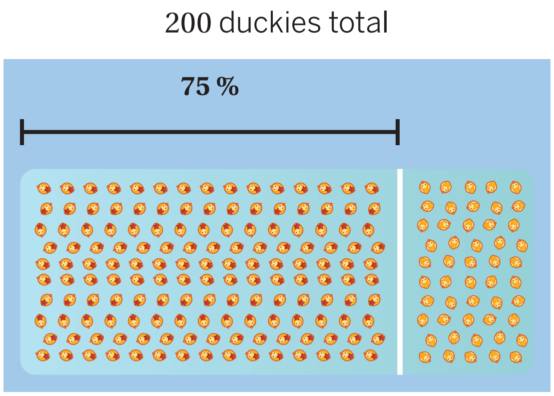
Percentages

Accelerated 6
Unit 6

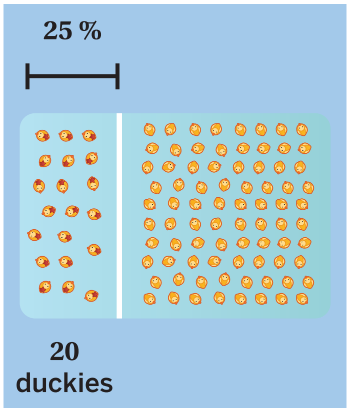
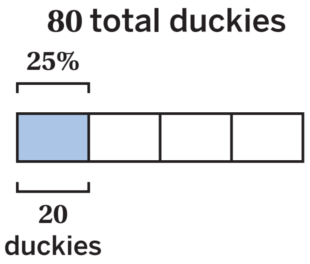
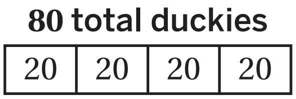
Synthesis

In your own words, explain what 75% of a number means.



Summary

Percent means for every 100. It is represented by the percent symbol, %. The different Ducky Games had certain percents of ducks with stars: 10%, 25%, 50%, or 75%. It can be helpful to use fractions and tape diagrams to interpret these benchmark percentage problems.

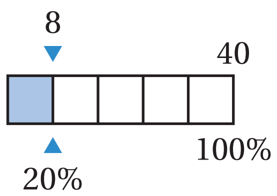
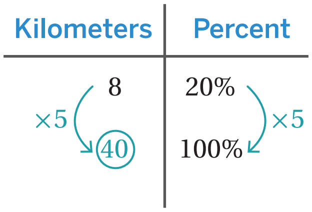
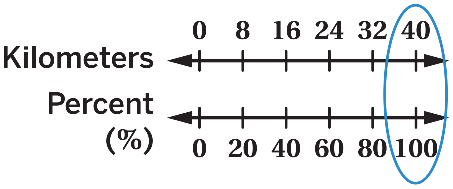
Percent example	Using fractions	Using a tape diagram
25% of 80 duckies have stars.  	25% of something means $\frac{25}{100}$ or $\frac{1}{4}$. $\frac{1}{4}$ of 80 duckies is 20 duckies.  	There are four 25s in 100, so the tape diagram is split into 4 pieces. The total duckies are also split into 4 parts, so there are 20 duckies in each section. 

Synthesis

Describe how solving problems with percentages is like solving problems with ratios. Use Activity 2 if it helps with your thinking.

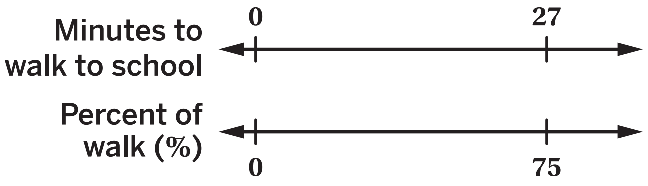
Summary

A **percentage** refers to a rate per 100 when the exact percent or quantity is not specified. Like ratios, percentages can be represented by tape diagrams, double number line diagrams, and tables. Using these tools and ratio reasoning helps to solve problems involving percentages. For example, if a biker has traveled 8 kilometers, which is 20% of their goal, multiple representations of ratio reasoning can be used to determine the biker's goal distance.

Tape diagram	Table	Double number line diagram
 	 	

Synthesis

Explain how this double number line can help you calculate the total time Eliza takes to walk to school.

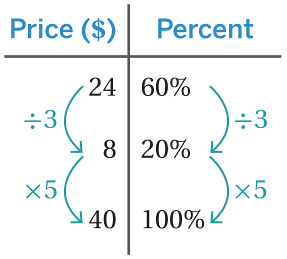
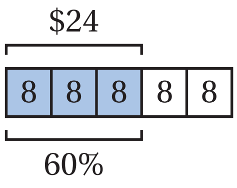
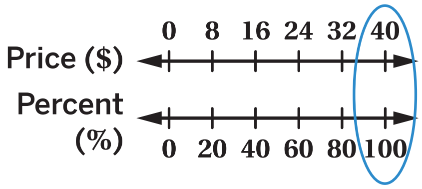


Summary

Tables, tape diagrams, and double number line diagrams can be used to determine unknown parts, percentages, and wholes. There are three basic categories of percentage problems:

- Given the part and the percentage, find the whole.
- Given the part and the whole, find the percentage.
- Given the percentage and the whole, find the part.

Consider this example: The sale price of a sweater is $24. The sweater is on sale for 60% of the original price. How much did the sweater cost before the sale? Here are three representations that can be used to find the whole (the original price of the sweater) given the part and the percent.

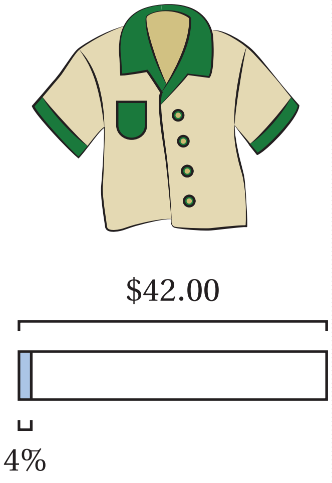
Table	Tape diagram	Double number line diagram
		

Synthesis

Describe a strategy for calculating a percentage of a number. Use the example if it helps you explain your thinking.

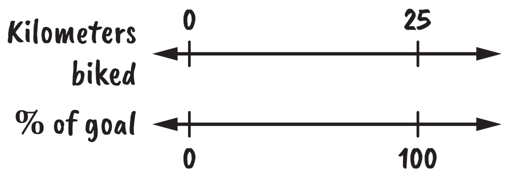
Summary

There are multiple strategies to calculate any percentage of a number. Consider this example: A button-up shirt costs $42.00. The factory's profit is 4% of the total cost. Calculate the factory's profit.

<table>
<tr><th>Tape diagram</th><th>Bao's strategy</th><th>Ada's strategy</th></tr>
<tr>
<td> </td>
<td><table>
<tr><th></th><th>Cost ($)</th><th>Percent</th><th></th></tr>
<tr><td></td><td>42</td><td>100%</td><td></td></tr>
<tr><td>÷100</td><td></td><td></td><td>÷100</td></tr>
<tr><td></td><td>$\frac{42}{100}$</td><td>1%</td><td></td></tr>
<tr><td>×4</td><td></td><td></td><td>×4</td></tr>
<tr><td></td><td>$\frac{42}{100} \cdot 4$</td><td>4%</td><td></td></tr>
<tr><td></td><td>$\boxed{\$2.70}$</td><td></td><td></td></tr>
</table></td>
<td>4% means 4 cents for every dollar, so that rate can be multiplied by the number of dollars to find the factory's profit.
$\frac{4}{100} \cdot 42$
$\boxed{\$2.70}$</td>
</tr>
</table>

In general, the expressions $\frac{A}{100} \cdot B$ or $\frac{B}{100} \cdot A$ can be used to calculate A% of B.

Synthesis



Here is what Basheera wrote to solve a bicycle challenge.

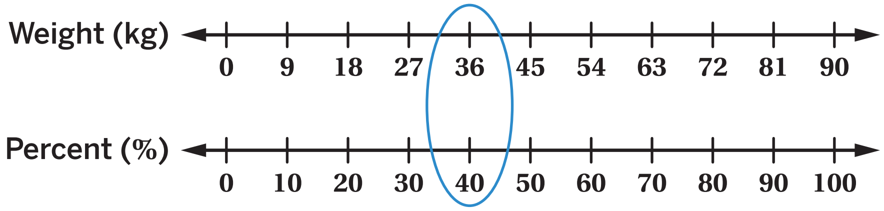
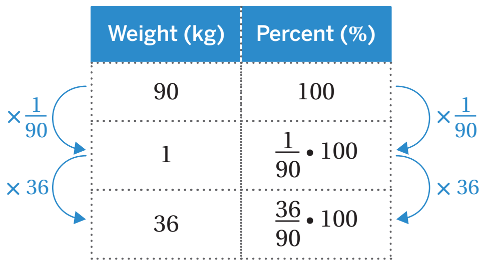
$\frac{31}{25} \cdot 100 = 124$

What do 31, 25, and 124 represent in this scenario?

- 31 represents . . .
- 25 represents . . .
- 124 represents . . .

Summary

Ratios can be used to determine what percent one amount is relative to another amount. For example, suppose an adult weighs 90 kilograms and a child weighs 36 kilograms. To determine the child's weight as a percent of the adult's weight, multiple methods could be used.

Double number line diagrams		
Ratio tables		Determine the unit rate (what percent matches 1 kilogram). Use the unit rate to determine the percent that corresponds with 36 kilograms.
Expressions	$36 \div 90 \cdot 100 = \frac{36}{90} \cdot 100 = 40$	Evaluate $\frac{p}{w} \cdot 100$ to determine the percent that one value p is of another value w.

Synthesis

How is working with percentages like working with a village of 100 people?

Summary

When looking at the population of a country, the numbers in each category are often very large, and sometimes very far apart. When this happens, it can be challenging to compare or get a sense of the real differences in the population.

Ratios, rates, and percentages serve two important purposes:

- They allow you to compare quantities that are on different scales because they describe things in terms of multiplying and dividing instead of adding and subtracting.
- Rates and percentages bring everything to the same scale, most commonly with a reference point of either 1 or 100, which makes comparing numbers more straightforward.

Instead of having to use very large and far-apart numbers for different populations in the world, determining percentages can help when comparing different groups to see what the distribution of people really looks like.

Synthesis

What are some strategies you can use to show that Haru's soup will have more carrots per liter than Mohamed's soup?

Mohamed's vegetable soup recipe	Haru's vegetable soup recipe
$\frac{1}{3}$ of a cup of carrots for every $\frac{1}{5}$ of a liter of soup	$\frac{7}{8}$ of a cup of carrots for every $\frac{1}{3}$ of a liter of soup

Summary

Proportional relationships may involve fractional amounts. You can solve problems involving fractions by using the same strategies you use to solve problems with whole numbers.

- To determine the constant of proportionality within a recipe, divide the amount of the specific ingredient by the total number of servings.
- Constants of proportionality can help to compare proportional relationships involving fractional quantities.

Here is a recipe for banana bread. To find the amount of sugar per serving, divide $\frac{3}{4}$ cups of sugar by 6 servings. This gives you $\frac{3}{4} \div 6$, or $\frac{1}{8}$ cups of sugar per serving.

Banana bread recipe

Number of servings: 6

- 2 lb of bananas
- $\frac{1}{2}$ cups of butter
- $\frac{3}{4}$ cups of sugar
- $2\frac{1}{2}$ cups of flour
- 1 tsp of baking soda

Synthesis

Describe how you can use a table displaying a proportional relationship to determine missing values. Use the table shown if it helps you with your thinking.

Height (in.)	Width (in.)
$2\frac{2}{3}$	$1\frac{1}{3}$
	$5\frac{1}{2}$

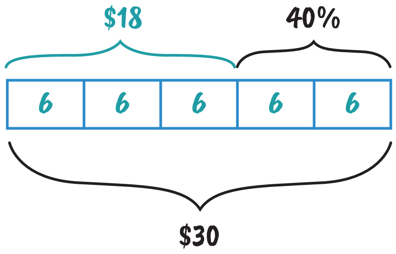
Summary

When a problem involves a proportional relationship, determining the constant of proportionality or scale factor can help unlock the answers to many questions about the situation.

In a table of proportional values, the constant of proportionality can help to determine any missing values. Usually, this means multiplying or dividing by the constant of proportionality, depending on which value is known and which is unknown.

Synthesis

Here are two different strategies to find the new price of a hat after a discount of 40%. The hat was originally $30.

Strategy 1	Strategy 2
	

Discuss How are these strategies alike and how are they different?

Summary

Tape diagrams and tables can help to make sense of problems involving **percent increase** and **percent decrease**.

In general, the terms *percent increase* and *percent decrease* describe an increase or decrease in a quantity as a percentage of the starting amount.

One method used to solve these types of problems is to start with the original amount and then either add or subtract the amount corresponding to the percent of increase or decrease.

Synthesis

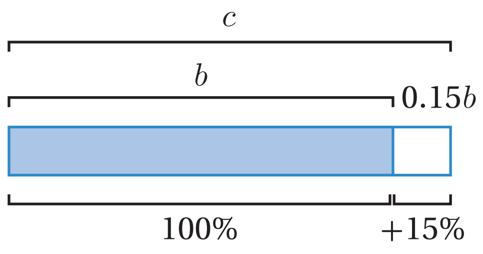
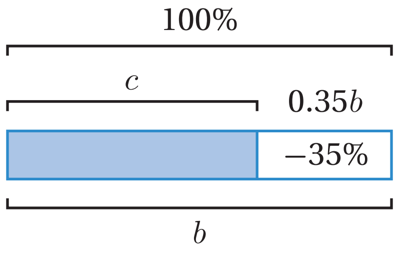
Here are two equations that could be used to solve a problem about *percent increase* or *percent decrease.*

$$1.25 \cdot b = c \qquad\qquad 0.80 \cdot b = c$$

Select an equation and write a story about a situation it could represent.

Summary

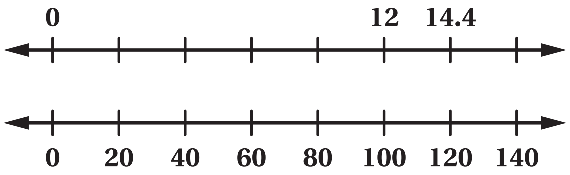
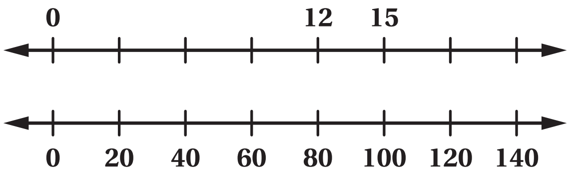
Equations can be used to reason about situations involving *percent increase* or *percent decrease*. Here are examples of equations representing a percent increase relationship and a percent decrease relationship.

	Relationship	Equations	Diagram
Percent increase	c is 15% more than b.	$c = b + 0.15b$ $c = (1 + 0.15)b$ $c = 1.15b$	
Percent decrease	c is 35% less than b.	$c = b - 0.35b$ $c = (1 - 0.35)b$ $c = 0.65b$	

Synthesis

Abdullah and Kanna are working on the same problem.

A juice box has 20% more juice in its new packaging. The original packaging held 12 fluid ounces. How many fluid ounces of juice does the new packaging hold?

Abdullah's double number line diagram	Kanna's double number line diagram
	

Whose double number line diagram is correct? Show your thinking.

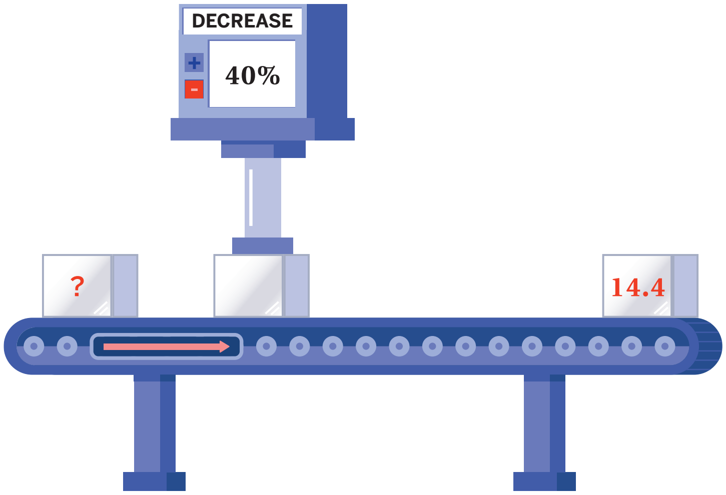
Summary

To solve percentage problems involving cost, first identify whether there is a markup or a markdown to help decide if the new amount will be greater than or less than the original amount.

Markup	Markdown
Example: A board game costs a retail store $12, but they sell it to consumers for $19.80 to cover their cost and make a profit. This is a *percent increase*. The new amount is 65% greater than the store's original cost of $12.	**Example:** A store sells a board game for $20, but they are offering a 15% off discount. This is a *percent decrease*. The new amount will be $17, which is less than the original cost of $20.

Synthesis

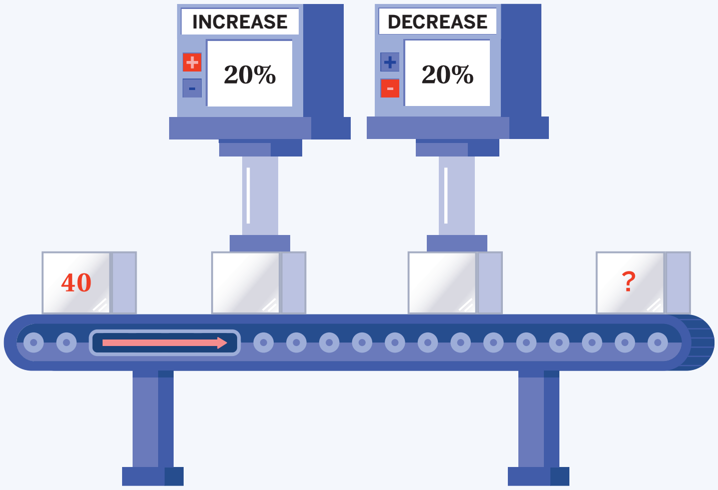
Describe a strategy for determining the input when you know the percent change and the output. Use this example if it helps you explain your thinking.



Summary

Percent machines take an input value and increase or decrease that value by a percentage to produce an output.

Increasing and then decreasing by the same percentage will not return the original input value.



For example:

- An input value of 40 is increased by 20%.
  $40 \cdot 1.2 = 48$
  or
  $40 + (40 \cdot 0.2) = 48$
- The new value is decreased by 20%.
  $48 \cdot 0.8 = 38.4$
  or
  $48 - (48 \cdot 0.2) = 38.4$

It may be surprising to discover the final result is *not* the original input value of 40! This is because the input value represents the whole when determining 20% of 40. However, in the second calculation the input value changes, so now we determine 20% of 48 and subtract that value from 48.

Synthesis

Describe a strategy for calculating the total for any item after a discount and tax.

Use the receipts if they help you with your thinking.

Item:	$12.00
20% Discount:	– $2.40
Subtotal:	$9.60
5% Tax:	+ $0.48
Total:	$10.08

Item:	$12.00
15% Discount:	– $1.80
Subtotal:	$10.20
15% Tax:	+ $1.53
Total:	$11.73

Item:	$12.00
10% Discount:	– $1.20
Subtotal:	$10.80
8% Tax:	+ $0.86
Total:	$11.66

Item:	$12.00
5% Discount:	– $0.60
Subtotal:	$11.40
6.25% Tax:	+ $0.71
Total:	$12.11

Summary

Sales tax is an amount of money that a government agency collects on the sale of certain items. Often, the tax rate is given as a percentage of the cost.

The total cost to the customer is the cost of the items plus the sales tax. This is a percent increase. For example, if the sales tax rate is 7.5%, the total cost to a customer is 107.5% of the cost of the item.

A tip, also known as a gratuity, is an amount of money that a person gives someone who provides a service, such as restaurant servers, hairdressers, and delivery drivers. It is customary in many restaurants to tip the server about 20% of the cost of the meal. If a person plans to leave a 20% tip on a meal, then the total cost with tip will be 120% of the cost of the meal.

Synthesis

Describe how you can determine how much a restaurant server makes in a week. Be as specific as you can.

Summary

Minimum wage is sometimes calculated differently for workers who receive tips.

Different types of wage systems — some with tips and some without — can be analyzed by comparing the earnings of different employees in different situations. Using your own sense of fairness, you decided whether the existing systems for paying restaurant servers are fair. You then created and proposed a fair wage system for servers.

Synthesis

Yosef has $150 in a savings account whose value increases by 2% every year from simple interest. What strategies could Yosef use to calculate how much his savings would be worth in one year? In 10 years?

Summary

Calculating percent change can be used to analyze data, make predictions, and make recommendations for real-world situations, such as comparing how the cost of college and minimum wage change over time.

Percent change can also help make comparisons about other real-world situations, such as weather or population changes over time. These calculations help us make informed decisions, predictions, or recommendations for the future.

Recall that one way to determine the percent increase or decrease is to:

- Find the difference between the original and new amount.
- Divide the result by the original amount. Round, if necessary.
- Multiply the final result by 100 to calculate the percentage.

What is the percent change from $12.00 to $15.50?

$$15.50 - 12.00 = 3.50$$

and

$$\frac{3.50}{15.50} \approx 0.23$$

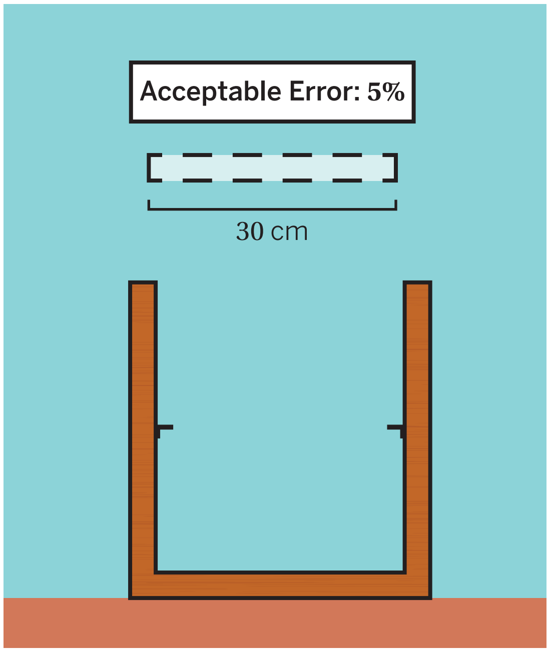
$$0.23 \cdot 100 = 23$$

The percent increase is 23%.

Synthesis

Describe a strategy for figuring out the range of values that are acceptable when you know the desired measurement and an acceptable percent error.

Use the example if it helps you with your thinking.



Summary

Percent error describes the difference between the actual and desired value, and is expressed as a percentage of the exact value.

For example, suppose a milk carton manufactured by a company is supposed to contain 16 fluid ounces, but it only contains 15 fluid ounces.

- The measurement error is 1 fluid ounce.
- The percent error is 6.25% because $\frac{1}{16} \cdot 100 = 6.25$.

It is important to remember that to determine the percent error, the amount of the error is always compared to the desired or expected value. You can use the following formula:

$$\text{percent error} = \frac{\text{(difference between actual value and desired value)}}{\text{desired value}} \cdot 100$$