Amplify Desmos Math CALIFORNIA

Grade 6

Volume 2: Units 5–8

Student Edition

About Amplify

Amplify is dedicated to collaborating with educators to create learning experiences that are rigorous and riveting for all students. Amplify creates K–12 core and supplemental curriculum, assessment, and intervention programs for today's students.

A pioneer in K–12 education since 2000, Amplify is leading the way in next-generation curriculum and assessment. All of our programs provide teachers with powerful tools that help them understand and respond to the needs of every student.

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Dear Student,

Welcome to Amplify Desmos Math California! We are excited to be partnering with you this year. You play an essential role in math class, so we wanted to reach out to introduce ourselves and tell you a bit about who we are.

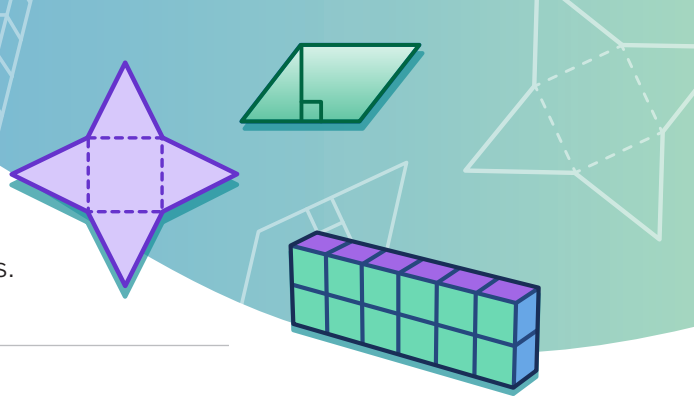
Amplify Desmos Math California is a team of math educators on a mission to support you and your classmates in learning math. We hope each lesson inspires you to use your creativity, ask questions, and discover connections between math concepts and the world around us.

Here is what you can expect this year:

- A blend of learning on both paper and devices.
- Interactive lessons that encourage you to ask questions, explore, settle disputes, create challenges for your classmates, and more!
- Digital activities that show what your ideas mean with plenty of chances for you to revise.
- Opportunities for you to engage with interesting and important big ideas in mathematics.

–The Amplify Desmos Math California Team





Unit 1 Area and Surface Area

In this unit, you will learn to calculate areas of polygons. You will also represent polyhedra with nets and calculate their surface areas.

Sub-Unit 1 Area 4

Explore: Tangram Paradoxes | How can you use tangram pieces to create tangram paradoxes? 5

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In this unit, you will be introduced to the concept of ratios. You will also represent ratios using double number lines, tables, and tape diagrams, and use ratio reasoning to solve problems.

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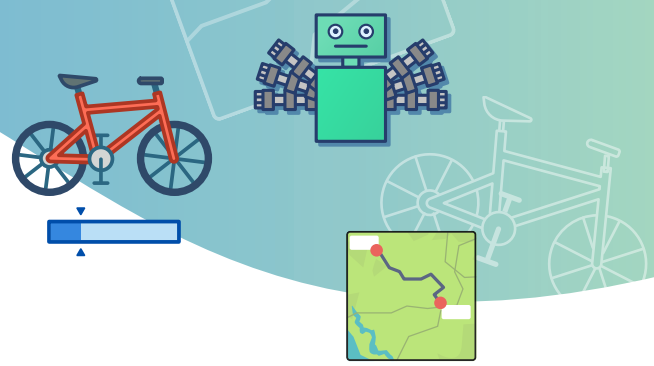
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In this unit, you will extend what you learned about dividing whole numbers to divide fractions by fractions. You will learn a variety of strategies, including making tape diagrams, creating common denominators, and rewriting equivalent multiplication problems using the reciprocal.

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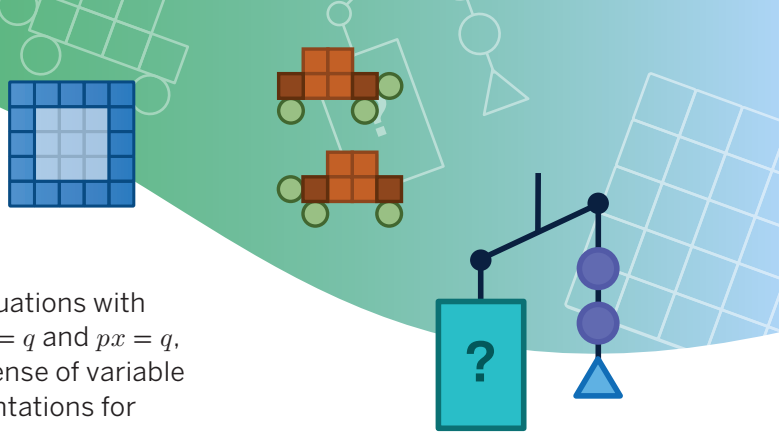
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Practice Day 4 629



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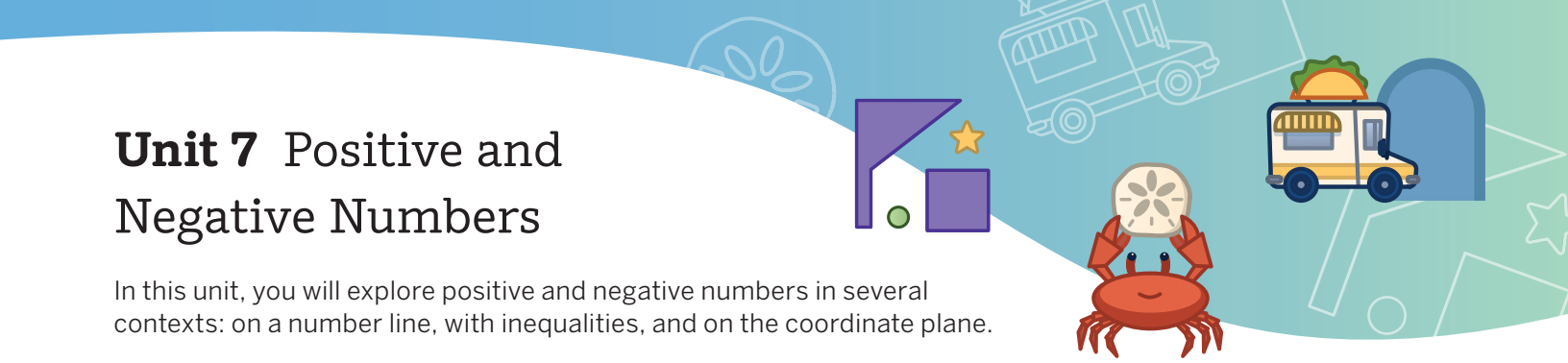
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In this unit, you will explore positive and negative numbers in several contexts: on a number line, with inequalities, and on the coordinate plane.

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In this unit, you will visualize data using dot plots, histograms, and box plots. You will also calculate measures of center and spread.

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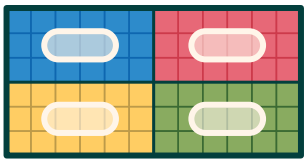
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Unit 5

Decimal Arithmetic

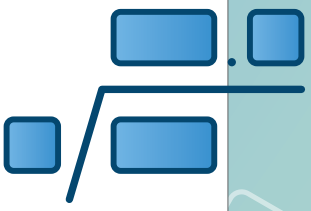


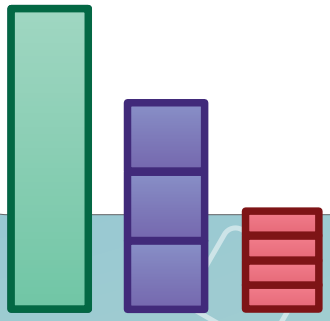
Big Ideas in This Unit

CC2 Fraction Relationships | Patterns Inside Numbers | CC3 Model the World

Questions for Investigation

- How do the place values in each decimal number in a calculation affect the place value of the result?
- How are strategies for multiplying decimals like strategies for dividing decimals? How are they different?

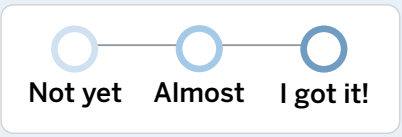




Explore: Precision and World Records
How do decimals set world records?

Watch Your Knowledge Grow

This is the math you'll explore in this unit. Rate your understanding to see how your knowledge grows!



I can . . .	Before	After
Use decimals to estimate and calculate with money.		
Represent decimals using tenths, hundredths, and thousandths.		
Add decimals using strategies such as diagrams, vertical calculations, and place value.		
Subtract decimals using strategies such as diagrams, vertical calculations, and place value.		
Write the sum of two values as one of their common factors times the sum of two whole numbers.		
Explain how numeric expressions related by the distributive property are equivalent.		
Use area models to reason about decimal multiplication.		
Multiply decimals using different strategies such as area models, fractions, and place value.		
Use different strategies to divide decimals.		
Make connections between decimal division and dividing fractions with common denominators.		

I can . . .	Before	After
Divide two multi-digit numbers with a remainder using the standard algorithm.		
Divide two multi-digit numbers without a remainder using the standard algorithm.		
Write an equivalent division expression in order to divide decimals.		
Use long division to divide multi-digit decimals.		
Compare strategies for dividing whole numbers and decimals.		
Add, subtract, multiply, and divide decimals to solve problems in context.		
Use ratio reasoning and decimal operations to solve real-world problems involving decimal values.		
Convert between percentages, fractions, and decimals.		
Make connections between percentages and decimals.		
Write and evaluate expressions with decimals to solve percentage problems.		

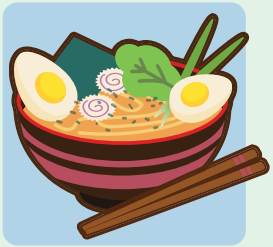
Sub-Unit

1

Adding and Subtracting Decimals



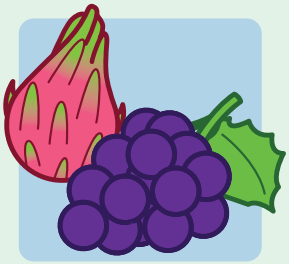
Explore
Precision and World Records



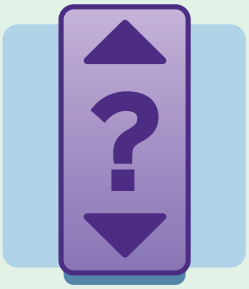
Lesson 1
Dishing Out Decimals



Lesson 2
Decimal Diagrams and Algorithms



Lesson 3
Fruit by the Pound



Lesson 4
Missing Digits

Name: ..... Date: ..... Period: .....

Model the World | Building Toward 6.NS.3, SMP.1, SMP.6

Explore: Precision and World Records

How do decimals set world records?

Warm-Up

1. A Swedish freeskier, Jesper Tjäder, set a world record by sliding along a rail on his skis.



a The number of meters recorded for the total distance is, 1 5 4 4 9 meters, but the decimal point is missing!

Where do you think the decimal point goes?

b As the decimal is "pinned," say the number aloud.

Name: ..... Date: ..... Period: .....

World's Longest Fingernails Challenge

At some point in 1979, Lee Redmond decided to let all of her fingernails continue to grow. Fast forward 30 years, and they would measure, in total, a staggering 8.65 meters (28 feet 4.5 inches). For nearly two decades, she held the World Record for the "Longest fingernails on a pair of hands ever (female)."



But wait! There's more.

In 2014, the World Record for the "Longest fingernails on a single hand ever" was recorded by Shridhar Chillal of India. They measured 909.6 centimeters (29 feet 10.1 inches) in total.

2. Whose fingernails are longer and by how many centimeters?

3. If Shridhar had let his nails grow to the exact same length on both hands, what would the total length of both hands have been?

Name: Date: Period:

World's Longest Fingernails Challenge (continued)

Your group will be assigned to recreate either Shridhar's fingernails or Lee's fingernails using strips of paper. Decide who in your group will wear the model.

Tape strips of paper together and then tape the long strips to this person's nails for a truly visual experience of a world record.

4. Record the measurements your group uses and creates for each fingernail in the table.

	Thumb	Pointer	Middle	Ring	Pinky	Total
Shridhar						
Lee						

Name: ..... Date: ..... Period: .....

Building Math Habits of Mind

Discuss:

- Which of these habits of mind did you strengthen during this activity?
- How did you use the one(s) you selected?

I can slow down and first make sense of a challenging problem before trying to solve it.

Not yet Almost I got it!

I can represent real-world problems using equations and inequalities and interpret their solutions within the context of the problem.

Not yet Almost I got it!

I can justify my thinking and ask questions to help me understand the thinking of others.

Not yet Almost I got it!

I can apply the math that I know to solve real-world problems, make assumptions and revise my thinking as needed.

Not yet Almost I got it!

I can select an appropriate tool to help me solve problems.

Not yet Almost I got it!

I can communicate my thinking and solutions clearly to others.

Not yet Almost I got it!

I can look for structure or patterns to help me solve problems.

Not yet Almost I got it!

I can look for repeated calculations and other repeated steps to make generalizations.

Not yet Almost I got it!

Name: ..... Date: ..... Period: .....

Model the World | Building Toward 6.NS.3, SMP.2, SMP. 3

Dishing Out Decimals

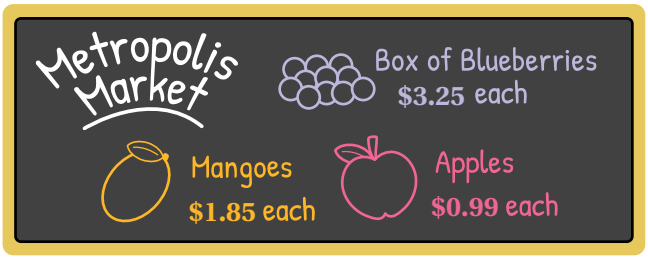


Let's use decimals to make estimations and rough calculations about money.

Warm-Up

1. Ramon has $10.00 to buy fruit for a smoothie.

Determine whether Ramon can buy each of the following.



a 2 mangoes, 2 boxes of blueberries, and 2 apples

b 1 mango, 1 box of blueberries, and 3 apples

c 3 boxes of blueberries

Name: ..... Date: ..... Period: .....

Cooking Challenge

In this cooking challenge, each contestant gets $20.00 to make a dish for 6 judges. Chef Fatima decided to make her signature recipe: Fatima's Flaming Fried Rice. Use the Metropolis Market Prices Sheet to help you answer the questions.



Fatima's Flaming Fried Rice
Serves 6 people

- 1 pound of brown rice
- 2.5 pounds of chicken thighs
- 2 heads of broccoli
- 3 jalapeños

2. Can Fatima make her recipe for less than $20.00? Explain your thinking.

3. Fatima wants to replace the 2.5 pounds of chicken with 2.5 pounds of tuna. Can Fatima still make her recipe for less than $20.00? Explain your thinking.

Activity 2

Name: ..... Date: ..... Period: .....

Make Your Own Dish

It's your turn to show off your culinary skills!

4. Choose ingredients from the Metropolis Market Prices Sheet to make a delicious dish for less than $20.00.

5. List each ingredient and the amount needed in the box below. Draw your dish and give it a name.

	Drawing of Your Dish
(Dish name) *Serves 6 people*	

6. Show or explain how you know that you can make your dish for less than $20.00.

7. About how much money will you have left over?

8. What additional ingredients can you afford with your leftover money?

You're invited to explore more.

9. You have $20.00 to spend on one ingredient from Metropolis Market. What would you buy? How much of it can you get?

Synthesis

10. One way to keep ingredients within a budget is to use estimation.

a Describe a strategy that is helpful for estimating costs at the grocery store.

b What other math can you do at the grocery store?

Summary 5.01

One of the most common, everyday places you might see decimals is at the grocery store. While you can use all sorts of strategies to calculate exact decimal values, estimation and rounding can help you do quick calculations in your daily life.

Let's see how estimation and rounding can help Adnan shop within his budget.

Adnan has $\$15.00$ to spend on groceries and wants to know if he can afford all of the ingredients for his favorite recipe.

- Spaghetti: $\$1.34$
- Diced tomatoes: $\$0.79$
- Chicken thighs: $\$4.99$
- Parmesan cheese: $\$4.38$

First, Adnan can round each price to a value that is easier to add using mental math.

- Spaghetti: $\$1.50$
- Chicken thighs: $\$5$
- Diced tomatoes: $\$1$
- Parmesan cheese: $\$4.50$

Then Adnan can add the rounded values to determine an estimate for the total cost.

$1.5 + 5 + 1 + 4.5 = \$12$

The ingredients will cost about $\$12$, so Adnan knows he can make his favorite recipe tonight!

Practice
5.01

Name: ..... Date: ..... Period: .....

1. Gabriela had $14.50. She spent $3.99 at the snack bar and $5.29 at the arcade. Does she have more or less than $7.00 left? Circle one.

More than $7.00 Less than $7.00

Show or explain your thinking.

Problems 2–3: Ishaan and his friends have $40.00 to spend at a restaurant. They are deciding between a pizza place and a salad place. A large pizza costs $7.50. A salad costs $6.00.

2. How many large pizzas can they buy? Show or explain your thinking.

3. How many salads can they buy? Show or explain your thinking.

Problems 4–5: Tickets to a show cost $5.50 each for adults and $4.25 each for students. A family is purchasing 2 adult tickets and 3 student tickets.

4. What is the total cost? Show or explain your thinking.

5. If the family pays $25.00, what is the exact amount of change they should receive? Show or explain your thinking.

Practice
5.01

Name: ..... Date: ..... Period: .....

Spiral Review

6. Aaliyah has $\frac{4}{5}$ gallons of juice. She wants to split it equally among 3 bottles. How much juice, in gallons, will there be in each bottle?

A. $\frac{1}{5}$ **B.** $\frac{4}{15}$ **C.** $\frac{12}{5}$ **D.** $\frac{16}{5}$

Problems 7–10: Use the symbols $<$, $>$, or $=$ to make each comparison true.

7. 0.19 ..... 0.91

8. 0.15 ..... 0.2

9. 1.75 ..... 1.750

10. 1.67 ..... 1.667

11. Before refrigerators existed, some people had blocks of ice delivered to their homes. The delivery wagon contained a storage box that was a rectangular prism. Consider a storage box that measures $7\frac{1}{2}$ feet by 6 feet by 6 feet. If the blocks of ice stored in the box were cubes with an edge length of $1\frac{1}{2}$ feet, how many blocks of ice could fit in the storage box?

A. 270

B. $\frac{33}{4}$

C. 80

D. 180

Name: ..... Date: ..... Period: .....

Model the World 6.NS.3, SMP.3, SMP.6

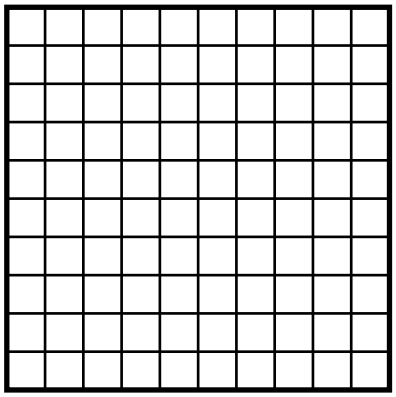
Decimal Diagrams and Algorithms

Let's add and subtract decimals.



Warm-Up

1 This large square represents 1.



Write 0.425 using different quantities of tenths, hundredths, and/or thousandths.

Write as many combinations as you can think of.

Tenths	Hundredths	Thousandths

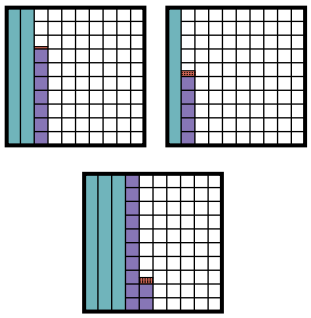
Activity 1

Name: ..... Date: ..... Period: .....

More Than One Way to Add

2 Prisha and Omar used different strategies to add $0.271 + 0.154$.

Prisha



= 0.271 + 0.154
= 0.425

Omar

$$\begin{array}{r} 0.271 \\ +\ 0.154 \\ \hline 0.3 \\ 0.12 \\ 0.005 \\ \hline 0.425 \end{array}$$

a Whose calculation is correct? Circle one.

Prisha's Omar's Both Neither

b Explain your thinking.

Name: ..... Date: ..... Period: .....

More Than One Way to Add (continued)

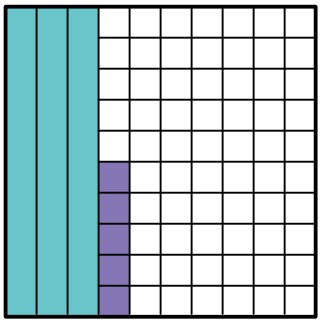
3 Group the representations that have the same value. You will have three groups.

a
$$\begin{array}{r} 0.048 \\ +\ 0.302 \\ \hline \end{array}$$

b
$$\begin{array}{r} 0.04 \\ +\ 0.010 \\ \hline \end{array}$$

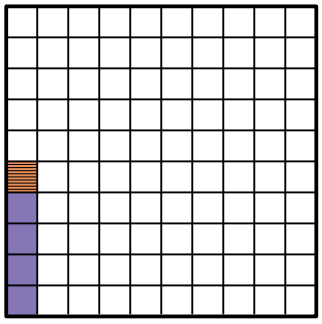
c
$$\begin{array}{r} 0.499 \\ +\ 0.001 \\ \hline \end{array}$$

d



e 50 hundredths

f



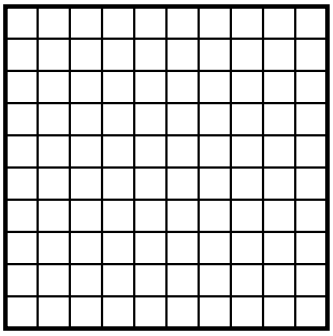
g 0.5

h 4 hundredths and 10 thousandths

i 0.35

Group 1	Group 2	Group 3

4 Determine the value of $0.275 + 0.135$.



Activity 2

Name: ..... Date: ..... Period: .....

Repeated Challenges

5 Determine the value for each sum or difference. Solve as many challenges as you have time for.

a $0.203 + 0.105$

b $0.15 + 0.1$

c $0.155 + 0.015$

d $0.5 + 0.151$

e $0.25 + 0.15$

f $0.01 + 0.002$

g $0.946 + 0.041$

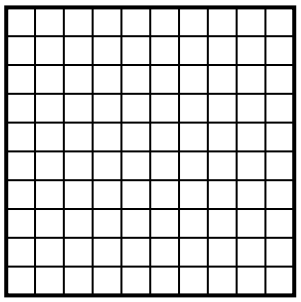
h $0.589 + 0.187$

Activity 3

Name: ..... Date: ..... Period: .....

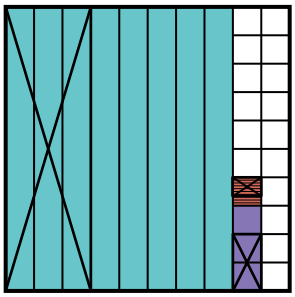
Decimal Differences

6 Determine the value of $0.2 - 0.11$.

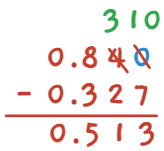


7 Adhara and Malik used different strategies to calculate $0.84 - 0.327$.

Adhara



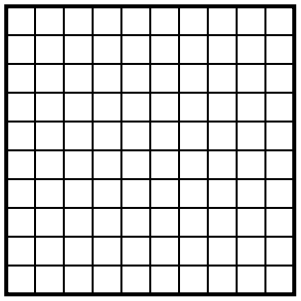
Malik



Discuss:

- How are these strategies alike? How are they different?
- Where do you see place value and regrouping in each strategy?

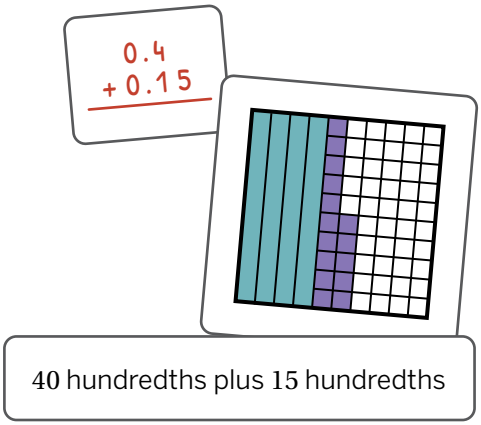
8 Determine the value of $0.562 - 0.17$.



9 Synthesis

Describe a strategy for adding decimals.

Use the example $0.4 + 0.15$ if it helps with your thinking.



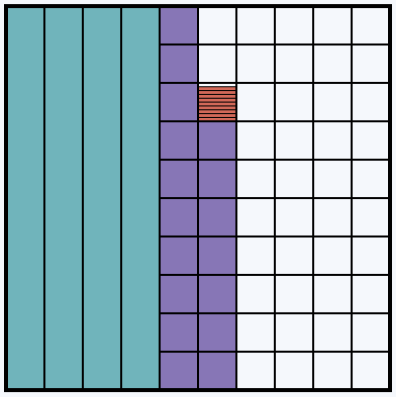
12 Summary 5.02

You can represent decimals in more than one way using words, diagrams, and decimal points. For example, six tenths, 0.6, sixty hundredths, and 0.60 all represent the same quantity.

Using multiple representations can help when you're adding or subtracting decimals.

Let's say we're calculating $0.189 + 0.39$.

Hundredths Chart



Vertical Calculation

$$\begin{array}{r} {\scriptstyle 1} \\ 0.189 \\ +\ 0.390 \\ \hline 0.579 \end{array}$$

Both the hundredths chart and vertical calculation show a total of 4 tenths, 17 hundredths, and 9 thousandths. 10 hundredths equals 1 tenth, so the final answer is 5 tenths, 7 hundredths, and 9 thousandths, or 0.579.

Practice
5.02

Name: ..... Date: ..... Period: .....

Problems 1–2: Determine each sum. Use a hundredths chart if it helps with your thinking.

1. $0.24 + 0.607$

2. $0.203 + 0.01$

Problems 3–4: Complete each calculation.

3.

$$\begin{array}{r} 1\ 4\ 2\ .\ 6 \\ -\quad 1\ .\ 4 \\ \hline \square\square\square.\square2 \end{array}$$

4.

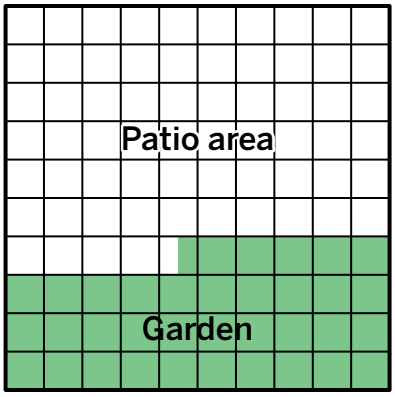
$$\begin{array}{r} 3\ 8\ .\ 6\ 0 \\ -\ 6\ .\ 7\ 5 \\ \hline \square\square.\square\square5 \end{array}$$

5. Determine the value of $0.15 - 0.08$. Use a hundredths chart if it helps with your thinking.

6. *Select all* the expressions that have a value greater than 1.

- ☐ **A.** $0.52 + 0.49$
- ☐ **B.** $0.7 + 0.04$
- ☐ **C.** $0.85 + 0.072$
- ☐ **D.** $0.903 + 0.09$
- ☐ **E.** $0.432 + 0.6$

7. This diagram represents a square backyard with a patio area and a garden. The shaded area represents the garden, and the unshaded area represents the patio. Each small square represents 1 hundredth of the total backyard area.



Which equation represents the difference between the size of the garden and the size of the patio area?

A. $1 - 0.645 = 0.355$

B. $0.355 + 0.645 = 1$

C. $0.355 + 0.355 = 0.71$

D. $0.645 - 0.355 = 0.29$

Practice
5.02

Name: ..... Date: ..... Period: .....

Spiral Review

Problems 8–10: A school band has 70 students. 50% of them are sixth graders, 30% are seventh graders, and the rest are eighth graders.

8. How many band members are sixth graders?

9. How many band members are seventh graders?

10. What percentage of the band members are eighth graders? Explain your thinking.

11. A car travels 75 miles in 60 minutes at a constant speed. How far does the car travel in 18 minutes?

Consider using the table to help with your thinking.

Minutes	Distance (mi)
60	75
6	
18	

Unit 5
Lesson
3

Name: ..... Date: ..... Period: .....

Model the World 6.NS.3, SMP.3, SMP.6

Fruit by the Pound

Let's use decimal addition and subtraction to solve problems.

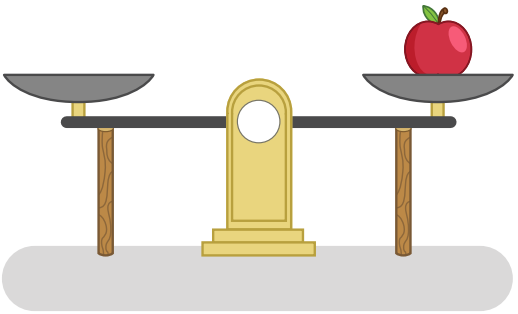


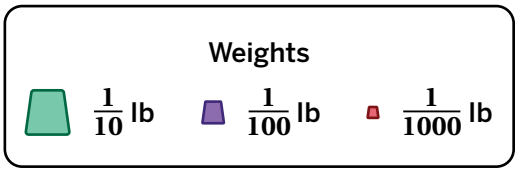
Warm-Up

1 An apple weighs 0.321 pounds.

Balance the scale by writing the quantity of each weight in the table.

Try to balance the scale in more than one way.





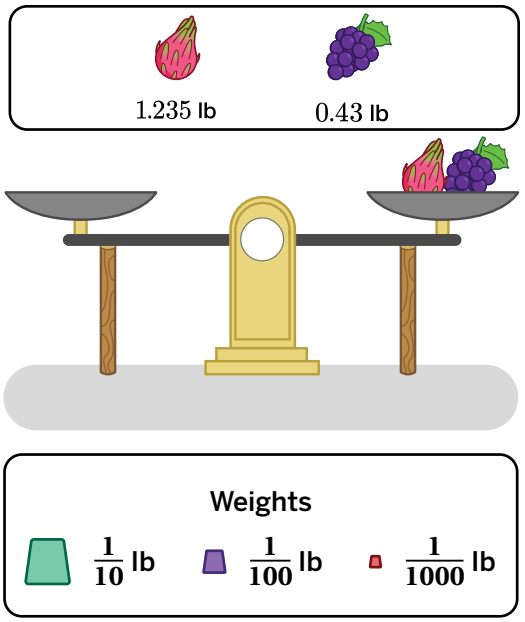
$\frac{1}{10}$ lb	$\frac{1}{100}$ lb	$\frac{1}{1000}$ lb

Name: ..... Date: ..... Period: .....

Weighing Fruit

2 Juliana bought a dragon fruit and a bunch of grapes.

What is the total weight of the fruits?



3 Santiago and Isaiah each set up vertical calculations to find $1.235 + 0.43$.

Whose calculation is correct? Circle one.

Santiago's Isaiah's Both Neither

Explain your thinking.

Santiago

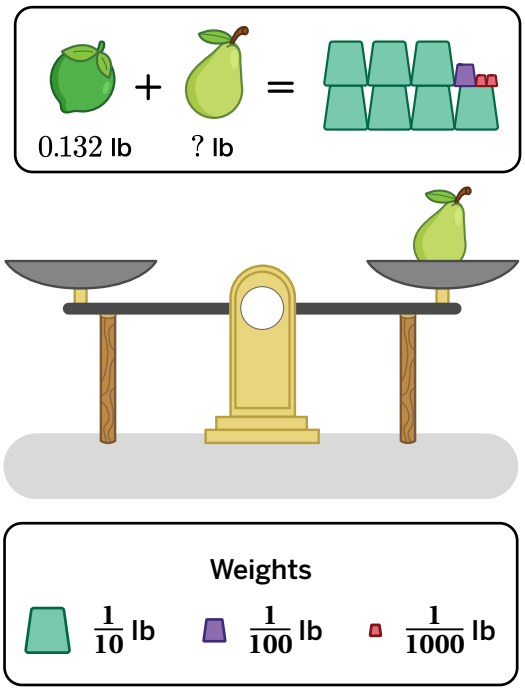
$$\begin{array}{r} 1.235 \\ +\ 0.43 \\ \hline 1.278 \end{array}$$

Isaiah

$$\begin{array}{r} 0.430 \\ +\ 1.235 \\ \hline 1.665 \end{array}$$

4 A lime and a pear weigh 0.712 pounds total.

If the lime weighs 0.132 pounds, how much does the pear weigh?



Activity 2

Name: ..... Date: ..... Period: .....

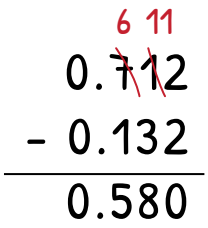
Fruit Challenges

5 Here is the work that Leo did to calculate $0.712 - 0.132$.

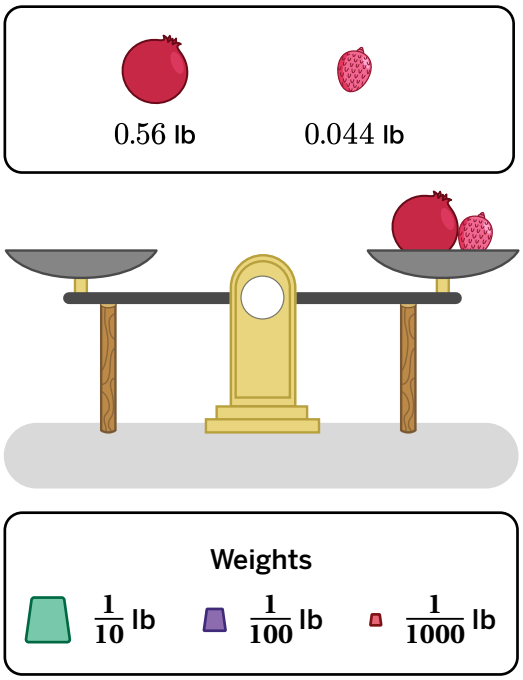


Discuss:

- What does the 6 represent in Leo's work?
- What does the 11 represent in Leo's work?

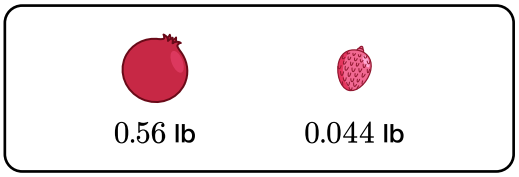


6 What is the total weight of the fruits?



7 Rafael says: 56 *plus* 44 *is* 100, *so the pomegranate and the lychee make a whole pound.*

Describe the error that Rafael made.



Activity 3

Name: ..... Date: ..... Period: .....

Repeated Challenges

8 Determine the value for each sum or difference. Solve as many challenges as you have time for.

a $0.204 + 0.106$

b $0.16 - 0.11$

c $0.145 + 0.025$

d $0.6 - 0.152$

e $0.957 - 0.032$

f $0.478 - 0.189$

g $0.35 + 0.25$

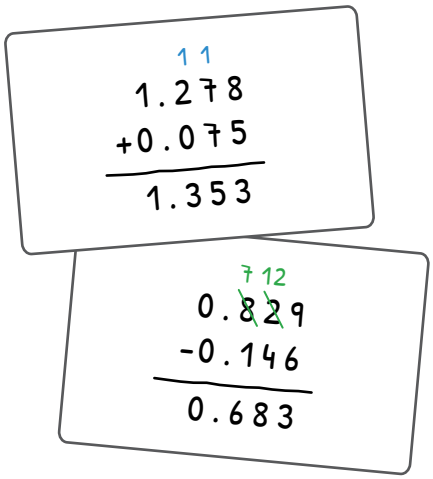
h $0.02 + 0.003$

9 Synthesis

Discuss: What are some advantages and disadvantages of using vertical calculations to add or subtract decimals?

Advantages:

Disadvantages:



12 Summary 5.03

When adding and subracing decimals, it can be helpful to organize the numbers by their place values.

Let's say a lemon and a strawberry together weigh 0.35 pounds, and the lemon on its own weighs 0.235 pounds.

How much does the strawberry weigh?

$$\begin{array}{r} 4\ 10 \\ 0.\ 3\ \cancel{5}\ \cancel{0} \\ -0.\ 2\ 3\ 5 \\ \hline 0.\ 1\ 1\ 5 \end{array}$$

You can use vertical calculations to rewrite the total weight, 0.35, as 3 tenths, 4 hundredths, and 10 thousandths. Now you can subtract the weight of the lemon from the total to determine the weight of the strawberry.

Name: ..... Date: ..... Period: .....

1. Circle the correct vertical calculation for $16 - 1.4$.

$$\begin{array}{r} 1\ 6.0 \\ -1.4\ 0 \\ \hline 0.2\ 0 \end{array} \qquad \begin{array}{r} 1\ 6.0 \\ -\ 1.4 \\ \hline 1\ 4.6 \end{array}$$

2. Explain how you know which calculation is incorrect in Problem 1.

Problems 3–6: Determine the value of each expression.

3. $0.27 + 0.04$

4. $0.35 - 0.1$

5. $0.43 + 0.09$

6. $0.755 - 0.405$

7. A student says: *We cannot subtract 1.97 from 20 because 20 does not have any hundredths to subtract from.*

Is the student correct? Explain your reasoning.

Practice
5.03

Name: ..... Date: ..... Period: .....

8. Gabriel needs 4 cups (0.946 liters) of boiling water to sterilize his lab equipment. Gabriel left the water boiling for several minutes and returned to find only 0.89 liters of water remaining in the container. To determine the amount of water that evaporated, Gabriel subtracts $0.946 - 0.89$. Which best explains Gabriel's work?

Gabriel's work

8 13 16
0.946
−0.89
0.857

A. Gabriel's work is incorrect. When he was done subtracting, he should have only moved the decimal point two places.

B. Gabriel's work is incorrect. He should have lined up the decimal points in both numbers to make sure he was only subtracting digits with the same place value.

C. Gabriel's work is incorrect. He should have added another 0 in front of 0.89 to make sure there were an equal number of digits.

D. Gabriel's work is correct. He correctly determined that 0.857 liters of water evaporated.

Spiral Review

Problems 9–10: Here is a diagram showing the prices at a store.



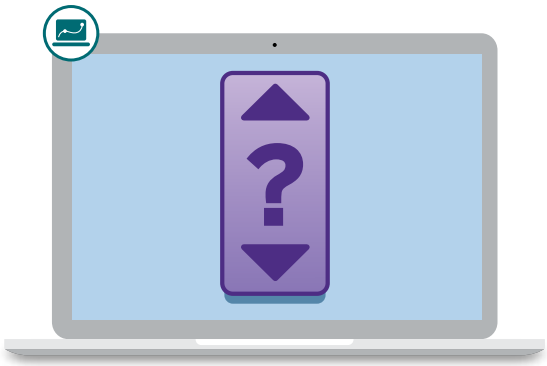
9. Hoang would like to buy a pencil, a hat, and 2 binders. He estimates that the total cost will be less than $20.00. Is the total cost less than $20.00? Explain your reasoning.

10. Estimate the number of pencils Hoang could buy with $5.00.

11. A rectangular prism measures $2\frac{1}{2}$ centimeters by 8 centimeters by $1\frac{1}{2}$ centimeters. What is its volume?

Name: ..... Date: ..... Period: .....

Model the World 6.NS.3, SMP.3, SMP.6, SMP.8



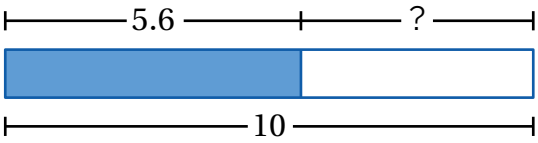
Missing Digits

Let's solve number puzzles.

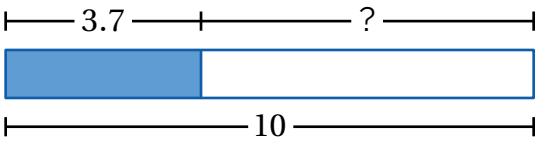
Warm-Up

1 Determine the missing number for each challenge.

a 5.6 plus what number equals 10?



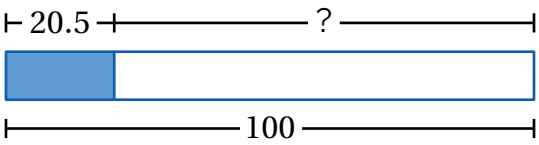
b 3.7 plus what number equals 10?



c 0.35 plus what number equals 1?



d 20.5 plus what number equals 100?

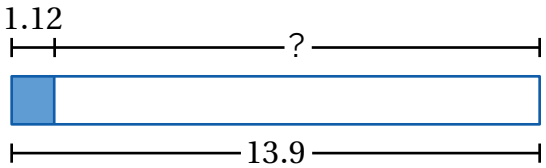


Activity 1

Name: ..... Date: ..... Period: .....

Vertical Calculations

2 1.12 plus what number equals 13.9?



3 Here is how Kayleen calculated 13.9−1.12.

What could you say to her to help her understand her mistake?

$$\begin{array}{r} 13.9 \\ -1.12 \\ \hline 2.7 \end{array}$$

Activity 2

Name: ..... Date: ..... Period: .....

Number Puzzles

4 Kayleen made a number puzzle.

Fill in the missing digit that makes the statement true.

$$\begin{array}{r} 2.5 \\ +7.? \\ \hline 9.8 \end{array}$$

5 Here is Yosef's number puzzle.

Fill in the missing digits that make the statement true.

$$\begin{array}{r} 0.0?85 \\ -0.00?9 \\ \hline 0.005? \end{array}$$

6 Match each number puzzle to its missing digit. Two digits will have no match.

1	2	3	5	6

a

$$\begin{array}{r} 0.58 \\ -0.322 \\ \hline 0.2?8 \end{array}$$

.....

b

$$\begin{array}{r} 59.4? \\ -40.6 \\ \hline 18.83 \end{array}$$

.....

c

$$\begin{array}{r} 9.874 \\ +0.?26 \\ \hline 10 \end{array}$$

.....

Name: ..... Date: ..... Period: .....

Number Puzzles (continued)

7 Here is Aba's puzzle.

Fill in the missing digits to make the statement true.

$5.8 - ?.3? = 3.?3$

8 Aba used addition to check her solution to her number puzzle.

Does this sum prove that her solution is correct? Circle one.

Yes No I'm not sure

Explain your thinking.

$$\begin{array}{r} {\scriptstyle 1} \\ 3.43 \\ +\ 2.37 \\ \hline 5.80 \end{array}$$

Activity 3

Name: _____ Date: _____ Period: _____

Repeated Challenges

9 Fill in the missing digits to make each statement true. Solve as many challenges as you have time for.

a 2.5 + ?.6 = 9.?

b 9.5 − 5.?5 = ?.15

c 0.404 + ?.??6 = 1.000

d ?4.?3 + 224.17 = 318.80

e 7 − ?.??33 = 3.4567

f 0.7 − 0.68? = 0.?12

You're invited to explore more.

10 How many different solutions are there to this problem? Explain your thinking.

$$\begin{array}{r} 5.? \\ -\ ?.?3 \\ \hline 2.2? \end{array}$$

11 Synthesis

Describe a strategy for determining the missing digits in this subtraction problem.

$$\begin{array}{r} 3 \\ -\,1.0\square \\ \hline \square.\square 8 \end{array}$$

14 Summary 5.04

One strategy that can help you make sense of decimal addition and subtraction is vertical calculations.

To use a vertical calculation, you just align numbers by place value so that you're adding or subtracting ones with ones, tenths with tenths, hundredths with hundredths, and thousandths with thousandths.

Here's a vertical calculation. To check if the calculation is correct, you could either estimate or use the opposite operation (addition).

$$\begin{array}{r} 6.2 \\ -\,2.5 \\ \hline 3.7 \end{array}$$

You could estimate that $6.2 - 3 = 3.2$, so your difference should be larger than 3.2.

You could also use addition to check your work, adding 2.5 to 3.7 to get 6.2.

Practice
5.04

Name: ..... Date: ..... Period: .....

Problems 1–8: Fill in the blanks with the digits that make each statement true.

1.

$$\begin{array}{r} 1.036 \\ +\ \square.\square\square\square \\ \hline 4.000 \end{array}$$

2.

$$\begin{array}{r} 38.60 \\ -\ 6.75 \\ \hline \square\square.\square 5 \end{array}$$

3.

$$\begin{array}{r} 241.76 \\ -\ 2.18 \\ \hline \square\square\square.\square 8 \end{array}$$

4.

$$\begin{array}{r} 0.404 \\ +\ \square.\square\square\square \\ \hline 1.000 \end{array}$$

5.

$$\begin{array}{r} 9.8765 \\ +\ \square.\square\square\square\square \\ \hline 10.0000 \end{array}$$

6.

$$\begin{array}{r} 0.7 \\ -\ \square.\square\square\square \\ \hline 0.012 \end{array}$$

7.

$$\begin{array}{r} 7 \\ -\ \square.\square\square\square\square \\ \hline 3.4567 \end{array}$$

8.

$$\begin{array}{r} 70 \\ -\ 6\square.\square\square\square\square \\ \hline 0.0089 \end{array}$$

Name: ..... Date: ..... Period: .....

Problems 9–10: The label on a bag of chocolates states that there are 0.384 pounds of chocolates. The actual weight of the chocolates is 0.3798 pounds.

9. Is the actual weight of the chocolates heavier or lighter than the weight written on the label? Explain how you know.

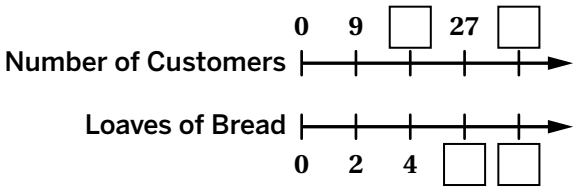
10. What is the difference, in pounds, between the weight on the label and the actual weight of the chocolates?

11. What is $99.22 - 78.095$?

Spiral Review

Problems 12–13: For every 9 customers, a chef prepares 2 loaves of bread.

12. The double number line and table show the number of loaves prepared by the chef for different numbers of customers. Complete each representation with the missing values.



Number of Customers	Number of Loaves
9	2
	4
27	
	14

13. If the chef prepares 20 loaves, how many customers can they serve?

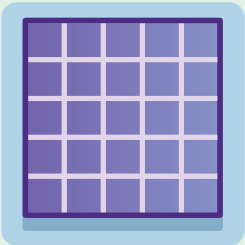
14. How many loaves does the chef prepare for 63 customers?

15. What fraction of a loaf is prepared for each customer?

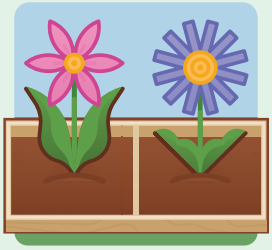
Sub-Unit

2

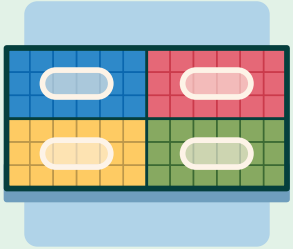
Multiplying Decimals



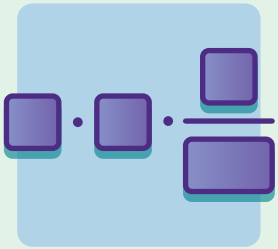
Lesson 5
Decimal Multiplication



Lesson 6
Garden Arrangements



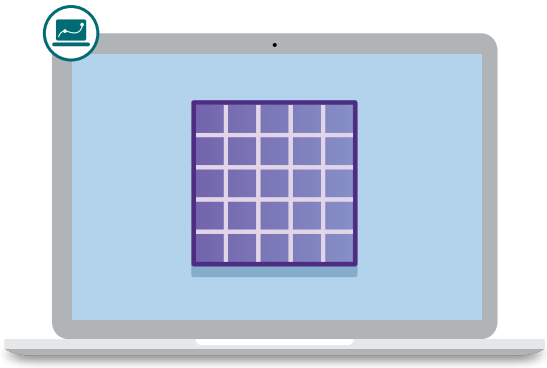
Lesson 7
Multiplying With Areas



Lesson 8
Multiplication Methods

Name: ..... Date: ..... Period: .....

Model the World 6.NS.3, SMP.1, SMP.8



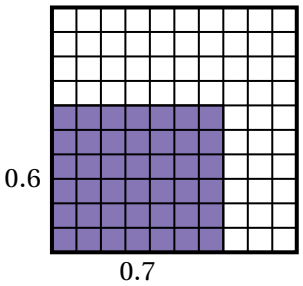
Decimal Multiplication

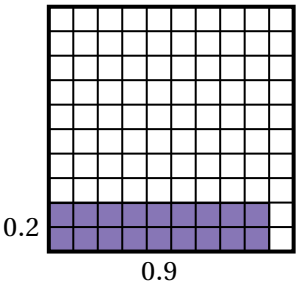
Let's explore decimal multiplication using place value.

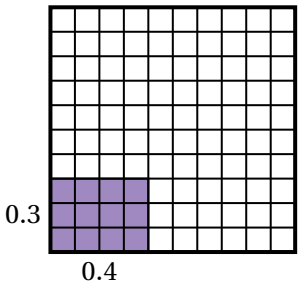
Warm-Up

1 The square has a length and a width of one.

a Take a look at these shaded rectangles with different lengths and widths.

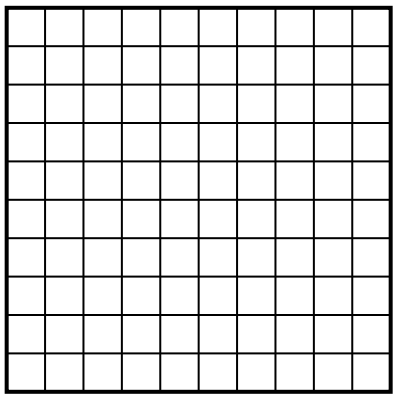






b Try to create shaded rectangles with the following areas:

Length	Width	Area
		0.16
		0.24
		0.30

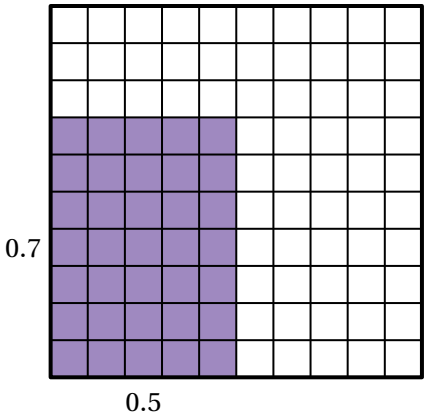


Name: ..... Date: ..... Period: .....

Keepin' It One Hundredth

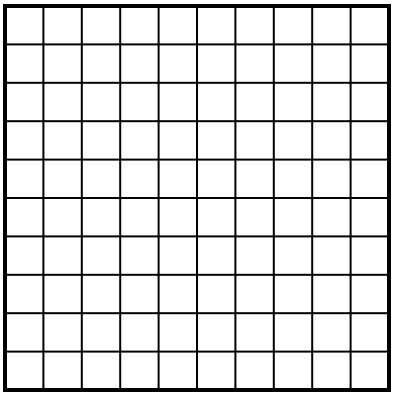
2 Makayla says that $0.5 \cdot 0.7 = 0.35$.

Use the diagram to show or explain why this makes sense.



3 Multiply $0.4 \cdot 0.2$.

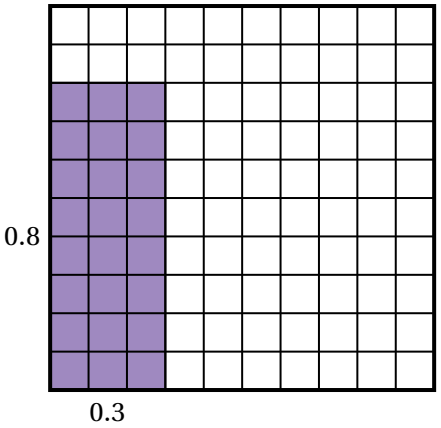
Write your answer as a decimal.

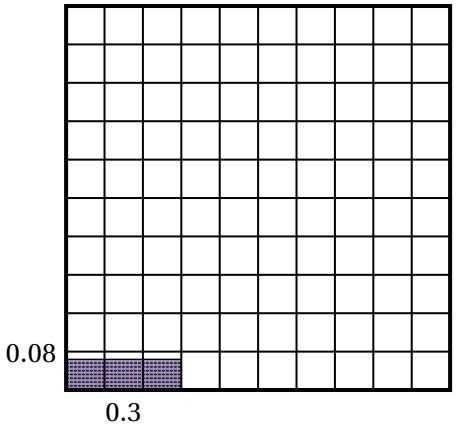


Name: ..... Date: ..... Period: .....

Keepin' It One Hundredth (continued)

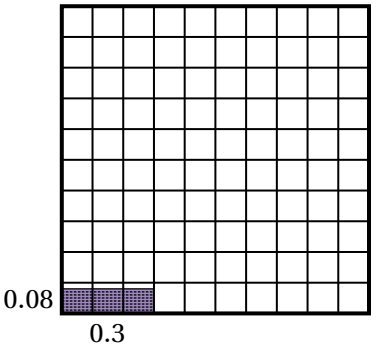
4 Here are two diagrams that represent different multiplication problems.





Discuss: How are the diagrams alike? How are they different?

5 What is $0.3 \cdot 0.08$?



6 This is Jayden's work for multiplying $0.3 \cdot 0.08$.

$$0.3 \cdot 0.08$$

$$\frac{3}{10} \cdot \frac{8}{100}$$

$$\frac{24}{1000} = 0.024$$

a **Discuss:** What is Jayden's strategy?

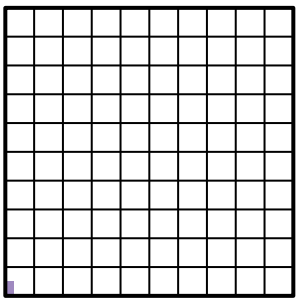
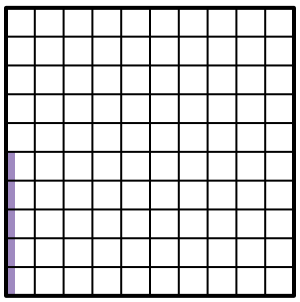
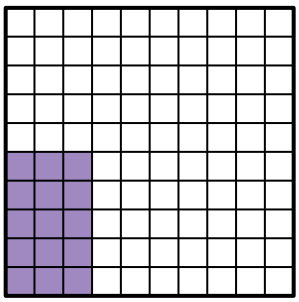
b Show or explain how you would use Jayden's strategy to multiply $0.02 \cdot 0.9$.

Name: ..... Date: ..... Period: .....

Clicking Into Place Value

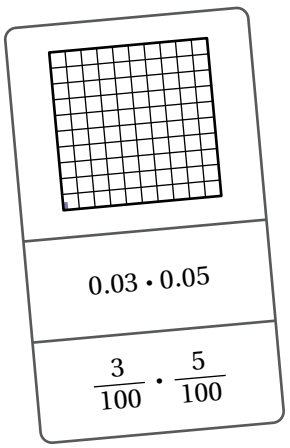
7 Match each area with its equivalent expressions.

$\frac{15}{100}$	$0.3 \cdot 0.5$	0.015
$0.03 \cdot 0.05$	$\frac{3}{100} \cdot \frac{5}{100}$	$0.03 \cdot 0.5$

Area 1	Area 2	Area 3
		

8 Jayden matched these cards on the previous screen and noticed that the product was missing.

Calculate $0.03 \cdot 0.05$.



Activity 3

Name: ..... Date: ..... Period: .....

Repeated Challenges

9 Solve as many challenges as you have time for.

a $0.04 \cdot 0.6$

b $0.03 \cdot 0.02$

c $0.2 \cdot 0.007$

d $0.003 \cdot 0.3$

e $0.5 \cdot 0.4$

f $0.001 \cdot 0.08$

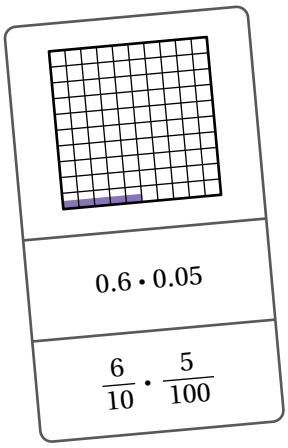
g $1 \cdot 0.05$

h $0.4 \cdot 0.005$

10 Synthesis

Show or describe how decimals, fractions, and the hundredths chart are related.

Use the example if it helps with your thinking.

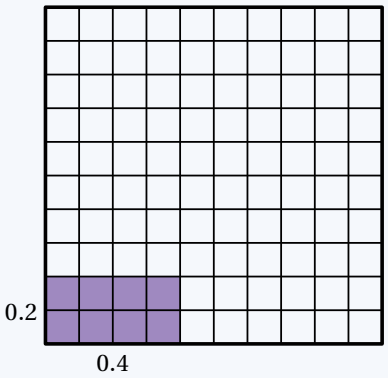


13 Summary 5.05

Here are two strategies for multiplying decimals:

Using an area model

This method allows you to visualize the product. For example, the hundredth chart shows $0.4 \cdot 0.2$. The total shaded area gives the product. However, area models can be challenging to use for decimal numbers extending to the thousandths place or smaller.



Area = length $\cdot$ width
$= 0.08$

Writing the decimals as equivalent fractions

This approach involves converting decimals to equivalent fractions. In this strategy you would:

- Rewrite 0.4 as $\frac{4}{10}$ and 0.2 as $\frac{2}{10}$
- Multiply the fractions: $\frac{4}{10} \cdot \frac{2}{10} = \frac{8}{100}$
- Convert the result back into a decimal: $\frac{8}{100} = 0.08$

Practice
5.05

Name: ..... Date: ..... Period: .....

Problems 1–4: Determine the value of each expression.

1. $20 \cdot 40$

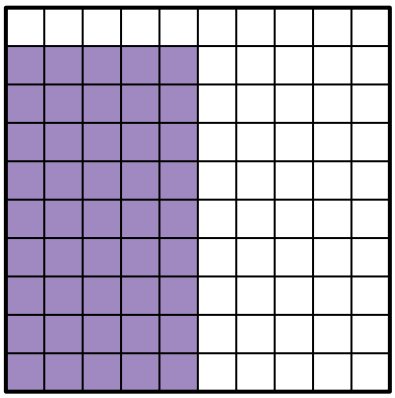
2. $200 \cdot 40$

3. $2 \cdot 40$

4. $2 \cdot 0.4$

Problems 5–6: Here is an expression: $0.5 \cdot 0.9$.

5. Explain why the diagram represents $0.5 \cdot 0.9$.



6. What is the value of $0.5 \cdot 0.9$?

7. Select *all* the expressions that have the same value as $0.05 \cdot 0.6$.

- ☐ **A.** $5 \cdot \frac{1}{100} \cdot 6 \cdot \frac{1}{10}$
- ☐ **B.** $5 \cdot 6 \cdot \frac{1}{1000}$
- ☐ **C.** $5 \cdot 0.001 \cdot 6 \cdot 0.01$
- ☐ **D.** 0.03
- ☐ **E.** 0.003

Name: ..... Date: ..... Period: .....

Problems 8–9: Emiliano attempted to calculate $0.003 \cdot 0.007$. Here is his work.

$$0.003 \cdot 0.007 = \frac{3}{1000} \cdot \frac{7}{1000}$$
$$= \frac{21}{1000} = 0.021$$

8. What do you think Emiliano's mistake might be?

9. What is the value of $0.003 \cdot 0.007$?

Problems 10–11: Determine the value of each expression.

10. $0.3 \cdot 0.2$

11. $1.2 \cdot 5$

Spiral Review

12. A plumber has 52.2 meters of PVC pipe to use on a job. On the first day, she used 21.863 meters of the pipe. How many meters of pipe does she have left after the first day?

A. 30.663 **B.** 30.337 **C.** 30.763 **D.** 30.237

Problems 13–14: Roberto's family is driving on a particular route between Guanajuato and San Miguel de Allende in Mexico. The distance of their route is 96.7 kilometers.

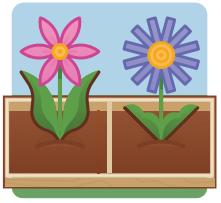
13. Roberto says they have already traveled 38.9 kilometers. How much farther do they need to travel?

14. The family needs to arrive in San Miguel de Allende for a wedding in 45 minutes. If they average 80 kilometers per hour, will they make it on time? Explain your thinking.

Name: ..... Date: ..... Period: .....

Patterns Inside Numbers 6.NS.4, SMP.1, SMP.3

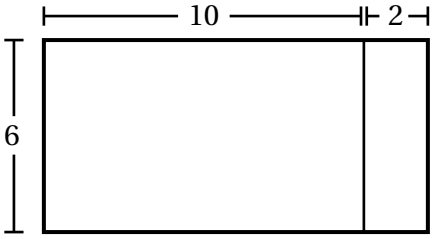
Garden Arrangements



Let's create expressions using area models.

Warm-Up

1. What is the area of this entire rectangle?



2. Write as many expressions as you can to represent the area of the entire rectangle.

Activity 1

Name: ..... Date: ..... Period: .....

Planting Flowers

3. Sai and Ichiro planted pink and white lilies in their family garden. They each wrote two different expressions to represent the total number of lilies.



Sai	Ichiro
$2(3 + 4)$	$6 + 8$

a **Discuss:** Select *one* person and explain how you think they came up with their expression.

b Are the values of the expressions the same? Show or explain your thinking.

4. Select *all* the expressions that represent the total number of tulips.



- ☐ **A.** $3(5) + 3(4)$
- ☐ **B.** $15 + 12$
- ☐ **C.** $3 \cdot 5 \cdot 4$
- ☐ **D.** $3(5 + 4)$
- ☐ **E.** $3(9)$

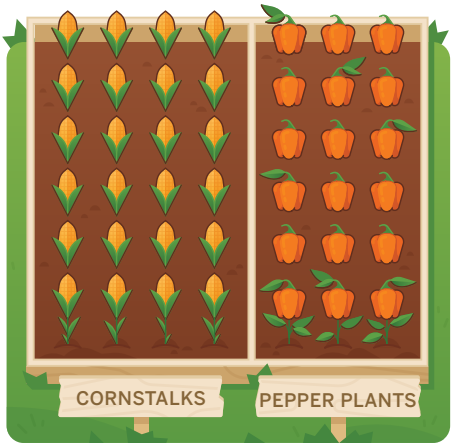
Name: Date: Period:

Garden Arrangements

Sai's family also has a vegetable garden. Sai wants to plant 24 cornstalks and 18 pepper plants and wants to have the same number of rows for both types of plant.

5. Sai created the plan shown as one option for organizing the garden.

Create as many other options as you can of ways Sai can create a garden with equally-spaced plants.



Name: _____ Date: _____ Period: _____

Garden Arrangements (continued)

6. Choose *one* of the options that you created. Write as many expressions as you can to represent that garden arrangement.

7. Fill in the blanks to represent the total number of cornstalks and pepper plants in Sai's garden.

3	4	6	18	24

_____ + _____ = _____ (_____ + _____)

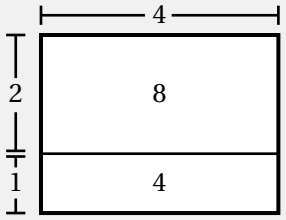
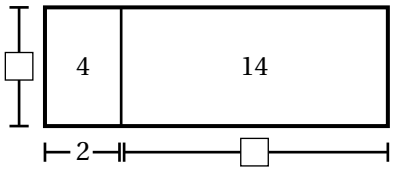
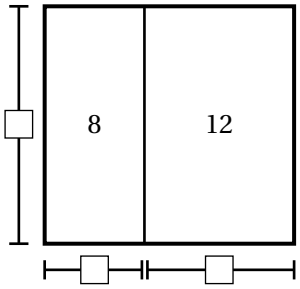
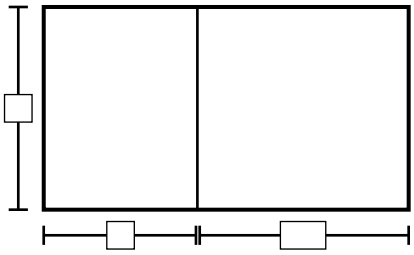
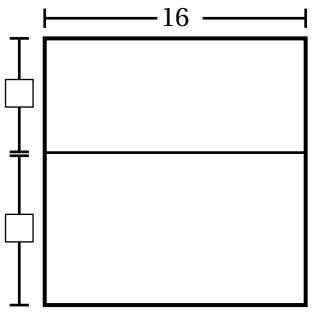
8. **Discuss:**

- What does the left side of the equation tell you about Sai's garden? The right side?
- When would it be helpful to use the left side of the equation? The right side?

Name: ..... Date: ..... Period: .....

Two Expressions, Same Area

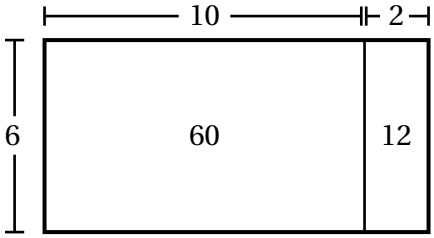
9. Fill in the missing values of each area model. Write two expressions to represent the total area.

	Area Model	Expression 1	Expression 2
		$8 + 4$	$4(2 + 1)$
a			
b			
c			$7(8 + 13)$
d		$48 + 64$	

Synthesis

10. **Discuss:** How can you write multiple expressions to describe the area of a rectangle?

Use this rectangle if it helps with your thinking.



Summary 5.06

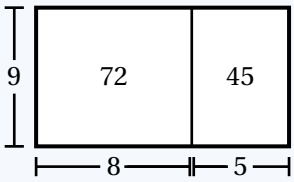
We can use common factors to create equivalent expressions.

For example, since 7 is a common factor of each term in the expression $21 + 14$, you can create the equivalent expression $7(3 + 2)$. This is an example of the **distributive property**.

The distributive property tells us that $a(b + c) = ab + ac$. This means that multiplying a number by the sum of two or more terms is equal to multiplying the number by each term separately before adding the products together.

We can use area models to determine equivalent expressions.

Let's write two expressions for this area model.



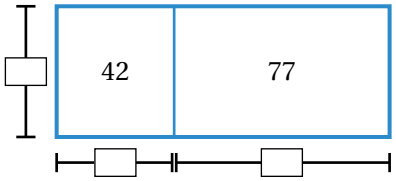
- $72 + 45$ represents the sum of the areas of the smaller rectangles.
- $9(8 + 5)$ represents the length of the whole rectangle multiplied by the width. This expression shows us that 9 is a common factor of 72 and 45.

Practice
5.06

Name: _____ Date: _____ Period: _____

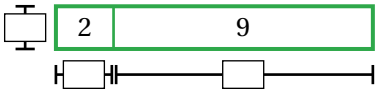
Problems 1–4: Use the area models to rewrite each expression using parentheses.

1. $42 + 77$



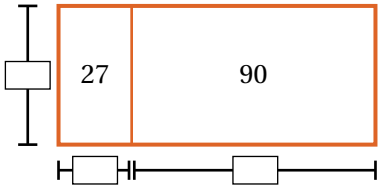
_____ (_____ + _____)

2. $2 + 9$



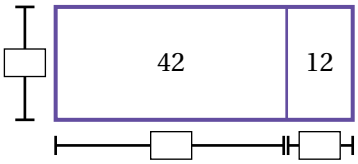
_____ (_____ + _____)

3. $27 + 90$



_____ (_____ + _____)

4. $42 + 12$

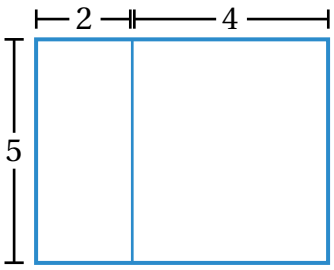


_____ (_____ + _____)

5. Select *all* the expressions that represent the area of the largest rectangle in figure A.

- ☐ **A.** $5(2 + 4)$
- ☐ **B.** $5 \cdot 2 + 4$
- ☐ **C.** $5 \cdot 2 + 5 \cdot 4$
- ☐ **D.** $5 \cdot 2 \cdot 4$
- ☐ **E.** $5 + 2 + 4$
- ☐ **F.** $5 \cdot 6$

Figure A



Name: _____ Date: _____ Period: _____

6. Which expression is equivalent to $54 + 66$ *and* uses the greatest common factor of the two numbers being added in the expression?

A. $2(27 + 33)$

B. $3(18 + 22)$

C. $6(9 + 11)$

D. $11(5 + 6)$

Spiral Review

Problems 7–10: Evaluate each expression.

7. $3.025 + 0.0268$

8. $6.105 - 1.0415$

9. $3.4 - 0.005$

10. $4.5 + 0.009$

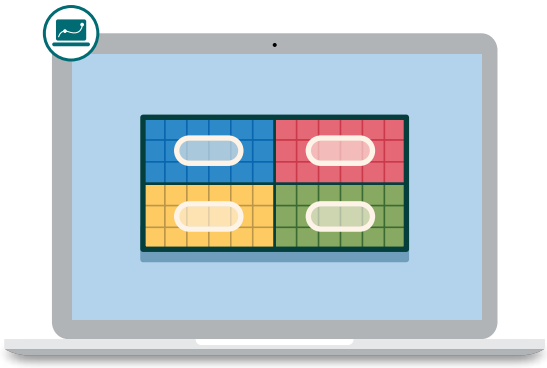
Problems 11–12: Determine the missing digits that will make each addition problem true.

11.
$$\begin{array}{r} 4.3\square \\ +\ \square.15 \\ \hline 6.\square 2 \end{array}$$

12.
$$\begin{array}{r} 1.5\square \\ +\ \square.38 \\ \hline 1.\square 4 \end{array}$$

Name: ..... Date: ..... Period: .....

Model the World 6.NS.3, SMP.1, SMP.3, SMP.6

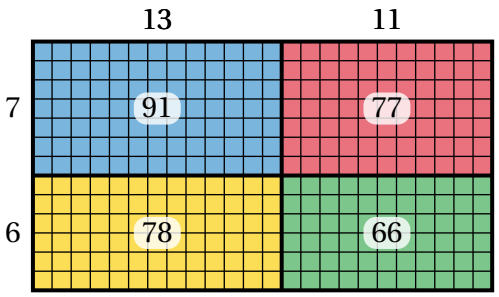


Multiplying With Areas

Let's use area models to multiply decimals.

Warm-Up

1 Diego likes using area models to multiply whole numbers.



a Let's look at different ways to split this 24-by-13 rectangle.

b What do you notice? What do you wonder?

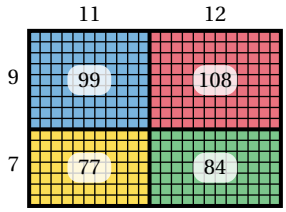
I notice:

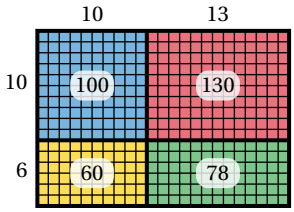
I wonder:

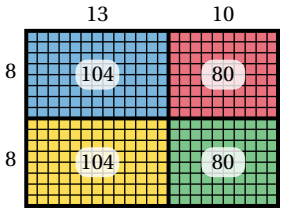
Name: ..... Date: ..... Period: .....

Creating an Area Model

2 Here are some new area models. Use them to multiply $23 \cdot 16$.



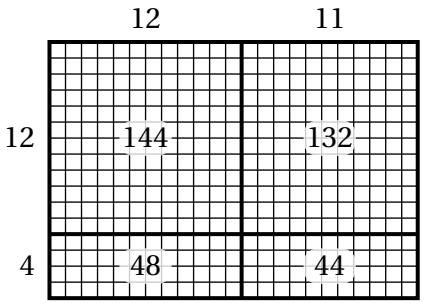




3 Mayra and Taylor made different area models to multiply $23 \cdot 16$.

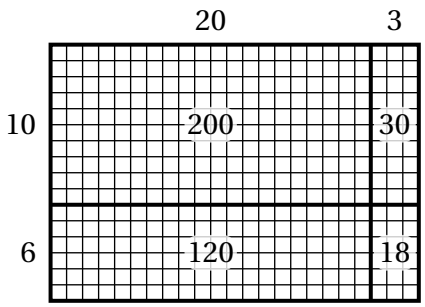
Discuss: Which area model is more helpful? Explain your thinking.

Mayra



144 + 132 + 48 + 44 = 368

Taylor



200 + 30 + 120 + 18 = 368

Name: ..... Date: ..... Period: .....

Creating an Area Model (continued)

4 Diego wonders if area models could also help him multiply decimals like $2.3 \cdot 1.6$.



a Show how you would split this rectangle into smaller parts to multiply $2.3 \cdot 1.6$.

b Explain why you split it that way.

5 Use your area model to calculate $2.3 \cdot 1.6$.

Name: ..... Date: ..... Period: .....

Calculating With an Area Model

6 **a** Use the digital activity to create an area model that helps you multiply $4.5 \cdot 2.9$.

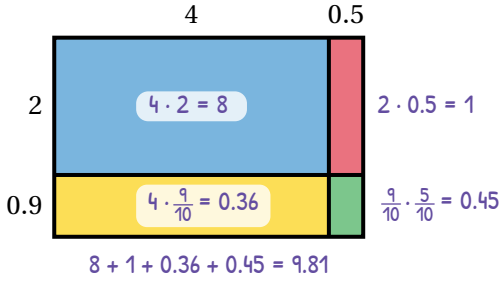
b Calculate the area of each part of your area model. The total area will be calculated for you.



7 Diego made an error while multiplying $4.5 \cdot 2.9$.

a Circle the error in Diego's work.

b What would you say to help him understand his mistake?



8 Multiply $3.4 \cdot 2.8$.

Use the diagram if it helps with your thinking.



Name: ..... Date: ..... Period: .....

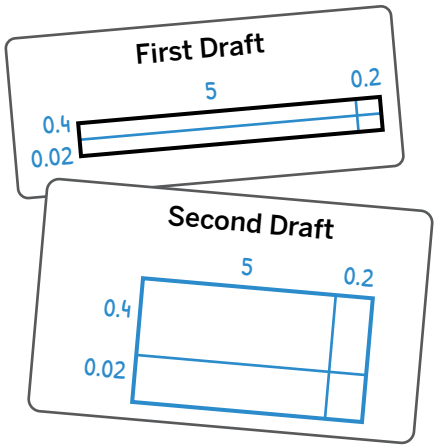
Calculating With an Area Model (continued)

9 Multiply $5.2 \cdot 0.42$.

Draw a diagram if it helps with your thinking.

10 Lucia made two drafts to calculate $5.2 \cdot 0.42$.

What are the advantages and disadvantages of Lucia's second draft?



Advantages:

Disadvantages:

You're invited to explore more.

11 Here is an area model that is missing some labels.

What multiplication problem could this help you solve? Explain your thinking.

	?	?
?	6	0.4
?	2.4	0.16

12 Synthesis

Discuss: How can you use an area model to multiply decimals like $2.7 \cdot 1.4$?

Draw on the diagram if it helps with your thinking.



15 Summary 5.07

One way to multiply two decimals is to use an area model.

To use an area model, separate the decimals into parts. This rectangle has side lengths measuring 3.4 and 1.2 units. Each side length has been split apart by place value: 3.4 has been split into $3 + 0.4$ and 1.2 has been split into $1 + 0.2$.

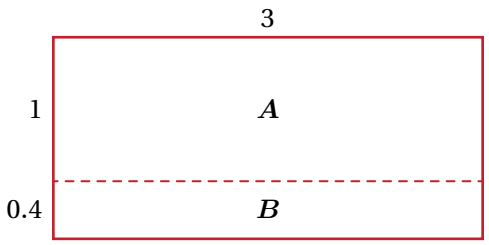
	3	0.4
1	3	0.4
0.2	0.6	0.08

The total area of the rectangle is equal to the sum of the areas of the four smaller rectangles: $3.4 \cdot 1.2 = 3 + 0.4 + 0.6 + 0.08 = 4.08$.

Practice
5.07

Name: ..... Date: ..... Period: .....

Problems 1–2: Here is a diagram that represents $3 \cdot 1.4$.

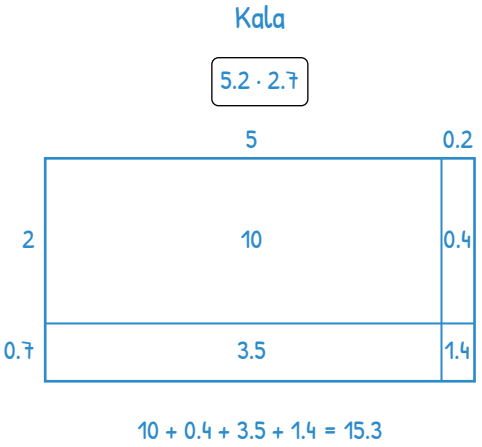


1. Determine the areas of A and B.

A	B

2. What is the value of $3 \cdot 1.4$?

Problems 3–4: Here is Kala's work for determining $5.2 \cdot 2.7$.



3. Kala made an error. Circle what you think the error is. Explain your thinking.

4. What is the value of $5.2 \cdot 2.7$? Show your thinking.

5. Draw an area diagram that represents $2.5 \cdot 1.4$.

6. Determine the product of $2.5 \cdot 1.4$. Use your area diagram from Problem 5 if it helps with your thinking.

Name: _____ Date: _____ Period: _____

7. Determine the value of $0.34 \cdot 0.02$.

Spiral Review

8. Tariq buys a granola bar that costs $\$1.59$. He pays with seven quarters, for a total of $\$1.75$. How much change should he get?

Problems 9–10: Fill in the blanks with the missing digits that make each problem true.

9.

$$\begin{array}{r} 4.3\square \\ +\ \square.15 \\ \hline 6.\square 2 \end{array}$$

10.

$$\begin{array}{r} 1.5\square \\ +\ \square.38 \\ \hline 1.\square 4 \end{array}$$

Problems 11–12: Determine the value of each expression.

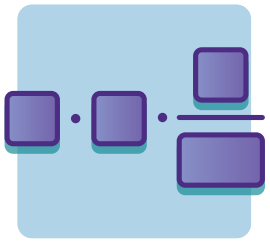
11. $0.47 - 0.103$

12. $0.3 - 0.09$

Name: ..... Date: ..... Period: .....

Model the World 6.NS.3, SMP.5, SMP.7, SMP.8

Multiplication Methods



Let's use place value to multiply decimals vertically.

Warm-Up

1. Calculate $84 \cdot 13$. Show your thinking.

2. Find someone who used a different strategy than you did. Discuss how you each calculated $84 \cdot 13$.

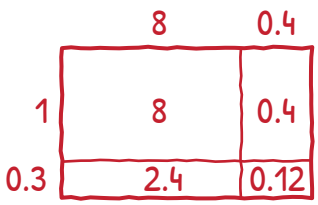
Activity 1

Name: Date: Period:

Multiple Methods

Kwame and Tiara used different strategies to multiply $8.4 \cdot 1.3$

Kwame



$8 + 0.4 + 2.4 + 0.12 = 10.92$

Tiara

$8.4 \cdot 1.3$

$84 \cdot 13 \cdot \frac{1}{10} \cdot \frac{1}{10}$

$$\begin{array}{r} 84 \\ \times 13 \\ \hline 12 \\ 240 \\ 40 \\ +800 \\ \hline 1092 \end{array} \cdot \frac{1}{100}$$

10.92

3. **Discuss:** How are their strategies alike? How are they different?

4. Show how each student might set up $8.4 \cdot 0.13$.

Kwame

Tiara

5. Use either strategy to finish calculating $8.4 \cdot 0.13$.

6. Calculate $0.352 \cdot 0.25$.

Name: Date: Period:

Multiple Multiplication Methods

7. Ayo used the strategy shown to calculate $2.9 \cdot 0.015$

$2.9 \cdot 0.015 \longrightarrow 29 \cdot 15$

$29 \cdot 15 = 435$

I know 2.9 is 29 tenths and 0.015 is 15 thousandths.

Tenths times thousandths = ten thousandths

$2.9 \cdot 0.015 = 435$ ten thousandths

$2.9 \cdot 0.015 = 0.0435$

Discuss: What do you notice about Ayo's strategy?

8. If $165 \cdot 12 = 1980$, then what is $16.5 \cdot 1.2$? Explain your thinking.

9. Select all the expressions that have a product of 0.024.

- ☐ **A.** $0.06 \cdot 0.4$
- ☐ **B.** $0.6 \cdot 0.04$
- ☐ **C.** $0.04 \cdot 0.06$
- ☐ **D.** $2 \cdot 0.012$
- ☐ **E.** $1.2 \cdot 0.02$

10. Write another expression that has a product of 0.024.

Name: ..... Date: ..... Period: .....

Scavenger Hunt

11. Start with any of the scavenger hunt sheets.

- Record the sheet shape, solve the problem, and write your answer.
- Look for your answer at the top of another scavenger hunt sheet. Solve that problem.
- Repeat until you make it back to your starting sheet.

Sheet: ..... **Answer** →	**Sheet:** ..... ↙ **Answer**
Sheet: ..... **Answer** →	**Sheet:** ..... ↙ **Answer**
Sheet: ..... **Answer** →	**Sheet:** ..... **Answer**

Synthesis

12. Describe a strategy that helps you multiply decimals like $0.039 \cdot 3.2$.

Summary 5.08

There are several different strategies you can use to multiply decimals, such as area models (like we saw in a previous lesson), fractions, and place value reasoning. Depending on the problem, one strategy might be more helpful than another.

Let's solve $2.4 \cdot 0.03$ using two strategies.

Strategy 1:
Converting Fractions

Rewrite factors as whole numbers and fractions.

$$2.4 \cdot 0.03 = (24 \cdot \frac{1}{10}) * (3 \cdot \frac{1}{100})$$

$$= (24 \cdot 3) \cdot (\frac{1}{10} \cdot \frac{1}{100})$$

Multiply.

$$72 \cdot \frac{1}{1000} = \frac{72}{1000}$$

Use the denominator to determine the place value.

$\frac{72}{1000}$ is 72 thousandths.

$$\frac{72}{1000} = 0.072$$

Strategy 2: Whole Numbers With Place Value Reasoning

Think of each term as a whole number, then multiply.

$$2.4 \cdot 0.03 \rightarrow 24 \cdot 3$$

$$24 \cdot 3 = 72$$

Think about the place value of each term.

2.4 is 24 tenths.
0.03 is 3 hundredths.

Determine the appropriate place value of the product.

tenths times hundredths = thousandths

$2.4 \cdot 0.03 = 72$ thousandths

$$2.4 \cdot 0.03 = 0.072$$

Practice
5.08

Name: ..... Date: ..... Period: .....

1. Explain how you could use $3 \cdot 65 = 195$ to determine $0.003 \cdot 0.65$.

2. Maia wrote this expression to help her calculate $4.5 \cdot 0.17$.

$45 \cdot 17 \cdot \frac{1}{10} \cdot \frac{1}{100}$

If $45 \cdot 17 = 765$, then what is $4.5 \cdot 0.17$?

3. Select *all* the expressions that have a product of 0.0042.

- ☐ **A.** $0.007 \cdot 0.6$
- ☐ **B.** $0.07 \cdot 0.06$
- ☐ **C.** $0.007 \cdot 0.06$
- ☐ **D.** $0.7 \cdot 0.06$
- ☐ **E.** $0.21 \cdot 0.02$

Problems 4–5: Determine the value of each expression using any strategy. Show or explain your thinking.

4. $5.4 \cdot 2.4$

5. $1.01 \cdot 0.00035$

6. A pound of blueberries costs $\$3.50$ and a pound of clementines costs $\$2.50$. What is the total cost of 0.6 pounds of blueberries and 1.8 pounds of clementines? Show or explain your thinking.

Practice
5.08

Name: ..... Date: ..... Period: .....

Spiral Review

Problems 7–14: Determine the value of each expression.

7. $20 \cdot 5$

8. $20 \cdot 0.8$

9. $20 \cdot 0.04$

10. $20 \cdot 5.84$

11. $23 \div 5$

12. $76.8 \div 12$

13. $230 \div 5$

14. $768 \div 12$

Problems 15–16: Amari bought 12 mini muffins for $5.40.

15. At this rate, what is the price of 4 mini muffins? Show or explain your thinking.

Number of Mini Muffins	Price ($)
12	5.40

16. How many mini muffins can Amari buy with $4.00? Explain your thinking.

Name: _____ Date: _____ Period: _____

Model the World 6.NS.3, 6.NS.4

Practice Day 1



Let's practice what you've learned so far in this unit!

You will use task cards for this Practice Day. Record all of your responses here.

Task A: The Timer Game

1. Closest: _____ Farthest: _____

2. _____ seconds **3.** _____ seconds

Task B: Sort and Solve

1.

Smallest **Largest**

2. Smallest: _____ Largest: _____

3. Area: _____ square units

Explanation:

4. Select *all* . . .

☐ **A.** ☐ **B.** ☐ **C.** ☐ **D.** ☐ **E.**

Name: _____ Date: _____ Period: _____

Practice Day 1 (continued)

Task C: It Takes Two

1. Select *all* . . .

☐ **A.** ☐ **B.** ☐ **C.** ☐ **D.** ☐ **E.**

2. **a.** Solution: _____ **b.** Solution: _____ **c.** Solution: _____

Task D: Show Me the Receipts

3. $ _____ **2.** $ _____

You're invited to explore more.

1.

$$\begin{array}{r} 0.\square \\ +\ 0.\square\square \\ +\ 0.\square\square\square \\ +\ 0.\square\square\square\square \end{array} \qquad \begin{array}{r} 0.\square \\ +\ 0.\square\square \\ +\ 0.\square\square\square \\ +\ 0.\square\square\square\square \end{array}$$

2.

$$\begin{array}{r} \square.\square\square \\ -\ \square.\square\square \\ \hline \square.\square\square \end{array} \qquad \begin{array}{r} \square.\square\square \\ -\ \square.\square\square \\ \hline \square.\square\square \end{array}$$

3. $0.\square \cdot 0.0\square = 0.\square\square\square$

$0.\square \cdot 0.0\square = 0.\square\square\square$

Sub-Unit

3

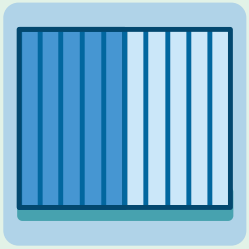
Division and Dividing Decimals



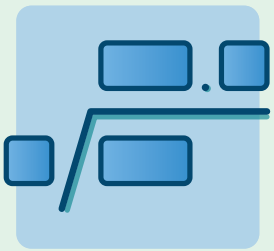
Lesson 9
Quotient Quest



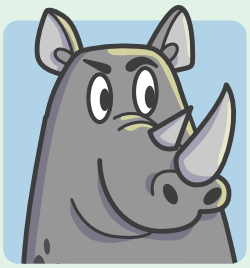
Lesson 10
Just Keep Dividing



Lesson 11
Division Diagrams



Lesson 12
Return of the Long Division



Lesson 13
Movie Time

Name: ..... Date: ..... Period: .....

Model the World 6.NS.2, SMP.3, SMP.6

Quotient Quest

Let's explore different ways to divide numbers.



Warm-Up

1. Abdullah owns a small flower farm. He has 375 sunflowers that he needs to plant in 3 equal rows.

How many sunflowers should be in each row? Show or explain your thinking.

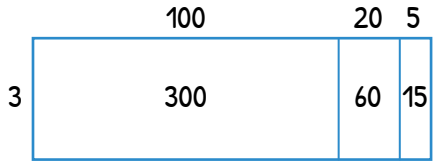


Activity 1

Name: ..... Date: ..... Period: .....

Strategies for Dividing

Abdullah and Camila each calculated $375 \div 3$ using different strategies.



$300 \div 3 = 100$

$60 \div 3 = 20$

$15 \div 3 = 5$

$100 + 20 + 5 = 125$

$375 \div 3 = 125$

Camila

125

$3\overline{)375}$

-300 $(3 \cdot 100)$

75

-60 $(3 \cdot 20)$

15

-15 $(3 \cdot 5)$

0

2. How are their strategies alike? How are they different?

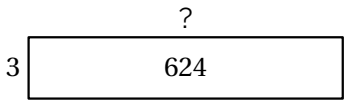
Alike: Different:

3. Try using Abdullah's and Camila's strategies to determine $624 \div 3$.

a **Discuss:** What might the first step look like for each strategy?

b Complete as much of the division as you can using each strategy.

Abdullah



Camila

$3\overline{)624}$

(3 groups of _____)

Name: ..... Date: ..... Period: .....

Partial Quotients

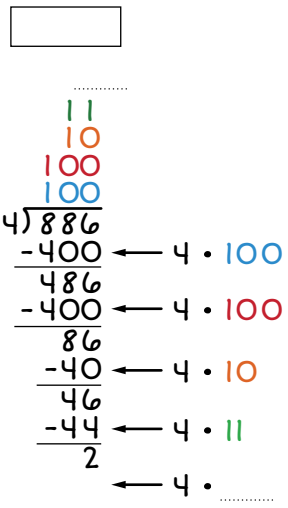
4. Camila used her strategy, which is also called the *partial quotients strategy*, to determine $884 \div 4$. Use Camila's strategy to determine $884 \div 4$ in two different ways.

Camila's Strategy	Partial Quotients 1	Partial Quotients 2
221 11 10 100 100 4)884 -400 ← (4 • 100) 484 -400 ← (4 • 100) 84 -40 ← (4 • 10) 44 -44 ← (4 • 11) 0	$4\overline{)884}$	$4\overline{)884}$

5. Camila tried to determine $886 \div 4$, but she wasn't sure how to finish the problem because she got a remainder of 2.

a What question could you ask Camila to help her finish the problem?

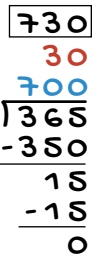
b Fill in the blanks to finish Camila's work and calculate the quotient of $886 \div 4$.



Name: ..... Date: ..... Period: .....

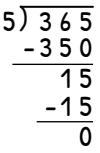
Partial Quotients (continued)

6. Matias is trying to determine $365 \div 5$, but he thinks the quotient is too large.



a What did Matias do well? What would you recommend he change in his work?

b Show the changes you could make to Matias's work to determine the quotient.



c **Discuss:** Why might it make sense for Matias to think about 36 divided by 5 instead of 365 divided by 50 as a first step? How could this strategy be helpful when working with other large dividends?

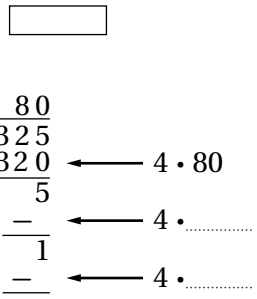
Activity 3

Name: ..... Date: ..... Period: .....

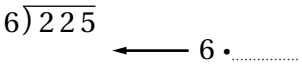
Delve Into Division

7. Use the partial quotients strategy to complete each division problem.

a $325 \div 4$



b $225 \div 6$



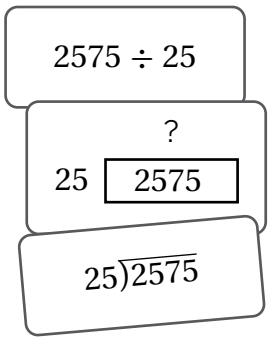
c $394 \div 8$

d $4875 \div 15$

Synthesis

8. Describe a strategy for dividing whole numbers.

Use the example if it helps with your thinking.



Summary 5.09

There are many ways you can divide numbers. Here are two strategies you can use to calculate $345 \div 3$.

Area Model

Draw a diagram with a width of 3 and an area of 345. Cut the diagram into smaller areas, such as 300, 30 and 15, to help find the missing lengths. Determine the sum of the missing lengths to determine the total quotient.



$$300 \div 3 = 100$$
$$30 \div 3 = 10$$
$$15 \div 3 = 5$$
$$100 + 10 + 5 = 115$$
$$345 \div 3 = 115$$

Partial Quotients

Starting with the hundreds place, determine the size of each group if 300 is divided by 3. Then subtract 300 from 345 to find out how much is remaining in the tens place, and determine the size of each group if it's divided by 3. Repeat with the ones place.

$$\begin{array}{r} \boxed{115} \\ 15 \\ 100 \\ \hline 3\overline{)345} \\ -300 \\ \hline 45 \\ -45 \\ \hline 0 \end{array} \quad \begin{array}{l} \\ \\ \\ \\ \leftarrow 3 \cdot 100 \\ \\ \leftarrow 3 \cdot 15 \\ \\ \end{array}$$

$$345 \div 3 = 115$$

Practice
5.09

Name: ..... Date: ..... Period: .....

Problems 1–2: Here is an incorrect calculation for $4809 \div 3$.

1. Explain how you know that $4809 \div 3$ is *not* $1{,}630$.

2. What is $4809 \div 3$?

1630
3
600
1000
3|4809
-3000
1809
-1800
09
- 9
0

Problems 3–4: Mariam is using partial quotients to calculate $623 \div 7$. She starts by dividing 620 by 7. The first partial quotient is 80.

Mariam

3. Write the number 80 where you think it belongs.

7|623

4. Complete Mariam's work to find the quotient.

Problems 5–7: Use partial quotients to calculate each quotient.

5. $5\overline{)465}$

6. $12\overline{)924}$

7. $3\overline{)1107}$

Name: ..... Date: ..... Period: .....

8. Camila calculated $10268 \div 4$ using partial quotients. Use her work to determine the quotient.

```
4)10268
 -8000
  2268
 -2000
   268
  -240
    28
   -28
     0
```

9. Evaluate $7600 \div 25$.

Spiral Review

10. The James Webb Space Telescope (2021) was designed to capture light from distant objects in space. Its primary mirror contains 18 hexagonal mirrors, each with an approximate area of 1.509 square meters.



Paopano/Shutterstock.com

The much older Hubble Space Telescope (1990) has a primary mirror with an area of approximately 4.5 square meters.

How much bigger is the area of Webb's primary mirror than Hubble's primary mirror? Show your thinking.

Problems 11–13: A school band has 40 students. 60% of them are sixth graders. 25% of them are seventh graders, and the rest are eighth graders.

11. How many band members are sixth graders?

12. How many band members are seventh graders?

13. What percentage of the band members are eighth graders?

Name: ..... Date: ..... Period: .....

Model the World 6.NS.2, SMP.1

Just Keep Dividing

Let's use long division to divide whole numbers when there is a remainder.



Warm-Up

1. Here is the work that Peter and Alina did to calculate $584 \div 4$.

Peter

```
      6
     40
    100
4 ) 584
   -400   <-- 4 · 100
    184
   -160   <-- 4 · 40
     24
    -24   <-- 4 · 6
      0
```

Alina

```
   146
4 ) 584
   -4
    184
   -16
     24
    -24
      0
```

Discuss:

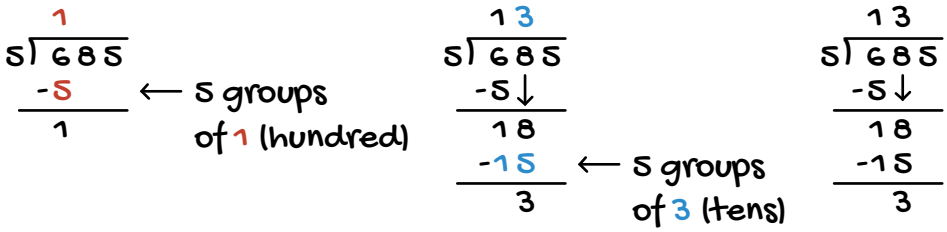
- Where do you see Peter's work in Alina's strategy?
- How is Alina's strategy different from Peter's partial quotients strategy?

Activity 1

Name: ..... Date: ..... Period: .....

Digit-by-Digit

2. Alina calculates $685 \div 5$ using a strategy called **long division**.



a **Discuss:** Alina explains her long division strategy as "dividing one digit at a time, from left to right." What do you think she means by this?

b Complete the work using Alina's strategy.

3. Use long division to complete each of these division problems.

a $792 \div 6$

$$\begin{array}{r} 1 \\ 6\overline{)792} \\ -6\downarrow \\ \hline 19 \end{array}$$

b $1656 \div 8$

$$\begin{array}{r} \\ 8\overline{)1656} \\ -16 \\ \hline \end{array}$$

c $2933 \div 7$

$$\begin{array}{r} \\ 7\overline{)2933} \\ \hline \end{array}$$

Name: ..... Date: ..... Period: .....

Dicey Decimals

4. Here is Peter's work for calculating $318 \div 4$.

$$\begin{array}{r} 79.5 \\ 4\overline{)318.0} \\ -28\downarrow \\ \hline 38 \\ -36\downarrow \\ \hline 20 \\ -20 \\ \hline 0 \end{array} \longleftarrow \text{4 groups of 5 (tenths)}$$

Peter says that 2 is the same as 20 tenths.

Discuss:

- Where do you see the 20 tenths represented in his work?
- Why do you think he might find it more helpful to divide 20 tenths by 4 instead of dividing 2 by 4?

5. Use Peter's strategy to find the quotient of each expression.

a $162 \div 5$

$5\overline{)162}$

b $70 \div 8$

$8\overline{)70.}$

6. Ali is working on $2369 \div 8$. Ali used Peter's strategy and wrote the remainder of 1 as 10 tenths, but still have a remainder of 2 (tenths).

What could Ali do to complete the division? Show or explain your thinking.

$$\begin{array}{r} 296.1 \\ 8\overline{)2369.0} \\ -16\downarrow \\ \hline 76 \\ -72\downarrow \\ \hline 49 \\ -48\downarrow \\ \hline 10 \\ -\ 8 \\ \hline 2 \end{array}$$

Name: Date: Period:

Division Detectives

7. Decide with your partner who will complete Column A and who will complete Column B. The solutions in each row should be the same. Compare your solutions, then discuss and resolve any differences.

Column A	Column B
$13 \div 4$	$26 \div 8$
$2166 \div 12$	$722 \div 4$
$11 \div 8$	$33 \div 24$
$4 \div 16$	$2 \div 8$

Name: ..... Date: ..... Period: .....

Division Detectives (continued)

Column A	Column B
$102 \div 25$	$204 \div 50$
$36 \div 32$	$18 \div 16$

You're invited to explore more.

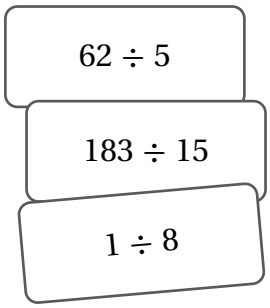
8. **a** Calculate the quotient for $46 \div 3$.

b Do you think this pattern will continue? Explain your thinking.

Synthesis

9. **Discuss:** What are some things you think are important to remember when dividing whole numbers that have remainders?

Use the examples if they help with your thinking.



Summary 5.10

When you get a remainder in a division expression, you can just continue to divide.

Here is how you can calculate $86 \div 4$ using **long division**.

$$\begin{array}{r} 21.5 \\ 4\overline{)\,86.0} \\ -8 \\ \hline 6 \\ -4 \\ \hline 20 \\ -20 \\ \hline 0 \end{array}$$

In this strategy, you break down the remaining 2 ones into 20 tenths by writing the dividend of 86 as 86.0.

This allows you to bring a 0 down to the right of the remaining 2 ones.

Then you add a decimal point to the right of the 1 in the quotient, to show that the resulting 5 is in the tenths place.

So $86 \div 4 = 21.5$.

long division A way to divide numbers. When we use long division, we determine the quotient one digit at a time, from left to right.

Practice
5.10

Name: ..... Date: ..... Period: .....

1. Here are two long division problems. Which one represents $6 \div 10$? Circle one.

$6\overline{)10}$ $10\overline{)6}$

Explain your thinking.

2. Here is a partially completed long division problem. Fill in the missing digits in each box to complete the calculations.

$$\begin{array}{r} 8\,\square \\ 6\overline{)5\;3\;4} \\ -\,\square\square \\ \hline \quad\square\square \\ -\,\square\square \\ \hline \quad\quad 0 \end{array}$$

3. Use long division to calculate the quotient of $99 \div 12$. Show your thinking.

Problems 4–5: Use long division to show that the fraction and decimal in each pair are equal.

4. $\frac{3}{50}$ and 0.06

5. $\frac{7}{25}$ and 0.28

Name: ..... Date: ..... Period: .....

6. A volleyball team has $90 to buy volleyballs. If each volleyball costs $16, what is the greatest number of volleyballs the team can buy?

A. 5 **B.** 6 **C.** 7 **D.** 8

Spiral Review

Problem 7–10: Determine each quotient.

7. $300 \div 3$

8. $12 \div 3$

9. $60 \div 3$

10. $372 \div 3$

11. The mass of one coin is 16.718 grams. The mass of a second coin is 27.22 grams. How much greater is the mass of the second coin than the first? Show or explain your thinking.

Problems 12–14: For every 7 customers, a baker prepares 3 loaves of bread.

12. How many loaves of bread does the baker prepare for 63 customers?

13. If the baker prepares 36 loaves of bread, how many customers can they serve?

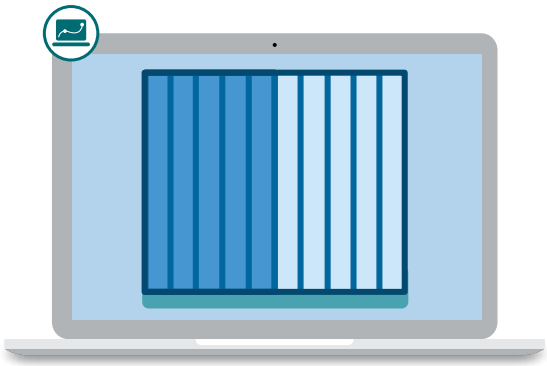
14. What fraction of a loaf is prepared for each customer?

Name: ..... Date: ..... Period: .....

Model the World 6.NS.3, SMP.1, SMP.3, SMP.5

Division Diagrams

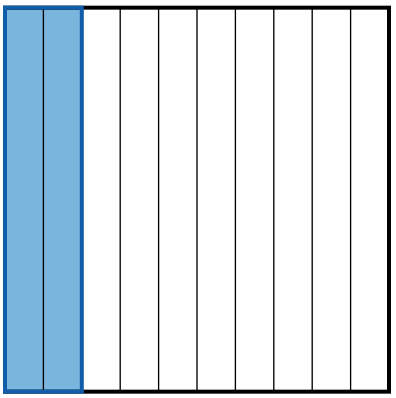
Let's divide decimals using hundredths charts and new expressions.



Warm-Up

1 This large square represents 1.

a Select *all* the equations you could use to determine how many blue pieces you need to fill the large square.



- ☐ **A.** $1 \div 0.2 = ?$
- ☐ **B.** $0.2 \div 1 = ?$
- ☐ **C.** $10 \div 2 = ?$
- ☐ **D.** $0.2 \cdot ? = 1$
- ☐ **E.** $2 \div 10 = ?$

b Pick *one* equation and explain how it represents the diagram.

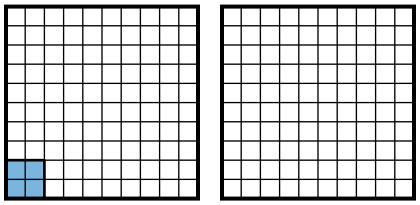
Name: ..... Date: ..... Period: .....

Decimal Division Strategies

2 Each large square represents 1.

Determine the value of $2 \div 0.04$. Use the hundredths chart if it helps with your thinking.

Hundredths



3 Diamond claims that $2 \div 0.04$ has the same value as $200 \div 4$.

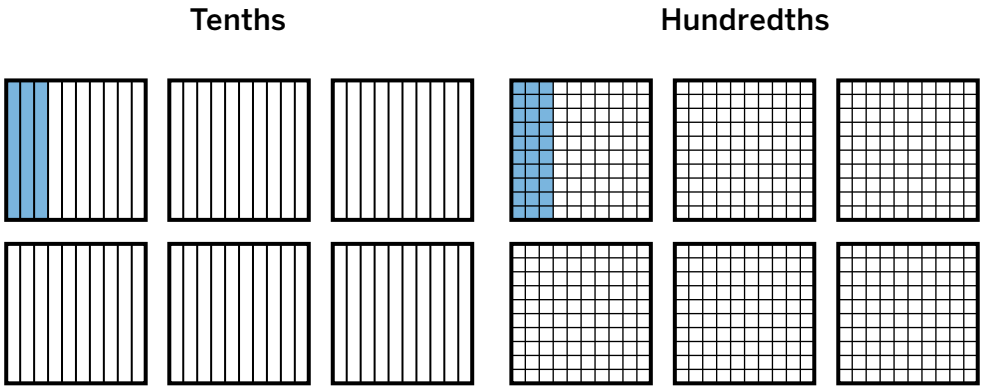
Explain why this makes sense.

Name: ..... Date: ..... Period: .....

Decimal Division Strategies (continued)

4 Each large square represents 1.

Determine the value of $6 \div 0.3$. Use the tenths or hundredths chart if it helps with your thinking.



5 Here is how Arjun determined the value of $6 \div 0.3$.

$$6 \div 0.3 = \frac{6}{1} \div \frac{3}{10}$$
$$= \frac{60}{10} \div \frac{3}{10}$$
$$= 60 \div 3$$
$$= 20$$

a **Discuss:** Describe Arjun's strategy.

b Use his strategy to determine the value of $5 \div 0.02$.

Name: ..... Date: ..... Period: .....

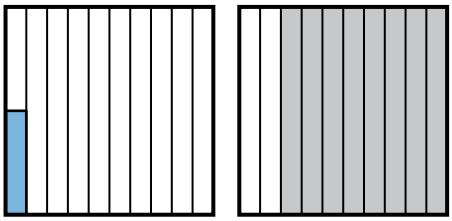
Different Expression, Same Value

6 Select *all* the expressions that have the same value as $1.2 \div 0.05$.

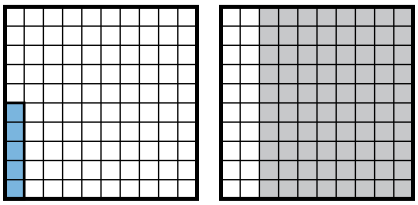
Use the tenths or hundredths chart if it helps with your thinking.

- ☐ **A.** $12 \div 5$
- ☐ **B.** $12 \div 0.5$
- ☐ **C.** $120 \div 5$
- ☐ **D.** $\frac{120}{100} \div \frac{5}{100}$
- ☐ **E.** $\frac{12}{100} \div \frac{5}{100}$

Tenths



Hundredths



7 Calculate $1.2 \div 0.05$.

Name: Date: Period:

Different Expression, Same Value (continued)

8 Write an expression with the same value as $3.6 \div 0.012$ using fractions.

9 Here is Raven's work for $3.6 \div 0.012$.

What would you say to help her understand her mistake?

$$3.6 \div 0.012 = \frac{36}{10} \div \frac{12}{1000}$$
$$= 36 \div 12$$
$$= 3$$

Activity 3

Name: ..... Date: ..... Period: .....

Card Sort

10 Match each division problem with its equivalent representations.

a $\frac{240}{100} \div \frac{8}{100}$

b $\frac{240}{100} \div \frac{80}{100}$

c $\frac{2400}{1000} \div \frac{8}{1000}$

d 30

e 300

f $\frac{2400}{1000} \div \frac{800}{1000}$

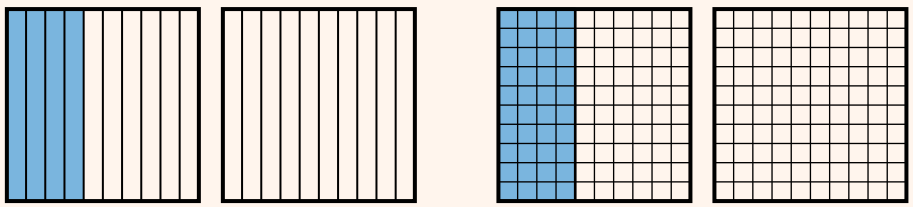
$2.4 \div 0.8$	$2.4 \div 0.08$	$2.4 \div 0.008$

You're invited to explore more.

11 The value of $2 \div 0.4$ is 5.

How many other decimal division expressions can you write that have a value of 5?

Tenths **Hundredths**



Record a dividend and divisor for each expression.

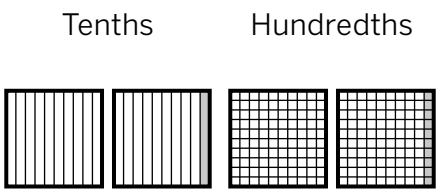
Dividend	2				
Divisor	0.4				

12 Synthesis

Circle an expression that has the same value as $1.9 \div 0.1$.

$\frac{19}{10} \div \frac{1}{10}$ $\quad$ $19 \div 1$ $\quad$ $190 \div 10$

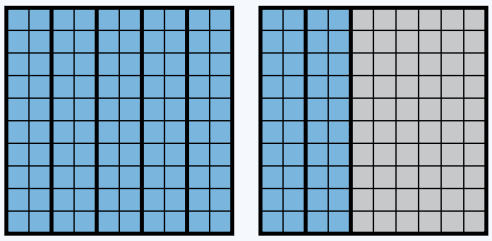
Explain how you know it has the same value.
Use the diagrams if they help with your thinking.



15 Summary 5.11

We can use hundredths charts, thousandths charts, and fractions to help us visualize and divide decimals.

This diagram represents the expression $1.4 \div 0.2$.



- Using the hundredths chart, you can count the number of groups of 2 tenths needed to fill the 1 whole and 4 tenths. It takes 7 groups, so $1.4 \div 0.2 = 7$.
- You can rewrite each decimal as an equivalent fraction, so 1.4 becomes $\frac{14}{10}$ and 0.2 becomes $\frac{2}{10}$. Now you can use your knowledge of fraction division to calculate the quotient.

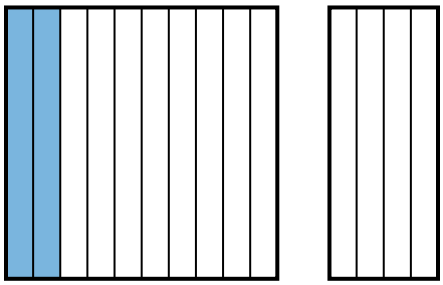
$$\begin{aligned} 1.4 \div 0.2 &= \frac{14}{10} \div \frac{2}{10} \\ &= 14 \div 2 \\ &= 7 \end{aligned}$$

Sometimes you will need to use common denominators to solve expressions with fractions. For example, $1.5 \div 0.03 = \frac{15}{10} \div \frac{3}{100} = \frac{150}{100} \div \frac{3}{100} = 150 \div 3 = 50$.

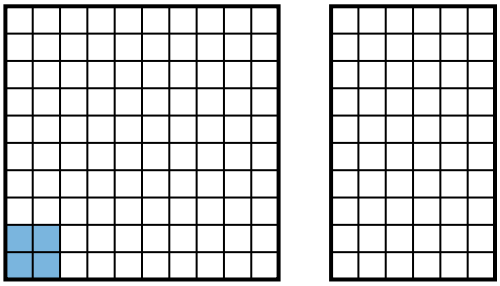
Practice
5.11

Name: ..... Date: ..... Period: .....

1. Determine the value of $1.4 \div 0.2$.



2. Determine the value of $1.6 \div 0.04$.



3. Select *all* the expressions that are equivalent to $3.5 \div 0.005$.

- ☐ **A.** $35 \div 5$
- ☐ **B.** $3500 \div 5$
- ☐ **C.** $35 \div 0.05$
- ☐ **D.** $\frac{35}{10} \div \frac{5}{100}$
- ☐ **E.** $\frac{35}{10} \div \frac{5}{1000}$

4. Calculate $3.5 \div 0.005$.

Problems 5–6: Remy says: *To determine the value of* $0.27 \div 0.003$, *I can divide* 270 by 3.

5. Do you agree with Remy? Explain your reasoning.

6. Calculate $0.27 \div 0.003$. Show or explain your thinking.

Name: ..... Date: ..... Period: .....

7. Calculate $0.225 \div 0.005$. Show or explain your thinking.

8. Rewrite $1.08 \div 0.4$ as a division expression using fractions.

9. Calculate $1.08 \div 0.4$. Show or explain your thinking.

Spiral Review

Problems 10–11: Xavier is multiplying $1.5 \cdot 0.82$. He knows that $15 \cdot 82 = 1230$.

10. What is $1.5 \cdot 0.82$? Show or explain your thinking.

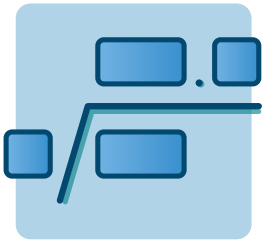
A. 0.0123 **B.** 0.123 **C.** 1.23 **D.** 12.3

11. What is $0.15 \cdot 0.82$?

Name: ..... Date: ..... Period: .....

Model the World 6.NS.2, 6.NS.3, SMP.5, SMP.7

Return of the Long Division



Let's use long division to divide decimals.

Warm-Up

1. Determine the value of each expression.

a $0.8 \div 4$

b $0.08 \div 4$

c $0.008 \div 4$

Name: ..... Date: ..... Period: .....

Ali's and Camila's Strategies

2. Use any strategy to calculate $26.5 \div 5$.

3. Let's look at Ali's and Camila's strategies for calculating $26.5 \div 5$.

Discuss: How are their strategies alike? How are they different?

4. Use any strategy to calculate $106 \div 0.8$.

5. Let's look at Ali's and Camila's strategies for calculating $106 \div 0.8$.

Discuss: Why was it helpful to rewrite this expression as $1060 \div 8$?

Name: ..... Date: ..... Period: .....

Ali's and Camila's Strategies (continued)

6. Calculate $19.8 \div 1.5$.

7. Let's look at Ali's and Camila's strategies for calculating $19.8 \div 1.5$.

Discuss: Why did they each write $198 \div 15$ instead of $1980 \div 15$?

8. Use any strategy to show that these equations are true.

a $0.7 \div 0.4 = 1.75$

b $22.5 \div 0.04 = 562.5$

Name: Date: Period:

Finding Expressions

You will use the Activity 2 Cards to complete this table. You can use each card more than once.

9. Write down at least *one* expression that . . .

. . . has a divisor	. . . has a quotient
greater than 1.	less than 1.
less than 1.	greater than 15.
in the hundredths place.	close to 10.

10. Work with a partner to calculate the value of at least three expressions each. Show all of your thinking. Make sure you and your partner select different expressions. When you're finished, compare your thinking with your partner.

Expression: _____ Expression: _____ Expression: _____

My work: My work: My work:

Synthesis

11. **Discuss:** What are some things you think are important to remember when dividing with decimals?

Use the examples if they help with your thinking.

$26.5 \div 5$

$57 \div 1.5$

$5.11 \div 0.05$

Summary 5.12

When dividing by a decimal, it can be helpful to rewrite the expression using whole numbers by multiplying by a power of 10.

For example, you can rewrite $7.65 \div 1.2$ as $765 \div 120$.

$$\begin{aligned} 7.65 \div 1.2 &= \frac{765}{100} \div \frac{12}{10} \\ &= \frac{765}{100} \div \frac{120}{100} \\ &= 765 \div 120 \end{aligned}$$

Once you have an expression with whole numbers, you can use long division to calculate the quotient.

$$\begin{array}{r} 6.375 \\ 120\overline{)765.000} \\ -720 \\ \hline 450 \\ -360 \\ \hline 900 \\ -840 \\ \hline 600 \\ -600 \\ \hline 0 \end{array}$$

Practice
5.12

Name: ..... Date: ..... Period: .....

1. Select *all* the expressions that are equivalent to $4.5 \div 0.08$.

- ☐ **A.** $\frac{45}{100} \div \frac{8}{100}$
- ☐ **B.** $45 \div 8$
- ☐ **C.** $\frac{450}{100} \div \frac{8}{100}$
- ☐ **D.** $450 \div 8$
- ☐ **E.** $45 \div 0.8$

2. What is the value of $4.5 \div 0.08$?

Problems 3–5: Use long division to calculate each quotient. Show your thinking.

3. $7.89 \div 2$

4. $176 \div 0.5$

5. $199.8 \div 0.8$

6. Four students set up a lemonade stand. By the end of the day, they earned $17.52. If they split the amount equally, how much money would each student get? Show or explain your thinking.

Name: ..... Date: ..... Period: .....

7. A bag of pennies weighs 5.1 kilograms. Each penny weighs 2.5 grams. Which of these is the best estimate for the number of pennies in the bag? Show or explain your thinking. (1 kilogram = 1000 grams)

A. 20 **B.** 200 **C.** 2,000 **D.** 20,000

8. Determine the quotient of $33.8 \div 32.5$.

Spiral Review

Problems 9–11: Fill in the blanks to make each subtraction problem true.

9.

$$\begin{array}{r} 5 \\ -\ \square\ \square\square\square \\ \hline 4.329 \end{array}$$

10.

$$\begin{array}{r} 1 \\ -\ \square\ \square\square\square \\ \hline 0.863 \end{array}$$

11.

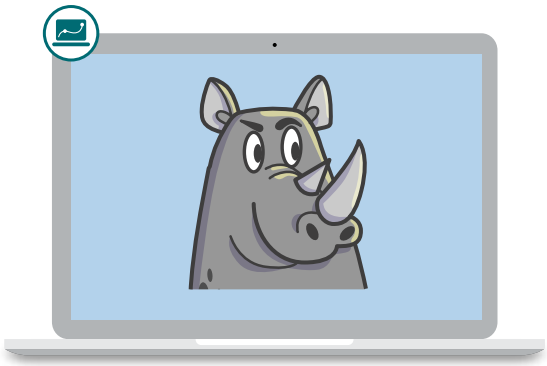
$$\begin{array}{r} 1 \\ -\ \square\ \square\square\square \\ \hline 0.015 \end{array}$$

Problems 12–14: Determine the quotients.

12. $7.28 \div 7$ **13.** $7.28 \div 0.7$ **14.** $0.728 \div 7$

Name: ..... Date: ..... Period: .....

Model the World 6.NS.2, 6.NS.3, SMP.1

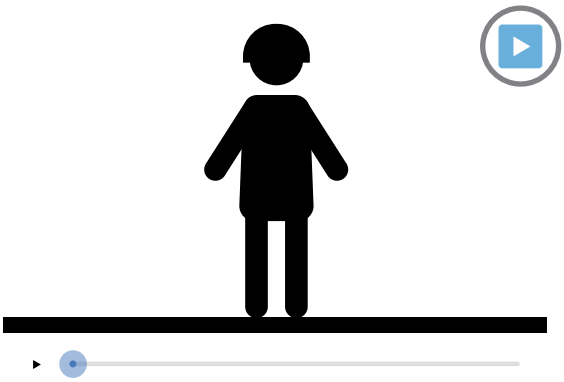


Movie Time

Let's use decimals to explore movies at different speeds.

Warm-Up

1 a Let's watch a short movie.



b Write a story about what you see.

Activity 1

Name: ..... Date: ..... Period: .....

Speedy Cinema

2 **a** Use the digital activity to select a speed, then press play. Try several speeds.

b **Discuss:** What do you notice? What do you wonder?

3 The original movie is 12 seconds long. Arnav wants to play the movie at a speed of 3x.

Which expression could he use to calculate how long it would take?

$12 - 3$ $12 \div 3$ $12 \cdot 3$ $3 \div 12$

Explain your thinking.

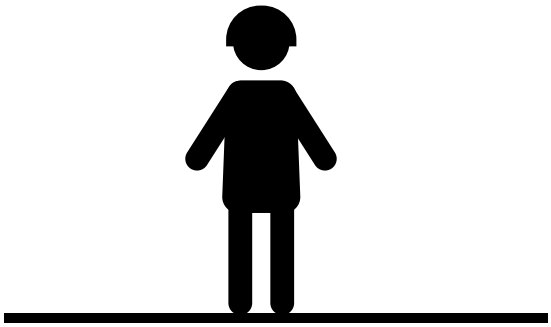
Speed	Duration (sec)
0.5x	24
1x	12
2x	6
4x	3
3x	?

4 The original movie is 12 seconds long.

a **Discuss:** Would it take more or less than 12 seconds to play the movie at a speed of 0.8x?

b How long would it take to play the movie at 0.8x?

Speed: 0.8x



Activity 2

Name: ..... Date: ..... Period: .....

Make Your Own Movie

5 Use the digital activity to make your own movie.

First, choose a character. Then select some dance moves and press "Play."

6 Use the digital activity to see the length of your movie.

a How long would it take to play your movie at $1.5x$? How do you know?

b Use the digital activity to check your answer.

7 Use the digital activity to see speeds and durations for your movie.

a Enter a new speed for your movie. Then enter the duration for that speed.

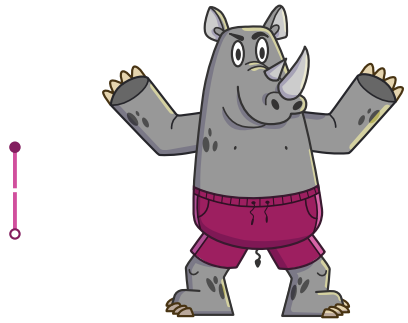
b Press "Try It" to watch the new movie.

Name: _____ Date: _____ Period: _____

Make Your Own Movie (continued)

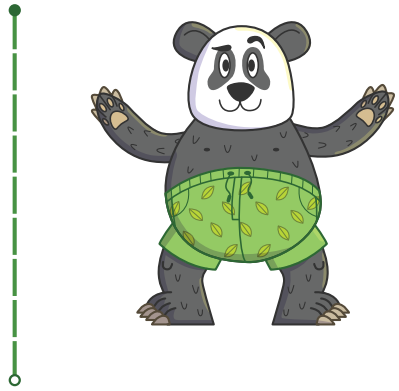
8 Roberto's movie is 2.4 seconds long. How long would it take to play his movie at 0.25x?

Playback speed: 0.25x



9 Dalia's movie is 12.25 seconds long. How long would it take to play her movie at 2.5x?

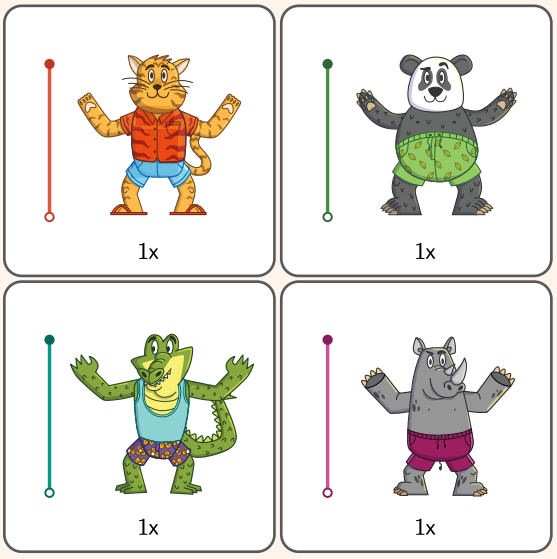
Playback speed: 2.5x



You're invited to explore more.

10 Here are four movies. Adjust the speeds so that all four movies take exactly 4.8 seconds to play.

Original Movie Length (sec)	Speed
1.2	
2.4	
4.8	
18	

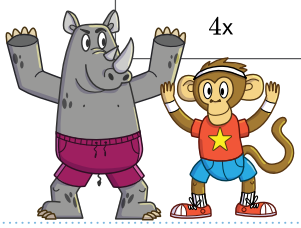


11 Synthesis

Discuss: How can dividing be useful when thinking about movie durations and speeds? Use the numbers in the table if they help with your thinking.

Original movie length: 12 seconds

Speed	Duration
0.5x	24 seconds
1x	12 seconds
2x	6 seconds
4x	3 seconds



14 Summary 5.13

To determine how long it will take to play a movie at a certain speed, you can divide the length of the original movie by the playback speed.

Let's say a movie is 5.6 minutes long.

- When the playing speed is doubled, it will take 2.8 minutes to watch the movie: $5.6 \div 2 = 2.8$.
- When the playing speed is halved, it will take 11.2 minutes to watch the movie: $5.6 \div 0.5 = 11.2$.

Practice
5.13

Name: ..... Date: ..... Period: .....

1. Select *all* the expressions that have a value of 8.

- ☐ **A.** $3.2 \div 0.4$
- ☐ **B.** $0.32 \div 4$
- ☐ **C.** $\frac{32}{10} \div \frac{4}{10}$
- ☐ **D.** $32 \div \frac{4}{100}$
- ☐ **E.** $0.032 \div 0.04$

Problems 2–3: Naoki recorded a video of his dog catching a frisbee. The video is 10 seconds long.

2. Naoki plays the video in slow motion at 0.5x speed. How long does it take?

3. How long does the video take if Naoki plays it at 2.5x speed?

Problems 4–5: Farah is listening to an audiobook. To get through the book faster, she listens at 1.2x speed.

4. Complete the table to determine how long it will take to listen to each of the first three chapters.

	Duration at Regular Speed (min)	Duration at 1.2x Speed (min)
Chapter 1	12	
Chapter 2	10.5	
Chapter 3	39.3	

5. It takes Farah 8.5 minutes to listen to Chapter 4 at 1.2x speed. What is the length of Chapter 4 at regular speed?

- **A.** 10.2 minutes
- **B.** 9.7 minutes
- **C.** 7.3 minutes
- **D.** 7.08 minutes

Name: _____ Date: _____ Period: _____

6. Estimate whether $9 \cdot 0.8$ is greater than or less than 10. Explain your thinking.

7. Estimate whether $3.9 \cdot 3.9$ is greater than or less than 10. Explain your thinking.

8. Estimate whether $4.25 \cdot 2.5$ is greater than or less than 10. Explain your thinking.

Spiral Review

Problems 9–10: Plant B is $6\frac{2}{3}$ inches tall. Plant C is $4\frac{4}{15}$ inches tall. Complete these sentences:

9. Plant C is _____ times as tall as Plant B.

10. Plant B is _____ times as tall as Plant C.

Name: _____ Date: _____ Period: _____

Model the World 6.NS.2, 6.NS.3

Practice Day 2

Let's practice what you've learned so far in this unit!



You will use problem cards for this Practice Day. Record all of your responses here.

Card 1

$ _____ per gallon

Card 2

Cost: $ _____

Card 3

Circle one: Yes or No

Explanation:

Card 4

Mauricio bought _____ gallons.

Card 5

Quotient: _____

Card 6

Quotient: _____

Card 7

Quotient: _____

Card 8

Expression: _____

Name: _____ Date: _____ Period: _____

Practice Day 2 (continued)

Card 9

Circle all that apply: A B C D E

Card 10

Circle one: Expression 1 or Expression 2

Explanation:

Card 11

Quotient: _____

Card 12

Expression: _____

Card 13

Quotient: _____

Card 14

Quotient: _____

Card 15

Quotient: _____

Card 16

_____ miles per hour

You're invited to explore more.

Determine as many ways as you can to solve this problem:

Make this equation true by filling in each blank using the digits 0 to 9 only once.

$\square \div \square = \square \cdot \square$

Sub-Unit

4

Solving Problems With Decimals



Lesson 14
Budget Vehicles



Lesson 15
Coining New Representations



Lesson 16
Grocery Prices

Name: ..... Date: ..... Period: .....

Fraction Relationships | Model the World | 6.NS.3, 6.RP.3.b, SMP.1, SMP.2, SMP.4

Budget Vehicles



Let's use decimals to compare car options.

Warm-Up

1. What might a family consider when buying a car?



2. Which of these is most important to you?

Name: ..... Date: ..... Period: .....

Do Hybrid Cars Save Money?

3. Brielle's family wants to buy a brand new vehicle. They're choosing between the DesWagon, which is a gas-powered car, and the DesMobile, which is a hybrid car.

Use the Activity 1 Sheet to complete the table.

	Problem	DesWagon 	DesMobile 
a	Gas costs $3.50 per gallon in Brielle's neighborhood. How much would it cost to fill a tank of gas?		
b	How much would it cost to drive 1 mile based on the cost of gas and the range of each vehicle?		
c	How much would Brielle's family spend on gas yearly if they drove about 15,000 miles a year?		

4. If Brielle's family plans on driving the car for 5 years, which car do you think they should buy? Explain your thinking.

5. Why might someone recommend the other car?

Activity 2

Name: ..... Date: ..... Period: .....

Do Electric Cars Save Money?

An advertisement for an electric vehicle claims that the savings on gas will make up for the higher sale price of the car.



Kweku is deciding between the DesWagon and the DesZip Electric. He determines that:

- The DesZip Electric costs $11,900 more than the DesWagon.
- On average, the cost of electricity for the DesZip Electric is $0.03 per mile.
- On average, the cost of gas for the DesWagon is $0.12 per mile.

Kweku drives about 18,000 miles a year.

6. How much money would he save in a year buying electricity for the DesZip Electric compared to buying gas for the DesWagon? Show your thinking.

7. About how long would it take to make up for the higher sale price of the DesZip Electric? Show your thinking.

8. You have been hired to design a new advertisement for either the DesZip, the DesWagon, or the DesMobile. Use your work from this lesson to create a persuasive and accurate advertisement. Include any additional information that you think is important.

Synthesis

9. **Discuss**:

- How did you use operations with decimals to compare the costs of different types of cars?
- What factors influenced your decision about which car was the best choice?
- How could the choices people make when buying a gas-powered, hybrid, or electric car affect the environment?

Summary 5.14

Decimals show up in all sorts of real-world problems. We can use decimal operations to make well-informed decisions in these situations.

You might start by writing an equation to represent the situation, then determine its solution.

Problem	Equations
Apples cost $\$0.75$ per pound. I have $\$4.00$ to spend on apples. I need to know how many pounds of apples I can buy.	$4.00 \div 0.75 = 5\frac{1}{3}$ $5\frac{1}{3}$ pounds of apples
Callen is measuring wood for a wheelchair ramp. The ramp is 5.7 feet long, and Callen will place a nail every 6 inches. How many nails will they use?	$5.7 \cdot 12 = 68.4$ $68.4 \div 6 = 11.4$ 11 or 12 nails
Marco works at the grocery store and earns $\$11.50$ per hour. Jaylene works at a pet store and earns $\$12.85$ per hour. How much more money will Jaylene earn than Marco for a 5.5-hour shift?	$12.85 - 11.50 = 1.35$ $1.35 \cdot 5.5 = 7.425$ $\$7.43$

Practice
5.14

Name: ..... Date: ..... Period: .....

Problems 1–2: A standard sheet of paper in the United States is 11 inches long and 8.5 inches wide. (1 inch $= 2.54$ centimeters)

1. How long and wide is a standard sheet of paper in centimeters?

Length (cm)	Width (cm)

2. A standard sheet of paper in Europe is 29.7 centimeters long and 21 centimeters wide. Which standard sheet of paper has a greater area? Circle one.

United States' Europe's They have the same area.

Problems 3–4: Tyler's school is having a stair-climbing challenge for Better Health Week. Tyler can climb 135 stairs in 90 seconds.

3. If Tyler climbs stairs at a constant rate, how many stairs per second will that be? Show or explain your thinking.

4. Jin also participates in the challenge and can climb 75 stairs in 1 minute. Who climbs at a faster rate? Show or explain your thinking.

5. Which polygon has the greatest area?

A. A rectangle that is 3.25 inches wide and 6.1 inches long.

B. A square with a side length of 4.6 inches.

C. A parallelogram with a base of 5.875 inches and a height of 3.5 inches.

D. A triangle with a base of 7.18 inches and a height of 5.4 inches.

6. Mateo is using ribbon to wrap presents. He has a roll of ribbon that is 12 meters long. He plans to cut 9 pieces that are each 0.4 meters long. How much ribbon will he have left?

Practice
5.14

Name: ..... Date: ..... Period: .....

Spiral Review

7. Annika walked $\frac{1}{8}$ of a 30-mile walking trail. How many miles did she walk?

Problems 8–9: Rope A is $6\frac{2}{3}$ inches long. Rope B is $4\frac{4}{15}$ inches long.

8. How much longer is Rope A than Rope B?

9. How many times longer is Rope A than Rope B?

10. Select *all* the expressions that are equivalent to $1.44 \div 1.2$

- ☐ **A.** $\frac{144}{100} \div \frac{12}{100}$
- ☐ **B.** $\frac{144}{100} \div \frac{12}{10}$
- ☐ **C.** $\frac{144}{100} \div \frac{120}{100}$
- ☐ **D.** $144 \div 12$
- ☐ **E.** $144 \div 120$

11. What is the value of $1.44 \div 1.2$?

Name: ..... Date: ..... Period: .....

Fraction Relationships | Model the World | 6.NS.3, 6.RP.3.c, SMP.1, SMP.7, SMP.8

Coining New Representations



Let's explore the relationship between percentages, decimals, and fractions.

Warm-Up

1. Complete the table to show the values of these U.S. coins.









	Penny	Nickel	Dime	Quarter
Value (in cents)	1			
Fraction of $1	$\frac{1}{100}$			
Value (in dollars)	0.01			
Percent of $1	1%			

Name: ..... Date: ..... Period: .....

Percentages, Decimals, and Fractions

You will use the Activity 1 Cards for this activity.

2. Create groups of three by matching the percent, decimal, and fraction cards that are equivalent.

Percent	Decimal	Fraction

3. Choose any set of three cards and explain how you know they are equivalent.

4. Imani makes a card with 8% on it and writes its decimal equivalent as 0.8. What could be said to help Imani understand this mistake?

Name: ..... Date: ..... Period: .....

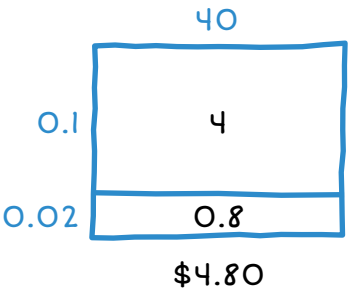
Resale Value

5. Ada and Bao keep 12% of the price of each shirt as a profit. Here is how they each calculated 12% of 40.

Ada

12% · 40

0.12 · 40



$4.80

Bao

12% · 40

$\frac{12}{100} \cdot 40$

$\frac{12}{100} \cdot 40$

$\frac{48}{10}$

$4.80

Discuss: How are their strategies alike? How are they different?

6. Oscar says that he can calculate 40% of 70 by writing the expression $\frac{2}{5} \cdot 70$. Is his expression correct? Explain your thinking.

7. Match each expression with a question. One expression will have no match.

a $\frac{4}{100} \cdot 70$	**b** $0.04 \cdot 14$	**c** $0.4 \cdot 48$
d $\frac{4}{100} \cdot 14$	**e** $0.04 \cdot 70$	**f** $0.04 \cdot 48$

What is 4% of $14?	What is 4% of $48?	What is 4% of $70?

Name: ..... Date: ..... Period: .....

Resale Value (continued)

8. Solve as many challenges as you have time for.

a The price of a space T-shirt is $22. 16.5% of every sale goes to material cost. Calculate the material cost.

b The price of a striped button-up shirt is $30. 9.4% of every sale goes to clothing company profit. Calculate the clothing company profit.

c The price of a striped long-sleeve T-shirt is $48. 8.5% of every sale goes to transport cost. Calculate the transport cost.

d The price of a blue pair of shoes is $65. 5.2% of every sale goes to factory profit. Calculate the factory profit.

Synthesis

9. **Discuss**: How you can convert between percentages, fractions, and decimals? Use the values in the chart if they help with your thinking.

Percent (%)	Fraction	Decimal
20	$\frac{20}{100}$	0.2
40	$\frac{2}{5}$	0.4
1.2	$\frac{1.2}{100}$	0.012

Summary 5.15

We can convert between percentages, decimals, and fractions to solve problems, including problems related to money.

We've learned that percent means "out of 100." This means that:

- We can write percentages as fractions with a denominator of 100. For example, 13% is equivalent to $\frac{13}{100}$.
- We can think of percentages as a number of hundredths, then determine an equivalent decimal value. For example, 13% is 13 hundredths, or 0.13.

It's important to think about place value when converting percentages into fractions and decimals. For example, 2.5% is equivalent to 2.5 hundredths $\left(\frac{2.5}{100}\right)$ or 25 thousandths $\left(\frac{25}{1000}\right)$, both of which can be represented by the decimal 0.025.

Practice
5.15

Name: ..... Date: ..... Period: .....

Problems 1–4: Determine the missing values in each row.

	Percent (%)	Decimal	Fraction
1.	5		
2.	400		
3.		0.5	
4.			$\frac{3}{5}$

5. Marc says that because percent means "out of 100," he needs to multiply 3.7 by $\frac{1}{100}$ to write a percentage that is equivalent to 3.7. Help Marc understand his mistake.

6. Match each percent with its decimal equivalent.

a	3.05%	..... 0.305
b	350%	..... 3.5
c	30.5%	..... 0.035
d	3.5%	..... 0.0305

7. Which value is 0.2 expressed as a percentage?

A. 2%

B. 0.02%

C. 20%

D. 0.002%

Practice
5.15

Name: ..... Date: ..... Period: .....

Spiral Review

Problems 8–10: Mentally evaluate each quantity. Explain your thinking.

8. 15 is what percent of 30?

9. 3 is what percent of 12?

10. 6 is what percent of 10?

For Problems 11–13: 460 of the students who attend Hypotenuse High walk to school every day.

11. The number of students who take public transit is 20% of the number of students who walk. How many students take public transit?

12. The number of students who bike to school is 5% of the number of students who walk. How many students bike to school?

13. The number of students who ride the school bus is 110% of the number of students who walk. How many students ride the school bus?

Name: ..... Date: ..... Period: .....

Fraction Relationships | Model the World | 6.NS.3, 6.RP.3.c, SMP.1, SMP.2, SMP.3

Grocery Prices

Let's explore the cost of groceries in different cities.



Warm-Up

In 2021, the average household in Austin, Texas, made about $1,500.00 per week.

1. **Discuss:** What percentage of their income do you think a family might spend on this list of groceries?

Groceries

Milk (1 gal)	Apples (1 lb)
Bread (1 loaf)	Bananas (1 lb)
Rice (1 lb)	Oranges (1 lb)
Eggs (1 dozen)	Tomatoes (1 lb)
Cheese (1 lb)	Potatoes (1 lb)
Chicken (1 lb)	Onion (1 lb)
Beef (1 lb)	Lettuce (1 head)

2. Complete this statement:

An average household in Austin would spend ..... % of their income on this grocery list.

3. Is this more or less than $100.00? Explain your thinking.

Activity 1

Name: Date: Period:

Grocery Prices

Tyani and Anika were asked to calculate approximately how much the average household in Austin, Texas, would spend on these groceries each week. They each wrote expressions to help them. Here is their work.

Tyani	Anika
2.2% of 1500 = 0.22 · 1500	2.2% of 1500 = 0.022 • 1500

4. Whose work do you agree with? Explain your thinking.

5. **a** Calculate 2.2% of $1,500.00 to determine how much an average household in Austin spends on groceries.

b **Discuss:** Does this amount seem reasonable based on what you know about grocery costs or household budgets?

Here is the approximate average weekly household income in 2021 for three other cities.

Honolulu, Hawaii	Seattle, Washington	Jackson, Mississippi
$1,700.00	$2,100.00	$700.00

6. If the average household in each city spent 2.2% of their weekly income on these groceries, how much would they spend? Complete the table and show your thinking.

	Honolulu	Seattle	Jackson
Expression for 2.2% of Income			
Money Spent			

Name: ..... Date: ..... Period: .....

Grocery Prices (continued)

You'll use the Activities 1 & 2 Sheet to complete this activity.

7. In which cities can someone buy all the groceries on the list using 2.2% or less of their weekly income? In which cities can they not?

8. **Discuss:**

a. Why do you think the cost of these groceries is different in different places?

b. What do you think the impact might be on families who spend more than 2.2% of their weekly income on these groceries?

Activity 2

Name: ..... Date: ..... Period: .....

Bought Milk?

You'll use the Activities 1 & 2 Sheet to complete this activity.

9. How much does a gallon of milk cost in Austin, Texas?

10. What percentage of the total cost of the grocery list is that? Show your thinking.

A. 0.0967% **B.** 0.967% **C.** 9.67% **D.** 96.7%

Alejandro says: *Milk is too expensive in Jackson. It's* 11% *of the total cost of that grocery list!*

11. Show or explain where the 11% comes from.

12. Do you agree with Alejandro? If you do, explain what you think would be a fair price for milk in Jackson. If you do not, explain why you think milk in Jackson is priced fairly.

13. Choose a different city. What percentage of the total grocery bill is milk? Do you think milk is too expensive in this city? Show or explain your thinking.

You're invited to explore more.

14. Choose a different food that has a price that you think is too high. Use percentages, decimals, rates, or ratios to propose a fairer price for that food.

Synthesis

15. Discuss:

a. What do you think is a fair way to determine the price for groceries in different places?

b. What can people do if they live in a place where the price of groceries is high?

c. What new questions do you have about food costs around the world?

Summary 5.16

You've used ratios, double number lines, and tape diagrams to model and solve percent problems. Another strategy you can use is to convert the percent to a decimal, then multiply or divide.

Let's say Isaiah's family spent 4% of their monthly income on groceries last week. Their monthly income is $\$4{,}000.00$. How much money did they spend on one week's worth of groceries?

To determine the answer, first write 4% as a decimal: $4\% = 0.04$.

Then multiply by the total monthly income: $0.04 \cdot 4000 = 160$.

That means Isaiah's family spent $\$160.00$ on their groceries last week.

Practice
5.16

Name: ..... Date: ..... Period: .....

Problems 1–4: Jada's family has a weekly income of $1,150.00. They try to spend no more than 9% of their weekly income on groceries.

1. Write an expression to represent how much money they spend on their weekly groceries.

2. How much money, at most, does Jada's family spend on groceries each week?

3. Jada's family puts 12% of their weekly income into a savings account. How much money do they save each week?

4. Jada's family recently had to spend $184.00 on a car repair. What percent of their weekly income did they spend on the car repair?

A. 0.16% **B.** 6.25% **C.** 16% **D.** 62.5%

5. Circle the expression that has a greater value.

7% of 250 70% of 25 They have the same value.

6. Oliver went to the store and purchased these items. Beef is the most expensive item. What percent of the total is it?

Items	Cost ($)
Milk (1 gal)	$3.61
Beef (1 lb)	$7.10
Apples (1 lb)	$2.39
Bananas (1 lb)	$0.91
Oranges (1 lb)	$1.99
Potatoes (1 lb)	$1.75
Total	$17.75

Practice
5.16

Name: ..... Date: ..... Period: .....

Spiral Review

7. One ounce of yogurt contains 1.2 grams of sugar. How many grams of sugar are in 14.25 ounces of yogurt?

A. 1.71 grams **B.** 11.875 grams **C.** 15.45 grams **D.** 17.1 grams

Problems 8–11: Determine the value of each expression. Show your thinking.

8. $4.4 - 0.72$

9. $4 + 1.3 + 0.56$

10. $4.34 \div 0.7$

11. $0.32 \cdot 4.7$

12. 0.15 is what percent of 0.6?

13. 0.15 is what percent of 3?

14. 0.15 is what percent of 30?

Name: _____ Date: _____ Period: _____

Fraction Relationships | Model the World | 6.NS.2, 6.NS.3, 6.RP.3.c

Practice Day 3



Let's practice what you've learned so far in this unit!

You will use task cards for this Practice Day. Record all of your responses here.

Task A: Fix It!

1. Explanation:

Correct solution: _____

2. Explanation:

Correct solution: _____

3. Explanation:

Correct solution: _____

4. Explanation:

Correct solution: _____

Task B: Calculation Station

1. Solution: _____ **2.** Solution: _____ **3.** Solution: _____

4. Solution: _____ **5.** Solution: _____ **6.** Solution: _____

Name: _____ Date: _____ Period: _____

Practice Day 3 (continued)

Task C: Rainy Days

1. _____ inches **2.** _____ inches **3.** _____ inches

4. Explanation:

Task D: Cupcakes for Sale

1. Shop A: $_____ per cupcake Shop B: $_____ per cupcake

Shop C: $_____ per cupcake Shop D: $_____ per cupcake

2. Circle one: Shop A Shop B Shop C Shop D

Explanation:

3. Explanation:

4. Price: $_____

Explanation:

You're invited to explore more.

Name: _____ Date: _____ Period: _____

Model the World 6.NS.2, 6.NS.3, SMP.6

Practice Day 4



Decide with your partner who will complete Column A and who will complete Column B. The solutions in each row should be the same. Compare your solutions, then discuss and resolve any differences.

	Column A	Column B
1.	$4.853 + 2.351$	$5.713 + 1.491$
2.	$1494 \div 18$	$1743 \div 21$
3.	$3.2 \cdot 2$	$1.6 \cdot 4$
4.	$9.6 \div 2.4$	$12.4 \div 3.1$
5.	$1620 \div 36$	$1260 \div 28$
6.	$7.1013 - 0.095$	$7.2253 - 0.219$
7.	$1.8 \cdot 0.002$	$1.2 \cdot 0.003$

Name: _____ Date: _____ Period: _____

Practice Day 4 (continued)

	Column A	Column B
8.	$30.735 \div 4.5$	$62.153 \div 9.1$
9.	$2178 \div 12$	$2904 \div 16$
10.	$9.623 - 2.45$	$8.923 - 1.75$
11.	$5.76 \cdot 1.4$	$4.8 \cdot 1.68$
12.	$45.6 \div 7.6$	$22.8 \div 3.8$
13.	$5.94 + 7.46$	$6.68 + 6.72$
14.	$625 \div 25$	$800 \div 32$
15.	$6.3 \cdot 1.25$	$2.5 \cdot 3.15$
16.	$11.7 \div 1.8$	$15.6 \div 2.4$

Notes:

Career Connection

To communicate decimal numbers, not everyone uses – or has used – a decimal "point."

When using decimal numbers, you need to show where the whole number part ends and the decimal part begins. This is often a symbol. Some countries, such as the United States, use a period (a "decimal point"). Other countries, such as many European and South American countries, use a comma. There are a few countries that use other symbols.

Yet, the history of these symbols didn't begin with points or commas. The concept of a decimal system can be traced back thousands of years. One of the earliest symbols used to represent decimal numbers was suggested by Arab mathematician Abu'l Hasan Ahmad ibn Ibrahim al-Uqlidisi in 952 CE. This symbol resembled a small vertical mark placed above a digit.

Translators convert written or spoken text from one language to another, while preserving the original meaning. When translating scientific or technical texts, they need to understand numbers and mathematical symbols, including how they are represented in various languages.



Meet Meet Abu'l Hasan Ahmad ibn Ibrahim al-Uqlidisi

Abu'l Hasan Ahmad ibn Ibrahim al-Uqlidisi was a mid-to-late 10th century Arab mathematician. One of his two major works, the *Kitab al-fusul fi al-hisab al-Hindi*, is the first known Arabic text describing the Hindu-Arabic number system, which we use today in the United States and around the world. His work is also the earliest known text that describes decimal fractions, a fraction whose denominator is 10, 100, 1000, etc.

Are you interested in studying how texts, including symbols, can be translated into different languages? What can you do to learn more?

Community Connection

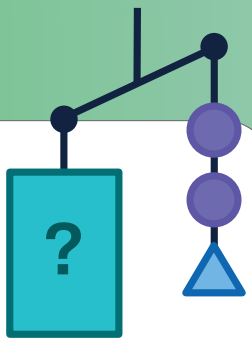
Design a new symbol that could be used to represent decimal numbers. Write 3 different multi-digit decimal numbers using your symbol. Show your symbol and decimal numbers with at least 3 people in your home or communities. Share why you chose that symbol.

Math Mindset

How is the opposite of a number different from its absolute value?

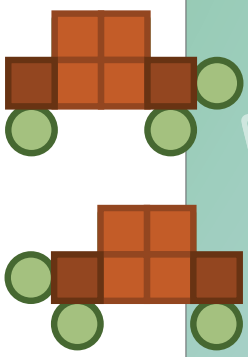
Unit 6

Expressions and Equations



Big Ideas in This Unit

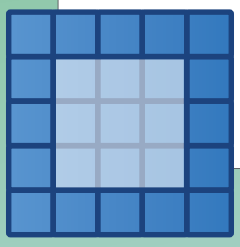
CC2 Fraction Relationships | Generalizing Multiple Representations | Patterns Inside Numbers | Relationships Between Variables | CC3 Model the World | Nets and Surface Area | NS Generalized Numbers Leading to Algebra | Ratios, Percents, and Proportional Relationships



Questions for Investigation

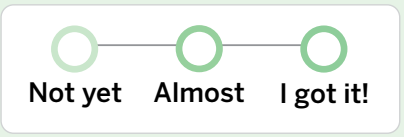
- How do we represent an unknown or changing value in a math problem?
- How can we use expressions and equations to represent different mathematical situations?
- How do independent and dependent variables relate to one another?

Explore: Detecting Counterfeit Coins
How can a balance scale help detect a counterfeit coin?



Watch Your Knowledge Grow

This is the math you'll explore in this unit. Rate your understanding to see how your knowledge grows!



I can . . .	Before	After
Use variables to represent numbers.		
Write an expression or equation to represent a mathematical or real-world problem.		
Use an equation to solve a mathematical or real-world problem.		
Use substitution to determine whether a given number makes an equation true.		
Evaluate expressions with letters standing for numbers.		
Rewrite verbal descriptions as expressions.		
Use ratios and rates to help solve real-world and mathematical problems.		
Use the order of operations to evaluate expressions.		
Apply the properties of operations to produce equivalent expressions.		
Identify when two expressions are equivalent.		

I can . . .	Before	After
Identify the parts of a mathematical expression.		
Plot pairs of values on the coordinate plane.		
Write numerical expressions involving whole-number exponents.		
Evaluate numerical expressions involving whole-number exponents.		
Write an equation and analyze the relationship between the dependent and independent variables using graphs and tables.		
Find the area of polygons.		
Find the volume of a right rectangular prism with fractional edge lengths.		

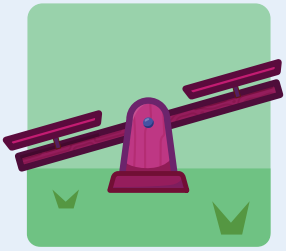
Sub-Unit

1

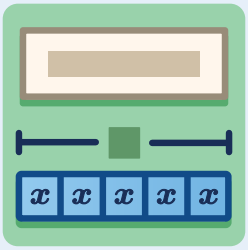
Solving Equations



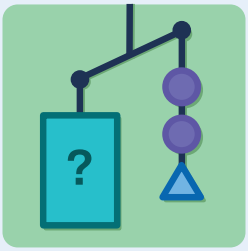
Explore
Detecting Counterfeit Coins



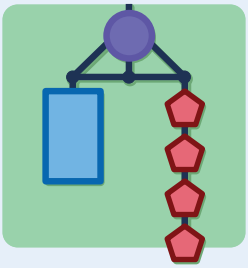
Lesson 1
Weight for It



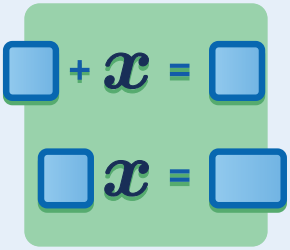
Lesson 2
Five Equations



Lesson 3
Hanging Around



Lesson 4
Hanging It Up



Lesson 5
Swap and Solve

Name: ..... Date: ..... Period: .....

Relationships Between Variables | Building Toward 6.EE.5, SMP.2, SMP.3, SMP.8

Explore: Detecting Counterfeit Coins

How can a balance scale help detect a counterfeit coin?

Warm-Up

1 Which one doesn't belong?

A.



Nimon/Shutterstock.com

B.



AndrewStarikov/Shutterstock.com

C.



Benton Frizer/Shutterstock.com

D.



Benton Frizer/Shutterstock.com

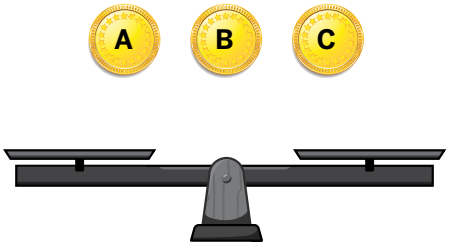
Explain your thinking.

Name: ..... Date: ..... Period: .....

Counterfeit Coin

Here are three identical coins. One of these coins is counterfeit, so it is a little lighter or heavier than the other coins. The balance scale helps determine the counterfeit coin.

2 Is it possible to know which coin is counterfeit after just one weighing? Will you always know which coin is counterfeit after just one weighing?



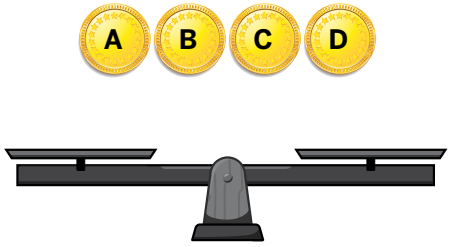
3 What is the smallest number of weighings you would need to perform to determine a coin is counterfeit?

Name: ..... Date: ..... Period: .....

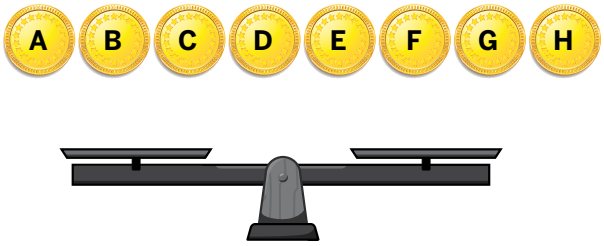
Counterfeit Coin (continued)

What is the smallest number of weighings you would need to perform so you could always say which coin is counterfeit if there are:

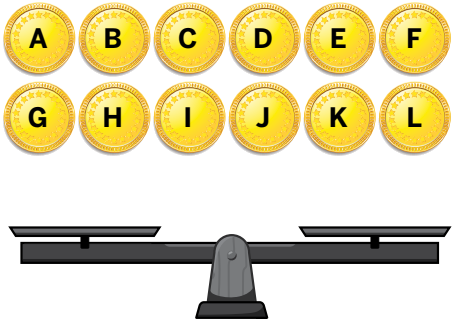
4 Four coins?



5 Eight coins?



6 Twelve coins?



Name: ..... Date: ..... Period: .....

Building Math Habits of Mind

Discuss:

- Which of these habits of mind did you strengthen during this activity?
- How did you use the one(s) you selected?

I can slow down and first make sense of a challenging problem before trying to solve it.

Not yet Almost I got it!

I can represent real-world problems using equations and inequalities and interpret their solutions within the context of the problem.

Not yet Almost I got it!

I can justify my thinking and ask questions to help me understand the thinking of others.

Not yet Almost I got it!

I can apply the math that I know to solve real-world problems, making assumptions and revising my thinking as needed.

Not yet Almost I got it!

I can select an appropriate tool to help me solve problems.

Not yet Almost I got it!

I can communicate my thinking and solutions clearly to others.

Not yet Almost I got it!

I can look for structure or patterns to help me solve problems.

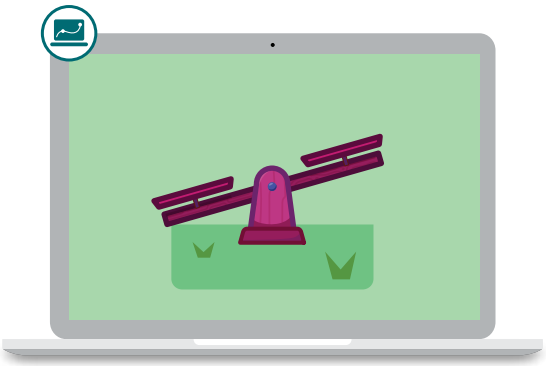
Not yet Almost I got it!

I can look for repeated calculations and other repeated steps to make generalizations.

Not yet Almost I got it!

Name: ..... Date: ..... Period: .....

Generalizing With Multiple Representations 6.EE.6, 6.EE.7, SMP.1, SMP.2

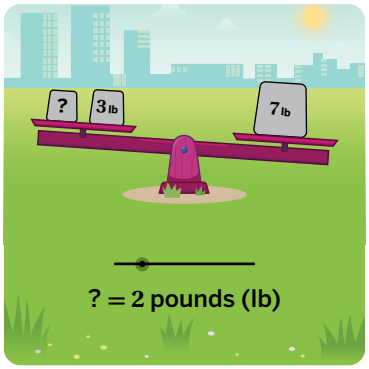


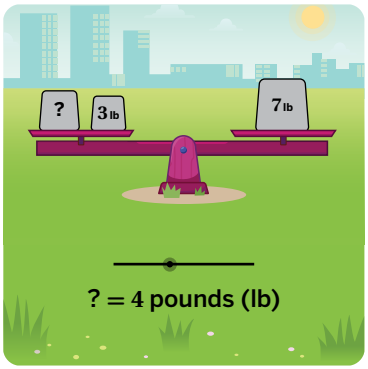
Weight for It

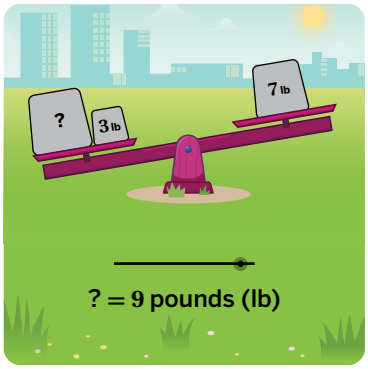
Let's use a seesaw to determine the weights of different animals.

Warm-Up

1 **a** Take a look at some weights on a seesaw.







b What do you notice? What do you wonder?

I notice:

I wonder:

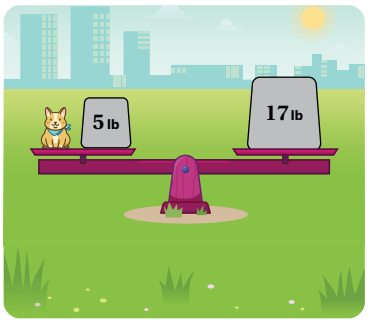
Activity 1

Name: ..... Date: ..... Period: .....

Equations and Tape Diagrams

2 This dog and a 5-pound weight balance a 17-pound weight.

How much does the dog weigh?



3 Tariq wrote an *equation* to represent the situation. He used the **variable** d to represent the dog's weight.

Explain how Tariq's equation is like the seesaw situation.

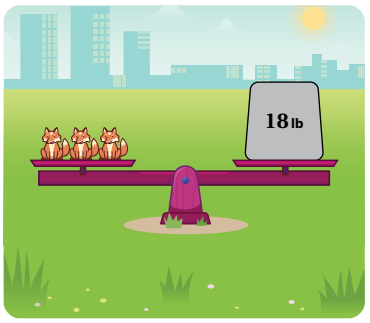
Tariq

$d + 5 = 17$

Name: ..... Date: ..... Period: .....

Equations and Tape Diagrams (continued)

4 These 3 foxes balance with an 18-pound weight. Each fox weighs the same amount.



a Choose an equation that represents this situation.

A. $3 + x = 18$

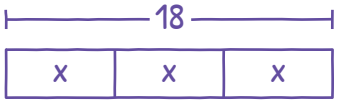
B. $3 \cdot x = 18$

C. $x + x + x = 18$

D. $3 + 18 = x$

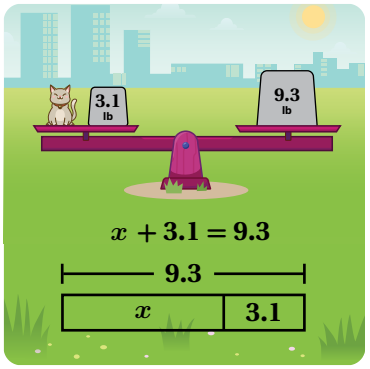
b How much does each fox weigh?

5 Tariq drew a tape diagram to determine the weight of each fox.



How are the tape diagram and the equation you chose in the previous problem alike?

6 This cat and a 3.1-pound weight balance a 9.3-pound weight.



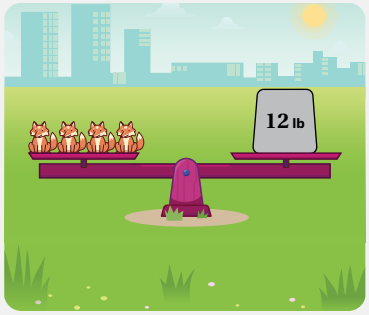
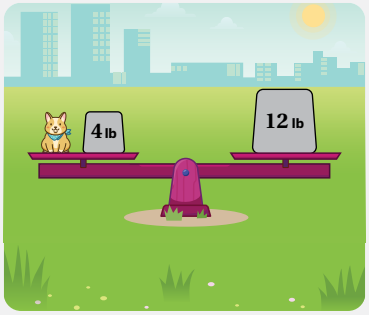
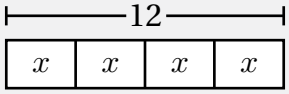
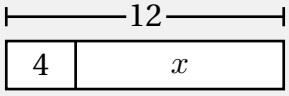
How much does the cat weigh? Use the tape diagram if it helps with your thinking.

Activity 2

Name: ..... Date: ..... Period: .....

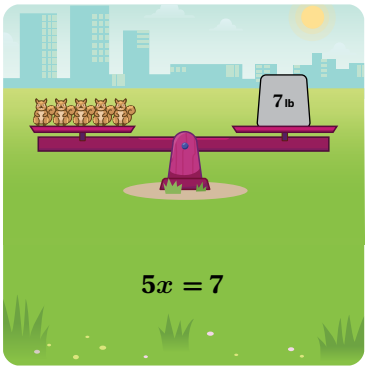
Determining Unknown Weights

7 For each equation or tape diagram, put a check mark under the balanced seesaw it represents.

		
$4 \cdot x = 12$		
$4 + x = 12$		
$x + x + x + x = 12$		
$x = 8$		
		
$x = 3$		
		

8 These 5 squirrels balance with a 7-pound weight. Each squirrel weighs the same amount.

How much does each squirrel weigh? Draw a tape diagram if it helps with your thinking.



Activity 3

Name: ..... Date: ..... Period: .....

Challenge Creator

9 You will use the Activity 3 Sheet to create your own seesaw challenge.

a. **Make It!** On the Activity 3 Sheet, create a balanced seesaw challenge.

b. **Solve It!** On this page, write an equation that represents your balanced seesaw problem. Then determine the weight of your animal. Draw a tape diagram if it helps with your thinking.

My Equation	Weight of My Animal

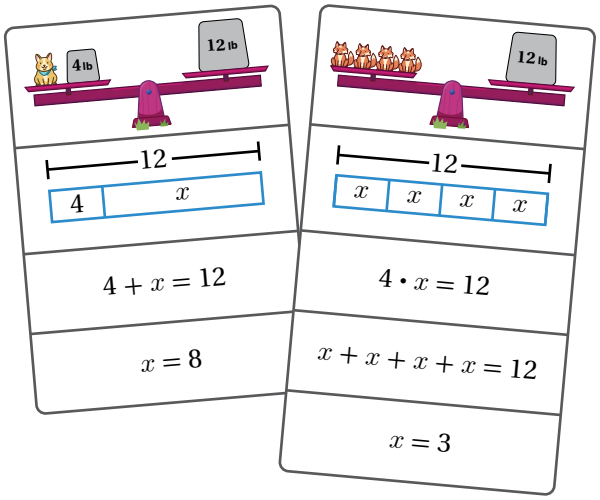
c. **Swap It!** Swap your challenge with one or more partners. Write your partner's equation, then determine the weight of their animal. Draw a tape diagram if it helps with your thinking.

	Equation	Weight of One Animal
Partner 1		
Partner 2		
Partner 3		
Partner 4		

10 Synthesis

Discuss:

- How can you tell if an equation and a tape diagram match?
- What does the **variable** stand for in each representation?

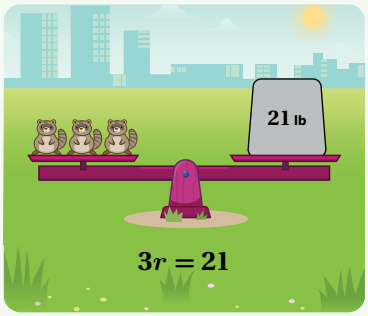


13 Summary 6.01

You can use seesaws and tape diagrams to represent *equations* and help determine unknown values.

We often use a letter, such as x or a, as a placeholder for an unknown number in tape diagrams and equations. This letter is called a **variable**.

For example, if 3 equal-weight racoons weigh a total of 21 pounds, you can represent the weight of each raccoon with r and write the equation $3r = 21$.

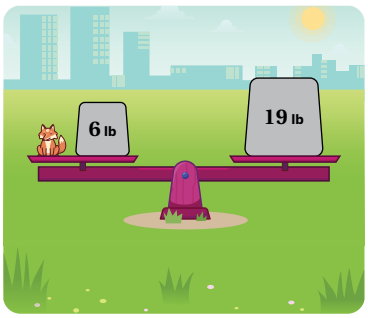


variable A letter or symbol that represents a value or set of values.

Practice
6.01

Name: ..... Date: ..... Period: .....

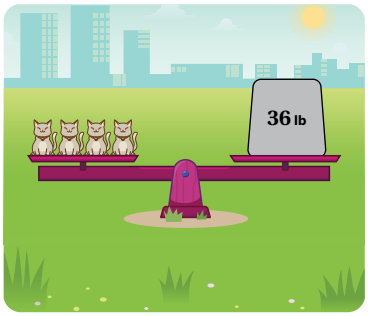
Problems 1–2: Let x represent the weight of a fox



1. Write an equation to represent the situation.

2. Determine the weight of 1 fox.

Problems 3–4: All 4 cats weigh the same amount. Let c represent the weight of a cat.

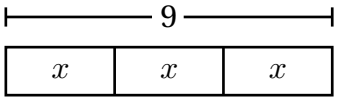


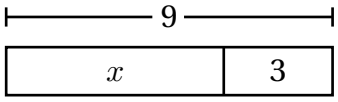
3. Write an equation to represent the situation.

4. Determine the weight of 1 cat.

5. Match each equation to the tape diagram it represents.

$3 + x = 9$ $x + x + x = 9$ $x = 9 \div 3$ $3 \cdot x = 9$ $x = 9 - 3$





Problems 6–7: Kwabena is trying to determine the value of n in the equation $5 \cdot n = 35$. He begins drawing a tape diagram, but isn't sure how to complete it.

n	n	n	n	n

6. Complete Kwabena's tape diagram so it represents the equation $5 \cdot n = 35$.

7. Determine the value of n.

Name: ..... Date: ..... Period: .....

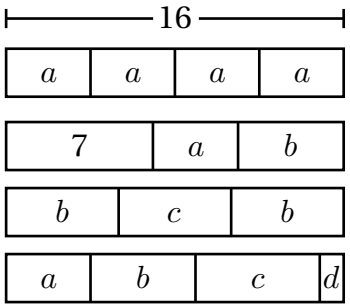
8. Determine the value of a, b, c, and d.

$a =$ .....

$b =$ .....

$c =$ .....

$d =$ .....



Spiral Review

Problems 9–10: Calculate the price per pound for each item.

9. $2.52 for 4.5 pounds of potatoes.

10. $7.75 for 2.5 pounds of broccoli.

Problems 11–16: Determine the missing value to create a true equation.

11. $7 + \square = 10$

12. $\square \cdot 5 = 45$

13. $23 - \square = 11$

14. $\square \div 4 = 8$

15. $\square \cdot \frac{1}{4} = 5$

16. $\square \div 10 = 12.5$

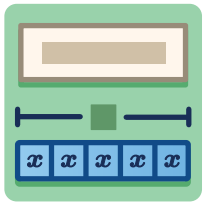
Name: ..... Date: ..... Period: .....

Generalizing With Multiple Representations | Relationships Between Variables

6.EE.5, 6.EE.7, SMP.1, SMP.2

Five Equations

Let's represent situations with equations and tape diagrams.



Warm-Up

1. Here is a situation. Let's make sense of it together as a class.

[redacted] cats weigh [redacted] pounds total.

Each cat weighs the same, so they each weigh [redacted] pounds.

a. **Discuss:** What is this situation about?

b. Let's look at the missing information. What does the variable c represent?

c. Create a tape diagram or sketch that represents this situation.

d. Use your tape diagram or sketch to determine the value of c.

Activity 1

Name: ..... Date: ..... Period: .....

Equations and Tape Diagrams

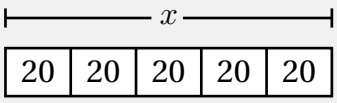
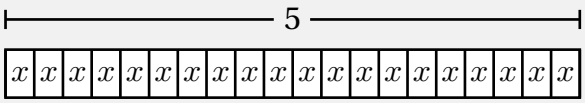
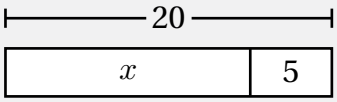
Here are five equations.

$x + 5 = 20$ $\quad 20 = x - 5$ $\quad 5 \cdot 20 = x$ $\quad 5x = 20$ $\quad 20x = 5$

2. Circle two equations that have something in common.

Discuss: How are these equations alike? How are they different?

3. Match each tape diagram with one of the equations. Two equations will not have matches.

Tape Diagram	Equation
	
	
	

4. Draw a tape diagram for an equation that did not have a match.

Tape Diagram	Equation

Name: ..... Date: ..... Period: .....

Equations and Tape Diagrams (continued)

5. For each equation, circle the value of x that makes the equation true.

Equation	Values
$x + 5 = 20$	4, 10, 15, 25
$20 = x - 5$	5, 15, 25, 30
$5 \cdot 20 = x$	4, 15, 100, 200
$5x = 20$	4, 5, 10, 15
$20x = 5$	$\frac{1}{4}$, $\frac{1}{2}$, 4, 100
$\frac{x}{5} = 20$	$\frac{1}{2}$, 10, 20, 100
$4x = 20$	$\frac{1}{2}$, 4, 5, 80
$\frac{x}{4} = 20$	5,20, 40, 80

6. **Discuss:** How did you determine which value makes each equation true?

Activity 2

Name: Date: Period:

Which Equation?

4 cats weigh 48 pounds total. Each cat weighs the same, so they each weigh c pounds.

Equation	Solution to the Equation	Solution's Meaning
$4c = 48$	$c = 12$	Each cat weighs 12 pounds.

7. What do you think a **solution to an equation** is?

You will use a set of description and situation cards.

8. Match each card with the equation that represents it.

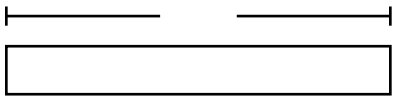
9. Circle the solution to each equation and write the solution's meaning for each situation.

	$x - 9 = 22$	$9x = 22$	$9 + x = 22$
Matching Cards			
Solution to the Equation ($x =$ ____)			
Solution's Meaning			

Synthesis

8. **Discuss:** How can you tell which equation represents a situation? Use the example if it helps with your thinking.

Kwasi rides the subway 20 stops to get to work. After x stops, he has 5 stops left.



$x + 5 = 20$ $5x = 20$

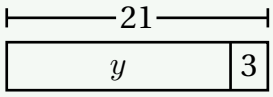
Summary 6.02

A tape diagram can help us visualize an equation and determine its solution.

When we work with an equation that represents a situation, it is important to determine what the variable represents when we determine the solution.

Here is an example.

Emmanuel needed \$21 to buy a gift. He had \$3 and borrowed the rest from his parents.

Equation	Tape Diagram	Solution to the Equation	Solution's Meaning
$3 + y = 21$		$y = 18$	Emmanuel borrowed \$18 from his parents.

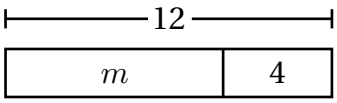
substitute To replace a variable with a value or other expressions.

solution to an equation A value of the variable that makes the equation true.

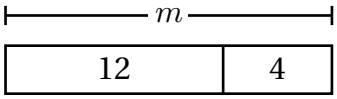
Practice
6.02

Name: ..... Date: ..... Period: .....

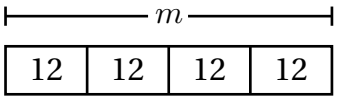
1. Match each equation to the tape diagram that represents it.

a 

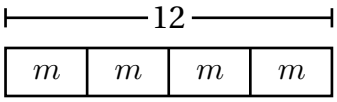
..... $12 = 4m$

b 

..... $m \div 4 = 12$

c 

..... $12 + 4 = m$

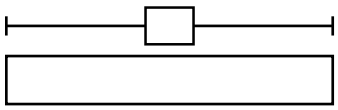
d 

..... $12 - 4 = m$

2. Which equations from Problem 1 has a solution of $m = 48$.

Problems 3–6: Aaliyah filled a water bottle with 24 ounces of water before school. She drank 15 ounces at lunch. There are x ounces of water left.

3. Draw a tape diagram to represent the situation.



4. Select *all* the equations that could represent this situation.

- ☐ A. $24 - 15 = x$
- ☐ B. $24 + 15 = x$
- ☐ C. $x + 15 = 24$
- ☐ D. $15x = 24$
- ☐ E. $24 \div 15 = x$

5. Determine the solution to one of the equations you selected in Problem 3.

6. Explain the solution's meaning in this situation.

Name: ..... Date: ..... Period: .....

Problems 7–8: Write an equation to represent each situation.

7. The sum of a number, x, and 7 is 12.

8. The product of 6 and a number, x, is 42.

Spiral Review

Problems 9–11: Fill in each blank to create a true equation.

9. $2.83 - 1.6 = \square$ **10.** $\square + 2.1 = 7$ **11.** $\frac{3}{4} \cdot \square = 8$

12. Fill in each blank using whole numbers from 0 to 9 so that x is the same value in each equation. Use each number only once.

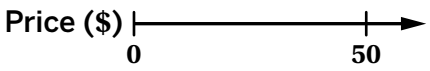
$x = \square \cdot \square$ $x = \square + \square$ $x + \square = \square$

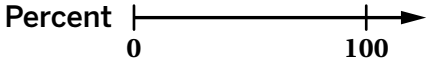
13. Select *all* the true equations.

☐ A. $5 + 0 = 0$ ☐ B. $15 \cdot 0 = 0$ ☐ C. $1.4 + 2.7 = 4.1$

☐ D. $\frac{2}{3} \cdot \frac{5}{9} = \frac{7}{12}$ ☐ E. $4\frac{2}{3} = 5 - \frac{1}{3}$

14. Hailey paid $40 for a jacket. The regular price was $50. What percent of the regular price did Hailey pay? Use the double number line if it helps with your thinking.

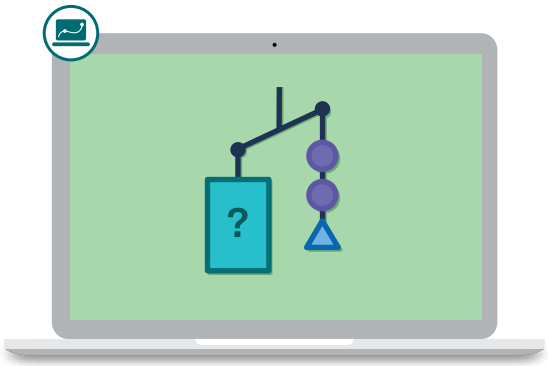




Name: ..... Date: ..... Period: .....

Generalizing With Multiple Representations | Relationships Between Variables

6.EE.5, 6.EE.6, 6.EE.7, SMP.6, SMP.7

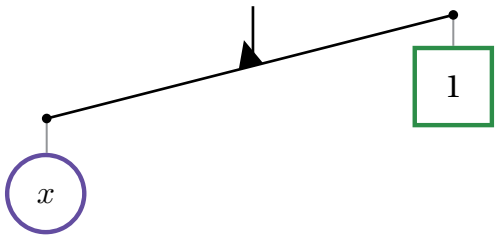


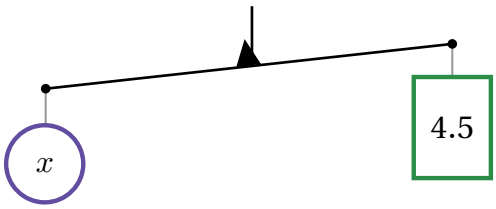
Hanging Around

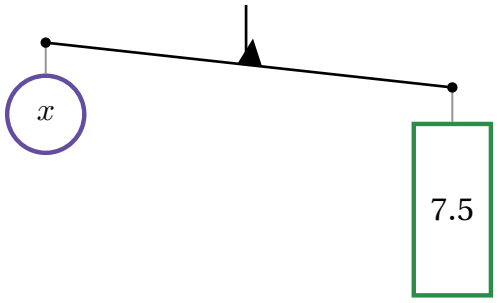
Let's use balanced hangers to solve equations.

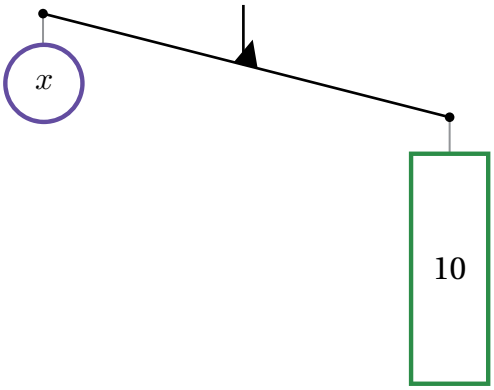
Warm-Up

1 a Take a look at the hangers with a circle of weight x on one side and a rectangle of different weights on the other side.







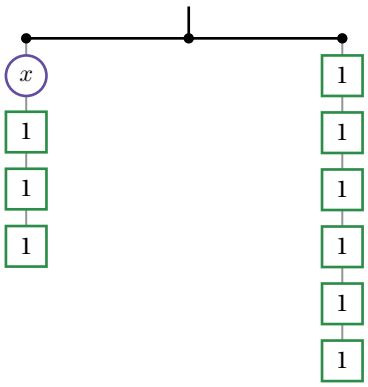


b **Discuss:** What is the weight of the circle? Explain your thinking.

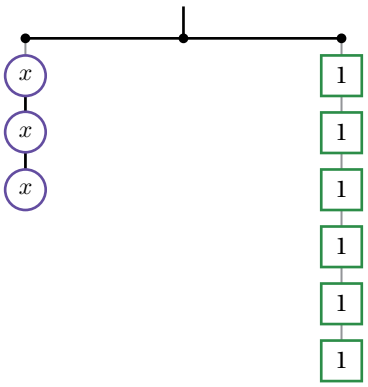
Name: ..... Date: ..... Period: .....

Connect It

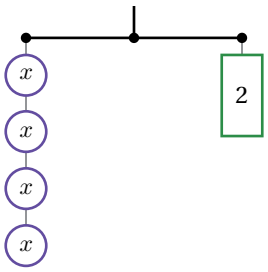
2 What value of x balances the hanger?



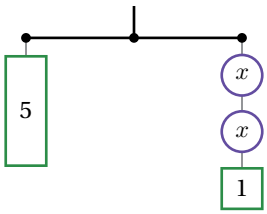
3 What value of x balances the hanger?



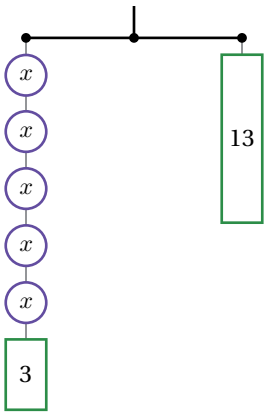
4 **a** Take a look at these hangers and the equations that represent them.



Equation: $4x = 2$



Equation: $5 = 2x + 1$



Equation: $5x + 3 = 13$

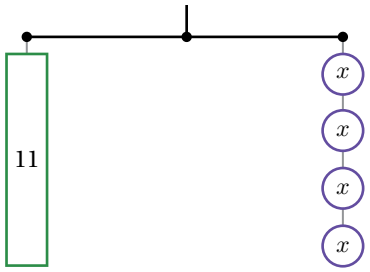
b Explain how an equation is like a hanger.

Activity 2

Name: ..... Date: ..... Period: .....

Make It, Solve It

5 Select an equation that represents this hanger.



A. $11 + x = 4$

B. $11 = 4x$

C. $11 = x + 4$

D. $11 = x + x + x + x$

Explain your thinking.

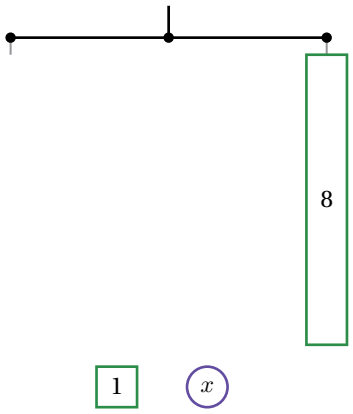
6 Use the hanger or the equation from the previous problem to determine the value of x that balances the hanger.

Name: ..... Date: ..... Period: .....

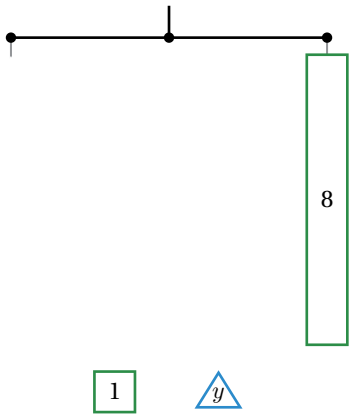
Make It, Solve It (continued)

7 Make a balanced hanger to represent each equation.

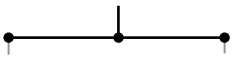
$5x = 8$



$5 + y = 8$



8 **a** Make a balanced hanger that represents $6 = x + 2$.





b What is the value of x that balances the hanger?

Activity 3

Name: ..... Date: ..... Period: .....

Challenge Creator

9 You will use the Activity 3 Sheet to create your own hanger challenge.

a **Make It!** On the Activity 3 Sheet, create your own balanced hanger challenge.

b **Solve It!** On this page, write the equation that represents your hanger and then determine the value of x that balances your hanger.

My Equation	Solution to My Equation

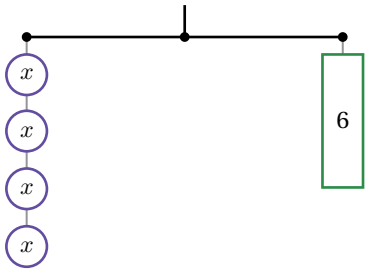
c **Swap It!** Swap your challenge with one or more partners. Write their equation, then determine the value of x that balances their hanger.

	Equation	Solution to Their Equation
Partner 1		
Partner 2		
Partner 3		
Partner 4		

10 Synthesis

How can a balanced hanger help determine the solution to an equation?

Use the hanger and equation if that helps you with your thinking.



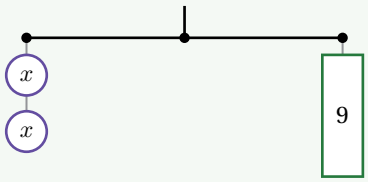
$4x = 6$

13 Summary 6.03

Hangers are a helpful way to represent equations. A hanger is balanced when the weight on both sides is equal.

Here is an example.

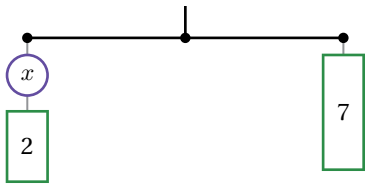
This hanger represents the equation $2x = 9$, or $x + x = 9$. The solution to this equation is the value of x that will keep the hanger balanced. The solution for this hanger is 4.5 because $4.5 + 4.5 = 9$ or $2(4.5) = 9$.



Practice
6.03

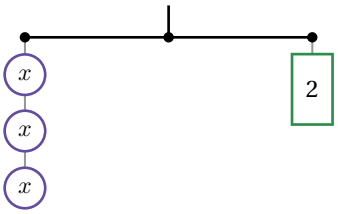
Name: ..... Date: ..... Period: .....

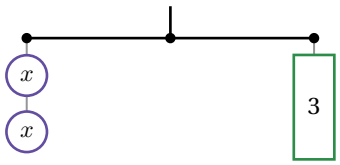
1. Anushka says that to balance this hanger the value of x must be 7. Is she correct? Explain your thinking.

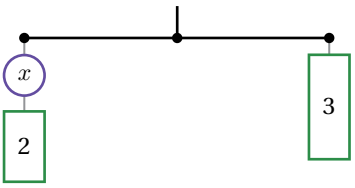


2. Match each equation to the hanger it represents. One equation will have no match.

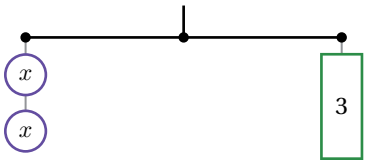
$2x = 3$	$2 + x = 3$	$3 + x = 2$	$3x = 2$

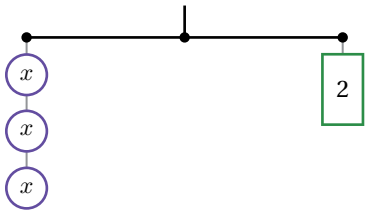






Problems 3–6: Write an equation to represent each hanger. Then, determine the value of x that balances the hanger.





3. Equation:

4. $x =$

5. Equation:

6. $x =$

Practice
6.03

Name: ..... Date: ..... Period: .....

Spiral Review

Problems 7–10: Determine the value of each expression.

7. $12 + 2.4$

8. $12 \cdot 2.4$

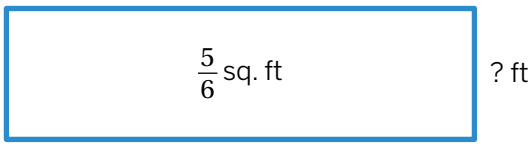
9. $12 - 2.4$

10. $12 \div 2.4$

11. Calculate the area of this rectangle.



12. Calculate the length of this rectangle.



13. Precious set a goal to save $\$20$ for a new game. Complete the table to show how much money Precious saved at different percentages of her goal.

Percentage of Goal (%)	Money Saved ($)
25	
75	
125	

14. Select *all* of the equations that have a solution of $m = 7$.

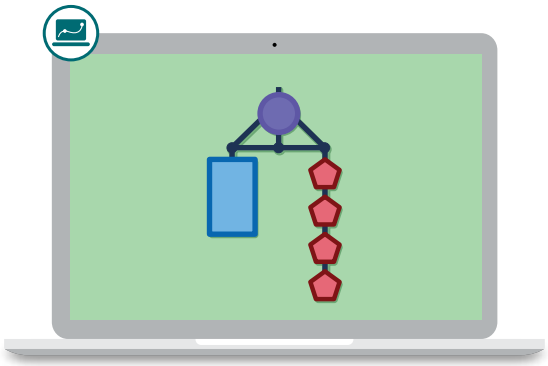
- ☐ **A.** $2m = 21$
- ☐ **B.** $m + 9 = 16$
- ☐ **C.** $3m = 21$
- ☐ **D.** $40 - m = 23$
- ☐ **E.** $m - 6 = 1$

Unit 6
Lesson
4

Name: ..... Date: ..... Period: .....

Fraction Relationships | Generalizing With Multiple Representations

6.EE.5, 6.EE.6, 6.EE.7, 6.NS.1, SMP.5, SMP.7



Hanging It Up

Let's use a variety of strategies to solve equations.

Warm-Up

Determine the value of each expression mentally. Try to think of more than one strategy.

1. $10 \div \frac{1}{3}$

2. $10 \div \frac{2}{3}$

3. $\frac{10}{3} \div \frac{1}{3}$

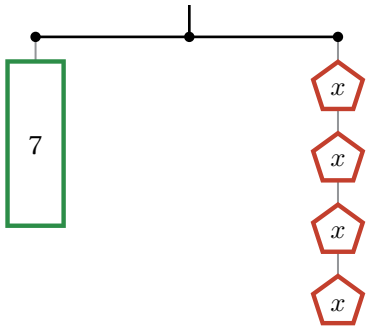
4. $\frac{10}{9} \div \frac{1}{3}$

Activity 1

Name: ..... Date: ..... Period: .....

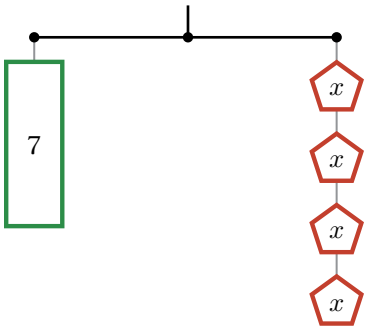
Methods for Solving

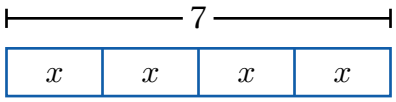
5 **a** Write an equation that matches the hanger.



b What is the *solution* to this equation?

6 Here are two different representations of the equation $7 = 4x$.





Discuss: How can you determine the solution using each representation?

7 Here is how Fabiana solved the equation $7 = 4x$. Describe her strategy.

Fabiana

$\frac{7}{4} = \frac{4x}{4}$

Name: ..... Date: ..... Period: .....

Solving and Solutions

8 Match each solution to its equation. One solution will not have a match.

13	4.8	$\frac{5}{3}$	$\frac{3}{5}$	$\frac{1}{8}$

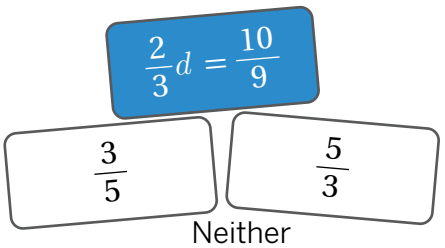
$2a = \frac{1}{4}$	$2.6 = \frac{1}{5}b$	$\frac{c}{4} = 1.2$	$\frac{2}{3}d = \frac{10}{9}$

9 Imani and Deiondre solved this equation.

Imani said the solution is $d = \frac{3}{5}$.

Deiondre said the solution is $d = \frac{5}{3}$.

Whose solution is correct? Circle one.



Imani's Diondre's Both Neither

Explain your thinking.

Name: ..... Date: ..... Period: .....

Solving and Solutions (continued)

10 Here are Fabiana's and Alejandro's strategies for solving $\frac{c}{4} = 1.2$.

Fabiana

$\frac{c}{4} = 1.2$

$\times 4 \quad \times 4$

$c = 4.8$

Alejandro

$\frac{c}{4} = 1.2$

$\frac{c}{4} = 1$

so . . .

$\frac{4.8}{4} = 1.2$

Discuss: How are their strategies alike? How are they different?

11 Use Fabiana's strategy to solve the equation $3.5 = x + 2.01$.

Activity 3

Name: _____ Date: _____ Period: _____

Repeated Challenges

12

- Decide with your partner who will complete Column A and who will complete Column B.
- The solutions in each row should be the same. Compare your solutions, then discuss and resolve any differences.
- Solve as many equations as you have time for. Sense-making is more important than speed.

	Column A	Column B
a	$36 = 4x$ $x =$ _____	$7x = 63$ $x =$ _____
b	$13 = x + 5$ $x =$ _____	$21 = x + 13$ $x =$ _____
c	$\frac{1}{3} = 2x$ $x =$ _____	$3x = \frac{1}{2}$ $x =$ _____
d	$x + 6.17 = 9$ $x =$ _____	$12.22 = x + 9.39$ $x =$ _____

Name: ..... Date: ..... Period: .....

Repeated Challenges (continued)

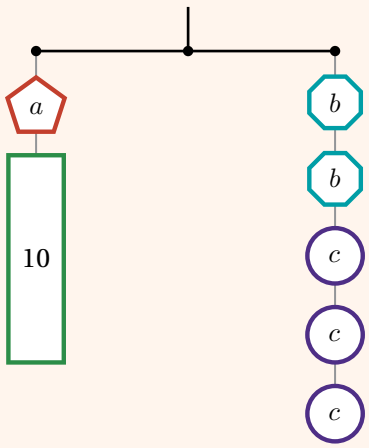
	Column A	Column B
e	$x + 1.8 = 14.7$ $x =$	$x + 5.3 = 18.2$ $x =$ .....
f	$\frac{1}{2}x = 16$ $x =$	$4 = \frac{1}{8}x$ $x =$ .....
g	$\frac{7}{8} = x + \frac{1}{4}$ $x =$	$x + \frac{1}{16} = \frac{11}{16}$ $x =$ .....

You're invited to explore more.

13 Complete the table with values for a, b, and c that will make the hanger balance.

Try to determine as many different sets of values as you can.

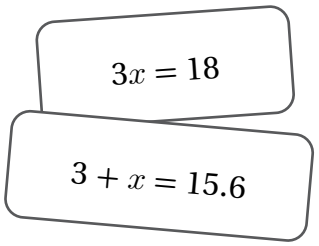
a	b	c



14 Synthesis

Discuss: What is a strategy for solving an equation?

Use the examples if they help with your thinking.



17 Summary 6.04

There are many strategies to solve equations, such as drawing models, using number sense to determine the value that makes an equation true, making a hanger balanced, or using inverse operations to isolate a variable.

Here are two examples that use inverse operations to solve an equation.

Equation	Explanation
$x + 1.5 = 3.25$	Original equation
$x + 1.5 - 1.5 = 3.25 - 1.5$	Subtract 1.5 from both sides.
$x = 1.75$	The solution to this equation is 1.75.

Equation	Explanation
$\frac{1}{2}y = 54.$	Original equation
$\frac{1}{2}y \div \frac{1}{2} = 54 \div \frac{1}{2}$	Divide both sides by $\frac{1}{2}$.
$y = 108$	The solution to this equation is 108.

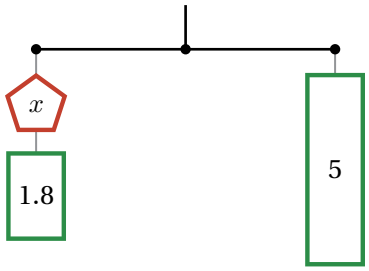
Practice
6.04

Name: ..... Date: ..... Period: .....

1. Select *all* the equations that have a solution of $n = 11$.

- ☐ **A.** $2n = 22$
- ☐ **B.** $23 - n = 12$
- ☐ **C.** $4n = 411$
- ☐ **D.** $n \div 1 = 11$
- ☐ **E.** $n - 7 = 4$

Problems 2–3: Use the hanger diagram if it helps with your thinking.



2. Determine the value of x in the equation $x + 1.8 = 5$.

3. Vihaan says the solution to $x + 1.8 = 5$ is $x = 6.8$. Explain how you know this is incorrect.

Problems 4–9: Solve each equation. Draw a hanger or tape diagram if it helps with your thinking.

4. $4m = 8$

5. $\frac{1}{2}a = \frac{5}{8}$

6. $10d = 32$

7. $w + 5.2 = 17$

8. $1.5x = 0.9$

9. $24.6 = 6.1 + c$

Name: _____ Date: _____ Period: _____

10. Fill in each blank using the numbers 0 to 9 so that x has the same value in each pair of equations. Each number can only be used once.

$x = \square$
$x + \square = \square$

$x = \square$
$x + \square = \square$

Spiral Review

11. Calculate each product.

Expression	Product
$212 \cdot 2$	
$21.2 \cdot 0.2$	
$21.2 \cdot 0.02$	

12. Kweku and Javier each used a different strategy to determine 25% of 60. Whose strategy is correct? Circle one.





Kweku's Javier's Both Neither

Explain your thinking.

Problems 13–14: A rectangle has an area of 14 square units. It has side lengths x and y. Determine the value of y based on the value of x.

13. $x = 2\frac{1}{3}$

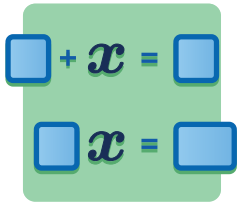
14. $x = 4\frac{1}{5}$

Name: ..... Date: ..... Period: .....

Generalizing With Multiple Representations 6.EE.6, 6.EE.7, SMP.1, SMP.2, SMP.6

Swap and Solve

Let's write and solve equations.



Warm-Up

1. Here is a situation with hidden information. Let's make sense of it as a class.

Takeshi has [redacted] to spend on laundry. It costs [redacted] to wash and dry each load. Takeshi can wash [redacted] loads of laundry.

2. Dhruv and Nyanna each wrote an equation to represent this situation.

Dhruv: $p = 21 \cdot 3.50$

Nyanna: $3.50p = 21$

Whose equation is correct? How do you know?

Activity 1

Name: ..... Date: ..... Period: .....

Solving Equations with a Partner

3. With your partner, decide who will be Partner A and who will be Partner B. For each equation, the person who has the variable on their side will go first! Share the operation to isolate the variable with your partner to solve each equation. Together, explain what you did and check your solution.

Partner A		Partner B	Rationale	Check
18 $\frac{18}{2}$ 9	$=$ $=$ $=$	$2x$ $\frac{2x}{2}$ x	**Partner _B_:** Because $2x$ represents 2 times x, I need to divide the side with the variable by 2. **Partner _A_:** To maintain equality both sides need to be divided by 2.	$18 = 2x$ $18 = 2 \cdot 9$ $18 = 18$ ✅
$\frac{x}{5}$	$=$		**Partner ____:** **Partner ____:**	
5	$=$	$x + 2.17$	**Partner ____:** **Partner ____:**	
17	$=$	$x - 9$	**Partner ____:** **Partner ____:**	

Activity 2

Name: Date: Period:

Trade and Solve

Takeshi has $21 to spend on laundry. It costs $3.50 to wash and dry each load. Takeshi can wash p loads of laundry.

Equation	Solution	Solution Check	Solution's Meaning
$3.50p = 21$	$p = 6$	$3.50 \cdot 6 = 21$	Takeshi can wash and dry 6 loads of laundry for $21.

4. What do you think a solution's meaning is?

5. Here is a new set of equations. Circle an equation and solve it for x.

Equations:

$x + \frac{3}{4} = 6$ $\frac{3}{4}x = 6$ $6x = \frac{3}{4}$ $x - 0.25 = 20$ $0.25x = 20$

6. Write a situation to match this equation. Make sure to include what the variable represents in your situation.

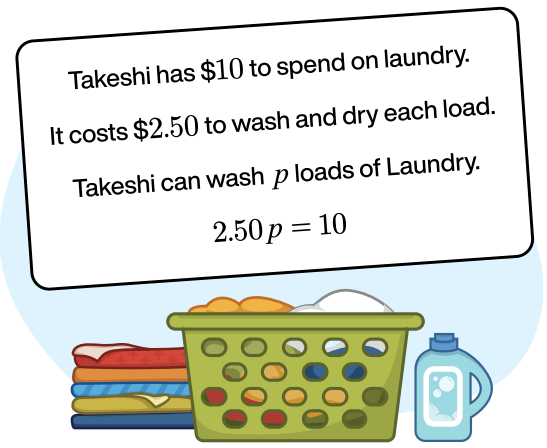
7. You will need two different partners for this problem. With each partner, trade the slips of paper with your situations and complete the table for their situation.

Partner's Name	Equation	Solution	Solution Check	Solution's Meaning

Synthesis

8. Discuss:

- What do you think is important to remember when writing equations to represent situations?
- How do the **properties of equalities** help you determine the solution to equations?



Summary 6.05

Writing an equation to match a situation is a helpful tool when trying to determine an unknown value. The **properties of equality** tell you the equation will still be true if you perform the exact same operations on both sides of the equal sign. The table shows the four properties of equality, using the variables a and b to represent any numbers.

Property of Equality	The equation will still be true, when the same
addition property of equality	number is added to both sides of the equation.
subtraction property of equality	number is subtracted from both sides of the equation.
multiplication property of equality	number is multiplied by both sides of the equation.
division property of equality	nonzero number is divided by both sides of the equation.

properties of equality Rules that apply to all equations such as properties of addition, subtraction, multiplication, and division. It states that if an equation is true, then performing the same operation to both sides of the equal sign will keep the equation true.

Practice
6.05

Name: ..... Date: ..... Period: .....

1. Anika buys 5 notebooks that contain 60 pages each. Select *all* the equations that represent the total number of pages, p.

- ☐ **A.** $p = 60 \div 5$
- ☐ **B.** $5 + 60 = p$
- ☐ **C.** $p = 5 \cdot 60$
- ☐ **D.** $p \div 5 = 60$
- ☐ **E.** $5p = 60$

2. Tiara buys a pack of paper with 200 sheets. She divides the sheets of paper equally into 5 binders. Select *all* the equations that represent the number of sheets of paper in each binder, b.

- ☐ **A.** $b = 200 \div 5$
- ☐ **B.** $200 \div b = 5$
- ☐ **C.** $b = 5 \cdot 200$
- ☐ **D.** $b \div 5 = 200$
- ☐ **E.** $5b = 200$

Problems 3–4: Here is an equation: $\frac{1}{2} + x = 4$.

3. Write a situation that the equation could represent.

4. Describe the meaning of the x in your situation.

Problems 5–6: A plant in Zahra's garden grows 0.8 inches taller each week. After x weeks, the plant has grown 6 inches.

5. Write an equation that could represent this situation.

6. Describe the meaning of the x in the situation.

7. Anika traveled 196 miles last week from Yosemite National Park to Lake Tahoe. This was four times as far as Tiara traveled, t.

Select *all* the equations that represent this situation. Then determine how far Tiara traveled?

- ☐ **A.** $4t = 196$
- ☐ **B.** $t = \frac{1}{4} \cdot 196$
- ☐ **C.** $196 \div 4 = t$
- ☐ **D.** $t = 196 \cdot 4$
- ☐ **E.** $\frac{t}{4} = 196$

$t =$ .....

Practice
6.05

Name: ..... Date: ..... Period: .....

Spiral Review

8. Use the numbers 0 to 9 to complete each equation so that the value of x is the same. Use each number only once.

$\square x = \square$ $\quad x + \square = \square$ $\quad x - \square = \square$

9. Select *all* the equations that have a solution of $c = 1.5$.

- ☐ **A.** $4c = 41.5$
- ☐ **B.** $150 \div c = 100$
- ☐ **C.** $13.5 - c = 10$
- ☐ **D.** $6c = 9$
- ☐ **E.** $0.2c = 0.3$

Problems 10–12: Solve each equation.

10. $6m = 33$

11. $p + 7.04 = 11.8$

12. $n + \frac{3}{5} = \frac{8}{10}$

13. Compare the information given about triangle C and triangle D.

Triangle C	Triangle D
Base $= 12$ inches	Base $= 15$ inches
Height $= 8$ inches	Height $= 6.5$ inches

Which triangle has the greater area?

Explain your thinking.

Notes:

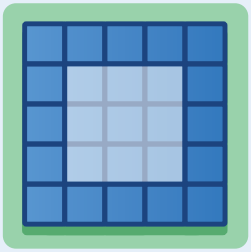
Sub-Unit

2

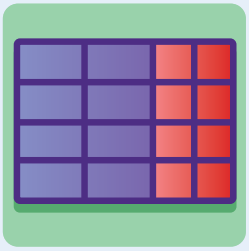
Equivalent Expressions



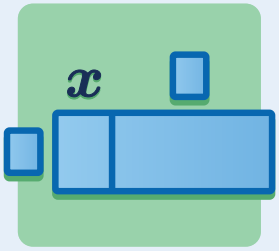
Lesson 6
Vari-apples



Lesson 7
Border Tiles



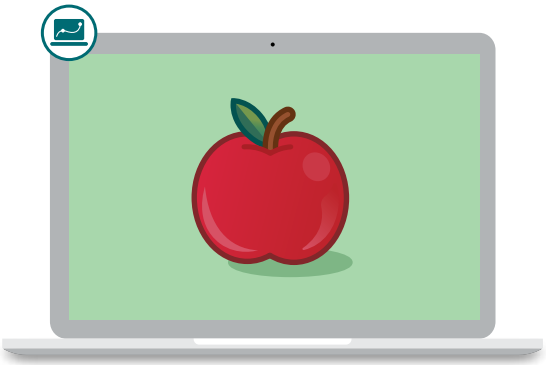
Lesson 8
Products and Sums



Lesson 9
Equivalent Expressions

Name: ..... Date: ..... Period: .....

Generalizing With Multiple Representations | 6.EE.2.a, 6.EE.2.b, 6.EE.6, 6.RP.3.a, SMP.2, SMP.3, SMP.6

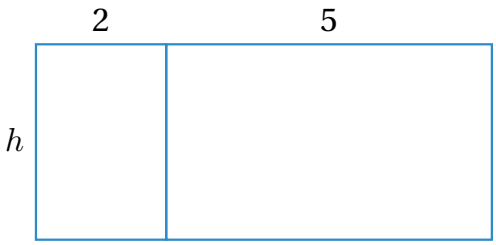


Vari-apples

Let's use variable expressions to represent situations.

Warm-Up

1 Write an expression for the area of the entire rectangle. Explain your thinking.



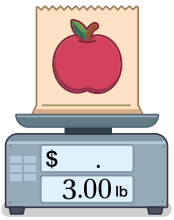
Activity 1

Name: ..... Date: ..... Period: .....

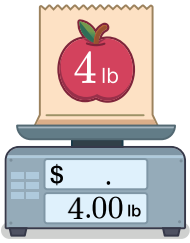
Intro to Variable Expressions

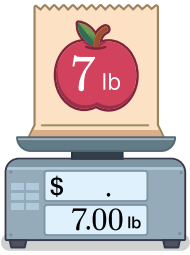
Apples at your store cost $1.50 per pound.

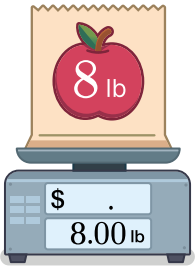
2 A customer orders 3 pounds of apples. How much should you charge them?



3 Here are three new orders. How much should you charge for each order?







Apples (lb)	Cost ($)
4	
7	
8	

4 Describe how to determine the cost of *any* number of pounds of apples.

Name: ..... Date: ..... Period: .....

Intro to Variable Expressions (continued)

5 Rudra and Sai each wrote an expression to describe the cost of p pounds of apples.

Rudra's Expression: p + 1.50

Sai's Expression: 1.50p

Whose expression is correct? Circle one.

Rudra's Sai's Both Neither

Explain your thinking.

Apples (lb)	Cost ($)
3	4.50
4	6.00
7	10.50
8	12.00
p	

6 A **constant** is a value that does not change.

A **coefficient** is a number multiplied by a variable.

1.50 is the constant in Rudra's expression.

a What is 1.50 in Sai's expression?

constant coefficient both

b **Discuss:** How did you decide which term to select?

7 Limes at your store cost $2.40 per pound.

How much should you charge for p pounds of limes?



Name: ..... Date: ..... Period: .....

Comparing Variable Expressions

8 For $5, you can get your groceries delivered. What is the total cost for each of these grocery deliveries?

Cost of Groceries ($)	Total Cost ($)
37.95	42.95
50.86	
72.11	
87.94	



Grocery cost:	$37.95
Delivery fee:	$5.00
Total:	$42.95

9 Write an expression for how much you should charge for g dollars worth of groceries, including delivery.

10 Match each situation with an expression that represents its cost. Two expressions will have no match.

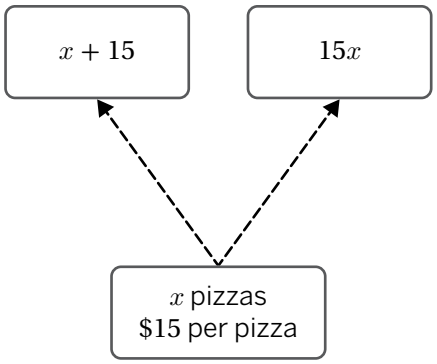
$10x$	$15x$	$x + 10$	$x + 15$

x pizzas $15 per pizza	x dollars of groceries $10 for delivery

Name: ..... Date: ..... Period: .....

Comparing Variable Expressions (continued)

11 Which expression represents this situation?



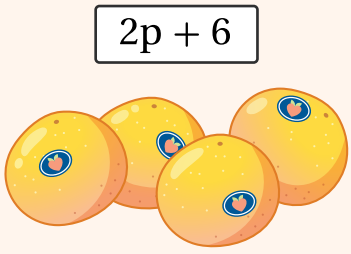
12 The expression $15x$ has one **term**. The expression $15 + x$ has two terms.

Select *all* the expressions that also have two terms.

- ☐ **A.** $y - 6$
- ☐ **B.** $\frac{1}{2}x$
- ☐ **C.** $3x + 2$
- ☐ **D.** $x + y$
- ☐ **E.** $3 \cdot 2$

You're invited to explore more.

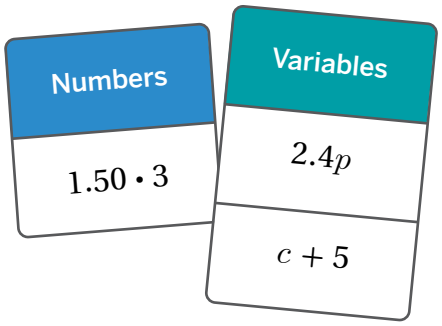
13 Describe a situation that could be represented by the expression $2p + 6$. Create a table if it helps with your thinking.



14 Synthesis

Here are two types of expressions: expressions with numbers and expressions with variables.

Discuss: When might each kind of expression be useful?



17 Summary 6.06

Expressions with variables can represent situations with known and unknown values. One or more parts together can be viewed as terms, coefficients, constants, sums, products, and quotients.

The cost of 1 pound of grapes is $2.25.

- Let p represent pounds of grapes.
- The expression $2.25p$ can be used to calculate the total cost for any number of pounds of grapes.

$2.25p$

Number of terms: 1
Factors: 2.25 and p

Coefficient: 2.25
Product: $2.25p$

A grocery store adds a $10 fee to the cost of groceries for delivery.

- Let c represent the cost of groceries.
- The expression $c + 10$ can be used to calculate the total cost for groceries and delivery.

$c + 10$

Number of terms: 2
Addends: c and 10

Constant: 10
Sum: $c + 10$

You have $8 to spend on bananas.

- Let b represent the cost of bananas per pound.
- The expression $8 \div b$ can be used to determine how many pounds of bananas you can buy.

$8 \div b$

Number of terms: 2
Dividend: 8
Divisor: b

Constant: 8
Quotient: $8 \div b$

constant A value that does not change.

coefficient A number that is multiplied by a variable.

term A part of an expression.

Practice
6.06

Name: ..... Date: ..... Period: .....

Problems 1–3: Oranges cost $\$1.25$ per pound. How much would it cost to buy:

1. 2 pounds of oranges?

2. 5 pounds of oranges?

3. x pounds of oranges?

4. You need red and blue ribbon for a craft project. The instructions say that the red ribbon should be 7 inches longer than the blue ribbon.

Complete the table to show how long the red ribbon should be for different lengths of blue ribbon.

Blue Ribbon (in.)	Red Ribbon (in.)
10	
27	
x	

5. 35 riders are on a bus, and n riders get off at the same stop. In this scenario, what does the expression $35 - n$ represent?

Problems 6–7: The variable s represents the number of students in one class in your school.

6. What does $\frac{1}{2}s$ represent?

7. What does $s + 1$ represent?

Problems 8–10: Evaluate the expression $3m + 5$ for each value of m.

Example

$m = 7$

$3(7) + 5 = 26$

8. $m = 8$

9. $m = 0.8$

10. $m = \frac{5}{6}$

Practice
6.06

Name: ..... Date: ..... Period: .....

Spiral Review

11. LaShawn's class raised $\$500$ for a fundraiser. The class used 10% of the money to cover the cost of materials, saved 20% for the next fundraising project, and donated the rest. How much money did LaShawn's class donate?

Explain your thinking.

12. A garbage bin can hold 50 gallons of waste.

Complete the table to show what percent of the bin would be filled for different amounts of waste.

Waste (gal)	Percent Filled (%)
5	
30	
45	

13. Choose any value for a.

$a =$ ☐

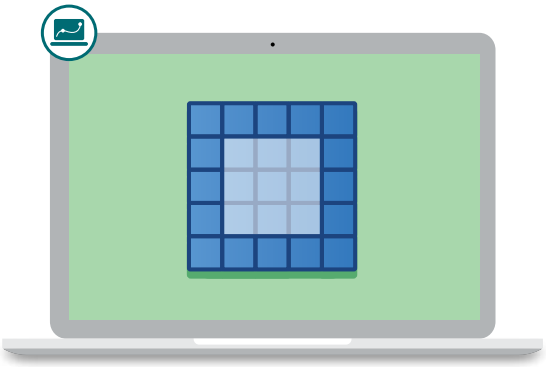
Use your value to complete the puzzle.

$3a$	$+$	4	$=$					
$-$				$+$				$-$
a		$4a$	$+$	$4a$	$-$		$=$	$5a$
$-$		$+$		$=$				$=$
a					$-$	$2a$	$=$	
$-$		$=$				$+$		
a	$+$	$6a$	$-$		$=$	$4a$		
$=$		$+$				$=$		
		a		a	$+$		$=$	

14. What is the price per pound of cashews if 3.8 pounds cost $\$15.96$?

Name: ..... Date: ..... Period: .....

Generalizing With Multiple Representations | 6.EE.2.b, 6.EE.2.c, 6.EE.3, 6.EE.4, SMP.3, SMP.7

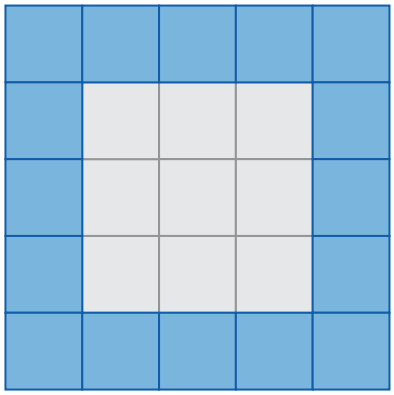


Border Tiles

Let's use diagrams to determine which expressions are equivalent.

Warm-Up

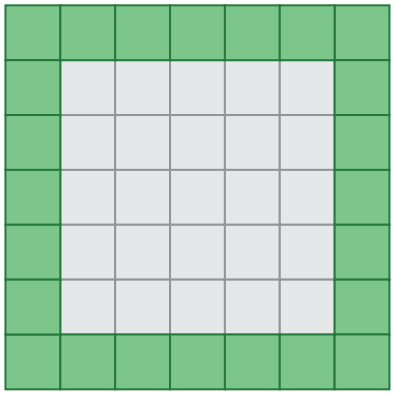
1 Here is a 3-by-3 square surrounded by border tiles.



a Without counting one by one, how many border tiles are there?

b Explain how you see it.

2 Here is a 5-by-5 square surrounded by border tiles. Without counting one by one, how many border tiles are there?



Activity 1

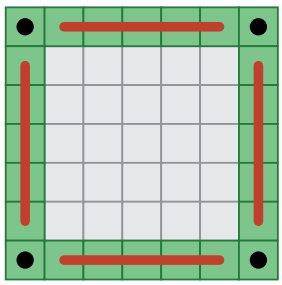
Name: ..... Date: ..... Period: .....

Border Tiles

3 **a** Take a look at Lucia's, Kyrie's, and Manuel's expressions for the 5-by-5 square.

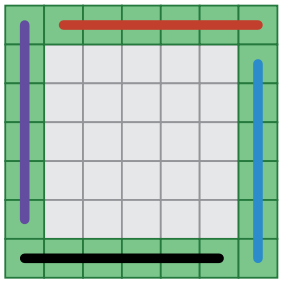
Lucia

$5 + 5 + 5 + 5 + 4$



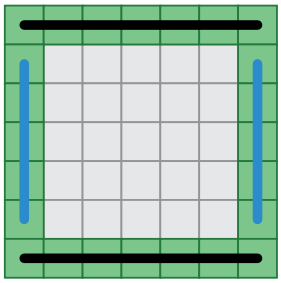
Kyrie

$4\,(5 + 1)$



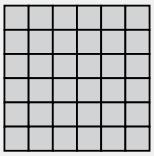
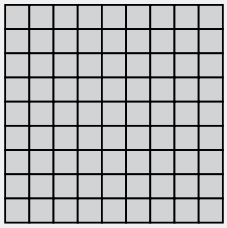
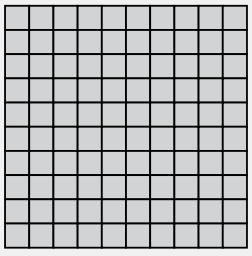
Manuel

$7 + 7 + 5 + 5$



b **Discuss:** How are all of their expressions alike?

4 Here are three new squares. Determine the number of border tiles for each square.

Model	Square	Border Tiles
	6-by-6	
	9-by-9	
	10-by-10	

5 How can you determine the number of border tiles for an n-by-n square? Use your table if it helps with your thinking.

Activity 2

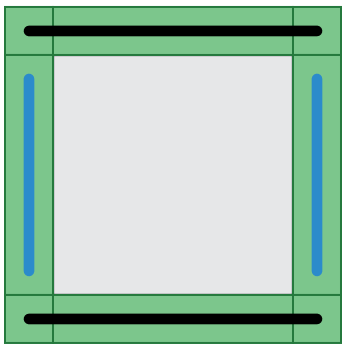
Name: Date: Period:

Equivalent Expressions

6 Manuel wrote this expression for the number of border tiles in an n-by-n square:

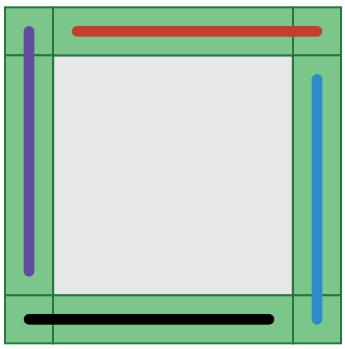
$(n + 2) + (n + 2) + n + n$

Show or explain how Manuel's expression is connected to his sketch.



7 Here is Kyrie's sketch for the same square.

What might Kyrie's expression be for the number of border tiles in an n-by-n square?



Name: ..... Date: ..... Period: .....

Equivalent Expressions (continued)

8 **Equivalent expressions** are expressions that are equal for every value of a variable.

Here are two expressions.

Kyrie: $4(n + 1)$

Manuel: $(n + 2) + (n + 2) + n + n$

a Solve their equations for each value of n in the table.

$n =$	Kyrie's Expression $4(n + 1)$	Manuel's Expression $(n + 2) + (n + 2) + n + n$
6		
7		
11		

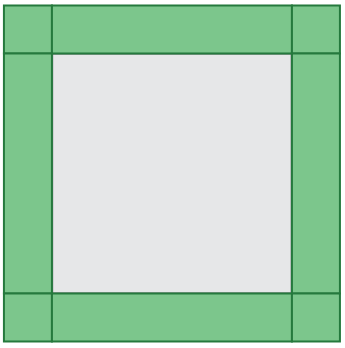
b **Discuss:** What do you notice about Kyrie and Manuel's expressions?

9 **Equivalent expressions** are expressions that are equal for every value of a variable. Which expression is also equivalent to Kyrie's and Manuel's expressions?

A. $(n + 1)(n + 1)$

B. $4n + 1$

C. $4n + 4$



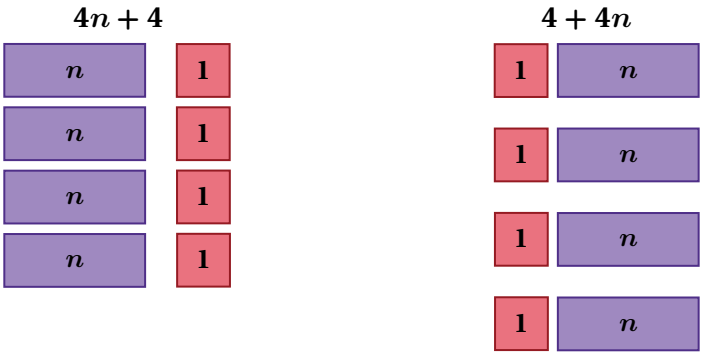
?

Activity 3

Name: Date: Period:

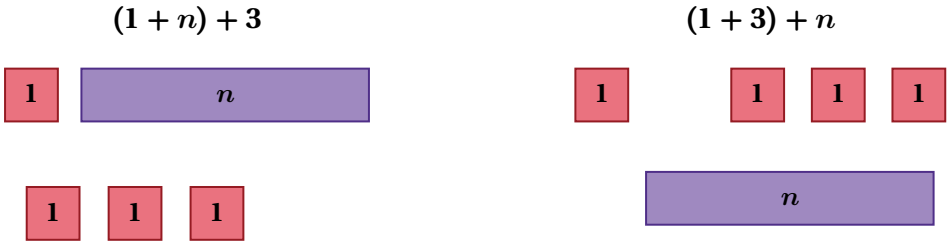
A New Diagram

10 Here is a new way of visualizing expressions.



Discuss: How do these diagrams show that $4n + 4$ and $4 + 4n$ are *equivalent*?

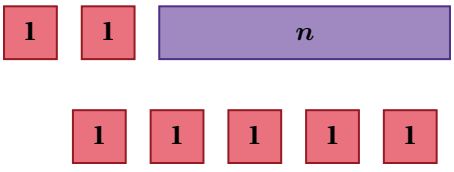
11 Here is another set of equivalent expressions.

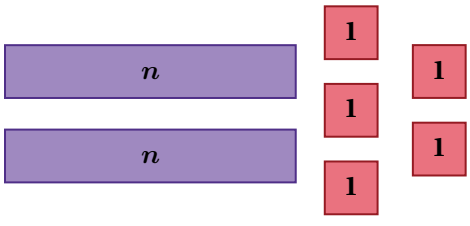


Discuss: How do these diagrams show that $(1 + n) + 3$ and $(1 + 3) + n$ are *equivalent*?

12 Match each expression with the diagram it represents. One expression will have no match.

$2n + 5$	$(2 + n) + 5$	$(n + 2) + (n + 2) + 1$	$2 + (n + 5)$
$n + n + 1 + 1 + 1 + 1 + 1$		$2(n + 1)$	$n + 7$

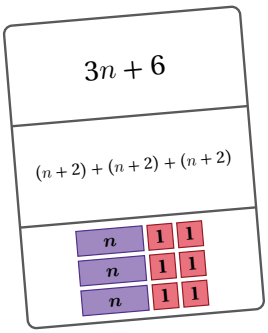




13 Synthesis

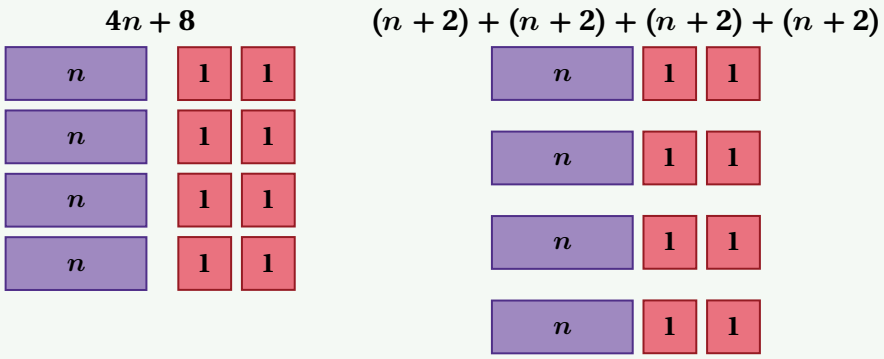
Discuss: How can you decide if two expressions are equivalent?

Use the example if it helps you explain your thinking.



16 Summary 6.07

Expressions that are equal for every value of a variable, such as $4n + 8$ and $(n + 2) + (n + 2) + (n + 2) + (n + 2)$, are **equivalent expressions.** Diagrams that represent these expressions can help us visually decide if the expressions are equivalent.



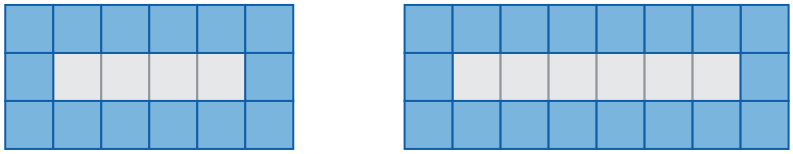
The diagrams for $4n + 8$ and $(n + 2) + (n + 2) + (n + 2) + (n + 2)$ both show 4 n-tiles and 8 one-tiles. Therefore, $4n + 8$ and $(n + 2) + (n + 2) + (n + 2) + (n + 2)$ are equivalent expressions because they are equal for every value of n.

equivalent expressions Expressions that are equal for every value of the variable.

Practice
6.07

Name: ..... Date: ..... Period: .....

Problems 1–3: Here are examples of an n-by-1 rectangle.



1. How many border tiles are there in the 4-by-1 rectangle?

2. How many border tiles are in the 6-by-1 rectangle?

3. Diego says $2n + 6$ represents the number of tiles needed for the border of an n-by-1 rectangle. Explain why his strategy is correct.

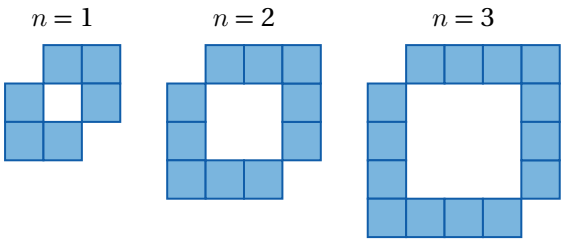
Problems 4–5: Here are five expressions.

$2 + 6n$ $\quad 2(n + 3)$ $\quad n + 3$ $\quad n + n + 6$ $\quad (n + 3) + (n + 3)$

4. Write *all* of the expressions that are equivalent to $2n + 6$.

5. Choose an expression that is *not* equivalent to $2n + 6$. Explain how you know it is not equivalent.

Problems 6–7: Here is a pattern.



6. Write an expression that describes the number of tiles for any stage, n.

7. Write an equivalent expression.

Practice
6.07

Name: ..... Date: ..... Period: .....

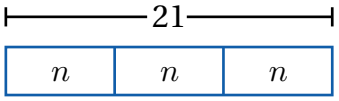
Spiral Review

Problems 8–9: Write an equation to represent each situation.

8. Aba's dog was $5\frac{1}{2}$ inches tall when it was a puppy but is now 14 inches tall. Aba's dog grew n inches.

9. Apples cost $\$1.10$ per pound. Darius bought x pounds of apples for a total cost of $\$2.75$.

Problems 10–11: Here is a tape diagram.



10. Write an equation to represent the tape diagram.

11. Determine the value of n.

Problems 12–15: Evaluate each expression for $b = 5$.

12. $3.5b$

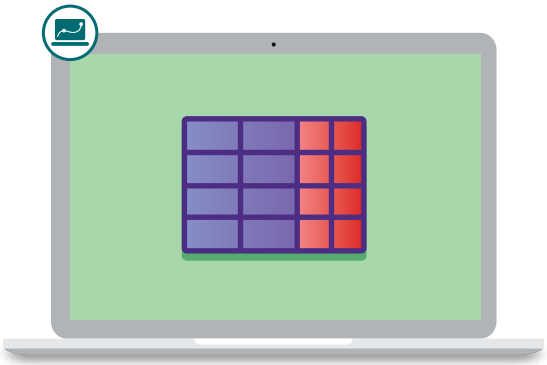
13. $6b + 1$

14. $\frac{1}{4} + b$

15. $\frac{1}{2}b$

Name: ..... Date: ..... Period: .....

Generalizing With Multiple Representations | 6.EE.2.b, 6.EE.3, 6.EE.4, SMP.3



Products and Sums

Let's explore equivalent expressions using rectangle areas.

Warm-Up

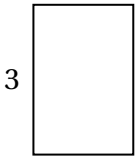
1 Write an expression for the area of each rectangle.

a



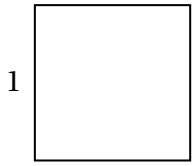
Expression: .....

b



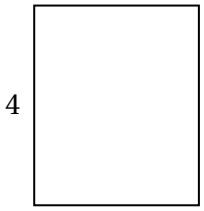
Expression: .....

c



Expression: .....

d



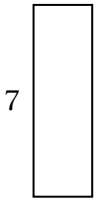
Expression: .....

e



Expression: .....

f



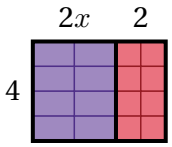
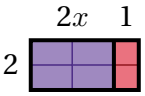
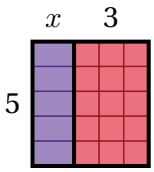
Expression: .....

Activity 1

Name: Date: Period:

Rectangles and Equivalent Expressions

2 Here are four rectangles and the *product* and *sum* expressions that represent their areas.

Rectangle				
Product	$4(2x + 2)$	$2(2x + 1)$	$1(3x + 5)$	$5(x + 3)$
Sum	$8x + 8$	$4x + 2$	$3x + 5$	$5x + 15$

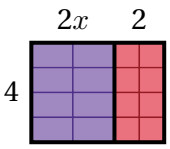
What do you notice about the product expressions?

What do you notice about the sum expressions?

3 When two (or more) expressions are multiplied together, both expressions are called **factors**. In the *product expression* $4(2x + 2)$, the factors are 4 and $(2x + 2)$

Select *all* parts of the product expression $3(x + 5)$ that are factors:

- ☐ **A.** 3
- ☐ **B.** $x + 5$
- ☐ **C.** $3x$
- ☐ **D.** x
- ☐ **E.** 5



Product	Factors
$4(2x + 2)$	$4, 2x + 2$

Name: ..... Date: ..... Period: .....

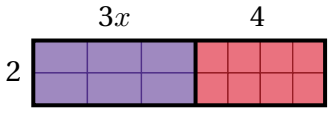
Rectangles and Equivalent Expressions (continued)

4 **a** Create a rectangle with an area of $2(3x + 4)$. Label the sides of your rectangle.

b Write an equivalent expression for the area.

5 How would you convince someone that $2(3x + 4)$ is *not* equivalent to $6x + 4$?

6 The **distributive property** says that multiplying a number by the sum of two or more terms is equal to multiplying the number by each term separately before adding them together.



$2(3x + 4)$

$2 \cdot 3x + 2 \cdot 4$

$6x + 8$

Discuss: What is happening in each step?

Activity 2

Name: ..... Date: ..... Period: .....

More Rectangles and Equivalent Expressions

7 Create a rectangle with an area of $6x + 12$.

8 Select *all* the expressions that are equivalent to $6x + 12$. Use the rectangle you created or draw a new rectangle if it helps with your thinking.

- ☐ **A.** $6(x + 2)$
- ☐ **B.** $6(x + 12)$
- ☐ **C.** $3(2x + 4)$
- ☐ **D.** $3(x + 4)$
- ☐ **E.** $2(3x + 6)$

9 **a** Here are three new expressions. Select the *two* expressions that are equivalent.

- ☐ **A.** $4(x + 2)$
- ☐ **B.** $4(x + 8)$
- ☐ **C.** $4x + 8$

b Create a drawing to convince someone that the two expressions you selected are equivalent.

Activity 3

Name: ..... Date: ..... Period: .....

Challenge Creator

10 You will use the Activity 3 Sheet to create your own rectangle challenge.

a. **Make It!** On the Activity 3 Sheet, create a rectangle challenge.

b. **Solve It!** On this page, write two expressions that represent your rectangle. Record one expression on your Activity 3 Sheet.

c. **Swap It!** Swap your challenge with one or more partners. Sketch your partner's rectangle and record their expression. Then create an equivalent expression.

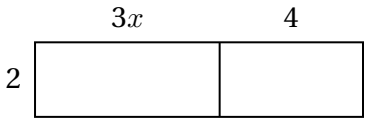
My First Expression	My Second Expression

	Sketch of Rectangle	Partner's Expression	Equivalent Expression
Partner 1			
Partner 2			
Partner 3			
Partner 4			

11 Synthesis

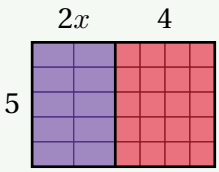
Describe how you can use the area of a rectangle to write two or more equivalent expressions.

Use the example if it helps to show your thinking.



14 Summary 6.08

You can use areas of rectangles to write equivalent expressions. Two expressions that are equivalent are a *product* expression and a *sum* expression because they refer to the same area. In a product expression, you can use the **distributive property** to multiply the two **factors**. No matter what value you substitute for the variable in a product or sum expression, the total area is the same.

Area Model	Product of Two Side Lengths	Sum of Two Areas
	$5(2x + 4)$ $10x + 20$	$5 \cdot 2x + 5 \cdot 4$ $10x + 20$

factor Each part of an expression in a product expression.

distributive property Multiplying a number by the sum of two or more terms is equal to multiplying the number by each term separately before adding them together.

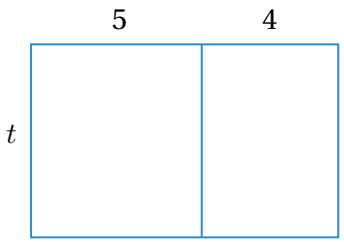
Practice
6.08

Name: ..... Date: ..... Period: .....

1. Select *all* the expressions that are equivalent to $4b$.

- ☐ **A.** $b + b + b + b$
- ☐ **B.** $b + 4$
- ☐ **C.** $b \cdot b \cdot b \cdot b$
- ☐ **D.** $2b + 2b$
- ☐ **E.** $4 \cdot b$

2. Select *all* the expressions that represent the area of the rectangle.

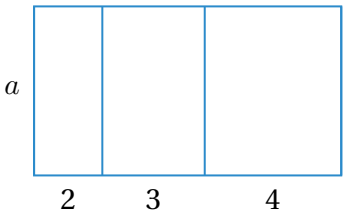


- ☐ **A.** $t + 5 + 4$
- ☐ **B.** $5t + 4t$
- ☐ **C.** $9t$
- ☐ **D.** $4 \cdot 5 \cdot t$
- ☐ **E.** $(5 + 4)t$

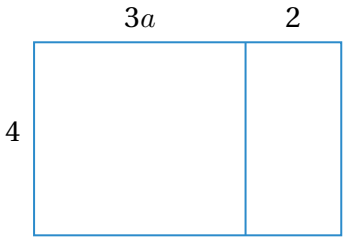
3. Zola wrote the total area of the rectangle as $2a + 3a + 4a$.

Amir wrote the total area as $(2 + 3 + 4)a$.

Explain why they are *both* correct.



4. Write two equivalent expressions that could be used to represent the area of the rectangle.

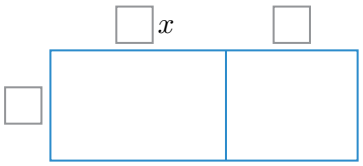


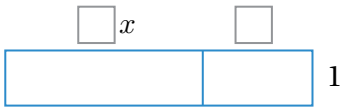
Expression 1: .....

Expression 2: .....

Name: ..... Date: ..... Period: .....

5. Use the numbers 0 to 9 to fill in each blank so that each rectangle has the same area. Use each number only once.





Spiral Review

Problems 6–8: Titus's aunt is 17 years older than he is.

6. How old will his aunt be when Titus is 15 years old?

7. How old will his aunt be when Titus is 30 years old?

8. How old will his aunt be when Titus is x years old?

Problems 9–11: Solve each equation. Show your thinking.

9. $10m = 25$

10. $13.65 = h + 4.88$

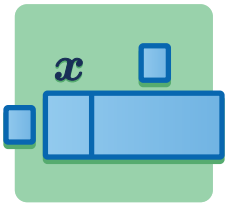
11. $k + \frac{1}{4} = 5\frac{1}{8}$

12. What is the value of m in the equation $2.5m = 30$?

Name: ..... Date: ..... Period: .....

Generalizing With Multiple Representations | 6.EE.3, 6.EE.2.a, 6.EE.2.b, 6.EE.4, SMP.3

Equivalent Expressions

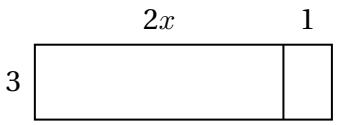


Let's explore equivalent expressions using area models and properties of operations.

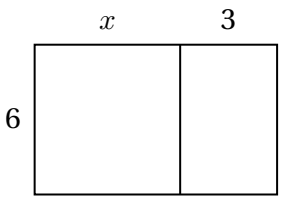
Warm-Up

1. Here are two rectangles.

Rectangle A



Rectangle B



a Which rectangle has an area of $6x + 3$?

Explain your thinking.

b What is the area of the other rectangle?

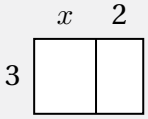
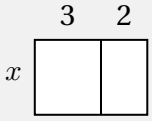
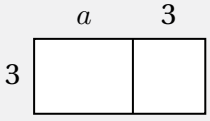
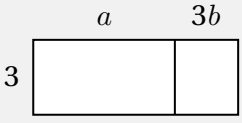
Activity 1

Name: ..... Date: ..... Period: .....

Card Sort

2. You will use a set of cards for this activity.

- Match each product or sum to its representation. Three expressions will be missing.
- Write in each missing expression.

	Representation	Product	Sum
a			
b			
c			
d			
e			
f	The product of 3 and the sum of a and b.		

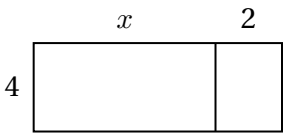
3. Explain how you determined one of the missing expressions. How do the terms, factors, or coefficients help you determine the missing expression?

Activity 2

Name: ..... Date: ..... Period: .....

Two Truths and a Lie

4. Two of these expressions represent the area of this rectangle.



a Which are they?

A. $3(x + 2) + 1(x + 2)$

B. $1 + 3(x + 2)$

C. $4(x + 2)$

b **Discuss:** Which expression is not equivalent to the others? How do you know?

5. In each row, two choices are equivalent and one is not. Circle the one that is *not* equivalent.

	Expression A	Expression B	Expression C
a	$6(2 + x)$	$2(6 + x)$	$6(x + 2)$
b	$16x$	$1(8 + x)$	$x + 8$
c	$3 + 2(3 + x)$	$5(3 + x)$	$9 + 2x$
d	The product of 5 and the sum of x and 2.	$5x + 2$	$5(x + 2)$
e	$4(6x + 3x)$	$36x$	$24x + 3x$
f	$7(x + 2)$	$3x + 4(x + 2)$	$7x + 8$

6. Pick one problem and explain how you decided which choice was not equivalent.

Synthesis

8. **Discuss:** How you can show that two expressions are equivalent?

Use these expressions if they help with your thinking.

Equivalent Expressions

$3x + 4(x + 2)$

$7x + 8$

Not Equivalent

$7(x + 2)$

Summary 6.09

There are many ways to decide if two expressions are equivalent. You can draw models, substitute values, or rewrite the expressions. If the expressions are equivalent, you can use the distributive property and other operations to rewrite one expression to look like the other.

Here is an example: Determine whether $2(x + 1) + 3(2x)$ is equivalent to $8x + 2$.

$$\begin{aligned} 2(x + 1) + 3(2x) &= 2x + 2 + 6x \\ &= 2x + 6x + 2 \\ &= 8x + 2 \end{aligned}$$

$2(x + 1) + 3(2x)$ and $8x + 2$ are equivalent expressions because after using the distributive property and adding the like terms, the expressions are the same.

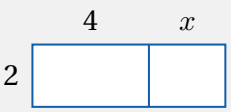
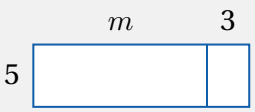
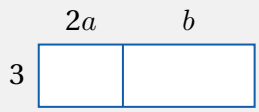
Practice
6.09

Name: ..... Date: ..... Period: .....

1. Select *all* the expressions that are equivalent to $4x + 8$.

- ☐ **A.** $4(x + 2)$
- ☐ **B.** $(4 + 8)x$
- ☐ **C.** $2(2x + 4)$
- ☐ **D.** $2(2x + 6)$
- ☐ **E.** $4 + 4(x + 1)$

2. Complete the table.

Rectangle			
Product		$5(m + 3)$	
Sum	$8 + 2x$		

3. Latifa and Joel are trying to rewrite $8y + 24$ as a product expression.

Latifa: $8(y + 3)$

Joel: $2(4y + 12)$

Are Latifa's and Joel's expressions both equivalent to $8y + 24$? Circle one.

Yes No

Explain your thinking.

Problems 4–6: Determine whether each pair of expressions are equivalent.

	Expression A	Expression B	Equivalent?
4.	$2(x + 8) + 3$	$2x + 19$	Yes No
5.	$5 + 2(y + 4)$	$7y + 28$	Yes No
6.	$3(z + 1) + 3(z + 1)$	$6(z + 1)$	Yes No

Name: ..... Date: ..... Period: .....

7. Complete the table.

Product Expression	Sum Expression
	$4x + 8$
$(6 + 8)d$	
	$10m + 7m$
$3(2b + 5)$	
$6(u + 2t)$	

8. The area of a rectangle is $30 + 12x$.

List three possibilities for the length and the width of the rectangle.

Length	Width

Spiral Review

Problems 9–11: Solve each equation. Show your thinking.

9. $x + 5 = 11$

10. $0.6y = 1.8$

11. $5w = 17.5$

12. Select *all* the equations that have a solution of $m = 3.2$.

- ☐ **A.** $1.4m = 4.48$
- ☐ **B.** $62 \div m = 19$
- ☐ **C.** $m + 8.7 = 11.9$
- ☐ **D.** $0.5m = 1.8$
- ☐ **E.** $7m = 22.4$

Name: ..... Date: ..... Period: .....

Generalizing With Multiple Representations | Relationships Between Variables

6.EE.2.a, 6.EE.2.b, 6.EE.3, 6.EE.4, 6.EE.5, 6.EE.6, 6.EE.7, SMP.7

Practice Day 1



Let's practice what you've learned so far in this unit!

Start with any of the scavenger hunt sheets

- Record the sheet shape, solve the problem, and write your answer.
- Look for your answer at the top of another scavenger hunt sheet. Solve that problem.
- Repeat until you make it back to your starting sheet.

Sheet: ..... Answer →	**Sheet:** ..... ↙ Answer
Sheet: ..... Answer →	**Sheet:** ..... ↙ Answer
Sheet: ..... Answer →	**Sheet:** ..... Answer ↗

continued on next page...

Name: Date: Period:

Practice Day 1

Sheet: Answer →	**Sheet:** ↙ Answer
Sheet: Answer →	**Sheet:** Answer

You're invited to explore more.

Here is a rectangle. Determine whole number values for A and for side lengths w, x, y, and z. Are there other values that will work? How many possibilities are there?

	y	z
w	A	24
x	18	72

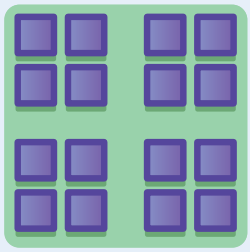
Diagram not to scale.

Notes:

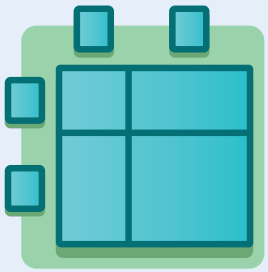
Sub-Unit

3

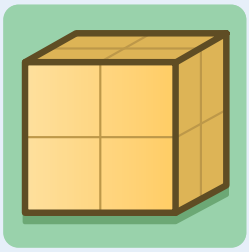
Expressions Involving Exponents



Lesson 10
Powers



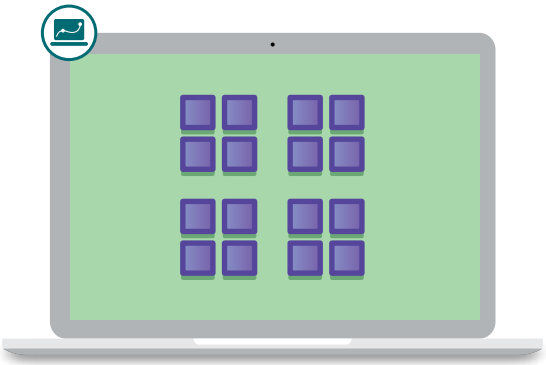
Lesson 11
Exponent Expressions



Lesson 12
Squares and Cubes

Name: ..... Date: ..... Period: .....

Generalized Numbers Leading to Algebra | 6.EE.1, SMP.3, SMP.7, SMP.8

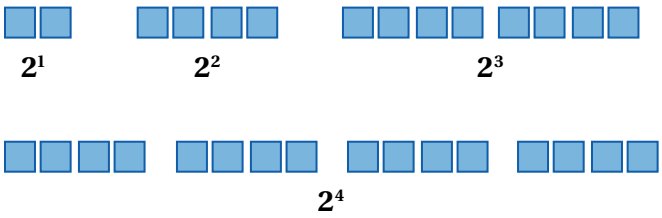


Powers

Let's see how exponents show repeated multiplication.

Warm-Up

1 Here are some images and their matching expressions.



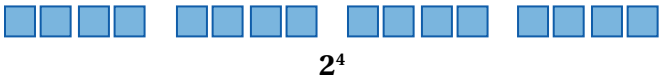
Discuss: What do you notice? What do you wonder?

Activity 1

Name: ..... Date: ..... Period: .....

Powers of 2

2 The expression 2^4 (2 to the power of 4) is equivalent to $2 \cdot 2 \cdot 2 \cdot 2$.



2^4

How could you determine the value of 2^5?

3 Write a number or expression that is equivalent to 2^5.

4 Group the equivalent expressions. One expression will not have a match.

$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$		$2 + 2 + 2 + 2 + 2$		1^3		$5 + 5$	
2^5	1^5	$2 \cdot 5$		5^2		$2^4 \cdot 2$	

Group 1	Group 2	Group 3

Name: ..... Date: ..... Period: .....

Powers of 2 (continued)

5 Miriam and Kira had different answers for the card sort.

Miriam says the 1^5 card and the 1^2 card can be paired together because they both have a value of 1.

Kira says 1^5 does not equal 1^2 because $1^5 = 5$ and $1^2 = 2$.

Who is correct? Explain your thinking.



Activity 2

Name: ..... Date: ..... Period: .....

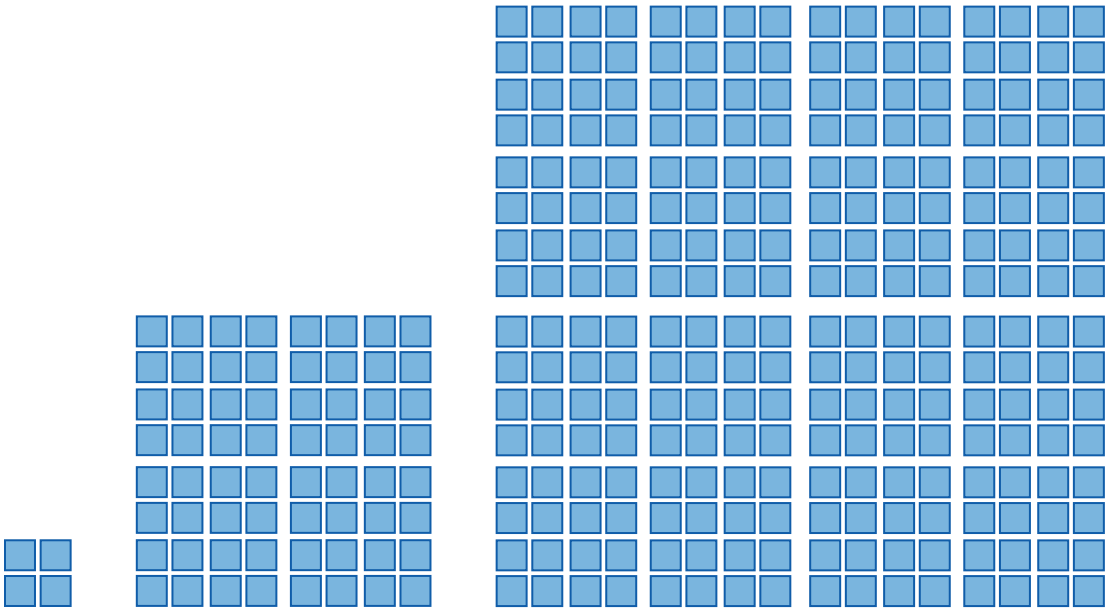
Exponents With Whole Number Bases

6 Select *one* expression that is equivalent to 3^4.

A. 12 **B.** $4 \cdot 4 \cdot 4$ **C.** $3 \cdot 3 \cdot 3 \cdot 3$ **D.** 81

Show or explain your thinking.

7 $4 \cdot 4$ is equivalent to 4^2, where 2 is the **exponent** and 4 is the **base**.



Write $4 \cdot 4 \cdot 4 \cdot 4 \cdot 4$ using at least one exponent.

8 Write a number or an expression that is equivalent to 4^3.

Activity 3

Name: ..... Date: ..... Period: .....

Exponents With Decimal and Fractional Bases

9 Here are some images and their matching expressions.



$(0.5)^1$ $(0.5)^2$ $(0.5)^3$

Write two things you know about $(0.5)^3$.

1.

2.

10 Victor wrote two expressions equivalent to $\left(\frac{1}{3}\right)^4$.

Write a different expression equivalent to $\left(\frac{1}{3}\right)^4$.

Victor

$\left(\frac{1}{3}\right)^4$

$\frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{3}$

$\frac{1}{9} \cdot \frac{1}{9}$

Explain your thinking.

11 Write an expression that is equivalent to $\left(\frac{1}{2}\right)^5$.

Name: ..... Date: ..... Period: .....

Exponents With Decimal and Fractional Bases (continued)

12 Determine the value of each expression. Complete as many as you have time for.

a 2^3

b 4^2

c $\left(\frac{1}{3}\right)^2$

d 3^4

e 5^1

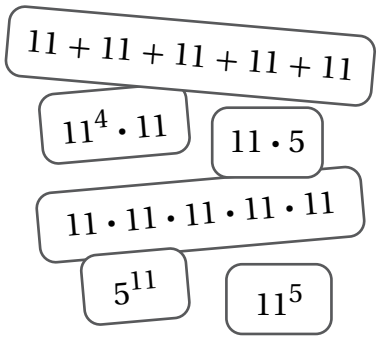
f $\left(\frac{1}{2}\right)^3$

g 1^8

h $\left(\frac{1}{4}\right)^3$

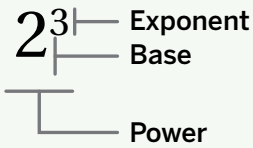
13 Synthesis

Discuss: How can you tell whether expressions with exponents are equivalent?



16 Summary 6.10

Exponents are used to represent repeated multiplication. In the expression 2^n, 2 is the **base**, and n is the **exponent**. If n is a positive whole number, it represents how many times 2 should be multiplied to determine the value of the expression.



Here are some examples.

$2^1 = 2$ $\quad\quad$ $2^3 = 2 \cdot 2 \cdot 2$

There are several different ways to say "2^3."

- "Two to the power of three."
- "Two raised to the power of three."
- "Two to the third power."
- "Two cubed."

base (of a power) The number that is raised to an exponent.

exponent The number that describes repeated multiplication of the base.

Practice
6.10

Name: ..... Date: ..... Period: .....

1. Determine the value of each expression.

Expression	Value
$3 + 3 + 3 + 3$	
$3 \cdot 3 \cdot 3 \cdot 3$	
$4(3)$	
3^4	

2. Complete the table. The first row has been completed for you.

Expression With Exponent	Expression Without Exponent
3^5	$3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$
	$2 \cdot 2 \cdot 2 \cdot 2$
4^3	
5^1	
	$\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2}$
$\left(\frac{1}{3}\right)^4$	

3. Select *all* the expressions that are equal to 16.

- ☐ **A.** 8^2
- ☐ **B.** 2^4
- ☐ **C.** 2^8
- ☐ **D.** 4^2
- ☐ **E.** 16^1

4. Circle the two expressions that have the same value.

$6 + 6 + 6$ 6^3 3^6 $3 \cdot 6$

Name: ..... Date: ..... Period: .....

5. Write the expression that represents each description, then determine its value.

Description	Expression	Value
Three to the third power		
Five to the second power		
Two to the power of five		
Three to the second power		

6. Determine a value of a that makes both of these equations true. Explain your thinking.

$a^2 = 2^a$ $a^4 = 4^a$

7. Select *all* the expressions that are equal to 3^2.

☐ **A.** $3 \cdot 3$ ☐ **B.** 9 ☐ **C.** 6 ☐ **D.** 5 ☐ **E.** 1

Spiral Review

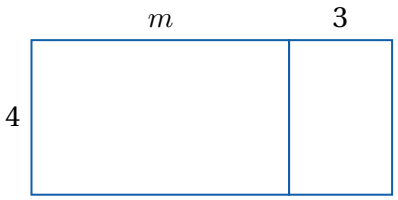
Problems 8–10: Solve each equation.

8. $a - 2.01 = 5.5$ **9.** $b + 2.01 = 5.5$ **10.** $10c = 13.71$

11. Write two expressions for the area of the rectangle.

Product: .....

Sum: .....



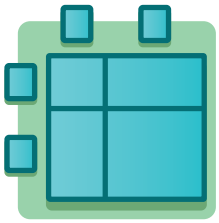
Unit 6
Lesson
11

Name: ..... Date: ..... Period: .....

Generalizing With Multiple Representations | Nets and Surface Area

6.EE.1, 6.EE.2.c, SMP.1, SMP.3, SMP.7

Exponent Expressions



Let's evaluate expressions with exponents.

Warm-Up

1. 4, 9, and 16 are examples of **perfect squares**. Here are some representations of these perfect squares

Perfect Square	4	9	16
Diagram			
Exponent Expression	2^2	3^2	4^2

a. **Discuss:** What do you notice? What do you wonder?

b. Do you think 49 is a perfect square? Explain your thinking.

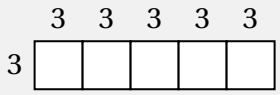
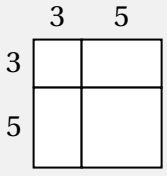
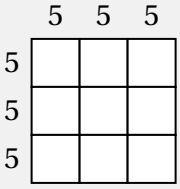
Activity 1

Name: ..... Date: ..... Period: .....

What's Missing?

2. You will use a set of cards for this activity.

- Work with a partner to group the matching diagrams, expressions, and values. There will be three blank spaces.
- Complete the table with the missing representations.

Diagram	Expression	Value
	$5 \cdot 3^2$	
		
		

3. **Discuss:** How did you use the area of each diagram to help you determine the value of the expression?

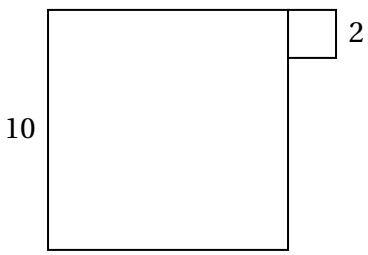
Activity 2

Name: ..... Date: ..... Period: .....

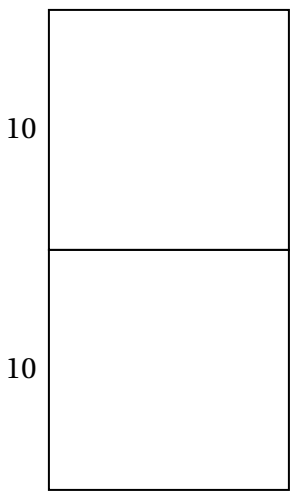
Evaluating Expressions

4. **Discuss:** Which diagram represents $2 + 10^2$?

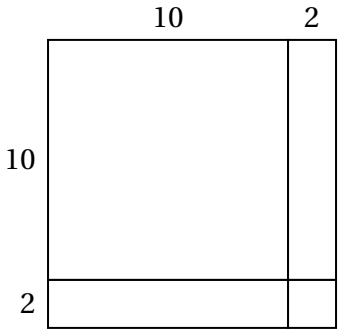
A.



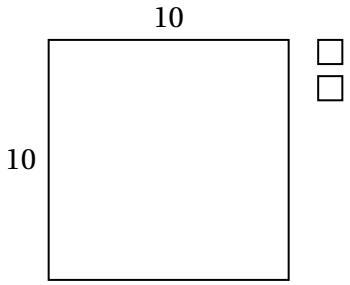
B.



C.



D.



5. Latifa and Nicolas each got a different value for $2 + 10^2$.

Latifa

$2 + 10^2$

12^2

144

Nicolas

$2 + 10^2$

$2 + 100$

102

Whose work is correct? Circle one.

Nicolas's Latifa's Both Neither

Explain your thinking.

Name: ..... Date: ..... Period: .....

Partner Problems

6. Decide with your partner who will complete Column A and who will complete Column B.

- The solutions in each row should be the same. Compare your solutions, then discuss and resolve any differences.

	Column A	Column B
a	$5^2 + 4$	$2^2 + 25$
b	$5^2 - 4$	$25 - 2^2$
c	$\frac{4^2}{2}$	$\frac{2^5}{4}$
d	$(7 - 2)^2 - 3^2$	$\frac{1}{4}(1 + 3)^3$
e	$\frac{8 - 2^2}{4}$	$\frac{8 - 2(3)}{2}$
f	$\frac{2^4 - (3^2 - 5)}{2^2}$	$\frac{1^5 + (6^2 - 10)}{3^2}$

You're invited to explore more.

7. Write an expression for your partner to evaluate. Swap problems, then write an expression with an exponent that has the same value as your partner's but uses different numbers.

Synthesis

8. What are some things to remember when determining the value of expressions with exponents?

Use these examples if they help with your thinking.

$5 \cdot 3^2$

$(3 + 5)^2$

$(3 \cdot 5)^2$

$5^2 + 3^2$

Summary 6.11

Exponents are used to represent repeated multiplication. When numbers have an exponent of 2 they are known as **perfect squares**.

There is a specific *order of operations* we use to evaluate expressions with more than one operation, like $5 \cdot 2^4$ or $(5 \cdot 2)^4$.

With Parentheses	Without Parentheses
Evaluate the operations in parentheses first:	Evaluate the term with the exponent first:
$(5 \cdot 2)^4$	$5 \cdot 2^4$
$(10)^4$	$5 \cdot (2 \cdot 2 \cdot 2 \cdot 2)$
$10 \cdot 10 \cdot 10 \cdot 10$	$5 \cdot 16$
$10{,}000$	80

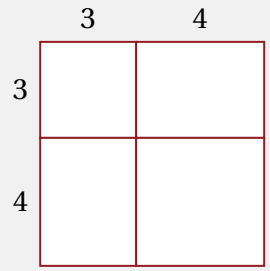
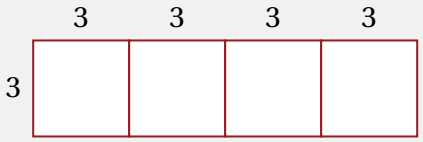
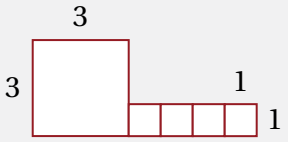
perfect square The product of a whole number multiplied by itself.

Practice
6.11

Name: _____ Date: _____ Period: _____

1. Match each expression with the diagram it represents. Evaluate the expression to determine the area of each diagram.

$4 \cdot 3^2$ $\quad$ $3^2 + 4$ $\quad$ $(3 + 4)^2$

Diagram	Expression	Area (sq. units)
		
		
		

Problems 2–3: Evaluate each expression.

2.

Expression	Value
$5 + 4^2$	
$(3 + 2)^3$	
$2^2 \cdot 5$	
$8 \cdot \left(\frac{1}{2}\right)^2$	

3.

Expression	Value
$42 - 9 \cdot 2^2$	
$\frac{12 - 3^2}{6}$	
$\frac{3 + (2 + 1)^2}{4}$	
$\frac{1}{4}(3 - 1)^3$	

4. What is the value of the expression $\frac{3 + 5^2}{4}$?

A. 16 $\quad$ **B.** 7

C. 4 $\quad$ **D.** $\frac{13}{4}$

Practice
6.11

Name: ..... Date: ..... Period: .....

Spiral Review

5. Use the numbers 0 to 9 to fill in each blank so that the values of the expressions are in order from *least* to *greatest*. Use each number only once.



Least **Greatest**

Problems 6–9: Determine the value of each expression.

6. $3(5)$

7. $3(5) + 2$

8. $3(5 + 2)$

9. $3(2 + 5)$

10. Select *all* the expressions that are equal to 3^4.

- ☐ **A.** $3 \cdot 3 \cdot 3 \cdot 3$
- ☐ **B.** 12
- ☐ **C.** $3 + 3 + 3 + 3$
- ☐ **D.** $9 \cdot 9$
- ☐ **E.** 81

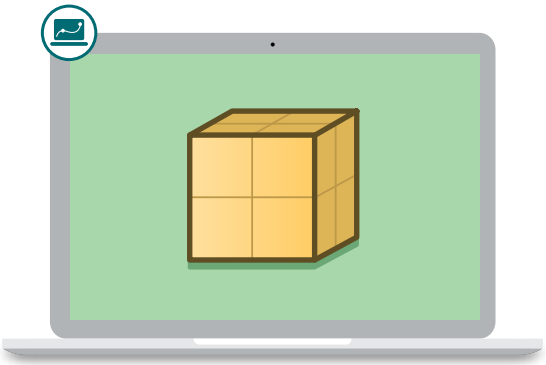
11. Determine the volume of a cube with side lengths of 3.25 inches. Round your answer to the nearest hundredth.

Unit 6
Lesson
12

Name: ..... Date: ..... Period: .....

Generalizing With Multiple Representations | Nets and Surface Area

6.EE.2, 6.EE.2.c, 6.G.1, 6.G.2, SMP.3, SMP.6, SMP.7



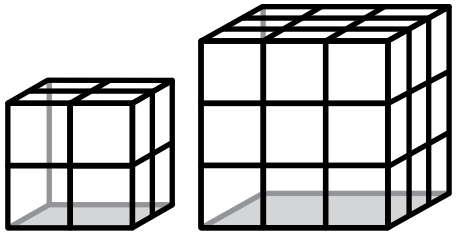
Squares and Cubes

Let's evaluate variable expressions.

Warm-Up

1 8 and 27 are examples of **perfect cubes**.

What other numbers do you think might be perfect cubes?



Name: ..... Date: ..... Period: .....

Variable Expressions With Area

2 Match each expression with its area model.

a $(x + 4)^2$

b $x^2 + 4$

c $4x^2$

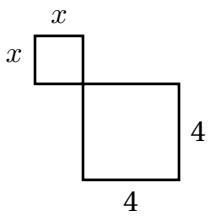
d $x^2 + 4^2$

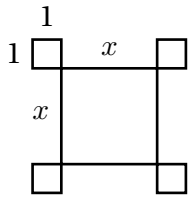
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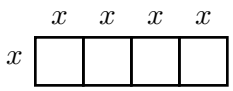
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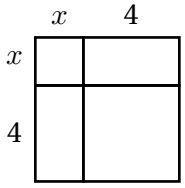
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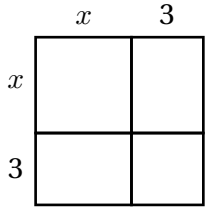
Name: Date: Period:

Variable Expressions With Area (continued)

3 The area of figure A is $(x + 3)^2$ square units.

What is its area when $x = 4$?

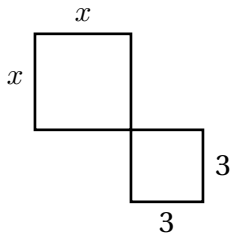
Figure A



4 The area of figure B is $x^2 + 3^2$ square units.

What is its area when $x = 4$?

Figure B



5 Amir says that $(x + 3)^2$ and $x^2 + 3^2$ are equivalent expressions. Help him understand why they are *not* equivalent.

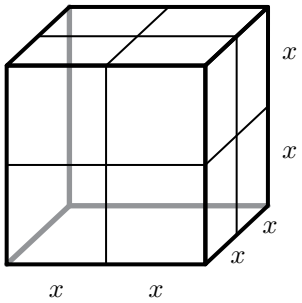
Use figures A and B if they help you show your thinking.

Name: ..... Date: ..... Period: .....

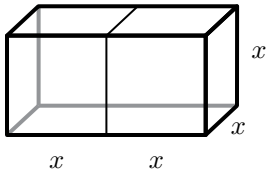
Cubes and Squares

6 Which prism has a volume of $(2x)^3$ cubic units?

Prism C



Prism D

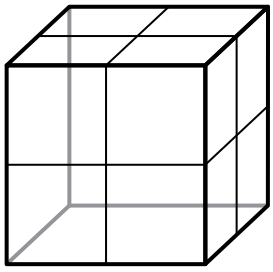


Prism C Prism D Both Neither

Explain your thinking.

7 What is the volume of prism C when $x = 3$?

Prism C



8 Here is a new expression: $2x^3$.

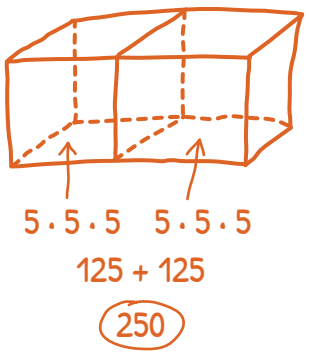
Evaluate $2x^3$ when $x = 5$. Show or explain your thinking.

Name: ..... Date: ..... Period: .....

Cubes and Squares (continued)

9 Here are Amir's and Chloe's strategies for evaluating $2x^3$ when $x = 5$.

Amir



Chloe

$2x^3$ when $x = 5$

$2(5)^3 = 2(5 \cdot 5 \cdot 5)$

$= 2 \cdot 125$

$= 250$

Discuss: How are their strategies alike? How are they different?

10 Use Chloe's strategy to evaluate $2x^3$ when $x = \frac{1}{2}$.

11 Use Chloe's strategy to determine the value of $5x^2 + 4x + 3$ when $x = 3$.

Activity 3

Name: ..... Date: ..... Period: .....

Repeated Challenges

12 Evaluate each expression for the given value of x.

	Expression	Value of x	Value of the Expression
a	$(x + 2)^2$	5	
b	$5x^2$	3	
c	$4x^3$	2	
d	$4 + x^2$	6	
e	$(2x)^3$	2	

You're invited to explore more.

13 Here are two expressions: $(x + 5)^2$ and $x^2 + 5^2$.

Amir says they *always* have the same value.
Chloe says they *never* have the same value.

Who is correct? Circle one.

Amir Chloe Both Neither

Explain your thinking.

14 Synthesis

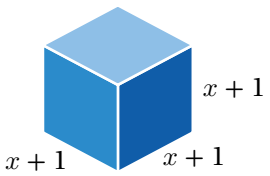
Discuss: How can you evaluate $(x + 1)^3$ when $x = 3$.

17 Summary 6.12

Writing variable expressions with exponents helps simplify the way we determine the *face area*, *surface area,* and *volume* of three-dimensional solids. For example, let x represent the side length of a cube.

- Its *face area*, $x \cdot x$, can be written as x^2.
- Its *surface area*, $6 \cdot (x \cdot x)$, can be written as $6x^2$.
- Its *volume*, $x \cdot x \cdot x$, can be written as x^3.

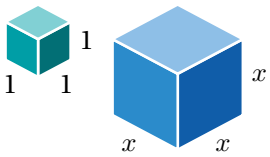
You can use area and volume models to evaluate expressions with exponents 2 and 3. For example, when $x = 2$.



$(x + 1)^3$ is

$(2 + 1)^3 = 3^3$

$= 27$



$x^3 + 1$ is

$2^3 + 1 = 8 + 1$

$= 9$

When evaluating expressions for a specific value of a variable using the order of operations evaluate the operations in parentheses first; in expressions without parentheses, exponents should be evaluated first.

You can use order of operations to evaluate expressions with exponents.

For example, when $x = 2$.

$(x + 1)^4$ is

$(2 + 1)^4 = 3^4$

$= 8^1$

$x^5 + 1$ is

$2^5 + 1 = 32 + 1$

$= 33$

perfect cube The cube of a whole number is called a perfect cube.

Practice
6.12

Name: ..... Date: ..... Period: .....

1. Determine the value of each expression when $x = 3$.

Expression	$x^2 + 6$	$4x^2$	2^x	$4 + 2^x$
Value when $x = 3$				

2. Determine the value of each expression when $x = 2$.

Expression	$x^4 + 5x$	$4x^3 + x - 10$	$1 + 3x^3$	$\left(\frac{1}{3}\right)^x$
Value when $x = 2$				

3. Determine the value of $2x^2 - 4x + 5$ when $x = 6$.

Problems 4–5: Here are two figures.

Figure A

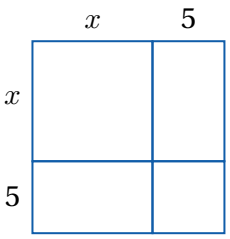
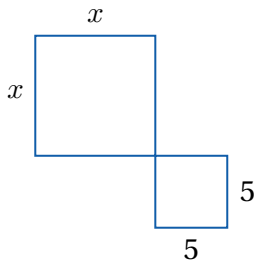


Figure B



4. Match each figure with an expression that represents its area. One expression will not have a match.

$x + 5^2$ $(x + 5)^2$ $x^2 + 5^2$

....

5. Explain why $(x + 5)^2$ and $x^2 + 5^2$ are *not* equivalent.

Practice
6.12

Name: ..... Date: ..... Period: .....

Spiral Review

Problems 6–7: Jalen built a tower out of 10 cubes. Each cube has a side length of 5 inches.



6. Which expression represents the volume of the tower?

A. $5(10)^3$

B. $10(5)^3$

C. $10 + 5^3$

D. $5 + 10^3$

7. Calculate the volume of the tower.

8. For each row, circle the expression with the greater value or circle that they have the same value.

Expression A	Expression B	Same Value
2^3	3^2	They have the same value.
1^{10}	10^1	They have the same value.
3^4	9^2	They have the same value.
$\left(\frac{1}{2}\right)^3$	$\left(\frac{1}{3}\right)^2$	They have the same value.

9. Use the numbers 1 to 9 to fill in each blank to create equivalent expressions. Use each number only once.

$\square \cdot \square^2$

$(\square + \square)^2$

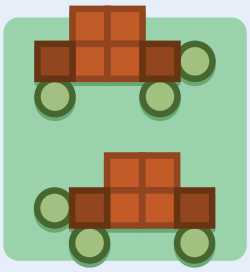
10. A number made by squaring a whole number is called a *perfect square*. A number made by cubing a whole number is called a *perfect cube*. Select *all* the numbers that are both perfect squares and perfect cubes.

☐ 1 ☐ 8 ☐ 9 ☐ 64 ☐ 81

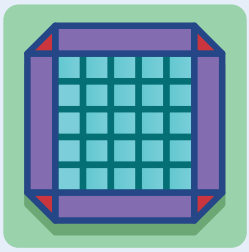
Sub-Unit

4

Introduction to Representing Relationships



Lesson 13
Turtles All the Way



Lesson 14
Representing Relationships



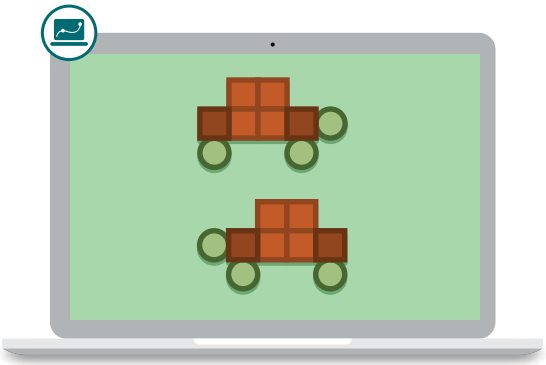
Lesson 15
Connecting Representations



Lesson 16
Subway Fares

Name: ..... Date: ..... Period: .....

Relationships Between Variables 6.EE.9, 6.EE.2.c, SMP.1, SMP.3

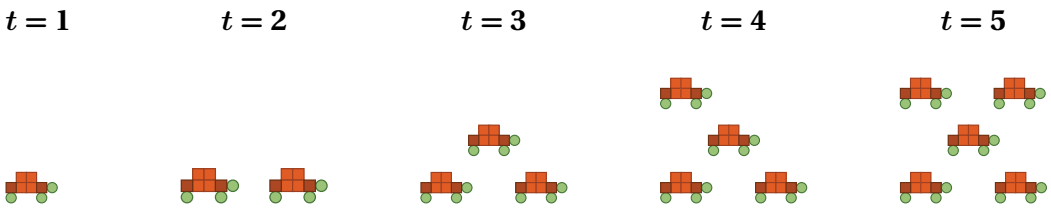


Turtles All the Way

Let's explore relationships between two variables.

Warm-Up

1 Here is a pattern of turtles. What different things can you count in this pattern?



Activity 1

Name: ..... Date: ..... Period: .....

Turtles, Turtles, Turtles

2 The variable t represents the number of turtles.

a Here are three other variables. Circle one.

s = number of square tiles h = height of a turtle g = number of green circles

b Describe how changing t affects the value of the variable you chose.

3 Saavni made a table to help her make sense of the relationship between t and g.

a Complete the table.

Number of Turtles, t	Number of Green Circles, g
1	3
2	
3	
4	
5	

b **Discuss:** What patterns do you see?

4 Saavni and Kadeem wrote equations to represent the relationship between t and g.

Saavni

$g = 3t$

Kadeem

$t = 3g$

Whose equation is correct? Circle one.

Saavni's Kadeem's Both Neither

Explain your thinking.

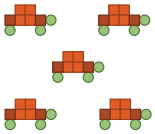
Name: ..... Date: ..... Period: .....

Dependent or Independent

5 The **independent variable** is the variable in a relationship that is the cause. The **dependent variable** is the effect or result.

In Saavni's example, the independent variable is the number of turtles, t, and the dependent variable is the number of green circles, g.

Image



Table

Number of Turtles, t	Number of Green Circles, g
1	3
2	6
3	9
4	12

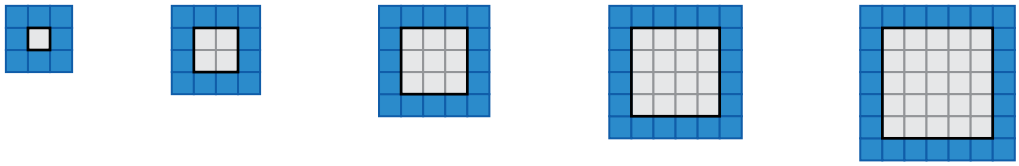
Equation

$g = 3t$

Discuss: If the independent variable is t, what other dependent variables could you explore?

6 Here is a pattern you may have seen before. The independent variable is n, the side length of the lighter square.

$n = 1$ $n = 2$ $n = 3$ $n = 4$ $n = 5$



a Circle one dependent variable to explore. Consider choosing one that's different than your partner's.

t = total area of the tiles g = area of the lighter tiles p = perimeter of the lighter square

b Describe how changing n affects the value of the dependent variable you chose.

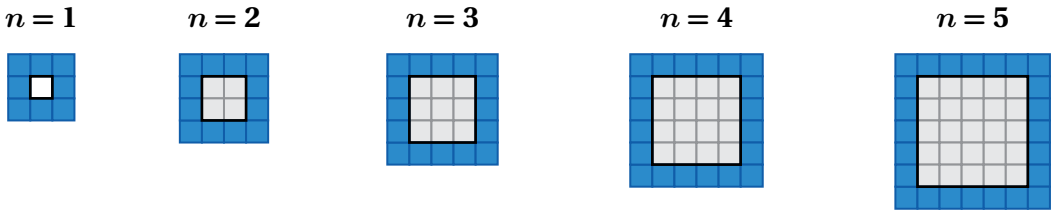
Activity 3

Name: ..... Date: ..... Period: .....

Border Tiles Revisited

Here is the pattern from the previous screen.

$n = 1$ $n = 2$ $n = 3$ $n = 4$ $n = 5$



7 **a** Complete the table for the variable you chose.

t = total area of the tiles

n	t
1	
2	
3	
4	
5	

g = area of the gray tiles

n	g
1	
2	
3	
4	
5	

p = perimeter of the gray square

n	p
1	
2	
3	
4	
5	

b **Discuss:** What patterns do you see?

8 Which equation represents your pattern? Explain your thinking.

t = total area of the tiles

A. $t = (n + 2)^2$

B. $n = (t + 2)^2$

C. $t = n^2 + 2$

g = area of the gray tiles

A. $g = n^2$

B. $n = g^2$

C. $g = 2n$

p = perimeter of the gray square

A. $p = 4n$

B. $p = 4(n + 2)$

C. $n = 4p$

9 What is the total area of the tiles when $n = 10$?

What is the area of the gray tiles when $n = 10$?

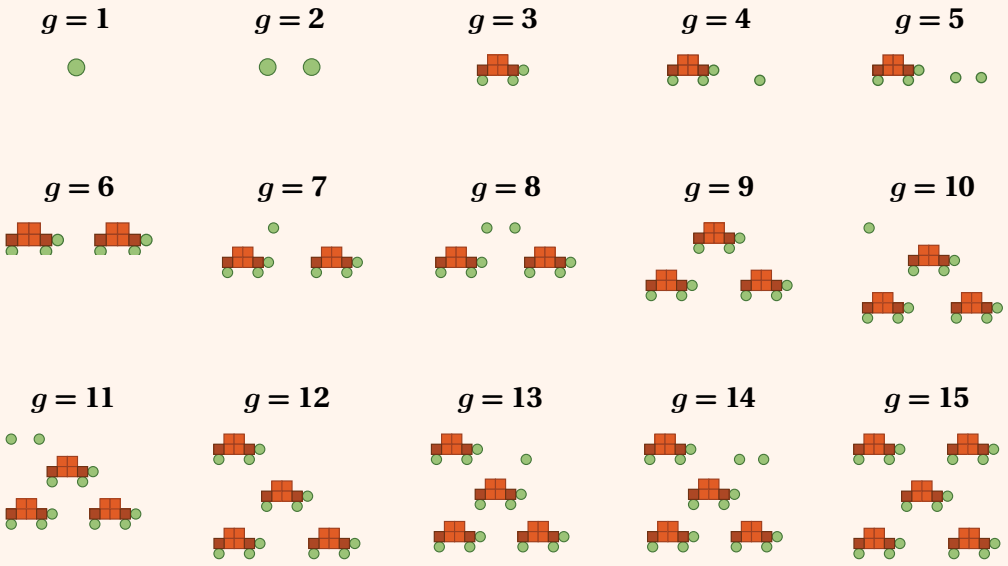
What is the perimeter of the gray square when $n = 10$?

Name: ..... Date: ..... Period: .....

Border Tiles Revisited (continued)

You're invited to explore more.

10 Here is a new pattern where g represents the number of green circles.



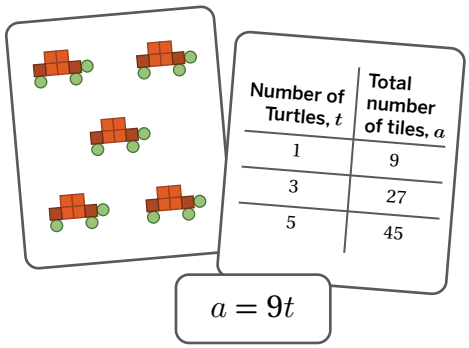
a What is the independent variable?

b Choose a dependent variable to explore.

c Describe the relationship between the independent variable and dependent variable you chose.

11 Synthesis

Discuss: How can we see relationships between **independent** and **dependent variables** in tables and equations?



14 Summary 6.13

Tables and equations can be used to represent and describe a relationship between **dependent** and **independent variables** or quantities.

For example, if a boat can travel 36 miles in 3 hours, then:

- The dependent variable is the distance traveled, d.
- The independent variable is the amount of time, t.

Table

t (hr)	d (mi)
3	36
1	12
2	24

The speed of the boat is 12 miles per hour.

Equation

$$d = 12t$$

In 6 hours, the boat will travel $d = 12 \cdot 6 = 72$ miles.

The boat can travel 120 miles in $t = 120 \div 12 = 10$ hours.

dependent variable The variable in a relationship that is the effect or result.

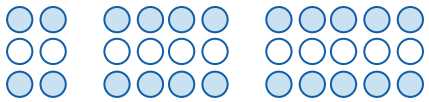
independent variable The variable in a relationship that is the cause.

Practice
6.13

Name: ..... Date: ..... Period: .....

1. The number of circles along the bottom, c, is the independent variable in this relationship.

List two to three dependent variables you could count or measure.



$c = 2$ $c = 4$ $c = 5$

Problems 2–4: Karima wants to help a community kitchen determine how much broth they need for their famous chicken soup. Each serving of soup uses 2 cups of broth.

2. Complete the table.

Number of Servings of Soup, s	Number of Cups of Broth, b
1	
2	
5	
	16

3. Which variable is the independent variable, s or b? Explain your thinking.

4. Karima and her sister each wrote an equation for the relationship.

Karima: $s = 2b$ Karima's sister: $b = 2s$

Who is correct? Circle one.

Karima Karima's sister Both Neither

Explain your reasoning.

5. This table shows the number of scooters a factory produces in different amounts of time.

Write an equation to represent the relationship between the number of hours, x, and the number of scooters, y, the factory produces.

Number of Hours, x	Number of Scooters, y
2	21
4	42
6	63
8	84

Practice
6.13

Name: ..... Date: ..... Period: .....

Problems 6–8: Saavni has a coupon for $4 off any item at a store.

6. How much would she pay if she buys an item that costs $10?

7. How much would she pay if she buys an item that costs $22?

8. How much would she pay if she buys an item that costs d dollars?

Problems 9–10: Here is a table.

n	a
1	5
2	10
3	15

9. Draw a visual pattern that represents this relationship.

10. What do the variables n and a represent in your pattern?

n represents . . .

a represents . . .

Spiral Review

11. Determine the value of each expression.

Expression	3^2	2^3	2^5	2^1
Value				

12. Determine the value of each expression.

Expression	$5^2 - (3^2 + 4^2)$	$\frac{1^2 + 1^3 + 1^4}{3}$	$\frac{2^5}{2^2}$	$\left(\frac{2}{3}\right)^3$
Value				

13. Determine the value of each expression when $x = 4$.

Expression	$(6 - x)^3$	$2(6 - x)^3$	$2^x - 6$	$\left(\frac{1}{x}\right)^3$
Value when $x = 4$				

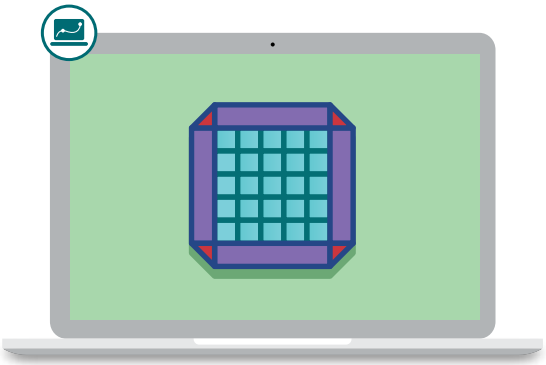
Unit 6
Lesson
14

Name: ..... Date: ..... Period: .....

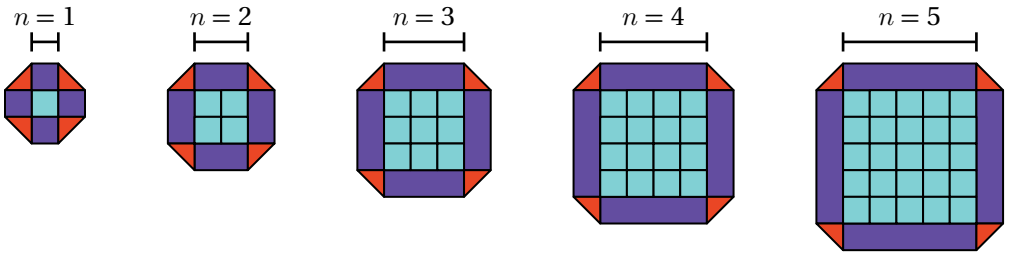
Relationships Between Variables | Patterns Inside Numbers | 6.EE.9, 6.RP.3.a, SMP.3, SMP.6

Representing Relationships

Let's use graphs to represent relationships between two variables.



Warm-Up



1 **Discuss:** What do you notice? What do you wonder?

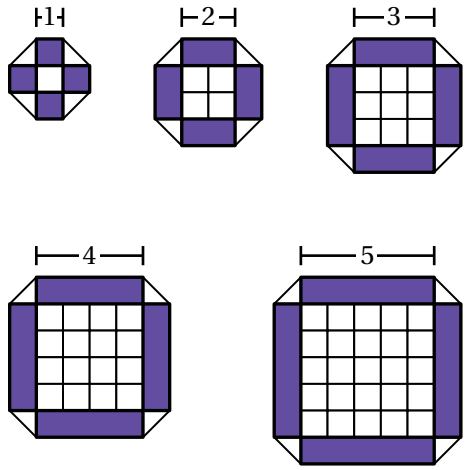
2 In this pattern, the independent variable is the side length of the purple rectangles, n. What dependent variables could you explore?

Name: ..... Date: ..... Period: .....

Introducing Graphs

3 Jayden chose to explore the relationship between the side length of the purple rectangles, n, and the total area of the purple rectangles, p.

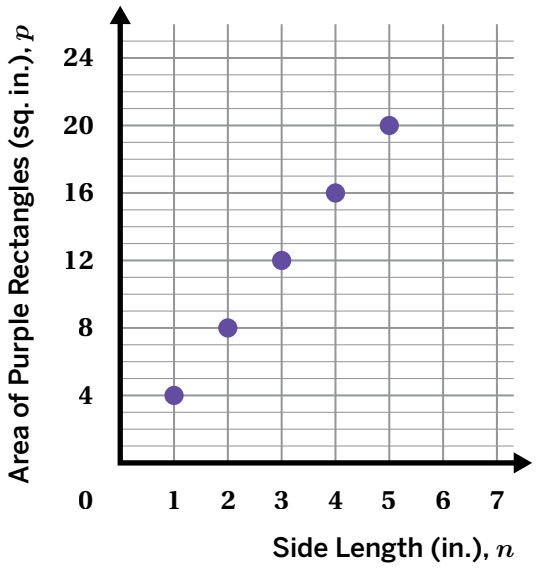
Complete the table to represent this relationship.



Side Length (in.), n	Area of Purple Rectangles (sq. in.), p
1	4
2	
3	
4	
5	

4 Rebecca represented the relationship between n and p with a graph.

Discuss: How does the graph show the same information as the images?



5 Explain how you know Jayden's table and Rebecca's graph represent the same relationship.

Name: ..... Date: ..... Period: .....

Introducing Graphs (continued)

6 Here is how Jayden and Rebecca connect two representations.

Discuss: How are their connections alike? How are they different?

Jayden

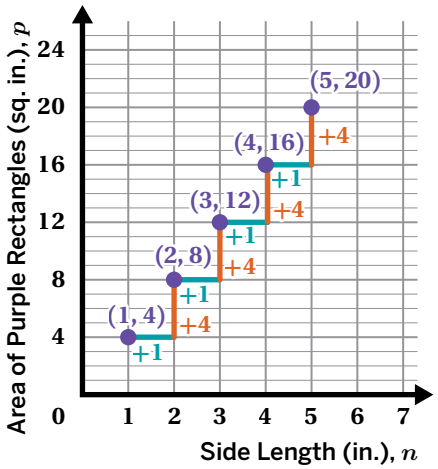
Side Length (in.), n	Area of Purple Rectangles (sq. in.), p
1	4
2	8
3	12
4	16
5	20

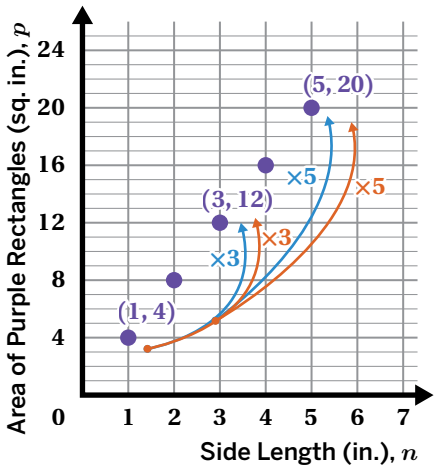
(+1 between each pair of side lengths; +4 between each pair of areas)

Rebecca

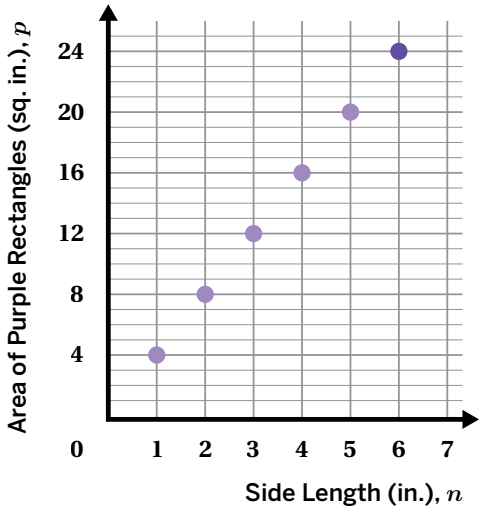
Side Length (in.), n	Area of Purple Rectangles (sq. in.), p
1	4
2	8
3	12
4	16
5	20

(×3 between 1 and 2; ×5 between 1 and 3; ×3 between 4 and 8; ×5 between 4 and 16; ×5 between 4 and 20)





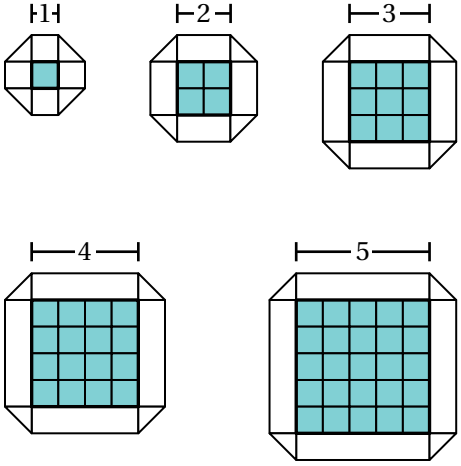
7 Rebecca added the *ordered pair* (6, 24) to the graph. Explain what each value represents in this context.



Name: ..... Date: ..... Period: .....

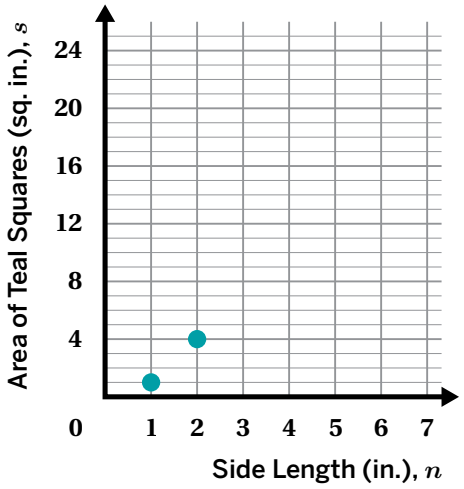
Plotting Points

8 Precious explored the relationship between the side length, n, and the total area of the teal squares, s. Complete the table to represent this relationship.



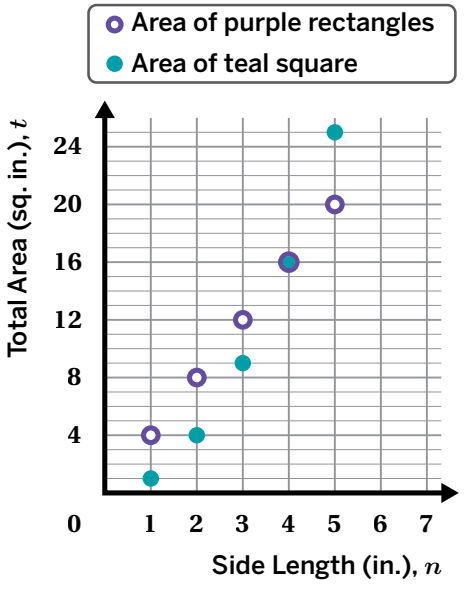
Side Length (in.), n	Area of Teal Squares (sq. in.), s
1	1
2	
3	
4	
5	

9 Plot points that represent the third and fourth rows in the table.



10 Precious noticed the point (4, 16) was on both graphs.

Discuss: What does this say about the area of purple rectangles and the area of the teal squares?

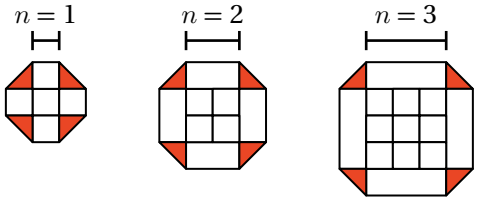


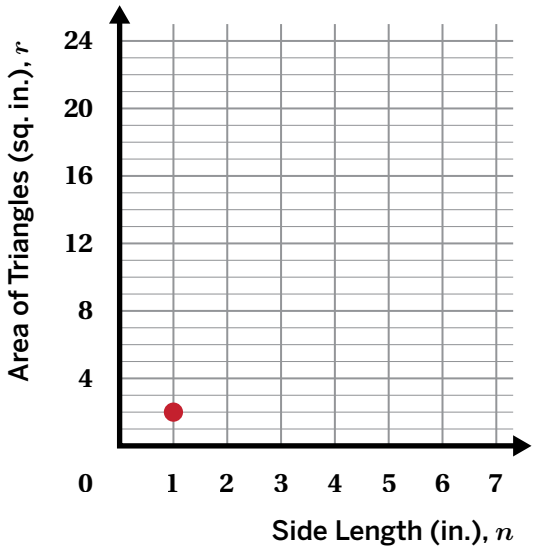
Activity 3

Name: ..... Date: ..... Period: .....

Comparing Relationships

11 Graph four points to represent the relationship between the side length, n, and the total area of the red triangles, r.





12 Let's look at a graph that shows all three relationships.

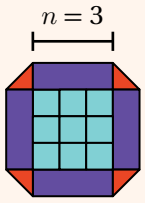
Discuss: How is the triangle relationship different from the other two relationships? How is it alike?

You're invited to explore more.

13 Draw an image to represent when:

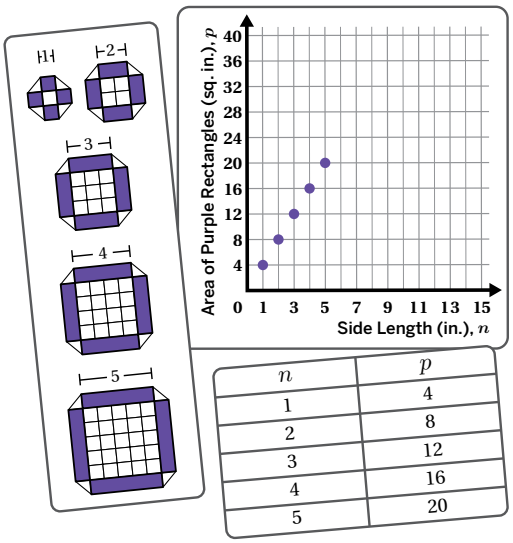
a $n = 0$

b $n = 10$



14 Synthesis

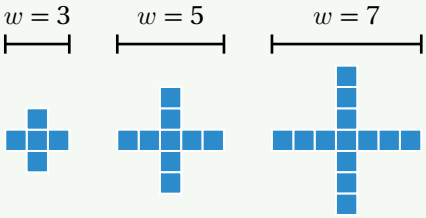
Discuss: How can you tell that a table, a graph, and an image show the same relationship?



n	p
1	4
2	8
3	12
4	16
5	20

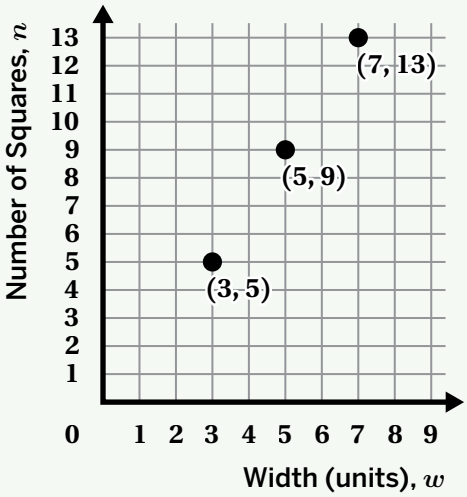
17 Summary 6.14

Like tables and equations, graphs are another way of representing the relationship between two quantities. For example, in this pattern, the independent variable is the width of the figure, w, and the dependent variable is the number of squares, n.



Here is the table and graph of this relationship.

Width of the Figure (units), w	Number of Squares, n
3	5
5	9
7	13



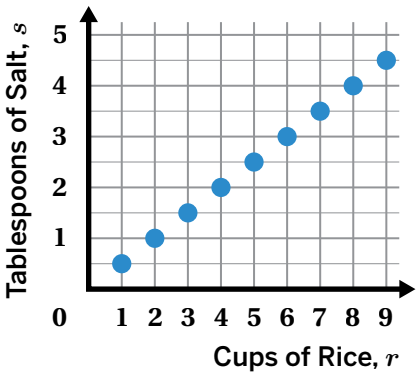
The numbers in each row of the table indicate an *ordered pair* on the coordinate plane. In the first row of the table, w is 3 and n is 5, which is represented by the point $(3, 5)$ on the graph.

While representing a relationship using a graph, the common practice is to use the x-axis for the independent variable.

Practice
6.14

Name: ..... Date: ..... Period: .....

Problems 1–3: This graph shows the number of tablespoons of salt, s, needed to make r cups of rice.



1. Complete the table to reflect some of the values on the graph.

Cups of Rice, r	Tablespoons of Salt, s
1	
4	
	3

2. What does the point $(8, 4)$ mean in this situation?

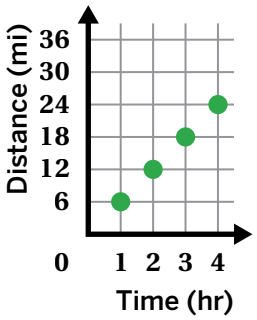
3. Which equation represents the relationship between the number of tablespoons of salt, s, and the number of cups of rice, r?

A. $r = \frac{1}{2}s$ **B.** $s = 2r$ **C.** $s = \frac{1}{2}r$ **D.** $s = \frac{1}{2} + r$

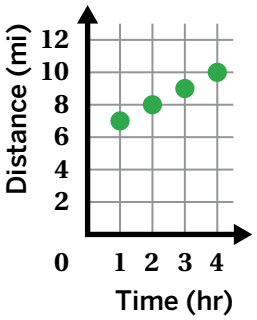
Explain how you know that equation is correct.

4. Rebecca rides her bike at a constant rate. The equation $d = 6t$ describes the relationship between the time, t, she rides in hours, and the distance, d, she rides in miles. Select the graph that represents the relationship between the amount of time and the distance she rides.

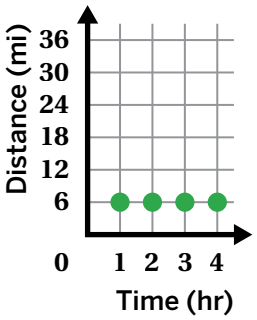
A.



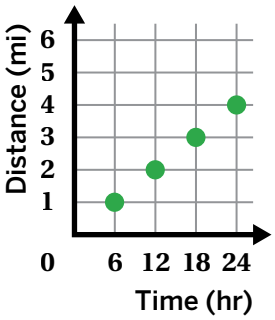
B.



C.



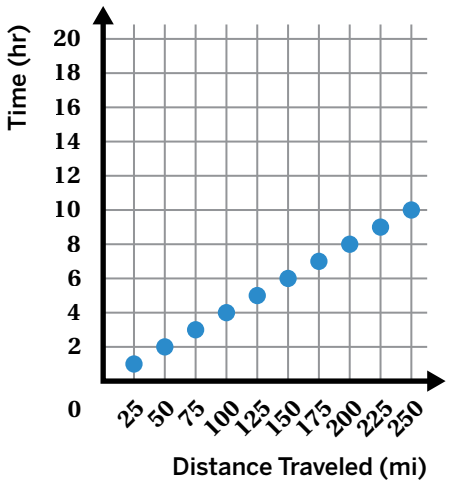
D.



Practice
6.14

Name: ..... Date: ..... Period: .....

Problems 5–7: The graph represents the amount of time in hours that it takes a ship to travel various distances in miles.

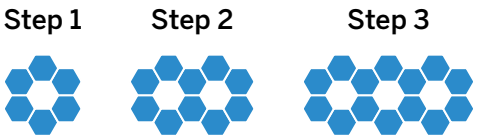


5. Select one point on the graph. Explain what that point means in this situation.

6. What is the independent variable?

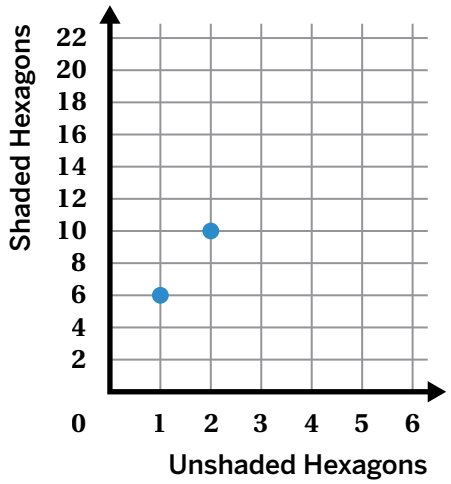
7. What is the dependent variable?

Problems 8–9: Here are the first three steps of a pattern.



8. Draw the fourth step of the pattern.

9. Plot the missing points that represent the third and fourth steps of the pattern.



Spiral Review

Problems 10–15: Determine the value of each expression.

10. 3^3

11. $2(3)^3$

12. $3^3 + 4$

13. $2 \cdot 3^3 + 4$

14. $2 \cdot 3^3 + 4 \cdot 3^2$

15. $\frac{2 \cdot 3^3 + 4 \cdot 3^2}{9}$

Name: ..... Date: ..... Period: .....

Relationships Between Variables | Generalizing With Multiple Representations | Model the World

6.EE.9, 6.RP.1, 6.RP.2, 6.RP.3, 6.RP.3.a, SMP.3, SMP.6

Connecting Representations



Let's make connections between different representations of the same relationship.

Warm-Up

1. Which table represents this graph?

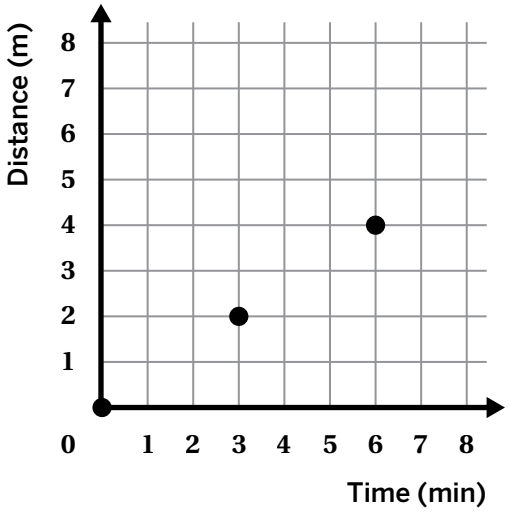


Table A

Time (min)	Distance (m)
0	0
2	3
4	6

Table B

Time (min)	Distance (m)
0	0
3	2
6	4

Explain your thinking.

Activity 1

Name: ..... Date: ..... Period: .....

What's Missing?

You will use a set of cards for this activity.

Paletas



Piraguas



2. With your partner, match each situation with the tables, graphs, and equations that show the same relationship.

3. Use Cards G, H, and I to complete the missing representations.

Situation	Table	Graph	Equation
Amanda sells paletas, p, for $\$2$ each. What is the total amount of money, m, she can earn?			
Tameeka sells paletas, p, for $\$2.50$ each. What is the total amount of money, m, she can earn?			
Esteban sells piraguas, p, for $\$3.50$ each. What is the total amount of money, m, he can earn?			

Name: ..... Date: ..... Period: .....

What's Missing? (continued)

4. Choose one of the previous situations. Show or explain where you see the price per item in the matching table, graph, and equation.

5. Sora wrote $p = \frac{m}{2}$ and Carlos wrote $m = 2p$ to represent the relationship between the number of paletas, p, Amanda sells and the total amount of money, m, she can earn. Whose equation is correct?

Sora Carlos Both

Explain your thinking.

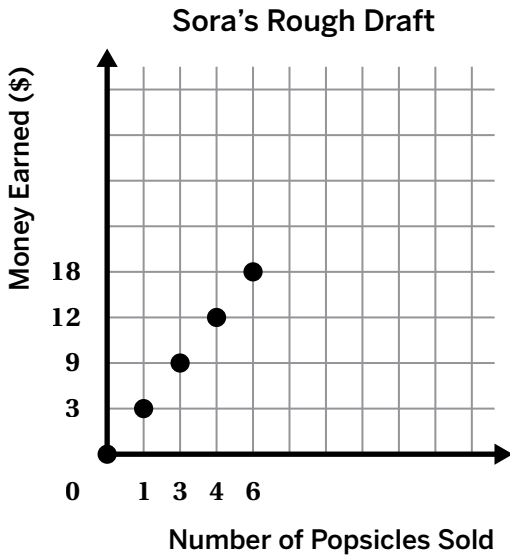
6. Ángel sells piraguas for $\$4.50$ each. How will Ángel's graph be different from Esteban's?

Name: Date: Period:

Critique, Correct, Clarify

Sora sells popsicles for $3 each. He made a table and a graph to help him understand the relationship between the number of popsicles he sells and the money he earns. His table is correct, but his graph is not quite correct.

Number of Popsicles Sold	Money Earned ($)
0	0
1	3
3	9
4	12
6	18

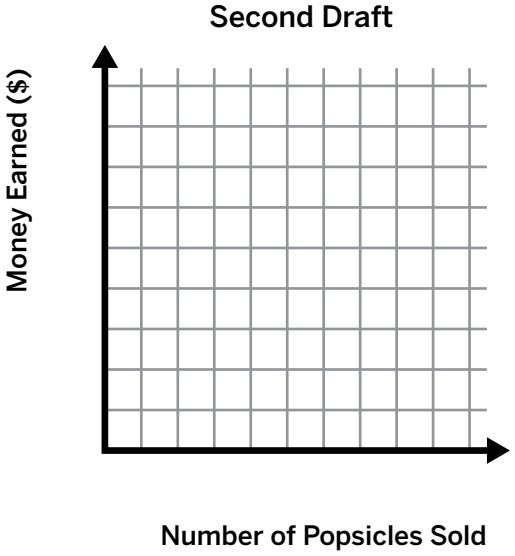


7. What do you think Sora did well in his graph?

8. What would you recommend Sora change about his graph?

9. Use Sora's table to create a second draft of the graph of the relationship between popsicles sold and money earned.

10. Circle one point on your graph. Explain what that point means in Sora's situation.

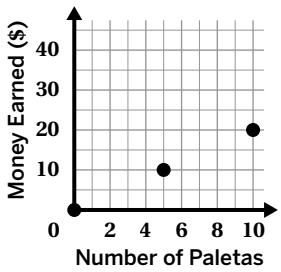


11. What are some other mistakes a person might make when they are creating a graph?

Synthesis

12. Explain how tables, equations, and graphs represent the same relationship.

Use the example if it helps with your explanation.

Situation	Table	Graph	Equation
Amanda sells paletas for $2 each.	p | m 0 | 0 5 | 10 10 | 20		$m = 2p$

Summary 6.15

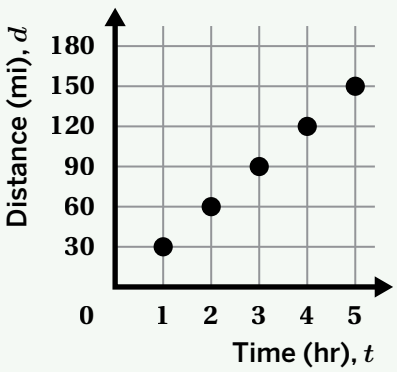
All three representations — tables, equations, and graphs — hold the same mathematical information described in a situation but display it in different ways.

For example, if a car travels 30 miles per hour at a constant speed, you can determine the distance the car traveled in 4 hours using a table, a graph, or an equation.

Table

Time, t (hr)	Distance, d (mi)
1	30
2	60
4	120

Graph



Equation

$d = 30t$

$d = 30(4)$

$d = 120$

In all three representations, we can see that the car traveled 120 miles in 4 hours.

Practice
6.15

Name: ..... Date: ..... Period: .....

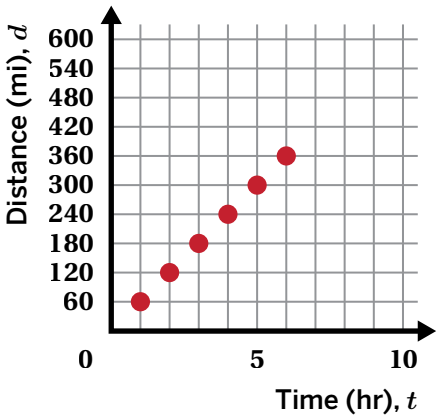
Problems 1–2: Melissa is selling little toy cars for $\$1.50$ each.

1. Write an equation that represents how much money Melissa earns, m, for selling any number of toy cars, c.

2. Complete the table that represents this situation.

c	m
2	
4	
	$\$16.50$
	$\$22.50$

Problems 3–4: This graph represents the distance a car traveled, d, over time, t.

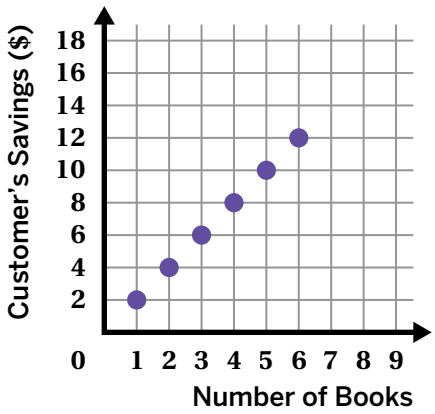


3. Create a table that matches the graph.

4. Nathan determined that the equation $d = 60t$ represents this situation. Explain how the parts of his equation relate to the situation.

5. A bookstore gives out coupons for $2 off each book. This graph represents the situation.

Select one point on the graph. Explain what that point means in this situation.



Practice
6.15

Name: ..... Date: ..... Period: .....

6. The equation $a = e - 4$ represents the relationship between Amanda's age, a, and Estaban's age, e. Which table represents the same relationship?

A.

a	e
10	14
11	15
12	16

B.

a	e
14	10
15	11
16	12

C.

a	e
10	6
11	7
12	8

D.

a	e
12	3
16	4
20	5

Problems 7–9: Camila is helping her team raise money for a new set of uniforms by selling school pennants. Pennants come in boxes of 15 and sell for $2 each.

7. Complete the table to show how much money is collected for the different numbers of boxes sold.

Number of boxes, b	Amount of money ($), m
1	30
2	
5	
10	

8. Write an equation for the amount of money, m, that will be collected if b boxes of pennants are sold.

$m =$ .....

9. Write an equation to determine the number of boxes that need to be sold, b, to raise m dollars.

$b =$ .....

Spiral Review

Problems 10–12: Calculate each percentage.

10. 25% of 40

11. 30% of 60

12. 45% of 90

Problems 13–14: Bao sells lemonade for $0.35 per cup.

13. If Bao earned $9.80, how many cups of lemonade did he sell?

14. Bao bought 50 paper cups for $0.05 each. How much did he spend on the paper cups?

Name: ..... Date: ..... Period: .....

Relationships Between Variables | Generalizing With Multiple Representations

Model the World | 6.RP.2, 6.RP.3, 6.EE.6, 6.EE.9, SMP.1, SMP.2, SMP.3

Subway Fares



Let's use tables, graphs, and equations to help customers compare subway fares.

Warm-Up

In Metropolis, there are three ticket options to ride the bus or subway.

Option 1
Regular Fare



Option 2
Unlimited 7-Day Pass



Option 3
Reduced Fare



For people who have low income, are 65 or older, or who have a qualifying disability.

1. For each option, how much will it cost to ride the subway 3 times in the same week?

Name: ..... Date: ..... Period: .....

Consider the Costs

2. The Metropolis Transit Association (MTA) is in charge of the public buses and subways in Metropolis. Your task is to help an MTA employee show customers how much each ticket option from the Warm-Up costs based on the number of rides.

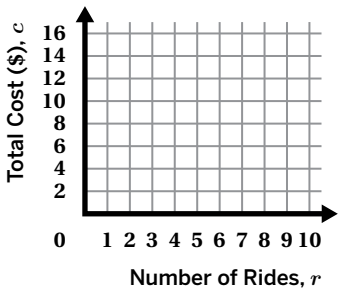
a As a group, work together to create a table, graph, and equation for each option.

Option 1 Regular Fare

Table

Number of Rides, r	Total Cost, c ($)

Graph



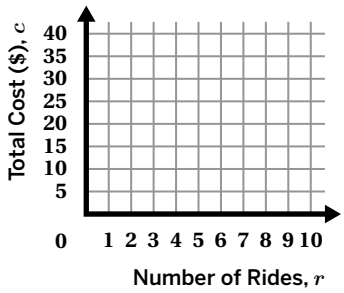
Equation

Option 2 Unlimited 7-Day Pass

Table

Number of Rides, r	Total Cost, c ($)

Graph



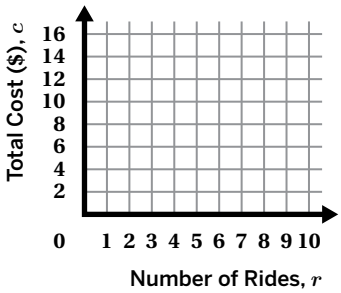
Equation

Option 3 Reduced Fare

Table

Number of Rides, r	Total Cost, c ($)

Graph



Equation

b Which variable is the independent variable? Which is the dependent variable?

c Write two to three sentences comparing and contrasting the graphs for the three options.

Activity 2

Name: ..... Date: ..... Period: .....

Helping Customers

3. Read about four subway customers and choose one to help. Make sure each person in your group chooses a different customer.

Eliza	Nikhil	Sydney	Bao
Eliza is 70 years old. She works at a daycare about 1.5 miles away from her house. Sometimes she walks to work and sometimes she takes the subway. She rides the subway between 2–8 times per week.	Nikhil is 23 years old. He uses a wheelchair and it takes him 20 minutes to get to the closest wheelchair -accessible subway station from his house. Nikhil works as a chef and uses the subway to get to and from work 5 days a week.	Sydney is 20 years old. Sydney is a college student and works part time. Sydney uses the subway to get to school and work. Sydney usually rides between 15–20 times per week.	Bao is 16 years old. He walks to school during the week and only uses the subway on the weekends to visit friends.

a Which fare option should your customer choose? Circle one.

Regular fare Unlimited 7-day pass Reduced fare

b Use the tables, graphs, and equations you made in Activity 1 to support your argument.

Activity 3

Name: Date: Period:

Increased Fares

The MTA needs more money to help maintain the subway service. The MTA leadership is thinking about raising the regular fare by $0.50.

4. Describe one advantage and one disadvantage of raising the regular fare. Explain your thinking for each.

One advantage of raising the regular fare is . . .

One disadvantage of raising the regular fare is . . .

5. Look back at your work in Activity 1 for the regular fare. How would raising the fare by $0.50 change the table, graph, and equation?

Table	Graph	Equation

6. Which of the four customers would be most impacted by the fare increase?

Explain your thinking.

7. If you were part of the MTA leadership, how would you adjust the fares to get the money you need to maintain service while also charging customers fairly?

Synthesis

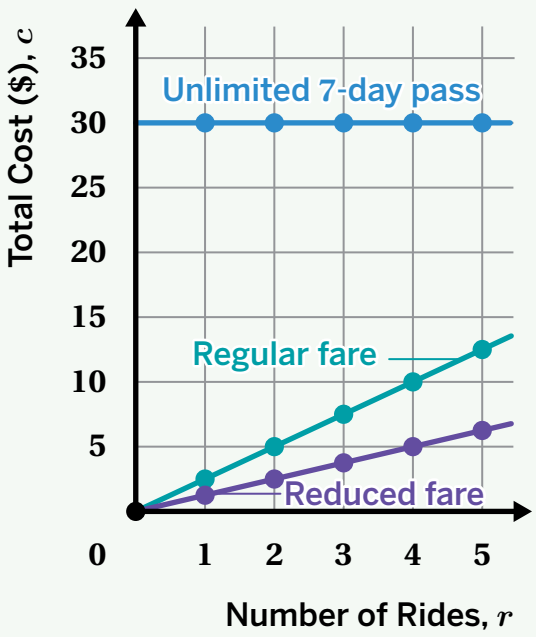
8. How can making a graph and a table help us understand relationships in the world, such as subway fares?

Summary 6.16

We can use data in tables, graphs, and equations to help make decisions in real-world situations. When it comes to analyzing subway ticket fares, these representations can help us make informed decisions about what type of transportation ticket to purchase.

Using the graph, we can see that the regular fare ticket is the best choice if we ride 5 times or less and do not qualify for the reduced fare. If we extend each line on the graph, we'll be able to determine when the price of an unlimited 7-day pass will be lower than the regular fare.

We can use these tools to make sure we get the best subway ticket for our needs.



Practice
6.16

Name: ..... Date: ..... Period: .....

1. Match each equation to the table it represents.

$p = 2n$ $\quad$ $p = \frac{1}{2}n$ $\quad$ $p = n + 2$

n	p
10	5
20	10
100	50

n	p
10	20
20	40
100	200

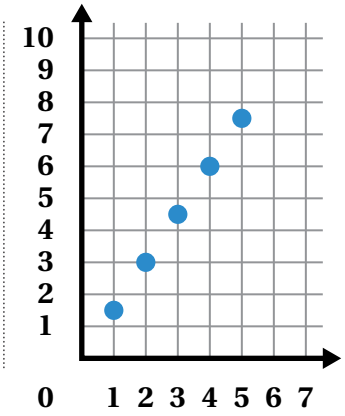
n	p
10	12
20	22
100	102

.....

.....

.....

Problems 2–4: Here is a graph.



.....

2. Write a situation that could be represented by the graph.

3. Label the axes on the graph to match your situation.

4. Complete the table using the points on the graph. Label each column with variables to match the graph.

.....	.....
1	1.5
2	
	4.5
4	

Practice
6.16

Name: ..... Date: ..... Period: .....

Problems 5–8: Riya's biking app says that she rides at a speed of 5 miles per hour.

5. At this speed, how far does Riya ride in 1 hour?

6. At this speed, how far does Riya ride in 3 hours?

7. Write an equation for the relationship between Riya's distance biked, d, and time, t.

8. Riya's speed last week could be represented by the equation $d = 3t$. What can you say about last week's speed compared to this week's speed? Explain your thinking.

Problems 9–12: A school supply store sells boxes of markers. Each box contains 16 markers.

9. Write an equation to represent the total number of markers, y, given x boxes.

Equation: .....

10. If $x = 5$ for one day of sales, use your equation to determine the total number of markers the supply store sells. Show your thinking.

11. If each marker is $\$1.50$, write an equation to represent the total cost, c, of y boxes of markers?

Equation: .....

12. If each marker is $\$1.50$, how many boxes of markers, y, does the supply store need to sell to gain $\$120$? Show your thinking.

Spiral Review

13. Select *all* the equations with a solution of $n = 3$.

- ☐ **A.** $10n = 103$
- ☐ **B.** $5n = 15$
- ☐ **C.** $\frac{1}{4} + n = \frac{13}{4}$
- ☐ **D.** $n \div 2 = 6$
- ☐ **E.** $\frac{1}{3}n = 3$

14. Using the numbers 1 to 9 only once, fill in each blank to make each inequality true.

$\square^{\square} < 2^{\square}$ $\quad \square^{\square} > 2^{\square}$ $\quad \square^{\square} < 3^{\square}$ $\quad \square^{\square} > 3^{\square}$

Name: ..... Date: ..... Period: .....

Generalizing With Multiple Representations | Relationships Between Variables | Model the World

6.RP.3, 6.RP.3.a, 6.EE.1, 6.EE.2.a, 6.EE.2.c, 6.EE.4, 6.EE.5, 6.EE.6, 6.EE.7, 6.EE.9, 6.G.1, SMP.2, SMP.7, SMP.8

Practice Day 2



Let's practice what you've learned so far in this unit!

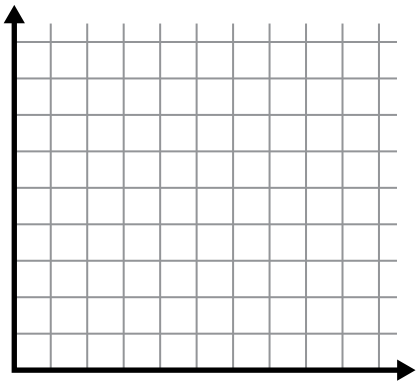
You will use task cards for this Practice Day. Record all of your responses here.

Task 1: Dance-a-Thon

1.

Time (min), t	Donation ($), d
1	
2	
4	6
10	

5.



2. Solution: ..... per minute

3. Equation: .....

4. Solution: $.....

6. Explanation:

Task 2: Equivalent Expressions

1. Circle one: A B C
New expression:

2. Circle one: D E F
New expression:

3. Circle one: G H I
New expression:

4. Expression 1: Expression 2: Expression 3:

Name: _____ Date: _____ Period: _____

Practice Day 2

Task 3: Solve 'em

1. Equation: _____ Equation: _____ Equation: _____

Solution: _____ Solution: _____ Solution: _____

2.

	Situation 1	Situation 2	Situation 3
Equation			
Solution			
Meaning			

Task 4: Rectangles and Expressions

1. Circle one: A B C D

2. Area: _____ square units

3. Area: _____ square units

4.

Expression			
Area Model			
Equivalent Expression			

You're invited to explore more.

1. $\square^2 + \square^2 = \square^2$

2. $\square^2 + \square^2 = \square^2$

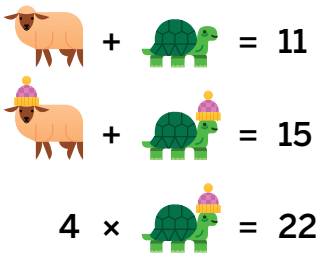
3.

Notes:

Career Connection

Sheep, turtles, and beanies – the making of a math problem!

Look at this equation puzzle. Can you figure out the value of a beanie?



You may have seen and solved equation puzzles like the one here. A lot of planning goes into creating these puzzles, including making sure it can be solved! You need to think about how many unknowns you want and then how many equations you need so that others can determine the value of each unknown.

Game designers create many aspects of games, including goals, characters, obstacles, levels, and more! They use math when planning their games, including making sure the game is fair for players.



Tanya Khovanova

Meet Tanya Khovanova

Born in Russia, Tanya Khovanova is an American mathematician. In high school, she won silver and gold medals at the International Mathematical Olympiad (only the second female gold medalist at the time). She teaches university students, coaches teams who compete in math competitions, contracts as an engineer and mathematician, and conducts research. Her recent work explores recreational math, creating and solving mathematical puzzles.

Are you interested in using math to create and solve puzzles? What can you do to learn more?

Community Connection

Now it's your turn to use math to create your own equation puzzle! Ask at least 5 people in your school or community to solve it. How many people were able to solve your puzzle?

Hint: Make sure you know the solution!

Math Mindset

How is an algebraic equation like a puzzle?

Unit 7



Positive and Negative Numbers

Big Ideas in This Unit

CC2 Relationships Between Variables | Generalizing With Multiple Representations

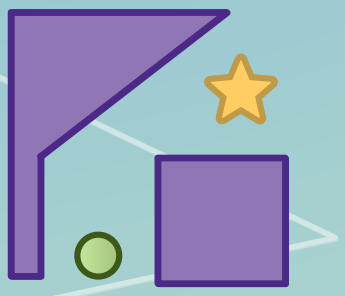
CC3 Nets and Surface Area | Model the World | CC4 Graphing Shapes

CC4 Distance and Direction | NS Number Line Understanding

Questions for Investigation

- What does it mean for a value to be less than zero?
- How can you represent all the numbers that are less than or greater than a value?
- How can you represent points with negative numbers on a coordinate plane?

Explore: How Far? Which Way?
How can magnitude and direction guide us as we move in a maze?



Watch Your Knowledge Grow

This is the math you'll explore in this unit. Rate your understanding to see how your knowledge grows!



I can . . .	Before	After
Use the symbol (-) and the word negative to describe numbers that are less than 0.		
Plot positive and negative numbers on the number line.		
Use positive and negative numbers to represent quantities in real-world contexts.		
Explain the meaning of 0 in different situations.		
Determine the absolute value of numbers.		
Order rational numbers and their absolute values.		
Write an inequality to represent a real-world or mathematical problem.		
Determine solutions to inequalities.		
Explain that an inequality has an infinite number of solutions.		

I can . . .	Before	After
Justify whether a value is a solution to a given inequality.		
Write coordinates for points on the coordinate plane.		
Estimate the location of a point on the coordinate plane using the signs of its coordinates.		
Draw a polygon on the coordinate plane given the coordinates for its vertices.		
Describe the meaning of points on a coordinate plane from a situation.		

Sub-Unit

1

Negative Numbers and Absolute Values



Explore
How Far? Which Way?



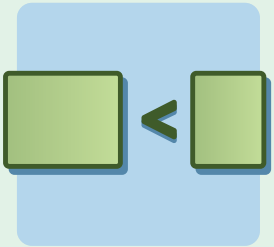
Lesson 1
Can You Dig It?



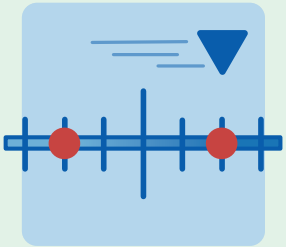
Lesson 2
Digging Deeper



Lesson 3
Sub-Zero



Lesson 4
Order in the Class



Lesson 5
Distance on the Number Line



Lesson 6
We've Got Game(s)

Name: ..... Date: ..... Period: .....

Distance and Direction | Building Toward 6.NS.5, 6.NS.8, SMP.6

Explore: How Far? Which Way?

How can magnitude and direction guide us as we move in a maze?



Warm-Up

1. Write down as many words or phrases as you can think of that relate to *magnitude* and *direction*.

Magnitude	Direction

Name: ..... Date: ..... Period: .....

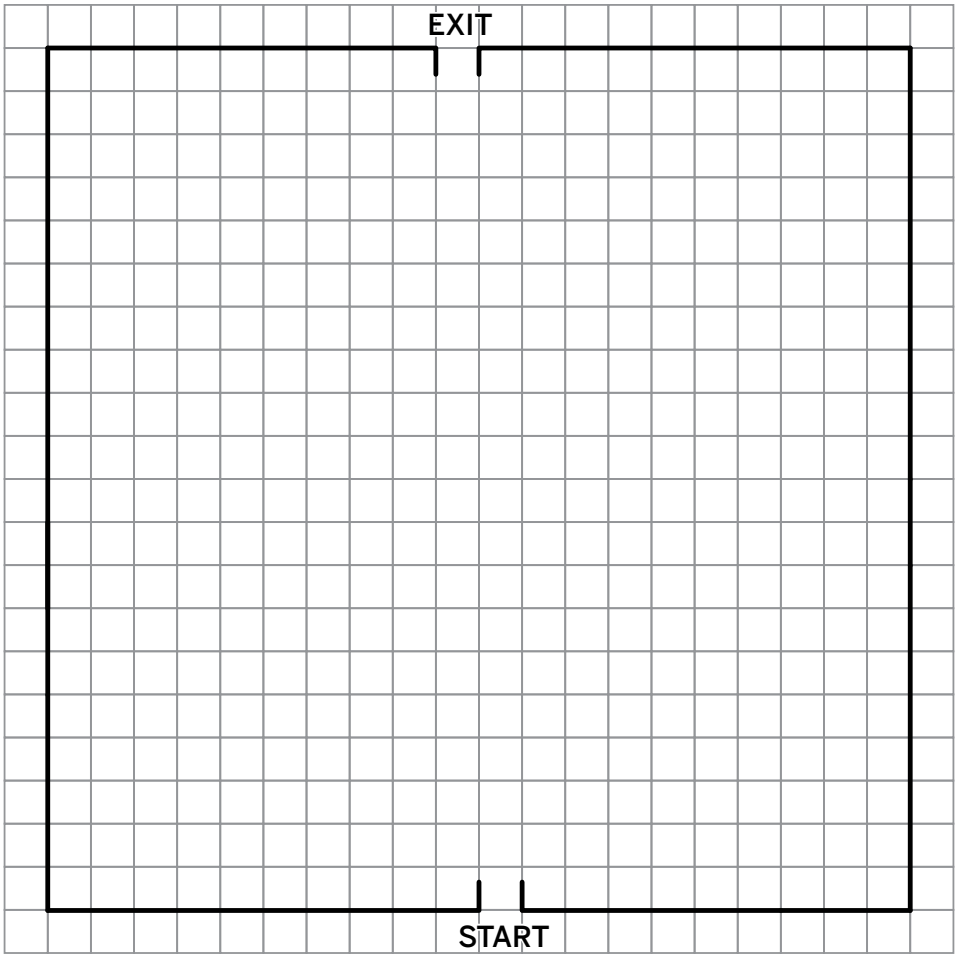
Invisible Mazes

2. You and your partner will take turns being the Maze Master and the Navigator.

Maze Master	Navigator
• You will be given an *Invisible Mazes* activity sheet. • Keep the sheet face down until you are told to turn it over. • Give your partner directions to help them move through the maze to the exit. • Record the time it takes to reach the exit.	• Use the invisible maze. • Place your pencil at the start of the maze. • Listen to your partner for directions and magnitude to move on the grid paper to reach the exit.

Maze 1

Time: ____minutes ____seconds



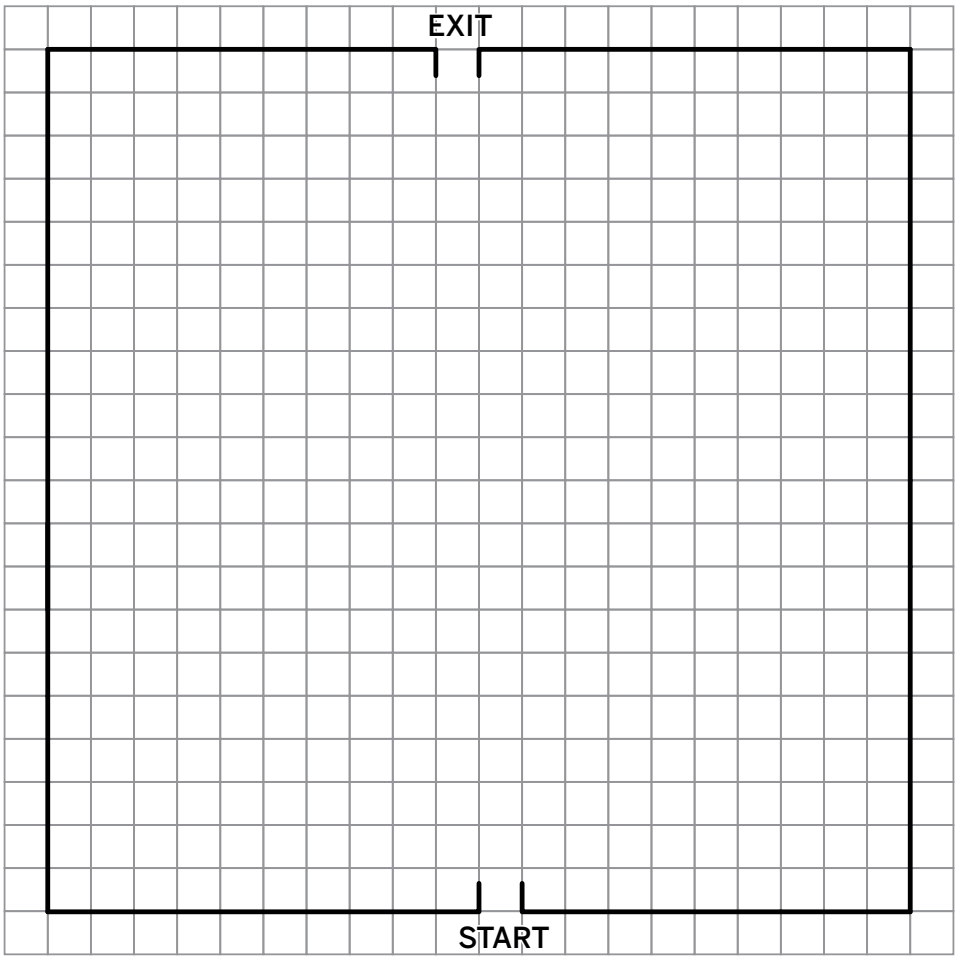
Name: ..... Date: ..... Period: .....

Invisible Mazes

Once you have successfully completed the first maze, switch roles and repeat with a new maze.

Maze 2

Time: ____minutes ____seconds



Name: ..... Date: ..... Period: .....

Building Math Habits of Mind

Discuss:

- Which of these habits of mind did you strengthen during this activity?
- How did you use the one(s) you selected?

I can slow down and first make sense of a challenging problem before trying to solve it.

Not yet Almost I got it!

I can represent real-world problems using equations and inequalities and interpret their solutions within the context of the problem.

Not yet Almost I got it!

I can justify my thinking and ask questions to help me understand the thinking of others.

Not yet Almost I got it!

I can apply the math that I know to solve real-world problems, making assumptions and revising my thinking as needed.

Not yet Almost I got it!

I can select an appropriate tool to help me solve problems.

Not yet Almost I got it!

I can communicate my thinking and solutions clearly to others.

Not yet Almost I got it!

I can look for structure or patterns to help me solve problems.

Not yet Almost I got it!

I can look for repeated calculations and other repeated steps to make generalizations.

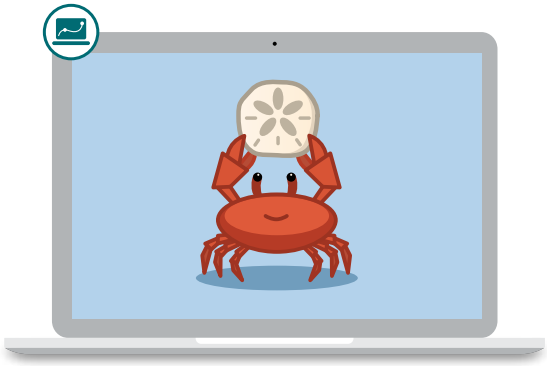
Not yet Almost I got it!

Name: ..... Date: ..... Period: .....

Distance and Direction 6.NS.5, 6.NS.6.c, SMP.3, SMP.6

Can You Dig It?

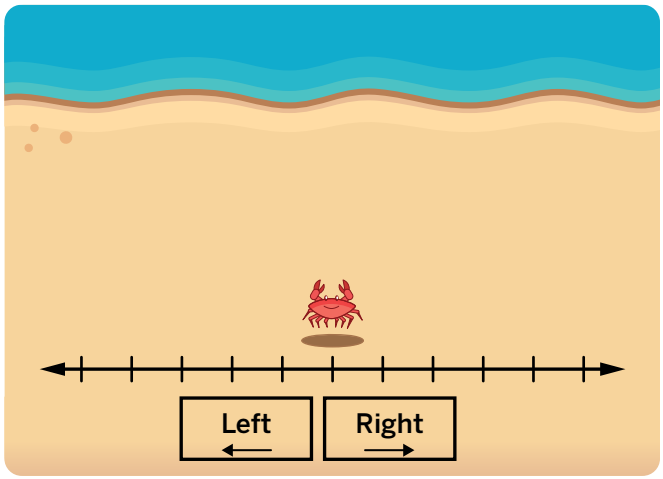
Let's find the hidden sand dollars.



Warm-Up

1 **a** Let's help the crab look for two sand dollars.

b Write a clue to help someone else find them.



Activity 1

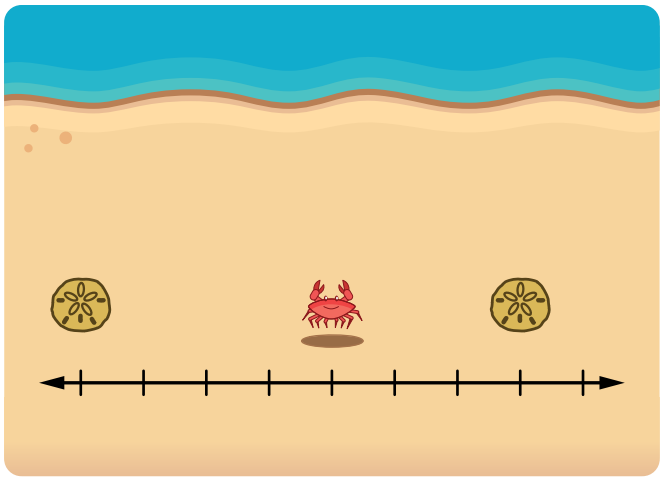
Name: ..... Date: ..... Period: .....

Finding Sand Dollars

2 Juliana and Kai each wrote correct clues.

Juliana: "One sand dollar is at positive 3 from the crab, and the other is at negative 4."

Kai: "Go 3 steps to the right of the crab to find the first sand dollar, then 7 steps to the left to find the other one."



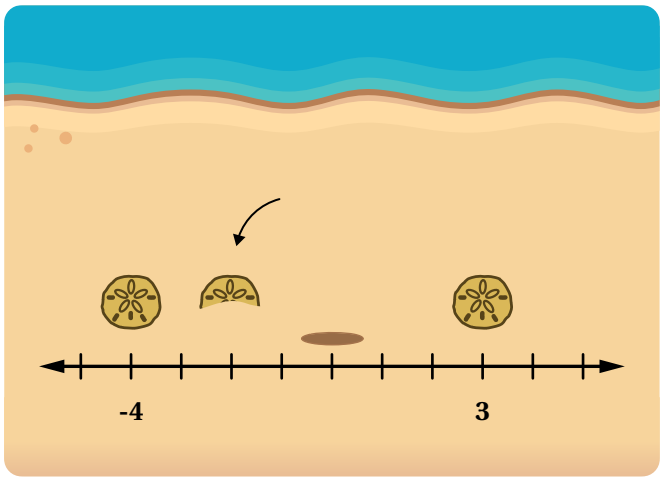
Discuss:

- How are their clues alike? How are they different?

- What do you think the "negative 4" in Juliana's clue means?

3 Here is a new sand dollar.

Write a clue that describes its location.

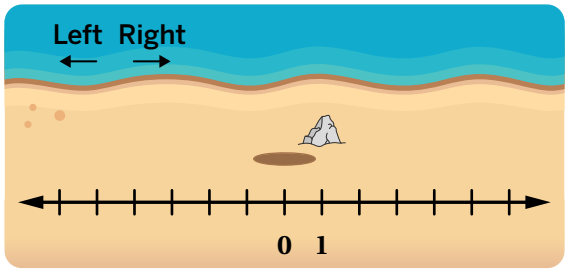


Name: ..... Date: ..... Period: .....

Sand Dollar Challenges

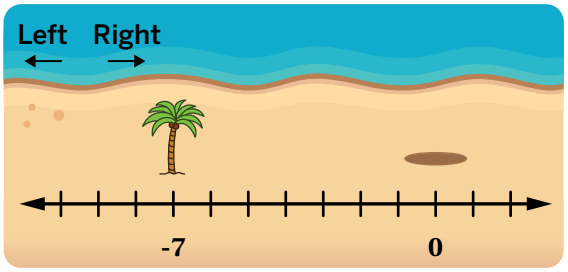
4 The rock is at 1 on the number line. A sand dollar is 4 units to the left of the rock.

Where is the sand dollar?



5 The palm tree is at -7 on the number line. A sand dollar is 3 units to the right of the palm tree.

Where is the sand dollar?



6 Juliana and Kai made mistakes on the previous problem.

Juliana said the sand dollar was at -5.
Kai said the sand dollar was at -10.

Circle your favorite mistake.

Juliana Kai

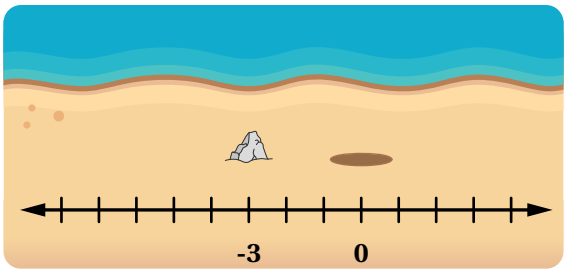
What could you tell this student to help them revise their answer?

Activity 3

Name: _____ Date: _____ Period: _____

More Sand Dollar Challenges

7 The rock is at -3 on the number line. A sand dollar is 2 units away from the rock.



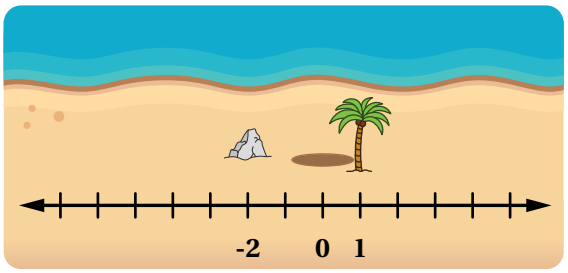
Lan says the sand dollar has to be at a **negative number**. Is she correct? Circle one.

Yes No I'm not sure

Explain your thinking.

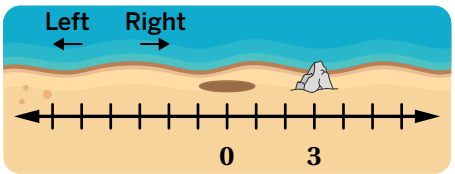
8 A sand dollar is 2 units away from the rock and 5 units away from the palm tree.

Where is the sand dollar?

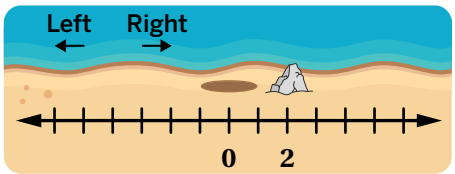


9 For each challenge, write the location of the sand dollar on the number line using the clue. Complete as many challenges as you can.

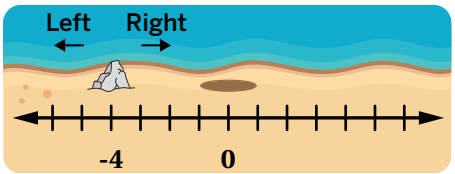
a 2 units to the right of the rock



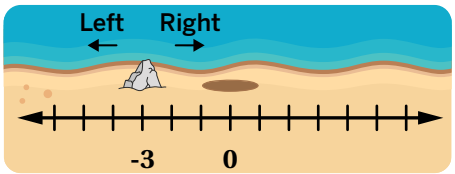
b 5 units to the left of the rock



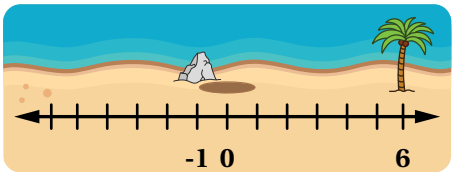
c 2 units to the left of the rock



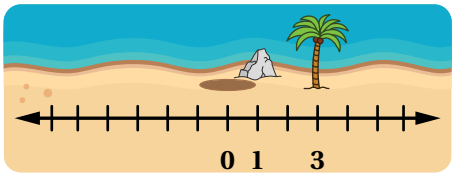
d 6 units to the right of the rock



e 3 units away from the rock and 4 units away from the palm tree



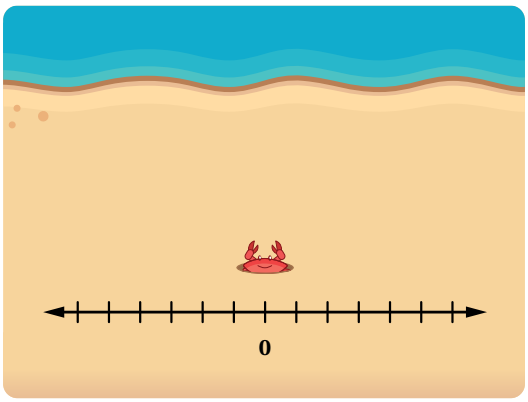
f 3 units away from the rock and 5 units away from the palm tree



10 Synthesis

List at least two things you know about **positive** and **negative numbers** on a number line.

Draw on the image if it helps to show your thinking.



13 Summary 7.01

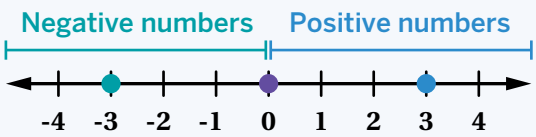
Positive numbers are numbers that are greater than 0. **Negative numbers** are numbers that are less than 0. Zero is neither positive nor negative. All whole numbers, whether positive, negative, or zero, are called **integers**.

You can extend a number line to the right of 0 to show positive numbers, and you can extend a number line to the left of 0 to show negative numbers.

For example:

The number 3 is 3 units to the right of 0 on the number line.

The number -3 is 3 units to the left of 0 on the number line.



3, 0, and -3 are all examples of integers. 0.3 and $\frac{1}{3}$ are not integers.

integers All positive whole numbers, all negative whole numbers, and zero.

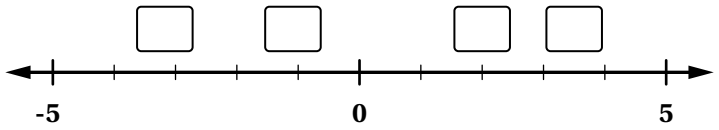
positive number A number that is greater than 0. On a horizontal number line, positive numbers are to the right of 0

negative number A number that is less than 0. On a horizontal number line, negative numbers are to the left of 0.

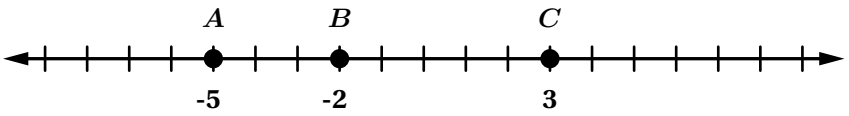
Practice
7.01

Name: ..... Date: ..... Period: .....

1. Fill in the blanks on the number line.



Problems 2–6: Here is a number line.



2. Describe where you would plot -100 on the number line.

3. Point D is 1 unit to the left of point A. Plot point D.

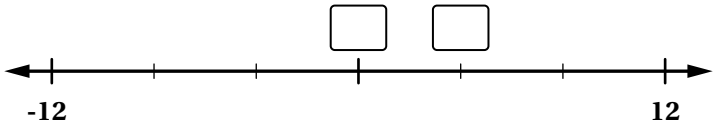
4. Point E is at 0. Plot point E.

5. List both locations that are 4 units away from point B.

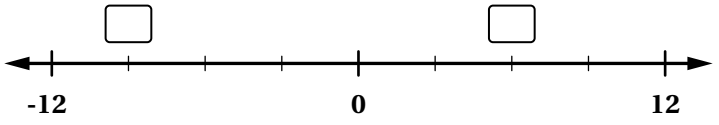
6. Point F is the same distance from point A and point C. Plot point F.

Problems 7–10: Fill in the blanks on the number lines.

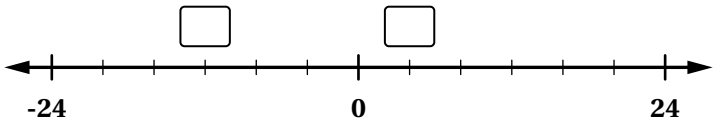
7.



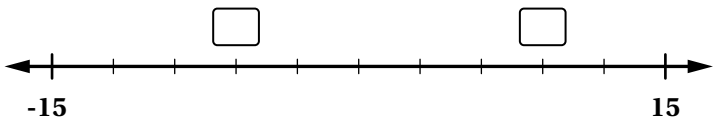
8.



9.



10.



Name: ..... Date: ..... Period: .....

11. Which pair of numbers are on opposite sides of zero on the number line?

A. 0 and 5

B. 2 and -5

C. -2 and -5

D. 2 and 5

Spiral Review

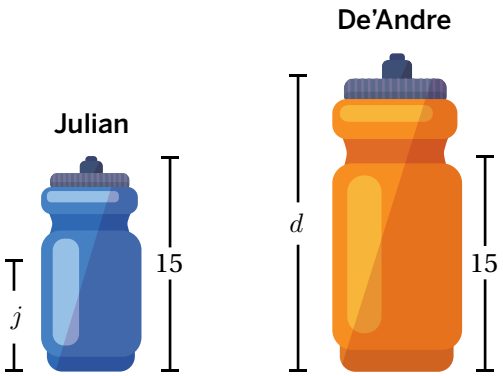
12. A rectangle has an area of 24 square centimeters. If one side is $2\frac{2}{5}$ centimeters long, how long is the other side?

$2\frac{2}{5}$ cm | 24 sq. cm

Explain your thinking.

Problems 13–15: Two friends at soccer practice are drinking from their water bottles. Julian drinks $\frac{3}{5}$ of his 15-ounce bottle. DeAndre drinks 15 ounces of water, which is $\frac{3}{5}$ of his bottle.

13. How are these two situations alike? How are they different?



14. Write an equation to represent the amount of water Julian drinks, j.

15. Write an equation to represent the total amount of water in DeAndre's bottle, d.

Name: ..... Date: ..... Period: .....

Distance and Direction 6.NS.6, 6.NS.6.a, 6.NS.6.c, SMP.1, SMP.7

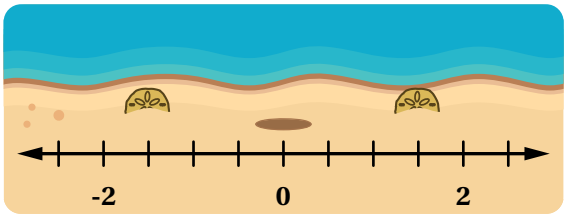


Digging Deeper

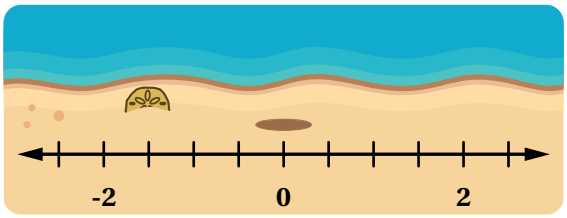
Let's plot positive and negative numbers on the number line.

Warm-Up

1 Here are two buried sand dollars. Write clues to describe their locations.



2 Select *all* the numbers that describe the location of the sand dollar.



- ☐ **A.** $-\frac{3}{2}$
- ☐ **B.** -1.5
- ☐ **C.** $-1\frac{1}{2}$
- ☐ **D.** -2.5
- ☐ **E.** -3

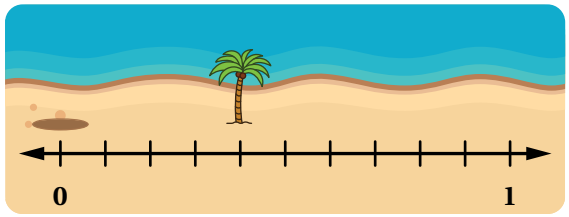
Activity 1

Name: ..... Date: ..... Period: .....

Rational Numbers and Their Opposites

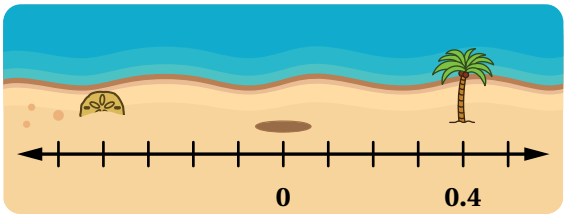
3 A sand dollar is buried under the tree.

Where is the sand dollar on the number line?



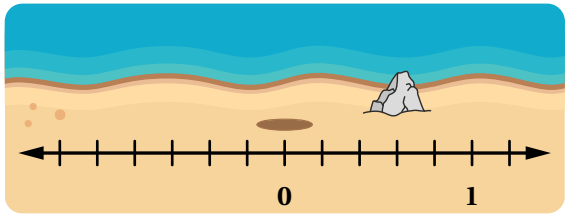
4 A new sand dollar is buried at the **opposite** of the palm tree.

What do you think opposite means here?



5 A new sand dollar is buried at the opposite of the rock.

Where is the new sand dollar?

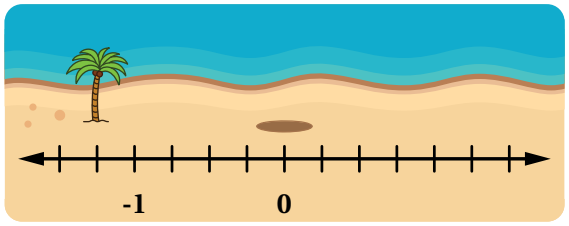


Name: ..... Date: ..... Period: .....

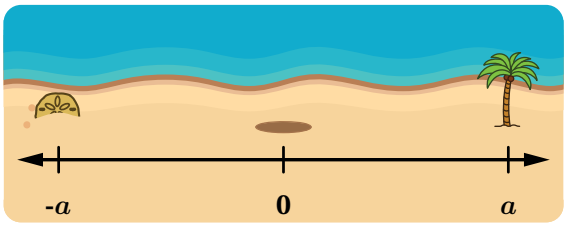
Rational Numbers and Their Opposites
(continued)

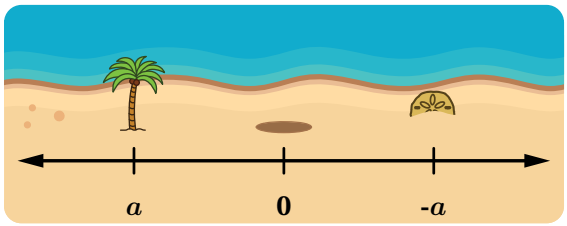
6 A new sand dollar is buried at the opposite of the palm tree.

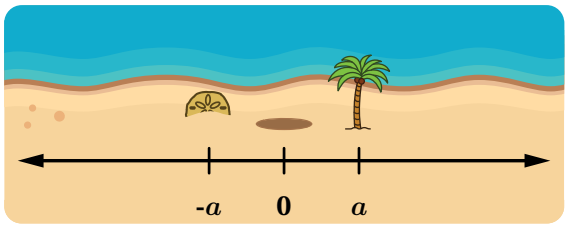
Where is the new sand dollar?

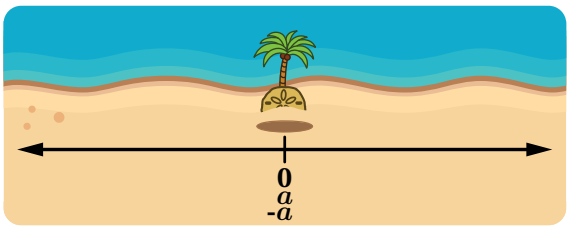


7 The palm tree is at a. The sand dollar is buried at the opposite of a, which you can write as $-a$. When a is in different locations, what do you notice about $-a$?









8 Pair opposite numbers together.

$-(-5)$	$\frac{3}{4}$	$\frac{4}{3}$	-0.75
$-1\frac{1}{3}$	-5	$\frac{1}{5}$	$-\frac{1}{5}$

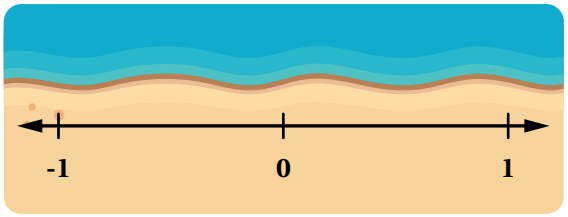
Pair 1	Pair 2	Pair 3	Pair 4

Activity 2

Name: ..... Date: ..... Period: .....

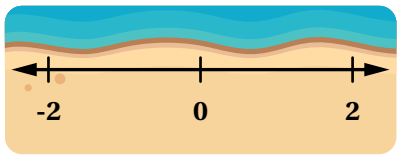
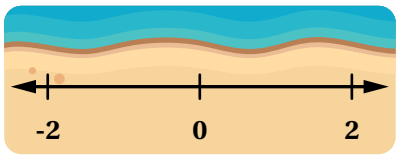
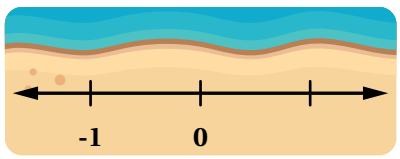
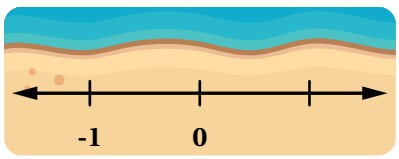
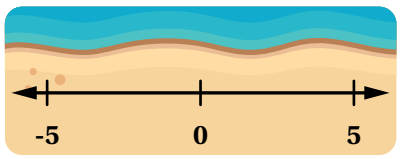
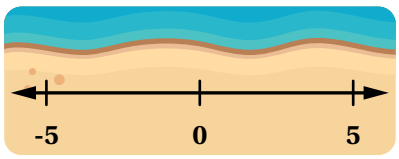
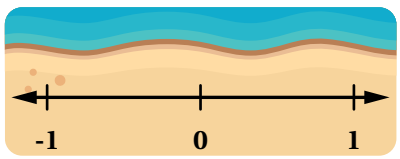
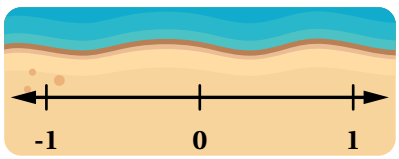
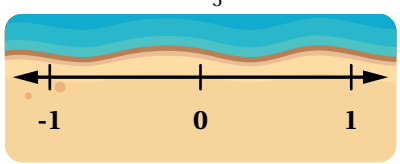
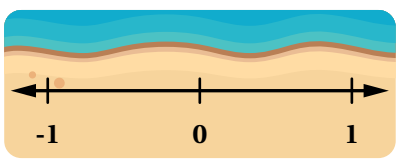
Rational Number Challenges

9 The crab is going to dig at $-\left(-\frac{3}{4}\right)$. Plot the sand dollar where the crab will find it.



10 Plot a point where the crab will dig to find the sand dollar.

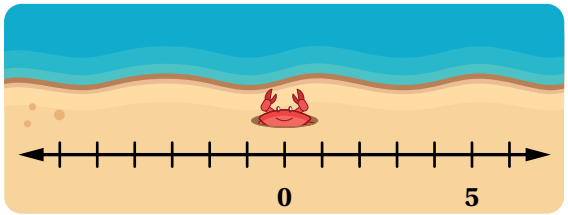
- Decide with your partner who will complete Column A and who will complete Column B.
- After plotting the location of each sand dollar, compare your solutions. The solutions in each row should be the same. Discuss and resolve any differences.

	Column A	Column B
a	The crab is going to dig at $\frac{3}{2}$. 	The crab is going to dig at 1.5. 
b	The crab is going to dig at -1.1. 	The crab is going to dig at the opposite of 1.1. 
c	The crab is going to dig at the opposite of -2. 	The crab is going to dig at $-(-2)$. 
d	The crab is going to dig at $-(-0.1)$. 	The crab is going to dig at 0.1. 
e	The crab is going to dig at the opposite of $-\frac{4}{5}$. 	The crab is going to dig at the opposite of -0.8. 

11 Synthesis

Discuss: Where is -3.2 on the number line?

Draw on the image if it helps to show your thinking.

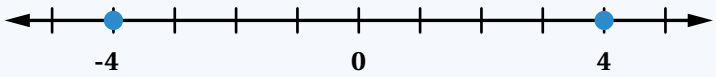


14 Summary 7.02

The minus sign (—) has several uses in mathematics;

- To represent subtraction, $24 - 5 = 19$
- To represent a negative number, as in -5.
- To represent the opposite of a number, as in, "-5 is the opposite of 5" or "-(-5) is the opposite of -5"

Two numbers are **opposites** if they are the same distance from 0 on different sides of the number line. For example, -4 and 4 are opposites because they are both 4 units away from 0.



Every number has an opposite, including fractions and decimals. 0 is its own opposite. The opposite of the opposite of a number is the number itself. For example, $-(-2) = 2$

All positive and negative whole numbers and 0 are a group of numbers called *integers*. All positive and negative numbers that can be written as fractions, including whole numbers, are called **rational numbers**.

2 and -2 are both integers and rational numbers.

8.3, -8.3, $\frac{3}{2}$ and $-\frac{3}{2}$ are rational numbers, but *are not* integers.

opposite Two numbers that are the same distance from 0 and on different sides of 0 on the number line are opposites.

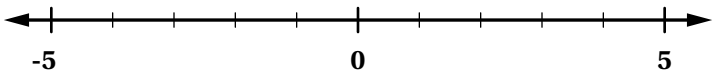
rational numbers All positive and negative numbers that can be written as fractions, including whole numbers, are called rational numbers.

Practice
7.02

Name: ..... Date: ..... Period: .....

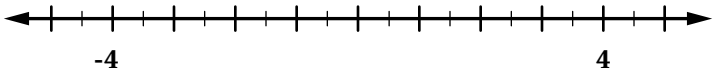
1. Plot each number in its approximate location on the number line.

-3	$\frac{3}{2}$	$-\frac{4}{3}$	$\frac{9}{4}$



2. Plot each number in its approximate location on the number line.

-2	$\frac{5}{4}$	-3.4	0



Problems 3–6: Complete each statement below.

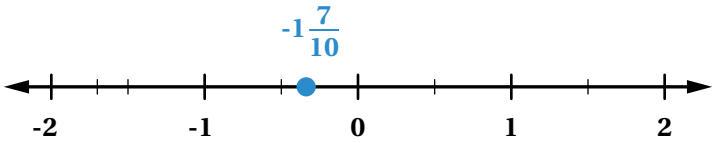
3. The opposite of -2 is ..... .

4. The opposite of $\frac{5}{4}$ is ..... .

5. The opposite of -3.4 is ..... .

6. The opposite of 0 is ..... .

Problems 7–8: Rebecca incorrectly plotted the point $-1\frac{7}{10}$ on the number line.

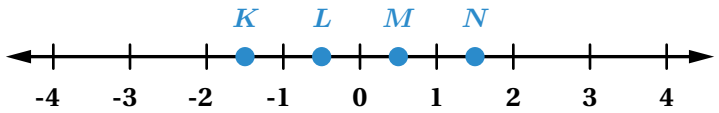


7. What question could you ask to help Rebecca understand her mistake?

8. Plot $-1\frac{7}{10}$ in the correct location on the number line.

Name: _____ Date: _____ Period: _____

9. Points K, L, M, and N are plotted on the number line.

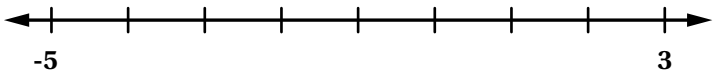


What point represents the location of -0.5?

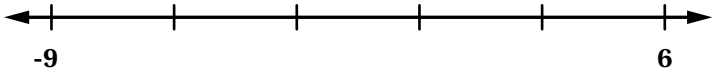
A. Point K **B.** Point L **C.** Point M **D.** Point N

Problems 10–12: Plot and label where zero is located on each number line.

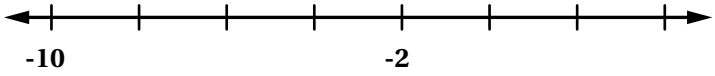
10.



11.



12.



Spiral Review

Problems 13–18: Solve each equation.

13. $3x = 6$

14. $\frac{1}{3}x = 6$

15. $\frac{1}{3} = 6x$

16. $\frac{1}{3} = \frac{1}{6}x$

17. $2x = \frac{2}{5}$

18. $x + \frac{2}{5} = 3$

Name: _____ Date: _____ Period: _____

Distance and Direction 6.NS.5, 6.NS.6.c, 6.NS.7.b, SMP.2, SMP.3, SMP.7



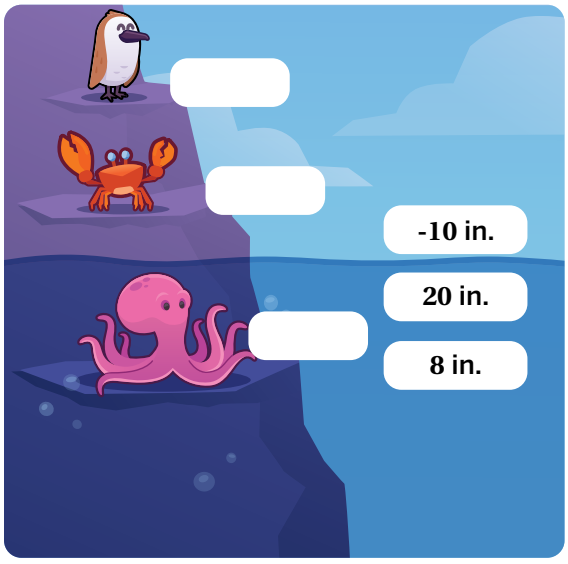
Sub-Zero

Let's use positive and negative numbers in context.

Warm-Up

1 Here are three animals sitting on rocks at different elevations.

Match each elevation with the correct rock.



2 What do you think 0 inches represents in this situation?

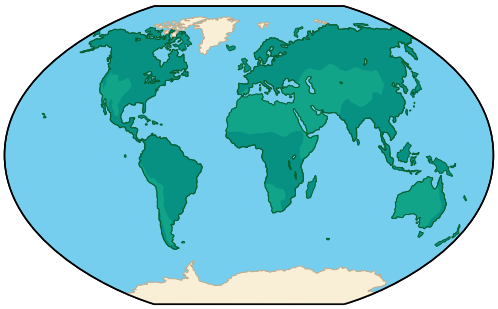
Activity 1

Name: ..... Date: ..... Period: .....

World Temperatures

3 Let's look at different temperatures around the world on one particular day.

Choose a location you'd like to visit and explain your choice to a classmate.

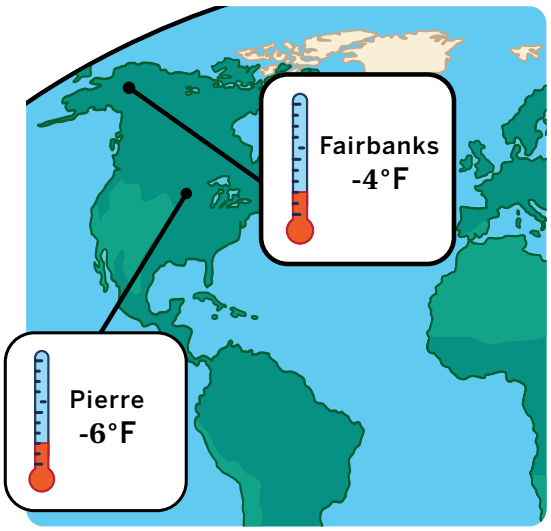


4 Neel says Fairbanks is warmer because $-4 > -6$. Mia says Pierre is warmer because 6 is greater than 4.

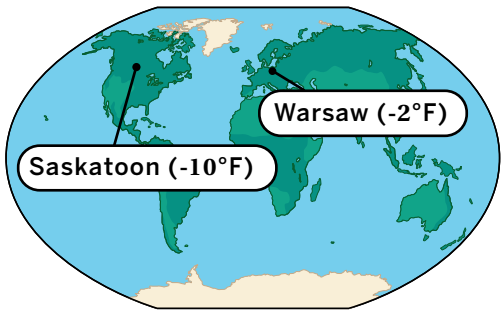
Whose thinking is correct? Circle one.

Neel's Mia's Both Neither

Explain your thinking.



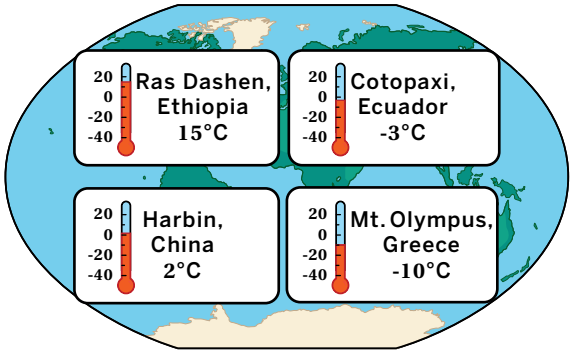
5 Write a temperature that is colder than Warsaw's and warmer than Saskatoon's.



Name: ..... Date: ..... Period: .....

Colder Temperatures

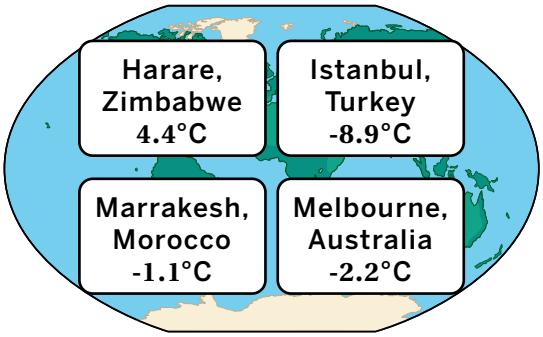
6 Here are the temperatures of different places around the world on another day.



Select *all* of the true statements.

- ☐ **A.** Ras Dashen is warmer than Cotopaxi.
- ☐ **B.** Cotopaxi is -3 degrees below 0.
- ☐ **C.** Mt. Olympus is the coldest.
- ☐ **D.** Mt. Olympus is 12 degrees colder than Harbin.
- ☐ **E.** Cotopaxi is colder than Mt. Olympus.

7 Here are some of the coldest recorded temperatures for four cities.



Order these temperatures from *warmest* to *coldest*.

Warmest 🔥

Coldest ❄️

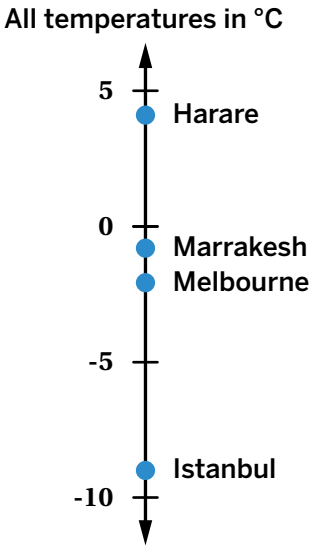
Name: ..... Date: ..... Period: .....

Colder Temperatures (continued)

8 This number line shows some of the lowest recorded temperatures for the same four cities.

One of the lowest recorded temperatures in Rome, Italy, is -7.2°C.

Plot the point for Rome at its approximate location on the number line.



9 One way to write that Harare's temperature is warmer than Marrakesh's is $4.4 > -1.1$.

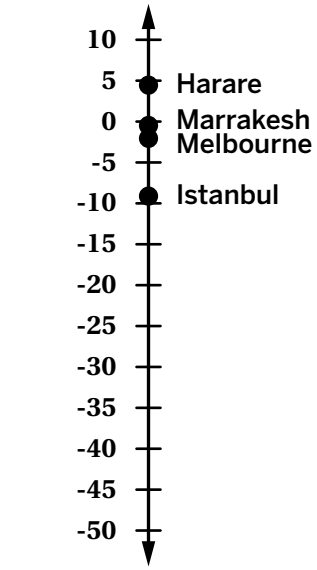
Write your own *inequality* comparing Rome's temperature to another city's temperature.

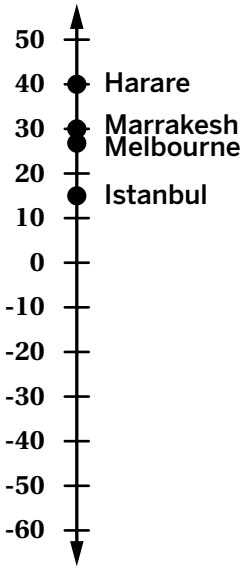
You're invited to explore more.

10 Look up the lowest recorded temperature in a city near you.

Plot and label a point to represent this temperature on one of the number lines.

Celsius	Fahrenheit
All temperatures in °C	All temperatures in °F

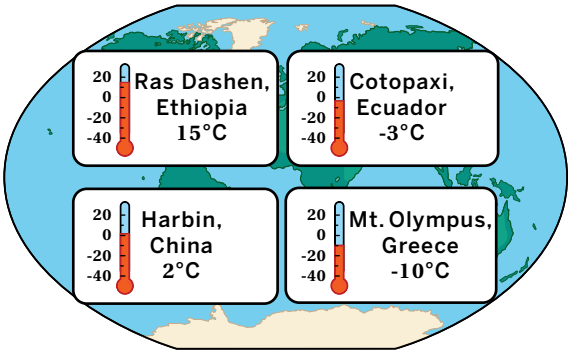




11 Synthesis

Discuss: When comparing two temperatures, how can you tell which is warmer and which is colder?

Use the examples if they help with your thinking.



14 Summary 7.03

You can use a vertical number line to represent positive and negative numbers. On a vertical number line, points above 0 are positive and points below 0 are negative.

When talking about elevation, 0 feet represents sea level. This means that a positive elevation is above sea level and a negative elevation is below sea level.

When talking about temperature, 0°C means the temperature is freezing. If the temperature in Mt. Olympus is -10°C, that means it has a temperature of 10°C below 0°C, or below freezing.

If the temperature in Cotopaxi is -3°C, you can write $-10 < -3$, which means that it is colder in Mt. Olympus than it is in Cotopaxi.

Practice
7.03

Name: ..... Date: ..... Period: .....

Problems 1–3: Determine if each statement is true or false.

Statement	True	False
1. An elevation of 35 feet is the same as an elevation of -35 feet.		
2. A city that has an elevation of -17 meters is closer to sea level than a city that has an elevation of -40 meters.		
3. A temperature of -4°F is the same as a temperature 4°F below zero.		

Problems 4–7: Here is a table that shows some elevations in several cities around the world.

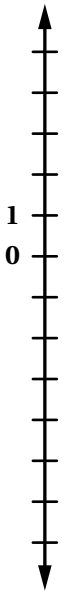
Location	Elevation (ft)
San Juan, Puerto Rico	-4
New Orleans, Louisiana	-7
Amsterdam, Netherlands	-2
Jakarta, Indonesia	3
Taipei, Taiwan	5
Tunis, Tunisia	0

4. A city, not listed in the table, has a higher elevation than San Juan, Puerto Rico.

Select *all* the numbers that could represent the city's elevation in feet.

- ☐ **A.** 0
- ☐ **B.** 4
- ☐ **C.** -2
- ☐ **D.** -10
- ☐ **E.** -7

5. Plot all of the elevations on the number line.



6. Mio says: *I know that 2 is less than 4, so -2 must be less than -4. This means Amsterdam has a lower elevation than San Juan.* Is Mio correct? Explain your thinking.

7. Nasir says: *3 is less than -7 because 3 is closer to 0 on the number line.* Is Nasir correct? Explain your thinking.

Name: _____ Date: _____ Period: _____

Problems 8–10: Here is a table that shows some of the lowest temperatures recorded in five U.S. locations.

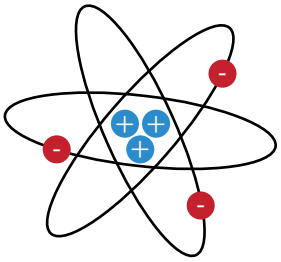
Location	Temperature (°F)
Boca, CA	-45
Coventry, CT	-32
Tallahassee, FL	-2
Mt. Carroll, IL	-38
CCC Fire Camp F-16, GA	-17

8. Which of these locations had the lowest record temperature?

9. Which location had a lower record temperature: Tallahassee, FL, or CCC Fire Camp F-16, GA? Write a statement using $<$ or $>$ to compare the recorded temperatures.

10. Which location had a lower record temperature: Coventry, CT, or Mt. Carroll, IL? Write a statement using $<$ or $>$ to compare the recorded temperatures.

Problems 11–13: All substances are made up of atoms, and atoms have protons and electrons.



- $+1$ represents the positive charge of a proton.
- -1 represents the negative charge of an electron.
- 0 represents the neutral charge which can be the combination of one positive and one negative charge.

11. A group of 4 protons and 4 electrons?

12. A group of 6 protons with no electrons?

13. A group of 4 electrons with no protons

Spiral Review

Problems 14–17: Determine the value of each expression.

14. $2^3 \cdot 4$

15. $\frac{2^3}{4}$

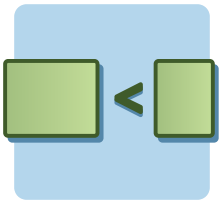
16. $2^4 - 4$

17. $2^3 + 4^3$

Name: ..... Date: ..... Period: .....

Distance and Direction CA CCSSM: 6.NS.6.c, 6.NS.6.a, 6.NS.7, 6.NS.7.a, SMP.2, SMP.7, SMP.8

Order in the Class



Let's compare positive and negative numbers.

Warm-Up

1. Which one doesn't belong?

A. $+3 > \frac{5}{4}$

B. $-\frac{5}{4} < 3$

C. $\frac{5}{4} > -\frac{5}{4}$

D. $-\frac{5}{4} > -3$

Explain your thinking.

Activity 1

Name: _____ Date: _____ Period: _____

Greater Than?

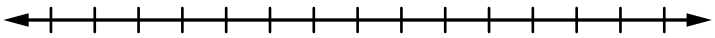
You will use your number card to compare numbers with your classmates.

My number: _____

2. For each round, compare your number with different classmates'. Use the number lines if they help with your thinking.

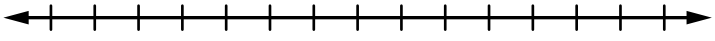
Round 1: Find a person whose number has a *different* **sign** than yours.

Partner's Name	Partner's Number	Comparison in Words	Comparison in Symbols
		_____ is greater than _____.	_____ > _____



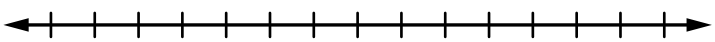
Round 2: Find a person whose number has the *same* sign as yours.

Partner's Name	Partner's Number	Comparison in Words	Comparison in Symbols
		_____ is greater than _____.	_____ > _____



Round 3: Find a person whose number is the opposite of yours.

Partner's Name	Partner's Number	Comparison in Words	Comparison in Symbols
		_____ is greater than _____.	_____ > _____



Name: _____ Date: _____ Period: _____

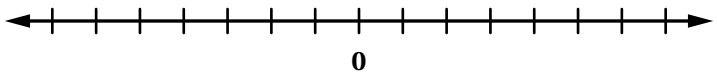
Least to Greatest

3. For each round, form a group of three or four people.

Round 4: Record the values from each of your cards: _____, _____, _____, and _____. Write three different inequalities comparing these numbers.

Inequality 1	Inequality 2	Inequality 3
_____ < _____	_____ < _____	_____ < _____

Round 5: Record the values from each of your cards: _____, _____, _____, and _____. Plot and label each number at its approximate location on the number line.



Round 6: Record the values from each of your cards: _____, _____, _____, and _____. Order the values from *least* to *greatest*.

Least **Greatest**

Round 7: Form a line with everyone in the class so that the numbers are in order from *least* to *greatest*.

Activity 3

Name: _____ Date: _____ Period: _____

Ordering Numbers

4. Here are four more numbers.

-2	$1\frac{1}{2}$	-1.5	-1

a Complete each number sentence using > or <.

-2 _____ $1\frac{1}{2}$ -1.5 _____ -1

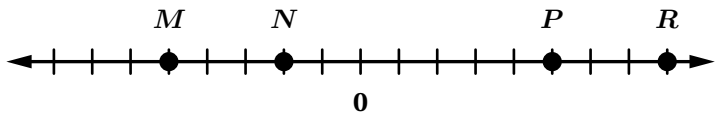
b Use two of the numbers to complete this sentence.

_____ is the opposite of _____ because . . .

c Order the numbers from *least* to *greatest*.

Least **Greatest**

5. Use the number line to complete each sentence.



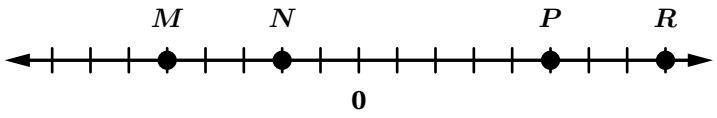
a _____ is the opposite of _____ because . . .

b _____ is greater than _____ because . . .

c _____ is the least of the numbers because . . .

Synthesis

6. Describe how to compare numbers on a number line. Use the example if it helps with your thinking.



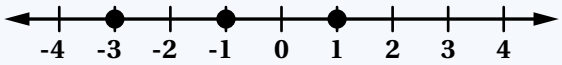
Summary 7.04

You can use a number line to compare numbers with different **signs** (like -4 and 3) or numbers with the same sign (like -4 and -3).

The order of numbers from least to greatest is the same order as they appear on the number line from left to right. This means that negative numbers farther from 0 are less than negative numbers that are closer to 0.

For example, let's say you want to compare -3 and -1. On a number line, -1 is to the right of -3. This means that -1 is greater than -3, or $-3 < -1$. This also makes sense because -1 is closer to 0 than -3 is.

A number line can also help you order numbers from least to greatest. 1 is greater than -1 and -3 because it is the farthest to the right on the number line.



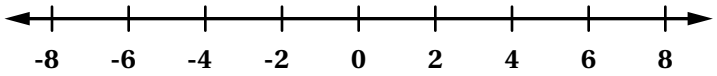
sign A number (other than 0) is either positive or negative.

Practice
7.04

Name: _____ Date: _____ Period: _____

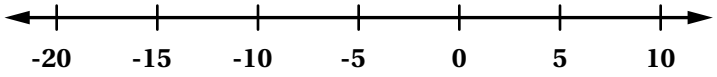
1. Complete each number sentence with a number that makes it true. Use the number line if it helps with your thinking.

a $\square < 5$ **b** $\square < -5$ **c** $-5 < \square$ **d** $-5 > \square$



2. Complete each number sentence with the symbol $<$, $>$, or $=$. Use the number line if it helps with your thinking.

a -5 _____ 2 **b** 5 _____ -5 **c** -12 _____ -15 **d** -12.5 _____ -12



3. Circle whether each statement is *true* or *false*.

Statement A: -8.4 is to the right of -8.7 on the number line. True False

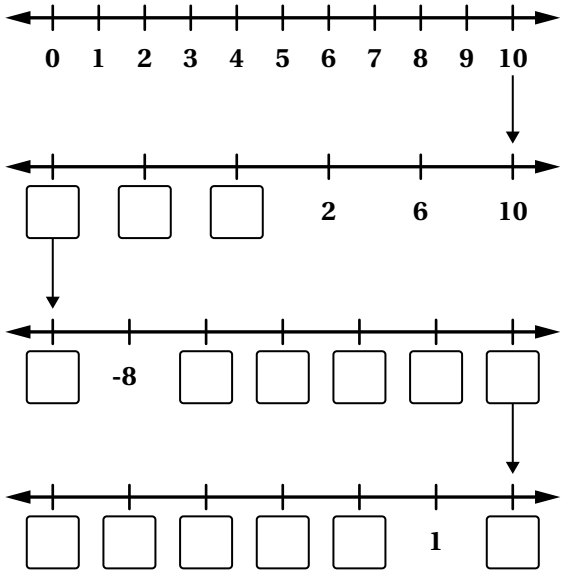
Statement B: -2.4 is greater than -2.3. True False

Statement C: $-\frac{11}{12} < -\frac{7}{12}$ True False

Choose one statement and explain your thinking.

4. Here is a number line maze.

Each number line has a different scale but at least one matching number, which is labeled with an arrow. Determine the missing values in this number line maze.



Name: _____ Date: _____ Period: _____

5. Here are five numbers: $-\frac{2}{5}$, -1, $\frac{4}{3}$, 1, $-\frac{3}{2}$.

These numbers are plotted on a horizontal number line. Which statement about the locations of the numbers is true?

A. $-\frac{2}{5}$ is the farthest to the left, and 1 is farthest to the right.

B. -1 is the farthest to the left, and 1 is farthest to the right.

C. $\frac{4}{3}$ is the farthest to the left, and $-\frac{3}{2}$ is farthest to the right.

D. $-\frac{3}{2}$ is the farthest to the left, and $\frac{4}{3}$ is farthest to the right.

Spiral Review

6. Solve each equation. Write each solution as a fraction and as a decimal.

Equation	Solution (Fraction)	Solution (Decimal)
$2x = 3$		
$5y = 3$		
$0.3z = 0.09$		

Problems 7–10: Each lap around a track is 400 meters. How many meters will someone run in:

7. 2 laps?

8. 5 laps?

9. x laps?

10. If Sol ran 7,600 meters, how many laps did Sol run? Explain your thinking.

11. Select *all* expressions that are equivalent to 6^2.

A. 36 **B.** 12 **C.** $6 + 6$ **D.** $2 \cdot 6$ **E.** $6 \cdot 6$

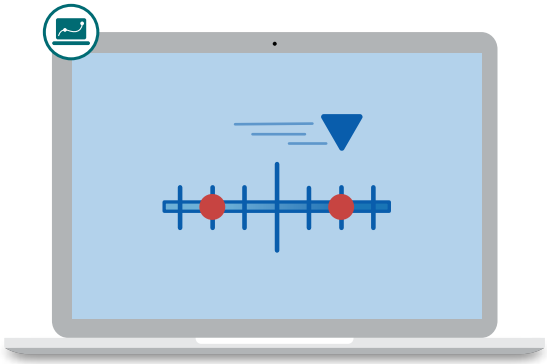
Unit 7
Lesson
5

Name: ..... Date: ..... Period: .....

Distance and Direction 6.NS.7.c, 6.NS.7, 6.NS.7.a, SMP.2, SMP.3, SMP.6

Distance on the Number Line

Let's explore the absolute value of rational numbers.



Warm-Up

1 Let's play a game to get a score.

Discuss: How do you think scores are determined?

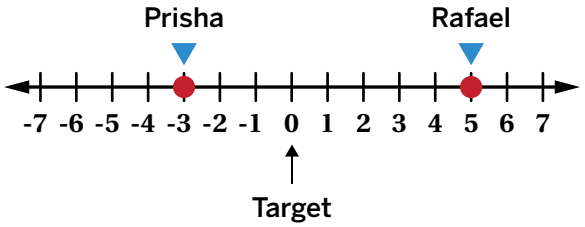
Activity 1

Name: Date: Period:

What Is Absolute Value?

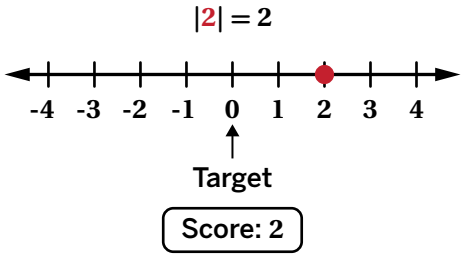
2 Prisha landed at -3. Rafael landed at 5. What was each student's score?

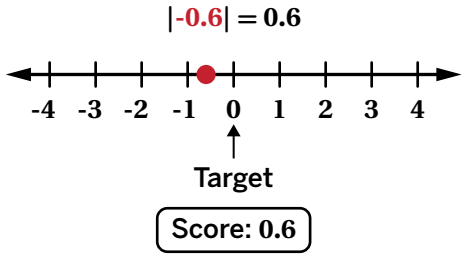
Student	Value
Prisha	
Rafael	

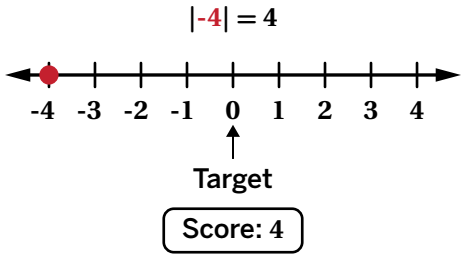


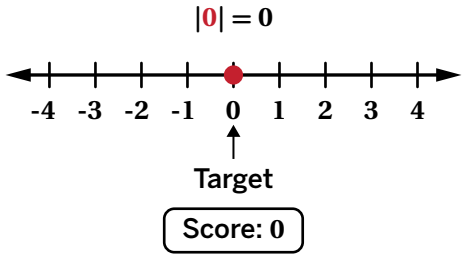
3 The game uses **absolute value** to calculate the score. We read |-2| as "the absolute value of negative 2."

a Take a look at some scores for different stopping points.









b Describe what you think absolute value means.

4 Prisha says $|x|$ means "the opposite of x." Rafael says $|x|$ means "how far x is from zero." Whose thinking is correct? Circle one.

Prisha's Rafael's Both Neither

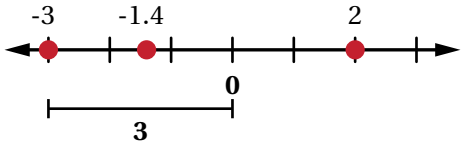
Explain your thinking.

Name: ..... Date: ..... Period: .....

Comparing Absolute Values

5 Determine the value of each expression. Use the number line if it helps with your thinking.

Expression	Value
$\lvert -3 \rvert$	3
$\lvert -1.4 \rvert$	
$\lvert 2 \rvert$	

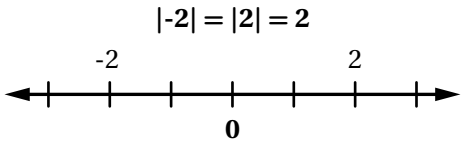


6 Determine whether each statement is true or false.

Statement	True	False
$\lvert 0 \rvert = 0$		
$\lvert 6 \rvert = -6$		
$\lvert 6 \rvert > \lvert -6 \rvert$		
$-5 < -4$		
$\lvert -5 \rvert < \lvert -4 \rvert$		
$\frac{3}{2} > -\frac{1}{2}$		
$\left\lvert \frac{3}{2} \right\rvert > \left\lvert -\frac{1}{2} \right\rvert$		

7 Isaiah says -2 and 2 have the same absolute value.

Explain why Isaiah's claim is correct.

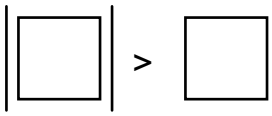


Activity 3

Name: ..... Date: ..... Period: .....

Absolute Value Puzzles

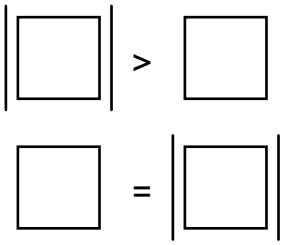
8 **a** Make a true inequality by using each number at most once.



-2 -1 1 2

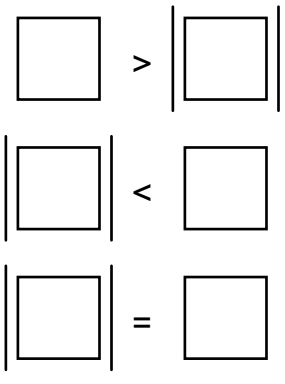
b Explain how you know your inequality is true.

9 Make true statements by using each number exactly once.



3 4 -4 -5

10 Make true statements by using each number exactly once.



-1 -2 -4

2 3 4

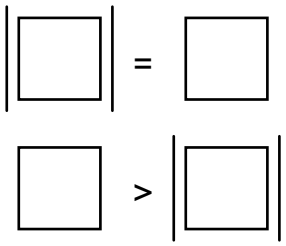
Name: ..... Date: ..... Period: .....

Absolute Value Puzzles (continued)

11 Is it possible to make true statements using each number exactly once? Circle one.

Yes No I'm not sure

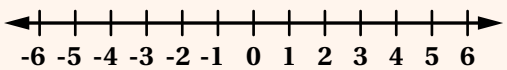
Explain your thinking.



-5 6 -6 -7

You're invited to explore more.

12 Use these clues to determine the values of A, B, and C. Use the number line to record your answers and to help you with your thinking.

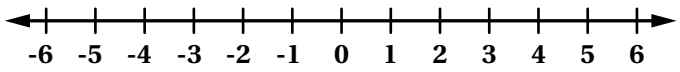


Clues

- The absolute value of A is 2.
- B is greater than A.
- Point B is closer to zero than point A is.
- C is positive.
- The distance between A and B is 1.
- The distance between B and C is 4.

13 Synthesis

Discuss: What are 2–3 things you know about absolute value. Use the number line if it helps to show your thinking?



16 Summary 7.05

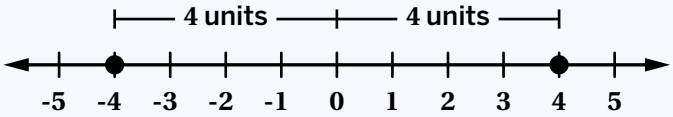
The **absolute value** of a number is a way to describe its distance from 0. For example:

The absolute value of -4 is 4, because it is 4 units away from 0.

$|-4| = 4$

The absolute value of 4 is also 4, because it is 4 units away from 0.

$|4| = 4$



The distance from 0 to itself is 0, so $|0| = 0$.

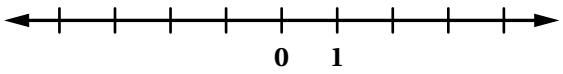
Absolute values are helpful when you are interested in the size of a difference or measurement but its direction is not important.

Absolute value The distance number is from 0 on the number line.

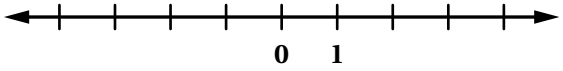
Practice
7.05

Name: _____ Date: _____ Period: _____

1. Plot and label all numbers that are 4 units away from 0.



2. Plot and label all numbers with an absolute value of $\frac{5}{2}$.



3. Ivory claims that a number and its opposite will always have the same absolute value. Is Ivory correct? Explain your thinking.

Problems 4–11: Complete each number sentence with the symbol <, >, or =.

4. -3.2 _____ 1.5

5. $|-3.2|$ _____ $|1.5|$

6. 2 _____ -1.5

7. $|2|$ _____ $|-1.5|$

8. $\frac{3}{2}$ _____ -1.5

9. $\frac{3}{2}$ _____ $|-1.5|$

10. $|-2.7|$ _____ $|-4.5|$

11. $|-2.7|$ _____ -4.5

12. Which list of absolute value expressions are ordered from *least* to *greatest* value?

A. $|1|, \frac{9}{4}, |-5|, |-7|$

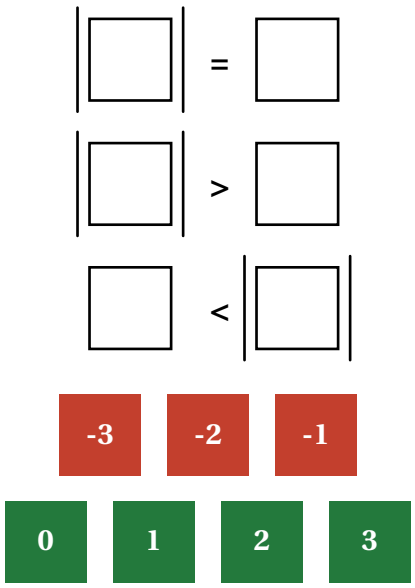
B. $|-7|, |-5|, |1|, \frac{9}{4}$

C. $|-7|, |-5|, \frac{9}{4}, |1|$

D. $\frac{9}{4}, |1|, |-5|, |-7|$

Name: ..... Date: ..... Period: .....

13. Make each statement true by using each number at most once.



Spiral Review

Problems 14–17: Determine the value of each quotient.

14. $24 \div 15$

15. $0.24 \div 0.15$

16. $0.24 \div 0.015$

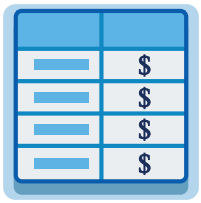
17. $0.024 \div 0.015$

18. On Monday, Martina made 24 cookies. On Tuesday, she made 75% as many cookies as she made on Monday. How many cookies did Martina make on Tuesday?

Name: ..... Date: ..... Period: .....

Distance and Direction CA CCSSM: 6.NS.7.c, 6.NS.7.d, 6.NS.5, 6.NS.7, SMP.2, SMP.3, SMP.4

We've Got Game(s)

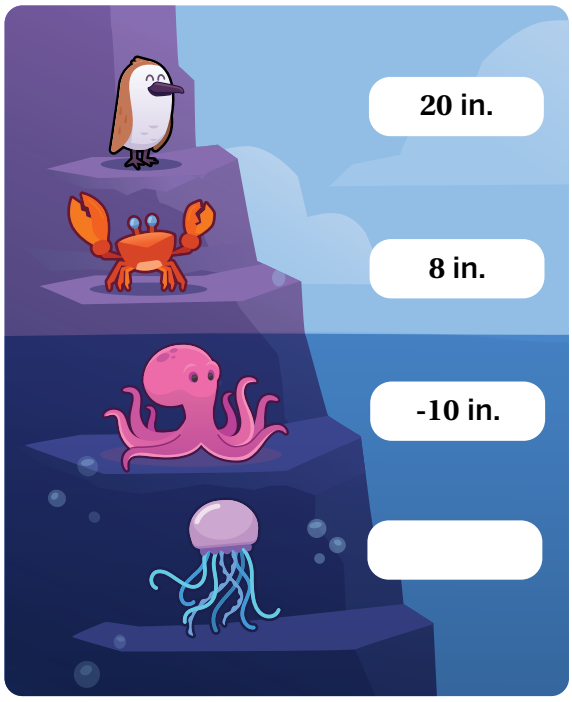


Let,s compare rational numbers and absolute values to make sense of contexts.

Warm-Up

1. The jellyfish is farther from sea level than the bird is. What could its elevation be?

Explain your thinking.



Activity 1

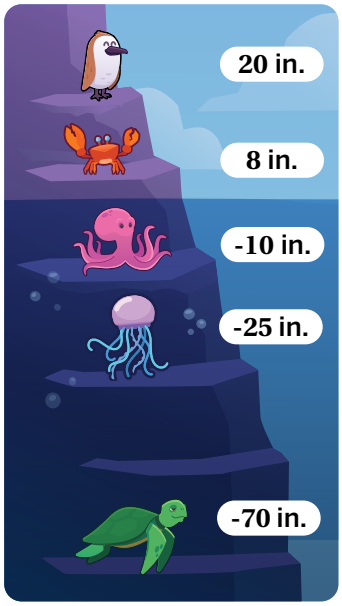
Name: _____ Date: _____ Period: _____

Comparing Distances From Sea Level

2. Which animal has the lowest elevation? The highest elevation?

Lowest elevation: _____

Highest elevation: _____



3. Which animal is farthest from sea level? Closest to sea level?

Farthest from sea level: _____

Closest to sea level: _____

4. Order the elevations of the animals from *closest* to sea level to *farthest* from sea level.

Closest to sea level **Farthest from sea level**

5. Here are two ways to compare the locations of the crab and the octopus. Match each inequality to the statement it represents.

a $-10 < 8$ _____ The crab is closer to sea level than the octopus is.

b $|8| < |-10|$ _____ The crab has a higher elevation than the octopus.

Name: Date: Period:

We've Got Game(s)

We've Got Game(s) sells board games.

6. The table shows the amount in their bank account (called an account balance) from January to June.

Discuss: What do you notice? What do you wonder?

Month	Account Balance ($)
January	-300
February	-200
March	-250
April	-100
May	0
June	50

7. Here is their account balance in July.
Two students described it in different ways.

Month	Account Balance ($)
July	-100

Alma says: *They had an account balance of* -$100.

Miko says: *They had a debt of* $100.

Whose description is correct? Circle one.

Alma Miko Both Neither

Explain your thinking.

8. Here is their account balance in August.
Two students described it in different ways.

Month	Account Balance ($)
August	300

Alma says: *They had an account balance of* $300.

Miko says: *They received a credit of* $400 *from July to August.*

Whose description is correct? Circle one.

Alma Miko Both Neither

Explain your thinking.

Name: Date: Period:

We've Got Game(s) (continued)

9. You will use a set of clue cards to estimate each account balance from August through December.

Month	Account Balance ($)
July	-100
August	
September	
October	
November	
December	

10. Card E says "Their account balance in December was twice as far from 0 as it was in July." Alma said the account balance in December was -$200. Miko said it was $200 dollars. Explain why they are both correct.

You're invited to explore more.

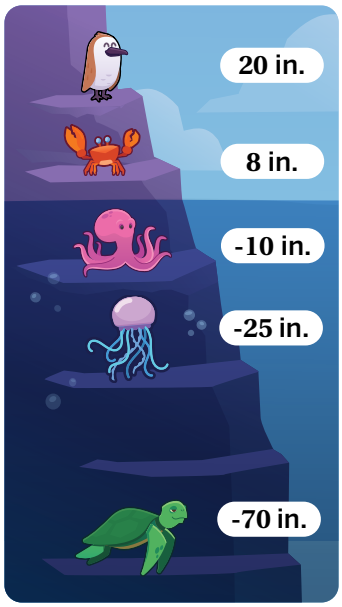
11. In the next year, the We've Got Game(s) account balance at the end of January was -$200. At the end of February it was $300. How much money was credited, or added, to their account from January to February?

Explain your thinking.

Synthesis

12. When is it helpful to use absolute value to compare numbers in context?

Use this example if it helps with your thinking.

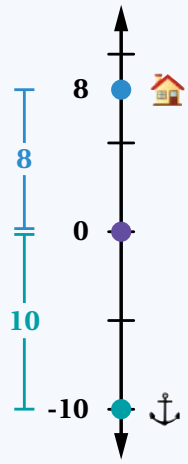


Summary 7.06

You can use absolute value to compare quantity changes or distances.

For example, suppose an anchor has an elevation of -10 meters and a house has an elevation of 8 meters.

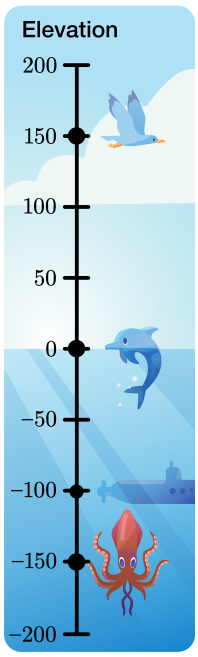
- To compare their elevations and describe the anchor as having a lower elevation than the house, you can write $-10 < 8$.
- To compare their distances from sea level and describe the anchor as being farther away from sea level than the house, you can write $|-10| > |8|$.



Practice
7.06

Name: ..... Date: ..... Period: .....

1. Use the number line to compare each statement. Determine which value is *greater*.



Statement A	Statement B	A	B	Same
The elevation of the submarine	The elevation of the giant squid			
The distance the submarine is from sea level	The distance the giant squid is from sea level			
The distance the seagull is from sea level	The distance the giant squid is from sea level			
Sea level	The elevation of the dolphin			

Problems 2–3: This table shows a bakery owner's bank account transactions.

Date	Item	Amount
May 3	Electricity	-$294.50
May 3	Catering order	$240.55
May 5	Baking supplies	-$147.95
May 6	Catering order	$428.30

2. What does the number -147.95 represent in this context?

3. Did the bakery owner receive more money or spend more money on May 3? Explain your thinking.

4. Anushka thinks her freezer is broken, so she tracks the temperature every 2 hours. The table shows the temperature at different times of the day. At what time was her freezer's temperature closest to freezing level (0°C)?

Time	Temperature (°C)
9 AM	0.4
11 AM	-0.3
1 PM	0.1
3 PM	-0.7

A. 9 AM

B. 11 AM

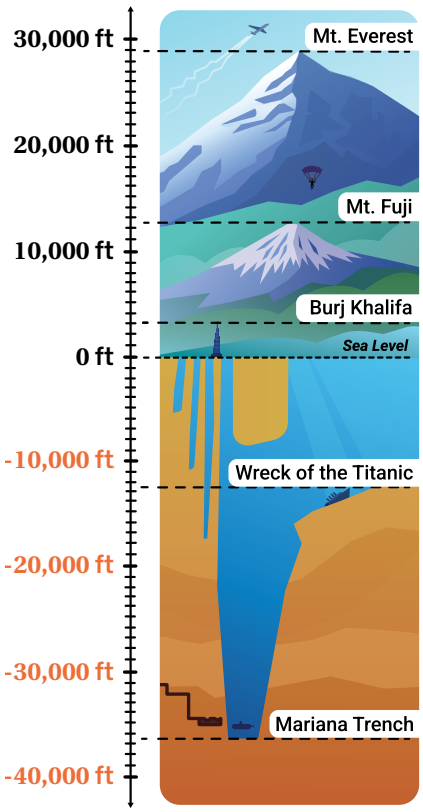
C. 1 PM

D. 3 PM

Practice
7.06

Name: ..... Date: ..... Period: .....

Problems 5–6: Here is a number line.



5. What is the approximate elevation of each location and its distance from sea level?

Location	Elevation (ft)	Distance from Sea Level (ft)
Mt. Everest		
Mt. Fuji		
Burj Khalifa		
Wreck of the Titanic		
Trench in Mariana Trench		

6. Select the two locations that are the same distance from sea level.

7. What does the number -36,500 represent in this context?

Spiral Review

Problems 8–13: Use the symbols <, >, or = to make true comparisons.

8. -32 ..... |15|

9. |-32| ..... |15|

10. 32 ..... |-15|

11. 5 ..... -5

12. 5 ..... |-5|

13. -5 ..... |-5|

Name: ..... Date: ..... Period: .....

Distance and Direction 6.NS.5, 6.NS.6, 6.NS.6.a, 6.NS.6.c, 6.NS.7, 6.NS.7.a, 6.NS.7.b, 6.NS.7.c, SMP.3, SMP.6

Practice Day 1



Let's practice what you have learned so far in this unit!

You will use problem cards for this Practice Day. Record all of your responses here.

1.



Left Right

2. a



Least Greatest

b

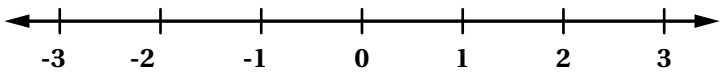
3. a Circle one: Always Sometimes Never

Explanation:

b Circle one: Always Sometimes Never

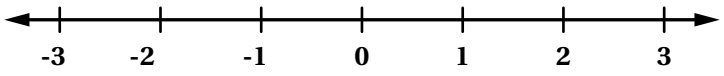
Explanation:

4.



5. a Absolute value is .....

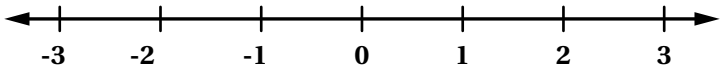
b



Name: _____ Date: _____ Period: _____

Practice Day 1

6.



7. **a**

Point	Fraction	Decimal
P		
Q		

b P _____ Q

8. **a**

Point	Fraction	Decimal
J		
K		

b J _____ K

9. **a** -3.5 _____ 2

b 6 _____ -6

c -12.25 _____ -13

d -1.25 _____ $-\frac{5}{4}$

10. **a** |-4.25| _____ |4.25|

b |-5| _____ -5

c |-5| _____ 4.5

d |-1.45| _____ |-1.5|

11.

Coldest Warmest

12.

Lowest Highest

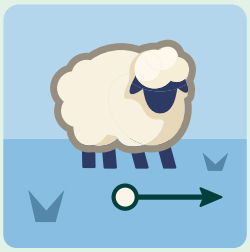
Sub-Unit

2

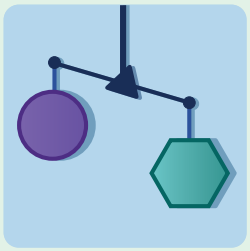
Inequalities



Lesson 7
Tunnel Travels



Lesson 8
Shira's Solutions



Lesson 9
Comparing Weights

Name: ..... Date: ..... Period: .....

Number Line Understanding 6.EE.8, SMP.2, SMP.3, SMP.8

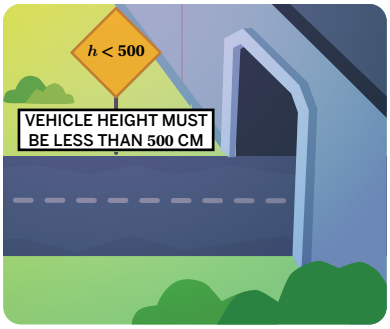


Tunnel Travels

Let's explore inequalities using words, symbols, and a number line.

Warm-Up

1 Select *all* of the vehicles that can fit in this tunnel.



☐ A.



☐ B.



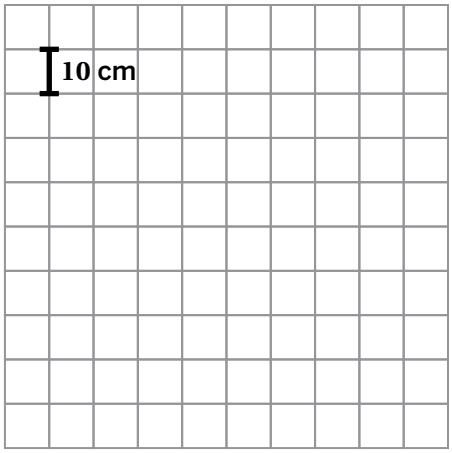
☐ C.



☐ D.



2 Sketch a vehicle that will fit in the tunnel.

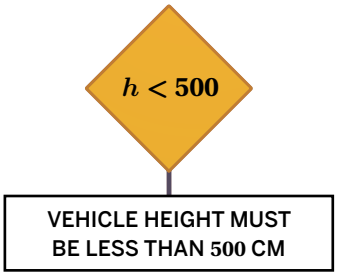


Name: ..... Date: ..... Period: .....

Inequalities in Context

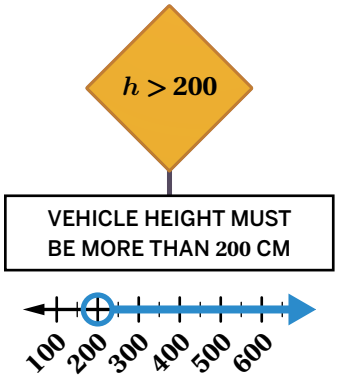
3 Here are the heights of several vehicles that fit in the tunnel.

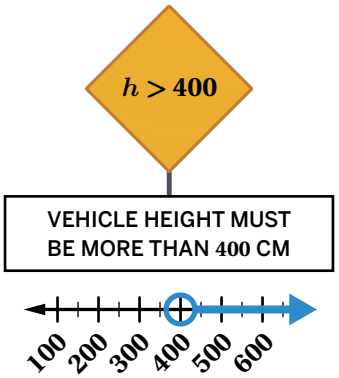


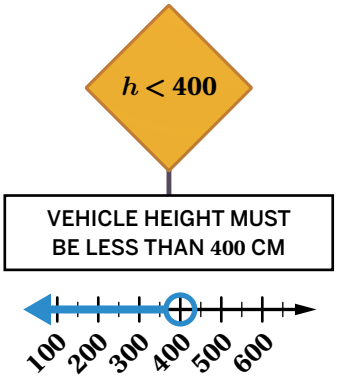


What do you think a graph of *all* the vehicle heights that fit would look like?

4 Here are three signs with inequalities and their number line graphs.



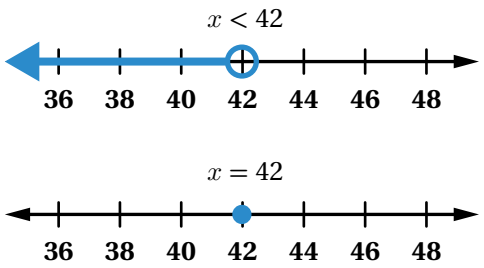




Discuss: What do you notice? What do you wonder?

5 Here are the graphs for $x < 42$ and $x = 42$.

Discuss: How are the graphs alike? How are they different?

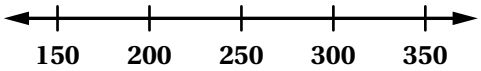


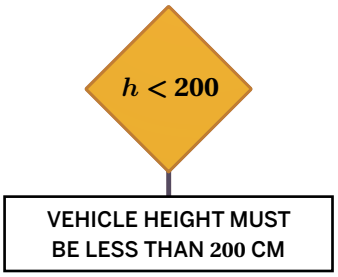
Name: ..... Date: ..... Period: .....

Inequalities in Context (continued)

6 Norma Merrick Sklarek was the first Black woman to become a licensed architect in California. In 1975, she designed the Pacific Design Center in Los Angeles, California. The parking garage at the Center can fit vehicles that are less than 200 centimeters tall.

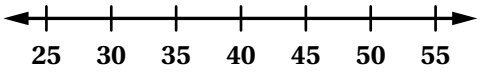
Graph all the possible vehicle heights that fit in this parking garage.

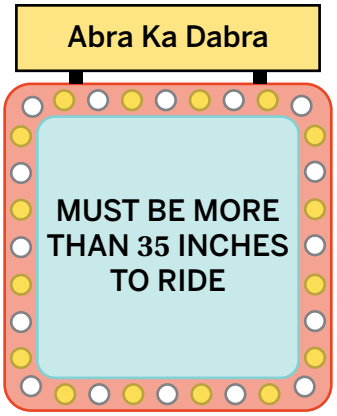




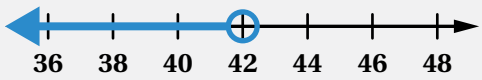
7 Fri Forjindam is co-owner and chief development officer of a company that develops theme parks. In 2016, she designed Bollywood Parks in Dubai. One ride at that park, Abra Ka Dabra, only allows passengers who are taller than 35 inches.

Graph all the possible heights for this ride.





8 Group the choices that represent the same situation. One choice will have no match.

A. 

B. 

C. You must be over 42 inches tall to ride The Whipper.

D. You must be under 42 inches tall to ride the kiddie swings.

E. You must be at least 42 inches to ride the roller coaster.

F. $x < 42$

G. $x > 42$

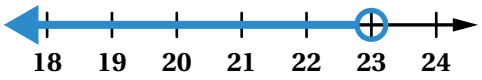
Group 1	Group 2

Activity 2

Name: Date: Period:

Inequalities Out of Context

9 Rewrite the inequality $x > 23$ so that it matches the graph.



10 To represent this graph:



- Martina wrote the inequality $20 < x$.
- Nasir wrote the inequality $x < 20$.

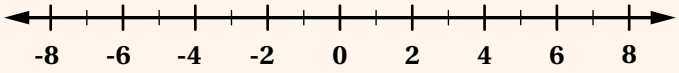
Whose inequality is correct? Circle one.

Martina's Nasir's Both Neither

Explain your thinking.

You're invited to explore more.

11 Here is a number line.



a Determine three possible values for x if $|x| < 5$.

b Plot these values on the number line.

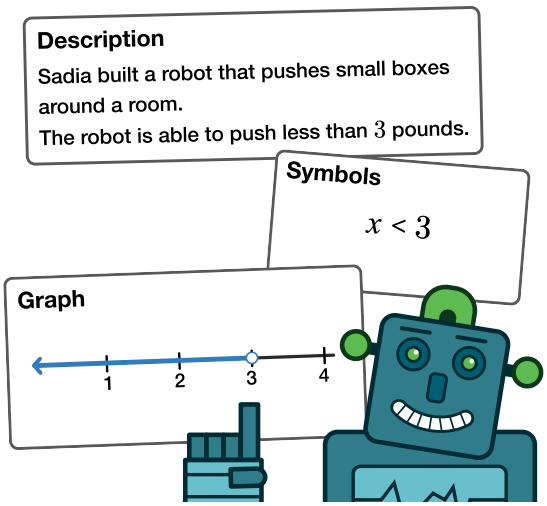
c Plot as many other possible values for x as you can.

12 Synthesis

Circle one representation and explain how it shows that Sadia's robot can push a 2-pound box.

Description Symbols Graph

Explain your thinking.

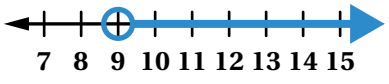


15 Summary 7.07

You can use variables, verbal descriptions, symbols, and number lines to represent inequalities related to real-world situations.

To represent an inequality on a number line, you can shade part of the number line to indicate that every point covered by the shaded region is part of the solution. This means every rational number in the shaded region is a solution. Then you can draw an arrow on one end of the number line to show the possible solutions continue on in that direction.

Here is an example.

Situation	Verbal Description	Inequality	Number Line
A two-year-old sleeps more than 9 hours a day.	Any value greater than 9.	$x > 9$	

Practice
7.07

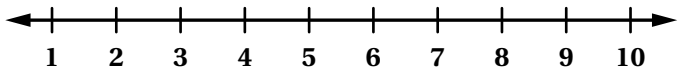
Name: ..... Date: ..... Period: .....

Problems 1–3: At a book sale, all books cost less than $\$5$.

1. List *three* possible prices for a book at this book sale.

2. Write an inequality to represent the cost of a book at the book sale, b.

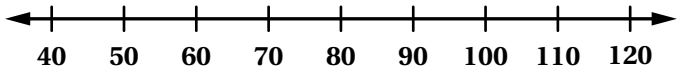
3. Graph all the possible prices of books at the sale.



Problems 4–7: n represents the number of eggs a sea turtle lays.

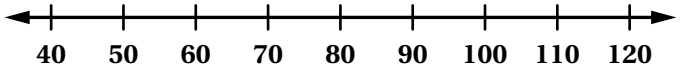
4. What does $50 < n$ mean in this situation?

5. Graph this inequality.



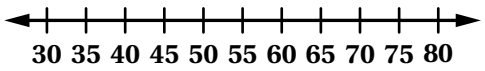
6. What does $n < 110$ mean in this situation?

7. Graph this inequality.

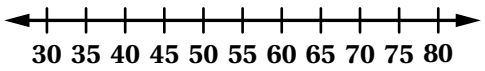


Problems 8–9: One day in Boston, the temperature was above 52°F and below 60°F.

8. Make two graphs, one to represent temperatures above 52°F and another to represent temperatures below 60°F.



Temperatures above 52°F



Temperatures below 60°F

9. Write two inequalities to represent the possible temperatures, T, on that day.

Practice
7.07

Name: _____ Date: _____ Period: _____

Spiral Review

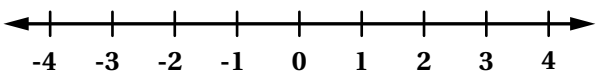
10. Select *all* of the temperatures that are warmer than -10°F.

☐ **A.** 10°F ☐ **B.** 0°F ☐ **C.** -5°F ☐ **D.** -11°F ☐ **E.** -20°F

11. Anika wrote the number sentence $|-5| = |5|$. Explain what it means in your own words.

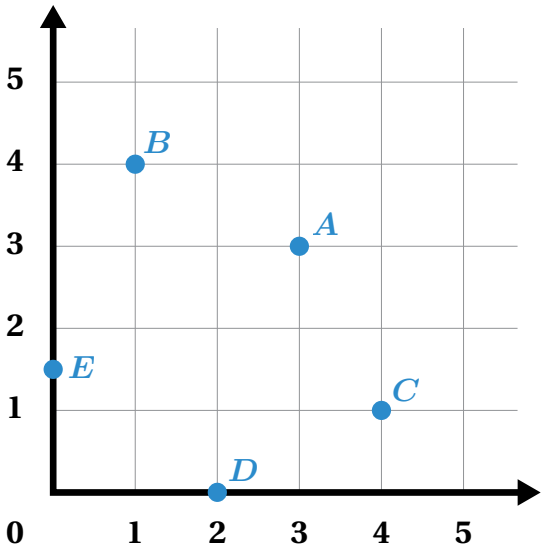
12. Complete each number sentence with the symbol <, >, or =. Use the number line if it helps with your thinking.

a $|-3|$ _____ -3 **b** 1 _____ $|-4|$ **c** 0 _____ $|-2|$ **d** $|0|$ _____ 0



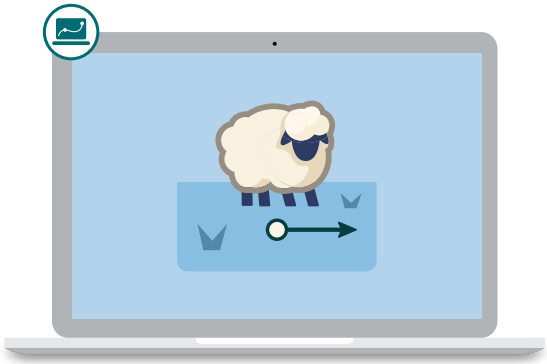
13. Write the coordinates of each point shown on the graph.

Point	Coordinates
A	
B	
C	
D	
E	



Name: ..... Date: ..... Period: .....

Generalizing With Multiple Representations | 6.EE.5, 6.EE.6, 6.EE.8, SMP.3, SMP.6



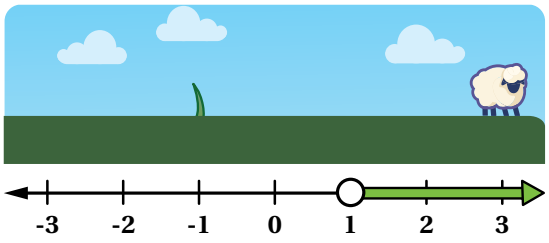
Shira's Solutions

Let's find solutions to an inequality using a number line.

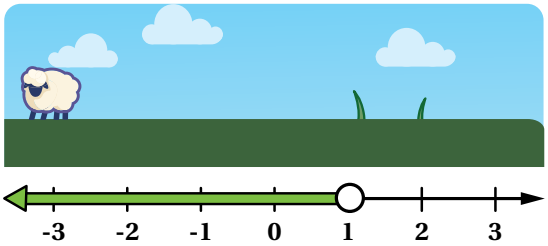
Warm-Up

1 Shira the Sheep loves eating all the blades of grass. These graphs show what happens when Shira eats grass based on different inequalities.

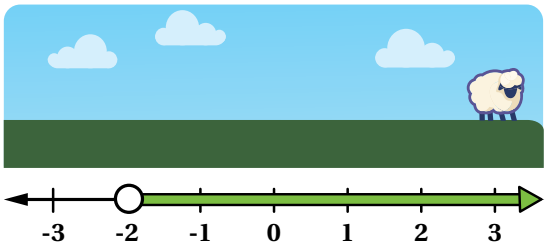
$x > 1$



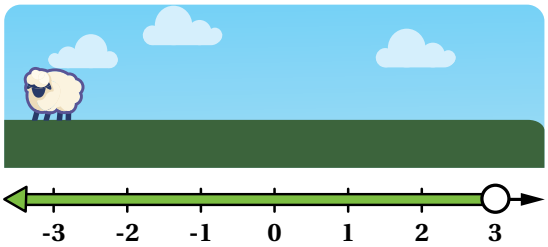
$x < 1$



$x > -2$



$x < 3$



Discuss: What do you notice?

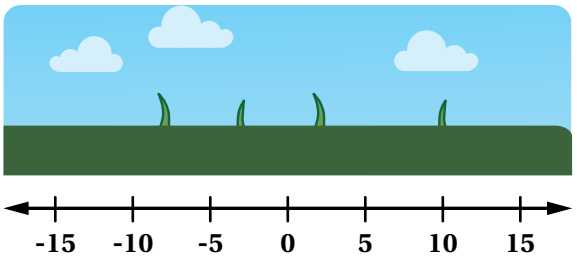
Name: ..... Date: ..... Period: .....

Connecting Graphs and Inequalities

2 Shira wants to eat these four blades of grass.

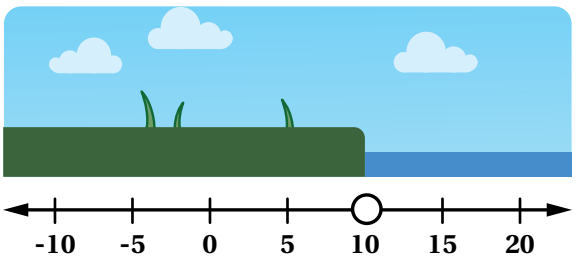
The inequality $11 < x$ did not work.

Fix this inequality to help Shira eat all the grass.



3 Shira the Sheep loves eating grass. She does not like water.

Write an inequality to help Shira the Sheep eat all the grass without falling in the water.



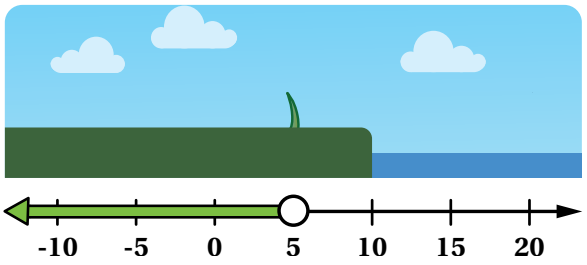
Explain your thinking.

Name: Date: Period:

Connecting Graphs and Inequalities
(continued)

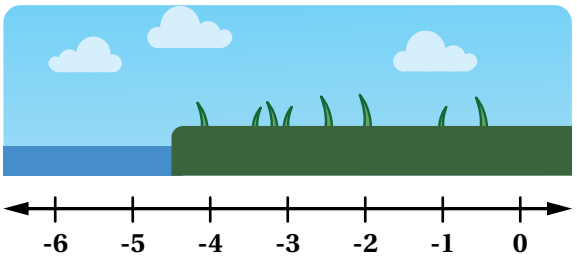
4 Kiana wrote $x < 5$ and was surprised that there was one blade of grass remaining.

Explain why 5 is not a **solution to the inequality** $x < 5$.

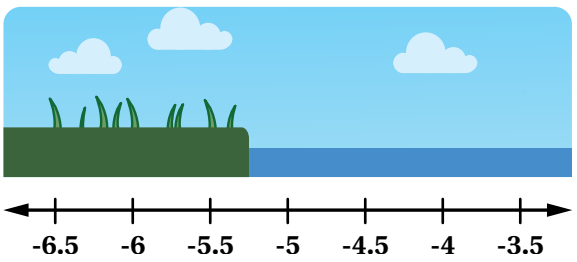


5 Write an inequality so that all the blades of grass are solutions and none of the water is.

Use the number line if it helps with your thinking.



6 Write an inequality so that all the blades of grass are solutions and none of the water is.

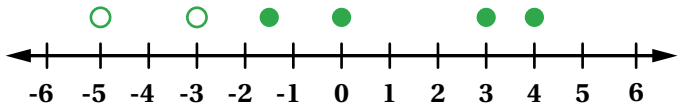


Activity 2

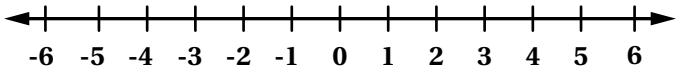
Name: ..... Date: ..... Period: .....

Solutions to an Inequality

7 Write an inequality so that all of the solid points are solutions and none of the open points are.



8 Write at least three solutions to the inequality $2.7 > x$.



Plot the solutions on the number line.

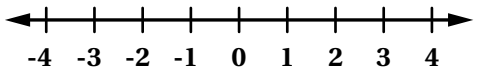
9 Let's look at other solutions to $2.7 > x$.

How many solutions does this inequality have? Explain your thinking.

Name: Date: Period:

Solutions to an Inequality (continued)

10 Make a graph of *all* the solutions to the inequality $x > -1.5$.



11 Match each inequality or solution with the graph that represents it.

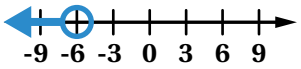
A. Some solutions: 5500, 6.5, -3

B. Some solutions: -100, 0.5, -6

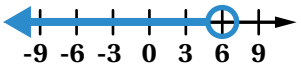
C. $x < -6$

D. $-6 > x$

E. $x < 6$

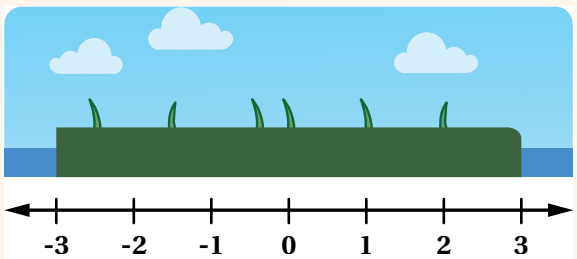






You're invited to explore more.

12 Write inequalities or describe how you can use inequalities to help Shira the Sheep eat all the blades of grass without falling in the water.

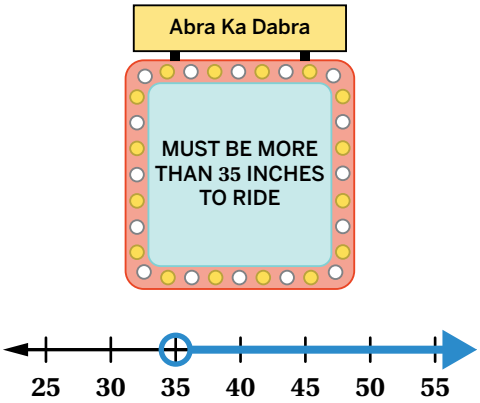


Name: ..... Date: ..... Period: .....

Revisiting Inequalities in Context

13 The sign and graph both show how tall passengers had to be to ride the Abra Ka Dabra ride at Bollywood Parks which opened in 2016.

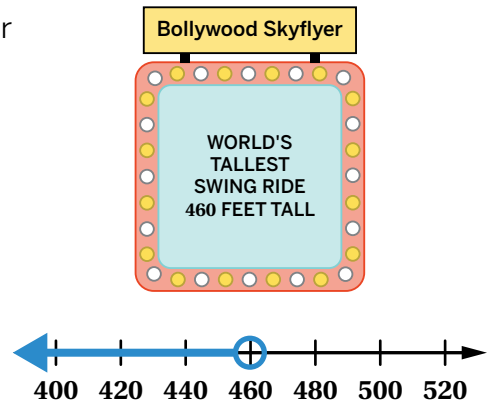
What is the shortest possible height you can be to ride this ride? What is the tallest possible height?



14 Another ride at Bollywood Parks is the Bollywood Skyflyer which opened in 2021. At 460 feet tall, it was once the tallest swing ride in the world.

Ivory created the graph shown to represent the possible heights from the ground of a person on this ride.

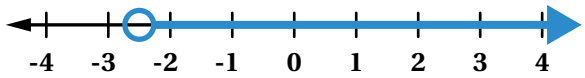
Do you think this graph is a good match? Explain your thinking.



15 Synthesis

Discuss: What does it mean for a number to be a solution to an inequality?

$x > -2.5$



16 Summary 7.08

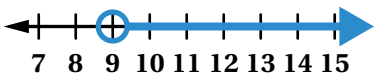
You can use a number line to represent the **solutions to an inequality**.

For example, for the inequality $c < 10$, you could say:

- 5 is a solution because $5 < 10$ is a true statement.
- 12 is not a solution because $12 < 10$ is not a true statement.

Some inequalities like $c < 10$ have an infinite number of solutions. We use inequality statements with variables and the symbols $<$ or $>$ to represent all the solutions.

Here is an example.

Inequality	Description	Possible Solutions	Number Line
$x > 9$	Any value greater than 9.	9.75, 10, 11.3, 82	

solutions to an inequality All values that make the inequality true.

Practice
7.08

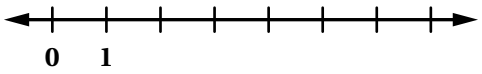
Name: ..... Date: ..... Period: .....

Problems 1–2: Here is the inequality $k > 5$.

1. Select *all* the values of k that are solutions to the inequality.

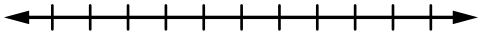
☐ **A.** 4.9 ☐ **B.** 5 ☐ **C.** 6 ☐ **D.** 5.2 ☐ **E.** -5.01

2. Make a graph of all the solutions to the inequality.

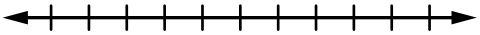


3. List three numbers that are solutions and three numbers that are not solutions to the inequality $x < -2.25$. Use the number line if it helps with your thinking.

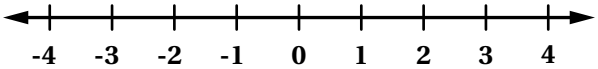
Solutions	Not Solutions



4. Write an inequality so that $-\frac{2}{3}$, -1.5, and -5 *are* solutions and 2, 0, and 100 *are not* solutions. Use the number line if it helps with your thinking.



5. Use the number line to create the graph of the inequality $x > -0.5$. Then describe the solution to the inequality.

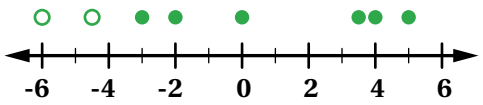


Practice
7.08

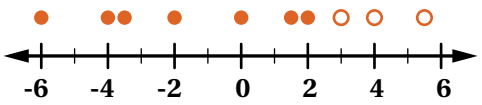
Name: _____ Date: _____ Period: _____

Problems 6–7: Write an inequality so that all of the solid points and none of the open points are solutions.

6.



7.



8. Use each number exactly once so that all of the statements are true.

-3 -2 -1 0 1 2 3

$x <$ ☐ **and** $x >$ ☐

Solutions to both inequalities:

☐ ☐ ☐

Not solutions to both inequalities:

☐ ☐

Spiral Review

Problems 9–12: Complete each number sentence with the symbol <, >, or =.

9. $\left|\frac{9}{20}\right|$ _____ -0.5

10. $|-0.5|$ _____ $\frac{9}{20}$

11. $\left|-\frac{9}{20}\right|$ _____ 0.5

12. $\left|-\frac{9}{20}\right|$ _____ $|-0.5|$

13. Different numbers of hot sauce bottles and their costs are given in the table.

What is the unit rate in dollars per bottle?

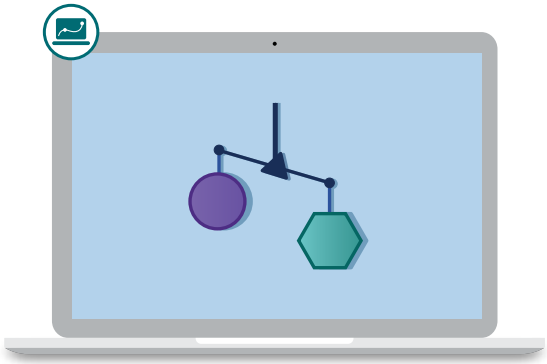
Number of Bottles	Cost
3	$10.62
5	$17.70
7	$24.78

Name: ..... Date: ..... Period: .....

Generalizing With Multiple Representations | 6.EE.6, 6.EE.8, SMP.2, SMP.3

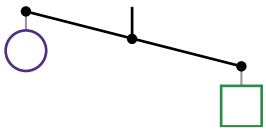
Comparing Weights

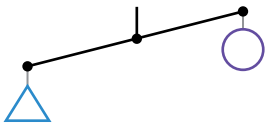
Let's write and interpret inequalities to describe unbalanced hangers.

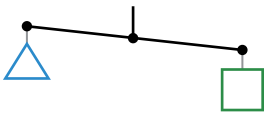


Warm-Up

1 Here are some hangers with different shapes.







Which shape is the heaviest? Circle one.

Circle Triangle Square

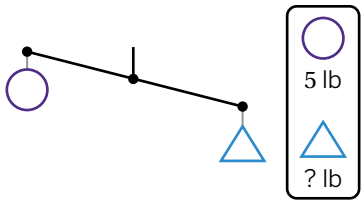
Explain your thinking.

Name: Date: Period:

One-Variable Inequalities

2 Here is an unbalanced hanger that Sora made. The circle weighs 5 pounds.

What is a possible weight for the triangle?



3 **a** Let's look at some hangers with a pentagon on one side and a square on the other.

b Describe all the possible weights that will make the square *heavier* than the pentagon.

4 Jasmine and Terrance wrote inequalities to describe the possible weights, s, that make the square heavier than the pentagon.

- Jasmine: $8 < s$
- Terrance: $s > 8$

Whose inequality is correct? Circle one.

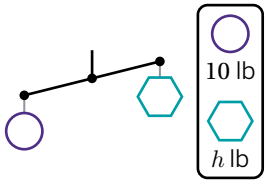
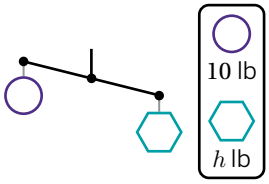
Jasmine's Terrance's Both Neither

Explain your thinking.

Name: Date: Period:

One-Variable Inequalities (continued)

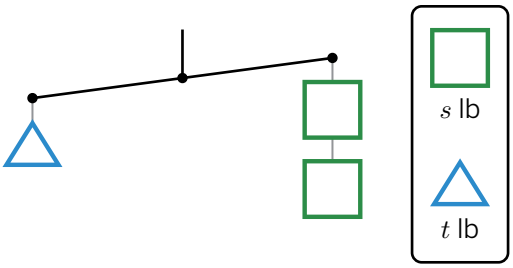
5 Match each representation with one of the unbalanced hangers by placing a checkmark in the appropriate column.

Representation		
10 pounds is less than the weight of a hexagon.		
One possible weight for this hexagon is 2 pounds.		
A hexagon weighs more than 10 pounds.		
10 pounds is greater than the weight of a hexagon.		
$10 > h$		
$10 < h$		
$h > 10$		

Name: Date: Period:

Many-Variable Inequalities

6 Which inequality represents this hanger?



A. $t < s$

B. $t < 2s$

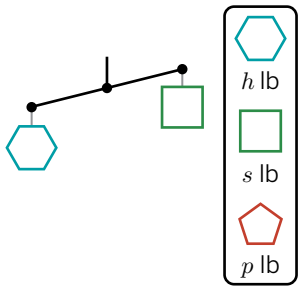
C. $t > 2s$

D. $t = 2s$

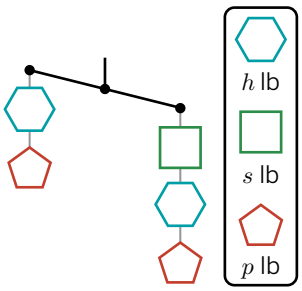
Explain your thinking.

7 Choose a hanger. Write an inequality that represents the hanger you chose.

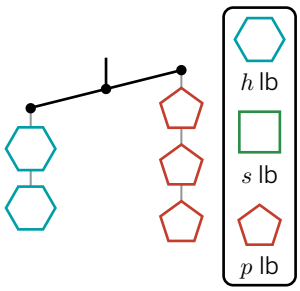
a



b



c



Activity 3

Name: ..... Date: ..... Period: .....

Challenge Creator

8 You will use the Activity 3 Sheet to create your own hanger challenge.

a. **Make It!** On the Activity 3 Sheet, create a hanger challenge.

b. **Solve It!** On this page, write an inequality that represents your hanger.

My Inequality

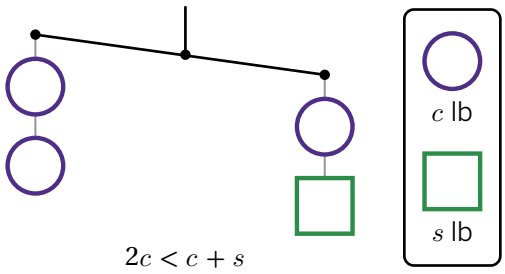
c. **Swap It!** Swap your challenge with one or more partners. Then write an inequality for your partners' hanger.

	Inequality
Partner 1	
Partner 2	
Partner 3	
Partner 4	

9 Synthesis

Describe how an inequality is like an unbalanced hanger.

Use the diagram if it helps with your thinking.

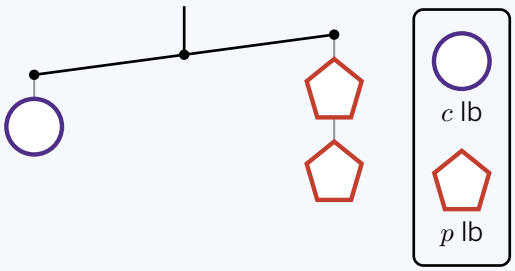


$2c < c + s$

12 Summary 7.09

You can represent inequalities with unbalanced hangers. Inequalities may include a variable as a placeholder for an unknown value.

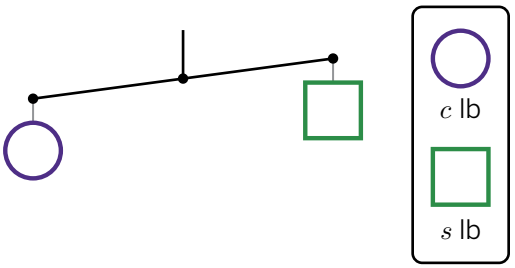
This hanger shows that the weight of the circle, c, is heavier than the weight of two pentagons, $2p$. This relationship can be represented by the inequality $c > 2p$ because these symbols mean that the circle has a greater weight than 2 pentagons.



Practice
7.09

Name: ..... Date: ..... Period: .....

Problems 1–4: Here is an unbalanced hanger.

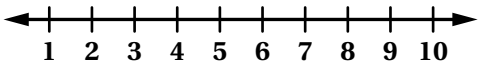


1. Write an inequality that compares the weights of the two shapes. Explain your inequality in words.

2. The square weighs 5 pounds. Rewrite your inequality to represent this.

3. List *three* possible weights for the circle. Use your inequality if it helps your thinking.

4. Graph all of the possible solutions for your inequality that represents the hanger.



Problems 5–7: There is leftover food that has been in Jin's refrigerator for d days.

5. What does the inequality $d < 7$ tell you about Jin's food?

6. What does the inequality $d > 0$ tell you about Jin's food?

7. List *three* possible values of d that make both $d < 7$ and $d > 0$ true.

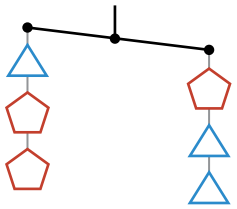
8. Gabriel is 12 years old. He has an older brother named Alejandro. Select *all* the inequalities that show the relationship between Gabriel's age, g, and Alejandro's age, a.

☐ **A.** $a < g$ ☐ **B.** $g < a$ ☐ **C.** $a > g$ ☐ **D.** $g > a$ ☐ **E.** $a > 12$

Name: ..... Date: ..... Period: .....

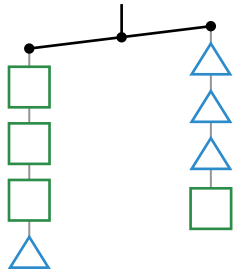
9. Write an inequality to represent Hanger 1. Use t to represent the weight of the triangle p to represent the weight of the pentagon.

Hanger 1



10. Write an inequality to represent Hanger 2. Use t to represent the weight of the triangle and s to represent the weight of the square.

Hanger 2



11. Which of the following is a solution to the inequality $3c > 12$?

A. $c = 1$ **B.** $c = 3$ **C.** $c = 4$ **D.** $c = 9$

Spiral Review

Problems 12–13: Angel's family is driving to their grandmother's house, which is 325 miles away.

12. What percent of the distance have they traveled after driving 26 miles?

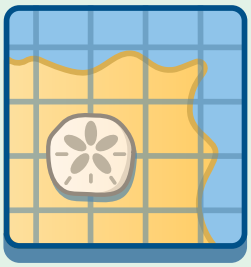
13. How far have they driven if they are 72% of the way to grandma's house?

Notes:

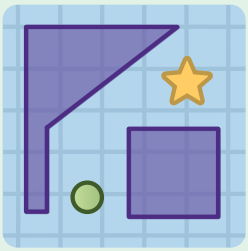
Sub-Unit

3

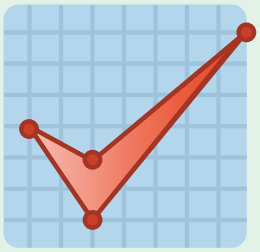
The Coordinate Plane



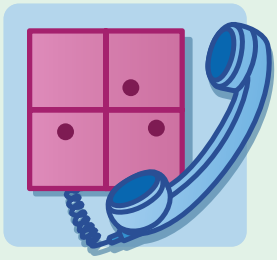
Lesson 10
Sand Dollar Search



Lesson 11
The A-maze-ing Coordinate Plane



Lesson 12
Polygon Maker



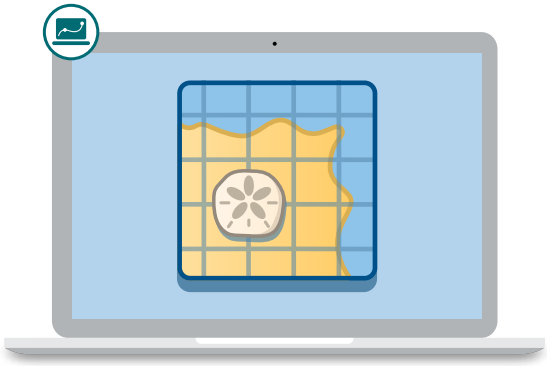
Lesson 13
Graph Telephone

Name: ..... Date: ..... Period: .....

Distance and Direction 6.NS.6, 6.NS.6.b, 6.NS.6.c, SMP.3, SMP.7

Sand Dollar Search

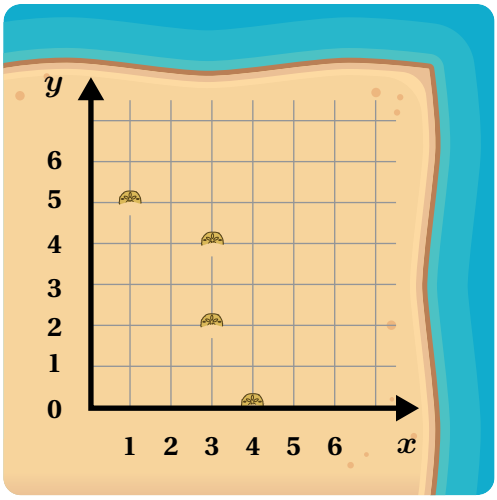
Let's explore negative numbers in the coordinate plane.



Warm-Up

1 Here is a map of part of an island.

Collect all four sand dollars by labeling each one with its coordinates.



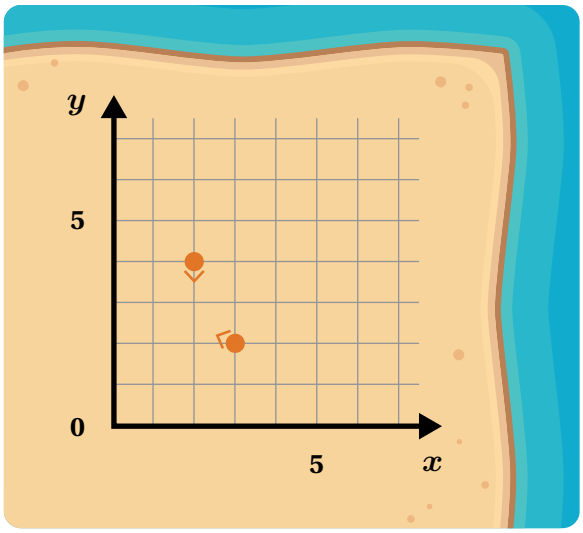
Name: Date: Period:

Find the Sand Dollar

2 Another sand dollar is buried on the island.

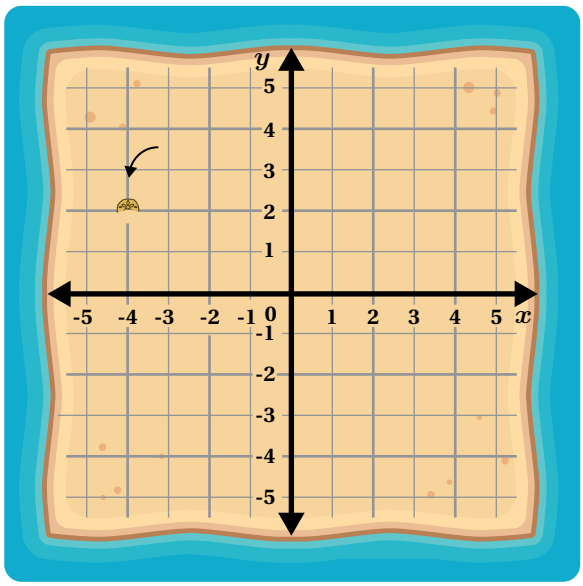
Here are two guesses for where the sand dollar is. Write coordinates for where you would search for it.

Coordinates (x, y)
$(3, 2)$
$(2, 4)$



3 You find a new map of the entire island.

A new sand dollar is visible. Write a clue that describes its location.



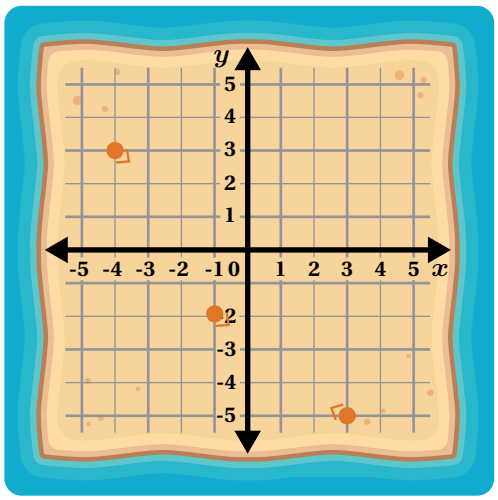
Name: Date: Period:

The Search Continues

4 There is another sand dollar buried somewhere on the island.

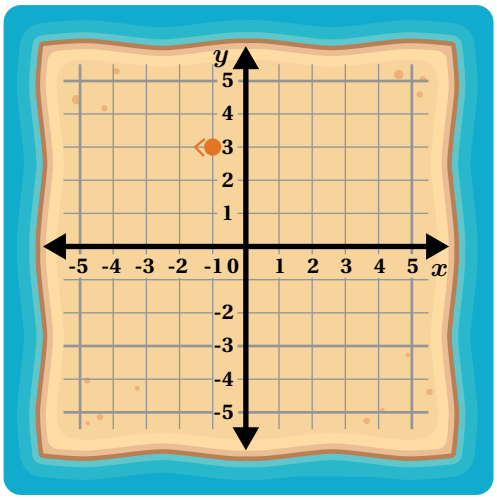
Here are three guesses for where the sand dollar is. Write coordinates for where you would search for it.

Coordinates (x, y)
(-4, 3)
(-1, -2)
(3, -5)



5 Esi was searching for another sand dollar. She entered (-1, 3) and the arrow pointed to the left.

What point would you recommend Esi try next? Explain your thinking.

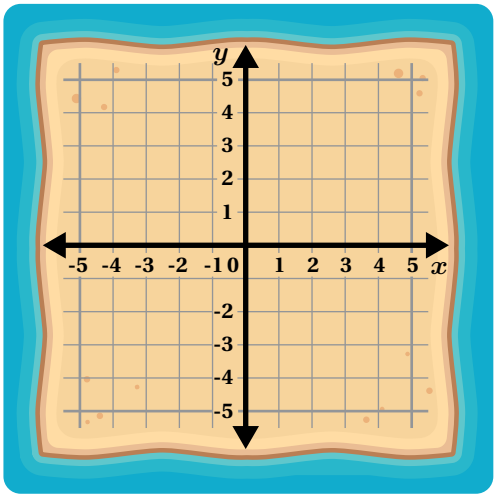


6 A sand dollar is hidden at a location that has two negative coordinates.

a Draw the sand dollar at a possible location.

Compare your point with a partner.

b **Discuss:** What do your points have in common?



Activity 3

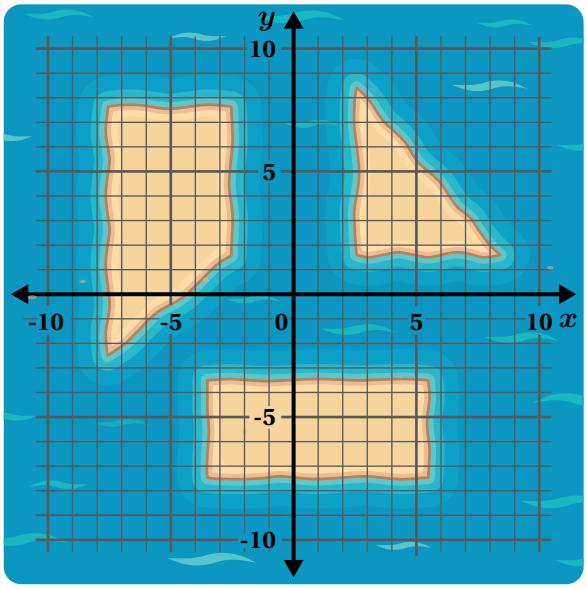
Name: ..... Date: ..... Period: .....

Find That Sand Dollar!

You will use support cards for this activity.

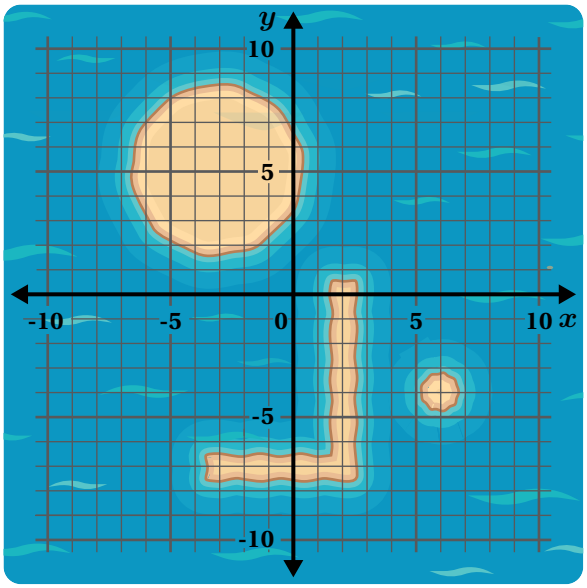
7 **Round 1:** There is a sand dollar on each of these islands.

When it is your turn to find the sand dollar, label each guess and the feedback you get until you find all three sand dollars.



8 **Round 2:** Here are three new islands.

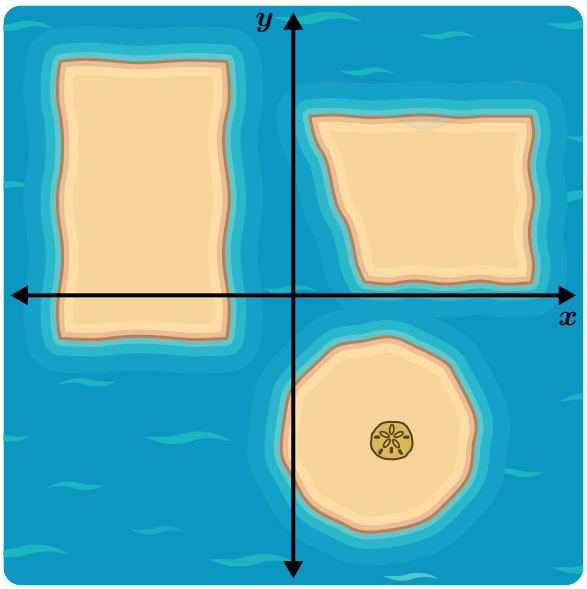
Label each guess and the feedback you get until you find all three sand dollars.



Name: ..... Date: ..... Period: .....

Find That Sand Dollar! (continued)

9 Which of these could be the location of the sand dollar?



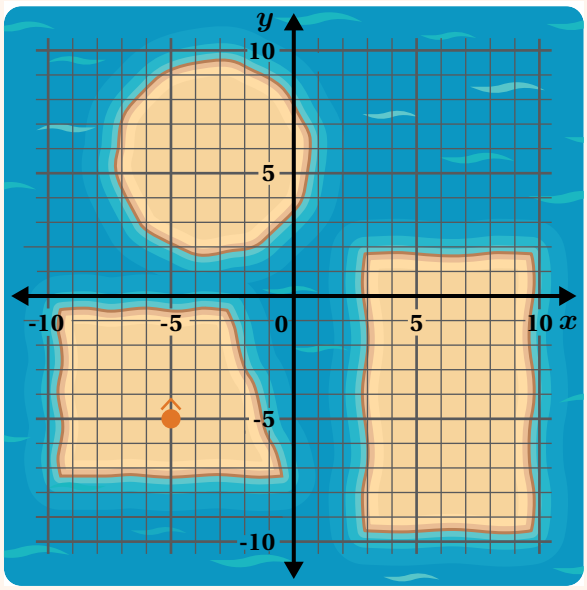
A. $(4, 6)$ **B.** $(4, -6)$

C. $(-6, -4)$ **D.** $(-6, 4)$

Explain your thinking.

You're invited to explore more.

10 Ariel was searching for a sand dollar and the arrow pointed up.



a What must be true about the sand dollar's x-coordinate?

A. $x = -5$

B. $x < -5$

C. $x > -5$

b What must be true about the sand dollar's y-coordinate?

A. $y = -5$

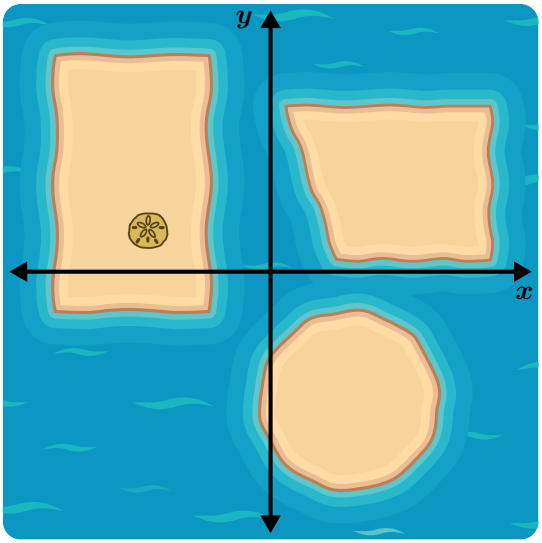
B. $y < -5$

C. $y > -5$

Explain your thinking.

11 Synthesis

Discuss: What do you know about the coordinates of this sand dollar?

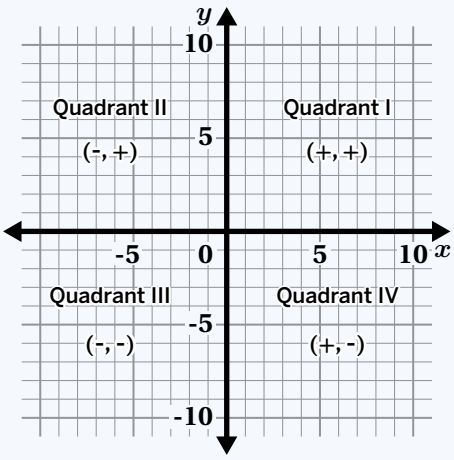


14 Summary 7.10

You can include positive and negative numbers along the x and y-axes, just like on a number line. The x- and y-axes cross at the origin, or the point $(0, 0)$. Coordinate pairs are written (x, y), where the x-value is the horizontal location (left and right) and the y-value is the vertical location (up and down).

The four regions of the coordinate plane are called **quadrants**. They are numbered 1–4 starting with the top right quadrant and going in a circle counter-clockwise.

The image shows each quadrant, along with the sign of the x- and y-values in that quadrant.

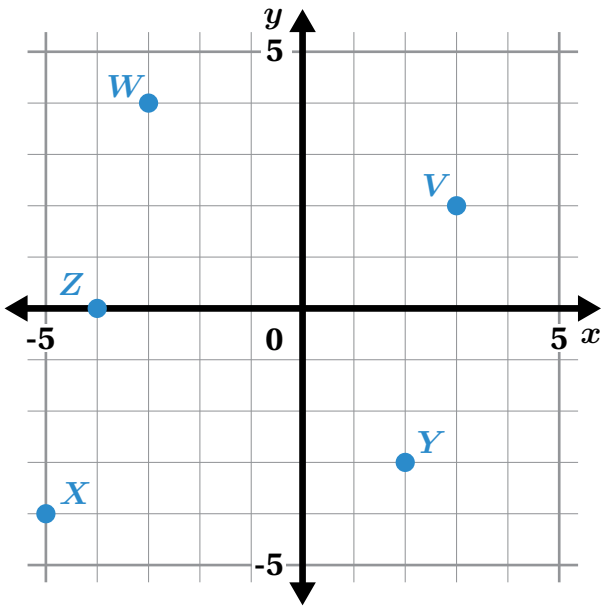


quadrant The coordinate plane is divided into 4 regions called quadrants.

Practice
7.10

Name: ..... Date: ..... Period: .....

Problems 1–6: Here is a coordinate plane.



1. Write the coordinates of each point in the table.

Point	Coordinates
V	
W	
X	
Y	
Z	

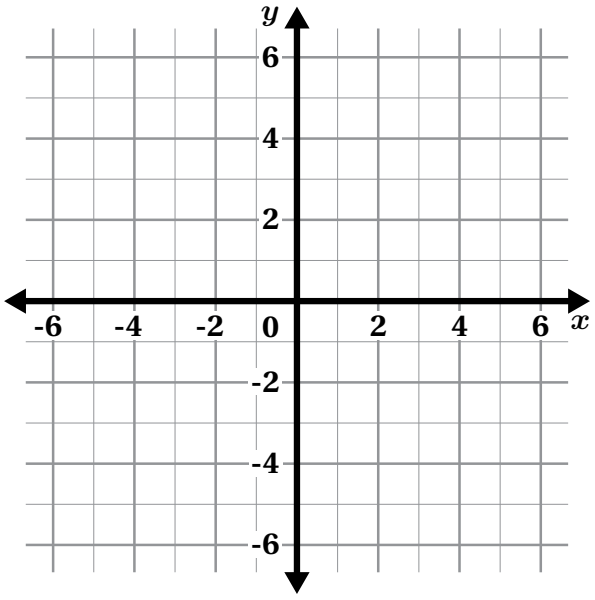
2. Plot and label point A at (-2, 1).

3. Plot and label point P at (1, -2).

4. Plot two points that are each 2 units away from point V. Label each point with its coordinates.

5. Point Q is more than 3 units directly to the left of point Y. Write at least one thing you know about the coordinates of point Q.

6. Draw the first letter of your name on the coordinate plane so that at least part of your drawing is in each quadrant.



Practice
7.10

Name: ..... Date: ..... Period: .....

Spiral Review

7. The height requirement for an amusement park ride is written as $h > 42$, where h represents a rider's height in inches. Write a sentence or sketch a sign that describes these rules as clearly as possible.

8. Select *all* the values of x that are solutions to the inequality $x > -2$.

- ☐ **A.** -1
- ☐ **B.** -2
- ☐ **C.** -3
- ☐ **D.** -2.1
- ☐ **E.** -1.8

Problems 9–14: Solve each equation.

9. $3a = 12$

10. $b + 3.3 = 8.9$

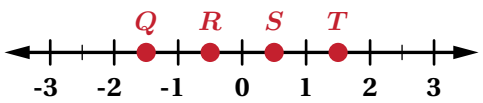
11. $1 = \frac{1}{4}c$

12. $2d = 6.4$

13. $4.5 = y - 0.7$

14. $\frac{2}{3} + z = 7$

15. Here is a number line with points Q, R, S, and T. Which point represents $-1\frac{1}{2}$?

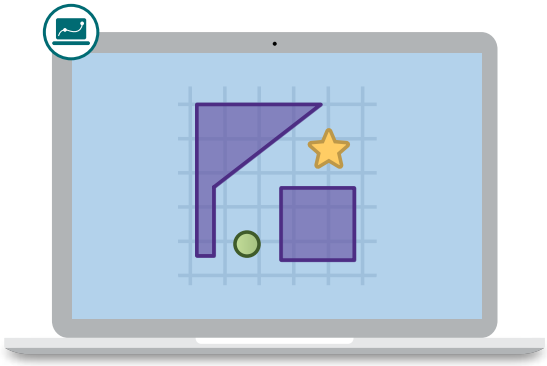


Name: ..... Date: ..... Period: .....

Relationships Between Variables 6.NS.6.b, 6.NS.6.c, 6.NS.8, SMP.3, SMP.6, SMP.7

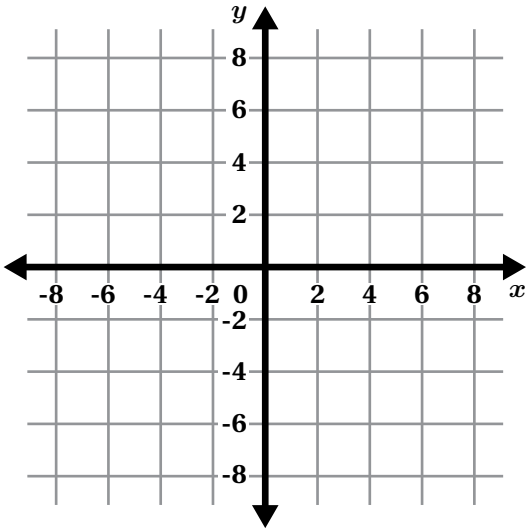
The A-maze-ing Coordinate Plane

Let's practice with the coordinate plane using mazes.



Warm-Up

1 a Plot a point at the coordinates $(3, -1)$.



b Explain how you chose where to plot the point.

Activity 1

Name: ..... Date: ..... Period: .....

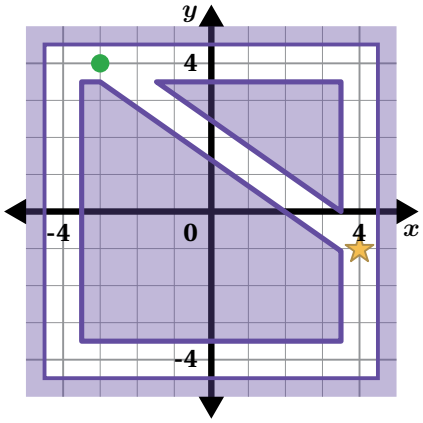
Maze Challenges

The goal of the maze is to move the ball to the star without hitting any walls.
For each challenge:

- Write the coordinates of a path to move the ball to the star. Your path can have several coordinate points.
- Pass your paper to a partner to draw the path using your coordinates.

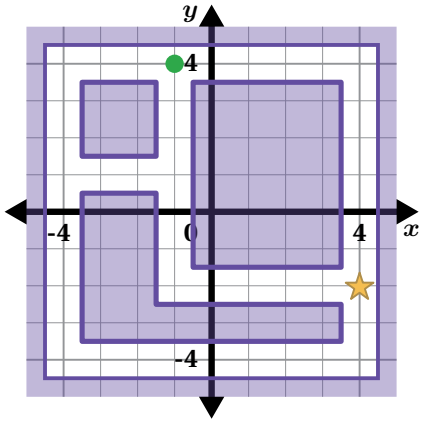
2 Let's watch the ball collect the star in this example. Finish writing the coordinates of the path.

Your Path
(-3, 4)



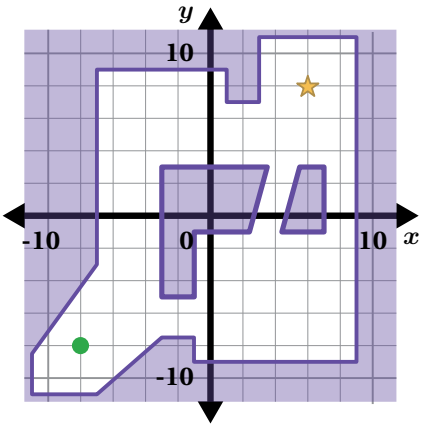
3 Write the coordinates of a path to move the ball to the star. Note: You do not need to use all the rows.

Your Path
(-1, 4)



4 Write the coordinates of a path to move the ball to the star.

Your Path
(-8, -8)



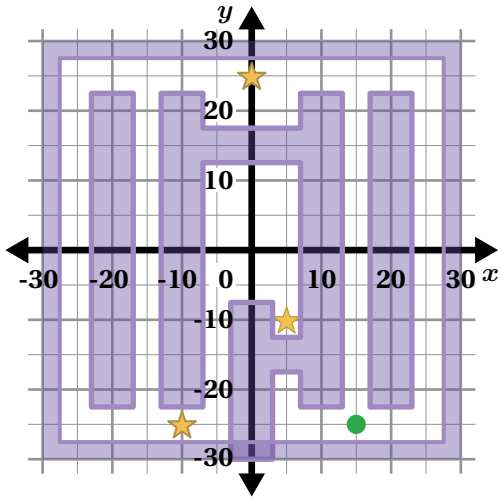
Name: ..... Date: ..... Period: .....

More Mazes

5 In this maze, there are multiple stars.

a Sketch a path to collect them all.

b Write the coordinates that create your path.



Your Path
(15, -25)

Name: Date: Period:

More Mazes (continued)

6 Felipe and Annika were working on the previous maze.

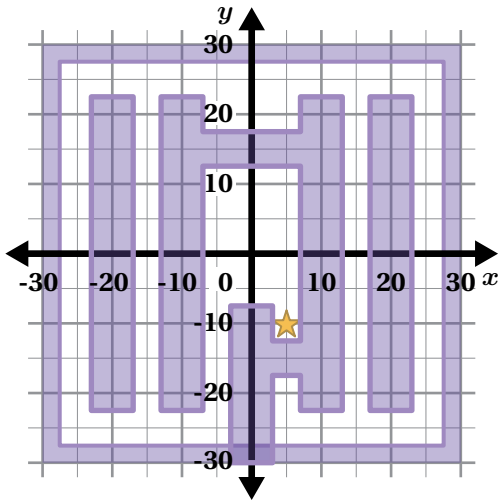
Felipe says: *The star's coordinates are* $(1, -2)$.

Annika says: *The coordinates are* $(1, -10)$.

Whose claim is correct? Circle one.

Felipe's Annika's Neither

Explain your thinking.



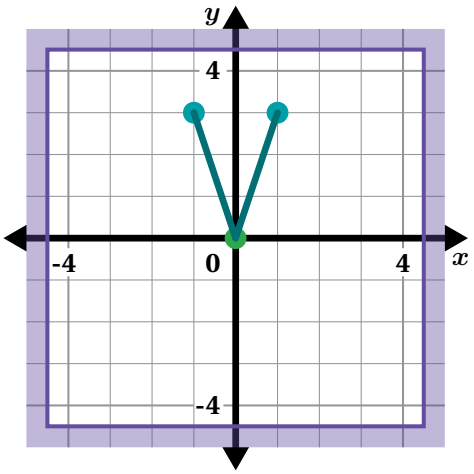
Name: ..... Date: ..... Period: .....

Reflections on the Plane

7 Let's look at a new way of collecting stars.

Discuss: What happens when we try the point $(1, 3)$?

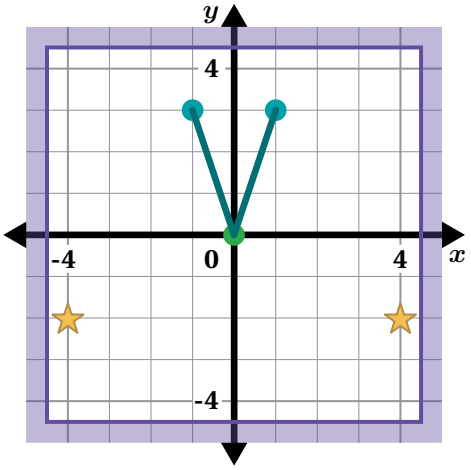
Your Path	Mirrored Path
$(0, 0)$	$(0, 0)$
$(1, 3)$	$(-1, 3)$



8 Here are two new stars!

a Complete the table to collect both stars.

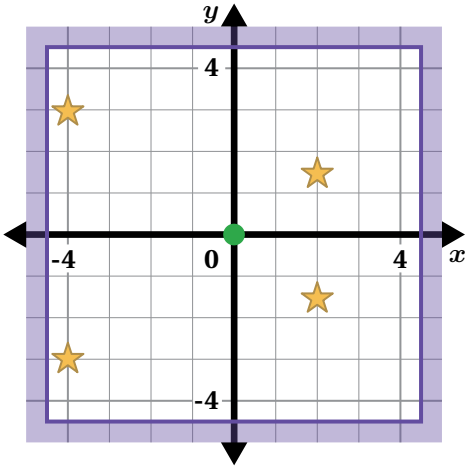
Your Path	Mirrored Path
$(0, 0)$	$(0, 0)$
$(1, 3)$	$(-1, 3)$



b **Discuss:** How are the coordinates of the two new stars alike? How are they different?

9 The ball is mirrored across the x-axis. Write coordinates to collect all the stars. Choose your two points wisely!

Your Path	Mirrored Path
$(0, 0)$	$(0, 0)$



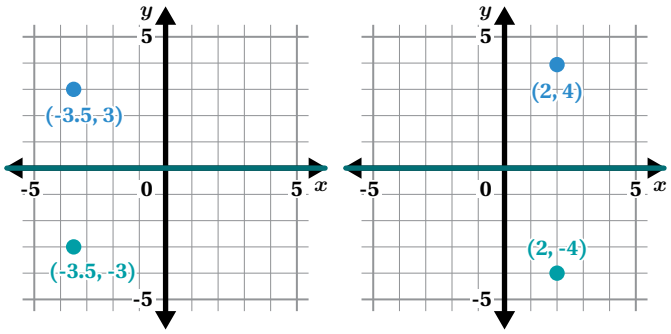
Name: ..... Date: ..... Period: .....

Reflections on the Plane (continued)

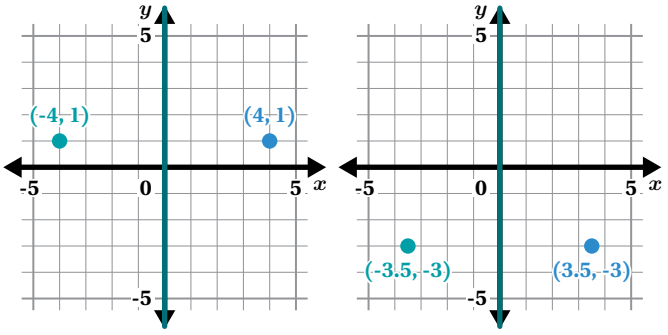
10 Here are several coordinates of a ball and its reflection.

Write at least two patterns about the coordinates that you found interesting.

Reflections Across the x-axis

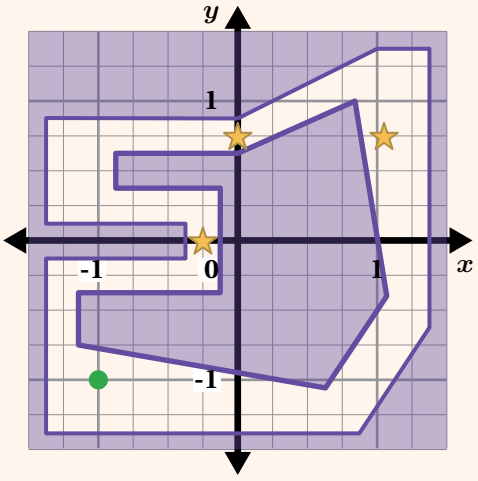


Reflections Across the y-axis



You're invited to explore more.

11 Sketch a path to collect all the stars. Then write the coordinates of your path.



12 Synthesis

Look back over the challenges and select the one you are most proud of solving.

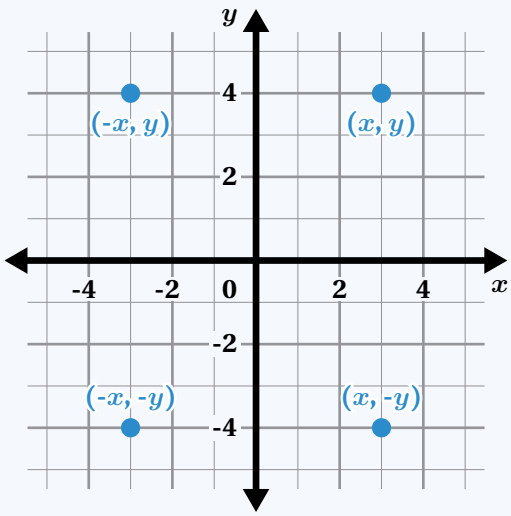
Write some advice for someone solving that challenge.

15 Summary 7.11

While the scale of 1 is often used, coordinate planes sometimes use different scales. Because of this, it is important to look at the axis scale before identifying points on the coordinate plane.

The points $(3, 4)$ and $(3, -4)$ have the same x-coordinate and the y-coordinates only differ by their sign. We can see on the graph that those points are a reflection, or a mirror, of each other across the x-axis.

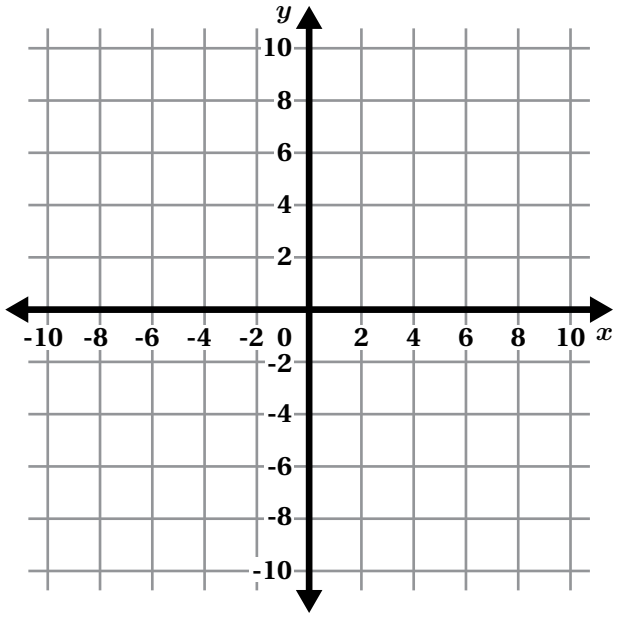
The points $(-3, -4)$ and $(3, -4)$ have the same y-coordinate and the x-coordinates only differ by their sign. We can see on the graph that those points are a reflection, or a mirror, of each other across the y-axis.



Practice
7.11

Name: ..... Date: ..... Period: .....

Problems 1–2: Here is a graph.

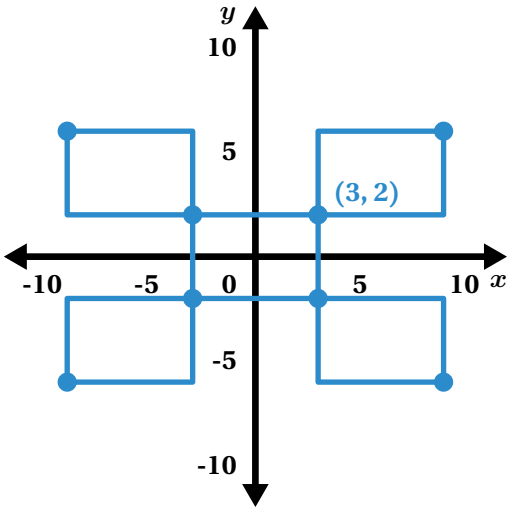


1. Plot and connect each point in order. Plot all of Column 1 first, then Column 2.

Column 1	Column 2
(-2, -9)	(-2, 4)
(-2, -3)	(0, 2)
(-4, -3)	(2, 4)
(-6, -1)	(2, -1)
(-6, 4)	(0, -3)
(-4, 2)	(-2, -3)

2. Describe your strategy for plotting the point (-2, -9) in Problem 1.

Problems 3–4: Here are identical rectangles on a coordinate plane. The origin is in the center of the middle rectangle.

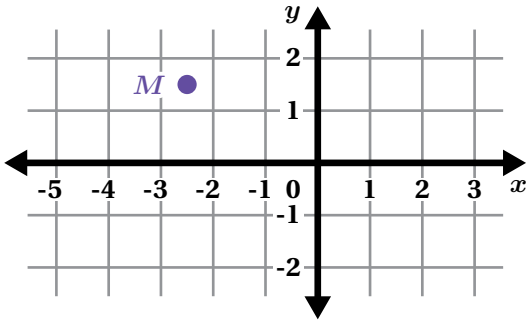


3. Label the coordinates of the remaining points.

4. What patterns do you notice?

Name: ..... Date: ..... Period: .....

5. What is the value of the x-coordinate of point M? Your answer should be a decimal rounded to the nearest 0.5.



6. Point A is located at $(3, n)$ in the coordinate plane. Point B is located at $(-3, n)$. What do you know about points A and B?

Spiral Review

Problems 7–10: Complete each number sentence with the symbol $<$, $>$, or $=$. Use a number line if it helps with your thinking.

7. $\left|-\frac{3}{2}\right| \square -\frac{2}{3}$

8. $-\frac{3}{2} \square -\frac{2}{3}$

9. $-\frac{3}{2} \square \frac{2}{3}$

10. $\frac{3}{2} \square \left|-\frac{2}{3}\right|$

Problems 11–13: DeShawn's dog weighs 34 pounds. Jacy's dog weighs 12 pounds more than DeShawn's dog.

11. Select *all* the equations that show the weight of Jacy's dog, j.

- ☐ **A.** $j = 34 + 12$
- ☐ **B.** $j = 34 - 12$
- ☐ **C.** $j + 12 = 34$
- ☐ **D.** $j - 12 = 34$
- ☐ **E.** $j = 34 \cdot 12$

12. Determine how much Jacy's dog weighs.

13. Karima adopts a dog that is twice the weight of DeShawn's dog. Write an equation to represent the weight of Karima's dog, k.

Name: ..... Date: ..... Period: .....

Nets and Surface Area | Graphing Shapes | CA CCSSM: 6.NS.8, 6.G.3, SMP.1, SMP.6, SMP.7



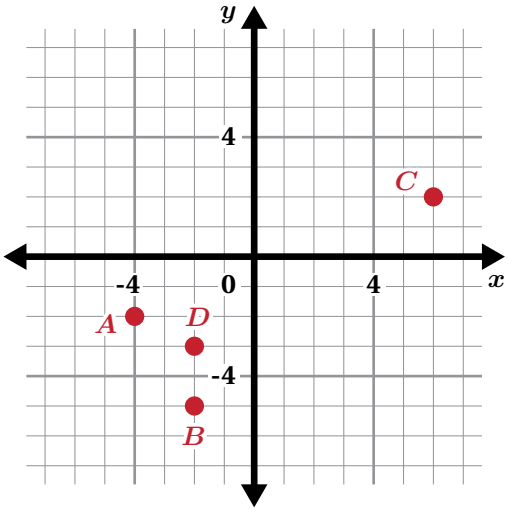
Polygon Maker

Let's explore polygons on the coordinate plane.

Warm-Up

1 Let's watch how a Polygon Maker works.

Point	Coordinates
A	(-4, -2)
B	(-2, -5)
C	(6, 2)
D	(-2, -3)



a Connect the coordinates to create a *polygon*. Tell a classmate what the polygon reminds you of.

b Change one coordinate and connect the points to make a new polygon.

Activity 1

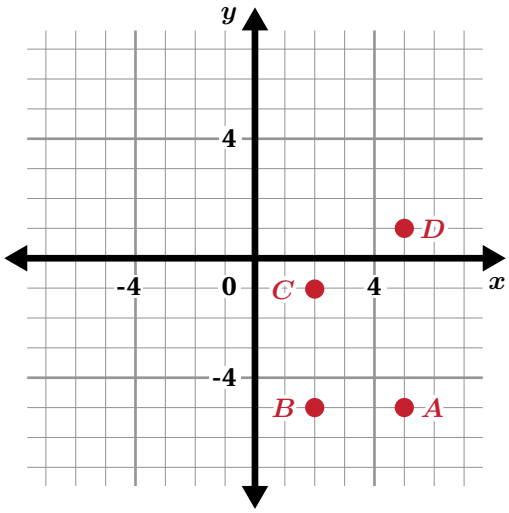
Name: ..... Date: ..... Period: .....

A Polygon

2 Melissa wants to make a rectangle.

Change some of her coordinates so that the Polygon Maker makes a rectangle.

Point	Coordinates
A	(5, -5)
B	(2, -5)
C	(2, -1)
D	(5, 1)



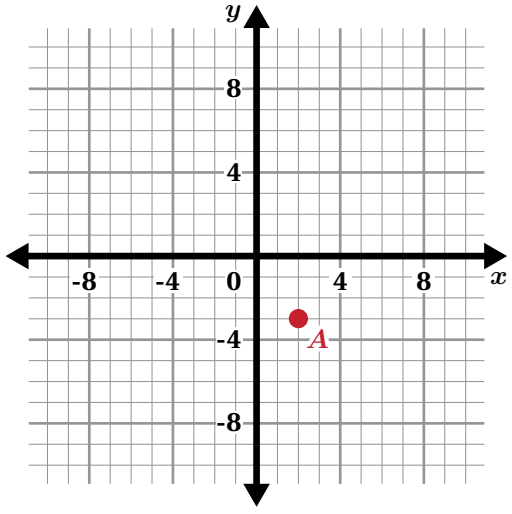
3 Calculate the perimeter of the rectangle you made.

Explain your thinking.

4 Hamza wants to make a square.

Complete the table with coordinates that will make a square. Sketch the square you made on the coordinate plane.

Point	Coordinates
A	(2, -3)
B	
C	
D	

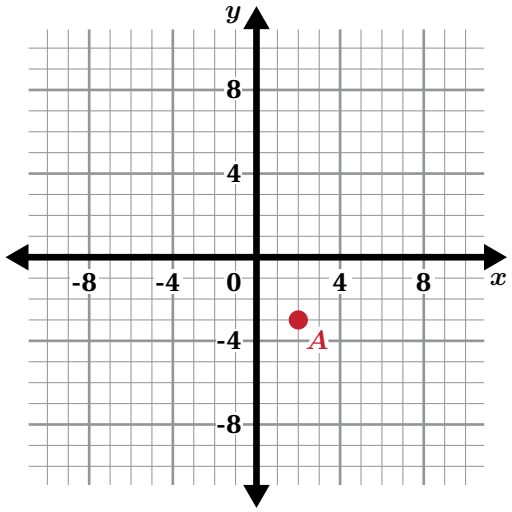


Name: ..... Date: ..... Period: .....

A Polygon (continued)

5 Here are Vicente's coordinates.

Point	Coordinates
A	(2, -3)
B	(2, 4)
C	(9, 4)
D	(9, -3)



Did he make a square? Circle one.

Yes No I'm not sure

Show or explain your thinking.

6 Vicente says: *My square has side lengths of 7.*

Sketch or describe how absolute value can be used to show 7 in the table and graph.

Point	Coordinates
A	(2, -3)
B	(2, 4)
C	(9, 4)
D	(9, -3)

Name: ..... Date: ..... Period: .....

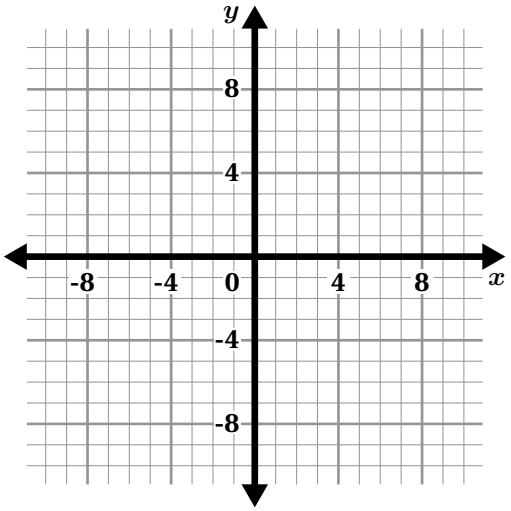
Make Your Own Polygon

7 Here are three descriptions:

- Looks like a house that's 15 units tall.
- Looks like a capital L that's 6 units wide.
- A rectangle with an area of 24 square units.

Choose one description. Write and sketch coordinates to make a polygon that matches that description.

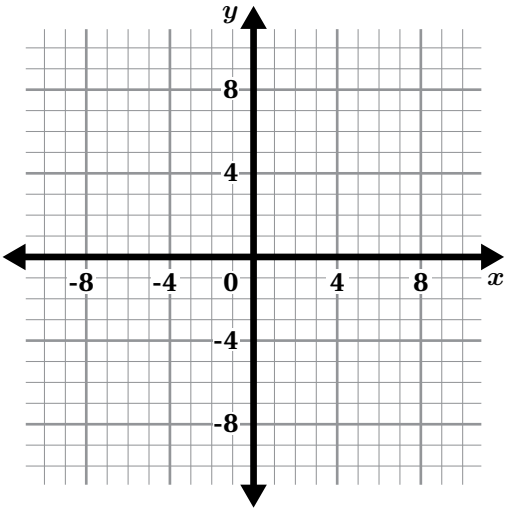
Point	Coordinates
A	
B	
C	
D	
E	



8 You will create your own polygon challenge.

- Write a description of a polygon on a slip of paper. Make it fun (like "looks like a pinecone") and include the size (like "must have a perimeter of 20 units").
- On this page, write the coordinates and sketch a polygon that matches your description.

Point	Coordinates
A	
B	
C	
D	



Name: ..... Date: ..... Period: .....

Make Your Own Polygon (continued)

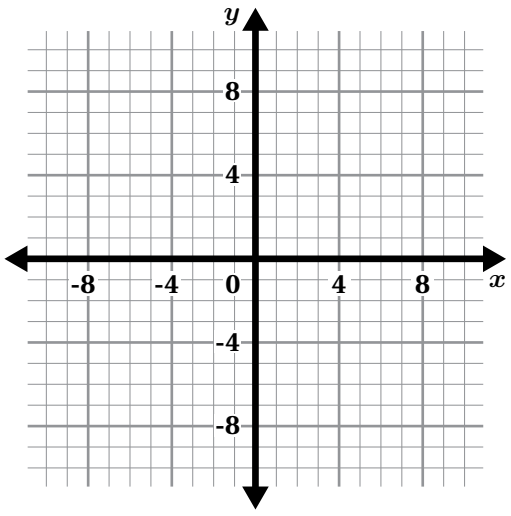
8 You will swap your polygon challenge with two classmates.

- Hand your slip of paper to a partner and invite them to use coordinates to create a polygon on their paper.
- When they finish, share your original polygon and compare.
- Write the coordinates and graph your partners' polygon challenges on this page.

Partner 1

Description:

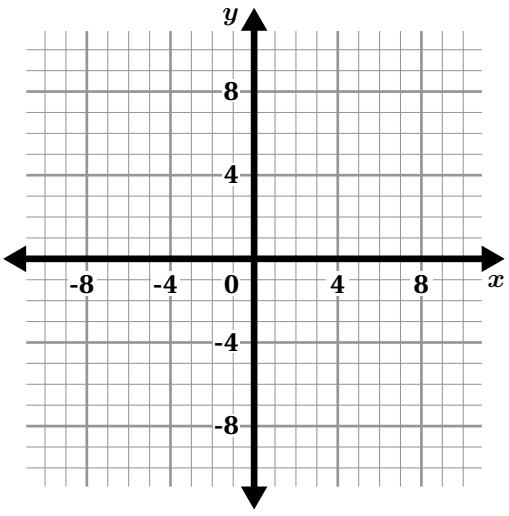
Point	Coordinates
A	
B	
C	
D	



Partner 2

Description:

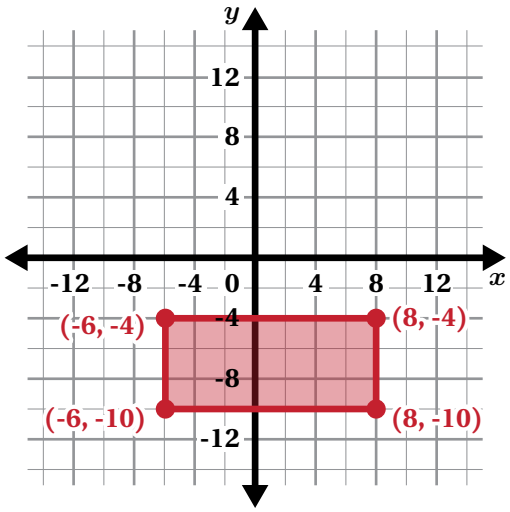
Point	Coordinates
A	
B	
C	
D	



9 Synthesis

Discuss: How can you use the coordinates to calculate the side lengths of a rectangle? Use the table and graph if they help with your explanation.

Point	Coordinates
A	(-6, -4)
B	(8, -4)
C	(8, -10)
D	(-6, -10)

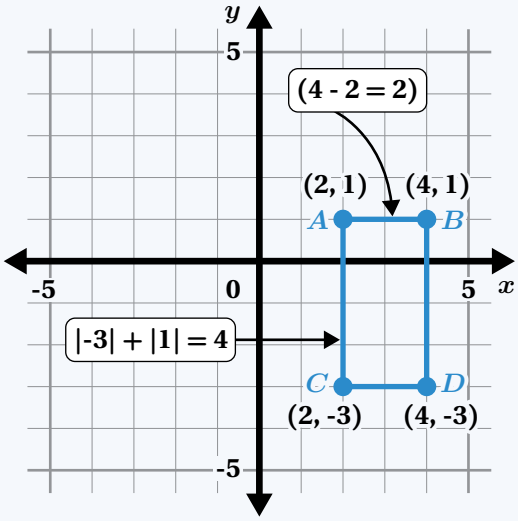


12 Summary 7.12

You can plot points on the coordinate plane to create *polygons*. When the vertices of a polygon are horizontally or vertically aligned in the graph, you can count the number of units between them to determine the length of that side.

You can also calculate the side lengths using the coordinates of each vertex. Here are two calculation strategies:

- If the coordinates are in the same quadrant, like points A and B, find the length by subtracting the coordinates that are different.
- If the coordinates are in different quadrants, like points A and C, use the absolute value to determine the distance each point is from the axis between them.



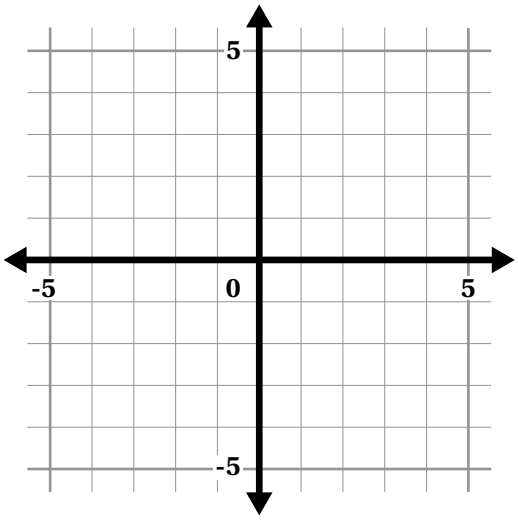
Practice
7.12

Name: ..... Date: ..... Period: .....

Problems 1–4: Here is a set of coordinates.

Point	Coordinates
A	(-3, 1)
B	(3, 1)
C	(3, -4)
D	(-1, -4)
E	(-1, -2)
F	(-3, -2)

1. Plot each point.



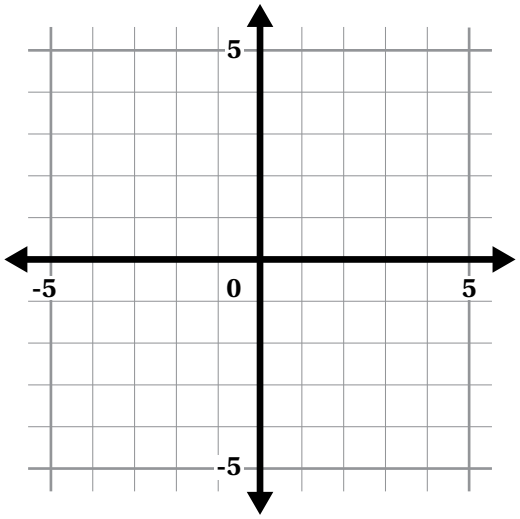
2. Connect the points to create polygon $ABCDEF$.

3. Determine the length of the segment between point A and point B.

4. Determine the perimeter of polygon $ABCDEF$.

Problems 5–7: Three points of a rectangle are (3, 0), (3, -5), and (-4, -5).

5. What are the coordinates of the missing point?



6. Sketch the rectangle and calculate its perimeter.

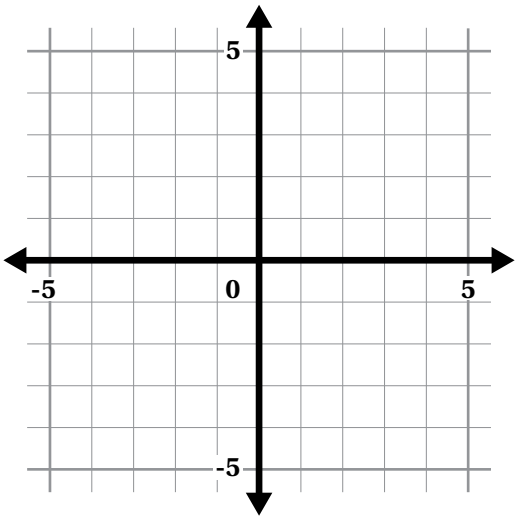
7. Calculate the area of the rectangle.

Name: _____ Date: _____ Period: _____

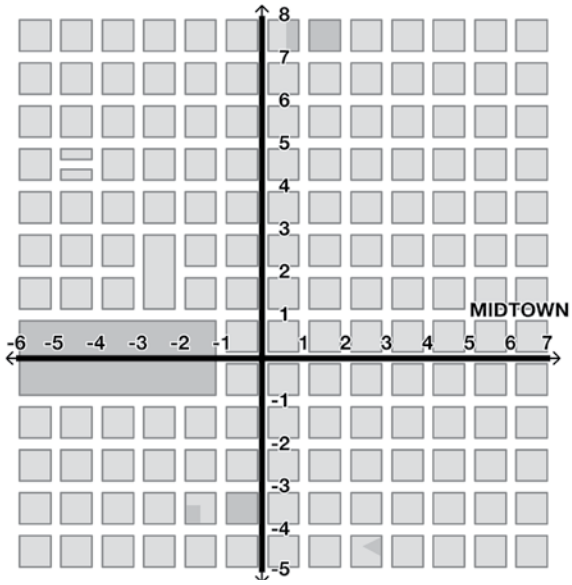
8. Gabriela drew a quadrilateral with these descriptions:

- It has exactly one pair of parallel sides.
- Two of the vertices are $(-4, 5)$ and $(2, -3)$.
- At least one side has a length of 5 units.

Create a quadrilateral that Gabriela could have made.



Problems 9–10: Here is a map of Ava's town.



9. Plot each of these locations on the map:

Library $(-1, 4)$
Coffee shop $(5, -3)$
School $(5, 4)$
Home $(-1, -3)$

10. If Ava leaves home and takes the shortest path traveling to the library, then school, then the coffee shop, and then home again, how many blocks will she travel?

Spiral Review

11. At top speed, an elephant can run 25 miles per hour and a giraffe can run 16 miles in $\frac{1}{2}$ hour. Which animal runs faster?

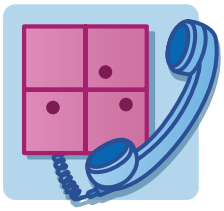
12. The bottle-nose dolphin can swim up to 11 miles in $\frac{1}{2}$ hour. The sei whale can swim up to 68 miles in 2 hours. Which sea animal can swim the fastest?

Name: ..... Date: ..... Period: .....

Relationship Between Variables | Model the World | Distance and Direction

6.NS.8, 6.NS.6.c, SMP.1, SMP.2, SMP.6

Graph Telephone



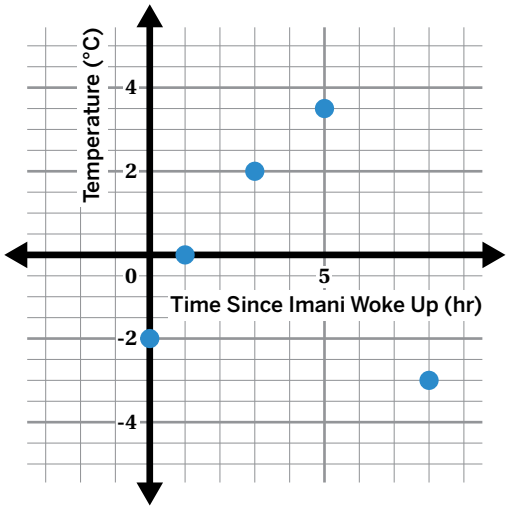
Let's plot and interpret coordinates in context.

Warm-Up

Imani woke up at 9:30 AM and tracked the temperature outside over the course of the day.

9:30 AM	10:30 AM	12:30 PM
-2°C	0°C	2°C

2:30 PM	5:30 PM
3.5°C	-3°C



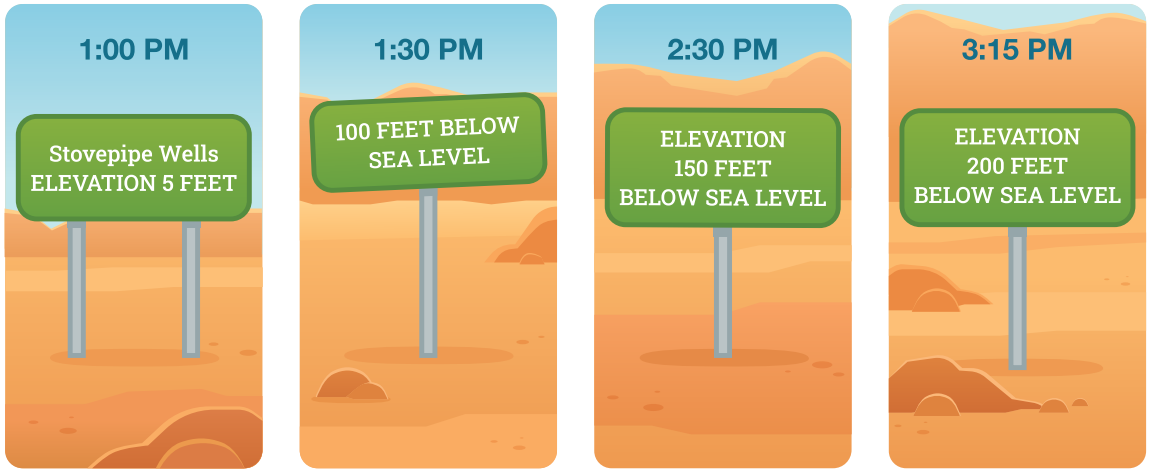
1. Write a story about the temperature over the course of the day.

2. It was -4°C at 8:30 AM. Plot this information on the graph.

Name: ..... Date: ..... Period: .....

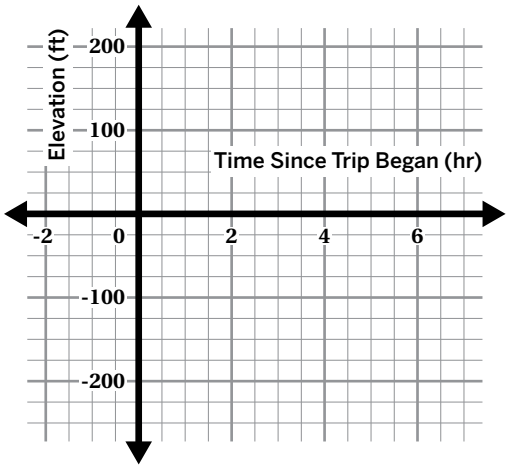
Elevation

Yona and her family took a road trip. One night, they stayed at a hotel where the elevation was 35 feet above sea level. They left their hotel at noon to drive through Death Valley, California. Yona took pictures along the way.



3. Plot five points on the graph to represent moments in Yona's day. Include a point that represents when Yona left the hotel.

4. Plot at least three new points to represent Yona's trip back to the hotel.



5. Swap papers with a partner. Tell a story about their graph so that another person can understand it.

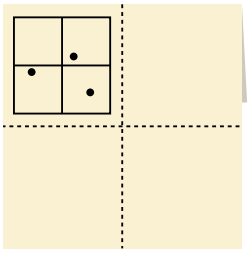
Name: ..... Date: ..... Period: .....

Graph Telephone

6. You will use one story from the Activity 2 Sheets.

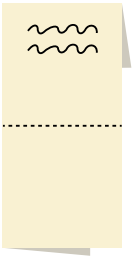
Round 1

- Read the story at the top of the page. Create a graph based on the story.
- When you're done, fold the paper to hide the story.
- When everyone is ready, pass your paper to the left.



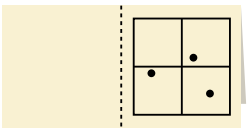
Round 2

- Write a story based on the graph your classmate made. Include enough details so that someone could recreate this graph.
- When you're done, fold the paper again so that only Rounds 2 and 3 are showing.
- When everyone is ready, pass your paper to the left.



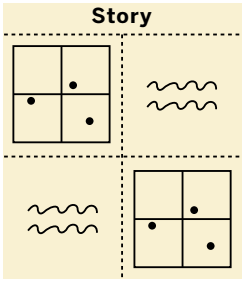
Round 3

- Create a graph based on the story your classmate wrote. When you are done, refold the paper so that only Rounds 3 and 4 are showing. Then pass your paper to the left.



Round 4

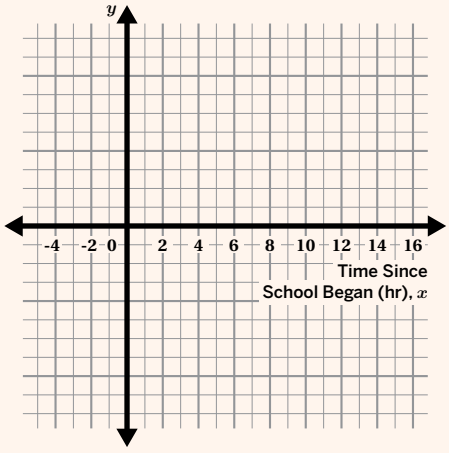
- Write a story based on the graph your classmate made in Round 3. When you're done, unfold the paper and look at how the story changed throughout the rounds. Share the stories and graphs.



You're invited to explore more.

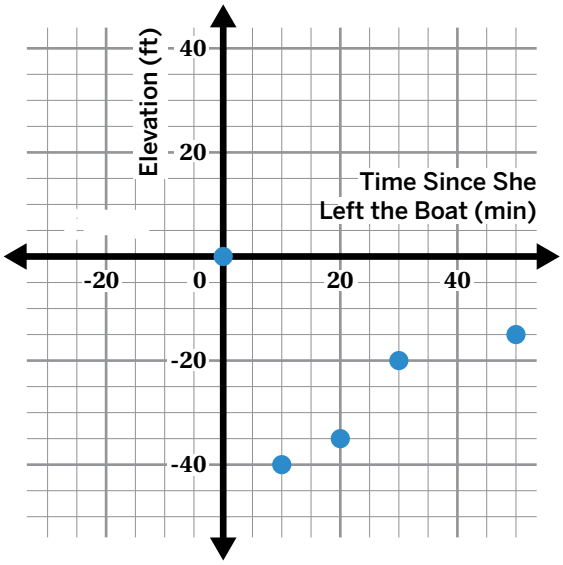
7. Think of a story about a school day, but don't write it down.

- Label and scale the y-axis based on your story.
- Create a graph to represent your story.
- Trade graphs with a classmate. Try to figure out what story their graph is telling.



Synthesis

8. Here is the graph of a situation from earlier when Ava jumped off a boat to go scuba diving. A point is added at $(-10, 5)$.



a Describe what this new point means for the story.

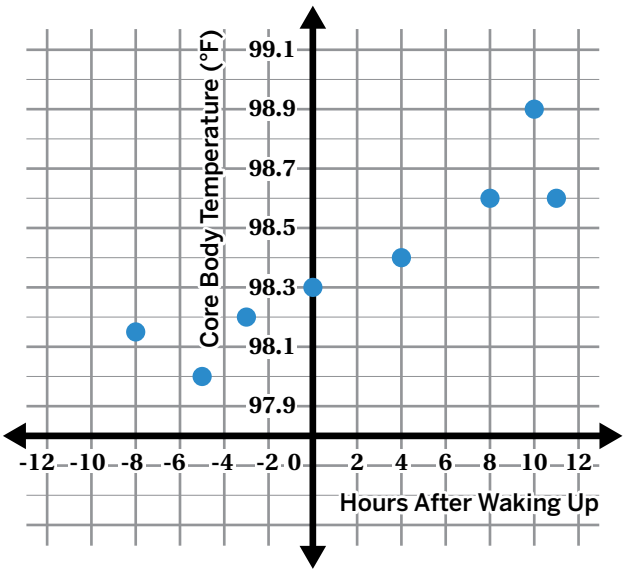
b Describe how points on a coordinate plane can help us understand more about a situation.

Summary 7.13

You can use points with positive and negative coordinates on a graph to make sense of different situations. Common contexts include elevation, time, temperature, and money.

Here's an example that uses temperature:

- Just like in the graph of Ava's elevation, time is represented on the x-axis.
- Temperature is represented on the y-axis.
- The points to the left of the y-axis represent temperatures recorded while the person was sleeping. The points to the right of the y-axis represent temperatures recorded after they woke up.

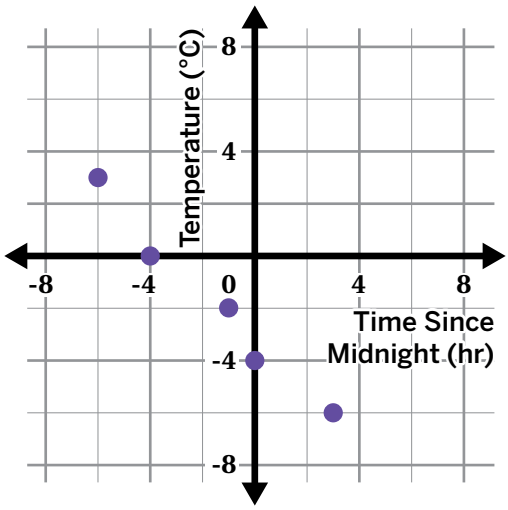


Coordinates on the graph tell different parts of the story. For example, the point $(-5, 98)$ indicates that 5 hours before they woke up, their temperature was 98°F.

Practice
7.13

Name: ..... Date: ..... Period: .....

Problems 1–5: This graph shows the temperatures on one night in Minneapolis, Minnesota. The x-axis shows the amount of time since midnight, or 12 AM.



1. The point $(0, -4)$ is on the graph. What does this point tell you?

2. The point $(-4, 0)$ is also on the graph. What does this point tell you?

3. At 10:00 PM, it was -1°C. Plot this point on the graph.

4. What was the coldest temperature at night?

5. The temperature got warmer between 3:00 AM and 4:00 AM. At 4:00 AM, it was below freezing. Plot a point to represent a possible temperature at 4:00 AM.

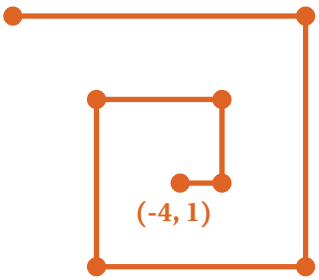
Practice
7.13

Name: ..... Date: ..... Period: .....

6. Felipe walked a path from point $(-4, 1)$ with these rules:

- Walk 1 meter and turn left.
- Walk 2 meters and turn left.
- Walk 3 meters and turn left.
- He followed this pattern 7 times.

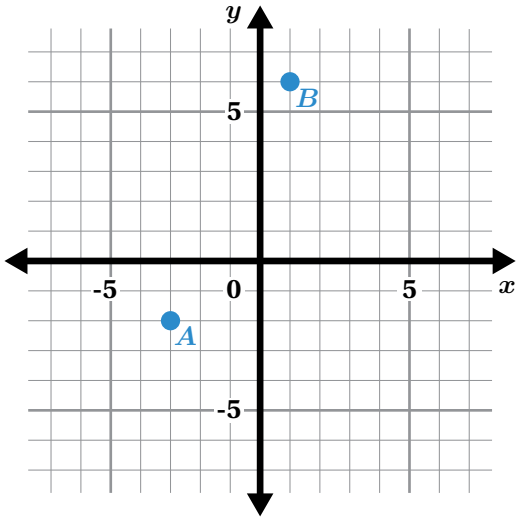
Felipe's path is shown. At what coordinates did Felipe end his walk?



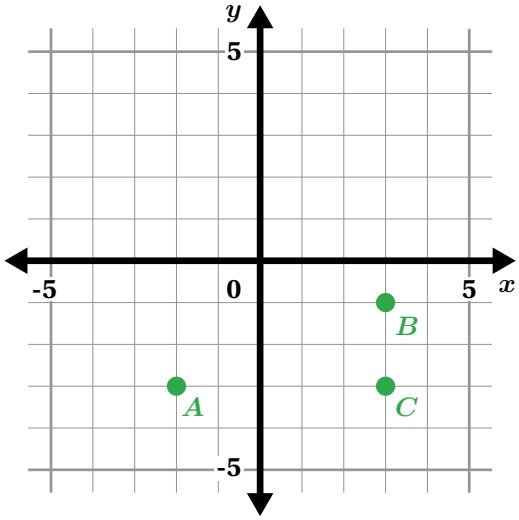
Spiral Review

7. Titus wants to make a right triangle. Write a coordinate pair to make a right triangle. Try to find as many different points as you can. (More than five are possible.)

Point	Coordinates
A	$(-3, -2)$
B	$(1, 6)$
C	



Problems 8–10: Here is a graph.



8. What is the distance between points B and C?

9. What is the distance between points A and C?

10. Identify the coordinates of a fourth point to create a rectangle.

Name: ..... Date: ..... Period: .....

Relationships Between Variables | Model the World | Distance and Direction | Graphing Shapes

6.EE.6, 6.EE.8, 6.G.3, 6.NS.5, 6.NS.6, 6.NS.6.b, 6.NS.6.c, 6.NS.8, SMP.2, SMP.7

Practice Day 2

Let's practice what you've learned so far in this unit!



You will use task cards for this Practice Day. Record all of your responses here.

Task A: Plot It

1. Solution: .....

2. Description:

Drawing:



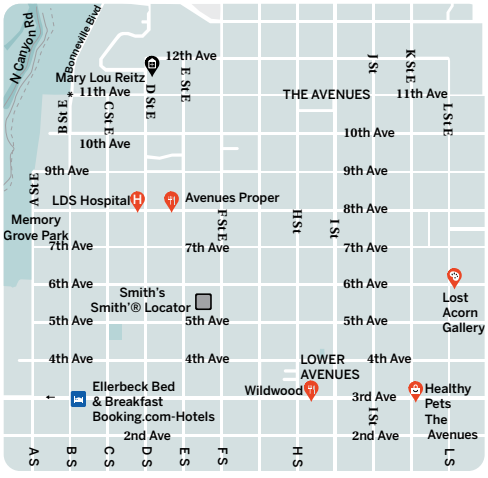
Description:

3. Drawing:

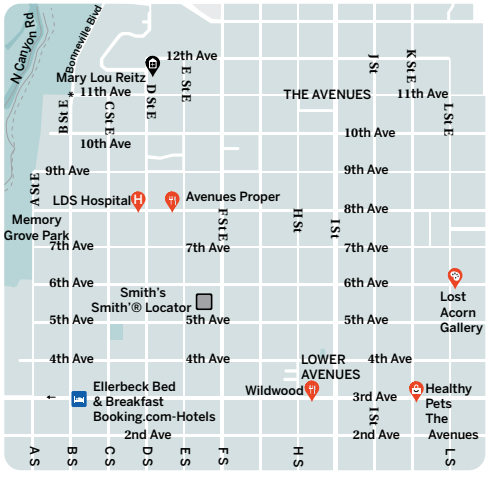


Description:

4. Drawing:



Drawing:



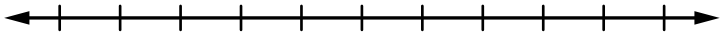
Name: ..... Date: ..... Period: .....

Practice Day 2 (continued)

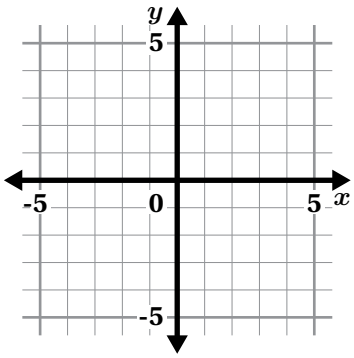
Task B: Finish It

1. Inequality: .....

2. Graph:



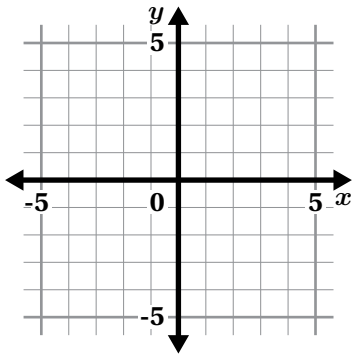
3. Graph:



Point D: .....

Explanation:

4. Graph:

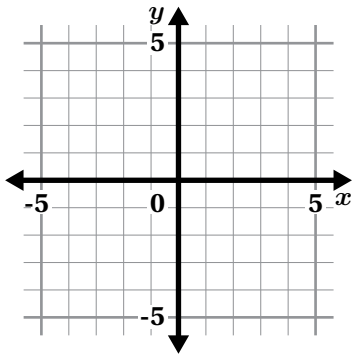


Point G: .....

Point H: .....

Explanation:

5. Graph:



Point J: .....

Point K: .....

Point L: .....

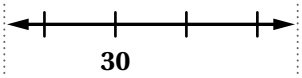
Explanation:

Name: _____ Date: _____ Period: _____

Practice Day 2 (continued)

Task C: Sort It

1. Organize your groups in this table.

Sentence	Inequality	Graph	Solutions
A			
B			
C			

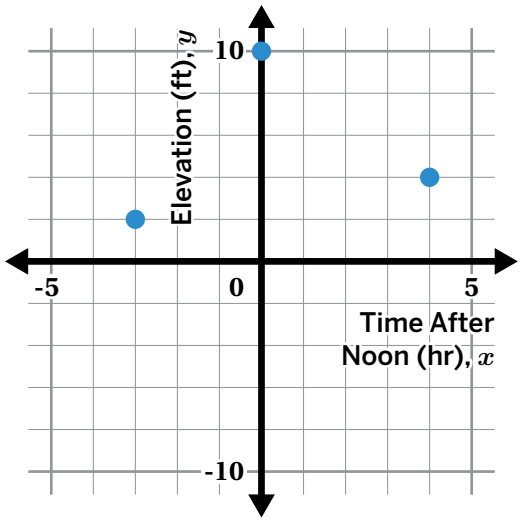
Name: _____ Date: _____ Period: _____

Practice Day 2 (continued)

Task D: Interpret It

1. Elevation: _____

Explanation:



2. Point: _____

3. Point: _____

4. Point: _____

5. Point: _____

Explanation:

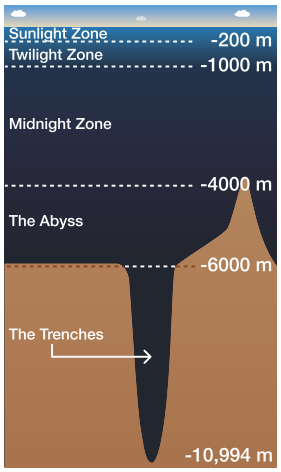
You're invited to explore more.

Career Connection

To boldly go where no human has gone before – that just might be parts of the ocean!

A whopping 98% of Earth's oceans' floors are unexplored. While most animals live in the Sunlight Zone, where light can still reach, Earth's oceans are much deeper than just that zone. While the exact number of animal species that live in our oceans is unknown, scientists have estimated that only about 9% have been classified.

Oceanographers study Earth's oceans, which can include the plants and animals that live within them. They often have to use negative numbers as they collect and analyze data from different depths.



Source: NOAA



"Buzjani, Persian mathematician and astronomer." Public Domain

Meet Abu al-Wafa' Buzjani

Abu al-Wafa' Buzjani is best known today for his discoveries in the area of mathematics called trigonometry, but in his own time he wrote about math for many different audiences. In 10th century Baghdad, astronomers, craftspeople, business workers, and accountants all could have picked up one of his "how-to" books to help them understand how math could support them in their field. One of Abu al-Wafa's books is the first known Arabic text where negative numbers are used.

One of Abu al-Wafa's books is the first and only recorded place where negative numbers were used in Arabic texts during this time period.

Are you interested in studying oceanography or the history of mathematics? What can you do to learn more?

Math in the World

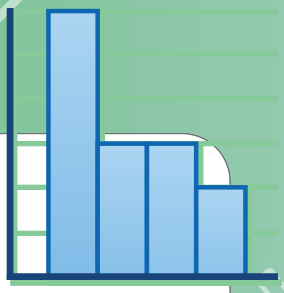
What's red *and* almost invisible? Some deep-sea animals! Because red light cannot reach beyond -100 meters, red animals appear black. The ocean gets darker the farther the depth. Imagine bumping into this guy in the dark!



Sailorr/Shutterstock.com

Math Mindset

Give an example of a negative number in a real-world situation and how you make sense of it.



Unit 8

Describing Data

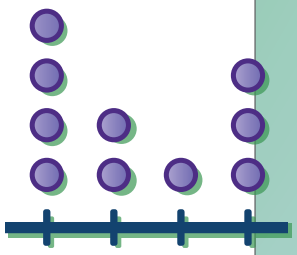
Big Ideas in This Unit

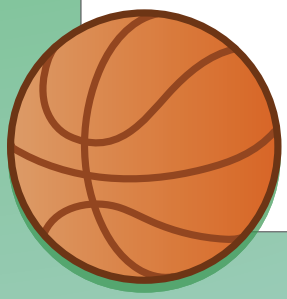
CC1 Variability in Data | The Shape of Distributions

CC2 Relationships Between Variables

Questions for Investigation

- What are statistical questions and how are they used?
- What are different ways to represent numerical data?
- How do we measure the center and spread of a data set?

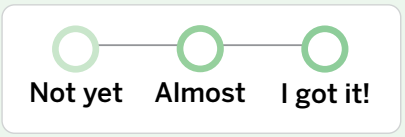




Explore: Spider Variations
How can we tell if a spider is just a rare variation or a new species?

Watch Your Knowledge Grow

This is the math you'll explore in this unit. Rate your understanding to see how your knowledge grows!



I can . . .	Before	After
Explain the difference between categorical and numerical data.		
Describe and interpret a dot plot to help answer a statistical question.		
Create a dot plot for a given set of data.		
Describe the center, spread, shape, and distribution of a data set.		
Describe and interpret a histogram that represents a data set.		
Create a histogram to visualize data.		
Calculate the mean of a data set.		
Calculate the mean absolute deviation of a data set.		
Explain what the mean and MAD says about a real-world data set.		
Determine and interpret the median of a data set.		

I can . . .	Before	After
Explain how the shape of a data set affects its mean and median.		
Determine and interpret the quartiles of a data set.		
Create a box plot to visualize a data set.		
Determine the range and IQR of a data set.		
Justify claims about data using box plots.		
Create a dot plot, histogram, or box plot to visualize a data set.		
Choose and calculate either the mean and MAD or median and IQR for a data set.		
Use mean and MAD or median and IQR to compare real-world data sets.		

Sub-Unit

1

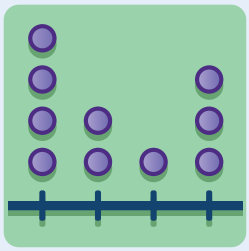
Visualizing Data



Explore
Spider Variations



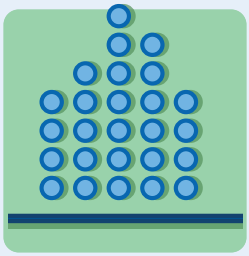
Lesson 1
Screen Time



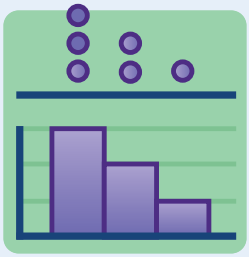
Lesson 2
Dot Plots



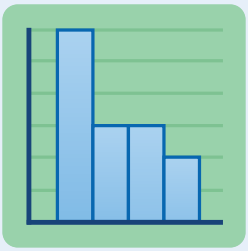
Lesson 3
Minimum Wage



Lesson 4
Lots More Dots



Lesson 5
The Plot Thickens



Lesson 6
DIY Histograms

Name: ..... Date: ..... Period: .....

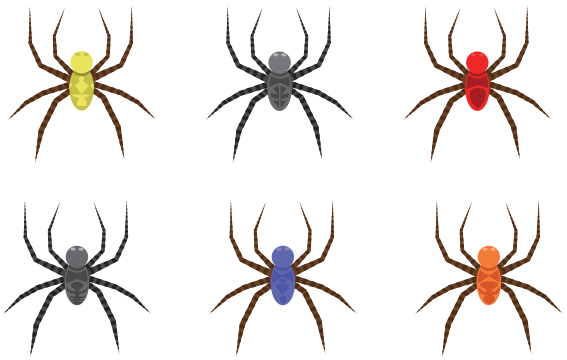
Variability in Data | Building Toward: 6.SP.1, 6.SP.5, SMP.1, SMP.3

Explore: Spider Variations

How can we tell if a spider is just a rare variation or a brand new species?

Warm-Up

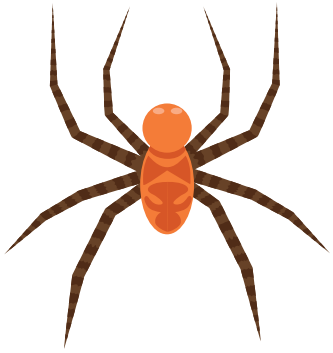
1. Analyze the images of 6 specimens of male spiders belonging to the *Loureedia* genus. Briefly describe what you notice and think might be true about the sizes, shapes, colors, markings, and any other features of this genus of spiders. Use words such as: *mostly, generally, typically, normally, sometimes, rarely, never,* etc.



Name: ..... Date: ..... Period: .....

Beyond a Reasonable Doubt?

You will be given fact sheets for three known species of spiders belonging to the Loureedia genus and one unclassified specimen, as seen here.



Part 1

2. With your group, decide whether you believe the unclassified specimen belongs to any of the three known species or is a new, previously unidentified species. Record your observations and the scientific or mathematical evidence you would use to support your claim.

Part 2

3. You will now join a new group and take turns sharing your claims and the related evidence, one at a time. For each claim that does not match yours, take turns providing counterarguments. For example, "The specimen is shorter than the average *Loureedia annulipes*."

4. Write two or three questions you could ask to help you come to a consensus about the classification of the specimen. These can be questions you already answered in making your claims, new questions that could also be answered using the data available to you, or even questions that would require additional information. If a question requires more information, be sure to state that extra information, as well.

Name: Date: Period:

Beyond a Reasonable Doubt? (continued)

You will be given another data sheet that provides some measurements of the pedipalps – the antenna-like sensory organ on a spider's head – for the unclassified specimen and the three known species.

Part 3

5. **Data Talk!** Analyze the measurements. Using that analysis and previously gathered information, investigate and make one final claim: Which species does the unclassified specimen belong to? Or, do you believe it is a new, previously unidentified species?

Name: ..... Date: ..... Period: .....

Building Math habits of Mind

Discuss:

- Which of these habits of mind did you strengthen during this activity?
- How did you use the one(s) you selected?

I can slow down and first make sense of a challenging problem before trying to solve it.

Not yet Almost I got it!

I can represent real-world problems using equations and inequalities and interpret their solutions within the context of the problem.

Not yet Almost I got it!

I can justify my thinking and ask questions to help me understand the thinking of others.

Not yet Almost I got it!

I can apply the math that I know to solve real-world problems, making assumptions and revising my thinking as needed.

Not yet Almost I got it!

I can select an appropriate tool to help me solve problems.

Not yet Almost I got it!

I can communicate my thinking and solutions clearly to others.

Not yet Almost I got it!

I can look for structure or patterns to help me solve problems.

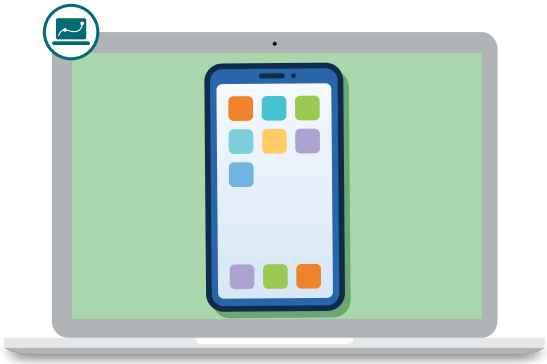
Not yet Almost I got it!

I can look for repeated calculations and other repeated steps to make generalizations.

Not yet Almost I got it!

Name: ..... Date: ..... Period: .....

Variability in Data | The Shape of Distributions | Building Toward: 6.SP.1, 6.SP.2, SMP.3, SMP.6



Screen Time

Let's learn about different types of data.

Warm-Up

1 Let's meet Antwon! Antwon is in 6th grade. He was wondering: *How much time do my friends spend on their phones?* What could Antwon do to answer his question?

2 To collect data about his question, Antwon asked each friend: *What activity do you use your phone for the most?*

Data Talk! How much time do you think Antwon's friends spend on their phones?

Friend	What activity do you use your phone for the most?
Maria	Texting
Julian	Watching videos
Carlos	Taking pictures
Laila	I don't have a phone
Troy	Playing games
Omar	I don't have a phone
Alina	Watching videos

Name: ..... Date: ..... Period: .....

Questions and claims

3 Antwon claims most of his friends spend more than 4 hours a day on their phones.

Do you agree with Antwon's claim? Circle one.

Agree Disagree I'm not sure

Explain your thinking.

Friend	What activity do you use your phone for the most?
Maria	Texting
Julian	Watching videos
Carlos	Taking pictures
Laila	I don't have a phone
Troy	Playing games
Omar	I don't have a phone
Alina	Watching videos

4 Antwon collected more data by asking his friends different questions. Use Antwon's data to answer: *Do you agree that most of Antwon's friends spend more than 4 hours a day on their phones?* Circle one.

Agree Disagree I'm not sure

Friend	How many hours do you spend on your phone?		Do you spend more than 4 hours a day on your phone?
	Each day	Each week	
Maria	2	12	No
Julian	5	25	Yes
Carlos	4.5	24	Yes
Laila	0	0	No
Troy	8	40	Yes
Omar	0	0	No
Alina	5	28	Yes

Name: ..... Date: ..... Period: .....

Questions and claims (continued)

5 Antwon claims most of his friends spend more than 4 hours a day on their phones.

Antwon is asking questions to determine if his claim is true. Order his questions from *most helpful* to *least helpful*.

	Most helpful
	Least helpful

a What activity do you use your phone for the most?

b How many hours are you on your phone each day?

c How many hours are you on your phone each week?

d Do you spend more than 4 hours a day on your phone?

6 Here are some more questions that Antwon asked his friends.

Friend	How many pets do you own?	What do you like to do after school?	What grade are you in?	What's your favorite type of music?
Maria	1	Basketball	6	K-pop
Julian	0	Skateboard	6	Hip-hop
Carlos	2	Paint	6	Jazz
Laila	2	Bake	6	Rock
Troy	5	Video games	6	Hip-hop
Omar	1	Watch T.V.	6	Pop
Alina	0	Swim	6	Banda

Make two claims based on the data about Antwon's friends.

Claim 1:

Claim 2:

Activity 2

Name: ..... Date: ..... Period: .....

Categorical and Numerical Data

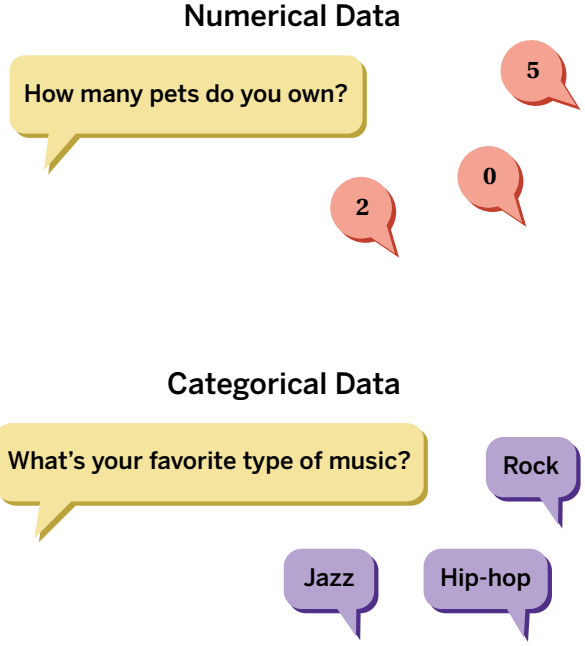
7 **Data Talk!** Asking different types of questions can produce different types of data.

One of these questions produces **numerical data** and the other produces **categorical data**.

Describe what you think these terms mean.

Numerical data:

Categorical data:



8 Match each question with the type of data it produces.

Question	Numerical Data	Categorical Data
Do you have 2 or more pets?		
How many hours do you spend outdoors each day?		
How old is the youngest person in your family?		
How much does your backpack weigh?		
What is your favorite vegetable?		

Activity 3

Name: ..... Date: ..... Period: .....

Challenge Creator

9 Think of something you want to learn about your classmates.

- **Make It!** Write a question for your classmates to answer.

My Question		

- Will the data from your question be numerical or categorical? Circle one.

Numerical Categorical

- **Solve It!** Ask some of your classmates your question and record their answers here.

Classmate's Name	Response to Question

10 Synthesis

Discuss: both questions, then select *one* and write your response.

- What is something you learned today?
- What do you want to learn more about?



13 Summary 8.01

You can use surveys and measurements to collect data to answer questions about a topic. This data can be either **categorical data** or **numerical data**. Some data that contain numbers, like addresses or dates, are categorical because the numbers are not quantities or measurements.

Here are some examples.

Categorical Data	Numerical Data
• Favorite color	• Number of people in each class
• Food people eat for lunch	• Weights of dogs
• People's phone numbers	• Ages of people in your school

categorical data Data that can be sorted into categories instead of counted.

numerical data Data that includes values that are numbers and can be measured and meaningfully compared.

Practice
8.01

Name: ..... Date: ..... Period: .....

1. Determine whether each question would produce numerical or categorical data.

Question	Numerical Data	Categorical Data
What is your favorite breakfast food?		
How did you get to school this morning?		
How many people live in your home?		
What is the last thing you ate or drank?		
How many minutes did it take you to get ready this morning, from waking up to leaving for school?		

2. Choose one question from the table that would produce numerical data. Explain your thinking.

3. Choose one question from the table that would produce categorical data. Explain your thinking.

Problems 4–6: A pizza shop owner wants to improve her dinner service, so she collects data from her sales. Here is a copy of a receipt she is examining.

4. Where do you see categorical data in this receipt?

5. Where do you see numerical data?

6. What questions could the pizza shop owner answer using the information from this receipt and others like it?

Booking type:	**Online**
Booking time:	**5:30 PM**
Server: Amir	**Party Size: 4**
1 × Pepperoni	**12.00**
2 × Vegetarian	**21.00**
1 × Chef's Special	**13.50**
Subtotal	**46.50**
Tax	**4.65**
Tip	**10.00**
Total	**$61.15**
Payment Method	**Credit Card**
Time	**6:47 PM**

Name: ..... Date: ..... Period: .....

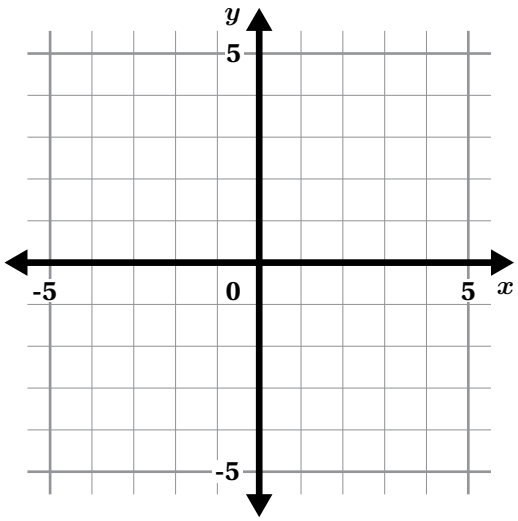
7. Lan claims that more students in her class walk to school than take the bus. Write a question that Lan could ask each of her classmates to investigate the claim.

8. What type of data would Lan be collecting with the question you wrote?

Spiral Review

Problems 9–11: Triangle DEF has vertices $D = (-4, -4)$, $E = (-2, -4)$, and $F = (-3, -1)$.

9. Plot the triangle on the coordinate plane and label the vertices.



10. What is the area of the triangle?

11. Which of these points would need to be added to the coordinate plane to make triangle DEF into a parallelogram $DEFG$?

A. $(-2, -1)$

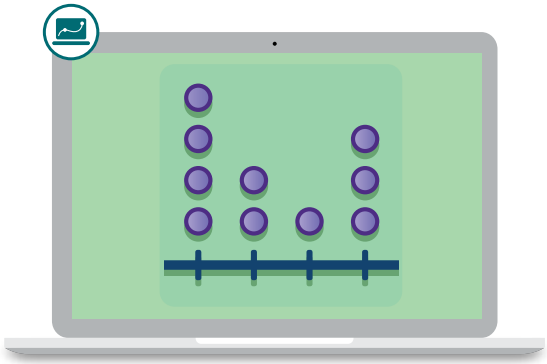
B. $(0, 0)$

C. $(-4, -3)$

D. $(-5, -1)$

Name: ..... Date: ..... Period: .....

Variability in Data 6.SP.1, 6.SP.4, 6.SP.5, 6.SP.5.a, 6.SP.5.b, SMP.1, SMP.3, SMP.6



Dot Plots

Let's turn data into dot plots.

Warm-Up

1 Antwon used a survey to collect data about some of his friends.

What question might he have asked?

Friend	?
Maria	20
Julian	22
Carlos	17
Laila	10
Troy	45
Omar	15
Alina	20

Activity 1

Name: ..... Date: ..... Period: .....

Dot Plots

2 **Data Talk!** Meet Antwon's classmates!

Antwon claims that most of these students spend more than 4 hours on their phones each day.

Do you agree? Circle one.

Agree Disagree I'm not sure

Explain your thinking.

How many hours do you spend on your phone each day?

Terrance 0 hr	Shanice 3 hr	Dylan 4 hr	Felipe 0 hr	Farah 0 hr
Omar 0 hr	Maria 4 hr	Esteban 5 hr	Emma 6 hr	Alma 2 hr
Diamond 5 hr	Mauricio 6 hr	Eva 2 hr	Kai 9 hr	Troy 3 hr
Maria 3 hr	Julian 7 hr	Carlos 4 hr	Laila 0 hr	Alina 8 hr

3 Let's look at another way to visualize numerical data: as a **dot plot**.

What do you notice? What do you wonder?

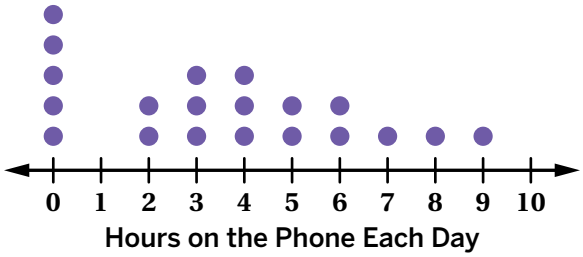
I notice:

I wonder:

Name: ..... Date: ..... Period: .....

Dot Plots (continued)

Here is the dot plot of the data from Antwon's classmates.



4 Raven is a new student in Antwon's class. She spends 3 hours on her phone each day. Draw a dot to add Raven's data to the dot plot.

5 Dylan spends 4 hours on his phone each day.

a Circle a dot on the dot plot that could represent Dylan.

b Is this the only dot that could represent Dylan?

Yes No I'm not sure

Explain your thinking.

6 **Data Talk!** Here is Antwon's claim from earlier.
He said: *Most students spend more than 4 hours on their phones each day.*

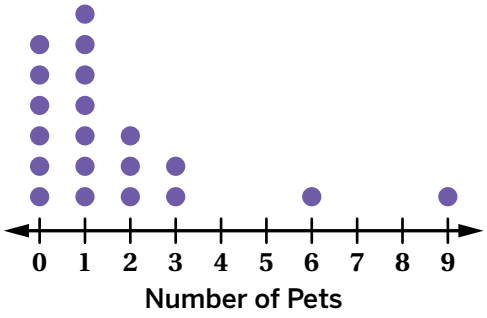
How can a dot plot help you decide if his claim is true?

Name: ..... Date: ..... Period: .....

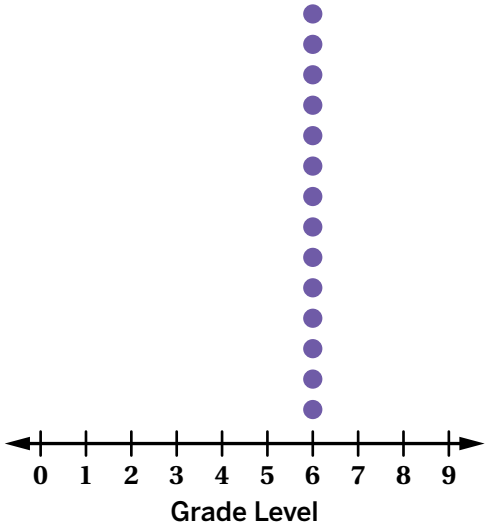
Statistical Questions

7 **Data Talk!** Antwon asked his classmates a few more questions and made dot plots of their answers.

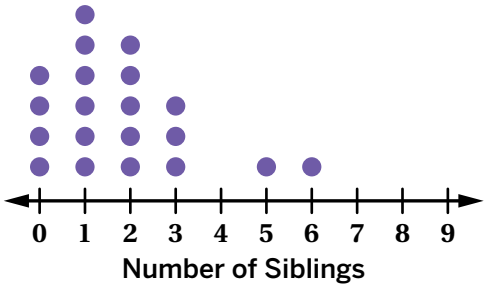
How many pets do you have?



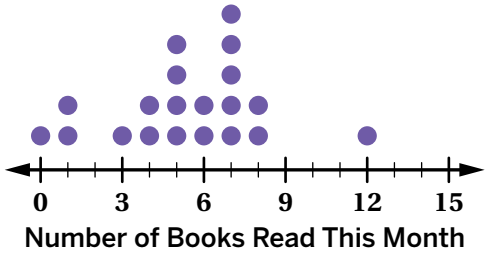
What grade are you in?



How many siblings do you have?



How many books have you read this month?



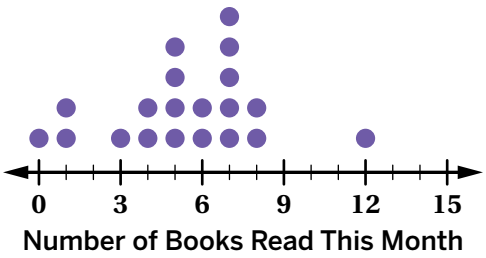
Discuss: What are two things you learned about Antwon's classmates from this data?

Name: ..... Date: ..... Period: .....

Statistical Questions (continued)

8 This dot plot shows the number of books Antwon's classmates read this month.

Based on the dot plot, how many classmates did Antwon survey?



9 **Data Talk!** Which question would a dot plot be *less* helpful in answering? Circle one.

How many books has Farah read this month?

Have most students read more than 3 books this month?

Explain your thinking.

10 A **statistical question** collects data and anticipates a variety of answers. Select *all* the statistical questions.

- ☐ **A.** How many books did each student in the class read this month?
- ☐ **B.** How many books has Farah read this month?
- ☐ **C.** What type of books do the students in this class prefer to read?
- ☐ **D.** What type of book did Julian read last?
- ☐ **E.** How long is the longest book each student in this class read?

11 Synthesis

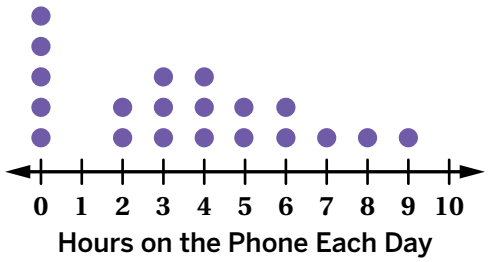
Discuss: What are the advantages and disadvantages of visualizing a data set as a dot plot?

Use the examples if they help with your thinking.

Data Set

0, 3, 4, 0, 0, 0, 4, 5, 6, 2, 5, 6, 2, 9, 3, 3, 7, 4, 0, 8

Dot Plot



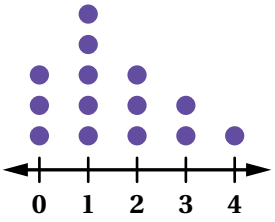
14 Summary 8.02

A **statistical question** is a question that needs more than one piece of data to answer it.

Here is an example:

- "Which classroom in your school has the most books?" is a statistical question because you need to know the number of books in *each* classroom to answer it.
- "How many books are in your classroom?" is not a statistical question because you only need to know the number of books in *one* classroom to answer it.

You can organize data that answers a statistical question into a list or a **dot plot**.

List	0, 0, 0, 1, 1, 1, 1, 1, 2, 2, 2, 3, 3, 4	Lists allow you to see all of the data. Lists can be used for both numerical and categorical data.
Dot Plot		Dot plots are a visual representation of numerical data and allow you to compare multiple data sets.

dot plot A way to visualize numerical data sets, where each data point is represented by a dot on a number line. Data points with the same value are stacked on top of each other.

statistical question A question that collects data and anticipates a variety of answers.

Name: _____ Date: _____ Period: _____

Problems 1–5: Five sixth-grade students at a school were each asked the following survey questions:

Question A: What grade are you in?

Question B: How many books did you read in the last year?

Question C: How many inches are in 1 foot?

Question D: How many dogs and cats do you have?

Their answers are shown in the table. Write the letter of the question that could have produced each line of data.

	Question	Diya	Peter	Tiana	Marc	Callen
1.		0	1	1	3	0
2.		12	12	12	12	12
3.		6	6	6	6	6
4.		11	5	18	20	9

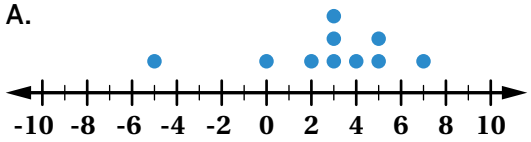
5. How are Questions A and C different from the other questions?

Problems 6–7: Anya asked 10 students how many minutes it takes them to get to school each morning.

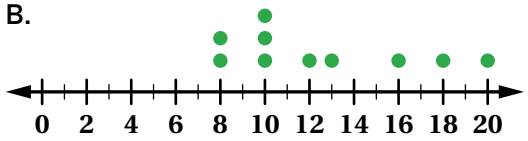
6. Which dot plot could represent the data that Anya collected?

7. Which dot plot could not represent Anya's data? Explain your thinking.

A.



B.



C.



Name: ..... Date: ..... Period: .....

8. Which question is a statistical question?

A. How old are you?

B. How old are the students in our school?

C. How old is your teacher?

D. What is the difference between your age and your teacher's age?

9. Deven likes to collect coins. Which of these is a statistical question someone could ask Deven about his coins?

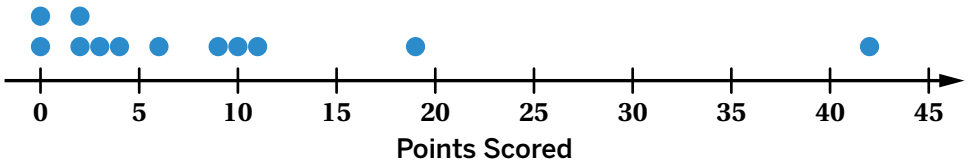
A. What coin in your bucket has the largest value?

B. How many coins are in your bucket?

C. How many of each coin do you have in your bucket?

D. What is a typical shape of the coins in your bucket?

10. This dot plot represents the points scored by 12 players on a basketball team during an important game.



Write a story about the basketball game.

Spiral Review

11. Order these values from *least* to *greatest*.

|-17|	|-18|	-18	|19|	20

Least **Greatest**

Problems 12–14: Determine each quotient.

12. $45052 \div 28$ **13.** $6052 \div 17$ **14.** $60.52 \div 1.7$

Name: ..... Date: ..... Period: .....

Variability in Data 6.SP.4, SMP.6

Minimum Wage

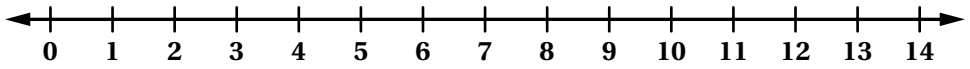


Let's create dot plots to represent real-world situations.

Warm-Up

1. Count the number of letters in your name.

2. As a class, let's make a dot plot using the number of letters in each of our names.



Activity 1

Name: ..... Date: ..... Period: .....

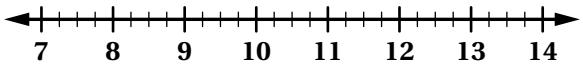
Minimum Hourly Wage

Data Talk! Here are the minimum wages in 7 states in the Southwestern U.S. from 2020.

Arizona	California	Nevada	New Mexico	Oklahoma	Texas	Utah
$12.00	$12.00	$9.00	$9.00	$7.25	$7.25	$7.25

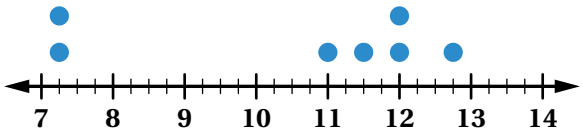
3. Why might someone be interested in visualizing this data?

4. Create a dot plot that shows the minimum hourly wages for these 7 Southwestern states.



Minimum Hourly Wage ($)

5. Here is a dot plot of the minimum hourly wages of 7 states in the Northeastern U.S. in 2020.



Minimum Hourly Wage ($)

Discuss:

- How are the two dot plots alike? Different?
- Why might there be differences between the Southwestern and Northeastern states?

6. The federal minimum wage in 2020 was $7.25 per hour. How many states in each region had minimum wages above the federal minimum wage?

Southwestern states: ..... Northeastern states: .....

Activity 2

Name: Date: Period:

Dot Plot Drafts

Data Talk! A student created a first draft of a dot plot to represent the data in four different situations.

Discuss:

- What is one thing they did well?
- What is one thing you would change about each dot plot?

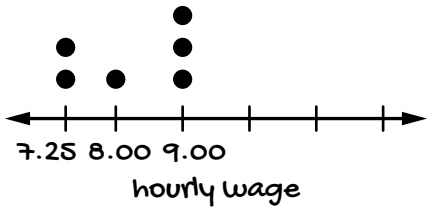
Create a second draft of each dot plot. Make sure to label your axis.

7. Situation 1: Pet Store

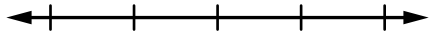
A new employee at a pet store asked 6 other employees: *What is your hourly wage?*

Employees' responses: $7.25, $8.00, $9.00, $7.25, $9.00, $9.00

First Draft



Second Draft

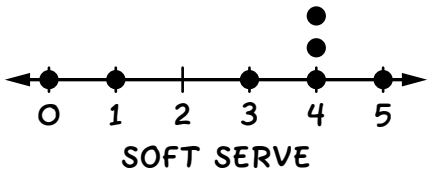


8. Situation 2: Soft Serve

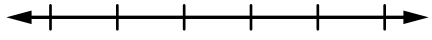
A journalist asked 7 employees at a soft serve shop: *How many hours did you work this week?*

Employees' responses: 5, 3, 4, 1, 4, 0, 4

First Draft



Second Draft



Name: Date: Period:

Dot Plot Drafts (continued)

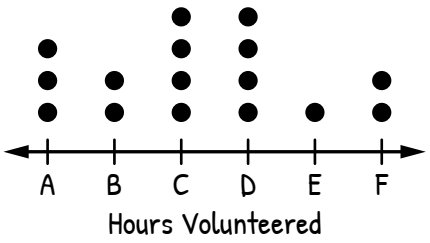
9. Situation 3: Tutoring Hours

A manager asked 6 of their volunteer tutors: *How many hours have you volunteered today?*

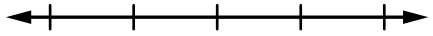
Volunteer responses:

Abby: 3 Bobby: 2 Carlo: 4 Dionte: 4 Esther: 1 Fatima: 2

First Draft



Second Draft

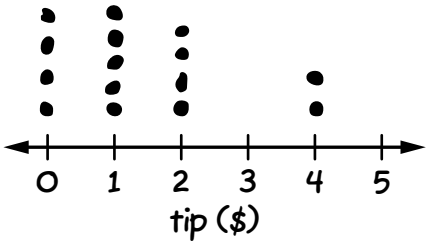


10. Situation 4: Pizza Shop

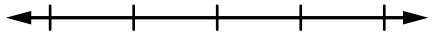
15 customers at a pizza shop took a survey that asked: *How much do you tip your delivery driver?*

Customer responses: \$0, \$2, \$1, \$2, \$2, \$0, \$4, \$0, \$1, \$1, \$1, \$4, \$2, \$0, \$1

First Draft



Second Draft



Synthesis

11. Here is a dot plot of the 10 U.S. states with the highest minimum hourly wages in 2020.



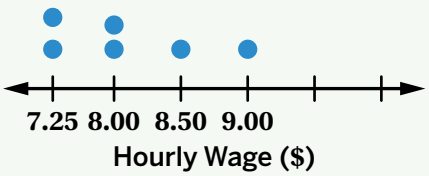
Discuss: What would you change or improve about this dot plot?

Summary 8.03

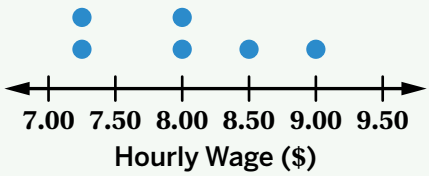
You can use dot plots to visualize data and compare different data sets. When creating a dot plot it is important to use a consistent scale on the number line and a consistent amount of vertical space between the dots.

Here is an example of two plots showing the same set of data, one with mistakes and one done accurately.

Dot Plot With Mistakes



Accurate Dot Plot



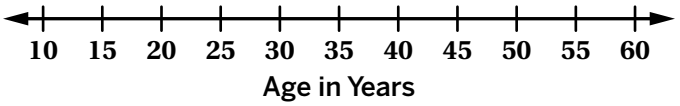
Notice:

- The scale of the dot plot with mistakes is inconsistent, but the scale of the accurate dot plot is consistent with $0.50 between each tick mark.
- The height of the dots above $7.25 and $8.00 should be the same because there are the same number of dots above each.

Name: ..... Date: ..... Period: .....

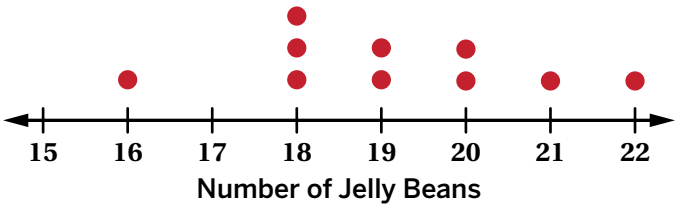
1. A soccer team of 15 middle school students had a practice one evening. 5 people came to watch them: most were parents, but some were siblings. A head coach and an assistant coach were also there.

Create a dot plot that could represent the ages of everyone at the soccer team practice.



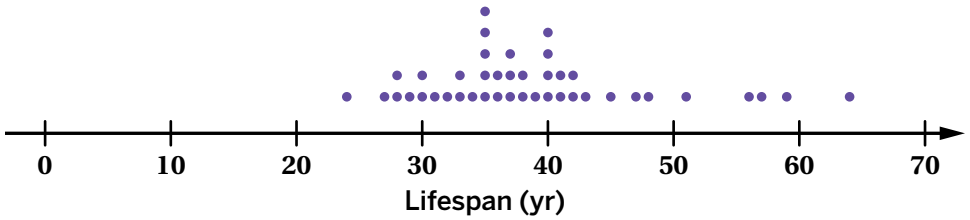
Problems 2–3: A teacher brought a bowl of 20 jellybeans to class and asked each student to estimate the number.

2. The teacher used a dot plot to record each estimate. How many students gave an estimate?



3. Do you think the students' estimates were accurate? Explain your thinking.

4. Here is a dot plot of the lifespans of 42 chimpanzees that lived in the wild.



How many chimpanzees lived fewer than 30 years?

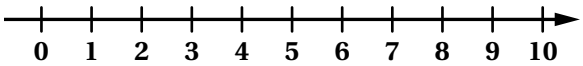
Practice
8.03

Name: ..... Date: ..... Period: .....

Problems 5–6: Melanie's basketball team is practicing their skills. Here is the number of successful shots each of her teammates recorded at the start of practice.

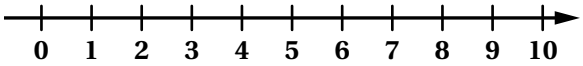
Number of Successful Shots			
3	2	1	2
3	4	1	0
5	2	0	2

5. Make a dot plot of this data.



Number of Successful Shots

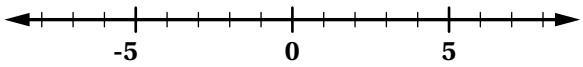
6. By the end of practice, most of Melanie's teammates improved their skills. Make a dot plot that could represent her teammates at the end of practice.



Number of Successful Shots (End of Practice)

Spiral Review

Problems 7–9: Plot each point on the number line.



7. -7 **8.** 1.75 **9.** $-\frac{5}{2}$

Problems 10–12: Solve each equation.

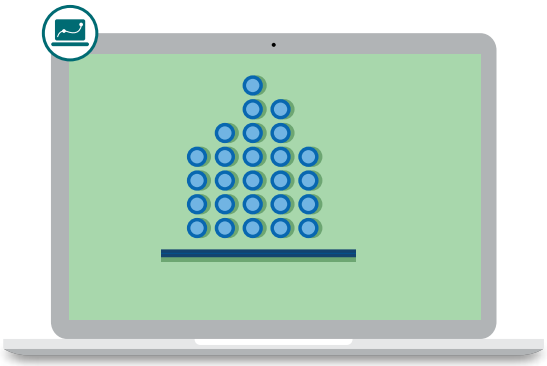
10. $9u = 1$ **11.** $1.5w = 7.5$ **12.** $2 = x + \frac{7}{10}$

Name: ..... Date: ..... Period: .....

Variability in Data | The Shape of Distributions | 6.SP.2, 6.SP.4, SMP.3

Lots More Dots

Let's describe dot plots.



Warm-Up

1 Play a few rounds of Polygraph with your classmates!

You will use a Warm-Up Sheet with dot plots for four rounds. For each round:

- You and your partner will take turns being the Picker and the Guesser.
- Picker: Select a dot plot from the Warm-Up Sheet. Keep it a secret!
- Guesser: Ask the Picker yes-or-no questions, eliminating dot plots until you're ready to guess which dot plot the Picker chose.

Record helpful questions from each round in this workspace:

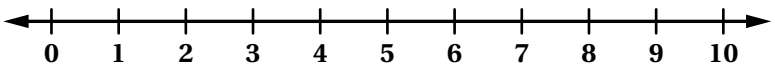
Activity 1

Name: ..... Date: ..... Period: .....

Exploring Center and Spread

2 Now it's your turn to make a dot plot!

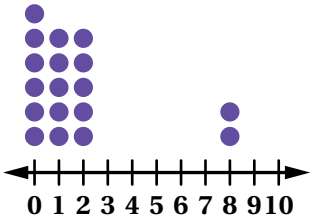
a Make a dot plot with at least 10 points.



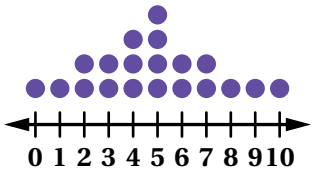
b Describe your dot plot in words.

Here are some dot plots from the Polygraph.

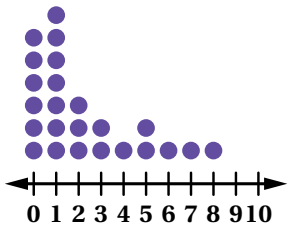
A.



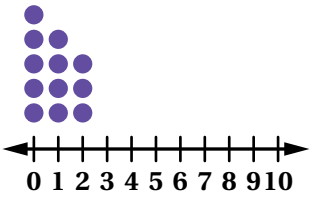
B.



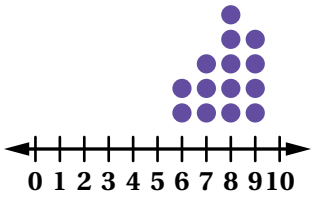
C.



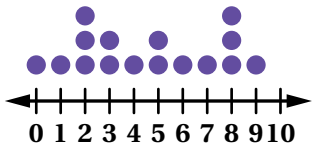
D.



E.



F.



3 Lola wrote: *My dot plot has a small* **spread**.

List *all* the dot plots that could match her description.

4 Brielle wrote: *My dot plot has a* **center** at 1.

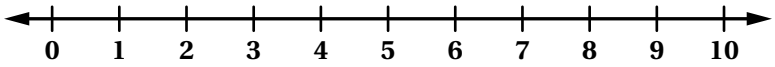
List *all* the dot plots that could match her description.

Name: ..... Date: ..... Period: .....

Exploring Center and Spread (continued)

5 Taylor described the **shape** of their data: *My dot plot has most of the data on the left.*

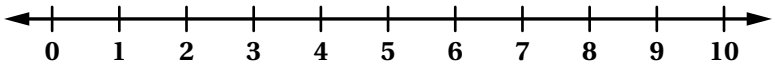
Create a dot plot that matches Taylor's description.



6 Ebony made a dot plot and wrote:

- *The center is at 7.*
- *There is a large spread.*
- *The shape looks like mountains.*

Create a dot plot that matches Ebony's description.



Activity 2

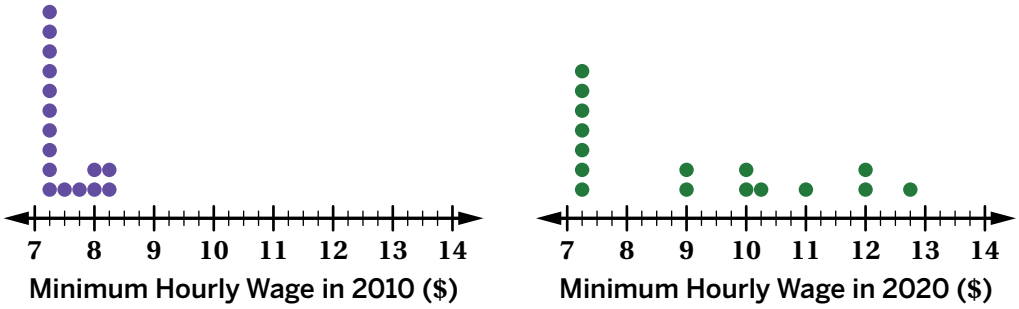
Name: ..... Date: ..... Period: .....

Comparing Dot Plots

7 The minimum hourly wage is the lowest amount that employers can pay their employees. Let's look at the minimum wages from 16 U.S. states in 2020. How do you think they compare to the minimum wages in 2010? Circle one.

Lower Higher About the same

8 **a** Take a look at two dot plots showing the minimum hourly wages of the same 16 states in 2010 and 2020.



b **Data Talk!**

- What do you think is the center of the 2010 data?
- What do you think is the center of the 2020 data?

9 Axel and Zoe studied the 2010 minimum wage data.

Axel said: *I think* $7.25 *is the center because it represents the most number of states.*

Zoe disagreed: *I think* $7.25 *is too low because some states have a higher minimum wage.*

Whose claim do you agree with? Circle one.

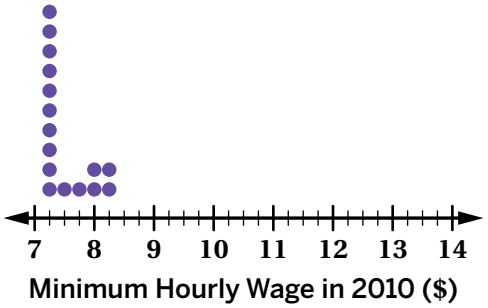
Axel's Zoe's Neither

Explain your thinking.

Name: Date: Period:

Comparing Dot Plots (continued)

10 **Data Talk!** Compare minimum wages from 2010 and 2020.





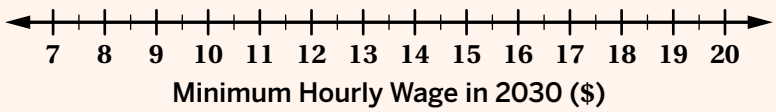
a Which year's data has a larger *spread*?

2010 2020 About the same

b What other questions do you have about this data?

You're invited to explore more.

11 Describe or show what you hope the minimum hourly wages of these states would be like in 2030.



12 Synthesis

Discuss: What do the terms *center*, *spread*, and *shape* tell you about a dot plot?

Use the example if it helps with your thinking.



Center:

Spread:

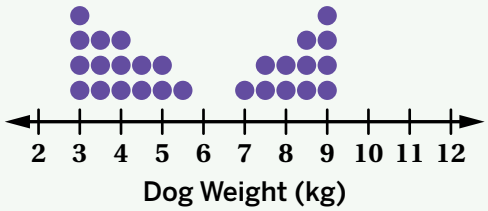
Shape:

15 Summary 8.04

You can use **center**, **spread**, and **shape** to describe the **distribution** of data on a dot plot.

For example, the distribution could be described as follows:

- There is a gap in the data between 5.5 and 7 kilograms.
- The data is spread out in two clumps, one with a peak at 3 kilograms and another peak at 9 kilograms.
- The center of the data is approximately 6 kilograms.
- The data is shaped like two triangles with the most dots on 3 kilograms and 9 kilograms.



center A single value that can be used to summarize the typical value in a data set.

distribution A collection of all of the data values and their frequencies. A distribution can be described by its features when represented visually, such as in a dot plot.

shape A description of the distribution (or pattern) of the data within a data set.

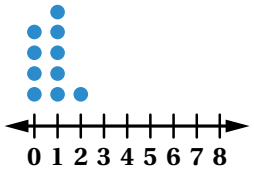
spread A description of how alike or different the values in a distribution are, often in relation to the center. The spread also describes the variability of a distribution.

Practice
8.04

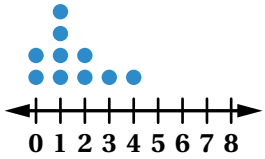
Name: ..... Date: ..... Period: .....

1. Which dot plot has a center at 6 and a small spread?

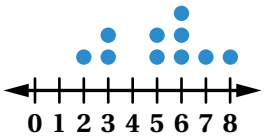
A.



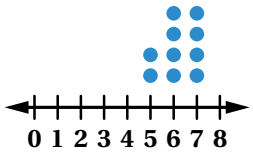
B.



C.

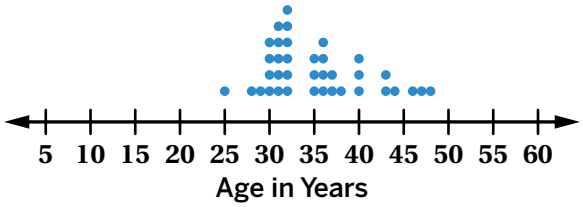


D.

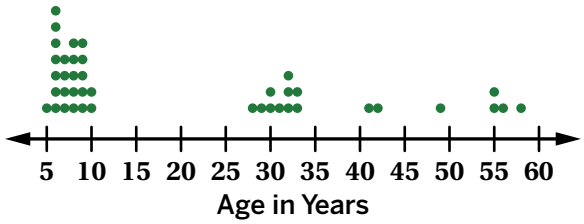


Problems 2–5: These dot plots represent the ages of people who watched two different movies at a movie theater.

Movie A



Movie B



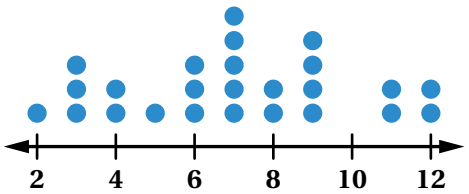
2. How are the dot plots alike?

3. How are they different?

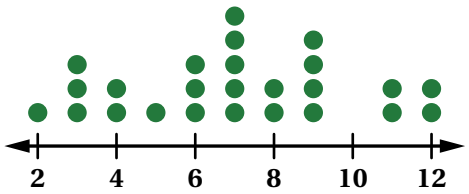
4. What is the typical age of people watching Movie A?

5. What kind of movie do you think Movie B is? Explain your thinking.

6. Cross off 5 points to shift the center to the *left*.



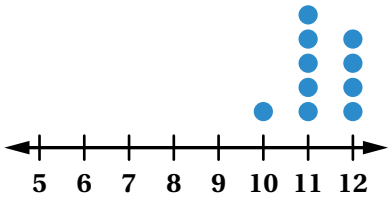
7. Cross off 5 points to shift the center to the *right*.



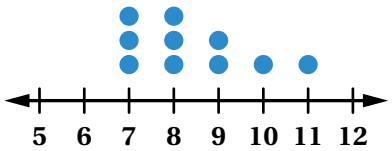
Name: _____ Date: _____ Period: _____

8. Jamir made three different dot plots about the students in a sixth-grade class, but he forgot to add labels. Match each dot plot to its missing label.

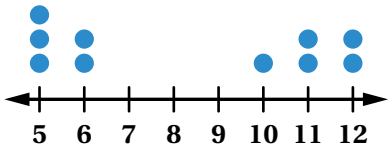
a



b



c



_____ Number of Books Read This Year

_____ Age in Years

_____ Number of Hours of Sleep on School Nights

Spiral Review

9. Select *all* of the numbers that are greater than 5.

☐ **A.** -9 ☐ **B.** 3 ☐ **C.** -6 ☐ **D.** |-9| ☐ **E.** 7

10. Select *all* of the numbers that are less than -8.

☐ **A.** -15 ☐ **B.** |-12| ☐ **C.** -9 ☐ **D.** 12 ☐ **E.** |-10|

Problems 11–14: Determine the value of each expression.

11. $3.72 + 1.38$

12. $3.72 - 1.38$

13. $6.05 \cdot 2.5$

14. $6.05 \div 2.5$

Name: ..... Date: ..... Period: .....

Variability in Data 6.SP.4, 6.SP.5, 6.SP.5.a, SMP.1, SMP.3

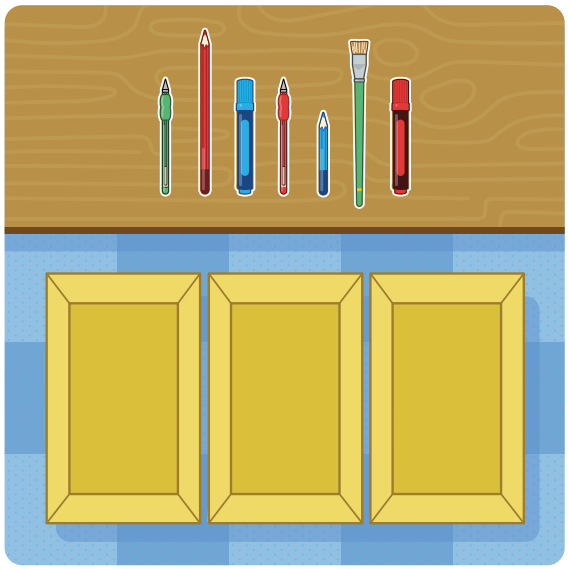


The Plot Thickens

Let's use histograms to represent data sets.

Warm-Up

1 a Organize the art supplies into the bins.



b Explain how you organized the supplies.

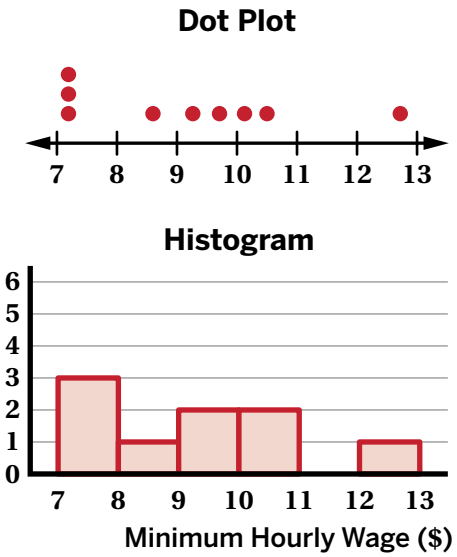
Activity 1

Name: Date: Period:

Introduction to Histograms

2 Here are two different ways to show the minimum wages of 9 U.S. states in 2020.

Data Talk! How do you think the **histogram** was made?

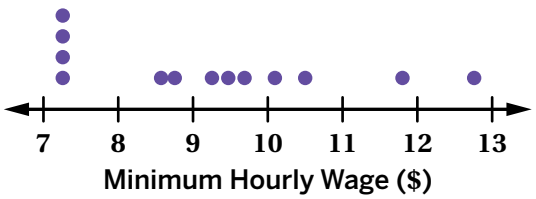


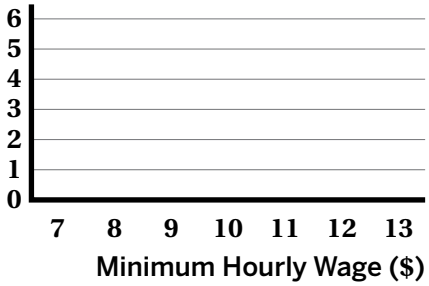
3 How are the histogram and the dot plot alike?

How are they different?

4 More states were added to the dot plot. Each bar in a histogram is called a **bin**.

Draw bins to make a histogram with this new data.





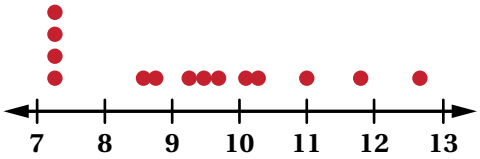
Activity 2

Name: ..... Date: ..... Period: .....

Creating Histograms

5 In statistics, each bin includes the smaller of the two numbers and goes up to but does not include the larger number. When a data point is on the edge between two bins, it goes into the bin with the larger values.

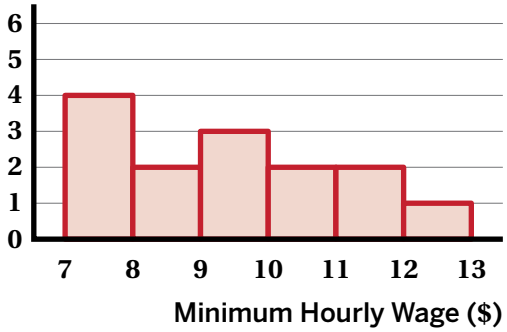
a $11 was added to this dot plot. Let's watch which bin the $11 goes into.



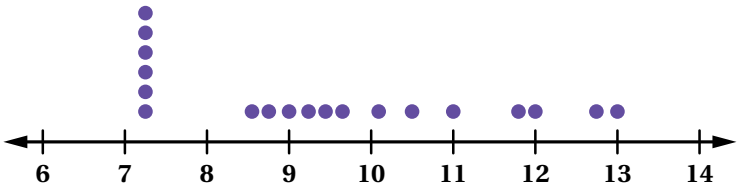
b Which bin would $8 go into? Circle one.

7 to 8 8 to 9 Somewhere else

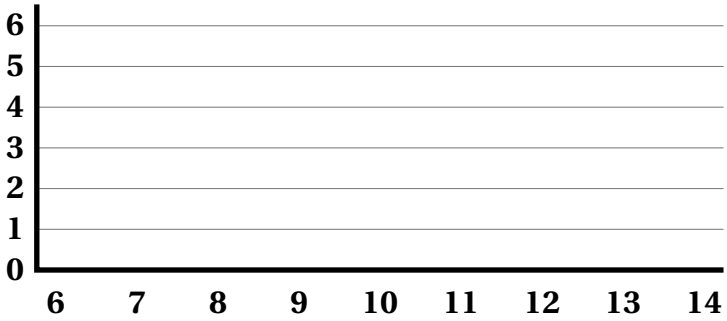
Explain your thinking.



6 So far, we've seen histograms with a bin size of 1. Here is a dot plot with some more minimum wages.



Make a histogram with a bin size of 2.



Activity 3

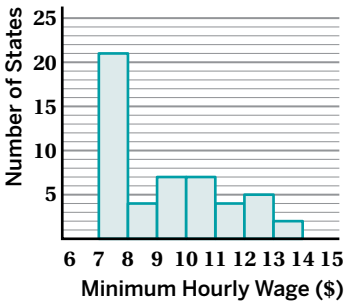
Name: Date: Period:

Using Histograms

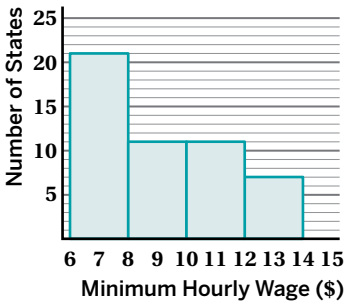
7 **Data Talk!** Let's look at the minimum wages for all 50 U.S. states. Why might these data points be difficult to visualize as a dot plot?

8 Each of these histograms show the minimum wages of all 50 states in 2020.

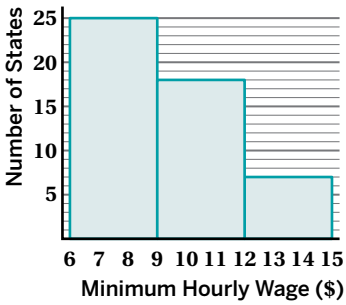
Bin Size 1



Bin Size 2



Bin Size 3



Discuss: What do you notice? What do you wonder?

9 Different visualizations like dot plots and histograms help answer different questions.

a Which of these questions can a histogram help answer?

A. Which state has the highest minimum wage?

B. What is the highest minimum wage?

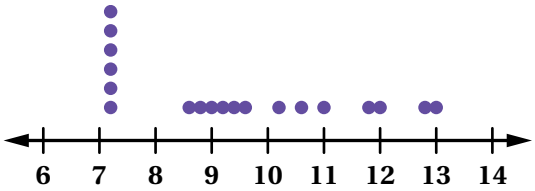
C. How many states have a minimum wage of $7.25?

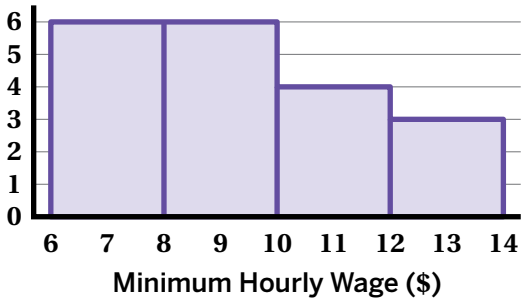
D. How many states have a minimum wage of $10 or more?

b Answer the question you chose.

10 Synthesis

What is important to remember about a histogram?

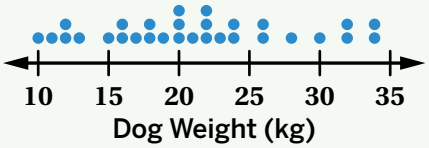




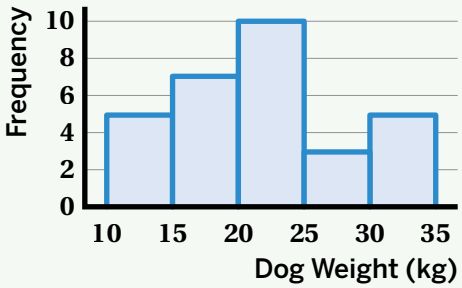
Summary 8.05

You can use dot plots and **histograms** to visualize numerical data. Here is an example of a data set of the weights of 30 dogs presented in a dot plot and in a histogram.

Dot Plot



Histogram



In a histogram, data values are grouped into bins that cover a range of values, and each **bin** represents the total number of values in that range, including the left boundary (least value) but excluding the right boundary (greatest value). For example, the height of the tallest bar, from 20 to 25, represents weights of 20 kilograms up to (but not including) 25 kilograms.

bin (of a histogram) The interval used to group data values in a histogram.

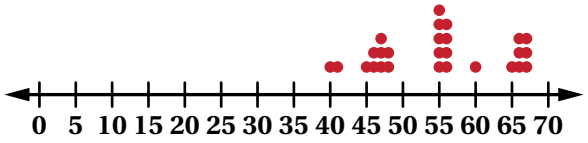
histogram A way to visualize numerical data where the data is grouped into bins represented by rectangles. The height of each rectangle shows how many values are in that bin.

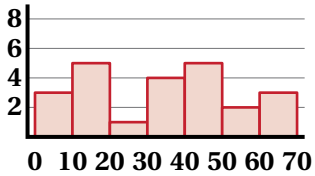
Practice
8.05

Name: ..... Date: ..... Period: .....

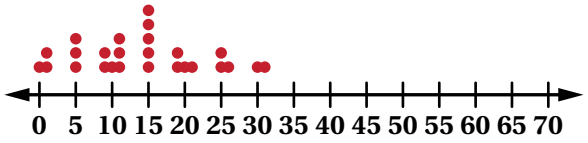
1. Match each histogram with the dot plot that represents the same data set.

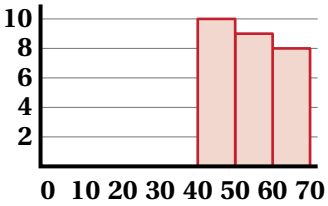
a



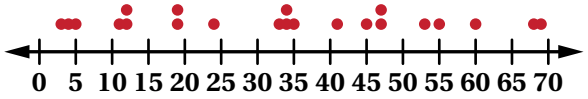


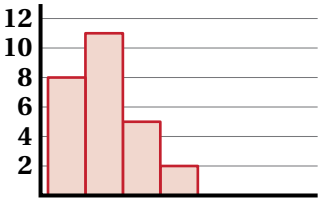
b



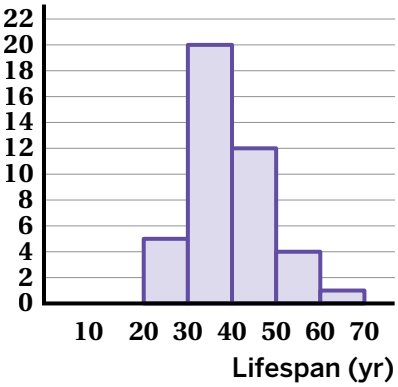


c

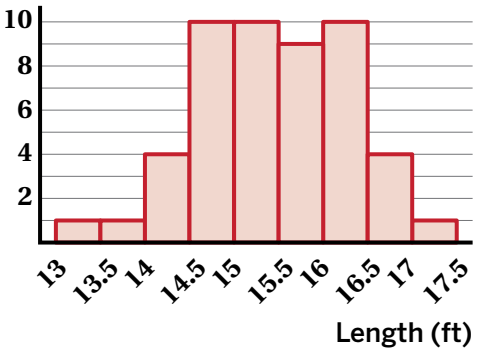




2. Here is a histogram showing the lifespans of 42 chimpanzees that lived in the wild. How many chimpanzees lived at least 50 years and less than 70 years?



Problems 3–4: A marine biologist is studying a group of sharks. She made a histogram of the lengths of a group of adult sharks.



3. Select *all* of the false statements

- ☐ **A.** A total of 9 sharks were measured.
- ☐ **B.** Two of the sharks are less than 14 feet long.
- ☐ **C.** A typical shark is about 14.5 to 16.5 feet long.
- ☐ **D.** The longest shark measured was 10 feet long.
- ☐ **E.** The smallest shark measured was 11 feet long.

4. Explain your thinking for one of the false statements.

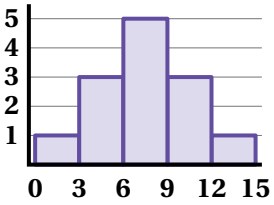
Name: ..... Date: ..... Period: .....

Problems 5–6: Here are three representations of the same data set.

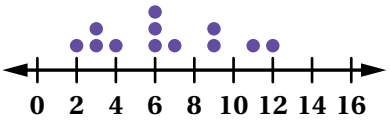
List

2, 3, 3, 4, 6, 6, 6, 7, 9, 9, 11, 12

Histogram



Dot Plot



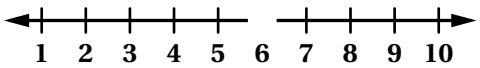
5. Circle the representation that doesn't match the other two.

6. Revise your circled representation to make it match the other two representations.

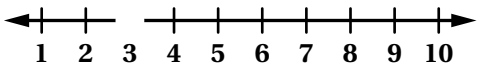
Spiral Review

Problems 7–8: Graph each inequality on a number line.

7. $m > 6$



8. $3 > n$



Problems 9–12: Determine the value of each expression.

9. $3.727 + 1.384$

10. $3.727 - 1.384$

11. $5.01 \cdot 4.8$

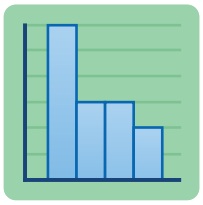
12. $5.01 \div 4.8$

13. Zoe has $9.78 to spend on snacks at the baseball game. She buys a bucket of popcorn for $3.50 and a sports drink for $4.75. How much money does she have left?

Name: ..... Date: ..... Period: .....

Variability in Data | The Shape of Distributions | 6.SP.2, 6.SP.4, 6.SP.5, 6.SP.5.a, SMP.1, SMP.2, SMP.3

DIY Histograms



Let's create histograms to represent and compare wages.

Warm-Up

In 2020, the minimum wage in Iowa was $7.25 an hour. The National Low Income Housing Coalition reported that the wage needed to afford rent for a two-bedroom apartment in Iowa was $ _____ an hour.

1. Make a prediction for the wage needed to afford rent for a two-bedroom apartment.

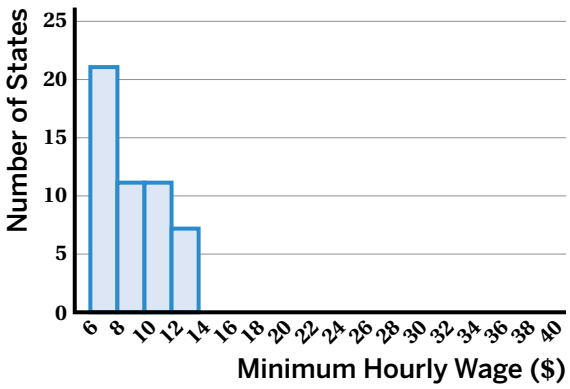
Let's look at the real rate together.

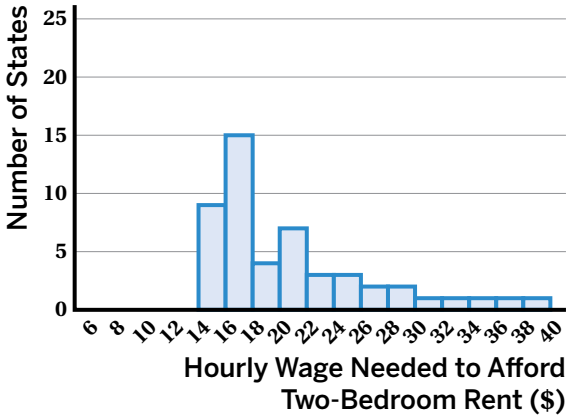
2. **Discuss:** How does the minimum wage compare to the cost to rent a two-bedroom apartment?

Name: ..... Date: ..... Period: .....

Out of Reach

3. **Data Talk!** These histograms show the minimum wages and the wages needed to afford rent for all 50 states in 2020.





a How would you describe the shape of each histogram?

b What do you think is the center of the histogram showing the minimum hourly wage? Explain what you think this number means.

c What do you think is the center of the histogram showing the wage needed to afford rent? Explain what you think this number means.

d Which histogram has a greater spread?

e Why do you think that might be the case?

4. This data comes from a report called "Out of Reach." Use the histograms to explain why it might be called that.

Activity 2

Name: Date: Period:

Comparing Regions

5. Yosef started to make a *frequency table* for this data set.

Frequency Table

Hourly Wages	Frequency
\$6 to less than \$8	𝍸
\$8 to less than \$10	|||
\$10 to less than \$12	

Hourly Wages		
\$7.25	\$11.00	\$9.25
\$7.25	\$11.00	\$7.25
\$8.56	\$11.99	\$7.25
\$7.25	\$9.00	\$10.50

a Complete the last row of Yosef's frequency table.

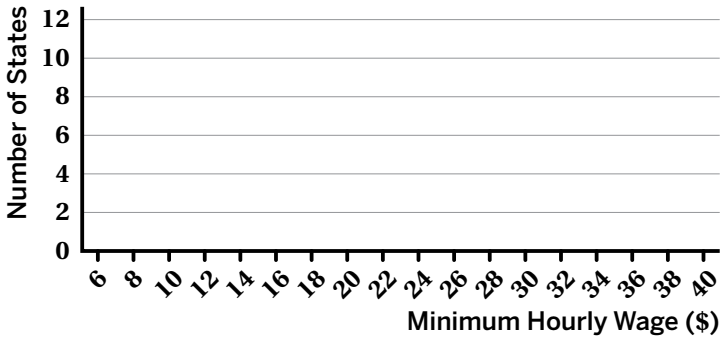
b How could a frequency table help Yosef make a histogram of this data?

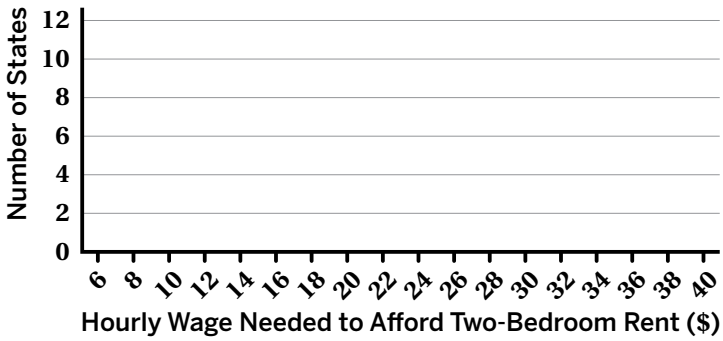
6. You'll use the Activity 2 Sheet to create two histograms with your partner.

Pick a region of the United States. Then create:

- A histogram representing the minimum hourly wage in that region.
- A second histogram representing the hourly wage needed to afford two-bedroom rent in that region.

Make a frequency table if it helps with your thinking.





Name: ..... Date: ..... Period: .....

Comparing Regions (continued)

7. **Discuss:**

- How are your histograms alike? How are they different?
- What do you hope these histograms will look like in the future?

8. **Data Talk!** Here are Tyrone's and Mayra's arguments.

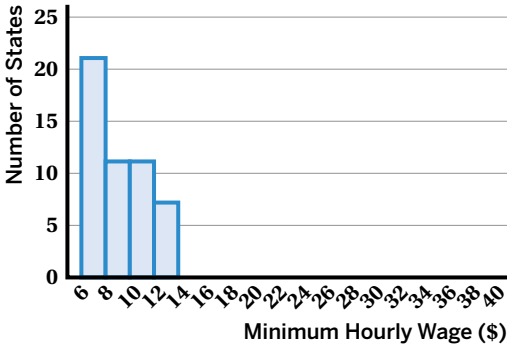
Tyrone says: *Wages would be more fair in the future if both histograms had the same center.*

Mayra says: *Wages would be more fair in the future if both histograms had the same spread.*

Who do you agree with more? Explain your thinking.

Synthesis

9. How does making histograms help us compare data sets?

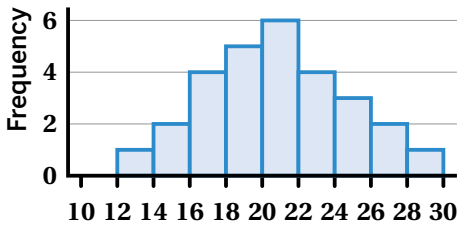




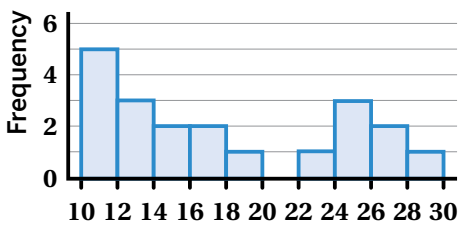
Summary 8.06

You can use histograms to compare numerical data sets using their shape, center, and spread. Here are two histograms with very different shapes and features.

Histogram A



Histogram B



Shape	Histogram A is shaped like a mountain. It is symmetric with a peak around 21.	Histogram B has two clusters, with peaks around 11 and 25. There is a gap in the data between 20 and 22.
Center	The center is approximately 21.	The center is approximately 17.
Spread	Most values are in the middle and there are roughly the same amount on each side 21.	Most values are on the left then get lower as it goes to the right.

Practice
8.06

Name: _____ Date: _____ Period: _____

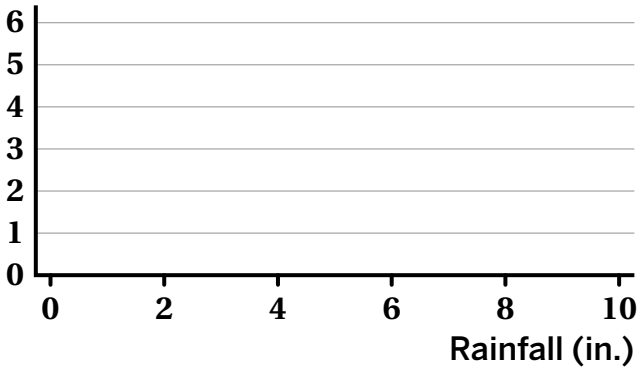
Problems 1–2: Here is the average amount of rainfall, in inches, for each month last year in Vicente's hometown.

Average Rainfall (in.)					
1.67	2	2.99	6.1	8.9	9.84
3.14	5.35	9.69	6	3.27	2.05

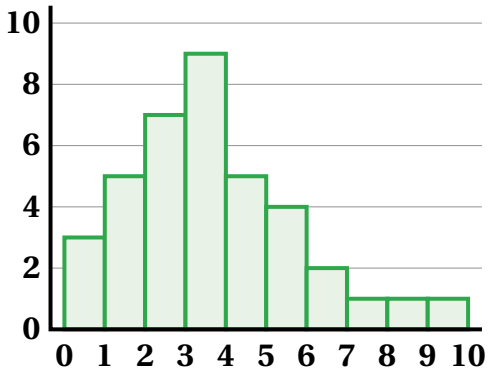
1. Complete the frequency table.

Rainfall (in.)	0 to less than 2	2 to less than 4	4 to less than 6	6 to less than 8	8 to less than 10
Frequency					

2. Create a histogram using the data about Vicente's hometown.



Problems 3–6: Here is a histogram.



3. Which of these situations could this histogram represent?

A. The height of a group of sixth-grade students (in inches).

B. The weight of students' backpacks (in pounds).

C. The temperature for 38 days in summer (in degrees).

D. The time it takes a group of students to walk to school (in hours).

Explain your thinking.

4. Describe the center of the histogram.

5. Describe the shape of the histogram.

6. Describe the spread of the histogram.

Practice
8.06

Name: ..... Date: ..... Period: .....

Spiral Review

7. Order these numbers from *least* to *greatest*.

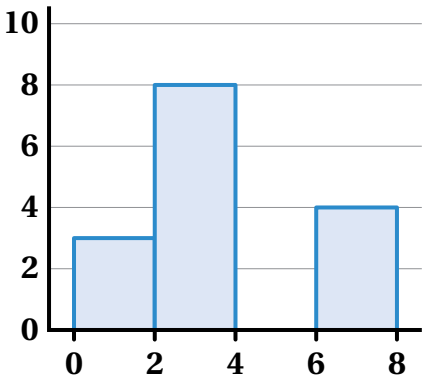
-6 $\quad 0.6 \quad$ $\frac{1}{6}$ $\quad 6 \quad -1.6$

Least **Greatest**

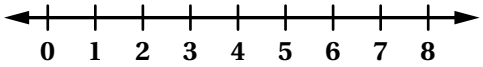
8. Rebecca drank 12 ounces of water from her water bottle, which is 60% of the volume of the bottle. How much water can the bottle hold when full? Use the double number line if it helps with your thinking.



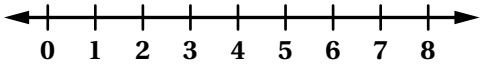
Problems 9–10: Create two possible dot plots for this histogram.



9.



10.



11. The point $(-2, 3)$ is one vertex of a square on a coordinate plane. Name three points that could be the other vertices.

12. The points $(-4, 0)$ and $(0, -3)$ are two vertices of a rectangle on a coordinate plane. Name two points that could be the other vertices.

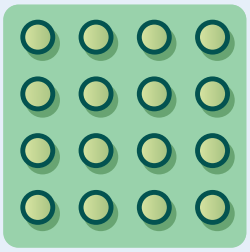
Sub-Unit

2

Measuring Data: Mean and MAD



Lesson 7
Snack Time



Lesson 8
Pop It!



Lesson 9
Hoops



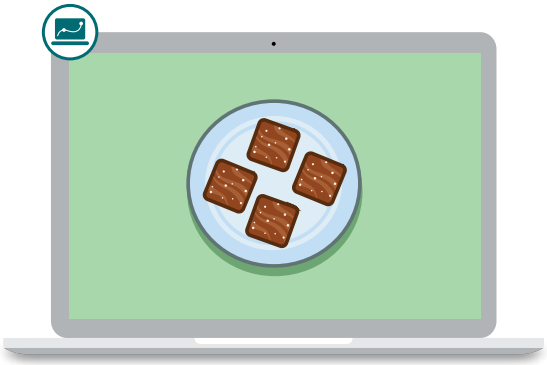
Lesson 10
Hollywood, Part 1

Name: ..... Date: ..... Period: .....

Variability in Data | The Shape of Distributions | 6.SP.5.c, 6.SP.3, 6.SP.5, SMP.3, SMP.6, SMP.8

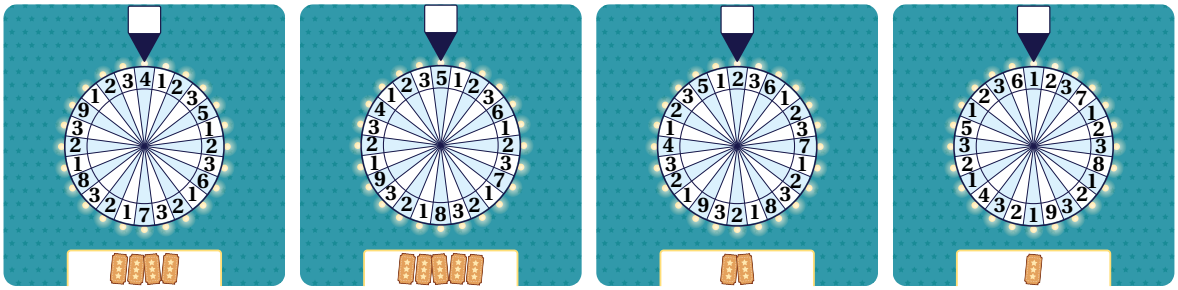
Snack Time

Let's explore the mean of a data set and what it tells us.



Warm-Up

1 Here are four images of an arcade game.



Discuss: What do you notice? What do you wonder?

Activity 1

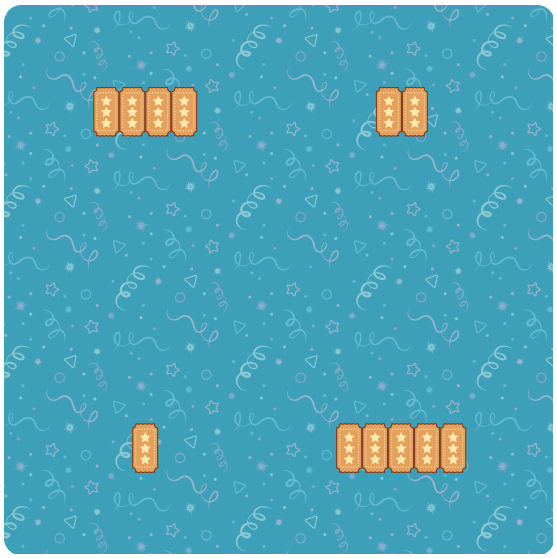
Name: ..... Date: ..... Period: .....

Mean as an Equal Share

2 4 friends played this game at the arcade. Here are the tickets each friend won.

They decided to share the tickets equally. How many tickets should each friend get?

Explain your thinking.



3 Here is Ava's work for determining how many tickets each friend gets.

Discuss What was Ava's strategy?

Ava

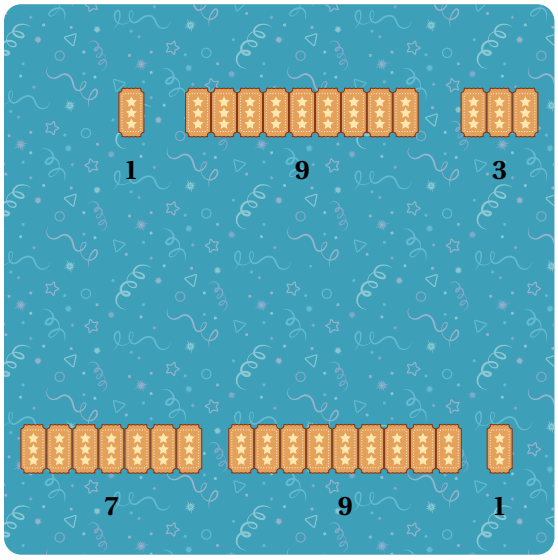
$4 + 2 + 1 + 5 = 12$

$12 \div 4$

4 The **mean**, or average, is the number of tickets each friend gets if the tickets are distributed equally.

Here are the tickets that 6 other friends won.

Calculate the mean number of tickets.



Name: ..... Date: ..... Period: .....

Mean as a Statistic

5 **Data Talk!** A **statistic** is a single number that measures something about a data set.

The mean is an example of a statistic.

What does the mean tell us about Ishaan's snacks?

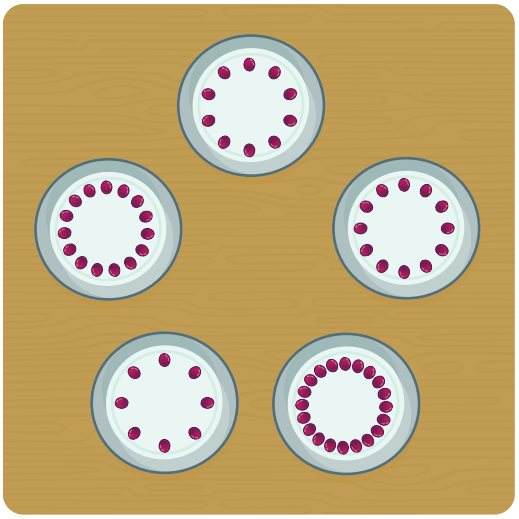
Ishaan's Snacks

Day	Number of Carrots
Monday	10
Tuesday	8
Wednesday	5
Thursday	12
Friday	10

Mean: 9 carrots

6 Here is how many grapes Joel ate each day last week. Calculate the mean of this data.

Day	Number of Grapes
Monday	10
Tuesday	12
Wednesday	20
Thursday	8
Friday	15



7 **Data Talk!** Ali and Makayla ate blueberries each day last week. Ali ate 12 blueberries per day. The table shows how many blueberries Makayla ate.

Who ate more blueberries per day on average? Circle one.

Ali Makayla They ate the same amount on average

Explain your thinking.

Ali's Snacks

12 blueberries per day

Makayla's Snacks

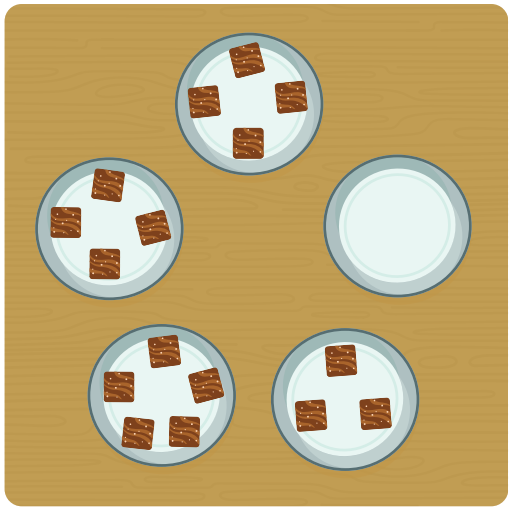
Day	Number of Blueberries
Monday	1
Tuesday	15
Wednesday	13
Thursday	11
Friday	15

Name: ..... Date: ..... Period: .....

Mean as a Statistic (continued)

8 Here is how many brownies Omari ate.

Day	Number of Brownies
Monday	4
Tuesday	0
Wednesday	3
Thursday	5
Friday	4

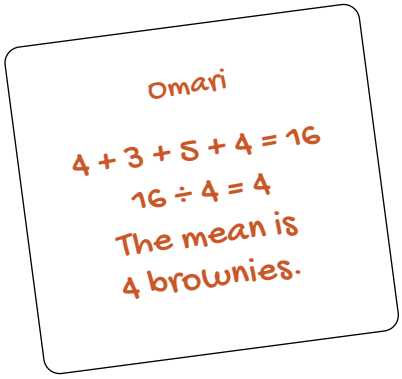


Calculate the mean of this data.

9 Omari calculated the mean incorrectly.

Discuss:

- What is correct about Omari's work?
- What is incorrect about Omari's work?



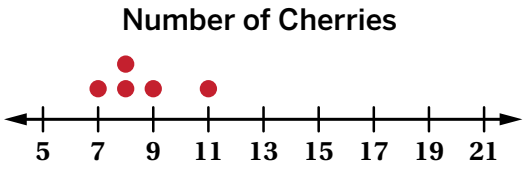
Activity 3

Name: ..... Date: ..... Period: .....

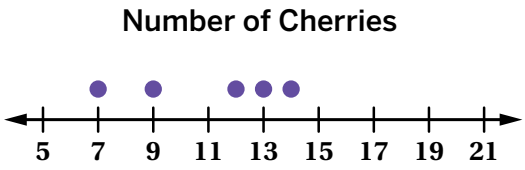
Mean as a Measure of Center

10 **Data Talk!** For 5 days, Oliver recorded the number of cherries he had as a snack. The mean for his data is 11 cherries. Circle a data set that could *not* be Oliver's.

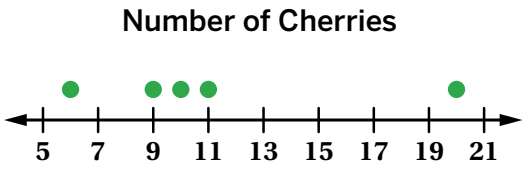
A.



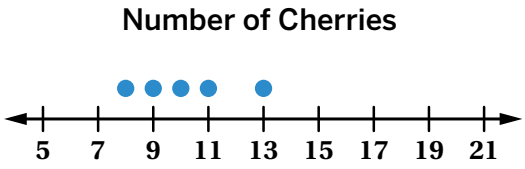
B.



C.



D.



Explain your thinking.

You're invited to explore more.

11 Add at least *four* more points to create a dot plot that has a mean of 7. Then check your work.

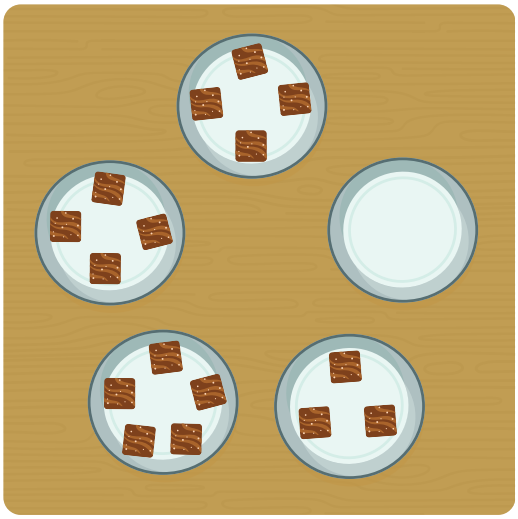
Create as many different dot plots as you have time for.



12 Synthesis

How can you determine the mean of a data set?

Use the example if it helps with your thinking.



15 Summary 8.07

A **statistic** is a single number that measures something about a data set. One way to measure the center of a data set is by determining the **mean**, or average, of all the data values. You can think of the mean as "an equal share."

For example, suppose this data set represents how many liters of water are in 5 bottles: $1, 4, 2, 3, 0$. To calculate the mean, you first add up all of the values to determine the total (10 liters), then divide that sum by the number of values (5 bottles). This example can be represented by the expression $(1 + 4 + 2 + 3 + 0) \div 5$, or $10 \div 5$. So, the mean amount of water in the 5 bottles is 2 liters (per bottle). The mean is a whole number in this example, but it is possible for the mean to be a decimal number.

mean A measure of center. If you equally distribute a set of items into different groups, the mean is the number of items in each group. It is also the balance point of a dot plot. To calculate the mean, you can add the values of all the data points, then divide by the number of data points.

measure of center A single number that summarizes all of the values in a data set. It is usually a typical value for a data set. Mean and median are measures of center.

statistic A quantity that is calculated from a data set, such as the mean, median, MAD, or IQR.

Practice
8.07

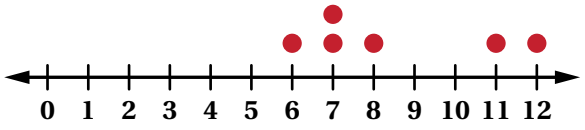
Name: ..... Date: ..... Period: .....

1. A preschool teacher plans to reorganize these 4 boxes of playing blocks so that each box contains an equal number of blocks. How could the teacher determine the number of blocks to put in each box?

Box 1	Box 2	Box 3	Box 4
32 blocks	18 blocks	41 blocks	9 blocks

2. 3 classes worked together to raise money for their classroom libraries. They agreed to share the money equally. The first class raised $25.50, the second class raised $49.75, and the third class raised $37.25. What is each class's equal share? Explain or show your thinking.

3. Kimaya guesses that 11 is the mean of this data set. Without calculating, determine if Kimaya's guess is correct. Explain your thinking.



Problems 4–5: For 12 days, Mai recorded how many minutes long her bus rides to school were. Here are the times she recorded.

Time on the Bus (min)											
9	12	6	9	10	7	6	12	9	8	10	10

4. Determine the mean for Mai's data. Explain or show your thinking.

5. What does the mean tell us about Mai's trip to school?

6. In her English class, Lan's teacher gives 4 quizzes, each worth 5 points. After 3 quizzes, Lan has scores of 4, 3, and 4. How many points does Lan need to get on the last quiz to have an average score of 4? Explain or show your thinking.

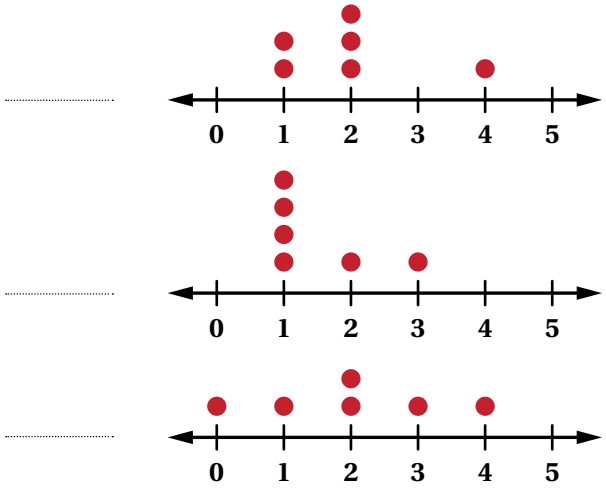
Name: _____ Date: _____ Period: _____

7. Select a dot plot that matches each statement. You can only select each dot plot once.

a The mean is 2.

b Only one value is greater than the mean.

c Two values are greater than the mean.



8. In a round of mini golf, Angel records the number of putts on each hole.

Number of Putts								
2	3	1	4	5	2	3	4	3

What is the mean number of putts per hole? Explain or show your thinking.

Spiral Review

Problems 9–11: Evaluate the expression $4x^3$ for each value of x.

9. 1 **10.** 2 **11.** $\frac{1}{2}$

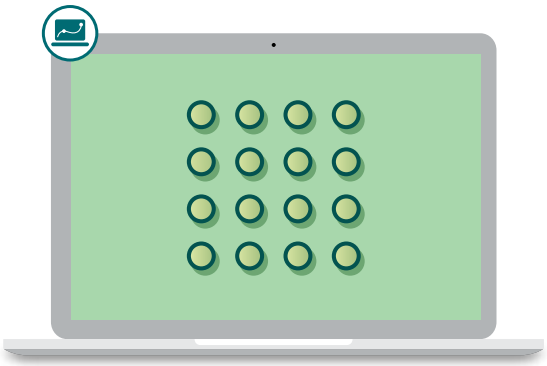
12. Select *all* of the values of x that are solutions to the inequality $x > -3$.

☐ **A.** -2 ☐ **B.** $|-4|$ ☐ **C.** -4 ☐ **D.** 3.5 ☐ **E.** -3.5

13. Explain how the solution to $x > -3$ compares to the solution to $x \geq -3$.

Name: ..... Date: ..... Period: .....

Variability in Data | The Shape of Distributions | 6.SP.3, 6.SP.5, 6.SP.5.c, SMP.2, SMP.3, SMP.6



Pop It!

Let's study distances between data values and the mean.

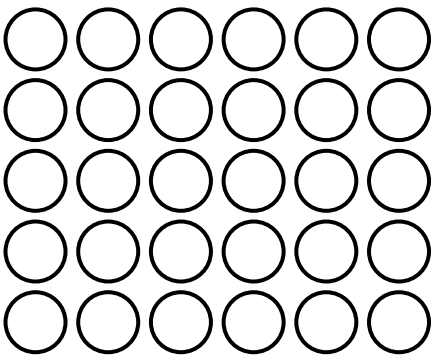
Warm-Up

1-2 Here are two sheets of bubble wrap.

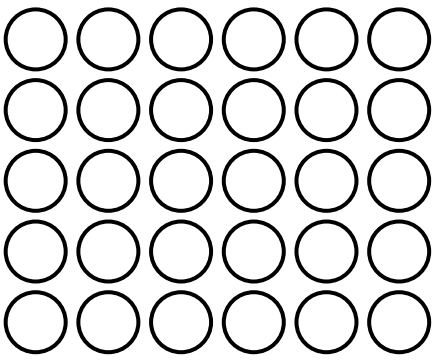
How many bubbles on Sheet 1 can you pop in five seconds?

- Fill in each bubble with a colored pencil or highlighter to "pop" it.
- Then try again with Sheet 2.
- Your partner will use a stopwatch to time you.

Attempt 1



Attempt 2



3 **Data Talk!** Let's look at the class data showing how many bubbles everyone popped in five seconds.

Discuss: Do you think the hollow red dot represents a person who is good at popping bubbles?

Activity 1

Name: ..... Date: ..... Period: .....

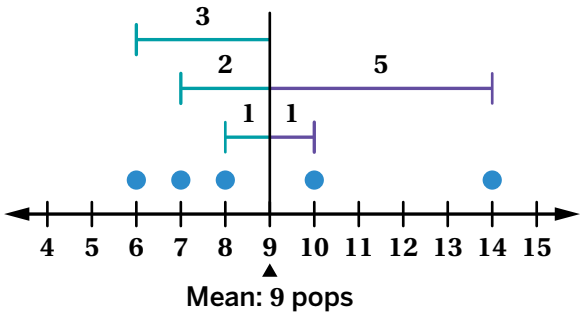
A New Property of the Mean

4 The dot plot shows how many bubbles Jalen and his friends popped in five seconds.

The distance between a data point and the mean is called an **absolute deviation**.

a Take a look at the absolute deviation for each data point.

b What do you notice? What do you wonder?



I notice:

I wonder:

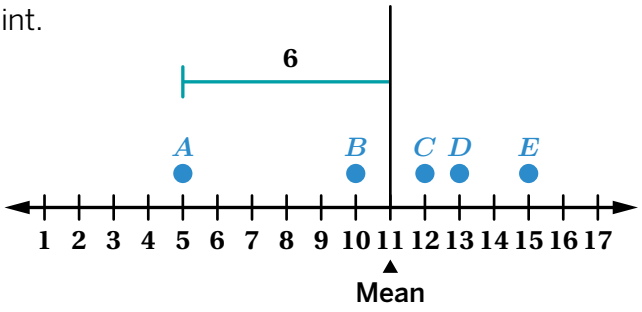
5 *The sum of the absolute deviations to the right of the mean is equal to the sum of the absolute deviations to the left of the mean.*

a Let's watch an animation to see what this looks like.

b Write the italicized sentence in your own words.

6 Determine the absolute deviation of each point.

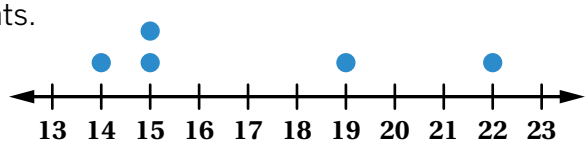
Point	Absolute Deviation
A	6
B	
C	
D	
E	



Name: ..... Date: ..... Period: .....

Put This Property to Work

7 **Data Talk!** Here is a data set with 5 points.

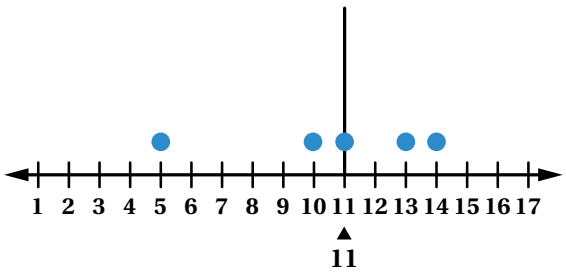


a. Mark the mean of the data set on the dot plot.

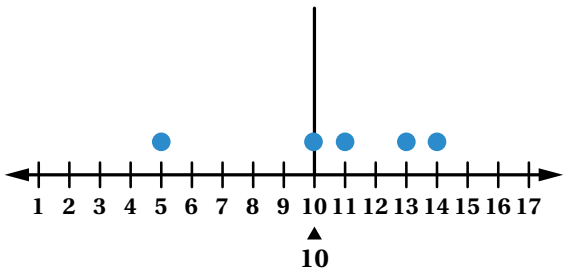
b. **Discuss:** How can absolute deviations help check if your mean is correct?

8 Demetrius and Josiah marked the mean of a data set.

Demetrius



Josiah



Whose mean is correct? Circle one.

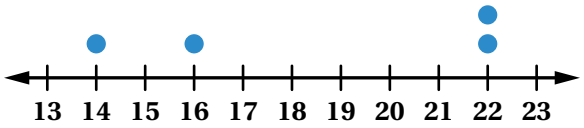
Demetrius's Josiah's Both Neither

Explain your thinking.

Name: ..... Date: ..... Period: .....

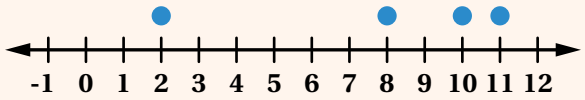
Put This Property to Work (continued)

9 Calculate the mean of this data. Use absolute deviations if it helps with your thinking.



You're invited to explore more.

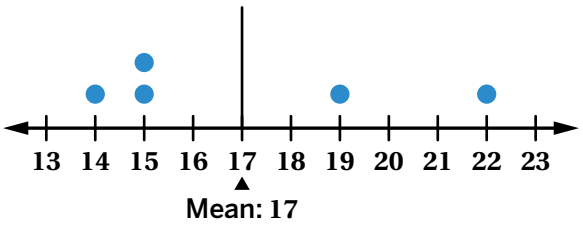
10 The mean of this data is not a whole number. If you add just one point, is it possible to make the mean equal to 1? 2? 3? . . . 10? Determine a point to add to the dot plot. Record each possible mean.



11 Synthesis

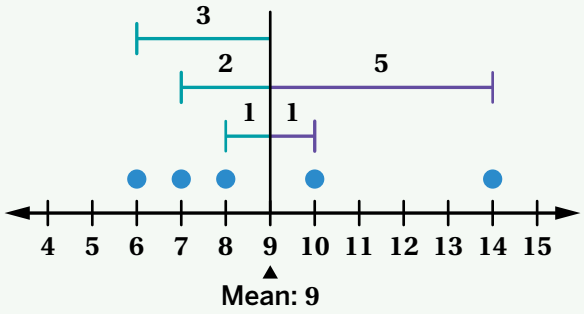
Discuss: How can you use absolute deviations to decide whether a number is the mean of a data set?

Use the example if it helps with your thinking.



14 Summary 8.08

The dot plot below shows a data set with a mean of 9. The absolute deviation of the point at 14 is 5 because 14 is 5 units away from 9.



The sum of the absolute deviations to the right of the mean is equal to the sum of the absolute deviations to the left of the mean. You can use this to check whether a number is the mean of a data set. For example, if 9 is the correct mean of the data set shown, the sum of the absolute deviations to the left of 9 should be equal to the sum on the right side.

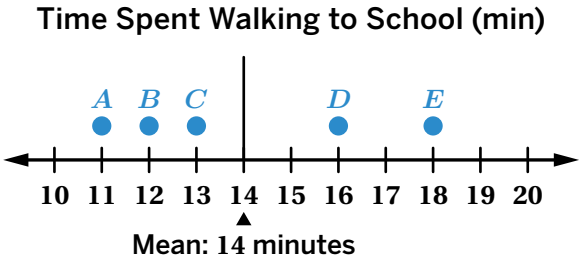
- The sum of the deviations on the left is $3 + 2 + 1 = 6$.
- The sum of the deviations on the right is $5 + 1 = 6$.
- The sums are equal so 9 is the correct mean.

absolute deviation The distance between a value and the mean of a data set.

Practice
8.08

Name: ..... Date: ..... Period: .....

Problems 1–2: The dot plot shows how long, in minutes, Gabriela spends walking to school on 5 different days.

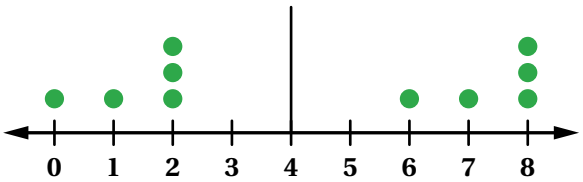


1. Record the distance between each point and the mean.

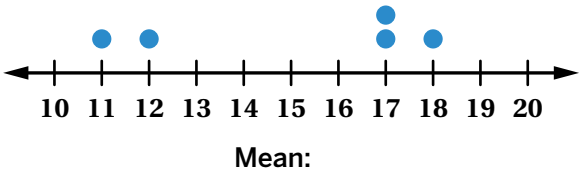
2. Explain or show how the distances are related to the mean.

Point	Distance From the Mean
A	
B	
C	
D	
E	

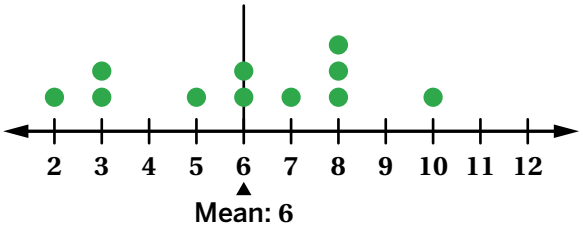
3. Amir says the mean of this data is 4. Is he correct? Explain or show your thinking using absolute deviations.



4. Calculate the mean of this data. Explain or show your thinking using absolute deviations.



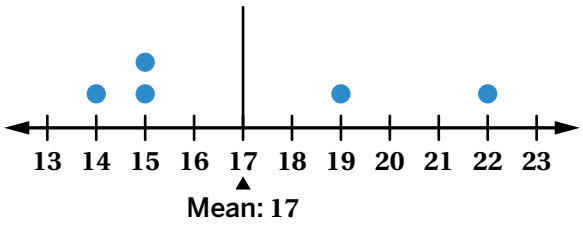
5. Add 5 points to this dot plot without changing the mean. Explain your thinking.



6. In a basketball game, LaShawn scored 20 points and Jacy scored 30 points. The mean number of points that LaShawn, Jacy, and Tyler scored for that game was 40. How many points did Tyler score in that game? Explain or show your thinking.

7. Determine whether each statement about the dot plot is true or false.

Statement	True	False
Adding a point at 15 will increase the mean.		
Adding a point at 20 will increase the mean.		
Adding a point at 17 will decrease the mean.		



Spiral Review

8. Raven thinks that $x = 3$ is the solution to the equation $x^2 = 8$. Is Raven correct? Explain your thinking.

9. Order these numbers from least to greatest.

-9	|-10|	|11|	12	-13

Least **Greatest**

Problems 10–13: Compare the numbers using >, <, or =.

10. $3 \square -4$ **11.** $|-4| \square |4|$ **12.** $|-15| \square |12|$ **13.** $-12 \square 15$

Name: ..... Date: ..... Period: .....

Variability in Data | The Shape of Distributions | 6.SP.5.c, 6.SP.2, 6.SP.3, 6.SP.5, SMP.2, SMP.6



Hoops

Let's explore how to measure the spread of a data set.

Warm-Up

1-2 You will use the Warm-Up Sheet for this activity.

How many baskets can you make?

- Flick the counter from each of the starting points. If the counter lands on top of the hoop, you made a basket.
- You get 8 shots per round.
- Try to make as many as you can!

Round 1: ..... baskets Round 2: ..... baskets

3 Let's look at the Round 1 and Round 2 results from the class.

Discuss: What do you notice?

Activity 1

Name: ..... Date: ..... Period: .....

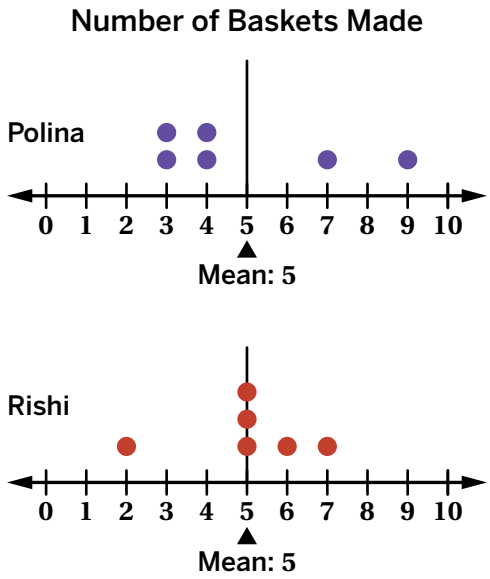
Mean Absolute Deviation

4 **Data Talk!** Polina and Rishi practice 10 free throws everyday. Each dot plot shows the number of baskets they made on 6 different days.

Which player is more consistent? Circle one.

Polina Rishi

Explain your thinking.



5 The **mean absolute deviation (MAD)** is a statistic that measures the spread of a data set. We can use this **measure of spread** to determine how consistent Polina is at making baskets. Polina's data has a MAD of 2 baskets.

Will the MAD of Rishi's data be larger than, smaller than, or equal to the MAD of Polina's data? Circle one.

Larger Smaller Equal

Explain your thinking.

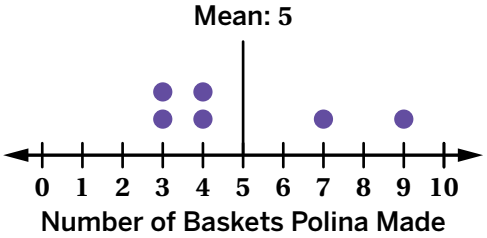
Activity 2

Name: ..... Date: ..... Period: .....

Calculating MAD

6 **Data Talk!** Here is Polina's data.

Number of Baskets	3	3	4	4	7	9
Absolute Deviation (distance from the mean, 5)	2	2	1	1	2	4



Mean Absolute Deviation (MAD)

Sum of absolute deviations:
$2 + 2 + 1 + 1 + 2 + 4 = 12$

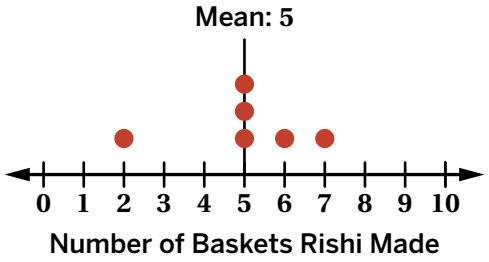
Average or mean of the absolute deviations:
$12 \div 6 = 2$

Discuss: What do you notice? What do you wonder?

7 Let's watch an animation to see how Polina calculated the *mean absolute deviation*. Then explain how to calculate the *mean absolute deviation* in your own words.

8 Here is Rishi's data.

Calculate the MAD. Use the table to help you organize your thinking.



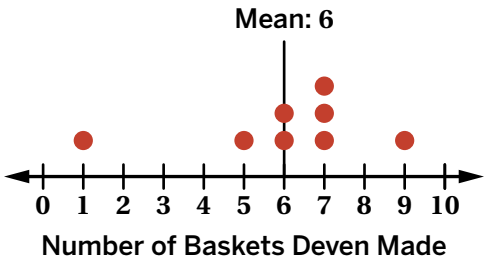
Number of Baskets	2	5	5	5	6	7
Absolute Deviation (distance from the mean, 5)						

Name: Date: Period:

Calculating MAD (continued)

9 Deven also practices free throws everyday.

Calculate the MAD of Deven's data. Use the table to help you organize your thinking.



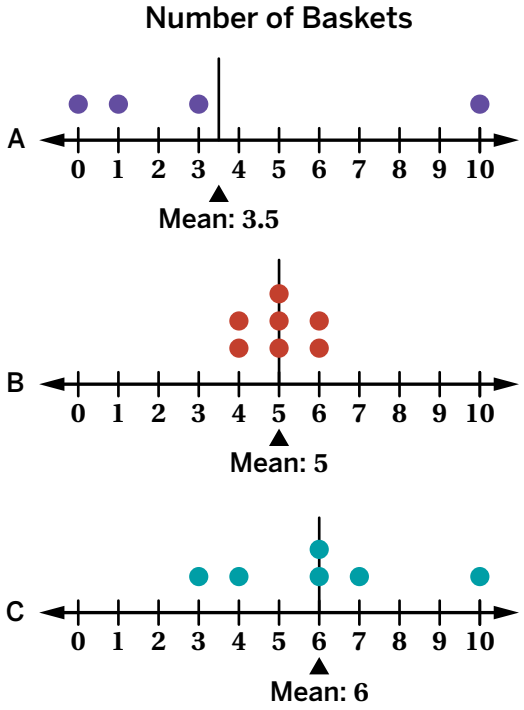
Number of Baskets	1	5	6	6	7	7	7	9
Absolute Deviation (distance from the mean, 6)								

10 Here are the number of baskets that three new players made.

Order the data sets from *smallest* to *largest* MAD.

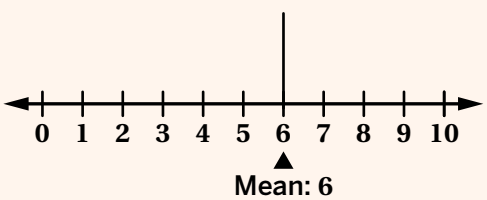
Smallest MAD

Largest MAD



You're invited to explore more.

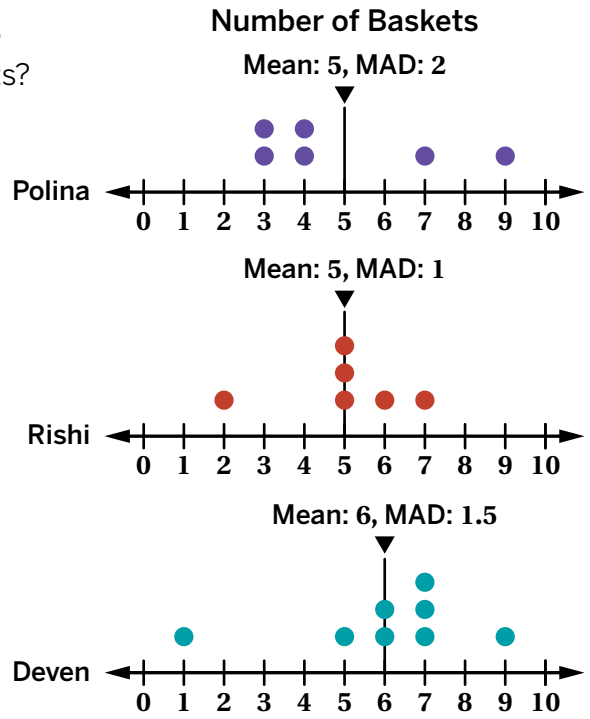
11 Create a dot plot with at least *four* points, a mean of 6, and a MAD of 1.



12 Synthesis

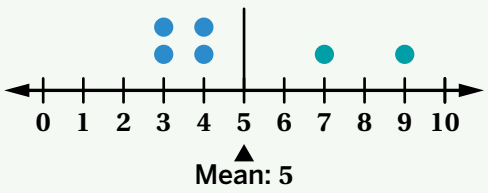
Discuss: How does the mean absolute deviation (MAD) help you compare data sets?

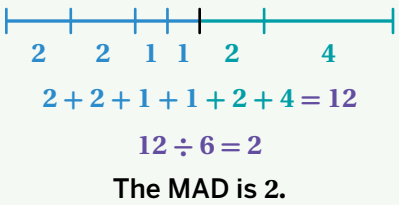
Use the examples if they help with your thinking.



15 Summary 8.09

You can describe how spread out the values in a data set are with a single number, the **mean absolute deviation (MAD)**. The MAD is calculated by determining the mean of the absolute deviations (e.g., the average of the distances between each data value and the mean).





The mean absolute deviation is an example of a **measure of spread**, (or **measure of variability**). A measure of spread is a way to measure the consistency of the values in a data set. The smaller the value of the MAD, the less spread out the data points are around the mean, and the more consistent the data is. The larger the MAD, the more spread out the data points are around the mean, and the less consistent the data is.

mean absolute deviation (MAD) A statistic calculated by adding the distances between each data value and the mean, then dividing by the number of data values in the set.

measure of spread (measure of variability) A single number that describes the spread of a data set. Range, interquartile range, and mean absolute deviation are measures of spread.

Practice
8.09

Name: ..... Date: ..... Period: .....

Problems 1–2: This table shows the amount of time it takes 6 students to get to school. Their mean travel time is 22 minutes.

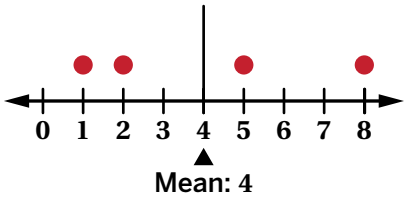
Time (minutes)	10	10	18	20	30	44
Absolute Deviation						

1. Complete the table by calculating the absolute deviation of each value from the mean.

2. Calculate the mean absolute deviation (MAD) of this data set.

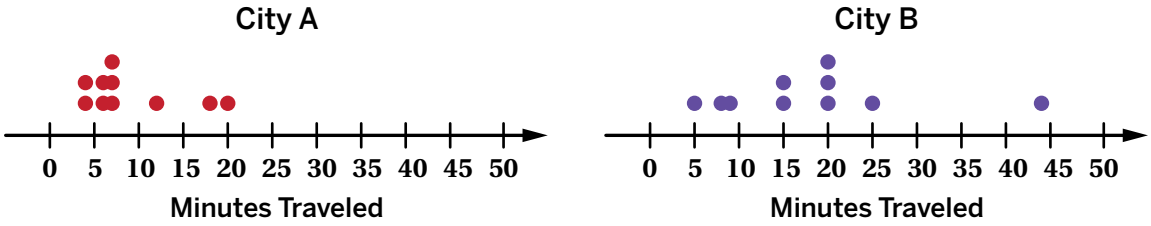
3. Calculate the MAD of this data set.

Use the table if it helps with your thinking.



Data Point				
Absolute Deviation				

Problems 4–5: These dot plots show the travel times for 10 students from two cities.



4. The MADs have been calculated for you. Match each MAD to the correct city.

MAD	City
4.4	
7.8	

5. Without calculating, explain how you matched the MADs to the data sets.

Practice
8.09

Name: ..... Date: ..... Period: .....

Problems 6–7: Here are two dot plots showing the recorded speeds of two manatees.

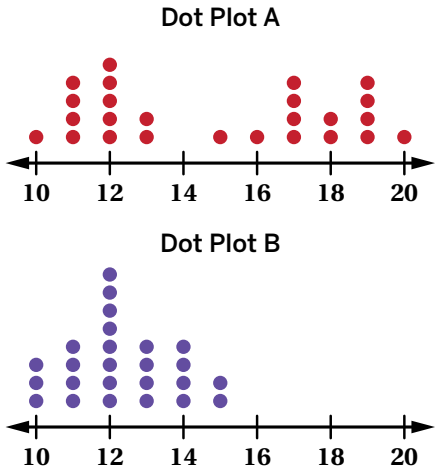
6. Which of these statements is true?

A. The data for Dot Plot B is more spread out than the data for Dot Plot A.

B. The manatee recorded in Dot Plot A swims at a more consistent speed.

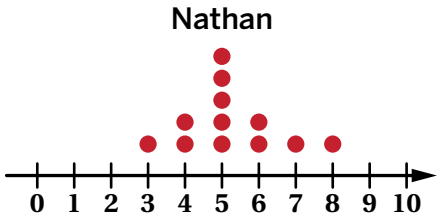
C. The MAD of Dot Plot A is greater than the MAD of Dot Plot B.

D. Both dot plots have approximately the same mean.



7. Which data set has the lesser mean? Explain your thinking.

Problems 8–9: Nathan and Esi recorded the number of baskets they each made out of 10 attempts. They each collected 12 data points. Nathan's mean number of baskets was 5.25, and his MAD was 1. Esi's mean number of baskets was 5.



Esi	7	8	9	0	1	2	1	3	2	10	9	8
Distance From 5												

8. Calculate Esi's MAD. Use the table if it helps with your thinking.

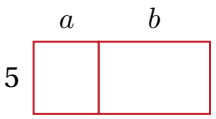
9. Which player is more consistent?

Spiral Review

10. Select *all* the expressions that are equivalent to $2(6 + 3x)$.

- ☐ **A.** $8 + 5x$
- ☐ **B.** $12x + 6$
- ☐ **C.** $12 + 6x$
- ☐ **D.** $6(2 + x)$
- ☐ **E.** $2(9x)$

11. Select *all* the expressions that represent the total area of the rectangle.



- ☐ **A.** $5(a + b)$
- ☐ **B.** $5 + ab$
- ☐ **C.** $5a + 5b$
- ☐ **D.** $2(5 + a + b)$
- ☐ **E.** $5ab$

Name: ..... Date: ..... Period: .....

Variability in Data | The Shape of Distributions | 6.SP.3, 6.SP.5, 6.SP.5.c, SMP.1, SMP.3

Hollywood, Part 1



Let's use mean and MAD to compare real-world data sets.

Warm-Up

1. Let's look at the salaries of two of the highest-paid movie actors.

Tay says: Movie actors are paid unequally because women are paid less than men.

Cho disagrees: Charlotte Johnson made more than Will Myth in 2019.

Who do you agree with? Explain your thinking.

Activity 1

Name: ..... Date: ..... Period: .....

Actor Salaries

Data Talk! Use the data on the Actor Salaries Sheet to help Tay and Cho settle their dispute.

2. One data set has a mean of $31.46 million dollars.

a Which data set is it?

b Calculate the mean of the other data set.

c Describe what the mean you calculated tells you about the data set.

3. The MAD of the salaries is about $8.65 million dollars for the actors who are women.

a Calculate the MAD for the actors who are men. Use the table to organize your thinking.

Salary (in millions)	89.5	76.5	66	65	58
Absolute Deviation (distance from .....)					

Salary (in millions)	57	57	43.5	41	35
Absolute Deviation (distance from .....)					

MAD: .....

b Tay says that since the MADs are alike, the salaries of the men and women in these data sets are also alike. What would you say to help Tay understand the mistake?

Name: ..... Date: ..... Period: .....

Actor Salaries (continued)

4. Tay and Cho revised their arguments.

Tay	Cho
Pay in acting is unequal because the highest paid women are paid less than the highest paid men.	Pay in acting is not unequal because some women actors are paid more than some men actors.

Whose claim do you agree with? Circle one.

Tay Cho Neither

Use the statistics you calculated to explain your thinking.

Name: ..... Date: ..... Period: .....

A Decade of Change?

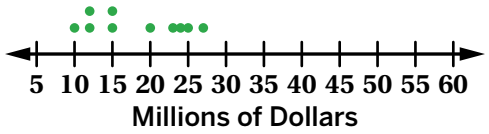
Data Talk! Tay and Cho decided to compare the salaries of the highest paid actors from 2009 and 2019.

Here are some of the statistics that they calculated.

Means	MADs
15 million dollars 18.3 million dollars 38.4 million dollars	5.5 million dollars 8.8 million dollars

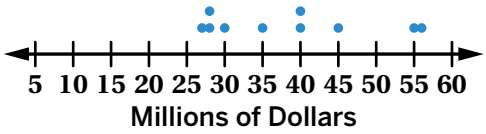
5. These dot plots show the salaries of the highest paid actors in 2009. Match each statistic with one of the dot plots. One statistic will have no match.

Women



Mean	MAD
.....	.....

Men



Mean	MAD
.....	.....

6. How has the average salary for the 10 highest paid women changed from 2009 to 2019?

7. Has the gap between the salaries of the highest paid men and women increased, decreased, or stayed the same? Use the data from 2009 and 2019 to explain your thinking, along with the means, MADs, and any other math you know.

Name: ..... Date: ..... Period: .....

Promoting Equal Pay

8. Tay and Cho are wondering how people in other industries work toward equal pay.

For instance, the U.S. women's national soccer team experienced a significant gender pay gap for years, even though they're more highly ranked than the men's national soccer team.

After a 6-year-long legal fight, they were able to win more equal pay. Here are some of the actions they took.

- In 2016, they filed a wage discrimination complaint with a U.S. agency (the Equal Employment Opportunity Commission).
- In 2017, they worked with their union to learn about labor law, so they could better fight for their rights.
- In 2019, they sued the U.S. Soccer Federation for gender discrimination.
- Also in 2019, they used social media to encourage soccer fans and the wider community to call out for change.
- In 2022, after two years in court, they came to an agreement with the U.S. Soccer Federation. They received $24 million dollars and a promise for more equal pay in future games, such as the World Cup.

Discuss:

- What surprises you about the U.S. women's national team's fight for equal pay?
- What other industries do you think might also have unequal pay for women and men?
- What could *you* do if you think people aren't being paid fairly?

Synthesis

9. Discuss:

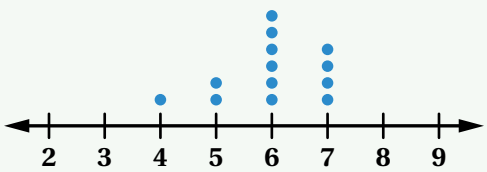
- How can calculating means and MADs help us make sense of topics like actors' salaries?
- What new questions do you have about salaries after exploring this lesson?

Summary 8.10

When comparing two data sets, the mean allows you to compare the average value of each set and the mean absolute deviation (MAD) allows you to compare their spreads.

Here is an example showing the ages of two groups of kids who started learning to read and who started learning to ride a bike.

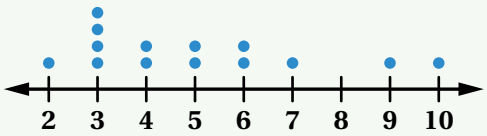
Learned to Read



Mean: 6 years **MAD:** ≈ 0.62 years

- On average, these kids started learning to read around 6 years old, which is older than the mean age for learning to ride a bike.
- The value of the MAD is smaller, meaning the data is less spread out.

Learned to Ride a Bike



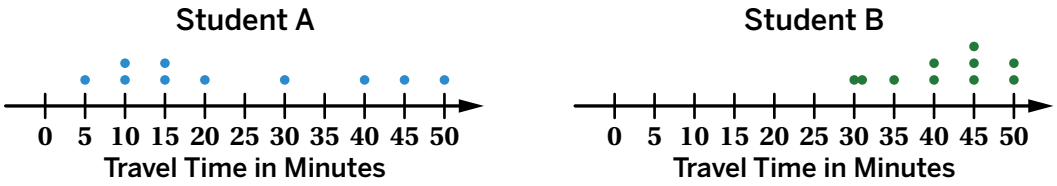
Mean: 5 years **MAD:** ≈ 1.86 years

- On average, these kids started learning to ride a bike around 5 years old, which is younger than the mean age for learning to read.
- The value of the MAD is greater, meaning the data points are more spread out.

Practice
8.10

Name: ..... Date: ..... Period: .....

Problems 1–2: The dot plots show the amount of time that it took two students to get to school for 10 days.



1. Without calculating, determine whose data set has the smaller mean. Circle one.

Student A Student B They are the same

Explain your thinking.

2. Without calculating, determine whose data set has the smaller MAD. Circle one.

Student A Student B They are the same

Explain your thinking.

Problems 3–4: Felipe played on the school basketball team for four years. Here are some statistics from his 9th grade and 12th grade seasons.

	9th Grade Season	12th Grade Season
Mean	6 points	11 points
MAD	5 points	3 points

3. Felipe's mean increased from 6 to 11. Describe what the increase could represent.

4. Felipe's MAD decreased from 5 to 3. Describe what the decrease could represent.

Name: ..... Date: ..... Period: .....

5. Amanda is comparing two basketball teams that have both won all of their games this season. She compared the mean and the MAD number of points scored each game.

	Sunnyside Bulls	Shadyside Unicorns
Mean	50 points	60 points
MAD	4 points	15 points

Which team's scores were the most consistent?

Explain your thinking.

Spiral Review

6. Which question is a statistical question?

A. Is the bison an endangered species?

B. How many legs does an adult bison have?

C. How much do adult male bison typically weigh?

D. What are adult female bison called?

7. The instructions for making pink paint say to mix 3 parts red paint with 2 parts white paint. Complete the table to show how much red and white paint you would need to make different amounts of pink paint.

Pink Paint (gal)	Red Paint (gal)	White Paint (gal)
50		
15		

Problems 8–10: Solve each equation.

8. $9v = 1$

9. $1.37w = 0$

10. $1 = \frac{7}{10}x$

Name: ..... Date: ..... Period: .....

Variability in Data | The Shapes of Distribution | 6.SP.3, 6.SP.4, 6.SP.5, 6.SP.5.a, 6.SP.5.c, SMP.1, SMP.2

Practice Day 1

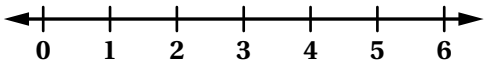


Let's practice what you've learned so far in this unit!

You will use task cards for this Practice Day. Record all of your responses here.

Task A: Pizza Toppings

1.



2. ..... toppings

3. Description:

You're invited to explore more.

Task B: State Populations

1. Circle one: True False Not enough information

Explanation:

2. Circle one: True False Not enough information

Explanation:

3. Circle one: True False Not enough information

Explanation:

4. Circle one: True False Not enough information

Explanation:

You're invited to explore more.

True:

False:

Not enough information:

Name: ..... Date: ..... Period: .....

Practice Day 1 (continued)

Task C: Soccer Goals

1. Mean of the first 5 games: ..... Mean of the last 5 games: .....

2. Circle one: Less than 2 Greater than 2 Equal to 2

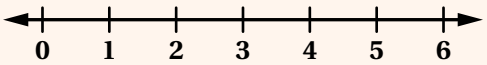
Explanation:

3. MAD of the first 5 games: .....

4. Circle one: First 5 games Last 5 games

Explanation:

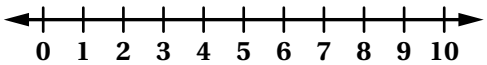
You're invited to explore more.



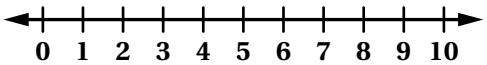
Number of Goals Scored

Task D: Dot Plot Maker

1.



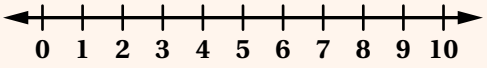
Dot Plot A



Dot Plot B

2. Description:

You're invited to explore more.

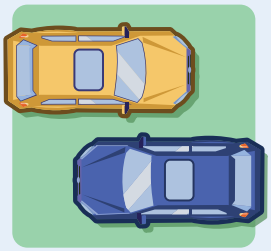


Notes:

Sub-Unit

3

Measuring Data: Median and IQR



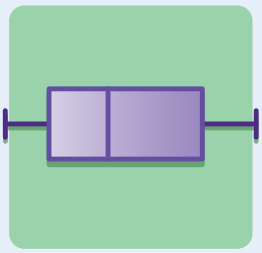
Lesson 11
Toy Cars



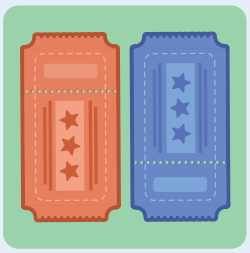
Lesson 12
In the News



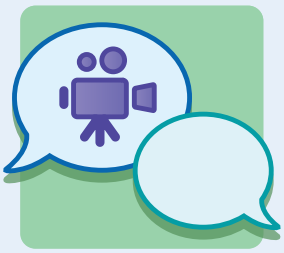
Lesson 13
Pumpkin Patch



Lesson 14
Car, Plane, Bus, or Train?



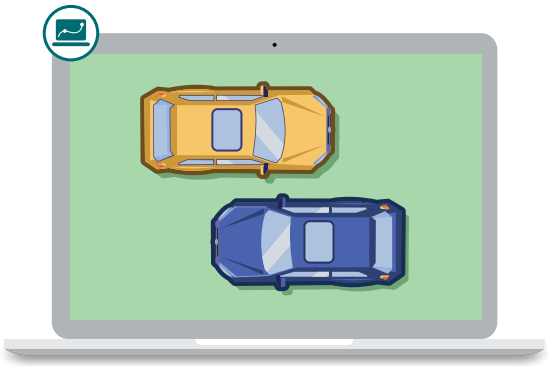
Lesson 15
Hollywood, Part 2



Lesson 16
Hollywood, Part 3

Name: ..... Date: ..... Period: .....

Variability in Data | The Shape of Distributions | 6.SP.4, 6.SP.5, 6.SP.5.c, SMP.2, SMP.3

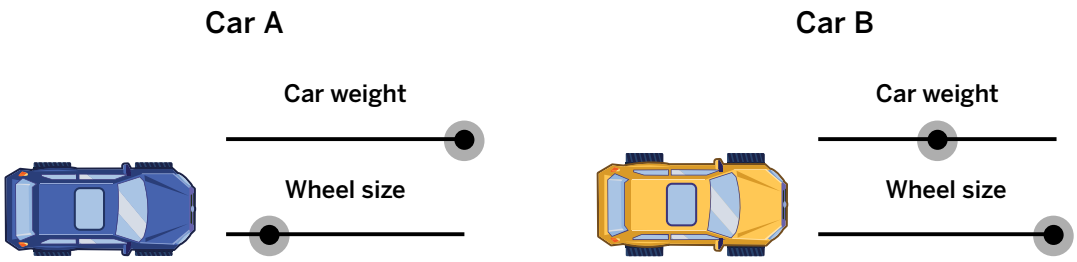


Toy Cars

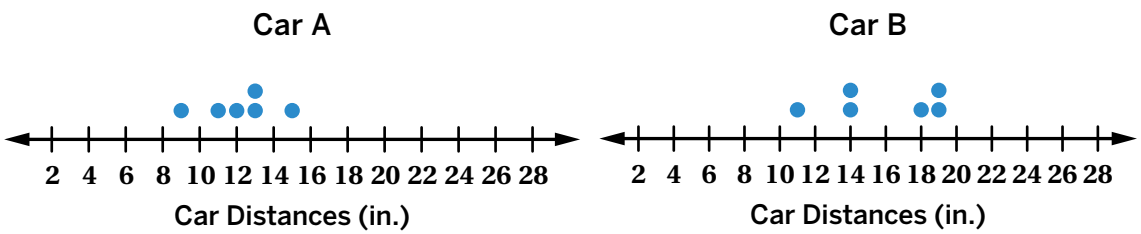
Let's explore the median of a data set and what it tells us.

Warm-Up

1-2 Here are two toy cars with different colors, weights, and wheel sizes.



a. Each car was launched 6 times. Compare their results.



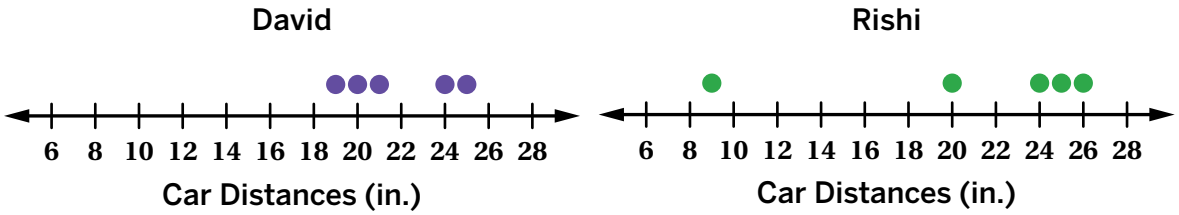
b. **Discuss:** Which car generally travels farther? How do you know?

Activity 1

Name: ..... Date: ..... Period: .....

A New Measure of Center

3 **Data Talk!** David and Rishi launched their cars 5 times each.



Which car do you think generally travels farther? Circle one.

David's car Rishi's car I'm not sure

Explain your thinking.

4 **Data Talk!** David and Rishi each think their car travels farther.

David says: *When we compare the mean distance for each car, my car travels farther.*

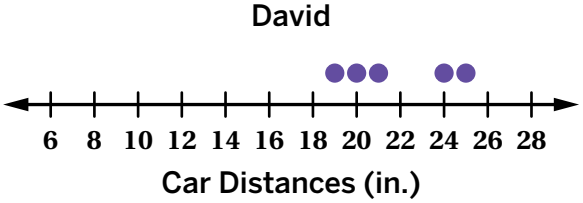
Rishi says: *When we compare the middle distance for each car, my car travels farther.*

Whose argument do you agree with? Circle one.

David's Rishi's Both Neither

Explain your thinking.

5 When the values are in numerical order, the middle value of a data set is called the **median**. The median distance for Rishi's car is 24 inches. What is the median distance for David's car?



Activity 2

Name: ..... Date: ..... Period: .....

Many Medians

6 **Data Talk!** Yona launched her car 7 times and recorded the distances, in inches, on a notepad. She calculated the median but made a mistake.

Yona

~~19~~, ~~14~~, ~~18~~, 28, ~~21~~, ~~12~~, ~~14~~

a. **Discuss:** Yona's strategy.

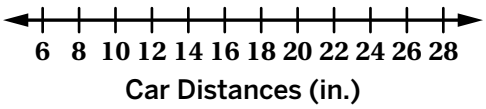
b. What would you recommend Yona to change in her work?

7 Here is Yona's data.

What is the median distance of this data set?

Create a dot plot if it helps with your thinking.

19, 14, 18, 28, 21, 12, 14

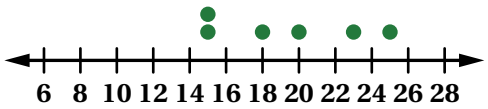


Car Distances (in.)

8 Amoli launched her car 6 times.

What do you think is the median of her data set? Explain your thinking.

Amoli



Car Distances (in.)

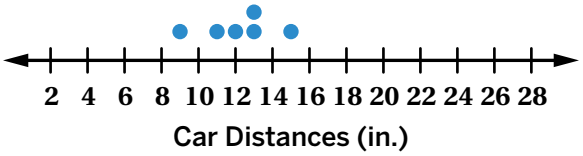
Name: ..... Date: ..... Period: .....

Many Medians (continued)

9 Statisticians agree that when there's an even number of data points, the median is the average of the middle two numbers.

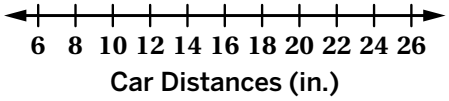
The median distance for Amoli's car is 19 inches because the average of 18 and 20 is 19.

The dot plot shows the data on Car A. What is the median distance that Car A traveled?



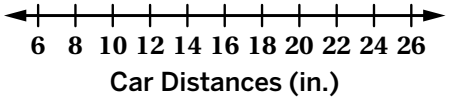
10 Calculate the median distance for each car. Complete as many problems as you have time for. Create a dot plot if it helps with your thinking.

a 14, 19, 15, 20, 17



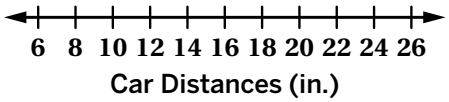
..... inches

b 25, 22, 19, 21, 14, 14



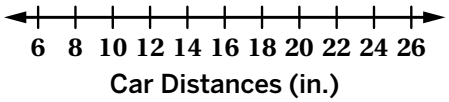
..... inches

c 18, 12, 24, 12, 18, 22, 22



..... inches

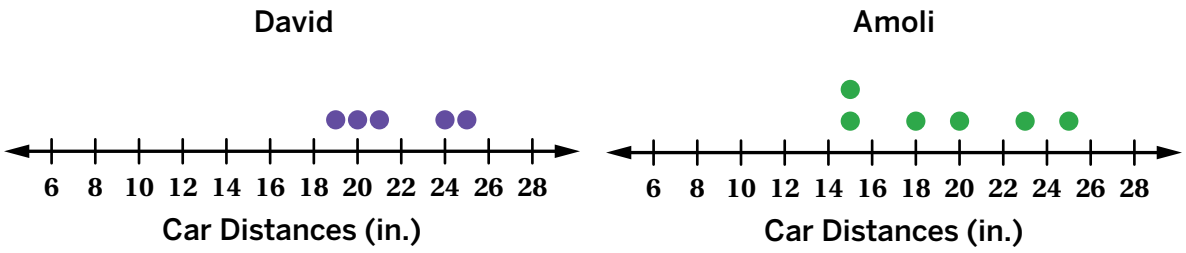
d 16, 6, 26, 14, 23, 27, 28, 19, 23, 17



..... inches

11 Synthesis

Discuss: How can you determine the **median** of a data set? Use the examples if they help with your thinking.



14 Summary 8.11

You can describe a data set using another measure of center called the **median**. Half of the data values are less than or equal to the median, and half of the data values are greater than or equal to the median.

To determine the median from an ordered representation of the data, you can repeat a process of eliminating the pairs of least and greatest values.

Here are some examples.

Odd Number of Values

~~0~~ ~~1~~ ~~1~~ (2) ~~2~~ ~~4~~ ~~5~~

Once all pairs have been eliminated, only one value remains in the middle, making it the median.

Median: 2

Even Number of Values

~~0~~ ~~1~~ ~~1~~ (1 2) ~~2~~ ~~4~~ ~~5~~

Once all pairs have been eliminated, two values remain.

Their average is the median.

$(1 + 2) \div 2 = 1.5$

Median: 1.5

median A measure of center. It is the middle value of a data set when the values are in numerical order. When there is an even number of data points, the median is the average of the two middle values."

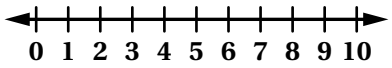
Practice
8.11

Name: ..... Date: ..... Period: .....

Problems 1–2: Here is a data set.

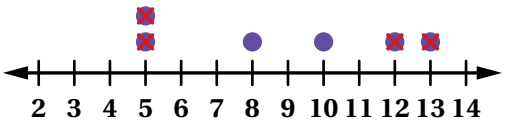
1	6	7	6	2	9	3

1. Create a dot plot for this data.



2. Determine the median of this data.

Problems 3–4: Kayla wants to determine the median of the data in this dot plot. She starts solving the problem but isn't sure what to do next.



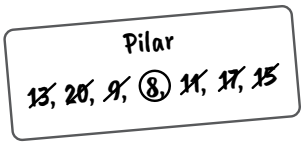
3. What could you say to help Kayla determine the median?

4. What is the median of this data?

5. The table shows Prisha's scores after attempting the first level of a video game 10 times. What is her median score?

130	150	120	170	130
120	160	160	190	140

6. Pilar recorded the number of points she scored in her last 7 basketball games. She says that her median score was 8 points. Is she correct?

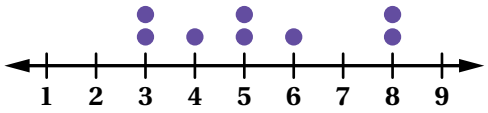


Explain your thinking.

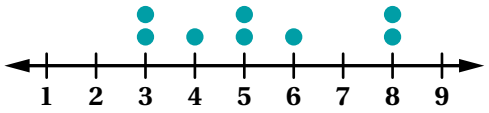
Practice
8.11

Name: ..... Date: ..... Period: .....

7. *Add* points to this dot plot to make the median 4.



8. *Remove* points from this dot plot to make the median 4.



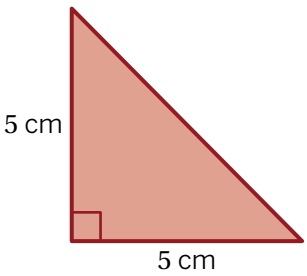
Spiral Review

9. Determine 25%, 50%, and 75% of 84.

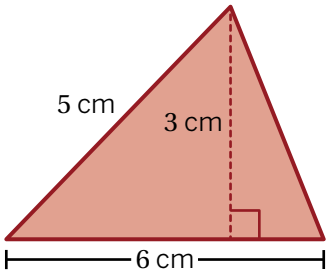
Use the double number line if it helps with your thinking.

Problems 10–13: Calculate the area of each triangle.

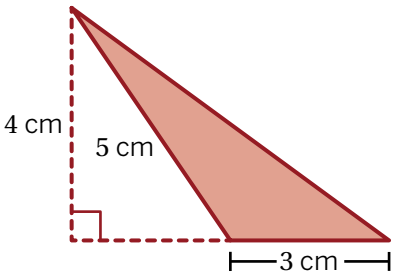
10.



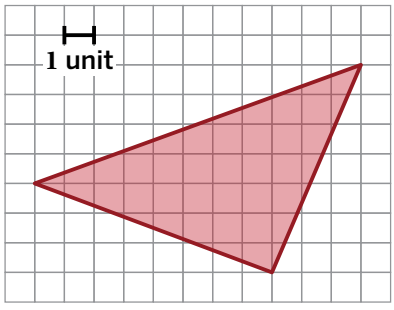
11.



12.

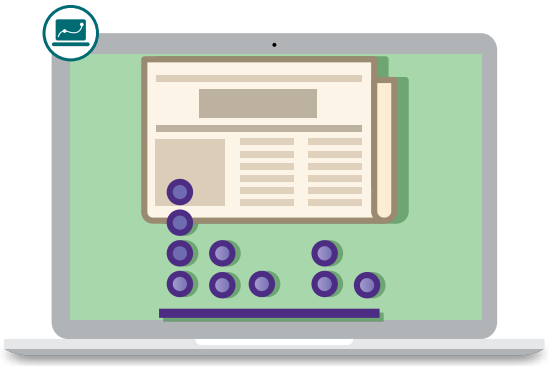


13.



Name: ..... Date: ..... Period: .....

Variability in Data | The Shape of Distributions | 6.SP.4, 6.SP.5, 6.SP.5.b, 6.SP.5.c, 6.SP.5.d, SMP.2, SMP.3, SMP6

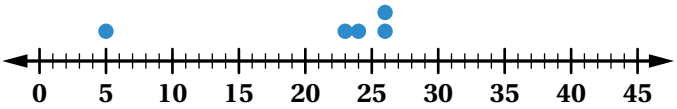


In the News

Let's compare the mean and median of data sets.

Warm-Up

1 **Data Talk!** Here is a data set.



a. Which is greater, the mean or the median? Circle one.

Mean Median They are the same

b. What could this data be about?

2 Let's observe how the mean and median of a data set changes as the value of the movable point changes.

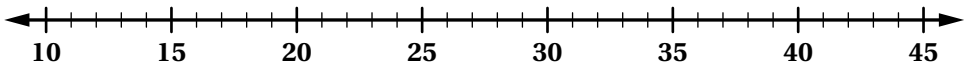
Discuss: What do you notice?

Name: ..... Date: ..... Period: .....

Mean vs. Median

3 Remy is throwing a 12th birthday party.

Create a dot plot to show the ages of people Remy might expect at the party.



Ages of Remy's Party Guests (yr)

Name: ..... Date: ..... Period: .....

Mean vs. Median (continued)

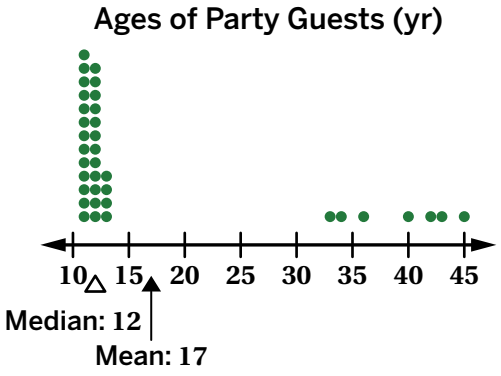
4 **Data Talk!** Here is a news headline about Remy's birthday party.

Let's look at the data the newspaper used.

Why is the headline misleading?

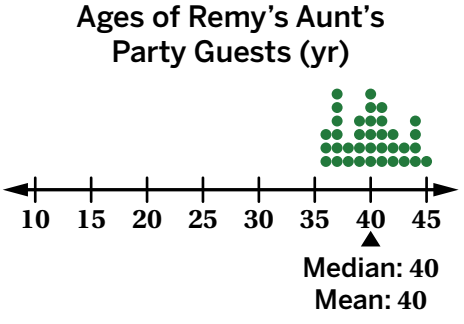


5 Why do you think the mean is higher than the median?



6 Remy's aunt also threw a party.

a Compare the dot plot for Remy's party guests with the dot plot for Remy's aunt's party guests.



b **Discuss:** Why do you think the mean and median ages are closer together for Remy's aunt's party than Remy's party?

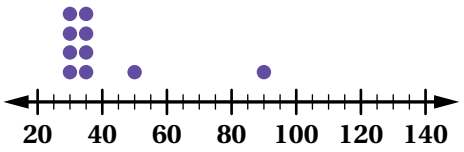
Activity 2

Name: ..... Date: ..... Period: .....

Which Would You Report?

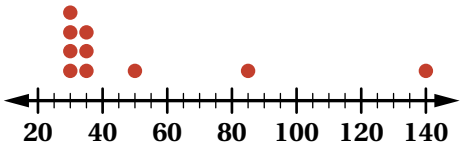
7 **Data Talk!** A reporter was wondering how salaries at the local grocery store have changed in recent years. They found data from five years ago and from today.

Five Years Ago



Employee Salaries (thousands of dollars)

Today



Employee Salaries (thousands of dollars)

What do you notice? What do you wonder?

I notice:

I wonder:

8 A local newspaper is considering two different headlines.

Headline 1



Headline 2



Discuss: Which headline do you think is more accurate? Why?

9 A different grocery store has a median salary that's greater than the mean salary. Create a dot plot with at least 10 points to represent what the salaries at this grocery store could be.



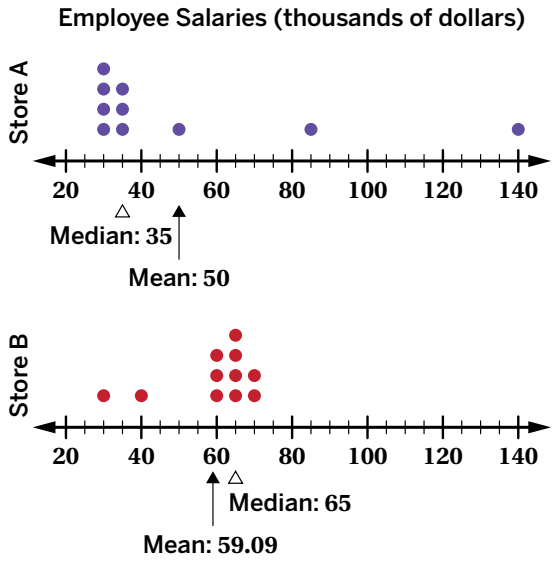
Employee Salaries (thousands of dollars)

Name: ..... Date: ..... Period: .....

Which Would You Report? (continued)

10 Here are two dot plots showing the current salaries of two local grocery stores.

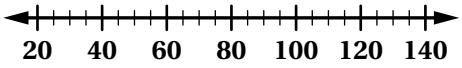
Discuss: How does the shape of a dot plot affect the relationship between the median and the mean of the data?



11 In 2020, BuyNSell released information about the pay rates of their U.S.-based employees.

- The median salary was $37,930.
- The mean salary was $48,345.

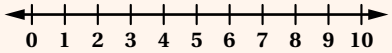
Why do you think many newspapers reported the median salary? Use the dot plot if it helps your thinking.



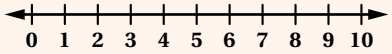
You're invited to explore more.

12 Create a dot plot that has:

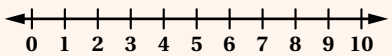
a
- A median of 7.
- A mean that is less than the median.



b
- A median of 6.
- A mean of 5.



c
- A median of 3.
- A mean of 5.

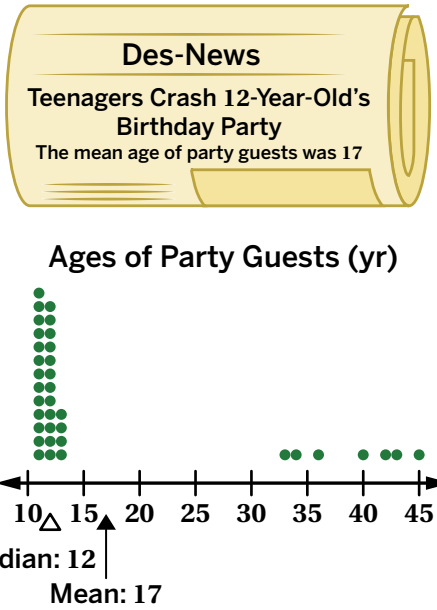


13 Synthesis

Discuss:

- When are the mean and median likely to be far apart?
- When might you choose to report the median instead of the mean?

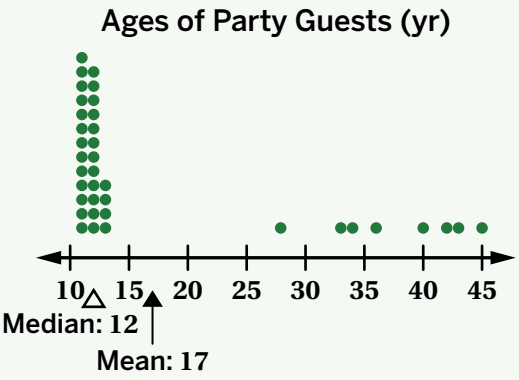
Use the example if it helps to show your thinking.



16 Summary 8.12

You can use mean and median to tell you different things about a data set. One measure might be more appropriate depending on the shape of the data and the situation.

Here is data from a 12-year-old's birthday party. The median of this data set is 12. The mean of this data set is 17. In this situation, the median is a better measure to represent the ages of the party guests because most guests are 11 to 13 years old. The mean has shifted away from where most of the data is because the older ages are added to the younger ages.

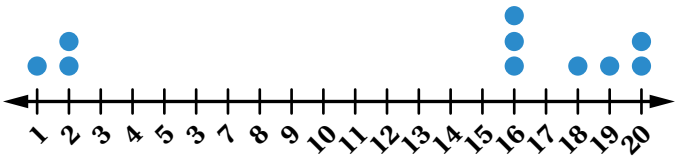


When most of the data is in one place but there are a few data points far away from the group (like in the dot plot shown), the mean and median are likely to be far apart.

Practice
8.12

Name: _____ Date: _____ Period: _____

1. Here are two statistics related to this dot plot: 13 and 16.



Without calculating, determine which number is the median and which number is the mean.

Median: _____ Mean: _____

2. Here are the scores for Sahana's history homework.

100	100	100	100	95	100	90	100	0

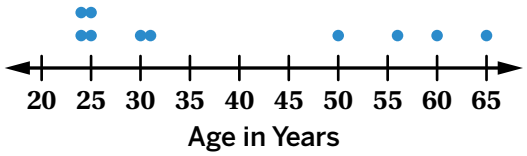
The history teacher uses the mean to calculate an overall grade for homework. How might Sahana argue that the median is a better way to measure her work?

3. Ayaan competed in a bowling tournament. Here are his scores for each game.

Game 1	Game 2	Game 3	Game 4	Game 5	Game 6
300	250	275	120	243	290

Do you think the tournament judges should use the median or the mean to calculate Ayaan's final score? Justify your choice.

4. Kweku surveyed 10 teachers at his school. The dot plot displays the ages of those teachers. Which of these statements is *true*?



A. The median is greater than the mean.

B. The median and the mean are the same.

C. The median is less than the mean.

D. The mean cannot be determined.

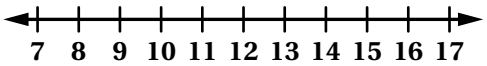
Explain your thinking.

Practice
8.12

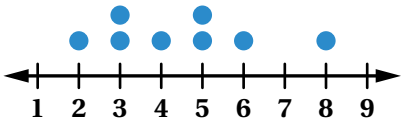
Name: ..... Date: ..... Period: .....

5. Create a dot plot with 5 data points that has:

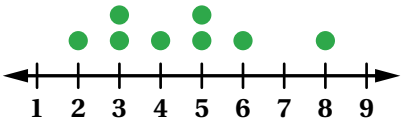
- A median of 14.
- A mean less than 14.



6. Add one point to this dot plot to make the median *greater* than the mean.



7. Add one point to this dot plot to make the median equal to the mean.

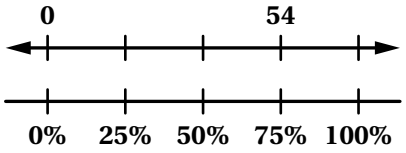


Spiral Review

8. The mean of three numbers is 8. When a fourth number is included, the average of all four numbers becomes 8.5. What is the value of the fourth number?

Explain your thinking.

9. 54 is 75% of a number. Determine 25%, 50%, and 100% of that number. Use the double number line if it helps with your thinking.



Problems 10–12: Convert each measurement using the information in the table.

1 kg = 1000 g	3 oz ≈ 85 g	4 kg ≈ 141 oz	11 lb ≈ 5 kg

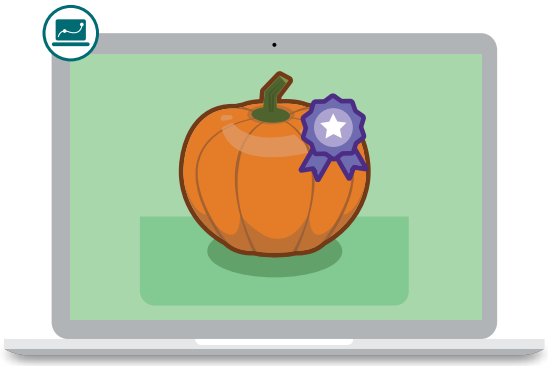
10. 30 ounces ≈ ..... grams

11. 12 kilograms ≈ ..... ounces

12. 6 pounds ≈ ..... kilograms

Name: ..... Date: ..... Period: .....

Variability in Data | The Shape of Distributions | 6.SP.3, 6.SP.5, 6.SP.5.c, SMP.3, SMP.6, SMP.8



Pumpkin Patch

Let's determine and interpret the quartiles of a data set.

Warm-Up

1 A farmer sells different sizes of pumpkins.

A customer wants a medium-sized pumpkin.

Circle the pumpkins the farmer could sell to this customer.



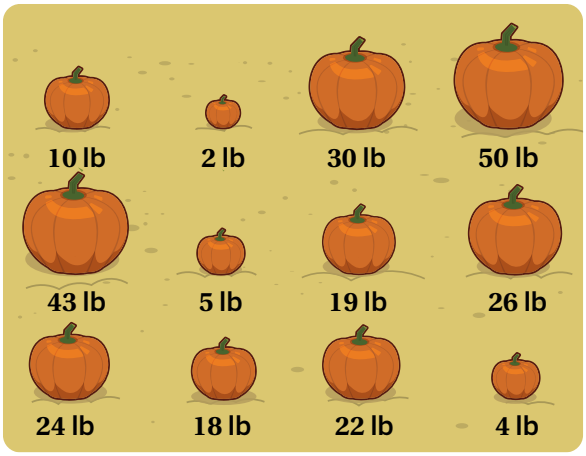
Name: ..... Date: ..... Period: .....

Introduction to Quartiles

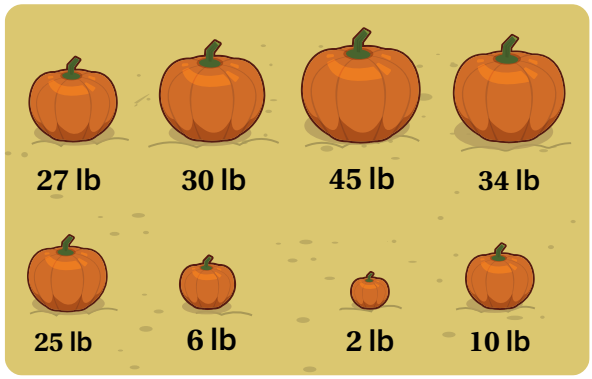
2 **Data Talk!** Farmer Na'ilah always marks the middle half of her pumpkins as medium-sized.

a Let's watch an animation to see how Na'ilah decided which pumpkins were in the middle half.

b Explain how she decided.



3 Here are some pumpkins on a different farm. Circle the pumpkins with weights that are in the middle half.

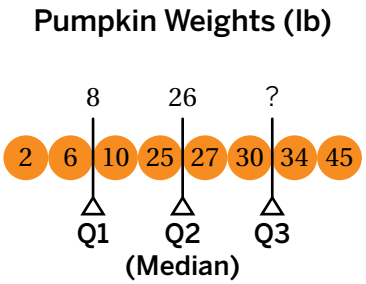


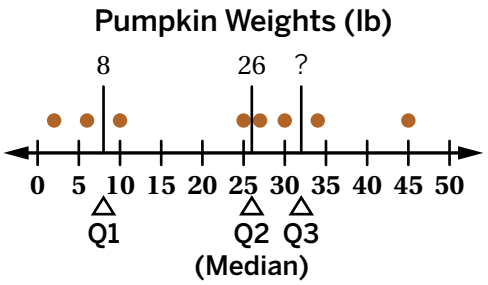
Name: Date: Period:

Introduction to Quartiles (continued)

4 **Data Talk!** Quartiles divide a data set into four sections. They can help us identify and describe the middle half of a data set.

Here are Quartile 1 (Q1) and Quartile 2 (Q2) for Na'ilah's pumpkins.





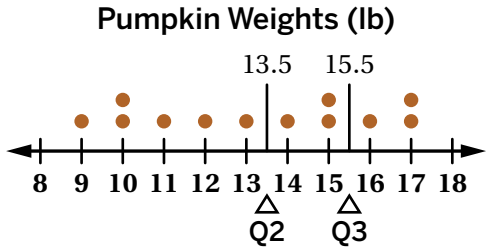
What do you think the value of Quartile 3 is?

Explain your thinking.

5 Quartile 1 is the median of the lower half of a data set.

Here are the weights of 12 pumpkins on Tasia's farm.

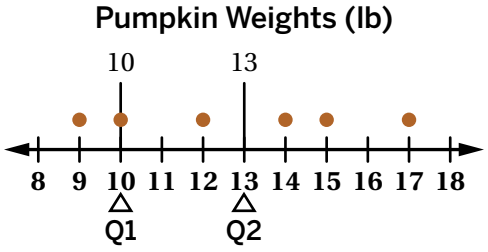
What is the value of Quartile 1?



6 Quartile 3 is the median of the upper half of a data set.

Here are the weights of 6 pumpkins on Tyler's farm.

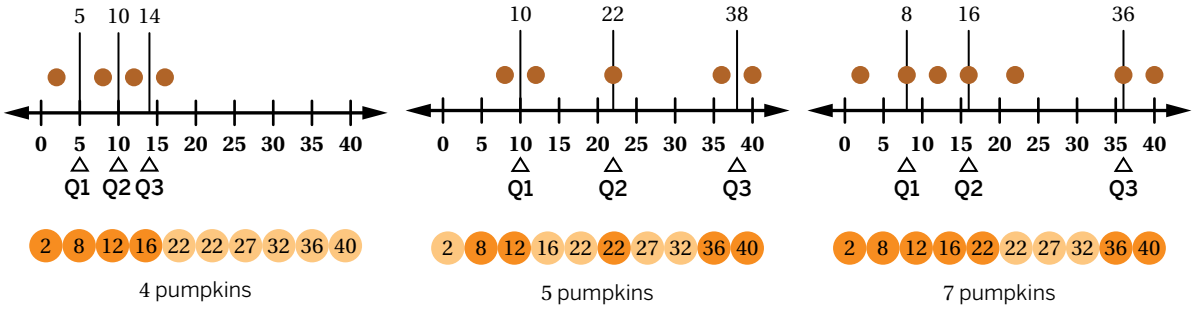
What is the value of Quartile 3?



Name: ..... Date: ..... Period: .....

Reasoning About Quartiles

7 Here are some dot plots with different pumpkin weights.



a **Discuss:** What do you notice?

b Here are several challenges. Select *all* the ones you think are possible. Use a device to check your answers.

- ☐ Q2 is not equal to a pumpkin weight.
- ☐ Q1, Q2, and Q3 are equal to pumpkin weights.
- ☐ Q1, Q2, and Q3 are not equal to pumpkin weights.
- ☐ Q1 is equal to a pumpkin weight, but Q3 is not.
- ☐ Q1 and Q2 have the same value.

8 **Data Talk!** Here are 9 pumpkins on Adrian's farm. What do you think the values of Q1 and Q3 are? Explain your thinking. Use the dot plot if it helps to show your thinking.

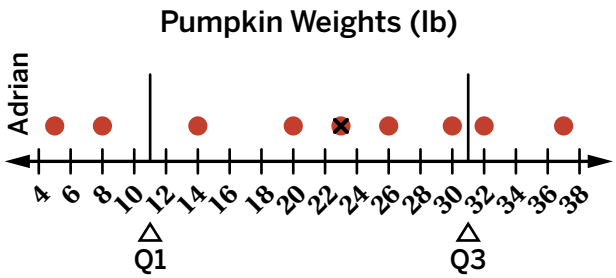
Pumpkin Weights (lb)



Name: ..... Date: ..... Period: .....

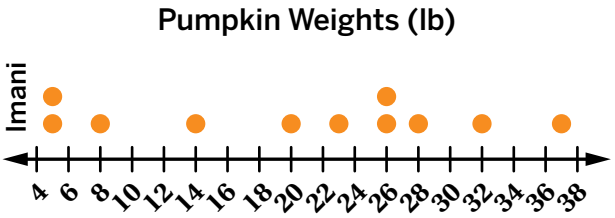
Reasoning About Quartiles (continued)

9 Statisticians agree that when you determine Q1 and Q3 for a data set that has an odd number of points, do *not* include the median in the lower or upper half of the data.



This dot plot shows the weight of 11 pumpkins on Imani's farm.

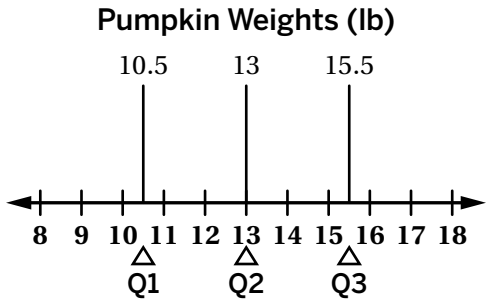
What is the value of Q3 for these pumpkin weights?



10 A store has 80 pumpkins for sale. Here are the values of the quartiles.

About how many of the 80 pumpkins would you expect to weigh *less* than 15.5 pounds?

Explain your thinking.



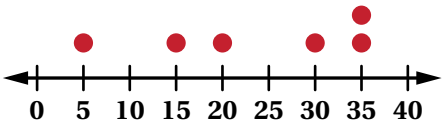
Activity 3

Name: ..... Date: ..... Period: .....

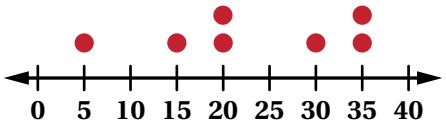
Repeated Challenges

11 Solve as many problems as you have time for. Sensemaking is more important than speed.

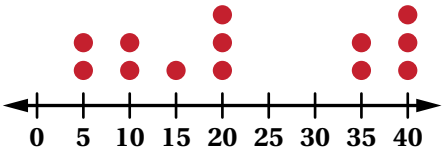
a The dot plot shows 6 data points. What is the value of Q1?



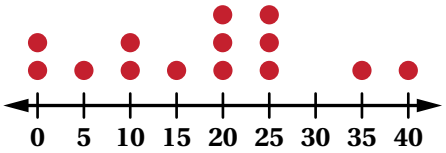
b The dot plot shows 7 data points. What is the value of Q2?



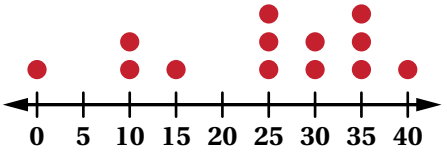
c The dot plot shows 13 data points. What is the value of Q1?



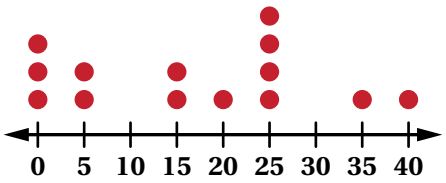
d The dot plot shows 14 data points. What is the value of Q3?



e The dot plot shows 13 data points. What is the value of Q2?



f The dot plot shows 14 data points. What is the value of Q3?

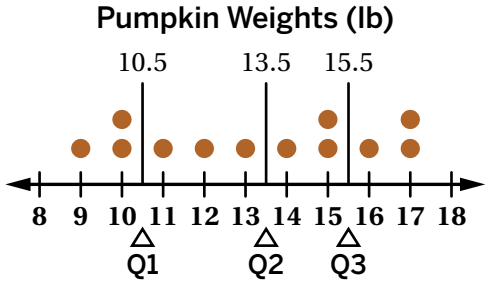


12 Synthesis

Discuss:

- How do quartiles relate to the middle half of a data set?
- How can you determine the value of the quartiles for a data set?

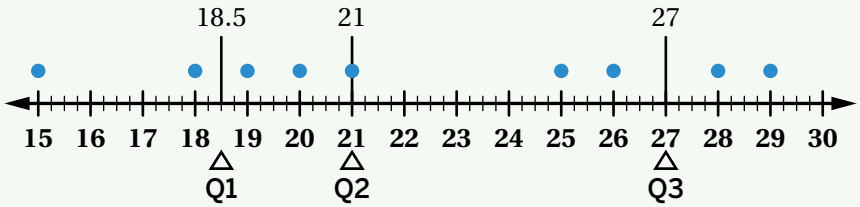
Use the example if it helps to show your thinking.



15 Summary 8.13

You can describe the middle half of a data set by dividing it into four equal sections called **quartiles**.

You can determine the value of the quartiles by splitting the entire data set in half and then splitting the halves again. The middle half is all the data points that are between Q1 and Q3. Representations such as dot plots are helpful for identifying quartiles to describe data sets.



The *first quartile (Q1)* is the median of the lower half of the data set.

The *second quartile (Q2)* is the median of the entire data set.

The *third quartile (Q3)* is the median of the upper half of the data set.

quartiles Quartiles divide an ordered data set into four equal sections. Quartile 1 (Q1) is the median of the lower half of the data. Quartile 2 (Q2) is the median. Quartile 3 (Q3) is the median of the upper half of the data.

Practice
8.13

Name: ..... Date: ..... Period: .....

Problems 1–2: Here are the ages of 20 people at a family reunion, ordered from youngest to oldest.

3, 8, 9, 10, 11, 11, 12, 18, 18, 28, 30, 35, 37, 40, 53, 54, 58, 65, 70, 72

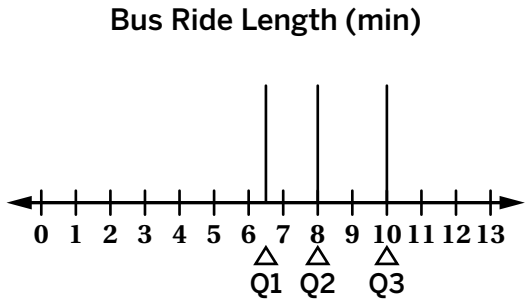
1. The value of Quartile 2 (Q2) is 29. What does that tell us about the people at the family reunion?

2. Determine the value of Q1 and Q3.

Quartile 1 (Q1): ..... Quartile 3 (Q3): .....

Problems 3–5: Haru recorded how long his bus ride to school took for 16 days. Here are the values of the quartiles of his data.

3. About how many rides would you expect to be less than 6.5 minutes long?



4. About how many rides would you expect to be less than 10 minutes long?

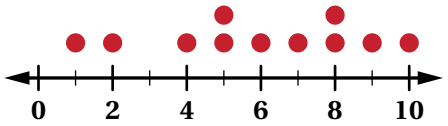
5. About what percent of the rides would you expect to be between 6.5 minutes and 10 minutes long?

6. The heights, in inches, of Javier's classmates are 60, 68, 56, 60, 62, 58, 55, 67, 59, 61, 62, 64, 63, 63, 59, 62, 66, and 61. Determine the values of Q1, Q2 (median), and Q3.

	Height (in.)
Q1	
Q2 (Median)	
Q3	

Name: _____ Date: _____ Period: _____

7. Makayla and Axel both try to determine the quartiles for this dot plot.



Makayla says:

Q1: 4.5 Q2: 6 Q3: 8

Axel says:

Q1: 4 Q2: 6 Q3: 8

Who is correct? Explain your thinking.

Spiral Review

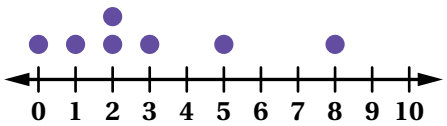
Problems 8–10: Calculate each percentage.

8. 25% of 40

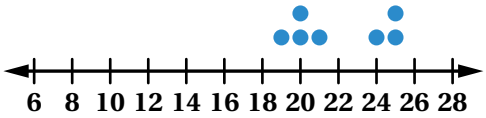
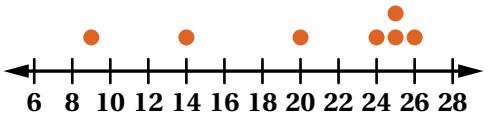
9. 25% of 120

10. 25% of 90

11. Add 5 points to this dot plot without changing the mean absolute deviation (MAD).



12. Adrian and Imani launched their cars 7 times each in a car race.

Adrian's car distances (in.)	Imani's car distances (in.)
	

Adrian says: *We need to compare the mean distances to determine the winner!*

Imani says: *We need to compare the median distances to determine the winner!*

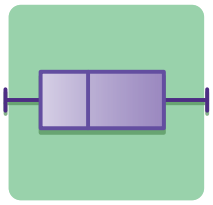
Why do you think Adrian and Imani choose to use different measures of center to determine the winner?

Name: ..... Date: ..... Period: .....

Variability in Data | The Shape of Distributions | Relationships Between Variables

6.SP.2, 6.SP.3, 6.SP.4, 6.SP.5, 6.SP.5.c, SMP.1, SMP.4

Car, Plane, Bus, or Train?



Let's explore box plots to visualize data.

Warm-Up

Jalen's family lives in St. Louis. They often visit relatives in Chicago.

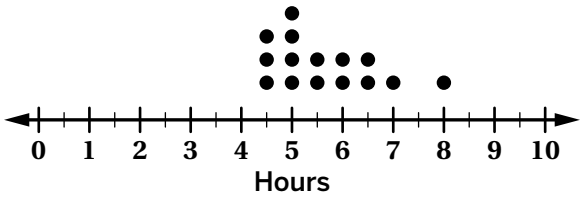


1. **Discuss:**

- What modes of transportation do you think Jalen's family could take?
- Which would *you* take? Why?

3. **Data Talk!** Jalen recorded how long it took to drive from St. Louis to Chicago the last 15 times his family drove there.

Determine the values of Q1, Q2, and Q3. Label them on the dot plot.



Activity 1

Name: ..... Date: ..... Period: .....

Car or Plane?

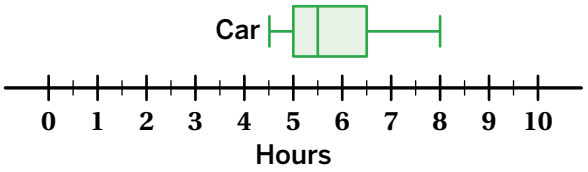
Data Talk! Jalen's family wonders: *What mode of transportation should we use to travel between Chicago and St. Louis?*

3. One way to compare data sets is by using **box plots**. Let's watch how a box plot is drawn.

a **Discuss:** What do you notice? What do you wonder?

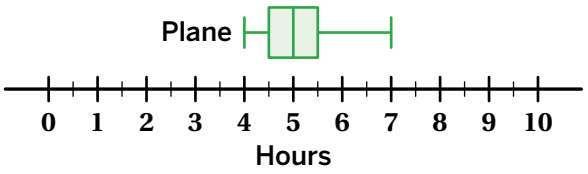
b Label the box plot with each statistic.

- Minimum (Min)
- Quartile 1 (Q1)
- Quartile 2 (Q2)
- Quartile 3 (Q3)
- Maximum (Max)



c What percent of the drives are between 5 and 5.5 hours? Explain your thinking.

4. Jalen's family sometimes takes a plane to get to St. Louis. Here is a box plot that represents some of their travel times when they flew.



a Determine each statistic for the plane data.

- Minimum: .....
- Quartile 1: .....
- Median: .....
- Quartile 3: .....
- Maximum: .....

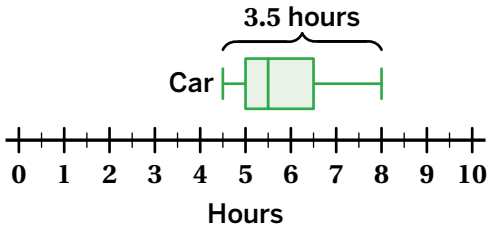
b What percent of the plane data was less than 5 hours long? Explain your thinking.

Name: ..... Date: ..... Period: .....

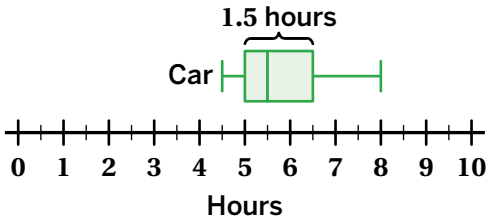
Measures of Spread

5. **Data Talk!** Jalen's family is also wondering: *How spread out are our travel times when we travel by car?*

Let's look at two ways to describe the spread of a box plot.



The **range** is the distance from the *minimum* to the *maximum*.



The **interquartile range (IQR)** is the distance from Q1 to Q3.

a Read the definitions of range and IQR.

How are they alike?

How are they different?

b Jalen says that one of these measures of spread describes the middle 50% of the data points.

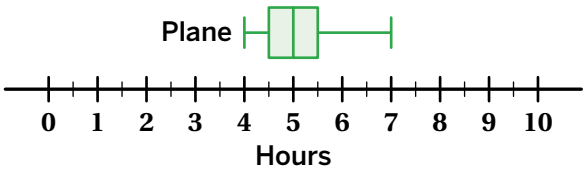
Which measure of spread do you think he is talking about? Circle one.

IQR Range Neither

Explain your thinking.

6. What is the range and IQR for the plane data?

Range: ..... IQR: .....



7. Use the car data and the plane data to help you answer: *Which is more consistent, driving or flying?* Explain your thinking.

Activity 3

Name: ..... Date: ..... Period: .....

Bus or Train?

8. **Data Talk!** Jalen's family has also traveled to Chicago by bus and by train.

You will use a set of cards for this activity.

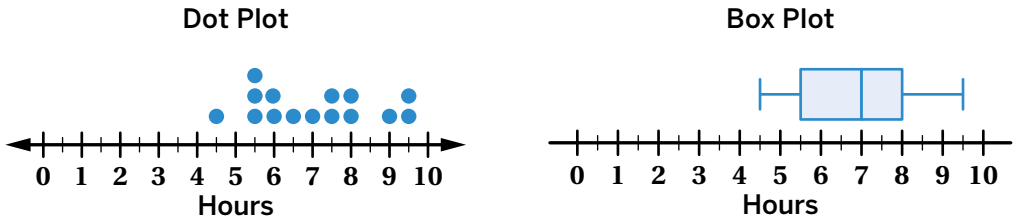
- Match each dot plot, box plot, and statistic to a mode of transportation. There will be two empty spaces in the table for you to complete.
- Draw a box plot on Card A and determine the values of the missing statistics to complete the table.

	Bus	Train
Travel Times (hours)	7.5, 10.5, 9.5, 9.5, 6, 7, 6.5, 7.5, 11, 8.5	6.5, 8.5, 6, 10.5, 6.5, 8, 7, 7, 9, 8
Dot Plot		
Box Plot		
Median		
IQR		
Range		

9. Which mode of transportation (car, train, bus, or plane) would you recommend Jalen's family use to travel between Chicago and St. Louis? Use evidence to support your argument.

Synthesis

10. **Discuss:** Which statistics are more visible in the dot plot? In the box plot? Why do you think that is?



Statistics			
Median: 7 hours	Number of Data Points: 15	Range: 5 hours	IQR: 2.5 hours

Summary 8.14

You can create a **box plot** to visualize a data set. While a box plot shows the same data as a dot plot, it gives us new information about the data. Rather than showing every data point, a box plot separates the data into quartiles. We can use box plots to describe the spread of the data in two ways.

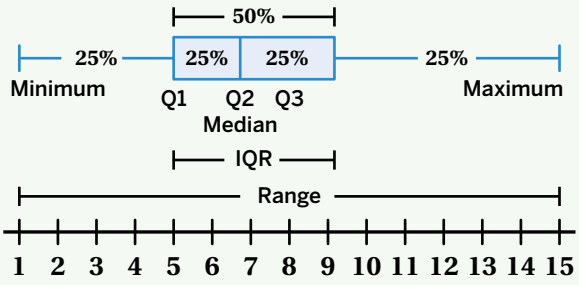
- The **range** describes the overall spread of the data.

  Range: $15 - 1 = 14$

- The **interquartile range (IQR)** describes how spread out the middle of the data is.

  IQR: $9.25 - 6.5 = 2.75$

Box plots do not show how many data points are in each set, or the values of any individual data points, except the minimum and maximum.



box plot A way to visualize numerical data sets. The data is divided into four sections using five values: the minimum, Q1, Q2 (or the median), Q3, and the maximum. A box is drawn between Q1 and Q3. The line inside the box represents the median.

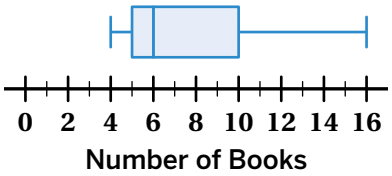
range A measure of spread. It is the difference between the maximum and minimum values in a data set.

Interquartile range (IQR) A measure of spread. The IQR is calculated as the distance from Q1 to Q3, or the width of the box in a box plot.

Practice
8.14

Name: ..... Date: ..... Period: .....

Problems 1–3: Each student in a class recorded how many books they read in a school year.



1. What is the greatest number of books that a student read this year?

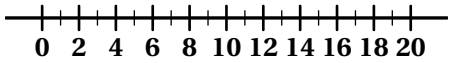
2. What is the median of this data?

3. What is the range of this data?

Problems 4–5: Here are five statistics about a data set.

Minimum	Q1	Median	Q3	Maximum
4	6	9	13	19

4. Create a box plot that represents this data set.



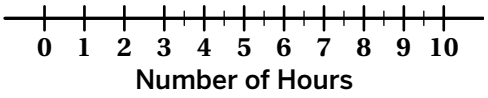
5. What is the IQR of this data set?

Problems 6–9: A group of seventh-grade students recorded the number of hours they spent doing homework in one week. Here is their data: 3, 4, 5, 5, 6, 7, 7, 9, 9, 10.

6. Determine each of the statistics.

Minimum	Q1	Median	Q3	Maximum

7. Create a box plot of this data.



8. Most seventh-graders do less than 5 hours of homework in a week. Circle one. True False

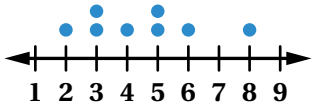
9. 25% of seventh-grade students spent between 9 and 10 hours doing homework in a week. Circle one. True False

Practice
8.14

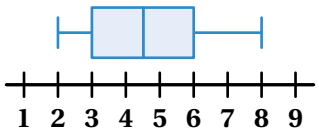
Name: ..... Date: ..... Period: .....

Problems 10–11: Here are three representations of the same data set.

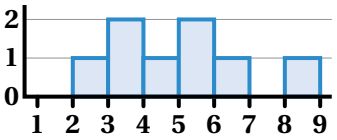
Dot Plot



Box Plot



Histogram



10. One representation has a mistake. Circle the incorrect representation.

11. Describe the mistake made in creating the representation you chose in the previous problem.

Spiral Review

12. Which of these has the largest value?

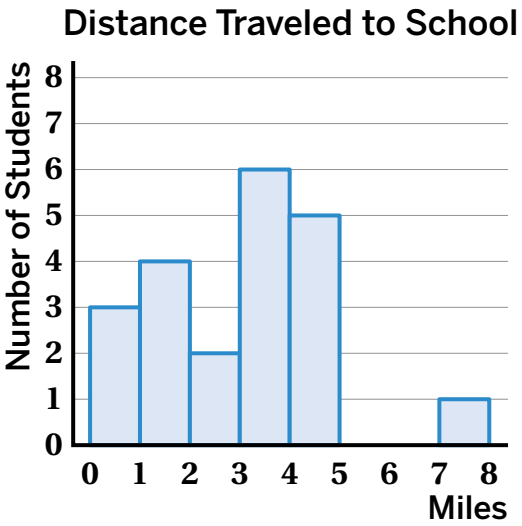
A. 25% of 80

B. 100% of 60

C. 75% of 100

D. 50% of 120

13. Nasir surveyed his classmates to determine how many miles they travel to school each day. This histogram shows the results. Select *all* the true statements.

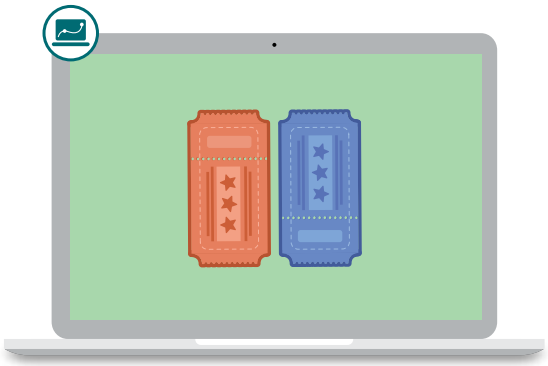


- ☐ **A.** Most of the data is between 4 and 7 miles.
- ☐ **B.** The median of the data values is between 3 and 4 miles.
- ☐ **C.** The interquartile range of the data values is greater than 5.
- ☐ **D.** No students travel between 2 and 3 miles.
- ☐ **E.** Most of the data is between 3 and 5 miles.

Unit 8
Lesson
15

Name: ..... Date: ..... Period: .....

Variability in Data | The Shape of Distributions | 6.SP.3, 6.SP.5, 6.SP.5.b, 6.SP.5.c, 6.SP.5.d, SMP.3, SMP.4



Hollywood, Part 2

Let's use box plots to analyze movies and sequels.

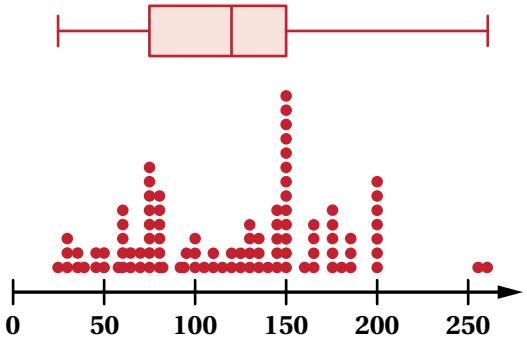
Warm-Up

1 **Data Talk!** The amount of money it takes to make a movie is called the *budget*.

This dot plot and box plot represent the budgets of 108 animated movies made in Hollywood between 2000 and 2021.

What questions could you use these graphs to answer?

Movie Budgets (millions of dollars)



Activity 1

Name: ..... Date: ..... Period: .....

Originals and Sequels

2 Here are the titles of some animated movies. Sort the movies into two or more groups and write a description for each group.

Racers 3	*The Browns vs. the Robots*	*Winter Sisters*	*Karate Koala 2*
Cocoa	*Figure Fables 4*	*Searching for the Swimmer*	*Searching for the Swimmer 2*

3 Let's look at how one student grouped the movies.

Out of 108 animated movies made between 2000 and 2021, 71 are originals and 37 are sequels.

a. **Discuss:** What do you know about originals and sequels?

b. Which do you think have higher budgets? Circle one.

Originals Sequels About the same

Name: ..... Date: ..... Period: .....

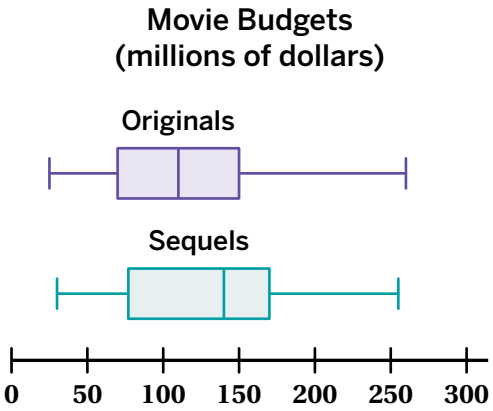
Originals and Sequels (continued)

4 **Data Talk!** These box plots represent the budgets of original movies and sequels.

Now, which do you think have higher budgets? Circle one.

Originals Sequels About the same

Use the box plots to justify your choice.



5 David said: *Original movies have higher budgets than sequels because the highest-budget movie is an original.*

What do you think about David's claim?

6 So far, you've looked at budgets for originals and sequels.

a Which movies do you think get better reviews? Circle one.

Originals Sequels About the same

b Which movies do you think sell more tickets? Circle one.

Originals Sequels About the same

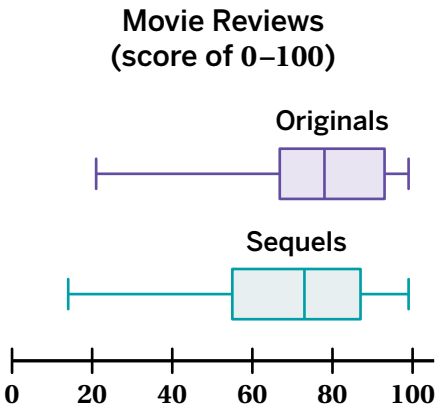
c Discuss your predictions with a classmate.

Activity 2

Name: ..... Date: ..... Period: .....

Movie Reviews and Ticket Sales

7 **Data Talk!** A website scores every movie on a scale of 0–100 based on its reviews. Here are two box plots that represent the scores.

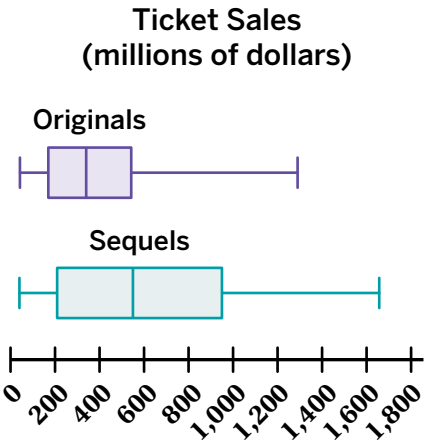


a. **Discuss:** Was your prediction correct?

b. Select *all* of the true statements.

- ☐ **A.** The lowest score a movie got was 14.
- ☐ **B.** The IQR for original movies is greater than the IQR for sequels.
- ☐ **C.** The range for sequels is 74.
- ☐ **D.** About 25% of original movies have a score of 93 or higher.

8 These box plots represent ticket sales.



a. **Discuss:** Was your prediction correct?

b. About what percentage of *originals* had ticket sales of $550 million or more?

c. About what percentage of *sequels* had ticket sales of $550 million or more?

9 Look at the IQRs for ticket sales of originals and sequels.

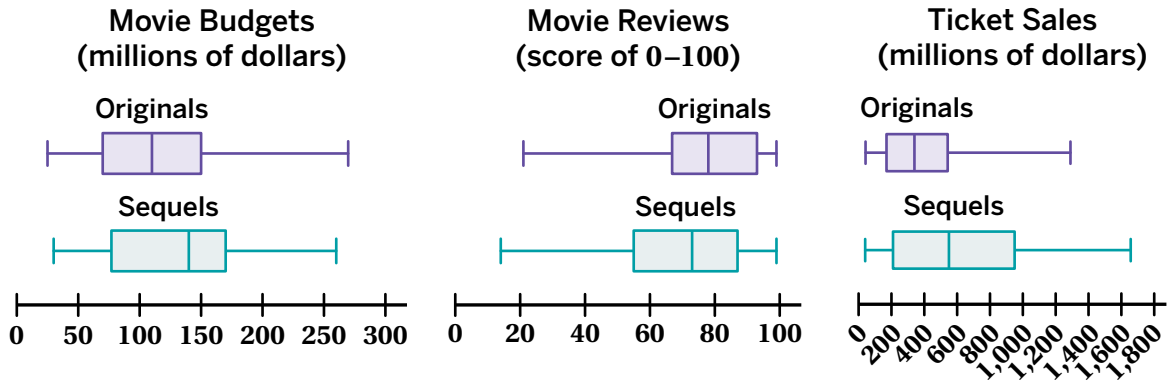
What do these numbers tell you about the movies?

Activity 3

Name: ..... Date: ..... Period: .....

You're in Charge!

10 Here are the three sets of box plots from this lesson.



a If you were in charge of making a movie, would you make an original or a sequel?

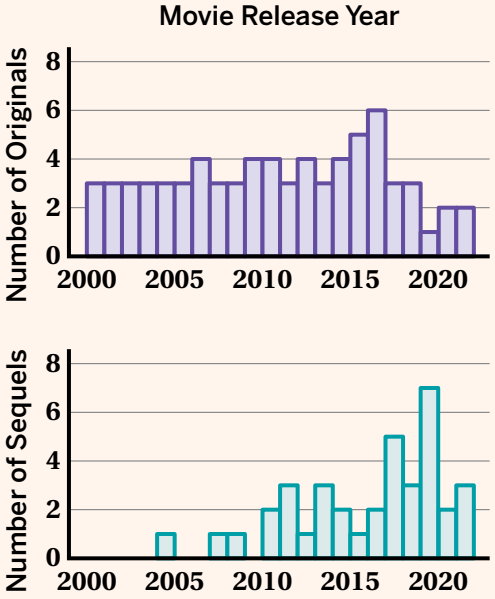
Original Sequel

b Explain your choice using evidence from one or more of the box plots.

You're invited to explore more.

11 These histograms represent the release years for originals and sequels.

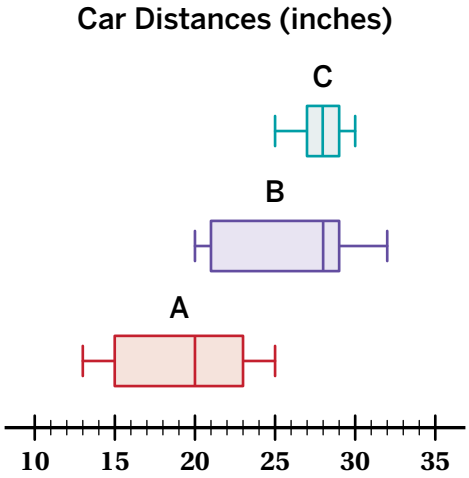
What do these histograms tell you about originals and sequels?



12 Synthesis

Discuss: What are some questions that you could use box plots to help you answer?

Use these box plots that represent toy car distances as examples.



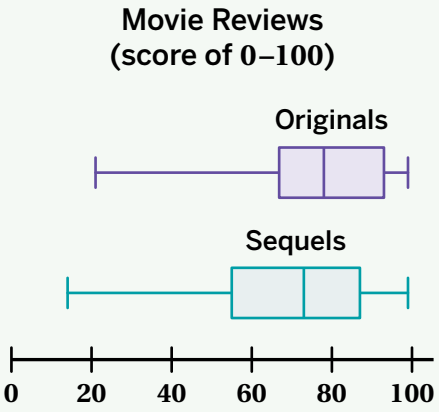
15 Summary 8.15

You can make many different claims about data using box plots.

For example, these box plots show data about movie review scores for originals and sequels.

You can use the IQR or the median to make claims about specific information, as well as general trends, including:

- The IQR for originals is 20 and for sequels is 30. This tells us that the middle 50% of the data is more spread out for sequels.
- The median score for originals is higher than the median score for sequels. This tells us that original movies typically had higher review scores than sequels.

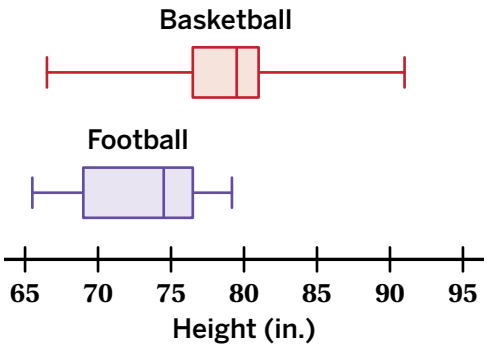


Originals	Sequels
Median: 80 IQR: 20	Median: 75 IQR: 30

Practice
8.15

Name: ..... Date: ..... Period: .....

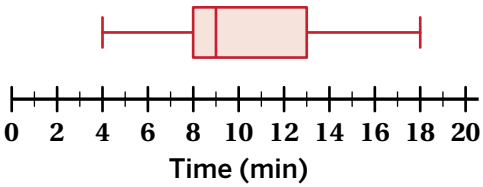
Problems 1–2: Here are box plots that represent the heights of 20 professional basketball and football players.



1. Liam says that basketball players have a greater spread of heights. Explain what statistic he used to make that claim.

2. Jayden says that football players have a greater spread of heights. Explain what statistic Jayden used to make that claim.

Problems 3–5: This box plot shows the delivery time, in minutes, that a pizza shop took to deliver 100 orders. Determine if each statement is *true* or *false*.

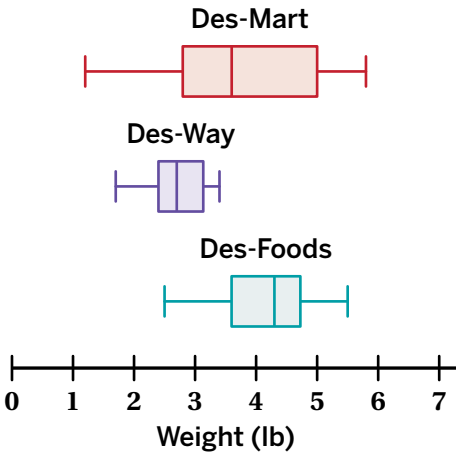


3. About 75% of the delivery times were 13 minutes or less.

4. About half of the delivery times were 5 minutes or greater.

5. The IQR is 14 minutes.

Problems 6–7: Three different grocery stores weighed the pineapples at their store. Here are three box plots that represent the weights in each grocery store.



6. Which grocery store has the greatest median weight?

7. Where would you shop if you needed pineapples that were many different weights?

Explain your thinking.

Practice
8.15

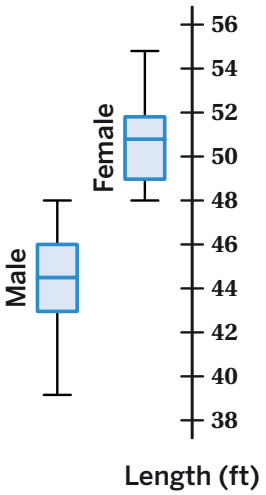
Name: ..... Date: ..... Period: .....

Spiral Review

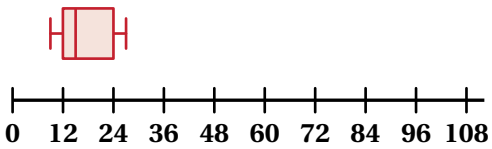
Problems 8–10: Researchers measured the lengths, in feet, of 20 male humpback whales and 20 female humpback whales. Here are two vertical box plots that summarize their data.

For each claim, state whether you agree or disagree. Explain your thinking.

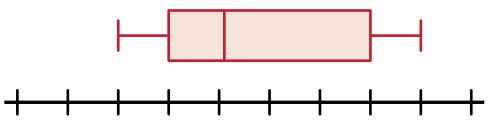
	Claim	Agree	Disagree	Explanation
8.	More than half of the female humpback whales measured were longer than 52 feet.			
9.	The male humpback whales were typically longer than the females.			
10.	The lengths of the male humpback whales vary more than the lengths of the females.			



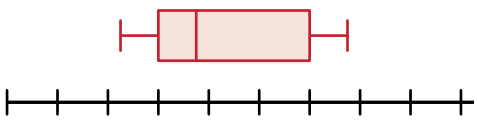
Problems 11–13: Here are three box plots that represent the same data on different axes. Complete the number lines for the second and third box plots.



11.



12.



13. What is the median of this data set?

Problems 14–17: Calculate each percentage.

14. 25% of 30

15. 75% of 30

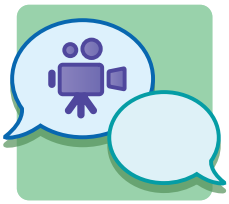
16. 75% of 150

17. 58% of 18?

Name: ..... Date: ..... Period: .....

Variability in Data | The Shape of Distributions | 6.SP.3, 6.SP.4, 6.SP.5, 6.SP.5.d, SMP.3, SMP.4

Hollywood, Part 3



Let's use all of our statistics tools to model and make claims.

Warm-up

Assemble: Finalgame was the top-grossing movie in 2019.

Tay and Cho watched the movie and noticed that the male characters had 381 minutes of screen time, but that female characters had 116 minutes of screen time.

1. Discuss:

- What do you think "top-grossing" means?
- Do you think gaps like this are common in movies?
- Do you think gaps like this have gotten smaller or larger over time?

Name: ..... Date: ..... Period: .....

Questions and Claims (continued)

Data Talk! You will use a copy of the Activities 1 & 2 Sheets with data from the 10 top-grossing films in 2010.

2. Take a look at the data.

a What do you notice? What do you wonder?

I notice: I wonder:

b **Discuss:** Why might someone want to collect and analyze this data?

c What do the numbers 24 and 76 tell us about *Toy Tales 3?*

3. Cho and Tay looked at the 2010 data.

Cho says: *Men speak more in these movies than women!*

Tay says: *Men don't speak more because 65% of the words in Alice's Whimsical Realm were spoken by women. That's more than half!*

a Who do you agree with? Explain your thinking.

b How could Cho and Tay use statistics to settle their dispute?

Name: ..... Date: ..... Period: .....

2010 Movies (continued)

4. Cho and Tay decided to compare the medians.

a Why might they have chosen to compare medians instead of means?

b Calculate the median percentages of words spoken by men and women in these ten movies from 2010.

Men: ..... Women: .....

5. Here are Tay and Cho's claims again.

Cho: *Men speak more in these movies than women!*

Tay: *Men don't speak more because 65% of the words in Alice's Whimsical Realm were spoken by women. That's more than half!*

Help Tay and Cho settle their dispute. Use the data displays (box plot and histogram) and any statistics you want to justify your answer.

6. Tay also said that the number of words spoken by women in these movies was more consistent. What statistic could Tay calculate to help back up the claim?

Activity 2

Name: ..... Date: ..... Period: .....

1990 Movies and 2010 Movies

Data Talk! You will use the Activities 1 & 2 Sheets again, looking at the data from the 10 top-grossing films in 2010 and 1990.

7. Choose *one* statistical question to investigate.

Question 1	Question 2	Question 3
How was the percentage of words spoken by *women* in the 10 top-grossing movies different in 1990 and 2010?	How was the percentage of words spoken by *men* in the 10 top-grossing movies different in 1990 and 2010?	What is the gap between the percentage of words spoken by men and women in the 10 top-grossing movies of 1990? How is this different from 2010?

8. Make a poster for the question you chose. Look at Activity 1 for any 2010 data or statistics that are helpful.

Your poster should include:

- ☐ The question you are trying to answer.
- ☐ A data display for each data set you need to answer your question (dot plot, histogram, or box plot).
- ☐ A measure of center and spread (mean/MAD or median/IQR) for each data set.
- ☐ An explanation of how you decided which measures of center and spread to use and what each number tells you about the data.
- ☐ Your answer to the question.
- ☐ Two new questions that you have after analyzing the data.

Synthesis

9. Use your knowledge of statistics and data visualizations from this unit to answer *one* of these questions.

a What could you do if you think the amount of words spoken by a group of people in a movie isn't fair?

b How can statistics and data displays help us make sense of topics like dialogue in movies?

c What other questions might you ask to investigate this topic more?

Summary 8.16

You can answer statistical questions using statistics and data displays. Here is data about how many hours of movies these students and teachers watched last month.

Students				
0	0	5	10	10
20	25	60	75	100

Teachers				
0	15	15	20	20
30	30	35	35	40

To make a claim about the movie habits of these students and teachers, you could create data displays like box plots or calculate measures of center (mean or median) and measures of spread (MAD or IQR).

Students	Teachers
Mean: 30.5 MAD: 28.7 Median: 15 IQR: 55	Mean: 24 MAD: 10 Median: 25 IQR: 20

From this data, you might claim that these teachers watched more movies because their median is higher. You might also claim that these students watch more movies because their mean is higher.

Practice
8.16

Name: ..... Date: ..... Period: .....

Problems 1–7: Saanvi recorded the number of math problems she did for homework each night for 10 days.

$$0, 5, 5, 4, 5, 10, 5, 6, 50, 0$$

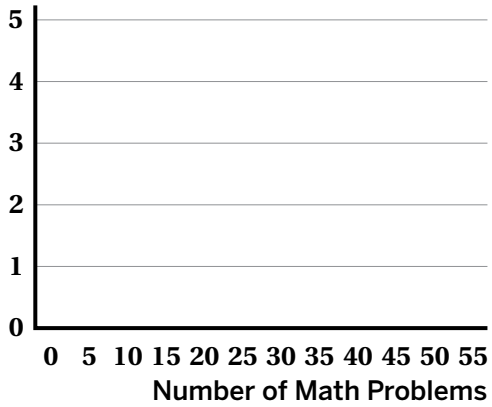
1. Calculate the mean for this data.

2. Calculate the MAD for this data.

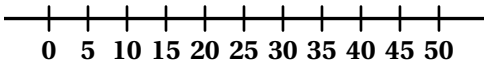
3. Calculate the median for this data.

4. Calculate the IQR for this data.

5. Create a histogram of Saanvi's data.



6. Create a box plot of Saanvi's data.



7. Which pair of measures (mean and MAD or median and IQR) would you use to describe this data?

Explain your thinking.

Practice
8.16

Name: ..... Date: ..... Period: .....

Problems 8–9: Kwame and Nikolai built paper airplanes. They each launched their paper airplanes 10 times and recorded the distance they traveled in yards. Here are the distances in order from shortest to longest.

Kwame

22	26	27	27	28	28	28	29	30	30

Nikolai

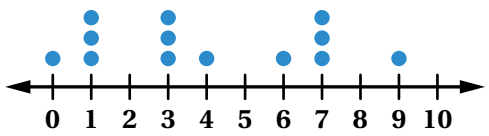
20	20	21	24	28	28	28	29	32	35

8. Nikolai says that his airplane typically flies farther than Kwame's. Is Nikolai's statement correct?

9. Whose airplane had a wider spread of distances? Use a statistic (mean, median, MAD, IQR, or range) to justify your answer.

Spiral Review

10. A data set is displayed in the dot plot shown. What is the mean of the data set?



11. How can the shape of the distribution help you compare the mean and median of this data set?

Problems 12–14: Calculate each percentage.

12. 50% of 17

13. 150% of 17

14. 75% of 17

Name: _____ Date: _____ Period: _____

Variability in Data | The Shapes of Distribution | 6.SP.3, 6.SP.4, 6.SP.5, 6.SP.5.c, SMP.3, SMP.6

Practice Day 2



Let's practice what you've learned so far in this unit!

You will use task cards for this Practice Day. Record all of your responses here.

Task A: Dot Plots and Box Plots

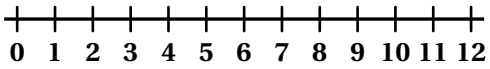
1. Quartile 1: _____

Median: _____

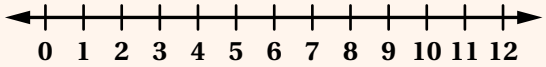
Quartile 3: _____

2.

3.



You're invited to explore more.



Task B: Homework

1. Kyrie's mean: _____ Tiara's mean: _____

2. Kyrie's MAD: _____ Tiara's MAD: _____

3. Circle one: Kyrie Tiara
Explanation:

4. Circle one: Kyrie Tiara
Explanation:

You're invited to explore more.

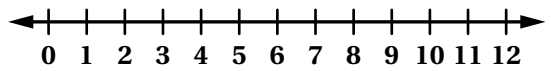
Name: _____ Date: _____ Period: _____

Practice Day 2 (continued)

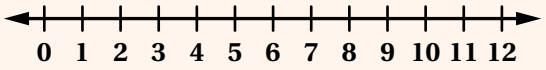
Task C: Dot Plot Maker

1. Dot Plot 1: Statement _____ Dot Plot 2: Statement _____

2.



You're invited to explore more.



Task D: Swimming

1. 1984 team: IQR: _____ Median: _____

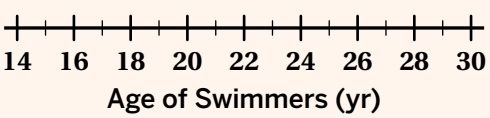
2016 team: IQR: _____ Median: _____

2.

a	1984 team	2016 team	Not enough information
b	1984 team	2016 team	Not enough information
c	1984 team	2016 team	Not enough information
d	1984 team	2016 team	Not enough information
e	1984 team	2016 team	Not enough information
f	1984 team	2016 team	Not enough information
g	1984 team	2016 team	Not enough information

3.

You're invited to explore more.



Notes:

Career Connection

From 2000 to 2023, the percentage of Internet use by U.S. adults almost doubled!

How do you think this information was determined? Pew Research Center, based in Washington, D.C, conducted surveys to determine the percentage of U.S. adults who say they use the Internet each year from 2000 to 2023. From 2000 to 2021, these surveys were conducted via phone. In 2023, the method changed to web and mail. In 2000, about 52% of U.S. adults said they used the Internet. In 2023, about 95% of U.S adults said they used the Internet.

Survey researchers design surveys, gather the data, and use statistical displays to analyze the data to draw conclusions. As they design the survey questions, they often think about whether the questions they are asking are statistical questions.



Meet Moon Duchin

Moon Duchin is a professor of mathematics and the John DiBiaggio professor of Citizenship and Public Service at Tufts University in Massachusetts. John DiBiaggio was the 11th president of Tufts and founded the University College of Citizenship and Public Service.

At Tufts, Moon Duchin runs a research lab that studies data science with many applications to civics and political processes.

Are you interested in a career in data science or survey research? What can you do to learn more?

Community Connection

Design a survey question for a topic you are interested in. Ask at least 15 people in your home and community your survey question. Record the data on a separate piece of paper. If applicable, create a statistical display to show your data.

What do you notice about the results of your survey? What do you wonder?

Math Mindset

What might be important to think about when designing a survey question?