

Unit 1

Volume

Essential Questions

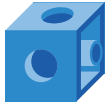
- How does volume help to describe and compare three-dimensional objects?
- How can you determine the volume of a figure that is composed of rectangular prisms?

Unit Story: Joyful Green

In this story, Trashville's excess trash is packed into cubes and shipped off to the town of Joyful Green, where the trash is given a new purpose.

Summary 5.02

Volume measures the amount of space a three-dimensional object takes up. It can be measured by counting the number of **unit cubes** needed to build the figure.

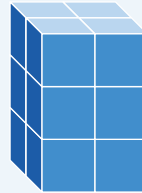
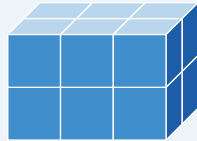
1 unit cube	Number of unit cubes used to build a figure	Volume of the figure
	7	7 unit cubes

Practice

- Which statement is true?
 - Volume is determined *only* by how long a figure is.
 - Volume is determined *only* by how tall and wide a figure is.
 - Volume is determined by the number of unit cubes needed to build a figure.
 - Volume is determined *only* by the length and width of a figure.
- Han used 12 unit cubes to build a figure. Determine the volume of the figure.

Summary 5.03

A rectangular prism is composed of identical layers, which can be used to determine its volume. Prisms that look different may have the same volume.

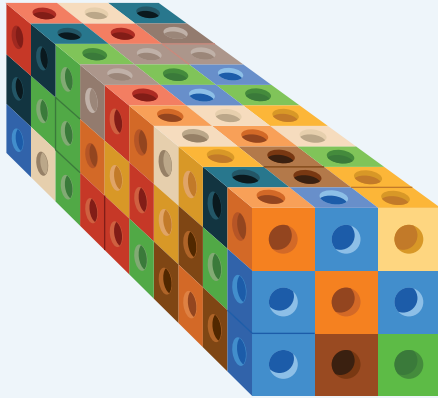


Practice

- 1** Diego used 12 unit cubes to build a rectangular prism. What could the prism look like? Select *all* that apply.
- A** There are 12 unit cubes in 1 layer. There are 3 layers.
 - B** There are 3 identical layers. In each layer, the unit cubes are arranged in 2 columns with 2 unit cubes in each column.
 - C** There are 3 layers. Each layer is arranged in 3 rows with 2 unit cubes in each row.
 - D** There are 2 layers of 3 columns with 2 unit cubes in each column.

Summary 5.04

The volume of a rectangular prism can be determined by counting equal groups of any 1 layer.



The prism has 10 layers of 9 cubes or 3 layers of 30 cubes.

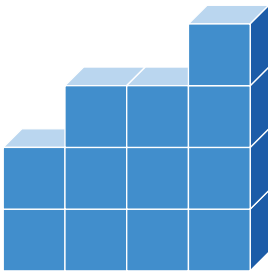
Practice

- 1 Shawn says that the volume of the rectangular prism is 45 unit cubes because there are 3 layers of 15 unit cubes. Do you agree with Shawn? Explain your thinking.

In this sub-unit . . .


- We defined **volume** as the amount of space a three-dimensional object takes up. It can be measured by packing an object with **unit cubes** so that there are no gaps or overlaps.

- We determined the volumes of three-dimensional figures by counting the number of unit cubes used to build the figures.

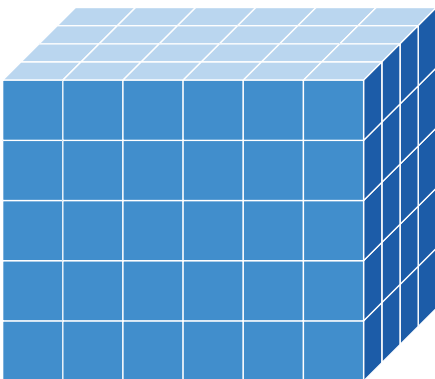


The volume is 12 unit cubes:

- 1 cube on top
- 3 cubes in the next row
- 8 cubes on the bottom arranged in 2 rows

 **Math tip:** Count more quickly by looking for equal groups of cubes in rows and columns.

- We saw that **rectangular prisms** are composed of layers, or equal groups of unit cubes. We described the rectangular prisms in different ways.

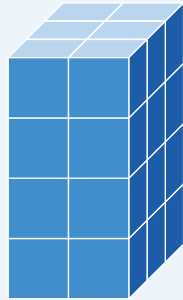


Face	Description
side	6 layers of 20 cubes
front	4 layers of 30 cubes
top	5 layers of 24 cubes

Summary 5.05

The volume of a rectangular prism is the product of the number of unit cubes in 1 layer, or the area of the base, and the number of layers. Because any face can represent the base, different multiplication expressions can represent the volume of the same prism.

Rectangular prism



Expressions

$$8 \times 3$$

$$(2 \times 4) \times 3$$

$$6 \times 4$$

$$12 \times 2$$

Practice

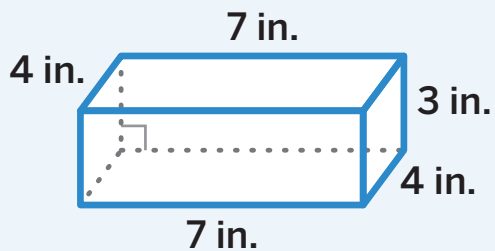
- 1 Determine the volume of the rectangular prism.

 Show or explain your thinking.

answer: _____

Summary 5.06

The volume of a rectangular prism is measured in cubic units, such as cubic inches. The formulas $V = \ell \times w \times h$ and $V = B \times h$ can be used to determine the volume of any rectangular prism.



$$V = \ell \times w \times h$$

- $V = 7 \times 4 \times 3$

$$V = B \times h$$

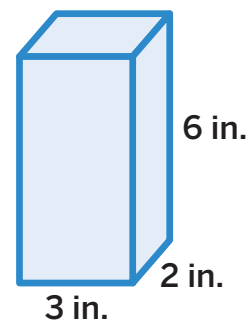
- $V = 28 \times 3$
- $V = 12 \times 7$
- $V = 21 \times 4$

The volume, V , is 84 cubic inches.

Practice

- 1 Determine whether each expression represents the volume of the rectangular prism in cubic inches. Place a check mark in the correct column.

	Yes	No
12×3		
$6 + 6 + 6$		
$(2 \times 3) \times 6$		
$3 \times 6 \times 3$		
6×6		



Summary 5.07

The volume of a rectangular prism is measured in **cubic units** and can be measured using different-sized standard units. While the measurement may be a different value, the volume is the same.



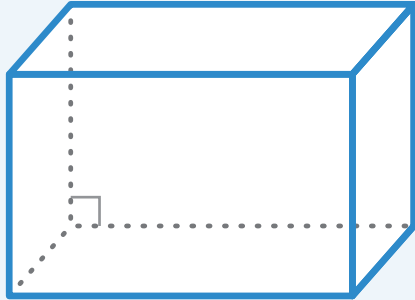
The volume is:
295 cubic centimeters
or
18 cubic inches
or
0.01 cubic feet

Practice

- 1 Which statements are true? Select *all* that apply.
- (A) The smaller the unit of measure used, the lesser the value representing the volume will be.
 - (B) The smaller the unit of measure used, the greater the value representing the volume will be.
 - (C) The volume of an object changes depending on the unit of measure.
 - (D) No matter which unit of measure is used, the volume of the object is the same, but the value representing the volume will be different.

Summary 5.08

The formulas for the volume of rectangular prisms can be used to solve real-world problems.



$$V = \ell \times w \times h$$

$$V = B \times h$$

Practice

For Problems 1 and 2, determine the volume of the object.

- 1 a box of milk that measures 4 centimeters by 10 centimeters by 30 centimeters

 Show or explain your thinking.

answer: _____

Sub-Unit 2 | Summary


In this sub-unit . . .

- We saw that the volume of a rectangular prism is the product of the number of unit cubes in 1 layer, which is the area of a base, and the number of layers.
- We saw that different multiplication expressions can represent the volume of the same rectangular prism.

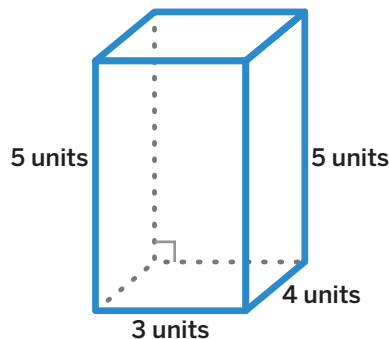


Expressions

- $2 \times 3 \times 4$
- $(2 \times 3) \times 4$
- 6×4
- $3 \times 4 \times 2$
- $(3 \times 4) \times 2$
- 12×2
- $2 \times 4 \times 3$
- $(2 \times 4) \times 3$
- 8×3

 **Math tip:** Use parentheses to show which factors represent the dimensions of the base. The other factor is the height.

- We determined the volumes of rectangular prisms by multiplying the dimensions, or multiplying the area of a base and its corresponding height.
- We used the formulas $V = \ell \times w \times h$ and $V = B \times h$.



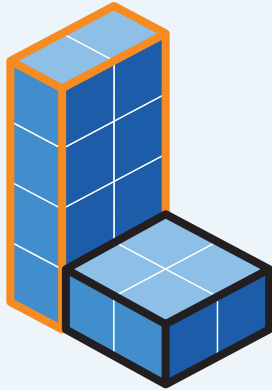
Expressions

- $3 \times 4 \times 5$
- 12×5
- 20×3
- 15×4

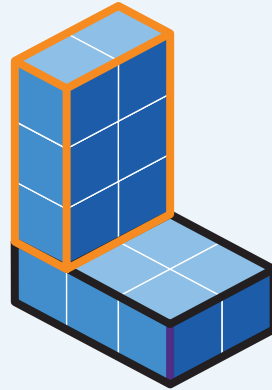
- We saw that we can use different-sized **cubic units** of measure, such as cubic inches and cubic feet, to measure the volume of an object. While the measurement may be a different number, the volume is the same.

Summary 5.09

Figures that are not rectangular prisms can be composed of rectangular prisms. The volume of the figure is the sum of the volumes of the prisms and is the same no matter how the figure is composed.



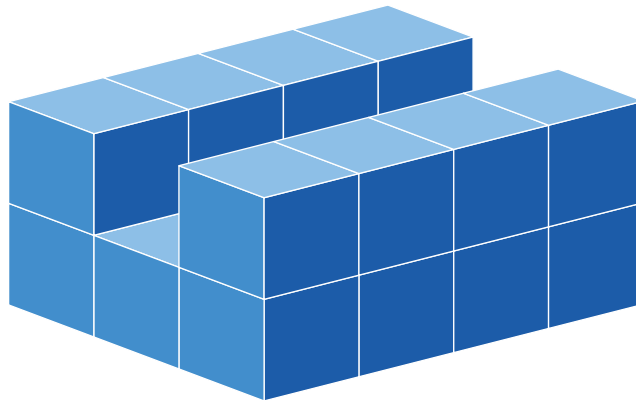
left prism: 8 unit cubes
right prism: 4 unit cubes
volume: 12 cubic units



top prism: 6 unit cubes
bottom prism: 6 unit cubes
volume: 12 cubic units

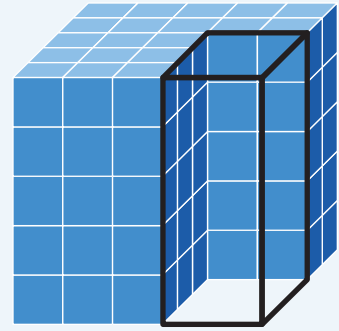
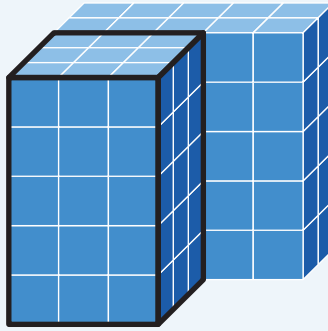
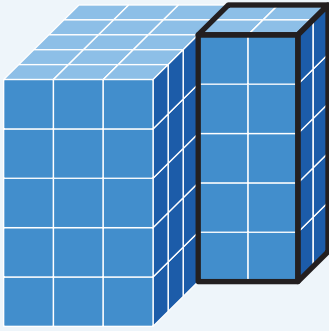
Practice

For Problems 1 and 2, use the figure.



Summary 5.10

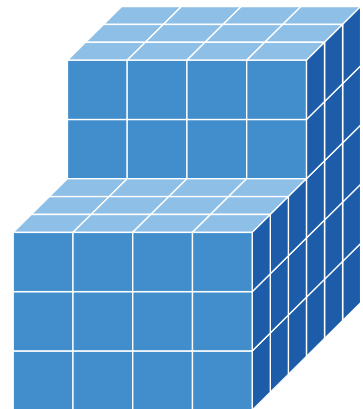
The volume of a figure composed of rectangular prisms can be determined by decomposing the figure into prisms. The volume can also be determined by filling an empty space, which is equivalent to composing a full prism.



Practice

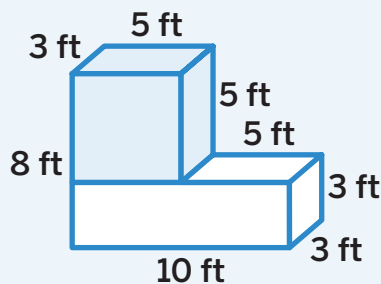
1 Determine the volume of the figure.

- (A) 24 cubic units
- (B) 96 cubic units
- (C) 106 cubic units
- (D) 144 cubic units

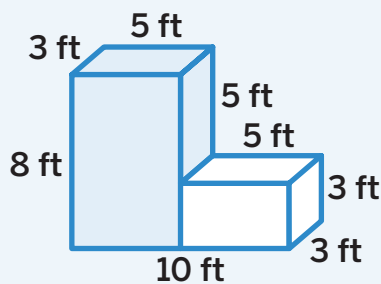


Summary 5.11

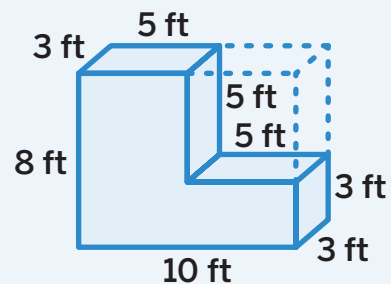
You can use the same strategies to solve a problem with volume whether you are given an image with or without unit cubes. Depending on the strategy you use, different expressions or equations will represent the volume of the figure.



$$(5 \times 3 \times 5) + (10 \times 3 \times 3) \\ 75 + 90 = 165$$



$$(5 \times 3 \times 8) + (5 \times 3 \times 3) \\ 120 + 45 = 165$$

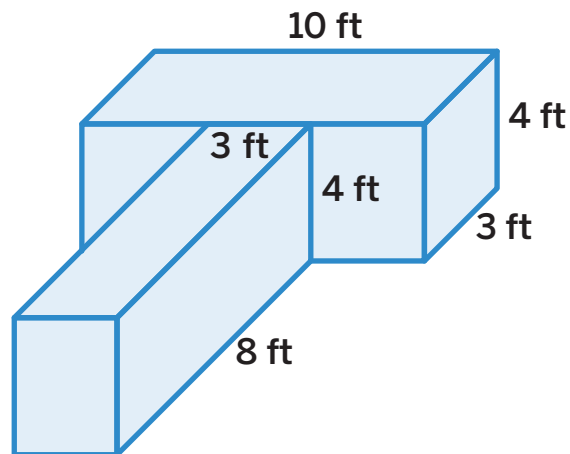


$$(10 \times 3 \times 8) - (5 \times 3 \times 5) \\ 240 - 75 = 165$$

Practice

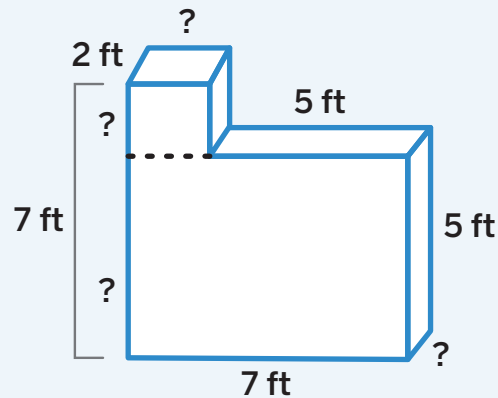
1 Select *all* the expressions that represent the volume of the figure.

- (A) $(40 \times 3) + (12 \times 8)$
- (B) $96 + 120$
- (C) $(4 \times 8 \times 3) + (4 \times 10 \times 3)$
- (D) $(30 \times 4) + (3 \times 32)$
- (E) $(10 \times 4 \times 8) + (9 \times 4)$



Summary 5.12

When determining the volume of a solid figure, you may need to determine some edge lengths. You can calculate the volume of the figure when you have the dimensions of your prisms.



volume: $(2 \times 2 \times 2) + (7 \times 2 \times 5)$

Practice

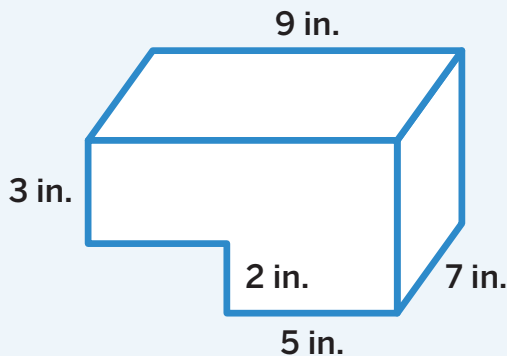
- 1 Determine the volume of the figure.

 Show or explain your thinking.

answer: _____

Summary 5.13

You can use what you know about the formula $V = \ell \times w \times h$ to explain how different expressions represent the volume of a figure composed of prisms.



$$(9 \times 7 \times 3) + (5 \times 7 \times 2)$$

$$(4 \times 7 \times 3) + (5 \times 7 \times 5)$$

$$(9 \times 7 \times 5) - (4 \times 7 \times 2)$$

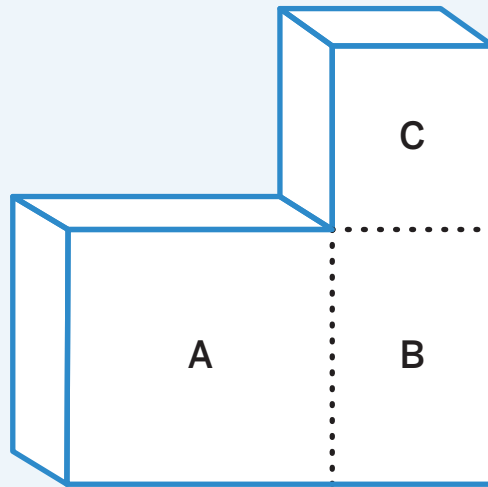
Practice

- 1 Show how the expression represents the volume of the figure.

 Show your thinking.

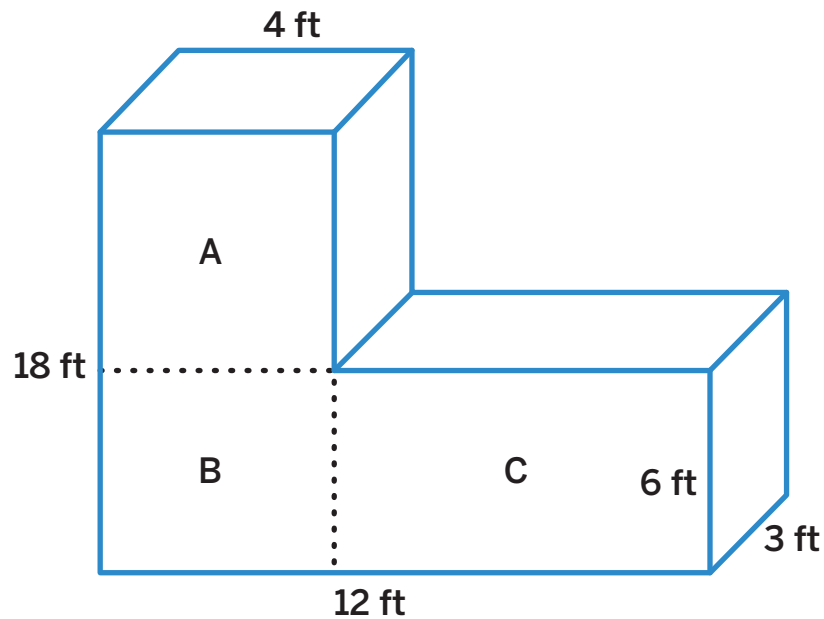
Summary 5.14

You can apply the known attributes of rectangular prisms to the formula $V = \ell \times w \times h$ to determine the volume of a solid figure made up of those prisms.



Practice

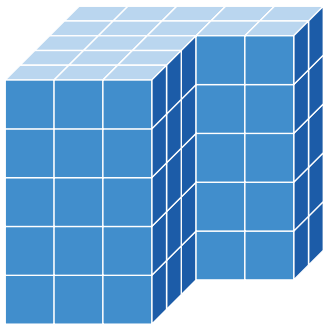
Use the figure for Problems 1 and 2.



Sub-Unit 3 | Summary

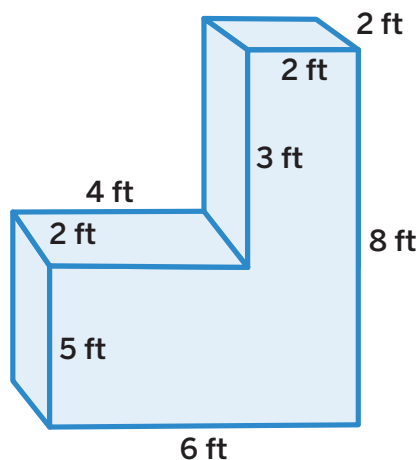
In this sub-unit . . .

- We saw that a figure made from 2 or more rectangular prisms can often be decomposed into rectangular prisms in different ways. The volume of the original figure is the sum of the volumes of the rectangular prisms.



Prism	Volume of the prism (cubic units)	Volume of the figure (cubic units)
left	$3 \times 3 \times 5 = 45$	$75 + 20 = 95$
right	$2 \times 2 \times 5 = 20$	
front	$3 \times 3 \times 5 = 45$	$45 + 50 = 95$
back	$5 \times 2 \times 5 = 50$	

- We wrote multiplication expressions to represent different strategies, such as decomposing the figure into rectangular prisms or filling the missing piece to make a completed rectangular prism. Sometimes, we need to determine some side lengths to calculate the volume in a way that reflects our chosen strategy.



Expressions

$$(6 \times 2 \times 5) + (2 \times 2 \times 3)$$

$$(16 \times 2) + (8 \times 5)$$

$$(8 \times 6 \times 2) - (4 \times 3 \times 2)$$

 **Math tip:** The volume of the original figure is the same, no matter which strategy you use.