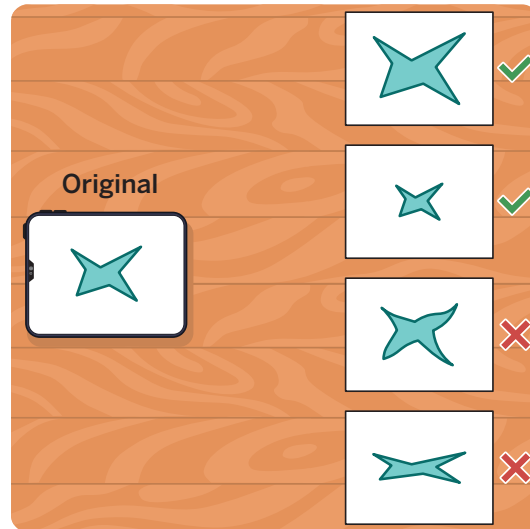
Describe how the printer is broken.

Printer Problems (continued)

5 Printouts 1 and 2 are scaled copies.

Printouts 3 and 4 are not.

Describe how you know if an image or a figure is a scaled copy.



6 Take turns sharing your first draft with a partner. Use these questions to help you give your partner feedback.

- How do you know that. . . ?
- What do you mean when you say. . . ?
- Can you say that another way?
- What is another example you could use?

7 Write a second draft that is stronger and clearer.

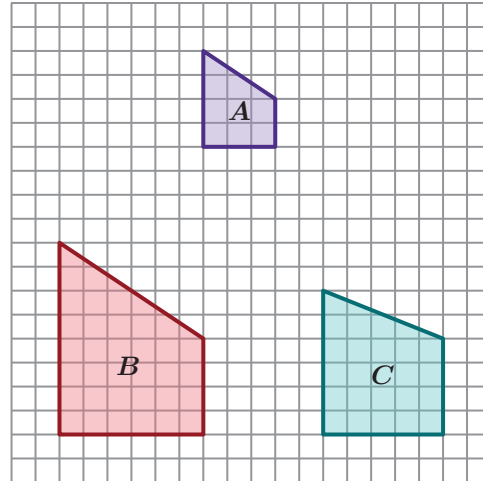
Scaled Copies

8 Here is figure *A* and two other figures.

Which is a scaled copy of figure *A*? Circle one.

Figure *B* Figure *C* Both Neither

Explain your thinking.



9 Scaled copies have **proportional** side lengths. This means that any two sides of the original shape form an *equivalent ratio* with the **corresponding** two sides of the copy. When part of an original figure matches up with part of a copy, we call them corresponding parts.

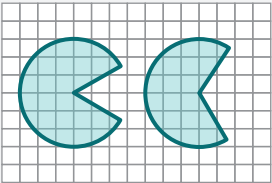
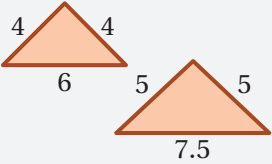
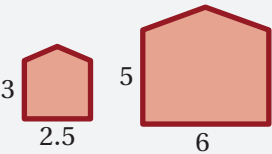
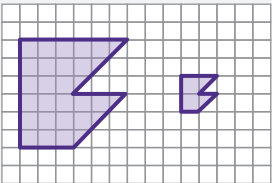
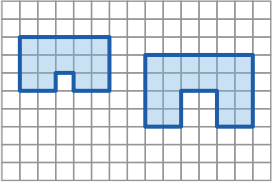
Here is one scaled copy of the shaded figure, along with its equivalent ratio.

Sketch and label a *different* scaled copy of the shaded figure.

Then sketch and label a figure that is *not* a scaled copy of the shaded figure.

Scaled Copies (continued)

10 For each row, put a checkmark to show whether the pairs of figures are scaled copies.

	Scaled Copies	Not Scaled Copies
		
		
		
		
		

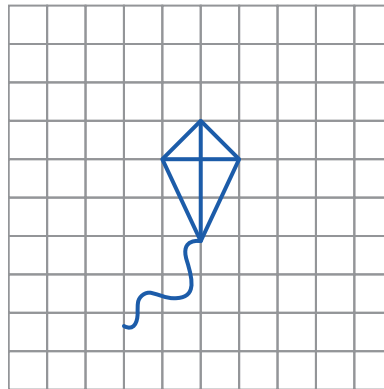
Machine Magic

11 Axel made a sketch.

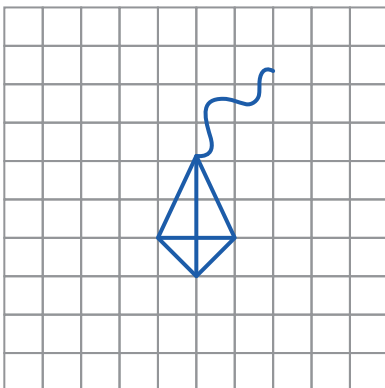
Here is what happened when four machines transformed his sketch.

Discuss: How does each machine work? Which machines make scaled copies?

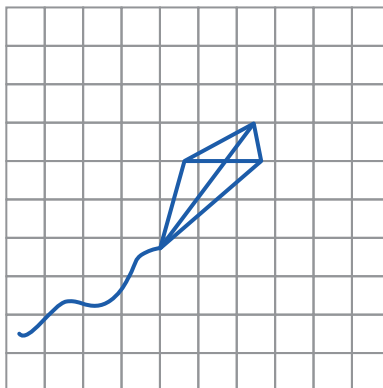
Axel's Sketch



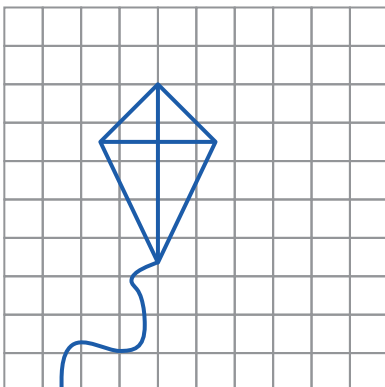
Machine 1



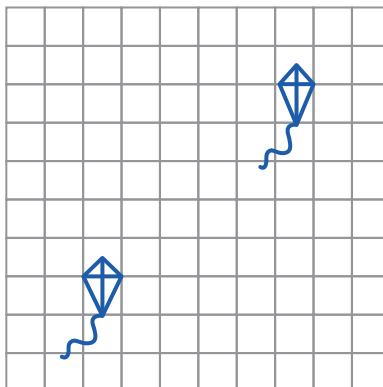
Machine 2



Machine 3

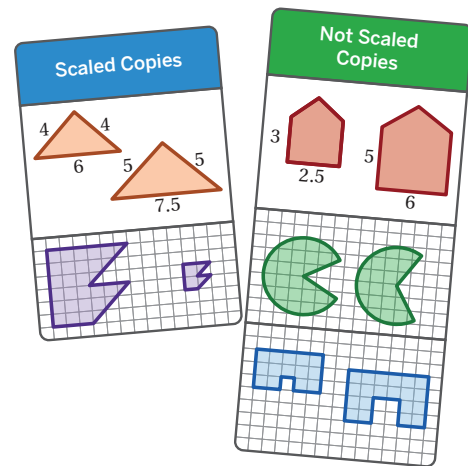


Machine 4



12 Synthesis

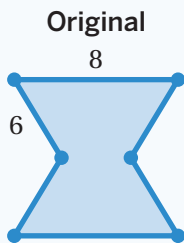
Discuss: How can you tell whether a figure is a scaled copy of another figure?



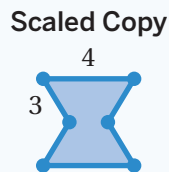
15 Summary 1.01

Scaled copies look similar to their originals. Though their overall size may change, their shape and angles do not. If a shape looks squished or stretched when compared to its original, it's likely not a scaled copy.

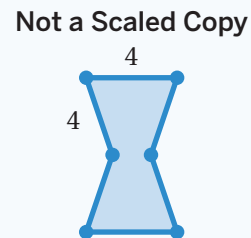
We know that two shapes are scaled copies when the lengths of their **corresponding** sides are **proportional** or form *equivalent ratios*.



Two of the sides in this figure have a ratio of $\frac{6}{8}$. Scaled copies should have an equivalent ratio.



The ratio here is $\frac{3}{4}$. This is a scaled copy because the ratio is equivalent to the original.



The ratio here is $\frac{4}{4}$. This is not a scaled copy because this ratio is not equivalent to the original.

correspond (corresponding parts) To correspond is to match. When part of an original figure matches up with part of a copy, we call them corresponding parts. These could be points, segments, angles, or distances.

proportional Quantities are proportional if they form equivalent ratios.

scaled copy A copy of an image that may change in size, but always maintains the shape and angle measurements of the original. If a shape looks squished or stretched when compared to its original, it is not a scaled copy. To create a scaled copy, we multiply every length in the original figure by the same number.

Practice 1.01

Name: _____ Date: _____ Period: _____

- Andre's grandma ordered school pictures. She thought she would receive the original portrait, but instead she got the following images. How is each image different from the original portrait of Andre?

Original Portrait



Image A



Image B



Image C

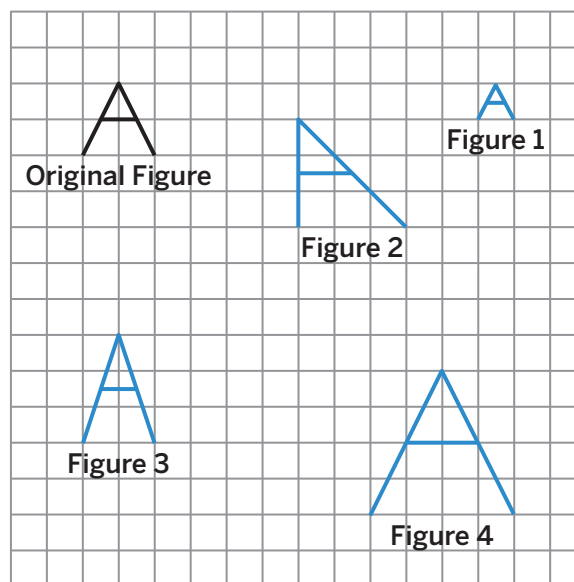


Image D



Problems 2–4: Here is the original figure along with several other figures.

- Which figures are scaled copies of the original figure?
- How does each scaled copy compare to the original?



- How do you know a figure is *not* a scaled copy?

Practice 1.01

Name: _____ Date: _____ Period: _____

Problems 5–6: Figures 1, 2, and 3 are scaled copies of each other. Each small square has a side length of 1 unit and an area of 1 square unit.

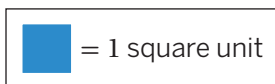


Figure 1

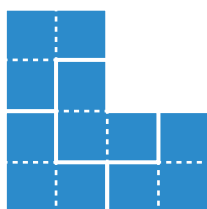


Figure 2

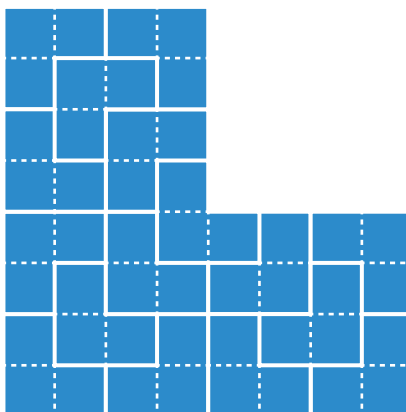


Figure 3

5. Complete the table.

	Figure 1	Figure 2	Figure 3
Perimeter (units)	8		
Area (sq. units)	3		

6. Describe any patterns you see.

Spiral Review

Problems 7–12: Complete each equation with a number that makes it true.

7. $5 \cdot \square = 15$

8. $6 \cdot \square = 9$

9. $\square \cdot 4 = 32$

10. $\square \cdot 12 = 3$

11. $21 \cdot \square = 7$

12. $\square \cdot 6 = 15$

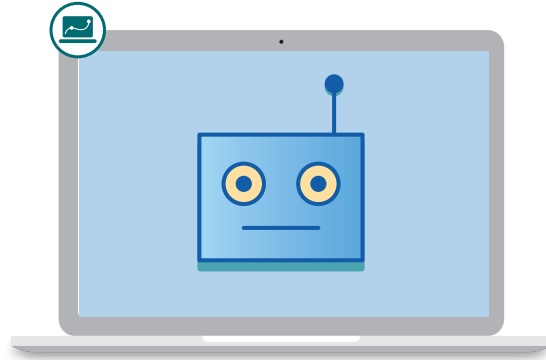
Unit 1
Lesson
2

Name: _____ Date: _____ Period: _____

Proportional Relationships Scale Drawings 📞 7.RP.2, 7.G.1, SMP.6, SMP.7

Scaling Robots

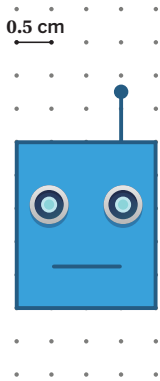
Let's explore scaled copies further.



Warm-Up

- 1 Create a robot face that you like. Your robot face should be a rectangle and include two eyes and an antenna. (See the example robot.) Then complete the table.

Example Robot



Height (cm)	2.5
Width (cm)	2
Eye Distance (cm)	1
Antenna (cm)	0.75

My Robot



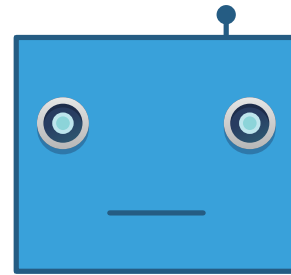
Height (cm)	
Width (cm)	
Eye Distance (cm)	
Antenna (cm)	

Scaling Robots

- 2** Here's Felipe's robot. He wants to make a scaled copy of his robot. Complete the table so that the new measurements represent a scaled copy of his original robot.

	Original Robot	New Robot
Height (cm)	5	10
Width (cm)	4	
Eye Distance (cm)	2.5	
Antenna (cm)	0.5	

Felipe's Robot



- 3** Here are Anh's and Imani's strategies for determining the lengths of the new robot, which is a scaled copy of the original robot.

Anh

	Original Robot	New Robot
Height (cm)	5	10
Width (cm)	3	6
Eye Distance (cm)	2	4
Antenna (cm)	1.5	3

$$\frac{\text{height}}{\text{width}} = \frac{5}{3} = \frac{10}{6}$$

$$\frac{\text{eye distance}}{\text{antenna}} = \frac{2}{1.5} = \frac{4}{3}$$

Imani

	Original Robot	New Robot
Height (cm)	5	10
Width (cm)	3	6
Eye Distance (cm)	2	4
Antenna (cm)	1.5	3

$$\frac{10}{5} = \frac{3}{1.5} = 2$$

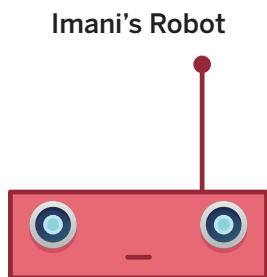
Discuss: How their strategies are alike? How are they different?

- 4** Anh's strategy compares ratios within each figure. Imani's strategy uses **scale factor**.

- a** Scaled copies always have a scale factor. What scale factor did Imani use?
- b** How did Imani calculate the scale factor?

Analyzing Robots

5-6 Imani made another robot and tried to make a scaled copy.



	Original Robot	New Robot
Height (cm)	2	8
Width (cm)	6	12
Eye Distance (cm)	4	10
Antenna (cm)	1	7


Do you think the new robot will be a scaled copy?

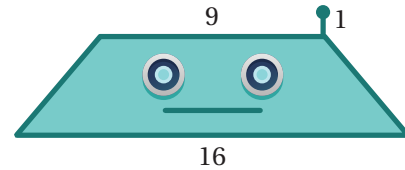
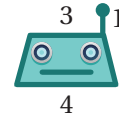
If yes, explain your thinking.

If no, cross out and replace one or more measurements so that the new robot *is* a scaled copy.

Analyzing Robots (continued)

7 Anushka made the small robot. Then she tried to make a scaled copy of the robot.

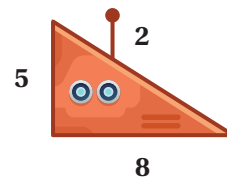
- a**  **Discuss:** How can you tell the two figures are not scaled copies?



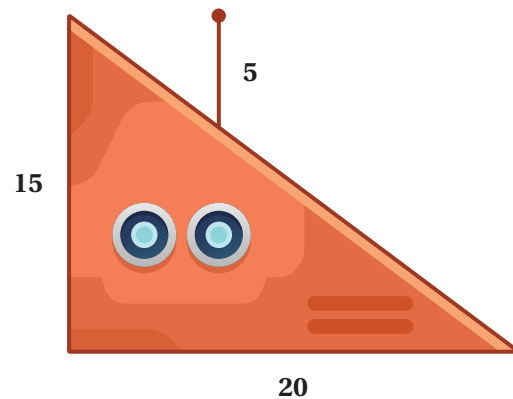
- b** How could you help Anushka revise her work?

8 Na'ilah drew a small and a large robot. Help her make the large robot a scaled copy.

- a** Cross out and replace one or more measurements on the large robot that could make it a scaled copy.



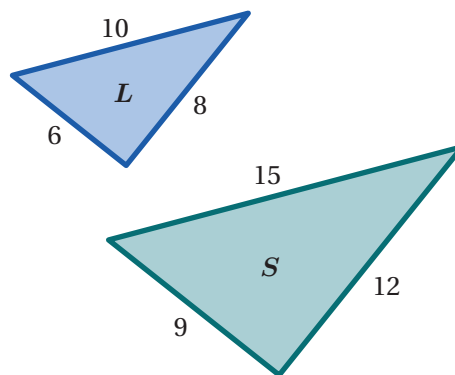
- b** What is the scale factor from the small to the large robot?



9 Synthesis

How can you use lengths to determine whether a figure could be a scaled copy of another figure?

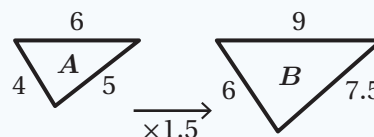
Use the example if it helps with your thinking.



12 Summary 1.02

When we create scaled copies, the **scale factor** is the number that every length of the original shape is multiplied by to produce the scaled copy.

For example, the scale factor from triangle *A* to triangle *B* is 1.5.



To find the scale factor, look for the ratio between *corresponding* sides in the two shapes. The ratio of the new length to the original length is the scale factor.

For triangle *A* and triangle *B*, the ratios are $\frac{9}{6}$, $\frac{7.5}{5}$, and $\frac{6}{4}$. Because those ratios are all equivalent, any one can be used as the scale factor. You can also use another equivalent ratio as the scale factor, like $\frac{3}{2}$ or 1.5.

Scaled copies always have pairs of corresponding sides, within each shape, that are proportional. For triangles *A* and *B*, the ratios could be $\frac{4}{5} = \frac{6}{7.5}$ and $\frac{5}{6} = \frac{7.5}{9}$. These equivalent ratios show that the shapes are scaled copies, but they do not give us the scale factor.

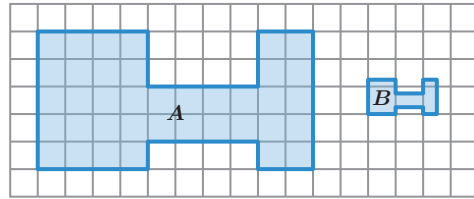
scale factor The number that every length of the original shape is multiplied by to produce the scaled copy.

Practice 1.02

Name: _____ Date: _____ Period: _____

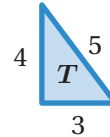
1. Figures *A* and *B* are scaled copies.

What scale factor takes *A* to *B*? Explain your thinking.



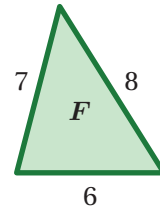
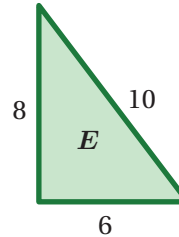
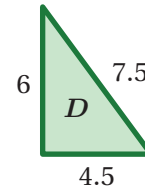
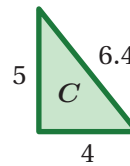
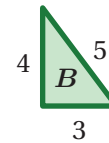
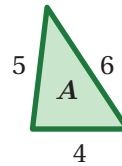
Problems 2–4: Here are seven triangles.

2. Which triangles are scaled copies of triangle *T*?



3. For each scaled copy, write the scale factor that takes triangle *T* to that triangle. Leave blank if it is not a scaled copy.

Triangle	Scale Factor
<i>A</i>	
<i>B</i>	
<i>C</i>	
<i>D</i>	
<i>E</i>	
<i>F</i>	



4. List two triangles whose lengths can be represented by the equivalent ratios $\frac{4}{8} = \frac{5}{10} = \frac{3}{6}$.

5. Using the digits 0 to 9 without repetition, fill in the blanks to make rectangles that are scaled copies.

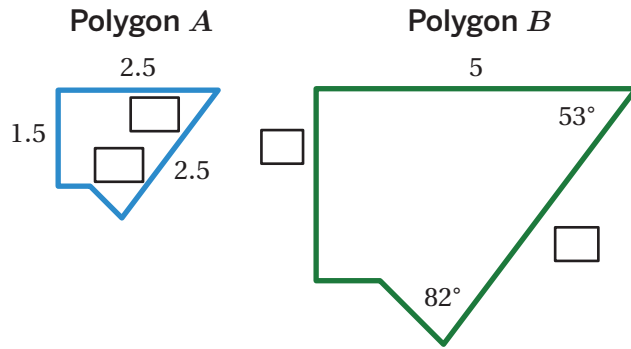


Practice 1.02

Name: _____ Date: _____ Period: _____

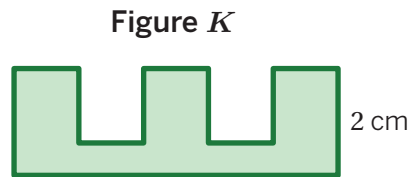
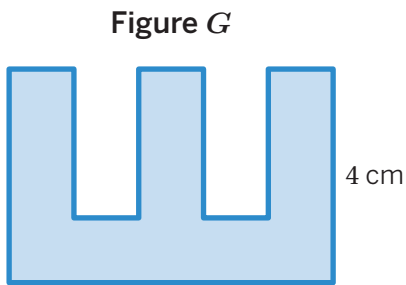
Problems 6–8: Polygon B is a scaled copy of polygon A .

6. What is the scale factor that takes polygon A to polygon B ? Explain your thinking.



7. Enter the missing lengths in polygon B . 8. Enter the missing angle measurements in polygon A .

9. Tyler says that figure K is a scaled copy of figure G . Is Tyler correct? Explain your thinking.



Spiral Review

Problems 10–13: Evaluate each expression.

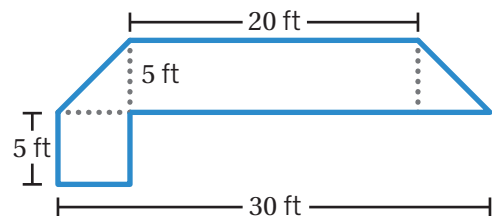
10. $\frac{1}{4} \cdot 32$

11. $\frac{1}{4} \cdot 5.6$

12. $7.2 \cdot \frac{1}{9}$

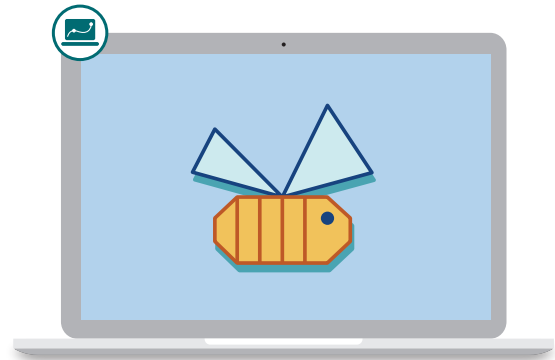
13. $2 \div \frac{1}{4}$

14. A patio in the shape of two triangles, a rectangle, and a square was built around the back of a house as shown. What is the area of the patio in square feet?



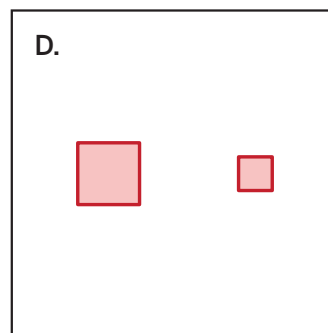
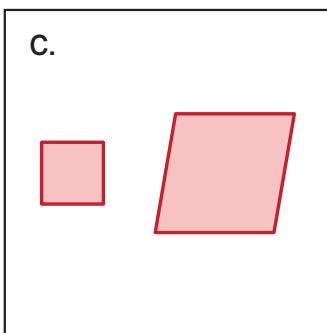
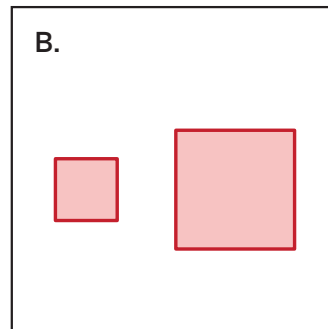
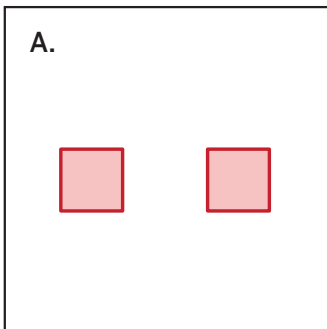
Make It Scale

Let's draw scaled copies.



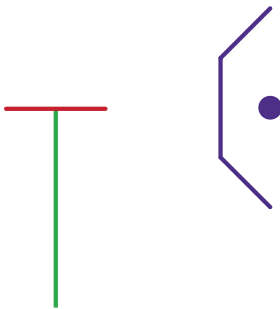
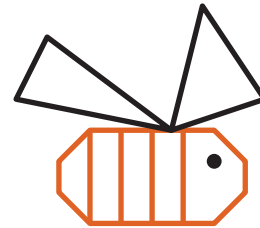
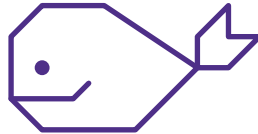
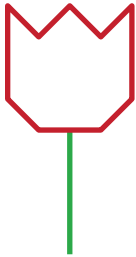
Warm-Up

1 Which pair doesn't belong? Explain your thinking.



Drawing Scaled Copies Without a Grid

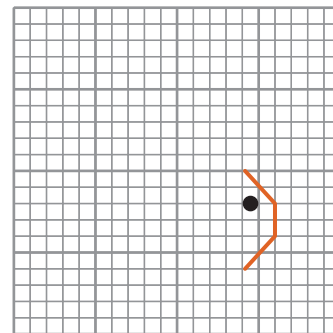
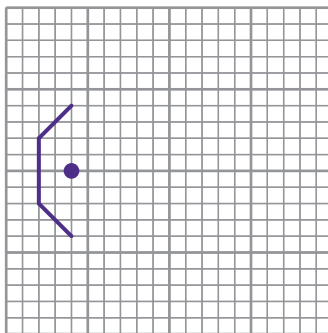
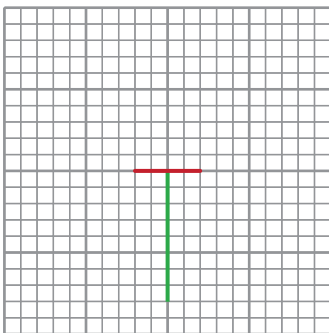
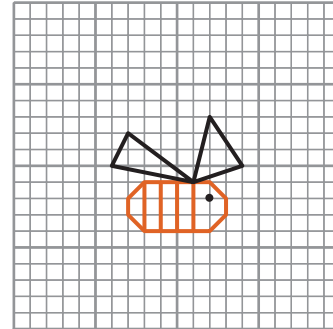
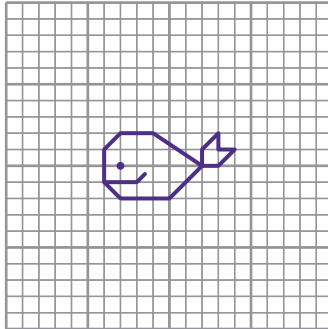
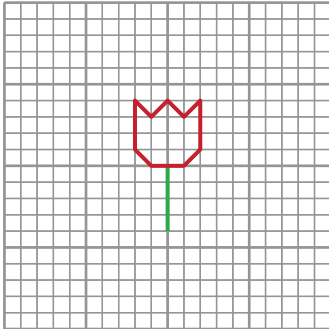
- 2-3** Choose *one* figure that you'd like to make a scaled copy of. Complete the scaled copy of the figure you chose. Use a scale factor of 2.



- 4** What might help you make a more accurate scaled copy?

Drawing Scaled Copies With a Grid

5 Make a new scaled copy of the figure you chose. Use a scale factor of 2.

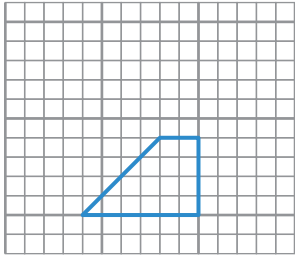


6 Explain the strategy you used to draw the new scaled copy.

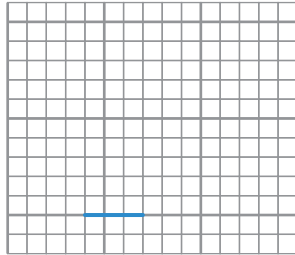
Drawing Scaled Copies With a Grid (continued)

7 Choose a scale factor of 0.5 or 1.5. Then complete the scaled copy.

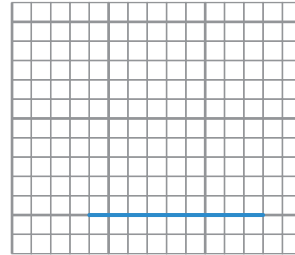
Original Figure



Scale factor: 0.5

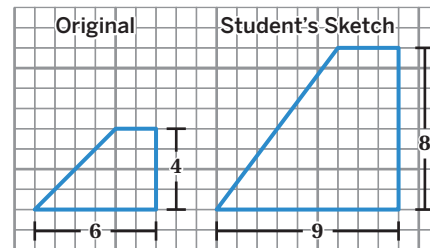


Scale factor: 1.5



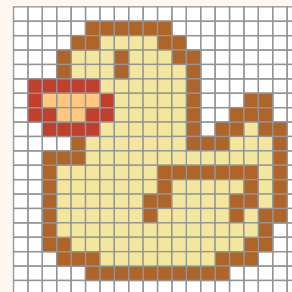
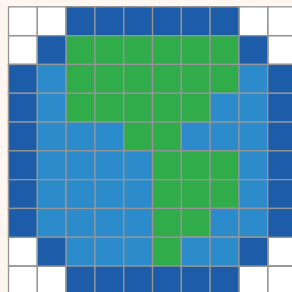
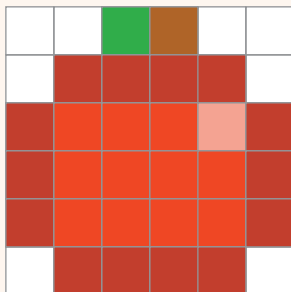
8 Here is one student's sketch. Sasha thinks the student used a scale factor of 2. Randy thinks the student used a scale factor of 1.5. Who is correct? Explain your thinking.

- A. Sasha B. Randy C. Neither




You're invited to explore more.

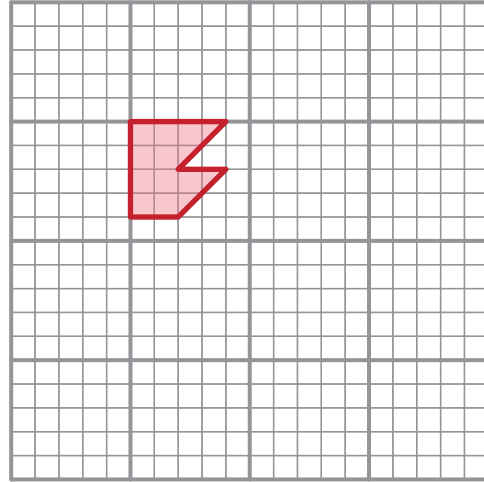
9 On a piece of graph paper, draw one of these images using a scale factor of 1.5. Or draw your own image and a scaled copy.



10 Synthesis

 **Discuss:** How can you draw a scaled copy?

Use the example if it helps with your thinking.

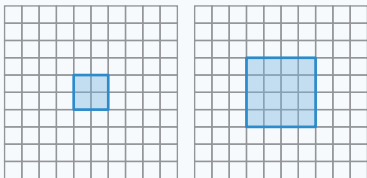


13 Summary 1.03

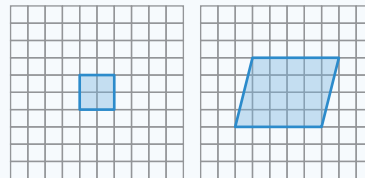
To create a scaled copy, we multiply all the side lengths in a shape by the same *scale factor*. This will create new side lengths, while keeping the angle measures and ratio between sides the same as the original.

To draw an accurate scaled copy, it's helpful to use measuring tools or a grid to make sure your drawing has the correct side lengths and angles.

Scaled Copies



Not Scaled Copies

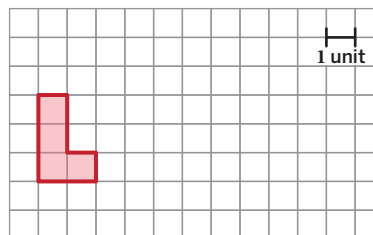


Practice 1.03

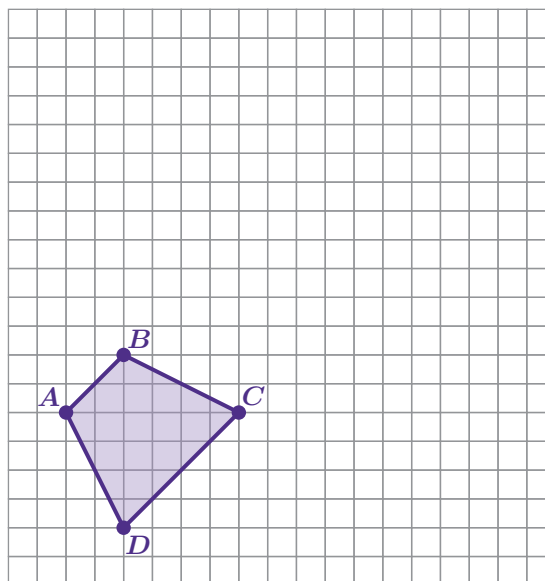
Name: _____ Date: _____ Period: _____

Problems 1–2: Here is a polygon.

1. Draw a scaled copy of the polygon using a scale factor of 2.
2. What is the area and perimeter of your scaled copy?

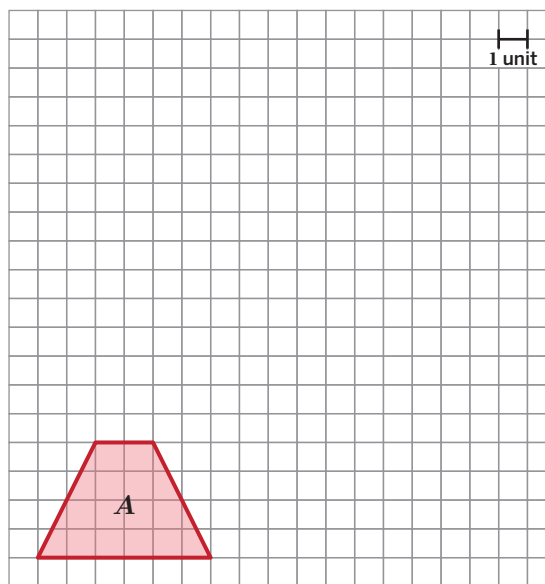


3. Draw a scaled copy of figure $ABCD$ using a scale factor of 1.5.



Problems 4–6: Imagine there is a quadrilateral B that is a scaled copy of quadrilateral A . Its shortest side is 5 units long.

4. What is the scale factor from quadrilateral A to B ?
5. What will be the measure of the longest side in quadrilateral B ?
6. Draw quadrilateral B .

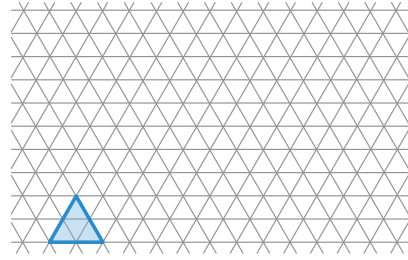


Practice 1.03

Name: _____ Date: _____ Period: _____

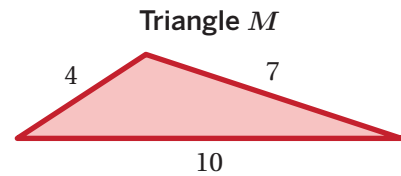
Problems 7–8: Here is an equilateral triangle.

7. Draw a scaled copy of this equilateral triangle using a scale factor of 3.
8. Equilateral triangles are always scaled copies. What other shapes are always scaled copies?

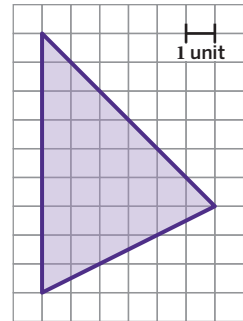


Spiral Review

9. Triangle Z is a scaled copy of triangle M . Select *all* sets of values that could be the side lengths of triangle Z .
 - A. 8, 11, and 14
 - B. 10, 17.5, and 25
 - C. 6, 9, and 11
 - D. 6, 10.5, and 15
 - E. 8, 14, and 20



10. Determine the area of this triangle.



Problems 11–13: Solve each equation. Show or explain your thinking.

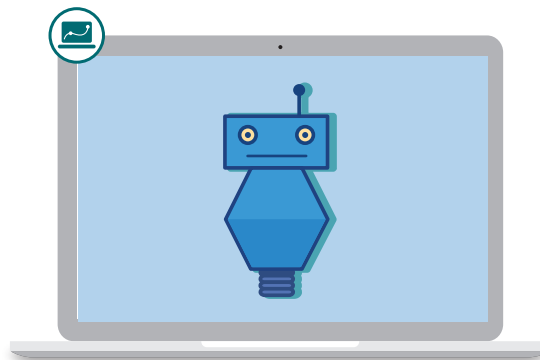
11. $6x = 156$

12. $16x = 8$

13. $\frac{1}{5}x = 1$

Scale Factor Challenges

Let's explore how scale factors affect the size of scaled copies.



Warm-Up

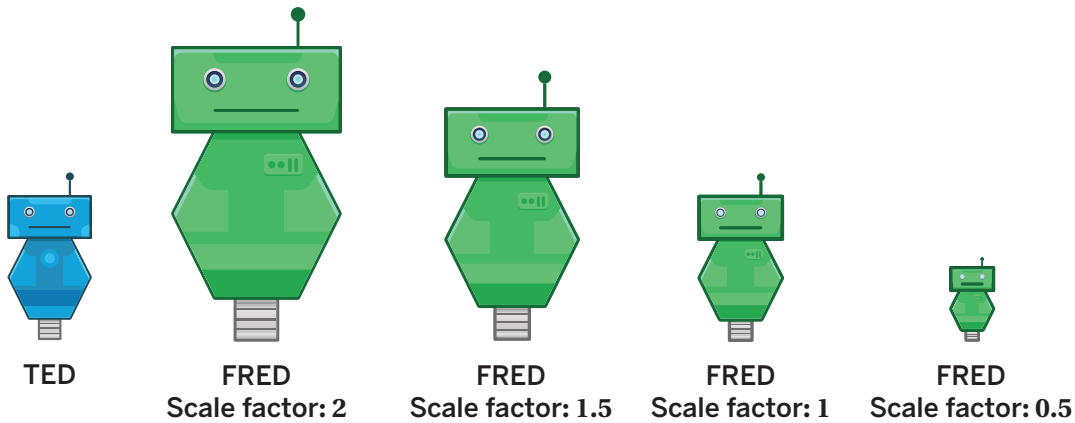
1 **a**  **Discuss:** How are these equations alike? How are they different?

- A. $8x = 80$
- B. $8x = 8$
- C. $8x = 1$
- D. $\frac{1}{8}x = 1$

b Use what you noticed to solve each equation mentally.

Exploring Scale Factors

2 Here is a robot: TED. The other robots are scaled copies of TED with different scale factors.



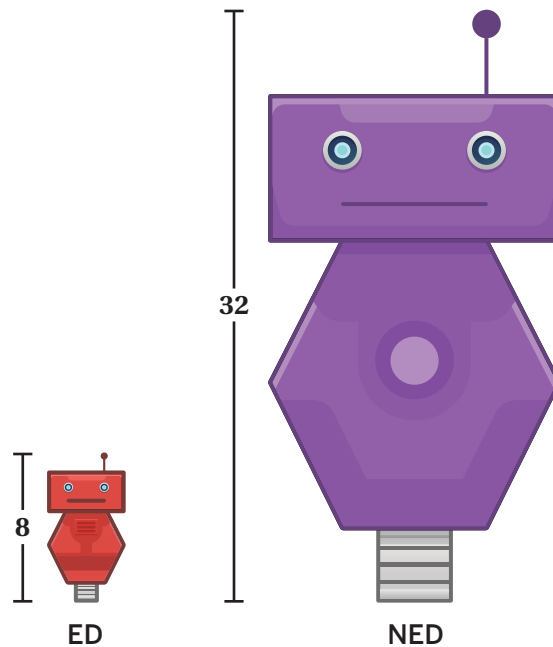
Discuss: What do you notice?

3 Here are two new robots: ED and NED. These robots are scaled copies of TED. In this lesson, all measurements are in grid units.

What scale factor will make ED match NED?

4 What scale factor will make NED match ED?

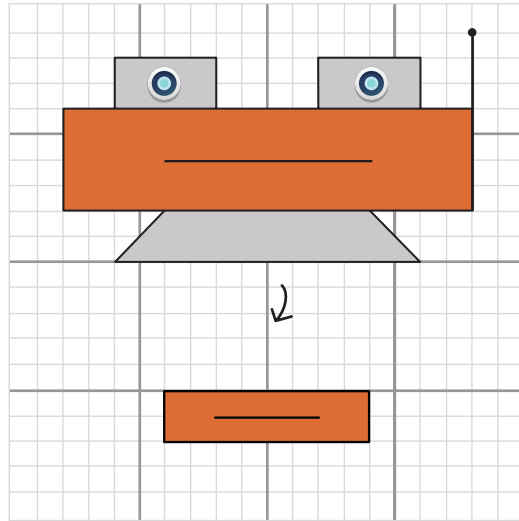
Explain your thinking.



Scaled Down and Back Again

5 Here is a robot called ROVER.

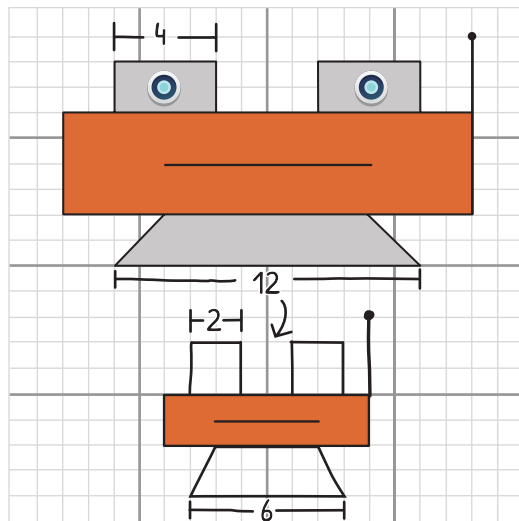
Complete the scale drawing of ROVER using a scale factor of $\frac{1}{2}$.



6 Here is Adhira's drawing from the previous problem.

a What is something Adhira did well?

b What is something Adhira can improve?



$$\frac{2}{4} = \frac{6}{12}$$

7 The scale factor from the original to the copy is $\frac{1}{2}$.

a What scale factor could you use to scale the copy back to the original?

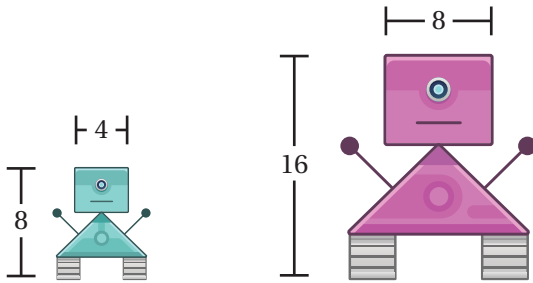
b How are these two scale factors related?

Practicing With Scale Factors

8 In this activity, all pairs of bots have corresponding measurements that are proportional.

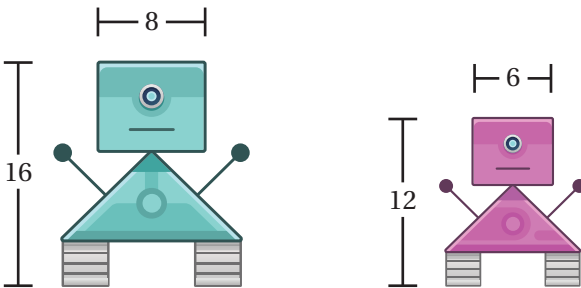
Determine a scale factor to make the bot on the left match the bot on the right.

a



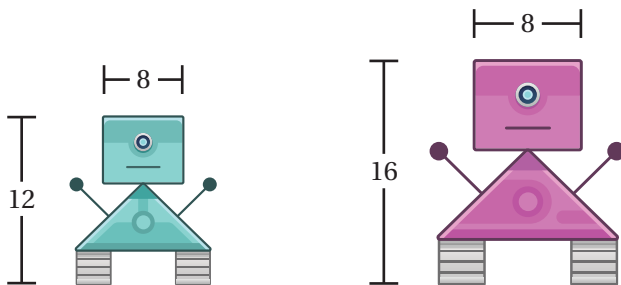
Scale factor:

b



Scale factor:

c



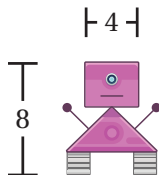
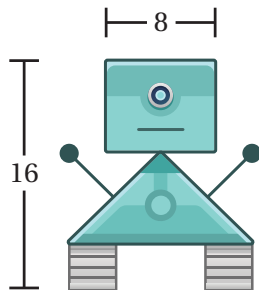
Scale factor:

Activity
3

Name: Date: Period:

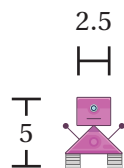
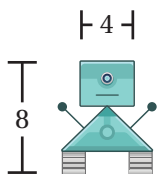
Practicing With Scale Factors (continued)

d



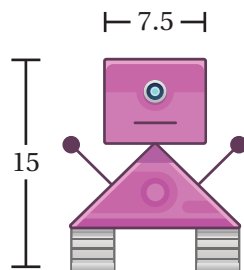
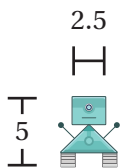
Scale factor:

e



Scale factor:

f




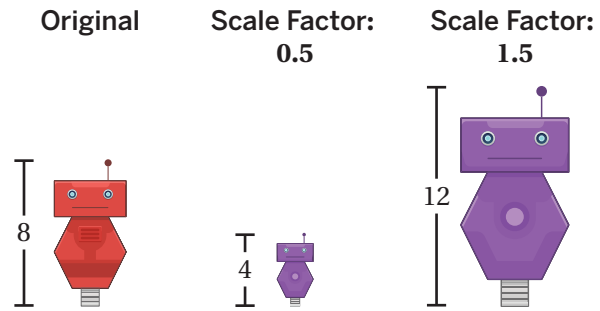
Scale factor:

You're invited to explore more.

- 9** Use the You're Invited to Explore More Sheet to design your own robot. Then complete its scaled copy.

10 Synthesis

 **Discuss:** How can you tell from the scale factor whether a scaled copy will be larger than, smaller than, or the same size as the original?

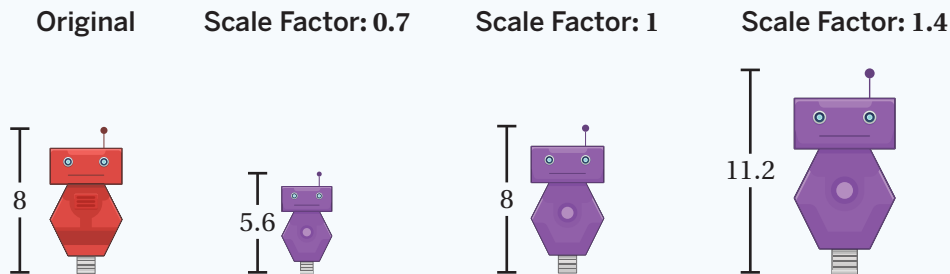


13 Summary 1.04

You can use different scale factors to create copies that are smaller, larger, or the same size as the original.

If the scale factor is:

- *Less than 1*, the copy will be *smaller* than the original.
- *Equal to 1*, the copy will be *the same size* as the original.
- *Greater than 1*, the copy will be *larger* than the original.



Between a pair of scaled copies, there are two possible scale factors. One scale factor takes the original to the scaled copy, and the other takes the copy back to the original. These two scale factors are reciprocals, so they always multiply to make 1.

Practice 1.04

Name: _____ Date: _____ Period: _____

- Rectangles P , Q , R , and S are scaled copies of one another. For each pair, state whether the scale factor that takes one figure to another is greater than 1, equal to 1, or less than 1.

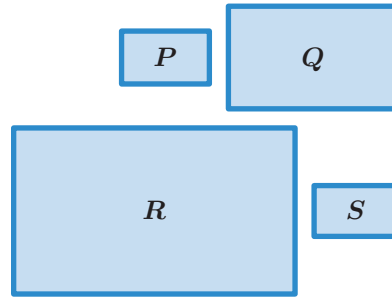
Rectangle P to rectangle R

Rectangle Q to rectangle S

Rectangle Q to rectangle R

Rectangle S to rectangle P

Rectangle R to rectangle P

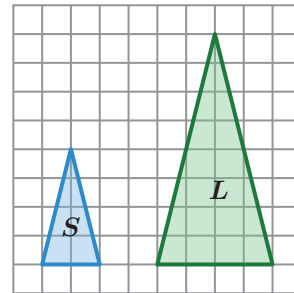


Problems 2–4: Triangle S and triangle L are scaled copies of one another.

- What is the scale factor that takes triangle S to triangle L ?

- What is the scale factor that takes triangle L to triangle S ?

- Triangle M (not shown) is also a scaled copy of triangle S . The scale factor that takes triangle S to triangle M is $\frac{3}{2}$. What is the scale factor that takes triangle M to triangle S ?

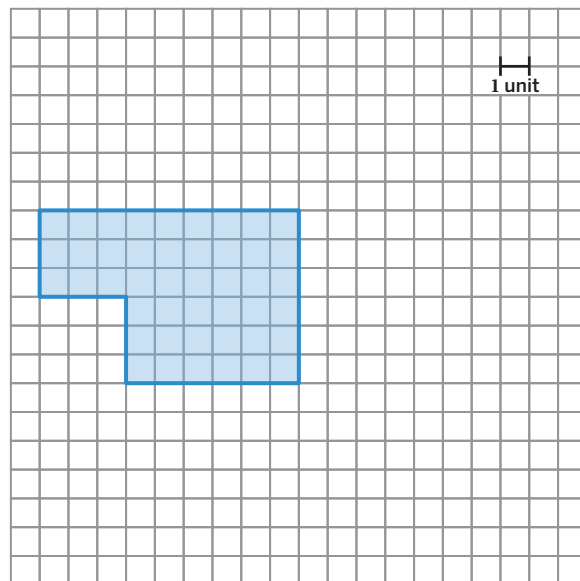


Problems 5–7: Here is a polygon.

- Draw a scaled copy of the polygon that has a perimeter of 10 units.

- What is the scale factor from the original polygon to your scaled copy?

- What is the scale factor from the scaled copy back to the original polygon?




Practice 1.04

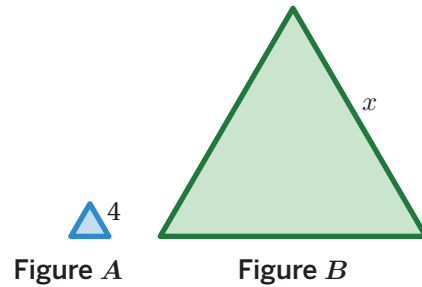
Name: _____ Date: _____ Period: _____

8. Will any two squares always be scaled copies of one another?

Explain your thinking.

 **Problems 9–10:** Figure B is a scaled copy of figure A with a scale factor of $5\frac{1}{2}$.

9. What is the value of x ?



10. What is the scale factor that would move Figure B to Figure A ?

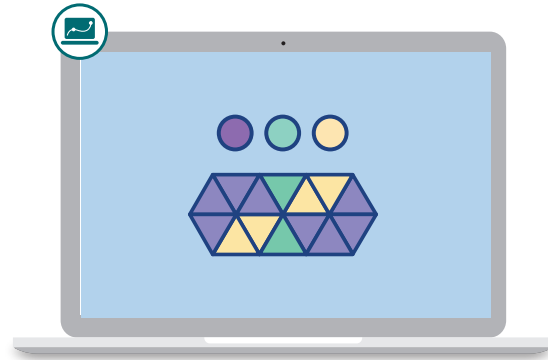
Spiral Review

11. Select *all* the ratios that are equivalent to $12 : 3$.

- | | |
|--------------------------------------|--|
| <input type="checkbox"/> A. 6 : 1 | <input type="checkbox"/> B. 1 : 4 |
| <input type="checkbox"/> C. 4 : 1 | <input type="checkbox"/> D. 24 : 6 |
| <input type="checkbox"/> E. 15 : 6 | <input type="checkbox"/> F. 1200 : 300 |
| <input type="checkbox"/> G. 112 : 13 | |

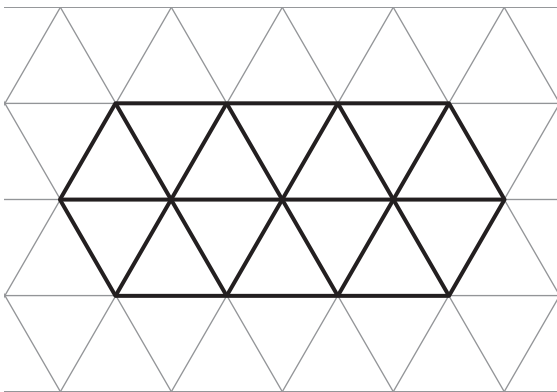
Tiles

Let's use tiles and mosaics to explore how area changes in scaled copies.



Warm-Up

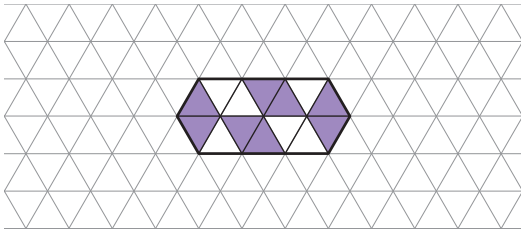
- 1 Create your own tile mosaic by coloring the triangles. Use at least two colors.



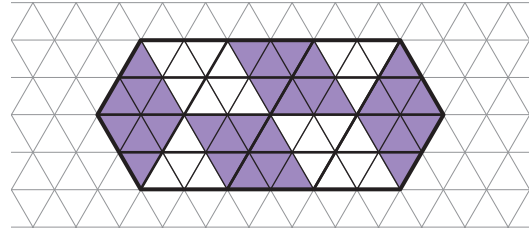
Scaling Mosaics

2 **a** Take a look at a mosaic and three scaled copies.

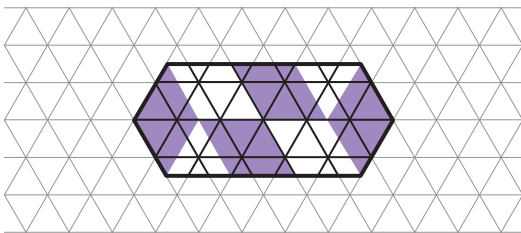
Original



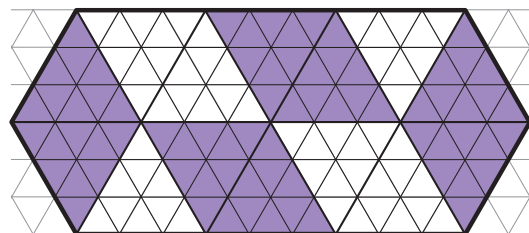
Scale factor = 2



Scale factor = 1.5



Scale factor = 3



b What do you notice? What do you wonder?

I notice:

I wonder:

3-4 Here is a table about the mosaic and one of the scaled copies.

a Complete the table.

b Describe any patterns you see.

Color	Number of Small Tiles in Original	Number of Small Tiles in copy (Scale Factor = 2)	Number of Small Tiles in copy (Scale Factor = 3)
Purple	8		
White	6		

Scaling Mosaics (continued)

5 Here is a new mosaic and its scaled copy.

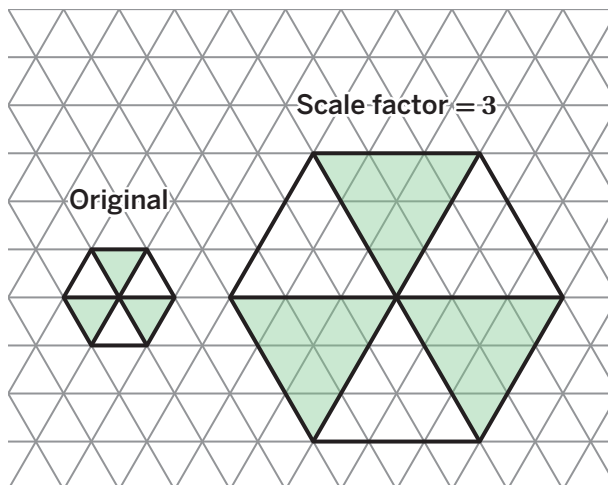
Lucy says the scaled copy has 3 times as many tiles as the original.

Malik claims the scaled copy has 6 times as many tiles as the original.

Whose claim is correct? Circle one.

Lucy Malik Both Neither

Explain your thinking.

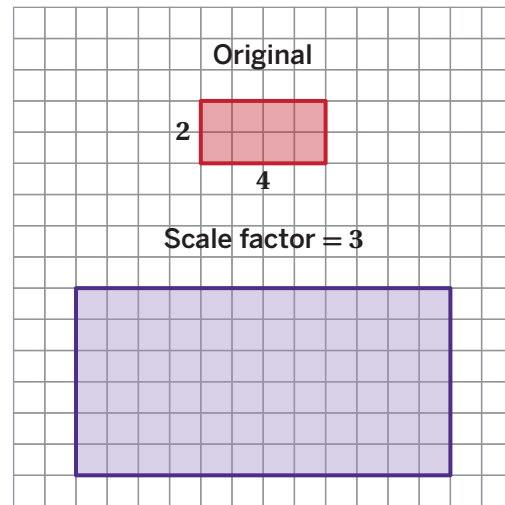


Area of Scaled Copies

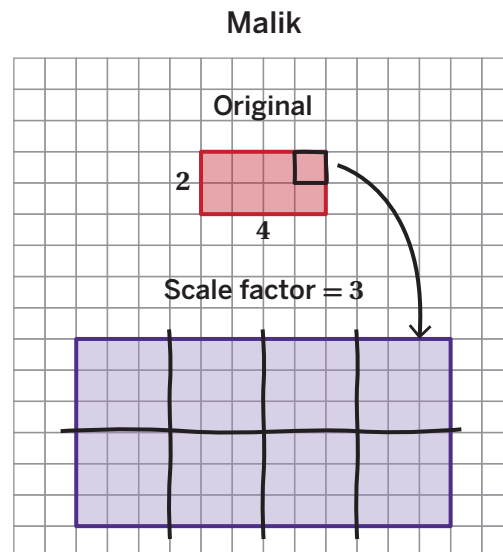
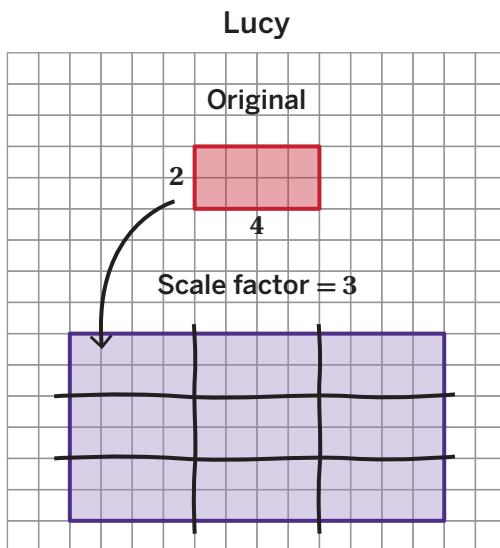
6 Here is a rectangle and a scaled copy.

The area of the original rectangle is 8 square units.

What is the area of the scaled copy?
Show or explain your thinking.



7 Here is Lucy's work and Malik's work for the previous problem.

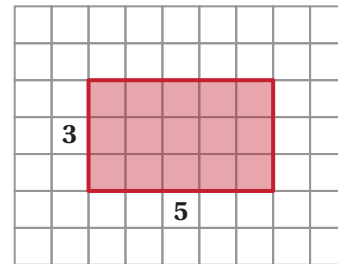


Explain how you think one student calculated the area.

Area of Scaled Copies (continued)

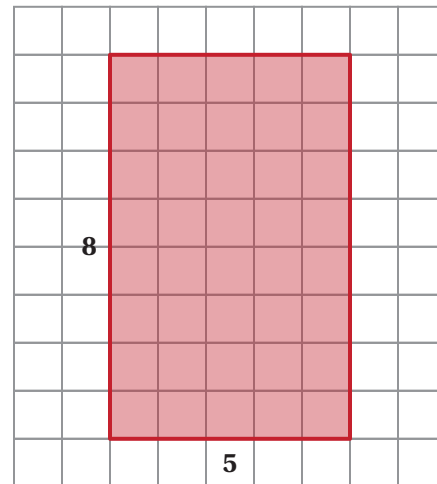
8 Imagine scaling this rectangle using a scale factor of 2.

What is the area of the scaled copy?



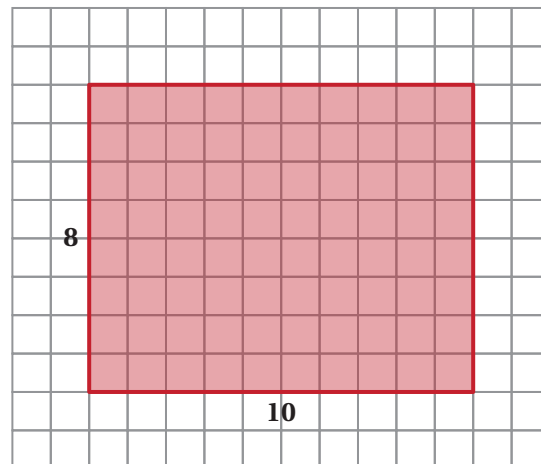
9 Imagine scaling this rectangle using a scale factor of 4.

What is the area of the scaled copy?



10 Imagine scaling this rectangle using a scale factor of $\frac{1}{2}$.

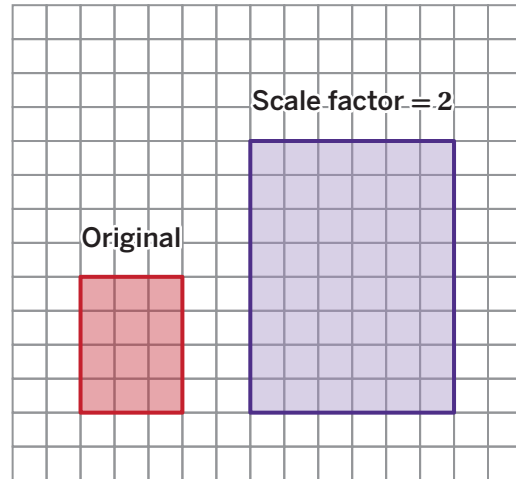
What is the area of the scaled copy?



11 Synthesis

Discuss: How can you determine the area of a rectangle's scaled copy?

Use the example if it helps with your thinking.



14 Summary 1.05

The scale factor describes how the side lengths of a shape change when it's scaled, but it doesn't directly describe how the area will change. You can use reasoning about the scale factor to find the area of a scaled copy. Here are some strategies to find the area of a scaled copy:

Scale the side lengths

1. Multiply each side length by the scale factor.

$$1 \times 2 = 2$$

$$3 \times 2 = 6$$

2. Calculate the area.

$$2 \times 6 = 12$$

Square the scale factor

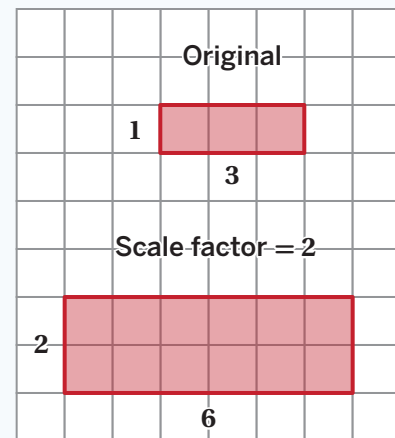
1. Determine the area of the original.

$$1 \times 3 = 3$$

2. Square the scale factor.

$$2 \times 2 = 4$$

3. Multiply the original area by the squared scale factor. $3 \times 4 = 12$



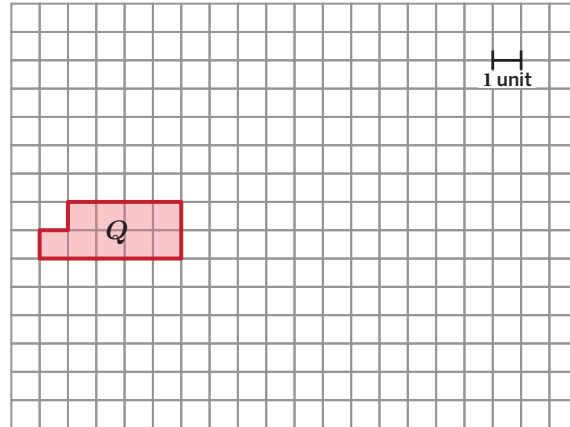
We can use either strategy to determine that the area of this scaled copy is 12 square units.

Practice 1.05

Name: _____ Date: _____ Period: _____

Problems 1–4: Here is polygon Q .


1. Draw a scaled copy of polygon Q using a scale factor of 2.
2. What is the area of the scaled copy?



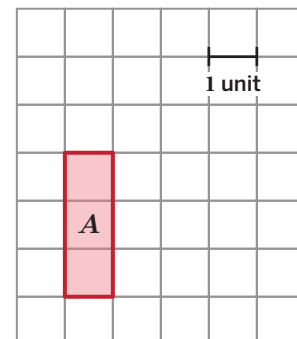
3. Complete the table for different scaled copies of polygon Q .

Scale Factor	Perimeter (units)	Area (sq. units)
1	14	9
	28	
3		

4. How is the relationship between the scale factor and perimeter different from the relationship between the scale factor and area?

 **Problems 5–6:** Ivory created a scaled copy of rectangle A . The area of Ivory's scaled copy is 48 square units.

5. How many times greater is the area of Ivory's scaled copy than the area of rectangle A ?
6. What scale factor did Ivory use to create the scaled copy? Show or explain your thinking.



Practice 1.05

Name: _____ Date: _____ Period: _____

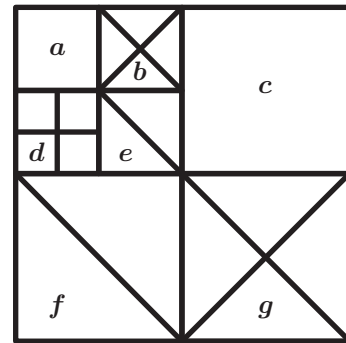
7. Imagine a right triangle with an area of 36 square units.

Scaled copies are made using different scale factors. Complete the table with the area of each scaled copy.

Scale Factor	Area (sq. units)
1	36
2	
3	
5	
$\frac{1}{2}$	
$\frac{2}{3}$	

8. The square shown is made up of scaled copies of a triangle and a square. The area of region d is 1 square unit. Determine as many different areas as you can.

Region	a	b	c	d	e	f	g
Area (sq. units)				1			

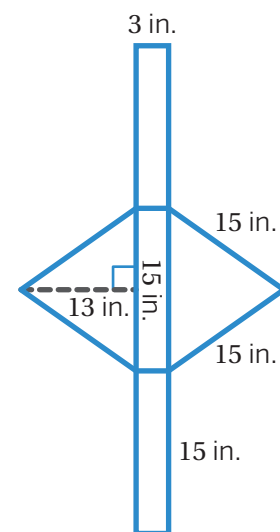


Spiral Review

9. This diagram shows the net of a triangular prism and its dimensions. What is the total surface area of the triangular prism?

- A. 135 square inches B. 195 square inches
C. 330 square inches D. 525 square inches

Show or explain your thinking.



10. Train A travels 70 meters in 2 seconds. Train B travels 150 meters in 3 seconds. Which train is faster?

Practice Day 1



Let's practice what you've learned so far in this unit!

You will use task cards for this Practice Day. Record all of your responses here.

Task A: Choose It!

Circle *all* that apply:

1. A B C D E

2. A B C D E

3. A B C D E

You're invited to explore more.

1. Explanation:

2. Explanation:

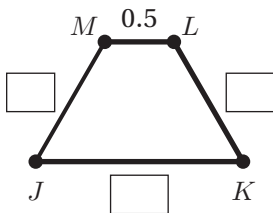
3. Explanation:

Task B: Fix It!

1. Explanation:

2. Advice:

3.



4. Response:

You're invited to explore more.

Solution: _____

Practice Day 1 (continued)

Task C: Draw It!

Figure *M*

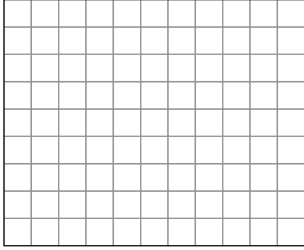


Figure *N*

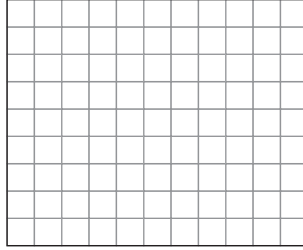
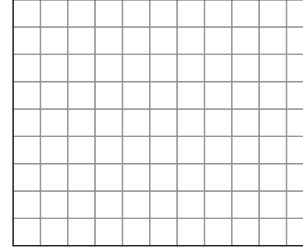
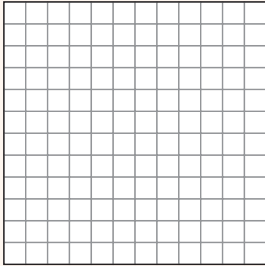


Figure *P*

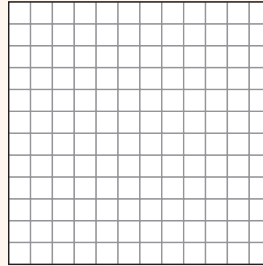


You're invited to explore more.

Original



Scaled Copy

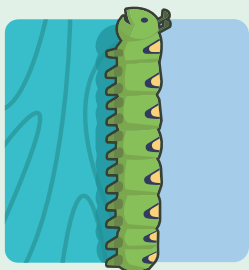


Task D: Reverse It!

1. From *A* to *B*
From *B* to *A*
2. From *A* to *B*
From *B* to *A*
3. From *A* to *B*
From *B* to *A*
4. Circle one: Yes or No
Explanation:

You're invited to explore more.

Scale Drawings



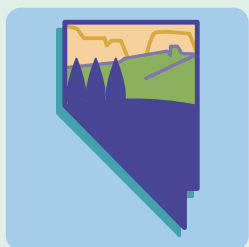
Lesson 6
Introducing Scale



Lesson 7
Will It Fit?



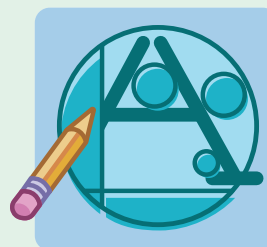
Lesson 8
Scaling States, Part 1



Lesson 9
Scaling States, Part 2



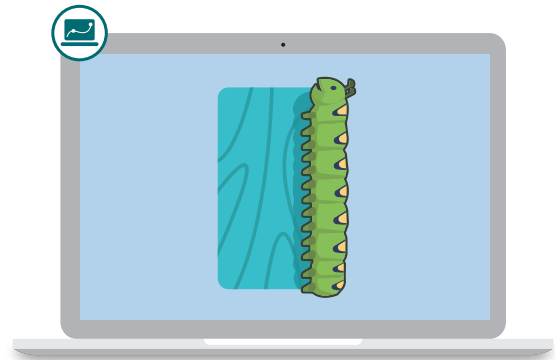
Lesson 10
Scaling Buildings



Lesson 11
Team Spirit

Introducing Scale

Let's explore scale diagrams.



Warm-Up

1-2 Each grid line is one centimeter.

<p>Object is actual size.</p>	<p>Real object is 0.016 times as large as drawing.</p>	<p>Real object is 10 times as large as drawing.</p>	<p>Real object is 2000 times as large as drawing.</p>
<p>Caterpillar</p>	<p>Poppy Seed</p>	<p>Dog</p>	<p>World's Tallest Tree</p>

What do you notice? What do you wonder?

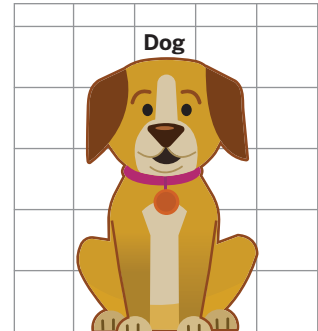
I notice:

I wonder:

Scale Factor

3 Estimate the dog's actual height. Remember that each grid line is 1 centimeter.

Real object is 10 times as large as drawing.

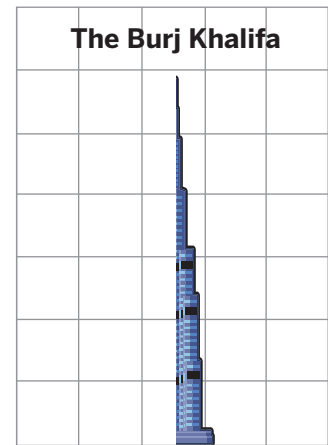
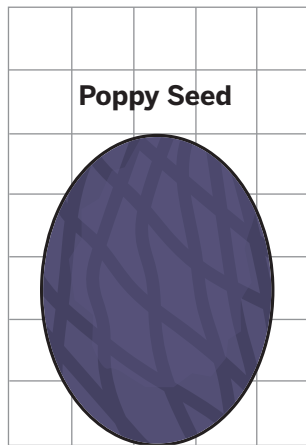


4 Estimate the actual heights of the poppy seed and the Burj Khalifa (the world's tallest building). Remember that each grid line is 1 centimeter.

Real object is 0.016 times as large as drawing.

Real object is 14000 times as large as drawing.

Object	Actual Height (cm)
Poppy seed	
The Burj Khalifa	

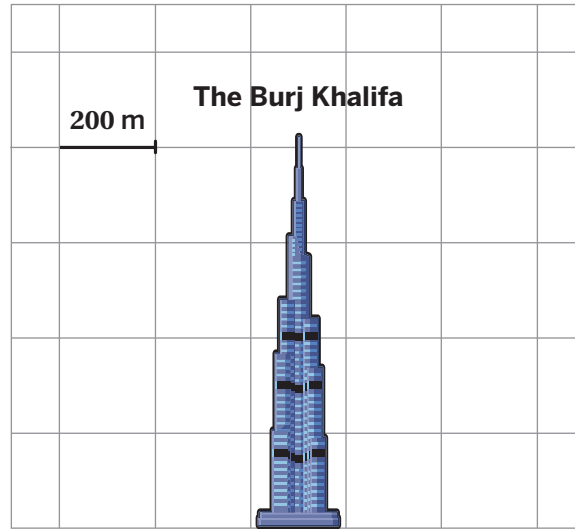


5 Using your estimate, how tall is the Burj Khalifa in meters? Remember that 1 meter is equal to 100 centimeters.

Scale

6 So far, the drawings in this lesson have used scale factor. Here is a new drawing that uses **scale**. The scale is 1 unit to 200 meters.

How can you tell from this new drawing that the Burj Khalifa is more than 800 meters tall?



7 The Activity 2 Sheet contains scale drawings of all the objects in the table.


Estimate the actual height of each object.

Object	Actual Height
Amoeba	
Poppy seed	
Ladybug	
Caterpillar	4 cm
Dog	
Person	
5-story building	
World's tallest tree	
The Burj Khalifa	828 m

You're invited to explore more.

8 Use the You're Invited to Explore More Sheet to create your own scale drawing.

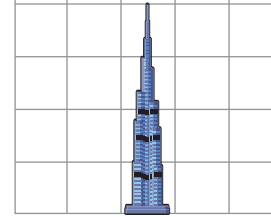
9 Synthesis

 **Discuss** How are scale factor and scale alike? How are they different?

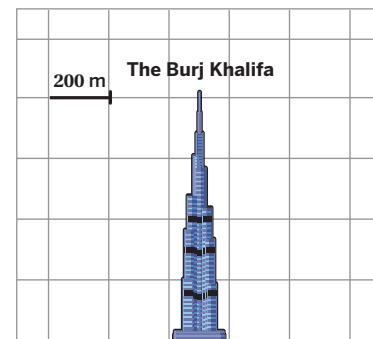
Scale Factor

Real object is 21000 times as large as drawing.

The Burj Khalifa



Scale

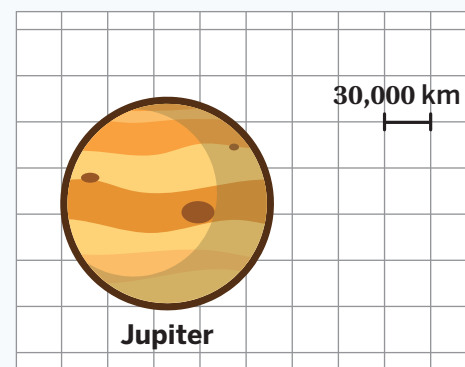


12 Summary 1.06

Some objects are so big or so small that it's hard to find the right scale factor to represent them in a drawing. Instead, we can use a **scale** to tell how actual measurements are represented in a drawing.

Scale is often shown with a line segment that indicates what distance 1 unit in the drawing represents in the actual object. Scales can also be written in units of measure, like inches or centimeters.

For example, this drawing of Jupiter uses a scale of 1 unit to 30,000 kilometers. This means that every 1 unit on the drawing represents 30,000 kilometers. Since the diameter of Jupiter is about 4.5 units on the grid, the actual diameter of Jupiter is about $30000 \cdot 4.5 = 135000$ kilometers.



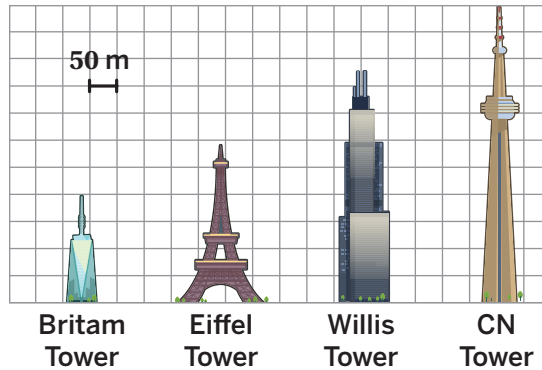
scale How the actual measurements of an object are represented in a drawing.

Practice 1.06

Name: _____ Date: _____ Period: _____

1. Here is a scale drawing of some of the world's tallest towers. Use the scale to estimate how tall each tower is.

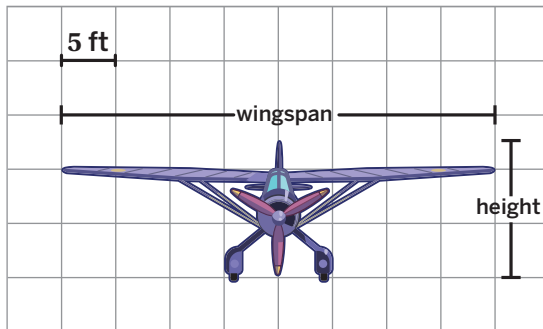
Tower	Estimated Height (m)
Britam Tower	
Eiffel Tower	
Willis Tower	
CN Tower	



Problems 2–3: The Westland Lysander was an aircraft used by the Royal Air Force in the 1930s. Here is a scale drawing that shows the Lysander.

Approximate the actual lengths.

2. The wingspan of the plane:

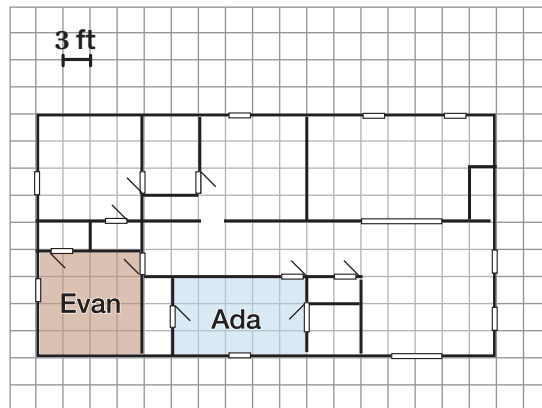


3. The height of the plane:

Ada and Evan are siblings. Here is a scale drawing of their house that shows their rooms.


4. Complete the table with the dimensions of their rooms.

	Length (ft)	Width (ft)
Ada's Room	15	
Evan's Room		



Practice 1.06

Name: _____ Date: _____ Period: _____

 **Problems 5–6:** Here is a map with a scale at the bottom.

5. Estimate how far Aki's house is from the pizza place.
6. Aki's friend's house is 600 feet away from his house. How many segments away will it appear on the map?



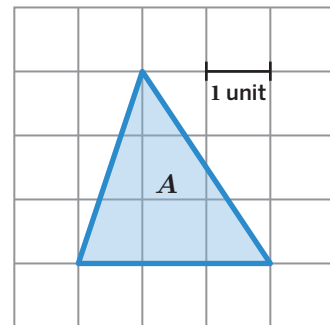
Spiral Review

7. Complete the table so that the two values in each row are related by the same ratio.

3	12
1.5	
4	16
	1

Problems 8–9: Tyrone created a scaled copy of triangle A with an area of 72 square units.

8. How many times greater is the area of the scaled copy compared to the area of triangle A ?
9. What scale factor did Tyrone apply to triangle A to create the scaled copy?



Unit 1
Lesson
7

Name: _____ Date: _____ Period: _____

Proportional Relationships Unit Rates in the World **Scale Drawings** Shapes in the World

 7.G.1, 7.G.6, 7.RP.3, SMP.2, SMP.4

Will It Fit?

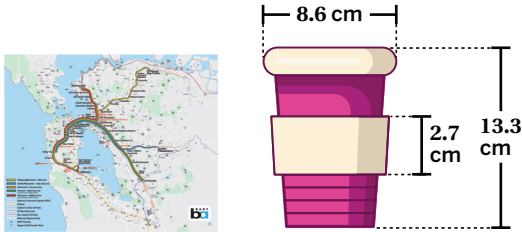
Let's analyze scale drawings.



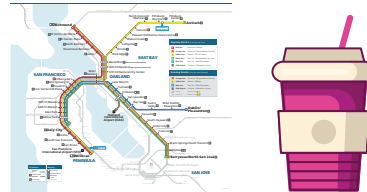
Warm-Up

Here are some drawings of the Bay Area Rapid Transit System (BART) and a coffee cup.

Scale Drawings



Might Not Be Scale Drawings

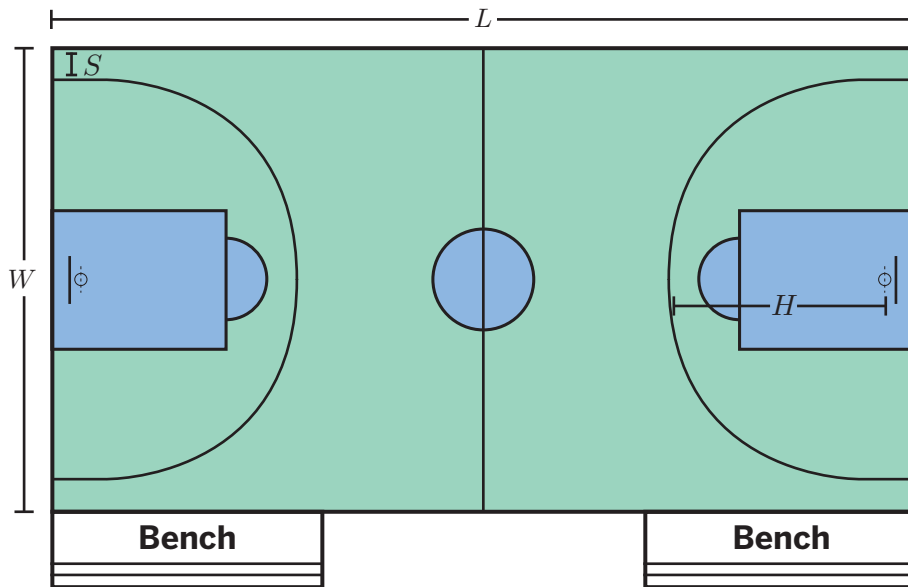


-  **Discuss:** What makes a drawing a scale drawing?

Will the Court Fit?

Karima heard from students that they would like a basketball court in their community park. When Karima presented the idea to the park's board of directors, they approved building the court in a 20-by-20-meter area of the park.

Here is the scale drawing that Karima presented.



- The scale for Karima's drawing is 1 centimeter to 2.5 meters. Explain what this means in your own words.

Will the Court Fit? (continued)

Will Karima's court fit in the 20-by-20-meter square area the park directors designated for the court?

3. Use your ruler to measure the scale drawing. Record the measurements in the table. Then determine the dimensions of the actual court.

	Length of Court, L	Width of Court, W	Hoop to 3-Point Line, H	3-Point Line to Side Line, S
Scale Drawing				
Actual Court				

4. Explain how you know whether the court will fit.

You're invited to explore more.

5. On an actual basketball court, the bench area is typically 9 meters long. Determine how long the bench area should be on the scale drawing.

Does your answer match Karima's drawing?

Synthesis

9. Explain how you could use Karima's scale drawing to calculate the actual distance across the center court circle. Remember that the scale for Karima's drawing was 1 centimeter to 2.5 meters.

	Distance Across the Center Court Circle
Scale Drawing	1.8 cm
Actual Court	

Summary 1.07

Scale drawings are two-dimensional representations of actual objects or places. Floor plans and maps are examples of scale drawings you might have seen before.

On a scale drawing:

- Every part of the drawing matches up with a part of the actual object.
- Distances on the drawing are proportional to their matching distances in real life.
- A *scale* tells you how actual measurements are represented on the drawing. For example, if a drawing has a scale of "1 inch to 8 feet," then a 0.5 inch line segment on that drawing would represent an actual distance of 4 feet.

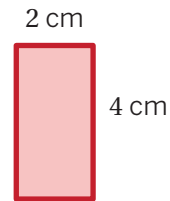
scale drawing A two-dimensional representation of an actual object or place. All the measurements in the drawing correspond to the measurements of the actual object by the same scale. Floor plans and maps are examples of scale drawings.

Practice

1.07

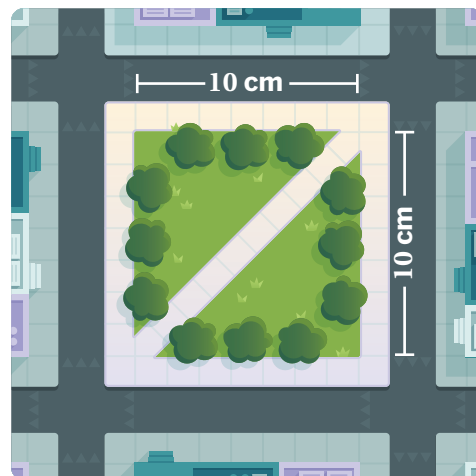
Name: _____ Date: _____ Period: _____

Problems 1–4: Here is part of a scale drawing of Zahra's office. The scale is 1 centimeter to 2.5 feet.



1. What are the dimensions of the actual office?
2. What is the actual area of her office?
3. Zahra wants to put a couch in her office that is 3 feet wide. How wide would the couch be if it were drawn on the scale drawing?
4. Fabiana's office is near Zahra's office and measures 4 centimeters by 8 centimeters in the scale drawing. Is Fabiana's office twice the area of Zahra's office? Explain your thinking.


Problems 5–7: Jada is looking at a map of a square park that has a scale of 1 centimeter to 40 feet. On the map, each side of the park is 10 centimeters long.

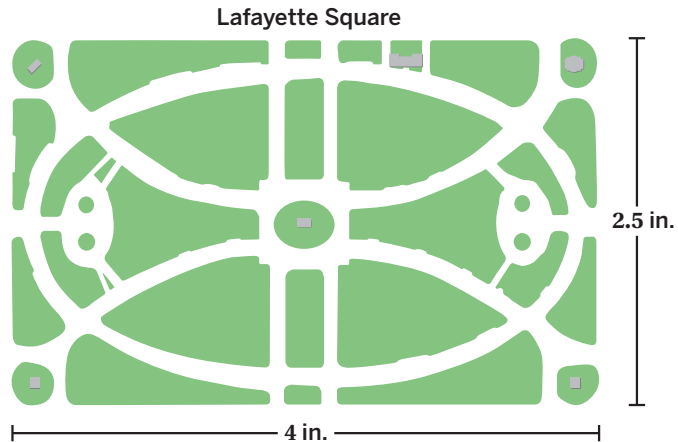


5. Jada lives 500 feet from the park. How long would this distance be on the map?
6. If Jada ran around the perimeter of the park once, what distance would she run?
7. Jada wants to run a mile (5,280 feet). About how many times would she need to run around the park in order to reach her goal?

Practice 1.07

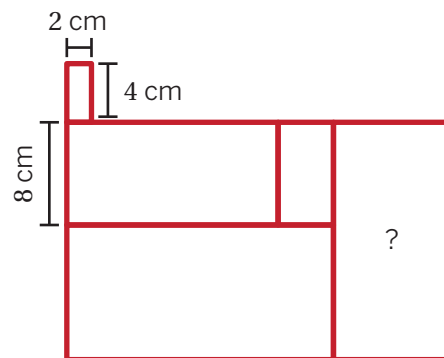
Name: _____ Date: _____ Period: _____

8.  Here is a scale drawing of Lafayette Square in Washington, D.C. The scale is 1 inch to 200 feet. Determine the actual dimensions of Lafayette Square.



9. Here is the whole floor that Zahra's office is on. Each room is a scaled copy of every other room. The scale for this blueprint is 1 centimeter to 2.5 feet.

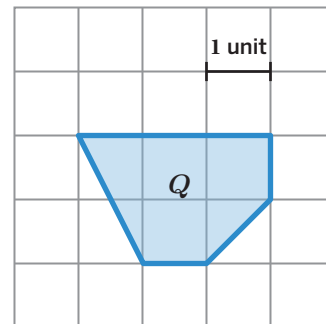
Use any strategy to determine the *actual area* of the room with the question mark.



Spiral Review

Problems 10–11: Here is polygon Q .

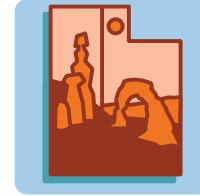
10. What is the area of polygon Q ?
11. Lola drew a scaled copy of polygon Q using a scale factor of 2. What is the area of Lola's polygon? Explain your thinking.



12. Titus is making hot chocolate for a winter gathering. For every 2 cups of hot chocolate mix, he needs 6 cups of milk. If he has 12 cups of hot chocolate mix, how much milk will he need? Complete the table to show your thinking.

Hot Chocolate Mix (cups)	Milk (cups)
2	
1	

Scaling States, Part 1



Let's explore how the scale affects the size of a scale drawing.

Warm-Up

As of 2024, there were 63 national parks in the United States and Utah has 5 of them. Marco is designing a poster with a *scale drawing* of Utah to promote Utah's national parks.



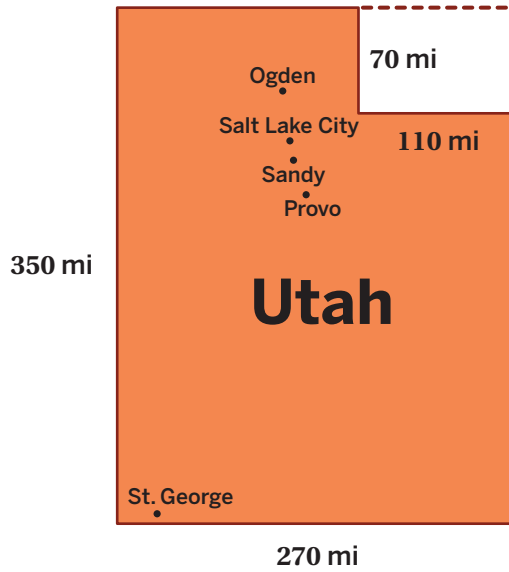
1. Describe the shape of Utah.

2. What information would you need in order to create a scale drawing of the state?

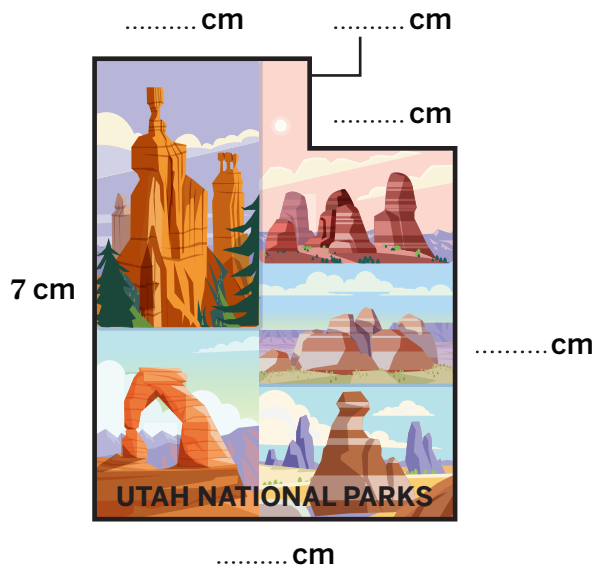
Calculating Scaled Distances

Marco researched the dimensions of Utah to help him create a scale drawing of the state and show the five national parks within it.

- To fit his scale drawing onto a poster, Marco began by drawing a 7-centimeter segment for Utah's western edge. The actual length of Utah's western edge is 350 miles. What scale did Marco use? Show or explain your thinking.



- Help Marco complete his scale drawing by labeling the missing lengths of Utah's outline on the drawing.



Scaling Utah

Marco will print his design on different products (T-shirts, sticky notes, phone cases, etc.). Work with your classmates to create different scale drawings of Utah's outline. You will need a ruler and a blank sheet of paper for this activity.

5. Select a scale for your scale drawing.

Scale A: 1 centimeter to 35 miles

Scale B: 1 centimeter to 70 miles


Scale C: 1 centimeter to 100 miles

6. Before you create your drawing, do you think your scale drawing will fit on a phone case? Explain your thinking.

7. On a blank piece of paper, create your own scale drawing of Utah. Include the following:

- Your scale drawing of Utah
- Labels for each length in the scale drawing
- The scale of your drawing

8. Compare your drawings with your classmates.

a  **Discuss:** How did your prediction from Problem 6 compare to your scale drawing?

b Use noticings from your drawings to describe how you would change the scale if Marco wants to print his design on a postage stamp.

Synthesis

9. What do you think will always be the same about scale drawings of the same object? What do you think can be different? Use the scale drawings created in Activity 2 to help with your thinking.

Summary 1.08

When creating scale drawings, you can select any scale that works for the space you have.

No matter what scale you select or the size of the space you are working with, an accurate scale drawing should always have the same shape and angles as the original figure.

When you select a scale, make sure to multiply or divide each length in your drawing by the same value.

Here are two ways to determine the measurements of a scale drawing of New Mexico using the scale 1 inch to 20 miles.



Actual length: 371 miles
 $371 \div 20 = 18.55$
Drawing length: 18.55 inches

Actual width: 344 miles
 $344 \div 20 = 17.2$
Drawing width: 17.2 inches

$1 \div 20 = 0.05$
The measurements in the scale drawing will be 0.05 times the actual length.

Actual length: 371 miles
 $371 \times 0.05 = 18.55$
Drawing length: 18.55 inches

Actual width: 344 miles
 $344 \times 0.05 = 17.2$
Drawing width: 17.2 inches

Practice

1.08

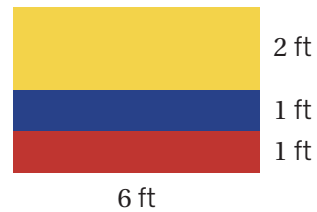
Name: _____ Date: _____ Period: _____

Problems 1–2: A book measures 6 inches wide and 9 inches tall. The publisher wants to display an image of the book on a billboard. The width of the book on the billboard is 36 inches.


1. How tall is the book on the billboard?
2. What scale is used for the image on the billboard?
Show or explain your thinking.

Problems 3–4: Here are the dimensions of the Colombian flag.

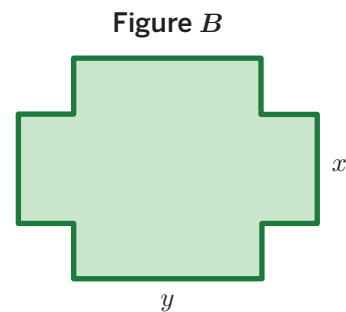
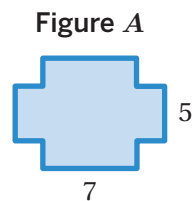
3. Create a scale drawing of the flag so that the top yellow stripe is 3 centimeters tall. Label the dimensions.



4. Create a scale drawing of the flag where 1 centimeter represents 2 feet.

 **Problems 5–6:** Figures *A* and *B* are scaled copies. The scale that takes figure *A* to figure *B* is 1 unit to $6\frac{3}{4}$ units.

5. What is the value of x ?
6. What is the value of y ?



Practice 1.08

Name: _____ Date: _____ Period: _____

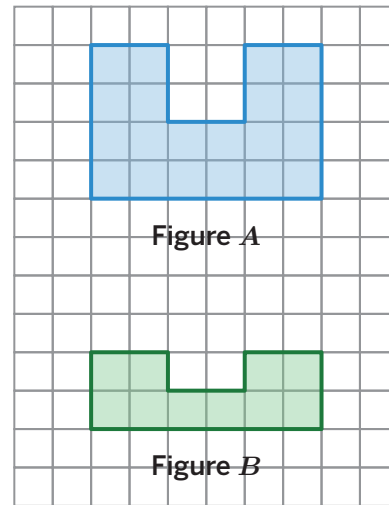
7. This map shows parts of Texas and Oklahoma.



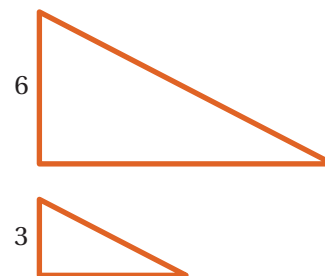
Approximately how far is it from Amarillo to Oklahoma City?

Spiral Review

8. Are figures *A* and *B* scaled copies? Explain your thinking.



9. The larger triangle is a scaled copy of the smaller triangle. The labeled side lengths are corresponding sides. The area of the smaller triangle is 9 square units. What is the area of the larger triangle? Show or explain your thinking.



Scaling States, Part 2



Let's create more scale drawings.

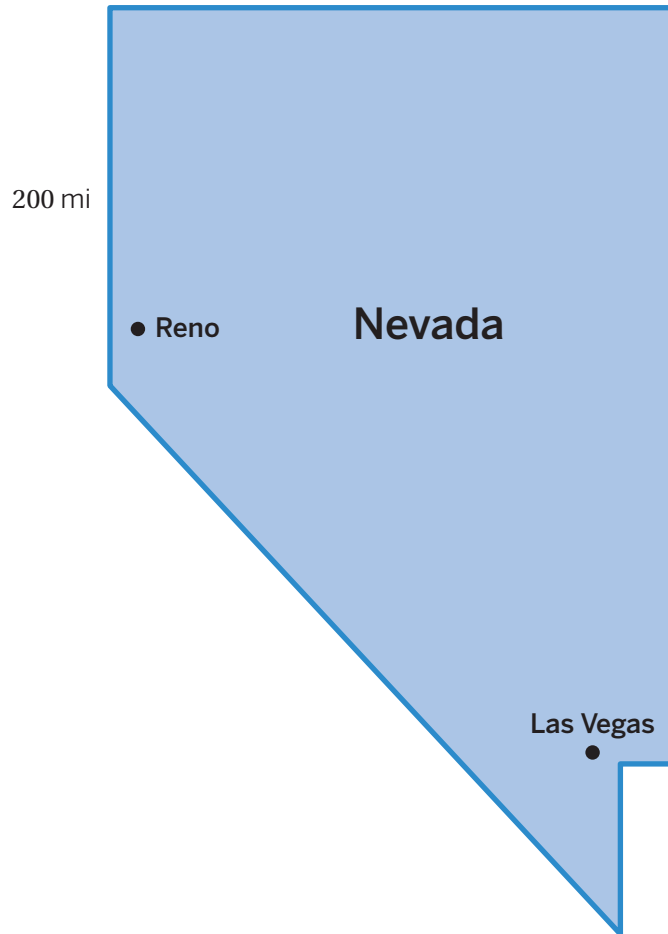
Warm-Up

Marco was very happy with his drawing of Utah and now wants to promote national parks in other states, beginning with Nevada.

The western edge of Nevada is approximately 200 miles.

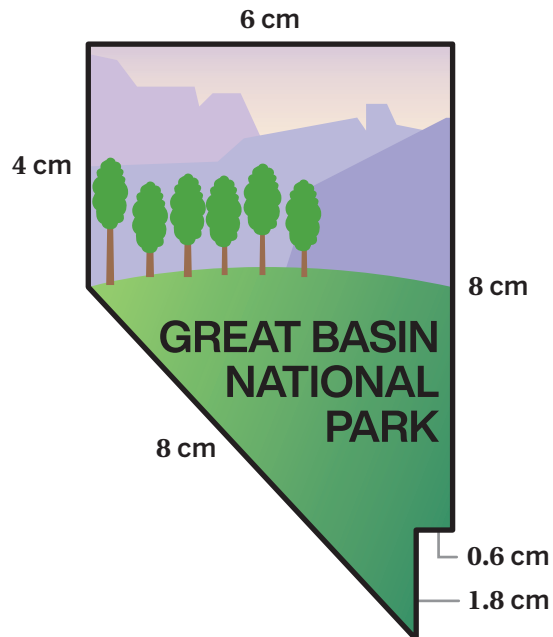
Marco created a scale drawing of Nevada where the western edge measured 5 centimeters.

1. What scale did Marco use? Explain your thinking.



Scaling Nevada

Marco is making T-shirts to feature Great Basin National Park in Nevada. His design is a scale drawing that uses a scale of 1 centimeter to 50 miles.



2. **a** If Marco uses a scale of 4 centimeters to 100 miles instead, are the distances going to be shorter, longer, or the same size as the distances in the original design? Explain your thinking.

- b** List the dimensions of Nevada using a scale of 4 centimeters to 100 meters.

3. If Marco uses a scale of 5 centimeters to 250 miles instead, are the distances going to be shorter, longer, or the same size as the distances in the original design? Explain your thinking.

Comparing Scales

Marco is exploring six different sizes for printing his Nevada design on T-shirts.

Size A

Scale:
1 cm to 70 mi

Size B

Scale:
3 cm to 70 mi

Size C

Scale:
1 cm to 14 mi

Size D

Scale:
6 cm to 150 mi

Size E

Scale:
2 cm to 35 mi

Size F

Scale:
1 cm to 25 mi

4. Order the design sizes from *smallest* to *largest*. Note that some sizes may be the same.

5. Describe your strategy for ordering the design sizes.

Synthesis

6. Describe how you can compare the scales used to create scale drawings of the same object to decide which scale drawing will be larger. Use the examples if it helps with your thinking.

Drawing A uses the scale 1 centimeter to 2 meters. Drawing B uses the scale 1 centimeter to 4 meters.

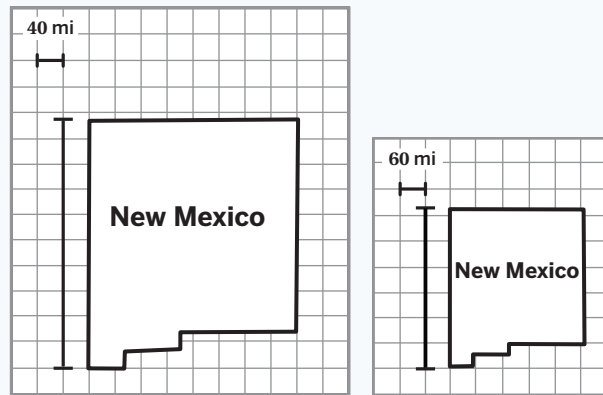
Summary 1.09

We can represent a place or object using many different scale drawings. Changing the scale will change the size of the scale drawing.

Here are two scale drawings of New Mexico.


The first drawing uses a scale of 1 unit to 40 miles, and the second drawing uses a scale of 1 unit to 60 miles.

When we represent a larger distance for each unit in the scale, the size of the drawing decreases. This is because it takes fewer units to represent the total distance.



Practice 1.09

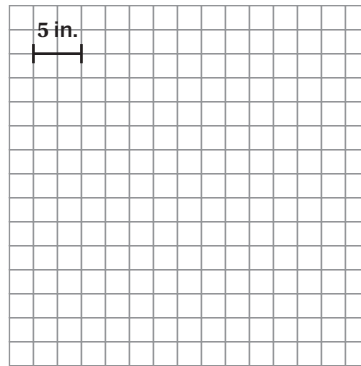
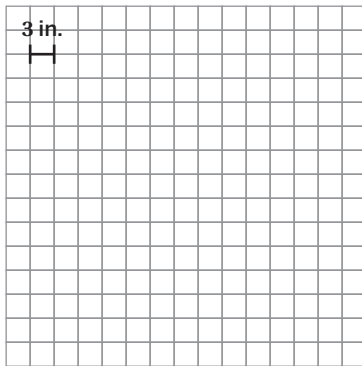
Name: _____ Date: _____ Period: _____

1.  Tyler made a scale model of Earth in science class. The scale that maps the scale model to the actual Earth is 1 inch to 1,070 miles. If the height in the scale model is 7.4 inches, what is the actual height of Earth?



Problems 2–4: Leonardo Da Vinci's famous painting, the *Mona Lisa*, measures 21 inches by 30 inches.

2. Create a scale drawing of the outline of the *Mona Lisa* where 1 unit represents 3 inches.
3. Create a scale drawing of the outline of the *Mona Lisa* where 2 units represents 5 inches.



4. Imagine a new scale drawing of the *Mona Lisa* where 3 units represent 1 inch. Is this drawing smaller, larger, or equal in size compared to your previous drawings? Explain your thinking.
5. The floorplan of a restaurant shows a scale of 1 inch to 12 feet. The floorplan shows the area of the restaurant as 60 square inches. Hoang says the actual area of the restaurant is 720 square feet. Is Hoang correct? Explain your thinking.

Spiral Review

Problems 6–9: Circle the phrase that makes each statement true.

6. The value of the expression $15 \cdot 0.75$ is . . .

Less than 15

Equal to 15

Greater than 15

7. The value of the expression $15 \cdot 1.3$ is . . .

Less than 15

Equal to 15

Greater than 15

8. The value of the expression $15 \cdot 1$ is . . .

Less than 15

Equal to 15

Greater than 15

9. The value of the expression $15 \cdot 0$ is . . .

Less than 15

Equal to 15

Greater than 15

Problems 10–13: Determine the missing value in each equation.

10. $32 \cdot \square = 4$

11. $24 \cdot \square = 3$

12. $16 \cdot \square = 2$

13. $8 \cdot \square = 1$

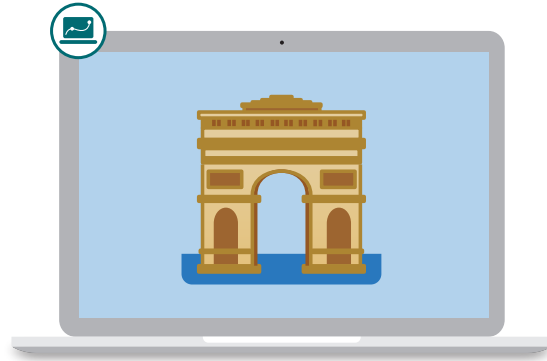
Unit 1
Lesson
10

Name: _____ Date: _____ Period: _____

Scale Drawings Shapes in the World 7.G.1, SMP.3, SMP.6, SMP.7

Scaling Buildings

Let's see how different scales can describe the same thing.

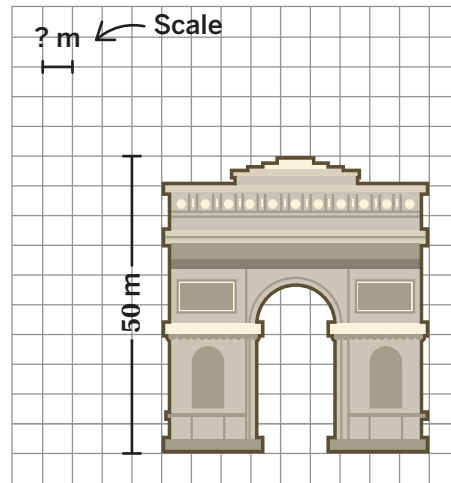


Warm-Up

1 Here is a scale drawing of the Arc de Triomphe in Paris, France.

The Arc de Triomphe is 50 meters tall.

What is the unknown value in the scale?



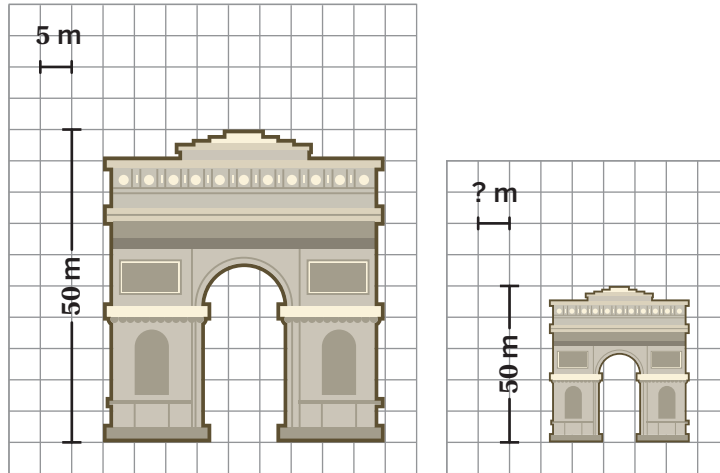
Same Object, Different Scale

Here are two scale drawings of the Arc de Triomphe: one from the Warm-Up and a new scale drawing.

2 On the new scale drawing, the unknown segment represents:

- A. More than 5 meters.
- B. Less than 5 meters.
- C. Exactly 5 meters.

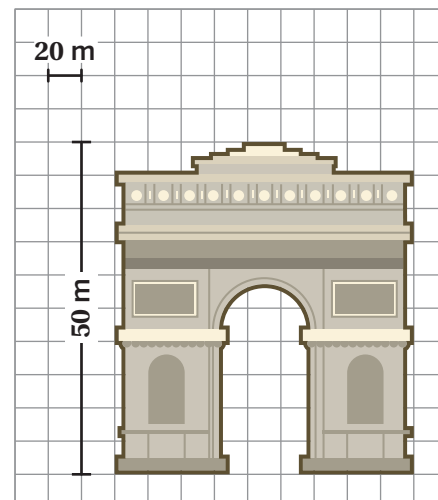
Explain your thinking.



3 What is the unknown value in the scale?

4 The scale in this scale drawing is the wrong size. Update the scale so that it shows the correct number of grid units for 20 meters.

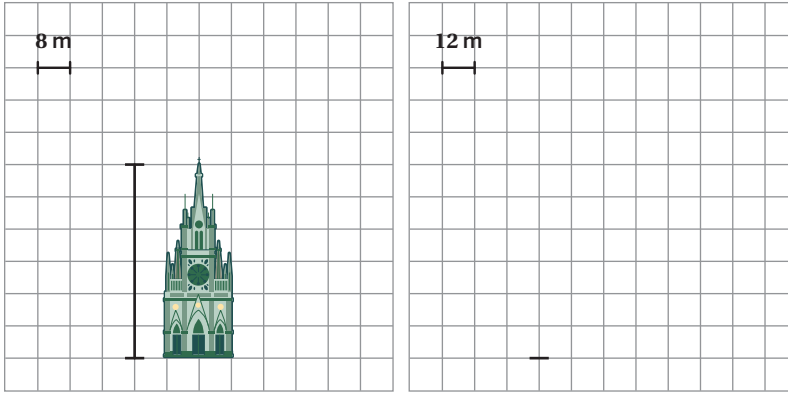
Explain your thinking.



Same Object, Different Scale (continued)

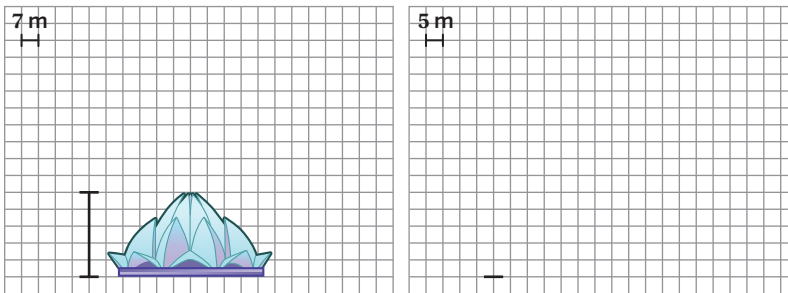
For each scale drawing, look at the building's height. Then show what the height of each building would be with the new scale.

5



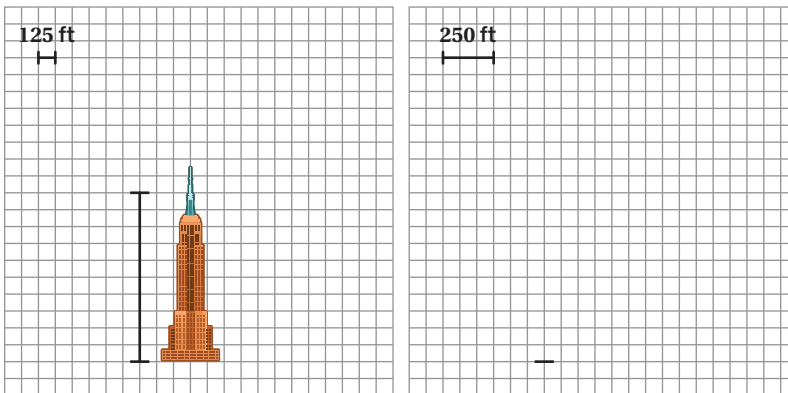
Las Lajas Sanctuary in Nariño, Colombia

6



Lotus Temple in Delhi, India

7



Empire State Building in New York City

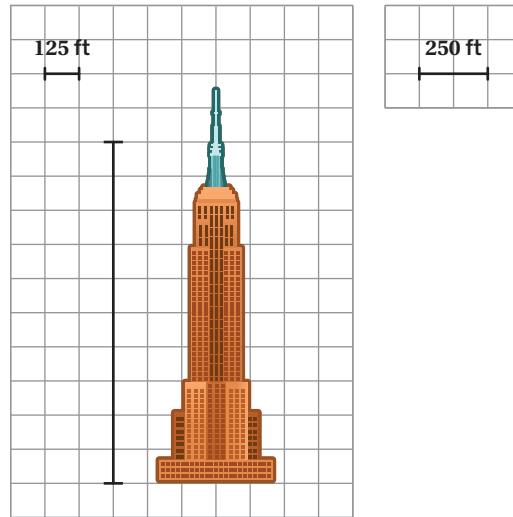
When Are Scales Equivalent?

- 8** Here is a scale drawing of the Empire State Building, with a scale of 1 unit to 125 feet. Imagine a new scale drawing with a scale of 2 units to 250 feet.

Will the new drawing be smaller, larger, or the same size as this drawing?

- A. Smaller
- B. Larger
- C. The same size

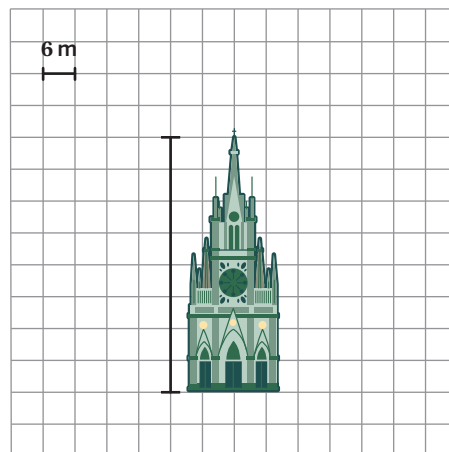
Explain your thinking.



- 9** Here is a scale drawing of Las Lajas Sanctuary with a scale of 1 unit to 6 meters.

Select *all* the scales that will make a scale drawing of the same size.

- A. 2 units to 3 meters
- B. 2 units to 6 meters
- C. 2 units to 12 meters
- D. 3 units to 9 meters
- E. 3 units to 18 meters



Activity
2

Name: Date: Period:

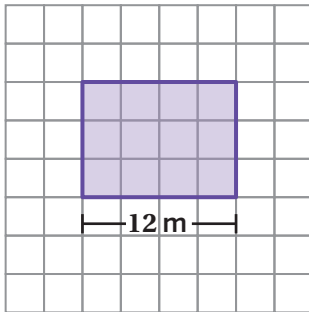
When Are Scales Equivalent? (continued)

- 10** Match each scale drawing with one or more possible way(s) of describing its scale.
One scale description will have no match.

Scale Drawing

Scale Description

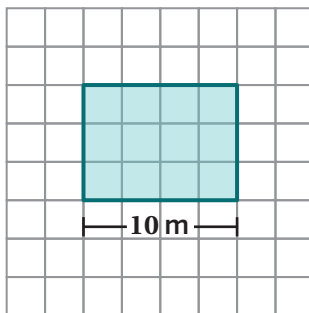
a.



..... 2 units to 10 meters

..... 2 units to 6 meters

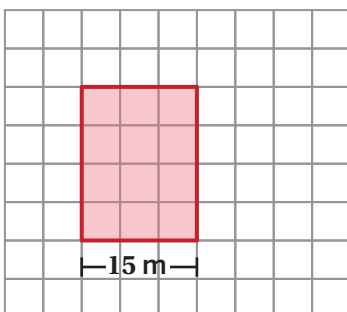
b.



..... 3 units to 9 meters

..... 1 unit to 2.5 meters

c.



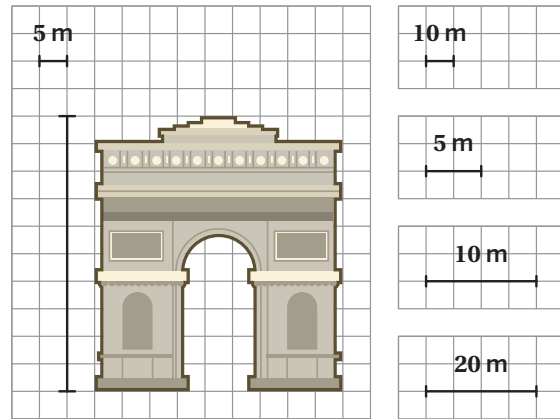
..... 3 units to 5 meters

..... 2 units to 5 meters

11 Synthesis

Discuss: How can you tell whether a new scale drawing will be smaller than, larger than, or the same size as the original just by looking at the scales?

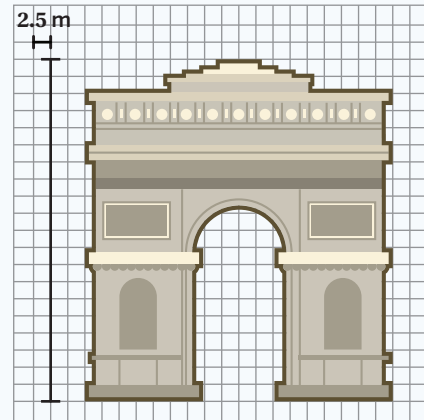
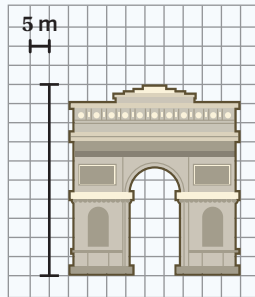
Use these examples if they help with your thinking.



14 Summary 1.10

Here is a scale drawing with a scale of 1 unit to 5 meters.

If you wanted to change this to a scale of 1 unit to 2.5 meters, you could do so in a couple of ways:



Strategy 1

- Use the original scale to find the actual dimensions.
The height in the drawing is 10 units.
 $10 \cdot 5 = 50$
The actual height of the building is 50 meters.
- Use the original dimensions and the new scale to find the dimensions of the new scale drawing. $\frac{50}{2.5} = 20$

The height in the new drawing should be 20 units.

Strategy 2

- Determine how the two scales are related. $2.5 \cdot 2 = 5$
Because $2.5 \cdot 2 = 5$, each length in the new drawing should be 2 times as long as they are in the original drawing.
- Use this relationship to calculate the dimensions for the new drawing.
The height in the original drawing is 10 units. $10 \cdot 2 = 20$

The height in the new drawing should be 20 units.

Practice

1.10

Name: _____ Date: _____ Period: _____

1. There are three scale drawings of a car that each use a different scale.

Note: There are about 1.1 yards in 1 meter, and 2.54 centimeters in 1 inch.

Scale drawing A	Scale drawing B	Scale drawing C
1 in. to 1 ft	1 in. to 1 m	1 in. to 1 yd

Order the scales from *smallest* to *largest*.

--	--	--

Smallest

Largest

Problems 2–5: Ali and Kiana buried a treasure together on their school's field. The field is 400 feet wide. Ali made a map that is 8 inches wide to record its location.

2. Write two possible scales Ali could have used to make her drawing.
3. Kiana made her own map using a scale of 1 inch to 20 feet. Whose map is larger: Kiana's or Ali's? Explain your thinking.
4. On Kiana's map, the treasure is 2 inches from the south edge of the field. How far is the treasure from the south edge on Ali's map?
5. On Kiana's map, the area of the field is 16 square inches. Kiana says that the actual area of the field is 320 square feet. Is Kiana correct? Explain your thinking.
6. Select *all* the scales that are equivalent to 3 centimeters to 15 meters.
- A. 3 inches to 15 inches B. 1 centimeter to 5 meters
- C. 3 meters to 15 centimeters D. 4 millimeters to 2 meters
- E. 1 inch to 5 feet

Explain your thinking for the scale(s) you selected.

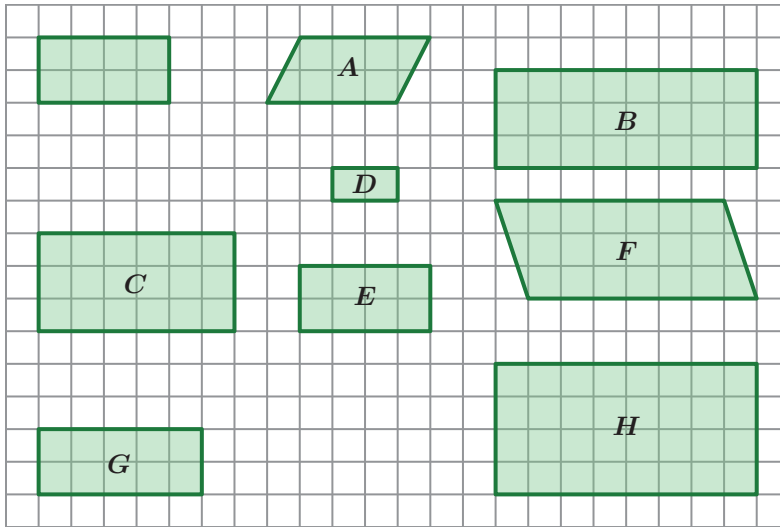
Practice 1.10

Name: _____ Date: _____ Period: _____

7. 📏 On a blueprint, the living room is 2.1 inches wide. The blueprint has a scale of 1 inch to 10 feet. How wide would the living room be on a blueprint that has a scale of 1 inch to 15 feet?
- A. 1.4 inches B. 2.1 inches C. 3.15 inches D. 21 inches

Spiral Review

8. Here is an unlabeled rectangle and several quadrilaterals that are labeled.

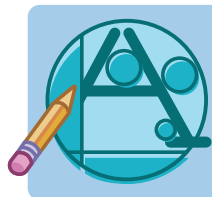


In the table, mark the quadrilaterals that are scaled copies of the unlabeled rectangle. Then, for all scaled copies, write the scale factor used to create it.

	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>	<i>E</i>	<i>F</i>	<i>G</i>	<i>H</i>
Is It a Scaled Copy?								
Scale Factor								

Team Spirit

Let's design and scale a team logo.

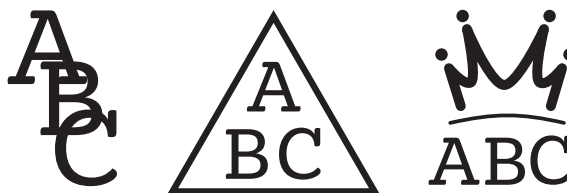


Warm-Up

Here are three logos that Anika designed.

1. What do you notice? What do you wonder?

I notice:



I wonder:

2. If you were to design a logo, where are some places you would put it?


Design Your Logo

You and your team are going to design a team logo.

3. Brainstorm and sketch at least two ideas for your team logo. Use simple shapes so that all of your teammates can replicate it. Your logo might also include letters.

4. Share your logos with your team.

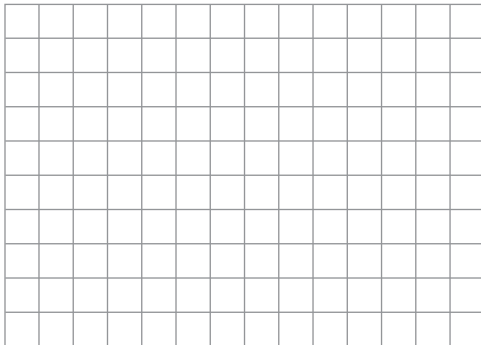
a  **Discuss** Which of the logos do you like best? Why?

b  **Discuss** Which logo do you think your team could replicate? Why?

5. As a team, select one logo to draw on the grid. Each grid line is $\frac{1}{2}$ centimeter.

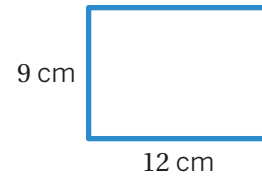
a Draw the logo. Work together to determine the measurements of each part of the logo.


b Label the height and width of the logo using centimeters.

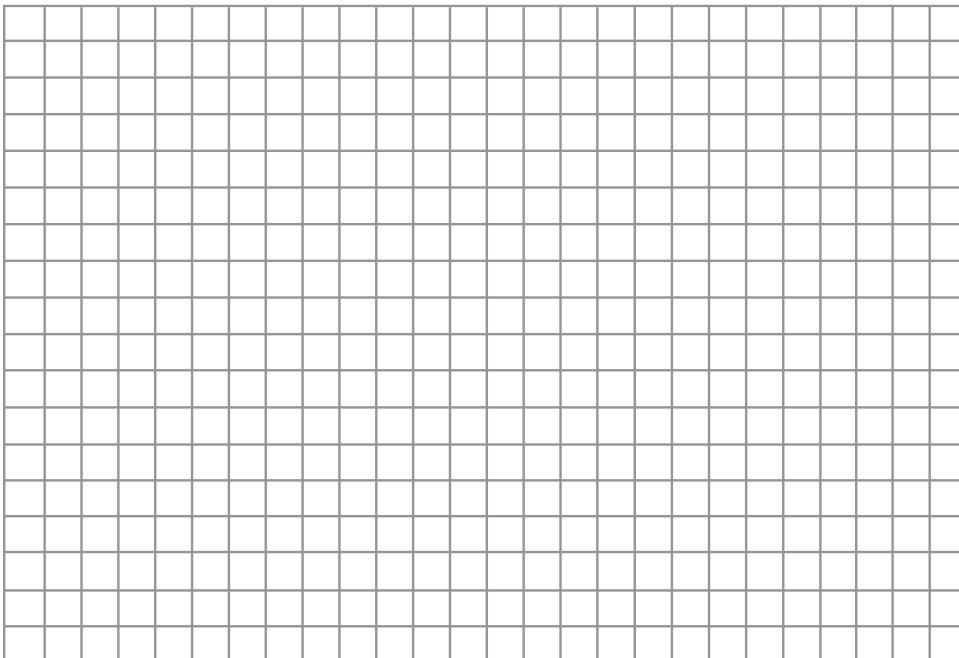


Sticker Stickler

Imagine that you decide to print your team logo on a sticker that measures 9 centimeters by 12 centimeters. You need to scale the logo to make sure it is a good size for the sticker.

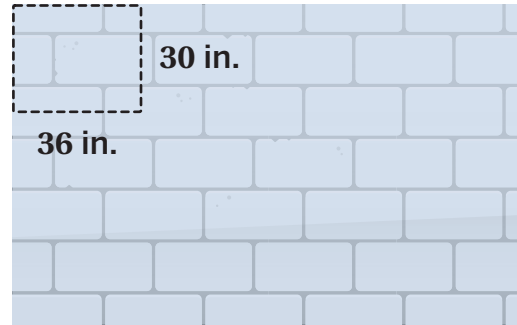


6.  **Discuss** What is a scale factor that would make your drawing *too large* to fit on the sticker? What is a scale factor that would make your drawing *too small* for the sticker?
7. What is a scale factor that would make your drawing *just right* for the sticker? Explain your thinking.
8. Use the scale factor you chose to draw a scaled copy of your logo within the 9-by-12 centimeter space below. (Each grid line is $\frac{1}{2}$ centimeter.) Label your logo's height and width.

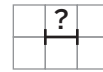


Mural Referral

Now imagine that your class creates a mural of all the team logos on a wall at your school. For your logo, you have a rectangular portion of the wall that is 30 inches tall by 36 inches wide.



9. Look back at your *original* team logo from Problem 5. What scale would you use for your mural drawing? Explain your thinking.

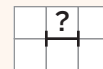


10. If you use the scale you chose in the previous problem, what would be the height of your logo on the mural wall? Show or explain your thinking.


11. What would be the width of your logo on the mural wall? Show or explain your thinking.

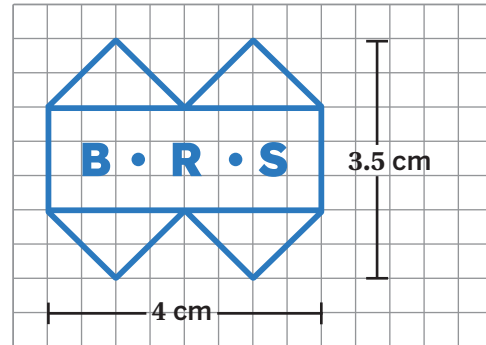
You're invited to explore more.

12. Imagine you get the *entire* wall to draw your team logo. The wall is 90 inches tall by 144 inches wide. Now what scale would you use for your mural drawing? Explain your thinking.



Synthesis

13.  **Discuss:** If you need to change the size of an object, when is it better to use a scale factor? When is it better to make a scale? Use the example if it helps with your thinking.



Summary 1.11

When you have a drawing, picture, or piece of art that needs to be a different size, you can communicate the size change with either a scale factor or a scale.

If you're using a scale factor:

- Measure and record relevant dimensions of the original object.
- Choose a scale factor that's appropriate for the situation.
- Multiply all the dimensions by that scale factor.

If you're using a scale:

- Determine the relevant dimensions of the original object in grid units.
- Consider the dimensions of the actual space.
- Choose a scale for the grid that makes the object the right size for the space. This makes a scale drawing.

Practice 1.11

Name: _____ Date: _____ Period: _____

1. 📍 The Empire State building in New York City is about 1,450 feet tall (including the antenna) and 400 feet wide. DeAndre wants to make a scale drawing of the building on a sheet of paper that is $8\frac{1}{2}$ -by-11 inches.

Which scale do you think is the most appropriate for the scale drawing?

- A. 1 inch to 50 feet B. 1 inch to 100 feet
C. 1 inch to 200 feet D. 1 inch to 400 feet

Explain your thinking.

Problems 2–5: This map shows a part of Philadelphia, Pennsylvania, near Logan Square. The Benjamin Franklin Parkway divides a plot of land into a right triangle.

2. Imagine making a scale drawing of this triangular plot of land that fits nicely on a sheet of paper. Write a scale that would make a scale drawing of appropriate size.

..... inch(es) to meter(s)



3. Using this scale, what are the dimensions of the triangle in the scale drawing?
4. What is the area of the triangular plot of land on your scale drawing?
5. How many square meters are represented by 1 square inch in your scale drawing?

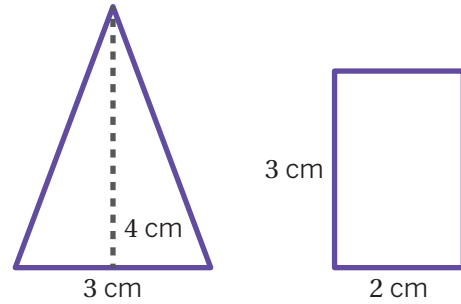
Explain your thinking.

Practice 1.11

Name: _____ Date: _____ Period: _____

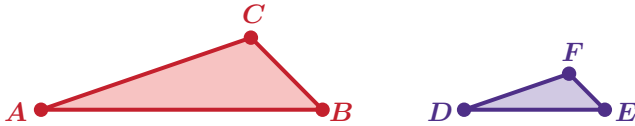
6. A triangle and a rectangle are shown. Determine the scale factor that will create the biggest triangle that can fit inside the rectangle.

Explain your thinking.



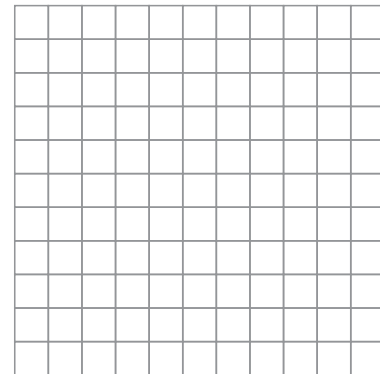
Spiral Review

Problems 7–10: Triangle DEF is a scaled copy of triangle ABC . For each of the parts of triangle ABC , identify the corresponding part for triangle DEF .



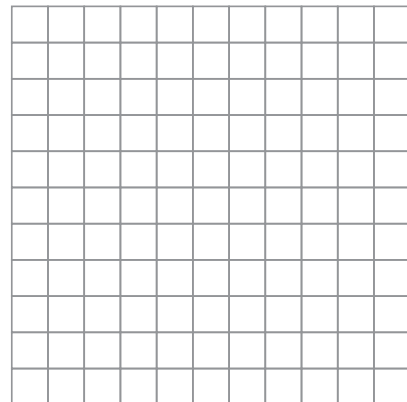
7. Angle ABC 8. Angle BCA 9. Side AC 10. Side BA

11. Draw a figure that has an area of 8 square units. Try to make a figure no other student will think of.



12. Imagine a scaled copy of your figure that uses a scale factor of 2. What would be the area of the scaled copy?

13. Make a scaled copy of your original figure using a scale factor of 0.5. What is the area of the new figure?



Practice Day 2



Let's practice what you've learned so far in this unit!

You will use task cards for this Practice Day. Record all of your responses here.

Task A: Two Buildings

Partner A

1. _____ meters
2. _____ centimeters wide

Explanation:

Partner B

1. _____ meters
2. _____ centimeters wide

Explanation:

Task B: Paper Towns

Use a ruler and label relevant length in centimeters.

1. Draw your scale drawing of Wyoming.
2. Draw your paper town.

Partner A

3. _____ square centimeters

Partner B

3. _____ square centimeters

Practice Day 2 (continued)

Task C: Dinosaurs

Partner A

1. Circle one: Binta's or Darius's
Explanation:

2. _____ centimeters

Partner B

1. Circle one: Binta's or Darius's
Explanation:

2. _____ centimeters

Task D: What's the Scale?

Partner A

1. Scale 1:

Scale 2:

2. Epidemiologist's first drawing:

Partner B

1. Scale 1:

Scale 2:

2. Epidemiologist's second drawing:

You're invited to explore more.

1. Scale(s) _____

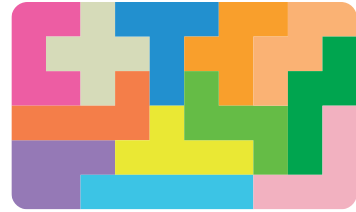
2. Scale(s) _____

Notes:

Career Connection

What could you call a domino if it had 5 squares instead of 2 squares?

Meet the “pentomino.” Solomon W. Golomb named this shape after imagining what a domino could be called if it had five (*pent-*) squares instead of two (*di-*) squares. You can create a scale drawing of each of the 12 pentominoes shown by using nine of the remaining shapes.



Game or puzzle designers often use scale factors and scale drawings to create game settings. If the game is online, they must make sure the scale factor is consistent across different screen sizes.



Brendan Hoffman/Getty Images

Meet Solomon W. Golomb

Mathematician and Professor Solomon W. Golomb used advanced mathematics to make important contributions to the field of digital communications, including to improve cell phone technologies, data encryption, and radios used in astronomy. He also had a knack for creating and naming math games and puzzles, such as “pentominoes.” Another of his creations, “Cheskers,” combines chess and checkers. His creation of “polyominoes” inspired the arcade game Tetris!

Are you interested in studying wildlife ecology? What can you do to learn more?

Math in the World

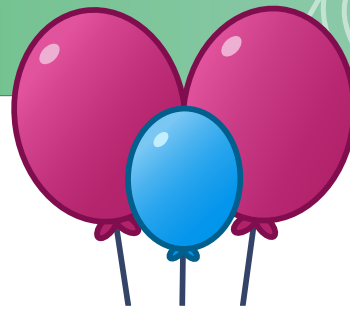
Here are two larger pentominoes that are made from 9 smaller pentominoes. Verify that each one is a scale drawing of its smaller, corresponding shape. What is the scale factor?



Math Mindset

What mathematical arguments can you use to defend whether a given figure is a scaled copy of another figure?

Unit 2



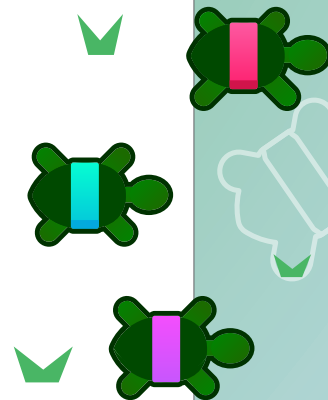
Introducing Proportional Relationships

Big Ideas in This Unit

- CC2 Graphing Relationships Proportional Relationships Unit Rates in the World
- NS Ratios, Percents, and Proportional Relationships

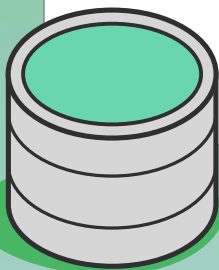
Questions for Investigation

- What does it mean for two things to be proportionally related? How can you tell?
- What are the different ways you can represent proportional relationships? How are these representations related?
- How can you make sense of the solutions of problems involving proportional relationships?



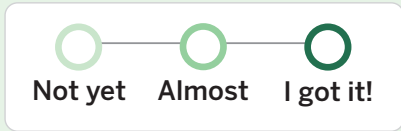
Explore: Currency Exchange

How can you convert between different currencies?









Watch Your Knowledge Grow

This is the math you'll explore in this unit. Rate your understanding to see how your knowledge grows!



I can . . .	Before	After
Tell whether ratios are equivalent.	<input type="radio"/> — <input type="radio"/> — <input checked="" type="radio"/>	<input type="radio"/> — <input type="radio"/> — <input checked="" type="radio"/>
Decide whether quantities are in a proportional relationship from a table.	<input type="radio"/> — <input type="radio"/> — <input checked="" type="radio"/>	<input type="radio"/> — <input type="radio"/> — <input checked="" type="radio"/>
Decide whether quantities are in a proportional relationship from an equation.	<input type="radio"/> — <input type="radio"/> — <input checked="" type="radio"/>	<input type="radio"/> — <input type="radio"/> — <input checked="" type="radio"/>
Decide whether quantities are in a proportional relationship from a graph.	<input type="radio"/> — <input type="radio"/> — <input checked="" type="radio"/>	<input type="radio"/> — <input type="radio"/> — <input checked="" type="radio"/>
Represent a proportional relationship between quantities using a table.	<input type="radio"/> — <input type="radio"/> — <input checked="" type="radio"/>	<input type="radio"/> — <input type="radio"/> — <input checked="" type="radio"/>
Represent a proportional relationship between quantities using an equation.	<input type="radio"/> — <input type="radio"/> — <input checked="" type="radio"/>	<input type="radio"/> — <input type="radio"/> — <input checked="" type="radio"/>
Identify the constant of proportionality from a table.	<input type="radio"/> — <input type="radio"/> — <input checked="" type="radio"/>	<input type="radio"/> — <input type="radio"/> — <input checked="" type="radio"/>
Identify the constant of proportionality from an equation.	<input type="radio"/> — <input type="radio"/> — <input checked="" type="radio"/>	<input type="radio"/> — <input type="radio"/> — <input checked="" type="radio"/>
Identify the constant of proportionality from a graph.	<input type="radio"/> — <input type="radio"/> — <input checked="" type="radio"/>	<input type="radio"/> — <input type="radio"/> — <input checked="" type="radio"/>

I can . . .	Before	After
Identify the constant of proportionality from a diagram and verbal description.		
Explain what a point (x, y) on the graph of a proportional relationship means.		
Solve real-world problems involving proportional relationships.		

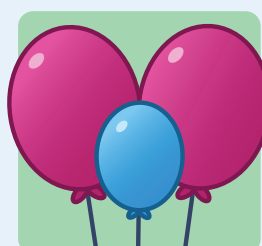
Proportional Relationships in Tables



Explore
Currency Exchange



Lesson 1
Paint



Lesson 2
Balloon Float



Lesson 3
Sugar, Spice, and
Everything Rice




Explore: Currency Exchange

How can you convert between different currencies?



Warm-Up

1. Here are two items and their price tags.

 **Discuss:** What do you notice? What do you wonder?



somkhana/Shutterstock.com



Linimages/Shutterstock.com



Using Tables to Convert Currency

Different countries around the world use different currencies (types of money). If you know the amount of money in one currency, you can use exchange rates to determine its equivalent value in another currency.

2. In 2024, the exchange rate from Chinese yuan (¥) to Philippine pesos (₱) was approximately 1 to 8.

- a Complete the table to determine the costs of several items in each currency.
- b Choose one item for which you calculated the cost. Describe your strategy.

	Chinese yuan	Philippine pesos
	1	8
Steamed dumplings	8	
Short taxi ride	2	
Bottle of water		40

3. In 2024, 4 U.S. dollars (USD \$) was equivalent to 3 British pounds (GBP £)

- a Complete the tables.
- b Show or explain your thinking for calculating the missing values.

USD (\$)	GBP (£)	GBP (£)	USD (\$)
1		1	
4	3	3	4
2		27	
100.20		90.75	

4. Your teacher will show you a list of places. Choose one that you might like to visit.

- a What is the currency there?
- b What amount in their currency has an equivalent value of \$1 (USD)?

**Using Tables to Convert Currency** (continued)

5. Several Native American tribes used wampum — strings of purple and white shell beads — to represent value and importance long before British and Dutch colonists arrived. It became the first currency used in the Massachusetts Bay Colony in 1630.

A steady exchange rate from pence — the coins used by the British — to wampum beads was established: 4 pence was equal in value to 10 purple wampum beads.

Number of pence	Number of wampum beads
1	
4	10
6	
8	

- a** Complete the table. Show or explain your thinking.
- b** What number of wampum beads have the same value as 100 pence? Explain your thinking.

6. In the video game Nortfite, the digital currency Dollar-Zs can be used to make in-game purchases. Here is a conversion table showing the price of Dollar-Zs in 2024.

Dollar-Zs	In-Game Price (USD, \$)
1000	8.99
2800	22.99
5000	?
13500	89.99

- a** How much do you think 5,000 Dollar-Zs would cost you?

- b** Your teacher will share the actual price of 5,000 Dollar-Zs.

Discuss: How does your prediction compare to the actual cost? If your prediction was incorrect, why do you think that is?



Building Math Habits of Mind



Discuss:

- Which of these habits of mind did you strengthen during this activity?
- How did you use the one(s) you selected?

I can slow down and first make sense of a challenging problem before trying to solve it.

— —
 Not yet Almost I got it!

I can represent real-world problems using equations and inequalities and interpret their solutions within the context of the problem.

— —
 Not yet Almost I got it!

I can justify my thinking and ask questions to help me understand the thinking of others.

— —
 Not yet Almost I got it!

I can apply the math that I know to solve real-world problems, make assumptions and revise my thinking as needed.

— —
 Not yet Almost I got it!

I can select an appropriate tool to help me solve problems.

— —
 Not yet Almost I got it!

I can communicate my thinking and solutions clearly to others.

— —
 Not yet Almost I got it!

I can look for structure or patterns to help me solve problems.

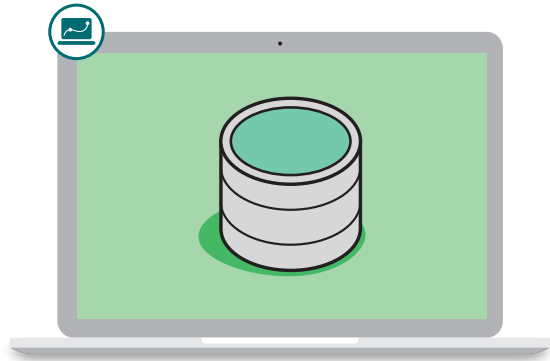
— —
 Not yet Almost I got it!

I can look for repeated calculations and other repeated steps to make generalizations.

— —
 Not yet Almost I got it!

Paint

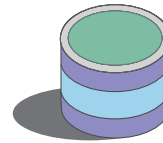
Let's explore equivalent ratios.



Warm-Up

- 1** Here is a color made from 5 cups of white paint and 7 cups of green paint.

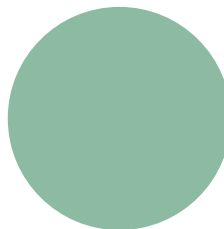
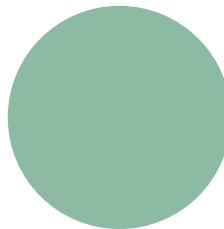
Discuss: What would you name this color? Why?



- 2** Brielle wants to match this color.

How many cups of green paint should she mix with 10 cups of white paint to make the same color?

White Paint (cups)	Green Paint (cups)
5	7
10	



5 white cups



7 green cups



10 white cups



? green cups

Color Match

3 Here are two paint mixtures:

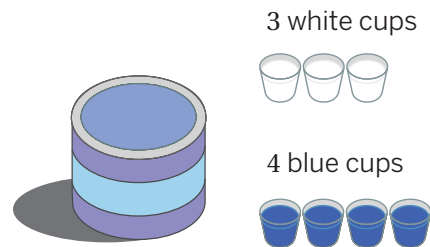
- 3 white cups and 4 blue cups
- 6 white cups and 8 blue cups

Both mixtures make the same color because they are *equivalent ratios*.

Which mixture would also make the same color?

- A. 5 white cups and 6 blue cups
 B. 1 white cup and $1\frac{1}{2}$ blue cups
 C. 4 white cups and 3 blue cups
 D. 1.5 white cups and 2 blue cups

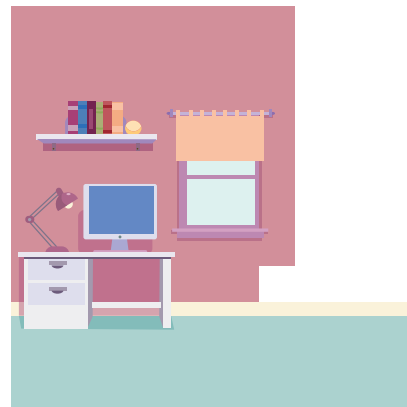
Explain your thinking.



4 Darryl mixed 4 cups of white paint with 6 cups of red paint, but he didn't have enough to finish painting his wall.

How much red paint would he need to add to 1 cup of white paint to match the color?

White Paint (cups)	Red Paint (cups)
4	6
1	



5 Here is Camila's and Habib's work for the previous problem.

$$\begin{aligned} &\text{Camilla} \\ \frac{\text{Red Cups}}{\text{White cups}} &= \frac{6 \cdot 0.25}{4 \cdot 0.25} = \frac{r}{1} \\ r &= 1.5 \end{aligned}$$

$$\begin{aligned} &\text{Habib} \\ \frac{6}{4} &= 1.5 \\ 1 \cdot 1.5 &= \boxed{1.5} \end{aligned}$$

Explain how you think one student determined how much red paint Darryl needed to match the color.

Paint Palooza

- 6** Brielle ran out of paint for her room.

Complete the table so that the new mixture matches the original paint color.

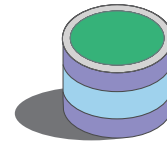
Blue Paint (cups)	Red Paint (cups)	White Paint (cups)
12	9	14
4		



- 7** Emika wants to make a green color. She mixes $\frac{1}{4}$ cups of white paint and $\frac{1}{2}$ cups of green paint.

How much white paint does she need to mix with 2 cups of green paint to make Emika's green color?

Explain your thinking.



- 8** **Discuss:** How much of each would Emika need to make 6 total cups of paint? How do you know?

Colorful Challenge

9 You will use the Activity 3 Sheet to create your own paint color challenge!

- a Make It!** Create your challenge on the Activity 3 Sheet.
- b Solve It!** On this page, record the number of cups of paint used in both your original mixture and the new mixture.

	Red Paint (cups)	Blue Paint (cups)	Green Paint (cups)	White Paint (cups)
Original Mixture				
New Mixture That Matches				


- c Swap It!**
- Swap your challenge with one or more partners.
 - Record the information about your partner's original mixture and their new mixture.
 - Fill in the missing amounts to complete the new mixture.

Partner 1	Red Paint (cups)	Blue Paint (cups)	Green Paint (cups)	White Paint (cups)
Original Mixture				
New Mixture That Matches				

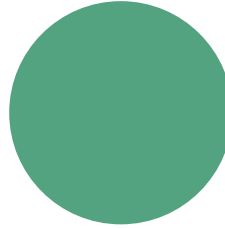
Partner 2	Red Paint (cups)	Blue Paint (cups)	Green Paint (cups)	White Paint (cups)
Original Mixture				
New Mixture That Matches				

Partner 3	Red Paint (cups)	Blue Paint (cups)	Green Paint (cups)	White Paint (cups)
Original Mixture				
New Mixture That Matches				

10 Synthesis

 **Discuss:** How can equivalent ratios help make matching paint colors?

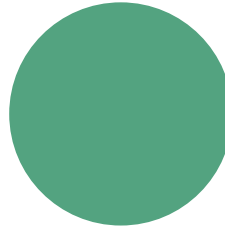
Use the example if it helps with your thinking.



1 white cup



4 green cups



2 white cups



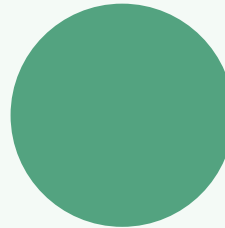
8 green cups



13 Summary 2.01

Ratios are said to be *equivalent ratios* when you can multiply the numbers in one ratio by the same factor to get the numbers in the other ratio.

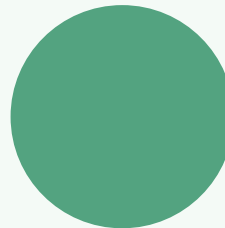
For example, 2 cups of white paint mixed with 3 cups of green paint creates the same color as 6 cups of white paint mixed with 9 cups of green paint. You can multiply the number of white cups and green cups in the first ratio by 3 to get the second ratio.



2 white cups



3 green cups



6 white cups



9 green cups



When determining if two ratios are equivalent, it can be helpful to use a table of ratios, like this one:

	White Paint (cups)	Green Paint (cups)	
×3	2	3	×3
	6	9	

Practice 2.01

Name: _____ Date: _____ Period: _____

Problems 1–5: The table shows an original recipe for orange-pineapple juice, as well as four other recipes.

Recipe	Orange Juice (cups)	Pineapple Juice (cups)
Original	10	15
Recipe A	4	6
Recipe B	3	2
Recipe C	9	12
Recipe D	6	9

- Select *all* the recipes that taste the same as the original.
 - A. Recipe A B. Recipe B
 - C. Recipe C D. Recipe D
- Choose one recipe that tastes the same as the original and explain your thinking.

- Choose a recipe that does not taste the same as the original. How do the two recipes compare?

- How much pineapple juice would you need to mix with 1 cup of orange juice to make a mixture that tastes the same as the original?

- Complete the table to create a different recipe that will also taste the same as the original.

Orange Juice (cups)	Pineapple Juice (cups)

- Brielle mixed 6 cups of blue paint with 3 cups of white paint to make the perfect color to paint her mural.


Complete the table to show several other ways to make this color.

Blue Paint (cups)	White Paint (cups)
6	3
8	
12	6
	15

Practice 2.01

Name: _____ Date: _____ Period: _____

7. Determine whether $\frac{6}{14}$ and $\frac{9}{21}$ are equivalent ratios.

8.  Select *all* the values that are *not* equivalent to $\frac{8}{10}$.

A. $\frac{16}{20}$

B. 0.8

C. $\frac{12}{20}$

D. $\frac{36}{40}$

E. $\frac{20}{25}$

9. Create equivalent ratios. Fill in each blank using the numbers 0 to 9 as many times as you want.

..... : = : = :

Spiral Review

10. A grocery store has three types of fish for sale. Carp costs \$5 for 3 pounds, mullet costs \$4 for 2 pounds, and herring costs \$3 for 2 pounds. Which fish costs the least per pound?

Show or explain your thinking.

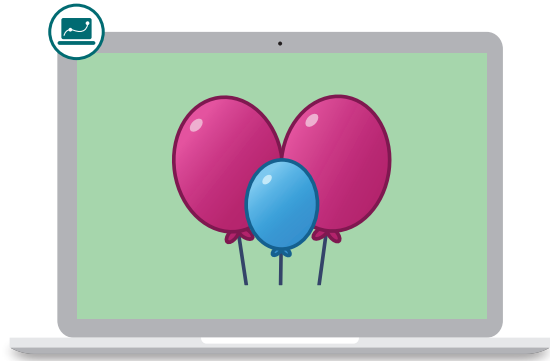
Problems 11–12: A square with side lengths that measure 7 units is being scaled. Decide whether the scaled figure be smaller or larger for each scale factor given.

11. 0.75

12. 1.5

Balloon Float

Let's explore proportional relationships in tables.



Warm-Up

1 This table shows a proportional relationship between the number of cases a store orders and how many rolls of paper towels they receive.

Number of Cases Ordered	Number of Rolls of Paper Towels
1	12
3	36
5	60
10	120

What do you notice? What do you wonder?

I notice:

I wonder:

Proportional Relationships

2 Here are two more tables.

Which of these two tables represents a proportional relationship? Circle one.

Table 1 Table 2 Both Neither

Explain your thinking.

Table 1

Weight (oz)	Number of Balloons
3	6
7	10
9	12
30	33

Table 2

Weight (oz)	Number of Balloons
4	12
6	18
42	126
8	24

3 Sort the tables into two groups based on whether they represent proportional relationships.

Table A

x	y
0	0
4	5
8	10
12	15

Table B

x	y
0	0
2	4
4	16
6	36

Table C

x	y
0	2
3	5
6	8
9	11

Table D

x	1	2	3	4
y	10	8	6	4

Table E

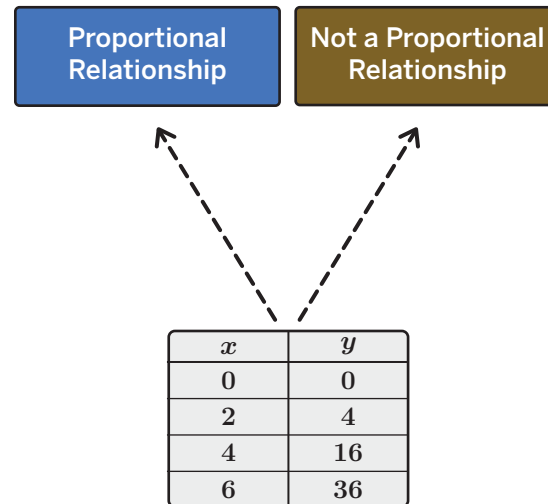
x	2	8	1	20
y	5	20	2.5	50

Proportional Relationship

Not a Proportional Relationship

Proportional Relationships (continued)

- 4** How did you decide whether this table represents a proportional relationship?



- 5** Select *all* the relationships you think are proportional.

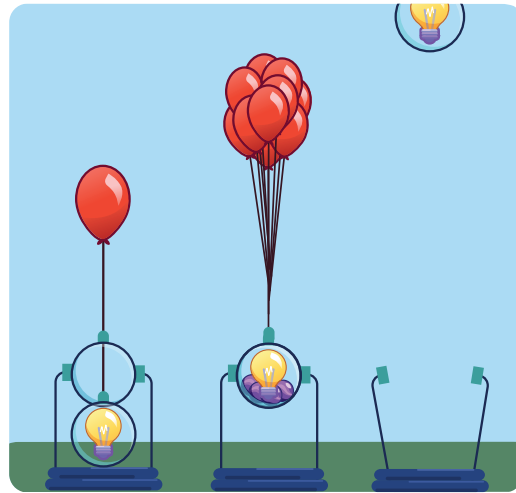
- A. A person's height in feet and their height in inches
- B. The number of cookies baked and the number of minutes they were in the oven
- C. The amount of bread baked and the number of grams of flour needed to bake it
- D. A person's time as they run a marathon and their total distance covered
- E. The gallons of gasoline purchased and their total cost

Balloon Float

Helium balloons can make objects float, but too many balloons will make objects fly away!

6 **a** Let's watch the *middle* light bulb float.

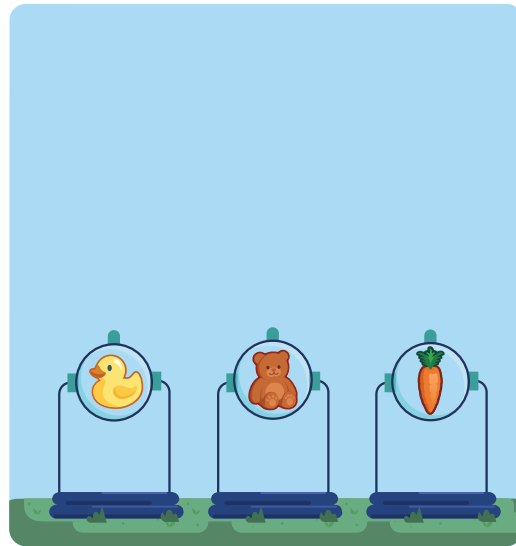
b **Discuss:** Why do you think the number of balloons matters?



7 In the previous problem, the light bulb weighed 2 ounces and needed 6 balloons to float.

If each balloon carries the same weight, how many balloons would you need to float each object?

Object	Weight (oz)	Number of Balloons
Light Bulb	2	6
Rubber Duck	10	
Toy Bear	6	
Carrot	3	



Balloon Float (continued)

8 Here are two strategies for determining the number of balloons needed to make the rubber duck float.

Ariel

Object	Weight (oz)	Number of Balloons
Light Bulb	2	6
Rubber Duck	10	30
Toy Bear	6	
Carrot	3	

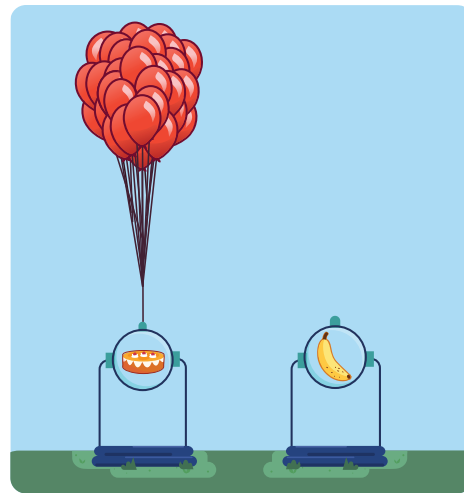
Emma

Object	Weight (oz)	Number of Balloons
Light Bulb	2	6
Rubber Duck	10	30
Toy Bear	6	
Carrot	3	

Discuss: How might Ariel and Emma use their strategies to finish their tables?

9 Here are some new objects. Complete the table so that each object floats.

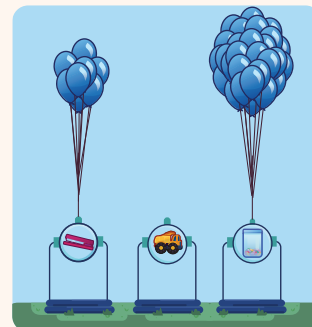
Object	Weight (oz)	Number of Balloons
Light Bulb	2	6
Cake		60
Banana	$3\frac{1}{3}$	



You're invited to explore more.

10 Blue balloons are different from red balloons. 8 blue balloons can float a 10-ounce stapler. Complete the table so that each object floats.

Object	Weight (oz)	Blue Balloons
Toy Truck	15	
Jelly Beans		28



11 Synthesis

Here are some relationships, some of which are proportional and some of which are not.

What determines whether a relationship is proportional?

Use the examples if they help with your thinking.

- A person's height in feet and their height in inches
- The number of cookies baked and the number of minutes they were in the oven
- The amount of bread baked and the number of grams of flour needed to bake it
- A person's time as they run a marathon and their total distance covered
- The gallons of gasoline purchased and their total cost

14 Summary 2.02

A **proportional relationship** is a set of *equivalent ratios*. The values for one quantity are each multiplied by the same number to get the values for the other quantity.

You can see this when moving between the columns of this table that shows the cost of varying amounts of soybeans. You can multiply the pounds of soybeans by 2 to get the cost.

When you multiply one quantity in a proportional relationship by a value, the other quantity will change by the same factor.

You can see this when moving between the rows of the table. When the pounds of soybeans is multiplied by 8, the cost for them is multiplied by the same number.

Soybeans (lb)	Cost (\$)
1	2
2	4
8	16
$\frac{1}{2}$	1
$\frac{1}{4}$	0.50

Diagram annotations: A blue arrow labeled $\times 2$ points from 1 to 2 in the first row. A blue arrow labeled $\times 2$ points from 2 to 4 in the second row. A blue arrow labeled $\times 8$ points from 1 to 8 in the third row. A blue arrow labeled $\times 8$ points from 2 to 16 in the third row.

proportional relationship Two variables are in a proportional relationship if the values for one variable are each multiplied by the same number to get the values for the other variable.

Practice 2.02

Name: _____ Date: _____ Period: _____

Problems 1–4: Complete the tables so that the relationship is proportional.

1.

x	y
30	3
120	
	10

2.

x	y
1	1.5
3	
	12

3.

x	y
15	45
1	
	0

4.

x	y
0.2	1
1	
	20

- 5.** Entrance to a state park costs \$6 per vehicle, plus \$2 per person. The table shows the entry cost for several recent groups of visitors. Is the relationship between the number of people and the total entrance cost a proportional relationship?

Number of People in Vehicle	Total Cost (\$)
2	10
3	12
4	14
10	26

Explain your thinking.

- 6.** A store charges \$4.80 for 16 ounces of bubble tea. Complete the table so that it shows a proportional relationship between ounces of tea and cost.

Tea (oz)	Cost (\$)
16	4.80
20	
	7.20

Practice 2.02

Name: _____ Date: _____ Period: _____

7. A bakery uses 8 tablespoons of honey for every 10 cups of flour to make bread dough. Complete the table so that it shows a proportional relationship between the amount of honey and the amount of flour. Show your thinking.

Honey (tbsp)	Flour (cups)
8	10
20	
13	
	12.5


Spiral Review

Problems 8–10: Solve each equation. Show your thinking.

8. $\frac{1}{2} + x = 2$

9. $\frac{2}{3}y = 6$

10. $3 = \frac{1}{4}b$

11.  A student makes a scaled copy of a rectangle. The dimensions of the original rectangle are 3.5 by 2.5 meters. Select *all* the possible dimensions of the scaled copy.
- A. 1.75 meters and 1.25 meters
 - B. 7 meters and 5 meters
 - C. 7 meters and 6 meters
 - D. 10 meters and 2.5 meters
 - E. 10.5 meters and 7.5 meters

Problems 12–13: Liam, Marc, and Victor run the 40 yard dash. Liam's time is 5.2 seconds, Marc's time is 5.02 seconds, and Victor's time is 5.12 seconds.

12. Who finishes first, second, and third?
13. Another runner finished less than a half a second behind the third-place finisher. What could their time have been?

Sugar, Spice, and Everything Rice

Let's explore proportional relationships some more.



Warm-Up

1. This table shows the amount of sugar in different volumes of soda.

What do you notice? What do you wonder?

I notice:

Soda (fl. oz)	Sugar (g)
1	3
7.5	22.5
12	36
20	60

I wonder:

Sugary Drinks

A 12-ounce bottle of orange juice contains 33 grams of sugar.

2. Explain why the relationship between the volume of orange juice and the amount of sugar is proportional.

	Orange Juice (fl. oz)	Sugar (g)
Glass	8	
Bottle	12	33
Carton	32	
Jug	128	

3. Complete the table to determine the amount of sugar in different-sized containers of orange juice.

When two quantities have a *proportional relationship*, you can multiply the value of one by a **constant of proportionality** to get the value of the other.

4. What is a constant of proportionality in the relationship between volume of orange juice and amount of sugar? What does it tell us about this situation?
5. The label on an orange juice box claims that it contains only 11 grams of sugar. How many fluid ounces must be in the orange juice box? Show or explain your thinking.
6. A 25-ounce bottle of apple cider contains 31 grams of sugar. Which drink is more sugary: apple cider or orange juice? Explain how you know.

Rice and Spice

Here are the instructions for cooking instant rice in the microwave.

Ingredients

Rice (cups)	Water (cups)
1	$1\frac{1}{2}$
2	3
3	$4\frac{1}{2}$

Cook Time


Rice (cups)	Time (min)
1	7
2	11
3	15

- Which relationships are proportional (if any)?
- For any proportional relationships, determine a constant of proportionality and explain what it means.


When making graham crackers, the relationship between the amount of whole wheat flour and the amount of cinnamon is proportional.

- Complete the table.

Whole Wheat Flour (cups)	2	4	5	
Cinnamon (tsp)	0.75	1.5		3.75

- What is a constant of proportionality in this relationship?
-  **Discuss:** If Xavier decides not to follow the recipe and uses 1 teaspoon of cinnamon for every 2 cups of wheat flour, how will that impact the taste of the graham cracker

Synthesis

12.  **Discuss:** How can you use a constant of proportionality to make sense of a proportional relationship?

Use the example if it helps with your thinking.

Corn (cups)	Protein (g)
1	5.1
3	15.3
5	25.5

Summary 2.03

In a *proportional relationship*, you can multiply the values of one quantity by a **constant of proportionality** to get the values of the other quantity.

For example, 12 is a constant of proportionality in the relationship between feet and inches.

Feet	1	2	6
Inches	12	24	72

That means that you can multiply the number of feet by 12 to determine the number of inches.

There are 12 inches for every 1 foot or $\frac{1}{12}$ foot for every 1 inch.

constant of proportionality In a proportional relationship, the number used to multiply the values for one quantity to get the values for the other quantity is called the constant of proportionality.

Practice 2.03

Name: _____ Date: _____ Period: _____

Problems 1–2: When Deven makes chocolate milk, he mixes 2 cups of milk with 3 tablespoons of chocolate syrup. This table shows how to make batches of different sizes.

1. Is there a proportional relationship between cups of milk and tablespoons of chocolate syrup?

Show or explain your thinking.

Milk (cups)	Chocolate Syrup (tbsp)
2	3
8	12
1	$\frac{3}{2}$
10	15

2. What is a constant of proportionality for this relationship?

Problems 3–4: When you mix two colors of paint in equivalent ratios, the color you get is always the same.

3. Complete the table so that each row makes the same color purple.

Blue (cups)	Red (cups)
2	6
1	

4. What is a constant of proportionality for this relationship?

Problems 5–7: Here is some information about the side lengths of two scaled copies, triangle *A* and triangle *B*.

5. Complete the table to determine the missing side lengths of each triangle.
6. What is a constant of proportionality in this relationship?

Side Length of Triangle <i>A</i> (in.)	Side Length of Triangle <i>B</i> (in.)
1	6
$\frac{1}{2}$	
	8

7. What does that constant of proportionality tell you about the triangles?

Practice 2.03

Name: _____ Date: _____ Period: _____

Problems 8–10: Table A shows different amounts of blue and white paint that can be mixed together to create a color called “Blue Breeze.” Sai wants to create a new color that is *bluer* than “Blue Breeze” and call it “Sea Glass Blue.”

8. Complete the first row of Table B to create a possible mixture for “Sea Glass Blue.”

Table A
Blue Breeze

Blue (cups)	White (cups)
1	3
2	6
5	15
8	24

Table B
Sea Glass Blue

Blue (cups)	White (cups)
4	
7	
	1
12	

9. Complete the rest of Table B so that the relationship between cups of blue and cups of white is proportional.

10. Explain how you know that the new color is bluer than “Blue Breeze”.

Spiral Review

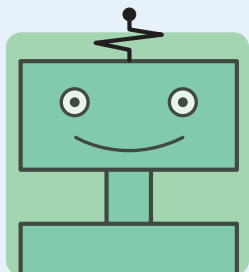
11.  Select *all* the scales that are equivalent to the scale 1 centimeters to 5 meters.

- A. $\frac{1}{2}$ cm to $2\frac{1}{2}$ m B. 1 cm to 5 km C. 5 cm to 1 km
 D. 5 cm to 25 m E. 10 cm to 50 m

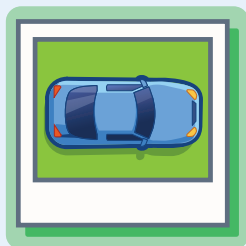
12. Select *all* of the ratios that are equivalent to 4 : 7.

- A. 8 : 15 B. 16 : 28 C. 7 : 4
 D. 20 : 35 E. 6 : 9

Proportional Relationships in Equations



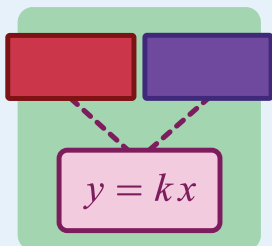
Lesson 4
Robot Factory



Lesson 5
Snapshots



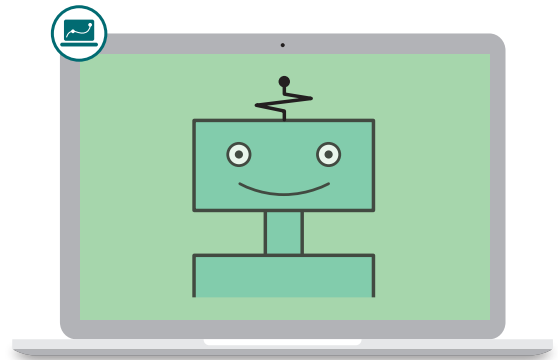
Lesson 6
Two and Two



Lesson 7
All Kinds of Equations

Robot Factory

Let's write equations for proportional relationships.



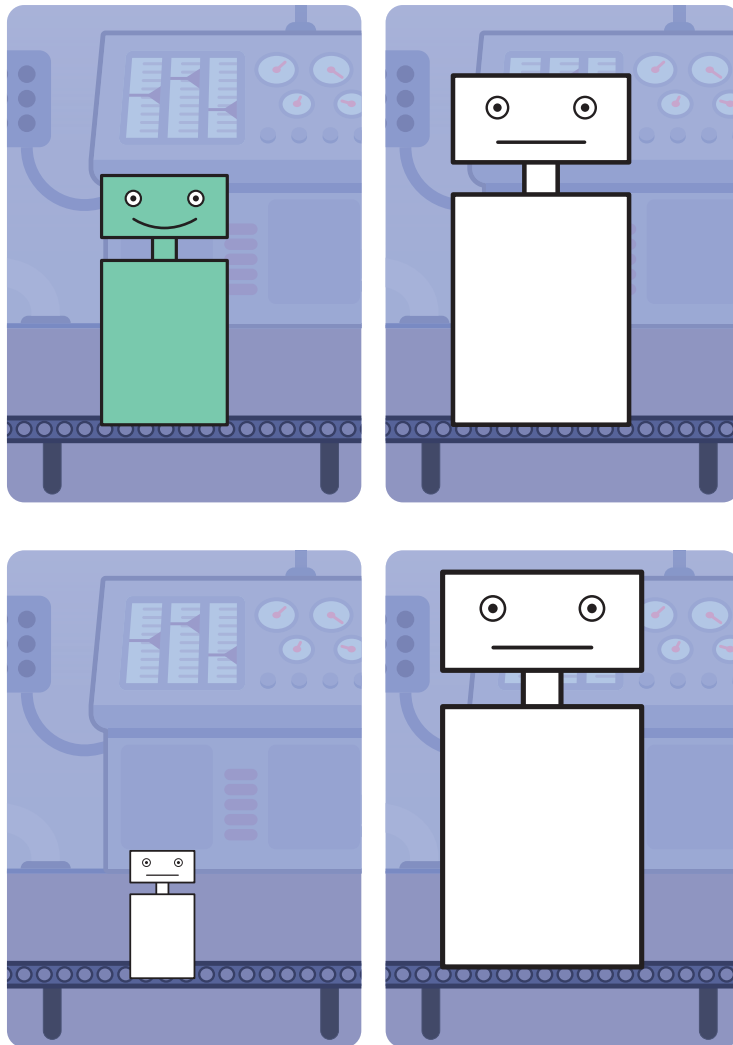
Warm-Up

1-2 The robot factory can make robots of all different sizes.

This robot's shade of green is made by mixing green and white paint using the amounts in the table.

Complete the table so that all four robots have the same shade of green.

Green Paint (cups)	White Paint (cups)
4	3
8	
1	
10	



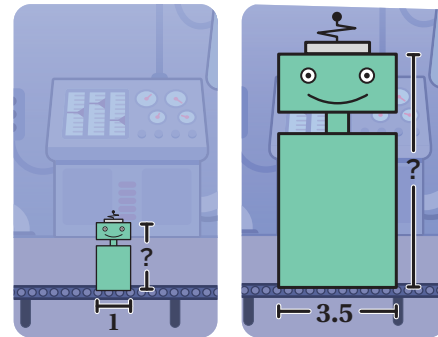
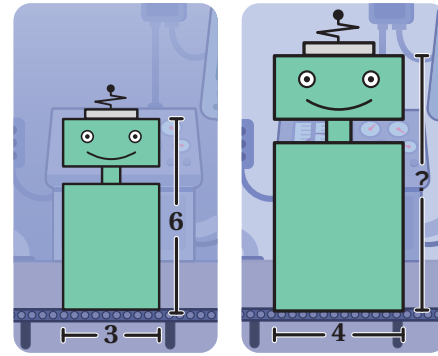
Robot Hats

3-4 Let's make *scaled copies* of this robot.

The width of this robot is 3 inches. Its hat is 6 inches off the ground.

Complete the table with the height for placing the hat on each robot.

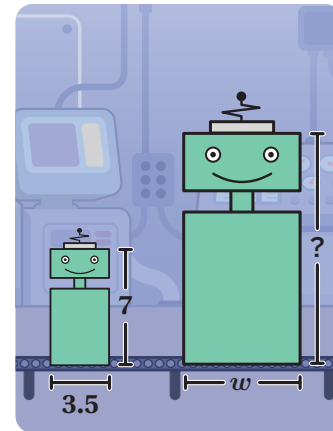
Robot Width (in.)	Height for Placing Hat (in.)
3	6
4	
1	
3.5	
2.18	
1.76	
3.425	



5 Here is a student's work from the previous problem.

Robot Width (in.)	Height for Placing Hat (in.)
3.5	$3.5 \cdot 2$
2.18	$2.18 \cdot 2$
1.76	$1.76 \cdot 2$

Help someone know how to finish the job. For any robot's width, how could you determine the height for placing its hat?



6 Instructions with words are useful for humans, but machines understand mainly numbers and symbols.

Which equation tells the factory the relationship between a robot's width, w , and the height for placing its hat, h ?

- A. $h = \frac{1}{2}w$
- B. $h = w + 3$
- C. $w = 2h$
- D. $h = 2w$

More Robot Parts

- 7-8** This robot is 3 inches wide. Its shoes are 1 inch tall.

Complete the table with the shoe height for each robot.

Robot Width (in.), w	Shoe Height (in.), s
3	1
6	
5	
1	

- 9** Write an equation the factory could use to put shoes on the rest of the robots.

Use s for the shoe height and w for the robot's width.

$$s = \underline{\hspace{2cm}}$$

- 10-11** This robot has a height of 9 inches. Its arms are 5 inches off the ground.

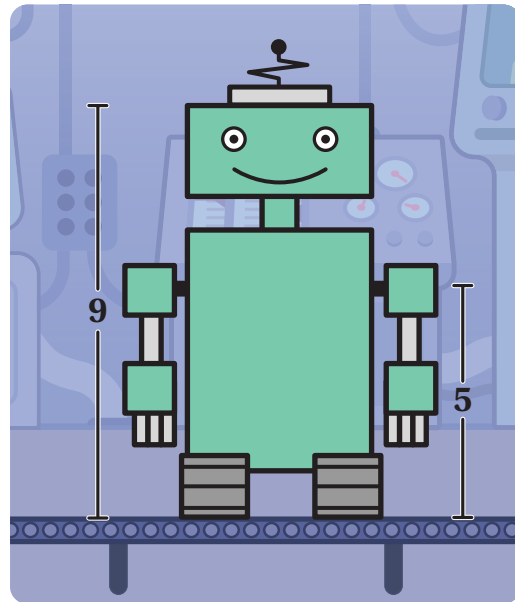
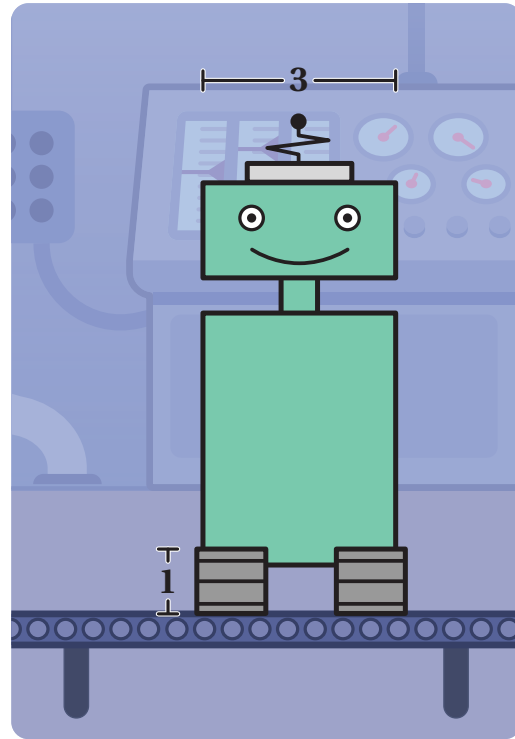
Write an equation that the factory could use to proportionally attach arms to the robots.

Use a for the height to place the arms and r for the robot height.


$$a = \underline{\hspace{2cm}}$$

Use this table if it helps with your thinking.

Robot Height (in.), r	Height for Placing Arms (in.), a
9	5
5	2.78
1	0.56



12 Synthesis

 **Discuss:** What is a strategy you can use to write an equation of a proportional relationship when given a table?

Use one or both of the examples if they help with your thinking.

Hat

Robot Width (in.), w	Hat Height (in.), h
3	6
1	2

The constant proportionality is 2.

An equation for this relationship is $h = 2w$.

Arms

Robot Width (in.), r	Arm Height (in.), a
9	5
1	$\frac{5}{9}$

The constant proportionality is $\frac{5}{9}$.

An equation for this relationship is $a = \frac{5}{9}r$.

15 Summary 2.04

Proportional relationships can be represented using the equation $y = kx$, where k is the *constant of proportionality*.

For example, the table shows the proportional relationship between the number of pounds of soybeans and the cost at a certain store.

- The cost of the soybeans is proportional to the weight with a constant of proportionality of 2.
- If c represents the cost and w represents the weight, then you can represent the proportional relationship with the equation $c = 2w$.


Weight (lb), w	Cost (\$), c
$\frac{1}{2}$	1.00
1	2.00
2	4.00
w	$2w$

Practice 2.04

Name: _____ Date: _____ Period: _____

1. The ceilings in many basements are made up of rectangular tiles. For one basement, each square meter of ceiling requires 10.75 tiles. Complete the table.

Area of Ceiling (sq. m)	Number of Tiles
1	
10	
	53.75
x	

 **Problems 2–3:** Each table represents a proportional relationship. Determine the constant of proportionality that completes each equation.

2.

s	P
2	8
3	12
5	20
10	40

$$P = \dots\dots\dots s$$

3.

d	C
2	6.28
3	9.42
5	15.7
10	31.4

$$C = \dots\dots\dots d$$

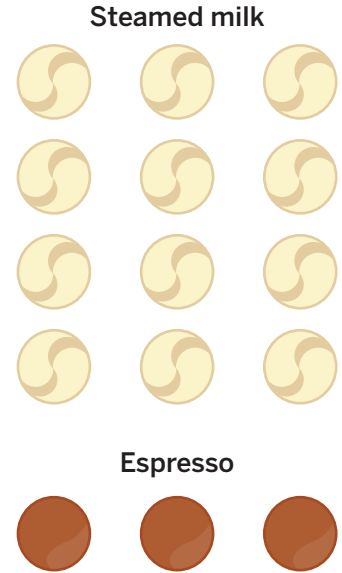
Problems 4–6: While she was traveling in 2023, Mai received 342.50 Norwegian kroner in exchange for 50 Australian dollars.

- How many Norwegian kroner would Mai have received in exchange for 1 Australian dollar?
- Write an equation to represent the amount of Norwegian kroner, k , received in exchange for a Australian dollars.
- Determine the number of Norwegian kroner Mai would receive in exchange for 120 Australian dollars.

Practice 2.04

Name: _____ Date: _____ Period: _____

Problems 7–9: Kiran, a barista, makes a latte with a particular ratio of espresso and steamed milk. His customers have told him that this ratio is a good balance — not too “milky” or too “espresso-y.”



7. Complete the table for the different-sized cups at his shop.

Size	Espresso (fluid oz)	Steamed milk (fluid oz)
Mini	0.5	
Small	1	
Medium	3	12
Large	5	
Extra large	6	24

8. What is the constant of proportionality? What does it represent?

9. Write an equation for the fluid ounces of steamed milk, m , for any number of fluid ounces of espresso, e .

Problems 10–11: A plane flew at a constant speed between Denver and Chicago. It took the plane 1.5 hours to fly 915 miles.

10. Complete the table.

11. How far would the plane fly in 10 hours at this speed?

Time (hr)	Distance (mi)
1	
1.5	915
2	
2.5	
t	

Spiral Review

Problems 12–13: A bicycle travels 21 meters in 3 seconds.

12. Complete the table.

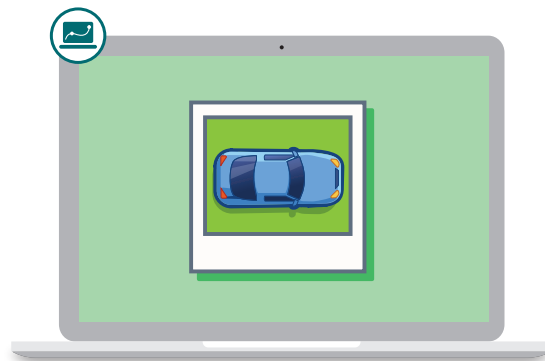
13. What is a constant of proportionality in this relationship?

What does this represent in the situation?

Time (sec)	Distance (m)
3	21
$1\frac{1}{2}$	
	$6\frac{3}{10}$

Snapshots

Let's use equations to make sense of proportional relationships in the world.



Warm-Up

- 1 Write a story about this car's trip.

6 seconds
180 meters

8.2 seconds
246 meters

17.5 seconds
525 meters

19.7 seconds
591 meters

Travel Times

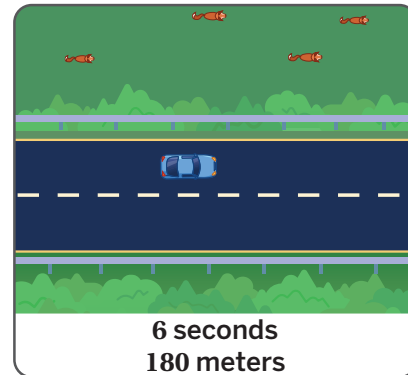
2 The car travels at a constant speed. After 6 seconds, it travels 180 meters.

- a** Write an equation for the car's distance, d , at any time, t .

$d = \dots\dots\dots$

- b** Enter the time and distance for three moments during the car's trip.

Time (sec), t	Distance (m), d



3 Ethan says that a row in a table is like a picture and an equation is like a video. Explain what he might be thinking.

4 Use the equation you wrote to complete the table.

Time (sec), t	Distance (m), d
6	180
3	
	60

Cakes


- 5** A cake recipe uses the equation $m = 6c$, where c is the number of cakes and m is ounces of milk.

Explain what the constant of proportionality means in this situation.

- 6** How many ounces of milk are needed to bake 12 cakes?



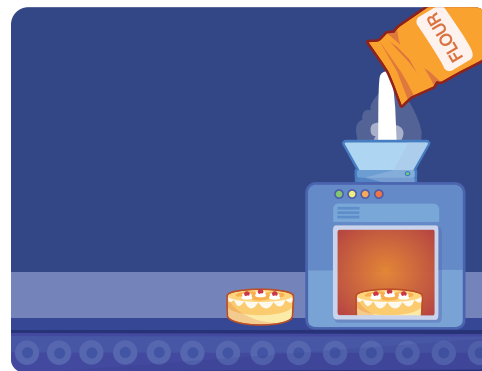
- 7** A cake recipe says to use 3 cups of flour for every 2 cakes.

- a**  **Discuss:** How much flour do you need for 1 cake?

- b** Write an equation to calculate the amount of flour needed, f , for any number of cakes, c .

$f = \dots\dots\dots$

- c** How much flour is needed to make 8 cakes?



2 cakes
3 cups of flour

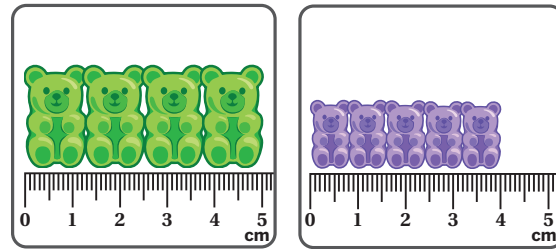
Gummy Bears

8 Here is an equation: $l = \frac{5}{4}g$. l is the total length and g is the number of gummy bears.

Which size gummy bear does the equation represent? Circle one.

Large Small Both Neither

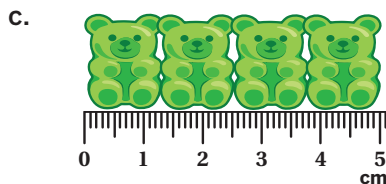
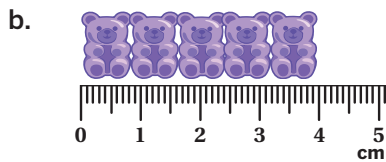
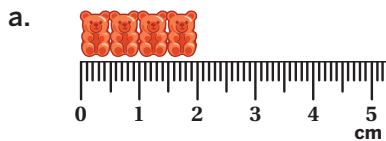
Explain your thinking.



$$l = \frac{5}{4}g$$

9 Match each image with one or more equations or descriptions. In each equation, l is the total length and g is the number of gummy bears.

Image



Equation or Description

..... $l = \frac{4}{5}g$

..... $l = \frac{1}{2}g$

..... $l = \frac{5}{4}g$

..... A line of 80 of these gummy bears is 100 centimeters long.

You're invited to explore more.

10 Use the You're Invited to Explore More Sheet to answer a question about a truck's trip.

11 Synthesis

Here is a snapshot from a situation represented by the equation $p = 8g$, where g represents a number of gallons and p represents a number of pints.

How can an equation of a proportional relationship represent a situation? Use the example if it helps you explain your thinking.



14 Summary 2.05

When a vehicle is traveling at a constant speed, there is a *proportional relationship* between the time traveled and the distance traveled. This is true for any person, animal, or object traveling at a constant speed.

For example, imagine someone running 5 meters per second. The table shows the distance they travel over different periods of time. The table also shows their speed, in meters per second.

The last row in the table shows that we can multiply the amount of time, t , with the *constant of proportionality*, 5, to determine the distance traveled, d .

The equation $d = 5t$ represents this relationship.

Time (sec)	Distance Traveled (m)	Speed (m/sec)
1	5	5
2	10	5
3	15	5
7	35	5
t	$5t$	5

Practice 2.05

Name: _____ Date: _____ Period: _____

Problems 1–4: A performer expects to sell 5,000 tickets for an upcoming concert. They plan to make \$311,000 from the sales of these tickets. Assume that all tickets have the same price.

1. What is the price for one ticket? Show or explain your thinking.
2. Write an equation to represent the relationship between the number of tickets sold, x , and the total amount of money (in dollars) that they make, y .
3. How much money do they make if 3,575 tickets are sold?
4. If they make \$379,420, how many tickets have they sold?

Problems 5–6: A car is traveling on a highway at a constant speed. The equation that represents the distance traveled in miles, d , for t hours is $d = 65t$.

5. What does the value 65 represent in this situation?
6. At this rate, how many miles will the car travel in 1.5 hours? Show your thinking.

Problems 7–8: On its way from New York to San Diego, a plane flew at a constant speed over Pittsburgh, Saint Louis, Albuquerque, and Phoenix.



7. This table shows the flight time and distance traveled for each segment of the flight. Complete the table.

Segment	Time (hr)	Distance (mi)	Speed (mph)
Pittsburgh to Saint Louis	1	550	
Saint Louis to Albuquerque	1.7		
Albuquerque to Phoenix		330	

8. Let t represent the time in hours and d represent the distance in miles. Write an equation that represents the distance traveled for t hours.


Practice 2.05

Name: _____ Date: _____ Period: _____

9. A train travels at a constant speed between Springfield and Chicago. The train travels $100\frac{1}{2}$ miles in $\frac{3}{4}$ hours. How far does the train travel in one hour at this same speed?
10. Na'ilah is making a pitcher of a lemon-flavored sports drink. The drink mix container says to mix $\frac{1}{4}$ cups of powdered drink mix with 2 quarts of water. She prefers her sports drink to taste *more* lemon-y than the recipe on the container. Complete the equation to represent a mixture of cups of drink mix, c , and quarts of water, w , that would be *more* lemon-y than the original mixture.

$c = \dots\dots\dots w$

Spiral Review

11.  Select *all* the tables that represent a proportional relationship between x and y .

A.

x	0	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{16}$
y	0	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$

B.

x	0	$\frac{1}{2}$	2	3
y	0	$\frac{1}{4}$	4	9

C.

x	0	4	8	12
y	0	2	4	6

D.

x	0	1	2	3
y	0	4	8	12

12. Is 4.5 a solution to the equation $1.5 + x = 6$?

Two and Two

Let's keep exploring constants of proportionality.



Warm-Up

1. Here are two tables. What do you notice? What do you wonder?

Table A

Length (m), x	Length (cm), y
1	100
0.25	25
1.6	160
57.1	5710

Table B

Length (cm), y	Length (m), x
100	1
450	4.5
78.2	0.782
123	1.23

I notice:

I wonder:

Table Equation Match-Up

Here are the tables from the Warm-Up again.

2. Match each table to an equation that represents the situation. Some equations may not have a match.

Table A

Length (m), x	Length (cm), y
1	100
0.25	25
1.6	160
57.1	5710

Table B

Length (cm), y	Length (m), x
100	1
450	4.5
78.2	0.782
123	1.23

Equations

$$y = 100x$$

.....

$$y = \frac{1}{100}x$$

.....

$$x = 100y$$

.....

$$x = \frac{1}{100}y$$

.....

3. Alexis thinks that the equation $y = 100x$ matches Table A. Carlos thinks that the equation $x = \frac{1}{100}y$ matches Table A. Whose claim is correct? Circle one.

Alexis's

Carlos's

Both

Neither

Explain your thinking.

Jayden's Cooler

4. It took Jayden 5 minutes to fill a cooler with 8 gallons of water from a faucet flowing at a steady rate.

Complete either Part 1 or Part 2.

Part 1

- a** Complete the table.
- b** What is a constant of proportionality in this relationship?
- c** Complete the equation for this proportional relationship.

$$w = \dots\dots\dots$$

Time (min), t	Water (gal), w
0	0
1	
2.5	
5	
t	

Part 2

- a** Complete the table.
- b** What is a constant of proportionality in this relationship?
- c** Complete the equation for this proportional relationship.

$$t = \dots\dots\dots$$

Water (gal), w	Time (min), t
0	0
1	
4	
8	
w	

5. Find a partner who completed the part you *didn't* complete.



Discuss: How are your responses alike? How are they different?

Jayden's Cooler (continued)

It took Jayden 5 minutes to fill a cooler with 8 gallons of water from a faucet flowing at a steady rate.

6. What is the relationship between the constants of proportionality that you and your partner identified?

7. What does $\frac{5}{8}$ tell you about the situation?


8. What does $\frac{8}{5}$ tell you about the situation?

You're invited to explore more.

9. **a** Describe a situation where $\frac{10}{3}$ is a constant of proportionality.
- b** What would the other constant of proportionality, $\frac{3}{10}$, mean in the situation you described?

Synthesis

Here are the proportional relationships from this lesson.

10.  **Discuss:** Why are there two constants of proportionality in a proportional relationship?

Situation	Constants of Proportionality	Equations
There are 100 centimeters, y , in every meter, x .	100	$y = 100x$
	$\frac{1}{100}$	$x = \frac{1}{100}y$
It took Jayden 5 minutes, t , to fill a cooler with 8 gallons of water, w , at a steady rate.	$\frac{5}{8}$	$t = \frac{5}{8}w$
	$\frac{8}{5}$	$w = \frac{8}{5}t$

Summary 2.06

When two quantities x and y are in a *proportional relationship*, you can represent the relationship in two ways:

- The equation $y = kx$, with k as the *constant of proportionality*.
- The equation $x = \frac{1}{k}y$, with $\frac{1}{k}$ as the constant of proportionality.

Each equation highlights the relationship between the two quantities.

For example, if one pound of soybeans costs \$2.00, then:

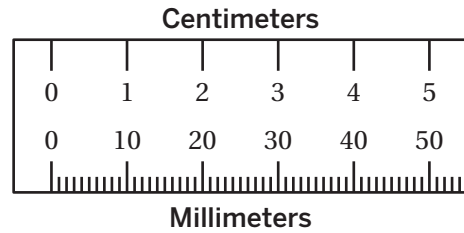
- The cost, c , is proportional to the weight, w . The equation $c = 2w$ represents the situation because you can multiply the weight by 2 to get the cost.
- The weight, w , is proportional to the cost c . The equation $w = \frac{1}{2}c$ represents the situation because you can multiply the cost by $\frac{1}{2}$ to get the weight.

reciprocal Two numbers whose product is 1. In this example, $\frac{3}{2}$ and $\frac{2}{3}$ are reciprocals because $\frac{3}{2} \cdot \frac{2}{3} = 1$.

Practice 2.06

Name: _____ Date: _____ Period: _____

Problems 1–3: There is a proportional relationship between a length measured in centimeters and the same length measured in millimeters.



1. Complete the table and then determine a constant of proportionality.

Length (cm)	Length (mm)
4	40
12	
50	
88.49	

Constant of proportionality: _____

2. Complete the table and then determine a constant of proportionality.

Length (mm)	Length (cm)
25	2.5
240	
4	
699.1	

Constant of proportionality: _____

3. How are these two constants of proportionality related to each other?

4. Amoli and Emika are converting measurements between inches and feet.

Amoli says that the constant of proportionality for the relationship between inches and feet is 12. Emika says it is $\frac{1}{12}$.

Whose claim is correct?

- A. Amoli's B. Emika's C. Both D. Neither

Explain your thinking.

Practice 2.06

Name: _____ Date: _____ Period: _____

Problems 5–8: A recipe for granola calls for $\frac{5}{2}$ cups of rolled oats and $\frac{3}{4}$ cups of shredded coconut.

5. Complete the table to show how many cups of coconut should be used for different amounts of oats.

Oats (cups)	Coconut (cups)
1	
5	
12	

6. What is a constant of proportionality in this relationship? What does it represent?

7. What is the *other* constant of proportionality? What does it represent?

8. t represents the number of cups of oats, and c represents the number of cups of coconut. Write two equations that represent the relationship between t and c .

Spiral Review

Problems 9–10: The Wrangell-St. Elias National Park and Preserve in Alaska has an area of 20,625 square miles. A scale drawing of the Wrangell-St. Elias National Park and Preserve has an area of 8.25 square inches.

9. What is the scale of the drawing?

10. Denali National Park and Preserve in Alaska has an area of 9,492 square miles. If you use the same scale, what will the area of the Denali National Park and Preserve be in the scale drawing?

11.  Which equation has a constant of proportionality equal to 5?

A. $5y = 5x$

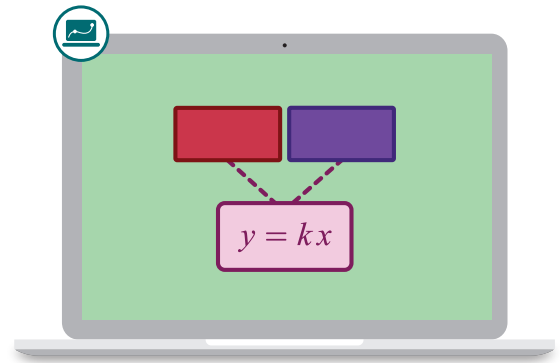
B. $5y = 15x$

C. $3y = 5x$

D. $3y = 15x$

All Kinds of Equations

Let's use equations to decide whether a relationship is proportional.



Warm-Up

1 This table represents the equation $y = 3x + 1$.

- a** Use the equation to complete the table.
- b** Does the equation represent a proportional relationship? Circle one.

Yes No Not enough information

Explain your thinking.

x	y
0	1
1	4
3	10
	16
2.5	

Stories, Equations, Tables

- 2** Use the story and equation to complete the table. Then decide whether the relationship is proportional. Complete either Story 1 or Story 2.

Story 1

Story: Lucia earns \$12 per hour.

Define Variables:

- Let x represent . . .
- Let y represent . . .

Equation: $y = 12x$

Is the relationship proportional?

Explain your thinking.

Time Worked (hr), x	Pay (\$), y
0	
1	
	30

Story 2

Story: A recipe recommends 1 banana for every 2 smoothies.

Define Variables:

- Let x represent . . .
- Let y represent . . .

Equation: $y = 0.5x$

Is the relationship proportional?

Explain your thinking.

Number of Smoothies, x	Number of Bananas, y
0	0
1	
	2.5

Find a partner who completed the other story.



Discuss: How are your responses alike? How are they different?

Stories, Equations, Tables (continued)

Complete either Story 3 or Story 4.

Story 3

Story: A cell phone costs \$500, plus \$35 per month for the plan.

Define Variables:

- Let x represent . . .
- Let y represent . . .

Equation: $y = 35x + 500$

Is the relationship proportional?

Explain your thinking.

Number of Months, x	Total Cost (\$), y
0	
1	
	605

Story 4

Story: The area of a square is the side length multiplied by itself.

Define Variables:

- Let x represent . . .
- Let y represent . . .

Equation: $y = x^2$

Is the relationship proportional?

Explain your thinking.

Side Length (units), x	Area (sq. units), y
0	
1	
	100

3 Here are the equations that represent the four stories.

a Select *all* the equations that represent a proportional relationship.

A. $y = 12x$

B. $y = 500 + 35x$

C. $y = \frac{1}{2}x$

D. $y = x^2$

b Explain one way to decide if an equation represents a proportional relationship.

Equations and Proportionality

- 4 Decide whether each equation, table, or story represents a proportional relationship by placing a checkmark in the appropriate column.

	Proportional	Not Proportional										
$4 + x = y$												
$y = 4x$												
Jacy walked 4 miles in 100 minutes at a steady pace.												
$0.04x = y$												
$y = \frac{x}{4}$												
$\frac{4}{x} = y$												
<table border="1"> <thead> <tr> <th>x</th> <th>y</th> </tr> </thead> <tbody> <tr> <td>1</td> <td>4</td> </tr> <tr> <td>2</td> <td>2</td> </tr> <tr> <td>3</td> <td>$\frac{4}{3}$</td> </tr> <tr> <td>4</td> <td>1</td> </tr> </tbody> </table>	x	y	1	4	2	2	3	$\frac{4}{3}$	4	1		
x	y											
1	4											
2	2											
3	$\frac{4}{3}$											
4	1											

- 5 Mai thinks $\frac{4}{x} = y$ is proportional. Karima thinks $y = \frac{x}{4}$ is proportional. Whose thinking is correct? Circle one.

Mai's Karima's Both Neither

Explain your thinking.

Proportional

$$\frac{4}{x} = y$$

$$y = \frac{x}{4}$$

You're invited to explore more.

- 6 Use the You're Invited to Explore More Sheet to answer questions about a relationship and proportionality.

7 Synthesis

- a What is one equation that represents a proportional relationship and one that does not?

Proportional Relationship	Not a Proportional Relationship

- b How do you know whether an equation represents a proportional relationship?

10 Summary 2.07

The structure of an equation representing the relationship between two quantities can tell us whether that relationship is proportional. An equation in the form of $y = kx$ has a *constant of proportionality*, k , which means it represents a proportional relationship.

Equations like $y = 3x + 1$ and $y = x^2$ do not have a *constant of proportionality*, so they do *not* represent proportional relationships.

Rewriting an equation in another form can help make a proportional relationship easier to see. For example, $y = \frac{x}{3}$ and $y = \frac{1}{3}x$ both represent the same proportional relationship.

Tables can help you determine whether an equation can be rewritten in the form of $y = kx$.

Practice 2.07

Name: _____ Date: _____ Period: _____

Problems 1–2: The relationship between a distance in yards, y , and the same distance in miles, x , is represented by the equation $y = 1760x$.

- Complete the table.
- Is there a proportional relationship between a distance in yards and the same distance in miles?

Distance (mi), x	Distance (yd), y
1	
5	
	3,520
	17,600

Explain your thinking.

Problems 3–6: Determine whether or not each relationship is proportional.

- | | Proportional | Not Proportional |
|---|--------------------------|--------------------------|
| 3. The remaining length, L , of a 120-inch rope after x inches have been cut off: $120 - x = L$. | <input type="checkbox"/> | <input type="checkbox"/> |
| 4. The total cost, t , after 8% sales tax is added to an item's price p : $1.08p = t$. | <input type="checkbox"/> | <input type="checkbox"/> |
| 5. The number of marbles, x , each sister gets after m marbles are shared equally among four sisters: $x = \frac{m}{4}$. | <input type="checkbox"/> | <input type="checkbox"/> |
| 6. The volume, V , of a rectangular prism whose height is 12 centimeters and whose base is a square with side lengths of s centimeters: $V = 12s^2$. | <input type="checkbox"/> | <input type="checkbox"/> |

Problems 7–8: Determine whether or not each relationship is proportional. Explain your thinking.

7.

x	y
2	5
3	7.5
6	15

8. $y = 3.2x + 5$

Practice 2.07

Name: _____ Date: _____ Period: _____

Problems 9–10: Determine whether each relationship is proportional or not proportional. Explain your thinking.

9. The weight of a stack of standard 8.5-by-11-inch paper and the number of sheets of paper.
10. The weight of a stack of different-sized books (where each book weighs a different amount) and the number of books in a stack.
11. Liam and Sadia are running a 60-meter race. Each of their distances can be represented by an equation in the form $y = kx$ where y is the distance in meters and x is the time in seconds.


Use this information to complete the table.

- Liam's distance is represented with the equation $y = 6x$.
- At 8 seconds, Liam is 16 meters ahead of Sadia.

Time (sec)	Liam's Distance (m)	Sadia's Distance (m)
0		
2		
4		
6		
8		

Spiral Review

Problems 12–13: The equation $y = 3.5x$ can be used to determine the total cost, y , in dollars, of x ounces of blueberries.

12.  What does the number 3.5 represent in the equation?
- A. The number of blueberries that \$1 can buy.
- B. The number of blueberries in x ounces.
- C. The cost of 1 ounce of blueberries.
- D. The cost of x ounces of blueberries.
13. Is the relationship between the number of ounces of blueberries and the total cost in dollars proportional? Explain your thinking.

Practice Day 1



Let's practice what you've learned so far in this unit!

You will use problem cards for this Practice Day. Record all of your responses here.

Card 1

Recipes: _____ and _____

Card 2

_____ cup(s) of red paint

Card 3

Explanation:

Card 4

Circle one:

Ashley Oscar Both Neither

Explanation:

Card 5

Circle one:

Proportional Not Proportional

Explanation:

Card 6

Response:

Card 7

Equation: _____

Card 8

Equation: _____

Practice Day 1 (continued)

Card 9

_____ grams

Card 10

_____ pounds

Card 11

Circle one:

Proportional Not Proportional

Explanation:

Card 12

Circle one:

Proportional Not Proportional

Explanation:

Card 13

Select *all* that apply:

- A. B. C.
 D. E.

Card 14

_____ pounds of peanuts

Card 15

Equation 1: _____

Equation 2: _____

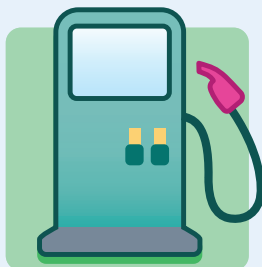
Card 16

Miles, m	Feet, f
1	
8	
	13,200
	52,800

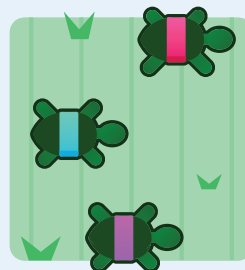
Proportional Relationships in Graphs



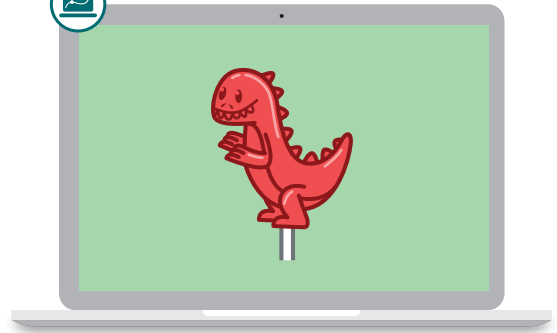
Lesson 8
DinoPops



Lesson 9
Gallon Challenge



Lesson 10
Three Turtles



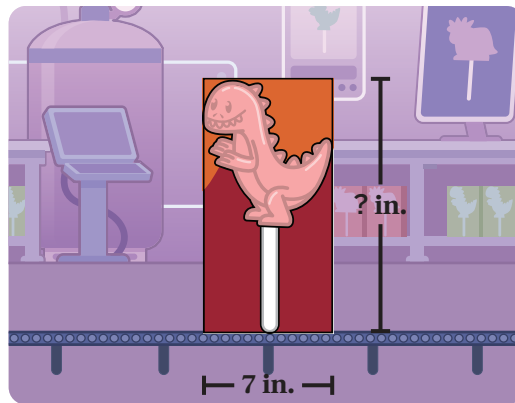
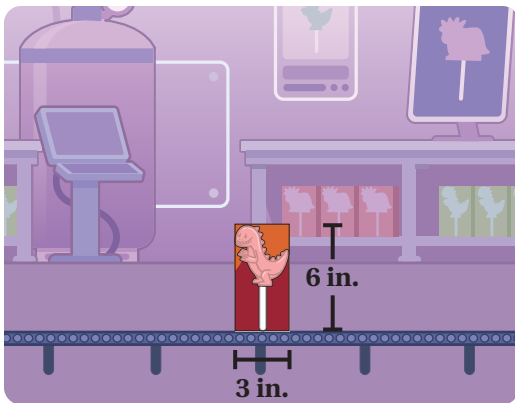
DinoPops

Let's explore what a proportional relationship looks like on a graph.

Warm-Up

1 Here are two DinoPops in their boxes.

DinoPops come in all sizes between 2 and 200 inches tall. They are always scaled copies of one another.

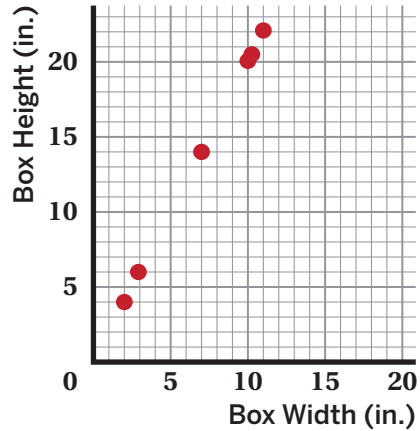
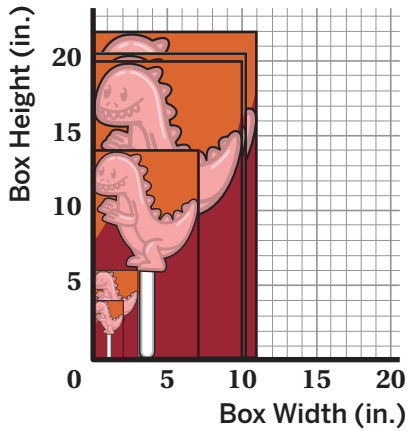


Complete the table to determine the height of the box for the large DinoPop.

Box Width (in.)	Box Height (in.)
3	6
7	

DinoPops

2 Here are some DinoPop boxes and a graph of some points.



What do you notice? What do you wonder?

I notice:

I wonder:

3 A box that is 5 inches wide and 10 inches tall is a perfect fit for a DinoPop. A box that is 7 inches wide and 14 inches tall is also a perfect fit.

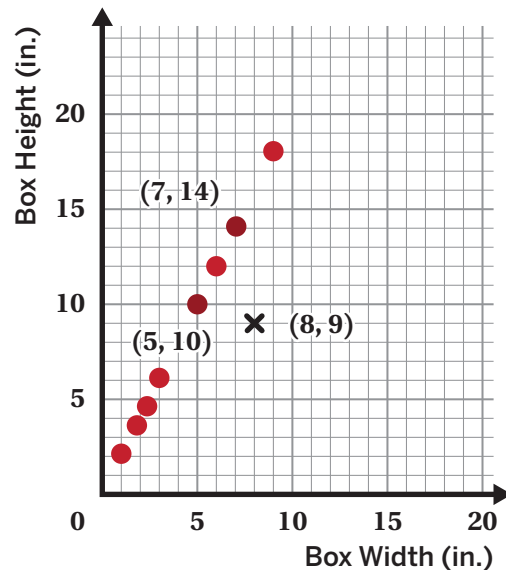
Write ordered pairs for two other boxes that are a perfect fit for a DinoPop.

(____, ____) (____, ____)

A box that is 8 inches wide and 9 inches tall is *not* a perfect fit for a DinoPop.

Write ordered pairs for two other boxes that are *not* a perfect fit for a DinoPop.


(____, ____) (____, ____)



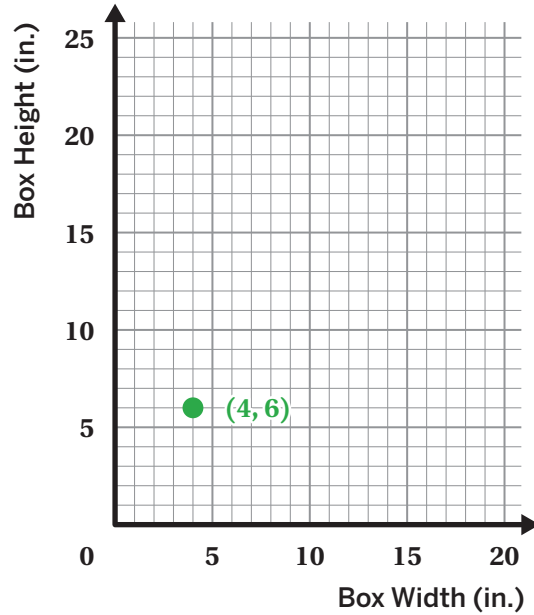
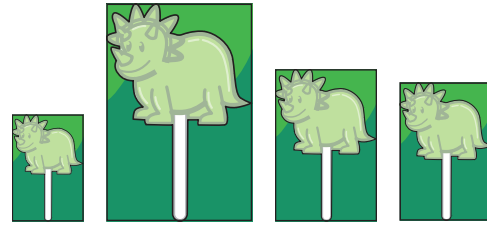
TriceraPops

4-5 Here are several TriceraPops in boxes.

The graphed point represents *one* of these boxes.

a  **Discuss:** What do you know about this box?

b Add at least *two* more points to the graph to represent other boxes that fit a TriceraPop.

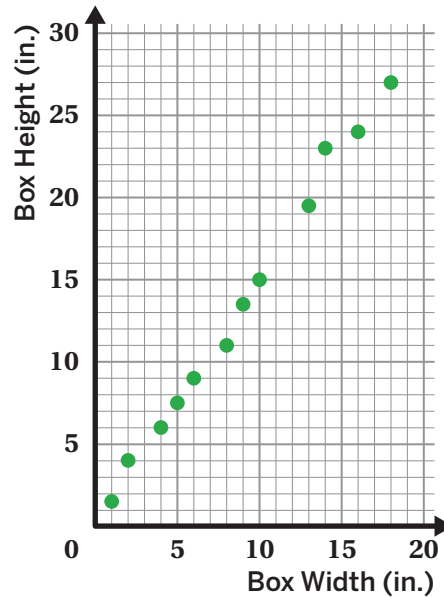


6 A student made some TriceraPop boxes, as shown on this graph.

A box with a width of 10 centimeters and a height of 15 centimeters was a good fit. Some of the boxes were not a good fit.

Describe how to use the graph to find the bad boxes.

Draw on the graph if it helps to show your strategy.



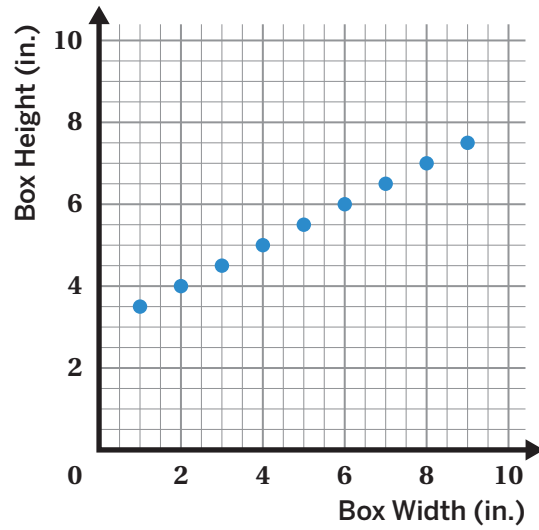
TriceraPops (continued)

7 Here is a graph of box sizes for a new lollipop.

Is there a proportional relationship between the height and the width of these boxes?
Circle one.

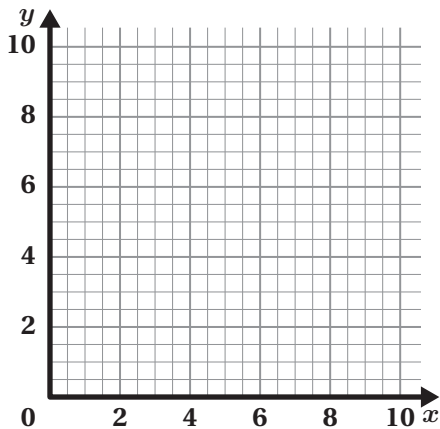
Yes No I'm not sure

Explain your thinking.

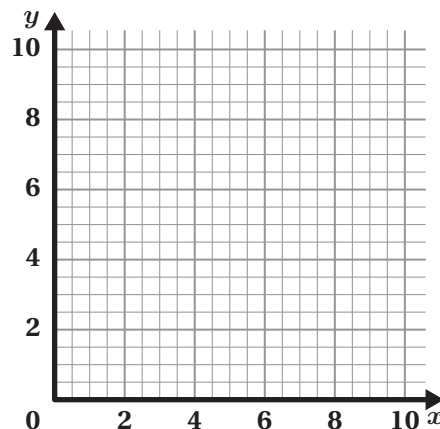


8 Make one graph that represents a proportional relationship and one graph that does not.

Proportional



Not Proportional

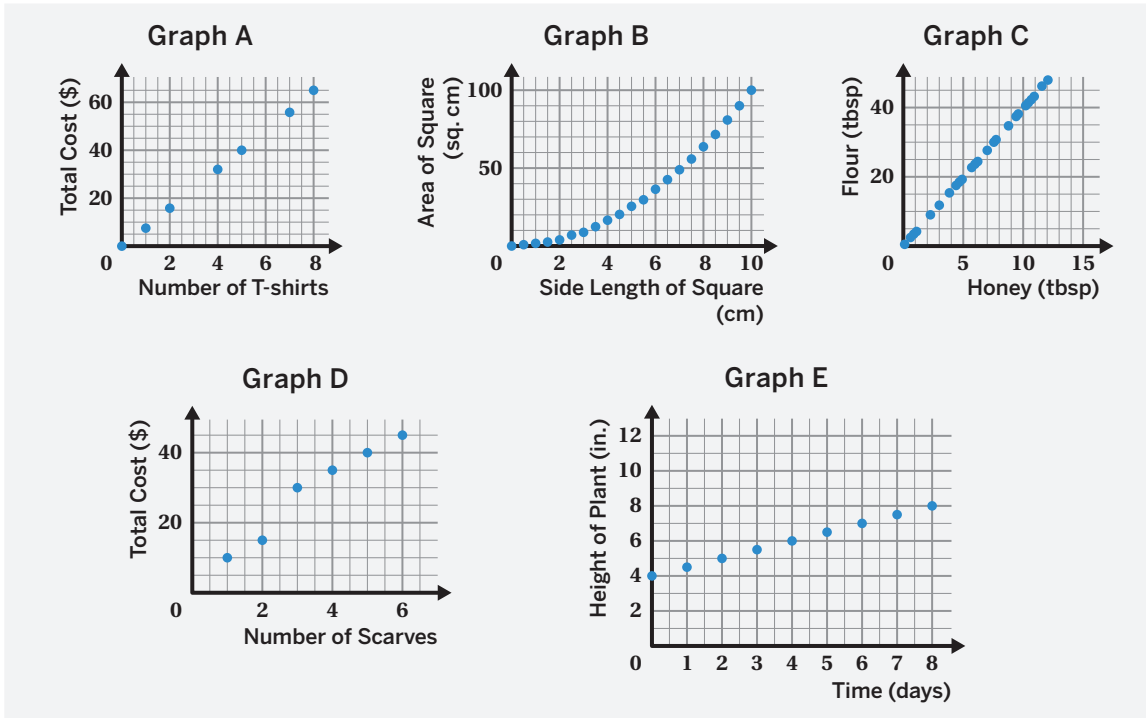


Activity 3

Name: _____ Date: _____ Period: _____

Graphs

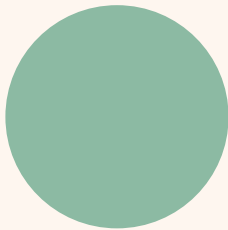
9 Decide whether each graph represents a proportional relationship.



Proportional	Not Proportional

You're invited to explore more.

10 This color green is made by mixing 3 cups of white paint and 2 cups of green paint.



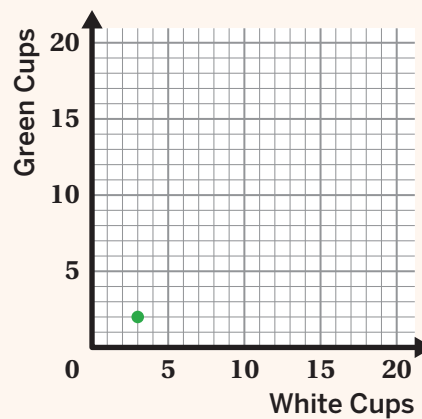
3 white cups




2 green cups



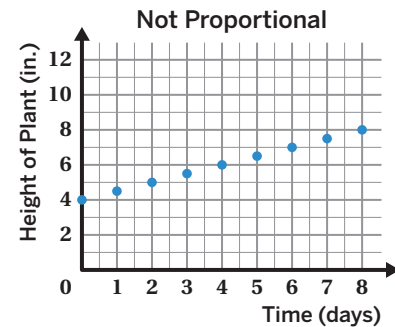
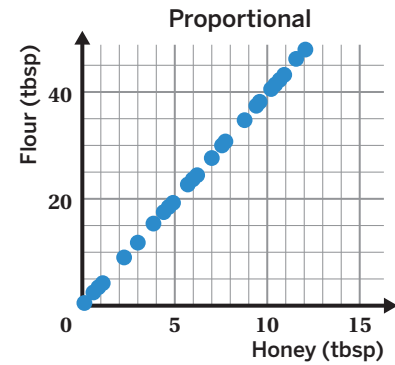
Graph points that represent *at least two* more ways to make this color.



11 Synthesis

 **Discuss:** How can you use a graph to decide whether a relationship is proportional?

Use the examples if they help with your thinking.



14 Summary 2.08

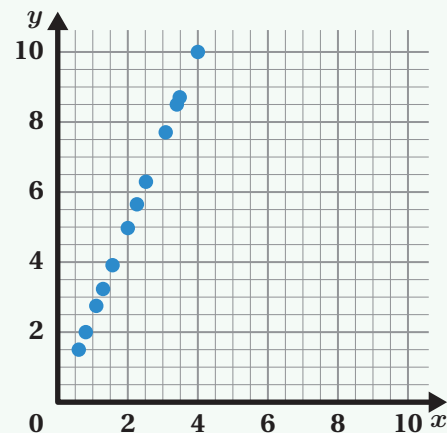
On a *coordinate plane*, if points all fall on a line that passes through $(0, 0)$, the relationship is proportional. The point $(0, 0)$ is known as the **origin**.

If it's unclear if the points form a line, you can test if the ratios of the coordinates are equivalent.

For example, the coordinates of two points on this line are $(2, 5)$ and $(4, 10)$.

$$5 \div 2 = 2.5 \text{ and } 10 \div 4 = 2.5$$

Since the ratio of the coordinates for both of these points is 2.5, these points are part of a proportional relationship.

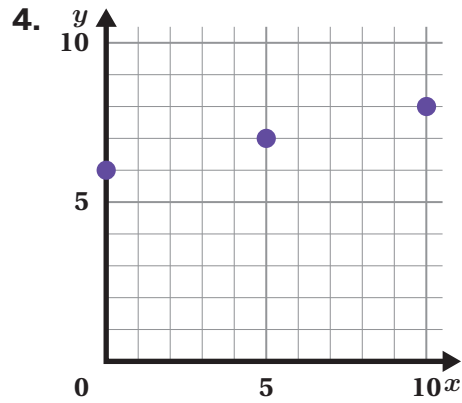
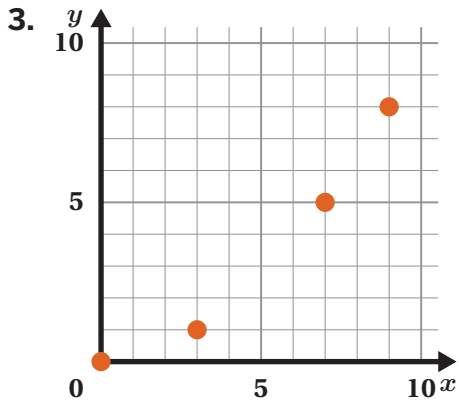
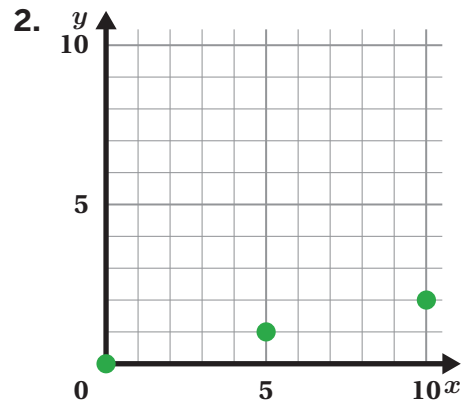
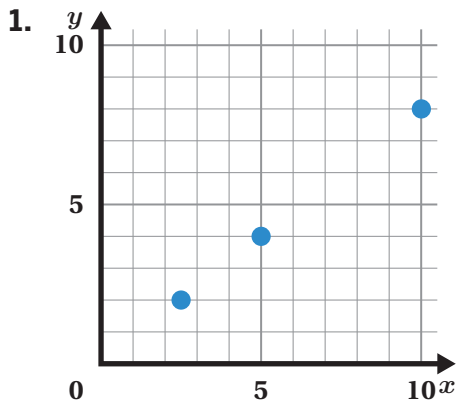


origin The point $(0, 0)$ in the coordinate plane. This is where the x -axis and the y -axis intersect.

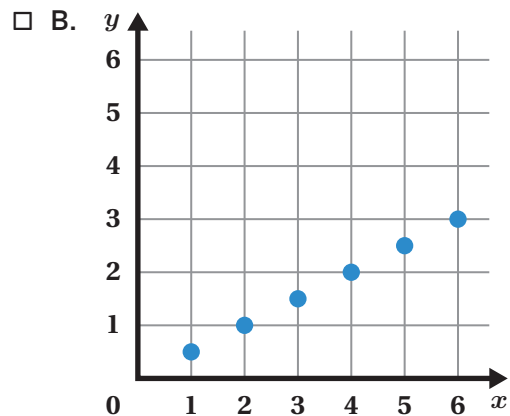
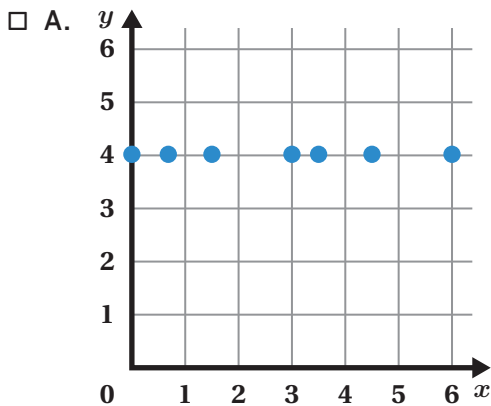
Practice 2.08

Name: _____ Date: _____ Period: _____

Problems 1–4: Determine if each graph represents a proportional relationship. Explain your thinking.



5. Select *all* the representations that show a proportional relationship.



C.

x	0	2	4	6
y	0	3	9	27

D.

x	0	2	4	6
y	0	12	24	36

Practice 2.08

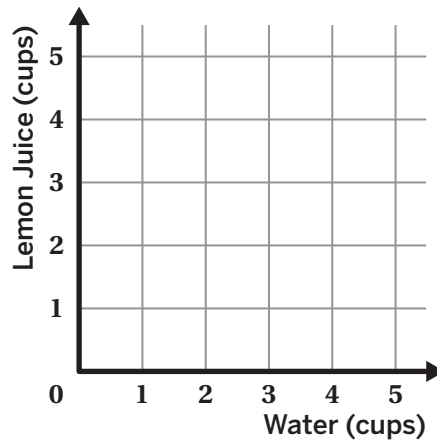
Name: _____ Date: _____ Period: _____

6. Arturo earns \$33.00 for babysitting for 4 hours. At this rate, how much will he earn if he babysits for 7 hours? Explain your thinking.

7. A lemonade recipe calls for a $\frac{1}{4}$ cup of lemon juice for every cup of water. The table shows different amounts of water and lemon juice you can use to make this recipe. Graph the ordered pairs to determine whether the relationship between water and lemon juice is proportional.

Water (cups)	Lemon Juice (cups)
1	$\frac{1}{4}$
2	$\frac{1}{2}$
3	$\frac{3}{4}$
4	1

Explain your thinking.



Spiral Review

8. A turtle is walking away from a rock. x represents the time in minutes that the turtle is walking. y represents the distance in meters between the rock and the turtle. If x and y are in a proportional relationship, select *all* the true statements.

- A. The equation $y = 3x$ could represent the distance that the turtle walks.
- B. The turtle walks for a bit and then stops for a minute before walking again.
- C. The turtle walks away from the rock at a constant rate.
- D. The equation $y = x + 3$ could represent the distance that the turtle walks.
- E. After 6 minutes, the turtle walks 18 meters, and after 10 minutes, the turtle walks 20 meters.

Problems 9–11: Solve each equation.

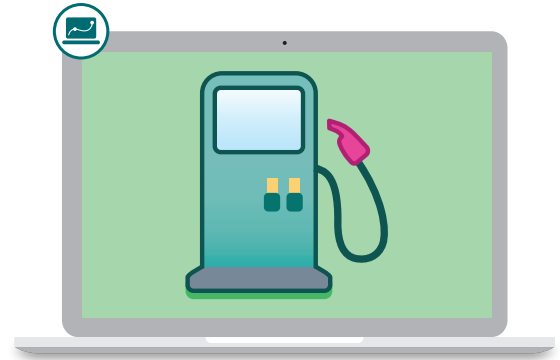
9. $8x = 32$

10. $6x = 2$

11. $x - 4 = 11$

Gallon Challenge

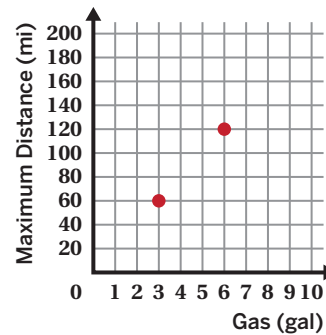
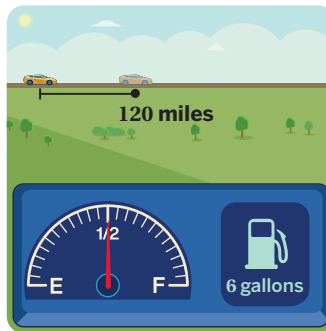
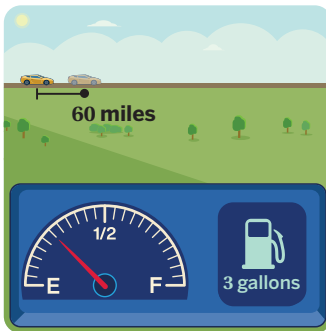
Let's identify constants of proportionality using a graph.



Warm-Up

1 Here are two images showing the amount of gas in a car's tank.

The graph represents the maximum distance the car can go using each amount of gas.

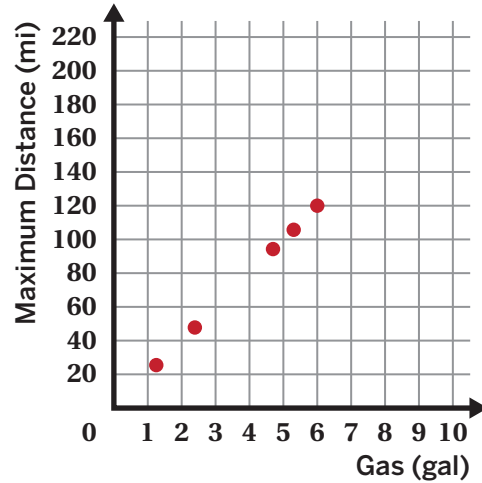


Discuss: What do you notice? What do you wonder?

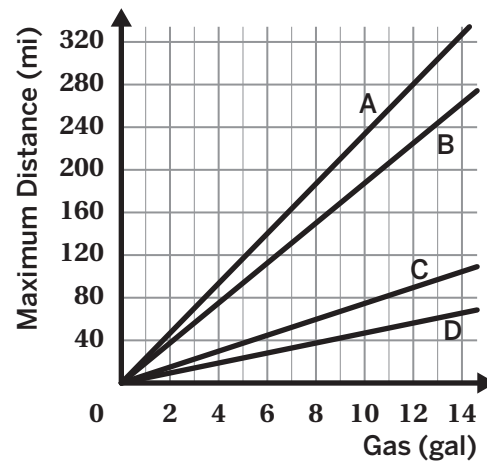
On the Road

2 Here are several points representing the maximum distance the car can go using different amounts of gas.

What would the graph look like if it included every possible point for this car?



3 The car can travel 240 miles with a full 12-gallon tank of gas. Which line represents this relationship?

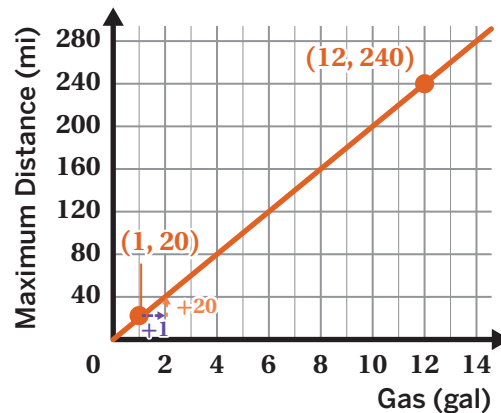


4 A car's *gas mileage* is the maximum distance it can go using 1 gallon of gas (measured in miles per gallon).

Based on the graph, what is this car's gas mileage?

_____ miles per gallon

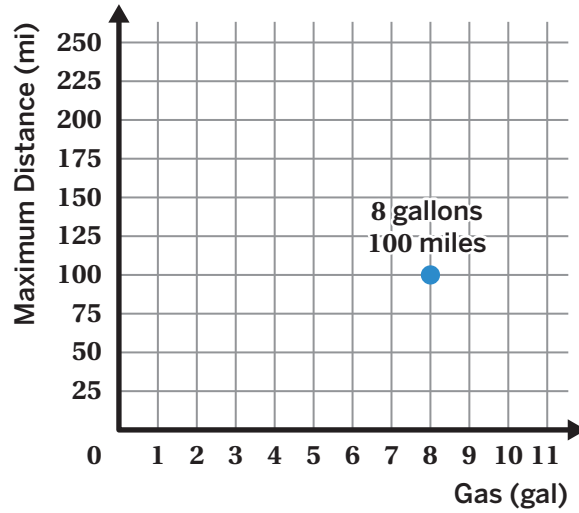
Explain your thinking.



Gas Mileage

5 Kaya's truck travels 100 miles using 8 gallons of gas.

What is the gas mileage for her truck?

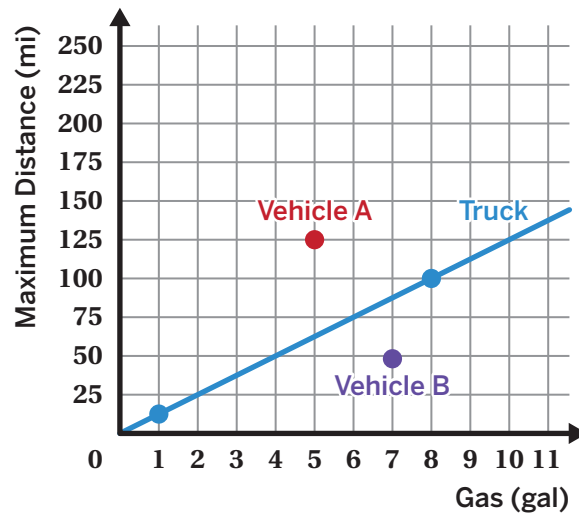


6 Kaya wants to buy a new vehicle that gets better gas mileage than her truck.

Which vehicle should she pick?
Circle one.

Vehicle A Vehicle B Either Vehicle A or Vehicle B

Explain your thinking.



Activity 2

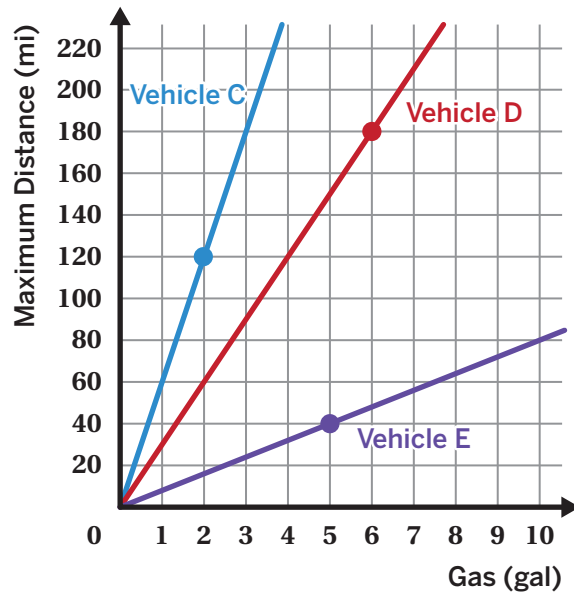
Name: _____ Date: _____ Period: _____

Gas Mileage (continued)

7 In the relationship between amount of gas and maximum distance, the vehicle's gas mileage is a *constant of proportionality*.

What is that constant of proportionality for each vehicle?

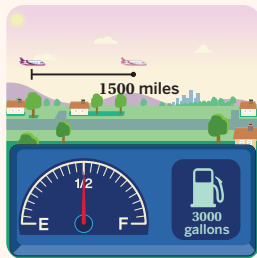
Vehicle	Constant of Proportionality (miles per gallon)
C	
D	
E	



You're invited to explore more.

8 Here is the maximum distance traveled by three vehicles using a certain amount of gas.

Plane



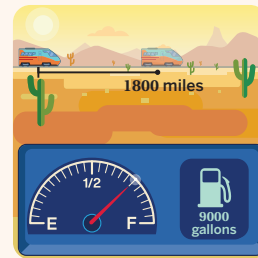
Note: The average plane has 100 passengers.

Cruise Ship



Note: The average cruise ship has 3000 passenger.

Train



Note: The average train has 300 passengers.

Compare each vehicle's environmental impact to the impact of driving a car.

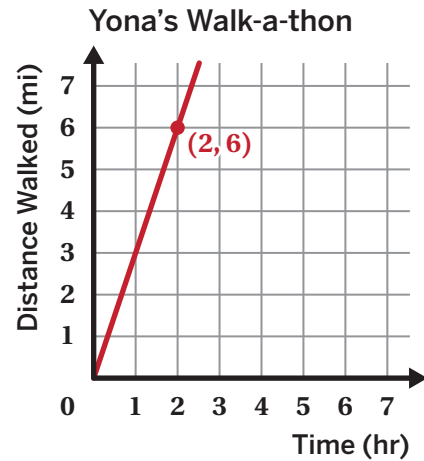
9 Synthesis

What are *two* different ways you can find a constant of proportionality using a graph?

Use the example if it helps with your thinking.

First method:

Second method:



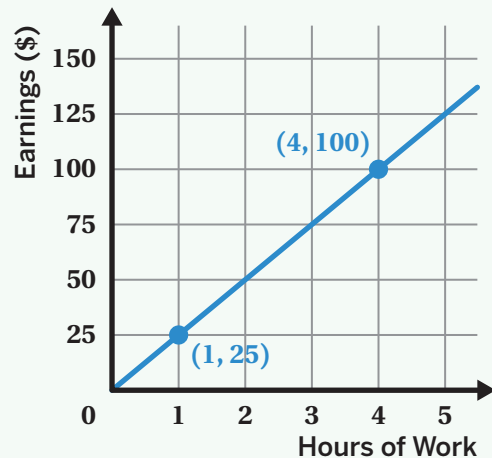
12 Summary 2.09

Each point on a graph of a proportional relationship tells a story using the quantities represented by x and y . You can determine a constant of proportionality from a graph by using:

- The value of y when x is equal to 1.
- The ratio of $\frac{y}{x}$ for any ordered pair.

For example, this graph shows a proportional relationship between hours worked, x , and money earned in dollars, y . One constant of proportionality is 25 because \$25 is earned for working 1 hour.

The ordered pair (4, 100) shows that \$100 is earned for 4 hours of work, which is an equivalent ratio to earning \$25 per hour.



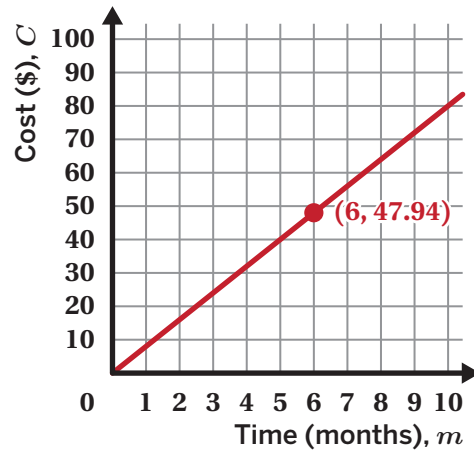
Practice 2.09

Name: _____ Date: _____ Period: _____

Problems 1–2: Here is a graph that shows a proportional relationship between the number of months Tiara had a streaming service subscription and the total amount of money she paid for the subscription.

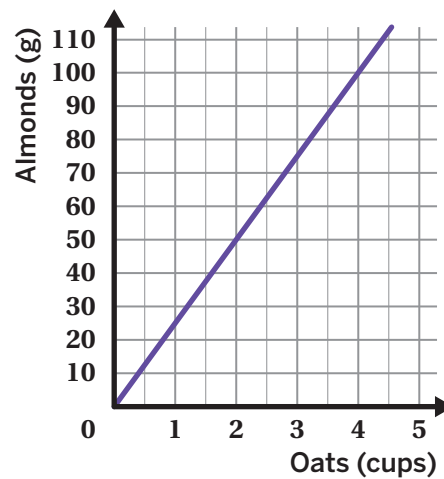
After 6 months, Tiara paid \$47.94.

1. What is a constant of proportionality in this relationship?
2. Write an equation that represents the relationship between C , the total cost of the subscription, and m , the number of months.



Problems 3–4: A recipe for granola calls for a mix of almonds and oats. The graph shows the amount of almonds, in grams, for different amounts of oats, in cups.

3. Determine a constant of proportionality for this relationship, then explain its meaning.



4. Label one place you see that constant of proportionality on the graph.
5. The graph shows the cost for two different varieties of apples. Which variety of apples has a higher cost per pound?

Explain your thinking.

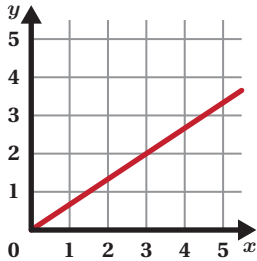


Practice 2.09

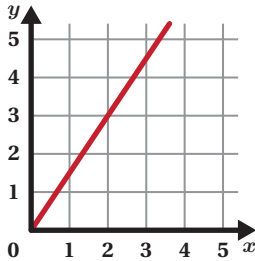
Name: _____ Date: _____ Period: _____

6. Which of these relationships has the same constant of proportionality as the equation $y = \frac{2}{3}x$? Select *all* that apply.

A.



B.



C.

x	y
0	0
3	2
6	4
9	6

D.

x	y
0	0
2	3
4	6
6	9

Spiral Review

7. Write an expression that is equivalent to $8(n + 6)$.

Problems 8–10: Haru and Irene were running laps around the track. Their coach recorded their times at the end of laps 2, 4, and 6.

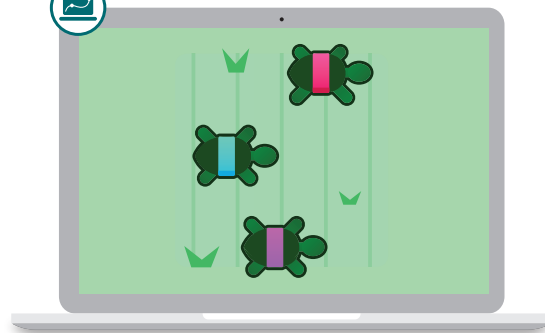
Haru's Run

Distance (laps)	Time (min)	Minutes Per Lap
2	4	
4	9	
6	15	

Irene's Run

Distance (laps)	Time (min)	Minutes Per Lap
2	5	
4	10	
6	15	

- Complete the tables with the minutes per lap for each run.
- Based on the table, is Haru running at a constant speed? Explain your thinking.
- Based on the table, is Irene running at a constant speed? Explain your thinking.

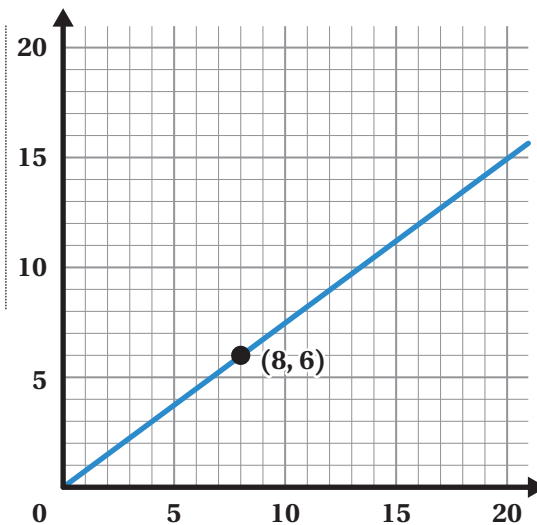


Three Turtles

Let's compare proportional relationships.

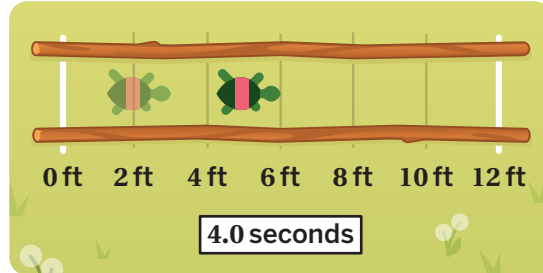
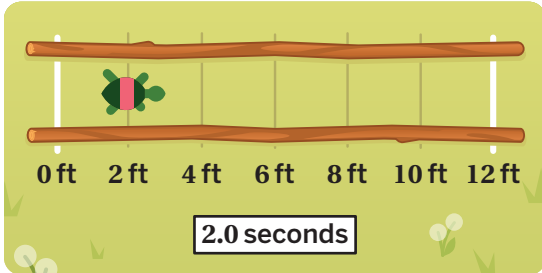
Warm-Up

- 1** Here is a graph that represents a proportional relationship.
 - a** Label the axes with any quantities you'd like.
 - b** Write a true statement about the quantities based on the graph.



Traveling Turtles

2 This turtle walks at a constant rate. The turtle's distance is measured at the front of its head.



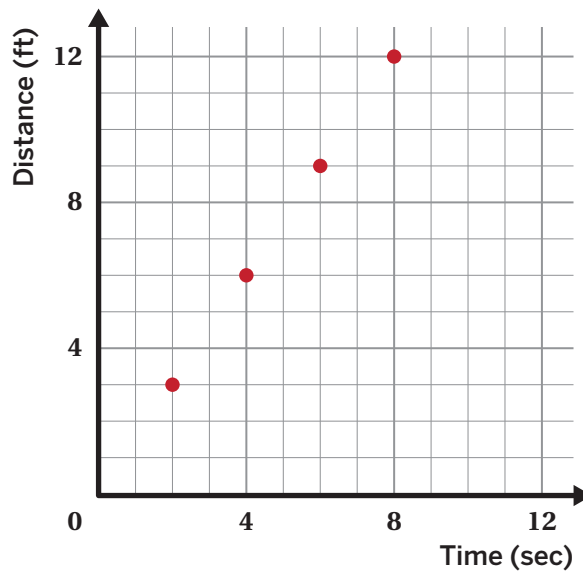
a Complete the table.

b What is a constant of proportionality for this relationship?

Time (sec)	Distance (ft)
2	3
4	6
6	
8	

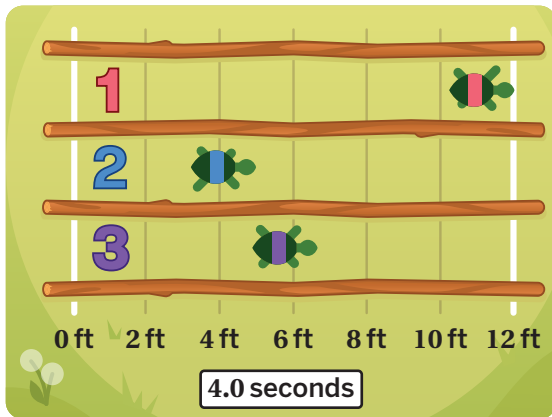
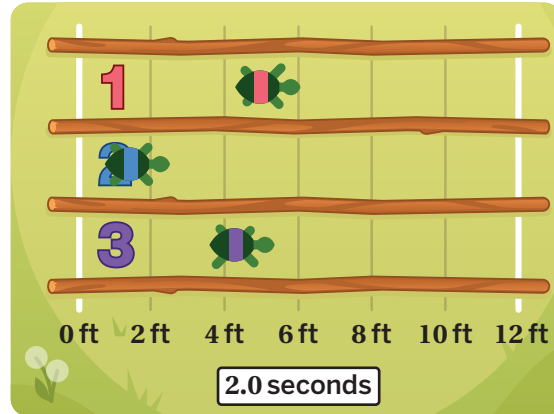
3 This graph shows the points from the table in the previous problem.

Write an equation for this relationship, using d for distance and t for time.



Three Turtles

4 These images show three turtles walking. Each turtle walks at a constant rate.

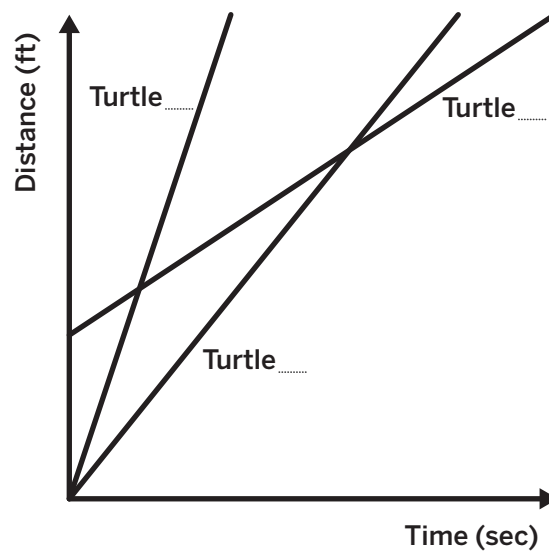


Label each line with the turtle it represents: Turtle 1, Turtle 2, or Turtle 3.

5 Match each equation to its graph.

Equation	Turtle
a. $d = 3t$ Turtle 1
b. $d = \frac{2}{3}t + 4$ Turtle 2
c. $d = 1.25t$ Turtle 3

Explain your thinking.



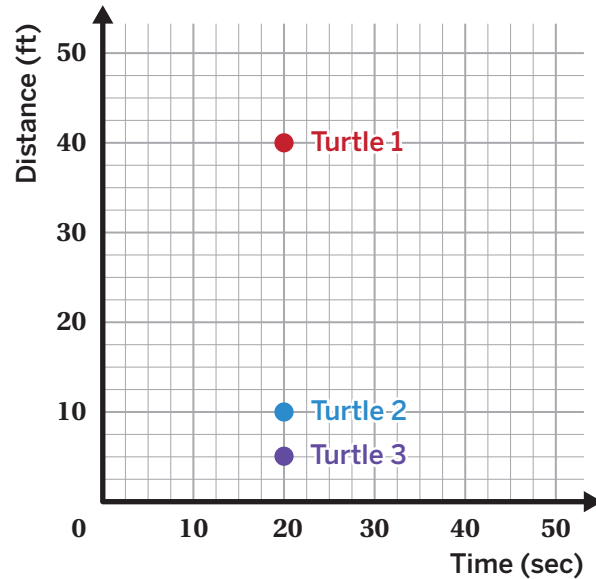
Turtle Challenges

6 Here are three new turtles and their distances at 20 seconds.

- Turtle 1 is 40 feet from the start.
- Turtle 2 is 10 feet from the start.
- Turtle 3 is 5 feet from the start.

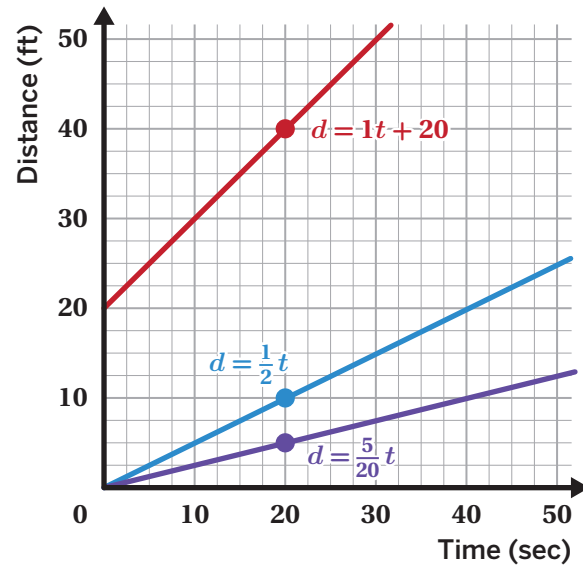
Write an equation for each turtle, using d for distance and t for time. (One equation has been written for you.)

Turtle	Equation
Turtle 1	
Turtle 2	$d = \frac{1}{2}t$
Turtle 3	



7 Irelle wrote three equations for the new turtles.

- a** Which of these relationships is *not* proportional?
- $d = 1t + 20$
 - $d = \frac{1}{2}t$
 - $d = \frac{5}{20}t$
- b** Describe this turtle's race.



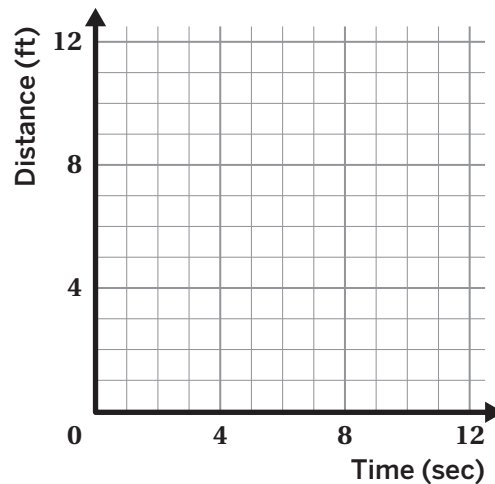
Turtle Challenges (continued)

8 Sort these cards into three groups that each represent the same turtle.

<p style="text-align: center;">Card A</p>	<p style="text-align: center;">Card B</p>	<p style="text-align: center;">Card C</p> <table border="1" style="width: 100%; border-collapse: collapse;"> <thead> <tr style="background-color: #0072bc; color: white;"> <th>Time (sec)</th> <th>Turtle Distance (ft)</th> </tr> </thead> <tbody> <tr><td>0</td><td>0</td></tr> <tr><td>1</td><td>4</td></tr> <tr><td>2</td><td>8</td></tr> <tr><td>4</td><td>16</td></tr> <tr><td>8</td><td>32</td></tr> </tbody> </table>	Time (sec)	Turtle Distance (ft)	0	0	1	4	2	8	4	16	8	32
Time (sec)	Turtle Distance (ft)													
0	0													
1	4													
2	8													
4	16													
8	32													
<p style="text-align: center;">Card D</p> <table border="1" style="width: 100%; border-collapse: collapse;"> <thead> <tr style="background-color: #0072bc; color: white;"> <th>Time (sec)</th> <th>Turtle Distance (ft)</th> </tr> </thead> <tbody> <tr><td>0</td><td>0</td></tr> <tr><td>1</td><td>$\frac{1}{2}$</td></tr> <tr><td>2</td><td>1</td></tr> <tr><td>4</td><td>2</td></tr> <tr><td>8</td><td>4</td></tr> </tbody> </table>	Time (sec)	Turtle Distance (ft)	0	0	1	$\frac{1}{2}$	2	1	4	2	8	4	<p style="text-align: center;">Card E</p> <p style="text-align: center;">At 8 seconds, the turtle's distance is 2 feet.</p>	<p style="text-align: center;">Card F</p> <p style="text-align: center;">At 2 seconds, the turtle's distance is 8 feet.</p>
Time (sec)	Turtle Distance (ft)													
0	0													
1	$\frac{1}{2}$													
2	1													
4	2													
8	4													
<p style="text-align: center;">Card G</p> <p style="text-align: center;">$d = 4t$</p>	<p style="text-align: center;">Card H</p> <p style="text-align: center;">$d = \frac{1}{4}t$</p>													
Group 1	Group 2	Group 3												

9 Create your own turtle race by sketching 3 lines that represent 3 different turtles. Your turtle race must include *at least two* of the following features:

- A turtle that stays still.
- A turtle that has a head start.
- Two turtles that finish at the exact same time.
- A turtle that travels backward.
- Two turtles that travel at the same pace.



You're invited to explore more.

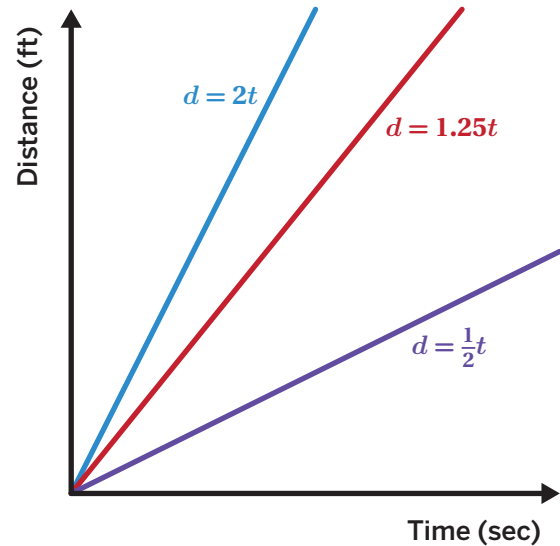
10 Use the You're Invited to Explore More Sheet to explore another turtle race.

11 Synthesis

This graph shows the distance traveled over time by three different turtles.

Discuss both questions. Then select *one* and write your response.

- How can you tell from the graphs which turtle moved the fastest?
- How can you tell from the equations which turtle moved the fastest?

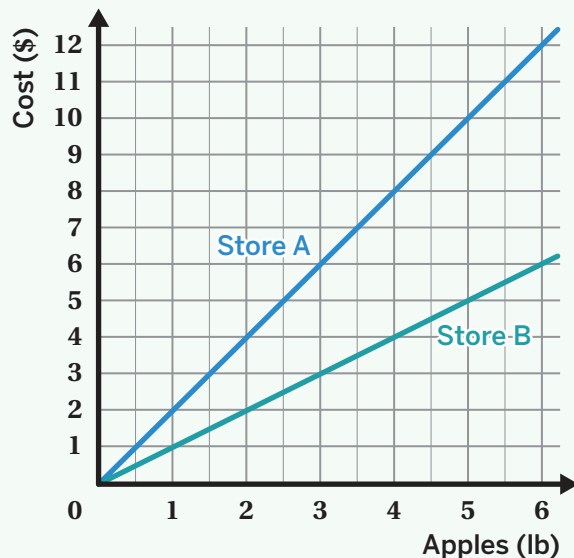


14 Summary 2.10

You can compare graphs of proportional relationships when they're on the same coordinate plane. The steeper the line, the greater the constant of proportionality.

For example, you can use this graph to compare the cost of apples at two different stores.

- The line representing Store A is steeper than the line representing Store B, so it has a greater constant of proportionality. This means Store A charges more per pound than Store B.
- Store A charges \$2 for one pound ($k = 2$), while Store B charges \$1 for one pound ($k = 1$). This is another way you can determine that the constant of proportionality at Store A is greater than that at Store B.



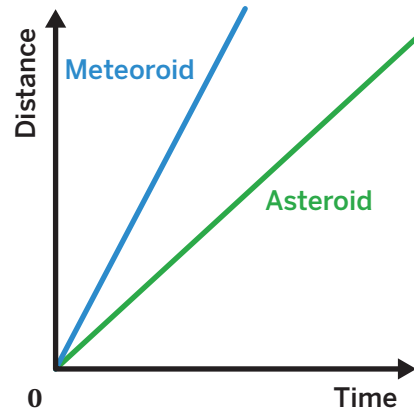
Practice 2.10

Name: _____ Date: _____ Period: _____

1. A meteoroid and an asteroid travel through the solar system. This graph shows how much distance they travel over time.

Does the asteroid travel faster or slower than the meteoroid?

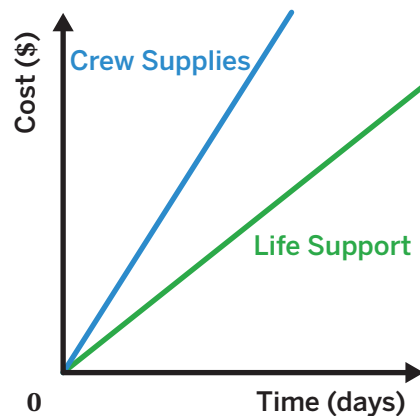
Explain your thinking.



2. Having a crew on the International Space Station requires life support and crew supplies. The graph shows the relationship between the cost of life support and crew supplies, and the number of days spent in space.

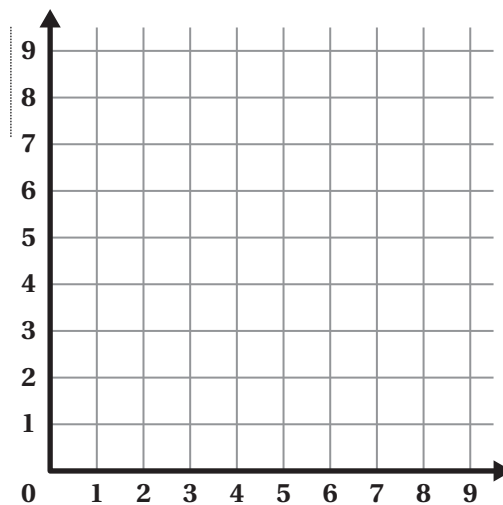
Which costs less, life support or crew supplies?

Explain your thinking.



Problems 3–6: At a supermarket, you can fill your own honey container and pay by the ounce. A customer buys 12 ounces of honey for \$5.40.

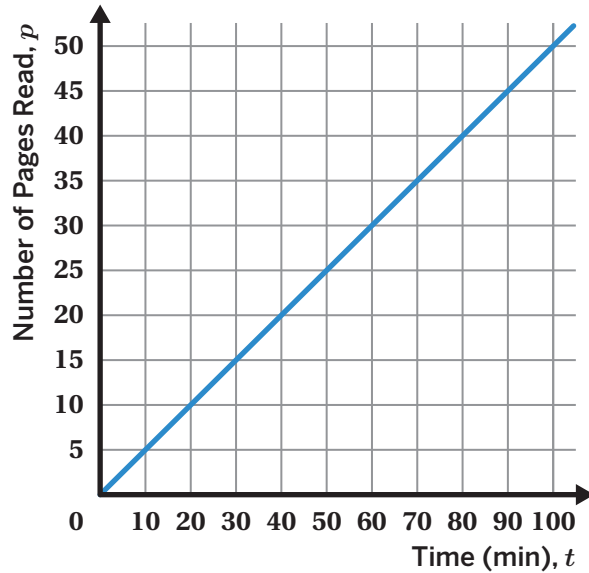
3. How much does the honey cost per ounce?
Show your thinking.
4. How much honey can be bought per dollar?
Show your thinking.
5. Write *two* different equations representing this situation. Use h for ounces of honey and c for cost in dollars.
6. Choose *one* equation and draw its graph on the coordinate plane. Be sure to label the axes.



Practice 2.10

Name: _____ Date: _____ Period: _____

Problems 7–10: The graph shows the relationship between time in minutes, t , and the total number of pages Joel has read, p .



7. Is this relationship proportional?

Explain your thinking.

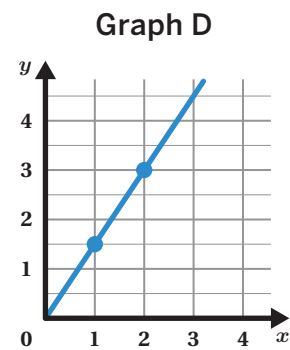
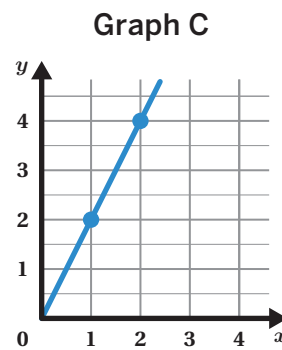
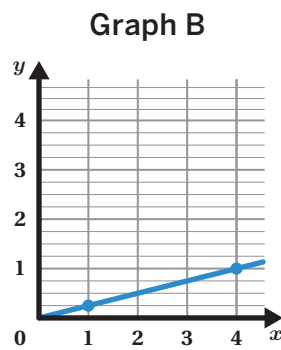
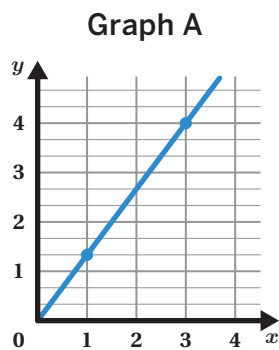
8. What is a constant of proportionality in this relationship?

9. What does this number mean in this situation?

10. Write an equation that represents the relationship between the time in minutes, t , and the total number of pages Joel has read, p .

Spiral Review

Problems 11–14: Match each equation with its graph.



11. $y = \frac{1}{4}x$

12. $y = \frac{3}{2}x$

13. $y = 2x$

14. $y = \frac{4}{3}x$

15. On a blueprint, the bedroom is 1.7 inches wide. The blueprint has a scale of 1 inches to 8 feet. How wide, in inches, would the bedroom be on a blueprint that has a scale of 1 inches to 12 feet?

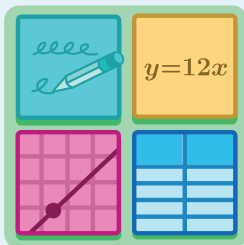
A. 0.7 inches

B. 1.13 inches

C. 2.55 inches

D. 13.6 inches

Using Proportional Relationships



Lesson 11
Four Representations



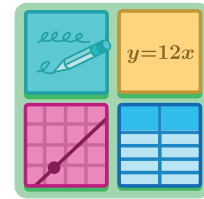
Lesson 12
Water Efficiency,
Part 1



Lesson 13
Water Efficiency,
Part 2

Four Representations

Let's explore connections between representations.



Warm-Up

- Here are four representations of a proportional relationship.

Description

Lucia works at a sporting goods store where she earns \$12 per hour.

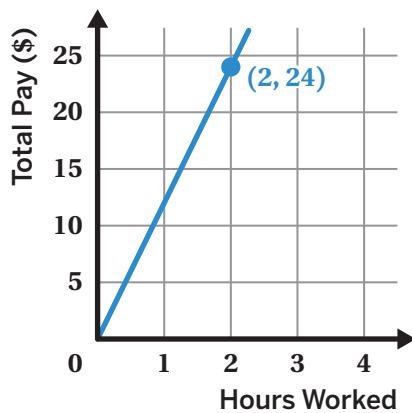
Equation

$$y = 12x$$

x represents the number of hours worked.

y represents the total pay in dollars.

Graph



Table

Hours Worked	Total Pay (\$)
0	0
1	12
5	60
8	96
10	120



Discuss: Show or explain the connections you see between the four representations.

Stronger and Clearer Each Time

2. Here are some lists of items and units of measurement.

Creatures		Units of Length		Units of Time	
starfish	centipedes	feet	centimeters	seconds	minutes
earthworms	dinosaurs	kilometers	miles	years	millennia
Body Parts		Units of Volume		Units of Weight	
legs	eyes	gallons	cups	ounces	pounds
toes	heads	cubic feet	teaspoons	kilograms	milligrams

- a** Choose *one* option from one list and *one* option from another list. You can also make up your own options.

..... and

- b** Write a description of a proportional relationship using your choices. Then use feedback from one or more partners to write a final draft of your description.

Description

First Draft:

Notes From Partner 1:

Notes From Partner 2:

Final Draft:

- c** Explain how you know the relationship you described is proportional.

Four Representations

3. Record the final draft of the description you wrote in Activity 1. Then create an equation, a graph, and a table for your relationship.

Description

Equation

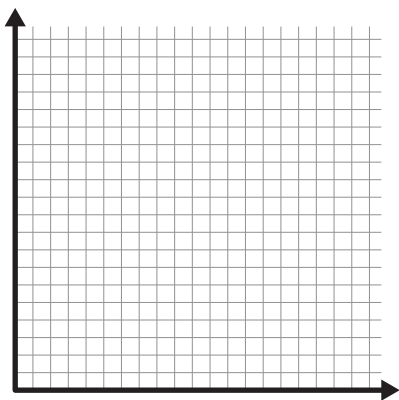
Define the variables.

Write the equation.

Graph

Table

Label each axis. Then graph the relationship.



Label each column. Then complete the table.

_____	_____

Synthesis

4. Here are the four representations from the Warm-Up.

Description

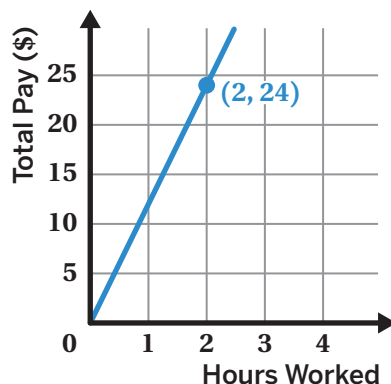
Lucia works at a sporting goods store where she earns \$12 per hour.

Equation

x represents the number of hours worked. y represents the total pay in dollars.


$$y = 12x$$

Graph



Table

Hours Worked	Total Pay (\$)
0	0
1	12
5	60
8	96

 **Discuss:** Where do you see a constant of proportionality in each representation?

Summary 2.11

You can determine the constant of proportionality, k , in different ways using different representations.

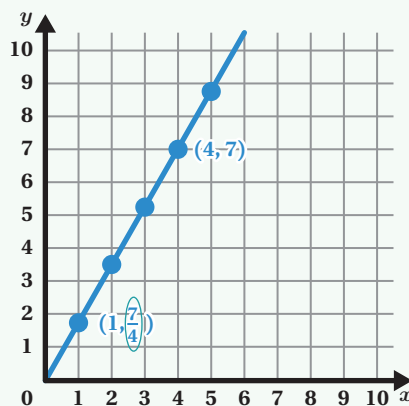
Equation

$$y = \left(\frac{7}{4}\right)x$$

Table

x	y
0	0
1	$\left(\frac{7}{4}\right)$
4	7

Graph



In a table or graph, the constant of proportionality is the y -value that is paired with the x -value of 1. The constant of proportionality is also $k = \frac{y}{x}$, where (x, y) is any ordered pair.

In an equation in the form of $y = kx$, the number multiplying x (called the *coefficient*) is the constant of proportionality.

Practice

2.11

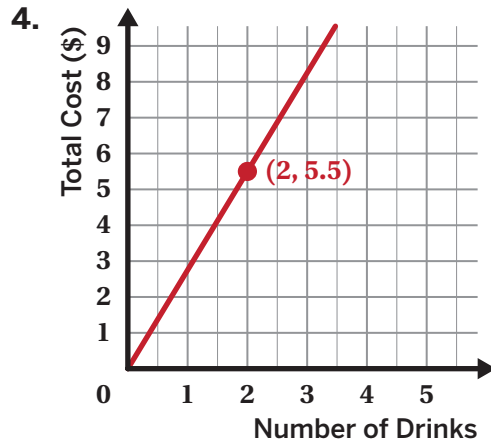
Name: _____ Date: _____ Period: _____

Problems 1–4: Determine a constant of proportionality in each representation. Show your thinking.

1. David walked 15 feet in 3 seconds. 2. $C = 4.8n$

3.

x	y
0	0
$\frac{7}{10}$	$\frac{28}{15}$



5. $(\frac{1}{2}, 3)$ is a point on a line that represents a proportional relationship. Select *all* the points that are also on this line.

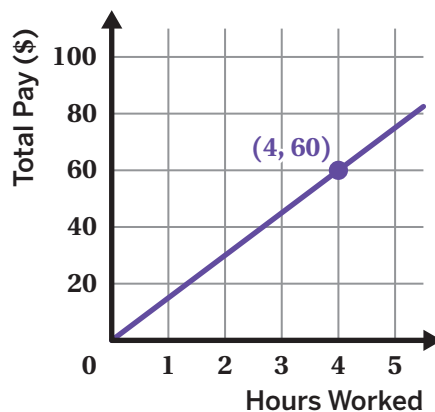
- A. $(9, \frac{3}{2})$ B. $(1, 6)$ C. $(1, 3)$ D. $(\frac{3}{2}, \frac{1}{4})$ E. $(\frac{7}{2}, 21)$

6. Here is a description, a graph, and a table.

Description

Yosef has to work 12 hours in a week to earn \$180.

Graph



Table

Hours Worked	Total Pay (\$)
0	0
2	25
6	75
10	125
20	250

Which one represents a different relationship than the others? Explain your thinking.


Practice 2.11

Name: _____ Date: _____ Period: _____

Problems 7–9: The equation $c = 2.95g$ gives the cost of gas on a particular day, where c represents the cost in dollars and g represents the number of gallons of gas purchased at a neighborhood gas station.

- Write four ordered pairs that fit this relationship, where the x -value is the number of gallons of gas and the y -value is the cost.
- What does the value 2.95 represent in this situation?
- Hailey's mom remarks, "You can get about a third of a gallon of gas for a dollar." Is she correct? Explain your thinking.

Spiral Review

10.  Select *all* the tables that represent a proportional relationship between x and y .

A.

x	y
0	0
$\frac{1}{4}$	$\frac{1}{8}$
$\frac{2}{4}$	$\frac{2}{8}$
$\frac{3}{4}$	$\frac{3}{8}$

B.

x	y
0	0
$\frac{1}{2}$	$\frac{1}{4}$
$\frac{1}{3}$	$\frac{1}{5}$
$\frac{1}{4}$	$\frac{1}{6}$

C.

x	y
0	0
1	1
2	4
4	16

D.

x	y
0	0
1	4
3	12
4	16

Water Efficiency, Part 1

Let's explore whether baths or showers use more water.



Warm-Up

*What uses more water: a bath or a shower?
And how much more water?*

1. What's your best guess for the answer to this question? Explain your thinking.



Synthesis

8. Why might someone care about the question you answered? How could you use your work to help them decide what to do?

Summary 2.12

You can use proportional relationships to model real-world questions, like whether you use more water when taking a bath or a shower.

Choosing *how* to represent your model is important because it affects how the information will be interpreted by others.

By creating a model, you can turn a problem without a clear answer into information that someone can actually use. That's important work!

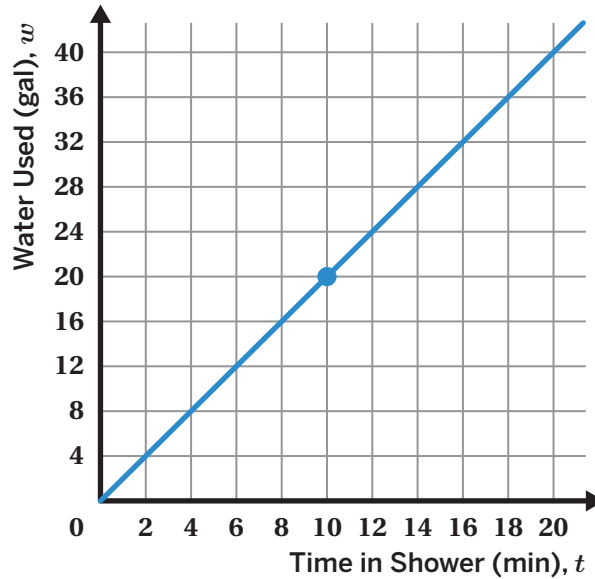


Practice

2.12

Name: _____ Date: _____ Period: _____

Problems 1–4: Here is a graph representing how many gallons of water, w , a shower used in t minutes.



1. Explain how you know that these quantities are in a proportional relationship.
2. What are the two constants of proportionality for this relationship?
3. What do these constants of proportionality mean in this situation?
4. Write two equations representing this relationship.

Problems 5–6: On its way from Los Angeles to Boston, a plane traveling at a constant speed flew over Las Vegas, Denver, Cedar Rapids, and Detroit.

5. Complete the table.

Segment	Time (hr)	Distance (mi)	Speed (mph)
Las Vegas to Denver	1	575	
Denver to Cedar Rapids	1.2		
Cedar Rapids to Detroit		460	

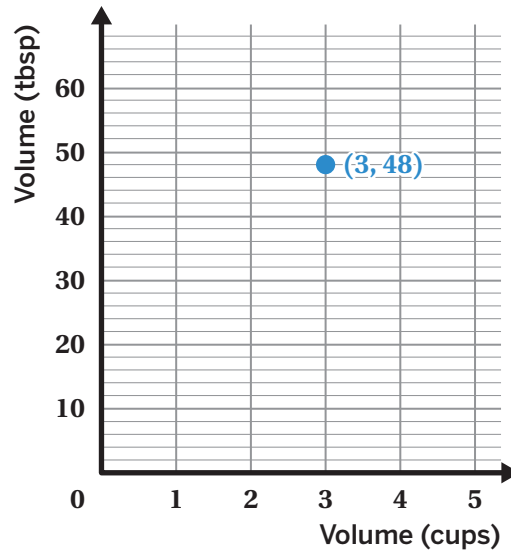
6. Write two equations representing the relationship between distance in miles, d , and time in hours, t , for this plane.

Spiral Review

7. 🌀 The length of a ruler is 12 inches. There are 2.54 centimeters in 1 inch. Which measurement is closest to the length of the ruler in centimeters?
- A. 3,048 centimeters B. 30.48 centimeters
C. 304.8 centimeters D. 3.048 centimeters

Problems 8–10: There is a proportional relationship between cups and tablespoons.

8. Plot and label *at least two* more points that fit this relationship. Then draw a line to represent the relationship.
9. For which value of y is the point $(1, y)$ on the line?



10. What is a constant of proportionality for this relationship?

11. Write two equations representing the relationship shown in the table.

x	y
3	36
5	60
12	144

Water Efficiency, Part 2



Let's explore whether baths or showers use more water.

Warm-Up

Look back at your work from the previous lesson.

1. What are you most proud of about your work so far?



Create a Poster

2. Create a poster displaying your work from the previous lesson.

Your poster should include:

- At least one* mathematical representation of a proportional relationship:
 - Graph
 - Equation
 - Table
- Your group's answer to the question: Under what conditions does a bath use more water than a shower?
- Your group's answer to the question: Under what conditions does a shower use more water than a bath?
- Your group's reasoning for each of these answers.

Gallery Tour

3. Take a look at your classmates' posters about water usage.
 - a What features of your classmates' posters helped you understand their thinking?
 - b Describe something you would change about your poster now that you have seen the work of other groups.
4. Use your thinking from the previous problem to make your poster stronger and clearer.
5. Add *at least one* additional representation to your poster (such as a graph, table, or equation).

Here are two reflection questions. Complete each problem independently.

6. What did you learn about water usage that was surprising or new?
7. An average showerhead uses 2.5 gallons of water per minute. In California, the law sets a maximum flow rate of 1.8 gallons per minute for new shower heads. How, if at all, would this affect your answer to the question which uses more water?

You're invited to explore more.

8. Kala and Luca are considering upgrading their showers to use less water.
 - Kala's shower can fill a 1-gallon jug in 20 seconds.
 - The equation $w = 2.5t$ represents the number of gallons, w , that Luca's shower fills up in t minutes.

Whose shower uses less water? How much less water? Show or explain your thinking.

Synthesis



Discuss:

- What was challenging about modeling water usage?
- What might you do differently when modeling situations in the future?

Summary 2.13

Identifying whether a relationship is proportional is an important step when you're working on a mathematical problem.

If the relationship is proportional, there are multiple ways of representing it. The representation you choose can affect how well your audience understands the relationship and the information you want to communicate about it.

Having someone else review and critique your work can help to make it stronger. Sometimes there are perspectives that you can't see until someone shares them with you.

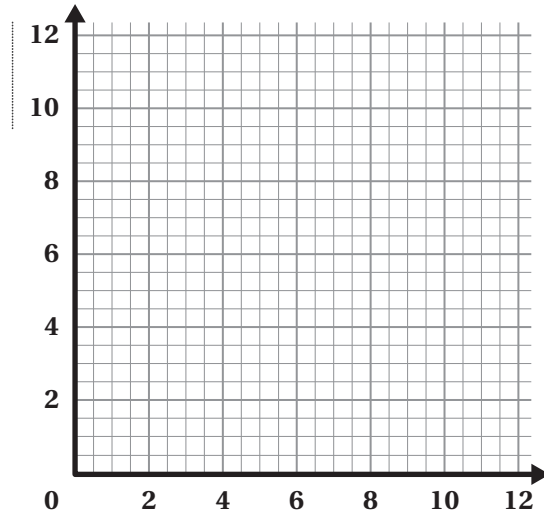
Practice

2.13

Name: _____ Date: _____ Period: _____


Problems 1–4: Elena goes to a store where customers can scoop their own popcorn and pay by the ounce. She buys 10 ounces of popcorn for \$2.50.

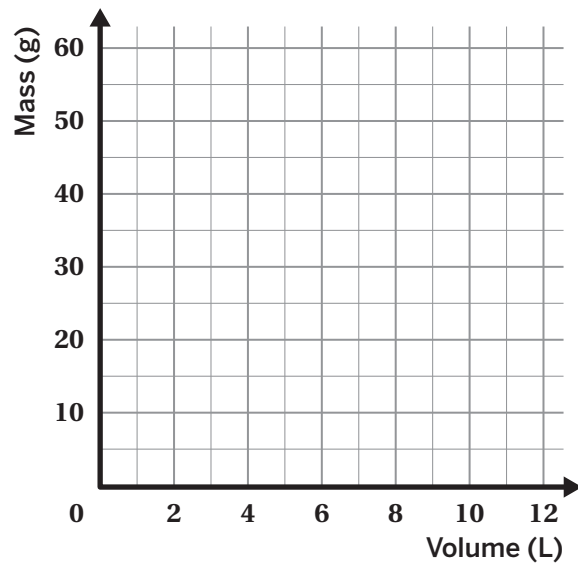
1. How much does the popcorn cost per ounce?
2. How much popcorn can Elena buy per dollar?
3. Write two different equations that represent this situation. Use p for ounces of popcorn and c for cost in dollars.



4. Graph the line of one of the equations. Be sure to label the axes.

Spiral Review

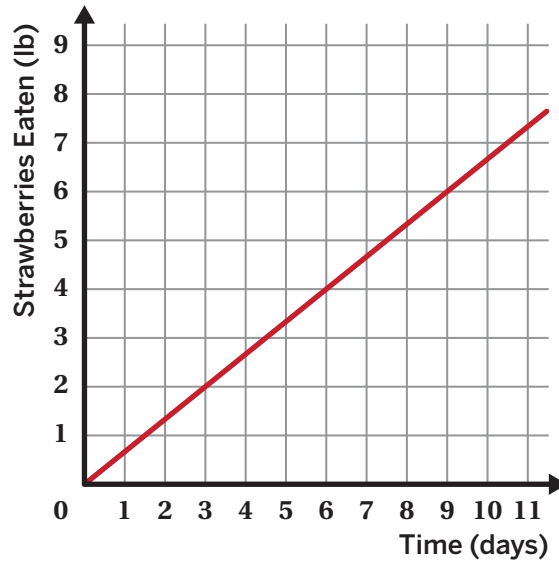
5.  A chemical substance has a proportional relationship between its mass and volume. One constant of proportionality is 2.5 grams per liter. Graph the proportional relationship.



Practice 2.13

Name: _____ Date: _____ Period: _____

Problems 6–8: This graph shows a proportional relationship. Represent this relationship in different ways:



6. In a table:

Time (days)	Strawberries Eaten (lb)

7. With an equation:

Use t for the time in days and s for the pounds of strawberries eaten.

8. With a story:

Problems 9–10: Each equation is represented on the graph.

9. Match each equation with its graph.

.....

$$y = \frac{2}{5}x$$

.....

$$y = 5x$$

.....

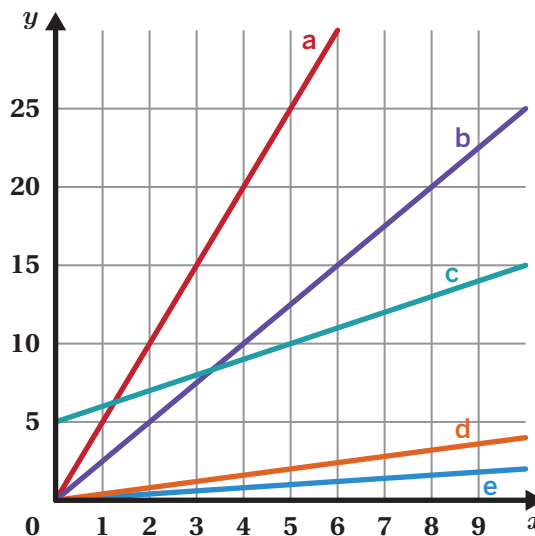
$$y = \frac{5}{2}x$$

.....

$$y = \frac{1}{5}x$$

.....

$$y = x + 5$$



10. How did you decide which equation to match with graph C?

Practice Day 2

Let's practice what you've learned so far in this unit!

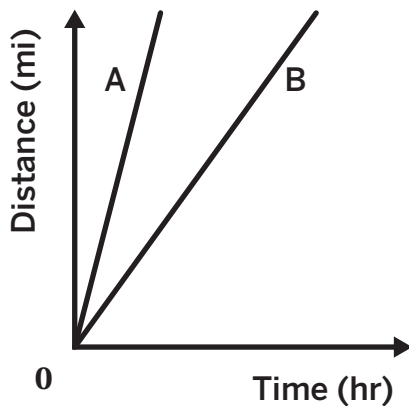


You will use task cards for this Practice Day. Record all of your responses here.

Task A: Comparing Relationships

1. **a** Circle one: Line A or Line B

b



Description:

2. **a** Circle one: 3-pack or 5-pack

Explanation:

b _____ dollars

Explanation:

3. **a** Machine 1

Time (min), t	Number of lollipops, l
3	30
5	50
	75

Machine 2

Time (min), t	Number of lollipops, l
2	16
4	
10	80

b Circle one: Machine 1 or Machine 2

Practice Day 2 (continued)**Task B: Equations**

1. Situation A: _____ Situation B: _____ Situation C: _____

2. Extra equation: _____

Situation:

3. Equation: _____

4. Circle one: A B C D

Task C: Two Truths and a Lie

1. Circle one: A B C

2. Circle one: A B C

3. Statement A:

Explanation:

Explanation:

Statement B:

Statement C:

Task D: Matching Representations

1. Table A: Graph _____ Table B: Graph _____ Table C: Graph _____

2. Matching Pair: _____

3. Relationships: _____

Equation: _____

Explanation:

4. Table Explanation:

Graph Explanation:

Equation Explanation:

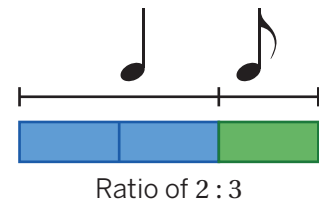
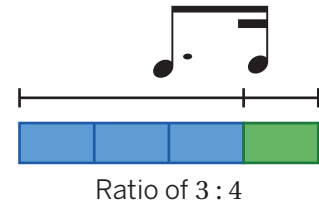
Notes:

Career Connection

How might ratios alter the mood of a song?

Eugenia Cheng, a mathematician and concert pianist, has helped to explain how math and music are connected. Both can involve patterns and ratios. The duration of the notes in a sound creates a rhythmic pattern based on ratios. Different rhythmic patterns can set the mood for a song. According to Eugenia Cheng's research, a rhythmic pattern where the first note takes up $3 : 4$ of the beat tends to sound more lively, while a rhythmic pattern where the first note takes up $2 : 3$ of the beat might sound more relaxed.

Musicians perform and create musical pieces for people to listen to and enjoy. They use ratios and proportional relationships to create sounds that when played together, are pleasing to the ear.



Paul Crisanti

Meet Eugenia Cheng

Born in the United Kingdom, Eugenia Cheng holds three degrees from the University of Cambridge and performs regularly both in Chicago and Europe. She is founder of the Liederstube in Chicago, a space where musicians can share music with each other and audiences in a relaxed setting. She enjoys explaining the many ways math is related to music, cooking, and the ways in which people relate to one another.

Are you interested in studying music or how music relates to math? What can you do to learn more?

Math in the World

Kheer is a South Asian pudding made from rice and milk. One recipe calls for $\frac{1}{4}$ cup of rice for every 4 cups of milk. If you want to use 10 cups of milk, how many cups of rice do you need?



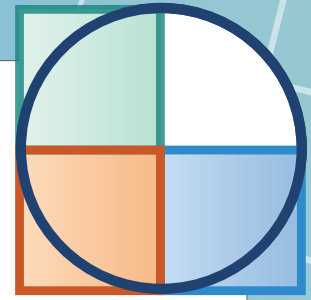
Lazartivan/Shutterstock.com.

Math Mindset

How can an equation representing a proportional relationship show the constant of proportionality?

Unit 3

Measuring Circles



Big Ideas in This Unit

- cc2 Graphing Relationships
- Proportional Relationships
- Unit Rates in the World
- cc3 2-D and 3-D Connections
- cc4 Shapes in the World
- Scale Drawings

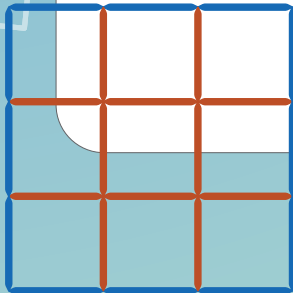
Questions for Investigation

- What is π and what does it have to do with circles?
- How can squares help you measure the area of a circle?
- How can I solve problems involving circles in the world?



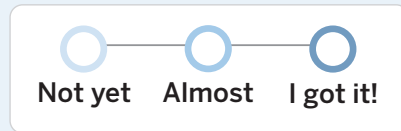
Explore: The Wandering Goat

How far can a goat on a rope roam?







Watch Your Knowledge Grow

This is the math you'll explore in this unit. Rate your understanding to see how your knowledge grows!



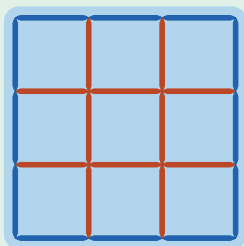
I can . . .	Before	After
Draw circles with given conditions using technology.	<input type="radio"/> — <input type="radio"/> — <input type="radio"/>	<input type="radio"/> — <input type="radio"/> — <input type="radio"/>
Draw circles with given conditions freehand.	<input type="radio"/> — <input type="radio"/> — <input type="radio"/>	<input type="radio"/> — <input type="radio"/> — <input type="radio"/>
Calculate the diameter when given the radius.	<input type="radio"/> — <input type="radio"/> — <input type="radio"/>	<input type="radio"/> — <input type="radio"/> — <input type="radio"/>
Calculate the radius when given the diameter.	<input type="radio"/> — <input type="radio"/> — <input type="radio"/>	<input type="radio"/> — <input type="radio"/> — <input type="radio"/>
Use the formula for the circumference of a circle to solve problems.	<input type="radio"/> — <input type="radio"/> — <input type="radio"/>	<input type="radio"/> — <input type="radio"/> — <input type="radio"/>
Determine the radius or diameter when given the circumference.	<input type="radio"/> — <input type="radio"/> — <input type="radio"/>	<input type="radio"/> — <input type="radio"/> — <input type="radio"/>
Explain how the circumference and area of a circle are related.	<input type="radio"/> — <input type="radio"/> — <input type="radio"/>	<input type="radio"/> — <input type="radio"/> — <input type="radio"/>
Use the formula for the area of a circle to solve problems.	<input type="radio"/> — <input type="radio"/> — <input type="radio"/>	<input type="radio"/> — <input type="radio"/> — <input type="radio"/>
Determine the radius or diameter when given the area.	<input type="radio"/> — <input type="radio"/> — <input type="radio"/>	<input type="radio"/> — <input type="radio"/> — <input type="radio"/>
Decide whether the relationship between the diameter and circumference of a circle is proportional.	<input type="radio"/> — <input type="radio"/> — <input type="radio"/>	<input type="radio"/> — <input type="radio"/> — <input type="radio"/>

I can . . .	Before	After
Decide whether the relationship between the radius and area of a circle is proportional.		
Identify the constant of proportionality, π , in the equation for circumference.		

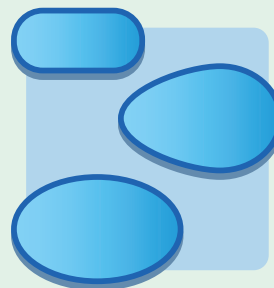
Circumference of a Circle



Explore
The Wandering Goat



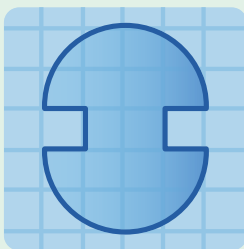
Lesson 1
Toothpicks



Lesson 2
Is It a Circle?



Lesson 3
Measuring Around



Lesson 4
Perimeter Challenges



Explore: The Wandering Goat

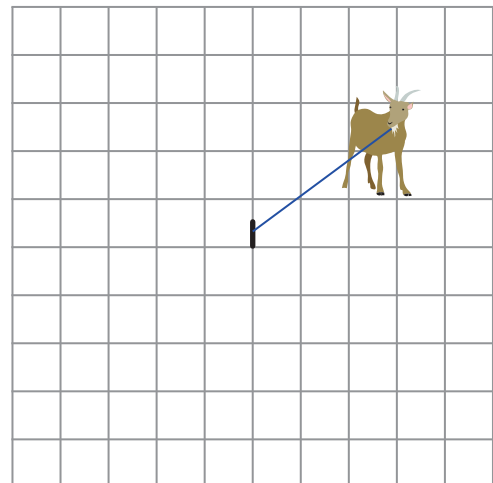
How far can a goat on a rope roam?



Warm-Up

1. A famous math problem - known as “The Goat Problem” - was first published in *The Ladies’ Diary* in 1748. A variation of this problem asks the question, “If a goat were tethered by a rope to a peg, what space could the goat occupy (In other words, where could it roam)?”

- a Choose one person to act as the goat and one person to act as the peg. Using the string provided by your teacher, explore what space the goat could roam in while it is attached to the peg.
- b Imagine this grid represents a field. Sketch the shape of the space where the goat would be able to roam while tethered to a rope that is attached to the peg.

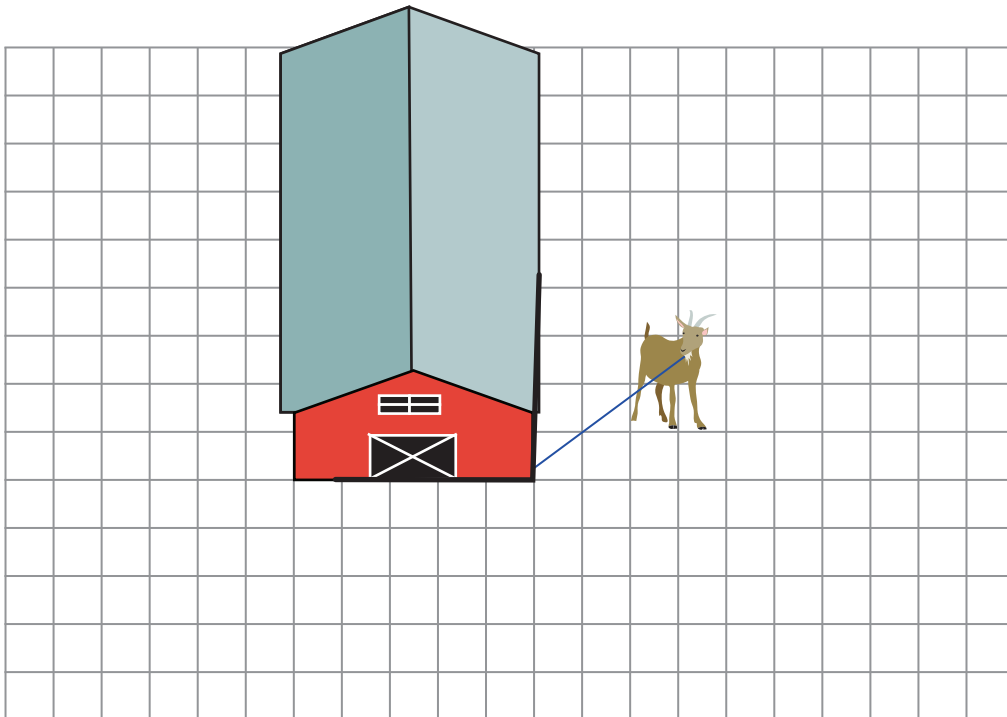




The Goat Problem

Now imagine that the goat is tethered to a rope that is attached to the corner of a rectangular barn. Assume the rope is shorter than both the length and the width of the barn. You will be given the materials needed for this activity.

- Using the materials provided and objects around the classroom, come up with a plan to model this situation. Describe your plan.
- Sketch the shape of the space in which the goat would be able to roam while tethered to a rope that is attached to the corner of the barn.

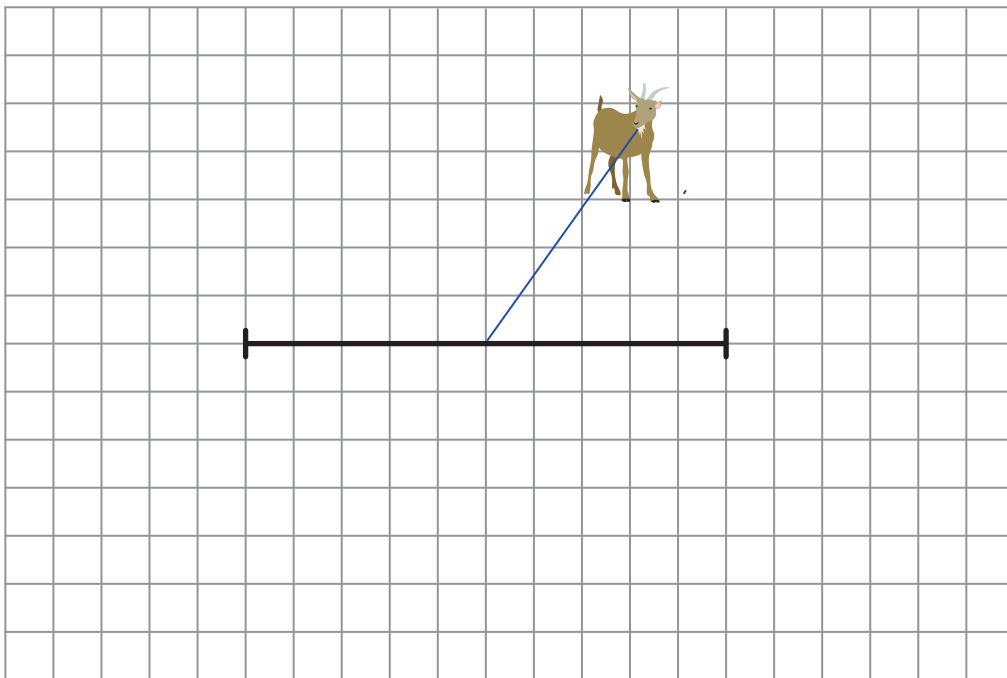




The Goat Problem (continued)

Now imagine that two pegs are in an open field and are connected by a bar. The goat's rope is attached by making a loop so that it can slide along the bar attaching the two pegs. You will have access to the same materials used in the previous activity.

- Using the materials provided and objects around the classroom, come up with a plan to model this situation. Describe your plan.
- Sketch the shape of the space in which the goat would be able to roam, while tethered to the rope between the two pegs.





Building Math Habits of Mind



Discuss:

- Which of these habits of mind did you strengthen during this activity?
- How did you use the one(s) you selected?

I can slow down and first make sense of a challenging problem before trying to solve it.

Not yet Almost I got it!

I can represent real-world problems using equations and inequalities and interpret their solutions within the context of the problem.

Not yet Almost I got it!

I can justify my thinking and ask questions to help me understand the thinking of others.

Not yet Almost I got it!

I can apply the math that I know to solve real-world problems, make assumptions and revise my thinking as needed.

Not yet Almost I got it!

I can select an appropriate tool to help me solve problems

Not yet Almost I got it!

I can communicate my thinking and solutions clearly to others.

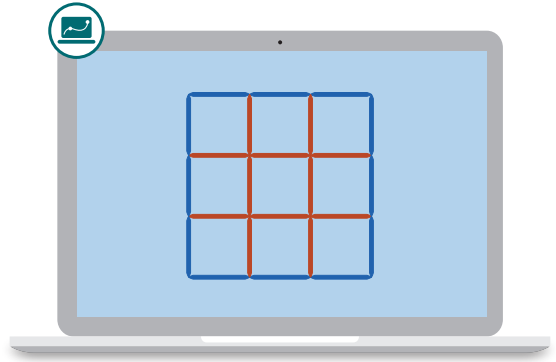
Not yet Almost I got it!

I can look for structure or patterns to help me solve problems.

Not yet Almost I got it!

I can look for repeated calculations and other repeated steps to make generalizations.

Not yet Almost I got it!



Toothpicks

Let's examine relationships between side length, diagonal length, and the perimeter of a square.

Warm-Up

1 These two figures were built with toothpicks.

Figure A

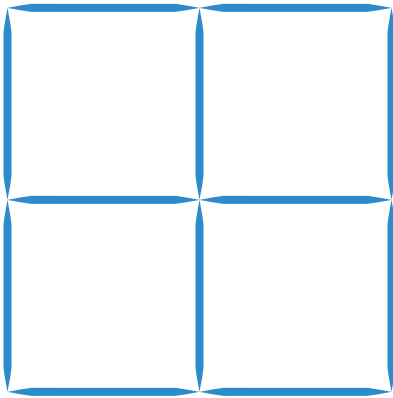
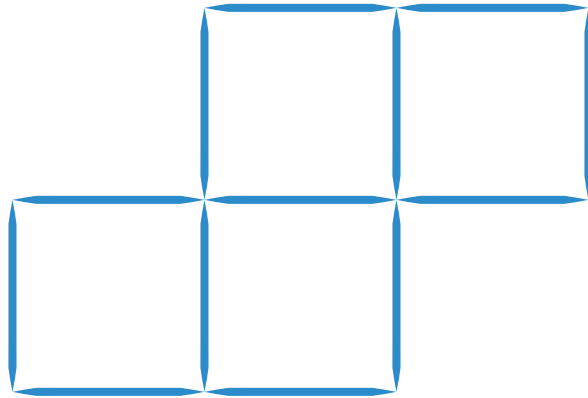


Figure B



Which figure used more toothpicks? Circle one.

Figure A

Figure B

They used the same

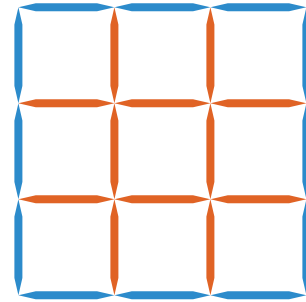
Explain your thinking.

Proportional or Not?

- 2** To determine the number of toothpicks needed to build this figure, we count the blue perimeter toothpicks and the orange interior toothpicks.

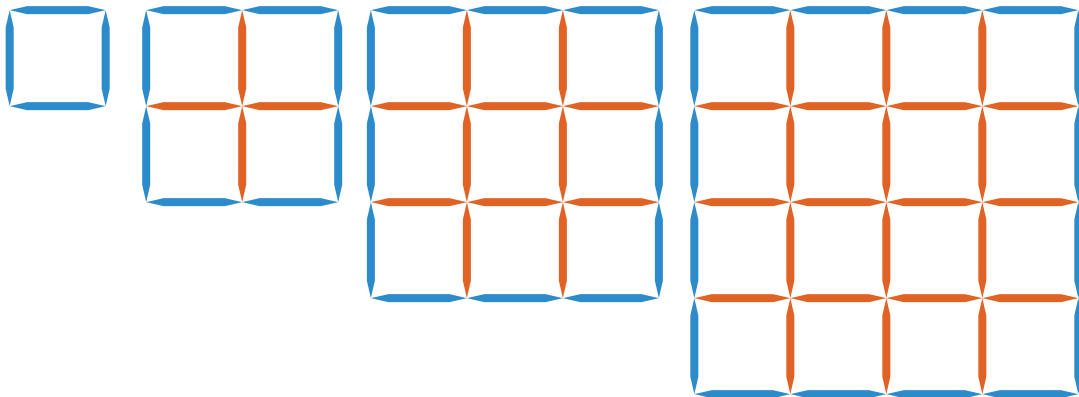
Write the number of each type of toothpick.

Perimeter (blue toothpicks)	Interior (orange toothpicks)



- 3** Let's explore how changing the size of the square changes the number of toothpicks.

- a** Take a look at this square and its scaled copies.



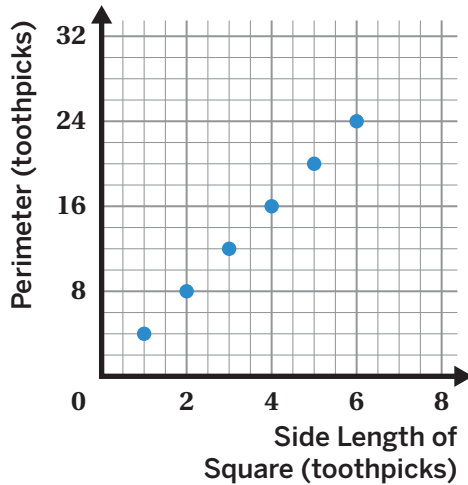
- b** **Discuss:** Why is each new square a scaled copy of the original?
- c** Write the number of each type of toothpick needed to build each square.

Side Length (toothpicks)	Perimeter (blue toothpicks)	Interior (orange toothpicks)
1		
2		
3		
4		

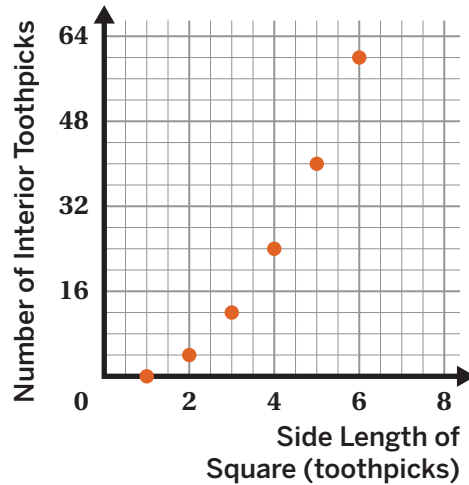
Proportional or Not? (continued)

4 Here are two graphs based on the toothpick squares from the previous problem.

Side Length vs. Perimeter



Side Length vs. Interior



Use the graphs to decide which relationships are proportional. Circle one.

- A. Side length of a square vs. perimeter B. Side length of a square vs. number of interior toothpicks C. Both D. Neither

Explain your thinking.

5 a For any relationship you said was proportional, what is its constant of proportionality?

b What does the constant of proportionality mean in this context?

You're invited to explore more.

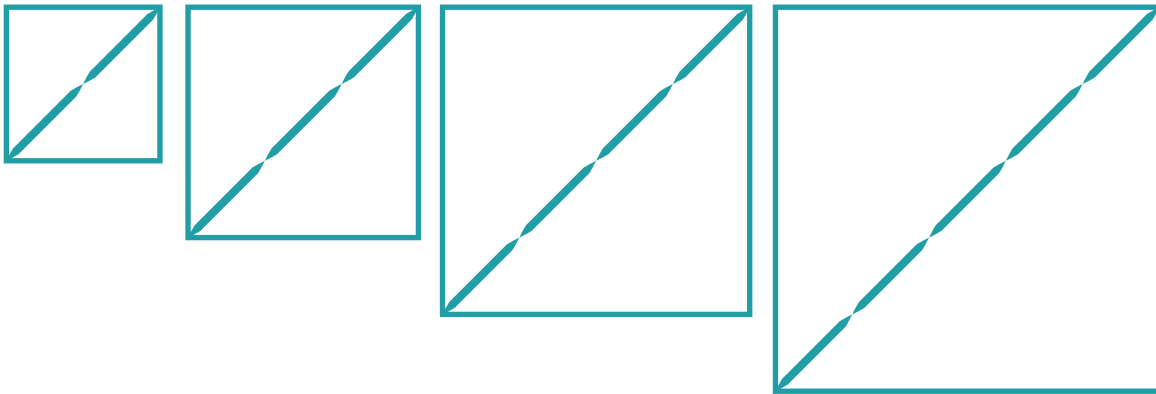
6 Use the You're Invited to Explore More Sheet to further explore these toothpick relationships.

Across and Around

- 7** This square has a diagonal that is 1 toothpick long. About how many toothpicks long is its perimeter? Parts of toothpicks are allowed. Get as close as you can.




- 8** Here are squares with different diagonal lengths.



Complete the table to show the number of toothpicks you would need to build the perimeter of each square.


Diagonal Length (toothpicks)	Perimeter (toothpicks)
1	
2	5.66
3	
4	
5	14.15

- 9 a**  **Discuss:** Is the relationship between diagonal length and perimeter proportional? How do you know?

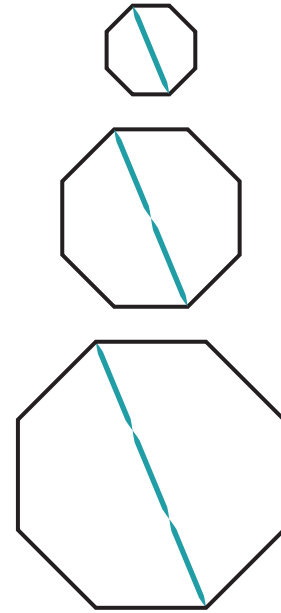
- b** How can you estimate the perimeter of a square with a diagonal of 100 toothpicks.

10 Synthesis

This table shows the relationship between the diagonal length and perimeter of an octagon.

 **Discuss:** How can you tell if two quantities in a geometry situation are proportional?

Diagonal Length (toothpicks)	Perimeter (toothpicks)
1	3.06
2	6.12
3	9.18
4	12.24



13 Summary 3.01

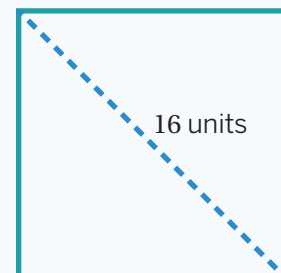
Proportional relationships can appear in geometry when you look at scaled copies of shapes.

Some examples of these proportional relationships include:

- The side lengths of a square and its perimeter
- The diagonal length of a square and its perimeter
- The diagonal length of an octagon and its perimeter

Because the relationship between the diagonal length and the perimeter of a square is proportional, you can determine one of the measurements if you know the other.

For example, to determine the perimeter of a square whose diagonal length is 16 units, you can multiply by the constant of proportionality, approximately 2.83. $16 \cdot 2.83 \approx 45$ units.



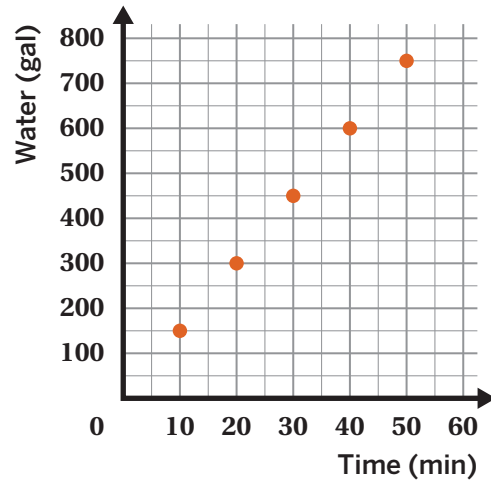
Practice

3.01

Name: _____ Date: _____ Period: _____

Problems 1–3: This graph shows the amount of water in a swimming pool as it fills up.

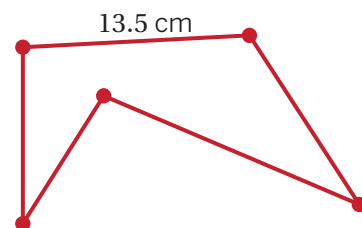
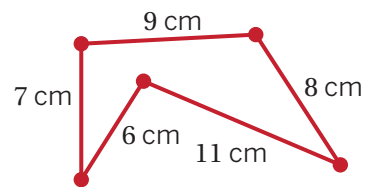
1. Explain how the graph shows that the relationship between time and the amount of water is proportional.
2. Calculate a constant of proportionality for this relationship.
3. This pool is safe to swim in when it contains 9,000 gallons of water. How long will it take to fill the pool to this level?



Problems 4–6: An equilateral triangle is a triangle where all three sides have the same length.

4. Complete the table to show the side lengths and perimeters for several equilateral triangles.
5. Explain why the relationship between the perimeter and side length for an equilateral triangle is proportional.
6. What is a constant of proportionality for the relationship?
7. These polygons are scaled copies. Determine the perimeter of the larger polygon. Show or explain your thinking.

Side Length	Perimeter
3	
10	
	81
	315



Spiral Review

Problems 8–9: Here are two recipes for making sparkling lemonade. For each recipe, determine how many tablespoons of lemonade mix are required *per cup* of sparkling water. Show or explain your thinking.

8. 4 tablespoons of lemonade mix and 12 cups of sparkling water

9. 4 tablespoons of lemonade mix and 6 cups of sparkling water

10. Select *all* of the coordinate pairs that could be part of a proportional relationship with a constant of proportionality of 4.

A. (3, 12)

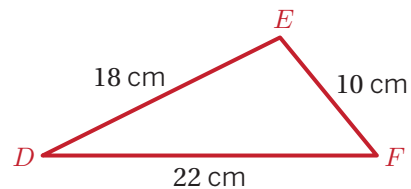
B. (4, 8)

C. (3.5, 14)

D. (6, 10)

E. $(\frac{1}{2}, 2)$

11. 🌀 Here is triangle DEF and its dimensions. Which measurements represent the dimensions of a triangle that is a scaled copy of triangle DEF ?



A. 20 cm, 12 cm, 24 cm

B. 9 cm, 5 cm, 10 cm

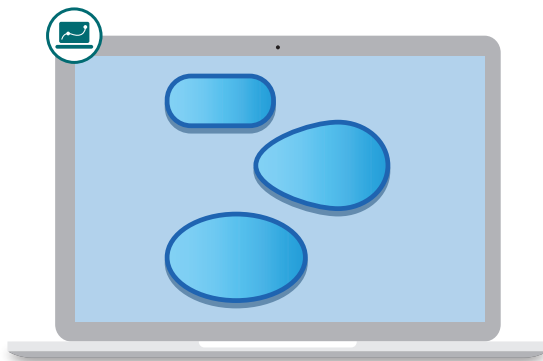
C. 27 cm, 16 cm, 33 cm

D. 13.5 cm, 7.5 cm, 16.5 cm

12. Diego created a graph of two quantities that he measured and said, “The points all lie on a line except one, which is a little bit above the line. This means that the quantities cannot be proportional.” Do you agree or disagree with Diego? Explain your thinking.


Is It a Circle?







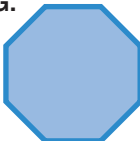

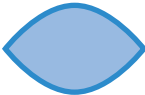
Let's explore what makes a circle a circle.



Warm-Up

1 Circle the shapes you would describe as round.

 **Discuss:** Which shape(s) would you call a *circle*? Why?

<p>A.</p> 	<p>B.</p> 	<p>C.</p> 
<p>D.</p> 	<p>E.</p> 	<p>F.</p> 
<p>G.</p> 	<p>H.</p> 	<p>I.</p> 

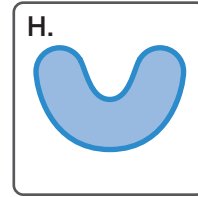
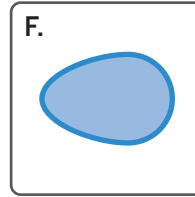
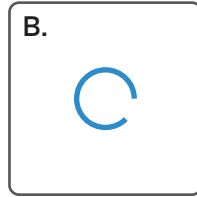
Circle Sizes

2 Here are some shapes students said were round.

Choose one that isn't a circle.

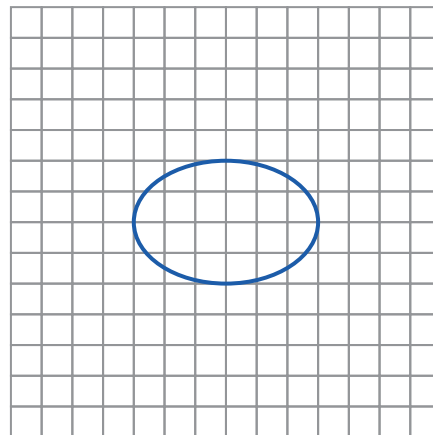
Explain your thinking.

Shape _____ is not a circle because . . .



3 What is one reason someone might say this figure is not a circle?

Draw if it helps to show your thinking.



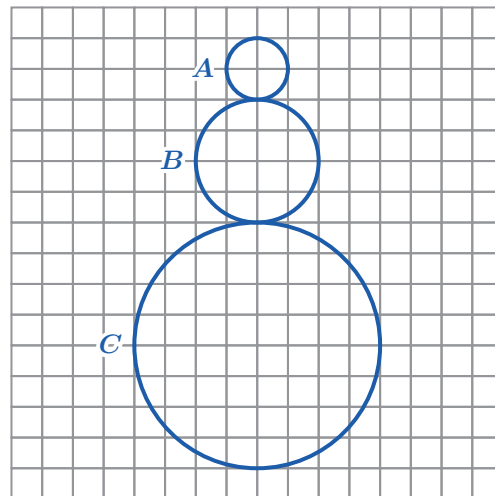
4 Adhira made a drawing using circles.

She said: *One of the circles measures 4 units.*

Which circle do you think Adhira is describing?
Circle one.

Circle *A* Circle *B* Circle *C*

Explain your thinking.

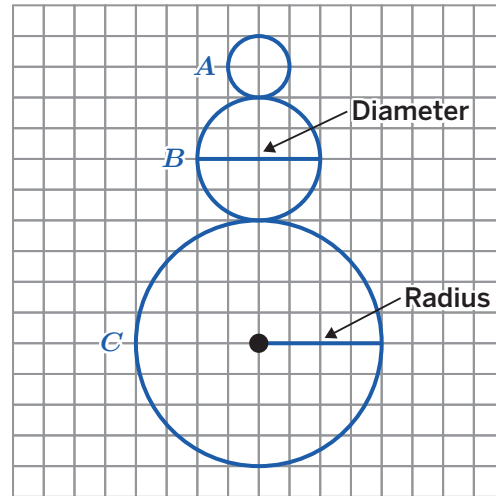


Circle Sizes (continued)

5 We can measure circles using radius or diameter.

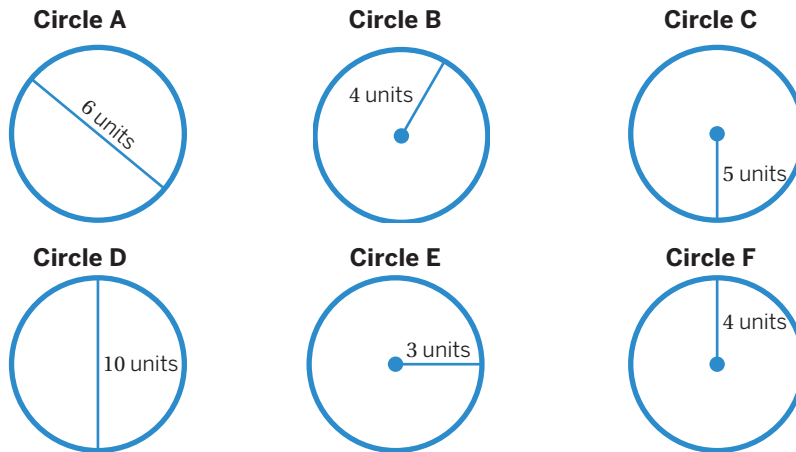
a Record the radius and diameter of each circle in the table. Some are done for you.

Circle	Radius (units)	Diameter (units)
A		
B		4
C	4	



b **Discuss:** How is the diameter of a circle related to its radius?

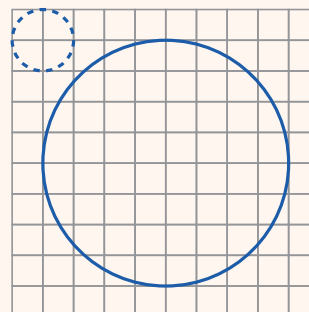
6 **Discuss:** Which images represent circles of the same size? **Note:** Circles are not drawn to scale.



You're invited to explore more.

7 How many full circles with a radius of 1 unit can you fit inside a circle of radius 4 units so that they don't overlap?

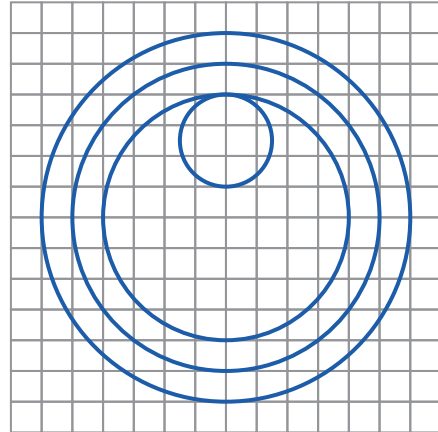
Sketch on the diagram if that helps with your thinking.



Circle Centers

- 8** Madison made a drawing using circles. Describe her drawing as precisely as you can so that someone who can't see her drawing could recreate it.

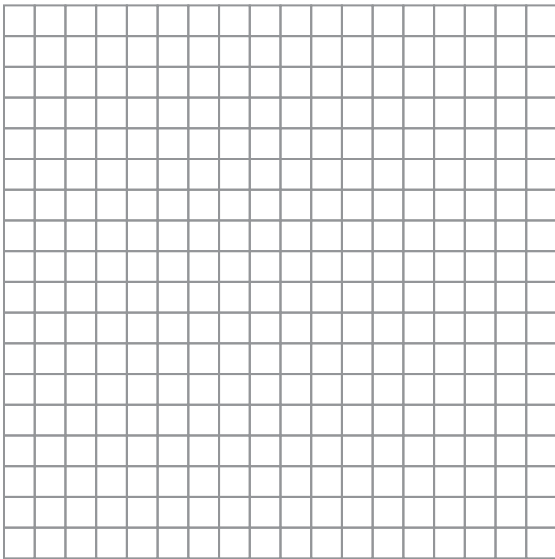
Draw if it helps with your thinking.



- 9** Callen also made a drawing using circles and wrote the following description:

- My drawing has 4 circles.
- All the circles touch at a single point.
- Each circle is a different size.
- The largest circle has a radius of 5 units.
- The smallest circle has a diameter of 2 units.

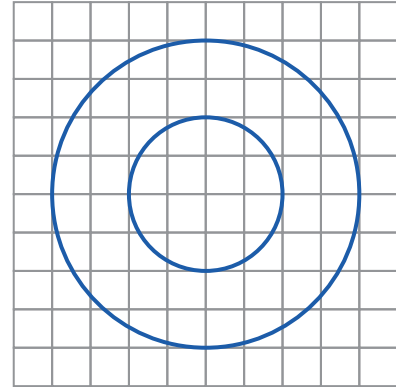
Draw circles on the grid or use the circles from the Activity 2 Sheet to recreate Callen's design.



10 Synthesis

Describe the drawing as precisely as you can so that someone who can't see it could recreate it.

Include the terms you used today in your description.



13 Summary 3.02

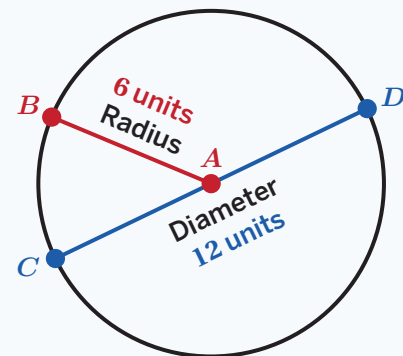
A *circle* is a shape made out of all the points that are the same distance from a center point.

You can measure a circle with its **radius**. A radius is a line segment that connects the center of a circle with a point on the circle. Every radius of a circle is the same length.

You can also measure a circle with its **diameter**, which is the distance from one point on a circle through the center to another point on the circle. It is also the longest distance across the circle.

For any circle, the length of the diameter is two times the length of the radius.

For example, here is a circle. The radius AB is 6 units. The diameter CD is 12 units.



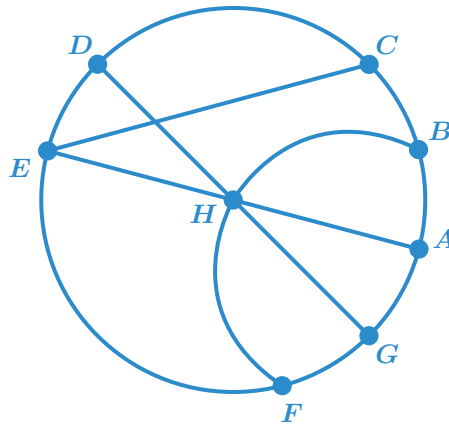
diameter The distance from one point on a circle through the center to another point on the circle. It is also the longest distance across the circle.

radius A line segment that connects the center of a circle with a point on the circle. Every radius of a circle is the same length.

Practice 3.02

Name: _____ Date: _____ Period: _____

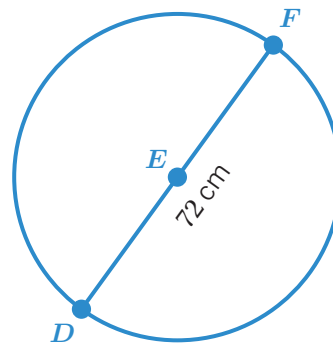
Problems 1–3: Here is a circle. Point H is the circle's center.



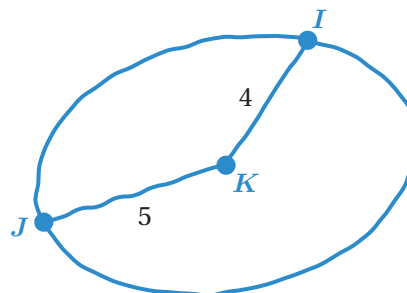
1. Trace a segment that is a radius.
2. Trace a segment that is a diameter.
3. Trace a segment that is neither a radius nor a diameter.
4. A circle has a radius of 4.5 units. What is its diameter?
5. Can a circle with a radius of 5.2 inches have a diameter of 13 inches?

Explain your thinking.

6. What is the length of the radius in the circle?



7. Kala's teacher asked her to create a circle and draw a diameter. Here is her drawing. Identify and describe her mistakes.

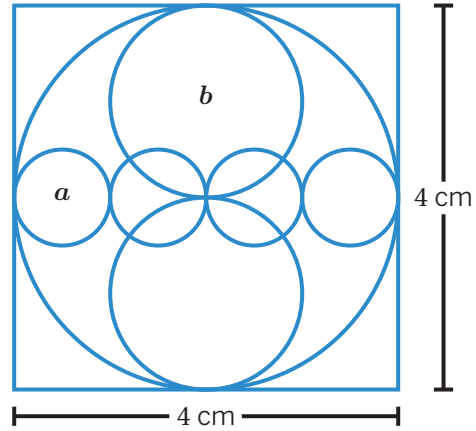


Practice 3.02

Name: _____ Date: _____ Period: _____

8. This square contains several circles. Determine the radius and diameter of circles *a* and *b*.

Circle	Radius (cm)	Diameter (cm)
<i>a</i>		
<i>b</i>		



Spiral Review

9. A lemonade recipe calls for $\frac{1}{4}$ cups of lemon juice, $\frac{1}{2}$ cups of sugar, and 1 cup of water. Complete the table to show the amount of lemon juice for different-sized batches.

Sugar (cups)	Lemon Juice (cups)
2	
10	
5	

10. 📞 The graph of a proportional relationship includes the point (3, 12).

What is one constant of proportionality for this relationship? Show or explain your thinking.

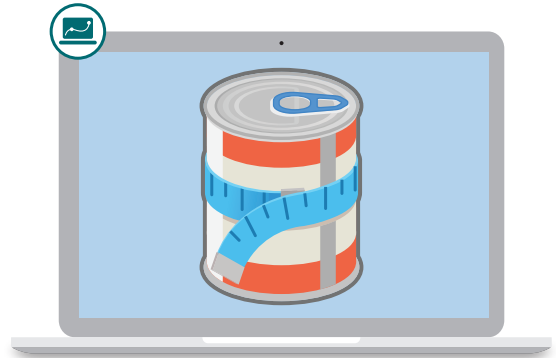
Problems 11–12. Search your home for these objects.

11. Two objects that you would describe as round, but not a circle. Explain your thinking.

12. Two objects that are round *and* circular. Explain your thinking.

Measuring Around

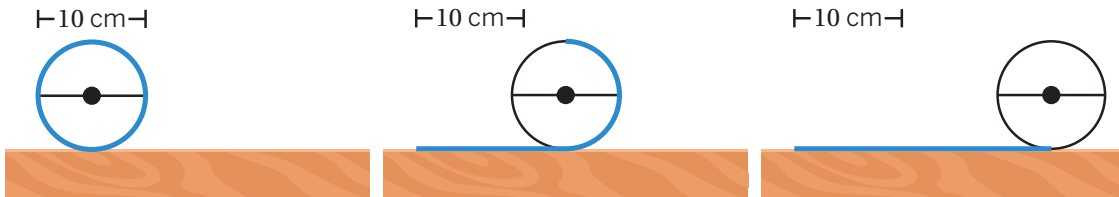
Let's explore the relationship between the diameter of a circle and its circumference.



Warm-Up

1 The **circumference** of a circle is the distance all the way around it.

a Imagine unrolling the circumference to show it as a line segment.



b Estimate the length of the circumference.

So Many Circles

2 Measure the diameter and the circumference of at least three circular objects. Record your results in the table.

Object	Diameter (cm)	Circumference (cm)
Toilet paper tube	4.1	12.9

3 This graph shows the diameter and circumference of several circular objects.

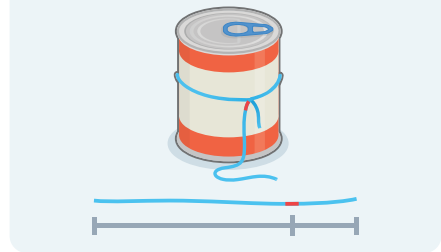
Do you think the relationship between circumference and diameter is proportional? Circle one.

Yes No I'm not sure

Explain your thinking.

Three Ways to Measure the Circumference

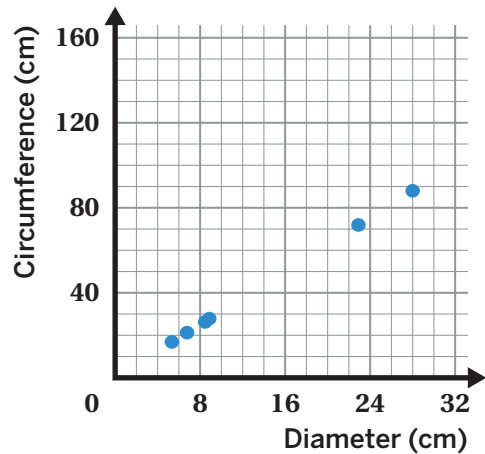
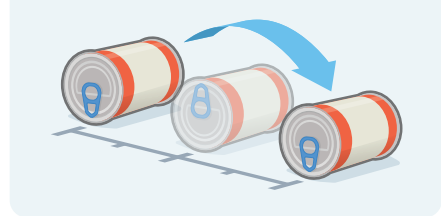
String



Tape Measure



Roll

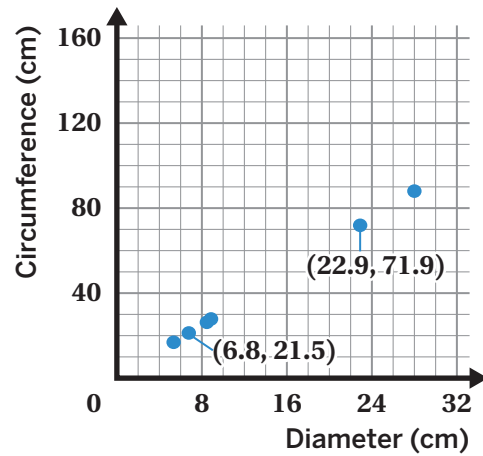


Introducing π

4 The relationship between circumference, C , and diameter, d , is proportional.

Estimate the *constant of proportionality* that completes the equation relating circumference and diameter.

$C = \dots\dots\dots d$

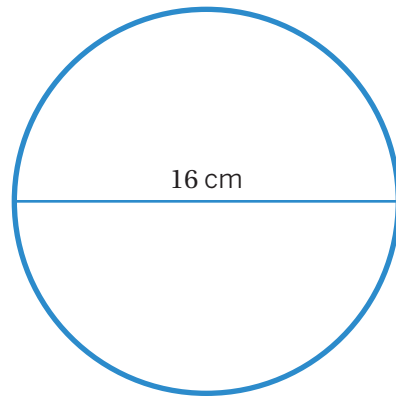


5 The constant of proportionality between a circle's diameter, d , and its circumference, C , is π (written as **pi** and pronounced "pie").

In other words, $C = \pi d$.

Common approximations for π are 3.14, $\frac{22}{7}$, and 3.14159. None of these are exactly π .

Calculate the circumference of a circle with a diameter of 16 centimeters.

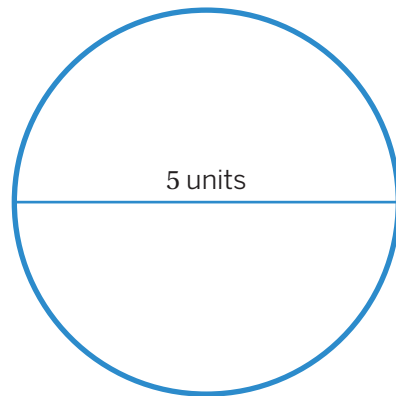


6 This circle has a diameter of 5 units.

Four students tried to calculate its circumference.

Order their answers starting with the farthest from the exact circumference to the closest to the exact circumference.

- 15.7 units 15 units 5π units 15π units



.....

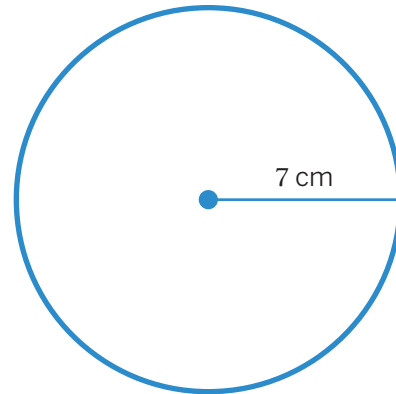
Farthest from exact circumference

Closest to exact circumference

Proportionality and π

- 7** The radius of this circle is 7 centimeters.

What is its circumference?



- 8** The constant of proportionality in the relationship between diameter and circumference is π .

- a** Complete the table.

Object	Radius (cm)	Diameter (cm)	Circumference (cm)
Cookie	3	6	
Small plate	9		
Quarter		2.4	
Frisbee			21π
Tennis ball can	$\frac{7}{2}$		
Vinyl record			100
Bike Wheel			135


- b** When you finish, compare your answers with a classmate.

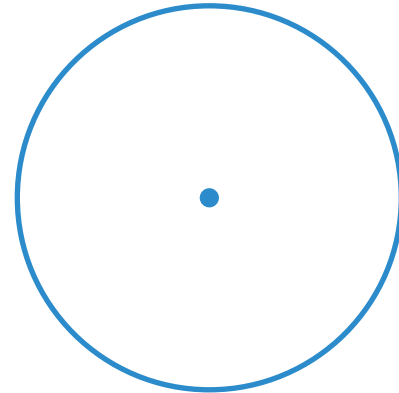
 **Discuss:** How are your answers alike? How are they different?

- 9** Consider the bike wheel.

- a** What distance in centimeters would it travel in one rotation?
- b** Two rotations?

10 Synthesis

 **Discuss:** What is the relationship between radius, diameter, and circumference?



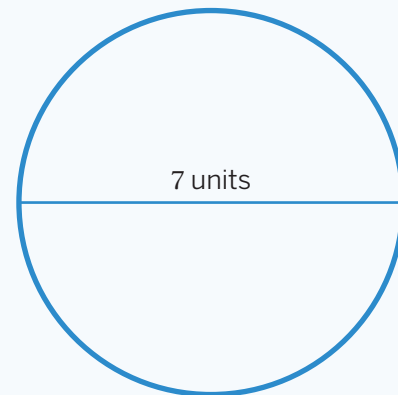
13 Summary 3.03

The distance around a circle is called the **circumference**. There is a proportional relationship between the *diameter* of a circle and its circumference. The *constant of proportionality* for that relationship is π (**pi**). π is often approximated as 3.14 or $\frac{22}{7}$.

For any circle, we can calculate the circumference, C , using the equation $C = \pi d$.

For example, if the diameter of a circle is 7 units, the circumference of the circle can be calculated approximately as $7 \cdot 3.14 = 21.98$. More accurately, $C = 7\pi$ units.

If we know the *radius* of a circle, we can calculate the circumference by first determining the diameter, then using the equation $C = \pi d$.



circumference The distance around a circle. If you imagine a circle as a piece of string, it is the length of the string.

The circumference of a circle, C , can be calculated with the formula $C = \pi d$, where d is the diameter of the circle, or $C = 2\pi r$, where r is the radius.

pi π A number that represents the constant of proportionality between the diameter and circumference of any circle. The symbol for pi is π . Some common approximations for π are 3.14 and $\frac{22}{7}$.

Practice 3.03


Name: _____ Date: _____ Period: _____

1. Lucy measured the diameter and circumference of several circular objects and recorded her measurements in a table.

Which object's circumference measurement do you think is the least accurate?

Explain your thinking.

Object	Diameter (cm)	Circumference (cm)
Half dollar coin	3	10
Flying disc	23	50
Jar lid	8	25
Flower pot	15	48

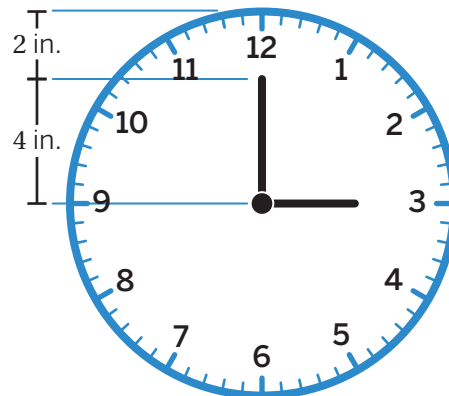
2.  Complete the table to determine the missing measurements for each object.

Object	Radius	Diameter	Circumference
Hula hoop		36 inches	
Circular pond			556 feet
Magnifying glass		5.2 centimeters	
Car tire			71.6 inches

3. The circumference of a circle is approximately 50.27 inches. Determine the length of the radius of the circle in inches. Round your answer to the nearest whole number.

Problems 4–5: The minute hand of a circular clock measures 4 inches. The distance from the end of the minute hand to the outer edge of the clock is 2 inches.

4. What is the length of the radius of the clock?
5. What is the circumference of the clock?



Practice 3.03

Name: _____ Date: _____ Period: _____

Problems 6–7: The size of a drum affects its pitch. When comparing drums, the one with a larger diameter has a lower pitch because the frequency of its sound waves is slower.

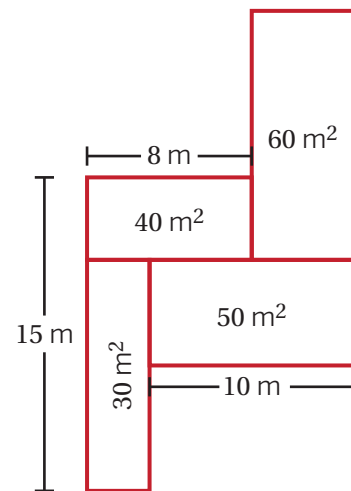
- If a drummer wants a drum with a lower pitch, should she choose a drum with a circumference of 18π inches or 22.5π inches?
- Determine the diameter of the drum she should choose.

Problems 8–10: Decide whether the measurement described is the diameter, radius, or circumference. Circle one.

- | | | | | |
|-----|---|----------|--------|---------------|
| 8. | The tires on a tractor are 4.5 feet tall. | Diameter | Radius | Circumference |
| 9. | The distance from the tip of a slice of pizza to the crust is 6 inches. | Diameter | Radius | Circumference |
| 10. | The length of the metal rim around a glass lens is 190 millimeters. | Diameter | Radius | Circumference |

Spiral Review

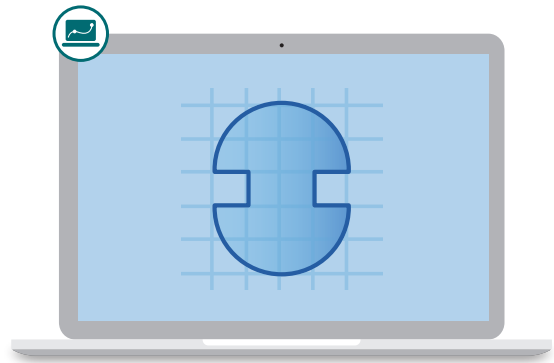
- This shape is made up of four rectangles. Determine the total perimeter of the shape.



- The scale factor from Polygon A to Polygon B is 3. The perimeter of Polygon B is 90 centimeters, what is the perimeter of polygon A? Show or explain your thinking.

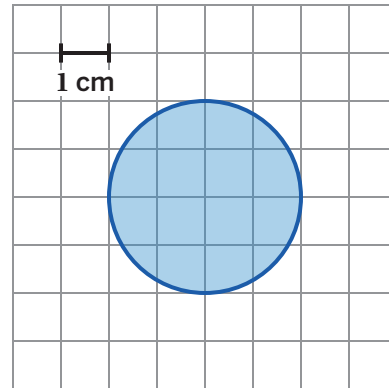
Perimeter Challenges

Let's calculate the perimeters of complex shapes.

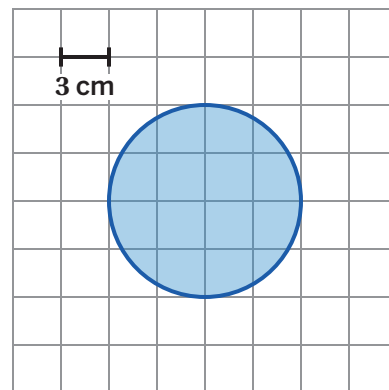


Warm-Up

1 What is the circumference of this circle?



2 Now, what is the circumference of *this* circle?



Calculating Perimeters

3 Dyani and Elena are in a walking club. They walk around a circular track.

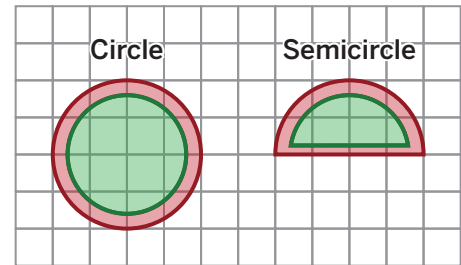
Dyani claims that by walking around the track in the shape of a semicircle, you can cover *half* the distance of the entire track.

Elena says the distances are *equal*.

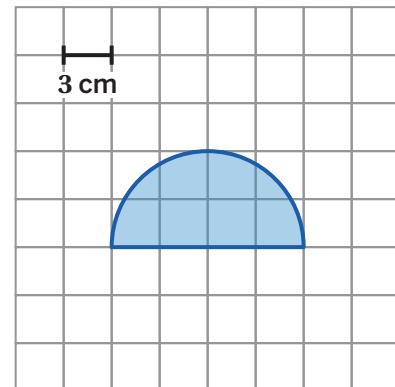
Whose claim is correct? Circle one.

Dyani's Elena's Both Neither

Explain your thinking.

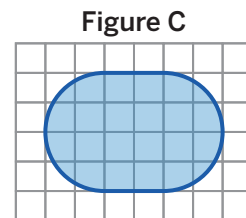
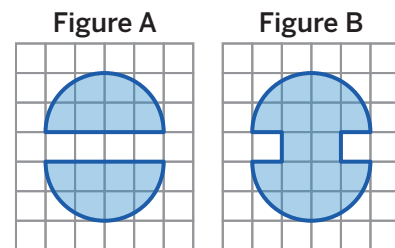


4 What is the perimeter of this semicircle?



5 Order the figures by their total perimeter.

	Least Total Perimeter
	Greatest Total Perimeter



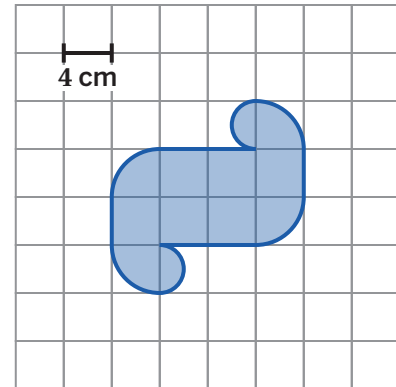
More Perimeters, More Problems

6 This shape is made of squares and parts of circles.

Aba said the perimeter of this shape is made of these parts:

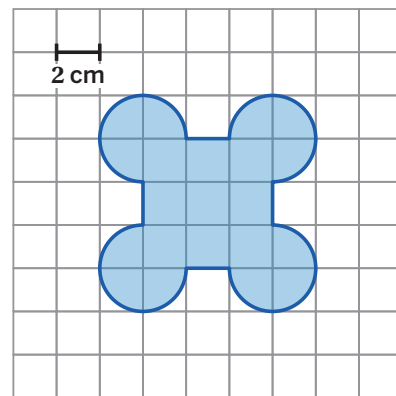
- Four quarter-circles with a radius of 4 cm
- Two half circles with a radius of 2 cm
- Two 4 cm pieces
- Two 8 cm pieces

Show or explain where Aba might see each of these parts.



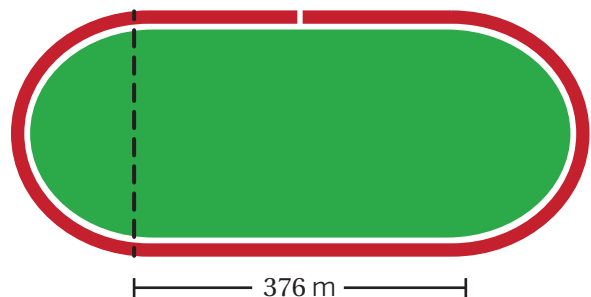
7 a **Discuss:** What parts of circles and squares do you see in this shape?

b Determine the perimeter of this shape.



8 Churchill Downs is one of the most famous horse racing tracks in the world. It is in Louisville, Kentucky where the Kentucky Derby is hosted each year.

The turf track of the Churchill Downs horse race is $\frac{7}{8}$ mile (1,408 meters) long. The straight sides of the track each measure 376 meters. If the rounded sides of the track are semicircles, what is the distance from one side of the track to the other? Show your thinking.




Sum of its Parts

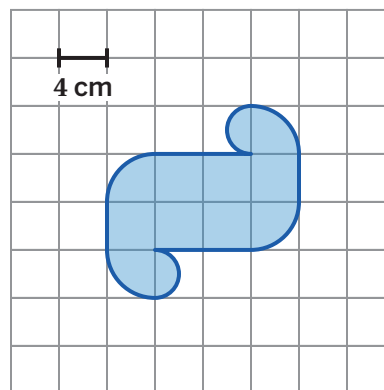
9 You will work with a partner.

- Decide who will complete Column A and who will complete Column B.
- Determine the perimeter of each shape. The perimeters in each row should be equal.
- Compare your solutions, then discuss and resolve any differences.

Column A	Column B
<p>1. Perimeter</p>	<p>1. Perimeter</p>
<p>2. Perimeter</p>	<p>2. Perimeter</p>
<p>3. Perimeter</p>	<p>3. Perimeter</p>

10 Synthesis

 **Discuss:** What is a strategy you can use to determine the total perimeter of a shape that is made of squares and parts of circles? Use this example if it helps you show your thinking.



13 Summary 3.04

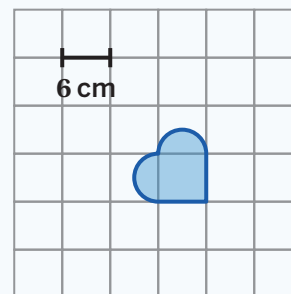
You can use what you know about the perimeter of squares and rectangles and the circumference of circles to find the perimeter of complex shapes.

- Determine what pieces it is made up of such as semicircles, quarter-circles, and straight pieces.
- Determine the length of each straight piece and the radius or diameter of each partial circle.
- Determine the perimeter or circumference of each piece.
- Add them together to get the total perimeter.

The perimeter of the heart shape is made up of 2 semicircles and 2 straight pieces. The *scale* is 6 centimeters.

Each semicircle has a diameter of 6 centimeters. Together they make up one entire circle with a diameter of 6 centimeters and a circumference of $6 \cdot \pi = 6\pi$ centimeters. Each straight edge is 6 centimeters long.

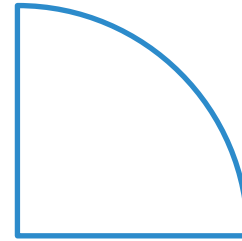
The total perimeter is $6\pi + 12$ centimeters.



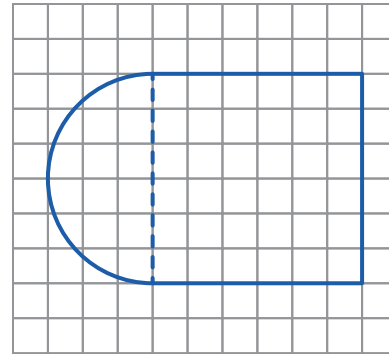
Practice 3.04

Name: _____ Date: _____ Period: _____

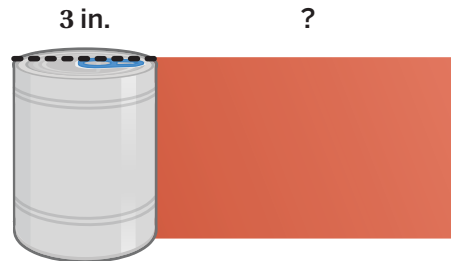
Problems 1–2: Here is a quarter-circle. It was created using a circle with a 12-inch diameter. The circle was folded in half and then folded in half again.



1. Label the quarter-circle with any important measurements.
2. What is the perimeter of this quarter-circle?
3. Here is a shape made of a semicircle and a square. What is the perimeter of this shape?

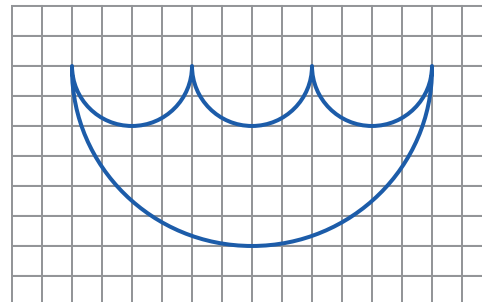


4. A soup can with a diameter of 3 inches has a wrapper that wraps exactly once around its outside. If the wrapper is rolled out, what is its approximate length? Show your thinking.



5. Determine the exact perimeter of this shape made from parts of circles.

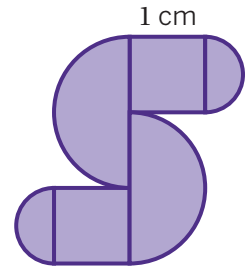
Show or explain your thinking



Practice 3.04


Name: _____ Date: _____ Period: _____

6. The figure shown is composed of two squares, each with side lengths of 1 cm, two larger semicircles, and two smaller semicircles. Select all the expressions that correctly calculate the perimeter of the shape, in centimeters. Show or explain your thinking.

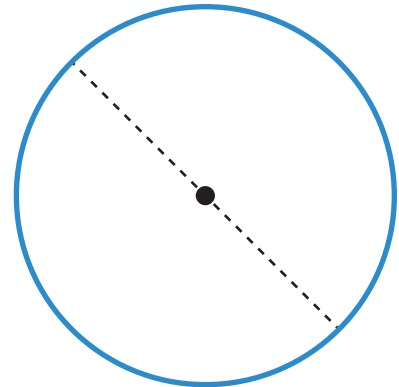


- A. 7
- B. 7π
- C. $4 + \pi + 2\pi$
- D. $4 + 3\pi$
- E. $4 + 7\pi$

Spiral Review

7.  This circle has a diameter measurement of 5 centimeters. Which measurement is closest to the circumference of the circle in centimeters?

- A. 6.25 centimeters
- B. 7.85 centimeters
- C. 15.7 centimeters
- D. 25 centimeters



Problems 8–10: Determine whether each measurement represents the radius, diameter, or circumference, and record it in the appropriate column of the table. Then determine the exact lengths of the other two measurements of the circle.

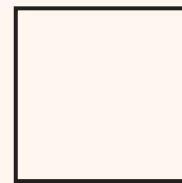
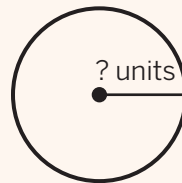
	Measurement	Radius	Diameter	Circumference
8.	The tires of a mining truck are 14 feet tall			
9.	The fence around a circular pool is 76 feet long			
10.	The center to the edge of a small plate measures 60 mm			

Practice Day 1 (continued)

<p>Sheet: _____</p> <p>Answer</p>	<p>Sheet: _____</p> <p>Answer</p>
<p>Sheet: _____</p> <p>Answer</p>	<p>Sheet: _____</p> <p>Answer</p>
<p>Sheet: _____</p> <p>Answer</p>	<p>Sheet: _____</p> <p>Answer</p>

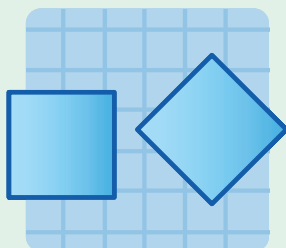
You're invited to explore more.

1. If the circumference of the circle is equal to the perimeter of the square, what is the radius of the circle?
2. The wheel of a stroller has a radius of 6 inches. Write an equation for the relationship between the number of revolutions the wheel makes, n , and the total distance the wheel travels, w .

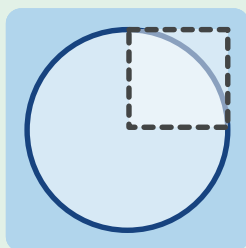


Notes:

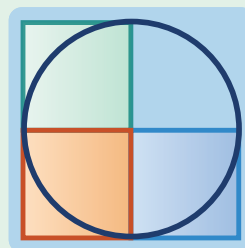
Area of a Circle



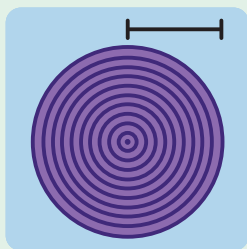
Lesson 5
Area Strategies



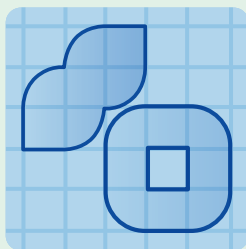
Lesson 6
Radius Squares



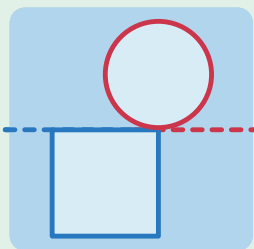
Lesson 7
Circle Area



Lesson 8
Why Pi?



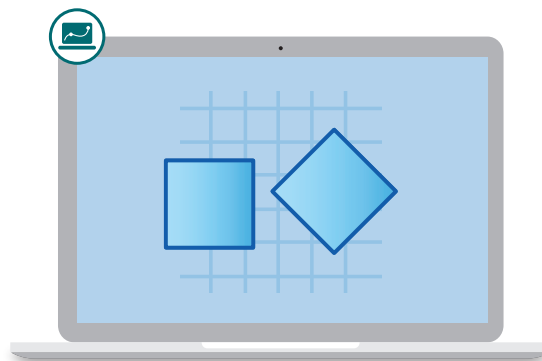
Lesson 9
Area Challenges



Lesson 10
Circle vs. Square

Area Strategies

Let's estimate and calculate areas.

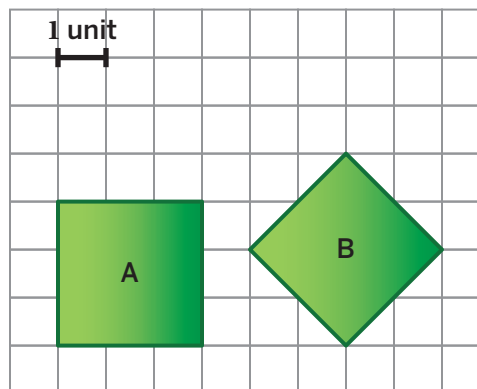


Warm-Up

1 A farmer is comparing his two fields. Which field has a larger area? Circle one.

Field *A* Field *B* They're the same

Explain your thinking.



2 The area of field *A* is 9 square units.

Work with a partner to determine field *B*'s area. Sketch if it helps with your thinking.

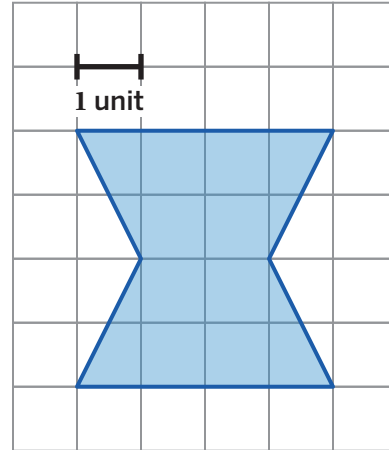
Be prepared to describe your strategy.

Calculating Area

3 What is the area of this shape?

Sketch to help show your thinking.

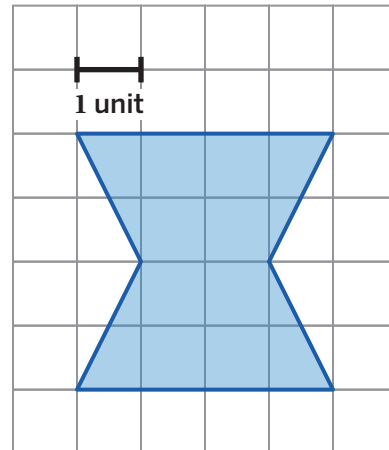
Explain your thinking.



4 There is often more than one way to determine area.

Sketch to show another way of determining the area of this shape.

Describe your strategy.

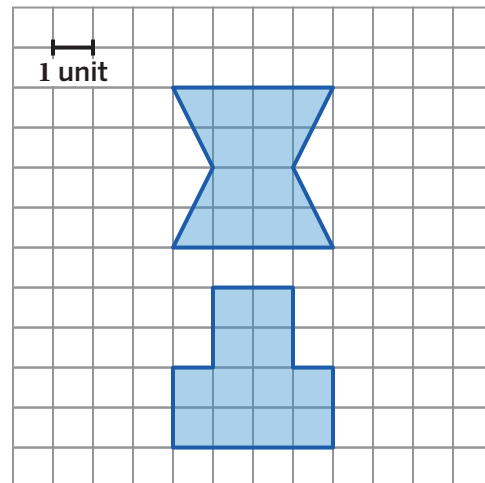


5 Victor says these two shapes have the same area.

Is Victor correct? Circle one.

Yes No

Explain your thinking.



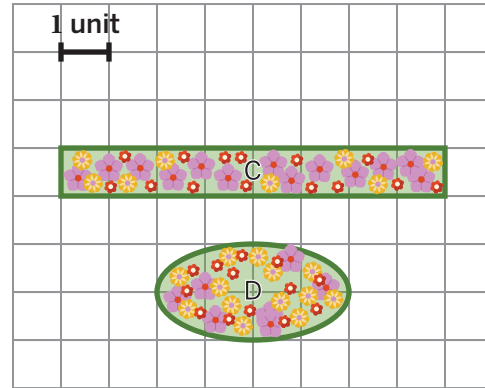
Estimating Area

6 Melissa is creating a new garden for spring. She is debating between these two garden designs and would like to choose the design with the larger area.

Which garden design should she choose?

Design *C* Design *D* They're the same

Explain your thinking.



7 What is the approximate area of shape *D*?

Sketch if it helps with your thinking.

You're invited to explore more.

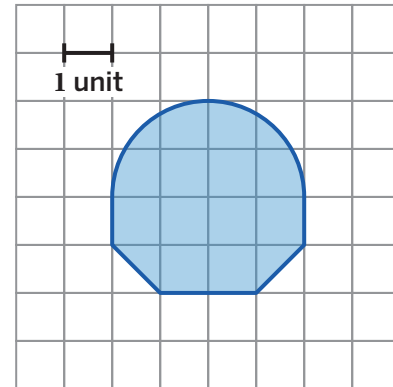
8 Draw an oval with an area that is approximately equal to each target area.

1 cm Target: 22 sq. cm

1 cm Target: 47 sq. cm

9 Synthesis

What is a helpful strategy for estimating the area of this shape? Show or explain your thinking.

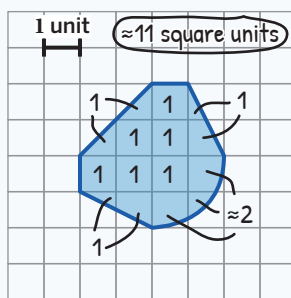


12 Summary 3.05

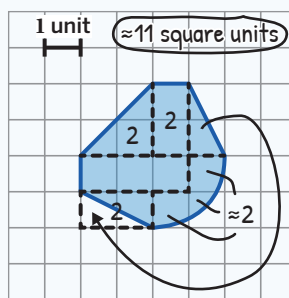
There are many ways you can find the *area* of a complex shape on a grid.

Here are some strategies you can use:

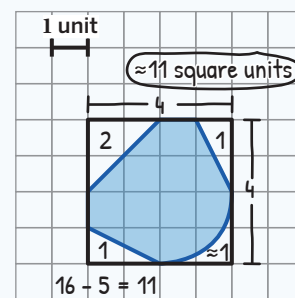
Counting whole and partial squares



Decomposing and rearranging




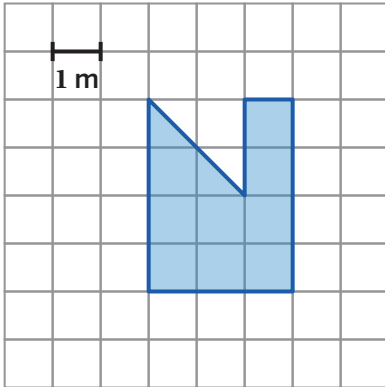
Surrounding and subtracting




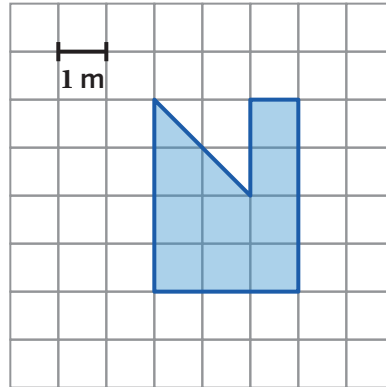
Practice 3.05

Name: _____ Date: _____ Period: _____

1.  What is the area of this shape? Show your thinking.

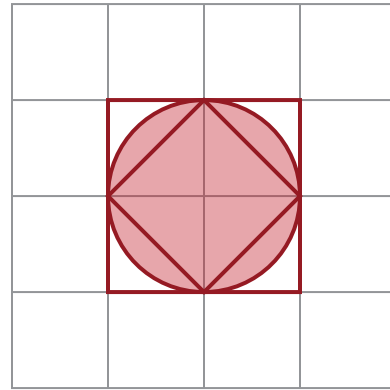


2.  Show or describe another way to determine the area of this shape.

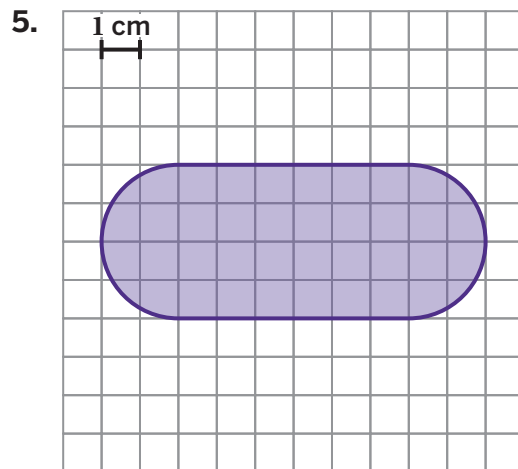
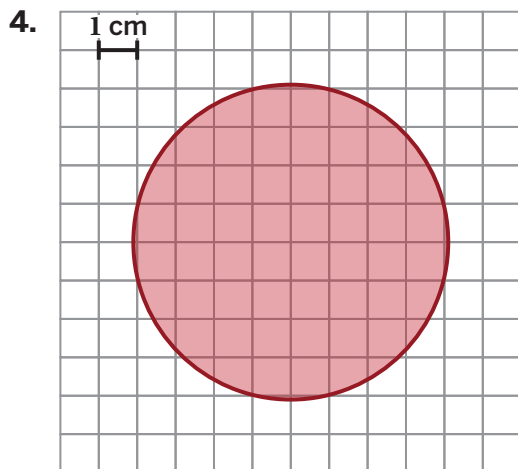


3. Here is a diagram of two squares and a circle.

Explain why the area of the circle is more than 2 square units but less than 4 square units.



Problems 4–5: Estimate the area of each shape.

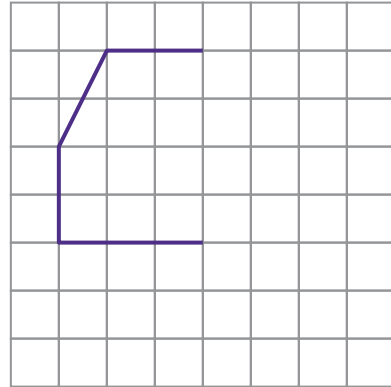


Practice 3.05

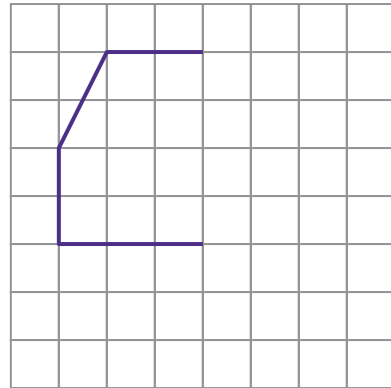
Name: _____ Date: _____ Period: _____

Problems 6–7: Basheera started drawing a polygon.

6. Complete Basheera's drawing so that the polygon has an area of 18 square units.



7. Complete Basheera's drawing in a different way so that the polygon has an area of 18 square units.



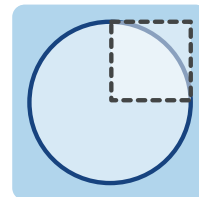
Spiral Review

8. Select *all* the expressions that are equivalent to 48.
- A. $3 \cdot 4^2$ B. $6 + 6 \cdot 4$ C. 50% of 96
- D. $(3 \cdot 4)^2$ E. $4(6 + 6)$
9. What is the greatest common factor of 48 and 64?
10. What is the least common multiple of 4 and 6?
11. The diameter of each wheel on a race car is 26 inches. If the tires must be changed after 150,000 rotations, how many miles will the race car travel on 1 set of tires? Show your thinking.
- Note:** 12 inches = 1 foot; 5280 feet = 1 mile



Radius Squares

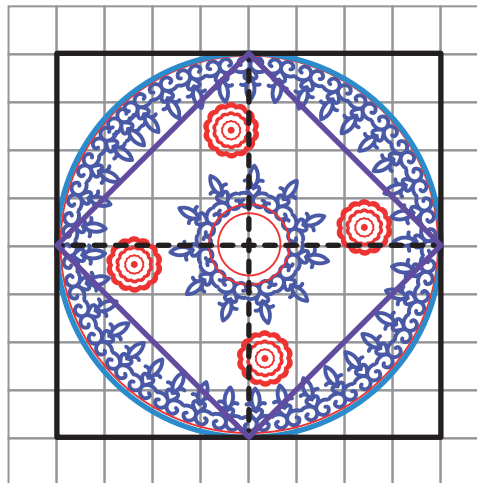
Let's start to explore the relationship between radius squares and circles.



Warm-Up

1. Ayaan used this drawing to estimate the area of a circular coaster. He said the area is between 32 and 64 square units.

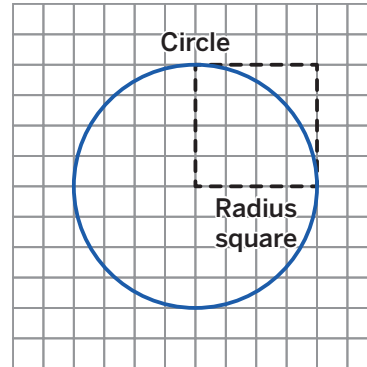
Where do you think the 32 comes from?
What about the 64?



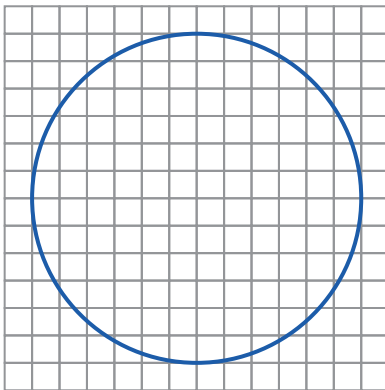
Estimating Circle Area

Every circle has a matching radius square.
Here is the radius square for Ayaan's circle.

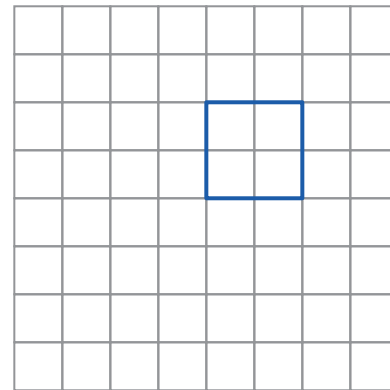
2. Why do you think this is called a radius square?



3. Draw a radius square for this circle.

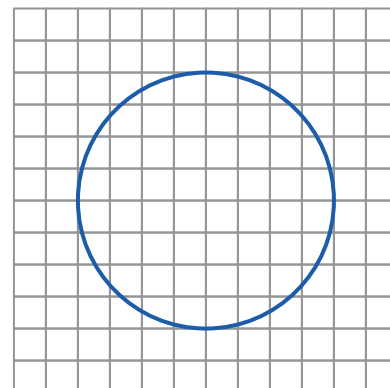


4. Draw a circle for this radius square.



5. Estimate the area of Ayaan's circle.

- a Record your own estimate and strategy.
- b Find a person who got a different estimate or used a different strategy, and record their estimate and strategy.



	Your Name	Your Partner's Name
Estimate		
Strategy		

c **Discuss:** How are the two strategies alike? How are they different?

Squares Cover Circle

Let's find out how many radius squares are needed to cover a circle.

You will get four circles and four sets of radius squares. For each circle, cut up the radius squares and rearrange the pieces to cover the circle.

6. Use your work to complete this table.

	Radius of the Circle (units)	Number of Radius Squares Needed to Cover the Circle
Circle A		
Circle B		
Circle C		
Circle D		

7. In general, how many radius squares do you think can cover a circle?

8. Here is a circle.

a Ayaan says the area of the radius square is 6^2 .

Do you agree? Circle one.

Agree

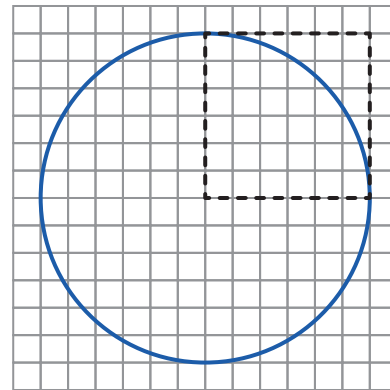
Disagree


I'm not sure

Explain your thinking.


b Estimate the area of the circle.

Explain your thinking.

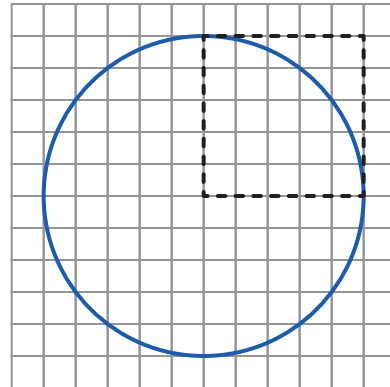


9.  **Discuss:** Why might it be helpful to know about how many radius squares are needed to cover a circle?

Synthesis

10.  **Discuss:** How can you use radius squares to estimate the area of a circle?

Use the example if it helps with your thinking.



Summary 3.06

You can use radius squares to estimate the area of a circle.

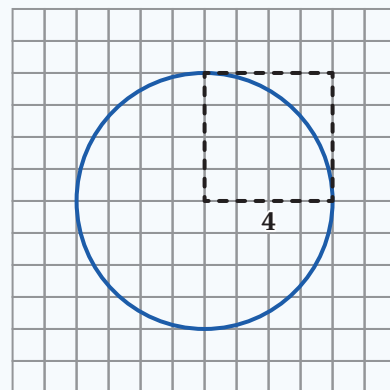
A radius square is a square whose side length is the same as the radius of the circle.

When you break apart and rearrange the radius squares, it takes a little more than 3 radius squares to cover a circle. This means that the area of the circle is a little more than three times the area of the radius square.

In the example, the radius is 4 units.

The area of the radius square is $4 \cdot 4$ or 4^2 , which is equal to 16 square units.

Since it takes a little more than 3 radius squares to cover a circle, the area of the circle is a little more than $3 \cdot 16$, or a little more than 48 square units.

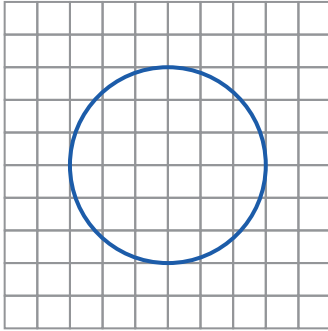


Practice

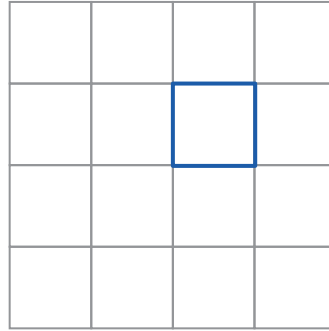
3.06

Name: _____ Date: _____ Period: _____

1. Draw a radius square for this circle.

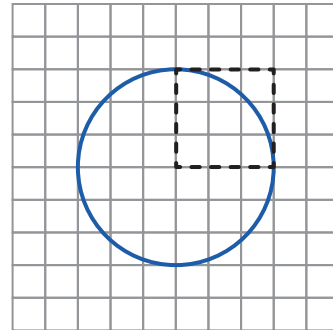


2. Draw a circle for this radius square.

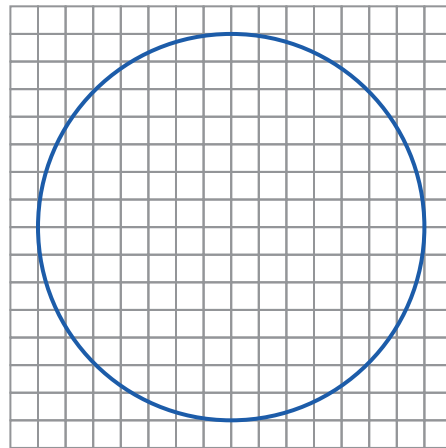


Problems 3–4: Prisha says the area of this circle is a little more than 27 square units.

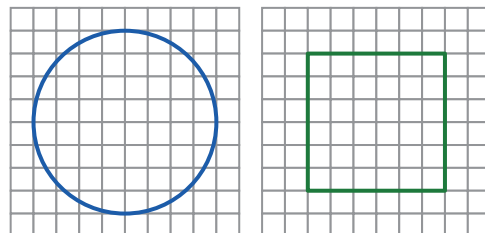
3. What is one way Prisha might have determined her estimate?



4. Estimate the area of this new circle. Show or explain your thinking.



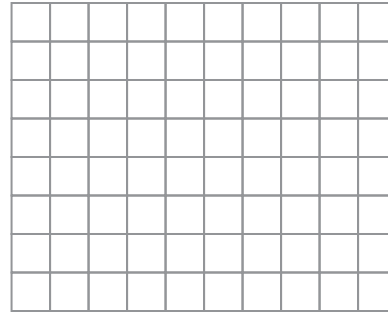
5. Which shape has a larger area: the circle or the square? Show or explain your thinking.



Practice 3.06

Name: _____ Date: _____ Period: _____

6. Use the grid to draw the best freehand circle you can with an area of about 12 square units. Explain your thinking.



Spiral Review

Problems 7–9: When the High Roller observation wheel opened in Las Vegas, NV, in 2014, it became the tallest observation wheel in the world, breaking the record set by the Singapore Flyer in Downtown Core, Singapore, in 2008.

Singapore Flyer




Sergey Peterman/Shutterstock.com

High Roller



Koby Dagan/Shutterstock.com

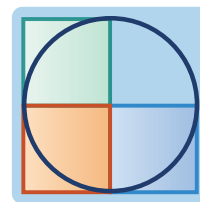
7. The Singapore Flyer has a diameter of 150 meters. If you rode all the way around the observation wheel, how far would you travel?
8. The High Roller has a diameter of 158.5 meters. If you rode all the way around the observation wheel, how far would you travel?
9. If you rode once all the way around each observation wheel, how much farther would you travel on the High Roller than on the Singapore Flyer?
10.  The cost for 16 ounces of strawberries is \$4.00.

Fill in the blank to write an equation that represents the relationship between the number of ounces of strawberries, s , and the cost, c .

$c = \dots\dots\dots s$

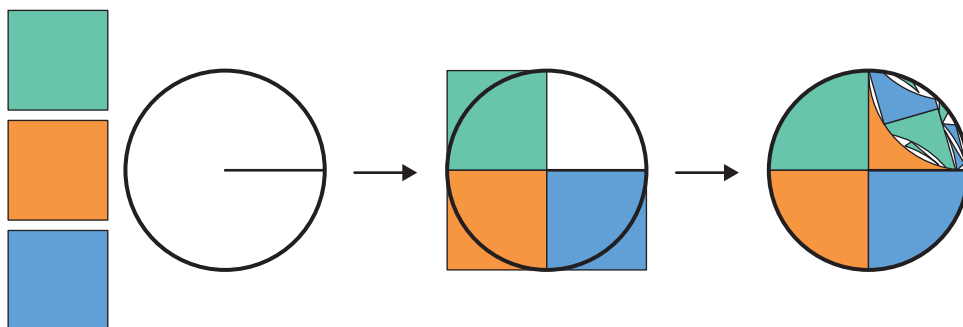
Circle Area


Let's determine a formula for the relationship between the radius of a circle and its area.



Warm-Up

Let's watch an animation.



1.  **Discuss:** What do you notice? What do you wonder?

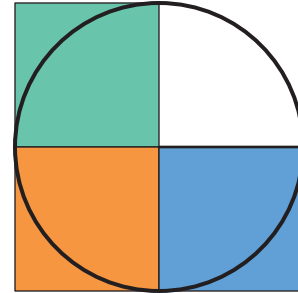
I notice:

I wonder:

Finding a Formula

2. Precious says that you can find the approximate area of a circle by calculating $3 \cdot r^2$.

What do you think each part of her expression means?
Draw on the diagram if it helps with your thinking.



3. Jaleel says that Precious' expression is an underestimate of the area of the circle.

Do you agree with Jaleel? Circle one.

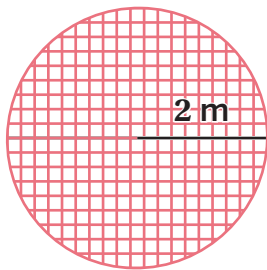
Yes No I'm not sure

Explain your thinking.

4. Here are four circles, their radius or diameter lengths, and their areas.

Write a formula that you think would help to calculate the area of a circle, A , with a radius, r . Use your work from Problems 2 and 3 if it is helpful.

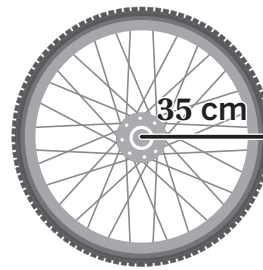
$A =$



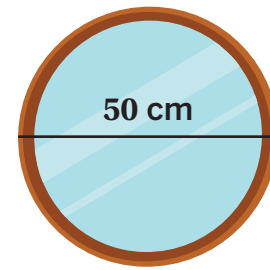
$A \approx 12.57$
square meters



$A \approx 50.27$
square inches



$A \approx 3848.45$
square centimeters



$A \approx 1963.50$
square centimeters

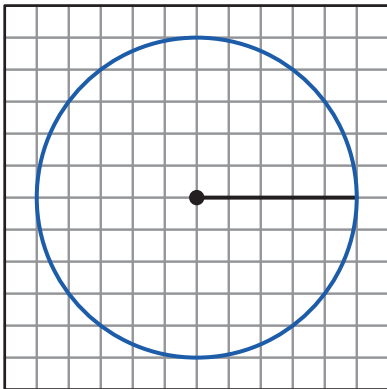
Circle Area

5. Rewrite the formula for the exact area of a circle you developed in Activity 1. Use A for the area of the circle and r for the length of the radius.

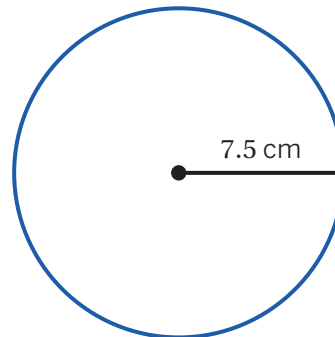
$A =$

6. Calculate the exact area of each circle.

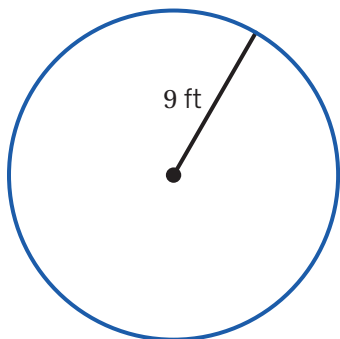
a Area



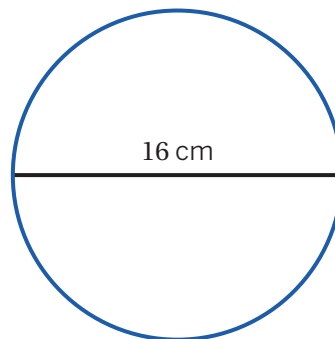
b Area



c Area



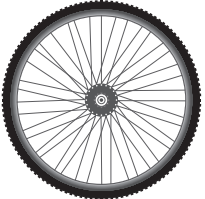
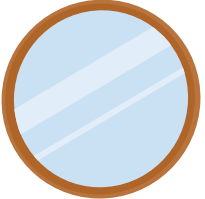


d Area



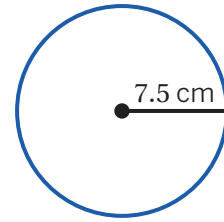
Area vs. Circumference

For each situation, determine whether you need to calculate the circumference or the area, then do the calculation.

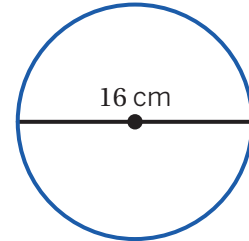
Situation	Area or circumference?	Calculation
<p>7. Kiandra is making a tablecloth for her circular table with a radius of 3 inches. How much fabric is needed for the top of the table?</p>		
<p>8. Pilar's Pizza offers personal pizzas. These pizzas have a diameter of 7 inches. How much room is there to put cheese on the top of the pizza?</p>		
<p>9. A bicycle wheel has a radius of 35 centimeters. What distance will the wheel travel in one revolution?</p>		
<p>10. A circular mirror measures 0.5 meters in diameter. How much wall space will it cover?</p>		

Synthesis

11. Describe a strategy to calculate the area of a circle if you know its *radius*.



How does your strategy change if you know a circle's *diameter*?



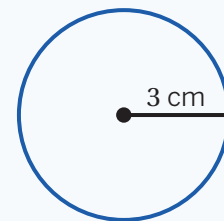
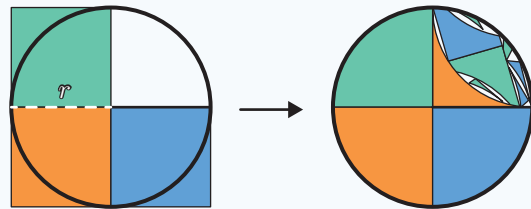
Summary 3.07

You can find the area of a circle if you know the length of its radius, r .

The *approximate* area of a circle is equal to the area of a little more than 3 radius squares. Each radius square has an area of r^2 .

The *exact* area of a circle is equal to the area of π radius squares. You can express this with the formula $A = \pi \cdot r^2$.

For example, to find the area of a circle with a radius of 3 centimeters, you can calculate 3^2 , then multiply the result by π . The area of the circle is 9π square centimeters.



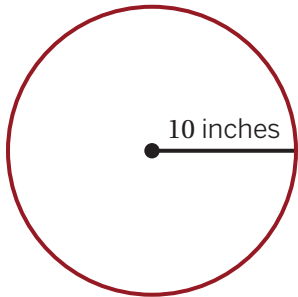
Practice

3.07

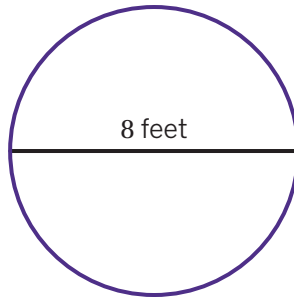
Name: _____ Date: _____ Period: _____

Problems 1–2: Find the exact area of each circle.

1.



2.



Problems 3–4: The table shows the diameters of 4 different coins.

3. To determine how much metal is on one face of a coin, it is more useful to use the area rather than the circumference. Explain why this is the case.

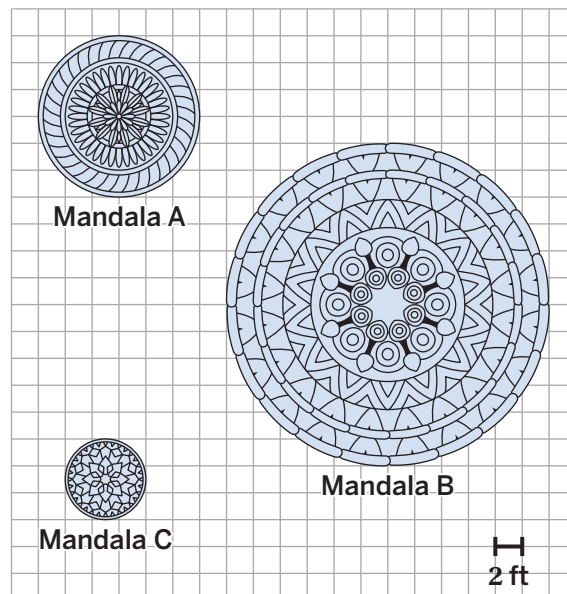
Coin	Diameter (cm)	Area (sq. cm)
Penny	1.9	
Nickel	2.1	
Dime	1.8	
Quarter	2.4	

4. Complete the table by calculating the exact area of each coin.

5. A mandala is a geometric figure that has spiritual relevance in many religions, including Hinduism and Buddhism. The word *mandala* is Sanskrit for *circle*.

Saanvi designed three mandalas. Determine the exact circumference and area for each mandala.


	Circumference (ft)	Area (sq. ft)
A		
B		
C		



Practice

3.07

Name: _____ Date: _____ Period: _____

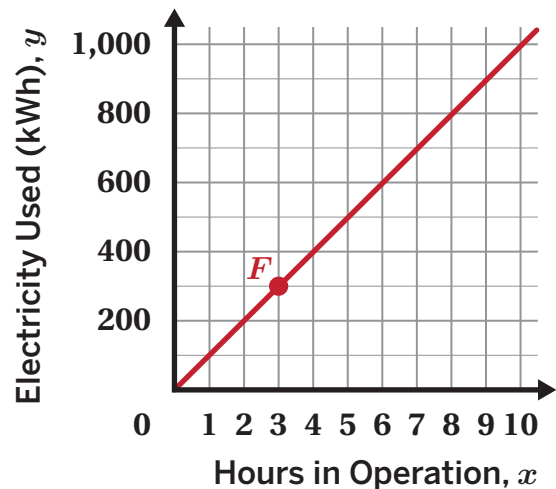
 **Problems 6–7:** The radius of Earth is approximately 6,400 kilometers. The equator is the circle around Earth that divides it into the northern and southern hemispheres.

- Is the circumference of a circle or the area of a circle more useful for finding the length of the equator?
- What is the length of the equator?
- A circle with a 12-inch diameter is folded in half and then folded in half again. What is the exact area of the resulting shape?

Spiral Review

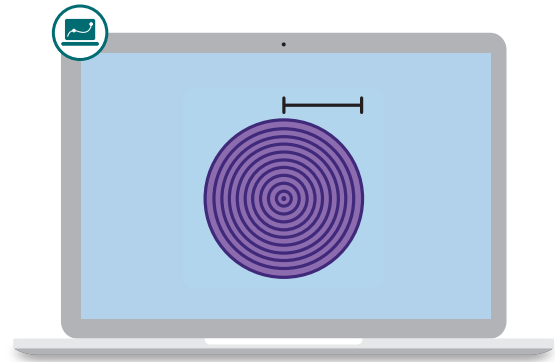
Problems 9–11: This graph shows a proportional relationship between the number of hours a manufacturing factory is in operation and the number of kilowatt-hours (kWh) of electricity used. Use the graph to determine whether each statement is true or false.

- Point F represents the number of kilowatt-hours of electricity used when the factory is in operation for 3 hours.
- The factory uses 6 kilowatt-hours of electricity when it is in operation for 600 hours.
- The factory uses 700 kilowatt-hours of electricity when it is in operation for 7 hours.



Why Pi?

Let's explore why the formula for the area of a circle makes sense.

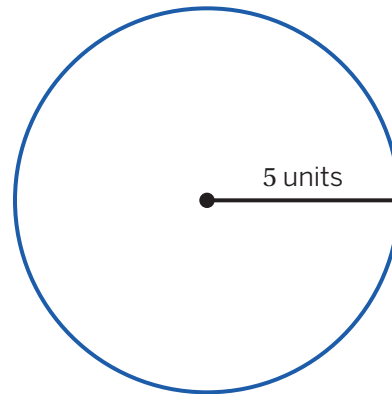


Warm-Up

1 This circle has a radius of 5 units.

Four students tried to calculate the area.

Order their answers starting with the farthest from the exact area to the closest to the exact area.



$25\pi^2$ square units	25π square units	10π square units	78.5 square units
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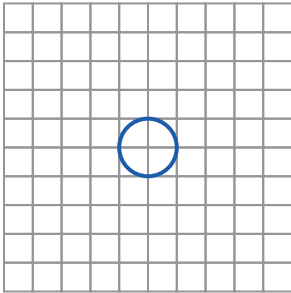
Farthest from exact area

Closest to exact area

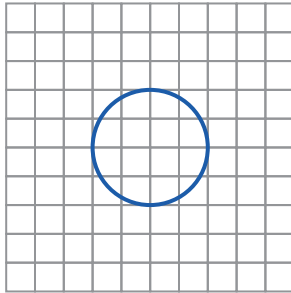
Proportional or Not?

2 Here are some circles.

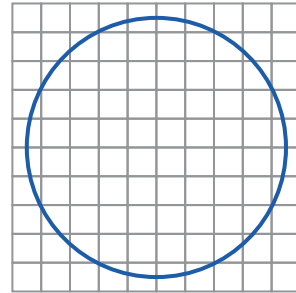
Circle *A*



Circle *B*



Circle *C*



Record the area of each circle in the table.

Circle	Radius (units)	Area (sq. units)
<i>A</i>	1	
<i>B</i>	2	
<i>C</i>	4.5	

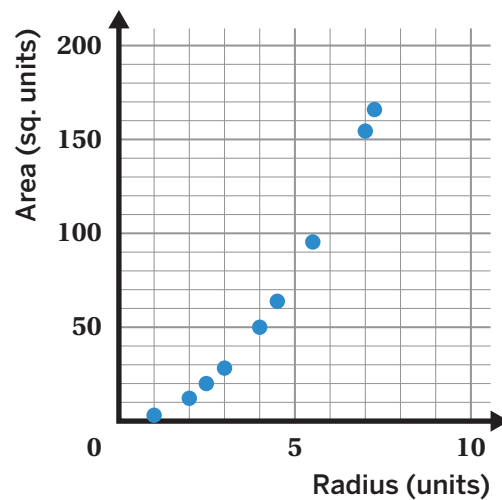
3 This graph shows the radius and area of several circles.

Is there a proportional relationship between the radius and the area of a circle?

Circle one.

Yes No I'm not sure

Explain your thinking.



Unrolling a Circle

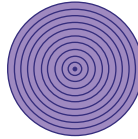
4 Let's watch an animation.



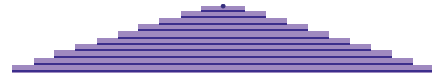
Discuss:

- How are the rolled and unrolled figures alike?
- How are they different?
- What parts of the original circle do you see in the unrolled figure?

Rolled



Unrolled

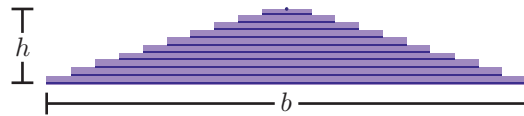
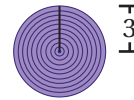


5 As the circle is split into thinner rings, the stack of unrolled rings looks closer to a triangle.

What is the base and the height of the triangle?

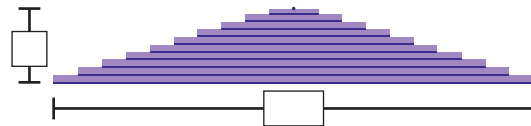
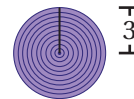
Base b : units

Height h : units



6 Label the diagram with the base and height you just determined.

What is the area of the triangle?

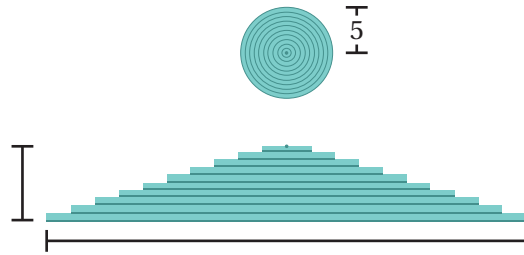


Unrolling a Circle (continued)

- 7** This diagram shows a new rolled and unrolled circle.

Calculate the area of the circle.

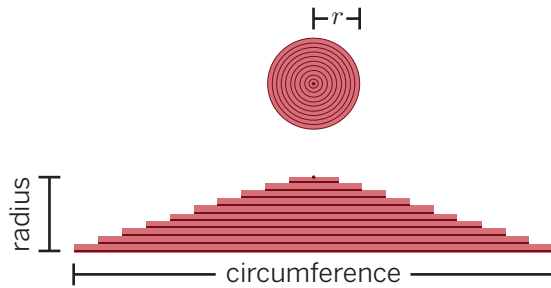
Write measurements on the triangle if it helps with your thinking.



- 8** Haruto says the area of the triangle can be calculated using this formula:

$$A = \frac{1}{2} \cdot \text{circumference} \cdot \text{radius}$$

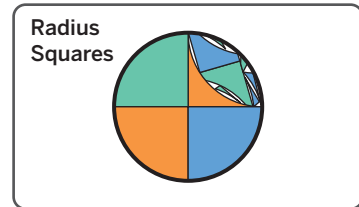
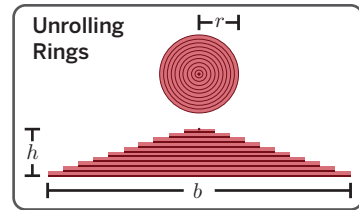
Use this fact to convince a friend that the area of a circle is $A = \pi \cdot r^2$.

**You're invited to explore more.**

- 9** Use the You're Invited to Explore More Sheet to explore a different way to find the area of a circle.

10 Synthesis

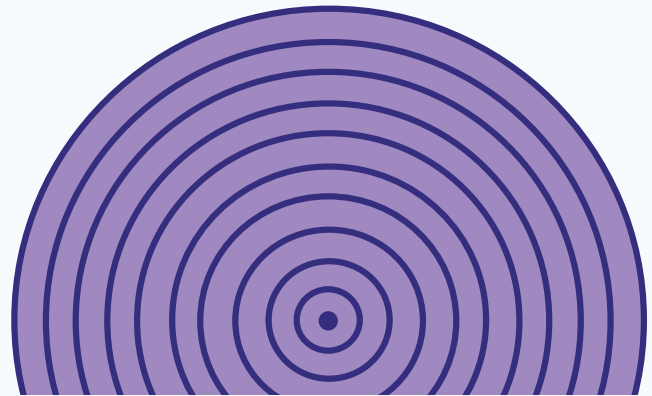
Discuss: Why can the area of a circle be calculated using the formula $A = \pi r^2$? Use the representations if they help with your thinking.



13 Summary 3.08

If you take apart a circle and rearrange it to resemble a triangle, the formula for the area of a circle can be related to the formula for the area of the triangle.

This helps us make sense of each part of the formula for the area of a circle.



- The radius of a circle is r and the circumference is $\pi \cdot d$, so you can substitute those values into the equation for the area of a triangle.
- You can replace d with $2 \cdot r$.
- $\frac{1}{2} \cdot 2 = 1$, which leaves $A = \pi \cdot r^2$.

$$A = \frac{1}{2} \cdot b \cdot h$$

$$A = \frac{1}{2} \cdot (\pi \cdot d) \cdot r$$

$$A = \frac{1}{2} \cdot \pi \cdot (2 \cdot r) \cdot r$$

$$A = \pi \cdot r \cdot r$$

$$A = \pi \cdot r^2$$

Practice 3.08

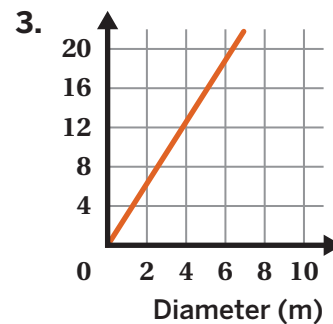
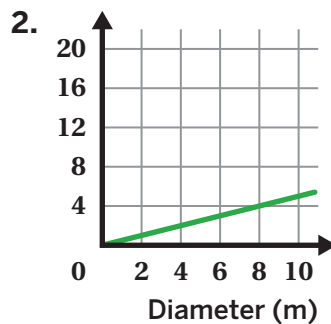
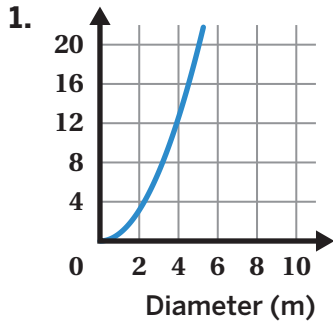
Name: _____ Date: _____ Period: _____

Problems 1–3: Select a phrase from the word bank that describes the relationship within a circle that each graph represents.

Radius vs. diameter

Circumference vs.
diameter

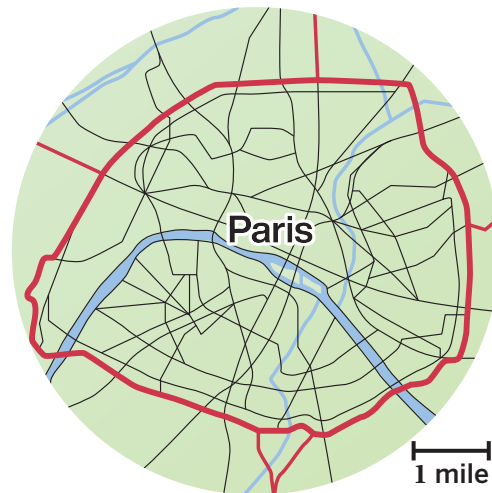
Area vs. diameter



Problems 4–5: The city of Paris, France, is surrounded by an almost circular road called the Périphérique.

4. Use the map and the scale to estimate the length of the road. Explain your thinking.

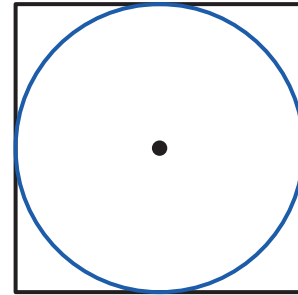
5. Use the map and the scale to estimate the total area enclosed by the road. Explain your thinking.



6. Select *all* the pairs of quantities that are proportional to each other. For the quantities that are proportional, write an equation that relates them.
 - A. The radius and diameter measurements of a circle.
 - B. The radius and circumference measurements of a circle.
 - C. The radius and area measurements of a circle.
 - D. The diameter and circumference measurements of a circle.
 - E. The diameter and area measurements of a circle.

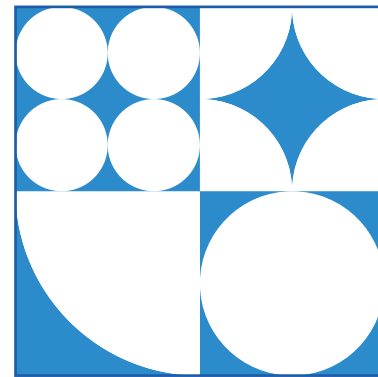
Spiral Review

7. Determine the length of the diameter of the circle.
Explain your thinking.



800 m

8. This 12-centimeter square contains squares, circles, and quarter-circles. Determine the exact area of the shaded region. Show or explain your thinking.



12 cm

 **Problems 9–10:** A circle has a diameter of 13 units.

9. What is the area of the circle to the nearest hundredth of a square unit?
- A. 40.84 B. 132.73 C. 530.93 D. 2,123.72
10. What is the circumference of the circle to the nearest hundredth of a square unit?
- A. 40.84 B. 132.73 C. 530.93 D. 2,123.72

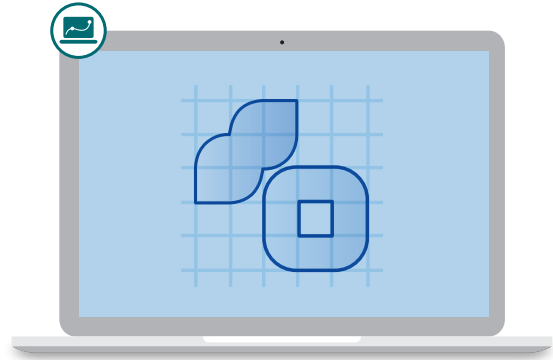
Problems 11–12: Solve each equation. Show your thinking.

11. $x - 2\frac{2}{3} = 6\frac{2}{3}$

12. $\frac{1}{2}x = 12$

Area Challenges

Let's calculate the areas of complex shapes.

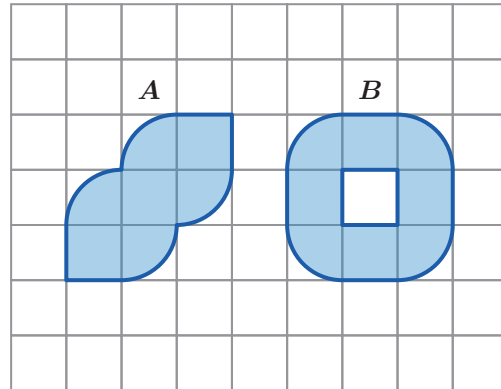


Warm-Up

1 Which shape has the greater area?
Circle one.

Shape *A* Shape *B* They're the same

Explain your thinking.

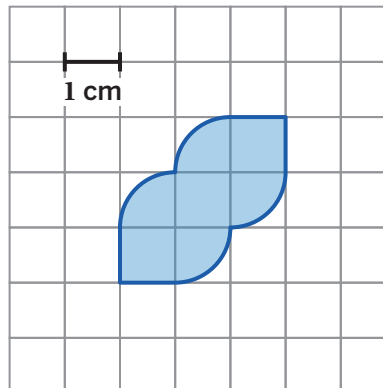


Calculating Areas

2 Here is shape *A* from the Warm-Up.

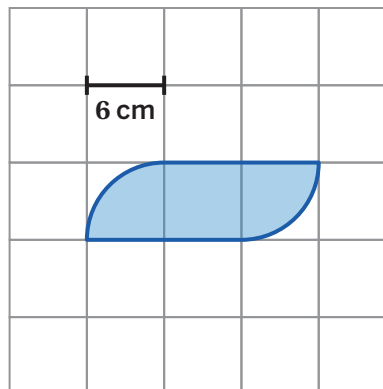
What is its area?

Explain your thinking.



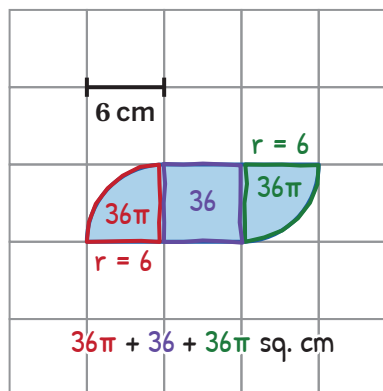
3 Here is a new shape with a new scale.

What is its area?



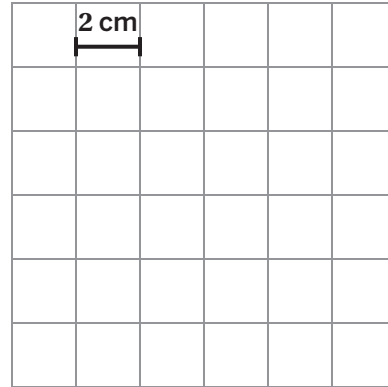
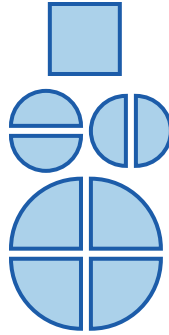
4 Taylor calculated the area of the shape using the expression $36\pi + 36 + 36\pi$, but made a mistake.

Identify the mistake and explain why it is incorrect.



Create a Shape

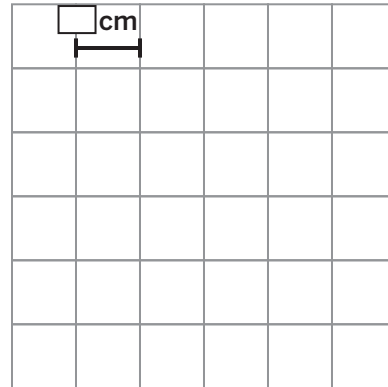
- 5** Create a shape made up of squares and parts of circles that has an area of $8 + 4\pi$ square centimeters.



- 6** Create your own challenge!

- a** Circle the scale you'd like to use, then label the diagram with your choice.
 2 cm 4 cm 6 cm 8 cm
- b** Draw a shape made up of squares and parts of circles.
- c** Determine the exact area of your shape.

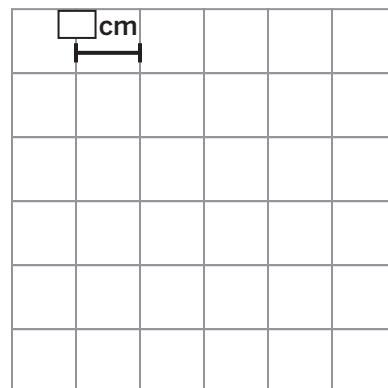
My Challenge



Now it's time to complete your partner's challenge!

- d** Ask your partner what scale they used, then label the diagram with what they chose.
- e** Ask your partner the area of their shape and record it:
 square centimeters
- f** Draw a shape made up of squares and parts of circles that has the same area as your partner's shape.
- g** When you're done, compare your shape with your partner's shape. How are they alike? How are they different?

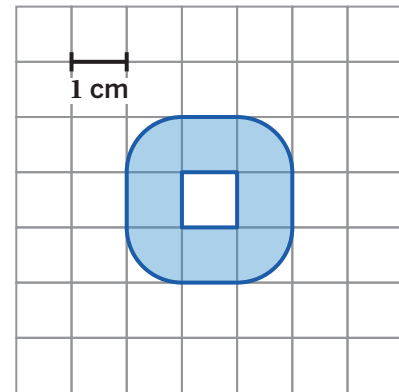
My Partner's Challenge



7 Synthesis

Describe a strategy for determining the area of a shape that's made up of squares and parts of circles.

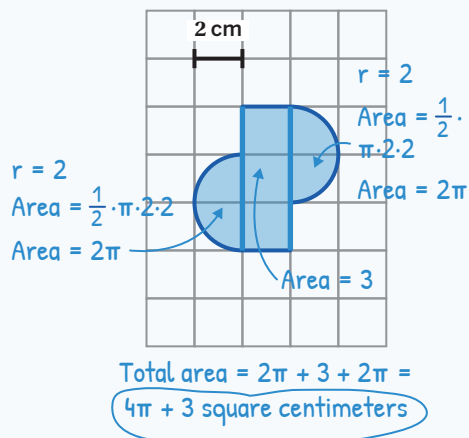
Use the example if it helps to show your thinking.



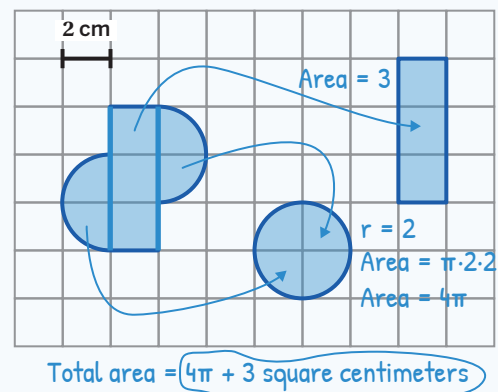
10 Summary 3.09

We can use different strategies and different ways of thinking to calculate the area of complex figures. Here are two ways we can determine the area of this shape:

Strategy 1



Strategy 2

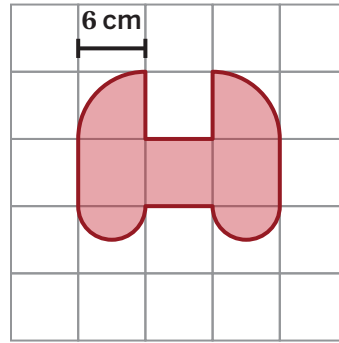


The path to the solution may not be obvious at first, but by breaking the shape down into squares, circles, and parts of circles, we can figure things out!

Practice 3.09

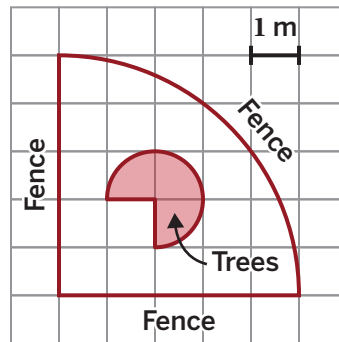
Name: Date: Period:

- Calculate the area of this shape.



Problems 2–4: Here is the architectural plan for a fenced-in garden. The shaded region represents an area for trees.

- What is the area of the tree region?
- The rest of the fenced-in garden is for flowers. How much area will be covered in flowers? Show your thinking.



- How long is the fence around the edge of the garden? (Note: The edge around the tree region is not a part of the fence.)
- Each of these squares has a side length of 12 units. Compare the areas of the shaded regions in these three figures. Which figure has the largest shaded region? Show or explain your thinking.

Figure A

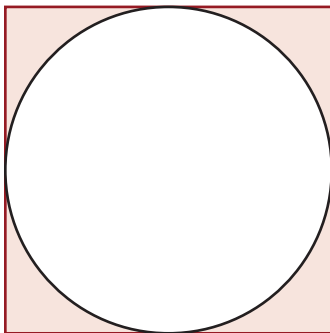


Figure B

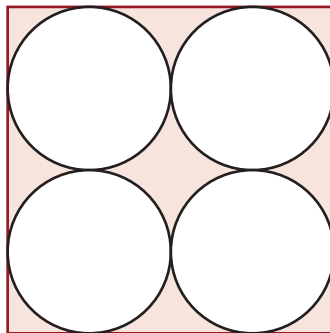
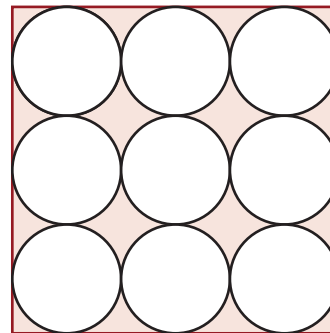


Figure C




Practice 3.09

Name: _____ Date: _____ Period: _____

6. A circle with a 12-inch long diameter is folded in half and then folded in half again. What is the exact area of the new shape? Write your answer in terms of π , and show your thinking.

Spiral Review

7.  A graffiti artist is spray painting a circular mural onto a wall. The mural will have a diameter that is 10 feet long. Each can of spray paint covers about 24 square feet. How many cans of spray paint will the artist need to create the mural? Explain your thinking.

Problems 8–10: Determine each value.

8. What is the length of the radius of a circle with a diameter that measures 10 units?
9. What is the length of the diameter of a circle with a radius that measures 10 units?
10. What is the radius of a circle with a diameter of 2.5 units?
11. Abena used wire fencing to form a border around a circular garden in her backyard. If the radius of the circular garden was 10 yards long, what was the total length of the border, rounded to the nearest tenth of a yard?

A. 31.4

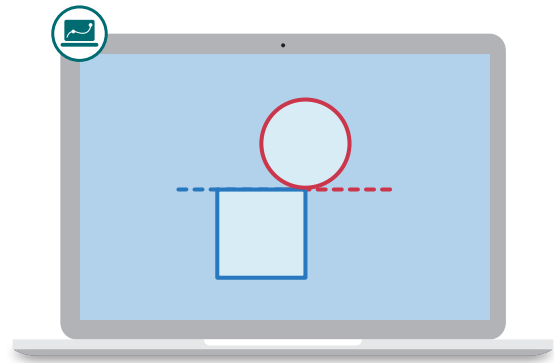
B. 62.8

C. 314.2

D. 628.3

Circle vs. Square

Let's solve challenges with circles and squares.



Warm-Up

1 Let's look at an animation.

What do you notice? What do you wonder?

I notice:

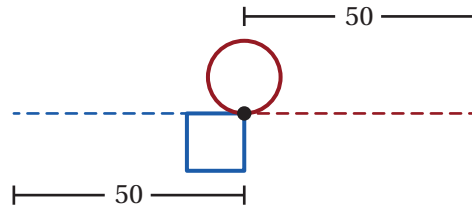
I wonder:

Which is Greater?

- 2** A farmer has 100 yards of fencing. He plans to use some of the fencing to create a circular pen for his horses, and some of the fencing to create a square pen for his pigs.

If he uses the same amount of fencing for each pen, which pen do you think has the greater area?

Square pen Circle pen They're the same



- 3** What information would you need to determine which shape has the larger area?

- 4 a** Calculate the area of each shape.

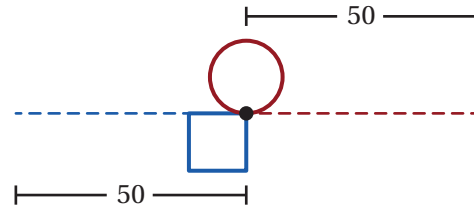
Square Area (square yards)	Circle Area (square yards)


- b** Describe your strategy for determining the area of the square and circle.

Greatest Total Area

5 When the farmer uses half of the 100 yards of fencing for the circle and half for the square, the total fenced area is about 355.19 square yards.

Let's explore if the total area stays the same if the 100 yards of fencing is split up in different ways.



- a** You and your partner will use a card set for this activity. Pick one of the cards.
- b** Calculate the new total area.
- c**  **Discuss:** How does the new total area compare to the 355.19 square yards from before?

- 6**
- a** Record the areas you calculated in the previous problem.
 - b** Complete the table to determine the greatest total area you can make from the circle and square pens.


Card	Left Segment Length (yards)	Square Area (sq. yards)	Circle Area (sq. yards)	Total Area (sq. yards)
1	100			
2	75			
3	60			
4	40			
5	25			
6	0			

You're invited to explore more.

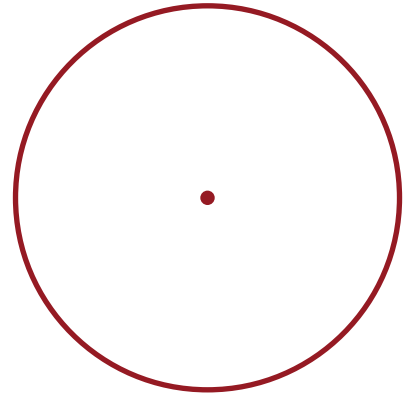
7 How might you distribute the 100 yards so that the circle pen's area and the square pen's area are as close to equal as possible?

Use blank paper to experiment with splitting up the 100 yards in different ways. Share your calculations with a classmate, then work together to see if you can get even closer!

8 Synthesis

 **Discuss:** How can you determine the area of a circle if you only know its circumference?

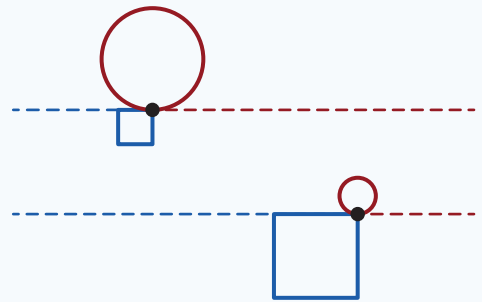
Make a drawing if it helps to show your thinking.



11 Summary 3.10

One of the fun things about math is how you can test predictions using calculations. You can make a guess about something, like when the total area of the square and circle will be greatest, and then determine how close your prediction is to the actual answer.

Sometimes your guess will turn out to be correct. Other times, you get to experience the surprise and wonder of discovering something unexpected!



Practice

3.10

Name: _____ Date: _____ Period: _____

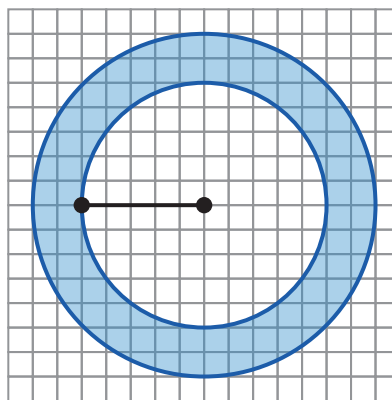
Problems 1–3: A circle's circumference is approximately 76 centimeters. Determine each of these measurements and show your thinking:


1. The length of the diameter of the circle.
2. The length of the radius of the circle.
3. The area of the circle.

4. The metal rim of a magnifying glass measures 10 inches around. What is the area of the surface of the glass inside? Show your thinking.



5. Determine the area of the shaded region. Show or explain your thinking.

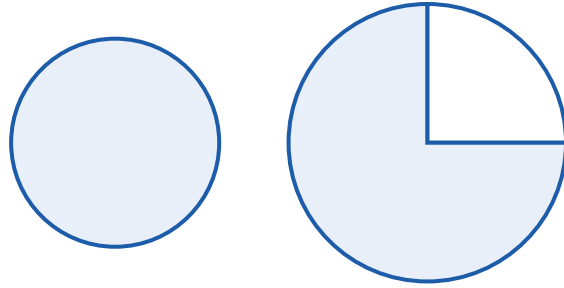


6.  The circumference of a circle is 30π centimeters. What is the area of the circle in terms of π ?
 - A. 15π square centimeters
 - B. 30π square centimeters
 - C. 225π square centimeters
 - D. 900π square centimeters

Practice 3.10

Name: _____ Date: _____ Period: _____

7. The diameter of the smaller circle is 4.5 centimeters long. The diameter of the larger circle is 6 centimeters long. Calculate the difference in the areas of the shaded regions of the two circles. Show your thinking.



The figures may not be drawn to scale.

Spiral Review

Problems 8–10: Complete the table by determining the unknown measurements of each circle.

	Radius (in.)	Diameter (in.)	Circumference (in.)	Area (sq. in.)
8.	12			
9.		12		
10.			12	

11. Jada paints a circular table that has a diameter of 37 inches. What is the exact area of the table, in terms of π ? Show or explain your thinking.

Practice Day 2

Let's practice what you've learned so far in this unit!



You will use problem cards for this Practice Day. Record all of your responses here.

- Sort the problem cards into two groups based on whether you would use the circumference or the area of a circle to answer the question.

Circumference:

Area:

- Sort the cards in each group from smallest measurement to largest based on your best estimate.

Circumference:

Area:

--	--	--	--

Smallest

Greatest

--	--	--	--

Smallest

Greatest

- Select one circumference card and one area card to examine more closely.

Circumference:

Area:

Card: _____

Card: _____

What information do you need?

What information do you need?

Estimates for this information:

Estimates for this information:

Practice Day 2 (continued)

4. Use your estimates to calculate an answer to your *circumference* question.

You are finished with this problem when your paper shows:

- A diagram of a circle labeled with your estimated measurements.
 - Your calculations, organized so that others can follow them.
 - Your answer in a sentence (both exact with units and rounded to the nearest hundredth).
 - A catchy title.
-

5. Use your estimates to calculate an answer to your *area* question.

You are finished with this problem when your paper shows:

- A diagram of a circle labeled with your estimated measurements.
- Your calculations, organized so that others can follow them.
- Your answer in a sentence (both exact with units and rounded to the nearest hundredth).
- A catchy title.

Notes:

Career Connection

Thanks to Emma Haruka Iwao, we now know the **100-trillionth decimal place of π is 0**.

π has been approximated for over 4,000 years. Ancient Babylonians estimated the area of a circle by using 3.125 for π . Ancient Egyptians estimated π as 3.16045. Later, Archimedes used polygons to determine π 's value to be between $3\frac{1}{7}$ and $3\frac{10}{71}$. Chinese mathematician Liu Hui determined the first 5 digits of π using a polygon with 96 sides, a figure that resembles a circle. Chinese mathematician Zhu Chongzhi also used polygons to estimate π as $\frac{355}{113}$.

Today, computers and the internet have significantly advanced calculations of π . In 2019, computer scientist Emma Haruka Iwao set the world record by using computers and the internet to calculate π to over 31.4 trillion digits. The calculations took 121 days to complete. But she didn't stop there! In 2022, she calculated 100 trillion digits of π , which took over 157 days.

Cloud software engineers develop, maintain, and improve software that operates “in the cloud” (over the internet). They might use math to develop computer algorithms.



Meet Emma Haruka Iwao

At the age of 12, Emma Haruka Iwao became fascinated with π and computers' abilities to calculate billions of its digits. In 2019, she used cloud computing to smash the previous record for computing digits of π by *9 trillion digits!* She became the first woman to hold the world record for calculating digits of π , with over 31.4 trillion digits (31,415,926,535,897 digits, to be exact).

Are you interested in cloud computing? What can you do to learn more?

Math in the World

While not a perfect circle, Earth's equator is called a Great Circle. The radius of Earth at its equator is about 6,378 kilometers. What is the approximate circumference around Earth at its equator?

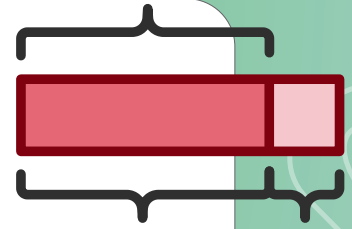


Kristi Blokhin/Shutterstock.com.

Math Mindset

To determine the exact circumference or area of a circle, how might you use π in the calculations?

Unit 4



Proportional Relationships and Percentages

Big Ideas in This Unit

- CC2 Proportional Relationships
- Unit Rates in the World
- CC4 Scale Drawings
- NS Number Line Understanding

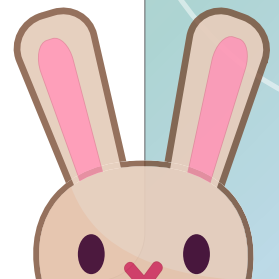
Questions for Investigation

- How are percentages and fractions related to proportional relationships?
- How do proportional relationships and percentages represent change in the real world?
- How does recognizing the unit rate allow us to solve real-world problems?



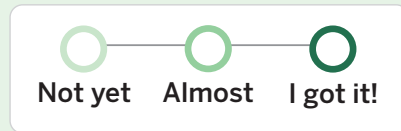
Explore: (Re)Presenting the United States

How can percentages help us to compare data about people in the United States?









Watch Your Knowledge Grow

This is the math you'll explore in this unit. Rate your understanding to see how your knowledge grows!



I can . . .	Before	After
Rewrite an expression in different forms.	<input type="radio"/> — <input type="radio"/> — <input checked="" type="radio"/>	<input type="radio"/> — <input type="radio"/> — <input checked="" type="radio"/>
Use variables to represent quantities in a real-world situation.	<input type="radio"/> — <input type="radio"/> — <input checked="" type="radio"/>	<input type="radio"/> — <input type="radio"/> — <input checked="" type="radio"/>
Construct simple equations to solve problems by reasoning about quantities.	<input type="radio"/> — <input type="radio"/> — <input checked="" type="radio"/>	<input type="radio"/> — <input type="radio"/> — <input checked="" type="radio"/>
Use simple equations to solve problems by reasoning about quantities.	<input type="radio"/> — <input type="radio"/> — <input checked="" type="radio"/>	<input type="radio"/> — <input type="radio"/> — <input checked="" type="radio"/>
Compute unit rates associated with ratios of fractions to solve problems.	<input type="radio"/> — <input type="radio"/> — <input checked="" type="radio"/>	<input type="radio"/> — <input type="radio"/> — <input checked="" type="radio"/>
Recognize proportional relationships between quantities.	<input type="radio"/> — <input type="radio"/> — <input checked="" type="radio"/>	<input type="radio"/> — <input type="radio"/> — <input checked="" type="radio"/>
Represent proportional relationships between quantities using equations.	<input type="radio"/> — <input type="radio"/> — <input checked="" type="radio"/>	<input type="radio"/> — <input type="radio"/> — <input checked="" type="radio"/>
Decide if the relationship between two quantities represented by an equation is proportional.	<input type="radio"/> — <input type="radio"/> — <input checked="" type="radio"/>	<input type="radio"/> — <input type="radio"/> — <input checked="" type="radio"/>
Use proportional relationships to solve multistep ratio and percent problems involving markups and markdowns.	<input type="radio"/> — <input type="radio"/> — <input checked="" type="radio"/>	<input type="radio"/> — <input type="radio"/> — <input checked="" type="radio"/>
Use proportional relationships to solve multistep ratio and percent problems involving gratuities, commissions, and fees.	<input type="radio"/> — <input type="radio"/> — <input checked="" type="radio"/>	<input type="radio"/> — <input type="radio"/> — <input checked="" type="radio"/>

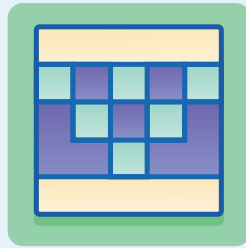
I can . . .	Before	After
Solve problems involving percent increase and decrease, and percent error.		
Convert a rational number to a decimal using long division.		
Recognize that the decimal form of a rational number terminates in 0s or eventually repeats.		

Percentages as Proportional Relationships



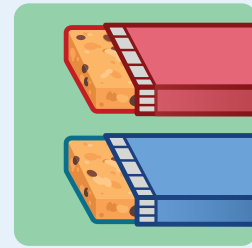
Explore

(Re)Presenting the United States



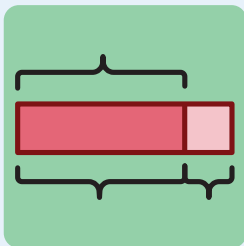
Lesson 1

Mosaics



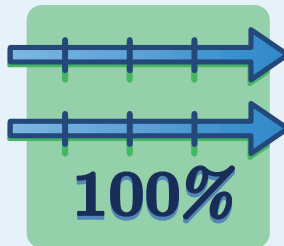
Lesson 2

More and Less



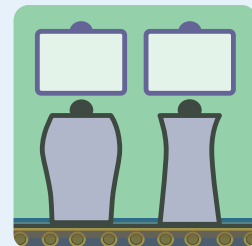
Lesson 3

All the Equations



Lesson 4

100%



Lesson 5

Percent Machines



Lesson 6

Back in My Day



Explore: (Re)Presenting the United States



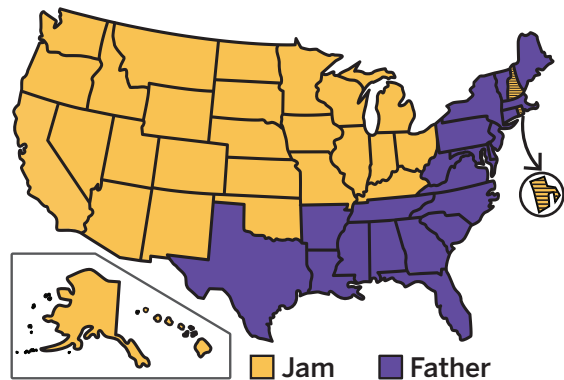
How can percentages help us to compare data about people in the 50 states?

Warm-Up

1. In 2003, the “Harvard Dialect Survey” was given to a group of people from each state to explore the different ways people speak. This map shows how the majority of people in each state pronounced the second vowel in the word pajamas.

What do you notice? What do you wonder?

I notice. . .



I wonder. . .



Analyzing State Data



Data Talk! You will use two cards with information about a state to complete the following problems. As a class, you will combine the data on your cards to represent information about the pronunciation of pajamas by people living in the 50 states.

First State:

- What percentage of the *total population* of people living in the 50 states does your state contain?
- How did you calculate this percent?
- Using the map from the Warm Up, do people in this state tend to pronounce the second vowel in pajamas like “jam” or “father”?

jam (yellow)

father (purple)

Second State:


- What percentage of the *total population* of people living in the 50 states does your state contain?
- How did you calculate this percent?
- Using the map from the Warm Up, do people in this state tend to pronounce the second vowel in pajamas like “jam” or “father”?

jam (yellow)

father (purple)



Analyzing State Data (continued)

8. Do most people in the 50 states pronounce the second vowel in pajamas like “jam” or “father”? How do you know?
9.  **Discuss:**
- Why do you think it is helpful to look at this data on the map?
 - Why do you think it is helpful to look at this data as percentages?



Building Math Habits of Mind

**Discuss:**

- Which of these habits of mind did you strengthen during this activity?
- How did you use the one(s) you selected?

I can slow down and first make sense of a challenging problem before trying to solve it.

— —
Not yet Almost I got it!

I can represent real-world problems using equations and inequalities and interpret their solutions within the context of the problem.

— —
Not yet Almost I got it!

I can justify my thinking and ask questions to help me understand the thinking of others.

— —
Not yet Almost I got it!

I can apply the math that I know to solve real-world problems, make assumptions and revise my thinking as needed.

— —
Not yet Almost I got it!

I can select an appropriate tool to help me solve problems.

— —
Not yet Almost I got it!

I can communicate my thinking and solutions clearly to others.

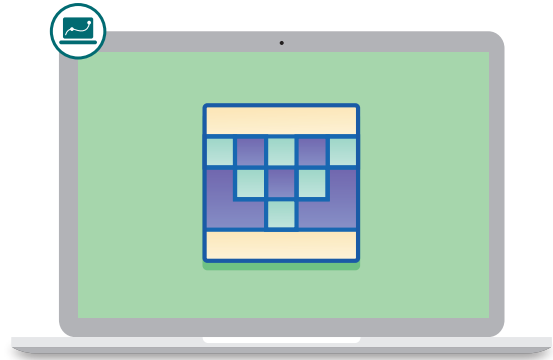
— —
Not yet Almost I got it!

I can look for structure or patterns to help me solve problems.

— —
Not yet Almost I got it!

I can look for repeated calculations and other repeated steps to make generalizations.

— —
Not yet Almost I got it!



Mosaics

Let's revisit relationships between percents and fractions.




Warm-Up

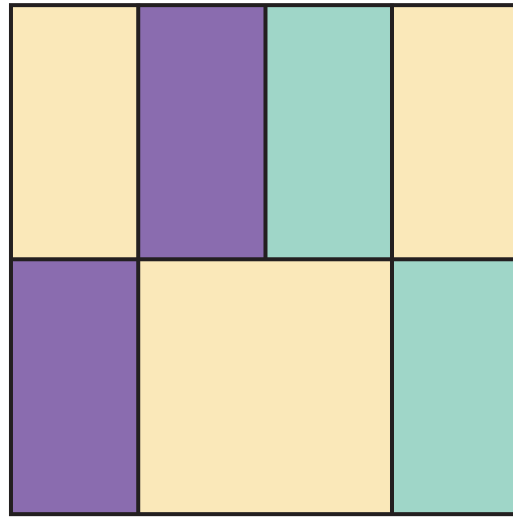
1 What percentage of each figure is shaded? Be prepared to explain your reasoning.

Figure	Percentage
<p>A</p>	
<p>B</p>	
<p>C</p>	

Percent Shaded

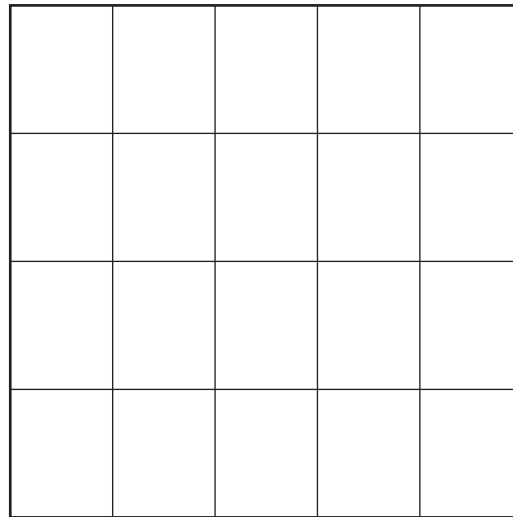
2 What percentage of this square is shaded in each color?

Color	Percentage
Purple 	
Teal 	
Yellow 	



3 Choose three colors and shade this square to match the percentages in the table.

Color	Percentage
	25%
	30%
	45%



Percent Shaded (continued)

4 Tiam and Ramón both determined how to shade 30% of the square teal. Here is their work.

Tiam

$100\% \div 20 = 5\%$

				5%
			5%	5%
			5%	5%
				5%

Ramón

20 boxes total

30% of 20

$0.30 \cdot 20$

6

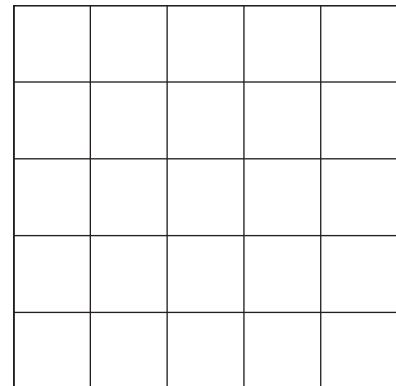
6 teal boxes

Discuss: How did each student determine the number of boxes to shade?

5 Let's create a new mosaic.

a Use Tiam's or Ramón's strategy to shade this square to match the percentages in the table.




Color	Percentage
	52%
	20%
	28%

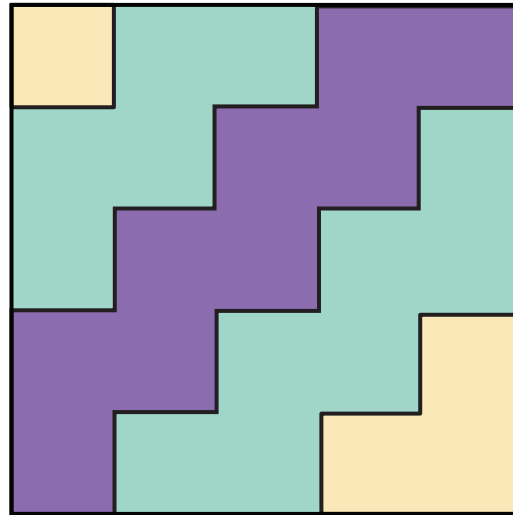


b Describe how you used Tiam's or Ramón's strategy.

My-saic

6 What percentage of this square is shaded each color?

Color	Percentage
Purple 	
Teal 	
Yellow 	



My-saic (continued)

7 You will use the Activity 2 Sheet to create your own mosaic.

a Make it!

- On the activity sheet, choose three colors and make your own design by shading parts of the square.
- In this table, write the percentage of your mosaic that is shaded each color.

My Mosaic

Color	Percentage

b Swap it!

- Share your mosaic with a partner. Look at each other's mosaics but keep your percentages a secret.
- What percentage of your partner's mosaic is shaded each color?


Partner 1's Mosaic

Color	Percentage

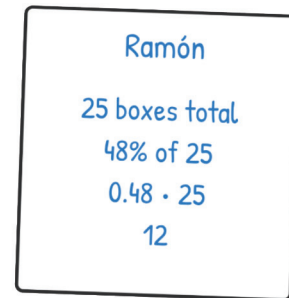
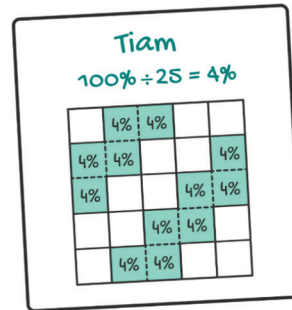
Partner 2's Mosaic

Color	Percentage

8 Synthesis

 **Discuss:** What strategy can you use to calculate 48% of 25?

Use Tiam's and Ramón's work if it helps with your thinking.



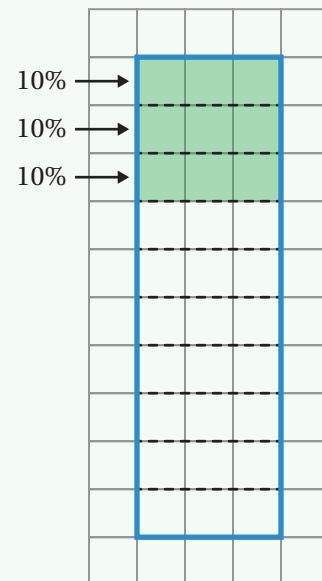
11 Summary 4.01

When determining the percent of a region that's shaded:

- Create a fraction by counting the number of shaded units (part) as the numerator and the total number of units (whole) as the denominator.
- Divide the part by the whole and then change to a percent by multiplying by 100.

When calculating a percent of a number:

- Multiply the percent, written as a decimal or fraction, by the whole, or
- Partition a grid in equal groups to help you determine the part. For example, when determining 30% of 30, break 30 into 10 groups of 3, each representing 10%. For 30%, you would need 3 groups of 3, or 9.

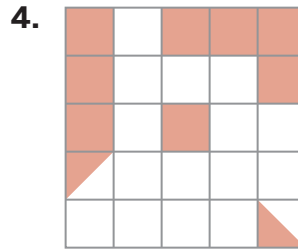
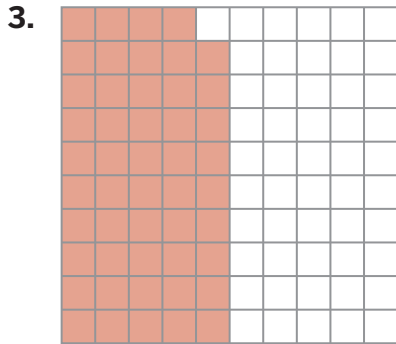


Practice 4.01

Name: _____ Date: _____ Period: _____

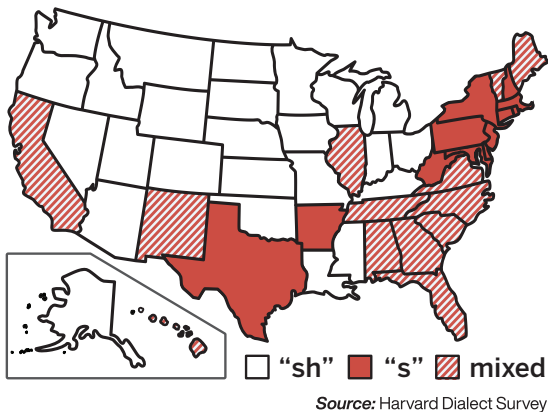
1. Explain how you can mentally determine 50% of a number.
2. 6 is what percent of 10?

Problems 3–4: Determine the percent of each grid that is shaded.

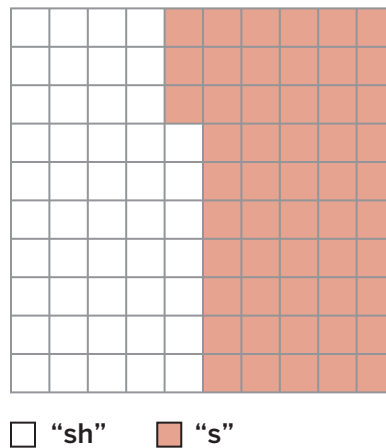


Problems 5–6: These two visuals show how people in the United States pronounce the letter *c* in the word *grocery*. Some pronounce the letter *c* using an “s” sound, while others use a “sh” sound.

Map of how people pronounce the word “grocery”



Percent of people who pronounce the word “grocery” with a “sh” versus an “s”

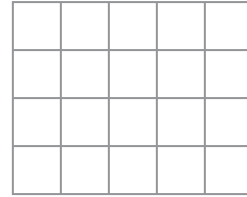


5. What information can you determine from the map?
6. What information can you determine from the percent grid?

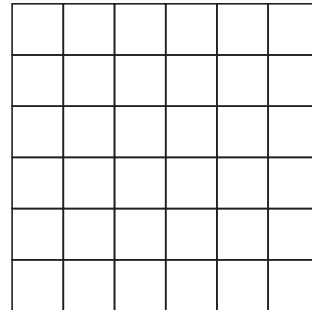
Practice 4.01

Name: _____ Date: _____ Period: _____

7. Shade 40% of the grid. Show or explain your thinking.



8. Shade 25% of the grid. Show or explain your thinking.



Spiral Review

 **Problems 9–10:** A circular mirror has a diameter of 10 inches.

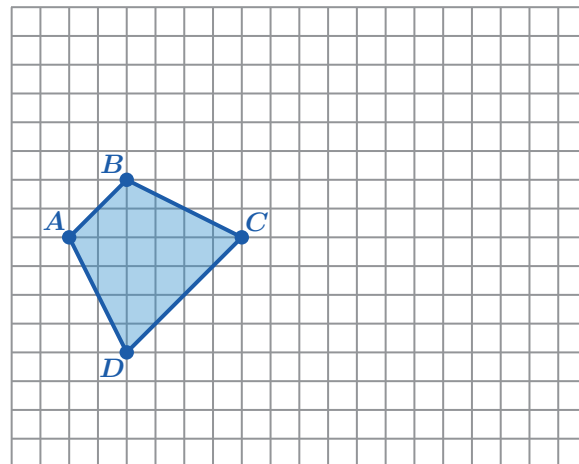
9. What is the area, in square inches, of the mirror?

A. 5π B. 10π C. 25π D. 100π

10. A circular frame that is 2 inches wide surrounds the mirror. What is the combined area, in square inches, of the circular mirror and the frame?

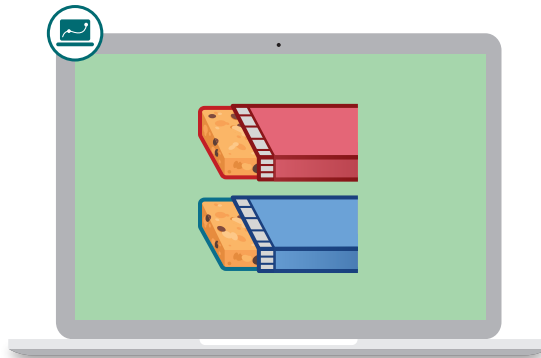
A. 4π B. 16π C. 29π D. 49π

11. Draw a scaled copy of quadrilateral $ABCD$ with a scale factor of $\frac{1}{2}$.



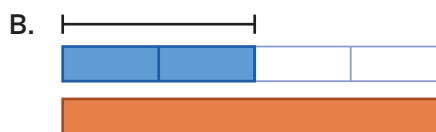
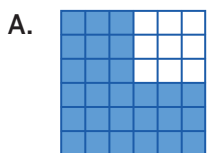
More and Less

Let's visualize what it means to increase or decrease by a percentage.



Warm-Up

1 Which one doesn't belong?



Explain your thinking.

Granola Bars

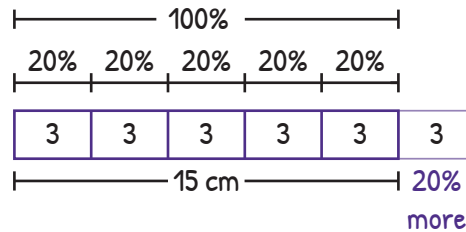
2 DesBest Granola bars are now 20% longer.

If the original bar was 15 centimeters long, how long is the new granola bar?



3 Let's take a look at how DeAndre found the new length of the 15-centimeter granola bar after a 20% increase.

Explain what he may have been thinking.



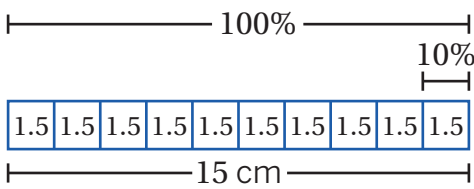
4 In order to make more money, DesWorst Granola bars are now 10% shorter.

If the original bar was 15 centimeters long, how long is the new granola bar?



Granola Bars (continued)

5 Here is how DeAndre and Afia determined the length of the 15-centimeter granola bar after a 10% decrease.

DeAndre	Afia									
 <p style="text-align: center; margin-top: 10px;">$15 - 1.5 = 13.5$</p>	<table style="margin: auto;"> <tr> <td style="border-right: 1px solid black; padding: 5px;">OLD</td> <td style="padding: 5px;">NEW</td> <td style="padding: 5px;">100%</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 5px;">100%</td> <td style="padding: 5px;">90%</td> <td style="padding: 5px;">$\frac{-10\%}{90\%}$</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 5px;">15 cm</td> <td style="padding: 5px;">13.5 cm</td> <td style="padding: 5px;"></td> </tr> </table> <p style="text-align: center; margin-top: 5px;">$\times 0.9$</p>	OLD	NEW	100%	100%	90%	$\frac{-10\%}{90\%}$	15 cm	13.5 cm	
OLD	NEW	100%								
100%	90%	$\frac{-10\%}{90\%}$								
15 cm	13.5 cm									

Discuss: How did each student determine the length?

6 Two different boxes of granola bars are the same price.

One box has 40% more bars. The other has 4 more bars.

Which is the better deal? Circle one.

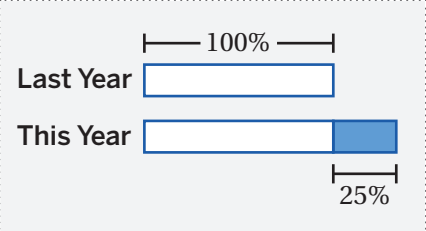
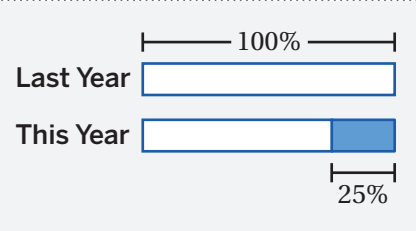
40% more bars 4 more bars Not enough information

Explain your thinking.



Fruitful Percentages

7 Decide which tape diagram could represent each situation by placing a check mark in the appropriate column.

		
This year's apple harvest is $\frac{3}{4}$ of last year's.		
There are $\frac{1}{4}$ more raspberries this year compared to last year.		
This year's plum harvest is 125% of last year's.		
This year's blueberry harvest is 75% of last year's.		
Compared to last year's strawberry harvest, this year's harvest is a 25% increase.		
Compared to last year, this year's peach harvest decreased 25%.		

8 Here's a situation from the previous problem.
If 80 pounds of peaches were harvested last year, how many pounds were harvested this year?

Explain your thinking.

Compared to last year, this year's peach harvest decreased by 25%.

Activity
3

Name: _____ Date: _____ Period: _____

Percent Practice

9 Order these values from *least* to *greatest*.

75% less than 60	25 more than 20	100% of 20	25% of 20	25% more than 20
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Least

Greatest

You're invited to explore more.

10 Evan can use three coupons to buy a \$25 shirt. The store will apply his coupons one at a time.

Order his coupons so that he gets the lowest price.

\$10 off	20% off	5% off
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	First Coupon
	Last Coupon

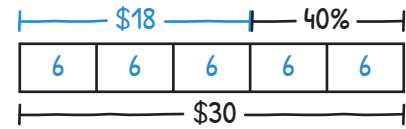
Explain your thinking.

11 Synthesis

Here are two different strategies to determine the new price of a hat after a discount of 40%.

The hat was originally \$30.

How are these strategies alike? How are they different?

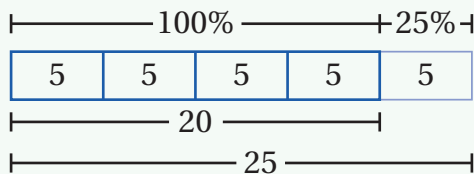


OLD	NEW	
100%	60%	100%
	$\times 0.6$	- 40%
\$30	\$18	60%
	$\times 0.6$	

14 Summary 4.02

Tape diagrams and tables can help us make sense of problems involving percent increase and percent decrease.

Using a Tape Diagram



You can divide a tape diagram into sections to determine the value represented by each section. Then this value can be added to or subtracted from the original amount.

In this example:

4 sections of 25% make 100%, so the 20 centimeters can be divided into 4 equal sections of 5 centimeters.

Add: $20 + 5 = 25$

Using a Table

Original	New
100%	125%
20 cm	25 cm

You can start with 100% and then add or subtract the percent increase or decrease. Then this percent (written as a decimal) can be multiplied by the original amount.

In this example:

Add: $100 + 25 = 125\% = 1.25$

Multiply: $1.25 \cdot 20 = 25$

percent decrease How much a quantity goes down, expressed as a percentage of the starting amount.

percent increase How much a quantity goes up, expressed as a percentage of the starting amount.

Practice 4.02

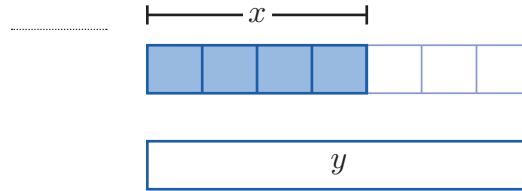
Name: _____ Date: _____ Period: _____

1. Match each situation with a diagram.

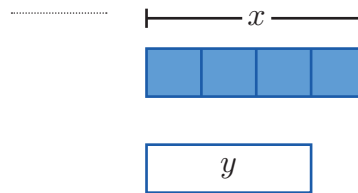
Situation

a. Hoang drinks x ounces of juice. Nekeisha drinks $\frac{1}{4}$ less than that.

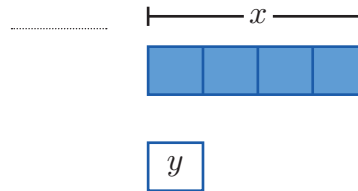
Diagram



b. Hoang runs x miles. Nekeisha runs $\frac{3}{4}$ more than that.




c. Hoang buys x pounds of almonds. Nekeisha buys $\frac{1}{4}$ of that.



Problems 2–3: Draw a diagram to represent each situation.

2. The amount of flour that the bakery used this month increased 40% compared to last month.

3. The amount of milk that the bakery used this month decreased 75% compared to last month.

4.  At the beginning of the month, there were 80 ounces of peanut butter in the pantry. Since then, a family has eaten 30% of it. Which expression represents the ounces of peanut butter left in the pantry?

A. $0.7 \cdot 80$

B. $0.3 \cdot 80$

C. $8 - 0.30$

D. $(1 + 0.3) \cdot 80$

Practice 4.02

Name: _____ Date: _____ Period: _____

Problems 5–7: Fill in the blanks to describe each increase or decrease as a percentage of the original amount.

5. This year, there was 40% more snow than last year.

The amount of snow this year is _____% of the amount last year.

6. This year, there were 25% fewer sunny days than last year.

The number of sunny days this year is _____% of the number of sunny days last year.

7. A restaurant adds a 93% markup to the price of the ingredients to set the menu price.

The menu price is _____% of the price of ingredients.

Spiral Review

Problems 8–10: A store sells strawberries for \$1.38 per pound.

8. Write an equation relating the cost, c , and the pounds of strawberries, p .

9. One strawberry order costs \$8.97. How many pounds did the person order?

10. A customer orders 8 pounds of strawberries. How much will their order cost?

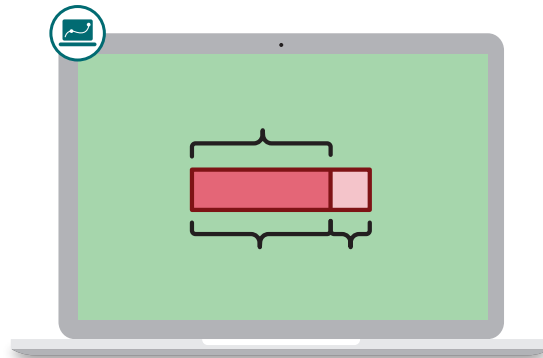
Problems 11–12: Solve each equation.

11. $x \cdot \frac{7}{3} = 1$

12. $1 \div \frac{11}{2} = x$

All the Equations

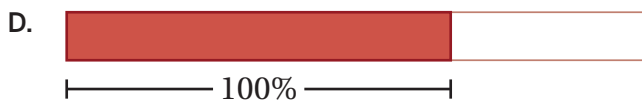
Let's use equations to represent percent increase or decrease.



Warm-Up

1 Here is a rectangle.

Which rectangle is 21% longer than the original rectangle? Circle one.

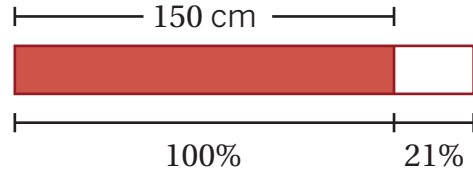
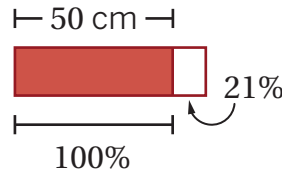
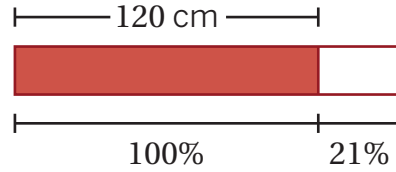


Percent Increase

Each new rectangle is 21% longer than the original.

2 Complete the table with the length of each new rectangle.

Original Rectangle Length (cm)	Length After 21% Increase (cm)
120	
50	
150	



3 We can use equations to represent relationships involving percent increase and decrease.

Write an equation to represent the relationship between the length of an original rectangle, b , and the length of a new rectangle, c .

$c = \dots\dots\dots$

Percent Decrease

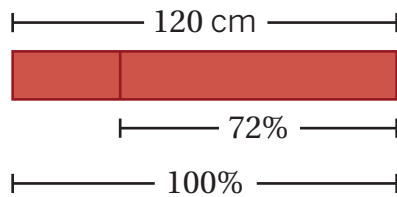
4 Here is a rectangle.

Which shaded rectangle is 72% shorter than the original rectangle? Circle one.



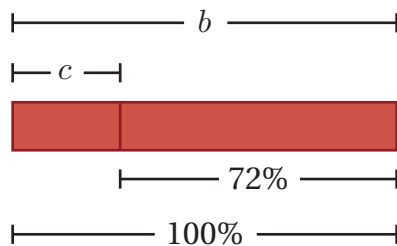
5 The original rectangle is 120 centimeters.

Calculate the length of a rectangle that is 72% shorter.



6 Select the *three* equations that represent the relationship between the length of the original rectangle, b , and the length of the new rectangle, c .

- A. $(100 - 72)b = c$
- B. $c = 0.28b$
- C. $0.72b = c$
- D. $(1 - 0.72)b = c$
- E. $c = b - 0.72$
- F. $c = 1b - 0.72b$



Percents and Equations

- 7** For each equation, put a check under the percent increase or decrease it represents. One equation will be neither.

Let b represent the original value and c represent the final value.

Equation	A Decrease of 88%	An Increase of 12%	Neither
$(1 - 0.88)b = c$			
$(1 + 0.12)b = c$			
$c = 0.12b$			
$1b - 0.88b = c$			
$0.88b = c$			
$c = 1.12b$			
$c = 1b + 0.12b$			

- 8** Manuel paid off 88% of his debt. He originally owed \$1,950. How much does Manuel owe now?

You're invited to explore more.

- 9** Use the You're Invited to Explore More Sheet to answer a question about some magical goo.

10 Synthesis

Here are two equations that could be used to represent a problem about *percent increase* or *percent decrease*.

Circle an equation and write a story about a situation it could represent.

$$1.25 \cdot b = c$$

$$0.80 \cdot b = c$$

13 Summary 4.03

We can use equations to help us make sense of situations involving *percent increase* or *percent decrease*.

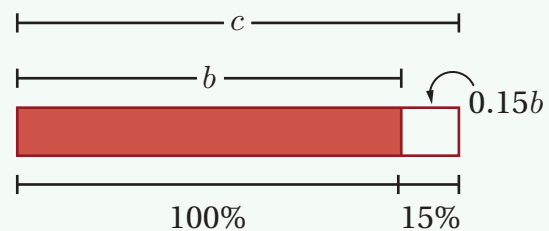
For example, c is 15% more than b .

Three equations can be written to model the relationship between b and c :

$$c = b + 0.15b$$

$$c = (1 + 0.15)b$$

$$c = 1.15b$$



The expressions $b + 0.15b$, $(1 + 0.15)b$ and $1.15b$ are equivalent and show that an increase by 15% is the same as multiplying by 1.15.

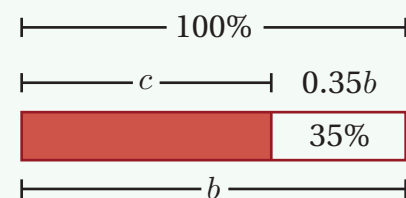
In this example, c is 35% less than b .

Three equations can be written to model the relationship between b and c :

$$c = b - 0.35b$$

$$c = (1 - 0.35)b$$

$$c = 0.65b$$



The expressions $b - 0.35b$, $(1 - 0.35)b$ and $0.65b$ are equivalent and show that a decrease of 35% is the same as multiplying by 0.65.

Practice 4.03

Name: _____ Date: _____ Period: _____

1. Draw a diagram that represents this situation:

The number of people in a town has increased by 50% in the past decade.

2. Write a story that matches the diagram.



3. Draw a diagram that represents this situation:

The amount of paper that the copy shop used this month decreased by 20% compared to what they used last month.

4. A new video game costs \$60 on the day it gets released. The price of a new game generally drops by about 12% within a few weeks. What will the price of this new game be a few weeks after release? Show or explain your thinking.

5. Kai and Amari both wrote an equation to represent an increase of 3%, where x represents the amount before the increase and y represents the new amount.

Kai wrote the equation $y = 1x + 0.03x$. Amari wrote the equation $y = 1.03x$.

Whose equation is correct? Circle one.

Kai

Amari

Both

Neither

Explain your thinking.


Practice 4.03

Name: _____ Date: _____ Period: _____

Problems 6–7: The population of Detroit, MI was approximately 1,027,027 in 1990. There was a 7.5% decrease in population from 1990 to 2000.


6. Write an equation you can use to determine the population in 2000.
7. What was the population of Detroit, MI in 2000?

Problems 8–9: A sneaker store raised its prices 15% compared to last year.

8.  If x is the price before the increase and y is the price after the increase, which equations are correct? Select *all* that apply.

- A. $y = 1.15x$ B. $y = x + 0.15$ C. $y = x + 0.15x$
- D. $y = 15 + x$ E. $y = (1 + 0.15)x$

9. A pair of designer sneakers was \$120 last year. What is the price this year?

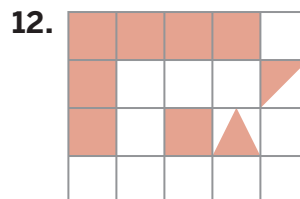
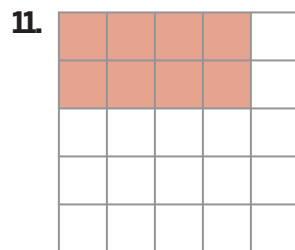
10.  Two stores each advertise a discount on the same type of sweatshirt. At both stores, the original price of the sweatshirt is \$25.
- Store A discounts the price of the sweatshirt by 20%.
 - Store B discounts the price of the sweatshirt by 15%.

How much less is the discounted price of the sweatshirt at Store A than the discounted price of the sweatshirt at Store B?

- A. \$0.75 B. \$1.00 C. \$1.25 D. \$1.75

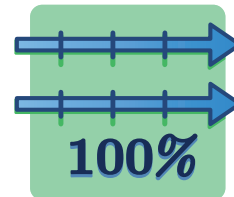
Spiral Review

Problems 11–12: Determine the percentage of each grid that is shaded.



100%

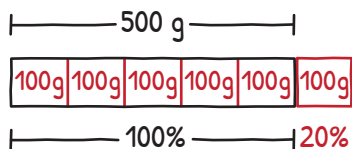
Let's make connections between double number line diagrams and percent problems.



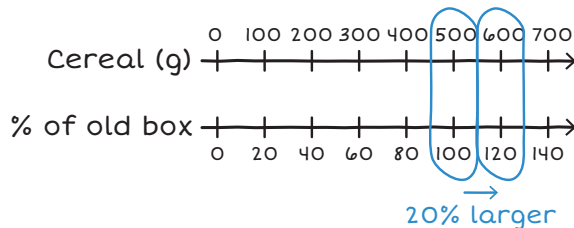
Warm-Up

1. A box of cereal used to weigh 500 grams. Now it's 20% larger. Sadia and Irene each worked to determine the new weight. What do you notice? What do you wonder?

Sadia



Irene



I notice:

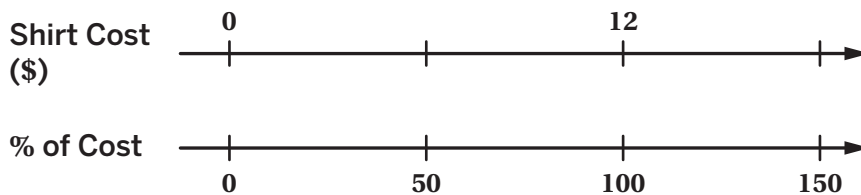
I wonder:

Double Number Lines

For each problem:

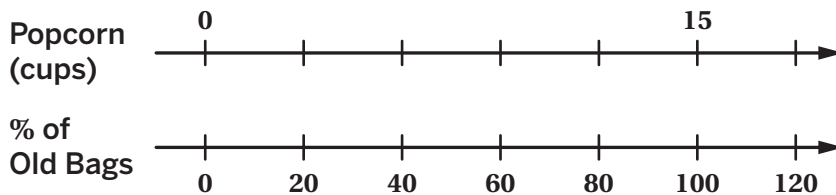
- Complete the double number line diagram to show the percentages that correspond to the original amount and to the new amount.
 - Answer the question.
2. A store sells all of its clothes for 50% more than it costs to make (also called a “markup”).

If a shirt costs \$12 to make, what is the price of the shirt?



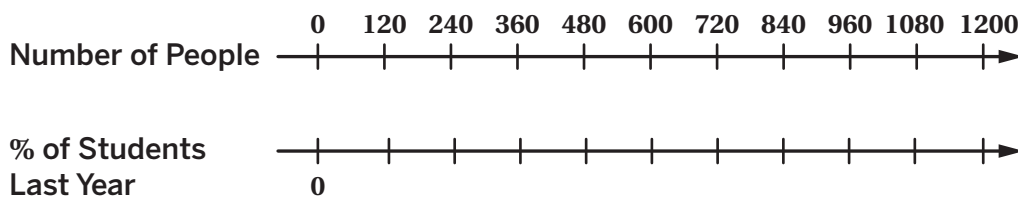
3. At the movie theater, the size of popcorn bags decreased by 20%.

If the original bags held 15 cups of popcorn, how much do the new bags hold?



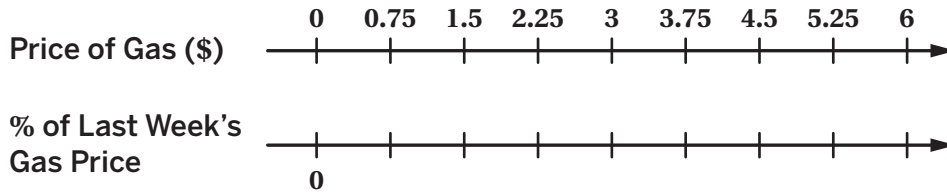
4. A school had 1,200 students last year and only 1,080 students this year.

What was the percent decrease in the number of students?



Double Number Lines (continued)

5. Last week, gas was \$3.75 per gallon. This week, gas was \$4.50 per gallon. What was the percent increase?



6. After a 20% discount (also called a “markdown”), the price of a T-shirt is \$24. What was the price before the discount?



7. A used car dealer has a 25% markup on its cars. If the dealer sells the car for \$6,600 after the markup, what was the cost before the markup?



Green Sea Turtles

Some beaches are protected sanctuaries so that green sea turtles can come to shore to lay eggs without being disturbed.

This year, there are 234 nesting turtles at a sanctuary. This is 10% less than the number of nesting turtles at the same sanctuary last year.

8. Create at least two representations that show how many nesting turtles were at the sanctuary last year.

Double Number Line

Table

Equation

9. How many nesting turtles were at the sanctuary last year?

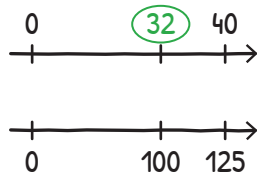
You're invited to explore more.

10. **a** If the population decreases another 10%, how many nesting turtles will be at the sanctuary next year? Explain your thinking.
- b** What do you think contributes to the decrease in the green sea turtles at the sanctuary?

Synthesis

11. A number increases by 25%. The new number is 40. Here are three strategies for finding the original number.

Double Number Line



Table

OLD	NEW
100%	125%
32	40

Arrows indicate: $100\% \xrightarrow{\times 1.25} 125\%$ and $32 \xrightarrow{\div 1.25} 40$.

Equation

$$\frac{40}{1.25} = \frac{1.25 \cdot b}{1.25}$$

$$32 = b$$

Choose one of the strategies.



Discuss: How can you explain this strategy in your own words?

Summary 4.04

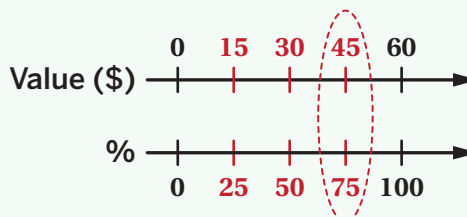
A double number line diagram is a helpful tool for understanding percentage problems.

When using double number lines, it helps to first identify which value aligns with which percentage. It may be helpful to think of the values as a new amount and an original, or old, amount.

Once the known values and percentages are aligned, filling in more values on both number lines can help you solve the problem. For percentage problems, the 0 on each number line should be aligned.

Example:

A furniture store offers 25% off every piece of furniture to make room in the warehouse. If a chair normally sells for \$60, what is its sale price?



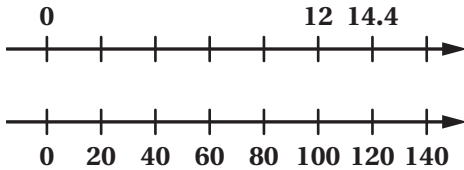
Practice 4.04

Name: _____ Date: _____ Period: _____

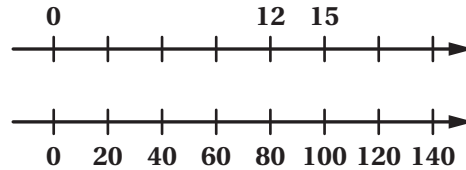
1. Abdullah and Kanna are working on this problem:

A juice box has 20% more juice in its new packaging. The original packaging held 12 fluid ounces. How many fluid ounces of juice does the new packaging hold?

Abdullah's Double Number Line



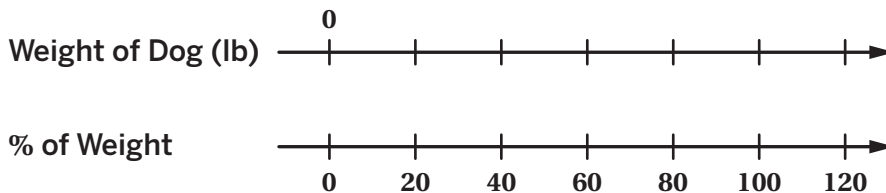
Kanna's Double Number Line



Whose double number line represents the problem? Explain your thinking.

Problems 2–3: Read each scenario and then answer the question. Complete the double number line diagram if it helps with your thinking.

2. A dog weighs 20% more than it did three months ago. It weighs 36 pounds now. How much did the dog weigh three months ago?




3. A bakery used 25% less butter this month than last month. The bakery used 240 kilograms of butter this month. How much did it use last month?



Practice 4.04

Name: _____ Date: _____ Period: _____

Problems 4–5: Next week, the price of oranges at the farmer's market will decrease by 20%.

4.  Select *all* the equations that represent the relationship between the price of oranges this week, x , and the price of oranges next week, y .

A. $y = \frac{1}{5}x$

B. $y = \frac{4}{5}x$

C. $y = x - \frac{1}{5}x$

D. $y = 0.2x$

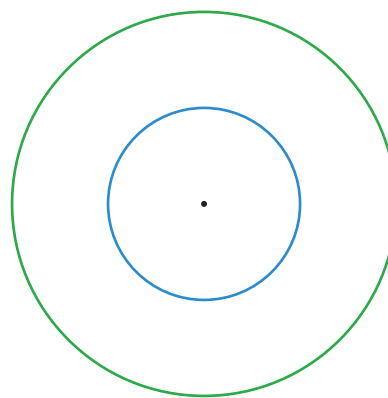
E. $y = 0.8x$

5. The price of oranges this week is \$4.50. What will the price be next week?
6. The number of fish in a lake decreased by 15% between last year and this year. This year, there are 51 fish in the lake. What was the population last year?
7. Without determining the actual percent increase, explain why it is not reasonable to say that a price increase from \$4.00 to \$5.00 for a bag of clementines represents a 125% increase.

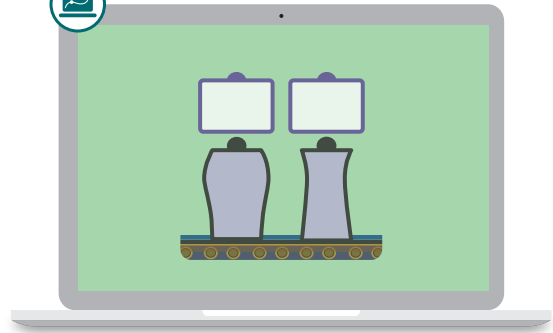
Spiral Review

Problems 8–9: Here is a circle and a scaled copy of the circle with a scale factor of 2.

8. How does the circumference of the scaled copy compare to the circumference of the original circle?
9. How does the area of the scaled copy compare to the area of the original circle?



10. Kiandra invests \$200 in a savings account that earns 5% in simple interest each year. Determine the total amount in the account after 1 year.



Percent Machines

Let's explore problems with multiple percent changes.

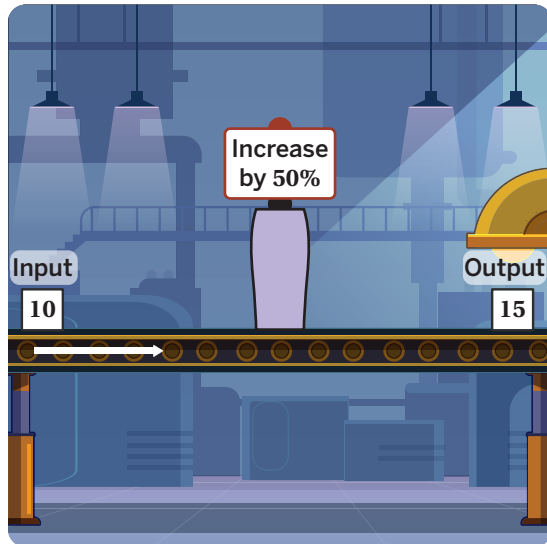
Warm-Up

1 This is a percent machine.

It takes an input and increases it by 50% to make an output.

What number is put into the machine?

What number comes out of the machine?



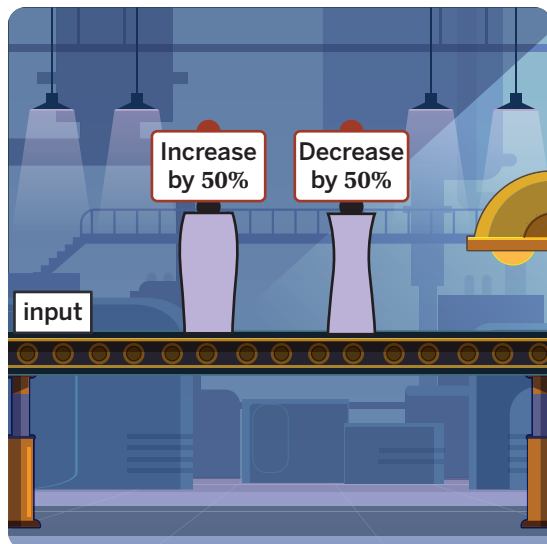
2 A machine was built to follow these rules:

- Increase by 50%
- Decrease by 50%

Do you think the machine's output will be less than, greater than, or equal to a positive input? Circle one.

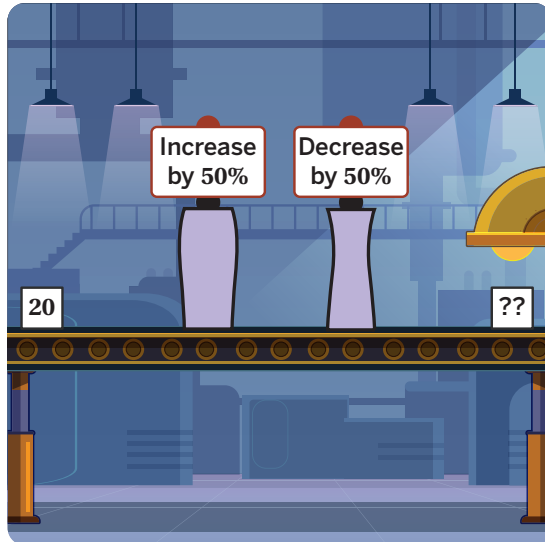
Less than Greater than Equal to

Explain your thinking.



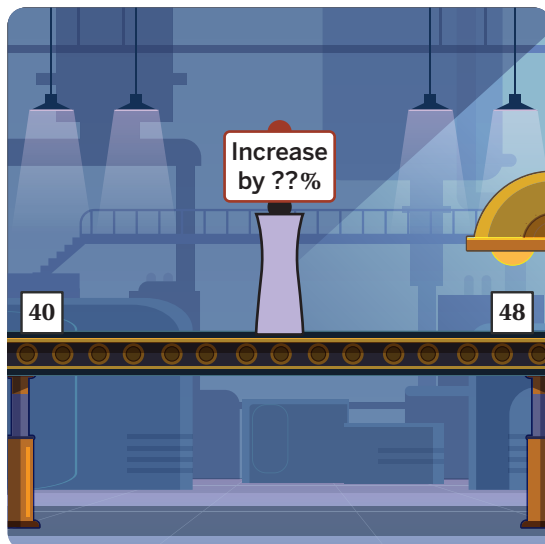
Putting Inputs In

- 3** If 20 is put into this machine, what will come out?



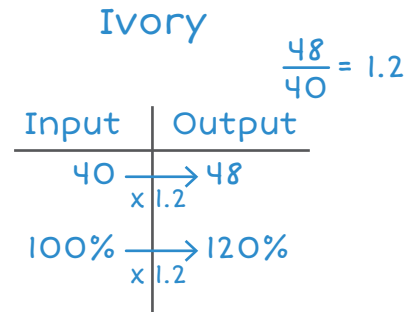
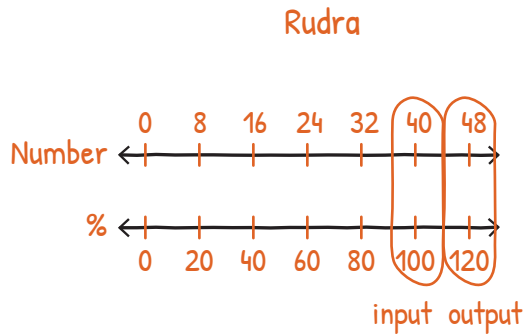
- 4** 40 went into this machine and 48 came out.

What percent increase does this machine use?



Putting Inputs In (continued)

5 Here are two strategies for determining the percent increase from the previous problem.

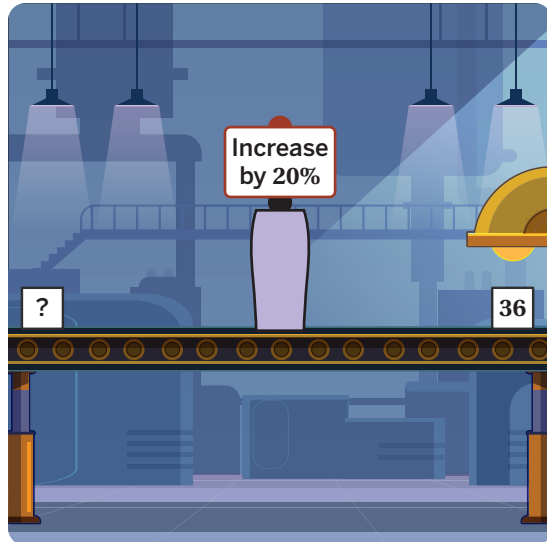


Discuss: How are Rudra's and Ivory's strategies alike? How are they different?

Getting Outputs Out

- 6** A number went into the machine and 36 came out.

What number went in?



- 7** Kai used a table to solve the previous challenge. He made an error.

a Describe something Kai did well.

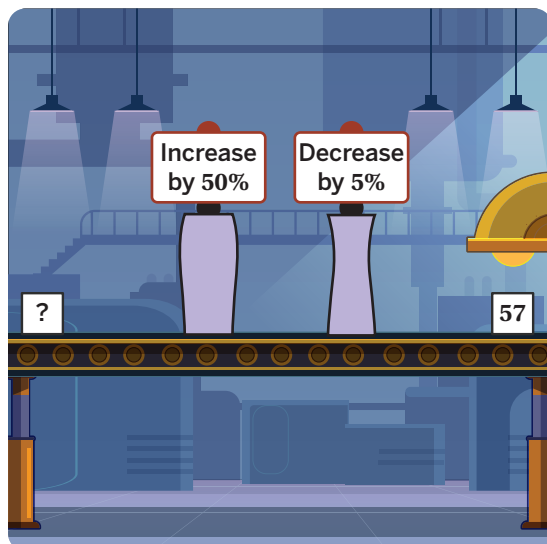
Input	Output
100%	120%
28.8	36

Red arrows indicate the relationships: from 100% to 120% with $\times 1.20$, and from 36 to 28.8 with $\times 0.80$.

b Describe what you would change.

- 8** A number went into this machine and 57 came out.

What number went in?



Activity
3

Name: Date: Period:

My Percent Machine

9 You will use the Activity 3 Sheet to create your own percent machine.

a Make It!

- On the activity sheet, create your own percent machine and choose an output.
- You and your classmates will determine the input that produces the output you chose for your machine.
- In this table, fill in the percent change(s) and the output for your machine. Then determine the input that will give you the output.

My Machine

Input	Step 1	Step 2 (optional)	Output

b Swap It!

- Share your machine with a partner. Look at each other's machines, but keep your own input a secret.
- What is your partner's input?
- As time allows, find more partners.

Partner 1's Machine

Input	Step 1	Step 2 (optional)	Output


Partner 2's Machine

Input	Step 1	Step 2 (optional)	Output

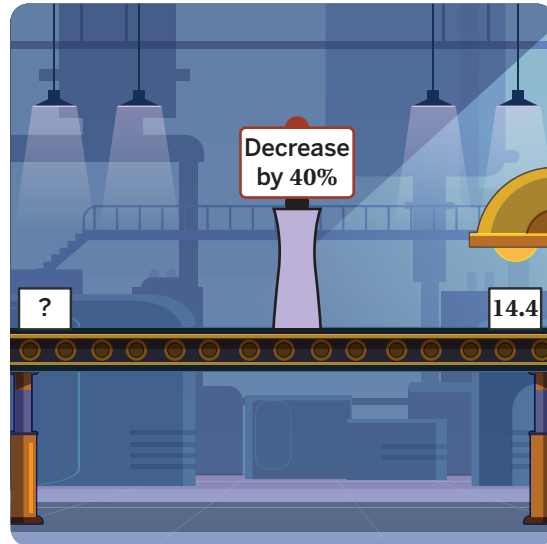
Partner 3's Machine

Input	Step 1	Step 2 (optional)	Output

10 Synthesis

 **Discuss:** What is a strategy you can use to determine the input when you know the percent change and the output?

Use this example if it helps you explain your thinking.



13 Summary 4.05

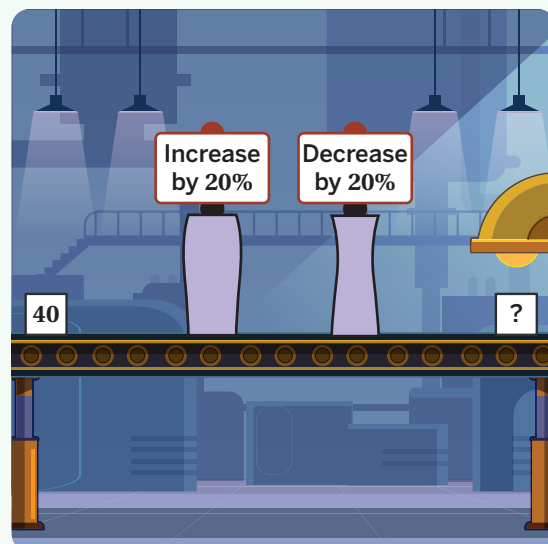
Percent machines take an input value and increase or decrease that value by a percentage to produce an output.

Increasing and then decreasing by the same percentage will not produce the original input value.

For example:

- An input value of 40 is increased by 20%.
 $40 \cdot 1.2 = 48$ or $40 \cdot 0.2 + 40 = 48$
- The new value is decreased by 20%.
 $48 \cdot 0.8 = 38.4$ or $48 - (48 \cdot 0.2) = 38.4$

It may surprise you to discover the final result is *not* the original input value of 40! This is because the input value represents the whole when you determine 20% of 40. But in the second calculation, the input value changes. You are now determining 20% of 48 and then subtracting that value from 48.



Practice 4.05

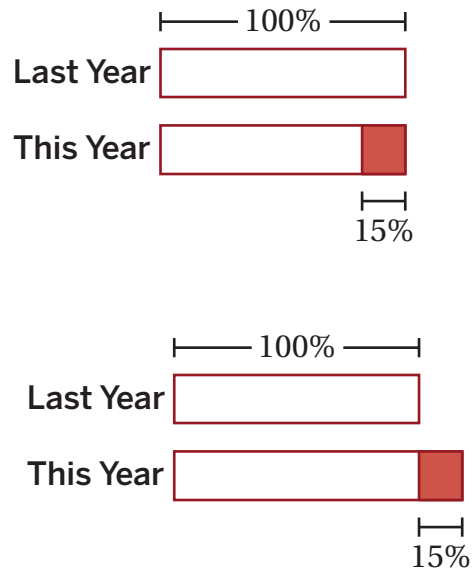
Name: _____ Date: _____ Period: _____

1. Match each situation with a diagram that represents it.

Situation

- a. The amount of apples this year decreased by 15% compared to last year's amount.
- b. The amount of cherries this year increased by 15% compared to last year's amount.
- c. The amount of pears this year is 85% of last year's amount.
- d. The amount of oranges this year is 115% of last year's amount.

Diagram




2. Mateo's aunt bought a trading card many years ago. The value of the card increased by 80%. Its value is now \$270. What was the value of the card when Mateo's aunt bought it?

Problems 3–5: A small town had a population of 4,000 in 1990.

3. By 2000, the population increased to 5,000. What was the percent change from 1990 to 2000? Show or explain your thinking.
4. In 2010, the population decreased back to 4,000. What was the percent change from 2000 to 2010? Show or explain your thinking.
5. Explain why the percent change in Problem 3 is not equal to the percent change in Problem 4.

Practice 4.05

Name: _____ Date: _____ Period: _____

 **Problems 6–7:** Tyler is paid by the hour. Last week he earned \$136 for 8 hours of work.

- This week, he got a 5% raise. What is his new hourly wage?
- Next year he gets another raise and is making \$19 an hour. What percentage was his new raise?

Spiral Review

Problems 8–9: Write an equation to represent each situation.

- A worker is paid the minimum wage in their state, which is \$10.00 per hour. What is their total earnings, t , for h hours worked?

- What is the value of y if it represents 40% of a given value x ?

- Eliza walked 12 miles. Then she walked $\frac{1}{4}$ of that distance. How many miles did she walk altogether? Select *all* that apply.

A. $12 + \frac{1}{4}$

B. $12 \cdot \frac{1}{4}$

C. $12 + 12 \cdot \frac{1}{4}$

D. $12\left(1 + \frac{1}{4}\right)$

E. $12 \cdot \frac{3}{4}$

F. $12 \cdot \frac{5}{4}$

Back in My Day

Let's explore how prices have changed over time.



Warm-Up

This table shows the average price of a movie ticket and a new car in different years.

Year	Movie Ticket	New Car
2000	\$5.39	\$21,000
2022	\$10.53	

1. The price of a new car increased 53% from 2000 to 2022. Complete the table.
2. By what percent did the price of a movie ticket increase from 2000 to 2022?
3. In your opinion, which item's price changed more? Explain your thinking.

Comparing the Changes

Let's compare the average price of a mattress in 2000 and in 2022. We'll also look at the federal minimum wage, which is the lowest amount of money a worker can earn per hour according to the U.S. government.

Year	Price of a Mattress	Minimum Wage (\$ per hour)
2000	\$995	\$5.15
2022	\$1,065	\$7.25

4. Complete this sentence: *From 2000 to 2022, the minimum wage increased by%, while the price of a mattress increased by%.*
5. Imagine that the price of a mattress increased by the same percent as the minimum wage.
- a Would the mattress price in 2022 be more or less than \$1,065? Why?
- b What would its price be?
- c For someone earning minimum wage, how many hours of work would it take to buy a mattress in the year ...

... 2000?

... 2022?

6.  **Discuss:**

- In what way have mattresses gotten *more expensive* since 2000?
- In what way have they gotten *cheaper* since 2000?

Change Over Time



Data Talk! Now let's look at the cost of one year of college, along with the minimum wage. In 2000, the cost of one year of college was \$3,400.

Year	Minimum Wage (\$ per hour)	Cost of One Year of College*
2000	\$5.15	\$3,400
2022	\$7.25	

*Average cost for tuition and fees at U.S. 4-year public universities

7. Let's think about the cost of a year of college in 2022. What are values that seem reasonable to you? Give at least two and explain your reasoning.

Value:

Explanation:

Value:

Explanation:

8. From 2000 to 2022, the cost of college actually increased by about 182%.

a What does it mean to have an increase of more than 100%?

b What was the cost of college in 2022? Show your thinking.

9. Imagine the minimum wage increased by the same percent as the cost of college. What would the minimum wage have been in 2022?

Change Over Time (continued)

Let's look at more price data.

Year	Minimum Wage (\$ per hour)	One Year of College	Monthly Rent	Dental Visit	Fast Food Hamburger
1978	\$2.65	\$670	\$211	\$24	\$0.90
2000	\$5.15	\$3,400	\$602	\$95	\$2.24
2022	\$7.25		\$1,128	\$200	\$5.15

- 10.** Use the previous page to complete the table. What do you notice? What do you wonder?

- 11.** Complete these sentences:


From 2000 to 2022, the cost of a dental visit increased by about _____ %.
If the minimum wage had increased by the same percent, the minimum wage in 2022 would have been about _____.

- 12.** Write a new comparison like the one above.

You're invited to explore more.

- 13.** Madam C. J. Walker was a business woman who made her fortune by developing a line of cosmetics and hair care for Black women. In 1919, her wealth was valued at approximately \$1,000,000. Every dollar in 1919 is worth 1,463% more in 2020. How much would Madam C.J. Walker's fortune be worth in 2020 dollars?

Synthesis

14.  **Discuss:** How can percentages help us make sense of how a price has changed over time?

Use the table if it helps you explain your thinking.

Year	Mattress	Monthly Rent	Minimum Wage (\$ per hour)
2000	\$995	\$602	\$5.15
2022	\$1,050	\$1,128	\$7.25

Summary 4.06

Prices generally go up over time, but *how much* prices go up matters. Dollar amounts and percent changes are two ways of describing price increases. Percentages are more useful when comparing price changes of two or more different things. Let's examine the price of bananas and ground beef.

Here is one way to calculate the percent increase in price for each item.

- **Bananas:** $\frac{0.63}{0.50} = 1.26$. This means 126%, or a 26% increase.
- **Ground beef:** $\frac{5.21}{2.56} = 2.04$. This means 204%, or a 104% increase.

Year	Pound of Bananas	Pound of Ground Beef
2004	\$0.50	\$2.56
2024	\$0.63	\$5.21

The price for both items increased during this period, but the ground beef price increased much more dramatically than the price of bananas.

It can be useful to see how prices change over time compared to people's income. The federal minimum wage is an example of a measure of income. From 2004 to 2024, minimum wage changed from \$5.15 to \$7.25, which is a 41% increase. That means that for someone making minimum wage in 2004 and 2024, bananas would feel like they have gotten cheaper with time, but ground beef would feel more expensive.

Practice 4.06

Name: _____ Date: _____ Period: _____

- Jayla purchased a brand new car for \$36,500. Her car is now worth \$16,800. What is the percent decrease between the car's original price and its value now?

Problems 2–4: The table shows the price of a tablet in 2021 and in 2022.

Year	Price (\$)
2021	155
2022	128.65

- Determine the percent decrease for the price of a tablet between 2021 and 2022.
- If the percent decrease continues for another year, what will the price of the tablet be in 2023?
- If the same percent decrease was true from 2020 to 2021, what was the price of a tablet in 2020?

Problems 5–6: Ecuador and Peru are neighboring countries in South America. Here are their populations in 2005 and 2020.

Year	Ecuador	Peru
2005	13,770,000	28,147,000
2020	17,589,000	33,305,000

- Circle the expression that could help you find the percent change in Ecuador's population from 2005 to 2020.

$$\frac{17589}{13770}$$

$$\frac{13770}{17589}$$

Explain your thinking.

- If the population of Peru had grown by the same percent as Ecuador from 2005 to 2020, which would be true about Peru's population? Circle one.


Greater than 33,305,000

Less than 33,305,000

Explain your thinking.

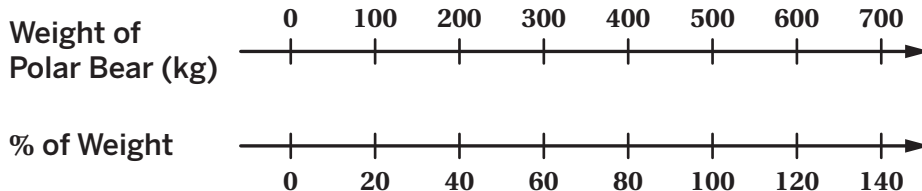
Practice 4.06

Name: _____ Date: _____ Period: _____

-  **Problems 7–8:** Caasi has money in a savings account that grows 1.5% every year.
7. Write an equation to represent the money in her account as it grows from one year to the next. Use x for this year's amount and y for next year's amount.
8. Caasi has \$500 in her savings account. How much did she have in the account last year?

Spiral Review

9. Write a situation that matches this diagram.



10. Sahana's total cost at the salon is \$59. She wants to pay using her credit card. The salon charges a 3% fee for using a credit card. What is the total amount that will be charged to Sahana's card?
11. A phone keeps track of the number of steps a person has taken and the distance traveled. Does the information in the table show a proportional relationship between the two quantities? Show or explain your thinking.

Number of Steps	Distance (km)
1,150	1
4,654	3.3
6,795	5.1

Practice Day 1



Let's practice what you've learned so far in this unit!

You will use problem cards for this Practice Day. Record all of your responses here.

Card 1

Circle one: True or False

Explanation:

Card 2

Circle one: True or False

Explanation:

Card 3

Solution: _____

Card 4

- a** Percent increase: _____ %
- b** New minimum wage: _____ dollars per hour

Card 5

Solution: _____ centimeters

Card 6

Solution: _____ nesting turtles

Practice Day 1 (continued)

Card 7

- a New price: _____ dollars
- b Equation: _____

Card 8

- a Cost: _____ dollars
- b Equation: _____

Card 9

Solution: _____

Card 10

Solution: _____

Card 11

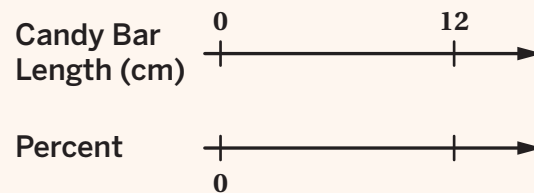
Description:

Card 12

Description:

You're invited to explore more.

A candy bar is 12 centimeters long. The king-size bar is 15 centimeters long. What percent of the original length is the king-size bar's length?



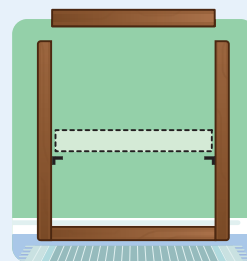
Applying Percentages



Lesson 7
Tax and Tip



Lesson 8
Plate Rate



Lesson 9
Bookcase Builder



Lesson 10
Population & Pollution



Tax and Tip

Let's solve multistep percent problems involving sales tax and tip.

Warm-Up

1-2 Customers are reporting problems with the receipts at the Des-Cafe.



Soup:	\$5.00
Tax:	+ \$0.35
Total:	\$5.35

Salad:	\$8.00
Tax:	+ \$0.56
Total:	\$8.56

Pizza:	\$18.00
Tax:	\$
Total:	\$

Sandwich:	\$13.00
Tax:	\$
Total:	\$

Here's the information from the two receipts that printed out correctly.

Item	Price	Tax	Total
Soup	5.00	0.35	5.35
Salad	8.00	0.56	8.56

What percent is the sales tax?
Explain your thinking.

Tax

3 The sales tax rate is 7%. The sales tax rate is proportional to the price of each item.

Determine the remaining values for these receipts.

Item	Price	Tax	Total
Soup	5.00	0.35	5.35
Salad	8.00	0.56	8.56
Pizza	18.00		
Sandwich	13.00		

Soup:	\$5.00
Tax:	+ \$0.35
Total:	\$5.35

Salad:	\$8.00
Tax:	+ \$0.56
Total:	\$8.56

Pizza:	\$18.00
Tax:	\$
Total:	\$

Sandwich:	\$13.00
Tax:	\$
Total:	\$

4 The Des-Cafe got a new cash register!

Write expressions that the cash register can use to determine the tax and total for any item.

Price	Tax	Total
\$5.00	\$0.35	\$5.35
\$8.00	\$0.56	\$8.56
c		

Soup:	\$5.00
7% Tax:	+ \$0.35
Total:	\$5.35

Soup:	\$8.00
7% Tax:	+ \$0.56
Total:	\$8.56

Tip

5 Customers at restaurants usually leave a tip for the server. Sometimes restaurants add a tip automatically.

Write expressions that the cash register can use to determine an 18% tip and the total after tip.

Total Before Tip	18% Tip	Total After Tip
\$5.35	\$0.96	\$6.31
\$8.56	\$1.54	\$10.10
t		

Total Before Tip:	\$5.35
18% Tip:	+ \$0.96
Total After Tip:	\$6.31

Total Before Tip:	\$8.56
18% Tip:	+ \$1.54
Total After Tip:	\$10.10

6 Kiran and Ava had a bill for \$100. The tax rate is 5% and they want to tip 20%.

- Kiran calculates the tip after tax is added to the bill.
- Ava calculates the tip *before* tax is added to the bill.

Whose strategy would result in a greater tip? Circle one.

Kiran's Ava's They're the same

Explain your thinking.

Kiran	
Bill:	\$100.00
5% tax:	\$
20% tip:	\$
Total:	\$??.??

Ava	
Bill:	\$100.00
5% tax:	\$
20% tip:	\$
Total:	\$??.??

Tip (continued)

7 A store offers a 20% off coupon.

The tax rate is 7.5% of the subtotal.

If an item is listed at \$15, what is the total after the coupon and tax have been applied?

Explain your thinking.

Price:	\$15.00
20% Off Coupon:	\$
Subtotal:	\$
7.5% Tax:	\$
Total:	\$??.??

Repeated Challenges

8 Find the total the customer pays for each receipt. Answer as many as you have time for.

- a** This meal's bill is \$28.00.

A 7% sales tax is applied, followed by an automatic tip of 18% (on the after-tax amount).

Bill:	\$28.00
7% Tax:	\$
Total Before Tip:	\$
18% Tip:	\$
Total With Tip:	\$?.??

- b** A store sells an item for \$18.00.

A 45% off coupon is applied, followed by a 7% sales tax on the subtotal.

Price:	\$18.00
45% Off Coupon:	\$
Subtotal:	\$
7% Tax:	\$
Total:	\$?.??

- c** A store sells an item for \$31.00.

A 15% off coupon is applied, followed by a 7% sales tax on the subtotal.

Price:	\$31.00
15% Off Coupon:	\$
Subtotal:	\$
7% Tax:	\$
Total:	\$?.??

- d** This meal's bill is \$23.00.

A 7% sales tax is applied, followed by an automatic tip of 18% (on the after-tax amount).

Bill:	\$23.00
7% Tax:	\$
Total Before Tip:	\$
18% Tip:	\$
Total With Tip:	\$?.??

You're invited to explore more.

- 9** A store sells an item for \$75.00.


Fatima uses a 20% off coupon and then tax is applied to the subtotal.

The total after coupon and tax is \$75.00.

What is the tax rate? Explain your thinking.

Price:	\$75.00
20% Off Coupon:	\$
Subtotal:	\$
?% Tax:	\$
Total:	\$75.00

10 Synthesis

 **Discuss:** What is a strategy you can use to calculate the total of an item after different discounts and tax rates?

Use the receipts if they help with your thinking.

Price:	\$12.00
20% Discount:	– \$2.40
Subtotal:	\$9.60
5% Tax:	+ \$0.48
Total:	\$10.08

Price:	\$12.00
15% Discount:	– \$1.80
Subtotal:	\$10.20
15% Tax:	+ \$1.53
Total:	\$11.73

Price:	\$12.00
10% Discount:	– \$1.20
Subtotal:	\$10.80
8% Tax:	+ \$0.86
Total:	\$11.66

Price:	\$12.00
5% Discount:	– \$0.60
Subtotal:	\$11.40
6.25% Tax:	+ \$0.71
Total:	\$12.11

13 Summary 4.07

The listed price and the total a customer ends up paying are often two different quantities. Tax, tip, and discounts are some of the reasons why the listed price and final price are different.

These changes are often calculated as percentages of the listed price.

For example, a tip is an amount of money that a person gives someone who provides a service, such as restaurant servers, hairdressers, and delivery drivers. If a person plans to leave a 20% tip, then the total cost with tip will be 120% of the bill.

Practice 4.07


Name: _____ Date: _____ Period: _____

Problems 8–10: A music store buys instruments and then sells them for 30% more than they paid.

8. If the store buys a guitar for \$45, what will the store sell it for?

9. If the price tag on a trumpet says \$104, how much did the store pay for it?

10. During a 20% off sale, the store offers a clarinet for \$93.60. How much did the store pay for the clarinet?

11.  A clothing store is having a sale: 25% off shirts and 30% off jeans.
 - Kyle bought a shirt with an original price of \$24.
 - Kyle bought a pair of jeans with an original price of \$32.Determine how much money Kyle saved.

Spiral Review

In 1969, the annual global mean temperature was 14°C . In 2019, the annual global mean temperature was 14.8°C . Two different news websites reported this change.

Website A

“In the last 50 years, the annual global mean temperature has increased by almost 6%.”

Website B

“There has been an increase in the annual global mean temperature of approximately 5% during the last half century.”

12. Which website more accurately reported the change? Explain your thinking.

Plate Rate

Let's use percent change to analyze an issue in society.



Warm-Up

The minimum wage is the lowest amount of money a worker can earn per hour.

In 2023, the federal minimum wage was \$7.25 per hour.



Data Talk! There is a different federal minimum wage for workers who receive tips (also known as gratuity). In 2023, this was \$2.13 per hour. However, if a worker does not receive enough tips to make the federal minimum wage on average, the employer must pay them the difference.

1. Abdel works at a restaurant that pays \$2.13 per hour plus tips.

One day, he had 10 tables during his 8 hour shift. Each table tipped him \$5.

- a How much did he earn during his shift?
- b How does this amount compare to what he would have earned if he worked for the federal minimum wage without tips?

Waiting Tables

Here is some information about four servers who work at different restaurants: Laila, Tiana, Peter, and Julian. Choose *one* server to complete the activity.

- Laila** is 35 years old. She is married and has two children. She has worked at the same restaurant for 7 years. She works 40 hours per week and makes \$2.13 per hour. In a typical week, she serves 75 tables. The average bill at the restaurant is \$41 per table, and she typically receives an 18% tip.
- Tiana** is 25 years old. She lives with a roommate and a dog. She has worked at a fancy restaurant for 6 months. She works 40 hours per week and makes \$2.13 per hour. She usually serves 45 tables per week. The average bill at the restaurant is \$130 per table, and she typically receives a 20% tip.
- Julian** is 29 years old. He is a single father with a 3-year-old son. He just finished his third year as a server. He works 40 hours per week and makes \$2.13 per hour. In a typical week, he serves 95 tables. The average bill at the restaurant is \$22 per table, and he typically receives a 15% tip.
- Peter** is 19 years old. He lives at home with his parents and goes to college part-time. He recently started as a server, working 40 hours per week. Where Peter lives, the minimum wage for tipped workers is \$7.25 per hour. In a typical week, he serves 90 tables. The average bill at the restaurant is \$21 per table, and he typically receives a 15% tip.

2. How much money does your server make in a typical week? Show or explain your thinking.

3. Let's say that customers at your server's restaurant raise their tips by 5 percentage points. (So for example, 18% becomes 23%.)

How much would your server make now in a typical week? And by what *percent* would their pay increase? Show your thinking.

Waiting Tables (continued)

4. Now, let's say that customers at the restaurant *lower* their tips by 5 percentage points (for example, 18% becomes 13%).

How much would your server make now in a typical week? And by what percent would their pay decrease (compared to Problem 2)? Show your thinking.

Gabriel gets paid \$11.75 per hour to work in the kitchen. Unlike servers, restaurant workers in the kitchen do not usually receive tips.

5. How much money does Gabriel earn in a 40-hour week?
6. Compare Gabriel's pay to the server you chose. Consider the server's pay in a typical week as well as when tips are high or low. Use percentages in your comparison.
7. Some restaurants have experimented with different ways of paying workers. One restaurant pays all workers \$16 per hour, but doesn't allow tipping.

How do you expect people might feel about this policy? Consider how restaurant workers, restaurant owners, and customers might feel.

You're invited to explore more.


8. Some restaurants have eliminated tipping and raised the price of menu items by 20%. They say this allows worker pay to be fairer. Do you think restaurants should do this? Explain your thinking.

A New Way to Pay

In other professions there are even more ways to get paid. Someone who works in a sales position, for example, may get paid by *commission*. Commission means money paid to an employee upon completion of a task, like selling a certain amount of goods or services. Commissions are usually a percentage of goods or services sold. A commission may be paid in addition to or instead of an hourly rate.

Here are two job postings for a hairstylist that will earn commission in addition to an hourly rate and tips.

The Style Lounge	Classy Cuts
<p>Are you passionate about hair styling? We're looking for stylists to join our brand new salon!</p> <p>We value creativity and a genuine love for making clients look and feel their best!</p> <p>Wage System: \$22 paid hourly + tips 30% commission on retail (no commission on services)</p>	<p>Join a locally owned salon and be one of the GREATS! Whether you're new to the industry or have years behind the chair, we welcome you!</p> <p>Come on board and help us elevate our clients' style.</p> <p>Wage System: \$10 paid hourly + tips 50% commission on all services (no commission on retail)</p>

9.  **Discuss:** Which job posting do you think would pay better?
10. Imagine you plan to work part-time at 20 hours per week in this job, selling a total of \$500 in services and \$200 in retail per week on average. How much would you make weekly in each position before tips?

Synthesis

11. Describe how you can determine how much a restaurant server makes in a week. Use the example if it helps with your thinking.

Tiana works 40 hours per week and makes \$2.13 per hour. She usually serves 45 tables per week. The average bill at the restaurant is \$130 per table, and she typically receives a 20% tip.

Summary 4.08

There are two different federal minimum wages — one for workers who receive tips and one for workers who don't.

For tipped workers, such as restaurant servers, pay depends not only on their hourly wage and the number of hours they work, but also the number of tables served, the average bill at those tables, and the average percent tip.

Let's say a restaurant server earns \$2.13 per hour and works 30 hours per week. They serve about 40 tables in a week, where the typical bill is \$75 and the tip is 18%. Here's an expression for how much this server earns in a typical week:

$$2.13 \cdot 30 + 40 \cdot 75 \cdot 0.18 = 603.90$$

We can compare this amount to other ways of paying servers, such as a simple rate of \$15 per hour ($15 \cdot 30 = \450 in a week), with no tips. Switching to this way would mean about a 25% decrease in pay for the restaurant server in our example, because

$$\frac{603.90 - 450}{603.90} = 0.2548.$$

Practice 4.08

Name: _____ Date: _____ Period: _____

Problems 1–2: A customer leaves a 15% tip on a \$20 meal.

1. Select the expression that represents the value of the tip.

- A. $15 \cdot 20$ B. $20 + 1.5 \cdot 20$
C. $1.15 \cdot 20$ D. $\frac{15}{100} \cdot 20$

2. Select the expression that represents the *total* bill.

- A. $15 \cdot 20$ B. $20 + 1.5 \cdot 20$
C. $1.15 \cdot 20$ D. $\frac{15}{100} \cdot 20$

Problems 3–4: Mauricio is a server at a restaurant. In an average 8-hour work day, he serves 10 tables, with an average bill of \$45 per table. He typically receives a 20% tip on each bill and earns \$7.25 per hour.

3. How much money does Mauricio earn in a typical day?

4. Let's say the typical tip increased to 23% of the bill. By what percent would Mauricio's earnings increase? Show or explain your thinking.

Problems 5–7: Here is some information about three parks. Complete each sentence.

5. Golden Gate Park is about _____ % larger than Central Park.
6. Longview Lake Park is about _____ % larger than Golden Gate Park.
7. If the size of Central Park increased by 200%, would it be larger than Longview Lake Park? Explain your thinking.

Park	Area (acres)
Central Park (New York City)	843
Golden Gate Park (San Francisco)	1,017
Longview Lake Park (Kansas City)	2,381

Spiral Review


8. Match each situation with an equation.

Situation

- a. Tay sleeps for x hours. Omar sleeps for 20% less than that.
- b. Tay practices piano for x hours. Omar practices for 40% less than that.
- c. Tay drinks x ounces of juice. Omar drinks 130% more than that.
- d. Tay spends x dollars. Omar spends 25% less than that.
- e. Tay completes x puzzles. Omar completes 60% more than that.
- f. Tay eats x grams of almonds. Omar eats 40% more than that.

Equation

- $y = 0.6x$
- $y = 0.75x$
- $y = 0.8x$
- $y = 2.3x$
- $y = 1.4x$
- $y = 1.6x$

9.  The radius of circle S is half the radius of circle L . The radius of circle L is 10 millimeters. Which measurement is closest to the area of circle S ?

- A. 78.54 square millimeters
- B. 31.42 square millimeters
- C. 314.16 square millimeters
- D. 15.71 square millimeters

Problems 10–12: Solve each equation.

10. $\frac{2}{3}x = \frac{8}{15}$

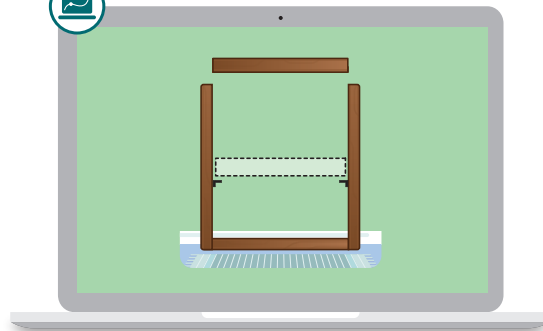
11. $1.8 + x = 7.2$

12. $5\frac{4}{5} = 3\frac{2}{3} + x$

13. Select all the scales that are equivalent to 5 centimeters to 15 meters.

- A. 5 inches to 15 inches
- B. 1 centimeter to 3 meters
- C. 5 meters to 15 centimeters
- D. 1 centimeter to 300 centimeters
- E. 1 inch to 3 feet

Explain your thinking for the scale(s) you selected.



Bookcase Builder

Let's explore situations where errors occur and represent those errors using percentages.

Warm-Up

1 Dakota and Ebony are buying deli meat.

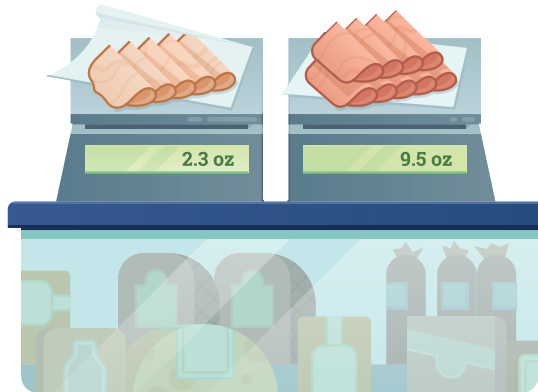
Dakota orders 2 ounces of ham, but ends up with 2.3 ounces.

Ebony orders 10 ounces of turkey, but ends up with 9.5 ounces.

Whose order had a bigger error?
Circle one.

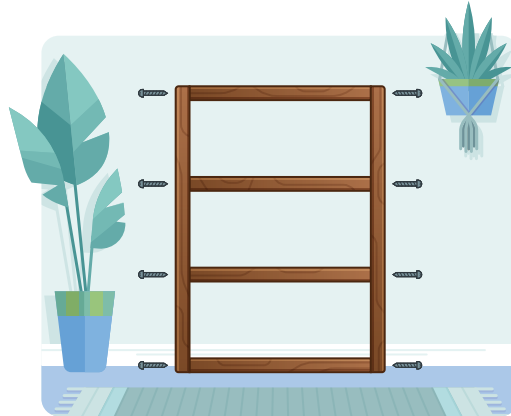
Dakota's order Ebony's order

Explain your thinking.



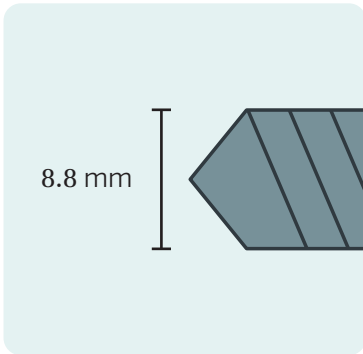
Bookcase Blunders

2 This bookcase was designed for 10 mm screws, but screws that are a little bigger or smaller are acceptable.

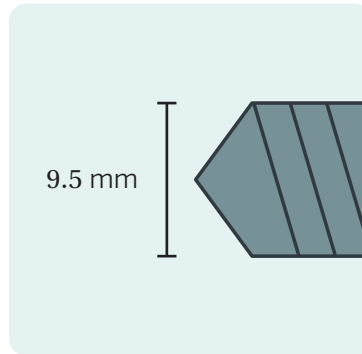


Let's watch the screen to find out which screws work for the bookcase.

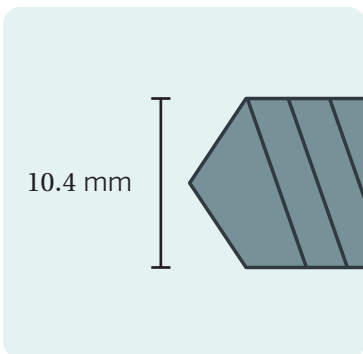
A.



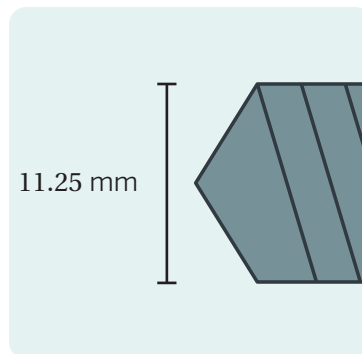
B.



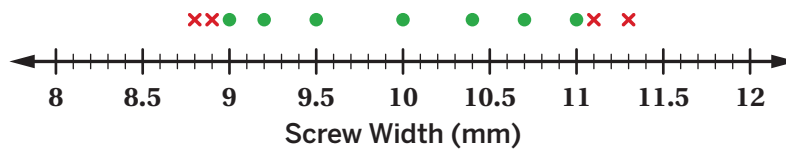
C.



D.



3 The screw widths that worked are represented with green dots on this number line. The widths that didn't work are represented with red Xs.

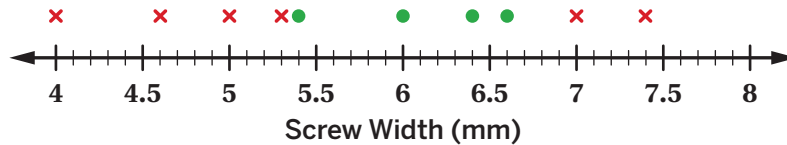


Discuss: What are the smallest and largest screws that will work?

Bookcase Blunders (continued)

4 Imagine a new bookcase designed for 6 mm screws.

Here is a number line that represents the widths of screws that work and don't work for the new bookcase.



What are the smallest and largest screw widths that work?

5 Here are the smallest and largest acceptable screw widths for the two previous bookcases.

How are the acceptable screw widths different in each row? How are they alike?

Desired Width (mm)	Smallest Acceptable Width (mm)	Largest Acceptable Width (mm)
10	9	11
6	5.4	6.6

Different:

Alike:

6 A factory tries to make a 6 mm screw. It ends up being 5.7 mm instead.

One way to describe the error is 0.3 mm.

Another way is to describe it as a **percent error**.

This screw has a percent error of $\frac{0.3}{6} = 5\%$.

Complete the table to decide if each screw will work (its percent error must be 10% or less).

Desired Width (mm)	Screw Width (mm)	Percent Error	Will It Work?
6	5.7	5%	Yes
10	10.9		
6	7.2		
15	13.8		

Challenge Your Shelf

- 7** Nikolai is making a shelf for his new bookcase.

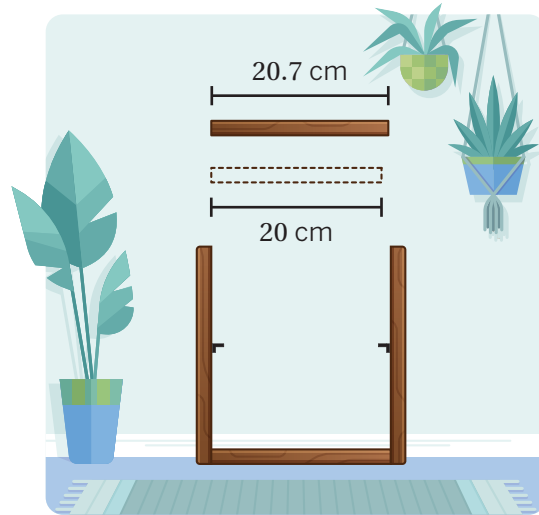
He wants it to be 20 centimeters long. It ends up being 20.7 centimeters long.

The acceptable percent error is 5% for a shelf to fit.

Will the shelf fit? Circle one.

Yes No

Explain your thinking.



- 8** If d represents the desired shelf length and s represents the actual shelf length, select *all* the equations that will let Nikolai to determine the lowest and highest acceptable lengths for the shelf.

- A. $d = 0.05s$ B. $s = 1.05d$ C. $s = 20d$
 D. $d = 0.95s$ E. $s = 0.95d$

- 9** Determine whether the shelf will fit given the acceptable error.

Answer as many as you have time for.

- a** • Shelf length: 19.3 cm
 • Desired shelf length: 20 cm
 • Acceptable error: 5%
- b** • Shelf length: 12 cm
 • Desired shelf length: 15 cm
 • Acceptable error: 5%

Will the shelf fit? Circle one.

Yes No

Will the shelf fit? Circle one.

Yes No

11 Synthesis

Explain how to calculate percent error.

Use the examples if they help with your thinking.

Dakota

- Orders 2 ounces of ham
- Ends up with 2.3 ounces

Ebony

- Orders 10 ounces of turkey
- Ends up with 9.5 ounces

14 Summary 4.09

Percent error describes the difference between a desired value and the actual value, expressed as a percent of the desired value.

For example, a milk carton is supposed to contain 16 fluid ounces, but it only contains 15 fluid ounces.

- The error is 1 fluid ounce.
- The percent error is 6.25% because $\frac{1}{16} \cdot 100 = 6.25$.

To determine the percent error, the amount of the error is compared to the desired value. You can use this formula:

$$\text{Percent error} = \frac{(\text{difference between actual value and the desired value})}{\text{desired value}} \cdot 100$$

In some situations, there is no clear “desired” value. In those cases, the denominator is the value that has no error. Here are some examples:

- For a thermometer that reads 73°, if the real temperature is 70°, the percent error for that thermometer’s reading is $\frac{3}{70}$, or 4.3%.
- For an estimate of 800 jelly beans in a jar, if the jar actually has 947 jelly beans, the percent error of the estimate is $\frac{147}{947}$, or 15.5%.

percent error A way to describe the difference between a desired value and the actual value, expressed as a percent of the desired value.

Practice 4.09

Name: _____ Date: _____ Period: _____

1. A baker needs 500 grams of flour for a batter, but accidentally only puts 300 grams of flour into the mixing bowl. What is the percent error?

2. Any can of seltzer should have 12 fluid ounces of liquid to be appropriately carbonated. One can on the assembly line contains 13.5 fluid ounces. What is the percent error?

3. A radar gun measured the speed of a baseball at 93 miles per hour. If the baseball was actually going 90 miles per hour, what was the percent error in this measurement?

Problems 4–5: In Burlington, all city departments get an annual budget. They are allowed to spend 1.5% over their budget. If they spend more than that, the mayor conducts a review.


4. The Department of Health has a budget of \$90,000. What is the largest acceptable amount for their spending? Show or explain your thinking.

.

5. The Department of Parks and Recreation has a budget of \$30,000, and they spent \$31,000. Will the mayor conduct a review of their spending? Show or explain your thinking.

.

Spiral Review

6.  Hoang is buying 5 concert tickets that cost \$95.00 each. He finds a coupon to receive a 10% discount. A sales tax of 8% is applied after the discount. How much did he pay after the discount and sales tax?
- A. \$92.34 B. \$427.50 C. \$461.70 D. \$513.00
7. Ahmed bikes x kilometers. Kimaya bikes $\frac{3}{10}$ less than that. Using y for Kimaya's distance, write an equation that describes the relationship between the two quantities.

Problems 8–11: Complete each blank using the symbols $>$, $<$, or $=$.

8. $-\frac{3}{4}$ _____ $\frac{4}{3}$

9. 3.24 _____ $-(-3.24)$

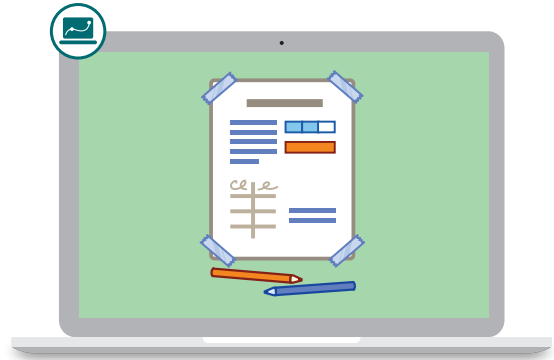
10. $-\frac{2}{3}$ _____ $-\frac{5}{6}$

11. $-|4|$ _____ $-|-4|$

12. Commercial planes typically fly at the maximum altitude of 38000 feet and a minimum altitude of 31000 feet. This range allows them to avoid other aircraft and weather conditions during the flight. Write one or more inequalities to describe the altitude at which a plane typically flies. Be sure to define your variable.

Population & Pollution

Let's use what we know about percentages to generate and answer questions about our world.



Warm-Up

1 Here is some information.



Data Talk! Write a question that you could figure out using this information and whose answer is not already given.

There are about 336 million people living in the United States.

The U.S. makes up about 4.2% of the global population.

In 1900, the population of the U.S. was about 76 million people, and the population of the world was 1.6 billion people.

Pollution Percentages

- 2** Population increase is not the only change the planet has seen. Here is some information about two other topics.

Plastics

Plastic has countless uses, from household items to food packaging to medical instruments. But plastic has always been tricky to dispose of.

After use, plastic waste might be recycled or sent to a landfill; or it might be mismanaged, like being dumped into a body of water or burned.

Fast Fashion

Fast fashion is a business model where companies quickly produce new and low-priced clothes in large quantities.

To cut costs, the companies often use cheaper materials, such as polyester, and methods that have negative consequences for the environment and for workers.



Discuss:

- What do you know about each topic?
- What might you want to know?

- 3** **a** Select one topic. Read through some more information about that topic.


Plastics

- The amount of plastic waste generated in the U.S. grew by **400%** from 1980 to 2019.
- In 1980, the U.S. recycled **20,000** tons of plastic waste. In 2019, it was **3.1** million tons.
- In 2019, **73%** of U.S. plastic waste went to landfills, **0.5%** ended up in the ocean, and **4%** was recycled.
- In 2019, the U.S. generated **72.8** million tons of plastic waste. This was **55%** more waste than in 2000.
- In 2000, the U.S. mismanaged **3.6** million tons of plastic waste. In 2019, the number was **33%** lower.

Fast Fashion

- Globally, **80** billion new items of clothing were bought in 2015. This was **400%** more than was bought in 1995.
- In 2015, the fashion industry emitted **1.2** billion tons of carbon dioxide.
- Doubling the life of clothing from one year to two years reduces carbon dioxide emissions by **24%**.
- In 2015, **60%** of new clothes contained polyester. Clothes makers used **21.3** million tons of polyester, which was a **157%** increase from 2000.
- The average person bought **60%** more clothes in 2015 than in 2000.

Pollution Percentages (continued)

- b**  **Data Talk!** Write two questions about the topic you selected that you could figure out using the given information and whose answer is not already given.

Question 1:

Question 2:

- 4**  **Discuss** the questions you wrote with two partners:

- Would the answer to this question be interesting or useful?
- Can you answer this question using only the information given?
- Is the answer to the question not obvious from the information given?

Revise your questions after each conversation.

<p>Partner 1 Conversation Notes</p>	<p>Revision of Both Questions</p> <p>Question 1:</p> <p>Question 2:</p>
<p>Partner 2 Conversation Notes</p>	<p>Final Version of Both Questions</p> <p>Question 1:</p> <p>Question 2:</p>

7 Synthesis



Discuss:

- What did you learn about the topic you chose?
- What questions do you still have about your topic, pollution, or population?

10 Summary 4.10

Situations involving percent increase and decrease are everywhere in our society.

For example, news articles often contain facts and statistics about pollution, such as:

In 2019, the U.S. generated 72.8 million tons of plastic waste.
This was 55% more waste than in 2000.

Information like this can be used to generate interesting questions. When writing these questions, it's important to be precise with language. We could ask: *How much waste was there in 2000?* But a more precise question might be: *How many tons of plastic waste did the U.S. generate in 2000?*

We can use strategies from this unit, such as equations, double number line diagrams, tables, and tape diagrams, to answer these kinds of questions.

Practice 4.10

Name: _____ Date: _____ Period: _____

Problems 1–2: A city has a 5% sales tax.



1. A toothbrush costs \$3.40 before tax. How much does it cost including tax?
2. A book costs \$32.55 after tax. How much did it cost before tax?

Problems 3–5: Oliver went to the store and purchased these items.

3. Oliver set a monthly grocery budget for himself. This grocery list costs 6% of the budget. What is his monthly budget?

Milk (1 gal)	\$3.61
Beef (1 lb)	\$7.30
Apples (1 lb)	\$2.39
Bananas (1 lb)	\$0.99
Oranges (1 lb)	\$1.96
Potatoes (1 lb)	\$1.75
Total	\$18.00

4. Complete each sentence: *Milk made up about _____% of Oliver's spending on this trip. This was about _____% of his grocery budget for the month.*

5. Use the given information to write another sentence about Oliver's spending. Include a percentage.
6.  A greeting card costs \$6 before tax. A customer has a coupon for a 10% discount. Then a 5% sales tax is added. How much will the customer pay for the greeting card?
7.  The price of gold is often reported per ounce. At the end of 2005, gold was \$513 per ounce. At the end of 2015, it was \$1,060 per ounce. By what percent did the price increase?

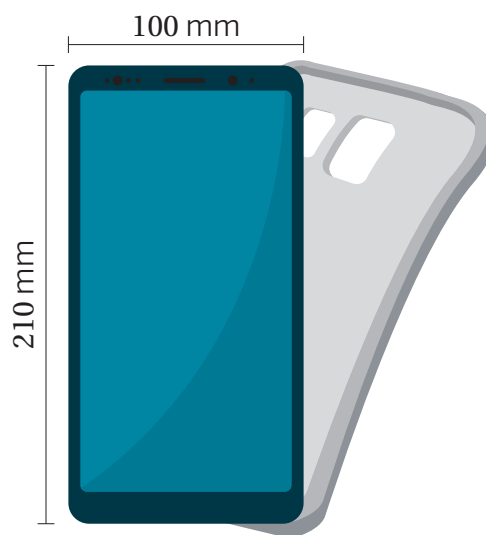
Practice 4.10

Name: _____ Date: _____ Period: _____

8. A grocery store allows customers to use multiple coupons when checking out. Suppose you have a \$5 off coupon and a 10% off coupon. The register will calculate the new price after each coupon is used. Does the order you use the coupons make a difference? Explain your thinking.

Spiral Review

9. Here are the dimensions of a phone case designed by the Soft Shield company. The size of a case can be up to 1% off and still fit a phone. Determine the largest and smallest width and height of the case that will fit a phone.
10. What is the decimal equivalent of $\frac{13}{100}$?



Notes:

Proportional Relationships With Fractions and Decimals



Lesson 11
Sticker Sizes



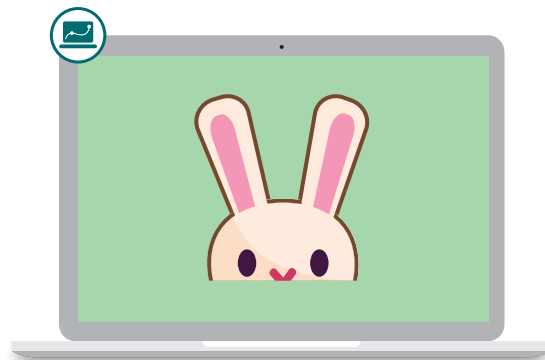
Lesson 12
Peach Cobbler



Lesson 13
Decimal Deep Dive

Sticker Sizes

Let's use constants of proportionality to determine unknown values in proportional relationships.



Warm-Up

- 1** StuckStickers is a company that makes stickers of all different sizes.

Which size will make a scaled copy of this logo? Circle one.

4 in.

3 in.



- A. 6 in. by 7 in.

- B. 6 in. by 8 in.

- C. 5 in. by 5 in.

7 in.

8 in.

5 in.

6 in.



6 in.



5 in.



Explain your thinking.

- 2** Aditi determined that a sticker measuring $2\frac{1}{2}$ by $3\frac{1}{3}$ inches would also make a scaled copy of the logo. She checked her work in two ways.

$$3\frac{1}{3} \div 2\frac{1}{2} = ? \quad = \frac{10}{3} \div \frac{5}{2} \quad = \frac{10}{3} \div \frac{5}{2}$$

$$= \frac{20}{6} \div \frac{15}{6} = \frac{20}{15} \text{ or } \frac{4}{3} \quad = \frac{10}{3} \cdot \frac{2}{5} = \frac{20}{15} \text{ or } \frac{4}{3}$$



Discuss: What do you notice about how Aditi checked her work?

Scaling Stickers

3 Here's a logo that Aditi is making into stickers.

- a** Fill in the missing values so that each sticker is a scaled copy of the logo. (Measurements are in centimeters.)

The image shows three stickers of a bunny in a pot. The first sticker has a height of 4 and a width of 2. The second sticker has a height of 6 and a width of $2\frac{3}{4}$. The third sticker has a height of $6\frac{2}{5}$ and a width of $1\frac{1}{3}$. There are also empty boxes for labels on the second and third stickers.

- b** Describe your strategy.

4 Hamza wants to use this design to create a bumper sticker.

If the bumper sticker has a height of $3\frac{1}{2}$ inches, what will the width of this bumper sticker be?

The bumper sticker design features a purple snake with a yellow and black pattern, surrounded by green leaves and colorful flowers. The height is labeled as 2 in. and the width is labeled as 5 in.

Scaling Stickers (continued)

5 Polina and Jamir both used tables to determine the width of Hamza's bumper sticker.

Polina

Height (in.)	Width (in.)
2	5
$3\frac{1}{2}$	

Note: Red arrows and the multiplier $1\frac{3}{4}$ indicate scaling from 2 to $3\frac{1}{2}$ and from 5 to the unknown width.

Jamir

Height (in.)	Width (in.)
2	5
$3\frac{1}{2}$	

Note: Blue arrows and the multiplier $2\frac{1}{2}$ indicate scaling from 2 to $3\frac{1}{2}$ and from 5 to the unknown width.

Discuss: How did each student determine the width?

6 Here is Alex's logo. Alex wants to use the logo to create a sticker.

If the sticker is $\frac{3}{5}$ inches wide, what does the height need to be?

Height (in.)	Width (in.)
3	$4\frac{1}{2}$
	$\frac{3}{5}$

3 in.



$4\frac{1}{2}$ in.

Sticker Sheets

- 7** Alex is thinking about buying stickers by the sheet. Four sheets cost \$14.

How much would $11\frac{1}{2}$ sheets cost?

Number of Sheets	Total Cost (\$)
4	14
$11\frac{1}{2}$	



- 8** Alex has \$70 to spend on stickers.

How many sheets of stickers can Alex buy?

Number of Sheets	Total Cost (\$)
4	14
	70

Sticker Sheets (continued)

- 9** Select *all* the equations that represent the relationship between the total cost, t , and number of sheets of stickers, s .

A. $t = \frac{7}{2}s$

B. $s = \frac{7}{2}t$

C. $t = \frac{2}{7}s$

D. $s = \frac{2}{7}t$

You're invited to explore more.

- 10** It takes about $1\frac{1}{4}$ seconds for light to travel from Earth to the Moon.

The Moon is about 238,000 miles from Earth.

The Sun is about 94,500,000 miles from Earth.

Determine approximately how long it would take for light to get from the Sun to Earth.

11 Synthesis

Describe how to use a table of a proportional relationship to determine missing values.

Use the example if it helps with your thinking.

Height (in.)	Width (in.)
$2\frac{2}{3}$	$1\frac{1}{3}$
	$5\frac{1}{2}$

14 Summary 4.11

When a problem involves a proportional relationship, determining the constant of proportionality or scale factor can be helpful.

You can see this relationship between the columns of this table. The heights are multiplied by $2\frac{1}{2}$ to get the widths.

When you multiply one quantity in a proportional relationship by a value, the other quantity will change by the same factor whether or not the values are whole numbers.

You can see this relationship between the rows of the table. When the height is multiplied by $1\frac{3}{4}$, the width is multiplied by the same number.

Height (in.)	Width (in.)
2	5
$3\frac{1}{2}$	$8\frac{3}{4}$

Height (in.)	Width (in.)
2	5
$3\frac{1}{2}$	$8\frac{3}{4}$

Practice

4.11

Name: _____ Date: _____ Period: _____

Problems 1–3: A snail is moving away from a rock at a constant rate. This table shows the distance the snail is from the rock at certain times.

Distance (in.)	Time (min)
0	0
1	$1\frac{1}{3}$

- How many minutes does it take for the snail to reach a distance of 9 inches from the rock?
- How far will the snail be from the rock after 9 minutes?
- Select *all* the equations that represent the relationship between the distance in inches, d , and time in minutes, t .
 - A. $d = \frac{4}{3}t$
 - B. $d = \frac{3}{4}t$
 - C. $t = \frac{4}{3}d$
 - D. $t = \frac{3}{4}d$

Problems 4–5: At a deli counter, a customer buys:

- $1\frac{3}{4}$ pounds of ham for \$14.50.
 - $2\frac{1}{2}$ pounds of turkey for \$26.25.
 - $\frac{3}{8}$ pounds of roast beef for \$5.50.
- Which deli meat is the least expensive per pound? Show or explain your thinking.
 - Which deli meat is the most expensive per pound?


Practice 4.11

Name: _____ Date: _____ Period: _____

6. To make a shade of paint called Jasper Green, mix 4 quarts of green paint with $\frac{2}{3}$ of a cup of black paint.

How much green paint should be mixed with 4 cups of black paint to make Jasper Green?

Spiral Review

 **Problems 7–8:** This table shows a relationship between x and y .

x	y
1	8
2	16
3	24

7. What is a constant of proportionality in the relationship?

8. Write an equation to represent the relationship.

9. Angel checks out 12 library books, and Inola checks out $\frac{1}{3}$ less than that. How many books does Inola check out?

Problems 10–12: Use what you know about percentages to complete these problems. Show your thinking.

10. Which is greater? Circle one.

40% of 12

12% of 40

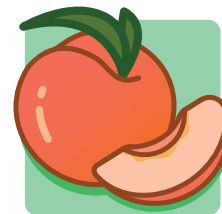
They are the same

11. 12 is 40% of what number?

12. What is 40% of 12?

Peach Cobbler

Let's use a constant of proportionality to compare relationships that involve fractional quantities.



Warm-Up

Determine each quotient mentally. Be prepared to share your strategy.

1. $\frac{\frac{1}{1}}{\frac{1}{3}}$

2. $\frac{\frac{5}{1}}{\frac{1}{3}}$

3. $\frac{\frac{1}{2}}{\frac{1}{3}}$

4. $\frac{2\frac{1}{2}}{\frac{1}{3}}$

Which Recipe?

Amara is making peach cobbler. She has three recipes and is deciding which one to make.

Recipe A	Recipe B	Recipe C
Number of servings: 9	Number of servings: 12	Number of servings: $4\frac{1}{2}$
<ul style="list-style-type: none"> • $2\frac{7}{10}$ lb of peaches • $\frac{1}{2}$ cups of butter • 1 cup of flour • $1\frac{1}{8}$ cups of sugar • $\frac{1}{2}$ tsp of lemon juice 	<ul style="list-style-type: none"> • 4 lb of peaches • $\frac{3}{4}$ cups of butter • $\frac{3}{4}$ cups of flour • $1\frac{1}{3}$ cups of sugar • $\frac{1}{2}$ tsp of lemon juice 	<ul style="list-style-type: none"> • $1\frac{4}{5}$ lb of peaches • $\frac{1}{4}$ cups of butter • $\frac{2}{3}$ cups of flour • $\frac{3}{4}$ cups of sugar • 1 tsp of lemon juice

Amara wants to make a recipe that isn't too sweet.

5. She thinks Recipe C will be the least sweet because it has the least amount of sugar. Do you agree? Explain your thinking.

6. In a recipe, the relationship between the amount of sugar and total number of servings is proportional.



Discuss: Which recipe should Amara make?

7. Which recipe might be called Super Peachy Cobbler? Explain your thinking.

Adjusting a Recipe

Jamar wants to make peach cobbler using Recipe B.

8. Determine how much of each ingredient he needs for one serving. Show your thinking.

Recipe B

Number of servings: 12

- 4 lb of peaches
- $\frac{3}{4}$ cups of butter
- $\frac{3}{4}$ cups of flour
- $1\frac{1}{3}$ cups of sugar
- $\frac{1}{2}$ tsp of lemon juice

Single Serving of Recipe B

Number of servings: 1

- lb of peaches
- cups of butter
- cups of flour
- cups of sugar
- tsp of lemon juice

9. Jamar plans to make just enough for 3 adults and 3 children. The children will eat less than the adults.

- a How many servings should he make? Explain your thinking.
- b Use the number of servings you chose to adjust Recipe B.


Recipe B for Jamar's Family

Number of servings:

- lb of peaches
- cups of butter
- cups of flour
- cups of sugar
- tsp of lemon juice

10. Jamar has a measuring spoon that is $\frac{1}{8}$ teaspoons. How many spoonfuls of lemon juice does he need to make this recipe? Show your thinking.

Synthesis

11.  **Discuss:** What are some strategies you can use to show that Haru's soup will have more carrots per pint than Mohamed's soup?

Mohamed's Vegetable Soup Recipe

$\frac{1}{3}$ of a cup of carrots for every

$\frac{1}{5}$ of a pint of broth

Haru's Vegetable Soup Recipe

$\frac{7}{8}$ of a cup of carrots for every

$\frac{1}{3}$ of a pint of broth

Summary 4.12

Proportional relationships may involve fractional amounts. You can solve problems involving fractions by using the same strategies you use to solve problems with whole numbers.

- To determine the constant of proportionality within a recipe, divide the amount of an ingredient by the total number of servings.
- Constants of proportionality can help to compare proportional relationships involving fractional quantities.

Here is a recipe for banana bread. To find the amount of sugar per serving, divide $\frac{3}{4}$ cups of sugar by 6 servings. This gives you $\frac{3}{4} \div 6$, or $\frac{1}{8}$ cups of sugar per serving.

Banana Bread Recipe

Number of servings: 6

- 2 lb of bananas
- $\frac{1}{2}$ cups of butter
- $\frac{3}{4}$ cups of sugar
- $2\frac{1}{2}$ cups of flour
- 1 tsp of baking soda

Practice

4.12

Name: _____ Date: _____ Period: _____

1. A recipe calls for $\frac{1}{2}$ cups of sugar and 1 cup of flour. Complete the table to show how much sugar and flour is needed for different batches of the recipe.

Sugar (cups)	Flour (cups)
$\frac{1}{2}$	1
$\frac{3}{4}$	
	$1\frac{3}{4}$
1	
	$2\frac{1}{2}$

Problems 2–4: A punch recipe calls for $1\frac{1}{2}$ quarts of sparkling water and $\frac{3}{4}$ quarts of grape juice.

2. How much sparkling water would you need to mix with 9 quarts of grape juice?
3. How much grape juice would you need to mix with $3\frac{3}{4}$ quarts of sparkling water?

4. How much of each ingredient would you need to make 75 quarts of punch?

Sparkling Water (qt): _____

Grape Juice (qt): _____

Problems 5–6: To make a specific color of green paint, a painter mixes $\frac{1}{2}$ of a gallon of blue paint with $\frac{4}{5}$ of a gallon of yellow paint.

5. How many gallons of yellow paint are needed to mix with 3 gallons of blue paint?

6. How many gallons of each color are needed to make 26 total gallons of this color?

Blue Paint: _____

Yellow Paint: _____

Practice 4.12

Name: _____ Date: _____ Period: _____

Problems 7–8: Evaluate the following expressions.

7. $\frac{10}{\frac{1}{2}}$

8. $\frac{\frac{1}{5}}{\frac{1}{10}}$

Spiral Review

9. Complete the table based on $y = \frac{2}{3}x$.

x	y
12	
	16

10. Select *all* the ratios that are equivalent to 4 : 5.


A. 2 : 2.5

B. 3 : 4

C. 3 : 3.75

D. 8 : 10

E. 14 : 27.5

11.  On a map, DeAndre's house is 2.5 inches from his grandma's house. The map has a scale of 1 inch to 20 miles. How far apart, in inches, would DeAndre's house be from his grandma's house on a map that has a scale of 1 inch to 80 miles?

A. 0.5

B. 0.625

C. 1.60

D. 1.75

12. What is 40% of 160?

13. What is 160% of 40?

14. What is 20% of 15?

15. 15 is 20% of what number?

Decimal Deep Dive

Let's convert fractions to decimals using long division.



Warm-Up

1. These are decimal representations of some unit fractions.

a What do you notice? What do you wonder?

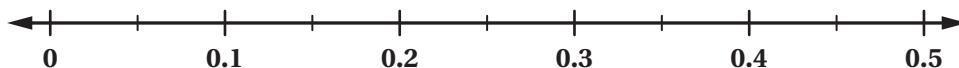
$$\frac{1}{2} = 0.5 \quad \frac{1}{4} = 0.25 \quad \frac{1}{6} = 0.166\dots \quad \frac{1}{8} = 0.125 \quad \frac{1}{10} = 0.1$$

$$\frac{1}{3} = 0.33\dots \quad \frac{1}{5} = 0.2 \quad \frac{1}{7} = 0.14285714\dots \quad \frac{1}{9} = 0.11\dots \quad \frac{1}{11} = 0.0909\dots$$

I notice:

I wonder:

b Plot each of these unit fractions on the number line.



Comparing Calculations

A student started converting these fractions to decimals using long division.

2. Finish writing $\frac{5}{8}$ as a decimal.

$$\begin{array}{r} 0.6 \\ 8 \overline{)5.0} \\ -0 \\ \hline 50 \\ -48 \\ \hline \end{array}$$

3. Finish writing $\frac{2}{11}$ as a decimal.

$$\begin{array}{r} 0. \\ 11 \overline{)2.00} \\ -0 \\ \hline \end{array}$$

4. Compare your calculations for $\frac{5}{8}$ and $\frac{2}{11}$. How are these alike? How are they different?

Alike:

Different:

5. For each completed conversion, determine whether the decimal is terminating or repeating.

$$\begin{array}{r} 0.222 \\ 9 \overline{)2.0000} \\ -0 \\ \hline 20 \\ -18 \\ \hline 20 \\ -18 \\ \hline 20 \\ -18 \\ \hline 20 \end{array}$$

$$\frac{2}{9} = 0.\overline{2}$$

Terminating Repeating

$$\begin{array}{r} 0.750 \\ 4 \overline{)3.0000} \\ -0 \\ \hline 30 \\ -28 \\ \hline 20 \\ -20 \\ \hline 00 \end{array}$$

$$\frac{3}{4} = 0.75$$

Terminating Repeating

$$\begin{array}{r} 0.6060 \\ 33 \overline{)2.0000} \\ -0 \\ \hline 200 \\ -198 \\ \hline 20 \\ -0 \\ \hline 200 \\ -198 \\ \hline 20 \end{array}$$

$$\frac{20}{33} = 0.\overline{60}$$

Terminating Repeating

Terminating or Repeating?

Use long division to write each fraction as a decimal. Then determine whether the decimal is terminating or repeating.

6. $\frac{3}{20}$

7. $\frac{3}{9}$

Terminating

Repeating

Terminating

Repeating

8. $\frac{3}{5}$


9. $\frac{5}{6}$

Terminating

Repeating

Terminating

Repeating


10.  **Discuss:** How can you determine whether a fraction will result in a terminating or repeating decimal?

Decimal Deep Dive

11. Find fractions that meet as many of these requirements as you can.

	Terminating decimal that ends after ...	Repeating decimal that repeats every ...
One Digit		
Two Digits		$\frac{2}{11} = 0.\overline{18}$
Three Digits	$\frac{5}{8} = 0.625$	
Four Digits		
Five Digits		
Six Digits		

Synthesis

12.  **Discuss:** What is important to remember when writing a fraction as a decimal?

Use the example if it helps with your explanation.

$$\begin{array}{r} 0.833 \\ 6 \overline{)5.000} \\ \underline{-0} \\ 50 \\ \underline{-48} \\ 20 \\ \underline{-18} \\ 20 \\ \underline{-18} \\ 2 \end{array}$$

Summary 4.13

Long division can be used to represent fractions as decimals. Sometimes, a decimal is a **terminating decimal**, which means that it ends. Other times, a decimal is a **repeating decimal**, where one or more of its digits (not all zeros) repeats forever. A repeating decimal can be written using bar notation over the digits that repeat or with the ellipses (. . .) at the end.

Examples

$$\frac{1}{3} = 0.333 \dots = 0.\overline{3}$$

$$\frac{14}{99} = 0.14141414 \dots = 0.\overline{14}$$

$$\frac{53}{90} = 0.5888 \dots = 0.5\overline{8}$$

Remember to put the bar *only* over the repeating digit(s).

repeating decimal A decimal with one or more digits (not all zeros) that eventually repeat forever. A repeating decimal can be written using bar notation over the digits that repeat or with the ellipses (. . .) at the end.

If the terminates in zeros, it would be called a terminating decimal.

terminating decimal A decimal with a finite number of non-zero digits after the decimal point.

Practice 4.13

Name: _____ Date: _____ Period: _____

1. Finish writing $\frac{3}{8}$ as a decimal. Then determine if the decimal representation is terminating or repeating. Explain your thinking.

$$\begin{array}{r} 0.3 \\ 8 \overline{)3.0} \\ \underline{-0} \\ 30 \\ \underline{-24} \\ \hline \end{array}$$

Problems 2–4: Use the fractions $\frac{9}{25}$, $\frac{11}{30}$, and $\frac{4}{11}$.

2. Match each fraction to its decimal equivalent.

Fraction

Decimal

a. $\frac{9}{25}$

..... 0.36

b. $\frac{4}{11}$

..... $0.3\overline{6}$

c. $\frac{11}{30}$

..... $0.\overline{36}$

3. What is the same about the decimal representations and what is different?

4. Which of these fractions has the greatest value? Explain how you know.

5. Diego was writing fractions as repeating decimals. He used long division and determined $\frac{52}{225} = 0.23111111$. He wrote his answer as $0.23\overline{1}$. Is Diego correct? Explain your thinking.

6. Complete the table.

Fraction	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{16}$	$\frac{1}{32}$
Decimal	0.5	0.25	0.125		
Number of Digits in Decimal	1	2	3		

How many digits will be in the decimal for the fraction $\frac{1}{256}$?

Spiral Review

7. Select *all* the true statements.

- A. $2.25 > 2$ B. $2.25 < 2$ C. $-2.25 > -2$
 D. $-2.25 < -2$ E. $2.25 > -2$

8. Irelle buys the following ingredients:

- $2\frac{1}{2}$ cups of chocolate chips for \$8.00
- 2 cups of raisins for \$6.50
- $\frac{2}{3}$ cups of shredded coconut for \$2.00

Which ingredient is the most expensive per cup? Circle one.

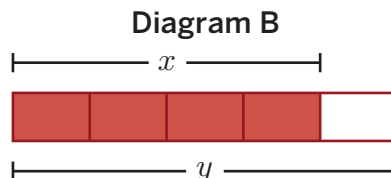
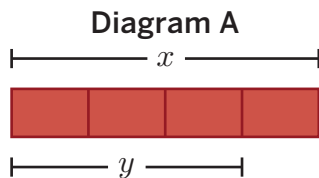
Chocolate Chips

Raisins

Shredded Coconut

Explain your thinking.

9. For each description or equation, decide whether it matches Diagram A, Diagram B, or neither. x represents the original value and y represents the final (new) value.



..... An increase by 25% $y = 1.6\bar{x}$ $y = 0.75x$

..... A decrease by 25% $y = 1.3\bar{x}$ $y = 1.25x$

10. 🍷 You can make dried cucumbers through the process of dehydration, which involves removing water. A cucumber weighs 14 ounces. After dehydration, the cucumber weighs 0.56 ounces. What is the percent decrease in the weight of the cucumber before and after dehydration?

Problems 11–12: Evaluate each expression.

11. $\frac{2\frac{1}{3}}{1\frac{1}{3}}$

12. $\frac{\frac{8}{5}}{\frac{7}{10}}$

Practice Day 2



Let's practice what you've learned so far in this unit!

You will use task cards for this Practice Day. Record all of your responses here.

Task A: Fix It!

1. Explain the error:

Correct solution: _____

2. Explain the error:

Correct solution: _____

3. Explain the error:

Correct solution: _____

Task B: Ice Cream

1. Quarts:

2. Servings:

3. Percent error:

Task C: Everything on Sale!

1. Circle one: Yes or No

Explanation: _____

2. Circle one: A or B

Explanation: _____

3. Final cost:

Practice Day 2 (continued)

Task D: Up and Down

1. Equation:

2. Equation:

3. Equation:

Original price:

4. Equation:

Next year:

After two years:

Task E: In The House

1. Percent increase:

2. Percent increase:

3. Solution:

4. Your question:

Career Connection

While money doesn't grow on trees, it can grow – in many different types of bank accounts.

Are you interested in having a part-time job in the future? What would you like to do with any money that you earn? Having a bank account can be very helpful for saving money, even at a young age, according to economics professor Ebonya Washington. Many banks offer both a variety of accounts to help you organize how you spend and save money. Some banks will pay you interest as a percentage of the amount in your account.



Luis Molinero/
Shutterstock.com.

Financial analysts study financial data and provide advice and recommendations for individuals and companies in making financial decisions. They might use percentages to calculate with interest rates and summarize overall financial goals and performance. Some financial analysts work in areas of public finance, which is the field of study concerning how governments make and use money to fund the needs of their societies.



Meet Ebonya Washington

Ebonya Washington is an economics professor and a professor of international and public affairs at Columbia University. She earned her PhD in Economics at the Massachusetts Institute of Technology and was a Yale University professor prior to joining Columbia University in 2022. One of her areas of speciality is public finance.

Are you interested in finance or public finance? What can you do to learn more?

Community Connection

Research the sales tax in your city and state. Then research an item that is marked down by a given percentage from its original price.

Determine the final cost of the item, including sales tax. Show someone in your home or communities how you determined it.

Math Mindset

A number is increased by 10%.
The result is then decreased by 10%.
How does the final result compare to the original number?