Amplify Math

Grade 8

Volume 1: Units 1–4

Student Edition

About Amplify

Amplify is dedicated to collaborating with educators to create learning experiences that are rigorous and riveting for all students. Amplify creates K–12 core and supplemental curriculum, assessment, and intervention programs for today's students.

A pioneer in K–12 education since 2000, Amplify is leading the way in next-generation curriculum and assessment. All of our programs provide teachers with powerful tools that help them understand and respond to the needs of every student.

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Hello, curious mind!

Welcome to Grade 8. The math you'll see is getting a little more formal . . . but it will be no less interesting.

You see, this year, you'll test some tessellations, make a piece of cardboard come alive, spot which work of art is a forgery, and even see how you can use a triangle to measure the shape of the Universe. And that's just Unit 1!

We promise you, it's all possible. This year, you'll see not only that math makes sense, but it can be fun, too.

Before you dig in, we want you to know two things:

This book is special! It's where you'll record your thoughts, strategies, explanations, and, occasionally, any award-winning doodles that might come to mind.

When you go online, you won't be mindlessly plugging numbers into your device . . . You'll be pushing, pulling, crawling, teleporting, melting . . . , well, let's just say you'll be doing a lot of things, and you'll be teaming up with your classmates as you do.

Let's dig in!

Sincerely,
The Amplify Math Team

Volume 1

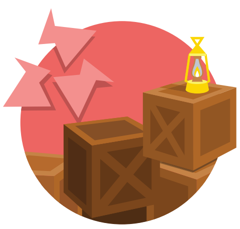
Unit 1 Rigid Transformations and Congruence

Unit Narrative:
The Art of Transformation

Shapes are all around us. We find them in art, architecture, and even in animated dancing frogs. In this unit, you will find out what happens when you slide, flip, and turn figures of all shapes and sizes. Plus, you may even create a masterpiece artwork along the way.

How do you make a piece of cardboard come alive?

Pack your geometry toolkits for a transformational journey into the movement of figures.



How can a crack make a piece of art priceless?

Something special happens when you perform rigid transformations on a figure.

What's got 10 billion galaxies and goes great with maple syrup?

Construct a triangle from a straight angle and cut two parallel lines to see what angle relationships you notice.

Unit 2 Dilations and Similarity

The way our brain interprets how objects appear — how big or small they are, how near or far — comes back to dilation. Learn to dilate figures and uncover the magic of this special type of transformation.

Unit Narrative:
More than Meets the Eye

Would you put poison in your eye?
Shrink and stretch objects on and off the plane and study the characteristics of the figures you dilate.

Do you really get what you pay for?
Learn how some companies use dilations to create similar, and slightly smaller, sized packaging, in a process called "shrinkflation."

Unit 3 Linear Relationships

How many cups tall is your teacher? Find out in this unit as you make connections between proportional and linear relationships.

Unit Narrative:
A Straight Change

How fast is a geography teacher?
On your mark, get set, go! Use your understanding of slope to show how a geography teacher shocked the world with her record-setting speed.

How did a coal mine help build America's most famous amusement park?
Use linear relationships to collect as many coins as you can at Honest Carl's Funtime World amusement park.

How did a 16-year-old take down a Chicago Bull?
Create equations from linear relationships and find how a 16-year-old was able to beat Michael Jordan in a game of basketball.

Unit 4 Linear Equations and Systems of Linear Equations

In this unit, you will learn the algebraic skills you need to find a solution for x. But wait, what do you mean there is no solution?

Unit Narrative:
The Path the Mind Takes

Who was the Father of Algebra?
When traders in 9th century Baghdad needed a better system for solving problems, a mathematician developed a new method he called “al-jabr” or algebra.

How is anesthesia like buying live lobsters?
Now that you have practiced solving equations, take a closer look at how you can use linear equations to solve everyday problems.

Volume 2

Unit 5 Functions and Volume

By studying functional relationships in this unit, you will soon be able to explain how height affects the volume of a sphere, calculate how the hare outran the tortoise, and produce your own version of the Happy Birthday song using a graph.

Unit Narrative: Pumping up the Volume on Functions

Who has the better camera: you or your grandparents?
Learn how functions can help you tell stories.

Who invented the waffle cone?
Use your prior knowledge about finding the volume of rectangular prisms to derive formulas for finding the volumes of cylinders, cones, and spheres.

Unit 6 Exponents and Scientific Notation

Unit Narrative:
From Teeny-Tiny to Downright Titanic

Imagine the *smallest* number you can think of. Now imagine the *largest* number you can think of. How can you write these numbers? How can you work with these numbers? In this unit, you'll learn about the power of exponents (pun intended), and how you can use them to work with numbers as small as the mass of a bacterium or as large as the number of atoms in the Universe.

How many carbs are in a game of chess?

You probably already know a thing or two about exponents, but what happens when you multiply or divide expressions with exponents?

Who should we call when we run out of numbers?

You'll work with numbers that are super small and incredibly large. But you won't waste your time writing pesky zeros!

Unit 7 Irrationals and the Pythagorean Theorem

Discover how three squares can prove something radical about triangles that has captivated mathematicians for centuries.

Unit Narrative:
The Mystery of the Pythagoreans

How rational were the Pythagoreans?
Find out if every number can be represented by a fraction.

What do the President of the United States and Albert Einstein have in common?
Uncover a special property of right triangles when you explore one of the nearly 500 proofs of the Pythagorean Theorem.

Unit 8 Associations in Data

Data literacy — being able to tell and interpret stories using data — is one of the most important skills you will ever need. In this unit, you will make sense of data in the world around you, represented in different forms. By the end of the unit, you will put your new data literacy skills to the test by examining the accuracy of newspaper headlines.

Unit Narrative:
Data and the Ozone Layer

Who is the biggest mover and shaker in the Antarctic Ocean?

Explore the ozone hole using scatter plots, while learning about the different kinds of associations data can have.

UNIT 1

Rigid Transformations and Congruence

Shapes are all around us. We find them in art, architecture, and even animated dancing frogs. In this unit, you will find out what happens when you slide, flip, and turn figures of all shapes and sizes. Plus, you may even create a masterpiece of art along the way.

Essential Questions

- What happens to a figure as you move it around a two-dimensional plane?
- What does it mean for two figures to be "the same"?
- Do the measures of the interior angles of a triangle really add up to 180°?
- *(By the way, can you spot a fraudulent painting of the Mona Lisa?)*

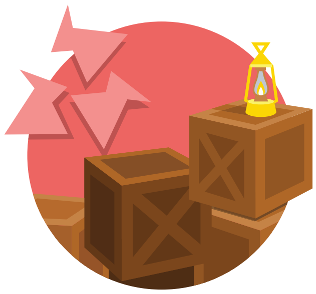
SUB-UNIT

1 Rigid Transformations

Narrative: The world's first animated feature film was created using geometric transformations

You'll learn . . .

- about translations, rotations, and reflections.
- how the structure of the coordinate plane adds precision to describing transformations.



SUB-UNIT

2 Rigid Transformations and Congruence

Narrative: Spotting forgeries of artistic works involves an understanding of congruent polygons.

You'll learn . . .

- what it means for two objects or figures to be "the same."
- how rigid transformations relate to congruence.

SUB-UNIT

3 Angles in a Triangle

Narrative: Discover what the sum of the angles in a triangle tells us about our Universe.

You'll learn . . .

- about transformations of lines and angles.
- what happens when parallel lines are intersected by another line.

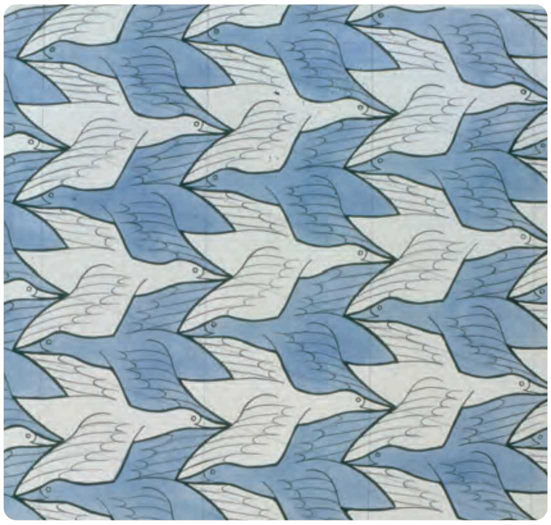
If the area of the larger square is 2, find the area of the smaller square.

Tessellations

Let's discover patterns with shapes.

Warm-up Notice and Wonder

The artwork shown was created by the Dutch artist Maurits C. (M.C.) Escher (1898–1972). What do you notice? What do you wonder?



M.C. Escher's "Two Birds"© 2020 The M.C. Escher Company-The Netherlands. All rights reserved. www.mcescher.com

1. I notice . . .

2. I wonder . . .

Log in to Amplify Math to complete this lesson online.

Name: ________________ Date: ________ Period: ________

Activity 1 Tessellate

You will be given a set of pattern blocks. Use them to create a tessellation of your own.

Draw a sketch of your tessellation here.

Activity 2 Triangle Tessellations

1. You will be given a plain sheet of paper and scissors. Draw a triangle and cut it out. Exchange triangles with your partner, and create a tessellation using your partner's triangle. Draw a sketch of your tessellation here.

2. Can you think of a triangle that does not work for making tessellations?

Collect and Display: As you share your response, your teacher will add the math language you use to a class display. You will continue to add and refer to this display throughout the unit.

Are you ready for more?

1. Draw a quadrilateral and cut it out. Exchange quadrilaterals with your partner, and create a tessellation using your partner's quadrilateral.
2. Can you think of a quadrilateral that does not work for making tessellations?

STOP

Unit 1 Rigid Transformations and Congruence

The Art of Transformation

Her name was Marjorie Rice, and she had a secret.

On the outside, she looked like any suburban housewife. She had a home in San Diego, a husband, and five children. But while her husband was at work, and the children were at school, Rice's other life emerged.

She'd already been caught several times, sketching at the kitchen counter. But what might have been dismissed as idle doodling was something more. Marjorie Rice was on the hunt for pentagons.

It began when she read an article in *Scientific American* on ***tessellations***. It had long been believed that every pentagon that could be tessellated had already been documented. But the article announced the discovery of new tessellating pentagons uncovered by a computer scientist living in California.

Inspired, and with barely any math education, Rice set off to work.

Over the 1975 holiday season, she worked tirelessly, at her kitchen table, hiding her work from family and friends. In February 1976, she wrote to the article's author, Martin Gardner, and showed him her discoveries. Gardner then forwarded her findings to Dr. Doris Schattschneider, a mathematician at Moravian College who confirmed her work:

Marjorie Rice had found a new tessellating pentagon.

By 1977, Rice would discover three more tessellating pentagons, and nearly 60 unique patterns for tiling them. Using her ingenuity and intuition on how to use transformations to duplicate shapes, Marjorie Rice shook the math world — reminding them there's always room for an amateur.

Welcome to Unit 1.

Practice

Name: ________________ Date: ________ Period: ________

1. Imagine a friend, family member, or future student who has not yet learned about tessellations. What would you do and say to teach them about tessellations?

2. Below is one example of the many complex tile patterns found in the famous 14th century Moorish palace of Alhambra, located in Spain. M.C. Escher visited the palace before making many of his drawings and paintings. Describe what you see.

Anneke Bart/SLU

Name: .... Date: .... Period: ....

3. Compare each of the following values using the symbols >, =, or <.

a. 4 .... -4

b. 11 15

c. -11 .... -15

d. $-(-6)$ 6

e. 8.01 8

f. -8.01 .... -8

g. -2.5 .... $-\frac{10}{4}$

h. $-\frac{2}{3}$ .... $-\frac{3}{2}$

i. $-\frac{1}{5}$ .... $-\frac{2}{10}$

j. $-\frac{3}{4}$ .... $\frac{3}{4}$

4. Rectangle $ABCD$ is drawn on a coordinate plane.

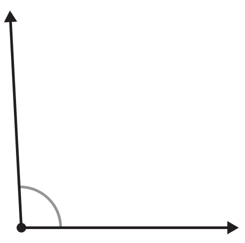
a. If point A is placed at $(2, 3)$, point B at $(4, 3)$, and point C at $(4, -3)$, what could be the location of point D?

b. Find the length of segment CD.

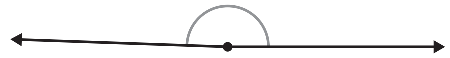
c. Find the area and perimeter of Rectangle $ABCD$.

5. Estimate the measure of each angle.

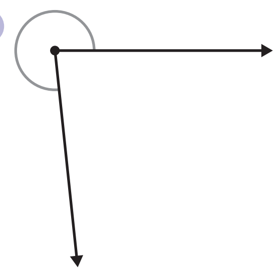
a.



b.



c.



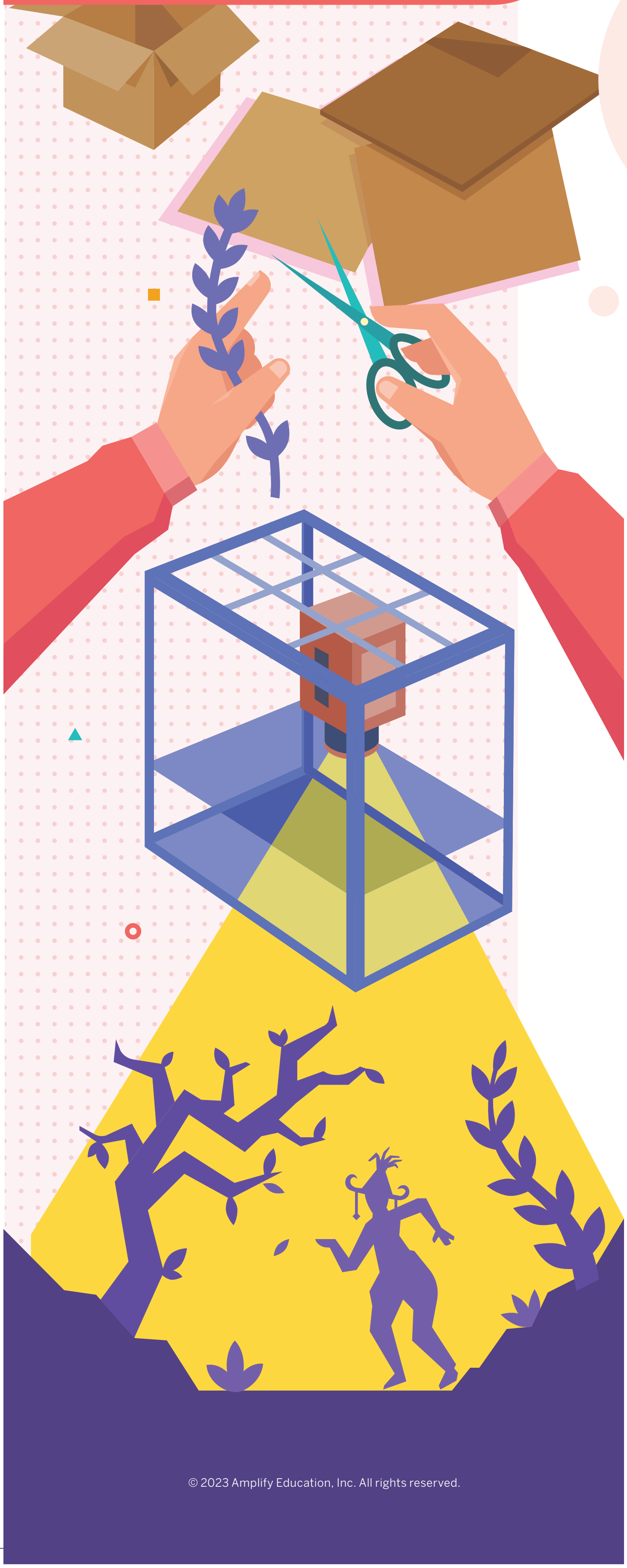
My Notes:

SUB-UNIT

1

Rigid Transformations

Narrative Connections



How do you make a piece of cardboard come alive?

Before Walt Disney, there was Lotte Reiniger.

As a girl living in Berlin, Reiniger was clever with a pair of scissors. She cut intricate figures out of the cardboard from old soap boxes. For many kids, this was a way to pass the time. But for Reiniger, it was something more.

Her interest in puppets led her into the world of German art and cinema. By the time she was twenty, she started making her own films.

Her most famous achievement is *The Adventures of Prince Achmed*. It was the world's first animated full-length feature film — ten years before Disney's *Snow White*.

With a staff of just five people, Reiniger constructed elaborate paper puppets. Then, using a camera of her own invention, she would lay the puppets out and change their position frame-by-frame. It was a long and tedious process, but when you ran the images through a film projector, it came out as a single fluid movement.

By changing the position of solid figures, Reiniger turned a piece of cardboard into a flapping wing, a gesturing arm, or a sorcerer casting a spell. With only a pair of scissors, her imagination, and clever uses of geometric transformations, Reiniger changed the world of animation forever.

Moving on the Plane

Let's describe ways figures can move on the plane.

Warm-up Notice and Wonder

You will be shown a short animation. What do you notice? What do you wonder?

1. I notice . . .

2. I wonder . . .

Log in to Amplify Math to complete this lesson online.

Name: ______ Date: ______ Period: ______

Activity 1 Frog Dance

Plan ahead: How will you use this opportunity to build a relationship with your partner?

You will be given a sheet with three sets of dancing frog images.

1. Arrange the sheet so that you and your partner can both see them right-side up. Choose one player to start the game.
   - The starting player mentally chooses Dance A, B, or C and describes the dance to the other player.
   - The other player identifies the dance as Dance A, B, or C, based on the starting player's description.
2. After one round, trade roles. When you have described all three dances, come to an agreement on the words or phrases you can use to describe the moves in each dance.
3. Complete the tables on this and the next page to write a final description of the moves for each dance.

Dance A:

From . . .	To . . .	Description of moves
Frame 1	Frame 2	
Frame 2	Frame 3	
Frame 3	Frame 4	
Frame 4	Frame 5	
Frame 5	Frame 6	

Activity 1 Frog Dance (continued)

Dance B:

From . . .	To . . .	Description of moves
Frame 1	Frame 2	
Frame 2	Frame 3	
Frame 3	Frame 4	
Frame 4	Frame 5	
Frame 5	Frame 6	

Dance C:

From . . .	To . . .	Description of moves
Frame 1	Frame 2	
Frame 2	Frame 3	
Frame 3	Frame 4	
Frame 4	Frame 5	
Frame 5	Frame 6	

Name: ________ Date: ________ Period: ________

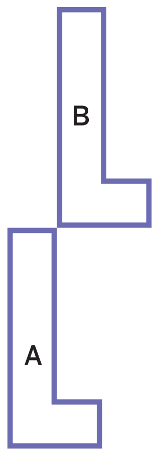
Activity 2 How Did You Make That Move?

For each problem, determine if Figure A maps onto Figure B using a *translation* or a *rotation*.

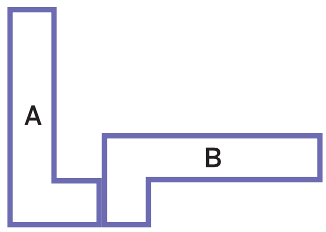
If the movement is a:

- **Translation:** Draw arrows to show the direction and corresponding vertices that are translated. Then write the distance the figure is translated, in centimeters.
- **Rotation:** Determine if the figure is rotated clockwise or counterclockwise, mark the center of rotation, and estimate the angle of rotation.

1.



2.



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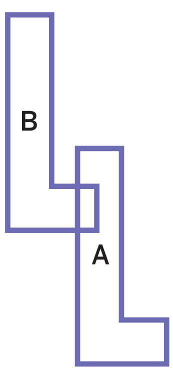

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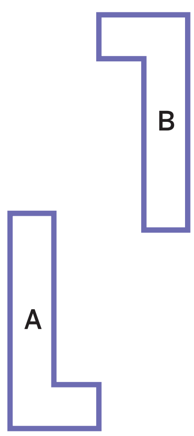


STOP

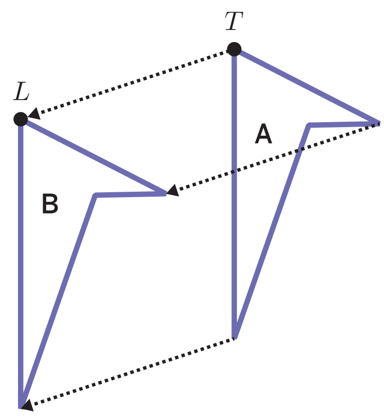
Summary

In today's lesson . . .

You described how a figure moves in a plane.

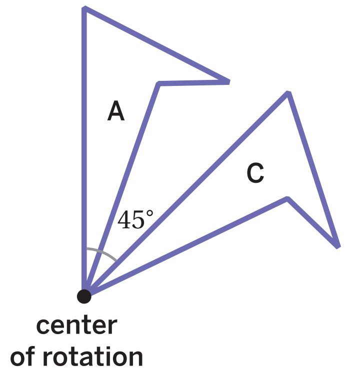
A ***translation*** slides a figure without turning it. Every point in the figure moves the same distance in the same direction. A translation can be described by two points.

- For example, if a translation maps point T onto point L, it moves the entire figure the same distance and direction as the distance and direction from point T to point L. The distance and direction of a translation can be shown by an arrow.
- Figure A was translated down and to the left, as shown by the arrows. Figure B is a translation of Figure A.



A ***rotation*** turns a figure about a point, called the ***center of rotation***. Every point on the figure travels along the path of a circle around the center of rotation to form the ***angle of rotation***. The rotation can be:

- **clockwise:** traveling in the same direction as the hands of a clock, or
- **counterclockwise:** traveling in the opposite direction as the hands on a clock.



A rotation can be described by an angle, a center, and the direction of the rotation.

- For example, Figure A was rotated $45°$ clockwise around the center of rotation shown. Figure C is a rotation of Figure A.

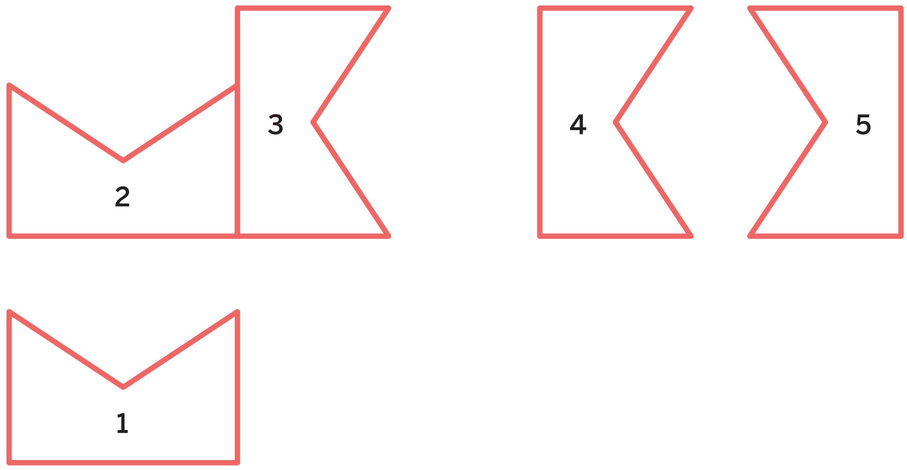
If one point on the original figure moves to another point on the new figure, they are *corresponding points*.

Reflect:

Practice

Name: .... Date: .... Period: ....

1. You will need a centimeter ruler. The diagram shows several moves of a figure. Determine if each statement is *true* or *false*.



a. From Figure 1 to Figure 2, the figure is translated 3 cm up.

b. From Figure 2 to Figure 3, the figure is rotated 90° counterclockwise.

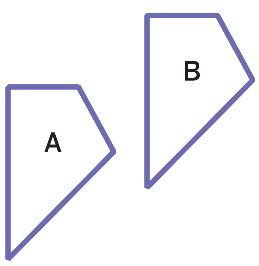
c. From Figure 3 to Figure 4, the figure is translated 4 cm to the right.

d. From Figure 4 to Figure 5, the figure is rotated 180° clockwise.

2. For each movement from Figure A to Figure B:

- Decide whether it shows a translation or rotation.
- Show or precisely describe the movement. Include the distance, in centimeters, and angle of rotation, if applicable.

a



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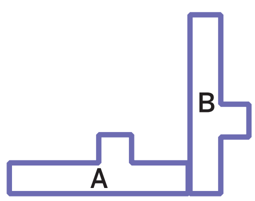
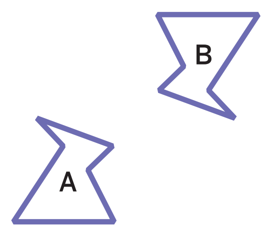


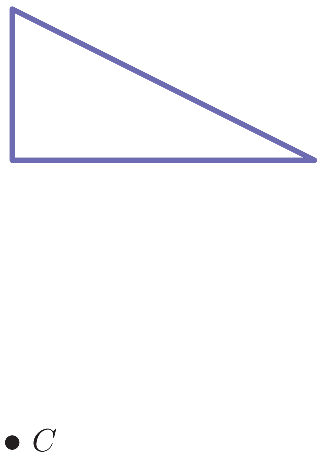
c



Practice

Name: ______ Date: ______ Period: ______

3. You will need a centimeter ruler. Translate the triangle shown 2 cm down, and then rotate the newly translated triangle $90°$ clockwise about point C.



4. Evaluate each expression.

a. $-5 \cdot (-2.4)$

b. $-7.4 \div 10$

c. $-\frac{4}{7} \div (-2)$

d. $4 \cdot \left(-\frac{3}{8}\right)$

5. Draw *all* the lines of symmetry for the figure shown. How many total lines of symmetry are there?

Name: ________________ Date: ________ Period: ________

Unit 1 | Lesson 3

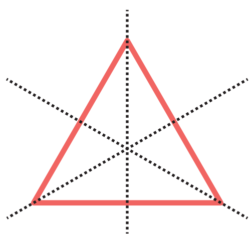
Symmetry and Reflection

Let's describe ways figures reflect on the plane.

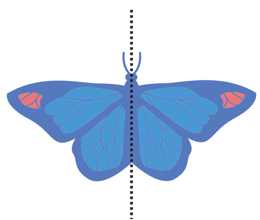
Warm-up Which One Doesn't Belong?

Study the figures. Which figure does not belong with the others? Explain your thinking.

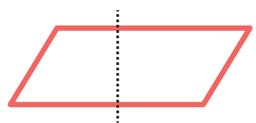
A.



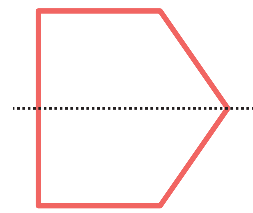
B.



C.



D.



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Activity 1 Mirror Image

You will need tracing paper from your geometry toolkit.

1. Use the tracing paper to complete the following steps.

- Draw a vertical line in the middle of the paper, and label it ℓ. On one side of the line, draw a triangle.
- Fold the paper along the line. Retrace the triangle on the other side of the line.
- Unfold the paper. You should now have two triangles that are mirror images of each other.
- Draw a dotted line segment to connect each of the corresponding points of the two triangles.
- Measure and label the distance from each point on the original triangle to its corresponding point on the new triangle.

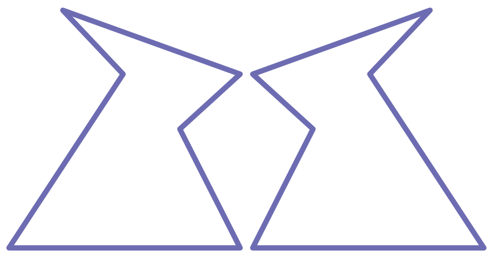
2. How is the line ℓ related to each dotted line segment you drew?

Name: ________ Date: ________ Period: ________

Activity 2 Flipping Figures

Study each pair of figures. For each pair, determine whether one figure is a reflection of the other. Write *yes* or *no*. If *yes*, draw a line of reflection.

a



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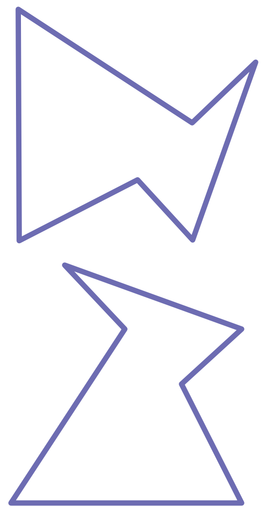

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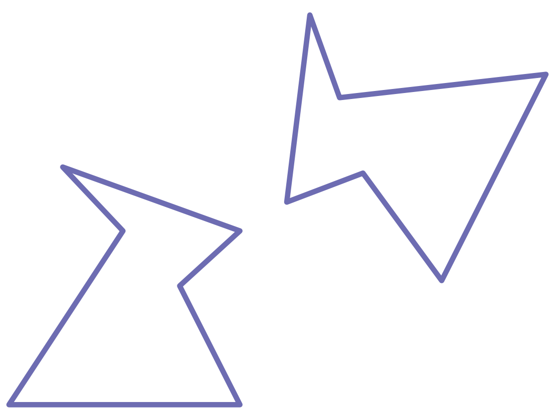

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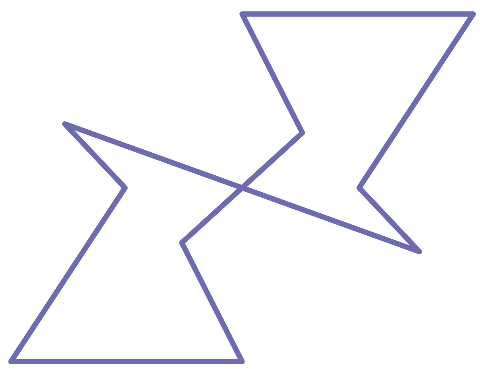

Activity 2 Flipping Figures (continued)

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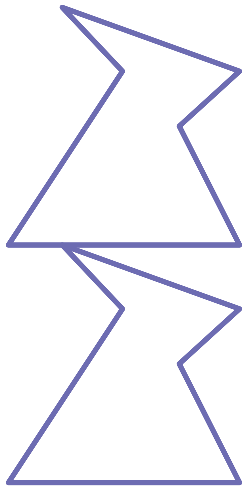
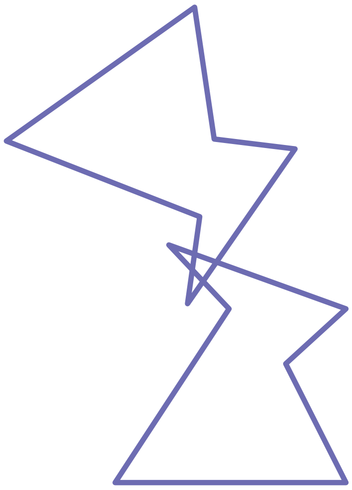


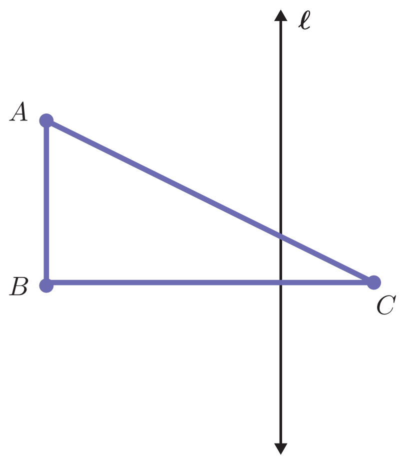
f



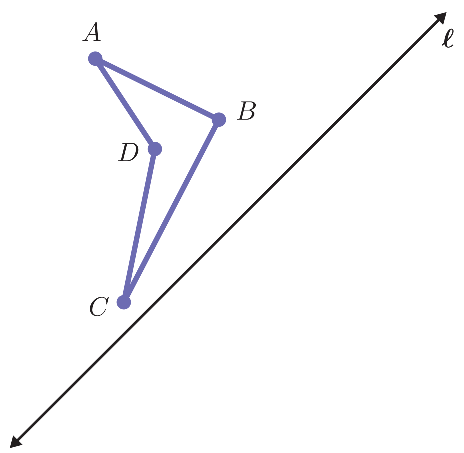
Name: .... Date: .... Period: ....

Activity 3 Drawing Reflections

1. Reflect Triangle ABC across line ℓ. Use A', B', and C' to indicate vertices in the image that correspond to the points A, B, and C in the preimage.



2. Reflect Polygon $ABCD$ across line ℓ. Use A', B', C', and D' to indicate vertices in the image that correspond to the points A, B, C, and D in the preimage.

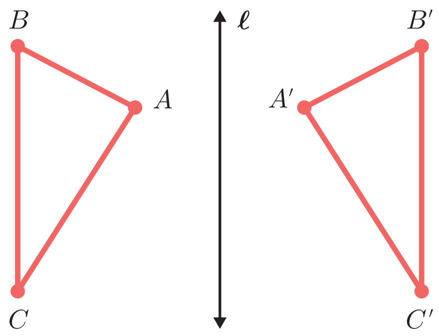


STOP

Summary

In today's lesson . . .

You explored how to precisely reflect a figure over a line. A ***reflection*** moves every point on a figure to a point directly on the opposite side of the ***line of reflection***. The new point is the same distance from the line as its corresponding point in the original figure. The ***orientation*** of the vertices is reversed.



The term ***image*** describes the new figure created by moving the original figure. The original figure is called the ***preimage***.

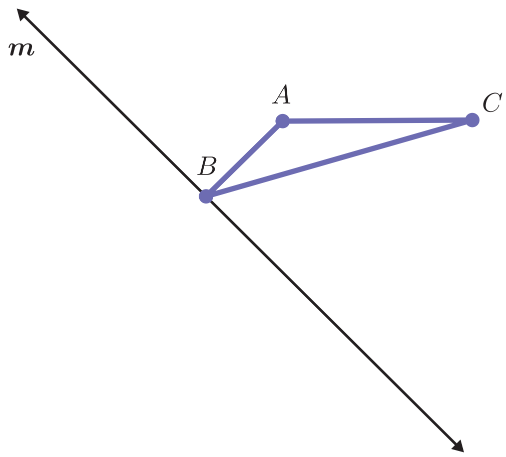
In the diagram, the vertices of the image are labeled using ***prime notation***, A', B', and C'. This notation is read "A prime", "B prime", and "C prime". These represent the vertices in the image and correspond to the vertices A, B, and C in the preimage.

Reflect:

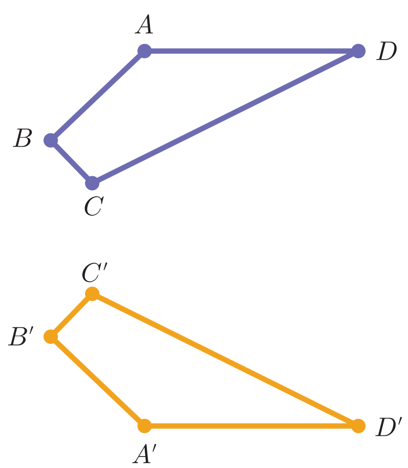
Name: ______ Date: ______ Period: ______

Practice

1. Reflect Triangle ABC across line m. Use A', B', and C' to indicate the vertices in the image that correspond to the points A, B, and C, in the preimage.



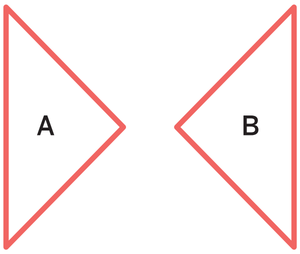
2. Polygon $A'B'C'D'$ is a reflection of Polygon $ABCD$. Draw the line of reflection and label it m. Explain your thinking.



Practice

Name: ______________________ Date: __________ Period: __________

3. Select *all* the ways Triangle A can map onto Triangle B.



A. Reflect Triangle A across a horizontal line.

B. Reflect Triangle A across a vertical line.

C. Translate Triangle A to the left.

D. Translate Triangle A to the right.

E. Rotate Triangle A $180°$ counterclockwise.

F. Rotate Triangle A $90°$ counterclockwise.

4. Write an operation in the box to make each equation true.

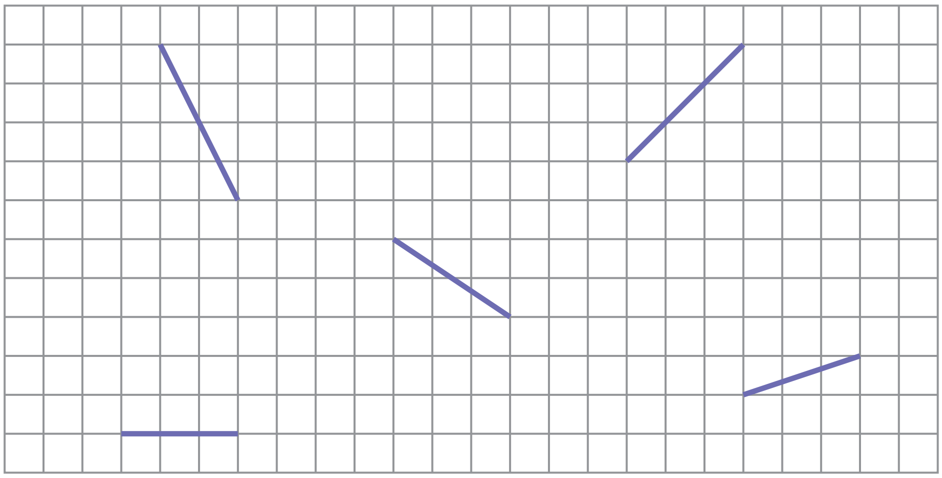
a $12 \boxed{} (-8) = 20$

b $-17 \boxed{} 9 = -26$

c $-14 \boxed{} 18 = 4$

d $24 \boxed{} (-29) = -5$

5. Draw a line connected to each line segment to form a right angle.



Name: ______ Date: ______ Period: ______

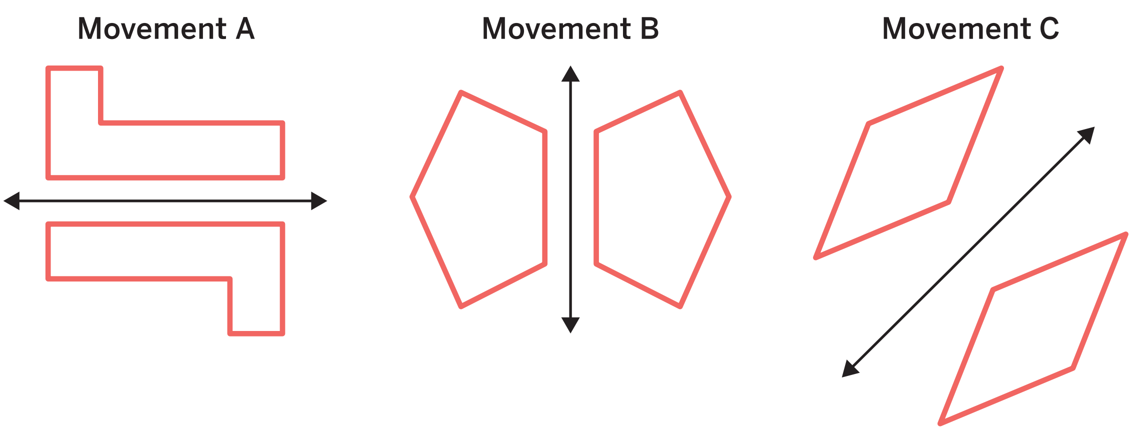
Unit 1 | Lesson 4

Grid Moves

Let’s transform some figures on grids.

Warm-up True or False

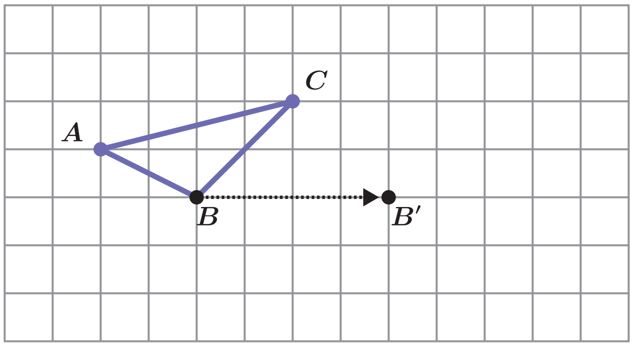
Shawn thinks the following movements are all examples of *reflections* across the line shown. Do you agree with Shawn? Be prepared to explain your thinking.



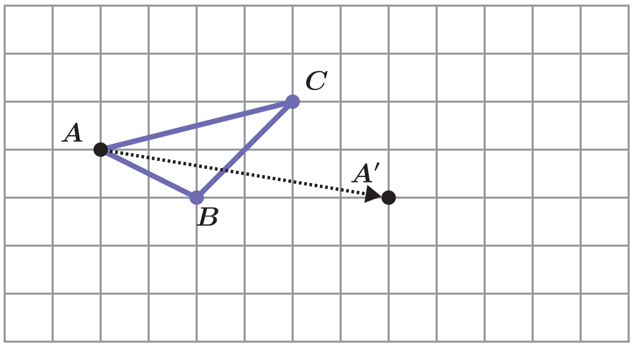
Log in to Amplify Math to complete this lesson online.

Activity 1 Transformation Information

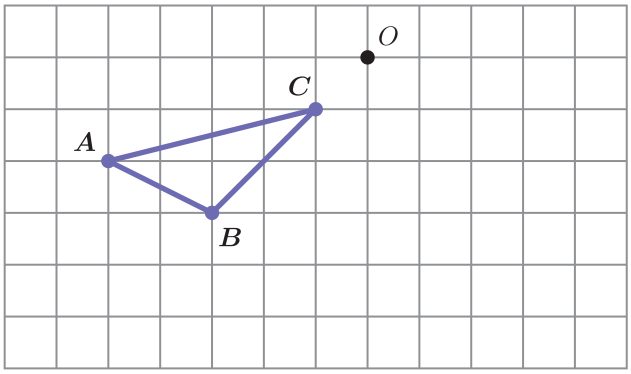
1. Translate Triangle ABC, so that point B maps onto point B'. Label the corresponding points on the image with A', B', and C'.



2. Translate Triangle ABC, so that point A maps onto point A'. Label the corresponding points on the image with A', B', and C'.



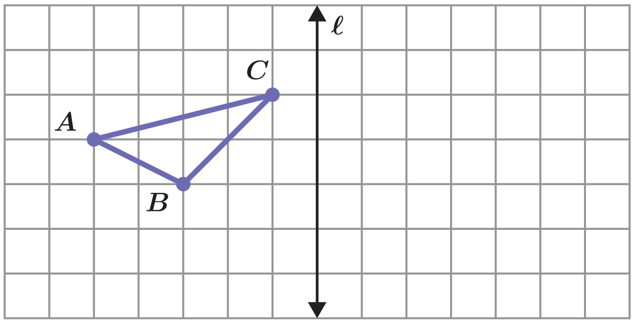
3. Rotate Triangle ABC $90°$ *counterclockwise* about point O. Label the corresponding points on the image with A', B', and C'.



Name: ______ Date: ______ Period: ______

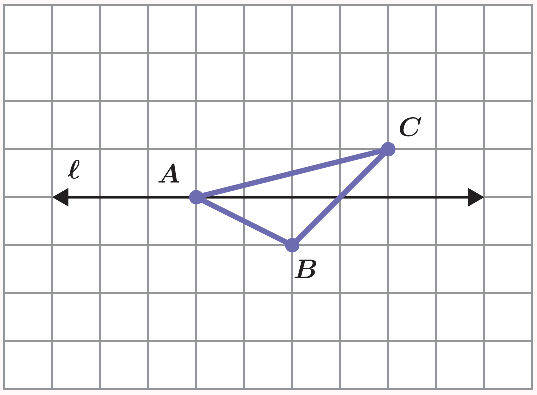
Activity 1 Transformation Information (continued)

4. Reflect Triangle ABC across line ℓ. Label the corresponding points on the image with A', B', and C'.

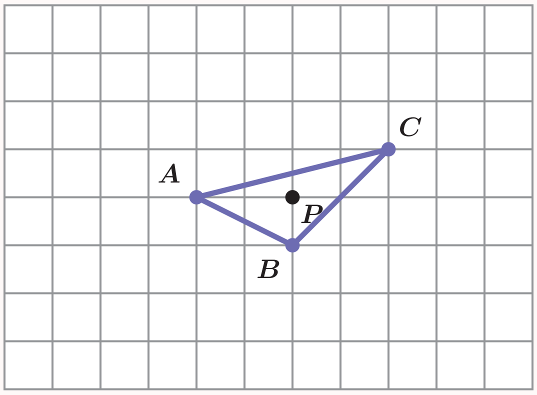


Are you ready for more?

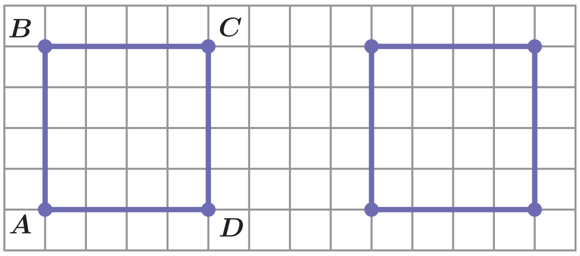
1. Reflect Triangle ABC across line ℓ. Label the corresponding points on the image with A', B', and C'.



2. Rotate Triangle ABC $90°$ *clockwise* about point P. Label the corresponding points on the image with A', B', and C'.



Activity 2 Identifying Transformations

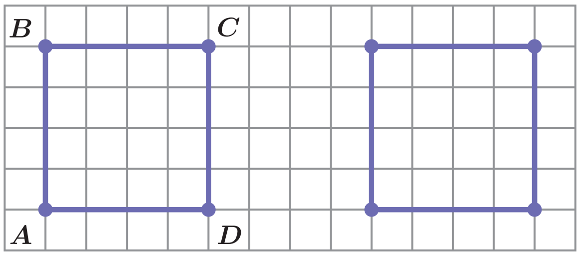


Square $ABCD$ is drawn on a grid, and a transformation has been applied.

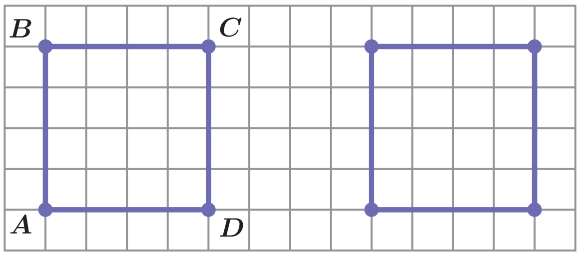
Kiran, Clare, and Noah each see different transformations. For each student:

- Label the vertices with the correct letters to show why each response is correct.
- Describe how each transformation maps the original figure onto the new figure.
- Be sure to include a line of reflection and a center of rotation, when necessary.

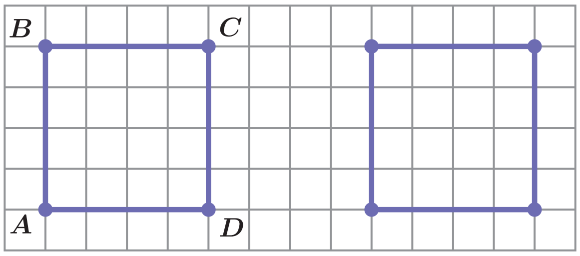
1. Kiran: "I see a translation."



2. Clare: "I see a reflection."



3. Noah: "I see a rotation."



STOP

Name: ________________ Date: ________ Period: ________

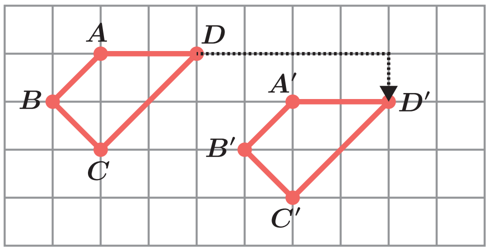
Summary

In today's lesson . . .

You saw that *translations*, *reflections*, and *rotations* are all examples of ***transformations***. When a figure is placed on a grid, you can use the structure of the grid to perform a transformation and describe it.

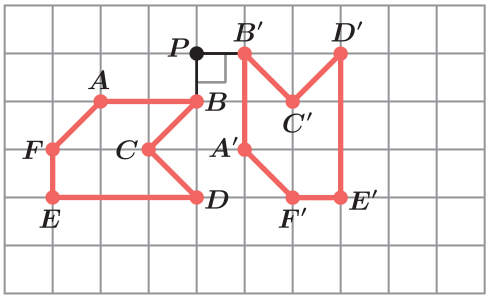
Quadrilateral $ABCD$ is translated to Quadrilateral $A'B'C'D'$.

- The grid shows that each point is moved to the right 4 units and down 1 unit.



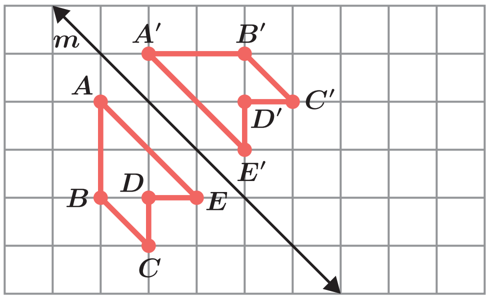
Hexagon $ABCDEF$ is rotated $90°$ counterclockwise about center P.

- The grid shows that the distance between the center of rotation and each vertex of the preimage is preserved in the rotated image.
- Each of these distances forms a $90°$ angle, in the counterclockwise direction.



Pentagon $ABCDE$ is reflected across line m.

- The grid shows that the distance from each vertex to the line of reflection in the preimage is maintained in the reflected image.



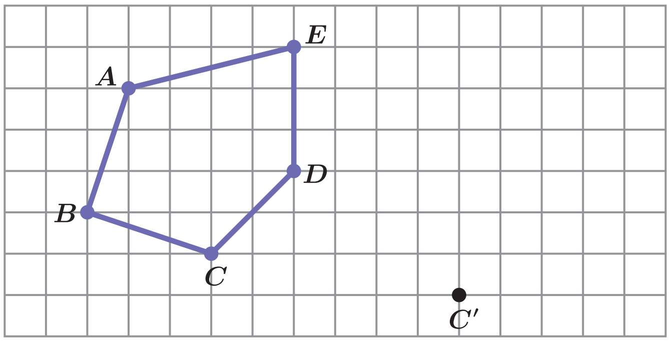
Reflect:

Name: .... Date: .... Period: ....

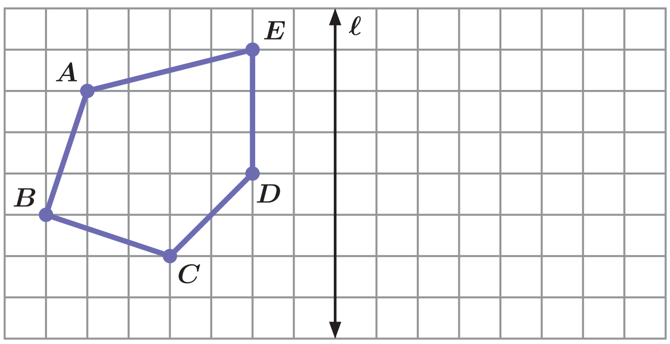
Practice

1. Complete each of the transformations described.

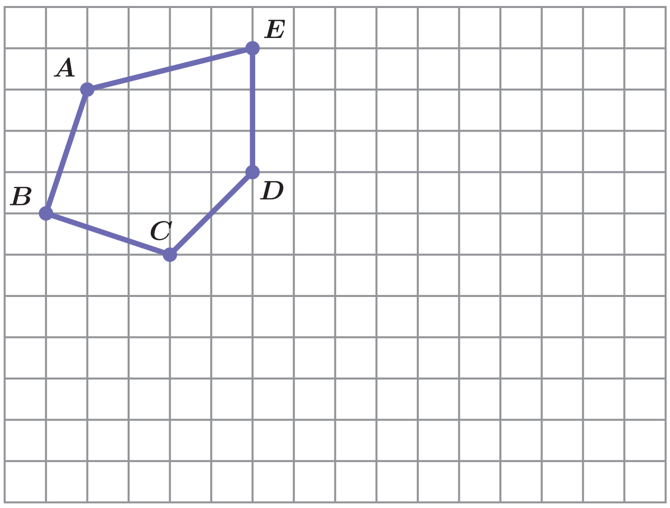
a Draw the translated image of Pentagon $ABCDE$, so that point C maps onto point C'. Label the corresponding points on the image with A', B', C', D', and E'.



b Draw the reflection of Pentagon $ABCDE$ across line ℓ. Label the corresponding points on the image with A', B', C', D', and E'.



c Draw the rotation of Pentagon $ABCDE$ clockwise $180°$ about point C. Label the corresponding points on the image with A', B', C', D', and E'. Tracing paper and a ruler may be useful here.

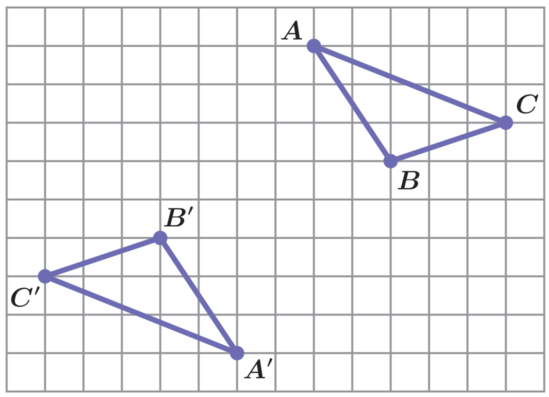


Practice

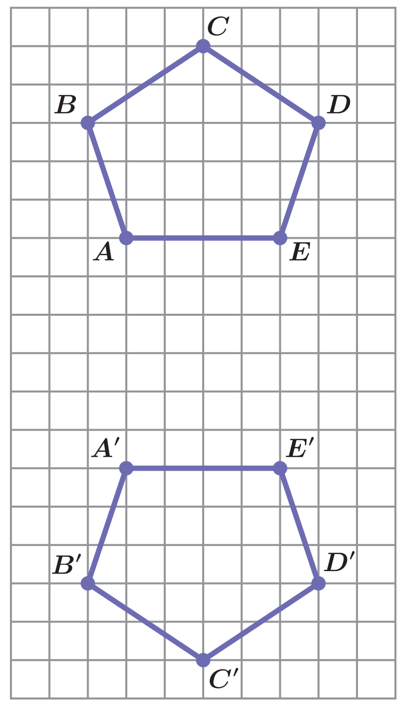
Name: ________ Date: ________ Period: ________

2. Describe each transformation that has occurred. Draw any points or lines that are used in each transformation.

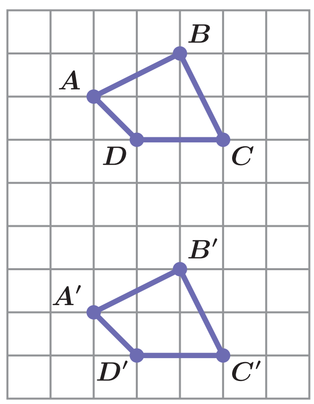
a



b



c



3. Match each expression with an equivalent expression.

Expressions	**Equivalent expressions**
a $-3x - 7$	________ $-0.9 + 5.7x$
b $-3.4 + 5.7x + 2.5$	________ $-\frac{1}{2}(6x - 14)$
c $1.8x - 5.9 + 3.9x$	________ $5.7x - 5.9$
d $-3x + 7$	________ $\left(-\frac{7}{2} - \frac{3}{2}x\right) \cdot 2$

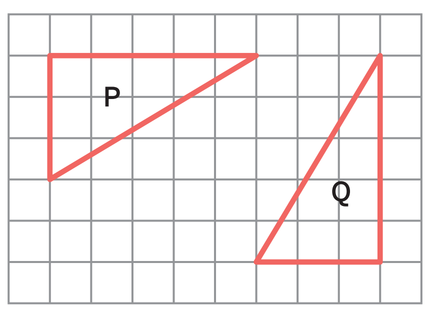
4. Identify the single transformation that will map Triangle P onto Triangle Q.

A. Translation

B. Reflection

C. Rotation

D. None of the above

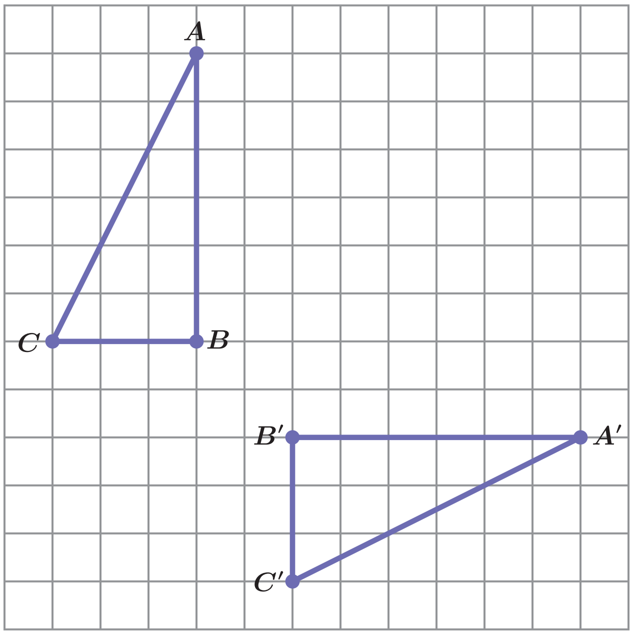


Making the Moves

Let's draw and describe translations, reflections, and rotations.

Warm-up Finding the Line of Reflection

Triangle ABC has been reflected to create Triangle $A'B'C'$. Draw the line of reflection.



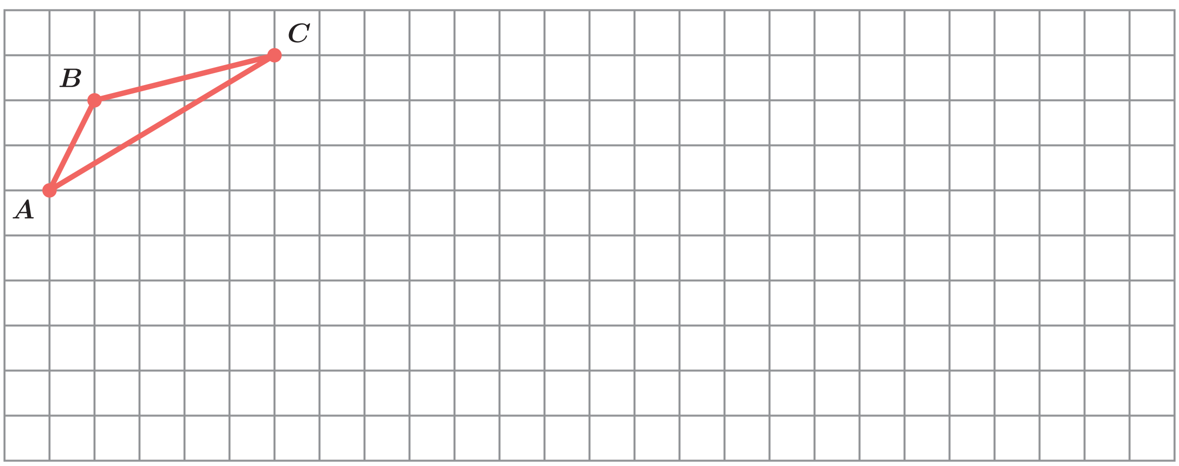
Log in to Amplify Math to complete this lesson online.

Name: ________ Date: ________ Period: ________

Activity 1 Make That Move

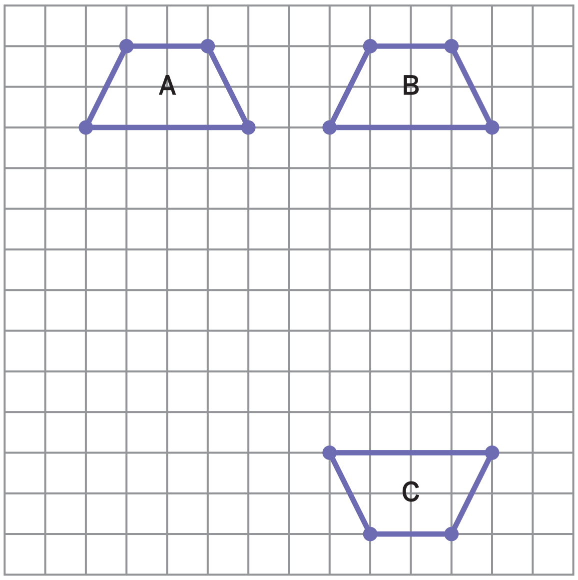
You will be given a set of cards. You and your partner will take turns describing and drawing transformations. Decide which student will draw first. The other student will describe the transformation on their card.

When describing . . .	When drawing . . .
• Your goal is to describe your transformation as accurately as possible using *only* your words. • You may offer verbal feedback to your partner once they have completed their initial sketch.	• Your goal is to listen carefully to your partner, and recreate the transformation they describe. • Draw the transformation your partner describes on the grid shown.



Activity 2 A to B to C

Refer to Figures A, B, and C. For each problem, describe a transformation that could map each figure onto the other. Draw any points or lines that are used in your transformation.



1. A transformation that maps Figure A onto Figure B.

2. A transformation that maps Figure B onto Figure C.

3. A transformation that maps Figure A onto Figure C.

Compare and Connect: After completing Problems 1–3, create a visual display of your chosen strategy or strategies. Then compare your strategy with a partner.

Are you ready for more?

Describe a *single* transformation that maps Figure A directly onto Figure C.

STOP

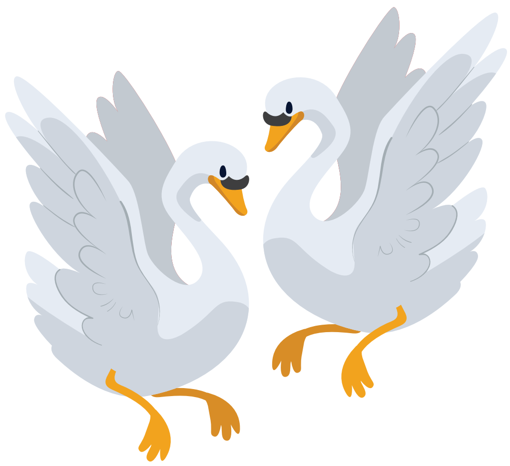
Name: .... Date: .... Period: ....

Summary

In today's lesson . . .

You described and performed transformations that map one figure onto another.

In some cases, mapping one image onto another requires more than one transformation. To map one bird onto the other bird in the following image, a reflection and a translation are needed.



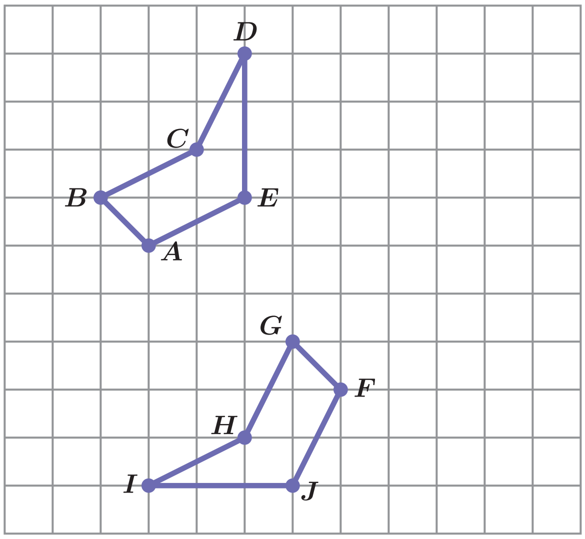
When more than one transformation is applied to a preimage, that series of moves is called a ***sequence of transformations***. There can be more than one sequence of transformations that maps a preimage to an image.

Reflect:

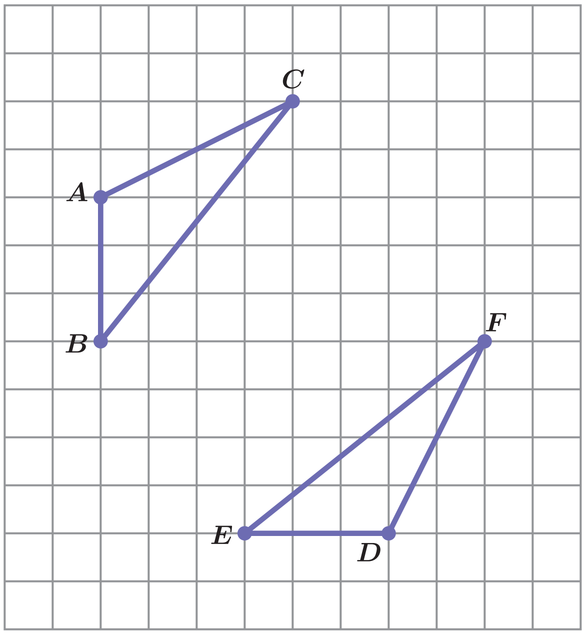
Name: ______________________ Date: __________ Period: __________

Practice

1. Describe a sequence of transformations that maps Polygon $ABCDE$ onto Polygon $FGHIJ$.



2. Describe a sequence of transformations that maps Triangle ABC onto Triangle DEF.



Name: .... Date: .... Period: ....

Practice

3. Select *all* the sequences of transformations that would return a triangle to its original position.

A. Reflect a triangle across line m, and then reflect the image across line m again.

B. Translate a triangle 1 unit to the right, then 4 units to the left, and then 3 units to the right.

C. Reflect a triangle across line ℓ, and then reflect the image across a different line.

D. Rotate a triangle 90° counterclockwise around point C, and then rotate the image 270° counterclockwise around the same point.

4. Solve each equation. Show your thinking.

a $12 + 0.5x = 21.5$

b $-5(x - 2) = 30$

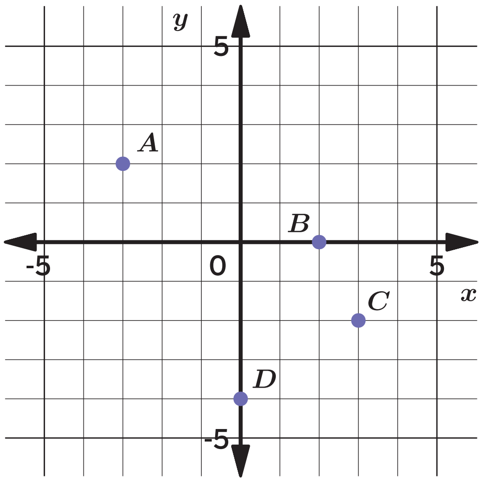
5. Identify the coordinates of the points graphed on the coordinate plane.

a Point A (.........., ...)

b Point B (.........., ...)

c Point C (.........., ...)

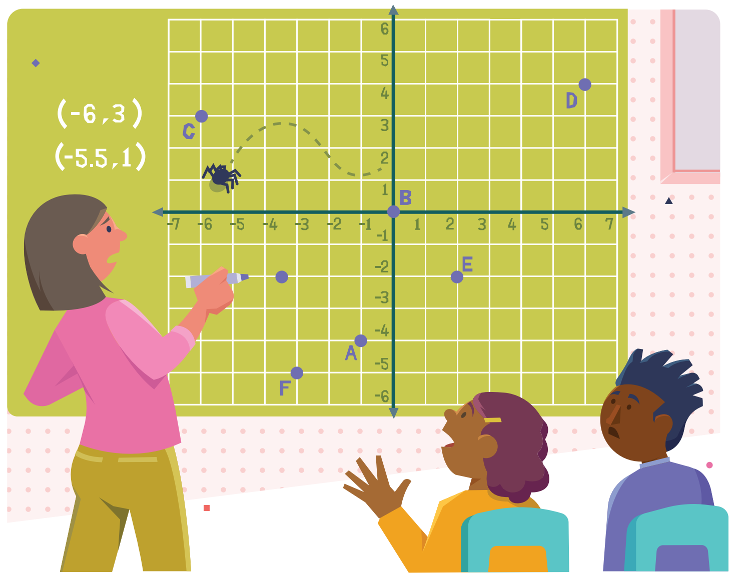
d Point D (.........., ...)



Unit 1 | Lesson 6

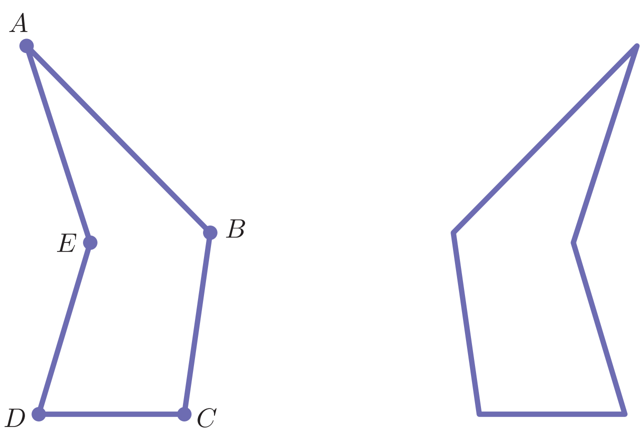
Coordinate Moves (Part 1)

Let's transform some figures and see what happens to the coordinates of the points.



Warm-up Getting Coordinated

Figure $ABCDE$ has been reflected. Label the corresponding points on the image with A', B', C', D', and E'.

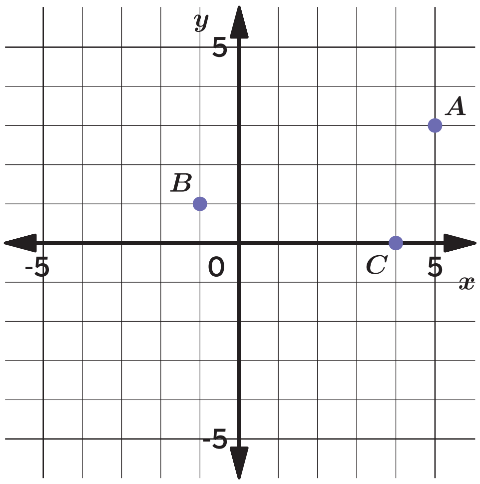


Log in to Amplify Math to complete this lesson online.

Name: .... Date: .... Period: ....

Activity 1 Translating Points on the Coordinate Plane

Refer to the graph showing points A, B, and C.



1. Translate points A, B, and C to the left 4 units and down 1 unit. Draw the image of these points in the graph. Label the points in the image as A', B', and C', respectively.

2. Write the coordinates of each point in the table.

Preimage coordinates		Image coordinates	
A		A'	
B		B'	
C		C'	

3. Compare the coordinates of the original points with the coordinates of their images. What do you notice?

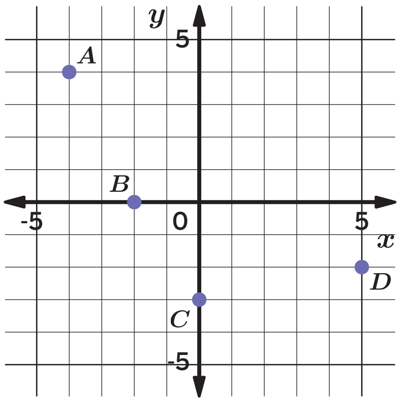
Are you ready for more?

How would the coordinates change if you translated the points 1 unit up and 4 units to the right instead?

Activity 2 Reflecting Points on the Coordinate Plane

Transforming points and figures using coordinates allows you to be very precise. When they studied which shapes of billiard tables resulted in special bouncing patterns for billiard balls, mathematicians Alex Wright and Maryam Mirzakhani first transformed the tables using multiple reflections.

1. Refer to the graph showing Points A, B, C, and D.



   a. Reflect points A, B, C, and D across the y-axis. Plot and label the resulting points A', B', C', and D', respectively.

   b. Write the coordinates of each point in the table.

   c. Compare the coordinates of the preimage points with the coordinates of their images. What do you notice?

Preimage coordinates		Image coordinates	
A		A'	
B		B'	
C		C'	
D		D'	

Featured Mathematician

Maryam Mirzakhani

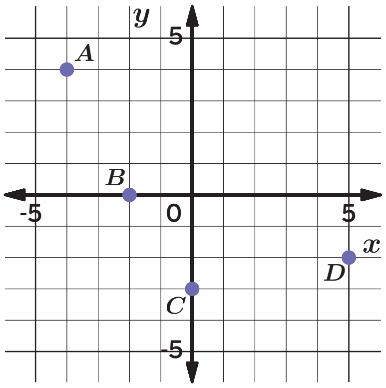
Maryam Mirzakhani grew up in Iran, where she was the first female student to earn a gold medal at the International Math Olympiad. She moved to the U.S. to complete her graduate work, becoming a professor at Princeton University and later Stanford University. She was awarded the Fields Medal in 2014, one of the highest honors in mathematics, for her study of moduli spaces. She and her colleagues used this work – along with geometric transformations and precise coordinates – to prove that certain quadrilateral shapes, such as billiard tables, have special "orbit closures." Mirzakhani died from breast cancer at the age of 40. Today, numerous schools, prizes, and other establishments bear her name.

Jan Vondrák

Name: .... Date: .... Period: ....

Activity 2 Reflecting Points on the Coordinate Plane (continued)

2. Refer to the graph showing points A, B, C, and D.



a Reflect points A, B, C, and D across the x-axis. Draw the image of these points in the graph. Label the points in the image as A', B', C', and D', respectively.

b Write the coordinates of each point in the table.

Preimage coordinates		Image coordinates	
A		A'	
B		B'	
C		C'	
D		D'	

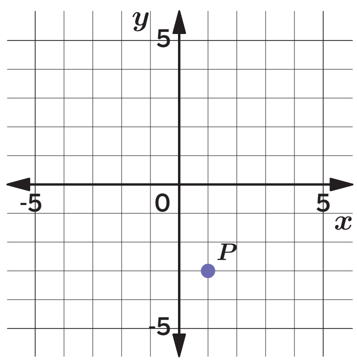
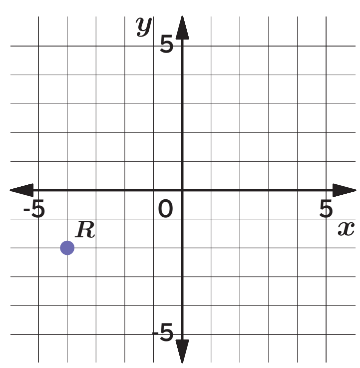
c Compare the coordinates of the preimage points with the coordinates of their images. What do you notice?

Activity 3 Partner Problems: Predicting Placement

With your partner, decide who will complete Column A and who will complete Column B. You each will work on the problems in your column; however, you should have the same responses as your partner. If you do not have the same responses, rework your partner's problem and discuss any errors.

Plan ahead: In what ways will you take on the perspective of your partner during this activity?

For each column, perform the transformation as described. Then write the coordinates of the image.

	Column A	Column B
1.	Translate point P to the right 2 units and then up 4 units, and label the resulting image as P'. What are the coordinates of point P'?  P'(........, ...)	Point P' is the result of translating point $P(1, -3)$ 2 units right and 4 units up. What are the coordinates of point P'? P'(........, ...)
2.	Point R' is the result of reflecting the point $R(-4, -2)$ across the y-axis. What are the coordinates of point R'? R'(........, ...)	Reflect point R across the y-axis and label the resulting image as point R'. What are the coordinates of point R'?  R'(........, ...)

STOP

Name: .... Date: .... Period: ....

Summary

In today's lesson . . .

You performed translations and reflections on the coordinate plane, and observed how the coordinates of the transformed points changed under each of these types of transformations.

You can use coordinates to describe the position of points and find patterns in the coordinates of transformed points.

You can describe a translation by expressing it as a sequence of horizontal and vertical translations.

Translating a point to the left or right . . .	Translating a point up or down . . .
changes the value of the x-coordinate. **Example:** Preimage: $(3, -5)$ • If the point is translated to the left 2 units, the image is $(1, -5)$. • If the point is translated to the right 2 units, the image is $(5, -5)$.	changes the value of the y-coordinate. **Example:** Preimage: $(3, -5)$ • If the point is translated up 2 units, the image is $(3, -3)$. • If the point is translated down 2 units, the image is $(3, -7)$.

Reflecting a point across an axis changes the sign of one coordinate.

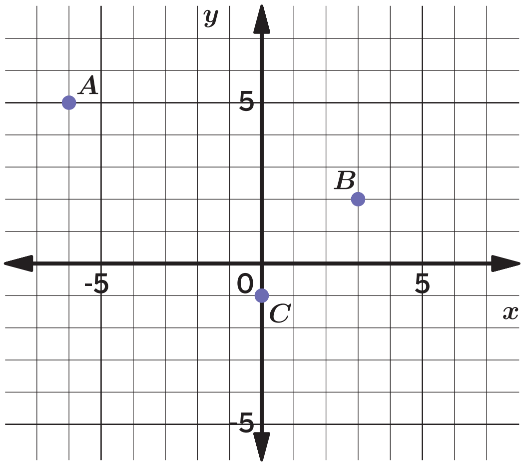
Reflecting a point across the x-axis . . .	Reflecting a point across the y-axis . . .
changes the sign of the y-coordinate. The x-coordinate remains the same. **Example:** Preimage: $(3, -5)$ Image: $(3, 5)$	changes the sign of the x-coordinate. The y-coordinate remains the same. **Example:** Preimage: $(3, -5)$ Image: $(-3, -5)$

Reflect:

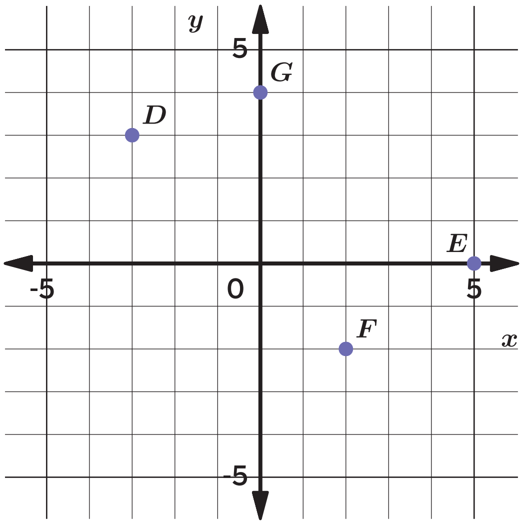
Practice

Name: ____________ Date: ____________ Period: ____________

1. Points $A(-6, 5)$, $B(3, 2)$, and $C(0, -1)$ are plotted on the coordinate plane. What are the coordinates of A, B, and C after a translation 4 units to the right and 1 unit up? Plot these points on the grid, and label them A', B', and C'. Include the coordinates of the images in your labels.



2. Points $D(-3, 3)$, $E(5, 0)$, $F(2, -2)$, and $G(0, 4)$ are plotted on the coordinate plane.



a. What are the coordinates of points D and E after a reflection across the y-axis? Plot these points on the grid, and label them D' and E'. Include the coordinates of the images in your labels.

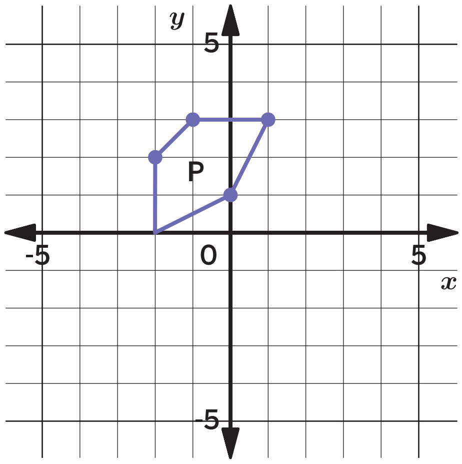
b. What are the coordinates of points F and G after a reflection across the y-axis? Plot these points on the grid, and label them F' and G'. Include the coordinates of the images in your labels.

Name: .... Date: .... Period: ....

Practice

3. Pentagon P is reflected across the x-axis. Predict the coordinates of the image by completing the table. Check your predictions by graphing the image.

Preimage coordinates	Image coordinates
(−2, 0)	
(−2, 2)	
(−1, 3)	
(1, 3)	
(0, 1)	



4. For each statement, explain how you would find the measure of the missing angle.

a. Two angles are complementary, and you are given the measure of one of these angles.

b. Two angles are supplementary, and you are given the measure of one of these angles.

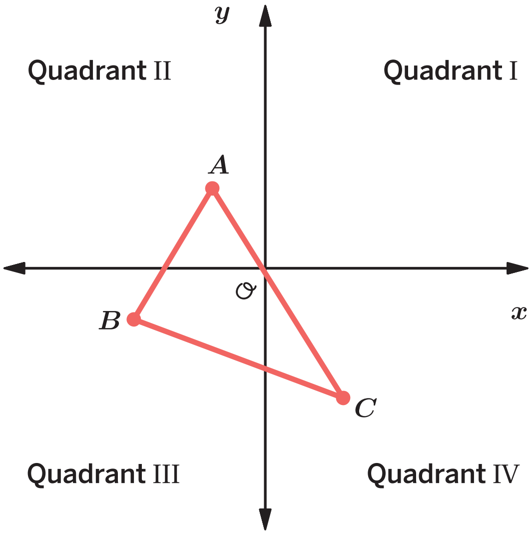
5. Triangle ABC is rotated 90° clockwise about the origin to create Triangle $A'B'C'$. In which quadrant would point C' be located?

A. Quadrant I

B. Quadrant II

C. Quadrant III

D. Quadrant IV



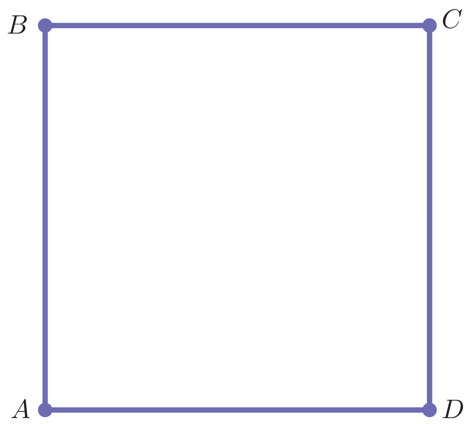
Unit 1 | Lesson 7

Coordinate Moves (Part 2)

Let's transform some more figures and see what happens to the coordinates of the points.

Warm-up Rotating Coordinates

Square $ABCD$ has been rotated in such a way that the image coincided with the preimage.



What could the angle of rotation be? Find as many possible answers as you can.

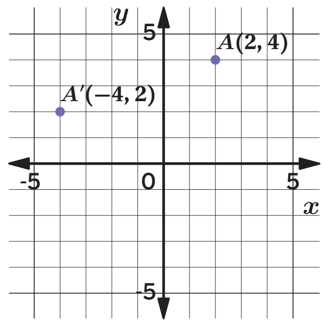
Log in to Amplify Math to complete this lesson online.

Name: ______ Date: ______ Period: ______

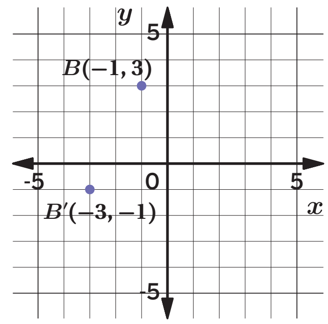
Activity 1 Rotations of a Point

1. Each of these points has been rotated $90°$ counterclockwise about the origin. Compare the coordinates of the original points with the coordinates of their images. What do you notice?

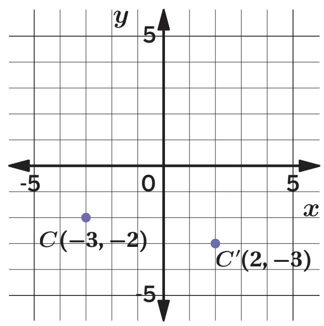
Point A



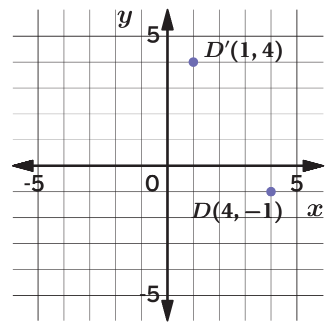
Point B



Point C

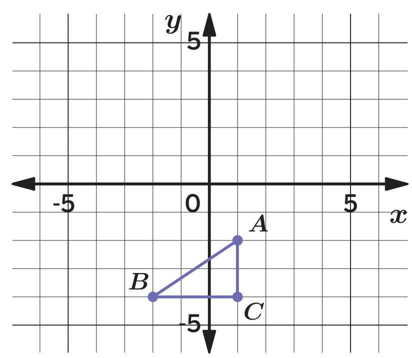


Point D



2. Use the pattern you noticed to predict the coordinates of Triangle $A'B'C'$ after rotating Triangle ABC $90°$ counterclockwise about the origin. Record your predictions in the table. Then check your predictions by plotting your points on the coordinate plane.

Preimage coordinates		Image coordinates	
A	$(1, -2)$	A'	
B	$(-2, -4)$	B'	
C	$(1, -4)$	C'	

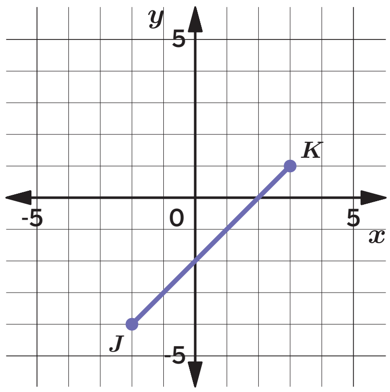


Critique and Correct:
Your teacher will display an incorrect rotation. WIth a partner, determine why it is incorrect and then correct it.

Activity 2 Rotations in Different Directions

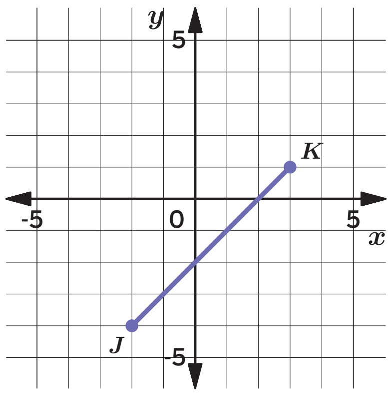
1. Rotate line segment JK as directed, and record the coordinates of the image in the table.

a 90° counterclockwise about the origin



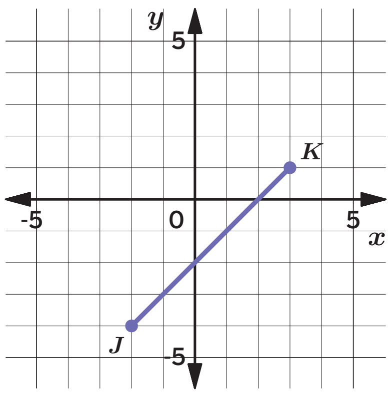
Preimage coordinates		Image coordinates	
J	$(-2, -4)$	J'	
K	$(3, 1)$	K'	

b 180° counterclockwise about the origin



Preimage coordinates		Image coordinates	
J	$(-2, -4)$	J'	
K	$(3, 1)$	K'	

c 270° counterclockwise about the origin

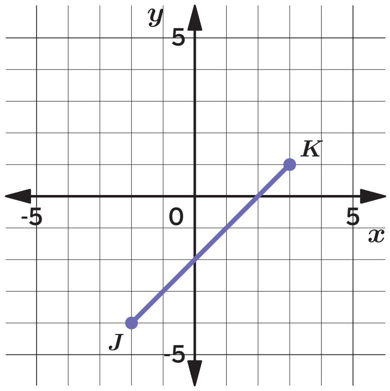


Preimage coordinates		Image coordinates	
J	$(-2, -4)$	J'	
K	$(3, 1)$	K'	

Name: ______ Date: ______ Period: ______

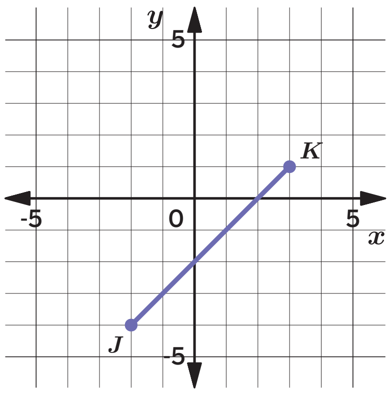
Activity 2 Rotations in Different Directions (continued)

d 90° clockwise about the origin



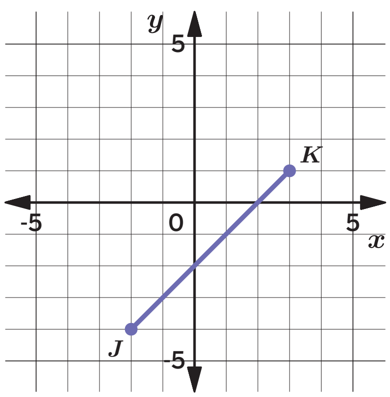
Preimage coordinates		Image coordinates	
J	$(-2, -4)$	J'	
K	$(3, 1)$	K'	

e 180° clockwise about the origin



Preimage coordinates		Image coordinates	
J	$(-2, -4)$	J'	
K	$(3, 1)$	K'	

f 270° clockwise about the origin



Preimage coordinates		Image coordinates	
J	$(-2, -4)$	J'	
K	$(3, 1)$	K'	

2. What observations can you make about the images and their coordinates?

STOP

Summary

In today's lesson . . .

You performed rotations on the coordinate plane, and observed the effects of these transformations on the coordinates of the transformed points.

You can use coordinates to describe points and find patterns in the coordinates of transformed points. Rotating a point about the origin results in an image whose coordinates are related to the coordinates of the preimage, as follows.

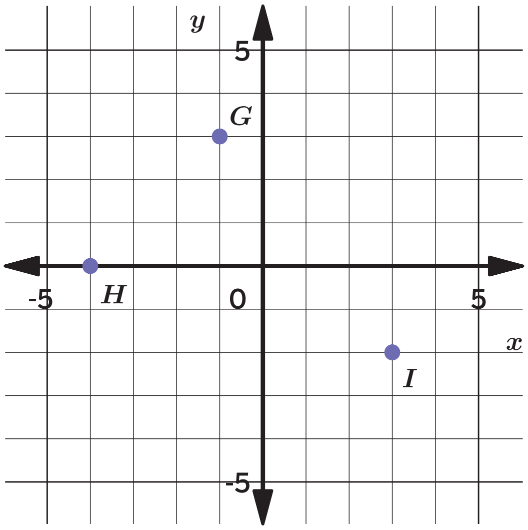
90° counterclockwise or 270° clockwise	90° clockwise or 270° counterclockwise	180° in either direction
The x- and y-coordinates switch places. The x-coordinate of the image has the opposite sign of the y-coordinate of the preimage. **Example:** Preimage: $(-3, 2)$ Image: $(-2, -3)$	The x- and y-coordinates switch places. The y-coordinate of the image has the opposite sign of the x-coordinate of the preimage. **Example:** Preimage: $(-3, 2)$ Image: $(2, 3)$	The order of the x- and y-coordinates of the image stay in the same place as the preimage, but have opposite signs. **Example:** Preimage: $(-3, 2)$ Image: $(3, -2)$

Reflect:

Name: ________ Date: ________ Period: ________

Practice

1. Points $G(-1, 3)$, $H(-4, 0)$, and $I(3, -2)$ are plotted on the coordinate plane. What are the coordinates of G, H, and I after a rotation $180°$ about the origin? Plot these points on the grid, and label them G', H', and I'. Include the coordinates of the images in your labels.



2. Point $P(5, 3)$ is rotated $270°$ clockwise about the origin, and the image is labeled P'. Which of the following are the coordinates of point P'?

A. $(-5, -3)$ **B.** $(-3, -5)$ **C.** $(3, -5)$ **D.** $(-3, 5)$

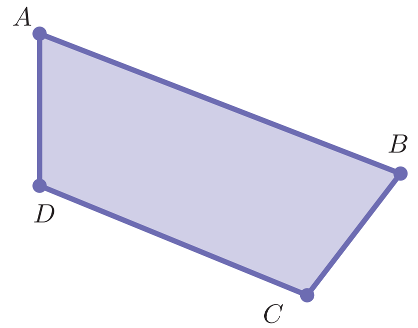
3. Triangle XYZ has been rotated about the origin to create Triangle $X'Y'Z'$. The following table shows the coordinates of the vertices. Indicate the degree and direction of the rotation that maps Triangle XYZ onto Triangle $X'Y'Z'$.

Preimage coordinates		Image coordinates	
X	$(0, 5)$	X'	$(5, 0)$
Y	$(-2, 1)$	Y'	$(1, 2)$
Z	$(4, 3)$	Z'	$(3, -4)$

Practice

Name: ______ Date: ______ Period: ______

4. Draw the image of Quadrilateral $ABCD$ after each transformation indicated.



a A translation that maps point B onto point D.

b A reflection across line segment BC.

5. Write five expressions that have a value of $\frac{3}{5}$, according to the following criteria.

a One expression must be a sum.

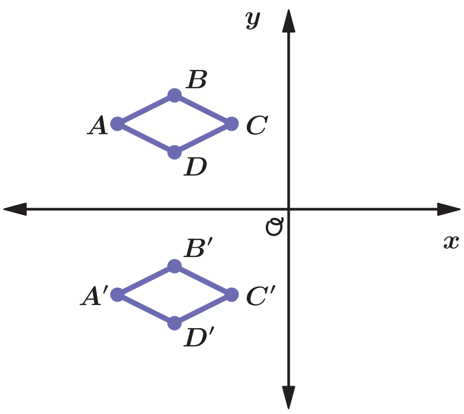
b One expression must be a difference.

c One expression must be a product.

d One expression must be a quotient.

e One expression must involve at least two operations.

6. Mai says that Quadrilateral $A'B'C'D'$ is the image of Quadrilateral $ABCD$ after a reflection across the x-axis. Do you agree with Mai? Explain your thinking.



Name: ______ Date: ______ Period: ______

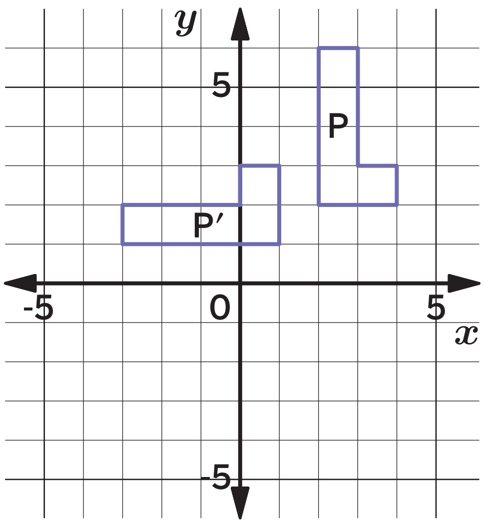
Unit 1 | Lesson 8

Describing Transformations

Let’s transform polygons on the coordinate plane.

Warm-up Center of Rotation

Andre performs a 90° counterclockwise rotation of Polygon P and creates the image Polygon P′, but he does not indicate the center of the rotation. What is the center of rotation?



Log in to Amplify Math to complete this lesson online.

Activity 1 Info Gap: Transformation Information

You will be given either a *problem card* or a *data card*.
Do not show or read your card to your partner.

If you are given the *problem card:*	If you are given the *data card:*
1. Silently read your card, and think about what information you need to be able to solve the problem.	**1.** Silently read your card.
2. Ask your partner for the specific information that you need.	**2.** Ask your partner, "What specific information do you need?" and wait for them to ask for information.
3. Explain how you will use the information to solve the problem. Continue to ask questions until you have enough information to solve the problem.	**3.** Before sharing the information, ask, "Why do you need that information?" Listen to your partner's reasoning, and ask clarifying questions.
4. Share the problem card with your partner, and solve the problem independently.	**4.** Read the problem card your partner shares with you, and solve the problem independently.
5. Read the data card your partner shares with you. Discuss the reasoning each of you used to solve the problem.	**5.** Share the data card with your partner. Discuss the reasoning each of you used to solve the problem.

Pause here so your teacher can review your work. You will be given a new set of cards to repeat the activity, this time trading roles with your partner.

Are you ready for more?

Sometimes two transformations, one performed after the other, can be described as a single transformation. For example:

- Translating a point 2 units up, followed by translating the image 3 units up can be described as translating the original point 5 units up.
- Rotating a point $20°$ counterclockwise about the origin, followed by rotating the image $80°$ clockwise about the origin can be described as rotating the original point $60°$ clockwise around the origin.

Find a single transformation that gives the same result as reflecting a point across the x-axis, followed by reflecting the image across the y-axis.

Name: ______ Date: ______ Period: ______

Activity 2 Transformation Rules

Complete the table with a partner. The first row has been completed for you.

Written description	Transformation rule	Example
Translation a units right and b units up	$(x, y) \rightarrow (x + a, y + b)$	$(-3, 2) \rightarrow (2, 9)$ Translate 5 units right and 7 units up
Translation a units left and b units down		$(-3, 2) \rightarrow (-8, -5)$ Translate 5 units left and 7 units down
Rotation $90°$ counterclockwise about the origin		
	$(x, y) \rightarrow (y, -x)$	
	$(x, y) \rightarrow (-x, -y)$	
Reflection across the x-axis		
		$(7, -1) \rightarrow (-7, -1)$

STOP

Summary

In today's lesson . . .

You described transformations using coordinates. You also discovered rules to describe the changes to coordinates created by these transformations.

Transformation	Rule
Translation a units right and b units up	$(x, y) \rightarrow (x + a, y + b)$
Translation a units left and b units down	$(x, y) \rightarrow (x - a, y - b)$
Reflection across the x-axis	$(x, y) \rightarrow (x, -y)$
Reflection across the y-axis	$(x, y) \rightarrow (-x, y)$
Rotation $90°$ clockwise about the origin	$(x, y) \rightarrow (y, -x)$
Rotation $90°$ counterclockwise about the origin	$(x, y) \rightarrow (-y, x)$
Rotation $180°$ about the origin	$(x, y) \rightarrow (-x, -y)$

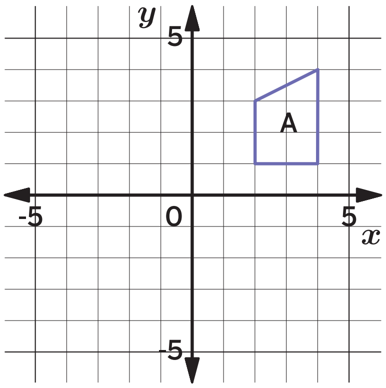
When you perform a sequence of transformations, the order of the transformations can be important. Two translations may be performed in any order, and the image is the same. However, when performing a translation and a reflection, changing the order of the transformations will change the location of the image on the coordinate plane.

Reflect:

Name: .... Date: .... Period: ....

Practice

1. Consider Trapezoid A.



a. Draw Trapezoid B, the reflection of Trapezoid A, using the y-axis as the line of reflection.

b. Draw Trapezoid C, the reflection of Trapezoid B, using the x-axis as the line of reflection.

c. Draw Trapezoid D, the reflection of Trapezoid C, using the y-axis as the line of reflection.

2. The point $(-4, 1)$ is transformed using the following rules. Write the coordinates of each image, and describe the transformation that has occurred.

a. $(x, y) \rightarrow (-y, x)$

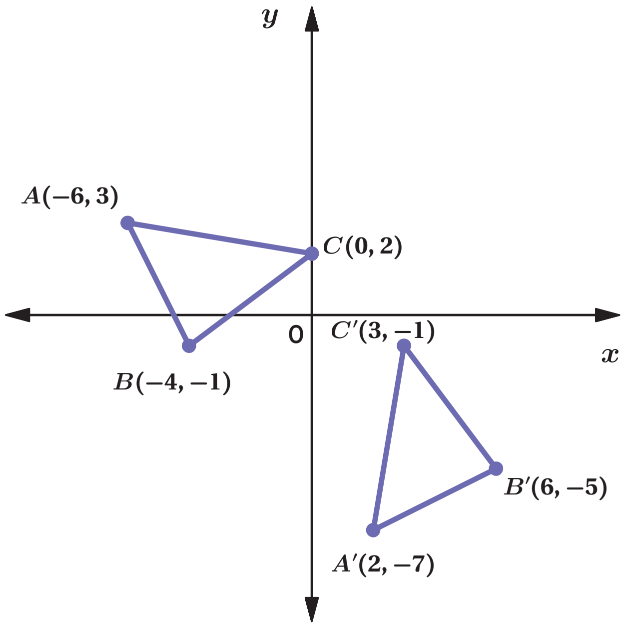
b. $(x, y) \rightarrow (-x, y)$

c. $(x, y) \rightarrow (x - 5, y + 7)$

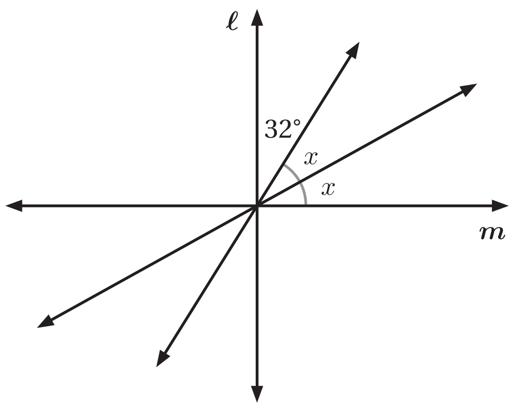
Practice

Name: ________ Date: ________ Period: ________

3. Describe a sequence of transformations that maps Triangle ABC onto Triangle $A'B'C'$.



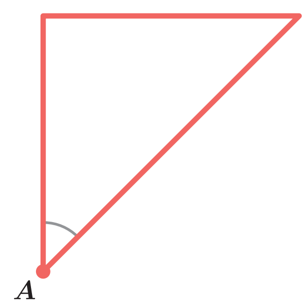
4. Line ℓ is perpendicular to line m. Find the value of x.



5. Use your ruler and protractor to make some measurements for the given triangle.

a What is the measure of angle A to the nearest degree?

b What is the perimeter of the triangle, to the nearest centimeter?



SUB-UNIT

2 Rigid Transformations and Congruence

Narrative Connections

How can a crack make a piece of art priceless?

As World War II came to a close, American troops stormed a Nazi-held salt mine. The mine was being used to store the stolen art collection of the Nazi marshal, Hermann Goering. Among Goering's most prized possessions was a painting from the 17th century Dutch master Johannes Vermeer.

The Allies traced the painting's sale to a Dutch art dealer, Han van Meegeren. Selling cultural treasures to the Nazis was punishable by death. But after his arrest, van Meegeren made an astonishing claim:

"It's not a *Vermeer*," he said. "I painted it myself!"

He claimed he was a master forger. For years he'd been passing off his own work as that of other artists, swindling Goering to the tune of $7 million.

To test van Meegeren's claim, a commission of experts examined the suspected forgery, studying the cracks on the painting's surface.

Over time, all paintings develop something called *craquelure*. It is a network of cracks that form on the paint as it dries. The chemicals in the paint, where the painting was made, even the material of the canvas all affect how the craquelure appears. This gives each painting its own sort of fingerprint.

By examining the craquelure and comparing it to the craquelure of authentic paintings from Vermeer's period, the commission was able to confirm the *Vermeer* was indeed a forgery. The courts dismissed van Meegeren's treason charge. Instead he was sentenced to a year in prison for committing forgery.

No Bending or Stretching

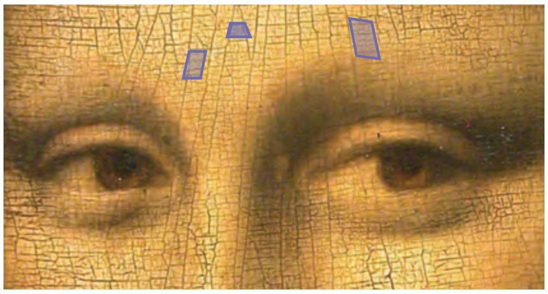
Let's compare measurements before and after translations, rotations, and reflections.

Warm-up Can You Spot the Fake?

Art experts and historians are always on the hunt for counterfeit art. One technique used to detect counterfeits is to study the *craquelure*, the natural patterns created by paint cracking over time. It is very difficult to fake craquelure!

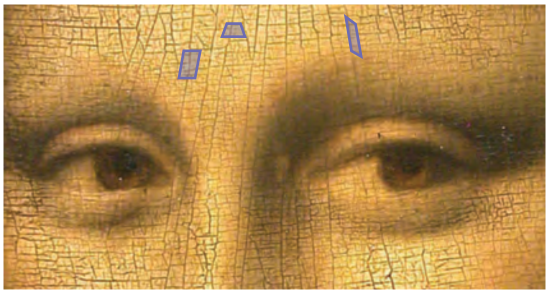
Consider the two images of the famous Mona Lisa painting. The first image is real, the second, a counterfeit. Focus your attention on the highlighted polygons formed by the cracking of the paint. How can you tell the second image is a counterfeit? Use any appropriate tool to support your claim.

Real



Public Domain

Counterfeit



Public Domain

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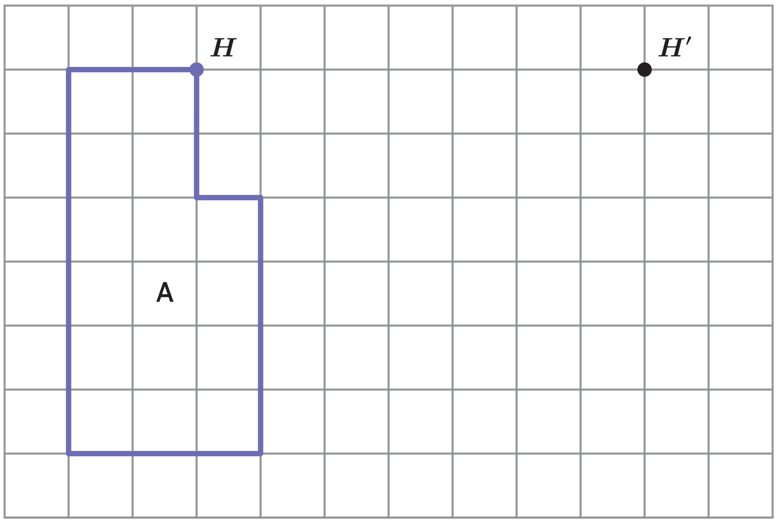
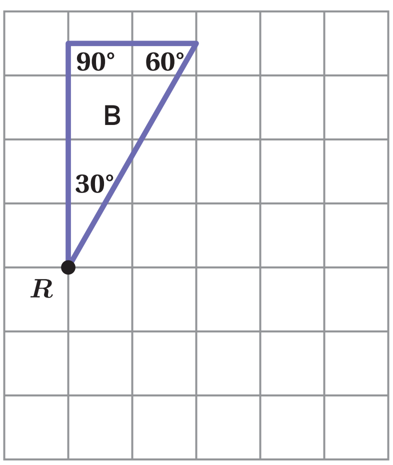
Name: ____________ Date: ____________ Period: ____________

Activity 1 Sides and Angles

1. Translate Polygon A so that point H maps onto point H'. In the image, label each side with its length, in grid units.



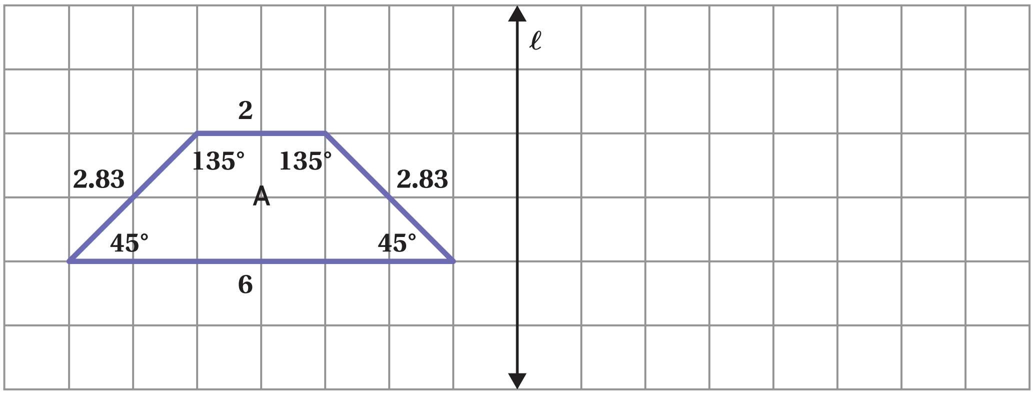
2. Rotate Triangle B $90°$ clockwise using point R as the center of rotation. In the image, label each angle with its measure, in degrees. Verify the angle measures using your protractor.



Reflect: How did using tools from your geometry toolkit help deepen your understanding of transformations?

Activity 1 Sides and Angles (continued)

3. Reflect Polygon A across line ℓ. In the image, label each side length, in grid units. Then label each angle measure, in degrees.

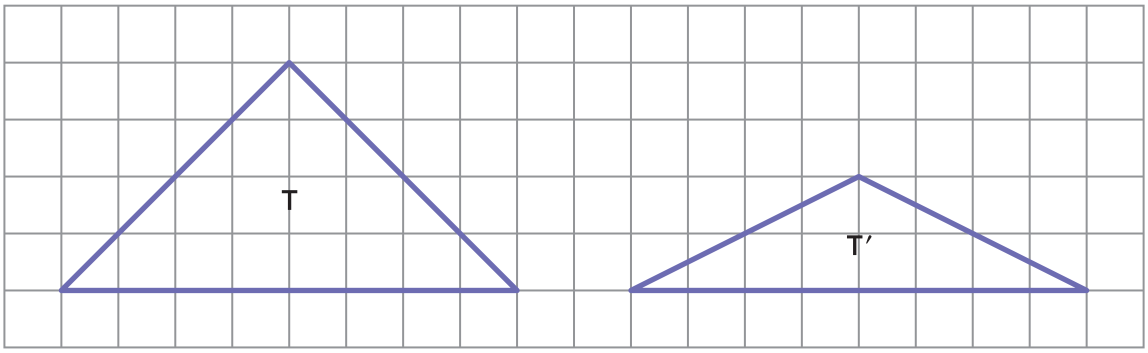


4. What did you notice about the side lengths and angle measures of each transformed polygon in Problems 1, 2, and 3? What conclusions can you make about the three types of transformations?

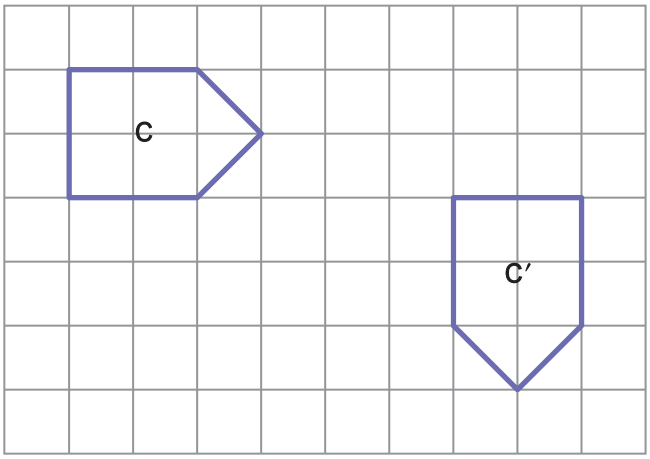
Name: ______ Date: ______ Period: ______

Activity 2 Rigid Transformations

1. Is there a sequence of ***rigid transformations*** that maps Triangle T onto Triangle T′? Explain your thinking.



2. Is there a sequence of rigid transformations that maps Pentagon C onto Pentagon C′? Explain your thinking.



STOP

Summary

In today's lesson . . .

You discovered that the translations, rotations, reflections, and sequences of these motions you have learned about so far are all examples of rigid transformations. A ***rigid transformation*** is a move that does not change measurements — side lengths or angle measures — from the preimage to the image.

Earlier, you learned that a preimage and its image have corresponding points. A preimage and its image also have corresponding sides and corresponding angles. When a preimage is transformed using a rigid transformation, corresponding sides have the same lengths and corresponding angles have the same measures.

Reflect:

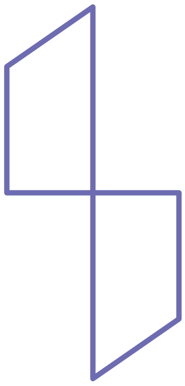
Name: ________ Date: ________ Period: ________

Practice

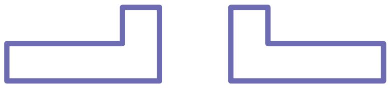
1. Is there a rigid transformation that maps Rhombus P onto Rhombus Q? Explain your thinking

2. For each of the following, determine whether a rigid transformation can map one figure onto the other. If so, explain how the rigid transformation can be performed.

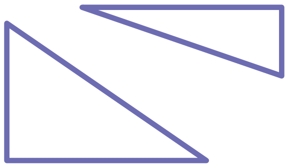
a



b



c

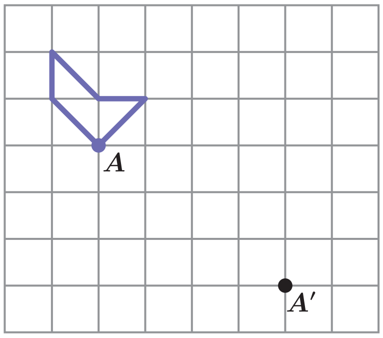


Practice

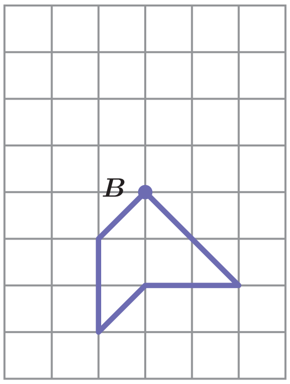
Name: .... Date: .... Period: ....

3. For each shape, draw its image after performing the transformation.

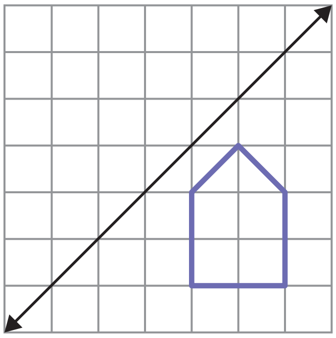
a Translate the preimage so that point A maps onto A'.



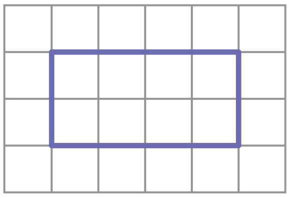
b Rotate the preimage $180°$ counterclockwise about point B.



c Reflect the preimage across the line shown.



4. Determine the area and perimeter of the rectangle. Show or explain your thinking.



Name: Date: Period:

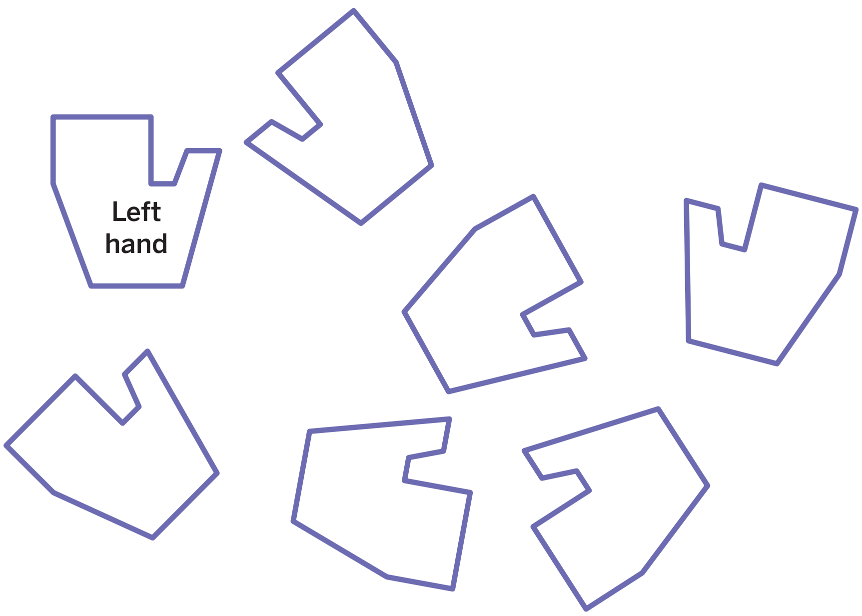
Unit 1 | Lesson 10

What Is the Same?

Let's decide whether shapes are the same.

Warm-up Find the Right Hands

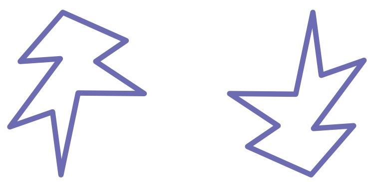
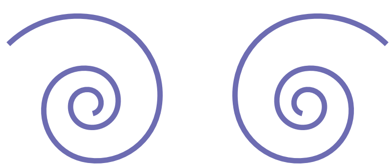
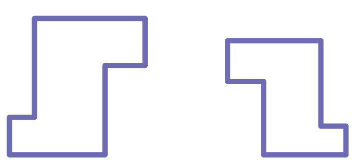
A person's hands are mirror images of each other. In the diagram, a left hand is labeled, where the palm of the hand is facing down. Shade all of the right hands where the palm is facing down.



Log in to Amplify Math to complete this lesson online.

Activity 1 Are They the Same?

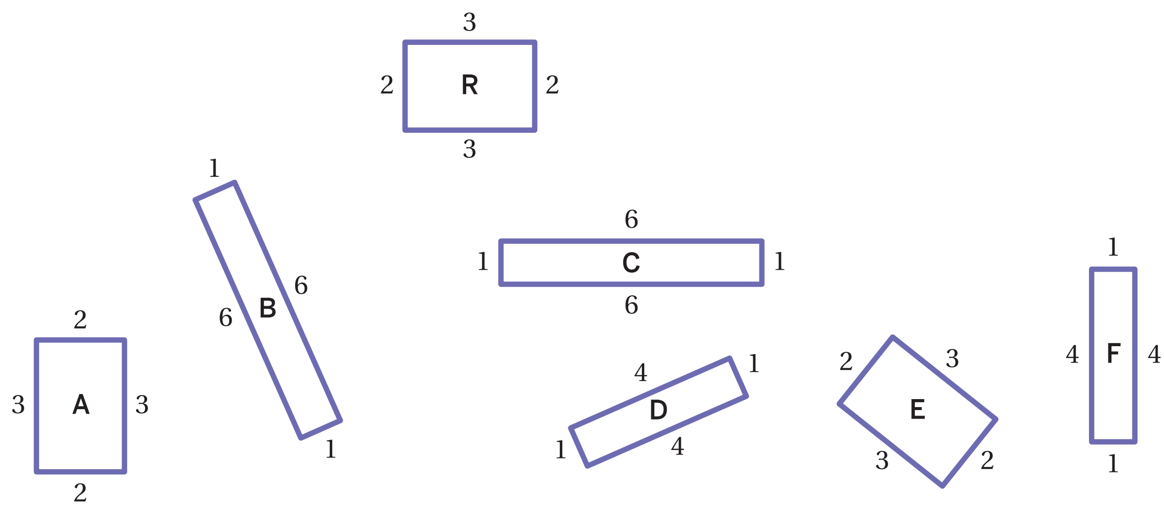
For each pair of figures, decide whether they are the same. Explain your thinking.

Pair	Are they the same?	Explain your thinking.
a 		
b 		
c 		
d 		

Name: ______________________ Date: ____________ Period: ________

Activity 2 Area, Perimeter, and Congruence

Study the rectangles shown. You will need access to your geometry toolkit.



1. Which of these rectangles have the same area as Rectangle R, but different perimeters? Explain your thinking.

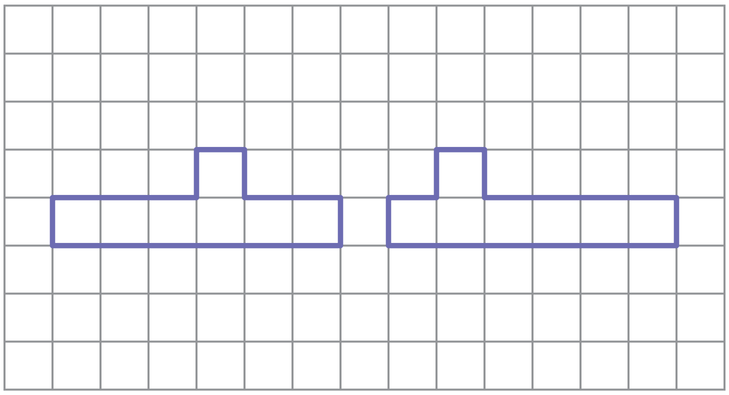
2. Which rectangles have the same perimeter as Rectangle R, but different areas? Explain your thinking.

3. Which have the same area and the same perimeter as Rectangle R? Explain your thinking.

4. Using your geometry tools, decide which rectangles are ***congruent***. Shade congruent rectangles with the same color.

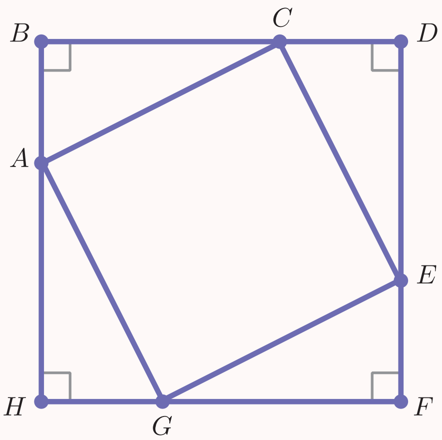
Activity 2 Area, Perimeter, and Congruence (continued)

5. These polygons have the same perimeter and the same area. Are they congruent? Explain your thinking.



Are you ready for more?

Figure $BDFH$ is a square. Points A, C, E, and G are selected and marked so that the lengths of the bold line segments are the same. Is Figure $ACEG$ also a square? Explain your thinking.



STOP

Name: ________________ Date: ________ Period: ________

Summary

In today's lesson . . .

You explored what it means for two figures to be congruent. This is a new term for an idea you already know about and have been using. Two figures are ***congruent*** if one figure maps onto the other figure exactly by using a sequence of rigid transformations. The congruence symbol $\cong$ can be used to show two figures are congruent. For example, $\triangle ABC \cong \triangle DEF$ means that the two triangles are congruent. The statement is read "Triangle ABC is congruent to Triangle DEF".

Here are some other facts about congruent figures:

- You do not need to check *all* the measurements to prove two figures are congruent. Instead, you can find a sequence of rigid transformations that maps one figure onto the other. If you can find such a sequence, then the figures are congruent.
- Two figures that are exact mirror images of each other are congruent. This means there must be a *reflection* in the sequence of transformations that maps one figure onto the other.
- Because two congruent polygons have the same area and the same perimeter, one way to show that two polygons are *not* congruent is to show that they have different perimeters or different areas.

Reflect:

Practice

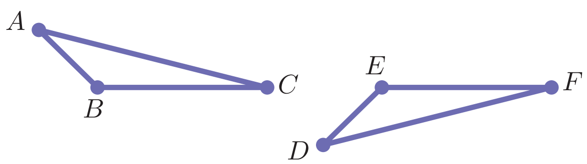
Name: ________ Date: ________ Period: ________

1. If two rectangles have the same perimeter, do they have to be congruent? Explain your thinking.

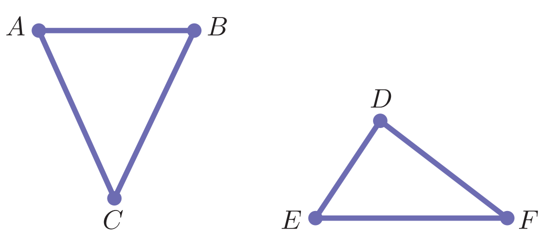
2. Draw two rectangles that have the same area, but are *not* congruent.

3. For each pair of triangles, decide whether the statement about congruence is *true* or *false*. Explain your thinking.

a $\triangle ABC \cong \triangle DEF$



b $\triangle ABC \cong \triangle DEF$



Name: ____________ Date: ________ Period: ________

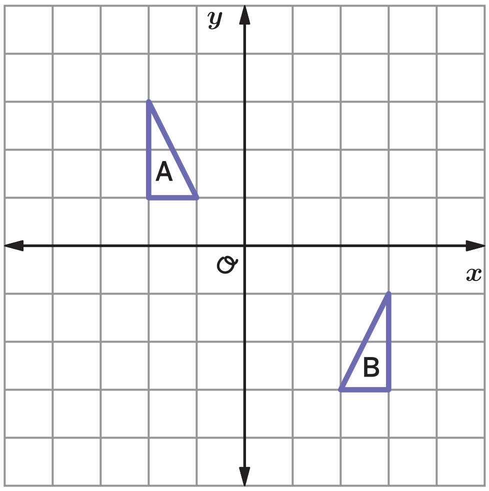
Practice

4. Find the coordinates of the image of point $A(2, -5)$ after each transformation.

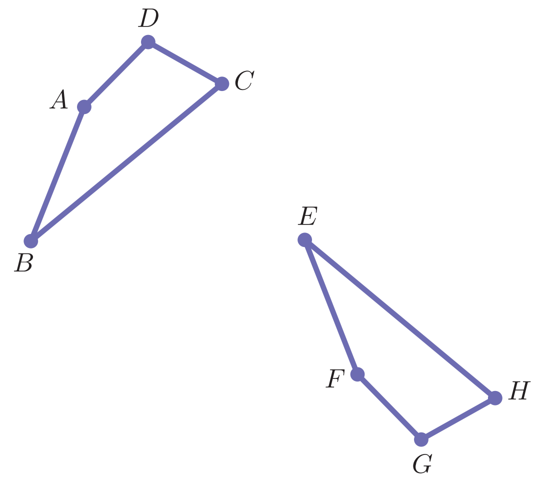
a. Point $A(2, -5)$ is reflected across the x-axis. What are the coordinates of the image?

b. Point $A(2, -5)$ is reflected across the y-axis. What are the coordinates of the image?

5. Prove that Triangle A and Triangle B are congruent by describing a sequence of rigid transformations that they could have used to map Triangle A onto Triangle B. Explain your thinking.



6. A series of rigid transformations was used to map one of these polygons onto the other. Which of the following statement(s) are true? Select *all* that apply.



A. $\angle A \cong \angle G$

B. $\angle A \cong \angle F$

C. Segment AD is congruent to segment FG.

D. Segment AD is congruent to segment GH.

E. Segment BD is congruent to segment EG.

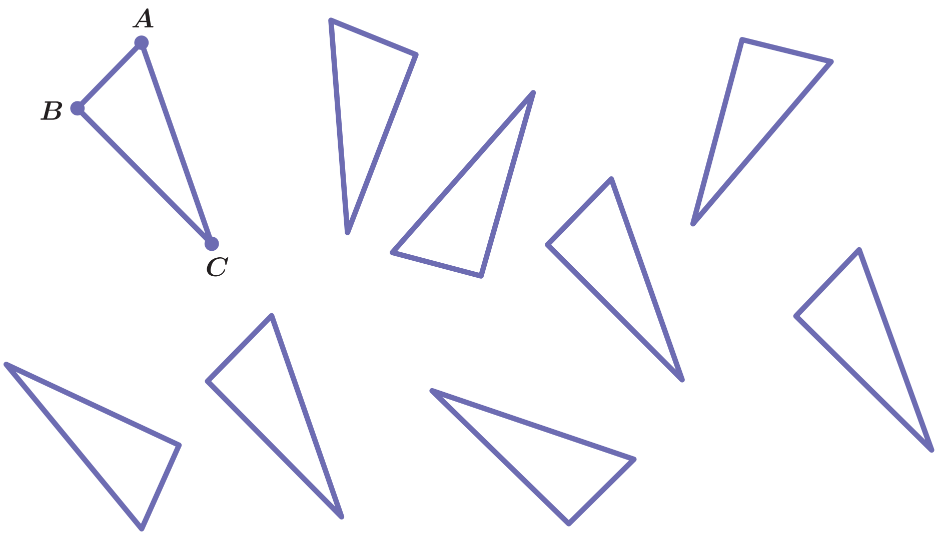
Unit 1 | Lesson 11

Congruent Polygons

Let's decide whether two figures are congruent.

Warm-up Translated Images

Study the triangles shown. All of these triangles are congruent to Triangle ABC, and all of the triangles were translated. Some of the triangles were also rotated and/or reflected.



1. Label triangles that were also rotated as "Ro."

2. Label triangles that were also reflected as "Re."

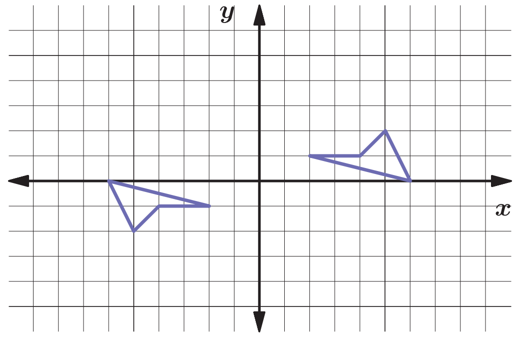
Log in to Amplify Math to complete this lesson online.

Name: .... Date: .... Period: ....

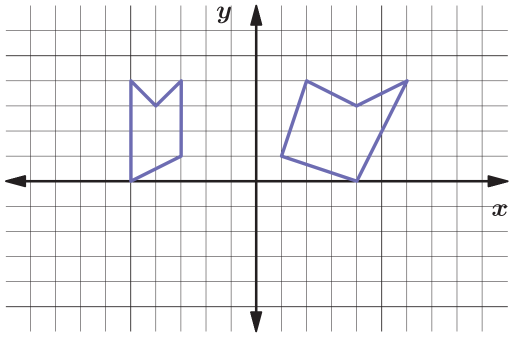
Activity 1 Congruent Pairs

For each pair of figures, decide whether they are congruent. Explain your thinking. If they are congruent, label the corresponding vertices.

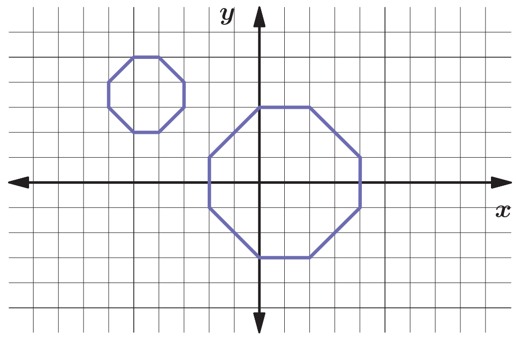
a



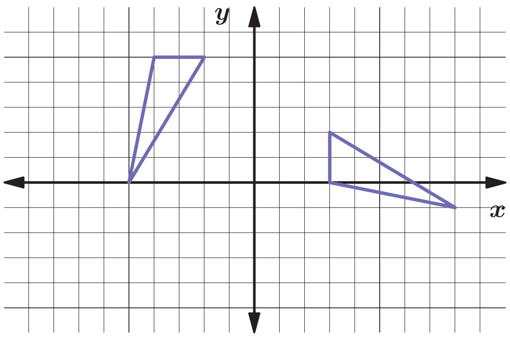
b



c



d

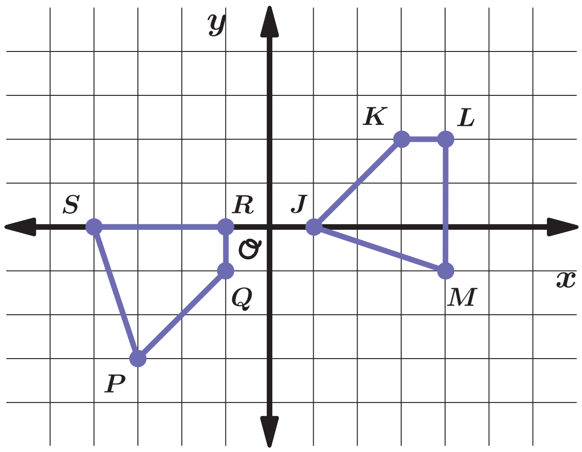


Discussion Support: Pay attention to the strategies you used and be ready to share them. As your classmates share, be ready to restate their ideas in your own words.

Activity 2 Are You Sure They Are Congruent?

Students in a different class are asked to determine if two polygons are congruent.

1. Priya is trying to persuade her classmates that the polygons shown are congruent. Which argument is most convincing?



A. Both figures have 4 sides and an area of 5.5 square units.

B. I can map one figure onto the other by translating Polygon $JKLM$ down 3 units and to the left 4 units. Then I can reflect the image across segment QP.

C. When I measure the side lengths of each polygon, I get the same measurements.

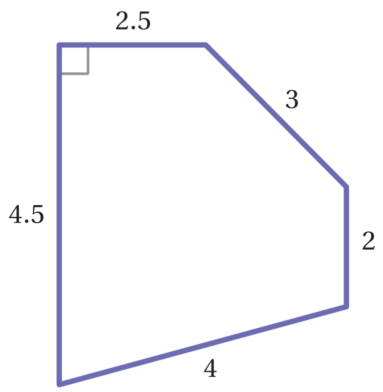
D. When I measure the angles of each polygon, the angle measurements show $m\angle S = m\angle J$, $m\angle R = m\angle K$, $m\angle Q = m\angle L$, and $m\angle P = m\angle M$.

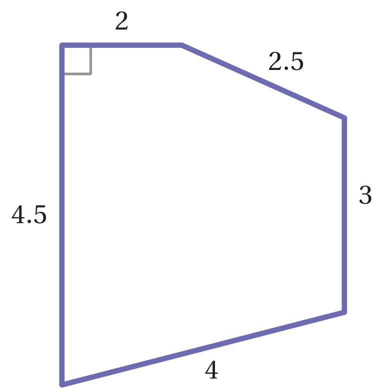
Explain your thinking.

Name: ______ Date: ______ Period: ______

Activity 2 Are You Sure They Are Congruent? (continued)

2. Andre studied the two figures shown and noticed that the side lengths of each figure are equivalent. Is this enough to claim the figures are congruent? Explain your thinking.

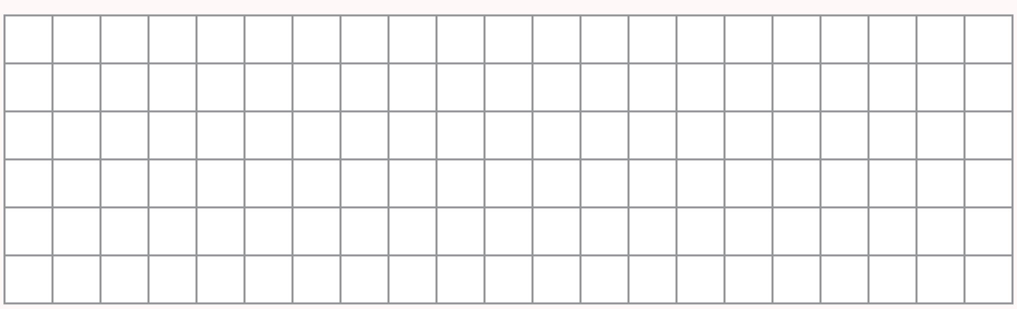




Are you ready for more?

A polygon has 8 sides: five sides that each have a length of 1 unit, two sides that each have a length of 2 units, and one side that has a length of 3 units. All sides lie on grid lines. Draw a polygon with these side lengths on the grid shown here.

Is there a second polygon, not congruent to your first, that also has these side lengths? If so, draw this polygon on the grid shown here.



STOP

Summary

In today's lesson . . .

You applied the definition of congruence to polygons. You learned that:

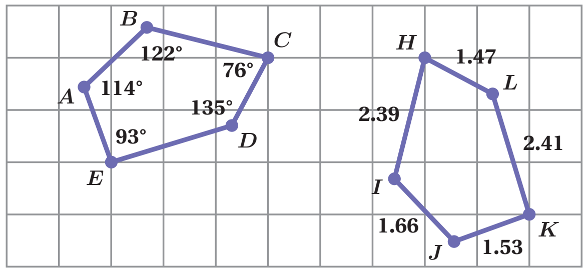
- Two polygons are *congruent* when there is a sequence of translations, rotations, and reflections that map one polygon onto the other.
- Two polygons are *not congruent* if they have different side lengths, different angle measures, or different areas.

Even if two polygons have the same side lengths, they might not be congruent. With four sides of the same length, for example, you can create many different rhombuses that are not congruent to one another because the angles may be different.

Reflect:

Name: .... Date: .... Period: ....

1. Refer to Pentagons $ABCDE$ and $JIHLK$.

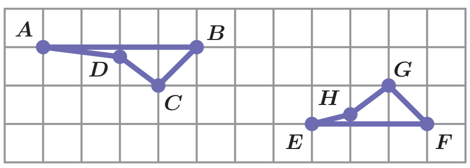


a Show that the two pentagons are congruent by describing a sequence of rigid transformations that can map one figure onto the other.

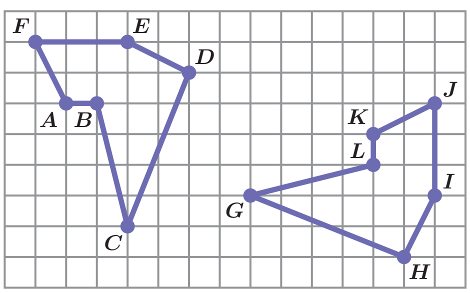
b Label the side lengths of Pentagon $ABCDE$ and the angle measures of Pentagon $JIHLK$.

2. For each pair of figures, decide whether they are congruent. Explain your thinking.

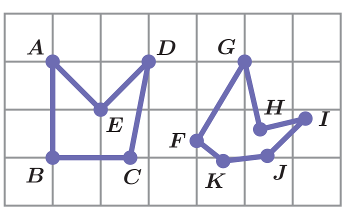
a



b



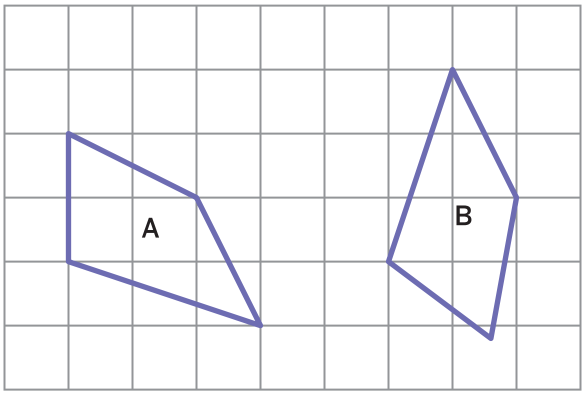
c



Practice

Name: .... Date: .... Period: ....

3. Lin says that she can map Polygon A onto Polygon B using *only* rotations and translations. Do you agree with Lin? Explain your thinking.



4. Point $(3, -5)$ was transformed using different transformations. Match the transformations described with the coordinates of the images.

Transformation	Image coordinates
a Translated 2 units up and 4 units to the left	.... $(-3, 5)$
b Reflected across the x-axis	.... $(5, -9)$
c Rotated $90°$ counterclockwise about the origin	.... $(3, 5)$
d Reflected across the y-axis	.... $(-3, -5)$
e Rotated $180°$ about the origin	.... $(5, 3)$
f Translated 4 units down and 2 units to the right	.... $(-1, -3)$

5. Kiran says it is more challenging to determine if two ovals are congruent than if two triangles are congruent. Do you agree with this statement? Explain your thinking.

Name: ________ Date: ________ Period: ________

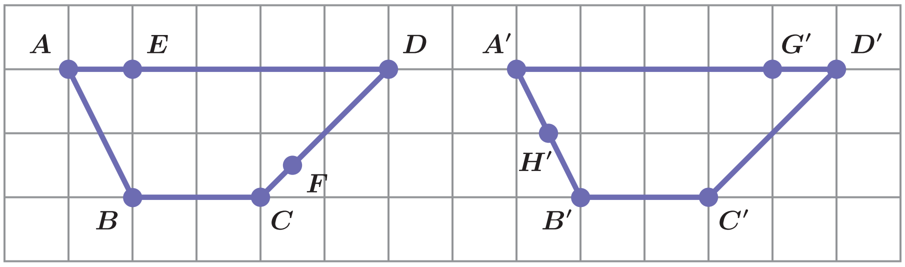
Unit 1 | Lesson 12

Congruence

Let's find ways to test congruence of polygons and other interesting figures.

Warm-up Not Just the Vertices

Trapezoid $ABCD$ is congruent to Trapezoid $A'B'C'D'$.



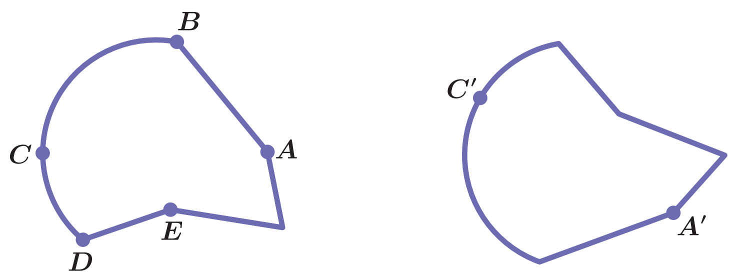
1. Draw and label the points on Trapezoid $A'B'C'D'$ that correspond to points F and E.

2. Draw and label the points on Trapezoid $ABCD$ that correspond to points G' and H'.

Log in to Amplify Math to complete this lesson online.

Activity 1 Corresponding Points in Congruent Figures

Here are two congruent shapes with some corresponding points labeled.



1. Label the points corresponding to B, D, and E with B', D', and E'.

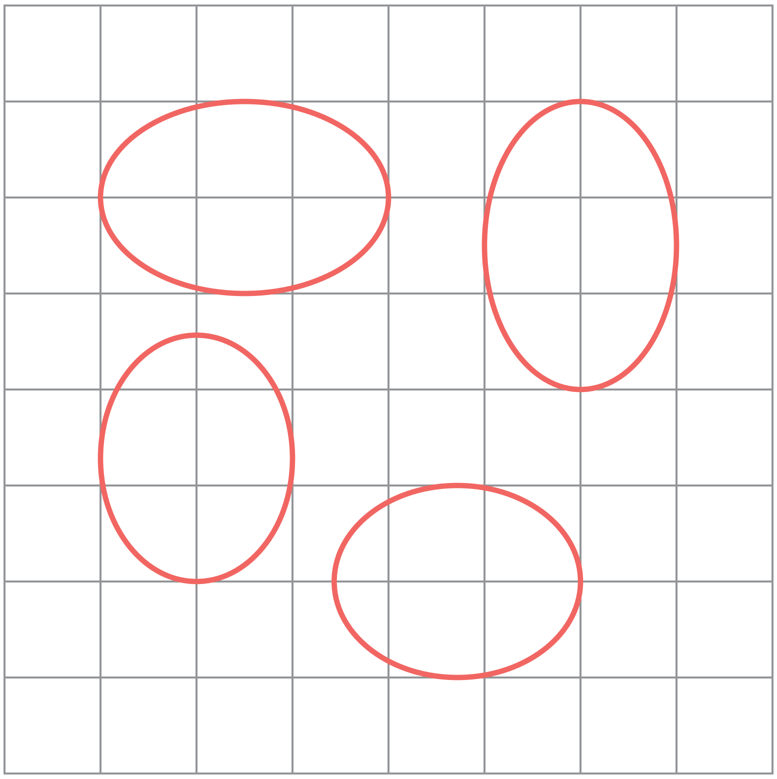
2. Draw line segments AD and $A'D'$ and measure them. Repeat for segments AE and $A'E'$ and for segments BC and $B'C'$. What do you notice?

3. Do you think there could be a pair of corresponding segments with different lengths? Explain your thinking.

Name: ____________ Date: ____________ Period: ____________

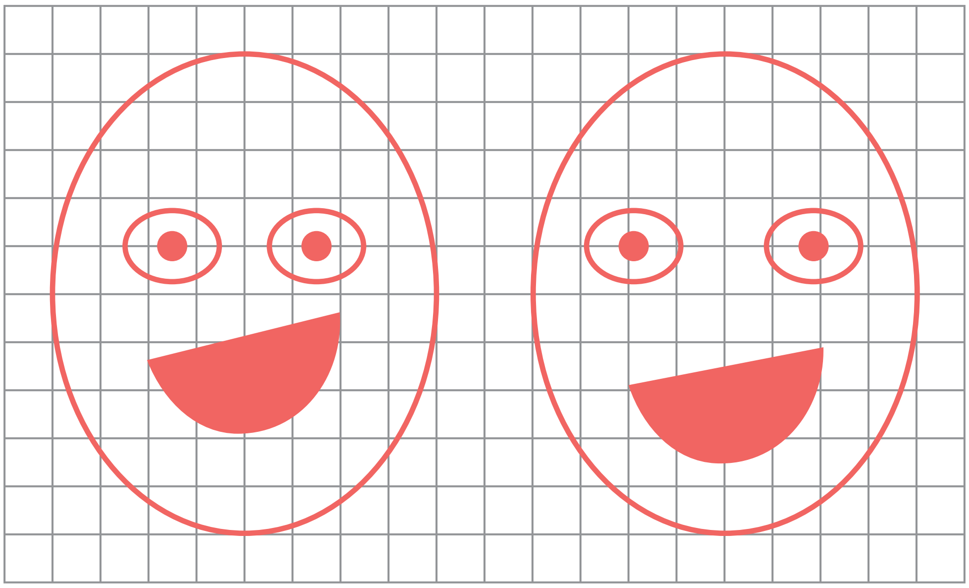
Activity 2 Congruent Ovals

Four ovals are shown. Are any of the ovals shown congruent to one another? Explain your thinking.



Activity 3 Astonished Faces

Are these faces congruent? Explain your thinking.



STOP

Name: ________ Date: ________ Period: ________

Summary

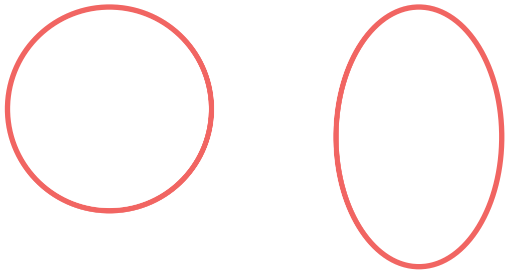
In today's lesson . . .

You explored different ways to show congruence between sets of polygons and other interesting figures.

To show that two figures are congruent, you can map one figure onto the other by a sequence of rigid transformations. This is true even for figures with curved sides. Distances between corresponding points on congruent figures are always equivalent, even for curved shapes.

To show two figures are *not* congruent, you can find parts of the figures that would correspond if the figures were congruent, but in reality have different measurements.

Here is an example of two figures that are *not* congruent.

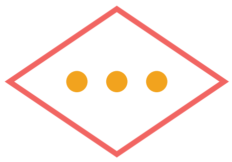


Reflect:

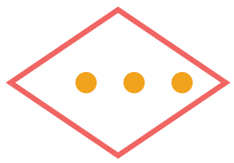
Practice

Name: ______________________ Date: ____________ Period: ____________

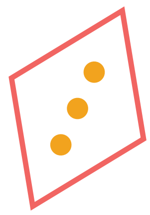
1. Which of these figures are congruent to the figure shown? Select *all* that apply.



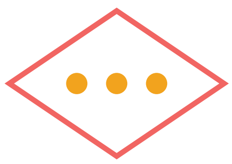
A.



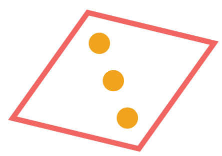
C.



B.



D.



2. Consider Figures A and B. Show, using measurements, that these two figures are not congruent.

Figure A

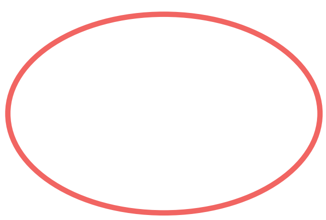
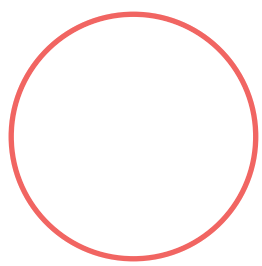


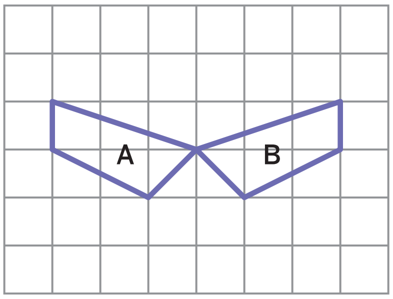
Figure B



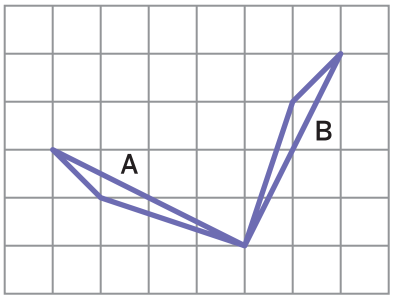
Name: .... Date: .... Period: ....

3. For each pair of polygons, describe the transformation that maps Polygon A onto Polygon B.

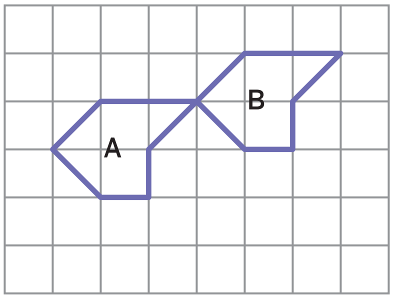
a



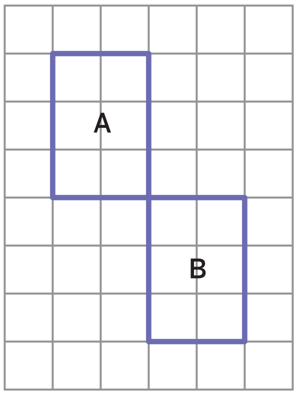
b



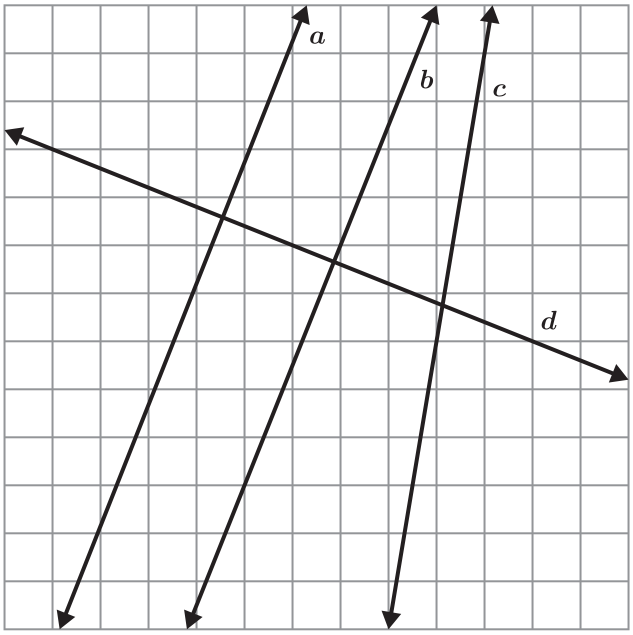
c



d



4. Refer to the four lines shown.



a Name a pair of lines that appear to be parallel.

b Name a pair of lines that appear to be perpendicular.

My Notes:

SUB-UNIT

3 Angles in a Triangle

Narrative Connections

What has 10 billion galaxies and goes great with maple syrup?

There have been a few theories about how our Universe is shaped. Some thought it was open like a saddle. Others thought it was round like a football. Some have even suggested that the Universe is shaped like a doughnut!

But, how do you actually find out?

Before we answer that, let's start with something a little bit smaller — Earth! Standing on Earth's surface, the world certainly *appears* flat. But there are ways to prove it isn't. For example, you could start walking (or swimming) in any direction, and eventually you'd wind up where you started.

Another way is to pick three points, thousands of miles apart, on Earth's surface. On a flat surface, the interior angles of the triangle formed by those points will always add up to — *spoiler alert!* — 180 degrees. But on a curved surface, like Earth's, the angles end up being something greater. The bigger the triangle on Earth, the greater the sum of its three angles.

Physicists have done the same thing with the Universe. With an assist from a specially designed spacecraft called the WMAP, NASA scientists effectively plotted a gigantic triangle across the Universe, then measured its angles. And what did they find?

It turned out the sum of the angles were very close to 180 degrees. So while the idea of a doughnut Universe sounds scrumptious, we'll have to settle for a Universe that's flat as a pancake (give or take a little curvature).

For now, let's see what else we can discover about the angles, lines, and triangles.

Line Moves

Let's transform some lines.

Warm-up Rotating a Triangle

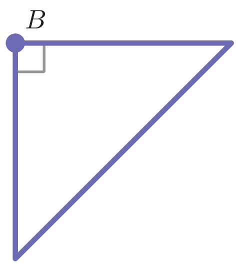
Refer to the isosceles right triangle shown.

1. Rotate the isosceles right triangle $90°$ clockwise about point B. Draw the image.

2. Rotate the original isosceles right triangle $180°$ about point B. Draw the image.

3. Rotate the original isosceles right triangle $270°$ clockwise about point B. Draw the image.

4. What do you notice?



Log in to Amplify Math to complete this lesson online.

Name: .... Date: .... Period: ....

Activity 1 Rotating a Segment

Plan ahead: What can you do to make sure you have an optimistic attitude before beginning this activity?

1. For each grid, draw and label a line segment AB. Then perform the indicated transformation.

a Rotate segment AB $180°$ about point B and label the resulting image $A'B'$.

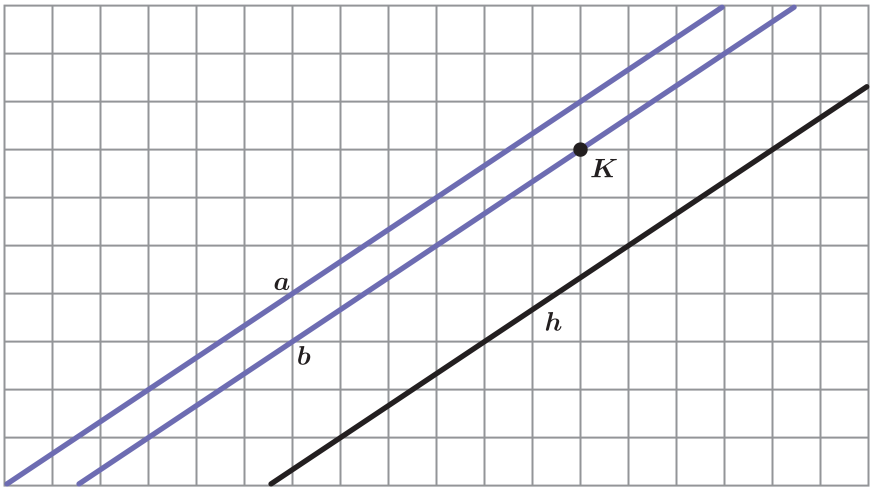
b Draw and label a point C that is not on the line segment AB. Rotate segment AB $180°$ about point C and label the resulting image $A'B'$.

c Rotate segment AB $180°$ about its midpoint and label the resulting image $A'B'$.

2. What do you notice when you rotate a line segment $180°$ about a point?

Activity 2 Parallel Lines

The diagram shows parallel lines a and b, point K, and line h.



1. Have each member in your group choose one of the following transformations to perform. Perform the transformation on the grid provided here.
   - Translate lines a and b 3 units up and 2 units to the right.
   - Rotate lines a and b $180°$ about point K.
   - Reflect lines a and b across line h.

2. Discuss each transformation with your group. What do you notice about the image of two parallel lines under a rigid transformation?

STOP

Name: ________________ Date: ________ Period: ________

Summary

In today's lesson . . .

You applied a 180° rotation to a line segment and discovered the following:

When the center of rotation is . . .

- the midpoint of the line segment, the segment maps onto itself, except the endpoints are switched.
- an endpoint of the line segment, the segment together with its image form a segment twice as long as the original.
- not a point on the line segment, the image is parallel to the original segment.

You also applied different rigid transformations to parallel lines. A rigid transformation of two parallel lines results in two parallel lines that are the same distance apart as the original two lines.

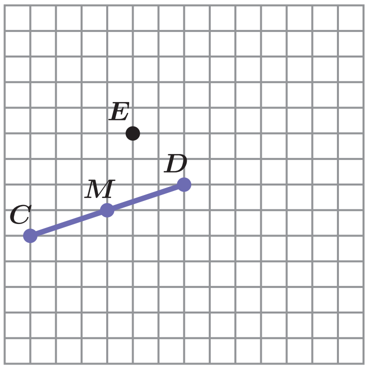
Reflect:

Practice

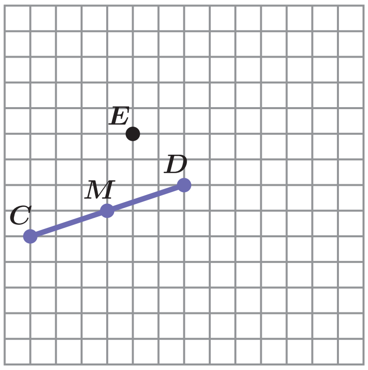
Name: ______ Date: ______ Period: ______

1. The diagram in each of parts a, b, and c shows line segment CD and point E that is *not* on CD.

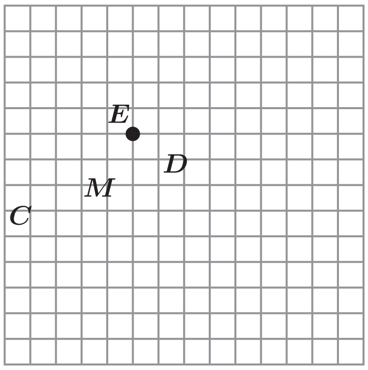
a Rotate segment CD $180°$ about point D and label the resulting image $C'D'$.



b Rotate segment CD $180°$ about point E and label the resulting image $C'D'$.



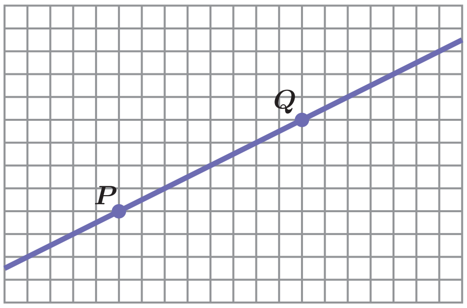
c Rotate segment CD $180°$ about point M and label the resulting image $C'D'$.



2. Points P and Q are plotted on the line shown.

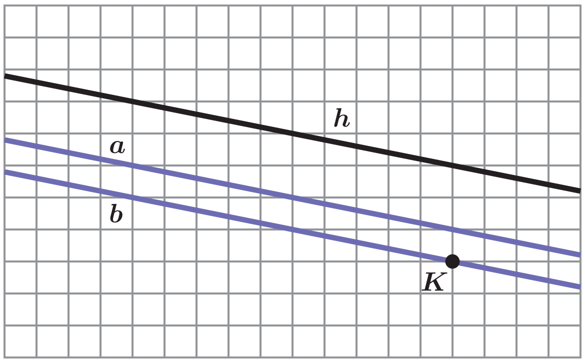
a Label point R so that a $180°$ rotation with center R maps point P onto point Q and maps point Q onto point P.

b Is there more than one point R that satisfies the conditions in part a?



Name: ______________________ Date: __________ Period: __________

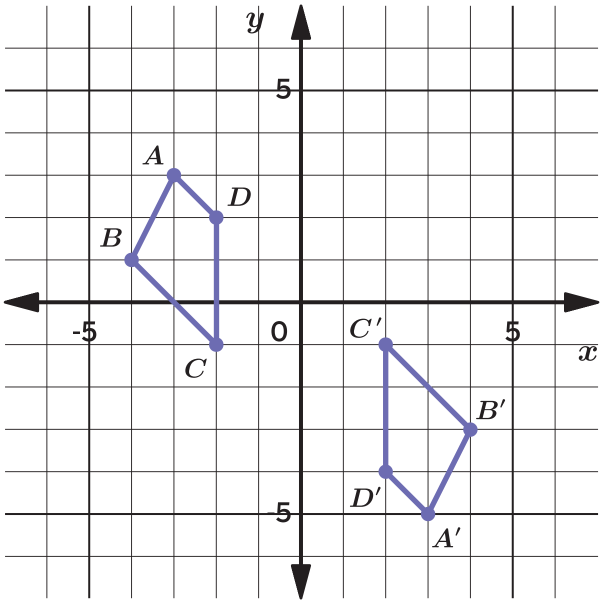
3. The diagram shows parallel lines a and b, point K, and line h.



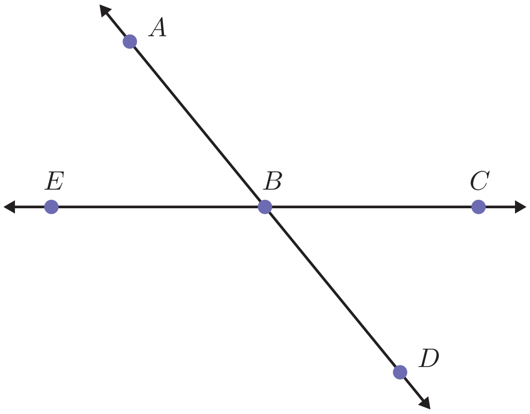
- Elena rotates line a about point K, and then reflects the image across line h. She labels the final image a'.
- Jada rotates line b about point K, and then reflects the image across line h. She labels the final image b'.

What is true about lines a' and b'?

4. The graph shows two quadrilaterals. Describe a sequence of transformations that maps Quadrilateral $ABCD$ onto Quadrilateral $A'B'C'D'$.



5. The diagram shows intersecting lines.



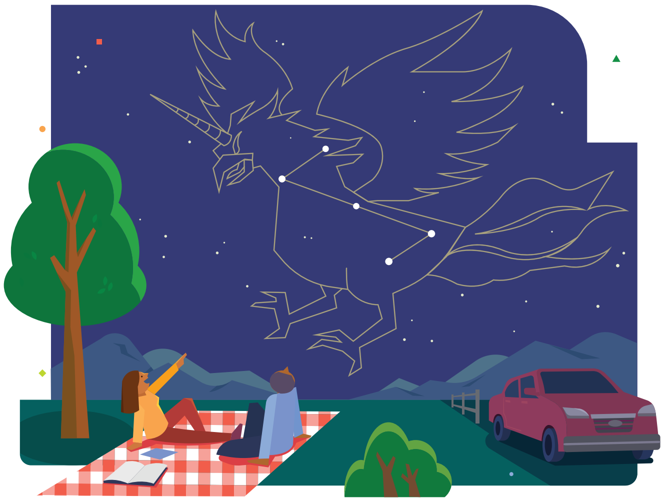
a. List *all* the pairs of vertical angles.

b. List *all* the pairs of supplementary angles.

Unit 1 | Lesson 14

Rotation Patterns

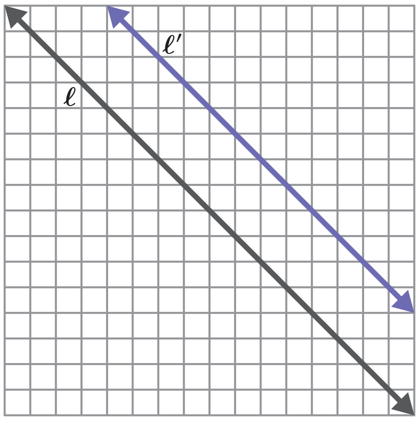
Let's rotate some angles.



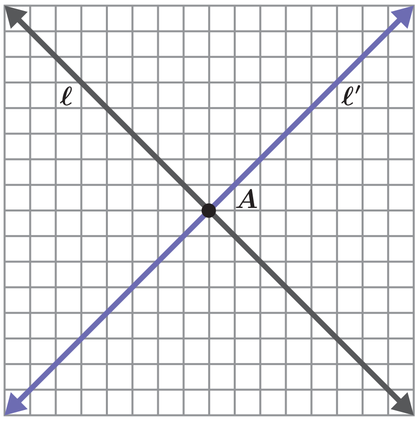
Warm-up How Many Ways?

1. For each diagram, describe a transformation that maps line ℓ onto line ℓ'. Describe as many transformations as you can.

   a. Line ℓ is parallel to line ℓ'.



   b. Line ℓ intersects line ℓ' at point A.



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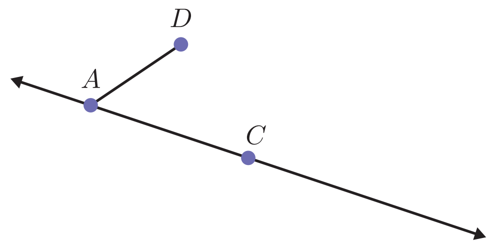
Name: ______ Date: ______ Period: ______

Activity 1 Let's Do Some 180s

1. The figure shows a line with points A and C on the line and a segment AD where point D is not on the line.

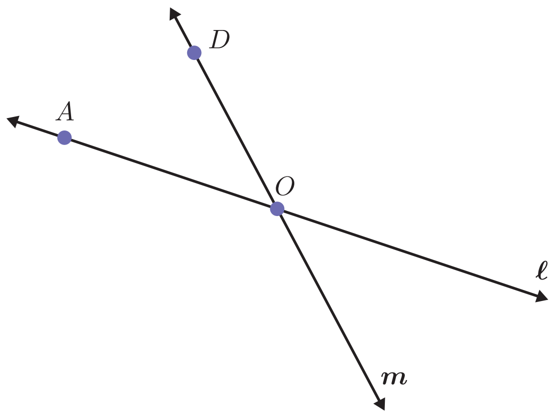


a. Rotate the figure $180°$ about point C. Label the points corresponding to A and D with A' and D'.

b. What do you know about the relationship between $\angle CAD$ and $\angle CA'D'$? Show or explain your thinking.

Activity 1 Let's Do Some 180s (continued)

2. The figure shows two lines ℓ and m that intersect at a point O. Point A is on the line ℓ and point D is on the line m.



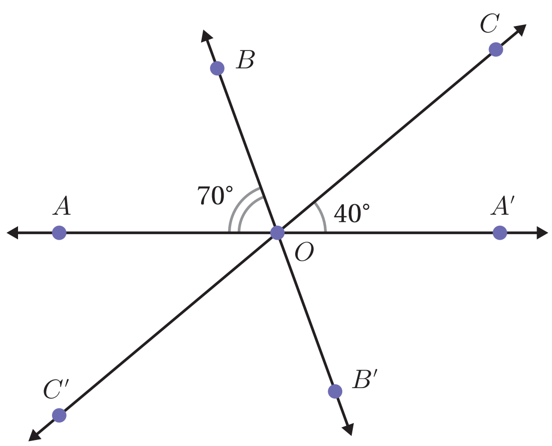
a. Rotate the figure $180°$ about point O. Label the points corresponding to A and D with A' and D'.

b. What do you know about the relationship between the angles in the figure? Explain or show your thinking.

Name: ______ Date: ______ Period: ______

Activity 2 Solving for Unknown Angles

Points A, B, and C are located at different distances from point O. The points A, B, and C are each rotated $180°$ about point O creating the images of points A', B', and C'.



The figure may not be drawn to scale.

1. Name a segment that has the same length as segment AO. Explain your thinking.

2. List all the angles that have a measure of $40°$. Explain your thinking.

3. List all the angles with a measure of $70°$. Explain your thinking.

STOP

Summary

In today's lesson . . .

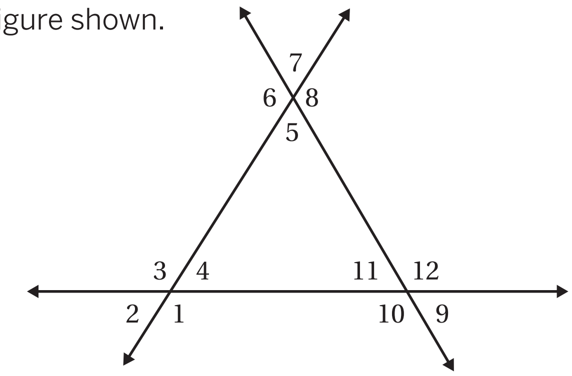
You rotated intersecting lines 180° about their point of intersection. Because a rotation is a rigid transformation that preserves angle measures, the vertical angles are congruent.

Reflect:

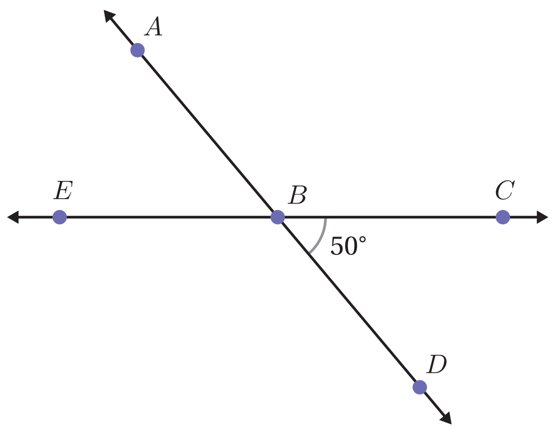
Name: .... Date: .... Period: ....

Practice

1. List *all* the pairs of congruent angles in the figure shown.



2. Use the figure to calculate the measure of each angle. Explain your thinking.



The figure may not be drawn to scale.

a. $m\angle ABC$

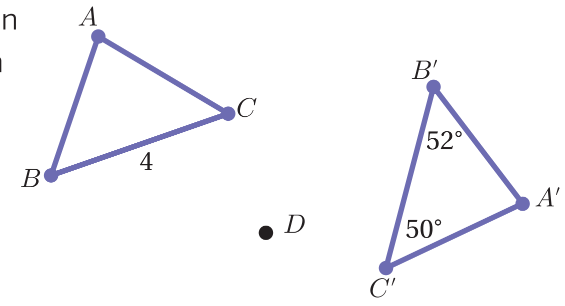
b. $m\angle EBD$

c. $m\angle ABE$

Practice

Name: .... Date: .... Period: ....

3. $\triangle A'B'C'$ is an image of $\triangle ABC$ after a rotation about point D. The figures may not be drawn to scale.



a What is the length of side $B'C'$? Explain your thinking.

b What is $m\angle ABC$? Explain your thinking.

c What is $m\angle ACB$? Explain your thinking.

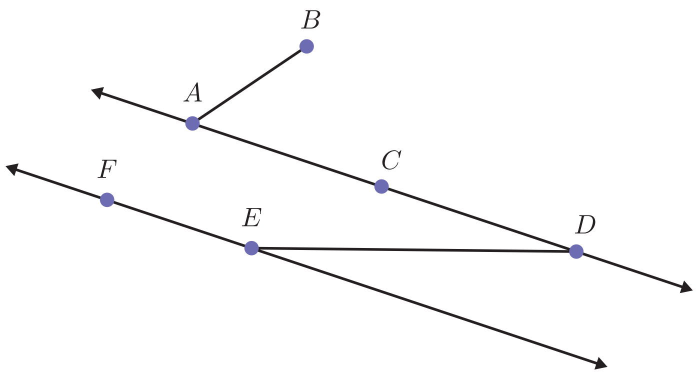
4. The point $(-4, 1)$ is rotated $180°$ counterclockwise about the origin. What are the coordinates of the image?

A. $(-1, -4)$ B. $(-1, 4)$ C. $(4, 1)$ D. $(4, -1)$

5. Refer to the figure shown.

a Highlight or shade line FE.

b Highlight or shade $\angle CDE$.



Name: ______ Date: ______ Period: ______

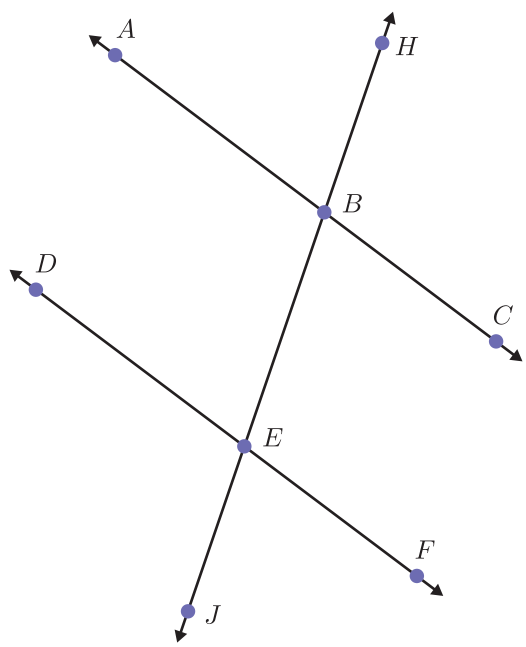
Unit 1 | Lesson 15

Alternate Interior Angles

Let's explore why some angles are always congruent.

Warm-up Notice and Wonder

Refer to the diagram. What do you notice? What do you wonder?



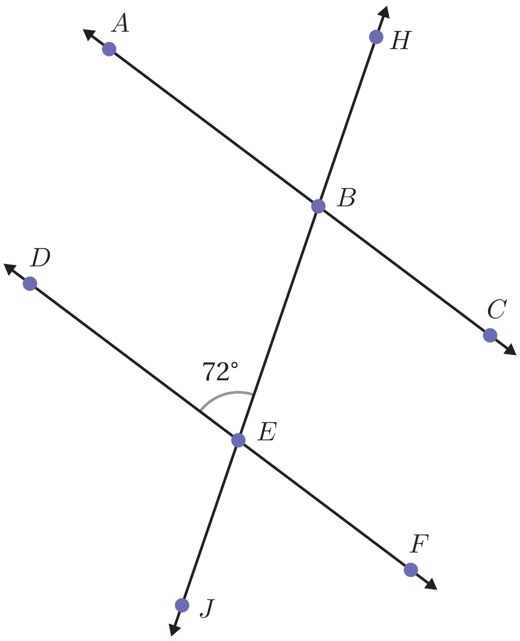
1. I notice . . .

2. I wonder . . .

Log in to Amplify Math to complete this lesson online.

Activity 1 Alternate Interior Angles

You will be given a protractor. Refer to the diagram. Lines AC and DF are parallel. They are intersected by transversal JH.



> **1.** Use your protractor to measure the seven missing angle measures.

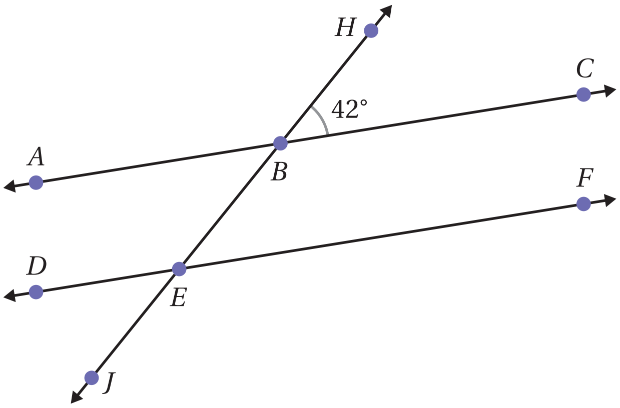
> **2.** What do you notice when a transversal intersects a pair of parallel lines?

Name: .... Date: .... Period: ....

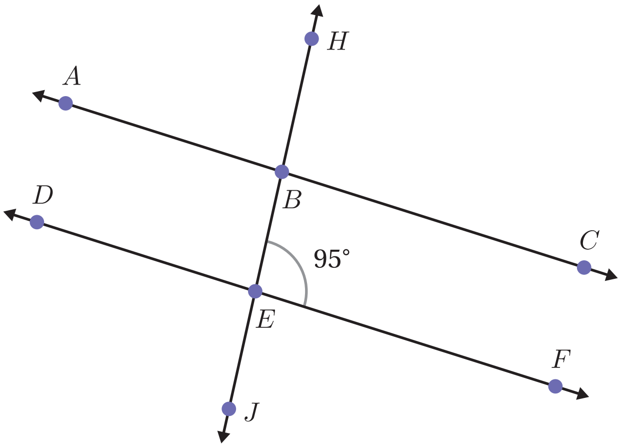
Activity 2 Three, Five, Seven

In each diagram, line AC is parallel to line DF. The lines are intersected by transversal HJ. The figures may not be drawn to scale.

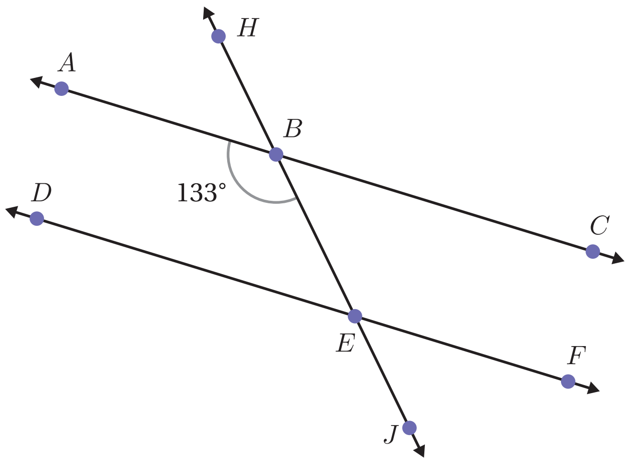
1. Determine *any three* angle measures that are not currently labeled.



2. Determine *any five* angle measures that are not currently labeled.

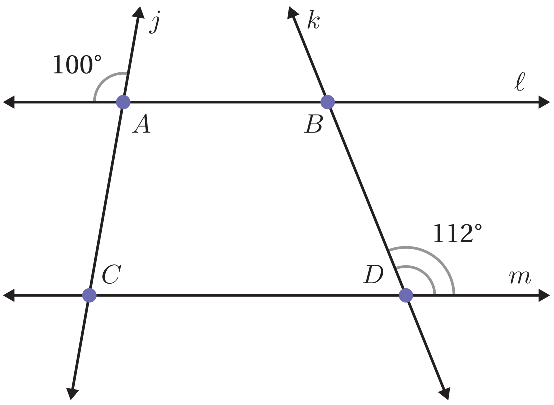


3. Determine *all* seven angle measures that are not currently labeled.



Activity 3 Double Transversals

Refer to the diagram. Lines ℓ and m are parallel. The lines are intersected by two transversals, j and k.

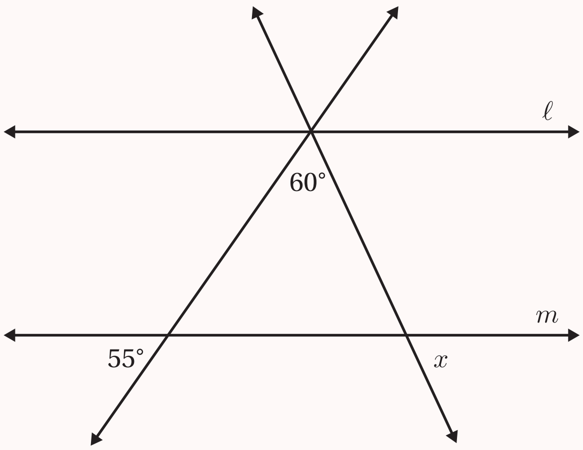


The figure may not be drawn to scale.

What is the sum of $m\angle CAB$, $m\angle ABD$, $m\angle BDC$, and $m\angle DCA$? Show or explain your thinking.

Are you ready for more?

Refer to the diagram. Parallel lines ℓ and m are intersected by two transversals that cross line l at the same point. Determine the measure of angle x.



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STOP

Name: ______ Date: ______ Period: ______

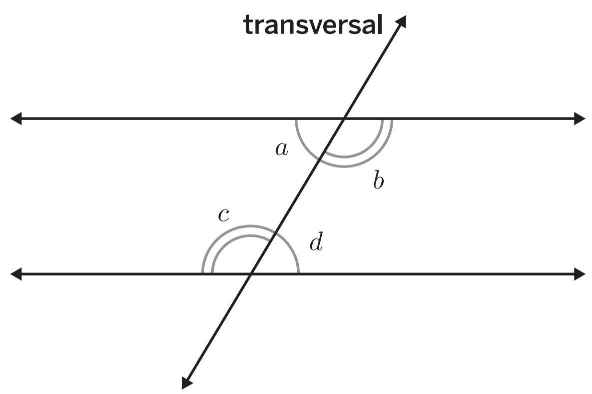
Summary

In today's lesson . . .

You explored the relationship between angles formed when two parallel lines are intersected by a transversal.

A ***transversal*** is a line that intersects two or more lines. ***Alternate interior angles*** are formed when a pair of parallel lines are intersected by a transversal. Interior angles are located inside the parallel lines and alternate angles are located on opposite sides of the transversal. So, alternate interior angles are located both inside the parallel lines and on opposite sides of the transversal. Alternate interior angles are congruent.

The diagram shows two pairs of alternate interior angles. Angles a and d are one pair of alternate interior angles, and are therefore congruent. Angles b and c are another pair of alternate interior angles, and are therefore congruent.

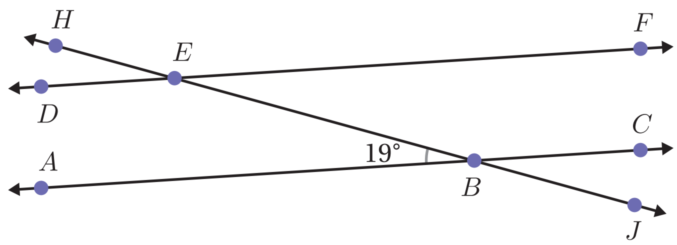


Reflect:

Practice

Name: .... Date: .... Period: ....

1. Line AC is parallel to line DF. The lines are intersected by transversal HJ.

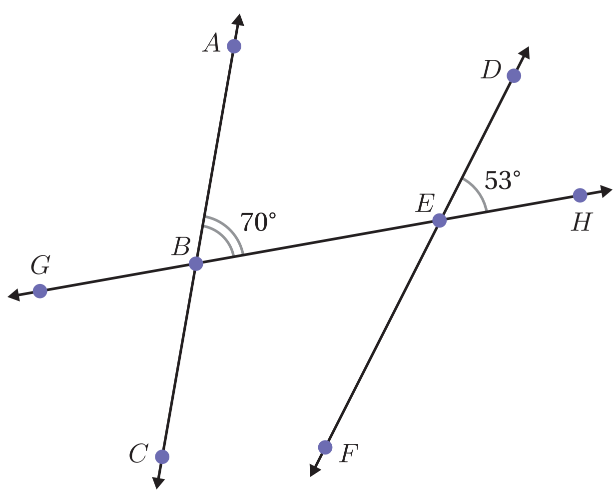


The figure may not be drawn to scale.

a What is $\mathrm{m}\angle FEB$? Explain your thinking.

b Explain why $\mathrm{m}\angle DEH$ has a measure of $19°$.

2. The diagram shows three lines with two given angle measures. Determine the measure of the six missing angles. Note that lines AC and DF are not parallel.

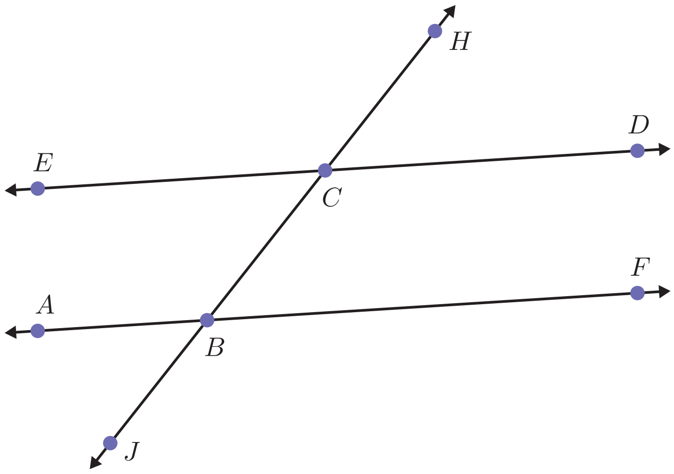


The figure may not be drawn to scale.

Name: .... Date: .... Period: ....

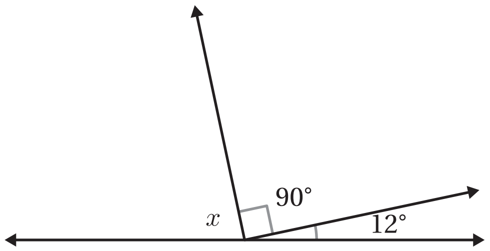
Practice

3. Line ED is parallel to AF. Lin claims that if she knows the measure of one angle, she is able to determine the measure of the remaining seven angles. Do you agree or disagree with Lin? Explain your thinking.



4. The point $(1, 3)$ is reflected across the x-axis, and then again across the y-axis. What are the coordinates of points of the final image?

5. Determine the measure of the missing angle.



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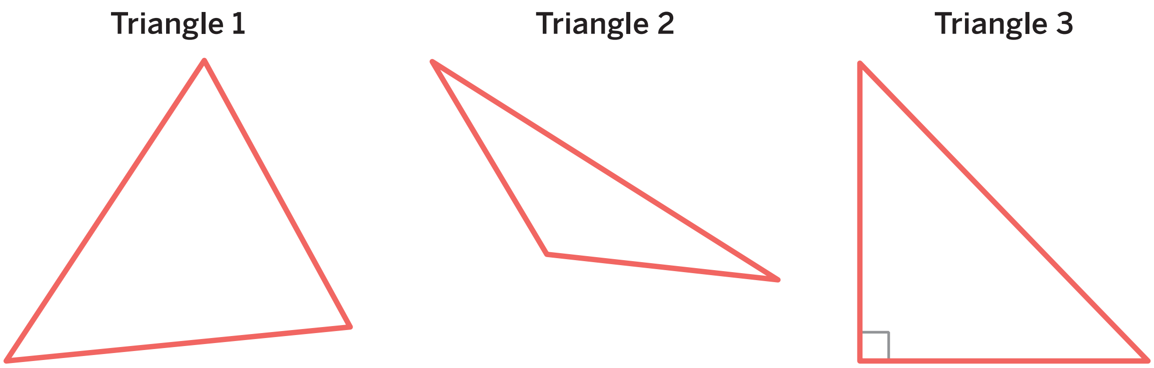
Unit 1 | Lesson 16

Adding the Angles in a Triangle

Let's explore the interior angles of triangles.

Warm-up Three Triangles

Refer to the three triangles shown.



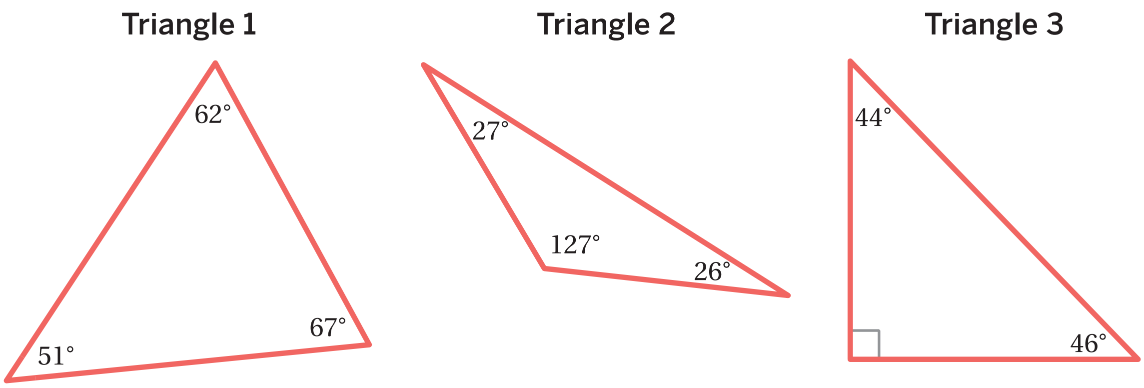
Which triangle do you think has the greatest sum of the three interior angle measures? Explain your thinking.

Log in to Amplify Math to complete this lesson online.

Name: ______________________ Date: __________ Period: __________

Activity 1 Find All Three

Refer to the same triangles from the Warm-up, now with their interior angle measures labeled. The figures may not be drawn to scale.



1. Determine the sum of the interior angle measures for each triangle.

   a. Triangle 1: b. Triangle 2: c. Triangle 3:

2. What do you notice about the sum of the interior angle measures?

3. Draw a different triangle. Make a prediction for the sum of the interior angle measures.

4. Measure the interior angles. Was your prediction correct?

Activity 2 Tear It Up

You will be given a set of two cards.

1. For Card A, complete the following tasks.

a Cut out the angles so that you have three separate angles.

b Can you create a triangle by placing the three angles together?

c Were the other members in your group able to make a triangle from the three angles they were given?

2. For Card B, complete the following tasks.

a Draw two line segments that each start from the given point so as to divide the straight angle into three angles. The angles do not have to be the same size. Try to create angles so that your partner *cannot* make a triangle from them.

b Label the interior of each angle with your initials. Then trade cards with your partner.

c Were you able to make a triangle using the three angles given to you by your partner?

d Were the other members in your group able to make a triangle from the three angles they were given?

3. What do you notice about the relationship between straight angles and the interior angles of a triangle?

Are you ready for more?

1. Draw a quadrilateral. Cut it out, tear off the angles, and place the angle so that they share a common vertex. What do you notice?

2. Repeat this for several more quadrilaterals. Make a conjecture about the angle measures.

Reflect: What constructive choices did you make about your behavior in order to complete this activity?

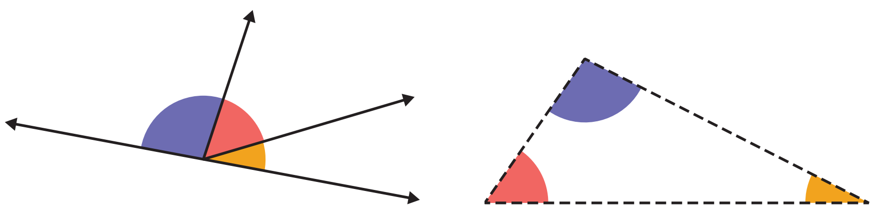
STOP

Name: ______ Date: ______ Period: ______

Summary

In today's lesson . . .

You investigated the interior angles of a triangle. You found that the sum of the angles inside the triangles you investigated in this lesson is $180°$. You may wonder if this relationship is true for *all* triangles and so, you will continue to explore this in the next lesson. You also found that any three angles that have a sum of $180°$ can be used to form a triangle.



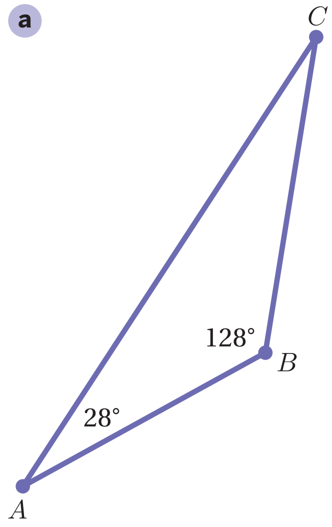
Reflect:

Practice

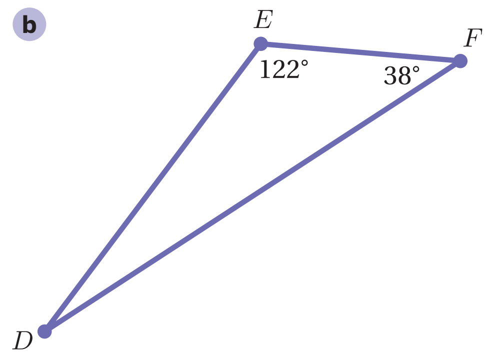
Name: ______________________ Date: ____________ Period: ____________

1. For each triangle, write a possible measure for the third angle. The figures may not be drawn to scale.

a



b



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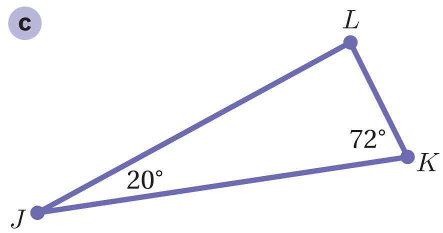
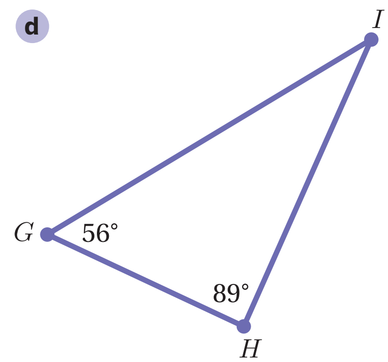


d



2. Which of the following sets of angles are possible for a triangle? Select *all* that apply.

A. $60^\circ, 60^\circ, 60^\circ$

B. $90^\circ, 90^\circ, 45^\circ$

C. $30^\circ, 40^\circ, 50^\circ$

D. $90^\circ, 45^\circ, 45^\circ$

E. $120^\circ, 30^\circ, 30^\circ$

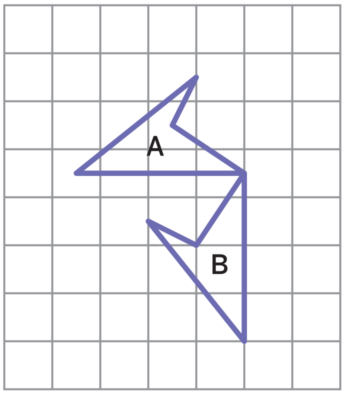
3. Can there be a triangle with two right angles? Explain your thinking.

Name: .... Date: .... Period: ....

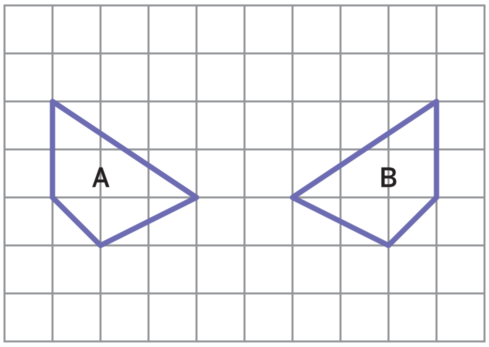
Practice

4. For each pair of polygons, describe the transformation that maps Polygon A onto Polygon B.

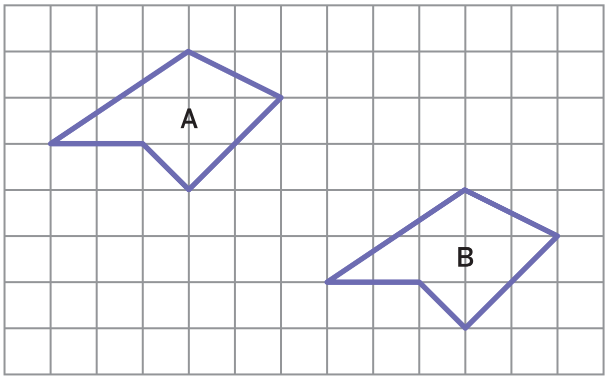
a



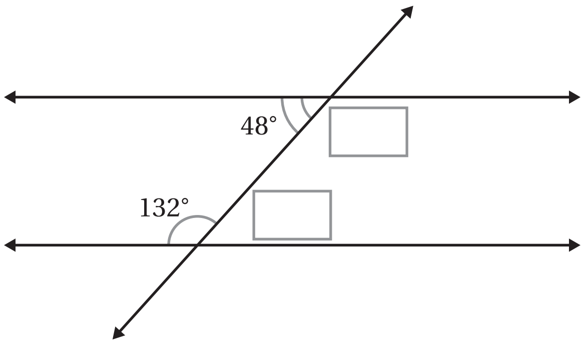
b



c



5. Refer to the figure, which shows two parallel lines intersected by a transversal. Determine the two missing angle measures indicated.



The figure may not be drawn to scale.

Unit 1 | Lesson 17

Parallel Lines and the Angles in a Triangle

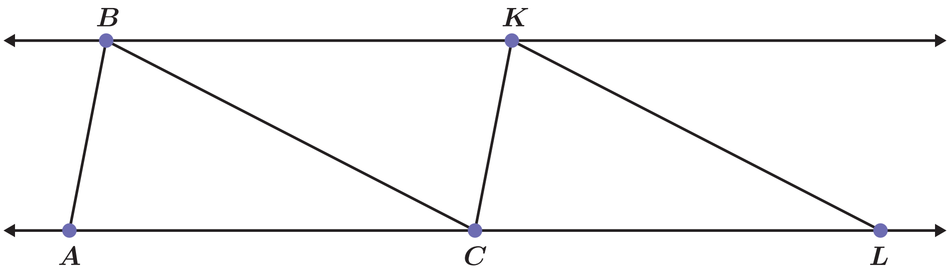
Let's investigate why the angles in a triangle add up to 180°.

Warm-up Matching Angles

You will need colored pencils for this Warm-up.

The figure shows two congruent triangles ABC and CKL placed between two parallel lines.

Shade *all* the congruent angles using different colors for each pair of congruent angles.

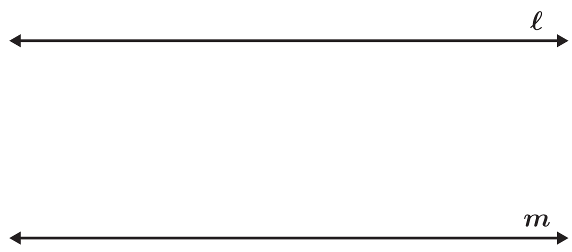


Log in to Amplify Math to complete this lesson online.

Name: ______________________ Date: ____________ Period: __________

Activity 1 Triangles and Parallel Lines

You will need a protractor. The figure shows two parallel lines, ℓ and m.



1. Draw two points on line m and one point on line ℓ. Connect the points to create a triangle. Measure and label the seven angles that are formed.

2. Compare your drawing with a partner. What patterns do you notice? List as many patterns as you can.

3. Explain how the figure demonstrates why the sum of the angle measures in *any* triangle is 180°.

Discussion Support: As your classmates share their observations, refer to the class display. Restate your classmates' ideas using the math language you are learning.

Are you ready for more?

Using a ruler, create at least three different quadrilaterals. Use a protractor to measure the four interior angles of each quadrilateral.

1. What is the sum of these four angle measures?

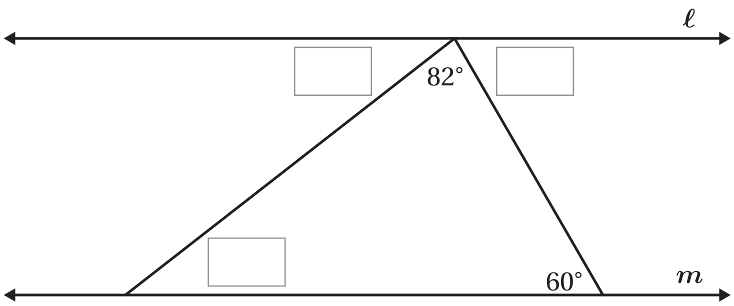
2. How can you use your knowledge about triangles to verify the sum of the angles in any quadrilateral?

Activity 2 Angle Puzzles

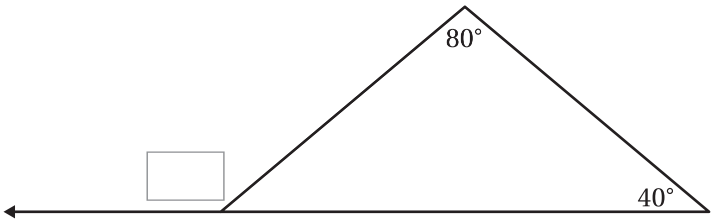
Determine the missing angle measures for each angle puzzle. The figures may not be drawn to scale.

Angle Puzzle 1:

Line ℓ is parallel to line m.



Angle Puzzle 2:

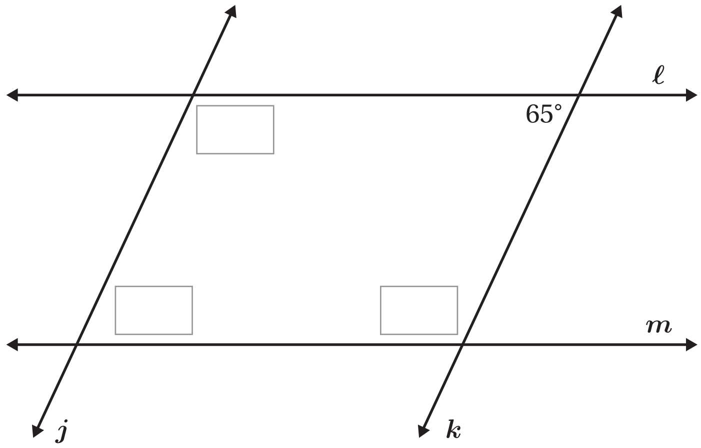


Name: ________ Date: ________ Period: ________

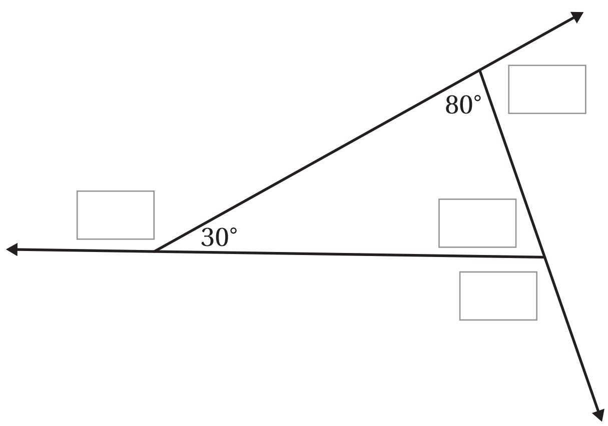
Activity 2 Angle Puzzles (continued)

Angle Puzzle 3:

Line ℓ is parallel to line m and line j is parallel to line k.



Angle Puzzle 4:



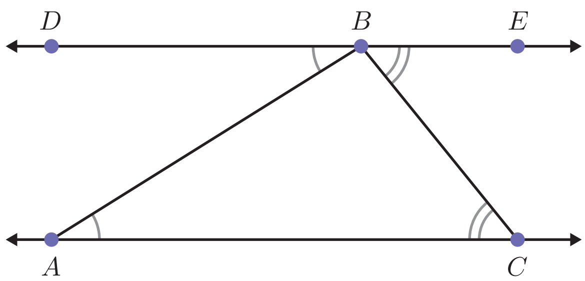
STOP

Summary

In today's lesson . . .

You applied what you learned about angle relationships, rigid transformations, and parallel lines to informally establish the ***Triangle Sum Theorem***. This theorem tells you that the sum of the three interior angles in *any* triangle is always $180°$.

Refer to parallel lines DE and AC. You know that $m\angle ABD = m\angle BAC$ and $m\angle ACB = m\angle CBE$ because the angles in each angle pair are alternate interior angles. You also know that angles $\angle ABD$, $\angle ABC$, and $\angle CBE$ form a straight angle, so their measures add up to $180°$. Therefore, the sum of the interior angles of any triangle is $180°$.

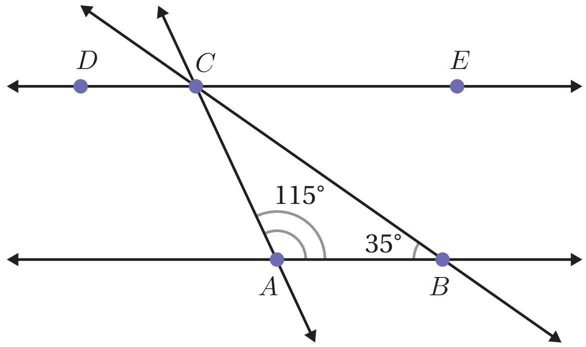


Reflect:

Name: ______ Date: ______ Period: ______

Practice

1. The diagram shows parallel lines AB and DE.



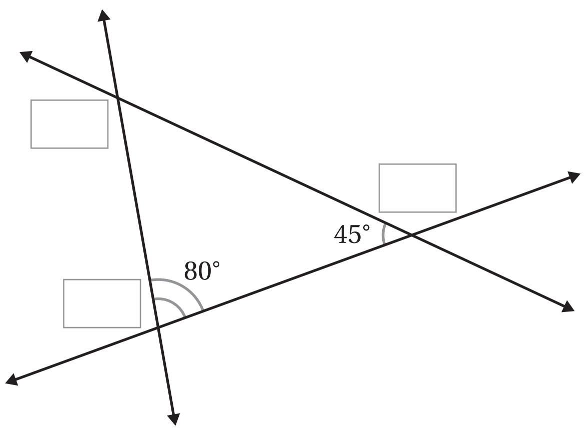
The figure may not be drawn to scale.

a What is $m\angle ACD$?

b What is $m\angle ECB$?

c What is $m\angle ACB$?

2. Three intersecting lines are shown.



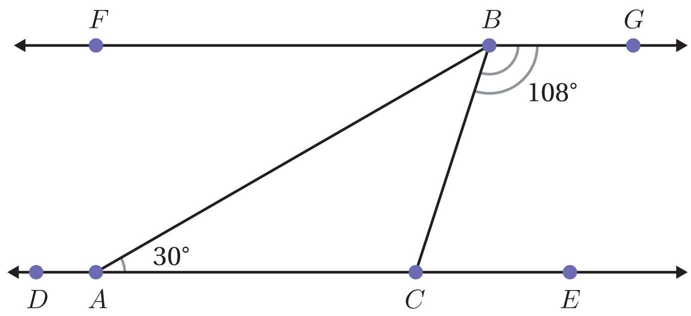
The figure may not be drawn to scale.

a Determine the three missing angle measures.

b What is the sum of these three angle measures?

Name: .... Date: .... Period: ....

3. Line DE is parallel to line FG. Is it possible to determine all five angle measures with the given information? Show or explain your thinking.

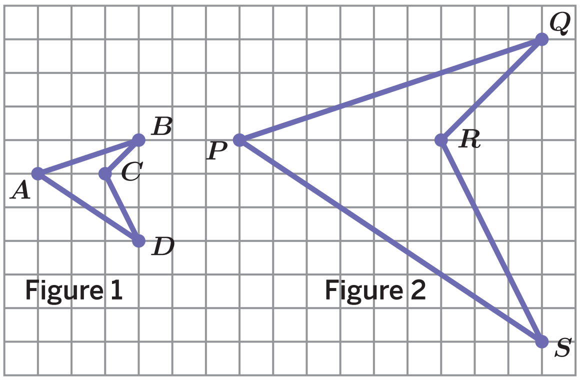


The figure may not be drawn to scale.

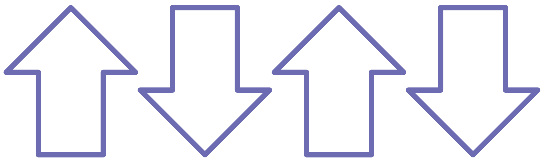
4. The two figures shown are scaled copies of each other.

   a. What scale factor is used to take Figure 1 to Figure 2?

   b. What scale factor is used to take Figure 2 to Figure 1?



5. Describe a transformation that maps the pattern shown onto itself. A protractor and a ruler may be useful here.



Name: ________ Date: ________ Period: ________

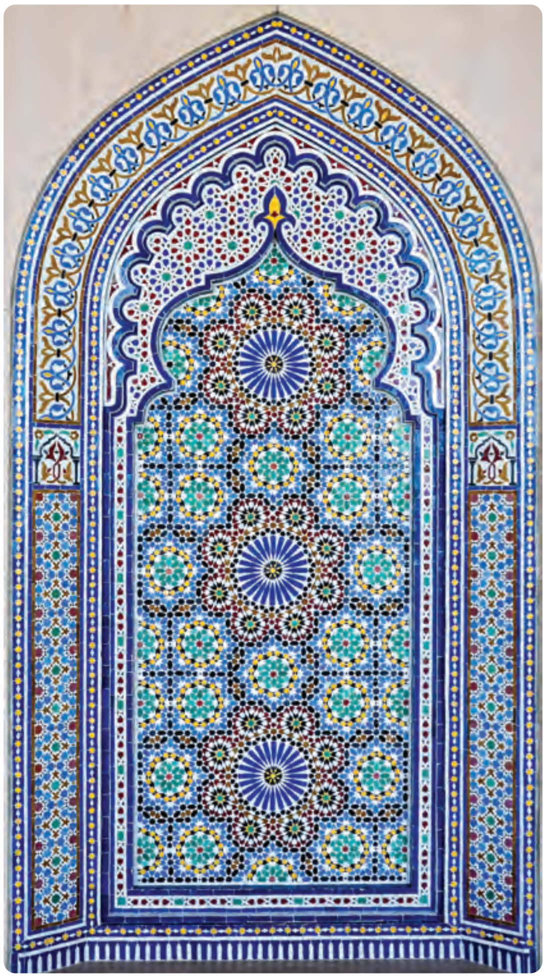
Unit 1 | Lesson 18 – Capstone

Creating a Border Pattern Using Transformations

Let's create borders using transformations.

Warm-up Notice and Wonder

Consider the image shown. What do you notice? What do you wonder?



Hussain Warraich/Shutterstock.com

1. I notice . . .

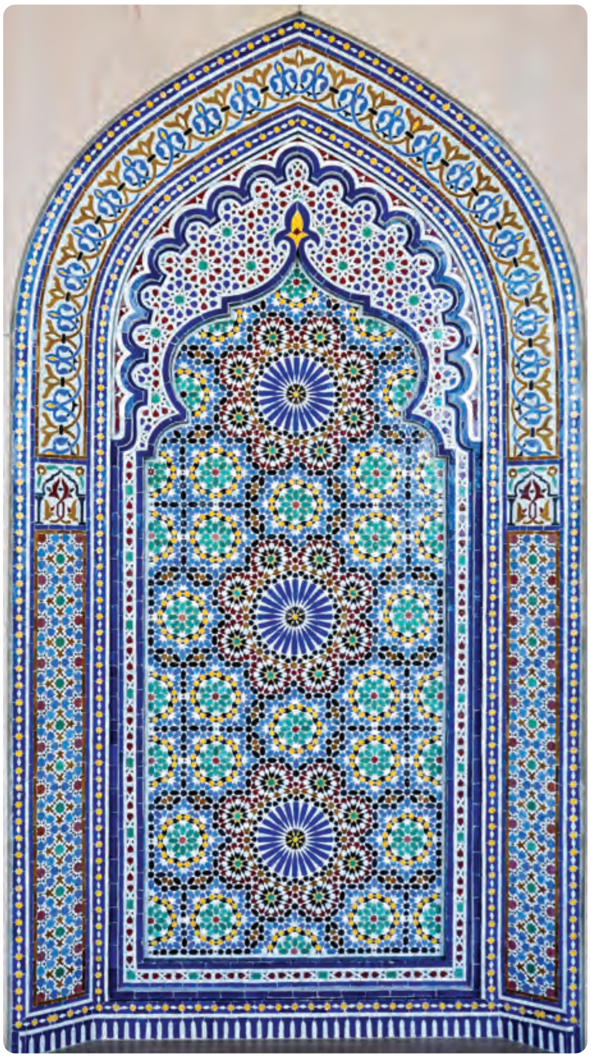
2. I wonder . . .

Log in to Amplify Math to complete this lesson online.

Activity 1 How Is It Made?

The pattern you saw in the Warm-up is from a wall in the Sultan Qaboos Grand Mosque in Muscat, Oman.

Find a single pattern or multiple patterns within the image that have rotation, translation, or reflection symmetry. Show or describe how each transformation is produced.



Hussain Warraich/Shutterstock.com

a rotation

b translation

c reflection

Name: ______ Date: ______ Period: ______

Activity 2 Designing a Border Pattern

In this activity, you will design your own border pattern. Border patterns have been studied by mathematicians, such as John Conway. You will be given a plain sheet of paper to use, starting with Problem 2.

1. Design a preimage for your border in this box.

2. Trace your preimage on the sheet of paper.

a. Apply a series of rigid transformations to create your border pattern. Draw a sketch of your border pattern here.

b. Trade borders with a partner. Describe a transformation of your partner's border pattern that maps the pattern onto itself. Write as many specific transformations as you can.

c. If there is time remaining in the activity, color your border pattern.

Featured Mathematician

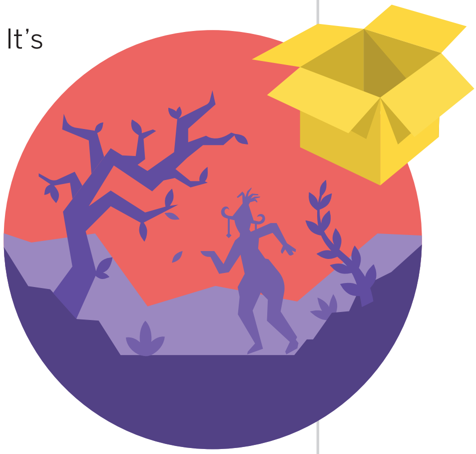
John Horton Conway

John Horton Conway was a British mathematician known for his playful attitude toward mathematics. Among his many contributions, he provided names for the seven groups of symmetric, infinitely long border (or "frieze") patterns: hop, step, sidle, spinning hop, spinning sidle, jump, and spinning jump. In 2020, Conway passed away due to complications from COVID-19.

STOP

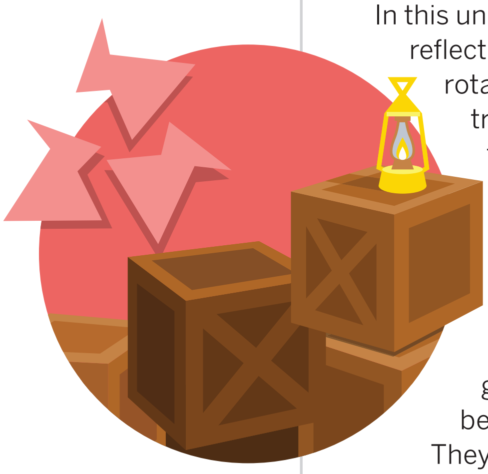
Narrative Connections

Unit Summary



Math isn't something that just sits on a page. It's not just a lifeless pile of numbers, diagrams, and figures. Math is flexible and dynamic. It has movement and energy. And nowhere is that movement more visible than in transformations.

Day-to-day, we use the word "transformation" to describe any dramatic change. Day transforms into night; flour transforms into a loaf of bread; and a much maligned duckling transforms into an ostentatious swan. In math, however, rigid transformations describe changes that don't affect the lengths or angles of a figure.



In this unit, you saw three different kinds of transformations: reflections—where a figure is flipped across a line of symmetry; rotations—where a figure turns around a point; and translations—where a figure is shifted in a direction. By themselves, they may not be that impressive, but when you combine them into a *sequence of transformations*, magic can happen.

These sequences allowed Lottie Reiniger to turn cardboard cutouts into living, breathing characters. They're what inspired Marjorie Rice to explore her groundbreaking tessellations. They govern the intricate beauty of Islamic tilework and Native American pottery. They're what gives the art of M.C. Escher its hypnotic and otherworldly qualities.

With transformations, and the rules that govern them, we can look at any shape and understand all the ways we might move it, without changing its inherent form.

See you in Unit 2.

Name: ____________ Date: ________ Period: ________

Practice

1. Refer to the trapezoid shown.

a. Use rigid transformations on the trapezoid to design a border pattern.

b. Describe the rigid transformations you used.

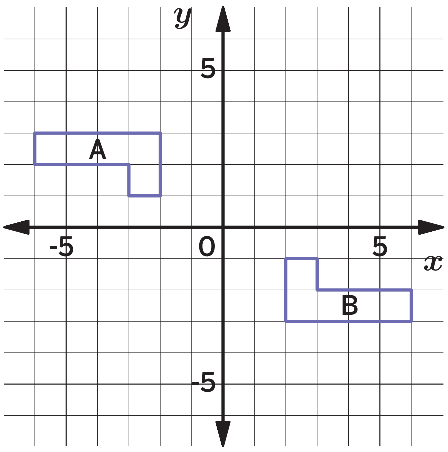
2. Refer to the national flag of Trinidad and Tobago. Describe a transformation that maps the triangle in the lower left corner onto the triangle in the upper right corner.

Public Domain

Practice

Name: ______ Date: ______ Period: ______

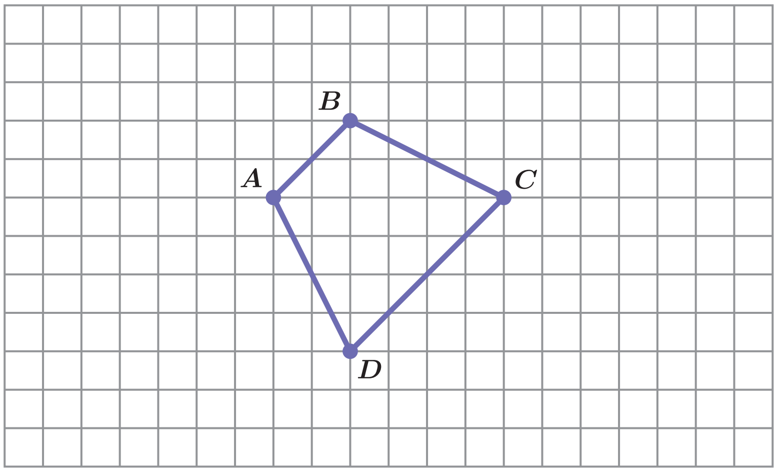
3. Polygon A is congruent to Polygon B. Describe a transformation or sequence of transformations that maps Polygon A onto Polygon B.



4. Write *all* of the possible combinations of three angle measures, from the following list, that can be the interior angle measures of a triangle.

60°	20°	100°	40°	110°	50°	30°

5. On the grid, draw a scaled copy of Quadrilateral $ABCD$, using a scale factor of $\frac{1}{2}$.



My Notes:

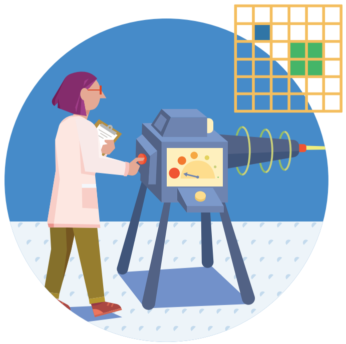
UNIT 2

Dilations and Similarity

The way our brain interprets how objects appear — how big or small they are, how near or far — comes back to dilation. Learn to dilate figures and uncover the magic of this special type of transformation.

Essential Questions

- What does it mean to dilate a figure?
- How can you identify whether two figures are similar?
- How can similar triangles be used to find the slope of a line?
- *(By the way, can you create an optical illusion that will trick your teacher's eyes?)*



SUB-UNIT

1 Dilations

Narrative: The pupils of your eyes dilate in response to light. But there is more to dilation than meets the eye.

You'll learn . . .
- how dilations are different from rigid transformations.
- how artists use dilations to create perspective drawing and illusions.

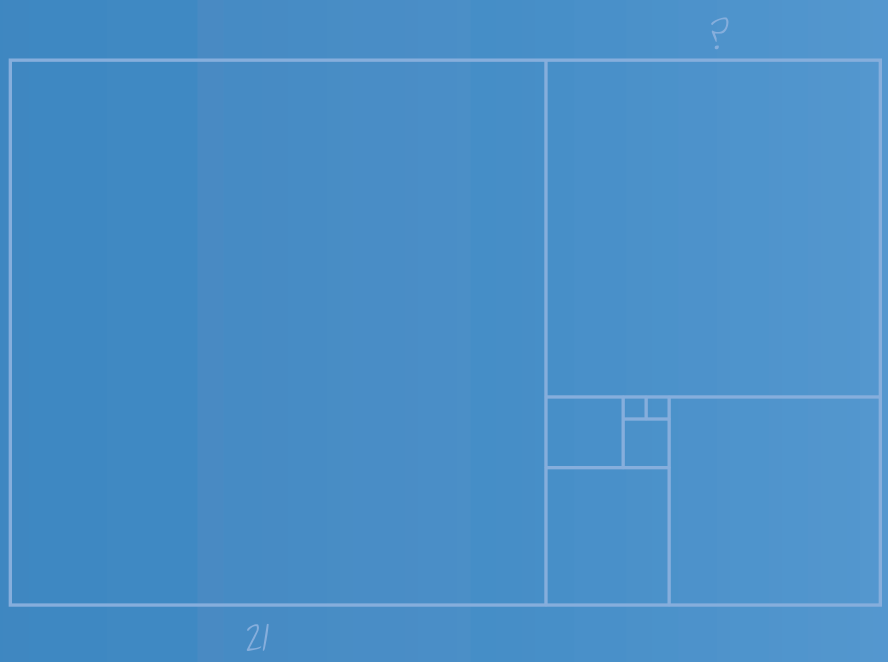
SUB-UNIT

2 Similarity

Narrative: Understanding similarity and proportional reasoning can help you combat *shrinkflation*.

You'll learn . . .
- how dilated figures are similar to each other.
- how similar right triangles help determine the slope of a line.

The following diagram is made of several squares. Find the missing side length, if the side length of the largest square is 21.



Projecting and Scaling

Let's explore scaling.

Warm-up Notice and Wonder

You will be given two sheets of paper. What do you notice? What do you wonder?

1. I notice . . .

2. I wonder . . .

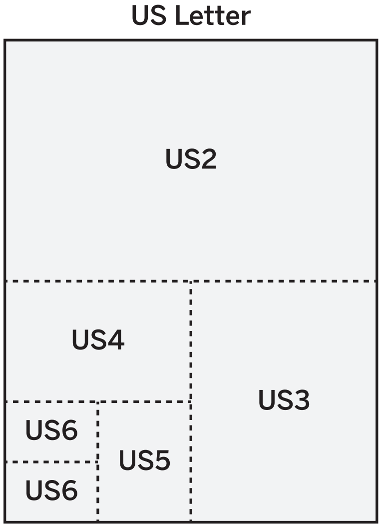
Log in to Amplify Math to complete this lesson online.

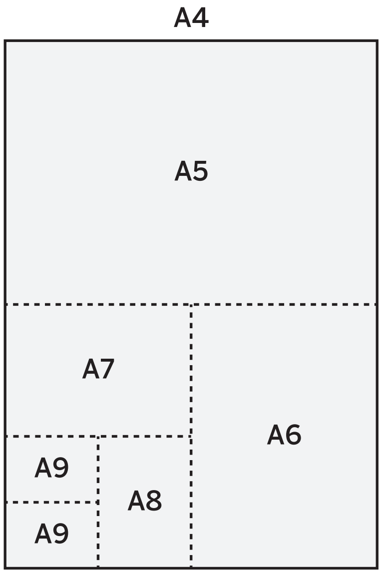
Name: ______ Date: ______ Period: ______

Activity 1 Sorting Rectangles

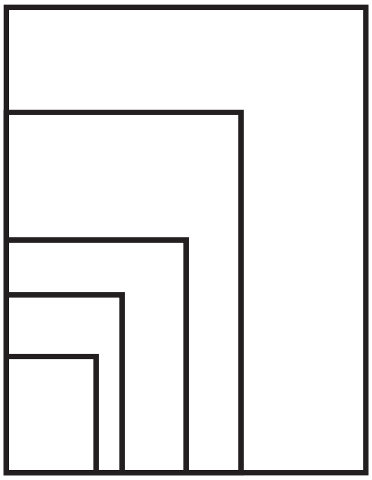
You will be given the materials for this activity.

1. Cut each sheet of paper in half. Then cut each sheet in half again. Continue cutting each sheet in half, as illustrated in the diagram. Label each rectangle as shown.





2. For each sheet of paper, stack the rectangles so that they all line up at a corner, as shown in the diagram. You will have two stacks of rectangles. What do you notice?



Activity 1 Sorting Rectangles (continued)

3. Record the dimensions, to the nearest millimeter, for each rectangle created from US Letter paper.

Rectangle	Length of short side (mm)	Length of long side (mm)	Ratio of long side to short side
Full sheet	216	280	1.3
US2			
US3			
US4			
US5			
US6			

4. Record the dimensions, in millimeters, for each rectangle created from A4 paper.

Rectangle	Length of short side (mm)	Length of long side (mm)	Ratio of long side to short side
Full sheet	210	297	1.4
A5			
A6			
A7			
A8			
A9			

5. Calculate each ratio of the long side to the short side. Write the ratios in the table, rounding to the nearest tenth.

6. What do you notice about the ratios for the rectangles?

Collect and Display: Your teacher will collect words and phrases you use. This language will be added to a class display for your reference.

STOP

Unit 2 Dilations and Similarity

More Than Meets the Eye

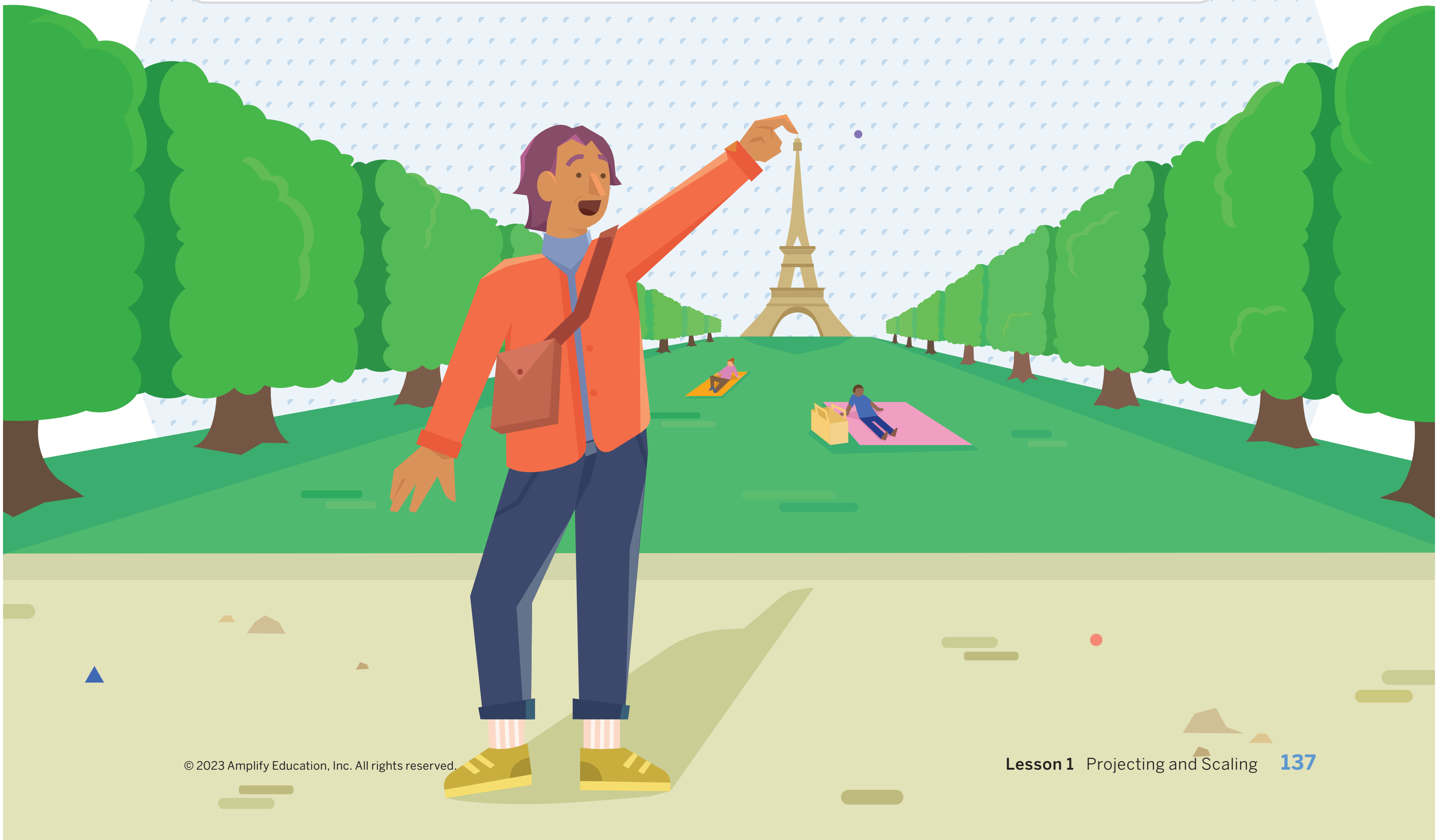
Be careful! Our eyes can play tricks on us when we least expect it. There are countless photos of tourists pinching the Eiffel Tower, or holding the Statue of Liberty in their palm. Although these monuments are huge, they can appear small thanks to an optical illusion called forced perspective. Without a frame of reference, our brain's ability to tell size and distance becomes confused. It tricks us into thinking things are bigger or smaller than they actually are.

The history of perspective in Western art begins with the Italian Renaissance. The architect Filippo Brunelleschi developed a system for showing objects in three-dimensional space. He realized that lines seemed to converge toward a single point on the horizon. Artists call this the vanishing point. Using this point, Brunelleschi could scale down a figure's size so that it appeared farther away.

This system revolutionized the art world, allowing artists to now depict figures realistically in space.

Whether it is a Renaissance painting or selfies by the Eiffel Tower, understanding how big or small something is comes down to understanding dilation.

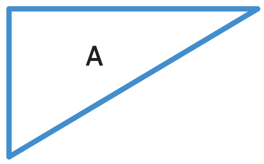
Welcome to Unit 2.



Practice

Name: .... Date: .... Period: ....

1. Refer to Polygon A.



a. Draw a scaled copy of Polygon A using a scale factor of $\frac{1}{4}$.

b. Draw a scaled copy of Polygon A using a scale factor of 2.

2. Triangle ABC is a scaled copy of Triangle DEF.
Side AB measures 12 cm and is the longest side of Triangle ABC.
Side DE measures 8 cm and is the longest side of Triangle DEF.

a. Triangle ABC is a scaled copy of Triangle DEF with what scale factor?

b. Triangle DEF is a scaled copy of Triangle ABC with what scale factor?

3. Rectangle A measures 12 cm by 3 cm. Rectangle B is a scaled copy of Rectangle A. Select *all* the measurement pairs that could be the dimensions of Rectangle B.

A. 6 cm by 1.5 cm

B. 10 cm by 1 cm

C. 18 cm by 4.5 cm

D. 6 cm by 1 cm

E. 80 cm by 20 cm

Name: ______ Date: ______ Period: ______

Practice

4. Which of these sets of angle measures could be the three interior angle measures of a triangle?

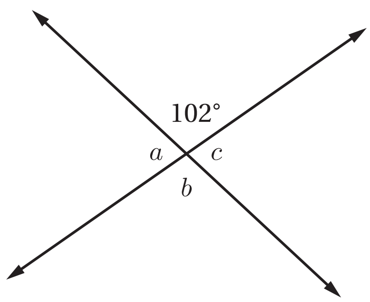
A. $40°, 50°, 60°$

B. $50°, 60°, 70°$

C. $60°, 70°, 80°$

D. $70°, 80°, 90°$

5. The diagram shows two intersecting lines. Determine the missing angle measures.

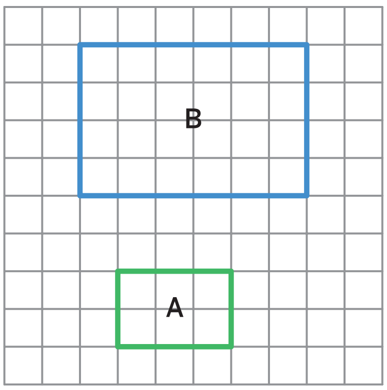


The figure may not be drawn to scale.

6. Refer to Rectangles A and B.

a What scale factor maps Rectangle A onto Rectangle B?

b What scale factor maps Rectangle B onto Rectangle A?



My Notes:

Would you put poison in your eye?

If you've ever had an eye exam, you've probably already heard the word *dilation*. The eyedrops the doctor dabs into your peepers are called "dilating drops." They cause your pupils — the black part within the iris — to widen, giving the doctor a chance to look inside and check the health of your eyeball.

The earliest dilating drops were made from a leafy shrub whose blossoms look like cherries. But keep these far away from your ice cream sundaes, since the plant is extremely toxic, causing paralysis or even death. Doctors used it for dilation precisely because it paralyzed the muscles in the iris, keeping the pupil from shrinking.

But it's not just eyes that can dilate. In these next few lessons, you'll learn how to dilate all kinds of figures. You'll discover the rules behind any dilation, and how it affects a figure's size. And the best part? No poison required!

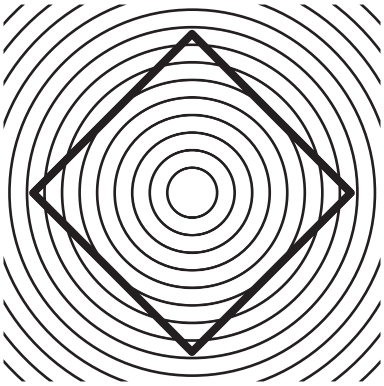
Unit 2 | Lesson 2

Circular Grids

Let's dilate figures on circular grids.

Warm-up Notice and Wonder

Consider the following optical illusion. What do you notice? What do you wonder?



1. I notice . . .

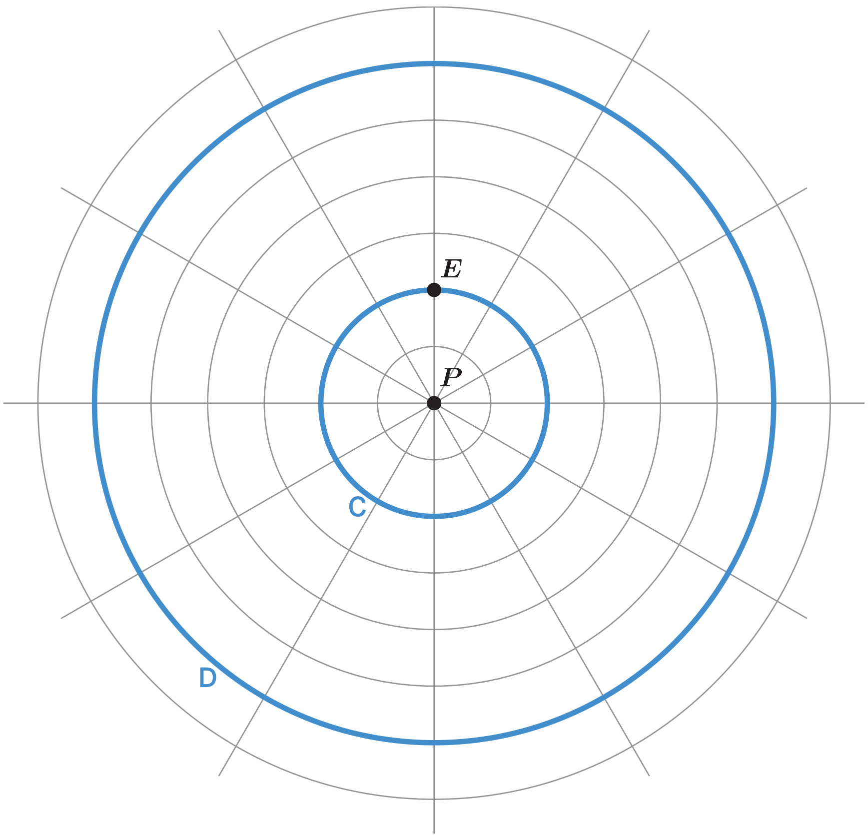
2. I wonder . . .

Log in to Amplify Math to complete this lesson online.

Name: ______ Date: ______ Period: ______

Activity 1 A Droplet on the Surface

In the circular grid, the distance from one circle to the next is the same. The radius of the innermost circle is 1 unit. The radius of each successive circle is 1 unit more than the radius of the previous circle. All the circles share the same center, point P. Circle C, Circle D, and point E are marked on the grid.



1. Plot two more points on Circle C. Label them F and G.

2. Draw rays from point P through the three points on Circle C. Extend the rays past Circle D.

3. Plot points where the ray intersects Circle D. Label the corresponding points E', F', and G'.

4. In the table, write the distance, in units, from point P to each point you drew.

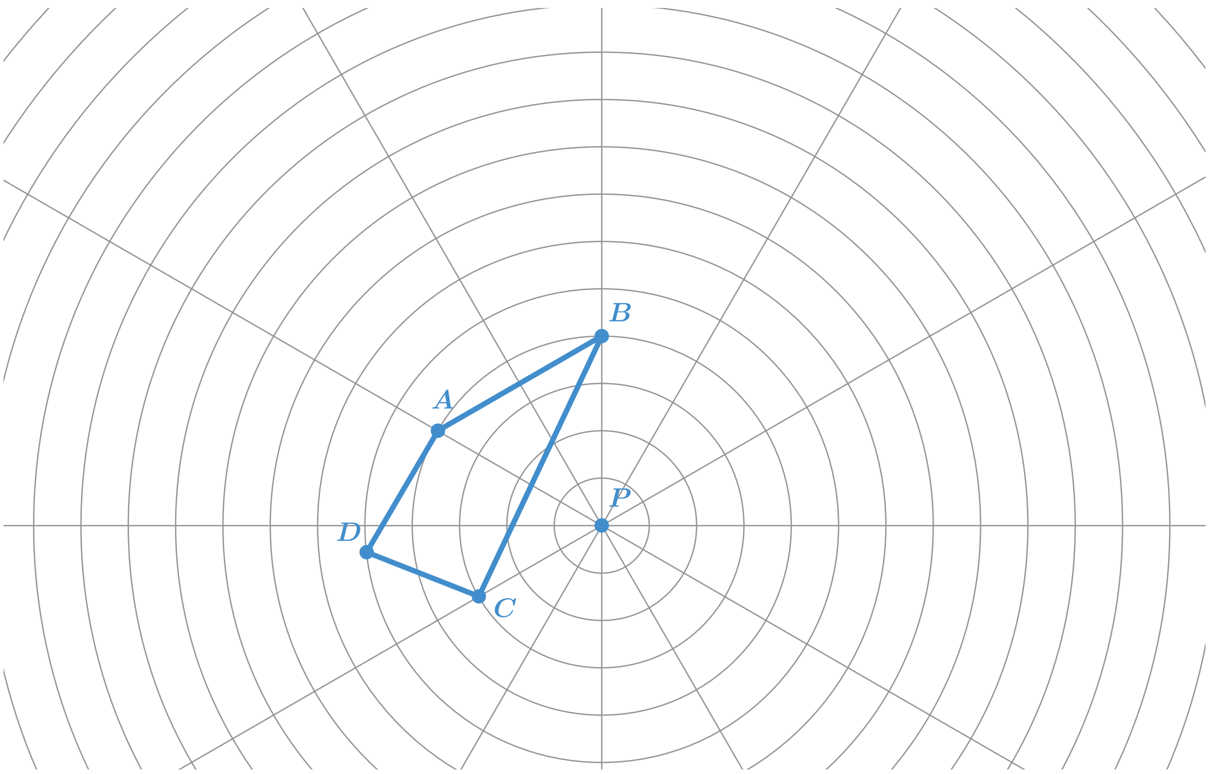
5. Find the ratio of the distances from the image of each point to the preimage point. What do you notice?

Distance from point P			
Point E		Point E'	
Point F		Point F'	
Point G		Point G'	

Activity 2 Partner Problems: A Quadrilateral on a Circular Grid

Consider Polygon $ABCD$. With your partner, decide who will complete Column A and who will complete Column B.

	Column A	Column B
>	**1.** Plot a point on any side of Polygon $ABCD$. Label the point E.	Plot a point on any side of Polygon $ABCD$. Label the point E.
>	**2.** Dilate points A, B, C, D, and E using point P as the center of dilation and a scale factor of 2.	Dilate points A, B, C, D, and E using point P as the center of dilation and a scale factor of $\frac{1}{2}$.
>	**3.** Draw segments between the dilated points to create a new polygon.	Draw segments between the dilated points to create a new polygon.
>	**4.** Measure the sides and angles of both polygons.	Measure the sides and angles of both polygons.



Name: .... Date: .... Period: ....

Activity 2 Partner Problems: A Quadrilateral on a Circular Grid (continued)

5. Compare your work with your partner. How does the dilation affect the polygon's size, side lengths, and angles? List as many observations as you can.

6. Andre says that to dilate Polygon $ABCD$, he can just dilate the vertices and connect them. Do you agree? Why or why not?

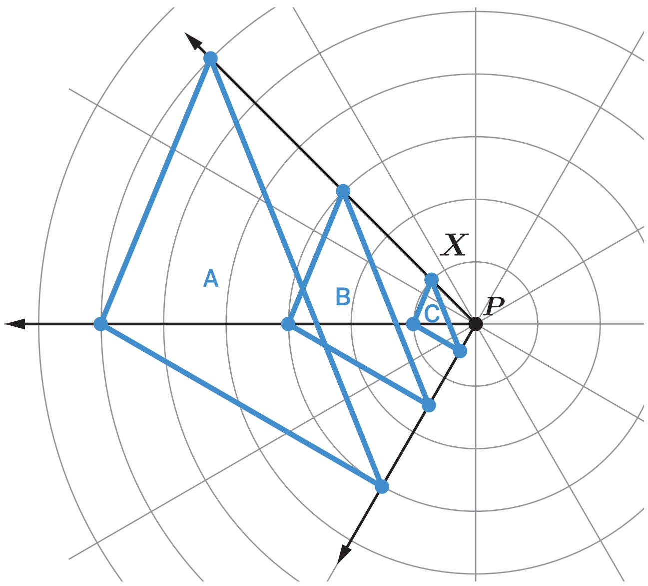
STOP

Summary

In today's lesson . . .

You performed dilations of a figure. A ***dilation*** is a transformation which is defined by a fixed point P, called the ***center of dilation***, and a scale factor k. In the figure shown, the dilation moves each point X into a point X' along ray PX such that its distance from a fixed point changes by the scale factor. A scale factor greater than 1 produces a larger scaled copy, and a scale factor less than 1 produces a smaller scaled copy.

- Triangle B is dilated using point P as the center of dilation and a scale factor of 2 to produce Triangle A.
- Triangle B is dilated using point P as the center of dilation and a scale factor of $\frac{1}{3}$ to produce Triangle C.

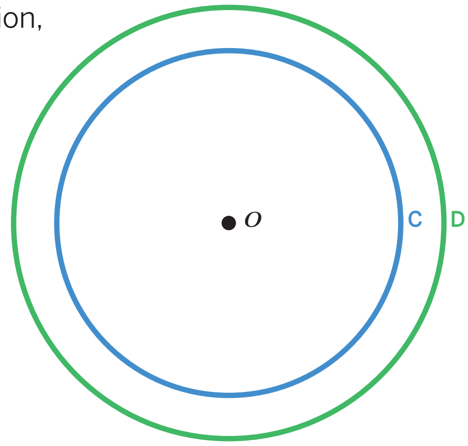


Reflect:

Name: ______ Date: ______ Period: ______

Practice

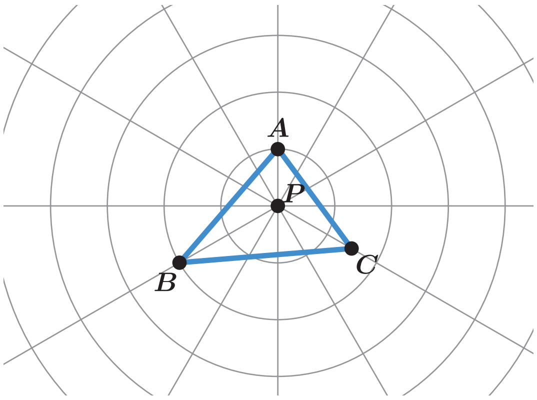
1. Refer to Circles C and D. Point O is the center of dilation, and the dilation maps Circle C onto Circle D.



a. Plot a point on Circle C and label it P. Plot point P', the image of P, after the dilation.

b. Plot a point on Circle D and label it Q'. Plot point Q so it will map onto point Q' after the dilation.

2. Refer to Triangle ABC.

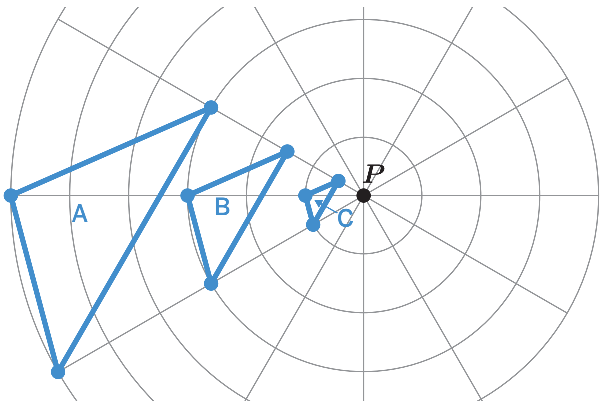


a. Dilate Triangle ABC using point P as the center of dilation and a scale factor of 2.

b. Dilate Triangle ABC using point P as the center of dilation and a scale factor of $\frac{1}{2}$.

c. How do the sides and angles of the two dilated triangles compare to each other? List as many observations as you can.

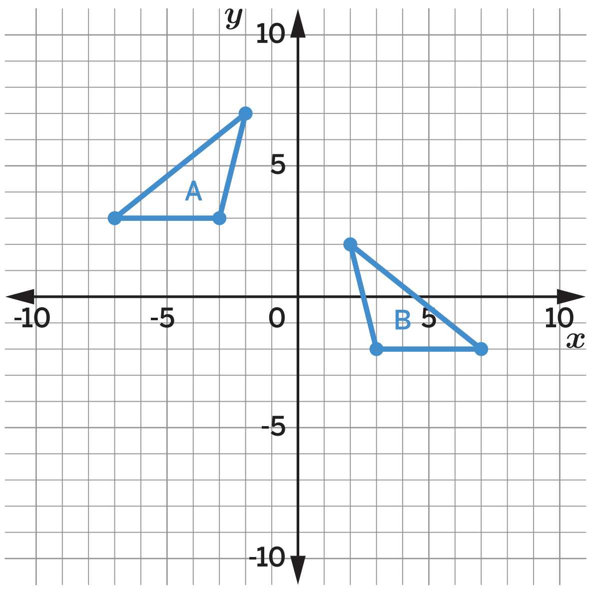
3. Triangles A, B, and C are scaled copies of each other. One of the triangles was dilated using point P as the center of dilation and a scale factor of 2. The same triangle was dilated using point P as the center of dilation and a scale factor of $\frac{1}{3}$. Which is the original triangle? Explain your thinking.



Practice

Name: .... Date: .... Period: ....

4. Describe a sequence of transformations that you could use to show that Triangles A and B are congruent.



5. Mai makes trail mix by combining 3 cups of raisins with 7 cups of oats.

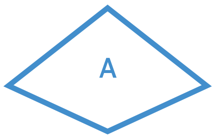
a How many cups of raisins should be added to 1 cup of oats?

Oats (cups)	Raisins (cups)
1	?
7	3

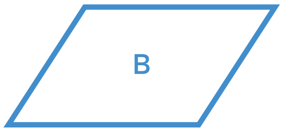
b What is the constant of proportionality?

6. Consider Polygons A and B.

a Draw a scaled copy of Polygon A using a scale factor of 2.



b Draw a scaled copy of Polygon B using a scale factor of $\frac{3}{2}$.



Name: ________ Date: ________ Period: ________

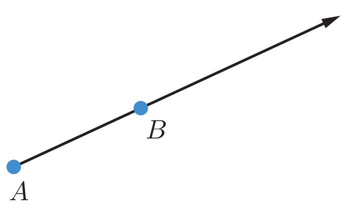
Unit 2 | Lesson 3

Dilations on a Plane

Let’s dilate figures without a grid.

Warm-up Dilating Along a Ray

Refer to the ray with points A and B plotted.



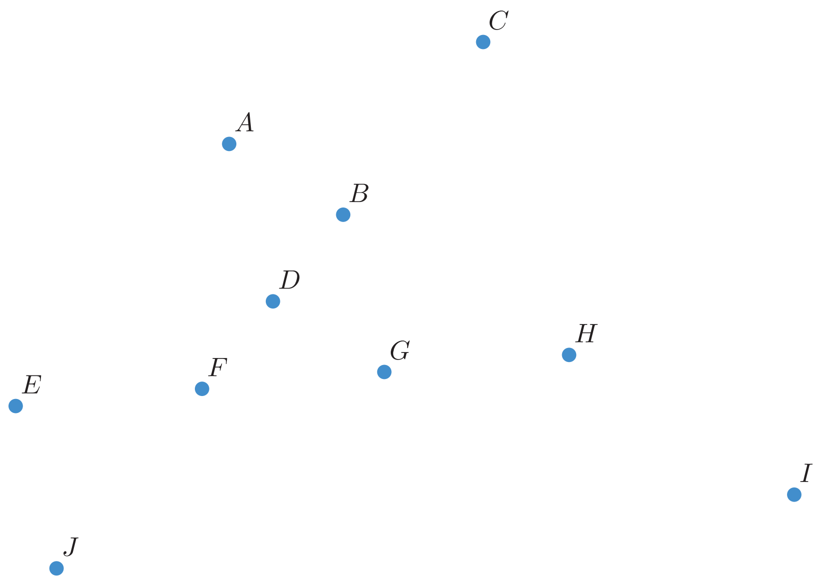
1. Find and label point C on the ray, whose distance from point A is twice the distance from point B to point A.

2. Find and label point D on the ray, whose distance from point A is half the distance from point B to point A.

Log in to Amplify Math to complete this lesson online.

Activity 1 Dilation Obstacle Course

Refer to the points A through J shown.



1. Dilate point B using point A as the center of dilation and a scale factor of 5. Which point is its image?

2. Dilate point G using point H as the center of dilation, so that its image is point E. What scale factor did you use?

3. Dilate point E using point H as the center of dilation, so that its image is point G. What scale factor did you use?

4. Using point B as the center of dilation, dilate point H so that its image is itself. What scale factor did you use?

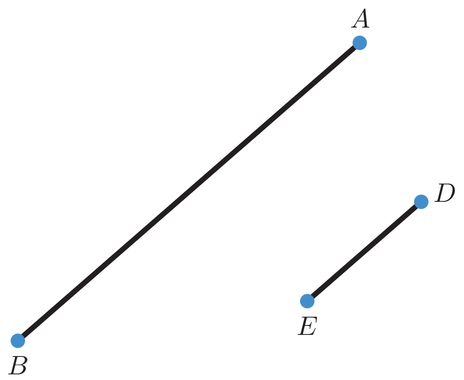
Are you ready for more?

Tyler and Diego want to find a center of dilation in order to dilate point F so that its image is point B. Tyler thinks the center is point J, while Diego thinks it is point C. Who is correct? Explain your thinking.

Name: .... Date: .... Period: ....

Activity 2 Dilating a Line Segment

Mai dilated line segment AB to create the image, segment DE, but erased her center of dilation.



1. Use a ruler to find and draw Mai's center of dilation. Label it point C.

2. What is the scale factor of the dilation? Explain or show your thinking.

3. Choose a point on segment AB and label it point F. Find the precise location of point F', the image of the dilation of point F. Explain or show your thinking.

4. How would the scale factor change if segment DE is the preimage and segment AB is the result of the dilation?

Activity 3 Perspective Drawing

A perspective drawing is an optical illusion that allows for an image printed or drawn on two-dimensional paper to have a three-dimensional look. You will use dilations to create a perspective drawing.

1. In the space provided at the bottom of this page, draw a polygon.
2. Choose a point outside the polygon to use as the center of dilation. Label it point C.
3. Using your center as point C and a scale factor of your choosing, dilate the polygon. Record the scale factor you use.
4. Draw a segment that connects each of the original vertices with its image. This will allow your diagram to look like a three-dimensional drawing! If time allows, you can shade the sides to make it look more realistic.
5. Compare your drawing with the drawings of other students. Talk about these questions.
   - What is the same, and what is different?
   - How do the choices you made affect the final drawing?
   - Was your dilated polygon closer to point C than to the original polygon, or farther away? How do you decide this?

Reflect: What good choices did you make about your personal behavior today?

STOP

Name: ______________________ Date: __________ Period: __________

Summary

In today's lesson . . .

You performed dilations on a plane using a center of dilation and a scale factor. You explored how to find the center and scale factor given a preimage and its dilated image.

You can determine the scale by finding the ratio of the distance between the image and the center of dilation, and the distance between the preimage and the center. Scale factors that are greater than 1 create images that are farther away from the center of dilation, while scale factors less than 1 create images that are closer to the center of dilation.

Reflect:

Practice

Name: .... Date: .... Period: ....

1. Segment AB measures 3 cm. Point O is the center of dilation. How long is the image of AB after a dilation with:

a A scale factor of 5?

b A scale factor of 3.7?

c A scale factor of $\frac{1}{5}$?

d A scale factor of s?

2. Refer to points A and B. Plot the points for each dilation described.

a Point C is the image of point B using point A as the center and a scale factor of 2.

b Point D is the image of point A using point B as the center and a scale factor of 2.

c Point E is the image of point B using point A as the center and a scale factor of $\frac{1}{2}$.

d Point F is the image of point A using point B as the center and a scale factor of $\frac{1}{2}$.

3. Rectangle A has a length of 12 in. and a width of 8 in. Rectangle B has a length of 15 in. and a width of 10 in. Rectangle C has a length of 30 in. and a width of 15 in.

a Is Rectangle A a scaled copy of Rectangle B? If so, what is the scale factor? If not, why not?

b Is Rectangle B a scaled copy of Rectangle A? If so, what is the scale factor? If not, why not?

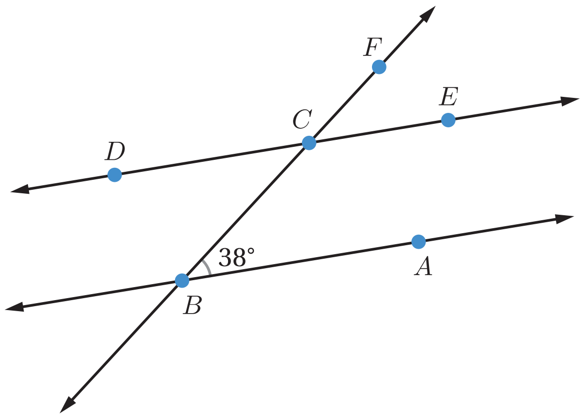
c Explain how you know that Rectangle C is *not* a scaled copy of Rectangle B.

d Is Rectangle A a scaled copy of Rectangle C? If so, what is the scale factor? If not, why not?

Name: .... Date: .... Period: ....

Practice

4. Consider parallel lines AB and DE and transversal BC. Calculate the measure of each of the indicated angles. Explain your thinking.



The figure may not be drawn to scale.

a $\text{m}\angle BCD$

b $\text{m}\angle ECF$

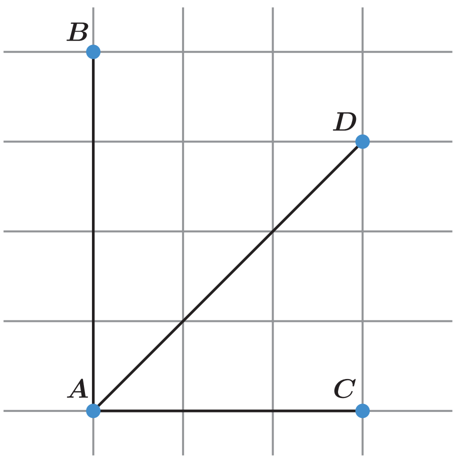
c $\text{m}\angle DCF$

5. Consider segments AB, AD, and AC.

a Plot the point in the middle of segment AB and label it point J.

b Plot the point in the middle of segment AC and label it point K.

c Plot the point in the middle of segment AD and label it point L.

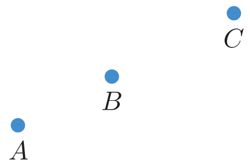


Dilations on a Square Grid

Let's dilate figures on a square grid.

Warm-up Estimating a Scale Factor

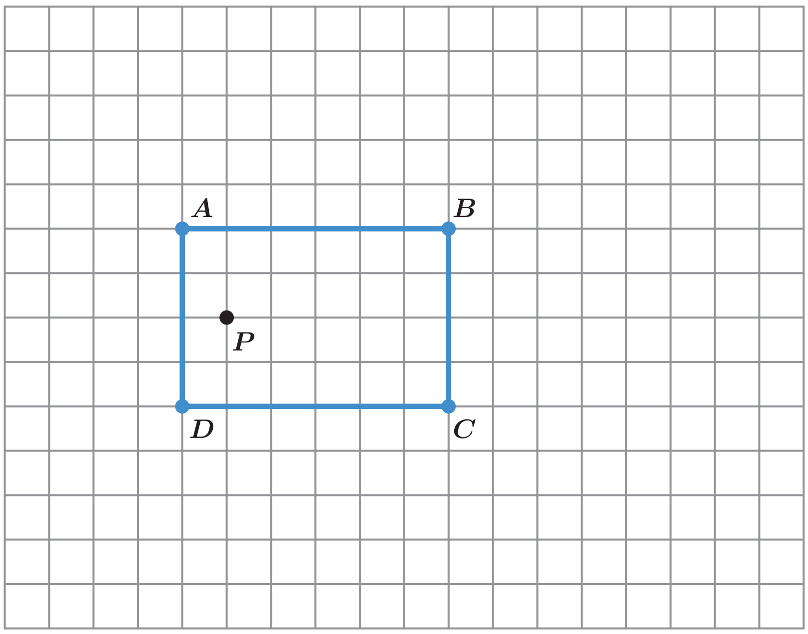
Point C is the dilation of point B, with point A as the center of dilation. Estimate the scale factor. Explain your thinking.



Name: ________ Date: ________ Period: ________

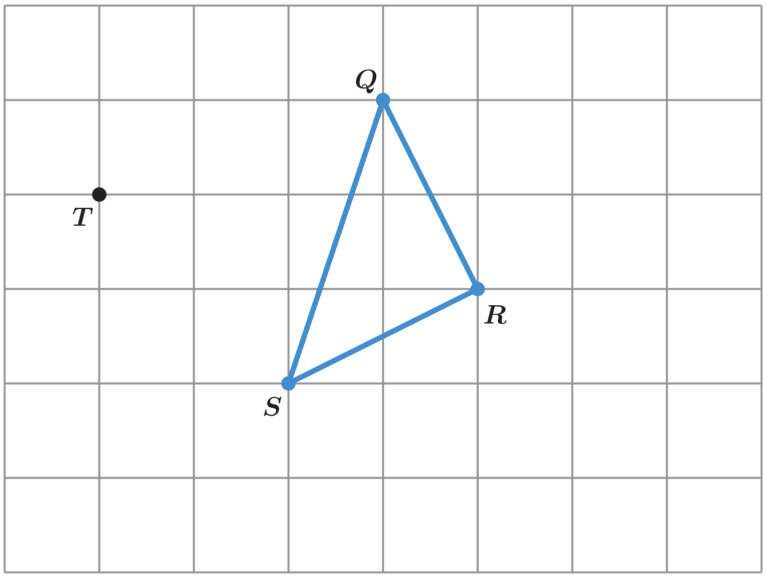
Activity 1 Dilations on a Grid

1. Dilate Rectangle $ABCD$ by a scale factor of 2 with point P as the center of dilation. Label the corresponding vertices in the image as A', B', C', and D'.



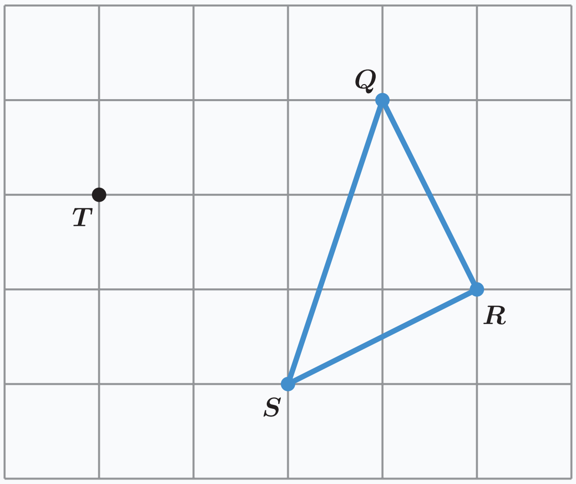
Activity 1 Dilations on a Grid (continued)

2. Dilate Triangle QRS by a scale factor of $\frac{1}{2}$ with point T as the center of dilation.



Are you ready for more?

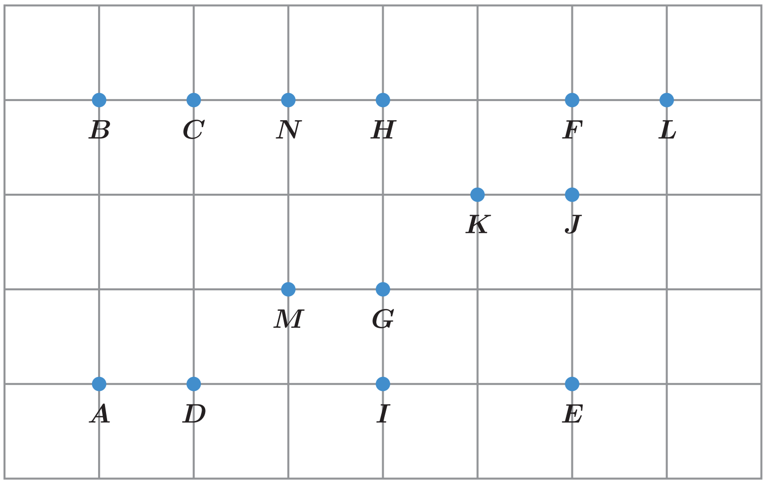
Dilate Triangle QRS by a scale factor of $\frac{1}{4}$ with point T as the center of dilation.



Name: ________________ Date: ________ Period: ________

Activity 2 Dilation Obstacle Course . . . on a Grid!

Refer to points A through L shown.



1. Using point I as the center of dilation, dilate point G so that its image is point H. What scale factor did you use?

2. Suppose point F is an image of point N after a dilation. Compare the scale factors with point B and point C as the centers of dilation.

3. To dilate point K so that its image is point A, what point could be the center of dilation, and what would be the scale factor?

4. Points D, E, and J can be used to form Triangle DEJ. Points D, I, and G can be used to form Triangle DIG. Identify the center of dilation and the scale factor that map Triangle DEJ onto Triangle DIG.

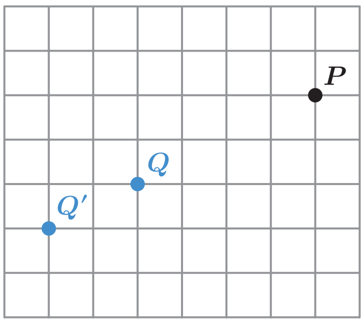
STOP

Summary

In today's lesson . . .

You explored how square grids can be useful for showing dilations. The grid is helpful, especially when the center of dilation and the point(s) being dilated are marked at the intersections of grid squares. Rather than using a ruler to measure the distance between the points, you can count grid squares.

For example, suppose you want to dilate point Q with a scale factor of $\frac{3}{2}$ and point P as the center of dilation.



Because point Q is 4 grid squares to the left and 2 grid squares down from point P, the dilated image will be 6 grid squares to the left and 3 grid squares down from point P. Can you see why?

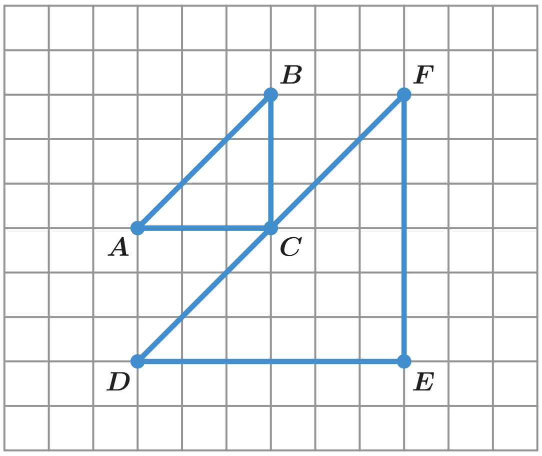
The dilated image is marked as point Q' in the grid shown.

Reflect:

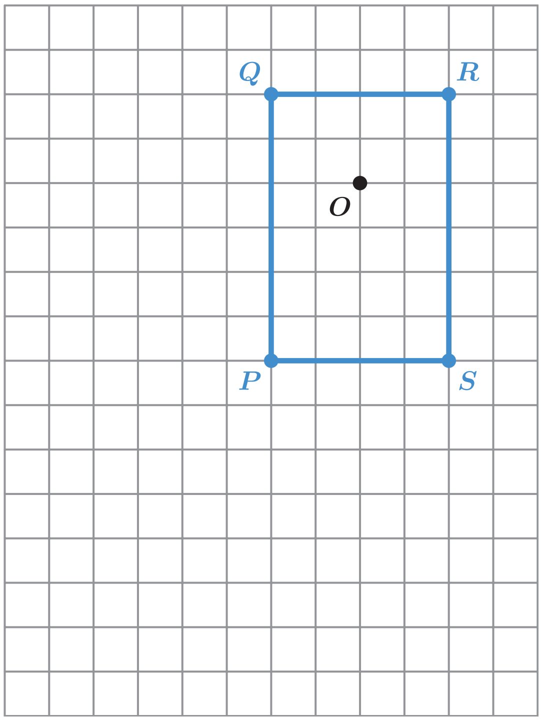
Name: ________________ Date: ________ Period: ________

Practice

1. Triangle ABC can be mapped to Triangle DFE using a dilation. What are the center and the scale factor of the dilation? Label the center as point P.



2. Consider Rectangle $PQRS$. Sketch the image of Quadrilateral $PQRS$ under the following dilations:

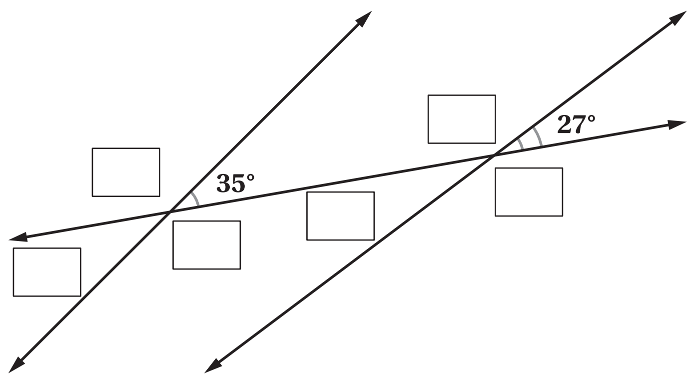


a. The dilation centered at point R with a scale factor of 2.

b. The dilation centered at point O with a scale factor of $\frac{1}{2}$.

Name: .... Date: .... Period: ....

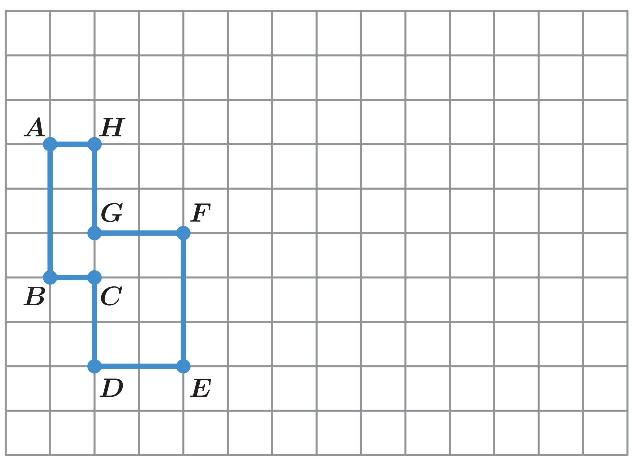
3. The diagram shows three lines, along with some angle measures. Determine the missing angle measures.



The figure may not be drawn to scale.

4. Construct an image of the polygon after performing the following sequence of transformations:

- Translate the polygon 5 units to the right and 1 unit down.
- Rotate the result $90°$ counterclockwise about point A'.



5. Point B has coordinates $(-2, -5)$. After a translation 4 units down, a reflection across the y-axis, and a translation 6 units up, what are the coordinates of the image?

Name: ______ Date: ______ Period: ______

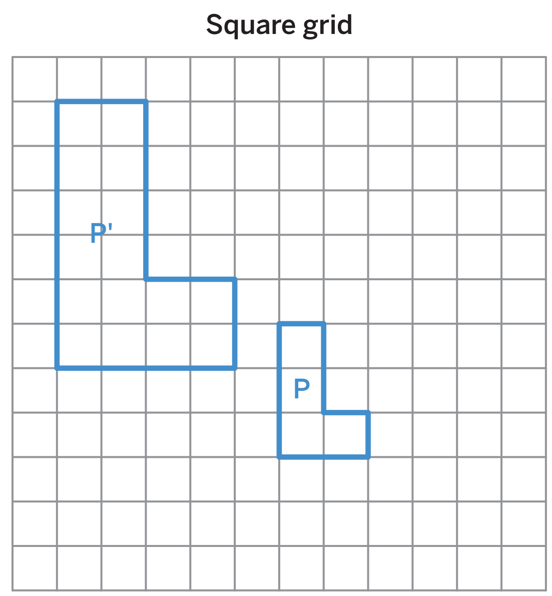
Unit 2 | Lesson 5

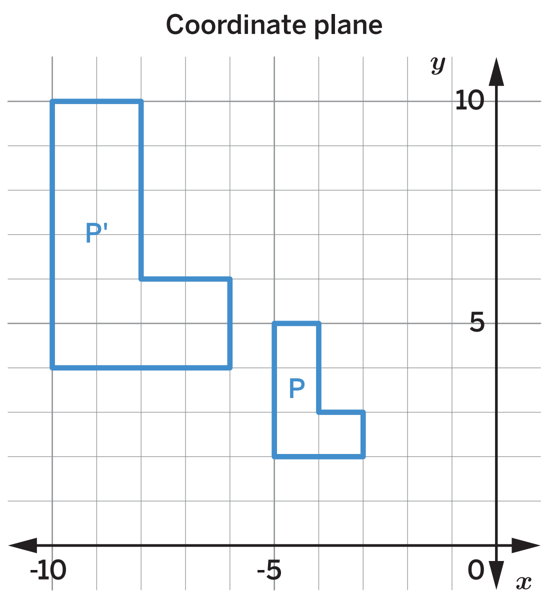
Dilations With Coordinates

Let's look at dilations on the coordinate plane.

Warm-up Describing Dilation

Consider the dilation of Polygon P on the square grid shown, and on the coordinate plane shown. Which one would you choose if you were asked to describe the dilation? Explain your thinking.





Log in to Amplify Math to complete this lesson online.

Activity 1 Info Gap: Make My Dilation

Plan ahead: How will you show that you are listening to your partner? When might you need to use clarifying questions?

You will be given either a *problem card* or a *data card*. Do not show or read your card to your partner.

If you are given a *problem card*:	If you are given a *data card*:
1. Silently read your card, and think about what information you need to be able to solve the problem.	**1.** Silently read your card.
2. Ask your partner for the specific information that you need.	**2.** Ask your partner, "*What specific information do you need?*" and wait for them to ask for information.
3. Explain how you are using the information to solve the problem. Continue to ask questions until you have enough information to solve the problem.	**3.** Before sharing the information, ask, "*Why do you need that information?*" Listen to your partner's reasoning, and ask clarifying questions.
4. Share the problem card and solve the problem independently.	**4.** Read the problem card, and solve the problem independently.
5. Read the data card and discuss your thinking.	**5.** Share the data card and discuss your thinking.

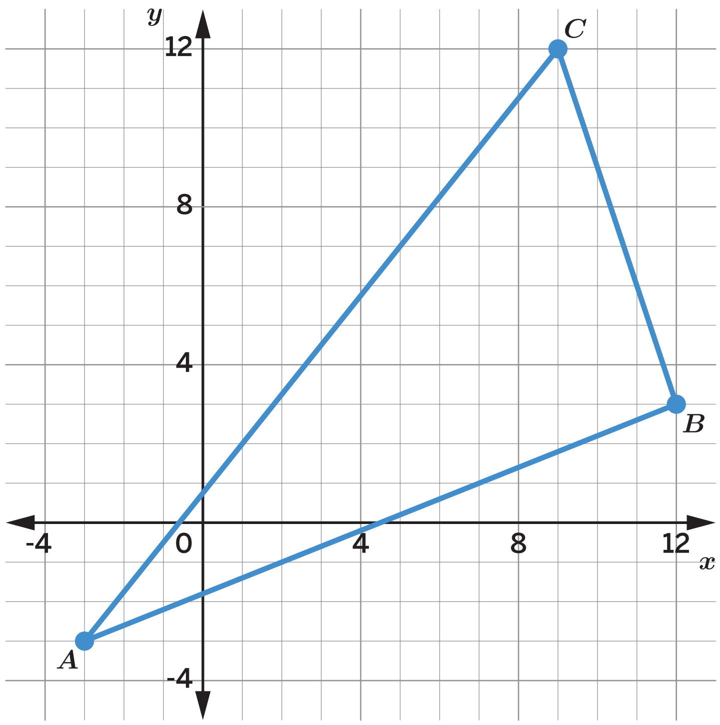
Pause here so your teacher can review your work. You will be given a new set of cards. Repeat the activity, trading roles with your partner.

Name: ______ Date: ______ Period: ______

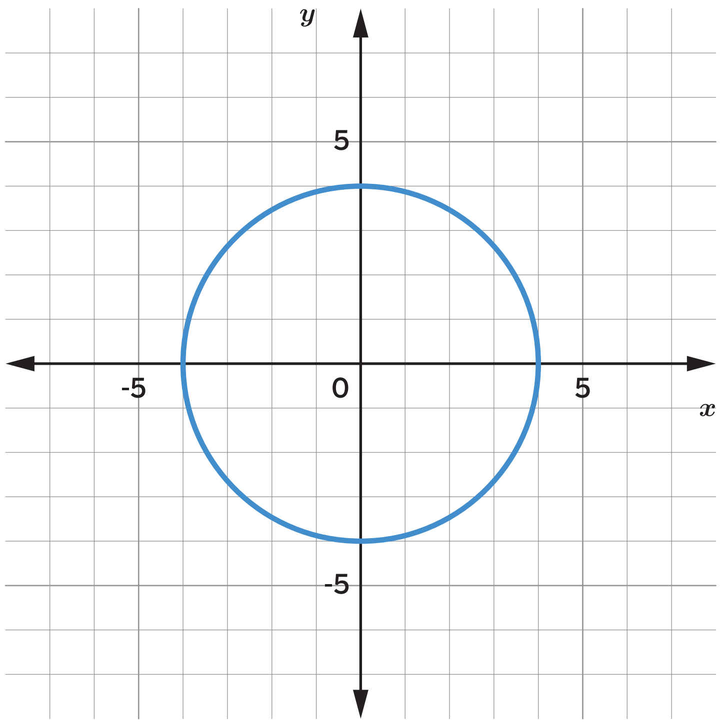
Activity 2 Dilate It!

You will be given access to your geometry toolkit. Choose tools from your geometry toolkit to perform the indicated dilations.

1. Dilate Triangle ABC using the origin as the center of dilation and a scale factor of $\frac{1}{3}$.



2. Dilate the circle shown using the origin as the center of dilation and a scale factor of $\frac{3}{2}$.



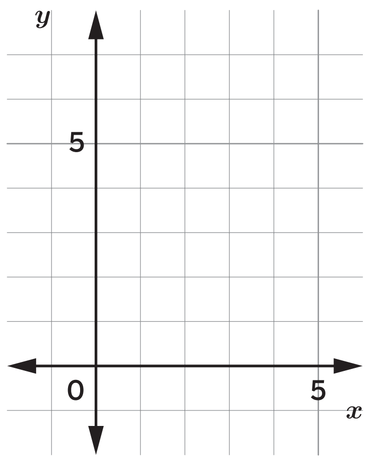
Activity 2 Dilate It! (continued)

3. Triangle ABC has vertices located at $A(0, 1)$, $B(2, 1)$, and $C(2, 3)$. Triangle $A'B'C'$ is the result of dilating Triangle ABC using the origin as the center of dilation and a scale factor of 2.

Preimage		Image	
Point A	$(0, 1)$	Point A'	
Point B	$(2, 1)$	Point B'	
Point C	$(2, 3)$	Point C'	

a Predict the coordinates of Triangle $A'B'C'$ and record them in the table.

b Draw Triangle ABC and its image on the coordinate plane. Use the images to verify whether your predictions were correct. Explain your thinking.

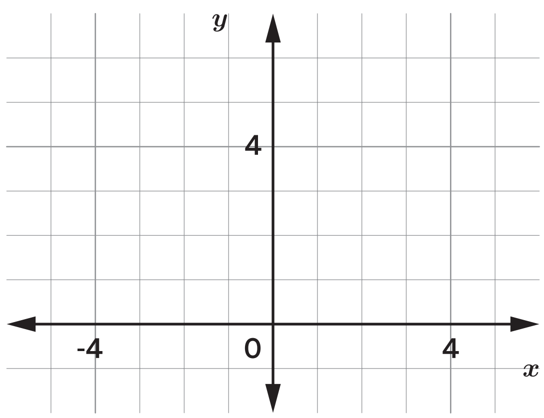


4. Parallelogram $ABCD$ has vertices $A(-5, 5)$, $B(0, 5)$, $C(3, 3)$, and $D(-2, 3)$. Parallelogram $A'B'C'D'$ is the result of dilating Parallelogram $ABCD$ using point A as the center of dilation and a scale factor of $\frac{1}{2}$.

Preimage		Image	
Point A	$(-5, 5)$	Point A'	
Point B	$(0, 5)$	Point B'	
Point C	$(3, 3)$	Point C'	
Point D	$(-2, 3)$	Point D'	

a Predict the coordinates of Parallelogram $A'B'C'D'$ and record them in the table.

b Draw Parallelogram $ABCD$ and its image on the coordinate plane. Use the images to verify whether your predictions were correct. Explain your thinking.



STOP

Name: ______ Date: ______ Period: ______

Summary

In today's lesson . . .

You dilated polygons on a coordinate plane.

Performing a dilation of a polygon requires three essential pieces of information:

1. The coordinates of the vertices
2. The coordinates of the center of dilation
3. The scale factor of the dilation

With this information, you can precisely describe any dilation of a figure.

Reflect:

Practice

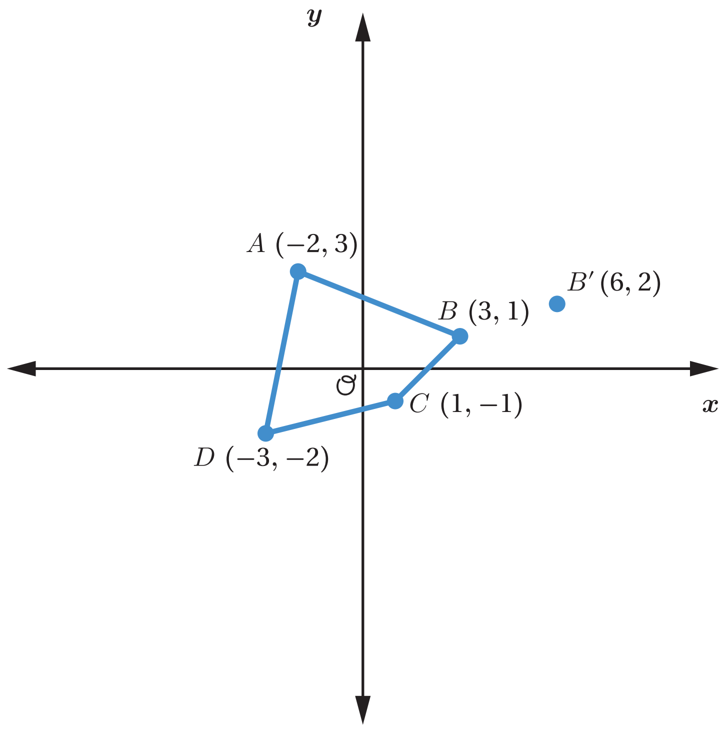
Name: ________ Date: ________ Period: ________

1. Quadrilateral $ABCD$ is dilated with the origin as the center of dilation, taking point B to point B'.

   a. What is the scale factor of the dilation?

   b. Draw Quadrilateral $A'B'C'D'$.

   c. Label the coordinate of points for A', C', and D'.

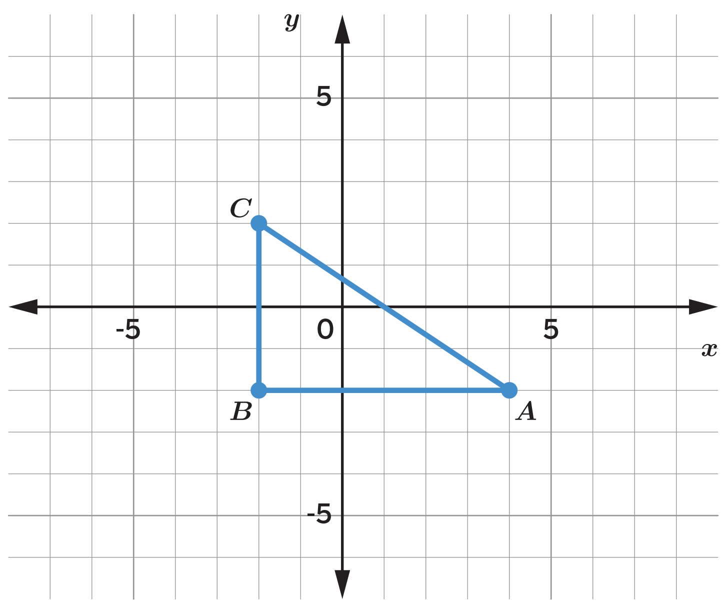


2. Consider Triangle ABC on the coordinate plane.

   a. Using the origin as the center and a scale factor of 2, draw the dilation of Triangle ABC. Label the image Triangle DEF.

   b. Using the origin as the center and a scale factor of $\frac{1}{2}$, draw the dilation of Triangle ABC. Label the image Triangle GHI.

   c. Is Triangle GHI a dilation of Triangle DEF? If yes, identify the center of dilation and scale factor.



Name: .... Date: .... Period: ....

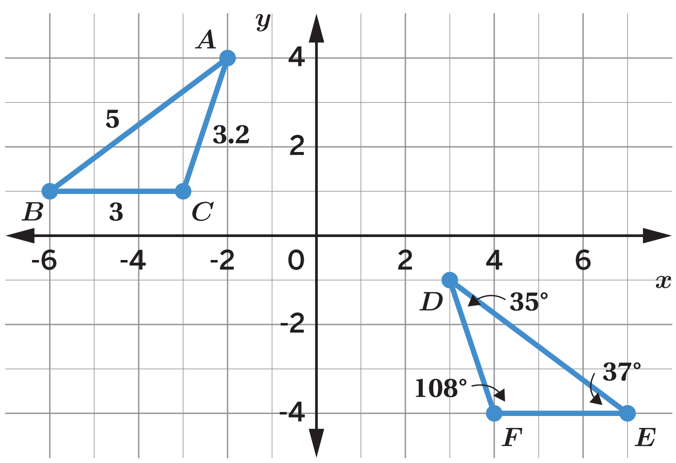
Practice

3. Use what you know about the interior angle measures of triangles to complete these problems.

a Triangle JKL is a right triangle, and the measure of angle J is $28°$. What are the measures of the other two angles?

b Triangle PQR is an obtuse triangle, and the measure of angle Q is $72°$. What are the measures of the other two angles?

4. Triangles ABC and DEF are shown on the coordinate plane.



a Show that $\Delta ABC \cong \Delta DEF$.

b Find the side lengths of Triangle DEF.

c Find the angle measures of Triangle ABC.

My Notes:

Do you really get what you pay for?

Roll through the aisles of your supermarket and you might think everything looks perfectly normal. But take a closer look and you might notice something a bit off about your favorite products.

For years, many manufacturers have been slowly reducing the size of their products. It's nothing dramatic. Maybe 10 percent here or there, shrinking the size of a box of cereal, or a carton of juice. But while the amount you get is shrinking, the price you pay actually remains the same.

This is called *shrinkflation*. Through subtle changes in packaging, manufacturers can get away with selling less of their product for the same cost. Most of the time, you'll find this in junk food and beverages, but it also happens with other household goods, like toilet paper.

A standard sheet of toilet paper used to be 4.5 by 4.5 inches. Now, many brands have shrunk that down to 4 by 4 inches. Some even make toilet paper that's 3.9 by 4 inches. This might not seem like a huge difference, but these shrinkages can really add up.

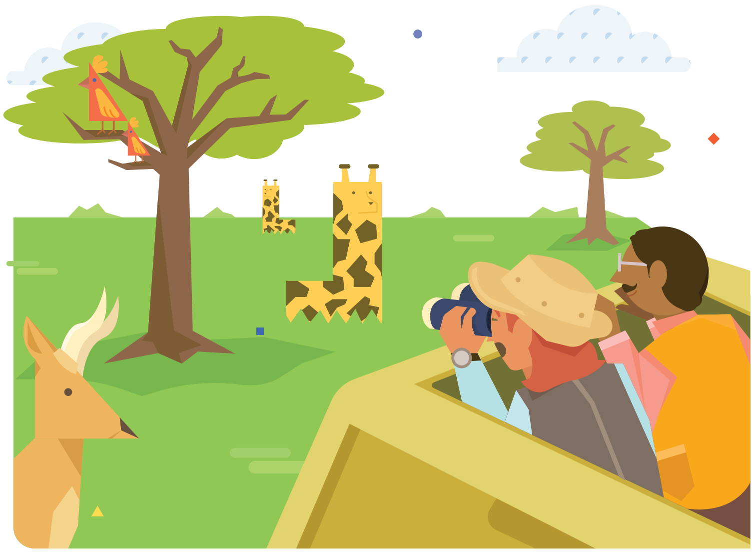
So how do we protect ourselves from overpriced, postage-sized TP? One strategy is to always check the unit price of a good. That way you know how much you're paying per ounce.

When we're dealing with tiny dilations in packaging, it's hard to know exactly when you're being duped. That's why it's important to have an accurate way of measuring, and knowing when things are exactly the same or when they are merely similar . . .

Unit 2 | Lesson 6

Similarity

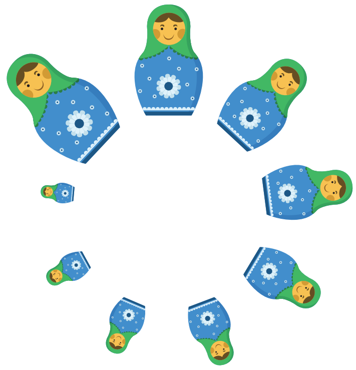
Let's explore similar figures.



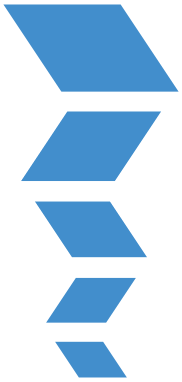
Warm-up Which One Doesn't Belong?

Study the images. Which image does not belong with the others? Explain your thinking.

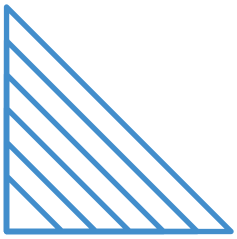
A.



C.



B.



D.

Log in to Amplify Math to complete this lesson online.

Name: .... Date: .... Period: ....

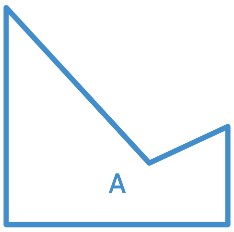
Activity 1 Creating Similar Figures

Complete the following problems using Figure A and the space provided on this page.

1. Sketch the image of Figure A using a reflection and a dilation with a scale factor greater than 1. Label your sketch Figure B.

2. Sketch the image of Figure A using a dilation with a scale factor less than 1. Label your sketch Figure C.

3. Sketch the image of Figure A using a translation and rotation, but no dilation. Label your sketch Figure D.

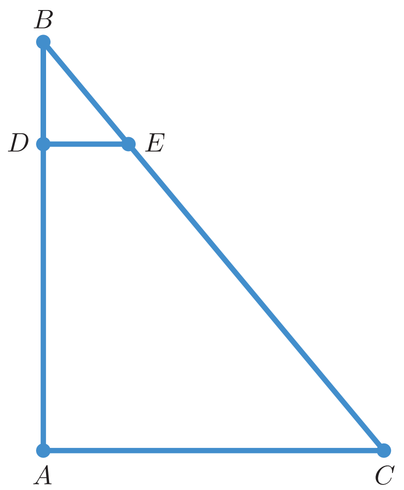


Compare and Connect:
Your teacher will display some of your classmates' images. What similarities do you see in the images sketched and the transformations that were used?

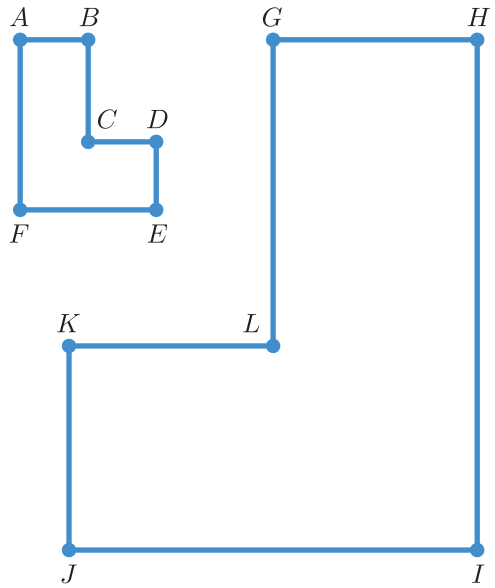
Activity 2 Are They Similar?

For Problems 1–6, determine whether each pair of figures is similar. Write *similar* or *not similar*. If a pair of figures is similar, write a sequence of transformations (translations, rotations, reflections, dilations) that maps one figure onto the other.

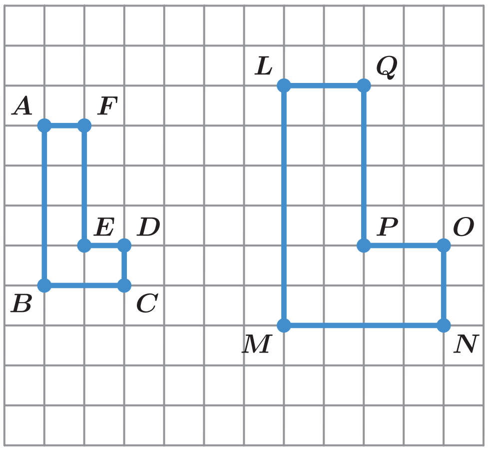
1.



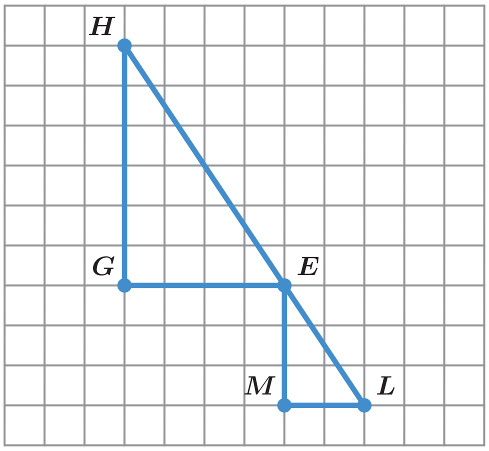
2.



3.



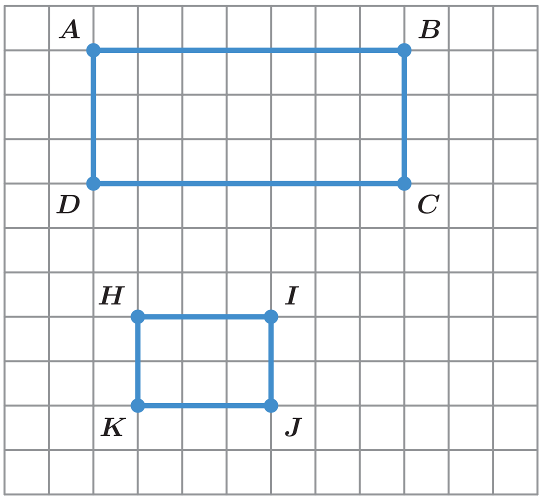
4.



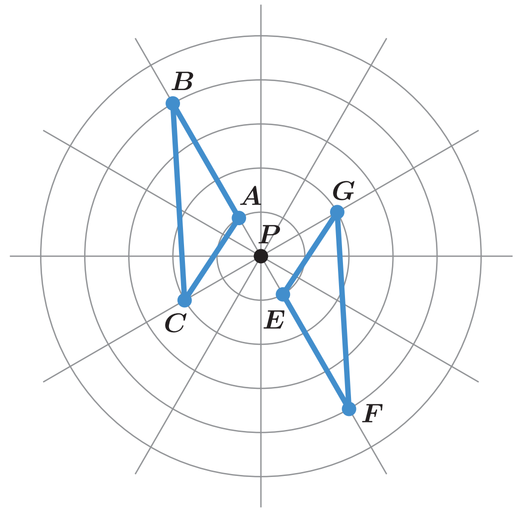
Name: ________________ Date: ________ Period: ________

Activity 2 Are They Similar? (continued)

5.

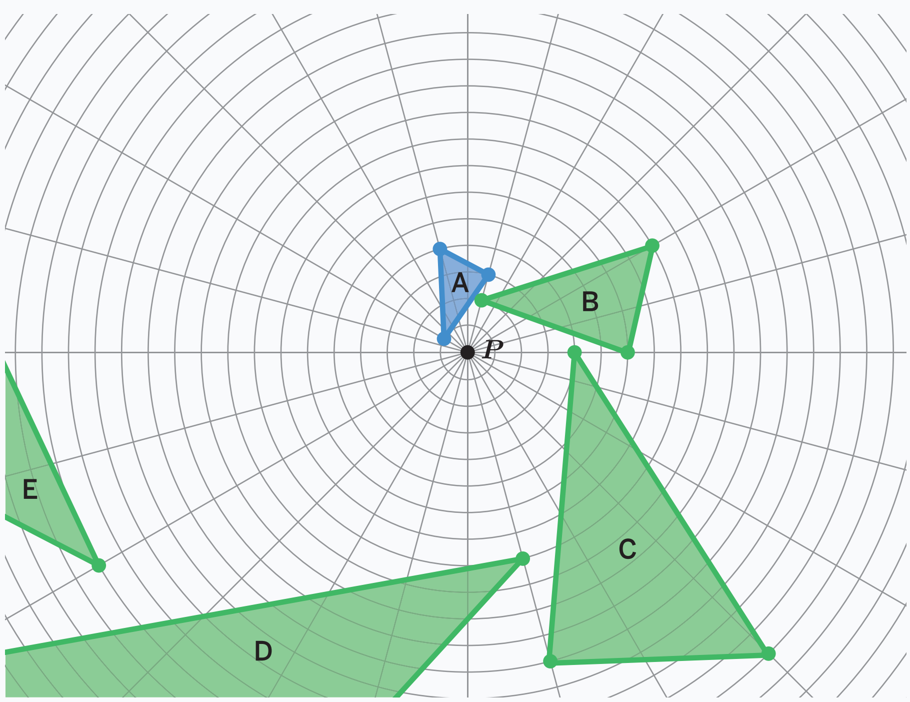


6.



Are you ready for more?

The same sequence of transformations that maps Triangle A onto Triangle B is used to map Triangle B onto Triangle C, and so on. Describe a sequence of transformations that could be this sequence.



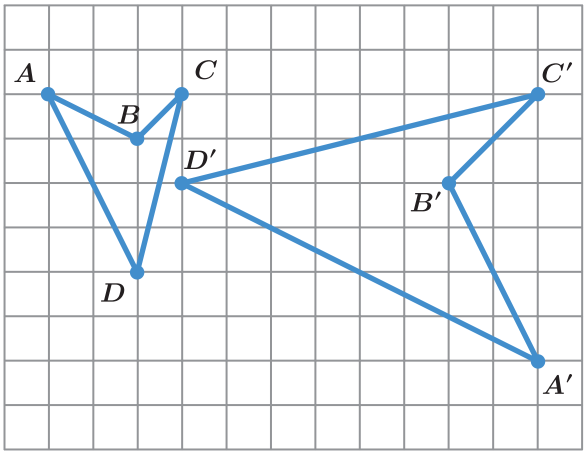
STOP

Summary

In today's lesson . . .

You saw that two figures are ***similar*** if one figure can be mapped onto the other by a sequence of transformations. There may be many correct sequences of transformations, but you only need to describe one to show that two figures are similar.

The symbol ~ indicates that two figures are similar. In the diagram, Polygon $ABCD \sim$ Polygon $A'B'C'D'$. Here is one sequence of transformations that maps Polygon $ABCD$ onto Polygon $A'B'C'D'$.



Step 1 Dilate Polygon $ABCD$ using point D as the center of dilation and a scale factor of 2.

Step 2 Translate the image so that point D maps onto point D'.

Step 3 Rotate the new image $90°$ clockwise about point D'.

Step 4 Reflect the new image across a horizontal line that contains points D' and B'.

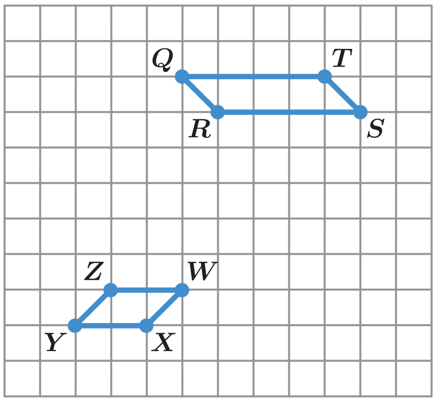
Reflect:

Name: ______ Date: ______ Period: ______

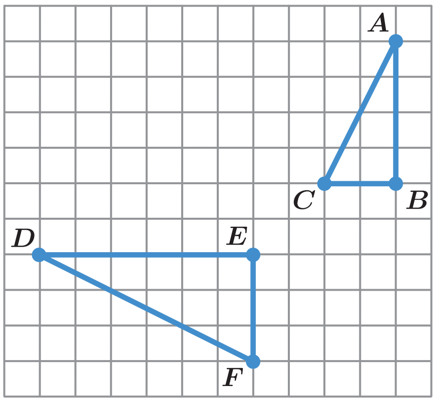
Practice

1. Which of the following shows a pair of similar figures?

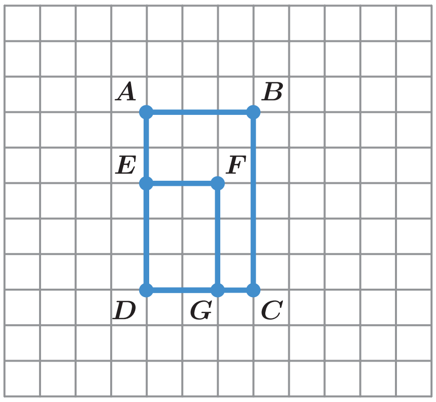
A.



C.

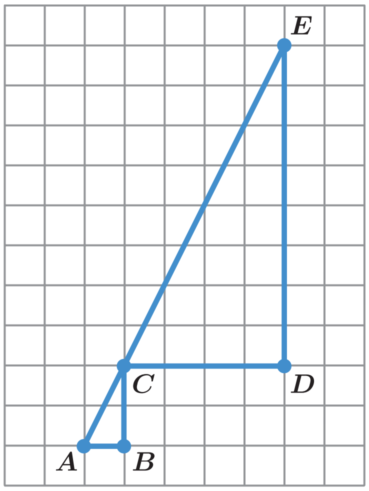


B.

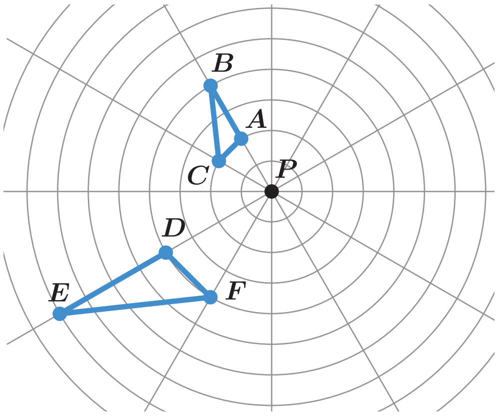


2. Show that each pair of figures is similar by identifying a sequence of transformations that maps the smaller figure onto the larger one.

a



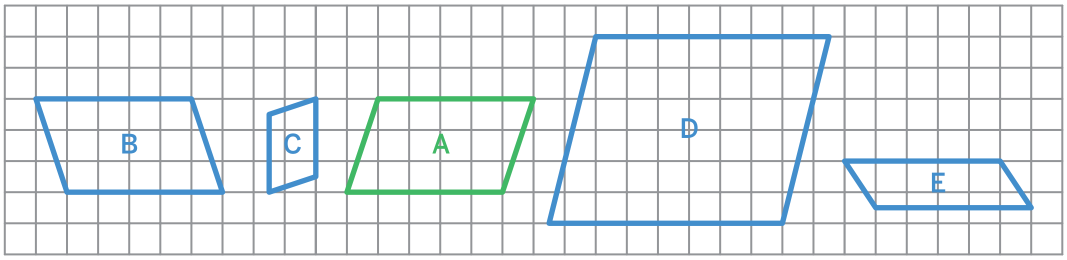
b



Practice

Name: .... Date: .... Period: ....

3. The Rhomboid Snack Group produces granola bars. They want to start producing granola bars in different sizes, but can only do so if they are *similar* to the original bar. Figure A shows the size of the original granola bar. Which granola bar design should they choose? List *all* that apply. Explain your thinking.



4. Each table represents a proportional relationship. Complete each table to show the missing values.

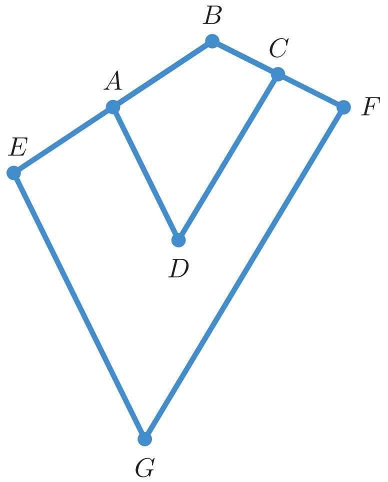
a

a	b
3.5	7
$\frac{1}{2}$	
	8.4

b

x	y
2	0.31
6	
18	

5. Polygon $ABCD$ is a scaled copy of Polygon $EBFG$ with a scale factor of $\frac{1}{2}$. Which of the following is *not* true?



A. Angle BCD is congruent to angle BFG.

B. Segment AD is half as long as segment EG.

C. The measure of angle EGF is twice as great as the measure of angle ADC.

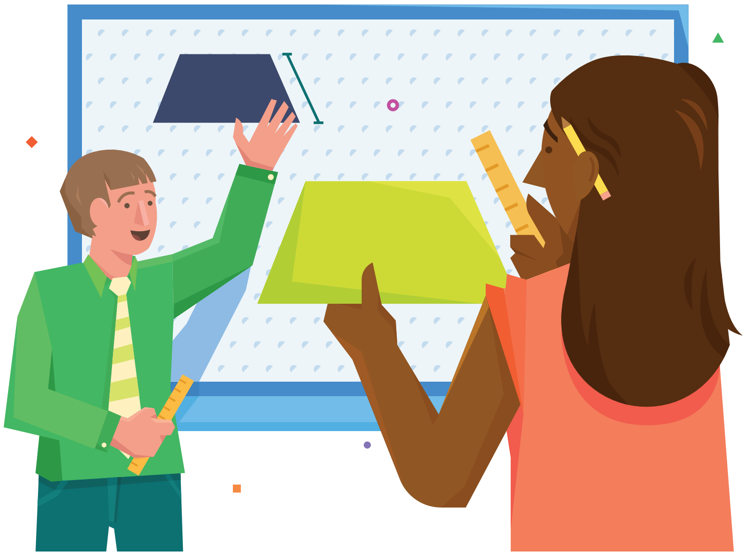
D. Segment EG is twice as long as segment AD.

Name: ______ Date: ______ Period: ______

Unit 2 | Lesson 7

Similar Polygons

Let's study the sides and angles of similar polygons.



Warm-up Sometimes, Always, Never

Determine whether each statement is *always*, *sometimes*, or *never* true.

1. If two figures are congruent, then they are similar.

2. If two figures are similar, then they are congruent.

3. If two figures are not congruent, then they are not similar.

4. If two figures are not similar, then they are congruent.

Log in to Amplify Math to complete this lesson online.

Activity 1 Different Dilations

Although it doesn't always seem that way, humans often behave in predictable ways. Dr. Hannah Fry, a research mathematician, has been fascinated with the idea of predicting human behavior such as how humans perceive similar but slightly different objects.

You and your partner will be given a set of cards with scaled copies of parallelograms and a plain sheet of paper.

1. Choose two of the cards and verify that they are scaled copies of each other. Explain your thinking here.

2. Glue your cards anywhere on the separate sheet of paper.

3. Switch papers with your partner, and ask them to show that your two parallelograms are similar.

Featured Mathematician

Hannah Fry

Have you ever wondered how streaming services know what shows to recommend? Many aspects of our lives are now influenced by algorithms designed to interpret and predict human behavior. Like so many of us, English mathematician Hannah Fry wants to understand why people do the things they do. She has worked with physicists, mathematicians, computer scientists, architects, and geographers to understand human behavior through pattern recognition. Her work studying the patterns of human behavior through mathematics has touched on many aspects of society, from shopping and dating to crime and terrorism.

Name: ________ Date: ________ Period: ________

Activity 2 Are You Sure They Are Similar?

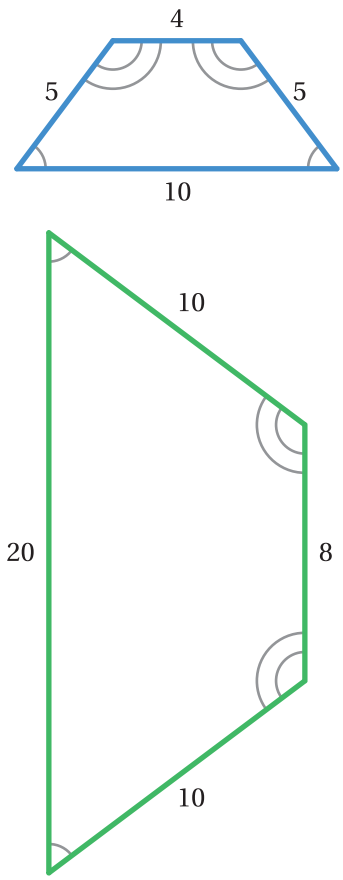
1. Clare is trying to persuade her classmates that the polygons shown are similar. Which argument is the most convincing? Explain your thinking.

   A. The corresponding side lengths are proportional.

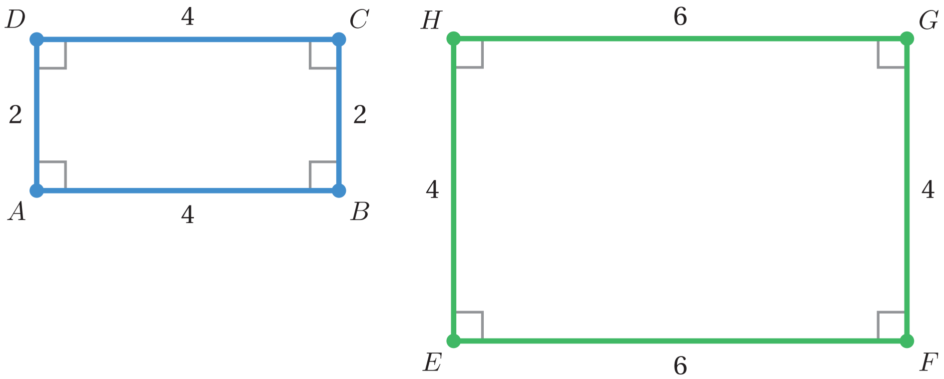
   B. The corresponding angles are congruent.

   C. The figures are scaled copies.

   D. The smaller figure has been rotated.



2. Jada studied the two figures shown and noticed that the angles are all congruent and the side lengths of each figure differ by 2 units. Is this enough to claim the figures are similar?



STOP

Summary

In today's lesson . . .

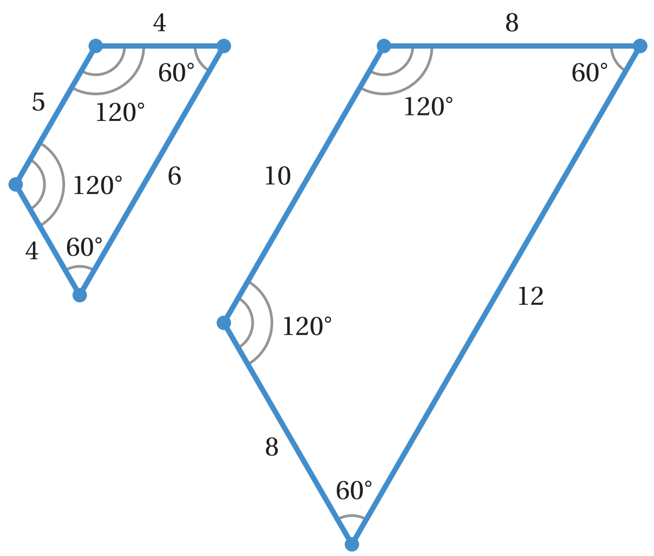
You explored the properties of figures to determine similarity.

When two polygons are similar:

- Every angle and side in one polygon has a corresponding part in the other polygon.
- All pairs of corresponding angles have the same measure.
- Corresponding sides are related by a single scale factor. Each side length in one polygon is multiplied by the scale factor to get the corresponding side length in the other polygon.
- A sequence of transformations can be applied to one polygon to map onto another polygon.

To show two polygons are similar, you can show they are scaled copies of each other.

- For example, you can examine the angle measures of these trapezoids and conclude that corresponding angles are congruent.
- Then you can determine that side lengths are proportional because each side length of the smaller trapezoid can be multiplied by 2 to get the corresponding side length of the larger trapezoid.
- Because these trapezoids meet the criteria for being scaled copies, they must be similar.



Reflect:

Name: ______ Date: ______ Period: ______

Practice

1. Triangle DEF is a dilation of Triangle ABC with scale factor of 2. In Triangle ABC, the greatest angle measures 82°. What is the greatest angle measure in Triangle DEF?

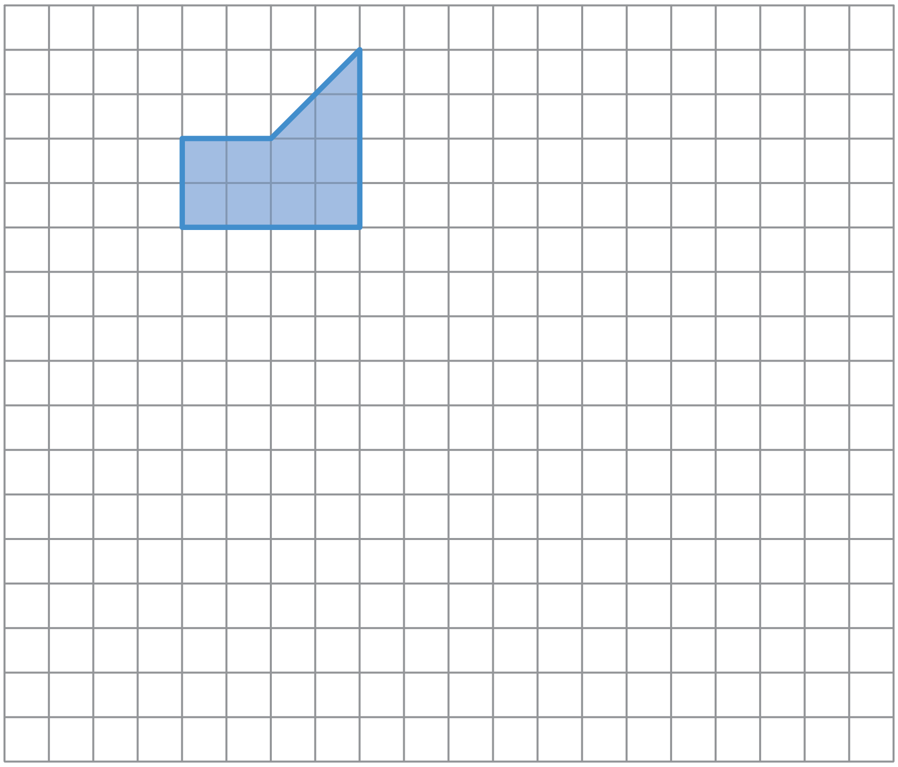
A. $41°$

B. $82°$

C. $84°$

D. $164°$

2. Consider the following polygon.



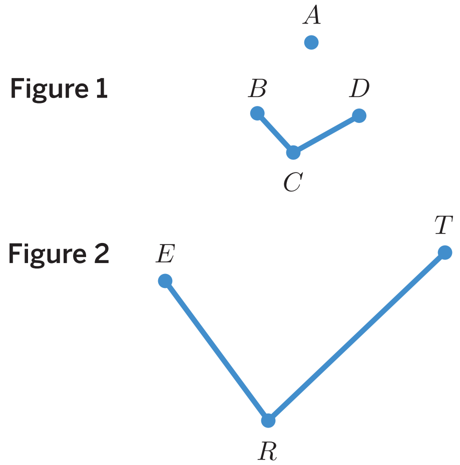
a Draw a polygon that is similar, but could be mistaken for being *not* similar.

b Draw a polygon that is *not* similar, but could be mistaken for a similar polygon.

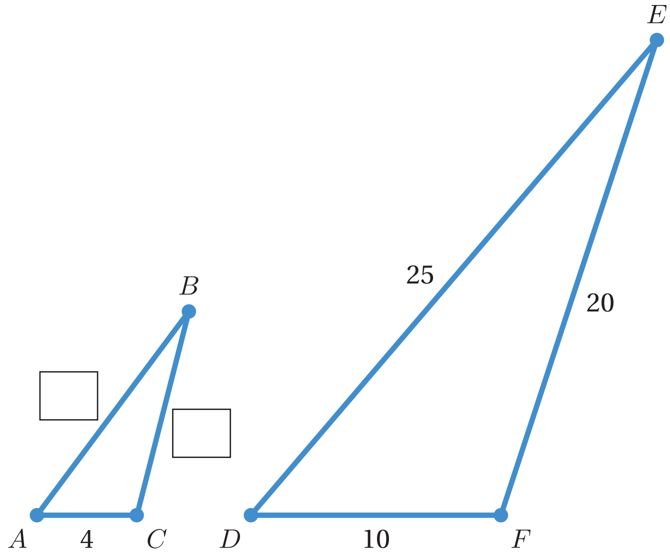
Practice

Name: .... Date: .... Period: ....

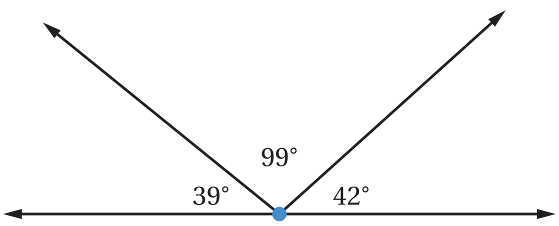
3. Lin claims that Figure 2 is a dilation of Figure 1 using point A as the center of dilation. What are some ways you can convince Lin that her claim is *not* true?



4. Triangle ABC is a scaled copy of Triangle DEF with a scale factor of $\frac{2}{5}$. Find the missing lengths of Triangle ABC.



5. The line shown has been partitioned into three angles. Is there a triangle with these three angle measures? Explain your thinking.



Name: ________ Date: ________ Period: ________

Unit 2 | Lesson 8

Similar Triangles

Let's explore similar triangles.

Warm-up Imagine a Triangle . . .

A triangle has an angle measure of $100°$. What else must be true about the triangle? Select *all* the true statements.

A. The triangle is isosceles.

B. The remaining two angle measures add to $80°$.

C. One angle has a measure of $20°$ and the other has a measure of $60°$.

D. The missing angles must each be acute.

E. It is an obtuse triangle.

Log in to Amplify Math to complete this lesson online.

Activity 1 Are Three Angles Enough?

1. Construct a triangle with angle measures of 40°, 55°, and 85°.

2. Compare your triangle to a partner's triangle.

   a. Are your triangles congruent?

   b. Are your triangles similar? Explain your thinking.

Name: ________________ Date: ________ Period: ________

Activity 2 Is One Angle Enough?

Andre drew a triangle with one angle that measured $65°$. Bard drew a triangle with one angle that measured $65°$. Can Andre and Bard guarantee they drew similar triangles? If yes, explain why. If not, show an example.

Activity 3 Card Sort: Similar or Not?

You will be provided with a set of cards. Each card contains two triangles.

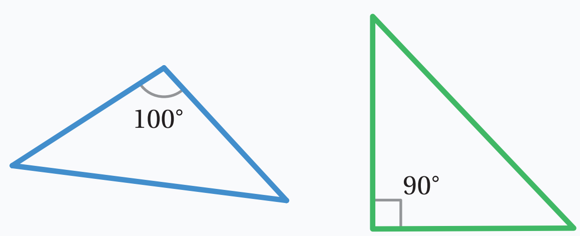
1. Sort the cards into three groups:
   - Triangles that are *similar*.
   - Triangles that are *not similar*.
   - Triangles for which you *do not have enough information* to determine whether they are similar.

Similar	Not similar	Not enough information

2. Select a card for which you decided did not have enough information to determine whether the triangles are similar. Explain what other information would be needed.

Are you ready for more?

Tyler and Elena wanted to determine in which category to place the following pair of triangles. Tyler said there was not enough information to determine whether they are similar. Elena says there is enough information and she knows the triangles are *not* similar. Do you agree with Tyler or Elena? Explain your thinking.



STOP

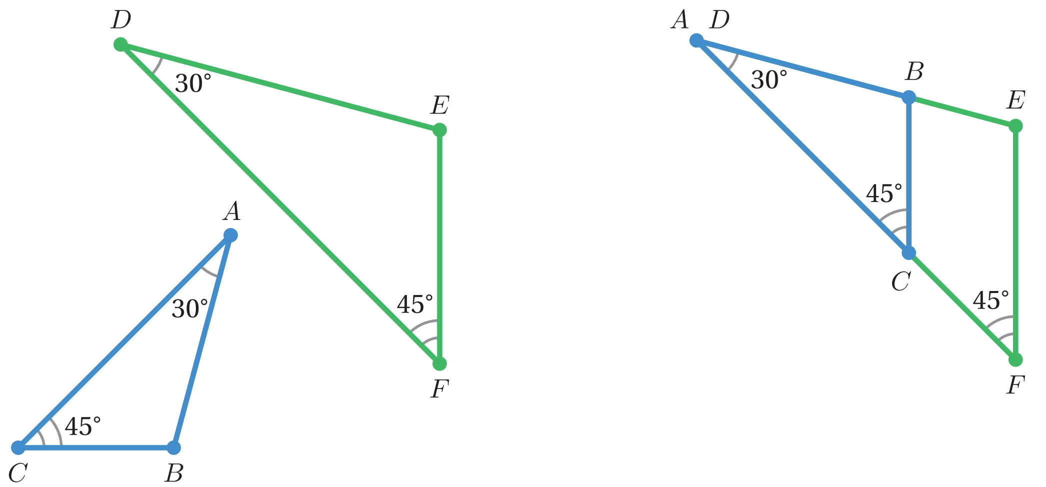
Name: ________________ Date: ________ Period: ________

Summary

In today's lesson . . .

You further developed your understanding that two polygons are similar when there is a sequence of translations, rotations, reflections, and dilations taking one polygon to the other. You saw that when the polygons are triangles, you only need to check that both triangles have two congruent angles to show they are similar.

For example, Triangle ABC and Triangle DEF each have a $30°$ angle and a $45°$ angle. You can translate Triangle ABC so that point A maps onto point D, and then rotate the resulting triangle so that the two $30°$ angles are aligned.



Now a dilation with center A and appropriate scale factor will map point C onto point F. This dilation also maps point B onto point E, showing that $\Delta ABC \sim \Delta DEF$.

Reflect:

Practice

Name: .... Date: .... Period: ....

1. In each pair of triangles, some of the angle measures are given. Determine whether the triangles are similar, not similar, or if there is not enough information to determine whether they are similar. Place a check mark in the appropriate column.

Triangle pairs	Similar	Not Similar	Not enough information
Triangle A: $53°$, $71°$ Triangle B: $53°$, $71°$			
Triangle C: $90°$, $33°$ Triangle D: $90°$, $57°$			
Triangle E: $63°$, $45°$ Triangle F: $14°$, $71°$			
Triangle G: $100°$ Triangle H: $70°$			

2. In the space provided, draw two equilateral triangles that are *not* congruent.

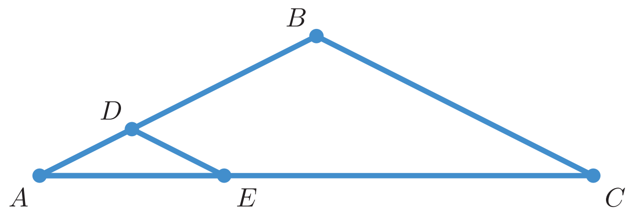
a. Measure the side lengths and angles of your triangles. Are the two triangles similar? Why or why not?

b. Do you think two equilateral triangles will *always*, *sometimes*, or *never* be similar? Explain your thinking.

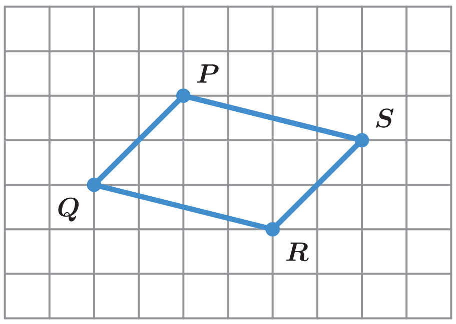
Name: ____________ Date: ________ Period: ________

Practice

3. In the figure, segment BC is parallel to segment DE. Explain why $\Delta ABC \sim \Delta ADE$.



4. Quadrilateral $PQRS$ is a parallelogram. Let Quadrilateral $P'Q'R'S'$ be the image of Quadrilateral $PQRS$ after performing a dilation centered at a point O (not shown) with a scale factor of 3. Which of the following is true?



A. $P'Q' = PQ$

B. $P'Q' = 3PQ$

C. $3P'Q' = PQ$

D. The relationship of segment PQ to segment $P'Q'$ cannot be determined from the information given.

5. Simplify each fraction.

a. $\frac{8}{12}$

b. $\frac{25}{10}$

c. $\frac{3}{3}$

d. $-\frac{9}{24}$

Unit 2 | Lesson 9

Ratios of Side Lengths in Similar Triangles

Let's use similarity to determine side lengths in similar triangles.

Warm-up Which One Doesn't Belong?

Study these ratios. Which ratio does not belong with the others? Explain your thinking.

A. $2 : 10$

C. $\frac{2}{5}$

B. -10 to -25

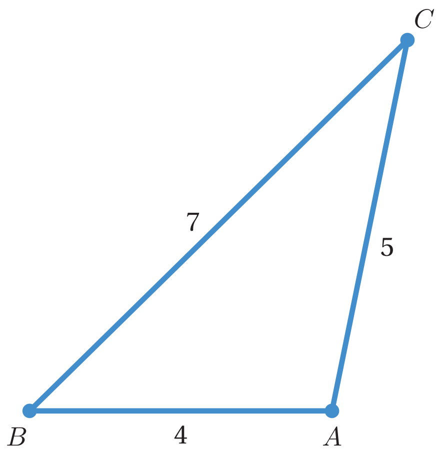
D. $\frac{4}{10}$ to $\frac{10}{10}$

Log in to Amplify Math to complete this lesson online.

Name: ________ Date: ________ Period: ________

Activity 1 Ratios of Side Lengths Within Similar Triangles

Triangle ABC is similar to each of Triangles DEF, GHI, and JKL. Note that Triangles DEF, GHI, and JKL are not shown.



1. The scale factor for the dilation that maps Triangle ABC onto each triangle is shown in the table. Determine the side lengths of Triangles DEF, GHI, and JKL. Record them in the table.

Triangle	Scale factor	Length of short side	Length of medium side	Length of long side
ABC	1	4	5	7
DEF	2			
GHI	3			
JKL	$\frac{1}{2}$			

Pause here so your teacher can review your work.

Activity 1 Ratios of Side Lengths Within Similar Triangles (continued)

2. With your group members, decide who will complete Column A, Column B, and Column C. For all four triangles, find and record the ratio of the indicated side lengths given for each column.

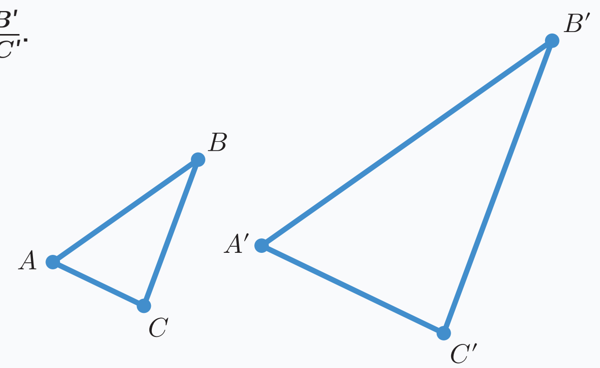
Triangle	Column A Ratio of long side to short side	Column B Ratio of long side to medium side	Column C Ratio of medium side to short side
ABC			
DEF			
GHI			
JKL			

3. What do you notice about the ratios?

4. Compare your results with your group members and then complete your table with your group's completed ratios.

Are you ready for more?

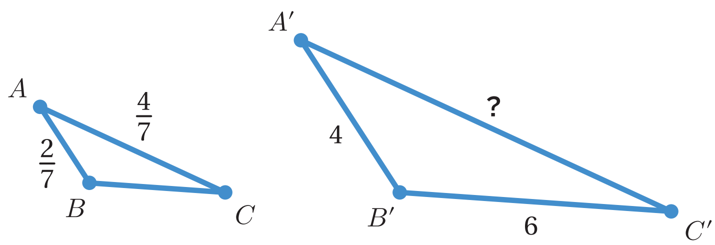
$\Delta ABC \sim \Delta A'B'C'$. Explain why $\frac{AB}{BC} = \frac{A'B'}{B'C'}$.



Name: .... Date: .... Period: ....

Activity 2 Completing the Missing Steps

In the diagram, $\triangle ABC \sim \triangle A'B'C'$. Bard and Elena both started to calculate the unknown side length $A'C'$. The first two steps for each student's method is shown.



The figures may not be drawn to scale.

1. Complete the missing Step 3 for each student.

Bard

Step 1: Create a ratio table.

Long side (AC)	$\frac{4}{7}$	?
Short side (AB)	$\frac{2}{7}$	4

$\times 14$

Step 2: Calculate the scale factor:

$$4 \div \frac{2}{7} = 14$$

Step 3:

Length of $A'C'$:

Elena

Step 1: Create a ratio table.

Long side (AC)	$\frac{4}{7}$	?
Short side (AB)	$\frac{2}{7}$	4

$\times 2$

Step 2: Determine the ratio of the long side to the short side in Triangle ABC:

$$\frac{4}{7} \div \frac{2}{7} = 2$$

Step 3:

Length of $A'C'$:

2. Use either Bard's or Elena's method to determine BC.

STOP

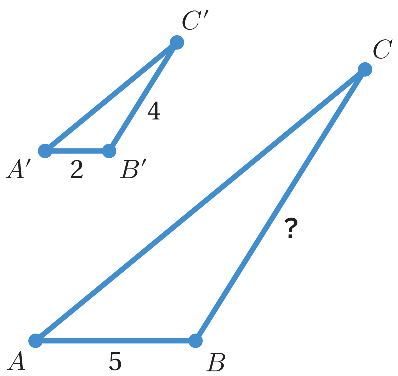
Summary

In today's lesson . . .

You discovered that the ratio of a pair of side lengths in one triangle is equal to the ratio of the corresponding side lengths in a similar triangle.

For a pair of similar triangles, you can calculate the missing side length by using the ratios of side lengths *within* a triangle or by using the scale factor *between* the triangles.

Suppose you know $\Delta ABC \sim \Delta A'B'C'$. Here are two methods you can use to determine side BC.



The figures may not be drawn to scale.

Method 1: Using the scale factor *between* the triangles

Because you need to determine the length of side BC, find the ratio of the lengths of the corresponding sides AB to $A'B'$ to determine the scale factor. The ratio is $5 : 2$, so the scale factor is 2.5. Multiply the length of side $B'C'$ by the scale factor to determine the length of side BC, $4 \cdot 2.5 = 10$.

Method 2: Using ratio of sides *within* one triangle

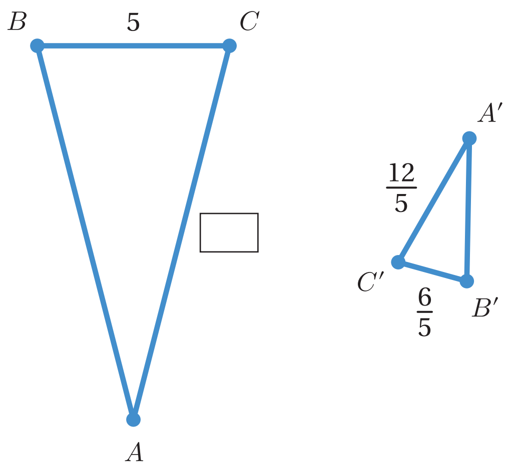
In Triangle $A'B'C'$, the ratio of the medium side to the short side is $4 : 2$, or 2. This means that the medium side is twice the length of the short side in both triangles. Therefore, the length of side BC is twice the length of side AB, $5 \cdot 2 = 10$.

Reflect:

Name: ________________ Date: ________ Period: ________

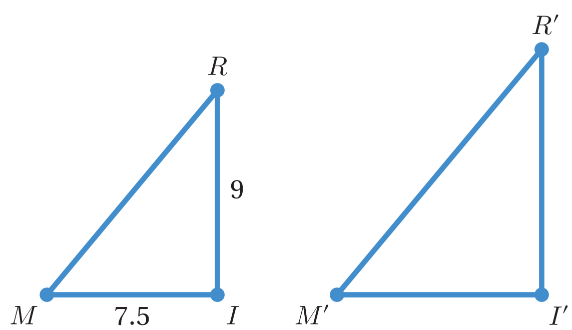
Practice

1. In the diagram, $\Delta ABC \sim \Delta A'B'C'$. Determine the length of side AC. Explain your thinking.



The figures may not be drawn to scale.

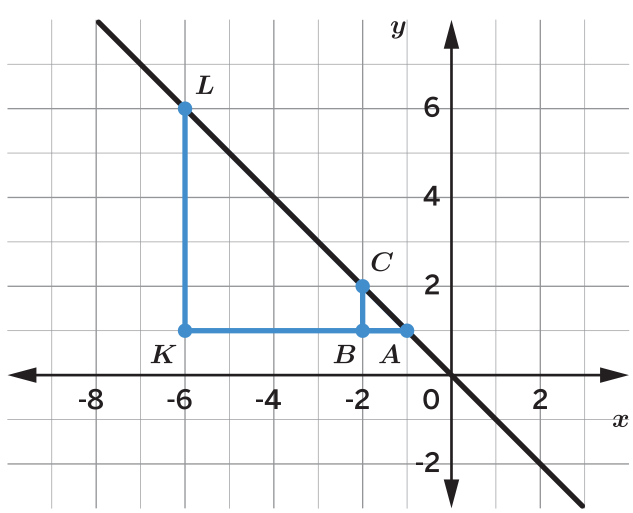
2. In the diagram, $\Delta MIR \sim \Delta M'I'R'$. Determine the ratio of $I'R'$ and $M'I'$. Explain your thinking.



3. Determine a center and a scale factor for a dilation that would map Triangle ABC onto Triangle AKL.

Center:

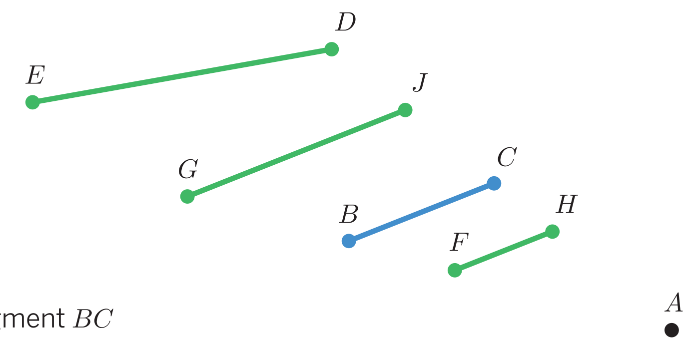
Scale factor:



Practice

Name: ______________________ Date: ____________ Period: ____________

4. Refer to the line segments shown.

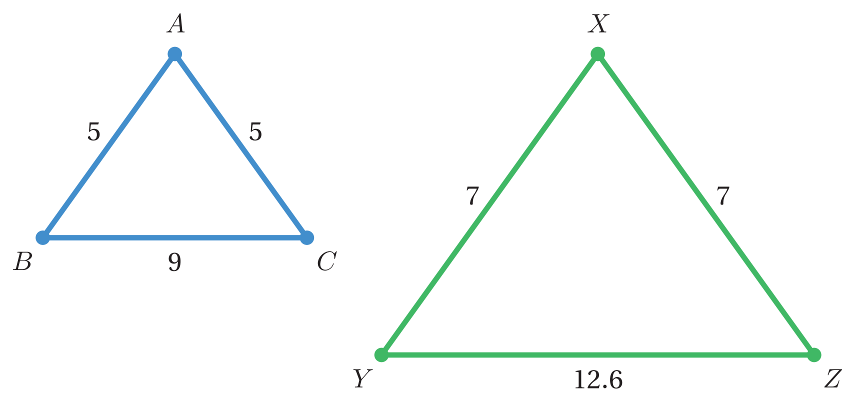


a Which segment is a dilation of segment BC using point A as the center of dilation and a scale factor of $\frac{2}{3}$?

b Which segment is a dilation of segment BC using point A as the center of dilation and a scale factor of $\frac{3}{2}$?

c Which segment is not a dilation of segment BC? Explain your thinking.

5. Triangle ABC is similar to Triangle XYZ. What is the scale factor that maps Triangle ABC onto Triangle XYZ? Explain your thinking.



The figures may not be drawn to scale.

Name: ____________ Date: ____________ Period: ____________

Unit 2 | Lesson 10

The Shadow Knows

Let's use shadows to determine the height of a figure.

Warm-up Notice and Wonder

Consider the following images. What do you notice? What do you wonder?

1. I notice . . .

2. I wonder . . .

Log in to Amplify Math to complete this lesson online.

Activity 1 Figures and Shadows

Study the image. The table lists the height of each person, dog, and lamppost, and their shadow.

	Height (in.)	Shadow length (in.)
Mocha	43	29
Mr. Mendez	72	48
Mai	51	34
Lamppost	?	114

The figures may not be drawn to scale.

1. What relationships do you notice between each person's or object's height and the length of its shadow?

2. Explain *why* the ratios of the height of each person or object to the length of their shadow are approximately the same.

3. Determine the height of the lamppost. Explain your thinking.

Historical Moment

Over 2,000 years ago, the ancient Greek mathematician Eratosthenes also studied shadows closely (in a slightly different way). He used his study of shadows to estimate the circumference of Earth with an error of less than 2%!

Name: ____________ Date: ____________ Period: ____________

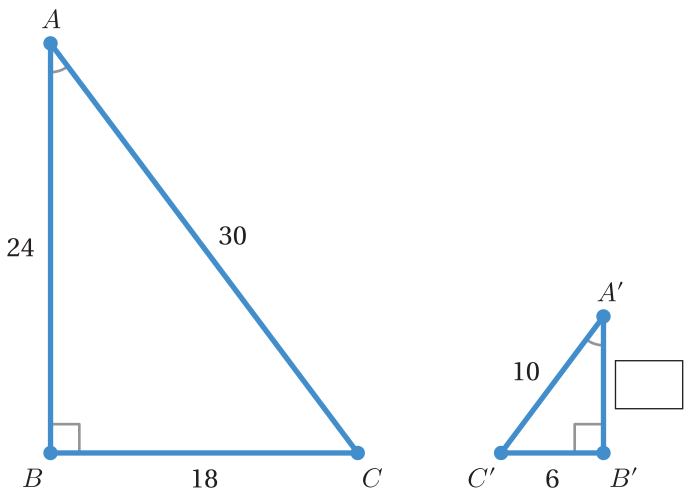
Activity 2 Four Challenges

Plan ahead: What will you do to stay focused on the challenges? How will you control your impulses?

For each challenge, determine the missing side length and explain your thinking. The figures may not be drawn to scale.

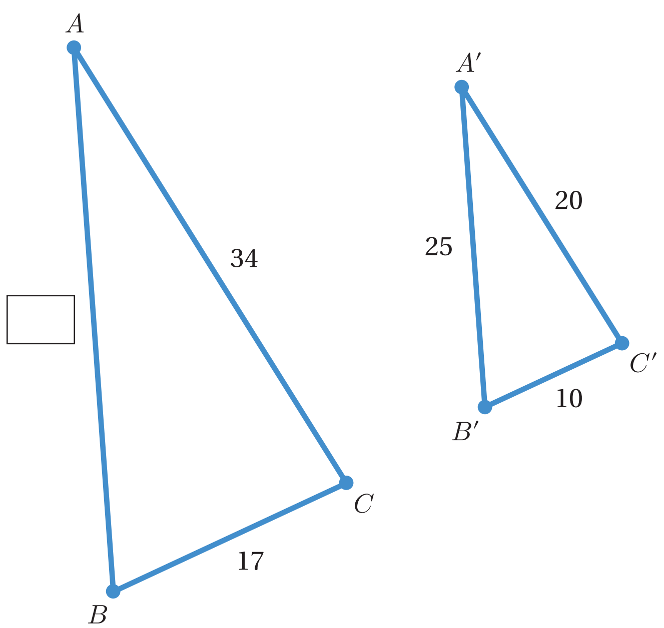
Challenge 1:

$\Delta ABC \sim \Delta A'B'C'$



Challenge 2:

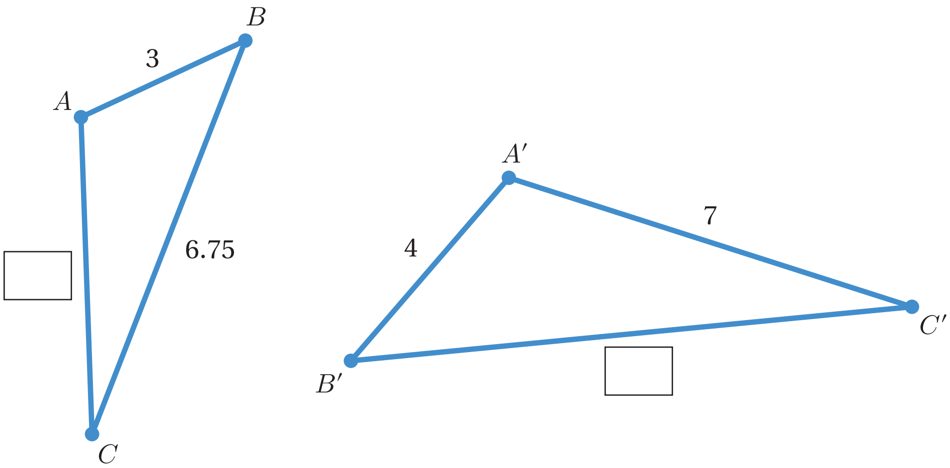
$\Delta ABC \sim \Delta A'B'C'$



Activity 2 Four Challenges (continued)

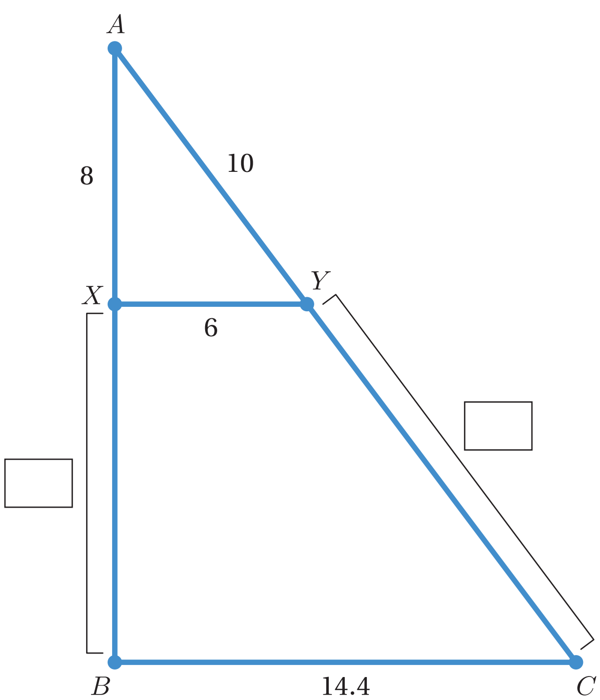
Challenge 3:

$\Delta ABC \sim \Delta A'B'C'$



Challenge 4:

Segment XY is parallel to segment BC.



STOP

Name: ______ Date: ______ Period: ______

Summary

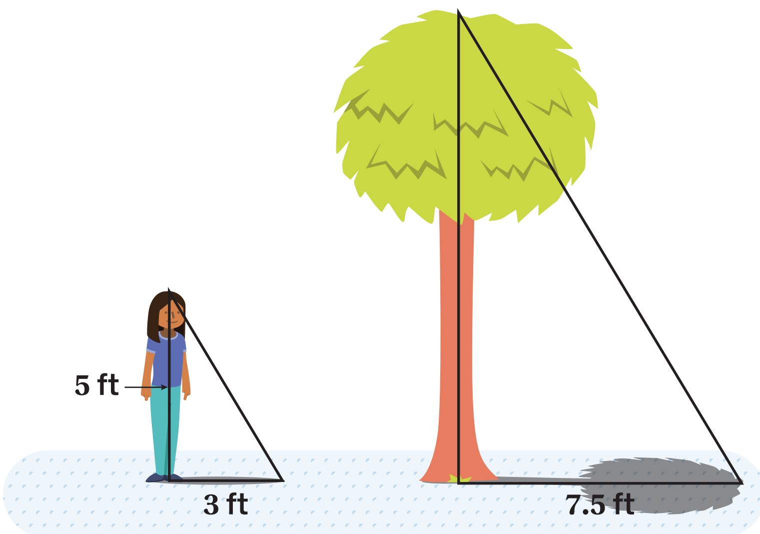
In today's lesson . . .

You examined the lengths of shadows of different figures. Because all of the figures were perpendicular to the ground and the sun's rays were cast on each figure at the same angle, you found that the right triangles that resulted from the figures and their shadows formed similar triangles.

The height of the tree shown can be determined by applying knowledge of similar triangles.

The ratio of the person's height to their shadow is $\frac{5}{3}$. This means the ratio of the tree's height to its shadow is also $\frac{5}{3}$.

So, you know the height of the tree is $\frac{5}{3} \cdot 7.5$, or 12.5 ft.



The figures may not be drawn to scale.

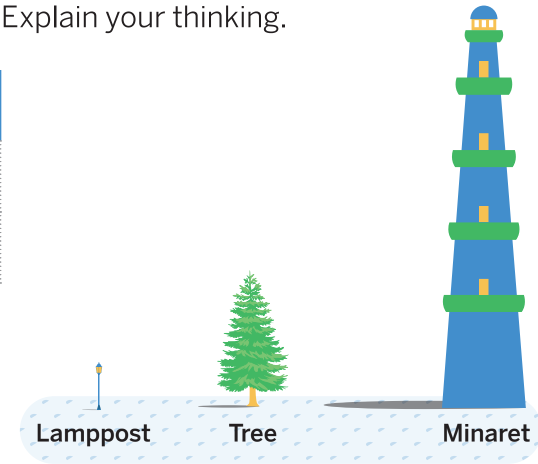
Reflect:

Practice

Name: ________________ Date: ________ Period: ________

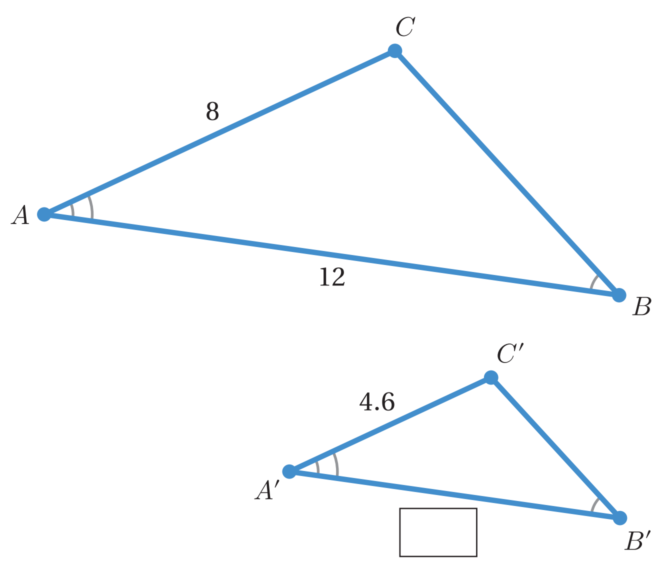
1. Determine the height of the tree and minaret. Explain your thinking.

	Lamppost	Tree	Minaret
Height (ft)	15		
Shadow (ft)	6	20	56



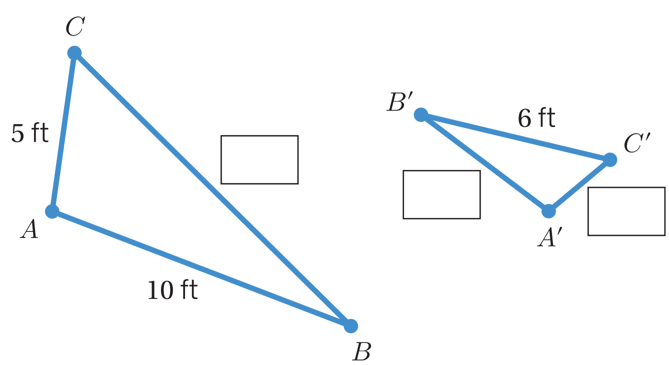
The figures may not be drawn to scale.

2. In the diagram, $\Delta ABC \sim \Delta A'B'C'$.
Determine the missing side length.
Explain your thinking.



The figures may not be drawn to scale.

3. $\Delta ABC \sim \Delta A'B'C'$. The scale factor that maps Triangle ABC onto Triangle $A'B'C'$ is $\frac{1}{2}$. Determine the missing side lengths.

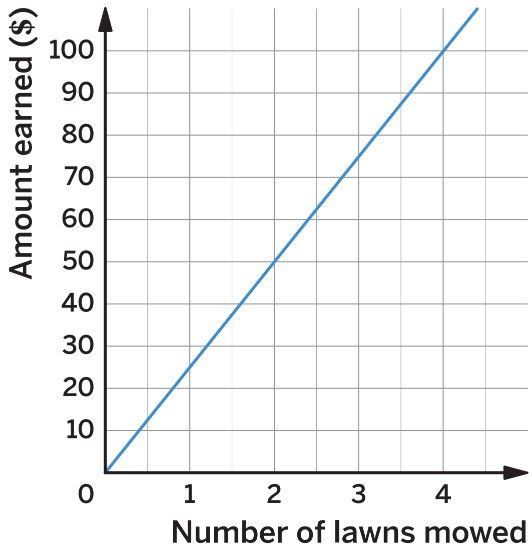


The figures may not be drawn to scale.

Name: ______________________ Date: ____________ Period: ____________

Practice

4. The graph shows the amount Tyler earns based on the number of lawns he mows. Label the point $(1, k)$ on the graph, find the value of k, and explain its meaning.



5. A rectangle has a length of 6 units and a width of 4 units. Which of the following statements tells you that Quadrilateral $ABCD$ is *not* similar to this rectangle? Select *all* that apply.

A. $AB = BC$

B. $m\angle ABC = 105°$

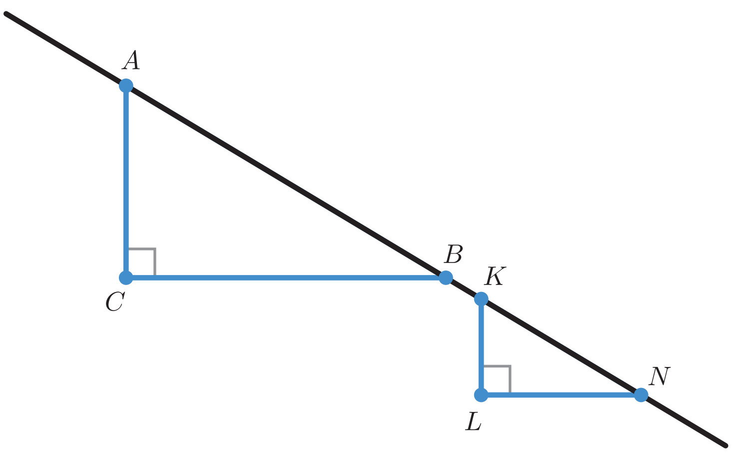
C. $AB = 8$

D. $BC = 8$

E. $BC = 2 \cdot AB$

F. $2 \cdot AB = 3 \cdot BC$

6. Segments AC and KL are parallel. Segments CB and LN are parallel. Show that $\Delta ABC \sim \Delta KNL$.



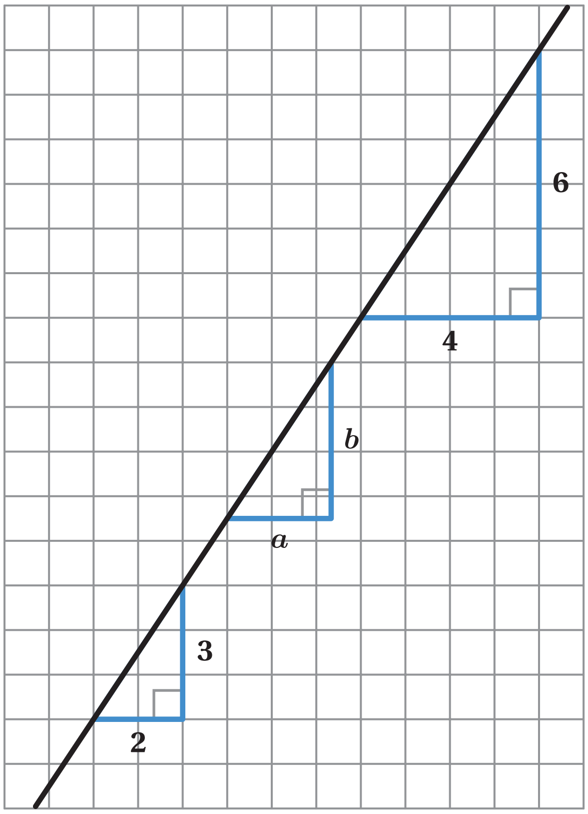
Unit 2 | Lesson 11

Meet Slope

Let's explore the slope of a line.

Warm-up Notice and Wonder

Study the image. What do you notice? What do you wonder?



1. I notice . . .

2. I wonder . . .

Co-craft Questions: Share your questions with a partner. Together, come up with 1–2 questions that you think you might be able to answer.

Log in to Amplify Math to complete this lesson online.

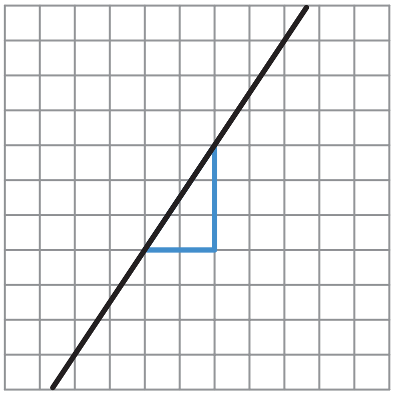
Name: ________________ Date: ________ Period: ________

Activity 1 Different Slopes, Different Lines

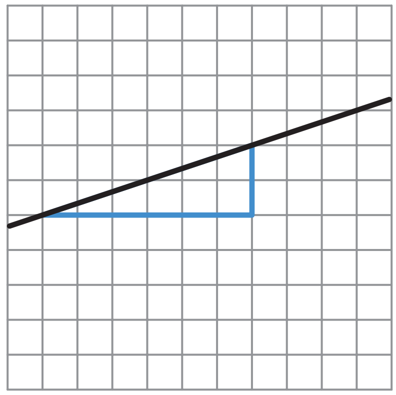
Study the images shown on each of these graphs. Match each line with its corresponding slope. Draw a line in the empty grid for Graph 6 that has a slope of $\frac{1}{5}$.

Slope	Graph
$\frac{1}{3}$	
2	
1	
0.25	
$\frac{3}{2}$	
$\frac{1}{5}$	6

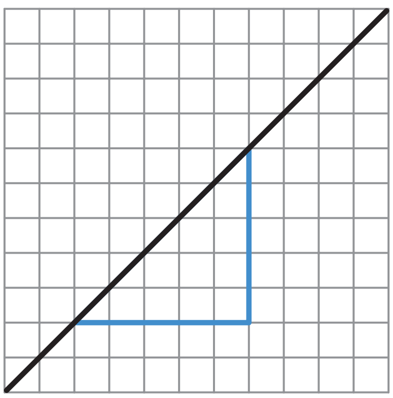
Graph 1



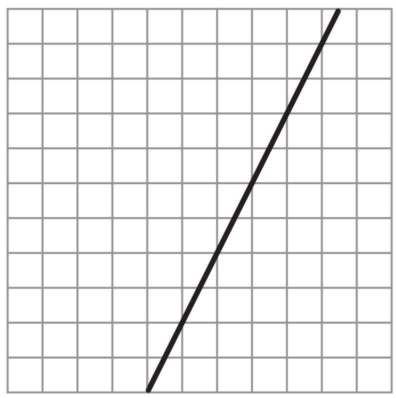
Graph 2



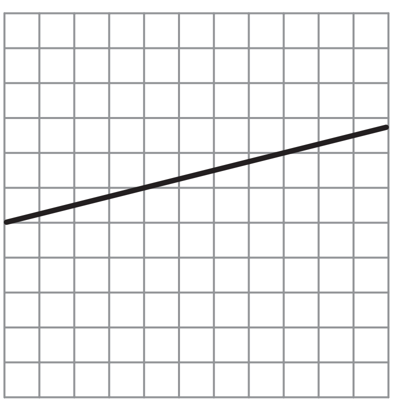
Graph 3



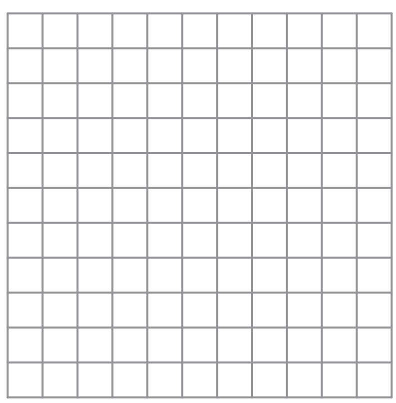
Graph 4



Graph 5

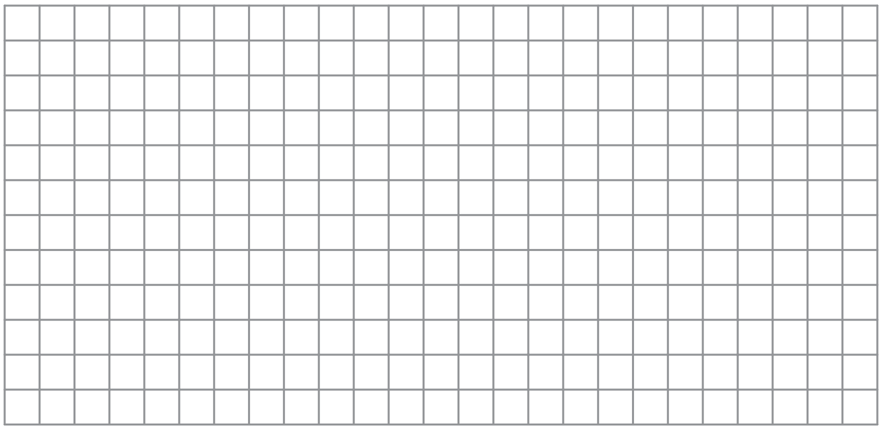


Graph 6

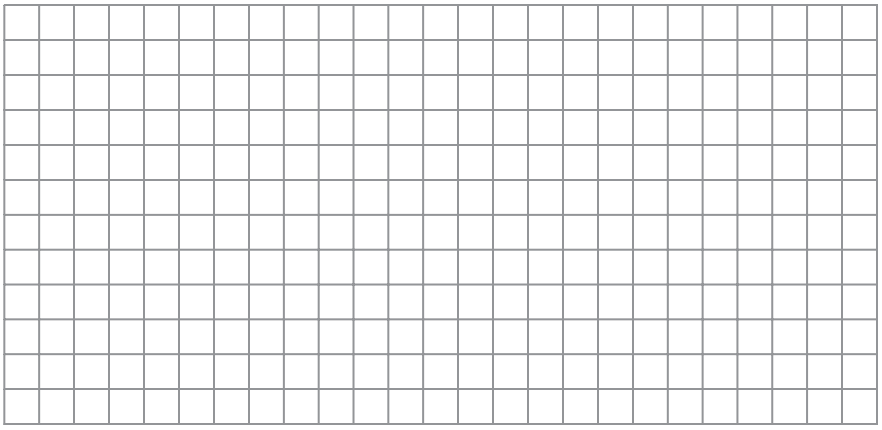


Activity 2 Multiple Lines with the Same Slope

1. Draw two lines that each have a slope of 3.



2. Draw two lines that each have a slope of $\frac{1}{2}$.



3. What do you notice about the lines you drew in Problems 1 and 2?

STOP

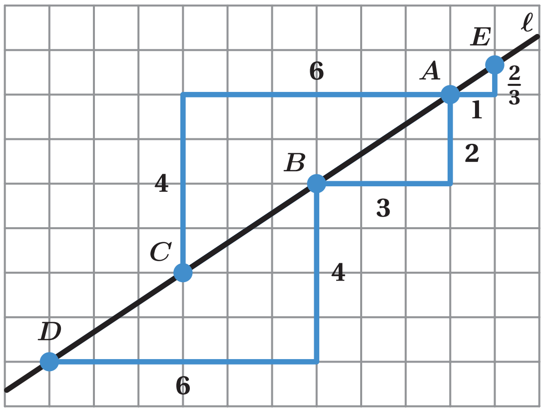
Name: .... Date: .... Period: ....

Summary

In today's lesson . . .

You used similar triangles to discover the slope of a line.

The four triangles shown are all examples of ***slope triangles***. One side of a slope triangle lies on the line ℓ, one side is a vertical line segment, and one side is a horizontal line segment.



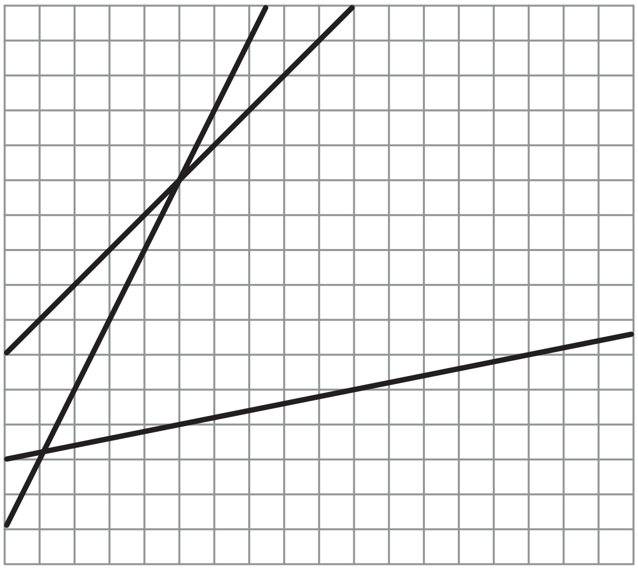
The ***slope*** of the line ℓ is the numerical value that represents the ratio of the length of the vertical side and the length of the horizontal side of any of these slope triangles. The slope of the line ℓ can be written as $\frac{4}{6}$, $\frac{2}{3}$, or any equivalent value.

Reflect:

Practice

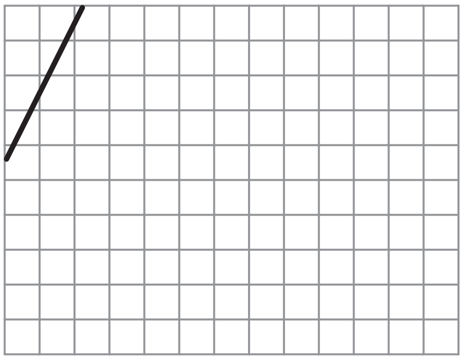
Name: ________________ Date: ________ Period: ________

1. Of the three lines shown, one has a slope of 1, one has a slope of 2, and one has a slope of $\frac{1}{5}$. Label each line with its slope.



2. Lin drew a line with a slope of $\frac{1}{3}$. Shawn drew a line with a slope of $\frac{1}{2}$. Who drew a steeper slope? Explain your thinking.

3. Refer to the line shown. Draw a second line with the same slope. What is the slope of each line? Explain your thinking.



Name: .... Date: .... Period: ....

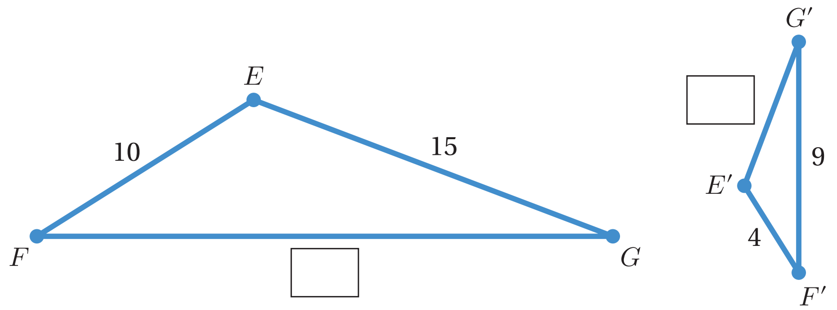
Practice

4. Triangle A has side lengths 3, 4, and 5. Triangle B has side lengths 6, 7, and 8.

   a. Explain how you know that Triangle B is *not* similar to Triangle A.

   b. Give possible sides lengths of a triangle that would be similar to Triangle A.

5. In the diagram, $\Delta EFG \sim \Delta E'F'G'$. Determine the missing values. Explain your thinking.

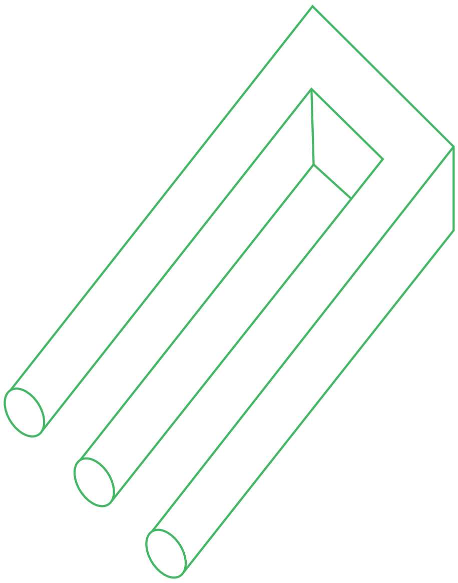


The figures may not be drawn to scale

6. The illustration shown is often referred to as an "impossible trident." What do you notice? What do you wonder?

   a. I notice . . .

   b. I wonder . . .



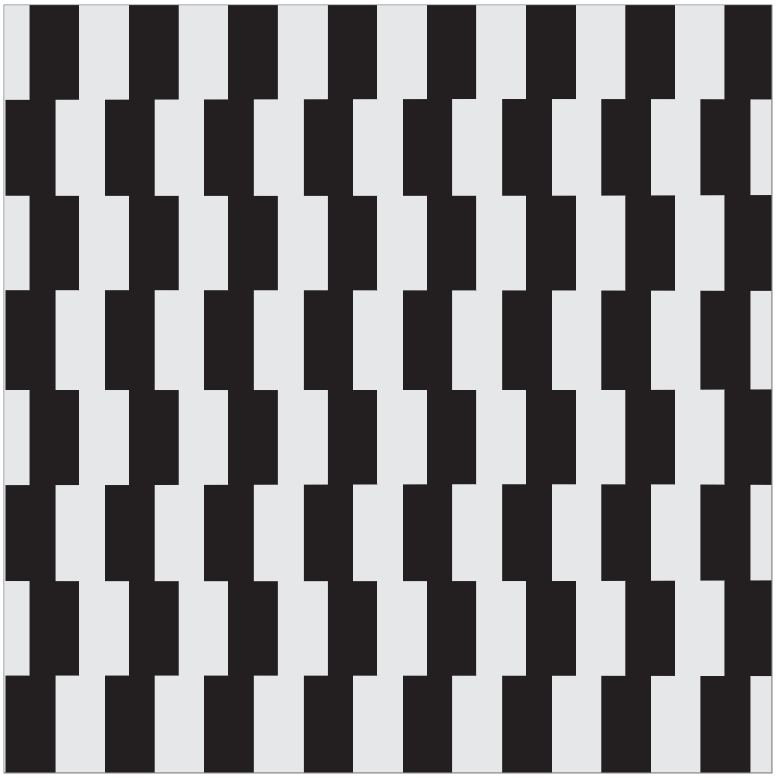
Unit 2 | Lesson 12 – Capstone

Optical Illusions

Let's create drawings that trick the eye.

Warm-up The Cafe Wall Illusion

Are the horizontal lines parallel or sloped? Explain your thinking and construct an argument to prove your response.

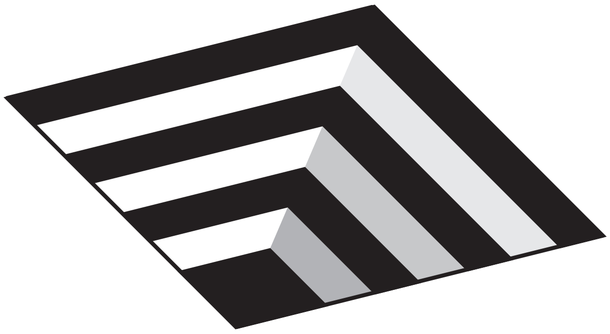


Log in to Amplify Math to complete this lesson online.

Name: ______ Date: ______ Period: ______

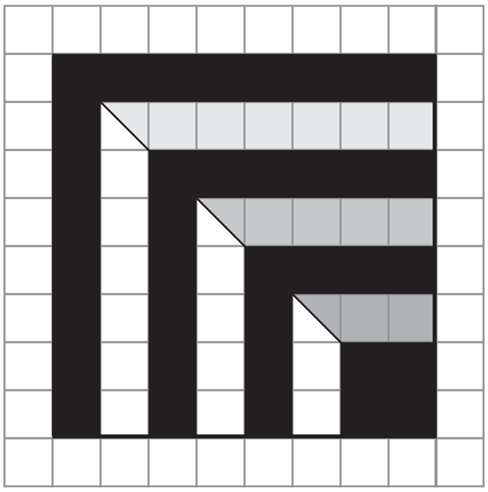
Activity 1 Is That a Hole in the Paper?

Consider this illustration.



1. Do you see an illusion? If so, describe the illusion and why you think it happens. If not, describe what you see.

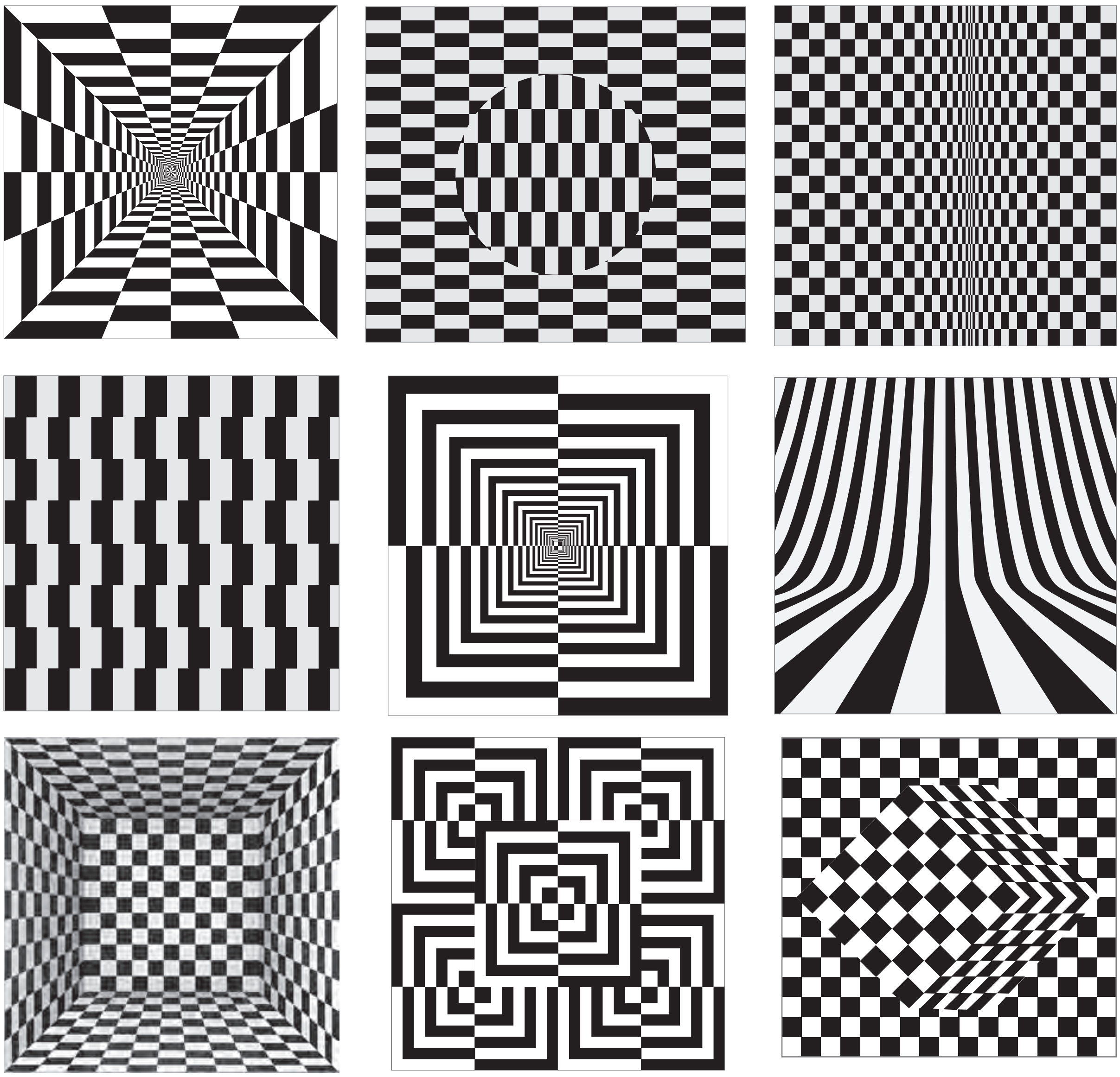
2. A grid was used to create the illusion. Study the grid. What math do you see?



3. How could a grid be useful in designing a pattern such as this?

Activity 2 Optical Illusions

Prominent mathematician and physicist Roger Penrose, along with many others in the fields of math, science, and art, have long tried to create optical illusions. And now you get to join them! Here are some examples of optical illusions to consider.



GreenBelka/Shutterstock.com, art_of_sun/Shutterstock.com, ScottMurph/Shutterstock.com, shooarts/Shutterstock.com, Master3D/Shutterstock.com

1. You will be given materials. Create your own optical illusion to see if you can trick your classmates' eyes!

Name: ______ Date: ______ Period: ______

Activity 2 Optical Illusions (continued)

2. You will now take part in a Gallery Tour of your peers' work. Record any notes in the table.

What patterns can you see in the artwork?	What makes optical illusions work best?

3. What connections do you see to topics you have learned about in Units 1 and 2?

Featured Mathematician

Sir Roger Penrose

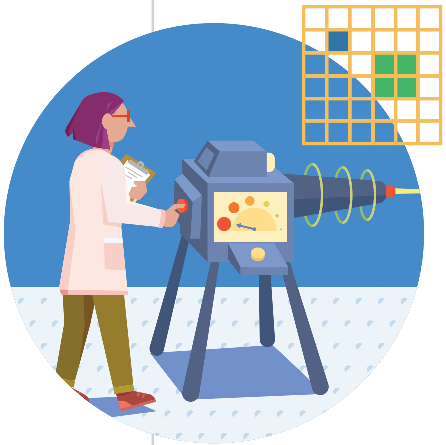
English mathematician and physicist Sir Roger Penrose turned just 27 in 1958, the year he and his father published a paper on an "impossible triangle," first devised by Swedish mathematician Oscar Reutersvärd in 1934. Now commonly known as the Penrose Triangle, the triangle appears to contain a combination of properties that cannot be realized by any three-dimensional object in space. Roger Penrose has since won many awards for his contributions on topics such as black holes or the relationship between consciousness and physics.

Reflect: How did you take the perspective of others as you created your optical illusion?

STOP

Unit Summary

Things are not always what they seem.



At the start of this unit, you met Renaissance architect Filippo Brunelleschi. He introduced a linear perspective into the world of art. With a system of grid lines that came together at a “vanishing point,” he could craft a sense of space and depth within a two dimensional canvas.

Using precise dilations, Brunelleschi blurred the line between what was real and what was a painting. Since Brunelleschi's time, dilation has enriched our culture. It has contributed to art, architecture, and design. But not all changes have been for the better.

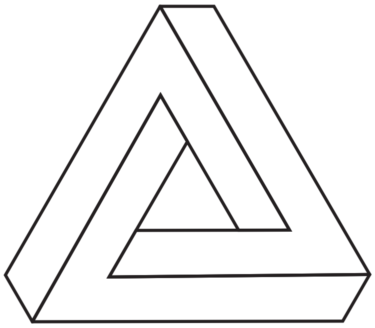
Dilations can also play tricks on us. Optical illusions can trick the brain into thinking things are farther or closer than they actually are. Additionally, consumer goods have been subject to “shrinkflation.” By using congruent angles and proportional corresponding sides, companies release packaging that is mathematically *similar* to the original, but is in fact imperceptibly smaller.

By understanding the rules for dilation, you can see through these tricks. While there is nothing wrong with using your eyes and trusting your intuition, math gives you a way to be precise in your measurements and to know exactly how and when the wool is being pulled over your eyes.

See you in Unit 3.

Name: ______________________ Date: __________ Period: __________

1. The triangle shown is a Penrose Triangle. What do you notice? What do you wonder?



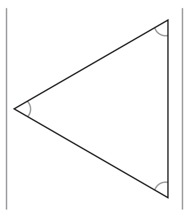
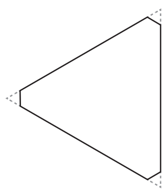
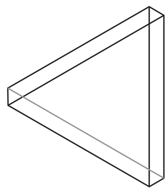
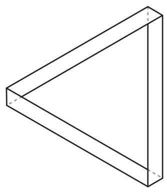
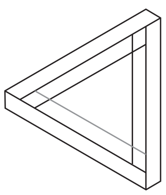
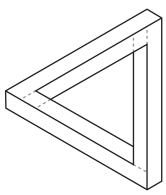
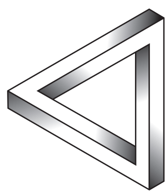
smx12/Shutterstock.com

a. I notice . . .

b. I wonder . . .

c. Create your own Penrose Triangle by following these steps:

Steps

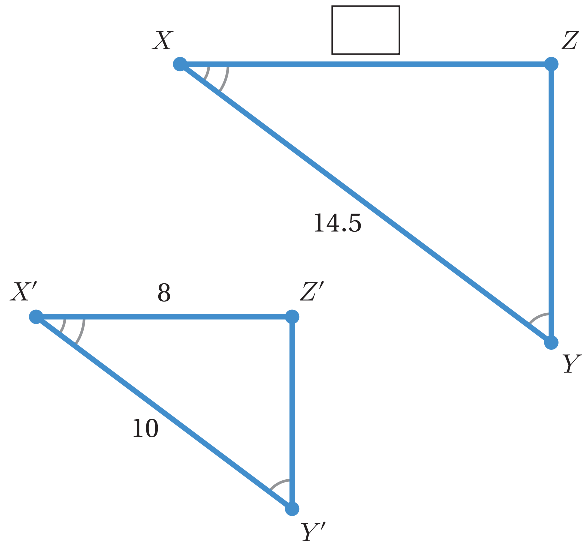
1 Draw an equilateral triangle. 	**2** Erase on the dotted lines. 	**3** Draw the three line segments shown. 	**4** Erase on the dotted lines. 
5 Draw three more line segments, as shown. 	**6** Erase on the dotted lines. 	**7** Shade the final result, as shown. 	

Draw your triangle here

Practice

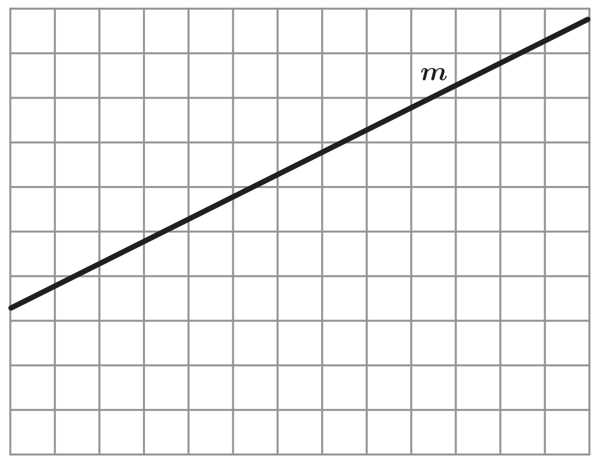
Name: .... Date: .... Period: ....

2. In the diagram, $\Delta XYZ \sim \Delta X'Y'Z'$. Find the missing length of side XZ. Explain your thinking.



The figures may not be drawn to scale.

3. What strategy can you use to determine the slope of line m shown? Explain your thinking.



My Notes:

UNIT 3

Linear Relationships

How many cups tall is your teacher? Find out in this unit as you make connections between proportional and linear relationships.

Essential Questions

- What does the slope of a line tell you about the line?
- What can proportional relationships teach you about linear relationships?
- What does it mean for an ordered pair to be a solution to a linear equation?
- *(By the way, did a 16-year-old really beat Michael Jordan in a game of one-on-one basketball?)*

SUB-UNIT

1 | Proportional Relationships

Narrative: Running at a constant rate results in a special kind of relationship between distance and time.

You'll learn . . .
- more about proportional relationships.
- how the slope of the line representing a proportional relationship relates to its unit rate.

SUB-UNIT

2 | Linear Relationships

Narrative: The thrill of a roller coaster ride is all about the slope between two points.

You'll learn . . .
- that not all relationships are proportional.
- how graphs, tables, equations, and verbal descriptions can represent nonproportional linear relationships.

SUB-UNIT

3 | Linear Equations

Narrative: Linear equations can help you sink the winning basket.

You'll learn . . .
- how an ordered pair can be a solution to a problem involving a linear relationship.
- how graphs, tables, and equations can be used to solve these problems.

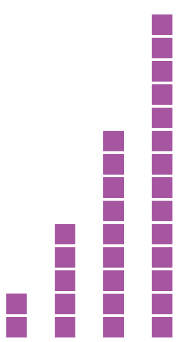
Visual Patterns

Let’s explore patterns in shapes and numbers.

Warm-up Which One Doesn’t Belong?

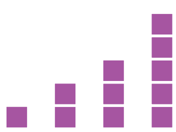
Examine the following patterns. Which pattern does not belong? Explain your thinking.

A.



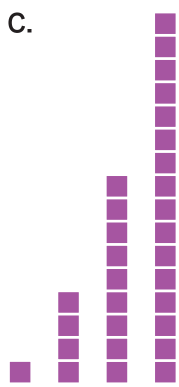
2, 5, 9, 14, . . .

B.



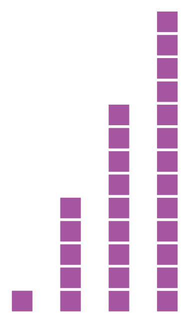
1, 2, 3, 5, . . .

C.



1, 4, 9, 16, . . .

D.



1, 5, 9, 13, . . .

Collect and Display: As you share your thinking, your teacher will collect the math language you use to add to a class display. You will continue to add and refer to this display throughout the unit.

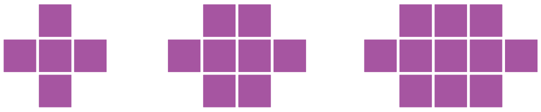
Log in to Amplify Math to complete this lesson online.

Name: Date: Period:

Activity 1 What Comes Next?

Consider the following pattern.

Figure 1 **Figure 2** **Figure 3** **Figure 4**



1. Study the pattern and draw a sketch of Figure 4. How many squares are in Figure 4?

2. What does Figure 10 look like? Draw or describe Figure 10 here. How many squares are in Figure 10?

Pause here while your class shares sketches.

Activity 1 What Comes Next? (continued)

3. Describe how the pattern is growing.

4. Complete the table to show the number of squares for different figure numbers.

Figure	1	2	3	4	10	26
Number of squares						

5. Write an expression that represents the number of squares in Figure n.

Are you ready for more?

What does Figure 0 look like? Draw or describe Figure 0 here.

Name: ______________________ Date: __________ Period: __________

Activity 2 Sketchy Patterns

You will be given a sheet with a new pattern on it and some problems about the pattern. Use the pattern to respond to the problems. Record your responses on the sheet.

After you have completed the problems on your sheet, compare your pattern with your group members. What is different? What is the same?

STOP

Unit 3 Linear Relationships

A Straight Change

Even as you read these words, things are changing: seas are rising, continents are breaking apart, and Earth is slowly, but surely, shifting underneath your feet. The very planet itself is changing, hurtling through space in its annual path around the Sun. Even inside you, new connections are being formed inside your brain.

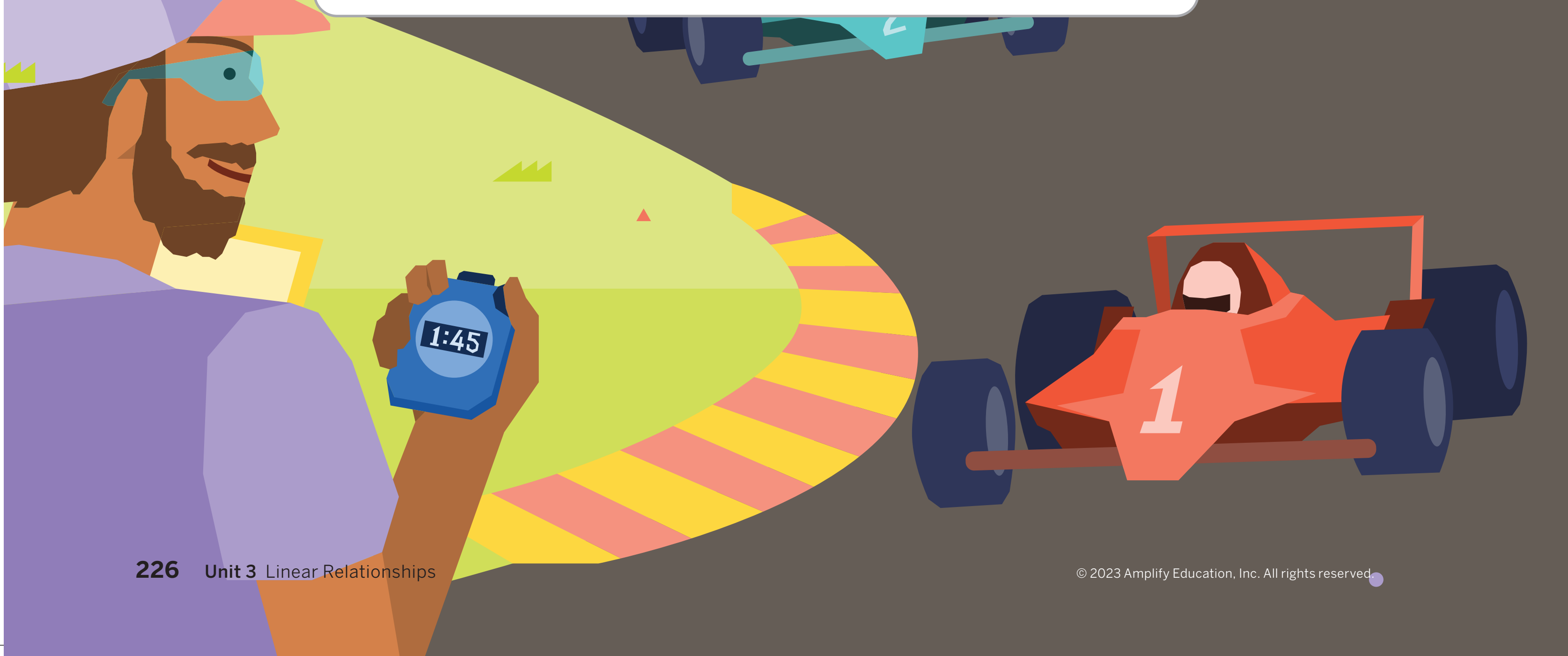
If there's one constant in the Universe, it's that things change. And not all changes are easy to see. Some are slow and can take eons to notice, while others happen so quickly you'll miss them if you blink. In both cases, it can be difficult to notice these changes with just the naked eye.

But math gives us a different way to observe and describe these changes. Consider a car speeding down a highway. With just your eyes, you wouldn't be able to say much about how fast that car was going. But once you *measure* how far that car traveled — and in what amount of time — suddenly you have a more precise way of expressing that car's speed.

By analyzing change mathematically, we can gain insights about the patterns and rules that make up these changes, giving us the ability to be precise and to make predictions about how something will behave.

Throughout your math career, you'll encounter many different kinds of changes. There's much more to come, but for now we begin with linear relationships.

Welcome to Unit 3.



Practice

Name: ____________ Date: ____________ Period: ____________

1. Kiran is describing a pattern. He says, "In Figure 1, there are 8 circles. In Figure 2, there are 11 circles. In Figure 3, there are 14 circles."

a Draw what Figures 1, 2, and 3 could look like.

b Write an expression that represents the number of circles in Figure n.

2. Design your own pattern of objects in which the number of objects in Figure n can be represented by the expression $2n + 4$. Draw Figures 1–3.

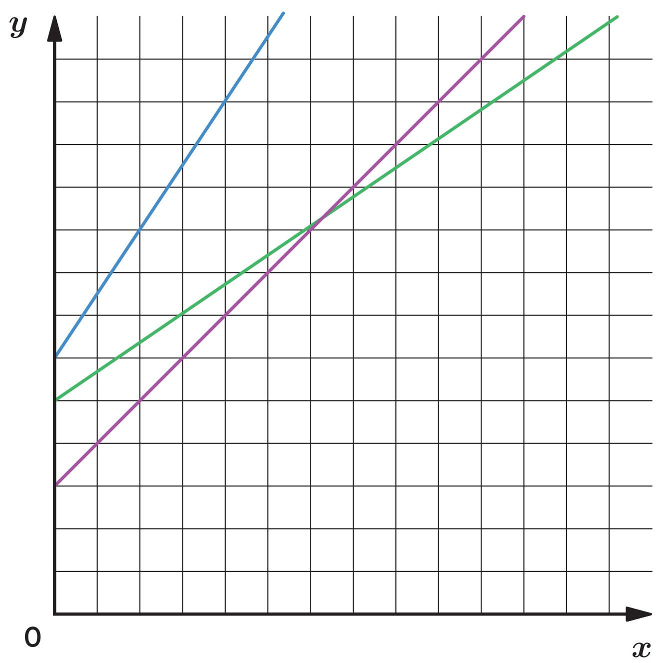
3. For each row, decide whether the expression in Column A is equivalent to the expression in Column B. If they are not equivalent, change the expression in Column B so that it is equivalent to the expression in Column A.

Column A	Column B	Equivalent?	If not equivalent, change Column B to . . .
$3x - 2x + 0.5$	$1.5x$		
$6(x + 4) - 2(x + 5)$	$2(2x + 7)$		
$3(x + 4) - 2(x - 4)$	$x + 4$		
$20\left(\frac{2}{5}x + \frac{3}{4}y - \frac{1}{2}\right)$	$\frac{1}{2}(16x + 30y - 20)$		

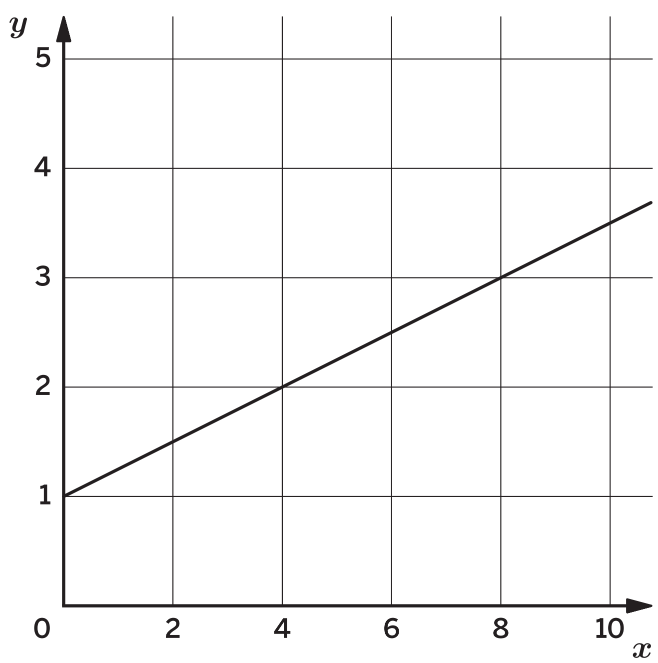
Practice

Name: .... Date: .... Period: ....

4. Of the three lines shown, one has a slope of 1, one has a slope of $\frac{2}{3}$, and one has a slope of $\frac{3}{2}$. Label each line with its correct slope.



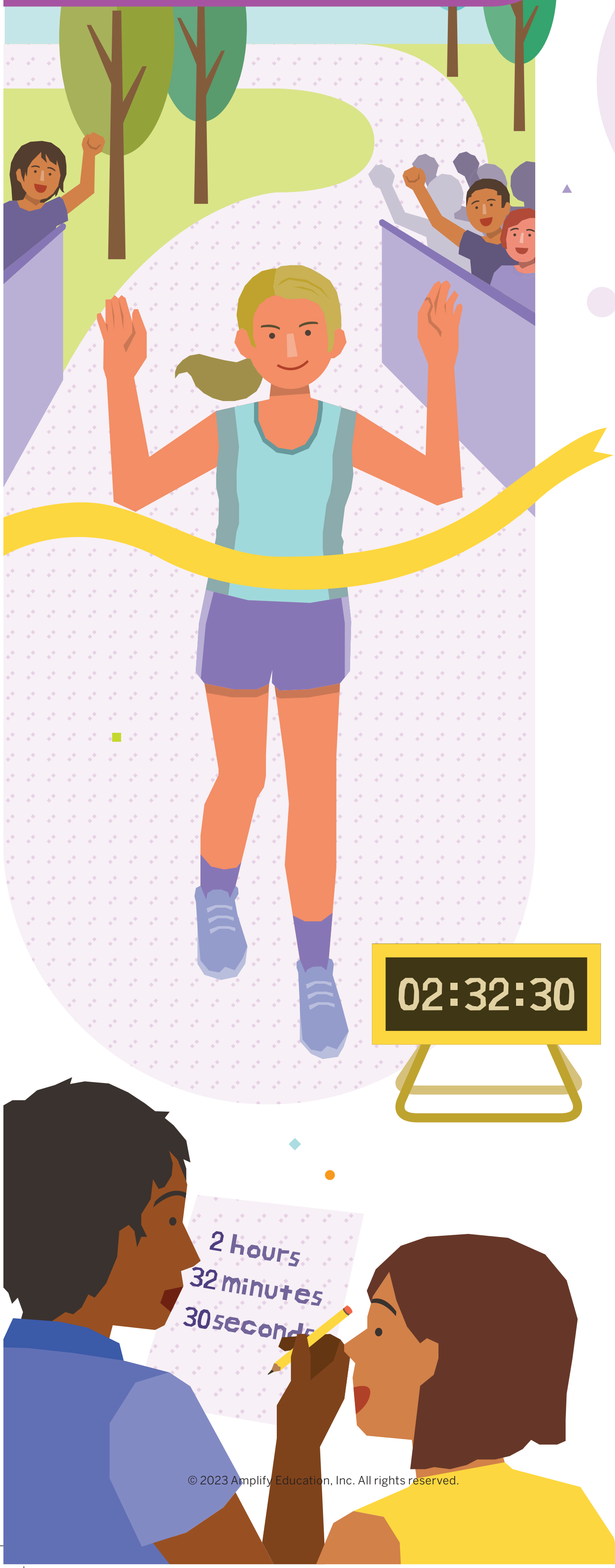
5. Determine the slope of the line shown. Show or explain your thinking.



SUB-UNIT

1 Proportional Relationships

Narrative Connections



How fast is a geography teacher?

On October 22, 1978, Grete Waitz ran her first marathon in New York City. A school geography teacher from Norway, Waitz had never run more than 13 miles at a time. But after some encouragement from her husband, she decided to tackle the New York City Marathon's 26.2 miles.

Few people had heard of Waitz since she hadn't yet raced in America. And when this newcomer appeared in the lead, spectators were shocked. She not only won the race by 10 minutes, she set a world record of 2 hours 32 minutes 30 seconds in the process.

Still, the last ten miles had tested Waitz's endurance. At the finish line, she took off her sneakers, chucked them at her husband, and vowed to never run a marathon again. But the very next year she broke that vow. She returned to New York to beat her previous world record, in a time of 2:27:33. She had become the first woman to run a marathon in under two-and-a-half hours.

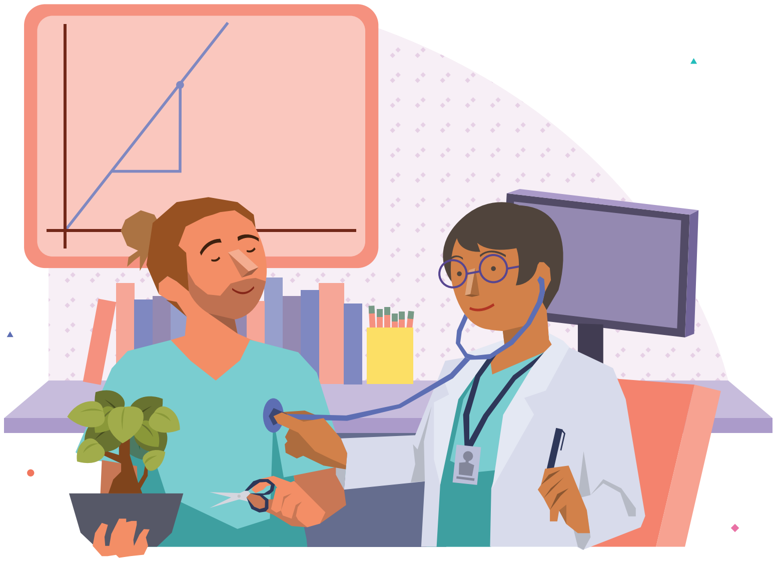
Grete Waitz's record-setting victories not only changed attitudes about women's racing —they changed the face of the New York City Marathon itself, and helped popularize it around the world. Her 10-minute margin of victory in 1978 transformed her from being a pacesetter to being a champion.

Whether you're calculating speed using a graph or an equation, figuring out the relationship between distance and time is essential for both the racer and the spectator. When the race begins, which one would you like to be?

Unit 3 | Lesson 2

Proportional Relationships

Let's explore the connection between points that lie on the line of a proportional relationship and the slope of the line.



Warm-up Heart Rate

1. Find your pulse. Count the number of heartbeats in 20 seconds and complete the first row in the table.

2. Assume the number of heartbeats per second remains constant. Based on your response to Problem 1, predict the number of heartbeats you will have in 1 minute.

Time	Number of heartbeats
20 seconds	
1 minute	

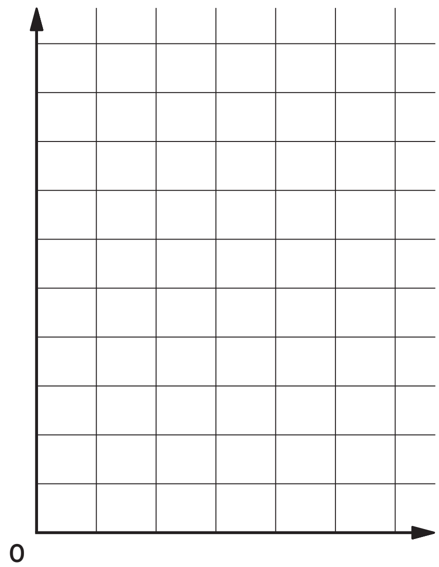
Log in to Amplify Math to complete this lesson online.

Name: ________ Date: ________ Period: ________

Activity 1 Graphing Heart Rates

When you are at rest, your heart pumps the least amount of blood you need because you are not moving or exercising. Your resting heart rate is normally between 60 (beats per minute) and 100 (beats per minute). Doctors care about heart rates because they can be indicators of good health.

1. Use your results from the Warm-up to graph the number of heartbeats you counted in 20 seconds and your prediction for the number of heartbeats in 60 seconds. Be sure to create a scale for your graph.

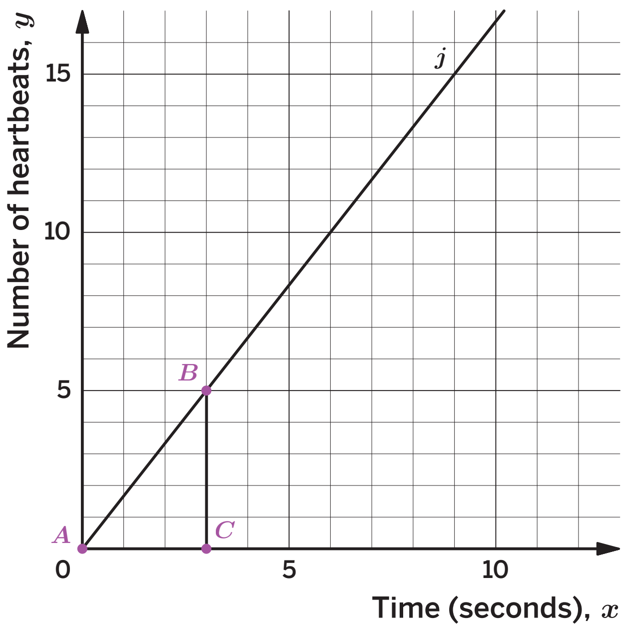


2. Draw a line connecting your two points. What is the slope of this line? What does it represent within the context of the scenario?

3. Plot an additional point on the line and label the coordinates of the point on the graph. What does this point represent within the context of the scenario?

Activity 2 The Equation of a Line

Kiran is training to swim in the 100-meter freestyle race. He measures his heart rate after he swims one length of the pool. He plots the point B as shown on the graph and draws line j to connect points B and the origin.



1. What are the coordinates of point B? What do they represent in context?

2. Is the point $(9, 16)$ on the line? Explain your thinking.

3. Is the point $(15, 25)$ on the line? Explain your thinking.

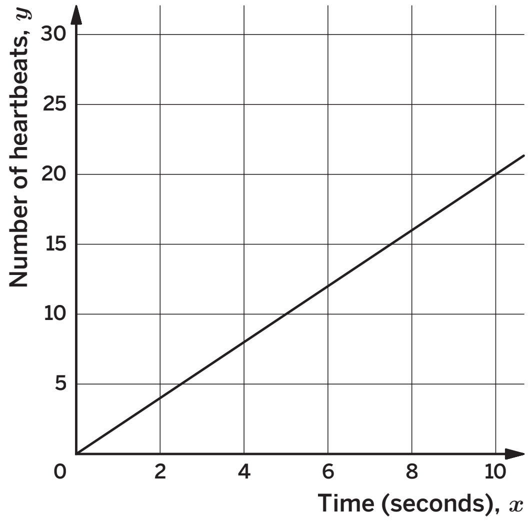
4. Suppose you know the x- and y-coordinates of a point. Write an equation that could be used to confirm the point is on line j.

Name: ______ Date: ______ Period: ______

Activity 3 Scale Factor

Kiran studied some data from similar swimmers with whom he might compete. Help him determine the slope and equation of each line.

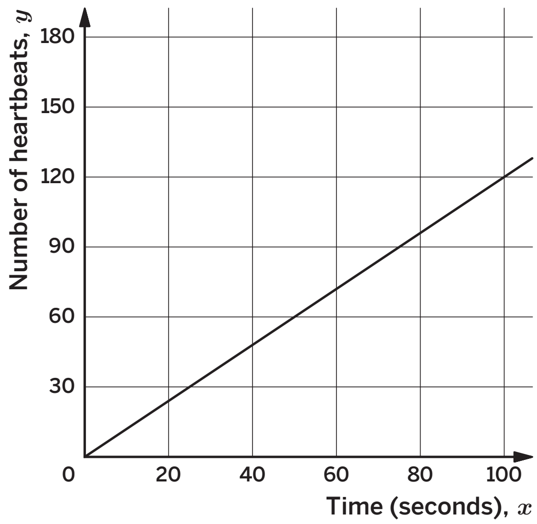
1. Jada's heart rate data



a Slope: ______

b Equation: ______

2. Tyler's heart rate data



a Slope: ______

b Equation: ______

STOP

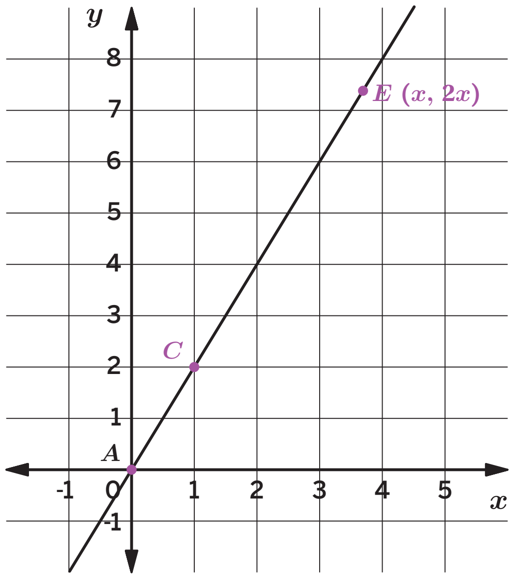
Summary

In today's lesson . . .

You found your resting heart rate. You collected your heart rate data in a table, and then represented it on the coordinate plane. The relationship between time and heartbeats was proportional and could be represented by a graph with the equation $y = kx$ where k is the constant of proportionality. For proportional relationships, the slope of the line that represents the relationship has the same value as the constant of proportionality. This value is also the unit rate.

Consider the line shown.

- The slope of the line shown is 2. For point C, the ratio of the vertical distance, 2, to the horizontal distance, 1, is equal to $2 : 1$, or 2.
- The constant of proportionality is 2 and is represented in the equation $y = 2x$. The equation tells you the y-values are always twice the x-values.
- The unit rate is 2 because the point $(1, 2)$ lies on the graph of the line.

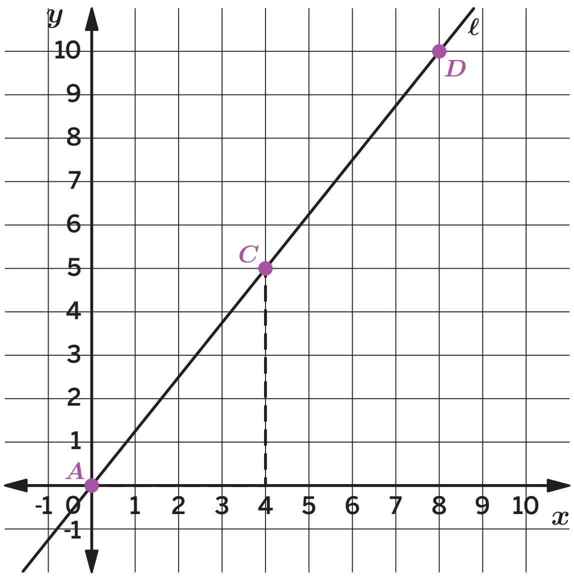


Reflect:

Name: .... Date: .... Period: ....

Practice

1. Line ℓ is shown in the coordinate plane.



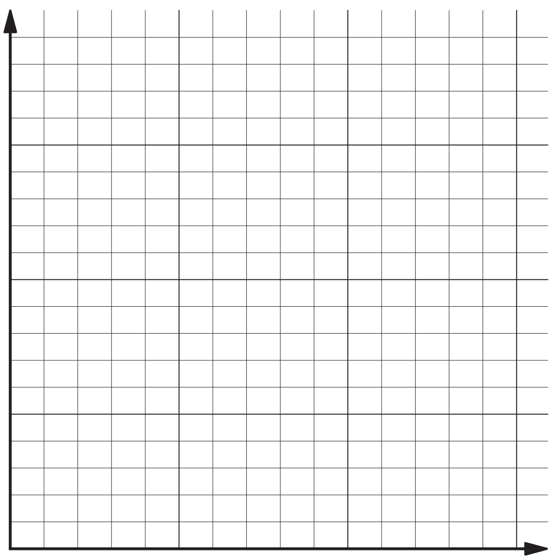
a. What are the coordinates of points C and D?

b. Is the point $(20, 24)$ on line ℓ? Explain your thinking.

c. Is the point $(16, 20)$ on line ℓ? Explain your thinking.

d. Write an equation that would allow you to test whether any point (x, y) is on line ℓ.

2. From rest, a car travels at a constant rate. After 2 hours, the car traveled 80 miles.



a. Graph the line showing the relationship between the car's distance and time. Be sure to create a scale and label the axes.

b. What is the slope of the line, and what does it represent in context?

c. What is the equation of the line?

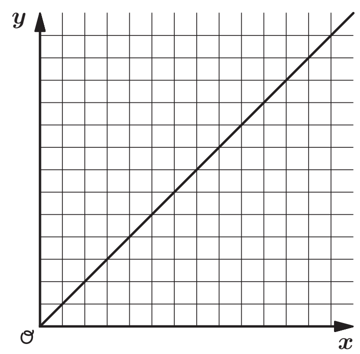
Practice

Name: .... Date: .... Period: ....

3. Which of the following *does not* represent a proportional relationship?

A. From rest, Clare walks at a constant speed of 3 mph.

B.



C.

x	y
2	4
5	10
6	12

D.

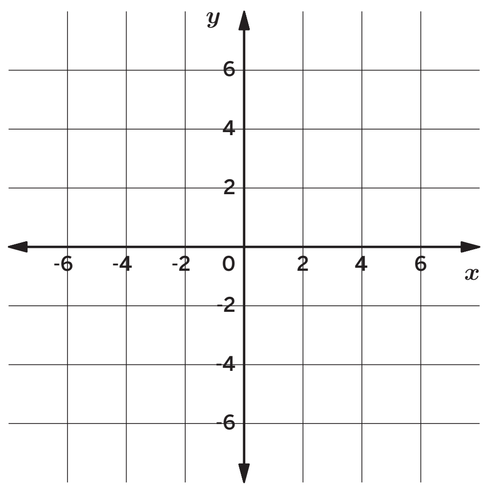
x	y
3	5
4	6
5	7

4. Draw and label a line that has each indicated slope.

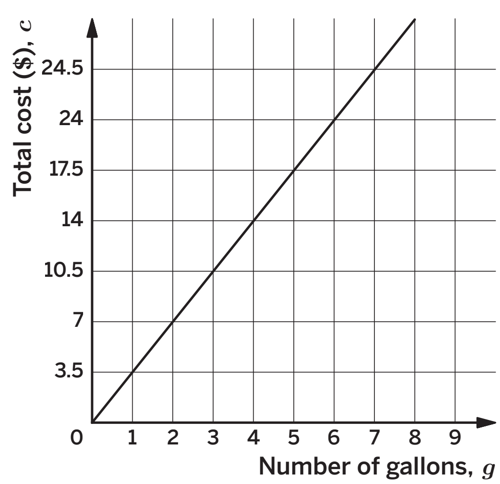
a Line a has a slope of 3.

b Line b has a slope of $\frac{1}{2}$.

c Line c has a slope of 5.



5. The graph shows the cost of gas per gallon at a local gas station. Write an equation that relates the total cost c to the number of gallons of gas g purchased.



Name: .... Date: .... Period: ....

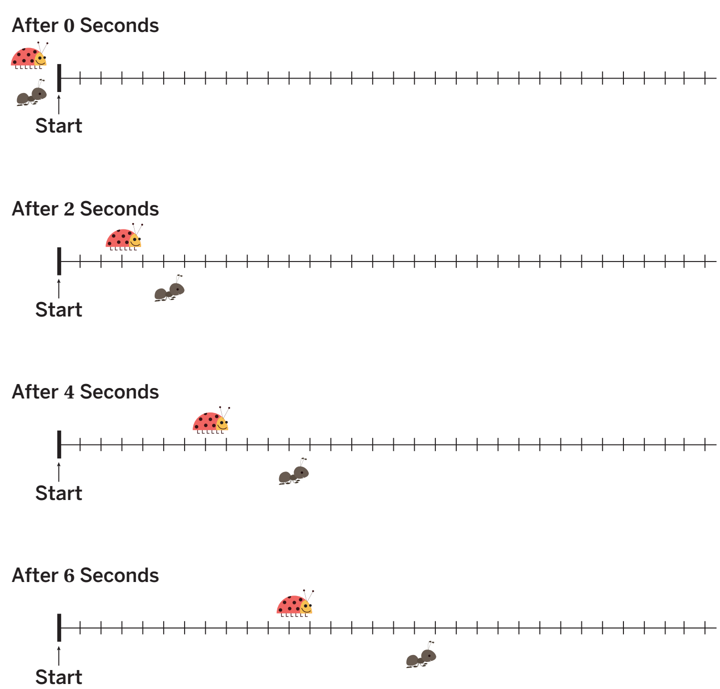
Unit 3 | Lesson 3

Understanding Proportional Relationships

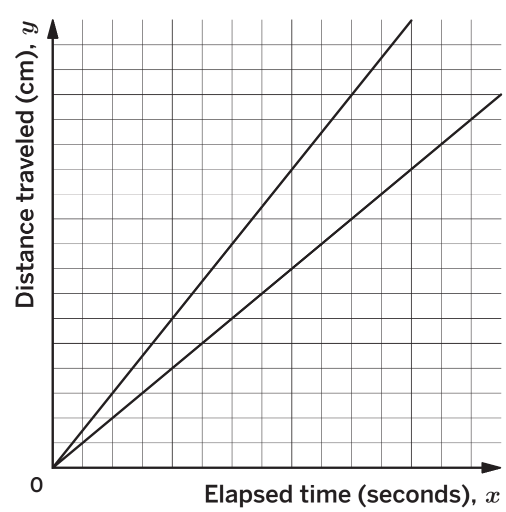
Let's study some graphs of proportional relationships.

Warm-up Traveling Bugs

Iowa State University's Department of Entomology hosts a traveling Insect Olympics with events that include Roach Races, Cockroach Pulls, and the Jumping Stick Jump. Lin decided to create her own race to see which insect travels faster, a ladybug or an ant. The diagrams with tick marks show the positions of the ladybug and the ant at different times. Each tick mark represents 1 cm.



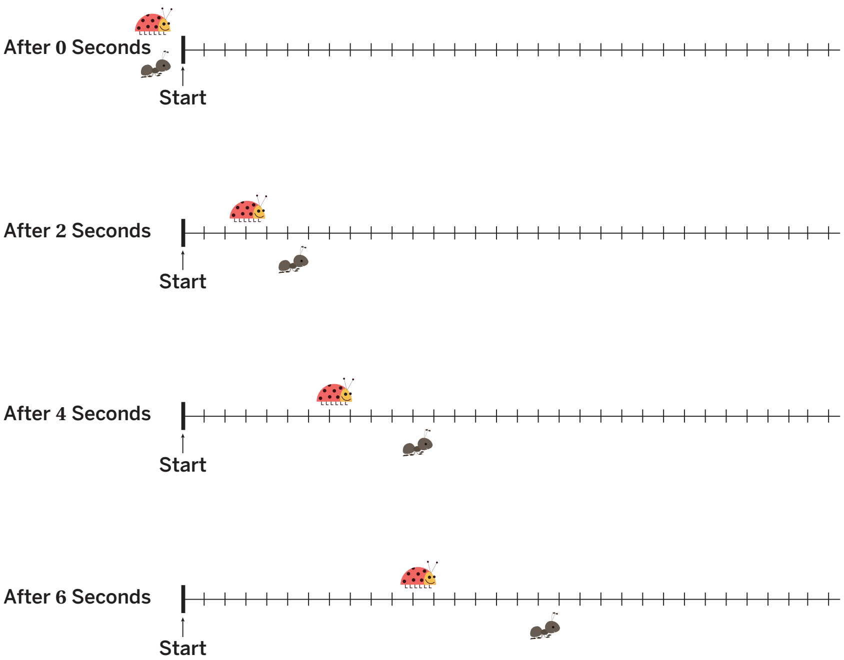
You will watch an animation that illustrates the relationship between distance and time for both insects. This relationship is represented by the following graph. Which line represents which insect? Label each line. Explain your thinking.



Log in to Amplify Math to complete this lesson online.

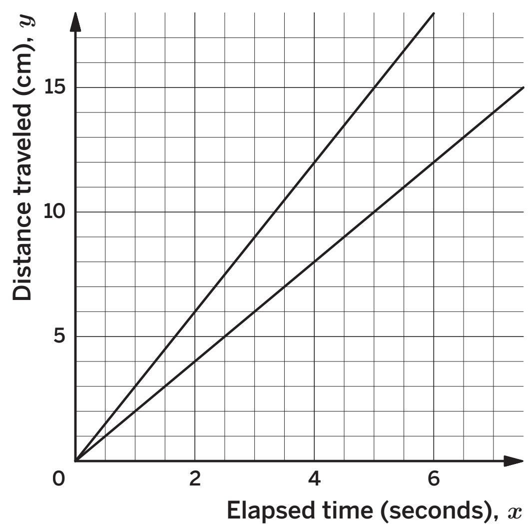
Activity 1 Moving Through Representations

These diagrams represent the same insects from the Warm-up.



1. Mark and label the point on each line that represents the time and position of each insect after traveling for 1 second.

2. Write the equation for each line. Be sure to define your variables.



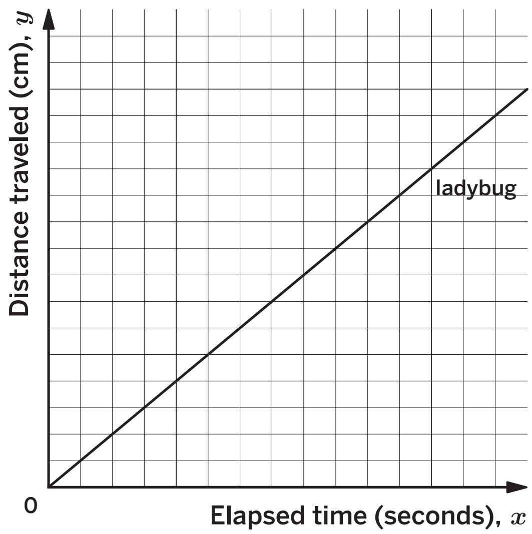
Are you ready for more?

Will there ever be a time when the ant is twice as far from the start as the ladybug? Explain or show your thinking.

Name: ________ Date: ________ Period: ________

Activity 2 Twice as Fast, Twice as Slow

Tyler provides two snails to race against the ladybug in Lin's race. Refer to the graph of the ladybug.



1. The first snail, Snaily McSnailface, travels twice as fast as the ladybug. Plot the snail's positions on the coordinate plane, label the line, and write an equation for the line.

2. The second snail, Sally Snailson, travels twice as slow as the ladybug. Plot the snail's positions on the coordinate plane, label the line, and write an equation for the line.

3. Order the equations from least constant of proportionality to greatest constants of proportionality.

Least **Greatest**

4. Compare the steepness of the lines and the constants of proportionality of their corresponding equations. What do you notice?

Reflect: How did you motivate yourself to stay focused throughout the activity?

STOP

Summary

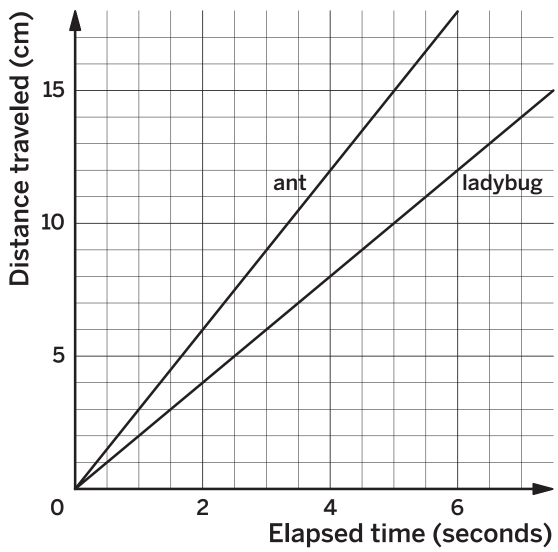
In today's lesson . . .

You made sense of the proportional relationship between distance and time using graphs.

When creating graphs to represent proportional relationships in context, it is important to label the axes and the scale. Without these, it is difficult to interpret the graphs in a meaningful way.

Consider the graph shown.

- Even without the scale, you can determine that the top line has a greater slope because it is steeper.
- With the scale, you can determine precisely how much faster one insect travels compared to the other insect. Then you can use these values to answer questions about the distance and length of time that each insect traveled.



Reflect:

Name: .... Date: .... Period: ....

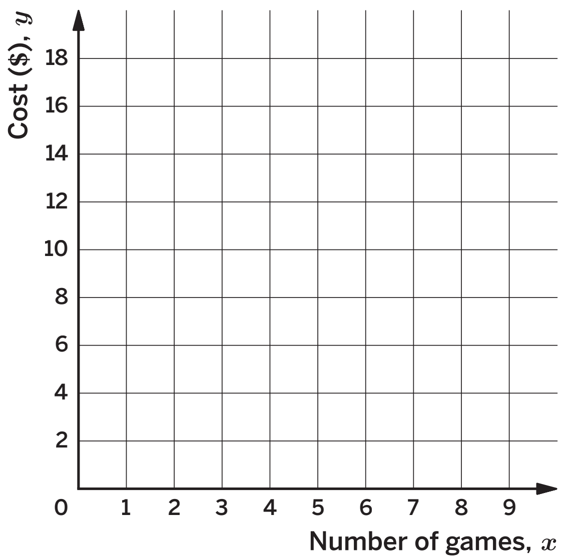
Practice

1. A mobile gaming service offers 6 games for $\$14.40$.

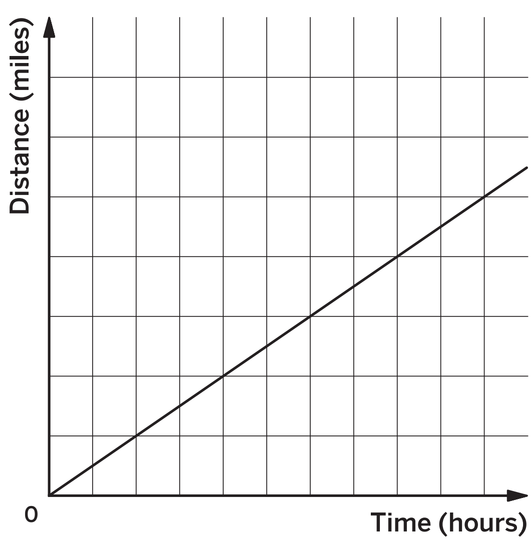
   a. If the total cost is proportional to the number of games purchased, write an equation that represents the relationship between total cost y and the number of games x. Show or explain your thinking.

   b. Graph this relationship on the coordinate plane.

   c. How many games can be purchased for $\$42$? Show or explain your thinking.



2. Priya jogs at a constant speed. The relationship between her distance and time is shown on the graph. Diego rides his bicycle at a constant speed that is twice as fast as Priya. Graph the relationship between Diego's distance and time on the same coordinate plane.



Practice

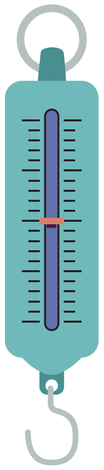
Name: .... Date: .... Period: ....

3. The table shows a proportional relationship between the weight on a spring scale and the distance the spring has stretched. Some of the values are missing.

a Complete the table.

b Describe the scales you could use on the x- and y-axes of a coordinate plane that would show all the distances and weights in the table.

Distance (cm)	Weight (newtons)
20	28
55	
	140
1	



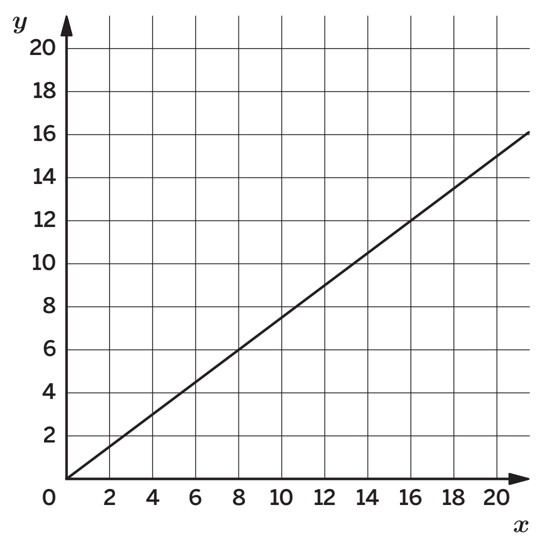
4. Solve each equation. Show your thinking.

a $2x + 60 = 100$

b $45 = 34 + \frac{1}{2}x$

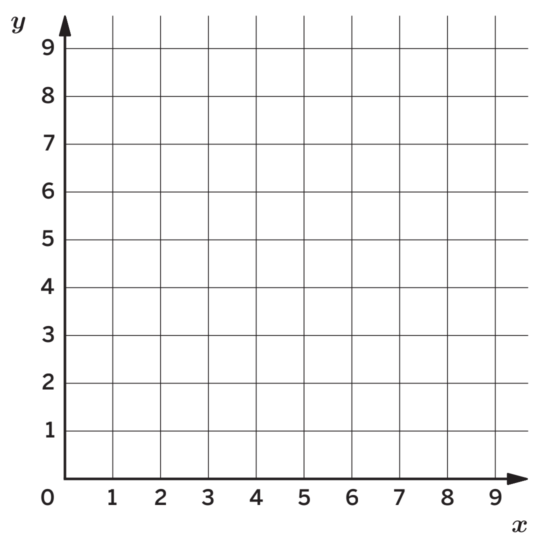
5. Think about what you have learned about graphs and equations of proportional relationships.

a Write an equation that represents the following graph.



Equation: ....

b Graph the relationship that is represented by the equation $y = 2x$.



Name: ______ Date: ______ Period: ______

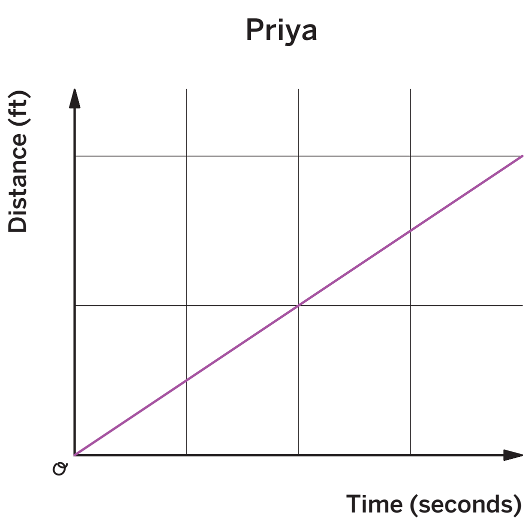
Unit 3 | Lesson 4

Graphs of Proportional Relationships

Let's think about scale.

Warm-up Would You Rather?

These graphs show the relationship between distance and time for two competitors running a race. Each runner maintains a constant speed.

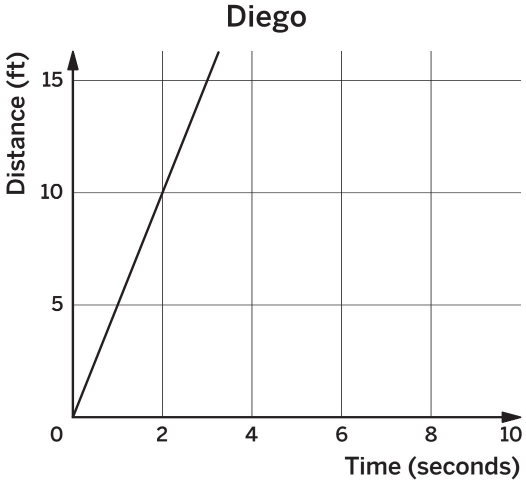


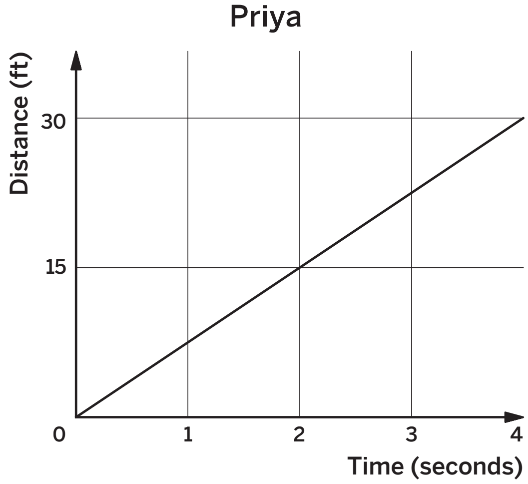
If you had to predict a winner based on these graphs, would you choose Diego or Priya? Explain your thinking.

Log in to Amplify Math to complete this lesson online.

Activity 1 Calculating the Rate

The graphs from the Warm-up are shown, now with additional information.





1. Calculate Diego's speed in feet per second, and use it to write an equation for the number of feet y traveled in x seconds. Show or explain your thinking.

2. Calculate Priya's speed in feet per second, and use it to write an equation for the number of feet y traveled in x seconds. Show or explain your thinking.

3. Does this new information change your thinking about who will win the race?

Pause here while your class shares responses.

4. Choose either Diego or Priya. Graph one runner's line on the other runner's graph, and compare the steepness of the lines. What do you notice?

Are you ready for more?

Han and Clare start out 1,000 ft apart and travel toward each other. Han is traveling at 20 ft per second, and Clare is traveling at 10 ft per second. How long will it take them to meet?

Name: ______________________ Date: __________ Period: __________

Activity 2 Card Sort: Proportional Relationships

You will be given 12 cards. Each card represents one of five possible proportional relationships.

1. Sort the cards into groups based on what proportional relationship they represent. Record your groupings in the table.

2. Write an equation for each group that can represent each card in the group. Record the equation in the table.

Group	Card(s) in this group . . .	Equation that can represent each card in the group . . .
1		
2		
3		
4		
5		

STOP

Summary

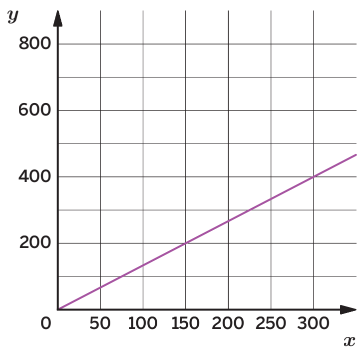
In today's lesson . . .

You explored how the scale of the axes can influence the appearance of a graph.

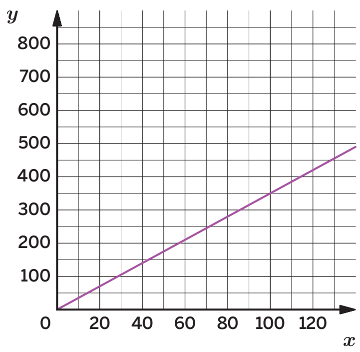
If you want to compare two proportional relationships, using graphs with different scales can be misleading about which proportional relationship has a greater constant of proportionality (or slope).

For example, consider these three graphs.

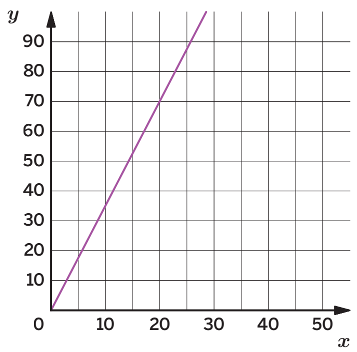
Graph A



Graph B



Graph C

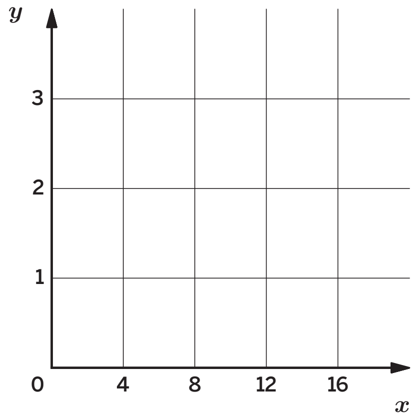


Without looking carefully, you might conclude that the slopes of the lines of Graphs A and B are much closer to one another than the slope of the line of Graph C. However, by finding the unit rate using a point from each line, you can determine that the slope of the lines of Graphs B and C are actually equivalent, and less than the slope of the line of Graph A.

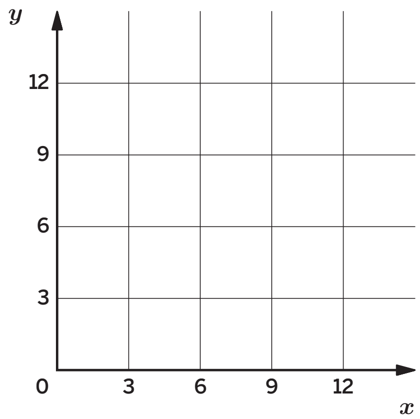
Reflect:

Name: ________________ Date: ________ Period: ________

1. Two coordinate planes with different scales are shown. Graph the equation $y = 0.75x$ on each coordinate plane. Explain your thinking.

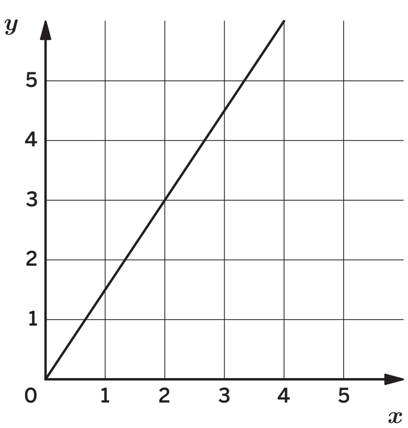
a 

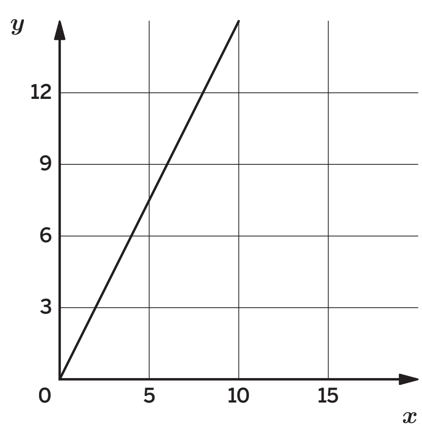
Explanation:

b 

Explanation:

2. A water tank is filled at a constant rate. The two graphs shown represent the same proportional relationship between the volume of water y and the amount of time x that has passed, in minutes.



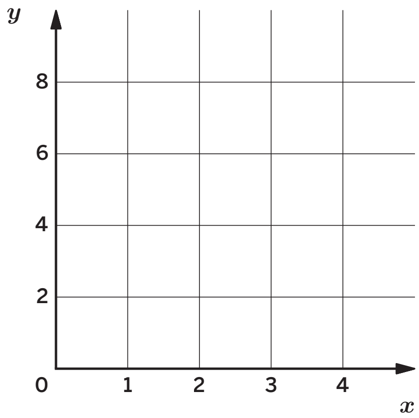


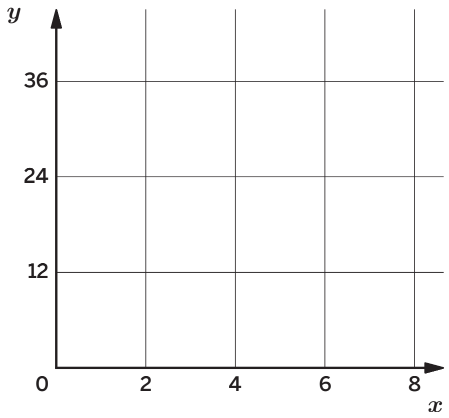
a Write an equation that represents the relationship between volume y and time x.

Practice

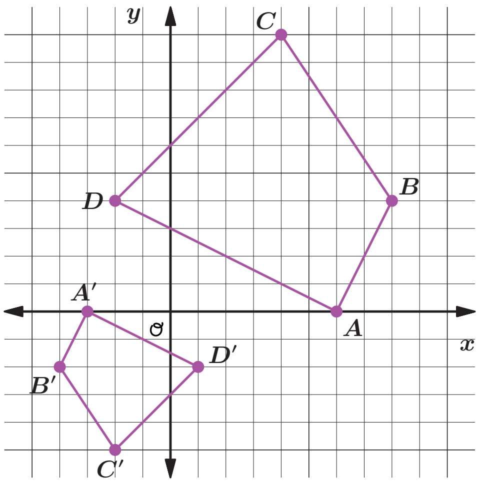
Name: .... Date: .... Period: ....

b Draw the missing lines on the following graphs to show the same proportional relationship as part a.





3. Describe a sequence of rotations, reflections, translations, and/or dilations that show that Quadrilateral $ABCD$ is similar to Quadrilateral $A'B'C'D'$. Be specific, stating the amount and direction of a translation, a line of reflection, the center and angle of a rotation, and the center and scale factor of a dilation.



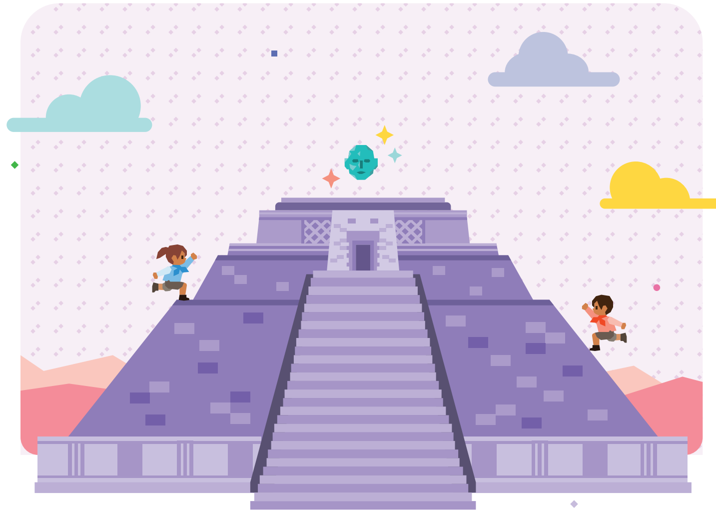
4. Given the graph of a line, describe how you can tell whether the line's slope is greater than 1, equal to 1, or less than 1.

5. Write an equation that represents a proportional relationship and whose line passes through the point $(25, 15)$. Show or explain your thinking.

Name: Date: Period:

Unit 3 | Lesson 5

Representing Proportional Relationships



Let's look at representations of proportional relationships.

Warm-up Defining Variables

Consider each of the following situations. For each situation, define variables to represent the quantities needed to calculate the indicated unit rate.

1. A rice pilaf recipe calls for 3 cups of water for every 2 cups of rice. Calculate the unit rate to describe how the amount of ingredients should change in order to prepare any amount of pilaf.

2. A tank is filled with water at a constant rate. After 20 minutes, there are 35 liters of water in the tank. Calculate the unit rate to describe how the amount of water should change after any amount of time.

3. While walking a dog, Shawn and the dog are both walking at a constant rate. When Shawn has walked 120 steps, the dog has walked 480 steps. Calculate the unit rate to describe how the amount of steps should change.

Log in to Amplify Math to complete this lesson online.

Activity 1 Representations of Proportional Relationships

Jada and Noah are practicing for the 100-meter dash. While each runs at a constant rate, they noticed they each take a different number of steps to travel the same distance. When Noah takes 10 steps, Jada takes 8 steps. When Noah takes 15 steps, Jada takes 12 steps. Solve these problems to describe the relationship between the number of steps Jada takes and the number of steps Noah takes.

1. Define your variables.

Let x represent ______________________________.

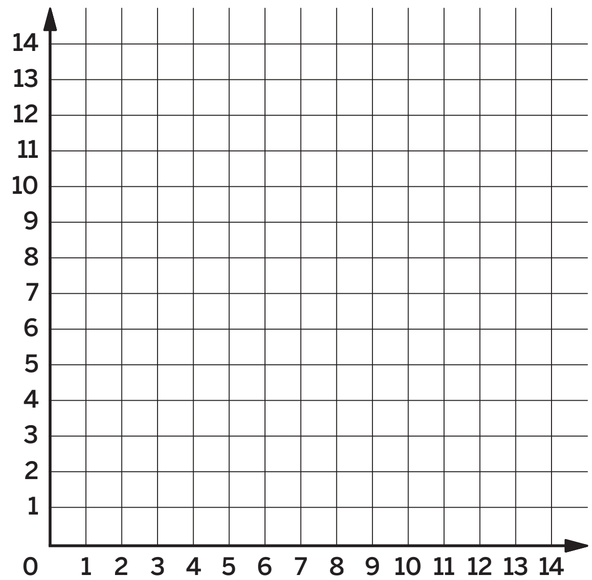
Let y represent ______________________________.

2. Create a table, a graph, and an equation to represent this situation.

Table:

Equation: ____________________

Graph:



3. Find the constant of proportionality in each representation. Explain your thinking.

Table:

Graph:

Equation:

4. What does the constant of proportionality mean in this context?

Name: ____________ Date: ________ Period: ________

Activity 2 Info Gap: Proportional Relationships

Competitions such as the Olympics or the New York City Marathon attract some of the best athletes in the world. You are about to learn more about two famous achievements of world-class athletes Jesse Owens and Grete Waitz.

lazyllama/Shutterstock.com

You will be given either a *problem card* or a *data card*. Do not show or read your card to your partner.

If you are given a *problem card*:	If you are given a *data card*:
1. Silently read your card, and think about what information you need to be able to solve the problem.	**1.** Silently read your card.
2. Ask your partner for the specific information that you need.	**2.** Ask your partner, *"What specific information do you need?"* and wait for them to ask for information.
3. Explain how you are using the information to solve the problem. Continue to ask questions until you have enough information to solve the problem.	**3.** Before sharing the information, ask, *"Why do you need that information?"* Listen to your partner's reasoning, and ask clarifying questions.
4. Share the problem card, and solve the problem independently in the space provided on this page.	**4.** Read the problem card, and solve the problem independently in the space provided on this page.
5. Read the data card and discuss your thinking.	**5.** Share the data card and discuss your thinking.

Pause here so your teacher can review your work. You will be given a new set of cards. Repeat the activity, trading roles with your partner.

STOP

Summary

In today's lesson . . .

You explored how proportional relationships can be represented in multiple ways.

Proportional relationships can be represented with written descriptions, equations, graphs, and tables. Which representation you choose depends on the purpose. The constant of proportionality can be determined in each representation. Remember the constant of proportionality has the same value as the slope of the line and the relationship's unit rate.

Written Description

A bakery recipe calls for 27 g of honey for every 6 g of flour.

Equation

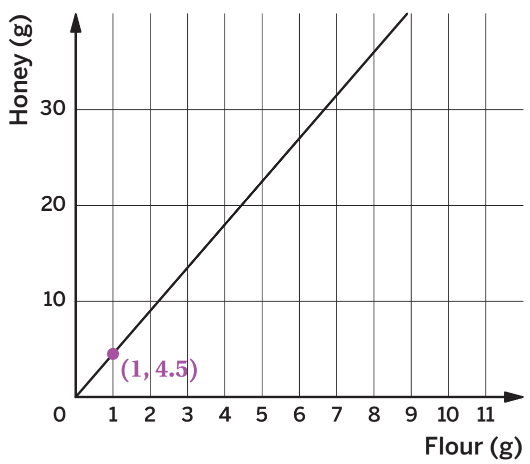
$y = 4.5x$, where y represents the number of grams of honey and x represents the number of grams of flour.

Constant of Proportionality 4.5

Table

Flour (g), x	Honey (g), y
1	4.5
6	27
10	45

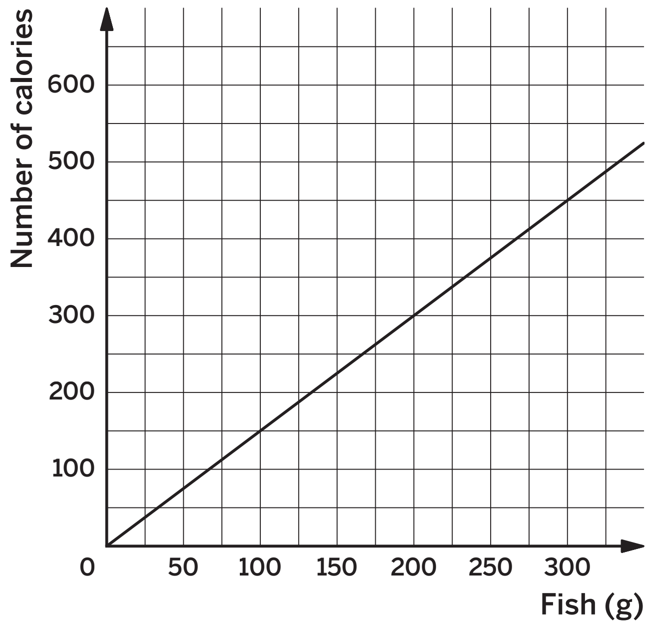
Graph



Reflect:

Name: ______________________ Date: __________ Period: __________

1. The graph shown represents the proportional relationship between grams of fish and number of calories.



a. Write an equation that represents this relationship. Let x represent the number of grams of fish, and y represent the number of calories. Show or explain your thinking.

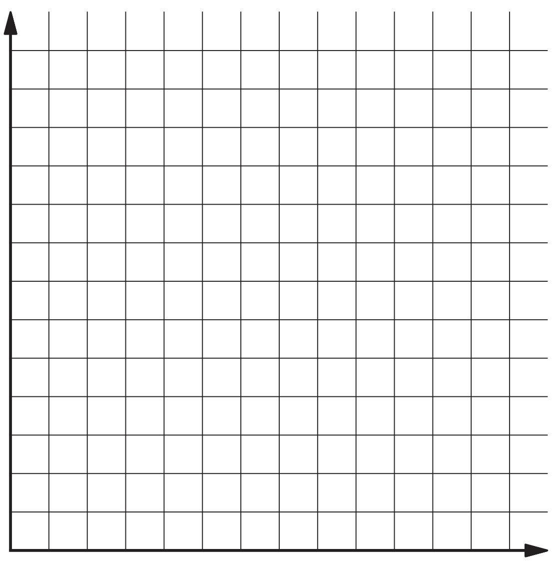
b. Use your equation to complete the table.

Fish (g)	Number of calories
$1,000$	
	$2,001$
1	

2. Students at a middle school are selling raffle tickets for a school fundraiser. They collect \$24 for every 10 raffle tickets they sell.

a. Let m represent the amount of money the students collect for selling r raffle tickets. Write an equation that represents the relationship between m and r.

b. Graph this relationship on the coordinate plane. Place r on the vertical axis and m on the horizontal axis. Label the axes and provide an appropriate scale. Make sure the scale is large enough to see how much money the students would raise if they sell $1,000$ tickets.



Practice

Name: .... Date: .... Period: ....

3. Solve each equation. Show your thinking.

a $\frac{x+5}{7}=\frac{3}{4}$

b $\frac{5}{2}=\frac{24-a}{8}$

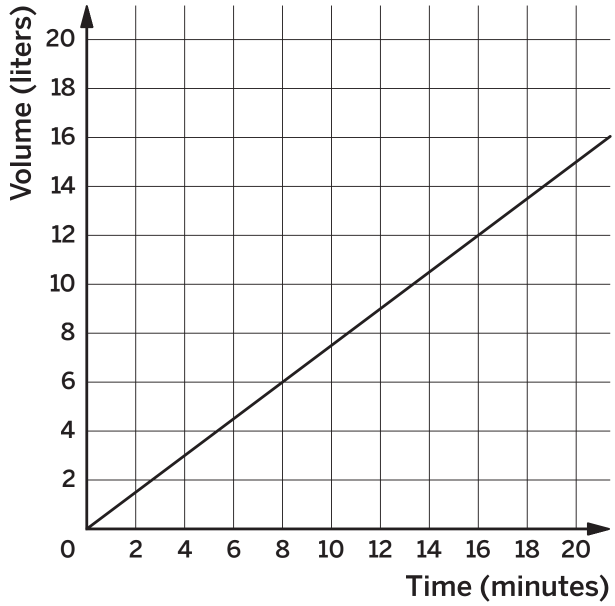
4. Write an equation for each of the following proportional relationships. Be sure to define your variables. Show or explain your thinking.

a Diego records his number of steps in the following table.

Number of steps	Time (minutes)
2,000	25
3,200	40

b A recipe calls for 3 cups of bulgur for every 2 cups of water.

c Water is filling a container at a constant rate, as shown in the graph.



Name: ________________ Date: ________ Period: ________

Unit 3 | Lesson 6

Comparing Proportional Relationships

Let's compare proportional relationships.

Warm-up Number Talk

Mentally find the value of each product.

1. $15 \cdot 2$

2. $15 \cdot 0.2$

3. $15 \cdot 0.5$

4. $15 \cdot 0.25$

5. $15 \cdot 2.25$

Log in to Amplify Math to complete this lesson online.

Activity 1 Gallery Tour

A high-tech toy company, E-Racers, is researching remote-controlled electric vehicles and drones. The company's designers have created some exciting prototypes. Each prototype has two models. The designers want to test the models against each other to determine the fastest model, and they need your help!

You will receive a sheet describing one of the prototypes. You will create a visual display that will be presented to the E-Racers board of directors (your teacher and classmates). The display should clearly demonstrate your thinking about which model is fastest, so be sure to use multiple representations in order to construct a convincing argument.

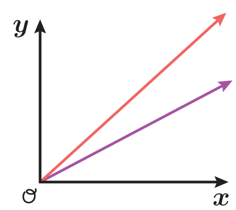
You and your classmates will participate in a Gallery Tour to inspect your display's accuracy.

When creating your visual display, consider the example shown.

Given Information

-
-
-

Graph



Questions

1.

2.

3.

STOP

Name: ________________ Date: ________ Period: ________

Summary

In today's lesson . . .

You compared proportional relationships using different representations.

When you are given more than one proportional relationship — even if they are represented differently — you can find the constant of proportionality (or unit rate) from each representation and use it to compare the relationships.

For example, let's compare Clare's earnings to Jada's earnings. Clare's earnings are represented by an equation and information about Jada's earnings are shown in the table.

Clare's earnings

$y = 14.5x$, where y represents the amount of money she earned for working x hours

Constant of proportionality: 14.5 (the coefficient of x)

Jada's earnings

Time worked (hours)	Earnings ($)
7	92.75
37	490.25

Constant of proportionality: 13.25 (the ratio of earnings to corresponding time worked, $92.75 : 7$, or 13.25)

Because $14.5 > 13.25$, Clare's earnings per hour are greater than Jada's earnings per hour.

Reflect:

Practice

Name: ______ Date: ______ Period: ______

1. A teacher wants to order soil for a school community garden. She collected information from two hauling companies.

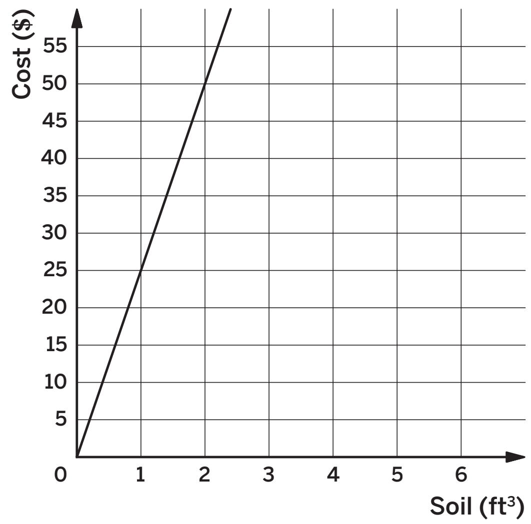
Green Garden Supplies

Green Garden Supplies provides their prices in the table shown.

Soil (ft^3)	Cost ($)
8	196
20	490
26	637

Happy Hauling Service

Happy Hauling Service provides their prices in the graph shown.



a. Calculate the constant of proportionality for each relationship. What do they mean for each company?

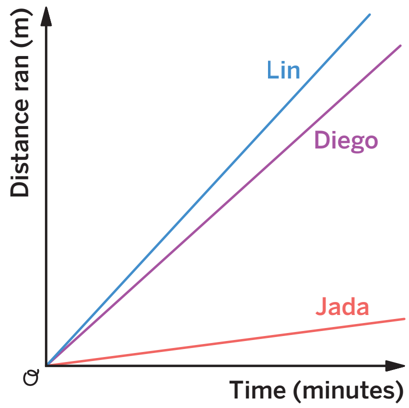
b. The teacher needs 40 ft^3 of dirt delivered and she has a budget of $\$1{,}000$. Which company should she hire? Show or explain your thinking.

2. Andre and Priya track the number of steps they walk, when they walk at a constant speed. Andre walks $6{,}000$ steps in 50 minutes. Priya writes the equation $y = 118x$, where y represents the number of steps and x represents the number of minutes she walks, to describe her rate. For one week, they each walk at their same constant speeds for a total of 5 hours. Who walks more steps? How many more steps? Show or explain your thinking.

Name: ____________ Date: ________ Period: ________

Practice

3. Lin runs twice as fast as Diego. Diego runs twice as fast as Jada. Could the following graph represent the speeds of Jada, Diego, and Lin? Explain your thinking.



4. The formula for converting temperature in degrees Celsius to degrees Fahrenheit is $C = \frac{5}{9}(F - 32)$. Use this formula to complete the table.

Temperature (°F)	Temperature (°C)
77	25
32	
	-18
-40	

5. Shawn deposits some money into a bank account every week. After one week, the total account balance is $11. After two weeks, the total account balance is $21. After three weeks, the total account balance is $31.

a. How much money does Shawn deposit each week?

b. How much money did Shawn have in the account initially, before the first week that money was deposited?

My Notes:

SUB-UNIT

2 Linear Relationships

Narrative Connections

How did a coal mine help build America's most famous amusement park?

Through the mountains of Carbon County, Pennsylvania there once ran a 9-mile stretch of railroad. This stretch was called the Mauch Chunk Switchback Railway. Here, cars filled with coal would roll down from the summit. Following them would be cars of mules. Their job was to drag the empty cars back to the top. Then someone at the coal company had an idea. What if instead of carrying coal, they carried passengers? So while the Mauch Chunk Switchback remained an operating coal rail during the day, in the evening it gave "pleasure rides" to visiting tourists.

Inventor LaMarcus Adna Thompson saw this and became fascinated. He applied the same design to something that required no mules and provided big thrills: a roller coaster!

Named the Switchback Railway, Thompson's invention opened to the public at Coney Island on June 16th, 1884. The novel experience drew huge crowds. Visitors came by the hundreds to climb a 45 ft tower, board a car, and roll along one track to another tower.

Modern roller coasters might be a far cry from the original model, with their death-defying heights and speeds exceeding 100 mph. But despite all the bells and whistles, the basic principles are the same. The speed of a car and the distance it travels depend on the steepness of the roller coaster's slopes. Learning to calculate the slope between two points can make the difference between a sleepy cruise and the ride of a lifetime.

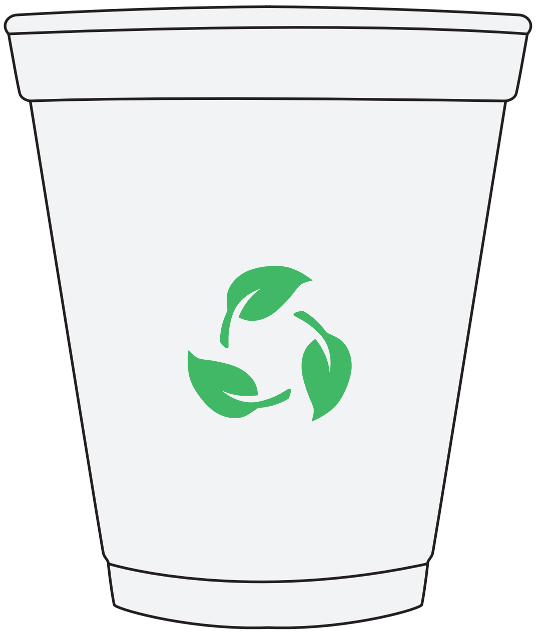
Unit 3 | Lesson 7

Introducing Linear Relationships

Let's explore some relationships between two variables.

Warm-up How Many Cups Tall Is Your Teacher?

This biodegradable foam cup is shown in actual size. About how many more biodegradable foam cups would you have to stack, starting from the ground, to reach the top of your math teacher's head?



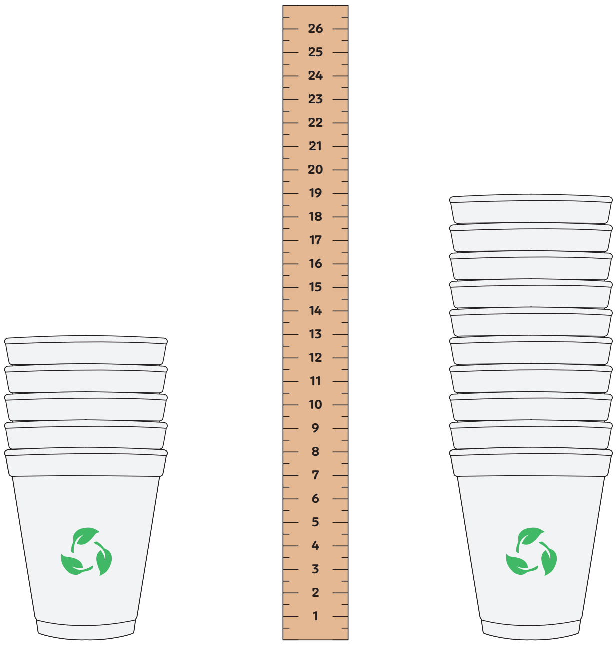
Log in to Amplify Math to complete this lesson online.

Name: .... Date: .... Period: ....

Activity 1 Stacking Cups

Let's compare two stacks of cups.

- **One stack has one cup with 4 additional cups nested inside one another and is measured to be 13 cm tall.**
- **The other stack has one cup with 9 additional cups nested and is measured to be 19 cm tall.**



1. Complete the table.

Number of additional cups	Height of stack (cm)
1	
2	
4	13
9	19
14	
19	
29	

Activity 1 Stacking Cups (continued)

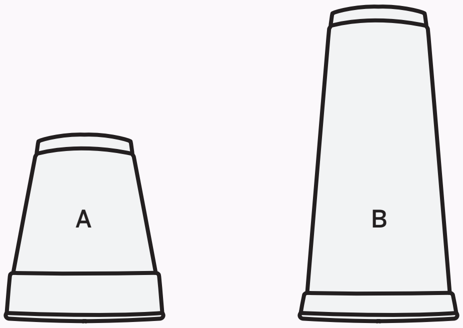
2. How much does each additional cup add to the height of the stack?

3. How many additional cups need to be stacked onto the original cup to reach a height of 1 m?

4. How many biodegradable foam cups would you have to stack, starting from the ground, to reach the top of your math teacher's head?

Are you ready for more?

Consider these two different cups. Imagine more cups, like each one, are stacked on top of these. Which stack will be taller after 3 additional cups are stacked onto the first cup? After 100 additional cups? Explain your thinking.

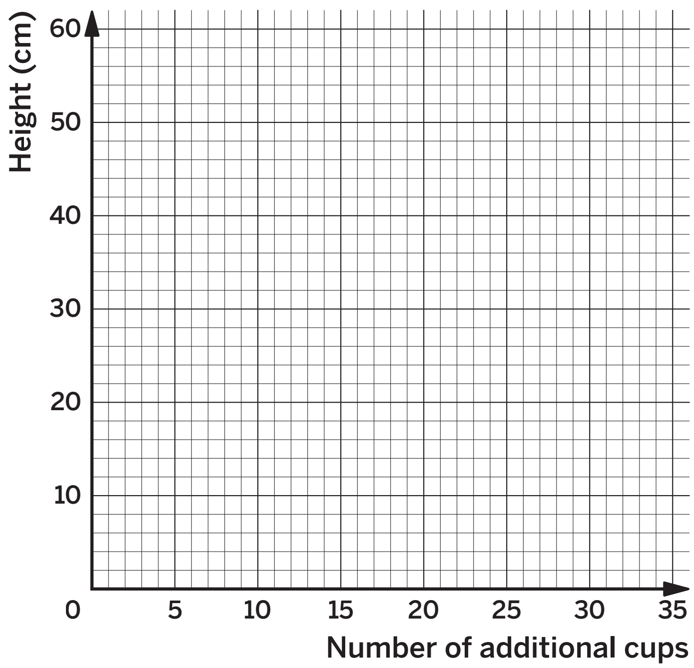


Name: .... Date: .... Period: ....

Activity 2 Graph It

Plan ahead: How can you clearly communicate the proof that your pattern works?

1. Refer to your table from Activity 1. Graph four points from your table on the coordinate plane shown.



2. What patterns do you notice?

3. Prove your pattern works by plotting a new point. Show how the point's coordinates can be found using the pattern you described in Problem 2.

Activity 2 Graph It (continued)

4. Connect the points on your graph to form a line. What is the slope of the line? What does the slope mean in this situation?

5. At what point does your line intersect the vertical axis? What do the coordinates of this point tell you about the stack of cups?

6. What are some ways that you can tell that the number of cups is *not* proportional to the height of the stack?

Discussion Support: How does the graph of the relationship support your response to Problem 6? How do the ordered pairs support your response?

STOP

Name: ________________ Date: ________ Period: ________

Summary

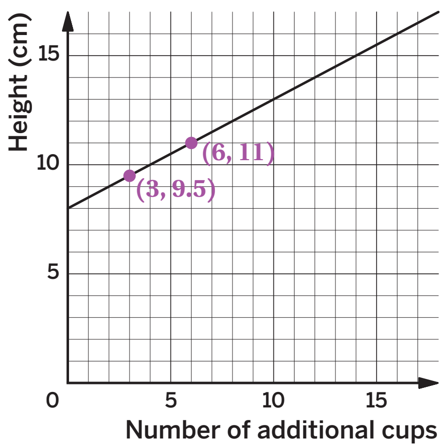
In today's lesson . . .

You encountered a linear relationship that was not proportional. A ***linear relationship*** is any relationship between two quantities in which there is a constant ***rate of change***. This means that when one quantity increases by a certain amount, the other quantity changes by a proportional amount.

A *proportional relationship* is a special type of linear relationship, but not all linear relationships are proportional.

For example, the graph displays the height, in centimeters, of the stacks for different additional numbers of cups.

- As the number of cups increases by 1, the height of the stack increases by 0.5 cm, which means the rate of change is 0.5 cm per additional cup.
- You can see the line intersects the vertical axis at the point $(0, 8)$. This means if 0 additional cups are added, the ***initial value*** of the cup has a height of 8 cm.



The relationship shown is linear, but it is not proportional because the line does not pass through the origin. You can also see that the ratios of the vertical distance to the horizontal distance of the points are not equivalent. $9.5 : 3$ is not equivalent to $11 : 6$.

Reflect:

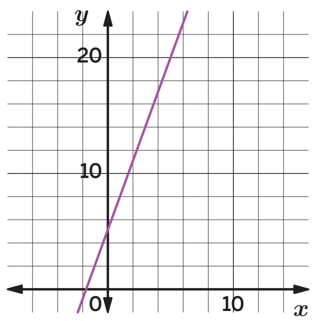
Name: ______________________ Date: ____________ Period: ____________

Practice

1. Choose *all* the relationships that are linear, but not proportional. Explain your thinking.

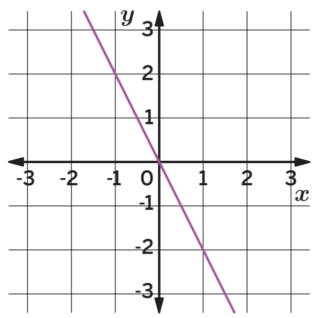
A. From rest, Diego walks at a constant speed of 5 km per hour.

B.



C. $y = 2x$

D.



E. A giraffe is initially 3 ft tall and grows 6 in. every month for a year.

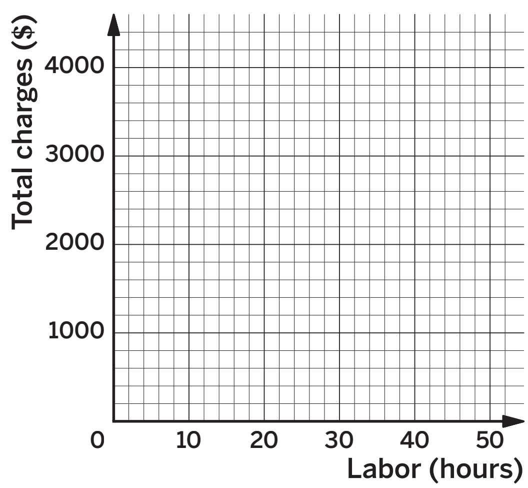
F.

x	y
2	4
3	5
5	7

2. To paint a house, a painting company charges a flat fee of \$500 for supplies, plus \$50 for each hour of labor.

a How much would the painting company charge to paint a house that requires 5 hours of labor? A house that requires 50 hours?

b Draw a line representing the relationship between x, the number of hours it takes the painting company to paint the house, and y, the total cost of painting the house. Label the two points from part a on the graph.



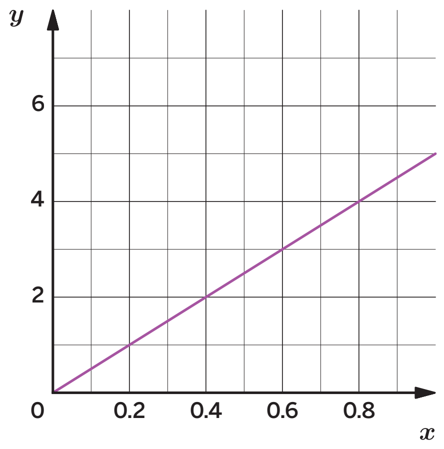
c Find the slope of the line you graphed. What does the slope mean in this context?

Name: ________ Date: ________ Period: ________

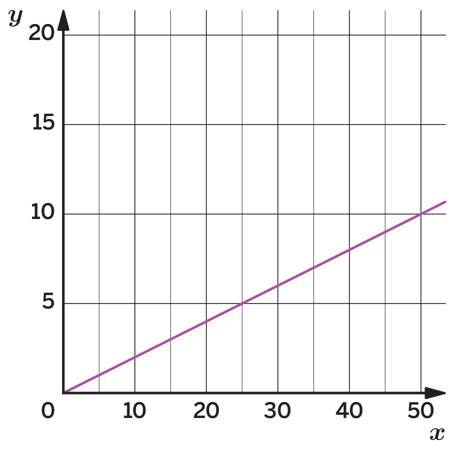
Practice

3. Which of these relationships has a slope that is *different* from the other three relationships? Explain your thinking.

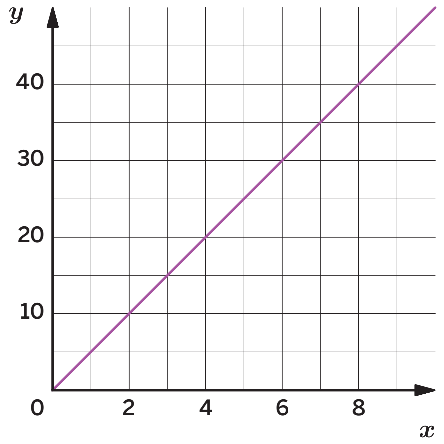
A.



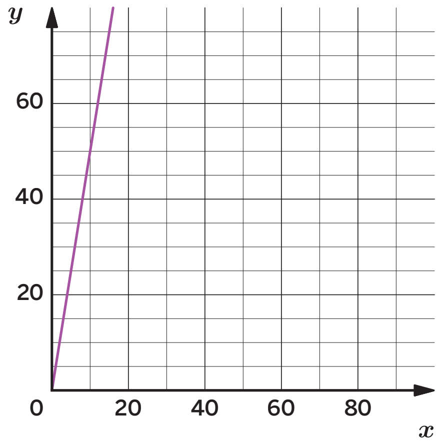
C.



B.



D.



4. Which of the following tables does *not* represent a proportional relationship? Explain your thinking.

A.

x	y
-2	-8
$\frac{1}{2}$	2
1	4

B.

x	y
-1	-1
1	1
2	2

C.

x	y
1	3
3	5
5	7

Unit 3 | Lesson 8

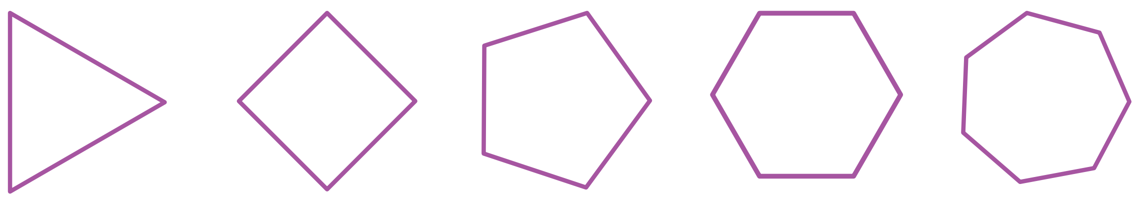
Comparing Relationships

Let's explore how linear relationships are different from other relationships.

Warm-up Diagonals

Consider the following regular polygons.

Triangle **Quadrilateral** **Pentagon** **Hexagon** **Heptagon**



1. How many diagonals are present in each polygon? Record your responses in the table.

2. Could the relationship between the number of sides of a polygon and the number of diagonals be linear? Explain your thinking.

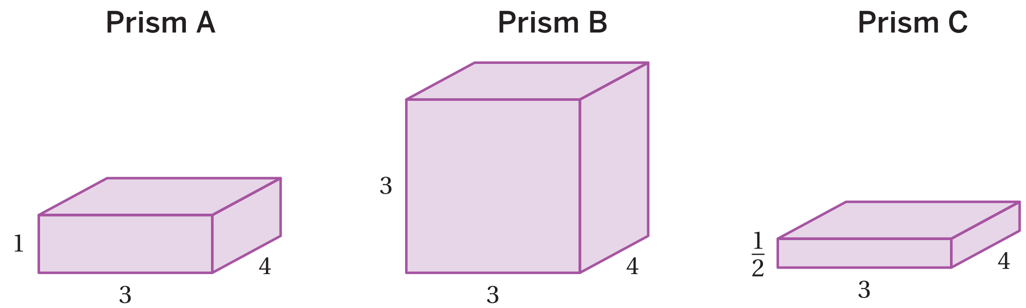
Polygon	Number of sides	Number of diagonals
Triangle	3	
Quadrilateral	4	
Pentagon	5	
Hexagon	6	
Heptagon	7	

Log in to Amplify Math to complete this lesson online.

Name: ______ Date: ______ Period: ______

Activity 1 Total Edge Length, Surface Area, and Volume

Consider the following rectangular prisms, each with different heights, but the same base dimensions of 3 units by 4 units.



1. For each prism, determine the length of each edge. Then determine the sum of the edge lengths for each prism. Record your responses in the table.

Prism	Height (units)	Total edge length (units)
A	1	
B	3	
C	$\frac{1}{2}$	
Any prism with base 3 units by 4 units	x	

2. What is the surface area of each prism? Record your responses in the table.

Prism	Height (units)	Surface area (square units)
A	1	
B	3	
C	$\frac{1}{2}$	
Any prism with base 3 units by 4 units	x	

Activity 1 Total Edge Length, Surface Area, and Volume (continued)

3. What is the volume of each prism? Record your responses in the table.

Prism	Height (units)	Volume (cubic units)
A	1	
B	3	
C	$\frac{1}{2}$	
Any prism with base 3 units by 4 units	x	

4. Consider these relationships for a rectangular prism with base dimensions 3 units by 4 units.

- The relationship between height and total edge length.
- The relationship between height and surface area.
- The relationship between height and volume.

Which of the relationships are linear? Explain your thinking.

Name: ______ Date: ______ Period: ______

Activity 2 Card Sort: Tables of Linear Relationships

You will be given a set of cards. Each card contains a table with information about a relationship.

1. Based on the information in each table, sort the cards by whether they represent *possible linear relationships* or *nonlinear relationships*. Record your card sort in the table.

Possible linear relationships	Nonlinear relationships

2. For each card that represents a possible linear relationship, determine the rate of change. Explain the meaning of the rate of change in context. You may not need all of the rows in the table.

Card	Rate of change	Explanation

STOP

Summary

In today's lesson . . .

You saw examples of linear and nonlinear relationships.

- *Linear relationships* have a constant rate of change.
- *Nonlinear relationships* do not have a constant rate of change.

Consider these tables which show the cost for bike rentals at two different companies. Bikes-R-Us charges a one-time rental fee *and* an hourly fee. Meadowland Bicycles posts their fees based on the number of hours.

Bikes-R-Us

Time (hours)	Cost ($)
1	14
3	26
5	38

$+2$, $+2$ (time); $+12$, $+12$ (cost)

At Bikes-R-Us, the rate of change is $6 per hour.

Based on the information in the table, the relationship between the total cost and time is *linear*.

Meadowland Bicycles

Time (hours)	Cost ($)
1	10
3	26
5	38

$+2$, $+2$ (time); $+16$, $+12$ (cost)

For Meadowland Bicycles, the cost for each additional hour varies.

There is no constant rate of change, so this is a *nonlinear relationship*.

Reflect:

Practice

Name: .... Date: .... Period: ....

1. Based on the information in each table, decide whether it could represent a linear relationship. Show or explain your thinking.

a. Money in a savings account

Time (months)	Money ($)
3	60
5	80
8	110

b. Cost of water usage

Volume of water (gallons)	Cost ($)
500	7.5
2,000	40
5,000	110

2. A taxi service charges $1.50 for the first $\frac{1}{10}$ mile, and then $0.15 for each additional $\frac{1}{10}$ mile.

a. Based on the description, do you think the relationship will be linear? Why or why not?

b. Complete the table with the missing information.

c. Determine if the relationship between distance traveled and total cost of the trip is a linear or nonlinear relationship. Show or explain your thinking.

Distance traveled (miles)	Total cost ($)
$\frac{1}{10}$	1.50
$\frac{2}{10}$	1.65
1	
$3\frac{1}{10}$	

Practice

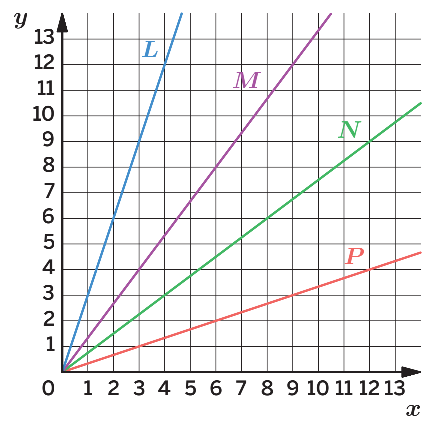
Name: .... Date: .... Period: ....

3. The equation $y = 4.2x$ could represent a variety of different real-world situations.

   a. Write a description of a real-world situation that could be represented by this equation. Decide what quantities x and y represent in your situation.

   b. Create a table and a graph that represent the situation.

4. Match each equation with the graph of its line.



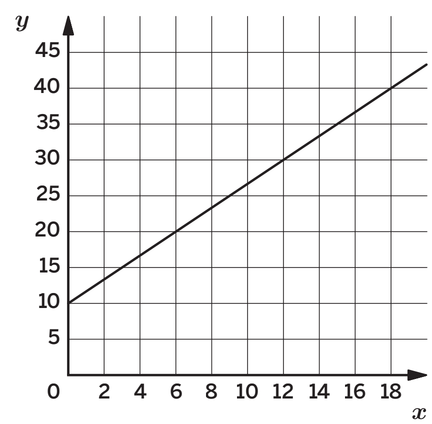
   a. $y = \frac{1}{3}x$ ....

   b. $y = \frac{4}{3}x$ ....

   c. $y = 3x$ ....

   d. $y = \frac{3}{4}x$ ....

5. Refer to the line graphed on the coordinate plane. What is the slope of this line?



   A. 10

   B. $\frac{1}{3}$

   C. $\frac{3}{5}$

   D. $\frac{5}{3}$

Name: .... Date: .... Period: ....

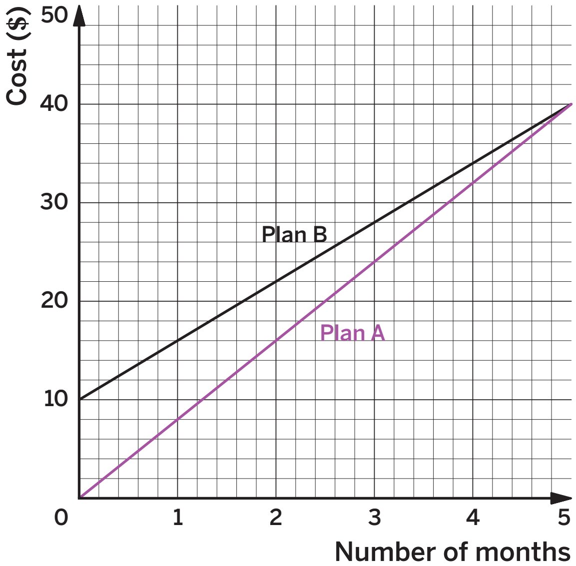
Unit 3 | Lesson 9

More Linear Relationships

Let's explore some more linear relationships and their equations.

Warm-up Would You Rather?

The lines on the graph show the cost of two different subscription plans from Audio Line, a music streaming service.



Which plan would you rather choose? Explain your thinking.

Log in to Amplify Math to complete this lesson online.

Activity 1 Let's Compare

Let's compare the two music subscription plans from the Warm-up.

1. Complete the table showing the total cost for the first five months of each plan from Audio Line.

Plan A: Pay $\$8$ every month.

Number of months, x	Cost (\$), y
1	
2	
3	
4	
5	

Plan B: Pay a one-time fee of $\$10$ to sign up, and then pay $\$6$ every month.

Number of months, x	Cost (\$), y
1	
2	
3	
4	
5	

2. Write an equation that represents the cost y, in dollars, after x months of service.

Plan A:

Plan B:

3. Diego wants to subscribe to one of these plans for 1 year. Which plan should he choose? Explain your thinking.

Name: .... Date: .... Period: ....

Activity 2 Card Sort: Slopes, Vertical Intercepts, and Graphs

You will be given six cards describing different real-world situations and six cards containing graphs.

1. Match each situation with its corresponding graph.

Situation A: .... Situation B: .... Situation C: ....

Situation D: .... Situation E: .... Situation F: ....

2. Select one proportional relationship and one nonproportional relationship. For each relationship you select, complete the following problems.

a How can you determine the slope from the graph? Show or explain your thinking.

Proportional: Situation	**Nonproportional:** Situation ....

b Explain what the slope represents in the situation.

Proportional: Situation	**Nonproportional:** Situation ....

c What is the vertical intercept? What does it tell you about the situation?

Proportional: Situation	**Nonproportional:** Situation ....

d Write an equation that represents the situation.

Proportional: Situation	**Nonproportional:** Situation ....

Compare and Connect:
After sharing your matches, examine Graphs 2 and 3. What does the value 40 represent in each situation? Discuss with your partner.

Activity 3 Matching Equations

The manager at Honest Carl's Funtime World compares two different roller coasters. The table gives the roller coasters' heights and speeds for the hill before the first drop.

Jack Rabbit	Thunderbolt
Starts from a platform with a height of 14 ft, and then climbs 4 ft per second.	Starts from a platform with a height of 4 ft, and then climbs 14 ft per second.

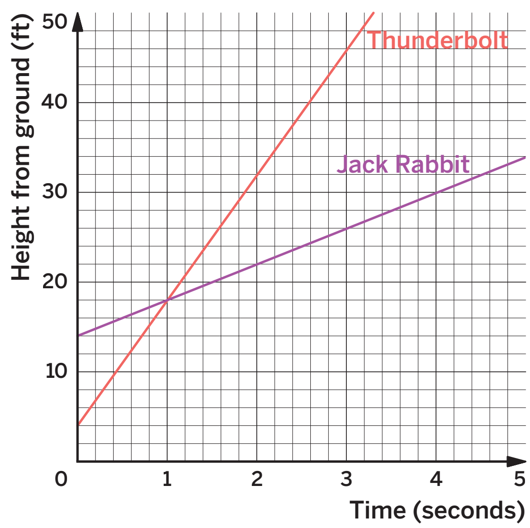
1. Match each roller coaster with its equation, where x represents the time in seconds and y represents the height, in feet, of the roller coaster above the ground.

Equation

a) Jack Rabbit .... $y = 4 + 14x$

b) Thunderbolt .... $y = 4x + 14$

2. The manager of Honest Carl's Funtime World wants to know the height of each roller coaster after 3 seconds. How can she use the equation or graph to determine this?



Historical Moment

***m* is for . . . slope?**

Why do we use the letter "*m*" to represent slope? Some speculate that it comes from the French word *monter*, which means to climb, while others speculate that it comes from the Latin word *montagne*, for mountain. Other countries even use different letters to represent the slope of a line! For example, Sweden uses the letter "*k*" and Uruguay uses the letter "*a*." Although the meaning of slope is the same across countries, the origin of why the letter "*m*" is used is still uncertain.

STOP

Name: ______________________ Date: __________ Period: __________

Summary

In today's lesson . . .

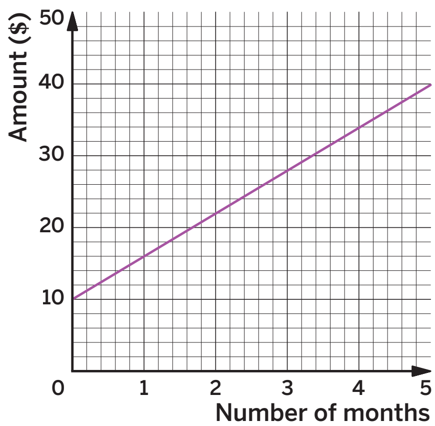
You used the slope and vertical intercept to interpret graphs of different real-world situations that represent linear relationships. The ***vertical intercept***, also called the ***y-intercept***, indicates where the line intersects the y-axis.

A linear equation can be represented using the form $y = mx + b$, where m represents the slope and b represents the y-intercept.

- For proportional linear relationships, the slope has the same value as the constant of proportionality (or unit rate).
- For nonproportional linear relationships, there is no constant of proportionality. The slope represents the constant rate of change.

Consider this graph of a line showing the amount of money paid for a music streaming service.

- The vertical intercept is $(0, 10)$. This means there was an initial cost of $\$10$ for the service.
- The slope, 6, represents the cost of the plan per month. The equation $y = 10 + 6x$ represents the cost y after x months.



Reflect:

Practice

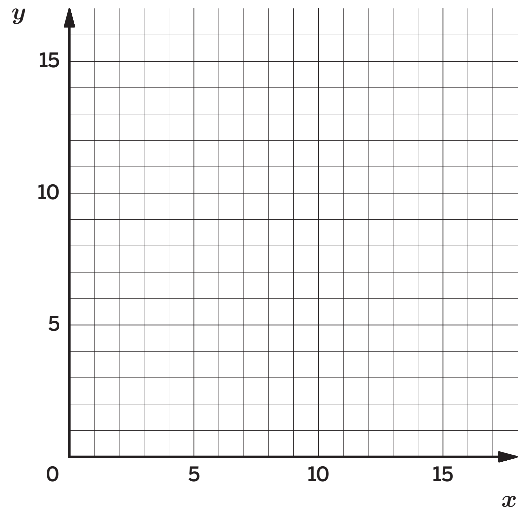
Name: .... Date: .... Period: ....

1. Create a graph that shows a linear relationship with each indicated slope. Then write an equation that represents the graph.

   a. Slope: $\frac{1}{4}$

   b. Slope: $\frac{3}{5}$

   c. Slope: $\frac{6}{5}$



2. Clare has a summer reading assignment. After reading the first 40 pages of a book, she plans to read 20 pages each day until she finishes the book. She creates the table and graph shown to track how many total pages she will read over the next few days. Clare claims after 7 days, she will have read 200 pages. Do you agree? Explain your thinking, based on the table and graph.

Number of days, x	Number of pages read, y
1	60
2	80
3	100
4	120
5	140

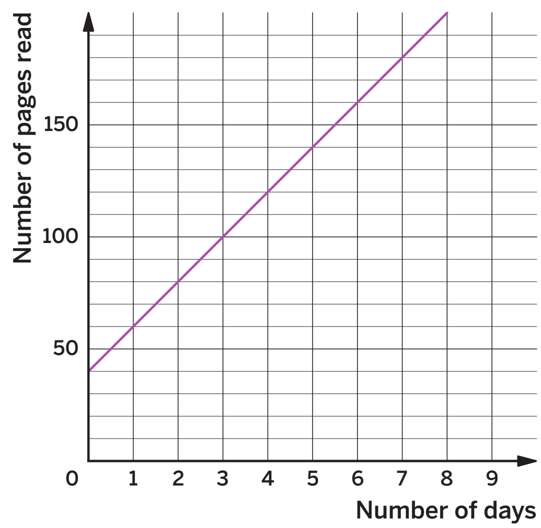


Table:

Graph:

Name: ______ Date: ______ Period: ______

Practice

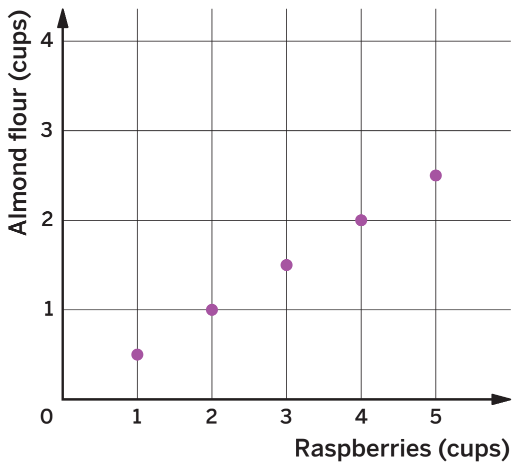
3. Explain what the slope and y-intercept represent in each real-world situation.

a. The amount of money y in a cash box after x tickets are purchased for carnival games. The slope of the line is $\frac{1}{4}$ and the y-intercept is 8.

b. Han is graphing the relationship between the cost y in dollars of a flower delivery and the number of flowers ordered, x. The slope of the line is 2, and the y-intercept is 3.

4. The table and graph show the amount of almond flour and fresh raspberries that are needed for each of Lin's and Noah's favorite raspberry lemon scone recipes.

Lin's scone recipe



Noah's scone recipe

Raspberries (cups)	Almond flour (cups)
$\frac{3}{2}$	1
3	2
$4\frac{1}{2}$	3

a. If you have 6 cups of almond flour for each recipe, how many cups of raspberries would you need to make each recipe?

Lin's recipe: **Noah's recipe:**

b. If you have 5 cups of raspberries for each recipe, how many cups of almond flour do you need to make each recipe?

Lin's recipe: **Noah's recipe:**

5. Write the equation of a line that has a slope of 2 and a vertical intercept of 8.

Unit 3 | Lesson 10

Representations of Linear Relationships

Let's write linear equations from context.

Warm-up Can You Guess the Game?

You will be shown an animation of a game. How do you think the game is played?

Log in to Amplify Math to complete this lesson online.

Name: ______ Date: ______ Period: ______

Activity 1 Rising Water Levels

You will be given the materials for this activity.

1. What is the volume, in milliliters, measured by the water level in the cylinder after you add:

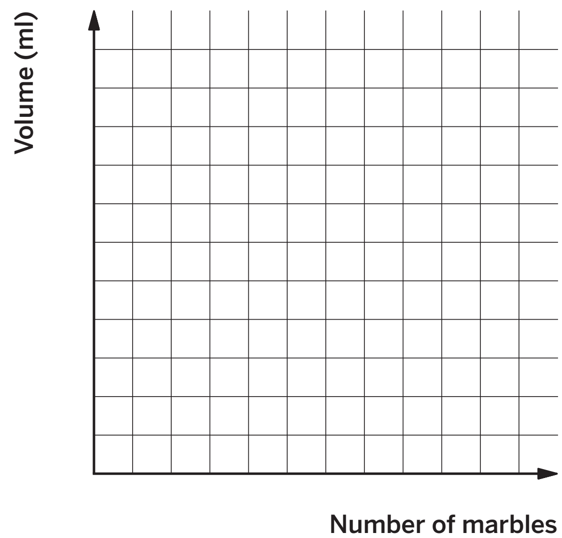
a 2 marbles?

b 5 marbles?

c 10 marbles?

d n marbles?

2. Use your responses from Problem 1 to plot three points on the graph. Then draw a line through the points. What patterns do you notice?

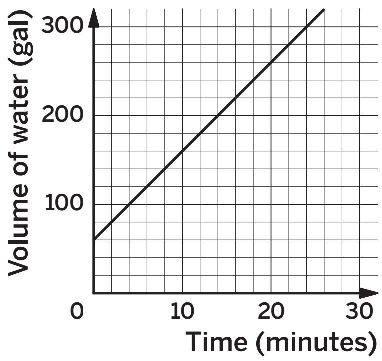
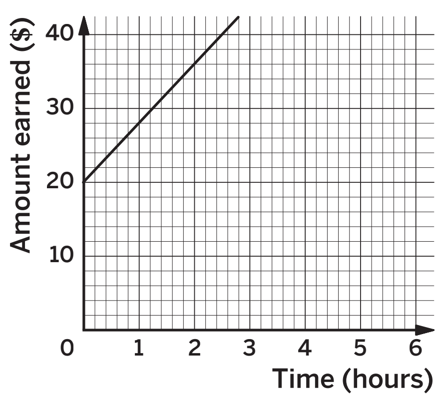


3. Write an equation for the volume v, after n marbles are added to the cylinder. Explain what each number in your equation represents in this situation.

4. If you wanted the water to reach the highest mark on the cylinder, how many marbles would you need to add? Explain your thinking.

Activity 2 Partner Problems

With your partner, decide who will complete Column A and who will complete Column B. After each row, share your response with your partner. Although the problems in each row are different, your responses should be the same. If they are not the same, work together to correct any errors or resolve any disagreements.

Column A	Column B
1. The manager of a carnival fills a dunk tank with water. The graph shows the volume of water in the tank as it is filled.  Write an equation that gives the volume of water w in the dunk tank after t minutes.	A dunk tank at a carnival has 60 gallons of water in it when the manager begins to fill the tank with a hose. Water fills the tank at a rate of 10 gallons per minute. Write an equation that gives the volume of water w in the dunk tank after t minutes.
2. Han operates the bumper cars at a carnival. On Independence Day, he earned $\$8$ per hour, plus a $\$20$ bonus for working on a holiday. Write an equation for the amount a Han earned for working h hours on Independence Day.	Han operates the bumper cars at a carnival. The graph shows the amount he earned working on Independence Day.  Write an equation for the amount a Han earned for working h hours on Independence Day.

STOP

Name: ______________ Date: ________ Period: ________

Summary

In today's lesson . . .

You wrote linear equations from different representations: verbal descriptions of real-world situations and graphs.

For example, in the marble activity, you graphed the relationship between the number of marbles and the volume of water in a cylinder. You interpreted the initial water volume as the y-intercept and the slope as the rate of change, or the amount the volume increased when one marble was added. Writing an equation helped you determine how many marbles are needed for the water to reach the top level of the cylinder.

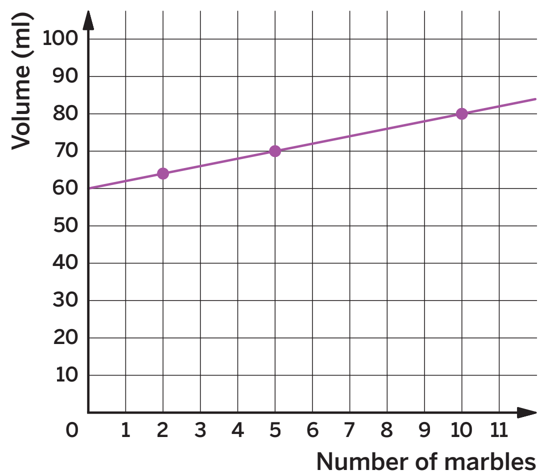
Scenario:

A cylinder contains 60 ml of water. Every marble that is added increases the volume of the water by 2 ml.

Equation:

$v = 2n + 60$, where n represents the number of marbles and v represents the volume of water, in milliliters.

Graph:

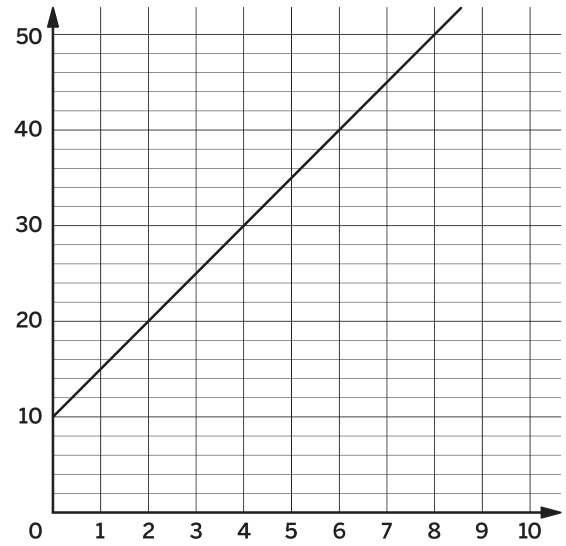


Reflect:

Name: ________________ Date: ________ Period: ________

Practice

1. Consider the graph of the line shown.

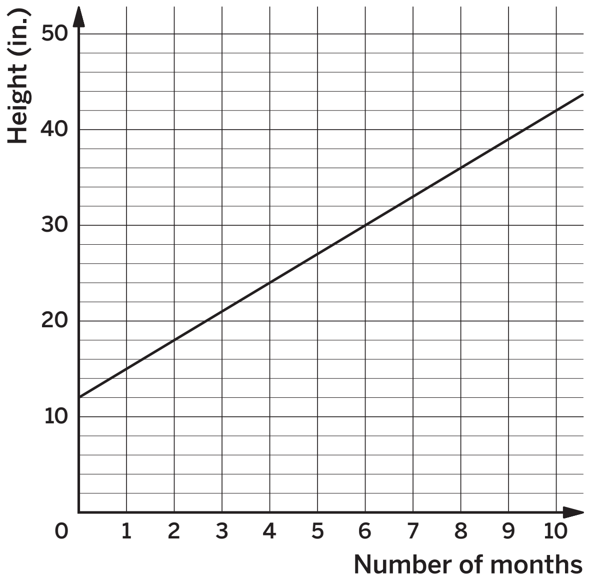


a. Write a real-world situation that could be represented by the line. Label the axes on the graph.

b. What equation describes the relationship between the two variables in your scenario?

c. Explain what each number and variable in your equation represents in this situation.

2. The graph shows the height h in inches of a bamboo plant n months after it has been planted.



a. Write an equation that gives the bamboo's height h after n months.

b. After how many months will the bamboo plant be 66 in. tall? Show or explain your thinking.

Name: ______________________ Date: ____________ Period: ____________

Practice

3. A taxi company charges an initial fee of $2.50, and then $2 per mile. Which equation represents the total cost c, after x miles? Select *all* that apply.

A. $c = 2 + 2.50x$

B. $c = 2x + 2.50$

C. $x = 2 + 2.50c$

D. $x = 2c + 2.5$

E. $c = 2.5 + 2x$

4. Tyler and Jada each have a favorite banana bread recipe using slightly different amounts of mashed bananas and honey. The number of cups of mashed bananas is proportional to the number of cups of honey.

Tyler's recipe:

Honey (cups)	Bananas (cups)
$\frac{1}{2}$	$\frac{3}{4}$
$2\frac{1}{2}$	$3\frac{3}{4}$
3	$4\frac{1}{2}$

Jada's recipe:

The relationship between the number of cups of mashed bananas y and the number of cups of honey x is represented by the equation $y = \frac{7}{4}x$.

a If you have 4 cups of honey, how many cups of mashed bananas would you need to make each recipe?

Tyler's recipe:

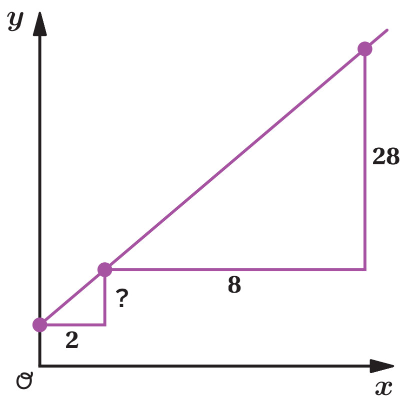
Jada's recipe:

b What is the rate of change for each recipe, and what does it mean within this context?

Tyler's recipe:

Jada's recipe:

5. Refer to the slope triangles shown. What is the unknown vertical side length? Explain your thinking.

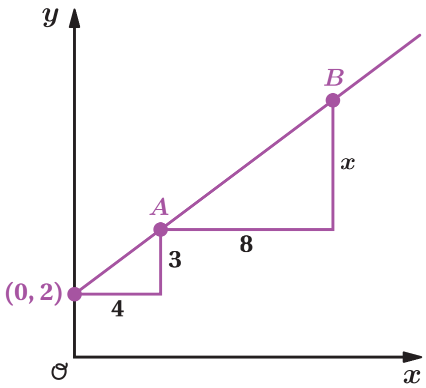


Writing Equations for Lines Using Two Points

Let's write an equation for a line that passes through two points.

Warm-up Coordinates and Lengths in the Coordinate Plane

Consider line AB and the two slope triangles shown.



1. Determine the vertical side length x of the larger triangle. Explain your thinking.

2. What are the coordinates of points A and B? Explain your thinking.

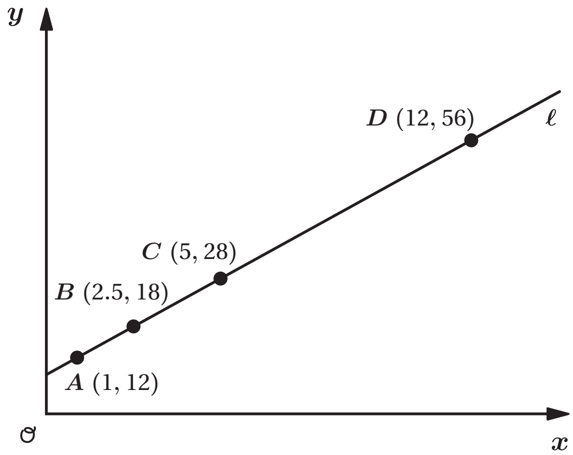
Log in to Amplify Math to complete this lesson online.

Name: ______________ Date: ______________ Period: ______________

Activity 1 Calculate the Slope

Plan ahead: How will you encourage your partner to do their best work?

Line ℓ is shown on the coordinate plane. Several points are marked on the line.



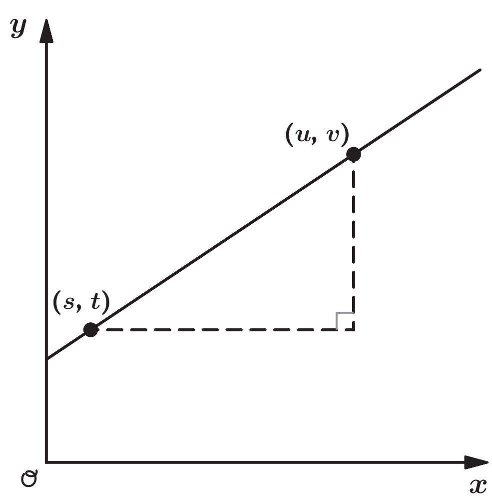
1. Choose two points on the line that are different from your partner. Using these two points, draw a slope triangle. Then determine the slope of the line ℓ.

   Points:

   Slope:

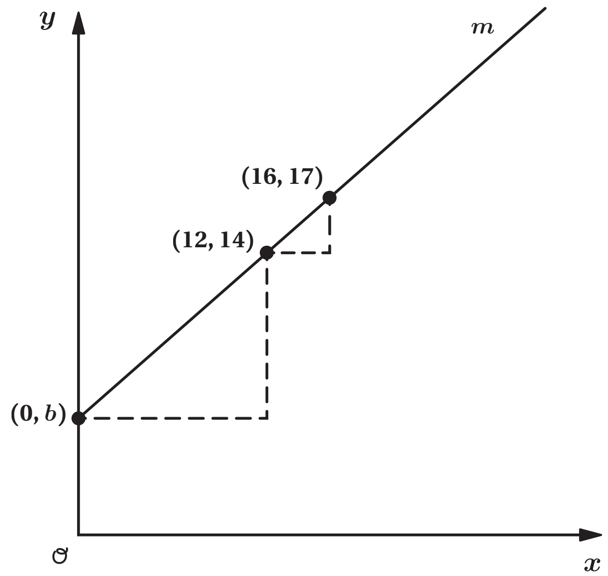
2. Compare your work with your partner. What do you notice?

3. Describe a method for calculating the slope between *any* two points on a line. Use the diagram if it helps your thinking.



Activity 2 Writing an Equation From Two Points

Line m is shown on the coordinate plane. Several points are marked on the line.



1. Label the horizontal and vertical side lengths of each slope triangle so that they have a number or expression representing their lengths.

2. Use what you know about similar triangles to calculate the value of b. Show or explain your thinking.

3. Identify the slope and y-intercept. Then write an equation for the line.

4. Are the following points on the line? Explain your thinking.

   a. $(24, 23)$

   b. $(100, 80)$

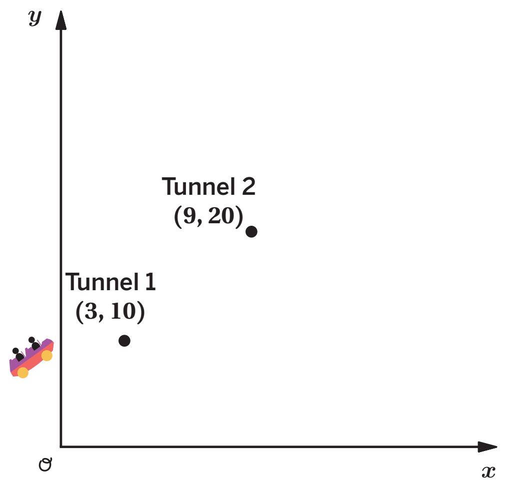
   c. $(60, 45)$

Name: ____________ Date: ________ Period: ________

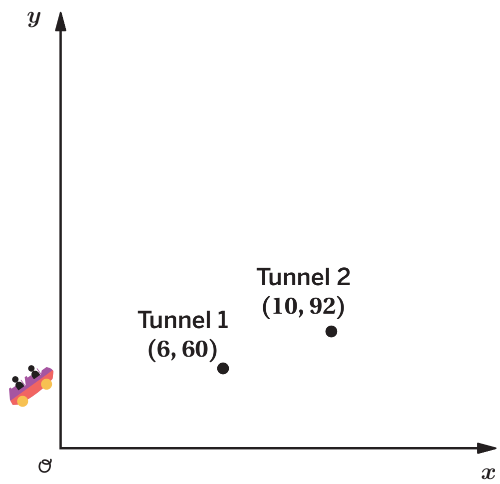
Activity 3 Through the Tunnel

For each graph, write an equation that represents the line so that the roller coaster passes through the tunnels marked by the coordinates of points. Show or explain your thinking.

1.



2.



Are you ready for more?

A line passes through the point $(2, 3)$ and has a slope of $\frac{1}{4}$. Write the coordinates for another point that lies on the same line. Then write an equation for the line.

STOP

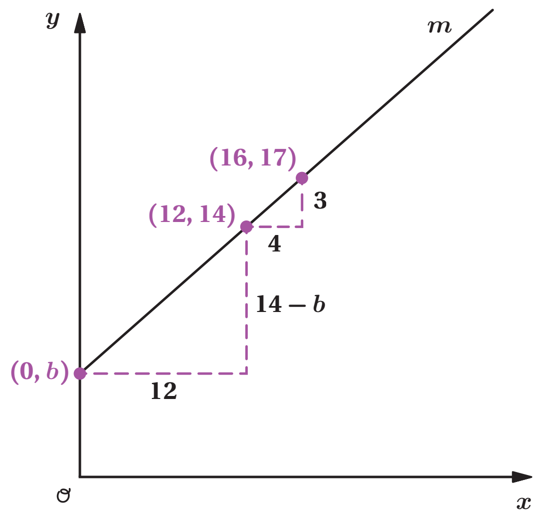
Summary

In today's lesson . . .

You discovered a method for calculating the slope between any two points. You also applied your understanding of similar triangles to write the equation of a line passing through two given points.

For example, because the two triangles shown are similar, the ratios of corresponding side lengths are equivalent, $\frac{3}{4} = \frac{14-b}{12}$. Because $\frac{3}{4} = \frac{9}{12}$, this means that $14 - b = 9$ and $b = 5$. The y-intercept is 5.

Now you can use the slope, $\frac{3}{4}$, and y-intercept, 5, to write an equation for the line: $y = \frac{3}{4}x + 5$.



Reflect:

Name: ________ Date: ________ Period: ________

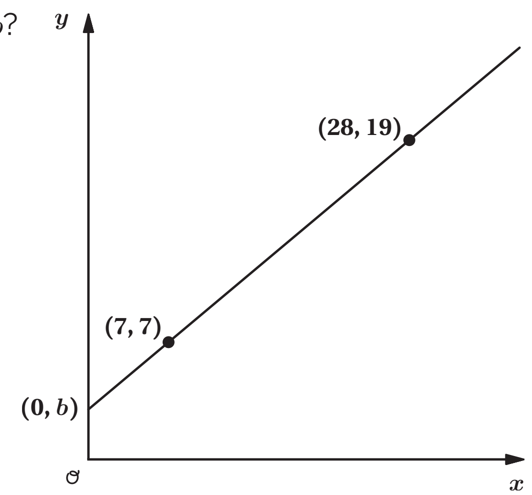
1. For each pair of points, calculate the slope of the line that passes through both points.

   a. $(1, 1)$ and $(7, 5)$

   b. $(1, 1)$ and $(5, 7)$

   c. $(2, 3)$ and $(5, 7)$

2. Refer to the graph shown. What is the value of b? Show or explain your thinking.



3. Select *all* the points that are on the line that passes through the points $(0, 5)$ and $(2, 8)$. Use the graph to help with your thinking.

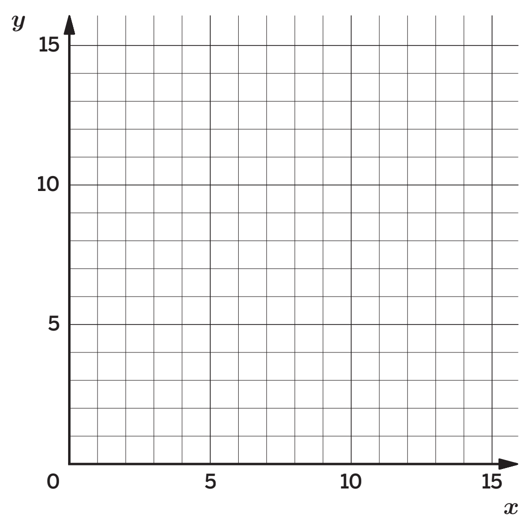
   A. $(4, 11)$

   B. $(5, 10)$

   C. $(6, 14)$

   D. $(30, 50)$

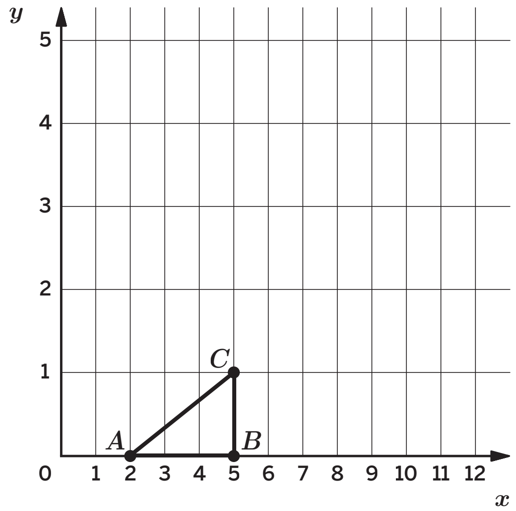
   E. $(40, 60)$



Practice

Name: .... Date: .... Period: ....

4. Consider Triangle ABC.



a Dilate Triangle ABC using A as the center of dilation and a scale factor of 2.

b Dilate Triangle ABC using A as the center of dilation and a scale factor of 3.

c Dilate Triangle ABC using A as the center of dilation and a scale factor of $\frac{1}{2}$.

d What are the coordinates of the image of point C when Triangle ABC is dilated using A as the center of dilation and a scale factor s?

5. Andre says, "I found two figures that are congruent, so they cannot be similar." Diego says, "No, they are similar! The scale factor is 1." Which friend is correct? Use the definition of similarity to explain your thinking.

6. Line segment AB has a length of 4 units. It is translated up 5 units. Which of the statements is true about the image, $A'B'$, after the translation is applied? Select *all* that apply.

A. Line segment $A'B'$ has a length of 5 units.

B. Line segment $A'B'$ has a length of 4 units.

C. Line segment AB is congruent to line segment $A'B'$.

D. Line segment AB is longer than line segment $A'B'$.

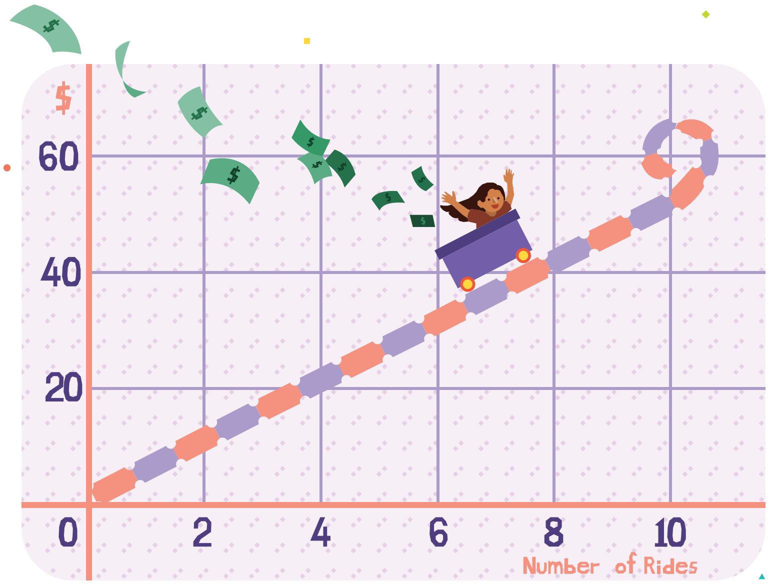
E. Line segment $A'B'$ is longer than line segment AB.

Name: ________ Date: ________ Period: ________

Unit 3 | Lesson 12

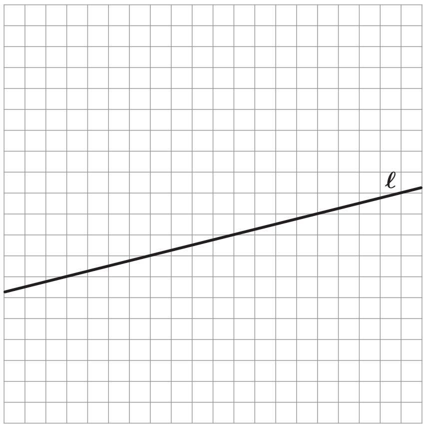
Translating to $y = mx + b$

Let's see what happens to the equations of translated lines.



Warm-up Translating a Line

Consider line ℓ.



1. Translate line ℓ using any translation you choose. Label the translated line n.

2. Determine the slope of each line.

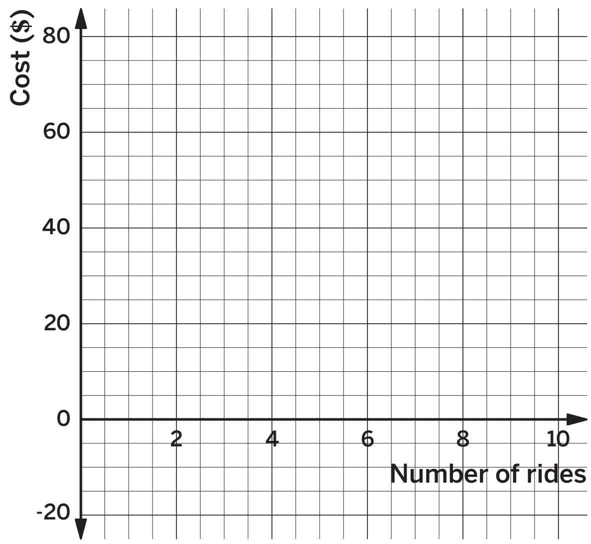
   Line ℓ: Line n:

3. Compare your responses with a partner. What do you notice about the lines?

Log in to Amplify Math to complete this lesson online.

Activity 1 How Much More?

1. Noah wants to go to Honest Carl's Funtime World amusement park on Saturday. On weekends, the amusement park charges an admission fee of $20 per person and then $5 for each ride. Graph the relationship that represents the amount of money y Noah would spend after x rides. Label the line e.



2. On weekdays, Honest Carl's Funtime World offers a special deal where they do not charge an admission fee. On the same coordinate plane, graph the relationship that represents the amount of money y Noah would spend after x rides, if he goes there on a Wednesday. Label the line d.

3. Compare the two lines. How much more money does Noah spend after 2 rides if he goes to Honest Carl's Funtime World on a Saturday instead of a Wednesday? 4 rides? 8 rides? x rides?

4. Write an equation for each line.

5. Noah goes to Honest Carl's Funtime World on Wednesday and he has a coupon that can be used for 2 free rides. On the same coordinate plane, graph the relationship that represent the amount of money y Noah would spend after x rides, if he uses the coupon. Label the line c. Then write an equation for the line.

Name: ______ Date: ______ Period: ______

Activity 2 Card Sort: Translating a Line

You will be given a set of cards. For each problem, determine the matching graph, equation, and table or description. Record the matching card numbers in the table.

1. The line $y = \frac{1}{2}x$ is translated up 1 unit.
2. The line $y = \frac{1}{2}x$ is translated down 1 unit.
3. The line $y = 2x$ is translated up 1 unit.
4. The line $y = 2x$ is translated down 1 unit.

	Graph	Equation	Table or description
1.			
2.			
3.			
4.			

Reflect: How did you make constructive decisions in order to complete the activity successfully?

STOP

Summary

In today's lesson . . .

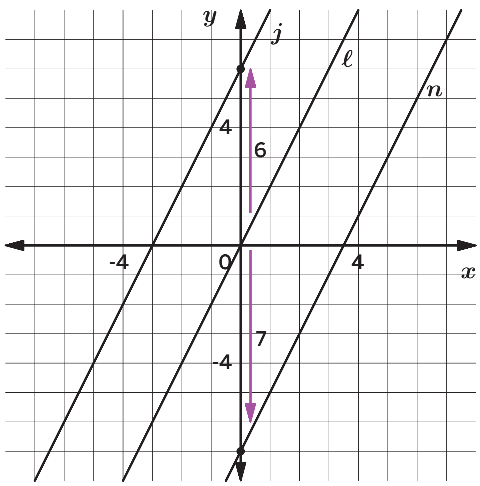
You investigated what happens to a line that represents a proportional relationship after a translation. A translation of a line that represents a proportional relationship creates a line that is parallel to the preimage, but changes the location of the vertical intercept.

The equation $y = mx$ represents a line that passes through the origin. The equation $y = mx + b$ represents a vertical translation of line $y = mx$ by b units.

- If $b > 0$, the line is translated up.
- If $b < 0$, the line is translated down.

For example, the equation of line ℓ is $y = 2x$.

- Line ℓ is translated 6 units up to produce line j. So, the equation of line j is $y = 2x + 6$.
- Line ℓ is translated 7 units down to produce line n. So, the equation of line n is $y = 2x - 7$.



Reflect:

Practice

Name: ____________ Date: ____________ Period: ____________

1. Select *all* the equations whose graphs have the same y-intercept.

A. $y = 3x - 8$

B. $y = -8x + 3$

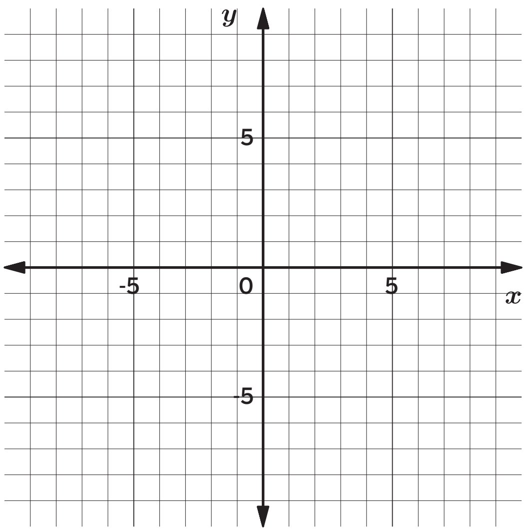
C. $y = 3x + 8$

D. $y = 5x - 8$

E. $y = 2x - 8$

F. $y = 13x - 8$

2. Refer to the coordinate plane.



a. Graph the equations $y = \frac{1}{4}x$ and $y = \frac{1}{4}x - 3$.

b. How are the graphs the same? How are the graphs different?

3. A cable company charges $70 per month for cable service to existing customers. For new customers, there is an additional one-time service fee of $100.

a. Write a linear equation representing the relationship between x, the number of months of service, and y, the total amount paid in dollars by a customer.

Existing customer:

New customer:

b. When the two equations are graphed on the coordinate plane, how are the graphs similar?

Practice

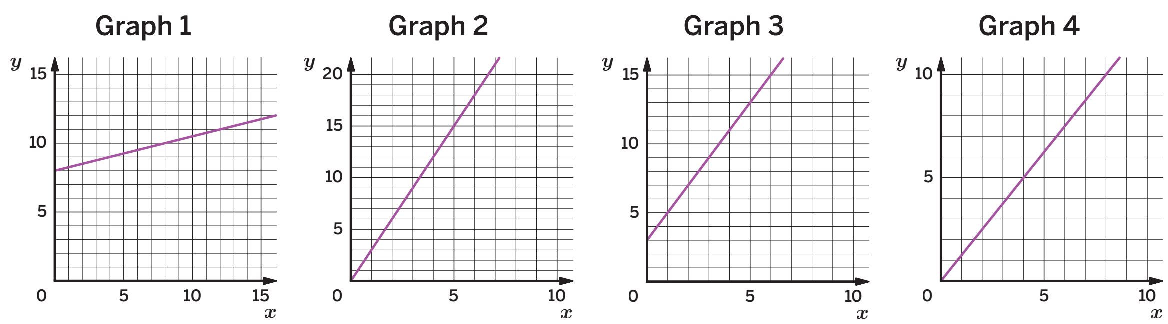
Name: .... Date: .... Period: ....

4. A mountain road is 5 miles long and gains elevation at a constant rate. After 2 miles, the elevation is $5{,}500$ ft above sea level. After 4 miles, the elevation is $6{,}200$ ft above sea level.

a Determine the elevation of the road at the point where the road begins.

b What are the coordinates of the point on a graph that represent your response in part a? Let y represent the elevation in feet and x represent the distance along the road in miles.

5. Consider Graphs 1–4 shown. For each real-world situation described, choose the graph that best represents it.



a The perimeter y, in units, for an equilateral triangle with a side length of x units. The slope of the line is 3.

b The amount of money y after x tickets are purchased. The slope of the line is $\frac{1}{4}$.

c The number of chapters y read after x days. The slope of the line is $\frac{5}{4}$.

d The cost y, in dollars, of x blueberry muffins ordered. The slope of the line is 2.

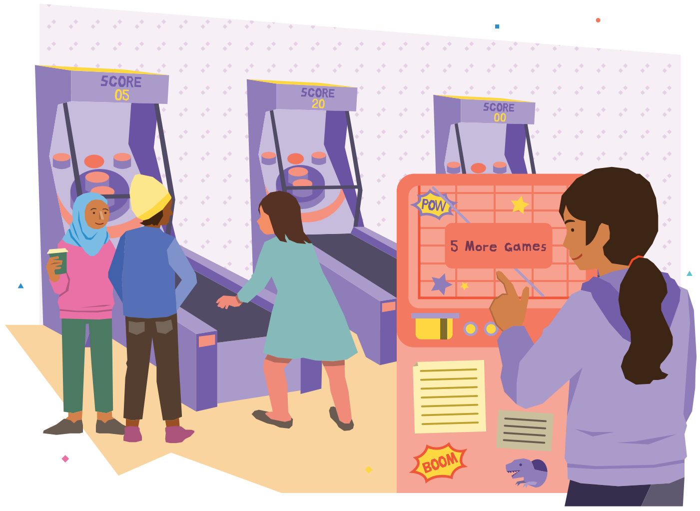
6. Calculate the slope of the line that passes through the points $(4, 5)$ and $(7, 6)$.

Name: ______ Date: ______ Period: ______

Unit 3 | Lesson 13

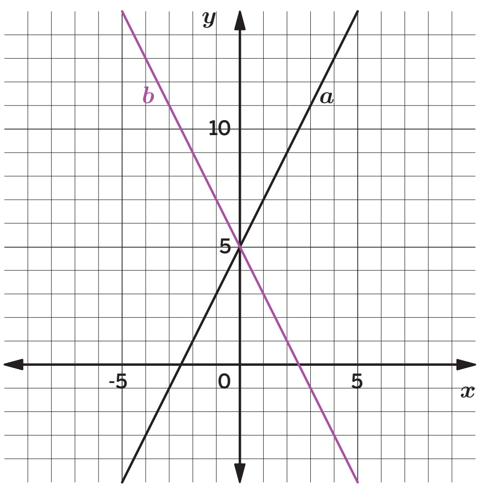
Slopes Don't Have to Be Positive

Let's find out what a negative slope means.



Warm-up Same and Different

Consider lines a and b.



1. What is the same about the two lines?

2. What is different about the two lines?

Log in to Amplify Math to complete this lesson online.

Activity 1 Noah's Game Card

Noah loads a game card with $40 for the arcade at Honest Carl's Funtime World. Every time he plays a game, $2.50 is subtracted from the amount available on his card.

1. How much money, in dollars, is available on his card after Noah plays:

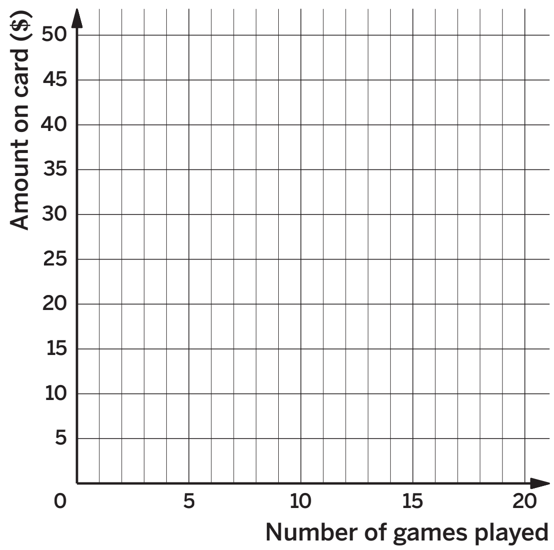
a 1 game?

b 2 games?

c 5 games?

d x games?

2. Use your responses from Problem 1 to plot three points on the graph. Then draw a line through the points. What patterns do you notice?

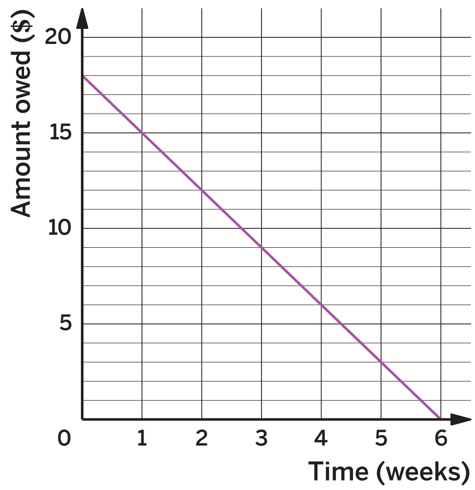


3. How many games can Noah play before the game card runs out of money? Where do you see this number of games on your graph?

Name: ______ Date: ______ Period: ______

Activity 2 Payback Plan

Elena borrowed some money from her brother. She pays him back by giving him the same amount every week. The graph shows how much she owes him after each week.



Co-craft Questions: Before completing this activity, preview Problem 3. What are some other questions you could ask about this graph? With your partner, think of 1–2 other questions you have about this situation.

1. Choose two points on the line and label the coordinates. Then calculate the slope of the line.

2. Write an equation for the dollar amount y owed after x weeks. Then explain what each number in your equation represents in this situation.

3. How much time will it take for Elena to pay back all the money she borrowed? Explain your thinking.

Activity 3 Info Gap: Making Designs

You will be given either a *design card* or a *blank graph card*. Do not show your card to your partner.

If you are given a *design* card:	If you are given a *blank graph* card:
1. Silently study the design and think about how you could communicate what your partner should draw. Think about ways that you can describe what a line looks like, such as its slope or the points that it passes through.	**1.** Listen carefully as your partner describes each line, and draw each line based on their description.
2. Describe each line, one at a time, and give your partner time to draw each one.	**2.** You are not allowed to ask for more information about a line other than what your partner tells you.
3. Do not show your design card to your partner until they have finished drawing all the lines.	**3.** Do not show your drawing to your partner until you have finished drawing all the lines.

When you and your partner are finished, place the drawing next to the card with the design, so that you and your partner can both see them. How is the drawing the same as the design? How is it different? Discuss any miscommunication that might have caused the drawing to look different from the design.

Pause here so your teacher can review your work. When you are given a new set of cards, trade roles with your partner and repeat the activity.

STOP

Name: Date: Period:

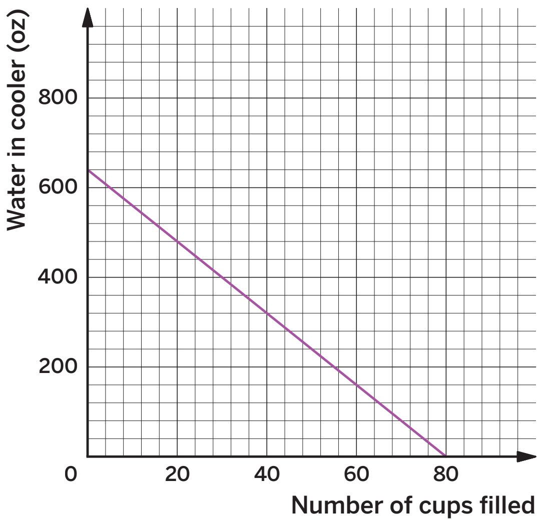
Summary

In today's lesson . . .

You saw that the slope of a line can have a negative value. When a linear relationship has a negative slope, this means that as the x-values increase, the y-values decrease at a constant rate.

For example, the equation $a = -8n + 640$ represents the amount a of water in a water cooler after n cups are filled with water.

- The vertical intercept, 640, represents the initial amount of water in the cooler.
- The slope, -8, tells you the rate of change in the amount of water each time a cup is filled. Because the slope is negative, the amount of water decreases.
- The ***horizontal intercept***, 80, tells you that it takes 80 cups of water to empty the water cooler.



Reflect:

Practice

Name: .... Date: .... Period: ....

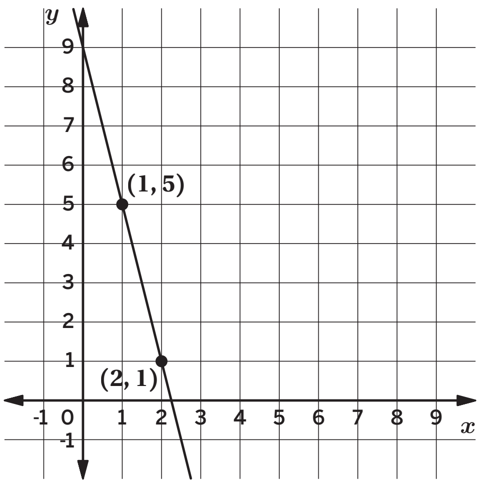
1. During its flight, the elevation e, in feet, of an airplane and its time since takeoff are related by a linear equation. Consider the graph of such an equation, with time in minutes represented on the horizontal axis and elevation in feet on the vertical axis. For each situation, determine whether the slope is positive or negative.

   a. The plane descends at a rate of $1{,}000$ ft per minute.

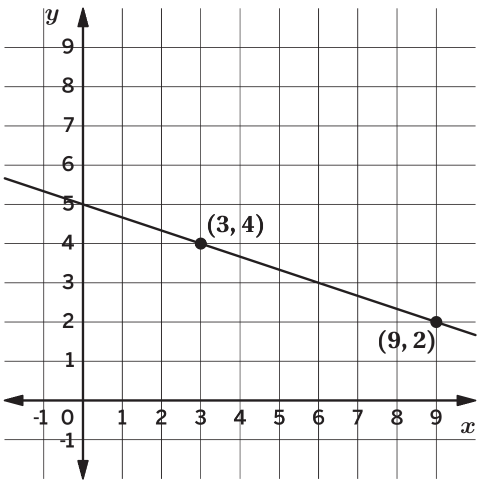
   b. The plane ascends at a rate of $2{,}000$ ft per minute.

2. Determine the slope of each line. Show your thinking.

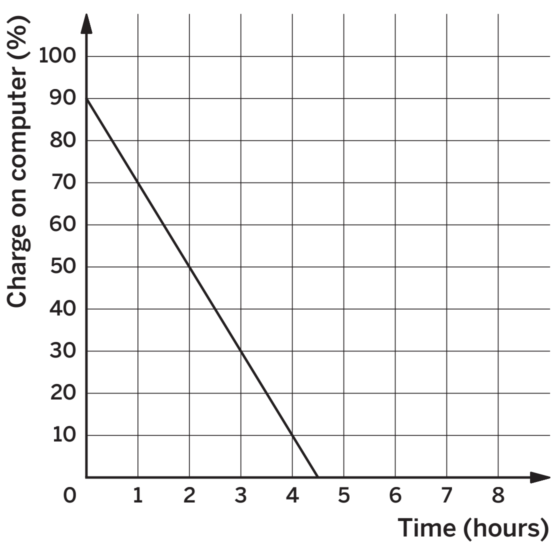
   a.



   b.



3. The graph shows the amount of charge c left on Lin's computer after h hours. Write an equation that describes the relationship between c and h. Show or explain your thinking.



Name: ______ Date: ______ Period: ______

Practice

4. Elena and Diego both have part-time jobs. Elena's aunt pays her $\$1$ for each call she makes to let people know about her aunt's new business. Diego washes his neighbor's windows and earns the same amount per window, as shown in the table. Select *all* the statements about their part-time jobs that are true.

Number of windows	Amount earned ($)
27	29.70
45	49.50
81	89.10

A. Elena makes more money for making 10 calls than Diego makes for washing 10 windows.

B. Diego makes more money for washing each window than Elena makes for making each call.

C. Elena makes the same amount of money for 20 calls as Diego makes for 18 windows.

D. Diego needs to wash 35 windows to make as much money as Elena makes for 40 calls.

E. The equation $y = 1.10x$, where y represents the number of dollars and x represents the number of windows, represents Diego's situation.

F. The equation $y = x$ represents Elena's situation, where y represents the number of dollars and x represents the number of calls.

5. A line passes through the points $(1, 1.5)$ and $(4, 6)$. Determine whether each point is also on the line. Place a check mark in the appropriate column.

	On the line	Not on the line
$(5, 7.5)$		
$(80, 50)$		
$(100, 150)$		

6. Which expression has a value of -25 when $a = -2$?

A. $-10a + 5$

B. $-10a - 5$

C. $10a + 5$

D. $10a - 5$

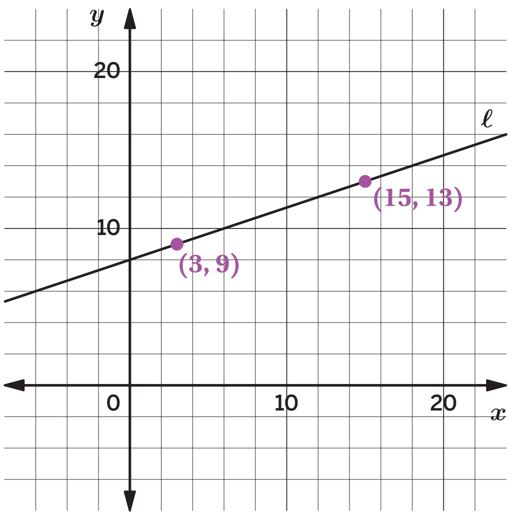
Unit 3 | Lesson 14

Writing Equations for Lines Using Two Points, Revisited

Let's write equations for lines.

Warm-up Two Truths and a Lie

Line ℓ is shown on the coordinate plane. Two points are labeled on the line. Which of these three statements is a lie?



A. The slope of the line can be calculated by evaluating the expression $\frac{13-9}{15-3}$.

B. The slope of the line can be calculated by evaluating the expression $\frac{9-13}{3-15}$.

C. The slope of the line can be calculated by evaluating the expression $\frac{13-9}{3-15}$.

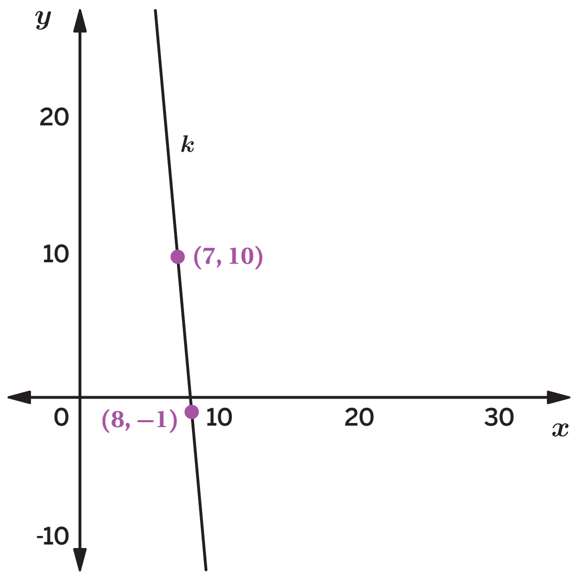
Explain your thinking.

Log in to Amplify Math to complete this lesson online.

Name: ______ Date: ______ Period: ______

Activity 1 Writing an Equation from Two Points, Revisited

Consider line k with the two labeled points as shown.



1. Calculate the slope of the line. Show your thinking.

2. Andre wants to write an equation for this line in the form $y = mx + b$, but he cannot see the y-intercept on the graph. He claims that because he knows the slope, he can calculate the value of b by substituting the coordinates of one of the points, $(7, 10)$, for x and y in the equation $y = -11x + b$. His unfinished work is shown.

   Andre's work:

   For $m = -11$ and point $(7, 10)$:

   $y = mx + b$

   $10 = -11(7) + b$

   Finish Andre's work by solving the equation for b. Show your thinking.

3. Use the slope and y-intercept to write the equation for the line in the form $y = mx + b$.

Activity 2 Coin Collector

The Coin Collector arcade game at Honest Carl's Funtime World requires a player to control a character that moves along a straight line to collect coins. The fewer lines a player uses, the more points they earn.

For each graph shown, draw lines to collect coins. Label each line with a number (1, 2, 3, etc.), and then write the equation for each line.

Note: You may not need to use all of the space provided for the equations. Additionally, you may add more equations, as needed.

Round 1:

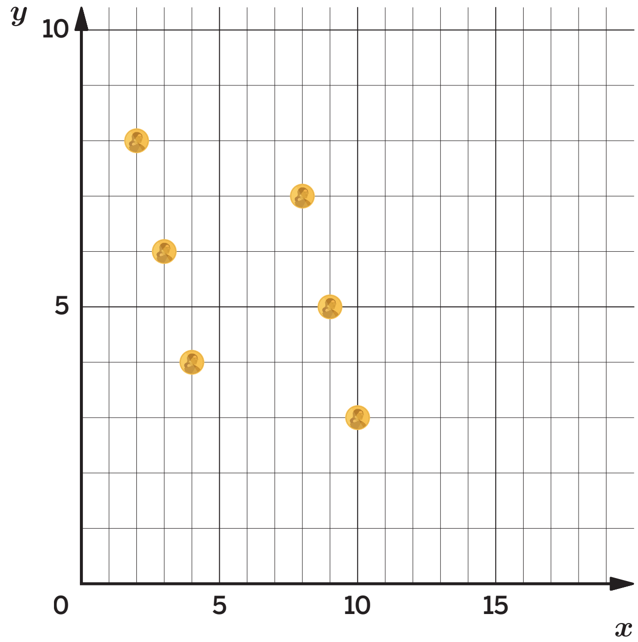
Equations:

Line 1:

Line 2:

Line 3:

Line 4:



Name: ______________________ Date: ____________ Period: ________

Activity 2 Coin Collector (continued)

Round 2:

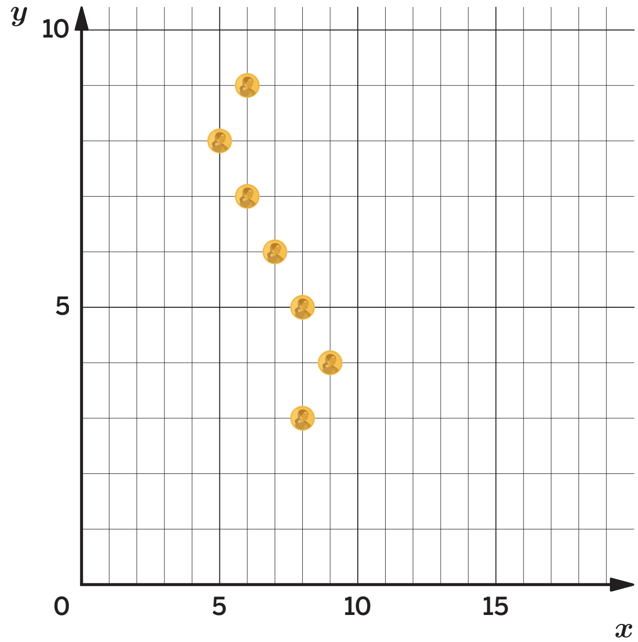
Equations:

Line 1:

Line 2:

Line 3:

Line 4:



Round 3:

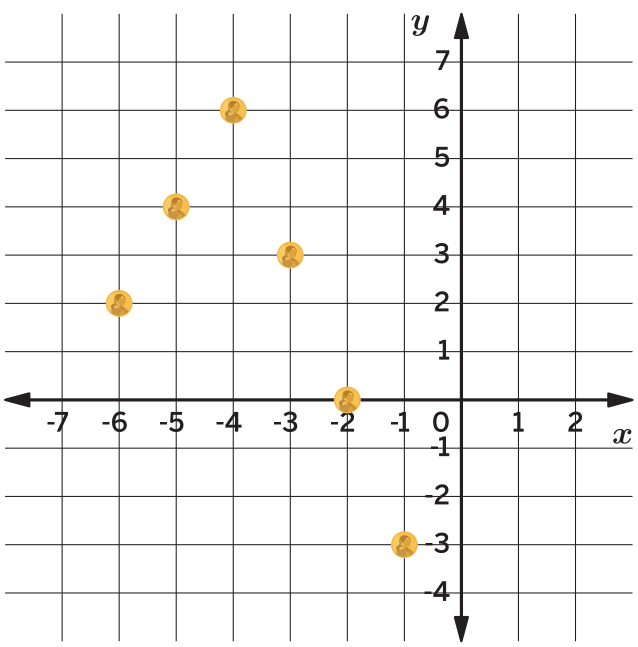
Equations:

Line 1:

Line 2:

Line 3:

Line 4:



STOP

Summary

In today's lesson . . .

You wrote the equation of a line that passes through two points, including lines with a negative slope.

For example, to write an equation of a line that passes through the points $(1, 7)$ and $(2, 4)$, you can follow these steps.

1. Calculate the slope by finding the ratio of the difference in the y-coordinates to the difference in the x-coordinates: $\frac{7-4}{1-2} = -\frac{3}{1} = -3$. The slope is -3.
2. Substitute the slope and the coordinates of one of the points, for example $(1, 7)$, into the equation $y = mx + b$. Then solve for b.

$7 = -3(1) + b$	The slope is -3. The point is $(1, 7)$.
$7 = -3 + b$	Multiply.
$10 = b$	Add 3 to both sides.

3. Write the equation in the form $y = mx + b$ using the slope, -3, and the y-intercept, 10.

   The equation is $y = -3x + 10$.

Even if you used the other point $(2, 4)$, you would arrive at the same equation. Try it!

Reflect:

Name: .... Date: .... Period: ....

Practice

1. Bard and Mai each write an equation of the line that passes through the points $(2, 9)$ and $(12, 14)$. They both calculate the slope as $\frac{1}{2}$.

- Bard substitutes the point $(2, 9)$ to determine the y-intercept.
- Mai substitutes the point $(12, 14)$ to determine the y-intercept.

Each student's work is shown. Review their work and solutions. Find and fix any errors in each person's work.

Bard's work:

$$y = \frac{1}{2}x + b$$
$$9 = \frac{1}{2}(2) + b$$
$$9 = 1 + b$$
$$b = 8$$

Mai's work:

$$y = \frac{1}{2}x + b$$
$$12 = \frac{1}{2}(14) + b$$
$$12 = 7 + b$$
$$b = 5$$

2. Write the equation of the line that passes through each pair of points. Show or explain your thinking.

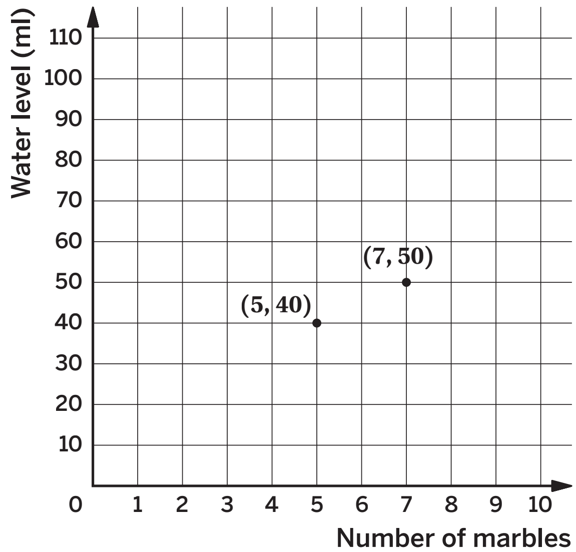
a. $(2, 14)$ and $(6, 26)$

b. $(-5, 7)$ and $(1, 1)$

Practice

Name: .... Date: .... Period: ....

3. Clare added marbles to a container of water. When she added 5 marbles, the water level was 40 ml. When she added 7 marbles, the water level was 50 ml. Write an equation for the water level y after x marbles are added.

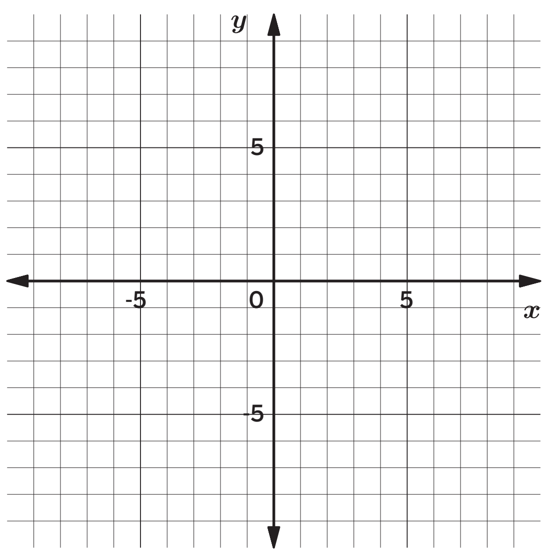


4. Graph each equation. Then label each line with its equation.

a **Equation A:** $y = 3x + 5$

b **Equation B:** $y = -\frac{2}{3}x + 1$

c **Equation C:** $y = -x - 5$



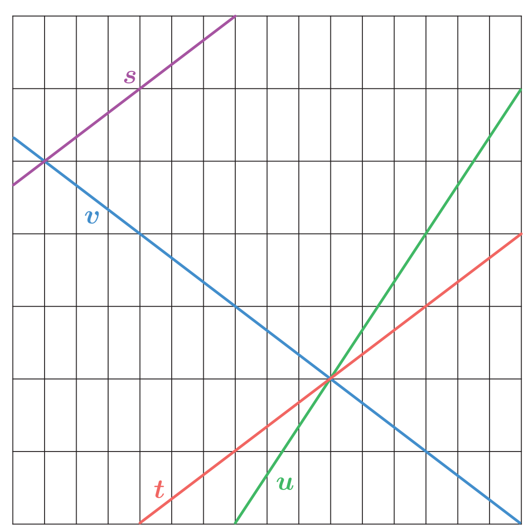
5. For each line, determine if the slope is positive or negative.

a line s

b line t

c line u

d line v

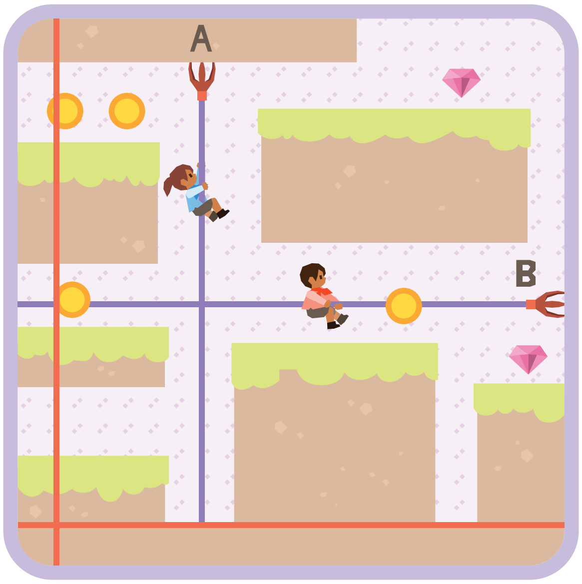


Name: ______________________ Date: __________ Period: __________

Unit 3 | Lesson 15

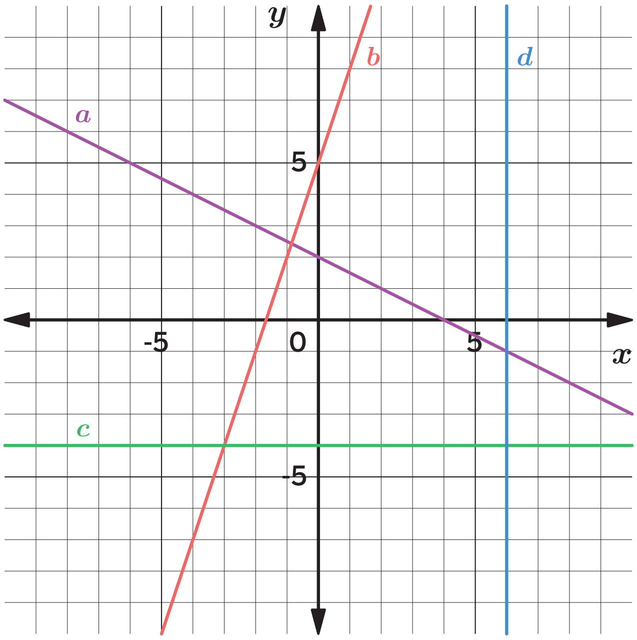
Equations for All Kinds of Lines

Let's write equations for vertical and horizontal lines.



Warm-up Which One Doesn't Belong?

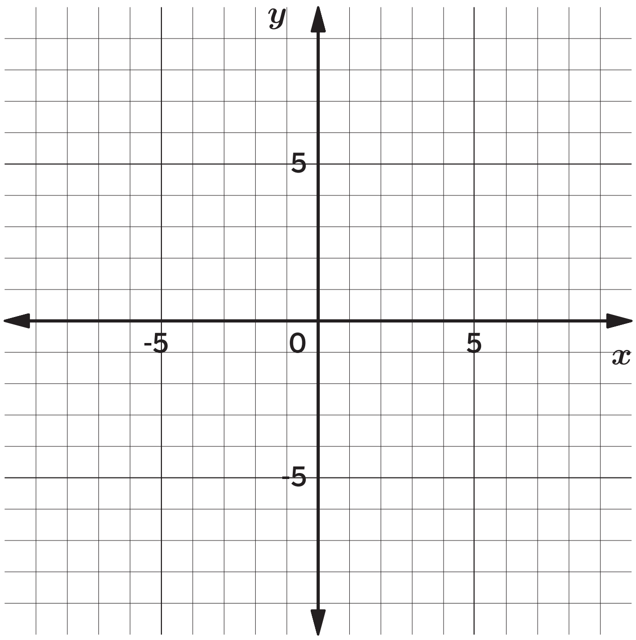
Study the four lines shown. Which line does not belong? Explain your thinking.



Log in to Amplify Math to complete this lesson online.

Activity 1 All the Same

Complete the following problems using the coordinate plane shown.



1. Plot at least 10 points whose y-coordinate is -7. What do you notice?

2. Study these equations. Which equation do you think represents *all* the points with a y-coordinate of -7?

   A. $x = -7$ **B.** $y = -7x$ **C.** $y = -7$ **D.** $x + y = -7$

3. Plot at least 10 points whose x-coordinate is 5. What do you notice?

4. Study these equations. Which equation do you think represents *all* the points with a x-coordinate of 5?

   A. $x = 5$ **B.** $y = 5x$ **C.** $y = 5$ **D.** $x + y = 5$

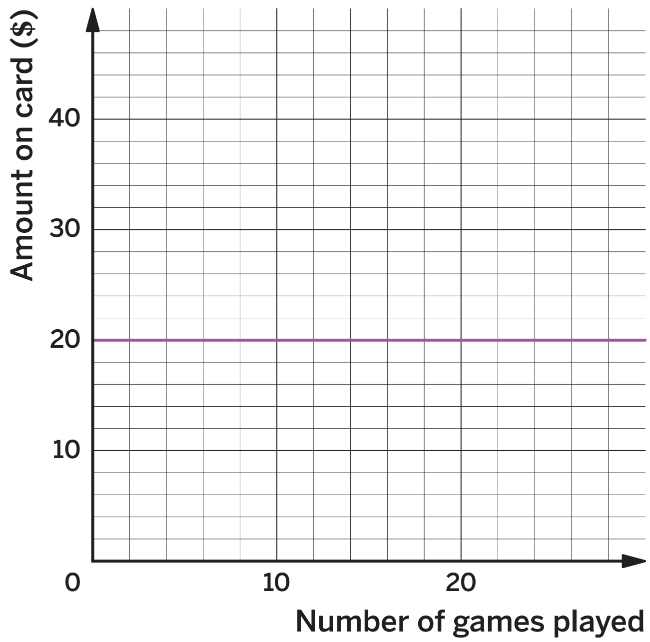
5. Graph and label the equation $y = 4$ on the coordinate plane.

6. Graph and label the equation $x = -8$ on the coordinate plane.

Name: ________ Date: ________ Period: ________

Activity 2 Han's Game Card

The graph shows the available amount, in dollars, on Han's arcade game card at Honest Carl's Funtime World for one day.



1. Describe what happened to the available amount on Han's game card as the number of games played increased.

2. What value makes sense for the slope of the line that represents the available amount on Han's game card? What does the slope represent in this situation?

3. Write an equation that represents the available amount y on the card after playing x games.

Activity 3 Coin Collector, Revisited

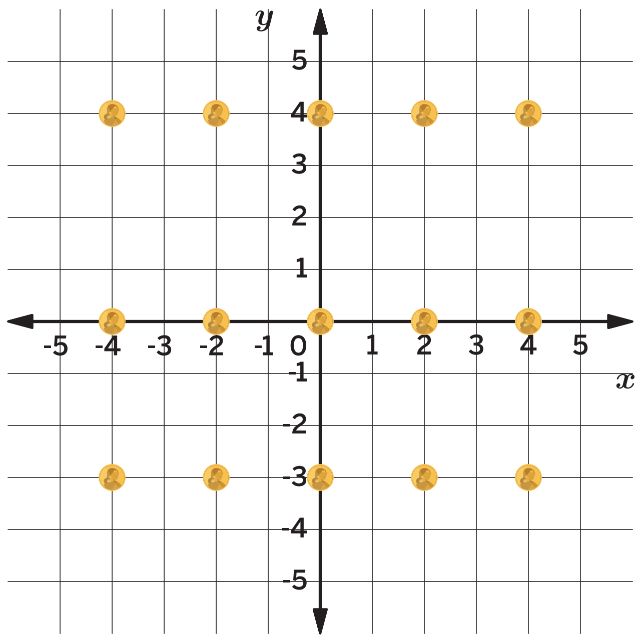
During a two-player game of Coin Collector, players take turns moving a character along a straight line to collect coins. With your partner, determine who will be Partner A and who will be Partner B.

For each round, take turns drawing lines and writing equations to collect the most coins. Then determine the total number of coins collected by each person.

Round 1

Partner A's equations	Number of coins
Line 1:	
Line 2:	
Line 3:	

Partner B's equations	Number of coins
Line 1:	
Line 2:	
Line 3:	



Partner A total: .... **Partner B total:** ....

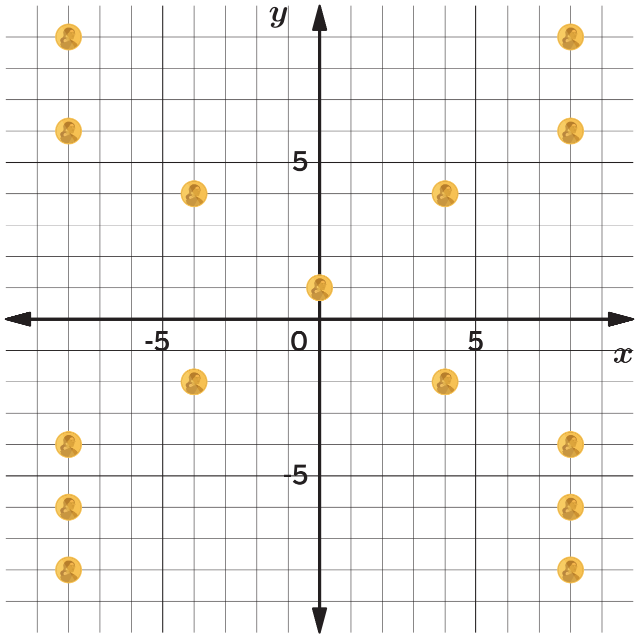
Name: ______ Date: ______ Period: ______

Activity 3 Coin Collector, Revisited (continued)

Round 2

Partner A's equations	Number of coins
Line 1:	
Line 2:	
Line 3:	

Partner B's equations	Number of coins
Line 1:	
Line 2:	
Line 3:	

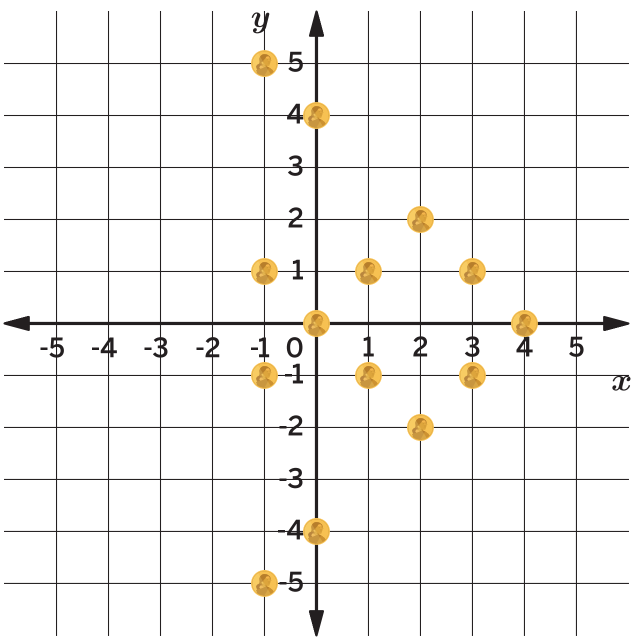


Partner A total: ______ **Partner B total:** ______

Round 3

Partner A's equations	Number of coins
Line 1:	
Line 2:	
Line 3:	

Partner B's equations	Number of coins
Line 1:	
Line 2:	
Line 3:	



Partner A total: ______ **Partner B total:** ______

STOP

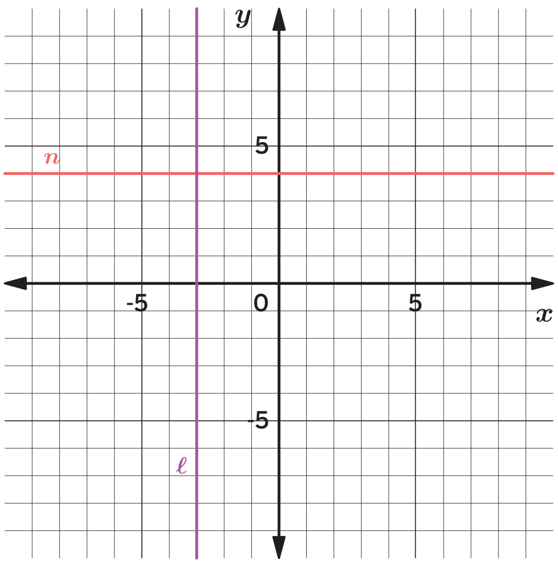
Summary

In today's lesson . . .

You wrote equations for horizontal and vertical lines. In the coordinate plane . . .

- Horizontal lines represent situations where the y-values do not change when the x-values change. Horizontal lines have a slope of 0.
- Vertical lines represent situations where the x-values do not change when the y-values change. Vertical lines have an undefined slope.

For example, the horizontal line n shown is represented by the equation $y = 4$. The vertical line ℓ shown is represented by the equation $x = -3$.

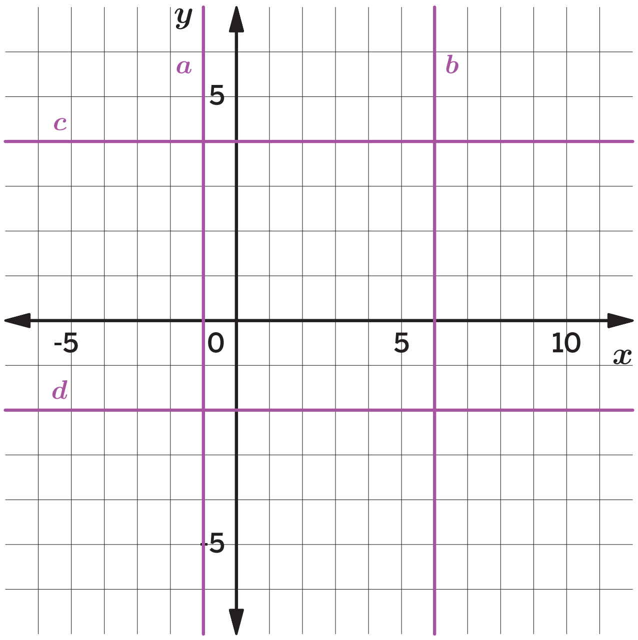


Reflect:

Name: ______ Date: ______ Period: ______

Practice

1. Consider the four lines shown on the coordinate plane. Write an equation for each line.



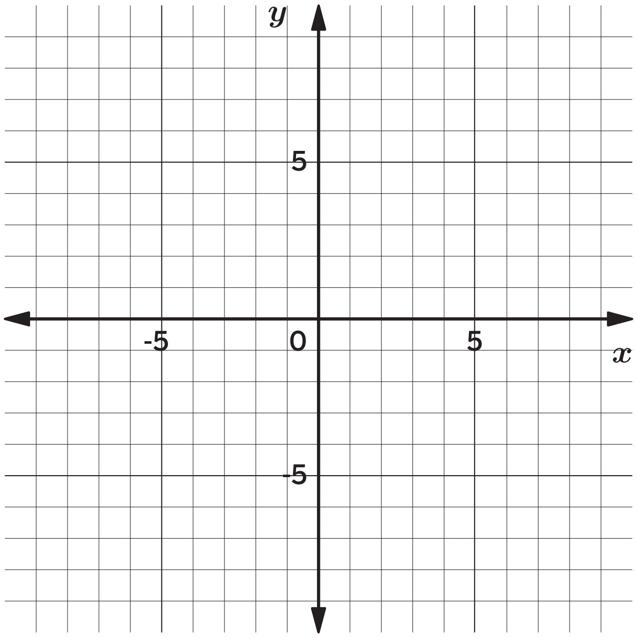
   a. Line a:

   b. Line b:

   c. Line c:

   d. Line d:

2. Use the coordinate plane to draw lines that meet the given criteria. Then write an equation for each line.



   a. Slope: 0
   y-intercept: -2

   b. Slope: 2
   y-intercept: -1

   c. Slope: -2
   y-intercept: 1

   d. Slope: $-\frac{1}{2}$
   y-intercept: -1

Practice

Name: .... Date: .... Period: ....

3. Lin's mom pays her smartphone bill each month. The table shows the amount of data used, in gigabytes (GB), and the total cost for several months. What equation relates the monthly cost y to the amount of data x Lin used?

	January	February	March	April	May
Amount of data used (GB), x	1.2	2	3.2	1.5	4.2
Cost ($), y	50	50	50	50	50

A. $y = 50$ B. $x = 50$ C. $y = 50x$ D. $x + y = 50$

4. A publisher wants to determine the thickness of a book they will print soon. The book has a front cover and a back cover, each of which have a thickness of $\frac{1}{4}$ in. The publisher can choose on what type of paper to print the book.

a. Bond paper has a thickness of $\frac{1}{4}$ in. per 100 pages. Write an equation that gives the width y of the book, if it has x-hundred pages printed on bond paper.

b. Ledger paper has a thickness of $\frac{2}{5}$ in. per 100 pages. Write an equation that gives the width y of the book, if it has x-hundred pages printed on ledger paper.

c. If the publisher selects front and back covers with a thickness of $\frac{1}{3}$ in. each, how would this change the equations you wrote in parts a and b?

5. Consider the equation $\frac{1}{2} = \frac{x}{y}$. What is a possible solution to the equation for x and y? Explain your thinking.

SUB-UNIT

3 Linear Equations

Narrative Connections

How did a 16-year-old take down a Chicago Bull?

In 1987, 16-year-old Eric Barber had written to the TV show "SportsWorld," regarding a featured segment called "Sports Fantasy," which gave viewers the chance to act out their greatest sports dreams. For Barber, it was a face-off against one of the greatest basketball players of all time, Michael Jordan.

The twist? They would play in wheelchairs.

Born with scoliosis, Barber lost the use of his legs at the age of three, but that didn't stop his love for the game. He learned to dribble and shoot on Chicago's playgrounds, and at the age of 13 he was introduced to wheelchair basketball.

Jordan accepted the challenge. The day of the match, Barber was confident his experience would shut Jordan out. And at the start, it looked like Barber was right. He was soon leading 16–4. But Jordan quickly caught up, closing the gap to 18–14.

Finally, Barber sank the winning basket, thereby besting the legend with a final score of 20–14. Barber would go on to become a two-time bronze medal winning member of the U.S. Paralympic wheelchair basketball team.

As in the NBA, points in wheelchair basketball are scored depending on where a shot is taken from. Shots from half-court and beyond are worth three points, while shots within half-court are two points. In Barber and Jordan's face-off, both competitors had several ways to score a winning basket. And linear equations can help model Barber's path to victory.

Unit 3 | Lesson 16

Solutions to Linear Equations

Let's think about what the solution to a linear equation with two variables means.

Warm-up Ordered Pairs

Choose a value for x and a corresponding value for y that makes the following equation true.

$3x + 2y = 24$

Log in to Amplify Math to complete this lesson online.

Name: .... Date: .... Period: ....

Activity 1 Barber vs. Jordan

Eric Barber competed in four Paralympic Games in Wheelchair Basketball, and won two bronze medals during his career. But he might be most known for a game of one-on-one he played against NBA basketball legend, Michael Jordan. For the match, it was decided the first player to score 20 points would win. The players could score baskets worth 2 points, two-pointers or 3 points, three-pointers. Both players would play while in a wheelchair.

1. Determine the number of points Eric Barber scored if he made:

   a. 5 two-pointers and 2 three-pointers.

   b. 4 two-pointers and 4 three-pointers.

   c. 3 two-pointers and 1 three-pointer.

2. Barber had an early lead and was winning 16 – 4, before Jordan began to catch up. What combinations of baskets could Barber have made to score his 16 points? Explain your thinking.

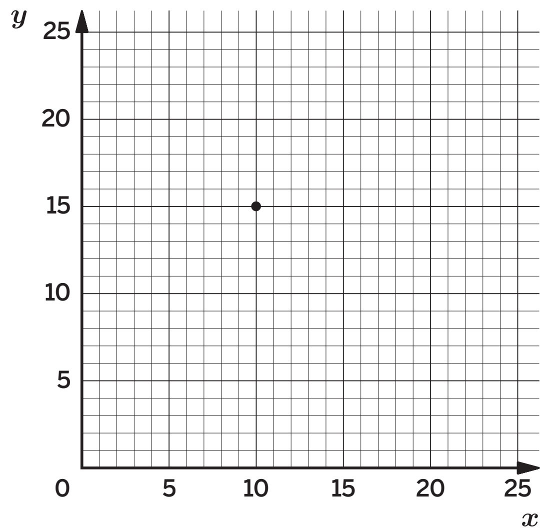
3. Eric Barber eventually won the game 20 – 14. Use two variables to write an equation that represents possible combinations of two-pointers and three-pointers that would equal a total score of 20 points. Be sure to define your variables.

Activity 2 Rectangles

1. There are many possible rectangles whose perimeter is 50 units. Complete the table with lengths x and widths y of at least five rectangles whose perimeter is 50 units.

Length, x					
Width, y					

2. A rectangle with a length of 10 and a width of 15 is represented by the point $(10, 15)$. Plot the lengths x and widths y of the other rectangles whose perimeter is 50 units. What do you notice?



3. Let x represent the length and let y represent the width of a rectangle whose perimeter is 50 units. Write an equation that represents the relationship between x, y, and 50.

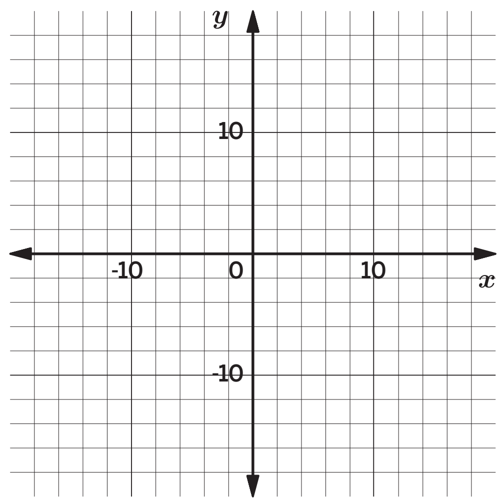
4. Could one of these rectangles have a width of 3.5 units? Explain your thinking using the graph and the equation.

Name: ________ Date: ________ Period: ________

Activity 3 Diophantine Equation

A Diophantine equation is an equation involving two or more variables, such as $Ax + By = C$, where the solutions of interest are pairs of integers. For example, the equation $3x + 2y = 24$ would be a Diophantine equation if the only solutions that made sense were integer solutions.

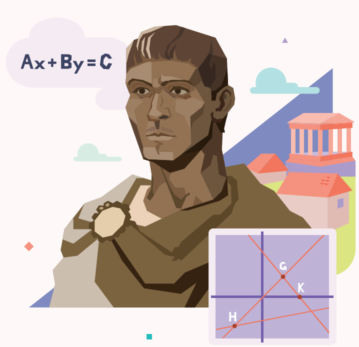
1. Plot at least five points that make the equation $3x + 2y = 24$ true. What do you notice about the points you have plotted?



2. List three points that do not make the statement true. Using a different color, plot and label each point on the same coordinate plane. What do you notice about these points compared to your first set of points?

3. Is the ordered pair $(-2, 15)$ a solution to the Diophantine equation? Explain your thinking.

Featured Mathematician



Diophantus

The term *Diophantine* comes from the 3rd century mathematician Diophantus of Alexandria. Diophantus was known for his published work, *Arithmetica*, in which he detailed the solutions to different types of algebraic problems. At the time, algebraic symbols and variables had not yet been invented, so Diophantus had to use words to describe the "first unknown" or "second unknown."

STOP

Summary

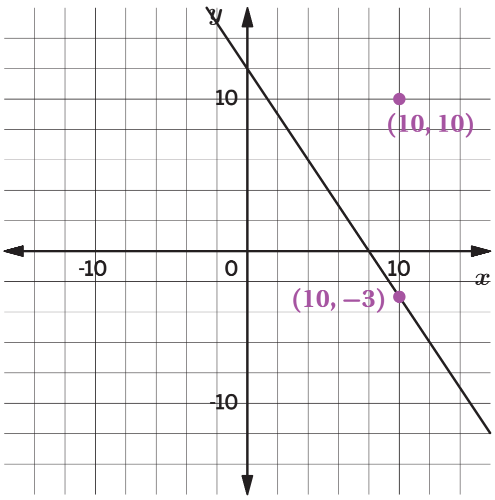
In today's lesson . . .

You saw that a solution to an equation with two variables is any ordered pair (x, y) that makes the equation true.

You can think of pairs of numbers that are solutions to a linear equation as ordered pairs (x, y) that represent points on the coordinate plane. These points form a line that represents all of the solutions to the equation. Only points that fall on the line are solutions to the equation. Points that do not fall on the line are *not* solutions to the equation.

For example, consider the linear equation $3x + 2y = 24$.

- The point $(10, -3)$ is on the line $3x + 2y = 24$. The ordered pair $(10, -3)$ is a solution to the equation $3x + 2y = 24$ because it makes the equation true; $3(10) + 2(-3) = 24$.
- The point $(10, 10)$ is not on the line. The ordered pair $(10, 10)$ is not a solution because $3(10) + 2(10) = 50$, not 24.



Reflect:

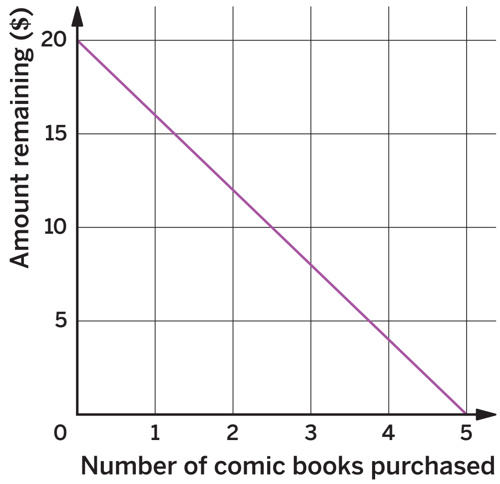
Name: ________________ Date: ________ Period: ________

Practice

1. Select *all* of the ordered pairs (x, y) that are solutions to the linear equation $2x + 3y = 6$.

A. $(0, 2)$
B. $(0, 6)$
C. $(2, 3)$
D. $(3, -2)$
E. $(3, 0)$
F. $(6, -2)$

2. The graph shows a linear relationship between x and y. Suppose x represents the number of comic books Priya buys at the store, all at the same price, and y represents the amount of money, in dollars, she has after buying comic books.



a. Find and interpret the x- and y-intercepts of this line

b. Find and interpret the slope of this line.

c. Write an equation of this line.

d. If Priya buys 3 comic books, how much money will she have left? Explain your thinking.

3. A container of fuel dispenses fuel at a rate of 5 gallons per minute. Let y represent the amount of fuel remaining in the container, and x represent the number of minutes that have passed since the fuel started dispensing. On the coordinate plane, will the slope of the line representing this relationship have a positive, negative, or zero slope? Explain your thinking.

Practice

Name: .... Date: .... Period: ....

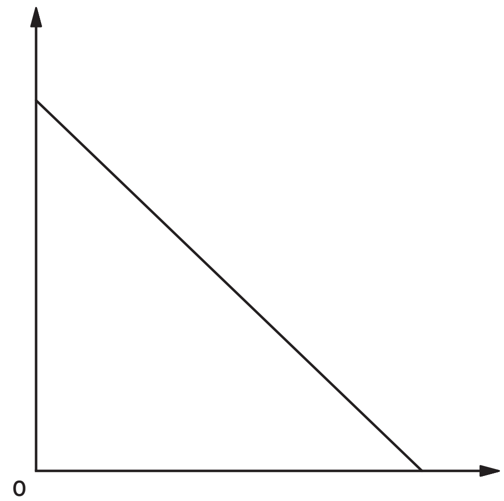
4. A sandwich store charges a delivery fee to bring lunch to an office building. One office pays \$33 for 4 turkey sandwiches. Another office pays \$61 for 8 turkey sandwiches. How much does each turkey sandwich add to the cost of the delivery? Explain your thinking.

5. Match each pair of points with the slope of the line that passes through them.

Pair of points	Slope
a $(9, 10)$ and $(7, 2)$	 4
b $(-8, -11)$ and $(-1, -5)$	.... -3
c $(5, -6)$ and $(2, 3)$	.... $-\frac{5}{2}$
d $(6, 3)$ and $(5, -1)$	.... $\frac{6}{7}$
e $(4, 7)$ and $(6, 2)$	

6. Refer to the graph shown.

a Write a story that matches the graph and label the axes

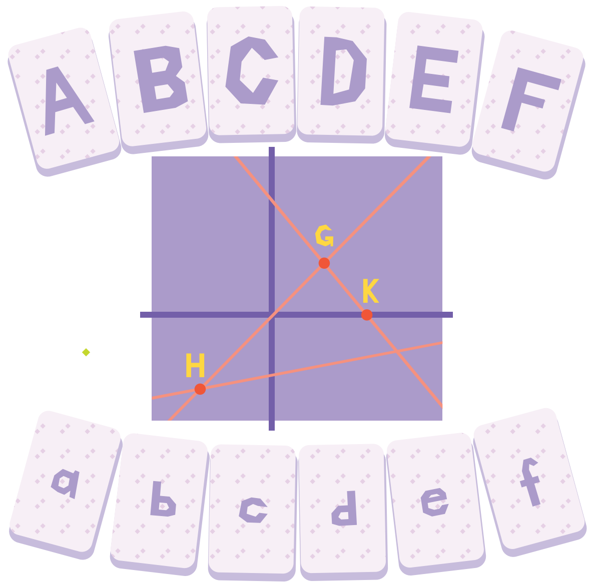


b Label two points on the line, one where $x = 0$ and one where $y = 0$. Then explain what each point means in the context of the story.

Name: ______ Date: ______ Period: ______

Unit 3 | Lesson 17

More Solutions to Linear Equations



Let's find solutions to more linear equations.

Warm-up Intercepts

Find the horizontal and vertical intercepts for the line represented by each equation.

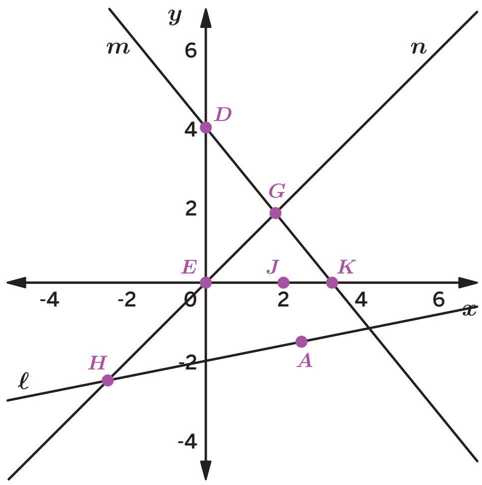
1. $y = -3x + 3$

2. $2x + 5y = -10$

Log in to Amplify Math to complete this lesson online.

Activity 1 True or False?

Refer to the diagram shown for this activity. For each statement, decide if it is *true* or *false*. Explain your thinking.



	True or False?	Explain your thinking.
1. The ordered pair $(4, 0)$ is a solution to the equation that represents line m.		
2. The coordinates of point G make both of the equations for line m and line n true.		
3. The ordered pair $(2, 0)$ makes both of the equations for line m and line n true.		
4. There is no solution to the equation represented by line ℓ that has a y-value of 0.		

Name: ________ Date: ________ Period: ________

Activity 2 I'll Take an X, Please

Plan ahead: Make a plan for how you will listen to your partner without interrupting or only being focused on sharing your thoughts.

You and your partner will be given six cards labeled A through F and six cards labeled a through f. In each pair of cards (for example, Cards A and a), there is an equation in one card and an ordered pair, (x, y), that makes the equation true on the other card.

If you are given an *equation* card:	If you are given an *ordered pair* card:
1. Ask your partner for either the x-value or the y-value. Explain why you want this particular value.	**1.** Provide the value your partner requests.
2. Use the value your partner provides to find the value of the remaining unknown variable. Explain each step as you go. Show your calculations on a separate sheet of paper.	**2.** After your partner finds the remaining unknown variable, tell them if they are correct or incorrect.
3. If your value is correct, move onto the next set of cards. If your value is incorrect, look through your steps to find and correct any errors.	**3.** If your partner's value is correct, move onto the next set of cards. If your partner's value is incorrect, look through their steps to find and correct any errors.

Keep playing until you have completed Cards A through F.

STOP

Summary

In today's lesson . . .

You saw that no matter the form a linear equation is given, you can always determine solutions to the equation by starting with one value, and then solving for the other value.

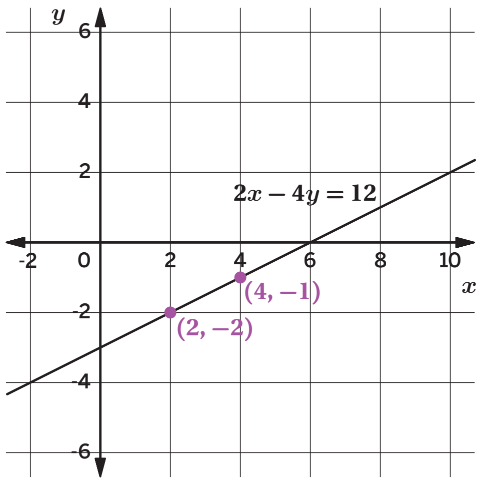
For example, consider the linear equation $2x - 4y = 12$.

- To determine a solution that has $x = 2$, you can substitute $x = 2$ into the equation and solve for y.

$$\begin{aligned} 2(2) - 4y &= 12 \\ 4 - 4y &= 12 \\ -4y &= 8 \\ y &= -2 \end{aligned}$$

- To determine a solution that has $y = -1$, you can substitute $y = -1$ into the equation and solve for x.

$$\begin{aligned} 2x - 4(-1) &= 12 \\ 2x + 4 &= 12 \\ 2x &= 8 \\ x &= 4 \end{aligned}$$



Reflect:

Name: ________ Date: ________ Period: ________

Practice

1. For each equation, determine the value of y when $x = -3$. Then determine the value of x when $y = 2$.

a $y = 6x + 8$

x	y
-3	
	2

b $y = \frac{2}{3}x$

x	y
-3	
	2

c $y + x = 5$

x	y
-3	
	2

d $y = \frac{3}{4}x - 2\frac{1}{2}$

x	y
-3	
	2

2. State whether the following is *true* or *false*. Show or explain your thinking.

The ordered pairs $(6, 13)$, $(21, 33)$, *and* $(99, 137)$ *all lie on the same line. The equation of the line is* $y = \frac{4}{3}x + 5$.

3. Consider the linear equation $y = \frac{1}{4}x + \frac{5}{4}$.

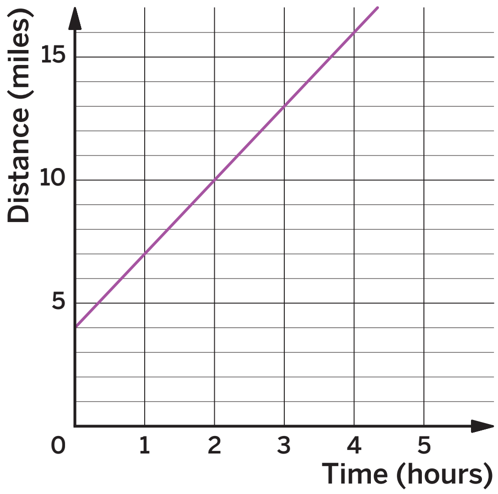
a Are the ordered pairs $(1, 1.5)$ and $(12, 4)$ solutions to the equation? Show or explain your thinking.

b Find the x-intercept of the graph of the equation. Show or explain your thinking.

Practice

Name: .... Date: .... Period: ....

4. A group of hikers park their car at a trailhead and walk into the forest to a campsite. The next morning, they head out on a hike from their campsite, walking at a constant rate. The graph shows their distance d in miles from their car after h hours of hiking.

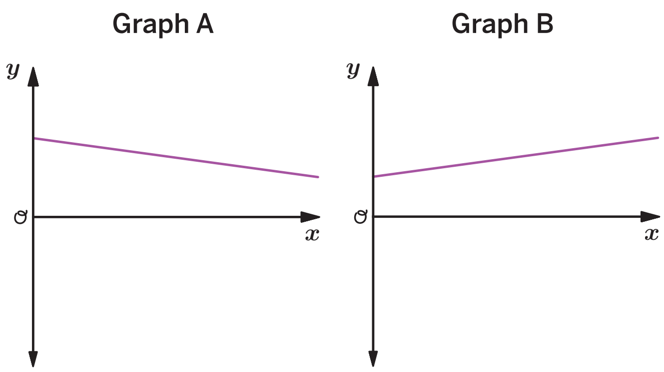


a. How far is the campsite from their car? Explain your thinking.

b. Write an equation that gives the distance d from their car for any number of hours hiked h.

c. After how many hours of hiking will they be 16 miles from their car? Explain your thinking.

5. Decide which graph best represents each of the following situations.



a. y represents the weight of a kitten x days after birth.

b. y represents the distance remaining in a car ride after x hours of driving at a constant rate toward its destination.

6. Write an equation to represent each relationship described.

a. Grapes cost $\$2.39$ per pound. Bananas cost $\$0.59$ per pound. You have $\$15$ to spend on g pounds of grapes and b pounds of bananas.

b. A savings account has $\$50$ in it at the start of the year and $\$20$ is deposited each week. After x weeks, there are y dollars in the account.

Name: .... Date: .... Period: ....

Unit 3 | Lesson 18

Coordinating Linear Relationships

Let's coordinate representations of linear relationships.

Warm-up Hunting for Ordered Pairs

Find four ordered pairs that are solutions to the equation, $\frac{1}{3}x + \frac{1}{2}y = 10$. Record the values in the table.

x	y

Log in to Amplify Math to complete this lesson online.

Activity 1 Representations of Linear Relationships

Lin and Mai went on a canoeing trip. From their starting point, they will paddle and *portage* — hike while carrying their canoe — a total of 20 miles to arrive at their campsite before dark. They can paddle at a speed of 1.5 miles per hour and portage at a speed of 2 miles per hour.

Complete the following problems to describe the relationship between the number of hours they can paddle and the number of hours they can portage.

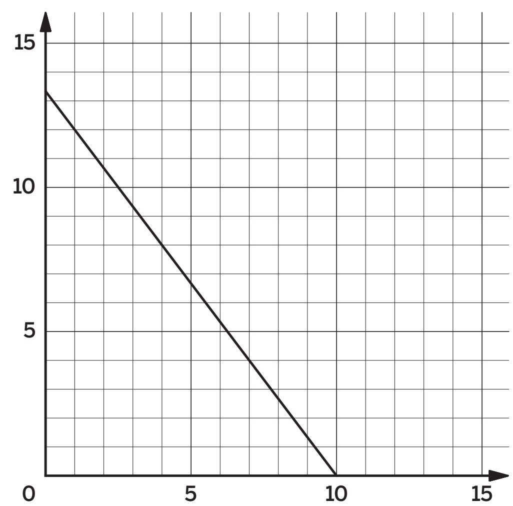
Meg Wallace Photography/Shutterstock.com

1. Define your variables.

2. Create a table, graph a line, and write an equation to represent the situation.

Table:

Graph:



Equation:

Name: ______ Date: ______ Period: ______

Activity 1 Representations of Linear Relationships (continued)

3. How can you find the rate of change using the table and the graph? What does the rate of change mean within the context of this problem?

4. Explain how you can tell that the equation, description, graph, and table all represent the same relationship.

Stronger and Clearer: Share your response to Problem 4 with 2–3 partners. Ask each other clarifying questions and offer suggestions for improvement. Then revise your original response based on their feedback.

Activity 2 Info Gap: Linear Relationships

Pictured here, rock climbers have set up their tents, suspended in air with ropes, as they rest and prepare to climb to the summit. You will be given either a problem card or data card describing a scenario related to rock climbing. Do not show or read your card to your partner.

Wollertz/Shutterstock.com

If you are given the *problem card:*	If you are given the *data card:*
1. Silently read your card, and think about what information you need to be able to solve the problem.	**1.** Silently read your card.
2. Ask your partner for the specific information that you need.	**2.** Ask your partner, "What specific information do you need?" and wait for them to ask for information.
3. Explain how you are using the information to solve the problem. Continue to ask questions until you have enough information to solve the problem.	**3.** Before sharing the information, ask, "Why do you need that information?" Listen to your partner's reasoning, and ask clarifying questions.
4. Share the problem card, and solve the problem independently in the space provided on this page.	**4.** Read the problem card, and solve the problem independently in the space provided on this page.
5. Read the data card, and discuss your reasoning.	**5.** Share the data card, and discuss your reasoning.

Pause here so your teacher can review your work. You will be given a new set of cards. Repeat the activity, trading roles with your partner.

STOP

Name: .... Date: .... Period: ....

Summary

In today's lesson . . .

You explored how linear relationships can be represented in multiple ways. Linear relationships can be represented with written descriptions, equations, graphs, and tables. Which representation you choose depends on the purpose. When creating representations, you can choose helpful values by paying attention to the context.

Written description:

An athlete wants to buy snack bars that cost \$2 each and hydration drinks that cost \$3 each. They have \$24 to spend.

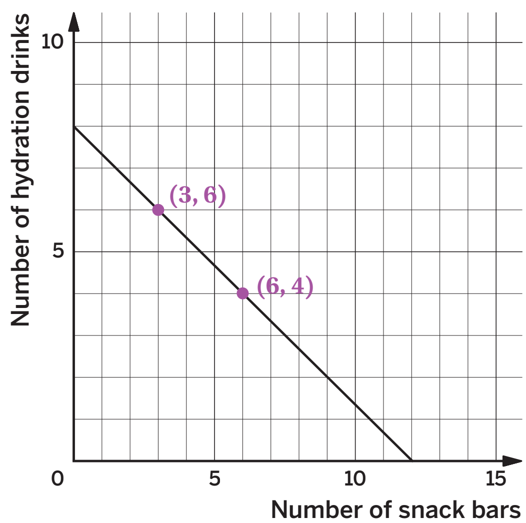
Equation:

$y = -\frac{2}{3}x + 8$ or $2x + 3y = 24$

Table:

Number of snack bars, x	Number of hydration drinks, y
6	4
3	6

Graph:

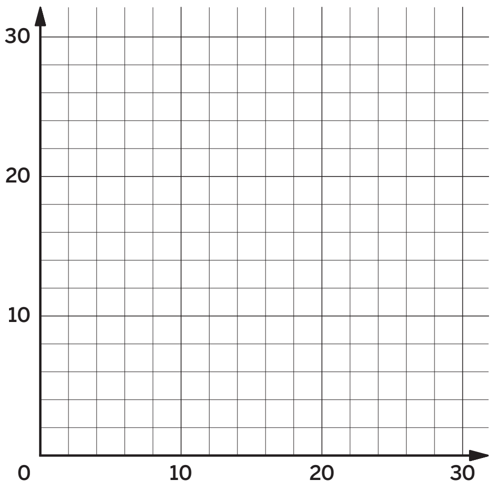


Reflect:

Practice

Name: .... Date: .... Period: ....

1. A high school theater is selling tickets to the school's spring musical. For family members of the cast, tickets cost $\$5$ each. For the general public, tickets cost $\$10$ each. The school earns $\$120$ from their ticket sales on opening night.



   a. Give two possible combinations of the number of family and general public tickets sold that earn a total of $\$120$ in sales.

   b. Write an equation to represent the relationship between the number of family tickets x sold and the number of general public tickets y sold.

   c. Graph this relationship on the coordinate plane. Label the axes.

2. Jada is planning a cookout for her family. She wants to buy veggie burgers and determines 20 lb will be enough for the cookout. Veggies-R-Us sells veggie burgers in 4-lb packages. Betabel Burgers sells veggie burgers in 3-lb packages.

   a. Write an equation to represent the relationship between the number of packages x Jada can buy from Veggies-R-Us, the number of packages y she can buy from Betabel Burgers, and the total amount of veggie burgers she needs.

   b. Is $(1.25, 5)$ a solution to both the equation and the problem? Explain your thinking.

Practice

Name: ________ Date: ________ Period: ________

3. Match each equation with the set of ordered pairs that are solutions to the equation. Some sets of ordered pairs may have no matching equation or more than one matching equation.

Equation	Sets of ordered pairs
a $y = 1.5x$	______ $(14, 21), (2, 3), (8, 12)$
b $2x + 3y = 7$	______ $(-3, -7), (0, -4), (-1, -5)$
c $x - y = 4$	______ $\left(\frac{1}{8}, \frac{7}{8}\right), \left(\frac{1}{2}, \frac{1}{2}\right), \left(\frac{1}{4}, \frac{3}{4}\right)$
d $3x = 2y$	______ $\left(1, 1\frac{2}{3}\right), (-1, 3), \left(0, 2\frac{1}{3}\right)$
e $y = -x + 1$	______ $(0.5, 3), (1, 6), (1.2, 7.2)$

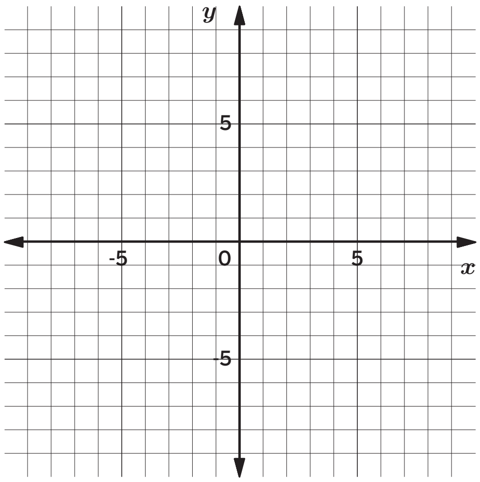
4. Use the coordinate plane to graph each equation.

a **Equation A:** $y = x - 5$

b **Equation B:** $y = 3x + 1$

c **Equation C:** $y = -\frac{1}{3}x - 2$

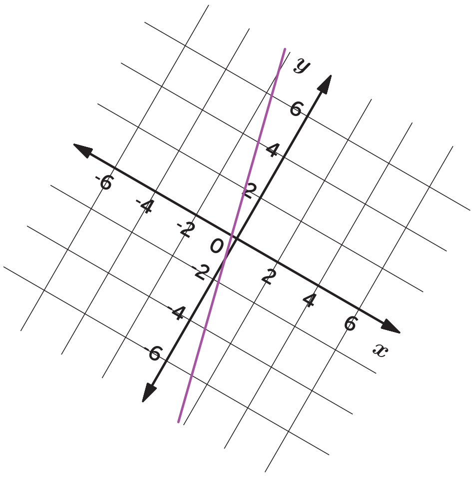
d **Equation D:** $y = 0.4x$



5. Consider the graph shown.

a What do you notice?

b What do you wonder?

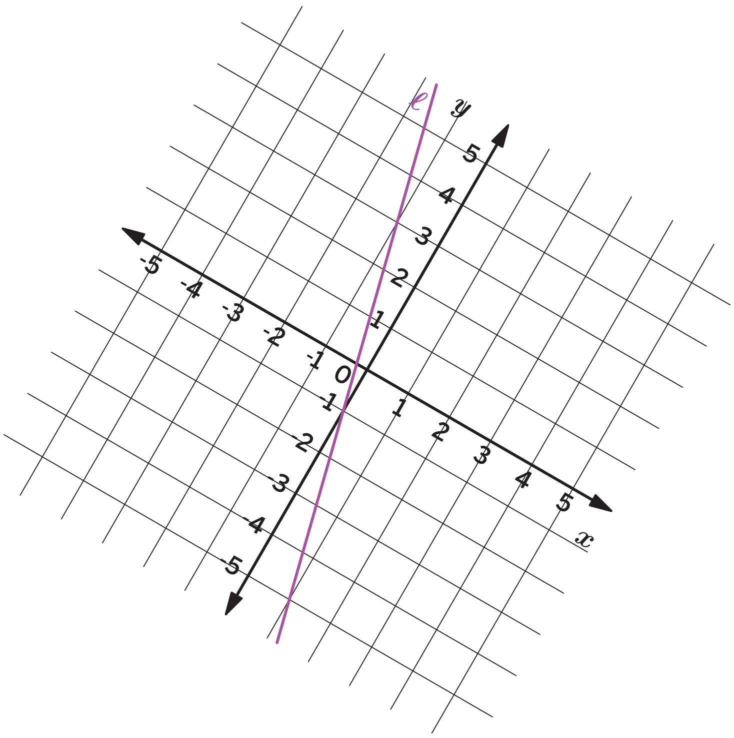


Rogue Planes

Let's see what happens when the coordinate plane acts in unusual ways.

Warm-up True or False?

Is it true that the slope of line ℓ is -4? Explain your thinking.



Log in to Amplify Math to complete this lesson online.

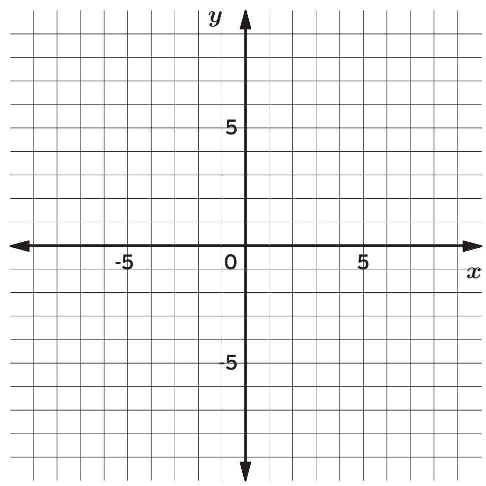
Name: Date: Period:

Activity 1 Something Weird Is Happening . . .

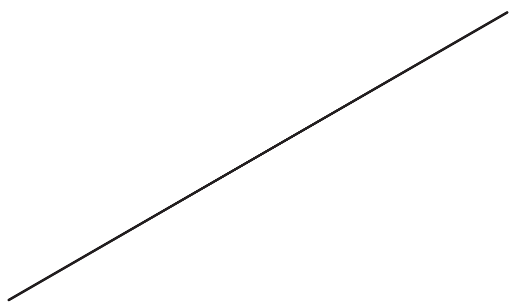
The coordinate plane we know and love has gone rogue!

Trace the coordinate plane on a piece of tracing paper. Orient the coordinate plane on each line such that the line matches the equation on the rotated, rogue coordinate plane.

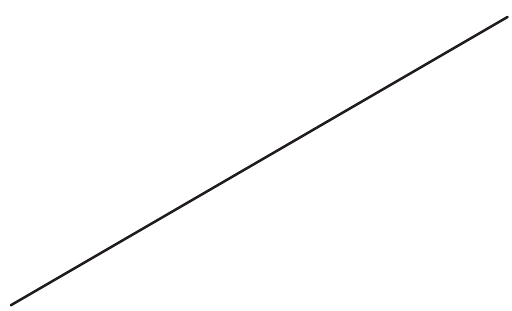
Sketch your graph on top of each line.



1. $y = x$

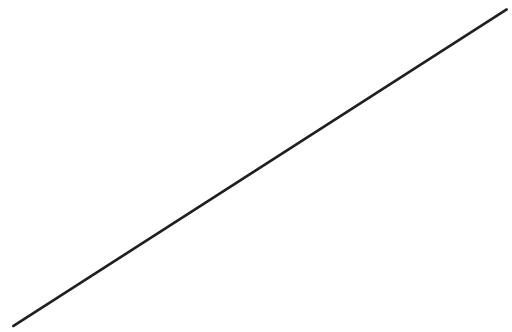


2. $y = 5x$

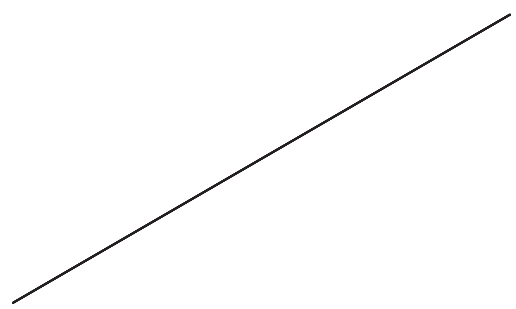


Activity 1 Something Weird Is Happening . . . (continued)

3. $y = -\frac{3}{2}x + 2$



4. $y = -\frac{1}{2}x - 4$

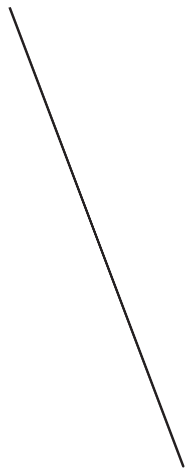


Name: ________________ Date: ________ Period: ________

Activity 2 Partner Planes

Up to this point, you have seen how lines can change on the coordinate plane. Today, you also encountered planes that were being transformed. Understanding changes to the coordinate plane, or “frame of reference,” for moving objects was key to the development of relativity by scientists and mathematicians like Albert Einstein and Emmy Noether.

Write an equation for the line on the space provided. Trade with a partner to see if they can place the coordinate plane on the line to correctly match your equation. Check their sketch to confirm they are correct.



Equation: ________________

Featured Mathematician

Emmy Noether

Born in Bavaria, Germany in 1882, Noether was a pioneer in abstract algebra. After completing her doctorate, she taught at a German university for seven years without pay due to sexism in academia. As the Nazis rose to power in the 1930s, Noether moved to the United States, teaching at Bryn Mawr and Princeton.

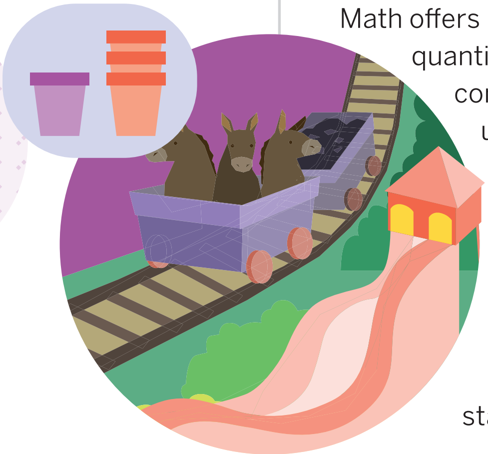
In 1915, she worked with David Hilbert and Felix Klein to further develop Albert Einstein’s theory of general relativity — a geometric interpretation of gravity. Noether proved that energy and momentum are indeed conserved in different physical systems, no matter how they are oriented — that is, whether or not their planes have gone rogue!

STOP

Public Domain

Unit Summary

Two people are running at different speeds toward the same destination — who will get there first? Which cell phone plan is more affordable, given their up-front and monthly costs? How high will an airplane be thirty seconds after takeoff?



Math offers us tools to measure these quantities precisely, to visualize and compare them. Many kinds of change can be understood using linear equations. And when these "linear" equations are plotted on a graph — surprise, surprise — they appear as straight lines.

Something as seemingly straightforward as a line in fact holds all kinds of useful information. Its slope conveys how fast or slow something is changing. Its vertical intercept shows a starting value.

Keep an eye out for things that move, change, or transform. You will notice that many of the changes you observe can be modeled as linear equations.

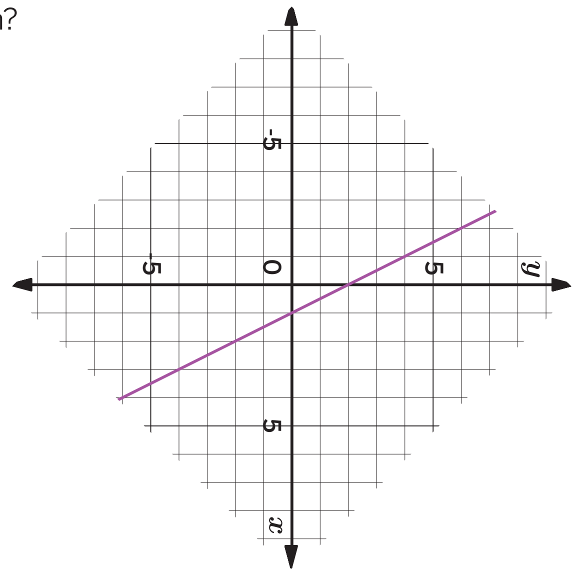
But some changes are different, and are not linear at all. Are there mathematical tools for describing these other kinds of change? Read on.

See you in Unit 4.

Name: .... Date: .... Period: ....

Practice

1. What is the equation of the line shown?



2. For each pair of points, find the equation of the line that passes through both points.

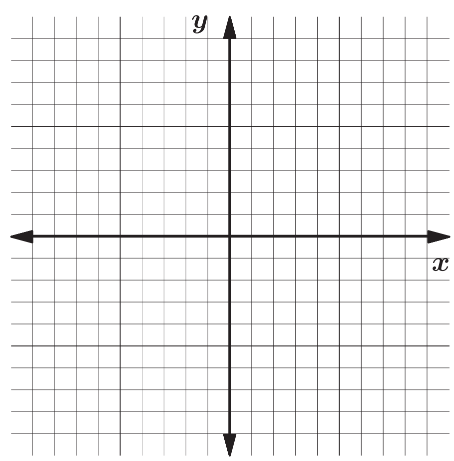
a $(0, 1)$ and $(2, 5)$

b $(1, 1)$ and $(7, 5)$

c $(-2, 11)$ and $(1, -1)$

d $\left(-5, \frac{3}{2}\right)$ and $\left(4, \frac{3}{2}\right)$

3. Graph the linear relationship with a slope of 4 and a negative y-intercept. Show how you know the slope is 4 and then write an equation for the line.



Practice

Name: .... Date: .... Period: ....

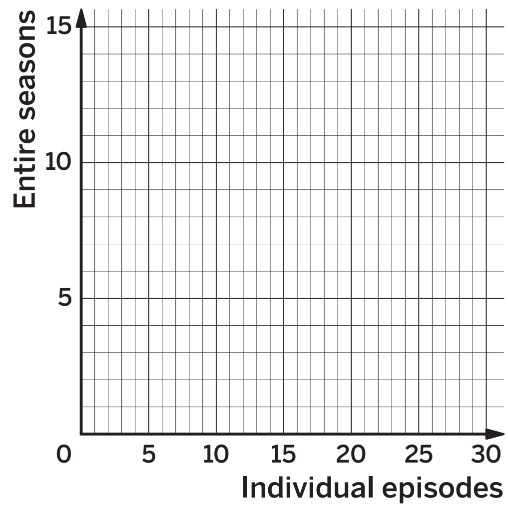
4. It costs $1.50 to download a television episode to watch and $12 to download the entire season of episodes. Jada has $42 to spend downloading television shows.

a. Complete the table showing three possible ways Jada can spend $42 downloading

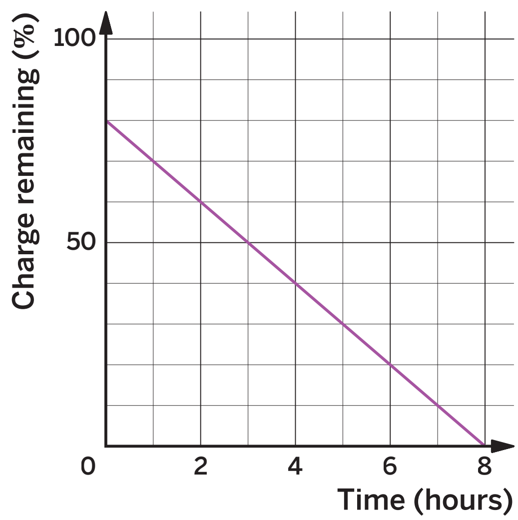
Individual episodes	Entire seasons

b. Write an equation relating the number of individual episodes x and the number of entire seasons y Jada can download.

c. Graph the relationship on the coordinate plane.



5. The graph shows the percentage of charge c remaining on Clare's cell phone after it has been in use for h hours.



a. Write an equation that gives the percentage of charge c remaining on her phone after h hours of use.

b. Explain what each number in your equation represents in this situation.

c. When will Clare's phone run out of charge? Where do you see this on the graph?

My Notes:

UNIT 4

Linear Equations and Systems of Linear Equations

In this unit, you'll learn the algebraic skills you need to find a solution for x. But wait, what do you mean there is *no* solution?

Essential Questions

- How can you determine the solution to an equation with variables on both sides?
- What does the number of solutions (none, one, or infinite) to a system of linear equations represent?
- How can systems of equations be used to represent situations and solve problems?
- (*By the way, how can you see solutions to systems of equations?*)

SUB-UNIT

1 Linear Equations in One Variable

Narrative: Without the work of mathematicians like Al-Khwarizmi, math might not be the universal language you know today.

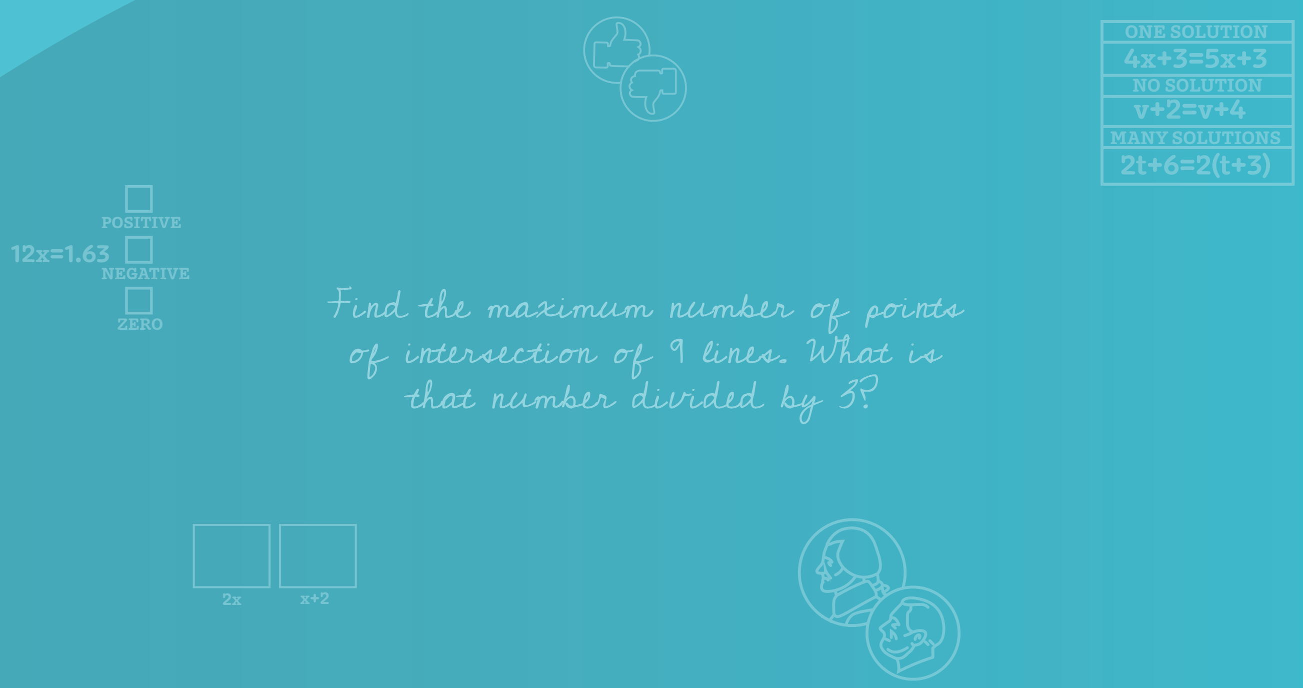
You'll learn . . .
- about solving linear equations using a variety of strategies.
- how to build fluency with these strategies.

SUB-UNIT

2 Systems of Linear Equations

Narrative: Discover how more than one equation can help you solve problems with more than one constraint.

You'll learn . . .
- what the solution to a system of linear equations means.
- how systems of linear equations can help solve everyday problems.



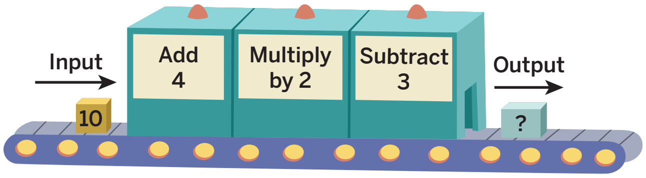
Unit 4 | Lesson 1 – Launch

Number Puzzles

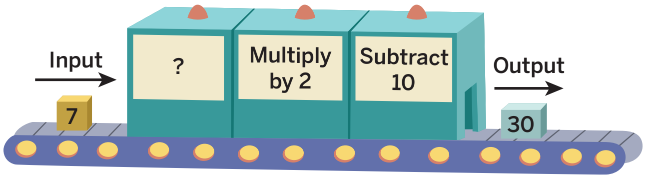
Let's solve some puzzles!

Warm-up Number Machine

1. Math, Inc. built a number machine. If 10 is the input, what number will be the output? Show or explain your thinking.



2. Advanced Algebra Corporation built their own number machine, but they lost the part that describes the first step. When 7 was the input, 30 was the output. Can you determine the missing part for the first step? Show or explain your thinking.

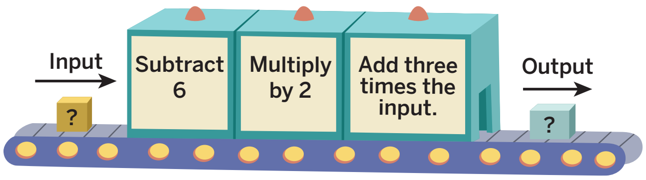


Log in to Amplify Math to complete this lesson online.

Name: .... Date: .... Period: ....

Activity 1 Think of a Number . . .

You will be given some sticky notes. Consider the number machine shown.



1. Think of a number to represent the input of the number machine. Record your input on a sticky note, without showing anyone.

2. When you put your number into the machine, what is the output? Record your output on a different sticky note, and share your output with a partner.

3. Record your partner's output and guess their input. Show your thinking and any calculations needed. Switch partners in your group until you have worked with all three partners.

   a. If Partner 1's output is ...., Partner 1's input is ....

   b. If Partner 2's output is ...., Partner 2's input is ....

   c. If Partner 3's output is ...., Partner 3's input is ....

Collect and Display:
As you discuss your results, your teacher will collect words and phrases you use. This language will be added to a class display for your reference.

4. How did you determine the input, when given the output?

Activity 2 Build Your Own Number Machine

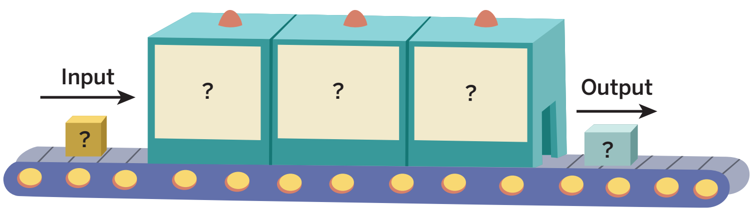
The number machine shows three steps. Create your own descriptions for the three steps. Using a sticky note, choose an input for your number machine and record the output.

My puzzle:

Step 1: ....

Step 2: ....

Step 3: ....



1. Trade puzzles and outputs with your partner to see if you can determine each other's inputs. Show your thinking. Then check whether your partner used your number machine correctly by confirming their input is correct.

2. With your partner, compare your solutions to each puzzle. Did each of you solve the puzzle the same way? If not, be prepared to share with the class which solution strategy you think is the most efficient.

Are you ready for more?

Consider a number machine with the following steps:

- Think of a number.
- Double the number.
- Add 9.
- Subtract 3.
- Divide by 2.
- Subtract the original number.
- The output should be 3.

Why does this always work?

STOP

Unit 4 Linear Equations and Systems of Linear Equations

The Path the Mind Takes

Riddles exist in almost every culture.

There is the ancient Greek tale of the Sphinx. The Sphinx was a fearsome creature with a human head, the body of a lion, and the wings of an eagle. It stood guard over the city of Thebes. Any traveler who wished to enter or leave the city had to answer the Sphinx's riddle. Those who failed were instantly devoured!

The Dagara people of West Africa tell *zupkai* to their children — a blending of folktales, riddles, and proverbs that provide valuable lessons through storytelling.

Meanwhile, in East Asia, the Zen Buddhist *kōans* present seemingly impossible scenarios — such as the sound of one hand clapping. These *kōans* were designed to inspire thought and reflection in the listener.

Across the globe, people have been fascinated by riddles in one form or another. But the most exciting part of any riddle is not the answer itself, but the path the mind takes to find the answer.

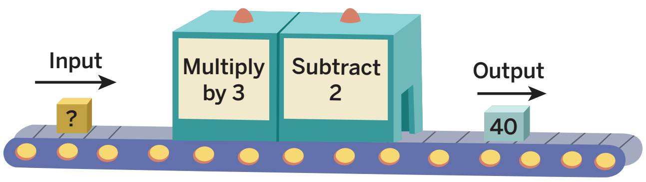
Math has plenty of riddles too, but they are not typically presented through stories, poems, or rhymes. Instead, they are posed using numbers and symbols, in a discipline called algebra. And much like the sphinx's riddle, the *zupkai*, or the *kōan*, the joy should not be in the answer itself, but the journey of finding the answer.

Welcome to Unit 4.

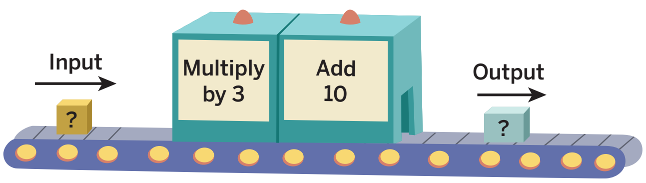
Practice

Name: .... Date: .... Period: ....

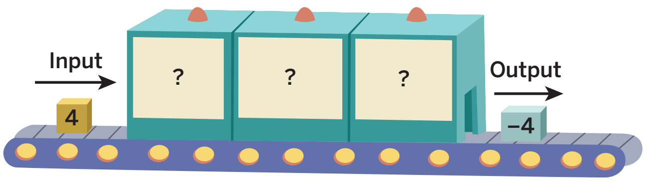
1. Diego chose a number to represent the input of the machine shown and 40 was the output. What was his input? Explain your thinking.



2. Clare wants to solve this puzzle represented by the number machine shown. She says, "If you take the output, then divide by 3 and subtract 10, the result is the same as the input." Do you agree? Explain your thinking.



3. The number machine shown has three steps. Create your own descriptions of these three steps so that an input of 4 gives an output of -4.



Step 1: ....

Step 2: ....

Step 3: ....

Name: ______________________ Date: __________ Period: __________

Practice

4. Solve each equation. Show your thinking.

a) $4x + 9 = 11$

b) $-3(x + 7) = -15$

5. Select *all* of the given points that lie on the graph of the linear equation $4x - y = 3$.

A. $(-1, -7)$

B. $(0, 3)$

C. $\left(\frac{3}{4}, 0\right)$

D. $(1, 1)$

E. $(5, 2)$

F. $(4, -1)$

6. Write each verbal description as a mathematical expression.

a) 5 more than x

b) k less than $\frac{1}{2}$

c) Half of r

d) The product of 12 and p

My Notes:

SUB-UNIT

1

Linear Equations in One Variable

Narrative Connections

Who was the Father of Algebra?

Situated along the banks of the Tigris river, the city of Baghdad was a bustling hub of business and commerce in the ninth century. But as economic activity grew, so too did disputes. Laborers needed wages. Inheritances needed to be split. Land had to be divided.

But resolving each dispute individually was time-consuming. But thanks to the Persian mathematician Muhammad ibn Mūsā al-Khwārizmī, a system was developed to help settle these disputes more efficiently.

Very little is known about Al-Khwārizmī's early life. By the age of 40, Al-Khwārizmī was invited by Caliph al-Ma'mun to Baghdad's House of Wisdom. This academic center hosted leading scholars and was considered the center of knowledge in the world at the time. There, Al-Khwārizmī was appointed as an astronomer and later as the head of the library.

Al-Khwārizmī's methods for settling disputes made up a great portion of his book, *The Compendious Book on Calculation by Completion and Balancing*, or *Hisab al-jabr w'al-muqabala*. The "al-jabr" in the title is where the word *algebra* is derived. Al-Khwārizmī's book brought together the geometry of Greeks and the algorithmic methods of Indian, Mesopotamian, and Chinese scholars.

It might seem like math is a universal language, but it wasn't always so. It took the work of mathematicians like Al-Khwārizmī to create the mathematical language and balancing methods we still use today to solve for unknowns.

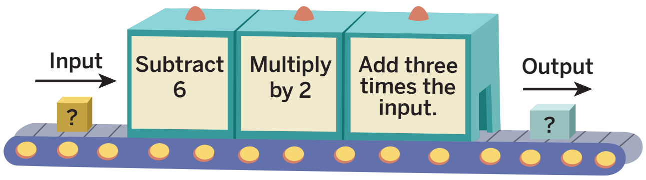
Unit 4 | Lesson 2

Writing Expressions and Equations

Let’s write expressions and equations.

Warm-up Think of a Number, Revisited

In Lesson 1, you worked with the number machine shown. Now, you will build an expression to represent the number machine.



1. The table shows the number machine steps. Let n represent any chosen input. Complete the table with a possible expression to represent the resulting value for each step.

Description from number machine	Expression
Let n represent the input.	n
Subtract 6.	
Multiply by 2.	
Add three times the input.	

2. How can you use the final expression you wrote in Problem 1 to:

a Determine the output if you use any input?

b Determine the input if you know the output?

Log in to Amplify Math to complete this lesson online.

Name: ______________________ Date: ____________ Period: ____________

Activity 1 Think of a Number, Revisited

Kiran used the following steps to find the input for the number machine from the Warm-up if the output is 17. Describe what Kiran did in each step.

Equation	Description
$2(n - 6) + 3n = 17$	Set the expression from the Warm-up equal to the output, 17.
$2n - 12 + 3n = 17$	
$2n + 3n - 12 = 17$	
$5n - 12 = 17$	
$5n - 12 + 12 = 17 + 12$	
$5n = 29$	
$5n \div 5 = 29 \div 5$	
$n = \frac{29}{5}$	This is the input, the solution to the equation.

Are you ready for more?

Consider a number machine that processes the following steps. Write an expression that represents the output, for any input. Define the variable you choose to use.

- Think of a number.
- Double the number.
- Add 9.
- Subtract 3.
- Divide by 2.
- Subtract the original number.

Activity 2 How Much Did Each Give?

In 1881, a local farmer from a village called Bakhsahli, a region in modern-day Pakistan, noticed a piece of birch bark buried in their field. Turned out, this was not some ordinary piece of bark. The bark was actually an ancient Indian mathematical text, the *oldest* known Indian mathematical text, now known as the Bakhshali manuscript. The manuscript is so old, researchers cannot say for certain when it was written. Some estimates suggest it was written as early as 224 CE.

Here is a similar problem to one written in the Bakhshali manuscript:

Of four coin donors, the second donor gave twice the first donor. The third donor gave three times more than the first donor and the fourth donor gave four more than the first. Together, all four donors gave 32 coins. How much did each give?

1. Choose a variable to represent the number of coins the first donor gave.

2. Write an expression that represents the number of coins each donor gave, based on the number of coins the first donor gave.

   a. First donor:
   b. Second donor:
   c. Third donor:
   d. Fourth donor:

3. Write an expression that represents how much the donors gave altogether.

4. Recall that together they donated 32. Write an equation that represents this statement.

5. Solve the equation you wrote in Problem 4. Show your thinking.

6. How many coins did each of the donors give? Explain your thinking.

Reflect: How can identifying and defining the variable help you to be more successful in solving this problem?

STOP

Name: ______ Date: ______ Period: ______

Summary

In today's lesson . . .

You wrote expressions and equations to represent scenarios. You then worked to write the expressions into fewer terms by using the Distributive Property and combining like terms. To solve the equations, you reviewed the properties of equality to ensure your equations were equivalent.

You will continue to practice solving equations for the remainder of this unit and develop strategies which will be useful for the rest of your mathematical career.

Reflect:

Practice

Name: .... Date: .... Period: ....

1. For each expression, combine like terms and write an equivalent expression with fewer terms. Show your thinking.

   a $4(x - 9) + 6$

   b $\frac{2}{3}(6x - 9) - (4x + 1)$

   c $3 - 2(x - 9) + 7$

2. Solve each equation. Show your thinking.

   a $-3y - 9y + 5y = 2.1$

   b $0.5x - \frac{1}{2}(4x - 6) = 7.5$

3. In a basketball game, Elena scored twice as many points as Tyler. Tyler scored 4 points fewer than Noah, and Noah scored three times as many points as Mai. If Mai scored 5 points, how many points did Elena score? Explain your thinking.

4. Triangle A is an isosceles triangle. One angle measures x degrees and another angle measures y degrees.

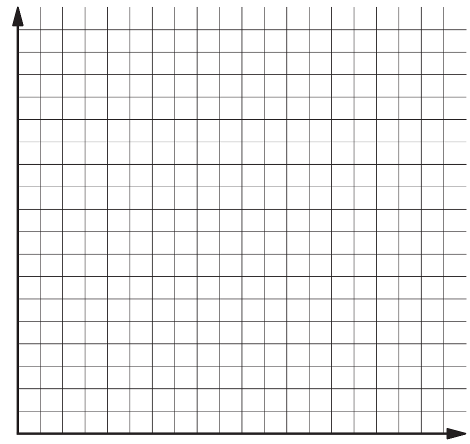
   a What values could x and y represent? Determine three pairs of values for x and y that could be the angle measures of the triangle.

   b Write an equation relating x and y.

Name: ______________________ Date: __________ Period: __________

Practice

5. A store is designing the space for rows of nested shopping carts. Each row has a starting cart that is 4 ft long, followed by the nested carts (so 0 nested carts means there is only the starting cart). The store measured a row of 13 nested carts to be 23.5 ft long, and a row of 18 nested carts to be 31 ft long.



a Create a graph of the situation. Remember to scale and label your axes.

b How many feet does each additional nested cart add to the length of the row? Explain your thinking.

c If the store design allows for each row of nested carts to have a maximum length of 43 ft, how many total carts can fit in a row?

6. Match each expression with an equivalent expression.

a $5(x - 7)$

b $-5(x - 7)$

c $2x - 30 - 5 - 7x$

d $3x + 2x - 5(-7)$

______ $5x + 35$

______ $5x - 35$

______ $-5x + 35$

______ $-5x - 35$

Unit 4 | Lesson 3

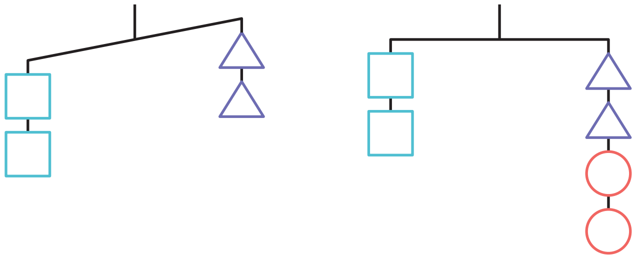
Keeping the Balance

Let's determine unknown weights on balanced hanger diagrams.

Warm-up What's True?

In the two hanger diagrams shown, all the triangles weigh the same as one another, all the squares weigh the same as one another, and all the circles weigh the same as one another.

Based on the diagrams, what is . . .



1. One thing that *must* be true?

2. One thing that *could* be true?

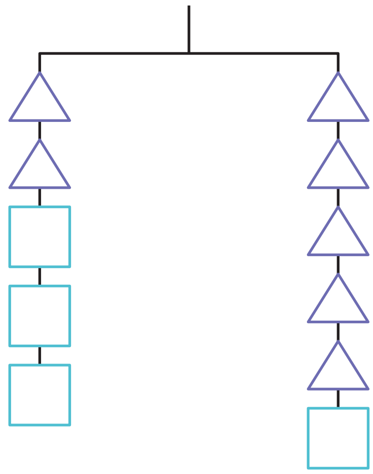
3. One thing that *cannot* be true?

Log in to Amplify Math to complete this lesson online.

Name: ____________ Date: ________ Period: ________

Activity 1 Hanging Blocks

The hanger diagram shown is balanced because the weight on each side is the same.



1. Which weight(s) can be removed so that the hanger remains balanced? Determine as many answers as possible.

2. If a triangle weighs 1 g, how much does a square weigh? Explain your thinking.

3. Determine another pair of measurements that keep the hanger diagram balanced.

Are you ready for more?

If the weight of a square is x grams and the weight of a triangle is 1 g, what equation could represent the hanger diagram?

Activity 2 Card Sort: Hanger Diagrams

You will be given a set of cards. Each card contains a hanger diagram.

1. For each card listed in the "Hanger 1" column of the table, match its hanger diagram with an equivalent hanger diagram. Record the card number in the "Hanger 2" column of the table.

2. Describe a possible move or moves that can be applied to Hanger 1 so that it will look like Hanger 2.

Hanger 1	Hanger 2	Possible Move
Card 1		
Card 2		
Card 3		
Card 4		
Card 5		

Are you ready for more?

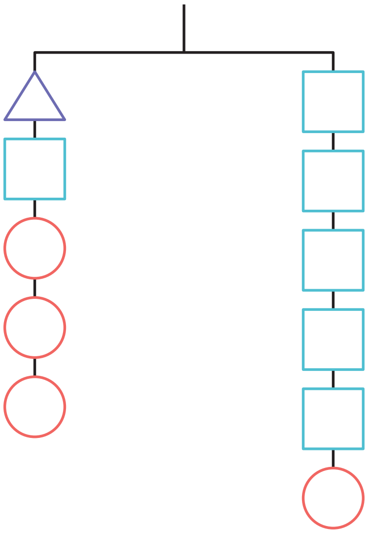
Bard has 24 pencils and 3 cups. The second cup holds one more pencil than the first cup. The third cup holds one more pencil than the second cup. How many pencils does each cup hold? Explain your thinking.

Name: .... Date: .... Period: ....

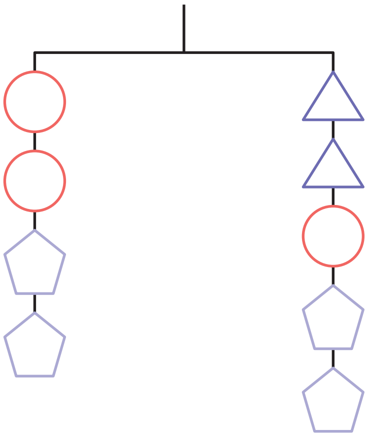
Activity 3 More Hanging Blocks

Consider the following hanger diagrams. Each triangle weighs 3 g, and each circle weighs 6 g.

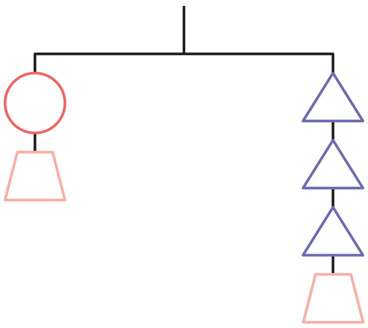
1. Determine the weight of 1 square. Show or explain your thinking.



2. Determine the weight of 1 pentagon. Show or explain your thinking.



3. Determine the weight of 1 trapezoid. Show or explain your thinking.



STOP

Summary

In today's lesson . . .

You balanced hanger diagrams and saw that adding or removing the same amount from each side kept the diagram balanced. You also saw that multiplying or dividing each side by the same amount kept the resulting hanger diagram balanced.

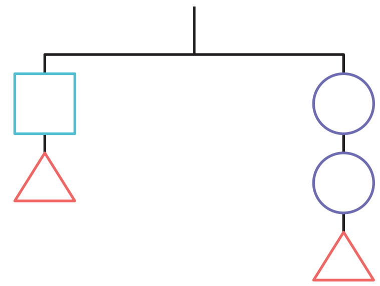
In the next lessons, you will connect this idea of balance to solving equations.

Reflect:

Practice

Name: ______ Date: ______ Period: ______

1. Which of these moves would keep the hanger diagram in balance? Select *all* that apply.



A. Add 2 circles to the left side and 1 square to the right side.

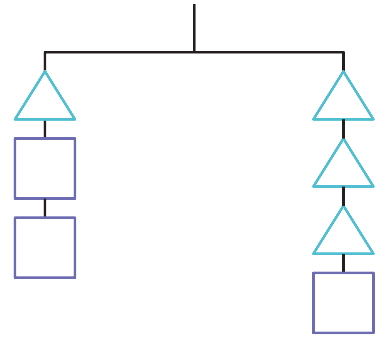
B. Add 2 triangles to each side.

C. Add 1 triangle to the left side and 1 square to the right side.

D. Remove 2 circles from the right side and 1 square from the left side.

E. Add 1 circle to the left side and 1 square to the right side.

2. What is the weight of 1 square if a triangle weighs 4 g? Explain your thinking.



3. Solve each equation. Show your thinking.

a. $5(2x - 3) = -10$

b. $4x - 5(3x - 2) = 10$

Name: .... Date: .... Period: ....

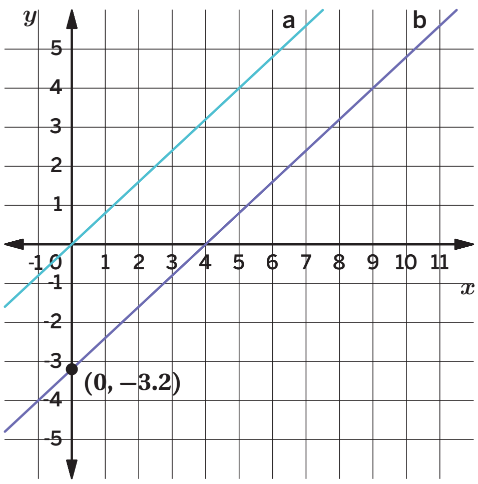
4. Andre created this puzzle: "I am three years younger than my brother, and I am two years older than my sister. My mom's age is one less than three times my brother's age. When you add all of our ages, you get 87. What are our ages?"

a To solve the puzzle, Jada writes the equation: $(x) + (x + 3) + (x - 2) + 3(x + 3) - 1 = 87$. What does the variable x represent in this scenario? Explain which parts of the equation represent Andre's brother's, sister's, and mother's ages.

b Write an equivalent equation in fewer terms.

c Solve the equation. How old is each member of Andre's family?

5. These two lines are parallel. Write an equation for each line.



6. Consider each statement.

a If the expression $6x + 9$ is divided by 3, which expression would be the result?

A. $6x + 3$ **B.** $2x + 9$ **C.** $2x + 3$ **D.** $x + 3$

b If the expression $3(6x + 9)$ is divided by 3, which expression would be the result?

A. $2x + 3$ **B.** $6x + 9$ **C.** $2x + 9$ **D.** $6x + 3$

Name: Date: Period:

Unit 4 | Lesson 4

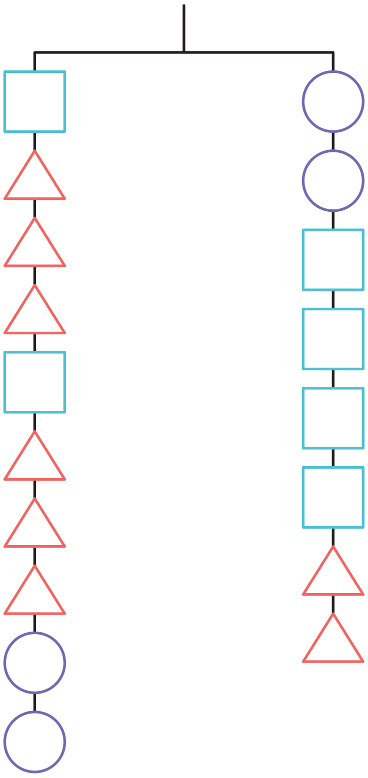
Balanced Moves (Part 1)

Let's rewrite equations, while keeping the same solutions.

Warm-up What's Being Represented?

Refer to the hanger diagram.

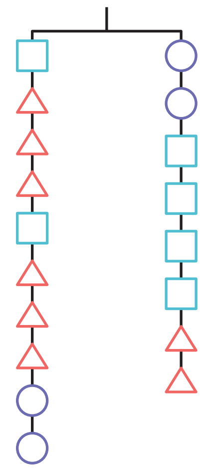
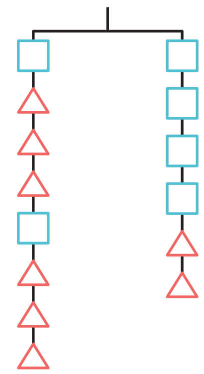
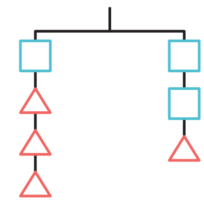
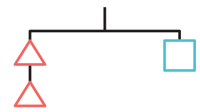
Let x represent the weight of 1 square, y represent the weight of 1 triangle, and z represent the weight of 1 circle. Describe the hanger diagram using these variables.



Log in to Amplify Math to complete this lesson online.

Activity 1 Matching Hangers

Hanger Diagrams 2, 3, and 4 show the result of simplifying the previous hanger diagram by removing equal weights from each side.

Diagram 1	Diagram 2	Diagram 3	Diagram 4
			
Equation 1	**Equation 2**	**Equation 3**	**Equation 4**
$2(x + 3y) + 2z = 2z + 4x + 2y$			

1. How does Equation 1 represent Diagram 1? Recall that x, y, and z represent the weight of a square, triangle, and circle, respectively.

2. Write an equation for the remaining hanger diagrams in the table.

Name: ____________ Date: ________ Period: ________

Activity 1 Matching Hangers (continued)

3. Explain what operation(s) were performed on each equation to create the next equation. Consider referencing the changes in the hanger diagrams to help with your thinking.

Equation 1 to 2	Equation 2 to 3	Equation 3 to 4

4. If the weight of 1 square is 6 g, what is the weight of 1 triangle? Which equation or diagram did you use to find this value?

Are you ready for more?

In a cryptarithmetic puzzle, the digits 0–9 are represented with letters of the alphabet. Use your understanding of addition to find which digits represent the letters A, B, E, G, H, L, N, and R if the following statement is true.

HANGER + HANGER + HANGER = ALGEBRA

Activity 2 Matching Equation Steps

The following shows a series of equations and possible moves or steps.

1. Match each set of equations with a possible step that turns the first equation into the second equation.
Note: You may not have a matching equation for every possible step listed.

Equations		Possible Steps
a	$3x + 7 = 5x$ $7 = 2x$	.... Divide each side by -3.
b	$-\frac{5x}{3} = 12$ $5x = -36$	.... Subtract $3x$ from each side.
c	$-3(4x - 3) = -15$ $4x - 3 = 5$	.... Add $3x$ to each side.
d	$4 - 3x = 12x$ $4 = 15x$	.... Subtract 3 from each side.
e	$10 - 6x = 4 + 5x$ $7 - 6x = 1 + 5x$	.... Multiply each side by 3.
f	$12x + 3 = 6$ $4x + 1 = 2$	.... Divide each term by 3.
		.... Multiply each side by -3.

Critique and Correct:
Your teacher will display an incorrect statement. Work with your partner to critique the statement, write a corrected statement, and clarify how and why you corrected it.

STOP

Name: ______ Date: ______ Period: ______

Summary

In today's lesson . . .

You saw how a balanced hanger diagram can be represented by an equation. An equation indicates that the two expressions on either side of the equal sign are equivalent. For example, if the expressions $2(x + 3y) + 2z$ and $2z + 4x + 2y$ are equivalent, you can write the equation $2(x + 3y) + 2z = 2z + 4x + 2y$.

You used hanger diagrams to see that mathematically valid moves create *equivalent equations*.

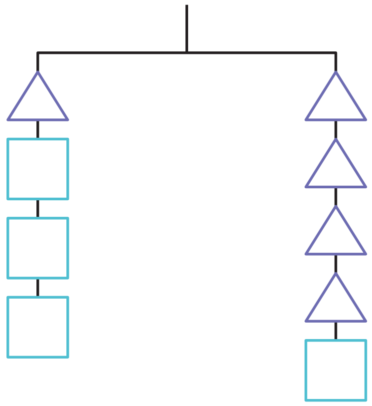
- If you add the same number to or subtract the same number from each side of the equation, the expressions on each side remain equivalent.
- If you multiply or divide the expressions on each side of an equation by the same nonzero number, the expressions on each side remain equivalent. **Note:** It is important that the number is not equal to zero because division by zero is undefined and multiplying each side by zero results in the equation $0 = 0$.
- Because expressions represent numbers, you can also add expressions to or subtract expressions from each side of an equation and maintain equality.

Reflect:

Practice

Name: .... Date: .... Period: ....

1. In this balanced hanger diagram, the weight of the triangle is x and the weight of the square is y.



a Write an equation using x and y to represent the hanger diagram.

b If $x = 6$, what is the value of y? Explain your thinking.

2. Andre and Diego were each trying to solve $2x + 6 = 3x - 8$. Describe the first step they each made to the equation.

a The result of Andre's first step was $-x + 6 = -8$.

b The result of Diego's first step was $6 = x - 8$.

3. Match each set of equations with a possible step that turns the first equation into the second equation.

Equations	Possible Steps
a $6x + 9 = 4x - 3$ $2x + 9 = -3$	.... Divide each side by -4.
b $-4(5x - 7) = -18$ $5x - 7 = 4.5$	.... Multiply each side by -4.
c $8 - 10x = 7 + 5x$ $4 - 10x = 3 + 5x$	.... Divide each side by 4.
d $-\frac{5}{4}x = 4$ $5x = -16$	.... Subtract $4x$ from each side.
e $12x + 4 = 20x + 24$ $3x + 1 = 5x + 6$	.... Subtract 4 from each side.

Name: .... Date: .... Period: ....

4. Consider the equation $3x + y = 15$.

a Complete the table with pairs of values for x and y which make the equation true.

x	y
2	
	3
6	
0	
3	
	0
	8

b Create a graph and plot the points in the table. Find the slope of the line that passes through the points.

5. Solve each equation. Show your thinking.

a $5(x + 2) = 30$

b $5x + 2 = 30$

Unit 4 | Lesson 5

Balanced Moves (Part 2)

Let’s rewrite some more equations, while keeping the same solutions.

Warm-up Is It a Solution?

Consider the equation $10x - 2x + 9 = 3(2x + 9)$. Is $x = 3$ a solution to the equation? Show or explain your thinking.

Log in to Amplify Math to complete this lesson online.

Name: .... Date: .... Period: ....

Activity 1 Step by Step by Step by Step

Bob Moses, a civil rights icon and algebra teacher, has dedicated his career to improving how algebra is taught and learned, especially for students who have not benefited from high-quality instruction. Being able to manipulate an equation using an algorithm is just one example of what Moses and others would consider to be important to studying algebra. An *algorithm* is a list of steps to follow and is particularly useful in solving equations.

1. The following table shows the description of steps for one method of solving the equation shown. Complete the table, using the steps shown, to solve the equation.

Description	Example
Original equation	$\frac{1}{2}(4x+7)+\frac{3}{2}=3(2x+5)+x$
Use the Distributive Property.	
Multiply each term by the least common denominator to eliminate the fractions.	
Combine like terms on each side.	
Add or subtract expressions so that the variable terms are on one side.	
Add or subtract expressions so that the constant terms are on the other side.	
Divide by the coefficient to isolate the variable.	
Solution	

Activity 1 Step by Step by Step by Step (continued)

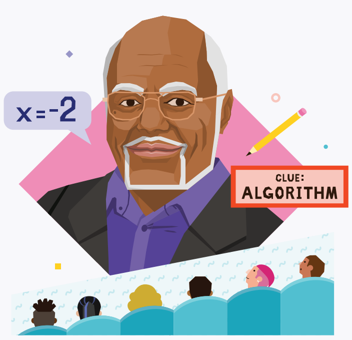
2. Solve the equation $10x - 2x + 9 = 3(2x + 9)$ from the Warm-up. Show your thinking and check your solution.

Check your solution:

3. Solve the equation $\frac{2}{3}(6x - 1) = -(x - 2)$. Show your thinking and check your solution.

Check your solution:

Featured Mathematician



Bob Moses

Bob Moses was an educator and was an activist in the Civil Rights Movement in the 1960s. He served as a leader of the Student Nonviolent Coordinating Committee, facing violence and intimidation as he helped register Black voters in Mississippi. In the 1980s, Moses founded the Algebra Project, which makes algebra more accessible to students, because "every child has a right to a quality education, to succeed in this technology-based society, and to exercise full citizenship." Bob Moses passed away in 2021.

Name: .... Date: .... Period: ....

Activity 2 Create Your Own Steps

Solve each equation. Show your thinking and check your solution. Correct any mistakes you may have made.

1. $8x + 7 = 6x - 13$

Check your solution:

2. $-3(a - 4) = 9a - 4$

Check your solution:

Activity 2 Create Your Own Steps (continued)

3. $m - 4 = \frac{1}{3}(6m - 54)$

Check your solution:

4. $\frac{12 + 6x}{3} = \frac{5x - 9}{2}$

Check your solution:

Are you ready for more?

A gaggle — or group — of geese are flying north after their summer migration. Half of the geese stop to rest on a lake while the other half continue the trip. When they pass another lake, half of the remaining geese stop at the lake, while the rest continue to fly. This continues until the geese are spread out over 7 lakes. What is the fewest number of geese in the gaggle? Show or explain your thinking.

STOP

Name: ________ Date: ________ Period: ________

Summary

In today's lesson . . .

You solved equations with variables on each side of the equal sign. How do you make sure your solution is correct? Accidentally adding when you meant to subtract, missing a negative sign when you distribute, forgetting to write an x from one line to the next — there are many possible mistakes to watch out for!

Fortunately, each step you take solving an equation results in a new equation with the same solution as the original. This means you can check your work by substituting the value of your solution into the original equation. If the resulting equation is true, you found the correct solution.

Reflect:

Practice

Name: .... Date: .... Period: ....

1. Mai and Tyler are each solving the equation $\frac{2}{5}b - 10 = 3b$. Mai's solution is $b = \frac{50}{17}$, and Tyler's solution is $b = -\frac{10}{13}$. Their work is shown. Do you agree with either of their solutions? Show or explain your thinking.

Mai's work:	**Tyler's work:**
$\frac{2}{5}b - 10 = 3b$	$\frac{2}{5}b - 10 = 3b$
$\frac{2}{5}b + 3b - 10 = 0$	$2b - 10 = 15b$
$\frac{17}{5}b - 10 = 0$	$-10 = 13b$
$\frac{17}{5}b = 10$	$-\frac{10}{13} = b$
$17b = 50$	
$b = \frac{50}{17}$	

2. Solve the equation $3(x - 4) = 12x$. Show or explain your thinking. Remember to check your solution.

Name: .... Date: .... Period: ....

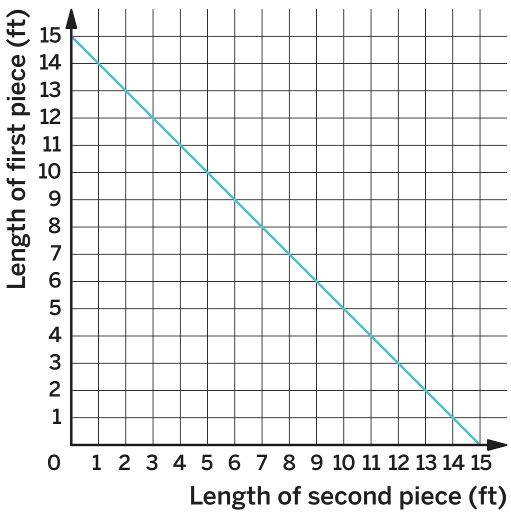
Practice

3. Andre solved the equation shown, but when he checked his solution, he realized it was incorrect. He knows he made a mistake, but he cannot find it. Find Andre's mistake, and then correctly solve the equation.

Andre's work:

$$-2(3x-5) = 4(x+3) + 8$$
$$-6x + 10 = 4x + 12 + 8$$
$$-6x + 10 = 4x + 20$$
$$10 = -2x + 20$$
$$-10 = -2x$$
$$5 = x$$

4. A length of ribbon is cut into two pieces to use in a craft project. The graph shows the length x of the second piece for each length y of the first piece.



a. How long is the ribbon? Explain your thinking.

b. What is the slope of the line?

c. Explain what the slope of the line represents in context of the scenario.

5. For each equation, determine whether $x = -3$ is a solution. Show or explain your thinking.

a. $\frac{2}{3}x = -2$

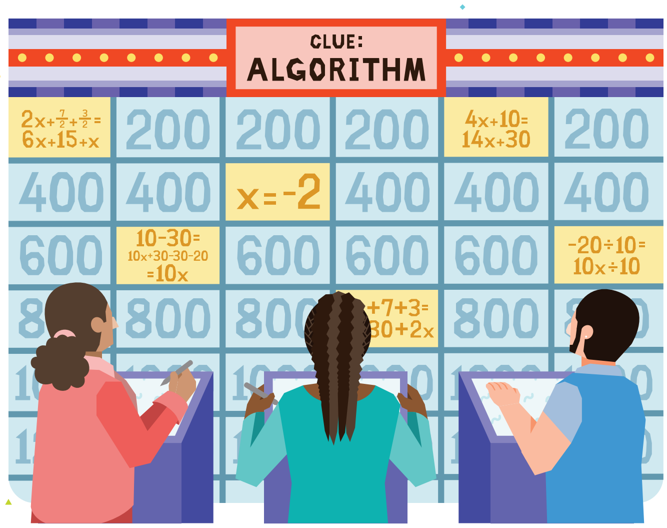
b. $4(x+7) - 9 = 7$

c. $-2(x+2) = -10$

Unit 4 | Lesson 6

Solving Linear Equations

Let's solve linear equations.



Warm-up Is It Equivalent?

Analyze each of the following equations.

- Place a checkmark next to each equation that is equivalent to the original equation.
- Circle the equation that you think represents the best next step for determining the value of b in the original equation.
- Solve the equation for b.

Original equation: $2b - 3 = \frac{1}{2}(2b - 3)$

☐ $2b - 3 = b - \frac{3}{2}$ ☐ $20b - 3 = b - 3$ ☐ $4b - 6 = 2b - 3$

☐ $2b = b$ ☐ $20b - 30 = 10b - 15$

Log in to Amplify Math to complete this lesson online.

Name: ________ Date: ________ Period: ________

Activity 1 Trading Equations

Plan ahead: As you trade and solve these equations, how will you use purposeful and precise communication?

You will be given a set of cards with equations on them. Follow these instructions.

1. Choose one card and have your partner choose a different card.
2. On your card, complete the first step in solving the equation. Fold your card so that only your step is visible and the original equation is hidden.
3. Trade cards with your partner and complete the next step for solving the equation on the card you received. Then fold the paper so only your step is visible.
4. Trade cards again. Complete the next step in solving the equation, fold the paper, and trade again.
5. Continue trading the cards back and forth after each step until the equations are solved.
6. Complete this process again with the remaining two equations and check your solutions.

Activity 2 Find and Fix

**Four equations are shown, with an attempt to solve each one. In each solution attempt, there may be one or more errors. If there are any errors, circle them, explain why they are errors, and then correct them.
If there are no errors, state whether you would solve the equation in the same way or take a different approach.**

1. **Equation 1:**

$$\begin{aligned} 4-2(3x-2) &= 14-x \\ 2\,(3x-2) &= 14-x \\ 6x-4 &= 14-x \\ 5x-4 &= 14 \\ 5x &= 18 \\ x &= \tfrac{18}{5} \end{aligned}$$

2. **Equation 2:**

$$\begin{aligned} \tfrac{1}{3}(12x-5) &= 10x-9x-6 \\ 4x-5 &= 10x-9x-6 \\ 4x-5 &= x-6 \\ 3x-5 &= -6 \\ 3x &= -1 \\ x &= -\tfrac{1}{3} \end{aligned}$$

Name: ________ Date: ________ Period: ________

Activity 2 Find and Fix (continued)

3. **Equation 3:**

$$\begin{aligned} 3x - 6 + 4\left(x - \frac{1}{2}\right) &= \frac{1}{4}(2x - 6) \\ 3x - 6 + 4x - 2 &= \frac{1}{2}x - \frac{6}{4} \\ 7x - 8 &= \frac{1}{2}x - \frac{6}{4} \\ 28x - 32 &= 2x - 6 \\ 26x &= 26 \\ x &= 1 \end{aligned}$$

4. **Equation 4:**

$$\begin{aligned} 1.1(x - 3) &= 0.1(2x - 6) \\ 11(x - 3) &= 1(2x - 6) \\ 11x - 33 &= 2x - 6 \\ 9x - 33 &= -6 \\ 9x &= 27 \\ x &= 3 \end{aligned}$$

STOP

Summary

In today's lesson . . .

You solved equations in one variable, and there are many ways to solve these types of equations. Generally, you want to perform steps which will get you closer to an equation where the variable is isolated, such as *variable* = *some number*.

Using the algorithm from Lesson 5 can help ensure the correct steps are taken to find the solution. However, the steps can be switched if it makes the process more efficient for you. Just remember to always maintain equality by using the properties of equality when moving terms across the equal sign.

Every time you solve an equation, remember you can always check your solution by substituting the value into the original equation and evaluating to see whether the resulting equation is true.

Reflect:

Name: .... Date: .... Period: ....

Practice

1. Solve each equation. Show or explain your thinking.

a $2(x+5)=3x+1$

b $3y-4=6-2y$

c $3(n+2)=9(6-n)$

2. Clare solved the equation shown, but when she checked her solution, she realized it was incorrect. She knows she made at least one mistake, but she cannot find it. Find Clare's mistake(s) and then correctly solve the equation.

Clare's work:

$$\begin{aligned} 12(5+2y) &= 4y-(5-9y) \\ 72+24y &= 4y-5-9y \\ 72+24y &= -5y-5 \\ 24y &= -5y-77 \\ 29y &= -77 \\ y &= -\frac{77}{29} \end{aligned}$$

3. Solve each equation. Show your thinking.

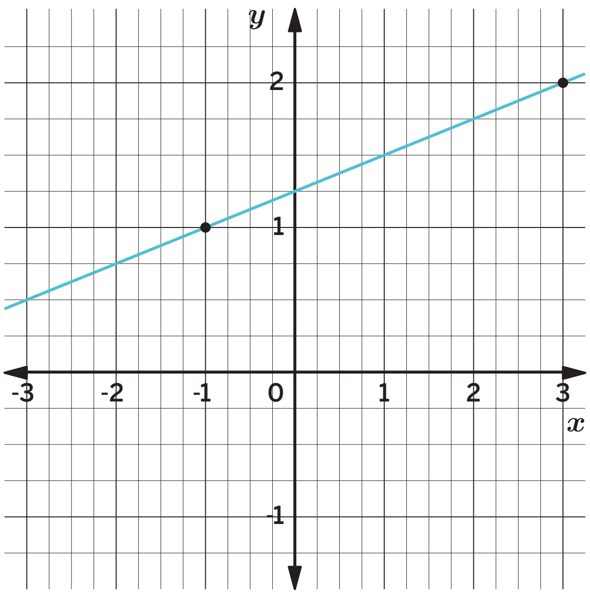
a $\frac{1}{9}(2m-16)=\frac{1}{3}(2m+4)$

b $1.5(5+0.2y)=0.4y-(0.6-0.9y)$

Practice

Name: .... Date: .... Period: ....

4. The graph of a linear equation is shown. Select *all* the true statements about this line and its equation.



A. One solution to the equation is the ordered pair $(3, 2)$.

B. One solution to the equation is the ordered pair $(-1, 1)$.

C. One solution to the equation is the ordered pair $\left(\frac{3}{2}, 1\right)$.

D. There are only 2 solutions.

E. There are infinitely many solutions.

F. The equation of the line is $y = \frac{1}{4}x + \frac{5}{4}$.

G. The equation of the line is $y = \frac{5}{4}x + \frac{1}{4}$.

5. Tyler invented a number puzzle. He asks Clare to choose any number, and then complete the steps shown.

Clare says the output is -3. Tyler says the input must have been 3. How did Tyler know that? Explain or show your thinking.

Steps:

- Triple the number.
- Subtract 7.
- Double the result.
- Subtract 22.
- Divide by 6.

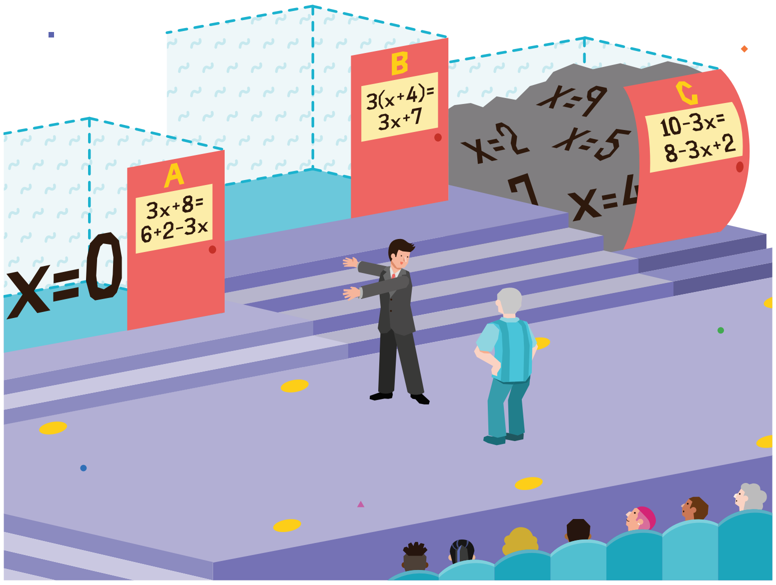
6. Solve the equation $2(x - 10) = -2(x + 8) - 4$. Show your thinking.

Name: ______ Date: ______ Period: ______

Unit 4 | Lesson 7

How Many Solutions? (Part 1)

Let's think about how many solutions an equation can have.



Warm-up True or False?

Choose any integer, decimal, or fraction to substitute for x in each equation. Determine whether your number makes the equation true or false by placing a check mark in each box.

My number: ______

	True	False
1. $3x - 10 = -3x + 5 + 15$	☐	☐
2. $3(x + 4) = 3x + 7$	☐	☐
3. $10 - 3x = 8 - 3x + 2$	☐	☐

Log in to Amplify Math to complete this lesson online.

Activity 1 Thinking About Solutions

The three equations from the Warm-up are shown. Solve each equation. Show or explain your thinking.

1. $3x - 10 = -3x + 5 + 15$

2. $3(x + 4) = 3x + 7$

3. $10 - 3x = 8 - 3x + 2$

Name: ________ Date: ________ Period: ________

Activity 2 Looking for Solutions

Determine whether each equation has *one solution*, *no solution*, or *infinitely many solutions*. Show or explain your thinking.

1. $v + 2 = v + 4$

2. $-4 + 3x = -4 + 3x$

3. $2t + 6 = 2(t + 3)$

4. $4x + 3 = -5x + 3$

5. $\frac{1}{2} + 5x = \frac{1}{3} + 5x$

6. $2(n - 1) = 10n + 6$

STOP

Summary

In today's lesson . . .

You discovered that some equations have one solution, no solution, or infinitely many solutions.

Here are some examples.

One solution:

$$\begin{aligned} 3x + 8 &= 6 + 2 - 3x \\ 3x + 8 &= 8 - 3x \\ 6x + 8 &= 8 \\ 6x &= 0 \\ x &= 0 \end{aligned}$$

This equation is only true when $x = 0$.

No solution:

$$\begin{aligned} 3(x + 4) &= 3x + 7 \\ 3x + 12 &= 3x + 7 \\ 12 &= 7 \end{aligned}$$

This equation is never true for any value of x.

Infinitely many solutions:

$$\begin{aligned} 10 - 3x &= 8 - 3x + 2 \\ 10 - 3x &= 10 - 3x \\ 10 &= 10 \end{aligned}$$

This equation is always true for any value of x.

Reflect:

Name: ____________ Date: ____________ Period: ____________

1. For each equation, decide if it has one solution, no solution, or infinitely many solutions. Show or explain your thinking.

a $x - 13 = x + 1$

b $x + \frac{1}{2} = x - \frac{1}{2}$

c $2(x + 3) = 5x + 6 - 3x$

d $-(7 - 5x) = 6x - 3$

2. Elena says the equation $3x + 10 = 5x - 4$ has no solution because when she substitutes each of the following numbers, all of the equations are false. Her work is shown.

Elena's work:

$x = 2$	$x = -6$	$x = 0.5$
$3(2) + 10 = 5(2) - 4$	$3(-6) + 10 = 5\,(-6) - 4$	$3(0.5) + 10 = 5(0.5) - 4$
$6 + 10 = 10 - 4$	$-18 + 10 = -30 - 4$	$1.5 + 10 = 2.5 - 4$
$16 = 6$	$-8 = -34$	$11.5 = -1.5$
False	False	False

Do you agree with Elena? Explain your thinking.

Practice

Name: ________ Date: ________ Period: ________

3. Kiran solved the equation $2(x - 3) = 8x - 6$ and got the answer "$x = 0$, no solution." Do you agree with Kiran's answer? Explain your thinking.

4. Solve each equation. Show or explain your thinking.

a $3x - 6 = 4(2 - 3x) - 8x$

b $\frac{1}{2}z + 6 = \frac{3}{2}(z + 6)$

5. The point $(-2, 3)$ is on a line that has a slope of 2.

a Write an equation for the line. Show or explain your thinking.

b Determine two more points on the line.

6. Several students are asked to identify the coefficient and constant in the expression $-5 + 3x + 12$. Select the statement that correctly identifies the coefficient and constant term.

A. Jada: "The coefficient is -5, and the constant is 12."

B. Shawn: "The coefficient is 3, and the constant is 7."

C. Diego: "The coefficient is 12, and the constant is -5.

D. Priya: "The coefficient is $3x$, and the constant is 7."

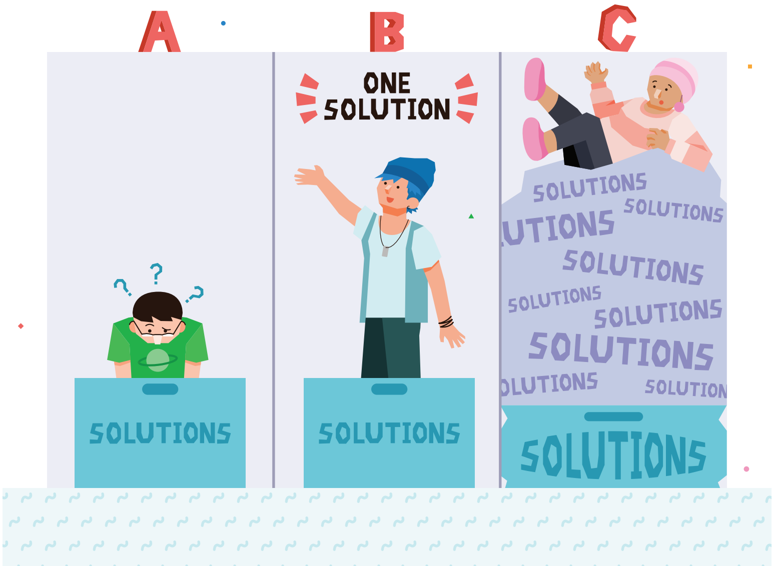
E. Clare: "The coefficient is 7, and the constant is 3."

Name: ____ Date: ____ Period: ____

Unit 4 | Lesson 8

How Many Solutions? (Part 2)

Let's solve equations with different numbers of solutions.



Warm-up Making Use of Structure

Refer to these equations from a previous lesson.

One solution	No solution	Infinitely many solutions
$4x + 3 = -5x + 3$	$v + 2 = v + 4$	$-4 + 3x = -4 + 3x$
$2(n - 1) = 10n + 6$	$\frac{1}{2} + 5x = \frac{1}{3} + 5x$	$2t + 6 = 2(t + 3)$
$3x + 8 = 6 + 2 - 3x$	$3(x + 4) = 3x + 7$	$10 - 3x = 8 - 3x + 2$

What patterns do you notice when a linear equation has:

a One solution?

b No solution?

c Infinitely many solutions?

Log in to Amplify Math to complete this lesson online.

Activity 1 Three Responses

With your group, decide who will complete Column A, who will complete Column B, and who will complete Column C.

For each problem, without solving each equation, determine whether there will be *one solution*, *no solution*, or *infinitely many solutions*. After each row, share your responses with your group. For each row, your group should have three *different* responses. If there is an error, work together to solve the equation and correct your responses.

	Column A	Column B	Column C
1.	$6x + 8 = 8 + 6x$	$6x + 8 = 6x + 13$	$6x + 8 = 7x + 13$
2.	$5x + 3x + 12 = 8x - 4$	$5x - 4x - 2 = -6x + 12$	$-5x + 2 - 3x = -8x + 2$
3.	$12r - 6 + 12r = -6$	$-3(4r - 2) = -12r + 6$	$\frac{1}{4}(12 - 4r) = 6 - r$
4.	$4n + 4n - 6 = 8n - 8$	$4n + 2(2n - 3) = 2(4n - 3)$	$4(2x - 2) = -8(x - 2)$
5.	$-6 + 9c + c = 10c - 6$	$c + 3(2 + 3c) = 4(c - 6)$	$c - 3(2 - 3c) = 2(5c + 3)$

Name: ________ Date: ________ Period: ________

Activity 2 Trading Equations, Revisited

You will be given an index card and a plain sheet of paper.

1. On the index card, complete each equation so that one equation has one solution, one equation has no solution, and one equation has infinitely many solutions. On the back of the index card, solve the equation that has one solution.

   a. $12x + \square = \square(4x + \square)$

   b. $2x - \square = \square x + \square$

   c. $\square x + 5 = \square x - \square x + 5$

2. Trade index cards with a partner, without telling them the number of solutions.

3. Using your partner's equations, decide which equation has *one solution*, *no solution*, and *infinitely many solutions*. For the equation that has one solution, solve the equation to determine the value of x that makes the equation true. Once you have made your decisions, check with your partner to see whether you are correct.

Are you ready for more?

Use whole numbers 1–10 to replace the boxes so that you create three different equations: one equation with one solution, one equation with no solution, and one equation with infinitely many solutions. Use each number only once for each equation.

$\square x + \square x + \square = \square(x + \square)$

One solution:

No solution:

Infinitely many solutions:

STOP

Summary

In today's lesson . . .

You discovered that the structure of an equation can tell you if the equation has one solution, no solution, or infinitely many solutions.

Infinitely many solutions: A linear equation has *infinitely many solutions* when the coefficients and constants are the same on each side. For example, the equation $2x + 5 = 2x + 5$ has infinitely many solutions.

No solution: A linear equation has *no solution* when the coefficients are the same, but the constants are not the same on each side. For example, the equation $2x + 5 = 2x + 10$ has no solution.

One solution: A linear equation has *one solution* when the coefficients are different on each side. The constants may or may not be the same on each side. For example, the equation $2x + 5 = 3x + 10$ has one solution.

Reflect:

Name: ______ Date: ______ Period: ______

Practice

1. Consider the unfinished equation $12(x-3)+18=$ ______.
Complete the equation so that it has:

a. One solution.

b. No solution.

c. Infinitely many solutions.

2. Decide whether each equation has *one solution*, *no solution*, or *infinitely many solutions*. If an equation has one solution, solve the equation to determine the value of x that makes the equation true.

a. $6x-4=-4+6x$

b. $4x-6=4x+3$

c. $-2x+4=-3x+4$

d. $10x+\frac{1}{4}=10x+\frac{2}{3}$

e. $3(2x-5)=4x+2x-15$

f. $-4\,(x-2)=-2\left(x-\frac{17}{2}\right)$

Practice

Name: .... Date: .... Period: ....

3. For each problem, determine whether you agree with the statement made by each person. Explain your thinking.

   a. Lin studied the equation $2x - 32 + 4(3x - 25) = 14x$. She said, "I can tell right away there is no solution because, on the left side, you will have $2x + 12x$ and a bunch of constants, but you have just $14x$ on the right side."

   b. Han studied the equation $6x - 4 + 2(5x + 2) = 16x$. He said, "I can tell right away there is no solution because, on the left side, you will have $6x + 10x$ and a bunch of constants, but you have just $16x$ on the right side."

4. The points $(-2, 2)$ and $(0, -6)$ lie on the graph of the same linear equation. Does the point $(2, 6)$ also lie on the graph of this linear equation? Explain your thinking.

5. What strategies can you use to determine whether $x = 5$ is a solution to the equation $2(x - 3) = 2x + 6$?

Name: ________ Date: ________ Period: ________

Unit 4 | Lesson 9

Strategic Solving

Let's practice solving linear equations.

Warm-up Predicting Solutions

Without solving, identify whether each equation has a solution that is *positive*, *negative*, or *zero*. Be prepared to explain your thinking.

1. $12x = 1.63$

2. $-2 + 11x = -24$

3. $\frac{x}{6} = \frac{3x}{4}$

Log in to Amplify Math to complete this lesson online.

Activity 1 Equations Scavenger Hunt

Begin with any of the scavenger hunt cards and solve the problem, using the space provided here. Then look for your answer at the top of another scavenger hunt card and solve the problem on that card. Note: Use the hanger diagrams in the last box, if it helps your thinking.

Answer:	Answer:
Answer:	Answer:
Answer:	Answer:

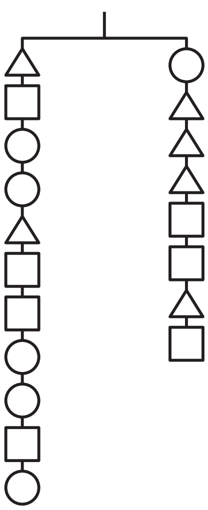
(continued)

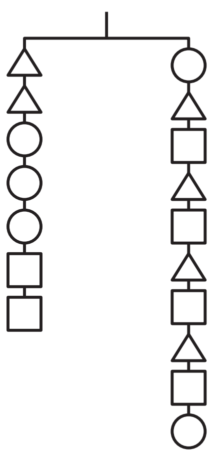
Name: ________ Date: ________ Period: ________

Activity 1 Equations Scavenger Hunt (continued)

Answer:	Answer:
Answer:	Answer:

Use these diagrams if they help you with your thinking.





Reflect: During the scavenger hunt, how well did you view this challenge as an opportunity to stretch your knowledge?

STOP

Summary

In today's lesson . . .

You showed how to apply strategies for solving linear equations that include fractions, decimals, negative values, and equations written with many terms.

For example:

$3(4-2x)+6=4-2x$	
$12-6x+6=4-2x$	Use the Distributive Property.
$18-6x=4-2x$	Combine like terms on each side.
$18=4+4x$	Add $6x$ to each side.
$14=4x$	Subtract 4 from each side.
$\frac{14}{4}=x$	Divide each side by the coefficient.

So, $x=\frac{7}{2}$.

Reflect:

Name: ________ Date: ________ Period: ________

Practice

1. Without solving, identify whether each equation has a solution that is *positive*, *negative*, or *zero*.

   a $7x = 3.25$

   b $-7x = 32.5$

   c $3x + 11 = 11$

   d $9 - 4x = 4$

   e $-8 + 5x = -20$

2. Solve each equation. Show or explain your thinking.

   a $2b + 8 - 5b + 3 = -13 + 8b - 5$

   b $2x + 7 - 5x + 8 = 3(5 + 6x) - 12x$

   c $5c + 3 = 2(6 - c) + 7c$

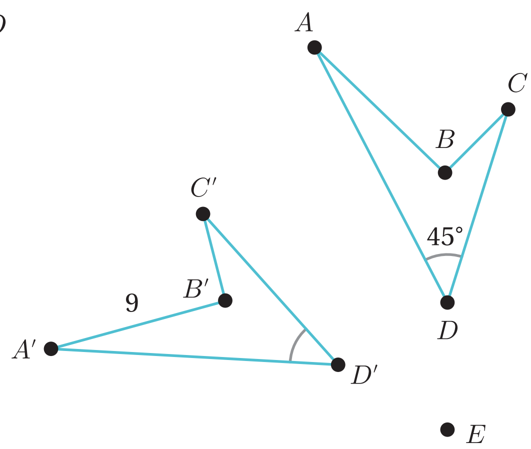
   d $1.3 + 6d = 2.7 - 8d$

3. Priya said the equation $9x + 15 = 3x + 15$ has no solution because $9x$ is greater than $3x$. Do you agree with Priya? Explain your thinking.

Practice

Name: ______________________ Date: __________ Period: __________

4. Polygon $A'B'C'D'$ is the image of Polygon $ABCD$ after a rotation about point E.



a What is the length of side AB? Explain your thinking.

b What is the measure of $\angle D'$? Explain your thinking.

5. Select *all* of the situations for which only zero or negative slopes make sense.

A. The height of a candle as it burns over an hour.

B. A bank account balance over a year.

C. The elevation above sea level of a hiker descending into a canyon.

D. The number of students remaining in school after 6:00 p.m.

E. The temperature in degrees Fahrenheit of an oven used on a hot summer day.

6. If you were babysitting, which option would you rather choose?

Option A: Charge \$5 for the first hour and \$8 for each additional hour.

Option B: Charge \$15 for the first hour and \$6 for each additional hour.

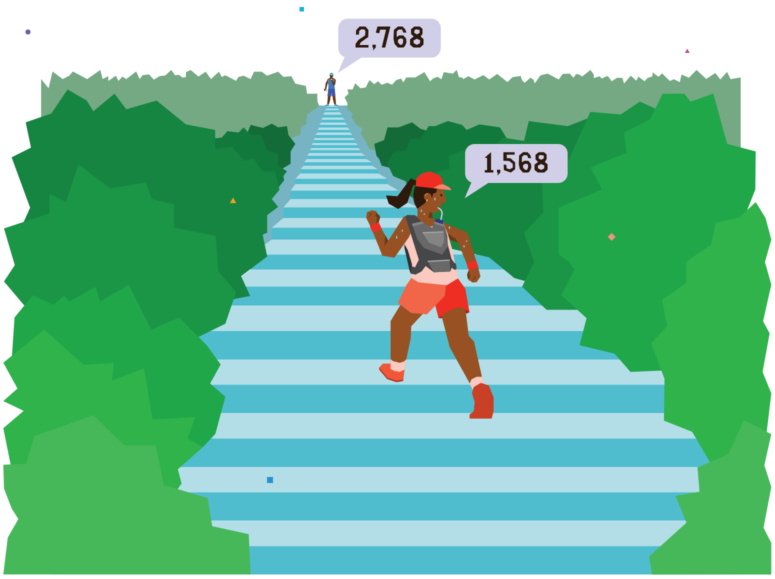
Explain your thinking.

Name: ______ Date: ______ Period: ______

Unit 4 | Lesson 10

When Are They the Same?

Let's use equations to think about situations.



Warm-up Perimeter Puzzle

Bard drew a square with side lengths of $2x$ units. Lin drew a square with side lengths of $x + 2$ units.

Bard says to Lin, "The two perimeters can never be equal for any number x because my square's side length multiplies x by 2, while your square's side length just adds 2 to x."

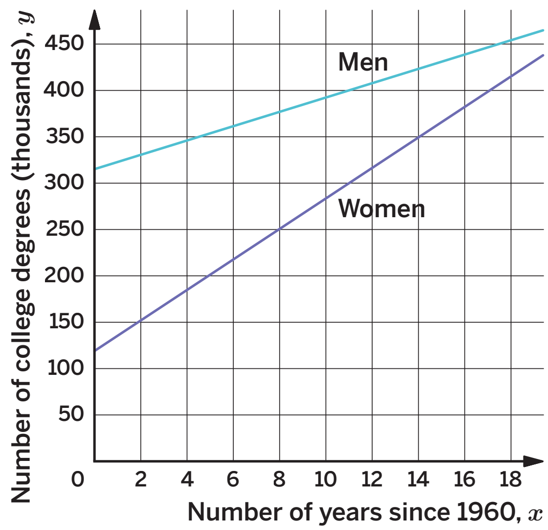
Lin responds, "I disagree. There must be some number x that makes these two perimeters equal."

Who do you think is correct? Explain your thinking

Activity 1 Education Gap?

Now, more than at any point in history, women are earning college degrees comparably to men. However, that wasn't always the case.

The graph shown can be used to model the trend in college degrees earned by men and women starting in 1960.



The line represented by the equation $y = 16.4x + 118.3$ shows the number of women that earned a college degree.

The line represented by the equation $y = 7.7x + 314.3$ shows the number of men that earned a college degree. In both equations, x represents the years since 1960 and y represents the number of people in thousands.

1. According to the trend line, how many women earned a degree in the year 1970? How many men?

2. During what year, if ever, do you think the number of women receiving a college degree will be the same as the number of men? Be as precise as possible.

3. Make a prediction based on the trend lines: will there be more men or more women earning a college degree in the year 2025? How many more?

Name: ______ Date: ______ Period: ______

Activity 2 Staircase to the Sky

Han and Priya organize a week-long trip to hike the Rocky Mountains in Colorado. On the trip, they decide to visit the Manitou Incline, a 2,768-step staircase, nearly 1 mile long, which serves as a popular destination for anyone looking for a good workout.

Han reaches the top before Priya and texts Priya from the top to say he is out of water. Han hopes to meet Priya so he can have a drink from Priya's water bottle. Han walks down as Priya continues walking up, both at constant rates.

The table shows how many stairs Han and Priya are from the start of the Manitou Incline, and the time, in minutes, after Han texted Priya.

Yobab/Shutterstock.com

Time (minutes after Priya's text)	Han (number of stairs from the start)	Priya (number of stairs from the start)
0	2,768	1,568
5	2,368	1,768
10	1,968	1,968
15	1,568	2,168
20	1,168	2,368
25	768	2,568

1. How many stairs per minute does Han walk down? Show your thinking.

2. How many stairs per minute does Priya walk up? Show your thinking.

Activity 2 Staircase to the Sky (continued)

3. Write an expression that represents the number of stairs Han is from the start x minutes after Priya's text.

4. Write an expression that represents the number of stairs Priya is from the start after x minutes after she texts Han.

5. When will they meet? Be as precise as you can. Show or explain your thinking.

STOP

Name: ______________ Date: ______________ Period: ______________

Summary

In today's lesson . . .

You saw how multiple expressions can be used to represent a scenario. You saw that you can set these expressions equal to each other to solve for an unknown variable.

For example, imagine two hikers walking towards each other on a mountain, one hiking down from the top and one on their way to the top.

Now imagine when the hikers meet each other on the mountain, when they are at the same altitude at the same time. To find out when this time is, you can write an expression representing the altitude of each hiker and set those expressions equal to each other.

Reflect:

Practice

Name: .... Date: .... Period: ....

1. Which story could be represented by the equation $-6 + 3x = 2 + 4x$?

   A. At 5 p.m., the temperatures recorded at two weather stations in Antarctica were -6 degrees at the first station and 2 degrees at the second station. The temperature changes at the same constant rate, x degrees per hour, throughout the night at both locations. The temperature at the first station, 3 hours after this recording, was the same as the temperature at the second station, 4 hours after this recording.

   B. Elena and Kiran play a card game. Every time they collect a pair of matching cards, they earn x points. At one point in the game, Kiran has -6 points and Elena has 2 points. After Elena collects 3 pairs and Kiran collects 4 pairs, they have the same number of points.

2. Priya and Han are biking in the same direction on the same path.

   a Han rides at a constant speed of 16 miles per hour. Write an expression that represents the number of miles Han has traveled after t hours.

   b Priya started riding a half hour before Han. If Han has been riding for t hours, how long has Priya been riding?

   c Priya rides at a constant speed of 12 miles per hour. Write an expression that represents the number of miles Priya has traveled after Han has been riding for t hours.

   d Use your expressions to determine when Han and Priya meet. Show your thinking.

3. Cell phone Plan A costs $\$70$ per month and comes with a free phone that is worth $\$500$. Cell phone Plan B costs $\$50$ per month, but does not come with a phone. Suppose you buy the $\$500$ phone and choose Plan B. After how many months is your total cost the same as it would have been had you chosen Plan A?

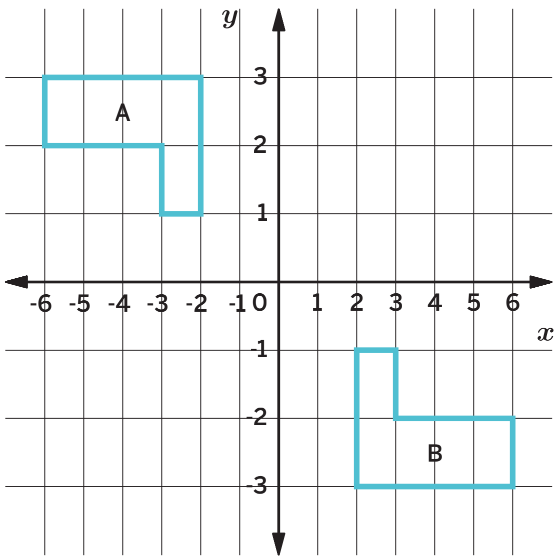
Name: .... Date: .... Period: ....

4. Solve each equation. Show or explain your thinking.

a $3d + 16 = -2(5 - 3d)$

b $2k - 3(4 - k) = 5k + 4$

5. Describe a rigid transformation that takes Polygon A to Polygon B.



6. Choose the equation for which the ordered pairs $(5, 7)$ and $(8, 13)$ are each solutions.

A. $3x - y = 8$

B. $y = x + 2$

C. $y - x = 5$

D. $y = 2x - 3$

My Notes:

SUB-UNIT

2 Systems of Linear Equations

Narrative Connections

How is anesthesia like buying live lobsters?

A patient lies on an operating table. Their anesthesiologist must take into account the patient's body mass before delivering just the right amount of anesthesia without endangering their life.

Elsewhere, a chef opens a restaurant in a beach town. As the weather gets warmer, crowds start to gather. Working from recipes that feed just ten people, the chef must now figure out how many clams, shrimp, and pounds of monkfish to order each week.

What do these scenarios have in common? They are two of the many everyday problems that can be solved with linear equations. Linear equations describe relationships where one value changes with another at a constant rate. Think about the way the amount of medicine changes with the weight of the patient. Or the number of clams for a recipe changes with the number of hungry diners.

But what happens when we need to use more than one linear equation at the same time? In these next lessons, you'll see how you can translate real-world problems into mathematical terms and solve them with a few handy methods. All with the help of a few linear equations!

Unit 4 | Lesson 11

On or Off the Line?

Let's interpret the meaning of points on the coordinate plane.

Warm-up Counting Coins

I have $2 in my pocket. If I have only nickels and dimes, how many nickels and dimes do I have?

Log in to Amplify Math to complete this lesson online.

Name: ______________________ Date: __________ Period: __________

Activity 1 Pocket Full of Change

Mathematicians like the renowned Zhang Quijian have long considered strategies for solving problems that lead to the Diophantine equations you explored in Unit 3. In this activity, you will work to find a strategy to help Jada and Noah solve the following problem:

Jada told Noah that she has $2 worth of nickels and dimes in her pocket and 31 coins altogether. She asked him to guess how many of each type of coin she has.

Use the table to find combinations of nickels and dimes that have a total value of $2. Use the number of nickels given to determine the number of dimes, and then complete the rest of the first two columns. Then complete the third column to find the total number of coins for each combination. Can you find a combination of nickels and dimes that uses a total of 31 coins?

Number of nickels	Number of dimes	Number of coins
0		
2		
4		
6		

Featured Mathematician

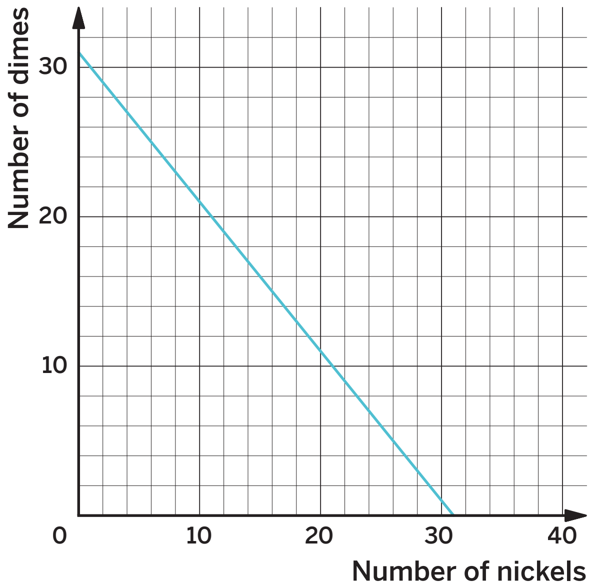
Zhang Qiujian

Little is known about the Chinese mathematician Zhang Quijian. His 5th century book, *Zhang Qiujian Suanjing*, is considered one of the most important mathematical texts in history. In this book, Zhang explores different mathematical methods and problems. Perhaps the most famous one is the "Hundred Fowls Problem." Can you solve it?

"Roosters cost 5 qian each, hens cost 3 qian each, and three chicks cost 1 qian. If 100 fowls are bought for 100 qian, how many roosters, hens and chicks are there?"

Activity 2 A New Way of Solving

Refer to the scenario from Activity 1. The graph shows the relationship between the number of nickels and the number of dimes if the total number of coins is 31.



1. Choose a point on the graph and explain what it means in context.

2. Using values from your table in Activity 1, sketch the graph of the line that shows combinations of nickels and dimes that have a total value of $2.

3. Label the point where the two lines intersect. What does this point represent?

4. Let x represent the number of nickels and y represent the number of dimes.

a Write an equation that shows that there are 31 total coins.

b Write an equation that shows that the total value of the coins is $2.

STOP

Name: ______ Date: ______ Period: ______

Summary

In today's lesson . . .

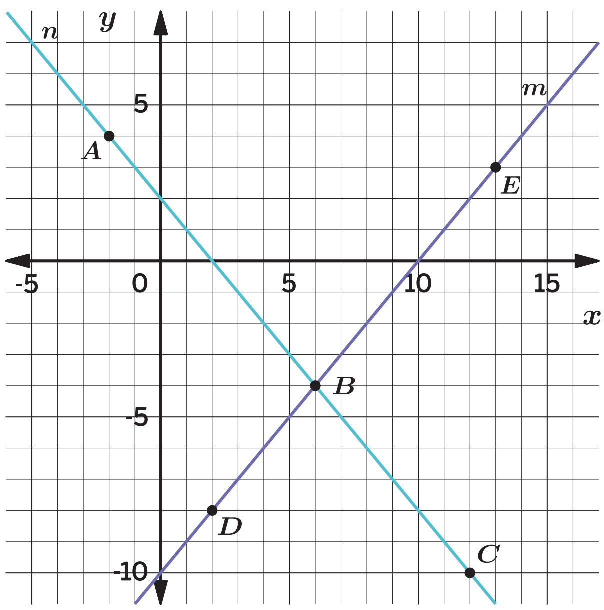
You saw an example of how you can use linear relationships to represent real-world scenarios. You saw that two equations can be used simultaneously to represent the same scenario, both graphed on the same coordinate plane. In some cases, this can be a more efficient way of finding a solution.

Reflect:

Practice

Name: ________ Date: ________ Period: ________

1. Refer to the coordinate plane.



a Which line, line m or line n, represents each statement?

- A set of points where the coordinates of each point have a sum of 2.
- A set of points where the y-coordinate of each point is 10 less than its x-coordinate.

b Which labeled point(s) represent each of the following statements?

- The coordinates are two numbers with a sum of 2.
- The coordinates are two numbers, where the y-coordinate of each point is 10 less than the x-coordinate.
- The coordinates are two numbers with a sum of 2 and where the y-coordinate is 10 less than the x-coordinate.

2. Mai earns \$7 per hour mowing her neighbors' lawns. She also earned \$14 for hauling away bags of recyclables from some neighbors.

Priya babysits her neighbor's children. The table shows the amount of money m she earns in h hours.

h	m
1	\$8.40
2	\$16.80
4	\$33.60

Priya and Mai have agreed to go to the movies the weekend after they have earned the *same* amount of money for the *same* number of hours worked.

a How many hours do they each have to work before they go to the movies?

b How much will each of them have earned?

Name: ________________ Date: ________ Period: ________

Practice

3. Diego has $\$11$ and begins saving $\$5$ each week toward buying a new phone. At the same time that Diego begins saving, Lin has $\$60$ and begins spending $\$2$ per week on supplies for her art class. Is there a week when they will have the same amount of money? How much do they have at that time? Show or explain your thinking.

4. Consider the equation $4x - 4 = 4x +$ ______. What value or expression could you write in the blank so that the equation would be true for:

   a. No values of x?

   b. All values of x?

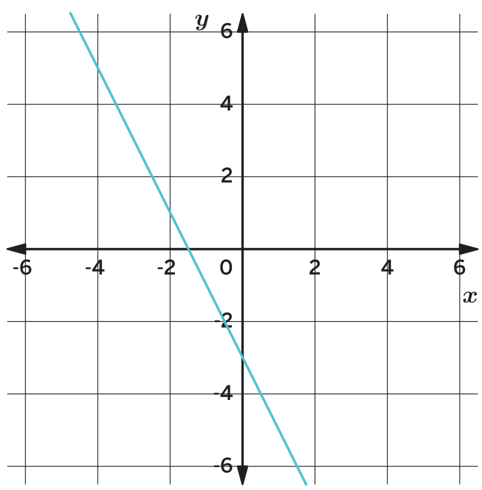
   c. One value of x?

5. Solve each equation. Show or explain your thinking.

   a. $\dfrac{3y-6}{9} = \dfrac{4-2y}{-3}$

   b. $0.3(x-10) - 1.8 = 2.7x$

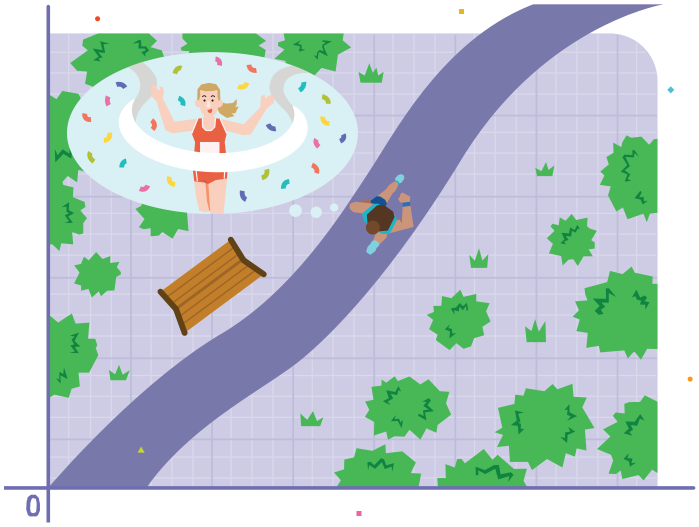
6. Draw a line with the same slope as the line given and a y-intercept of $(0, 4)$.



Unit 4 | Lesson 12

On Both of the Lines

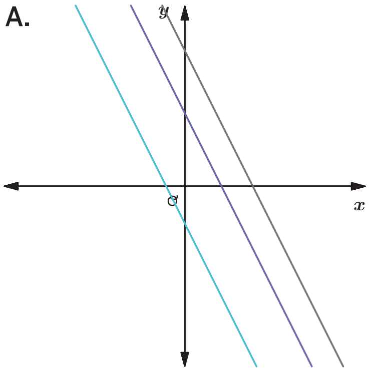
Let's use lines to analyze real-world situations.



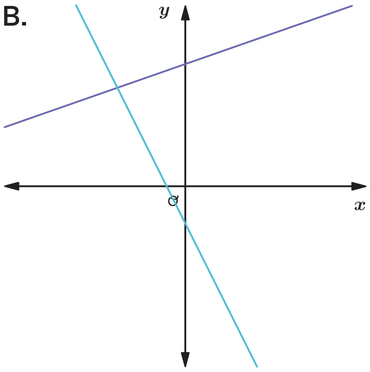
Warm-up Which One Doesn't Belong?

Consider the lines graphed. Which graph doesn't belong?

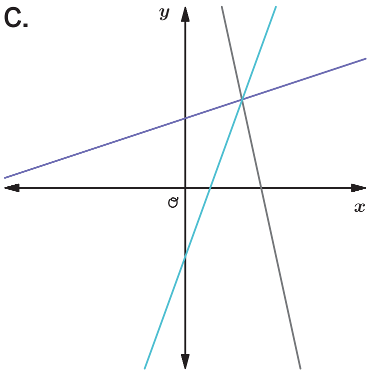
A.



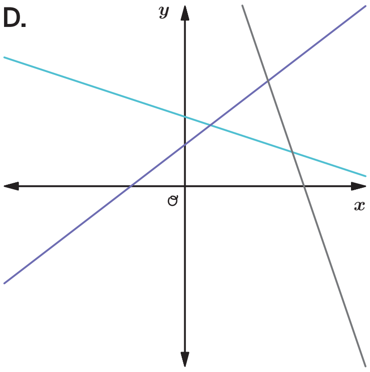
B.



C.



D.



Log in to Amplify Math to complete this lesson online.

Name: ______________________ Date: __________ Period: __________

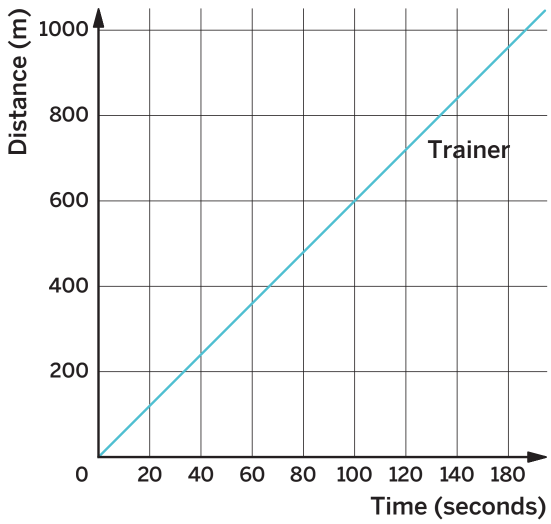
Activity 1 Can a Computer Science Teacher Run as Fast as Grete Waitz?

Ms. Hernández, a computer science teacher, wants to run a marathon as fast as Grete Waitz, the geography teacher who became a marathon legend.

Ms. Hernández starts training by running shorter races with her trainer. The graph shows the distance and time run by her trainer. Ms. Hernández hopes to beat his time. To add a challenge, Ms. Hernández starts the race 20 seconds after her trainer.

1. Ms. Hernández records her distance and time ran, from when her trainer starts, in the given table. Use the table to sketch the graph for Ms. Hernández.

Time (seconds)	Distance (m)
20	0
40	160
60	320
120	800



2. At what speed, in meters per second, is Ms. Hernández running? Show or explain your thinking.

3. At what speed, in meters per second, is Ms. Hernández's trainer running? Show or explain your thinking.

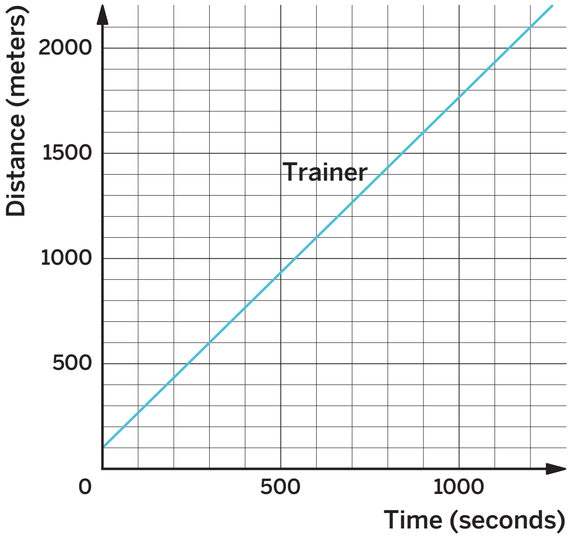
4. Estimate the coordinates of the point where the two lines intersect. Explain what the point means in context.

Activity 2 A Different Pace

To run a longer race, Ms. Hernández's trainer reminds her she will need to slow down to conserve energy. Ms. Hernández is now preparing to run a 5K, or 5,000 m race. Her trainer starts 100 m ahead of the start line so that Ms. Hernández can run a comfortable distance behind him, still within eyesight. The graph shows Ms. Hernández's trainer's distance y, related to the time, in seconds, x.

Eduard Moldoveanu/Shutterstock.com

1. Graph a line representing Ms. Hernández's distance if she runs at the same speed as her trainer, but starts at the starting line.



2. Write an equation to represent each line.

   Trainer:

   Ms. Hernández:

3. What do you notice about the two lines?

4. Ms. Hernández says that she will never catch up to her trainer at the pace they are both running. Does your graph support this? Explain your thinking.

5. Mr. Patel, an art teacher, who ran the same race, said that his graph looks exactly the same as Ms Hernández's graph. What do you think this could mean?

STOP

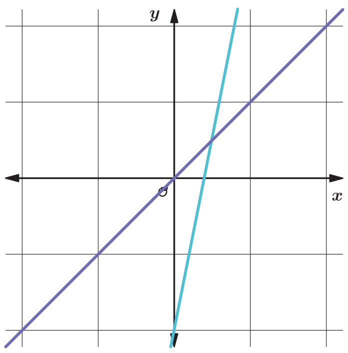
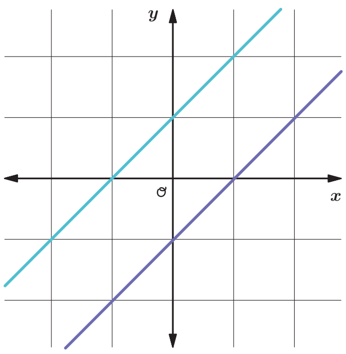
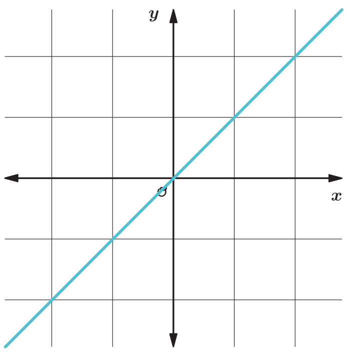
Name: Date: Period:

Summary

In today's lesson . . .

You continued to explore linear relationships. You saw that if you have two simultaneous equations, you can find if there is a solution to both equations by studying the lines of the equations on the same plane.

You saw examples of types of solutions for simultaneous equations:

One solution	No solution	Infinitely many solutions
		
There is one point of intersection, which is the solution.	There are no points of intersection. The lines are parallel.	There are infinitely many points of intersection. The lines are directly on top of each other.

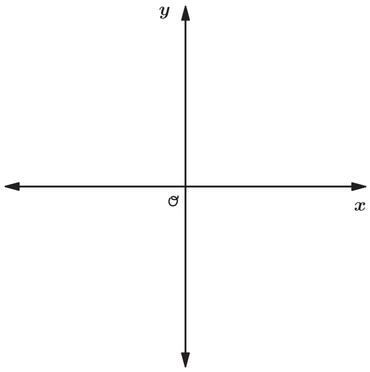
Reflect:

Practice

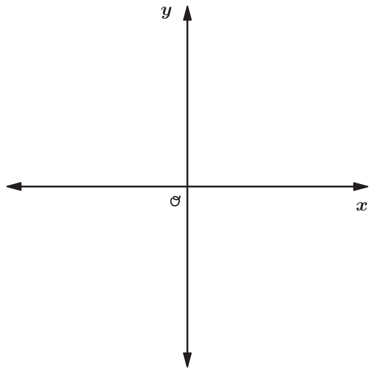
Name: .... Date: .... Period: ....

1. Sketch two lines that match each description. Then describe the number of solutions for each pair of lines.

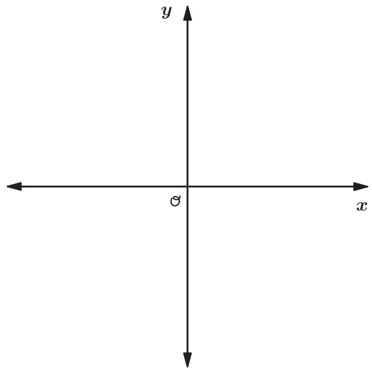
a Two lines with the same slope and different y-intercepts.



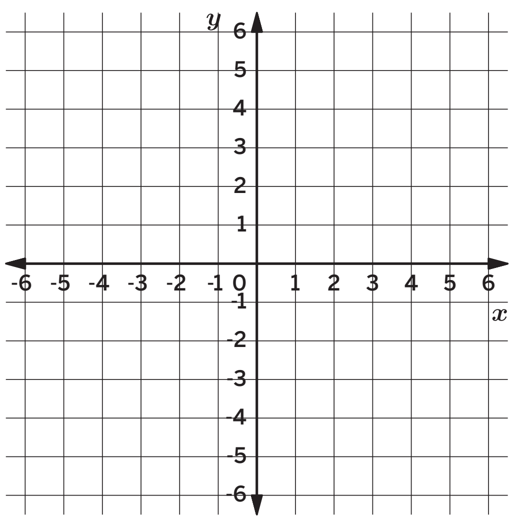
b Two lines with different slopes.



c Two lines with the same slope and same y-intercept.



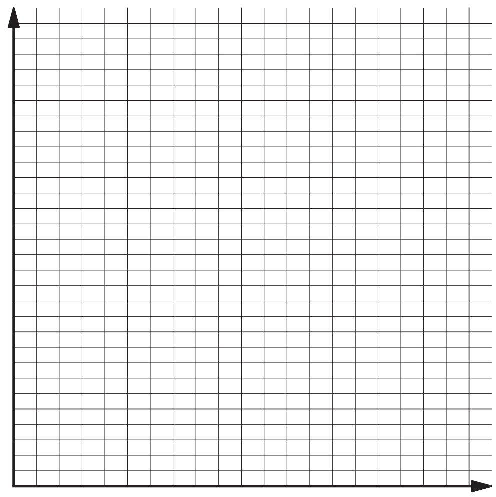
2. Draw a graph to find x- and y-values that make both of the equations $y = \frac{1}{4}x + 2$ and $y = 2x - 5$ true.



Name: .... Date: .... Period: ....

3. A stack of n small cups has a height h, in centimeters, that is represented by the equation $h = 1.5n + 6$. A stack of n large cups has a height h, in centimeters, that is represented by the equation $h = 1.5n + 9$.

a. Graph the equations for each stack of cups on the same coordinate plane. Make sure to label the axes and decide on an appropriate scale.



b. For what number of cups will the two stacks have the same height? Explain your thinking.

4. For what value of x do the expressions $\frac{2}{3}x + 2$ and $\frac{4}{3}x - 6$ have the same value?

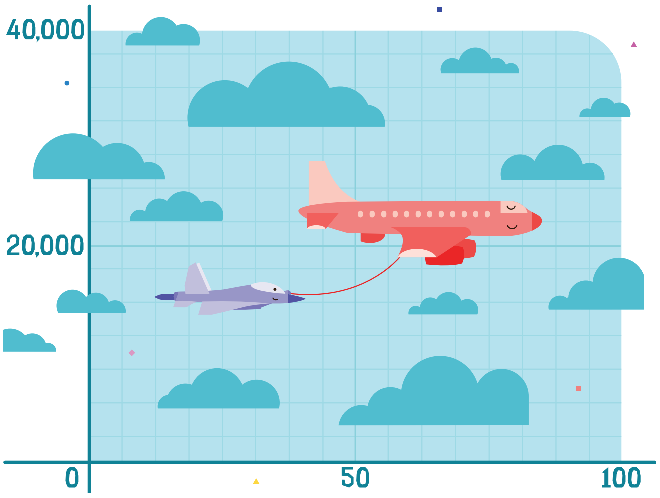
5. Write two equations that represent the following scenario. Be sure to define your variables.

A hiker descends a mountain at a rate of 3.5 miles per hour from a height of 0.5 miles above sea level. Another hiker ascends the mountain from the trailhead — which is located 0.1 miles above sea level — hiking at a rate of 2 miles per hour.

Unit 4 | Lesson 13

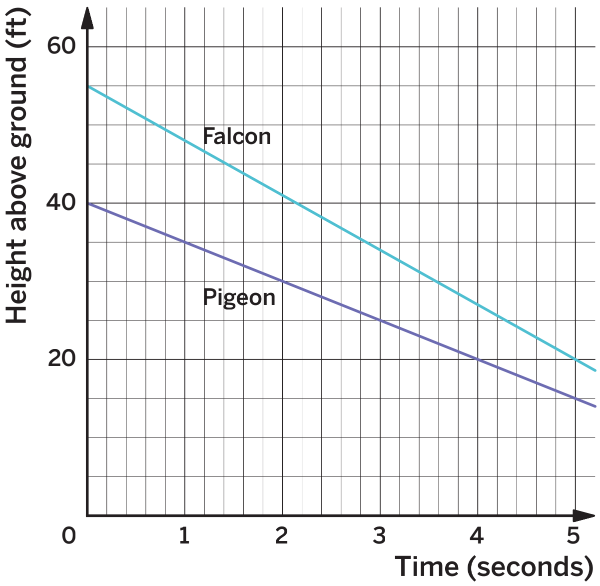
Systems of Linear Equations

Let's understand how a system of equations can be used to model a real-world context.



Warm-up Midair Meetup?

A falcon chases a pigeon in midair. The graph shows the height of each bird as the time passes. Will the falcon catch the pigeon? Explain your thinking.



Name: ____________ Date: ________ Period: ________

Activity 1 Time to Refuel

Occasionally, jet pilots need to refuel in the air. This complicated procedure is called *aerial refueling*. It requires mathematical precision and expert timing to be done safely.

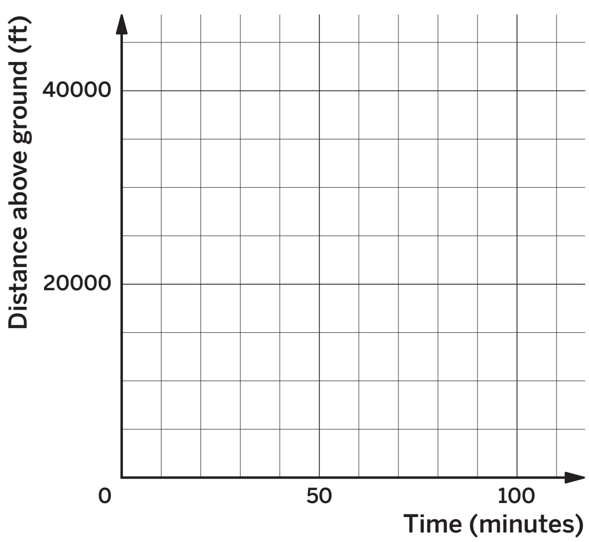
dmvphotos/Shutterstock.com

Suppose a pilot is flying a speed jet at an altitude of $30{,}000$ ft when she recognizes her jet is soon going to run out of fuel.

- **She requests an aerial refueling, and begins to descend at a constant rate of 100 vertical feet per minute to refuel at a safer altitude.**
- **The refueling plane takes off from ground level at the same time as the speed jet begins its descent, ascending at a speed of 400 vertical feet per minute.**
- **The refueling plane begins refueling the speed jet when the two planes reach the same altitude, at which point the refueling plane will position itself below the speed jet and connect to the jet's fuel tank.**

1. Write two equations in the form $y = mx + b$ to represent the situation. Be sure to define your variables.

2. Graph the lines representing each equation.



Activity 1 Time to Refuel (continued)

3. Find the point where the two graphs intersect each other. Estimate the coordinates of this point.

4. What do the coordinates represent in this situation?

Are you ready for more?

The Voyager, a refueling aircraft in the United Kingdom, can hold a little over 100 tons of fuel. The Voyager uses this fuel for its own engine *and* for aerial refuelling of other jets. Suppose the Voyager wanted to help out a fleet of American F-16 jets in need of aerial refueling. Each F-16 jet can hold 9.5 tons of fuel. If the Voyager burns about 6 tons of fuel per hour, and each refueling takes 0.5 hours, what is the greatest number of F-16 jets the Voyager could refuel completely and have fuel remaining to safely land at an airbase? Assume the Voyager needs 1 hour to land at an airbase.

Name: ______ Date: ______ Period: ______

Activity 2 Card Sort: System Sort

You will be provided with a set of cards, some of which have a system of linear equations written on them, and others that have a real-world scenario described. For each scenario, define your variables. Then match the scenarios with the systems. Some cards may not have a match.

1. List the card pairs you found that matched. Explain how you defined your variables.

Matching card pairs	Define your variables:

2. For which cards did you not find a match?

Are you ready for more?

For a card that did not have a match, describe a scenario that could be represented by the equations or write a system of linear equations that could represent the scenario.

STOP

Summary

In today's lesson . . .

You saw that a ***system of equations*** is a set of two equations with two variables where the variables represent the same unknown values. (In a later course, you will encounter systems with more than two equations and variables.)

A ***solution to a system of equations*** is an ordered pair that makes all equations in the system true.

For example, these equations make up a system of equations:

$$\begin{cases} x + y = -2 \\ x - y = 12 \end{cases}$$

One way to determine a solution to a system of equations is to graph both lines and locate the intersection point.

- If there is one point of intersection, you can conclude that the system of equations has one solution.
- If there is no point of intersection, you can conclude that the system of equations does not have a solution.
- If there are infinitely many points of intersection, you can conclude that the system has infinitely many solutions.

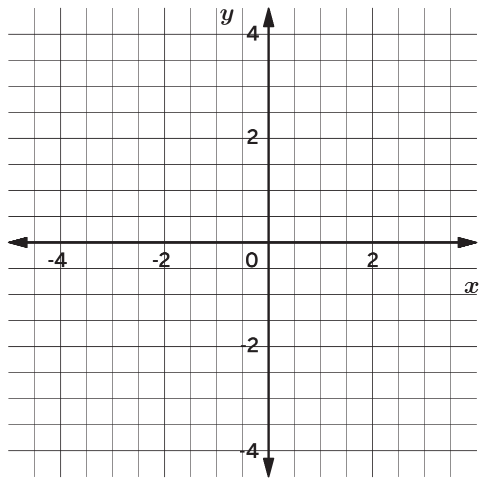
Reflect:

Practice

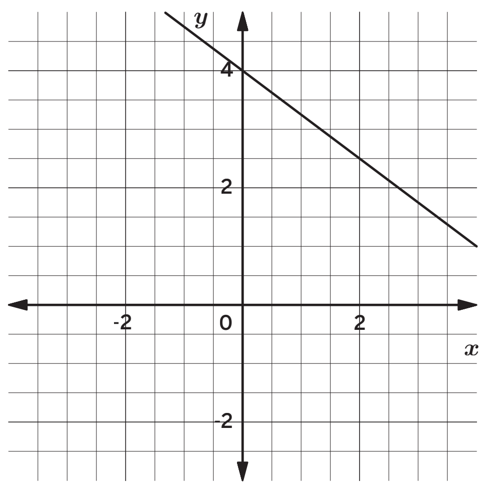
Name: .... Date: .... Period: ....

1. Graph the system of equations. Then estimate the coordinates of the ordered pair (x, y) that makes both equations true.

$$\begin{cases} y = x + 2 \\ y = -2x - 4 \end{cases}$$



2. Consider the graph shown. Suppose it represents one equation in a system of two equations. Write a second equation for the system that would satisfy each of the following conditions.



a The system has infinitely many solutions.

b The graph of the second equation passes through the point $(0, 1)$, and the system has no solution.

c The graph of the second equation passes through the point $(0, 2)$, and the system has one solution located at the point $(4, 1)$.

Practice

Name: ______________________ Date: ____________ Period: __________

3. The temperature in degrees Fahrenheit F is related to the temperature in degrees Celsius C by the equation $F = \frac{9}{5}C + 32$. There is one temperature for which the degrees Fahrenheit and degrees Celsius are the same, so that $C = F$. Use the expression from the equation, where F is expressed in terms of C, to find this temperature. Show or explain your thinking.

4. Decide whether each equation is true for *all*, *one*, or *no* values of x.

a $2x + 8 = -3.5x + 19$

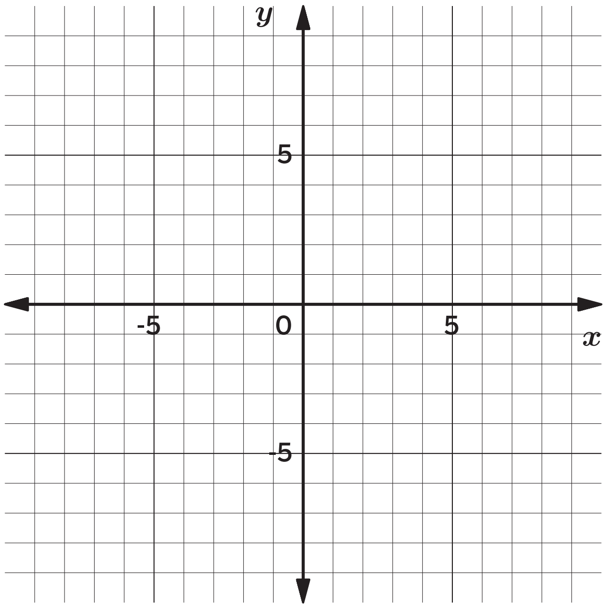
b $9(x - 2) = 7x + 5$

c $3(3x + 2) - 2x = 7x + 6$

5. Think about the equations $y = \frac{2}{3}x + 5$ and $x + y = 10$.

a Graph the equation $y = \frac{2}{3}x + 5$ and label its line as line a.

b Graph the equation $x + y = 10$ and label its line as b.



Name: .... Date: .... Period: ....

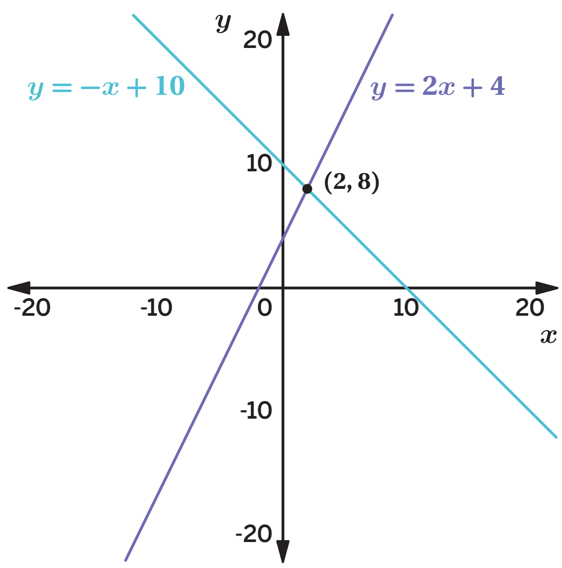
Unit 4 | Lesson 14

Solving Systems of Linear Equations (Part 1)

Let's solve systems of linear equations.

Warm-up True or False?

Consider the two lines represented by the equations $y = -x + 10$ and $y = 2x + 4$. Use the lines to decide whether each statement is true or false. Be prepared to explain your thinking.



a The ordered pair $(2, 8)$ is a solution to the equation $y = -x + 10$.

b The ordered pair $(8, 2)$ is a solution to the equation $y = 2x + 4$.

c The ordered pair $(8, 2)$ is a solution to both the equations $y = -x + 10$ and $y = 2x + 4$.

d The ordered pair $(2, 8)$ is a solution to both the equations $y = -x + 10$ and $y = 2x + 4$.

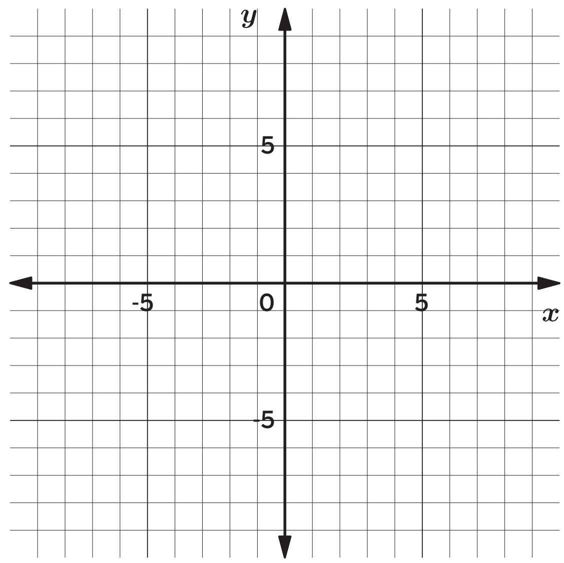
e There are no values of x and y that make the equations $y = -x + 10$ and $y = 2x + 4$ true at the same time.

Log in to Amplify Math to complete this lesson online.

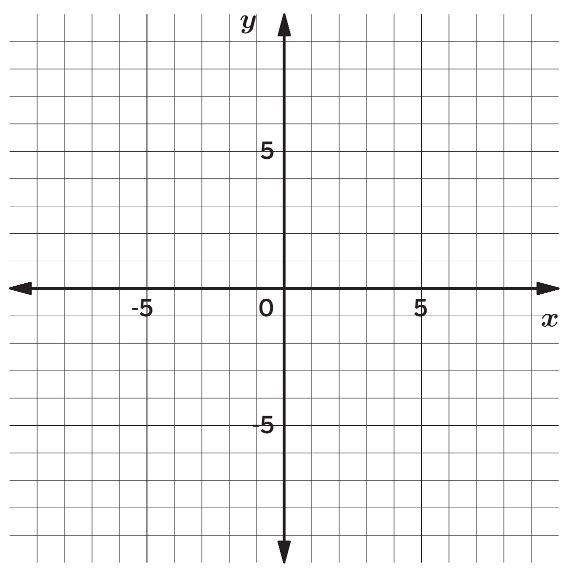
Activity 1 Graphing a System of Equations

Graph each system of equations. Then estimate the coordinates of the ordered pair (x, y) that makes both equations true.

1. $\begin{cases} y = \frac{1}{2}x + 6 \\ y = 4x - 1 \end{cases}$



2. $\begin{cases} y = 4x + 3 \\ x + y = -7 \end{cases}$



Name: ________ Date: ________ Period: ________

Activity 2 How Many Solutions?

You will need graphing technology to complete this activity.

1. Graph each system of equations. Determine whether the system of equations has *one solution*, *no solution*, or *infinitely many solutions* by placing a check mark in the appropriate box. If the system of equations has one solution, estimate the ordered pair that makes both equations true.

System of equations	One solution	No solution	Infinitely many solutions
$\begin{cases} y = 5(x-3) \\ y = 2x - 15 \end{cases}$	☐ ________	☐	☐
$\begin{cases} y = 2x + 3 \\ y = 2x - 5 \end{cases}$	☐ ________	☐	☐
$\begin{cases} y = -6x \\ y = -5x + 10 - x \end{cases}$	☐ ________	☐	☐
$\begin{cases} y = -4x + 6 \\ y = -4x + 6 \end{cases}$	☐ ________	☐	☐
$\begin{cases} y = -4x + 8 \\ y = -2x + 5 \end{cases}$	☐ ________	☐	☐
$\begin{cases} y = 6x + 3 - 4x \\ y = 2x + 3 \end{cases}$	☐ ________	☐	☐

2. What do you notice about the coefficient and constants in each system of linear equations when it has . . .

a One solution?

b No solution?

c Infinitely many solutions?

Activity 2 How Many Solutions? (continued)

3. Without graphing, determine whether each system of equations has *one solution*, *no solution*, or *infinitely many solutions*. Be prepared to explain your thinking.

a $\begin{cases} y = -\frac{4}{3}x + 4 \\ y = -1 - \frac{4}{3}x \end{cases}$

b $\begin{cases} y = -2x - 5 + 6x \\ y = -2x + 7 \end{cases}$

c $\begin{cases} y = 5x - 15 \\ y = 5(x - 3) \end{cases}$

d $\begin{cases} y = x + 6 \\ y = -(x + 6) \end{cases}$

Discussion Support: What math terms can you use during the discussion to explain how you determined the number of solutions?

Are you ready for more?

The graphs of the equations $x + y = \square$ and $y = \square x - 3$ intersect at the point $(2, 1)$. Determine the missing values in the equations. Show or explain your thinking.

STOP

Name: ________________ Date: ________ Period: ________

Summary

In today's lesson . . .

You graphed a system of linear equations to determine the solution to the system of equations. You found that you can identify the number of solutions for a system of equations by studying the coefficients and constants of the equations.

Here are some examples:

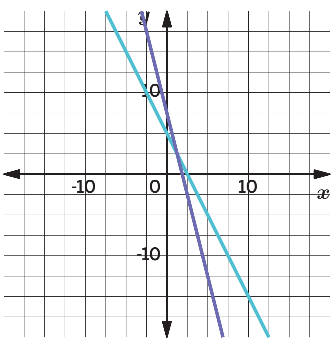
One solution

$$\begin{cases} y = -4x + 8 \\ y = -2x + 5 \end{cases}$$

Equations:

- Different coefficients
- Same or different constants

Graph:



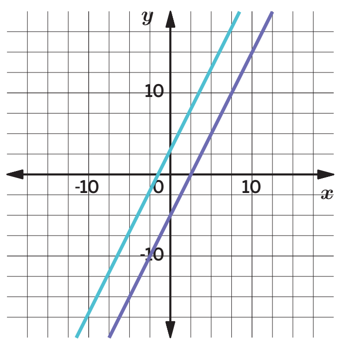
No solution

$$\begin{cases} y = 2x + 3 \\ y = 2x - 5 \end{cases}$$

Equations:

- Same coefficients
- Different constants

Graph:



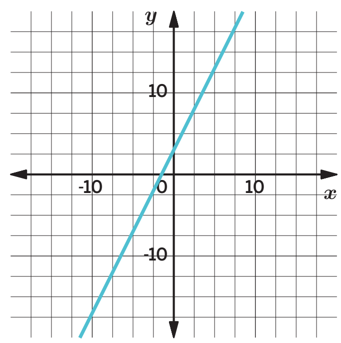
Infinitely many solutions

$$\begin{cases} y = 2x + 3 \\ y = 2x + 3 \end{cases}$$

Equations:

- Same coefficients
- Same constants

Graph:

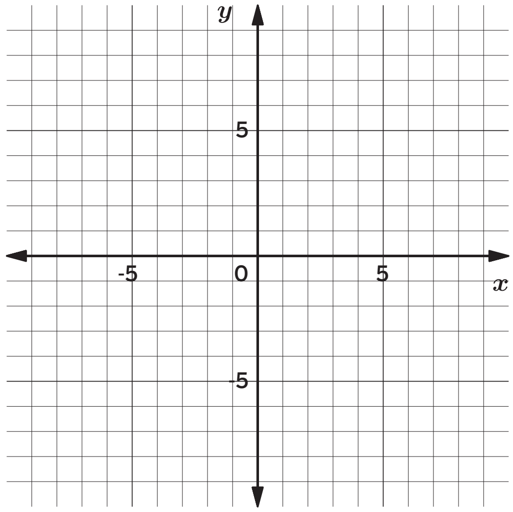


Reflect:

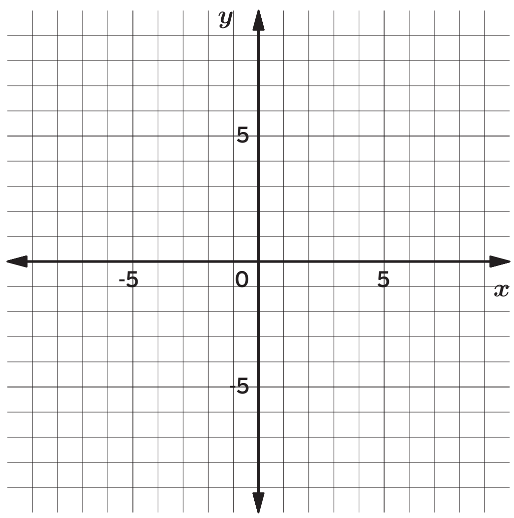
Name: .... Date: .... Period: ....

1. Graph each system of equations. Then estimate the coordinates of the ordered pair (x, y) that makes both equations true.

a $\begin{cases} y = 4x - 3 \\ y = -2x + 9 \end{cases}$



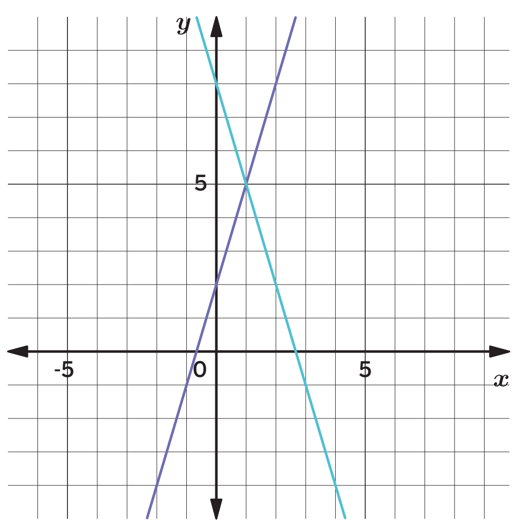
b $\begin{cases} y = \frac{1}{4}x - 2 \\ 3x + 2y = 10 \end{cases}$



2. Consider the two lines shown.

a Write an equation that can represent each line.

b Describe how to estimate the solution to the corresponding system by studying the graph. Then estimate the ordered pair that makes both equations true.



Name: .... Date: .... Period: ....

Practice

3. Which equation, together with the equation $y = -5x + 10$, creates a system with infinitely many solutions?

A. $y = 5 - 10x$

B. $y = 2x + -6 - 3x$

C. $y = 5(x - 2)$

D. $y = -5(x + 3) + 25$

E. $y = 10x + x - 5$

4. Solve each equation. Show or explain your thinking.

a $\frac{15(x-3)}{5} = 3(2x - 3)$

b $0.4(x + 7) = 0.2(x + 40) - 5.2 + 0.2x$

5. Determine the y-value for each equation if $x = 3$. What do you notice?

Equation 1:
$y = 3x + 9$

Equation 2:
$y = 7x - 3$

Unit 4 | Lesson 15

Solving Systems of Linear Equations (Part 2)

Let's solve systems of linear equations.

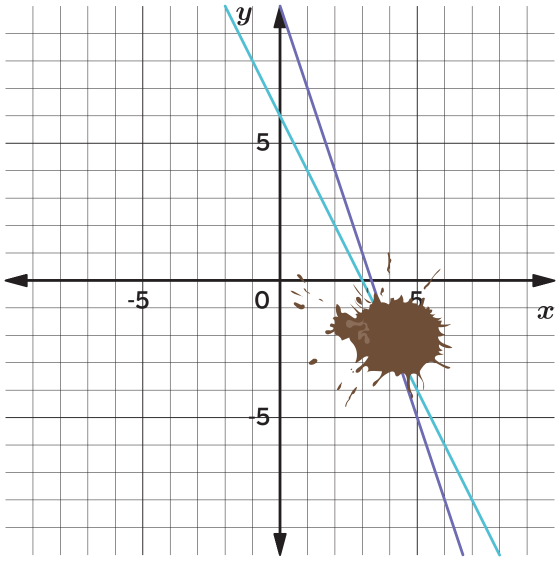
Warm-up Clean up on Quadrant Four

While Priya was doing her homework, her mom accidently spilled coffee on it! Priya was graphing a system of equations to determine the solution, but can no longer see where the lines intersect. Her work is shown.

Determine the ordered pair that makes both equations true.

$$\begin{cases} y = -3x + 10 \\ y = -2x + 6 \end{cases}$$

Show or explain another method Priya could use to determine the solution.



Log in to Amplify Math to complete this lesson online.

Name: ______ Date: ______ Period: ______

Activity 1 What's The Solution?

Elena solves the system of equations from the Warm-up. Some of her work is shown.

$$\begin{cases} \mathbf{y = -3x + 10} \\ \mathbf{y = -2x + 6} \end{cases}$$

Elena's work:

$$\begin{aligned} -3x + 10 &= -2x + 6 \\ -x + 10 &= 6 \\ -x &= -4 \\ x &= 4 \end{aligned}$$

1. Describe Elena's method for calculating the value of x.

2. Describe a method that Elena could use to calculate the value of y. Then use this method to determine the value of y.

3. What is the ordered pair that is a solution to the system?

Activity 2 Partner Problems

With your partner, decide who will solve the systems of equations in Column A and who will solve the systems of equations in Column B. After each row, share your responses with your partner. Although the problems in each row are different, your responses should be the same. If they are not the same, work together to correct any errors or resolve any disagreements

Column A	Column B
1. $\begin{cases} y = -3x + 9 \\ y = 2x + 4 \end{cases}$	$\begin{cases} y = -4x + 10 \\ y = 8x - 2 \end{cases}$
2. $\begin{cases} y = 5x + 7 \\ y = 6x + 4 \end{cases}$	$\begin{cases} y = -2x + 28 \\ y = -x + 25 \end{cases}$

Name: ________________ Date: ________ Period: ________

Activity 2 Partner Problems (continued)

	Column A	Column B
3.	$\begin{cases} y = -3x + 12 \\ x = 4 \end{cases}$	$\begin{cases} x = 4 \\ y = -3(x - 4) \end{cases}$
4.	$\begin{cases} 2x + y = 7 \\ 2x + y = 9 \end{cases}$	$\begin{cases} 3x - 2y = 11 \\ 3x - 2y = 7 \end{cases}$

STOP

Summary

In today's lesson . . .

You discovered that for an ordered pair to be a solution to a system of equations, the x- and y-values of the ordered pair must make both of the equations true.

For example, consider the following system of equations:

$$\begin{cases} y = 4x - 5 \\ y = -2x + 7 \end{cases}$$

To determine the solution to the system, you can write a single equation that sets the two expressions — for which y is equal to — equal to each other:

$$\begin{aligned} 4x - 5 &= -2x + 7 \\ 6x - 5 &= 7 \\ 6x &= 12 \\ x &= 2 \end{aligned}$$

Then you can use the solution for x and either of the original equations in the system to determine the value of y:

If $x = 2$, then $y = 4(2) - 5$, $y = 3$.

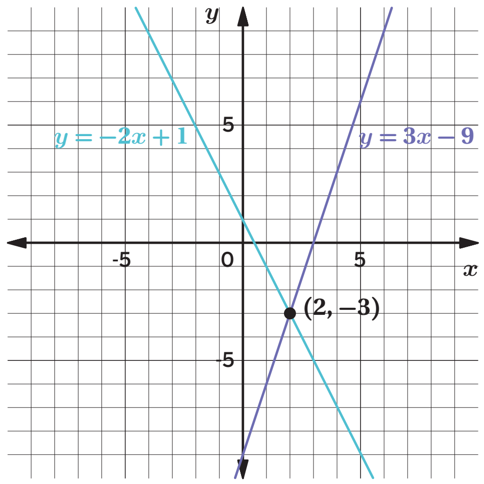
The ordered pair $(2, 3)$ is the solution to the system of equations.

Reflect:

Name: ______ Date: ______ Period: ______

Practice

1. Use the lines shown to decide whether each statement is true or false.



a. The solution to the equation $-2x + 1 = 3x - 9$ is $x = 2$.

b. The point $(2, -3)$ is a solution to the following system of equations:
$\begin{cases} y = -2x + 1 \\ x = 2 \end{cases}$

c. The point $(0, 1)$ is a solution to the equation $y = -2x + 1$.

2. Solve each system of equations. Show or explain your thinking.

a. $\begin{cases} y = 3x - 2 \\ y = -2x + 8 \end{cases}$

b. $\begin{cases} y = -3x - 5 \\ y = 4x + 30 \end{cases}$

c. $\begin{cases} y = 2x - 9 \\ y = 4 + 2x \end{cases}$

d. $\begin{cases} x = 2 \\ y = 3x - 1 \end{cases}$

Practice

Name: .... Date: .... Period: ....

3. The solution to a system of equations is $(1, 5)$. Select two equations that might make up the system.

A. $y = -3x + 6$

B. $y = 2x + 3$

C. $y = -7x + 1$

D. $y = x + 4$

E. $y = -2x + 9$

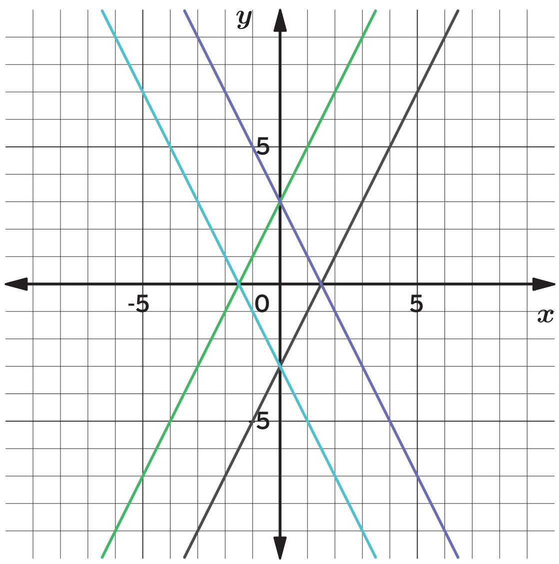
4. Label each line with its corresponding equation.

a $y = 2x + 3$

b $y = -2x + 3$

c $y = 2x - 3$

d $y = -2x - 3$



5. What is the slope of the line that passes through the points $(-3, 4)$ and $(1, 7)$?

6. Write an equation that represents each situation.

a A gym charges a $\$50$ one-time membership fee and then $\$15$ each month. Write an equation to represent the total cost c for m months of membership.

b Han purchases avocados and tomatoes. Each avocado costs $\$2$ and each tomato costs $\$1.50$. Write an equation to represent the number of avocados a and tomatoes t Han could buy with $\$15$.

Name: ____________ Date: ________ Period: ________

Unit 4 | Lesson 16

Writing Systems of Linear Equations

Let's write systems of equations to model real-world contexts.

Warm-up Algebra Talk

Consider the incomplete system of equations shown.

$$\begin{cases} y = \frac{2}{3}x - 6 \\ \boxed{\quad ? \quad} \end{cases}$$

Write a second equation so that the system of equations has:

a Exactly one solution.

b No solution.

c Infinitely many solutions.

Log in to Amplify Math to complete this lesson online.

Activity 1 Situations and Systems

Write a system of equations to model each scenario. Define the variables you choose to use. Without solving the system, interpret what the solution to the system would tell you about the scenario.

1. Elena plans a kayaking trip. Kayak Rental A charges a base fee of $15 plus $4.50 per hour. Kayak Rental B charges a base fee of $12.50 plus $5 per hour.

2. Diego works at a smoothie stand and prepares a batch of smoothies. The recipe calls for 3 cups of sliced strawberries for every cup of sliced apples. Diego uses a total of 5 cups of sliced strawberries and apples.

3. Andre orders some posters. At Store A, he can order 6 large posters and 4 small posters for $70. At Store B, he can order 5 large posters and 9 small posters for $81.

Are you ready for more?

For Problem 3, determine the price of one large poster and one small poster.

Name: ____________ Date: ____________ Period: ____________

Activity 2 Info Gap: Walking, Jogging, Running

You will be given either a *problem card* or a *data card*. Do not show or read your card to your partner.

If you are given the *problem card*:	If you are given the *data card*:
1. Silently read your card and think about what information you need to be able to solve the problem.	**1.** Silently read your card.
2. Ask your partner for the specific information that you need.	**2.** Ask your partner "What specific information do you need?" and wait for them to ask for information.
3. Explain how you will use the information to solve the problem.	**3.** If your partner asks for information that is not on the card, do not perform the calculations for them. Tell them you don't have that information.
4. Continue to ask questions until you have enough information to solve the problem.	**4.** Before sharing the information, ask "Why do you need that information?" Listen to your partner's reasoning and ask clarifying questions.
5. Share the problem card and solve the problem independently.	**5.** Read the problem card and solve the problem independently.
6. Read the data card and discuss your thinking.	**6.** Share the data card and discuss your thinking.

Pause here so your teacher can review your work. You will be given a new set of cards. Repeat the activity, trading roles with your partner.

Info Gap: To help you get started, ask yourself these questions:

- Do you know what the variables represent?
- Do you know each person's rate?

What else might you need to know?

STOP

Summary

In today's lesson . . .

You discovered that writing and solving systems of equations can help solve everyday problems. When writing a system of equations to model a given real-world problem, it is important to define your variables. After you have solved the system, you will know what the solution represents if you have clearly defined your variables.

Reflect:

Name: .... Date: .... Period: ....

Practice

1. Which story can be represented by the following system of equations?

$$\begin{cases} y = x + 6 \\ x + y = 100 \end{cases}$$

A. Diego's teacher creates a test worth 100 points. There are 6 more multiple choice questions than short answer questions.

B. Priya and her younger cousin each measure their heights. They notice that Priya is 6 in. taller, and their heights add up to exactly 100 in.

C. Kiran receives a $\$6$ allowance per week. At the end of the month, he saves $\$100$.

2. Clare and her brother play a game in which they earn the same number of points for each goal and lose the same number of points for each penalty. Clare earns 6 goals and has 3 penalties, ending the game with 6 points. Her brother earns 8 goals and has 9 penalties and ends the game with -22 points.

Write a system of equations to model this scenario. Define the variables you choose to use. Without solving the system, interpret what the solution to the system would tell you about the scenario.

3. Noah and his cousin each work during the summer for a landscaping company. Noah's cousin has been working for the company longer, so his pay is 30% more than Noah. Last week, his cousin worked 1 hour and Noah worked 3 hours. Together, they earned $\$36.55$. What is Noah's hourly pay? Show or explain your thinking.

Practice

Name: .... Date: .... Period: ....

4. Solve each system of equations. Show or explain your thinking.

a $\begin{cases} y = 6x - 8 \\ y = -3x + 10 \end{cases}$

b $\begin{cases} y = -8x + 2x - 4 \\ y = -6x - 4 \end{cases}$

5. Consider the incomplete system of equations shown. Create a second equation so that the system has no solution.

$$\begin{cases} y = \frac{3}{4}x - 4 \\ \boxed{\quad ? \quad} \end{cases}$$

6. Kiran read 182 pages of his new 232-page book. What percent of his new book has Kiran read? Round to the nearest tenth of a percent.

Name: ________ Date: ________ Period: ________

Unit 4 | Lesson 17 – Capstone

Pay Gaps

Let's learn about earning differences by gender

Warm-up Notice and Wonder

Women have been earning college degrees at a higher rate than men since 1981. Let's take a closer look at the types of majors men and women have been pursuing. The following data, taken from the National Center for Education Statistics and the U.S. Census Bureau in 2014, show some college majors with the highest and lowest earnings. What do you notice? What do you wonder?

1. I notice . . .

2. I wonder . . .

College majors with the highest/lowest earnings

Majors with highest earnings	Median earning	Percent female
Petroleum engineering	\$136K	14%
Pharmacy, pharmaceutical sciences	\$113K	59%
Metallurgical engineering	\$98K	23%
Mining and mineral engineering	\$97K	13%
Chemical engineering	\$96K	32%
Electrical engineering	\$93K	12%
Studio arts	\$42K	69%
Social work	\$42K	88%
Human/community services	\$41K	85%

Log in to Amplify Math to complete this lesson online.

Activity 1 Mind the Gap

The table shows the median annual earnings for veterinarians in the year 2018, according to data from the U.S. Census Bureau.

Men's median earnings ($)	Women's median earnings ($)	Women's median earnings as a percentage of men's
111,080	93,065	

1. Complete the table to calculate the women's median annual earnings as a percentage of men's for veterinarians. Round to the nearest tenth of a percent. Explain your thinking.

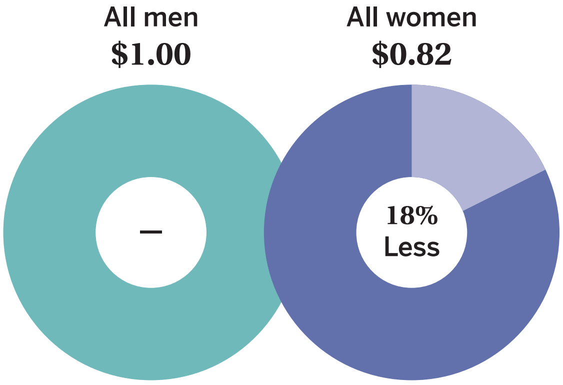
2. Your group will be given a sheet with data showing the median earnings for men and for women for ten different occupations. Calculate the women's median annual earnings as a percentage of the men's median annual earnings, for each occupation. Round to the nearest tenth of a percent. Record your responses in the table.

3. What conclusions can you draw from the data? What questions do you have?

Name: ______ Date: ______ Period: ______

Activity 2 Gender Pay Gap

The graphic shows what is commonly called the *gender pay gap*.

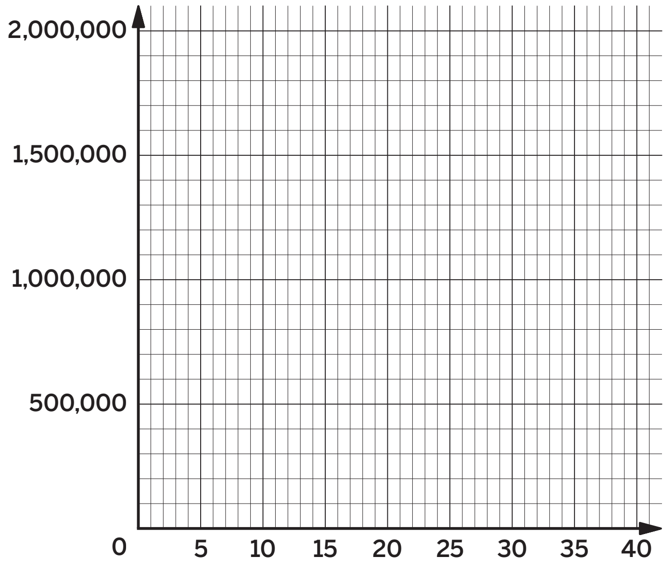


1. Describe the graphic in your own words.

2. On the coordinate plane shown, sketch two graphs, one labeled "Men" and one labled "Women," that show the impact of the gender pay gap over the course of a lifetime of earnings, assuming no changes in the gender pay gap. Be sure to label the axes.

   Here are some figures you may find helpful:

   - The median full-time annual salary for men in 2018 was just over $51,000.
     Note: You can use $50,000 for the purposes of this activity.
   - Assume the typical worker can expect to work full time for about 40 years.



3. For typical women to earn the same amount of lifetime earnings as typical men, how much more would they need to earn by the end of their 40-year careers?

STOP

Unit Summary

Algebra is a powerful tool. It gives us ways to talk about unknowns in ways that are concrete. You saw it in this very lesson with the gender pay gap. While it's already widely known that sexism has enabled men to earn more than women, algebra allows us to uncover exactly how wide the wage gap actually is.

Through a process of elimination and deduction that involves constants (numbers that stay the same) and variables (the numbers that change), you worked your way down to a solution. This algebraic process was first recorded by the Persian mathematician, Al-Khwārizmī, over a thousand years ago. By bringing together the work of ancient Greek, Chinese, Mesopotamian and Indian mathematicians, Al-Khwārizmī simplified the lives of Baghdad's merchants and traders. His detailed calculation methods allowed them to conduct business more efficiently.

Today algebra is just as useful as it was in Al-Khwārizmī's day. It offers a way to solve for unknowns through a process that's orderly and logical. Whether it's finding where two hikers will meet on a particular trail or describing the inequities in our society, algebra provides a path to uncovering the truth.

See you in Unit 5.

Name: ______ Date: ______ Period: ______

Practice

1. Select the situation that could be represented by this system of equations:

$$\begin{cases} y = x + 6 \\ x + y = 100 \end{cases}$$

A. Han and his younger cousin measure their heights. They notice that Han is 6 in. taller, and their heights add up to exactly 100 in.

B. Andre's teacher writes a test worth 100 points. There are 6 more multiple choice questions than short answer questions.

2. Solve each system of equations. Show your thinking.

a. $\begin{cases} y = 6x - 8 \\ y = -3x + 10 \end{cases}$

b. $\begin{cases} x = \frac{1}{2} \\ y = 3 - 4x \end{cases}$

c. $\begin{cases} y = 0.5x - 1 \\ y = 5 - 2.5x \end{cases}$

d. $\begin{cases} y = 2x + 1 \\ y = \frac{1}{2} + 2x \end{cases}$

Practice

Name: .... Date: .... Period: ....

3. Refer to the two lines graphed on the coordinate plane.

a Estimate the coordinates of the point where the two lines meet.

b Choose two equations that make up the system represented by the graph.

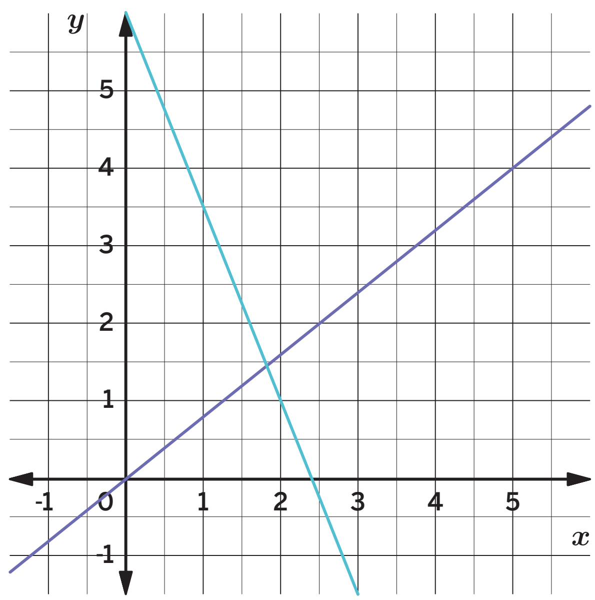
A. $y = \frac{5}{4}x$

B. $y = 6 - 2.5x$

C. $y = 2.5x + 6$

D. $y = 6 - x$

E. $y = 0.8x$



c Solve the system of equations. Round the coordinates of the solution to the nearest hundredth. Then confirm the accuracy of your estimate you made in part a.

4. A full 1,500-liter water tank springs a leak, losing 2 liters per minute. At the same time, a second tank contains 300 liters and is being filled at a rate of 6 liters per minute. When will the two water tanks have the same amount of water? Show or explain your thinking.

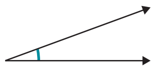
Glossary/Glosario

English | **Español**

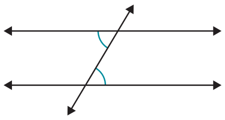
A

absolute value The value that represents the distance between a number and zero. For example, because the distance between -3 and 0 is 3, the absolute value of -3 is 3, or $|-3| = 3$.

acute angle An angle whose measure is less than 90 degrees.



alternate interior angles Alternate interior angles are created when a pair of parallel lines are intersected by a transversal. These angles lie inside the parallel lines and on opposite (alternate) sides of the transversal.

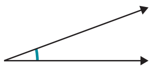


angle of rotation See the definition for *rotation*.

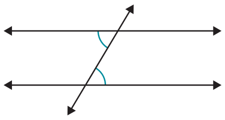
area The number of unit squares needed to fill a two-dimensional shape without gaps or overlaps.

valor absoluto Valor que representa la distancia entre un número y cero. Por ejemplo, dado que la distancia entre -3 y 0 es 3, el valor absoluto de -3 es 3, o $|-3| = 3$.

ángulo agudo Ángulo cuya medida es menor que 90 grados.



ángulos interiores alternos Se crean ángulos interiores alternos cuando un par de líneas paralelas son intersecadas por una transversal. Estos ángulos están dentro de las líneas paralelas y en lados opuestos (alternos) de la transversal.

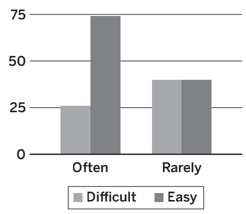


ángulo de rotación Ver *rotación*.

área Número de unidades cuadradas necesario para llenar una forma bidimensional sin dejar espacios vacíos ni superposiciones.

B

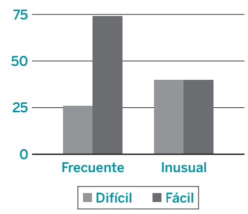
bar graph A graph that presents data using rectangular bars that have heights proportional to the values that they represent.



bar notation Notation that indicates the repeated part of a repeating decimal. For example, $0.\overline{6} = 0.66666\ldots.$

base The number that is raised to a power. The power indicates the number of times the base is multiplied by itself.

gráfica de barras Gráfica que presenta datos por medio de barras con alturas proporcionales a los valores que representan.



notación de barras Notación que indica la parte repetida de un número decimal periódico. Por ejemplo, $0.\overline{6} = 0.66666\ldots.$

base Número que se eleva a una potencia. La potencia indica el número de veces que la base debe multiplicarse por sí mismo.

C

center of dilation See the definition for *dilation*.

center of rotation See the definition for *rotation*.

circle A shape that is made up of all of the points that are the same distance from a given point.

circumference The distance around a circle.

clockwise A rotation in the same direction as the way hands on a clock move is called a *clockwise* rotation.

centro de dilatación Ver *dilatación*.

centro de rotación Ver *rotación*.

círculo Forma constituida por todos los puntos que están a la misma distancia de un punto dado.

circunferencia Distancia alrededor de un círculo.

en el sentido de las agujas del reloj Una rotación en la misma dirección en que se mueven las agujas de un reloj es llamada una rotación *en el sentido de las agujas del reloj*.

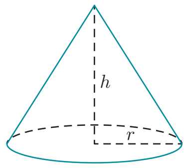
Glossary/Glosario

English

cluster A cluster represents data values that are grouped closely together.

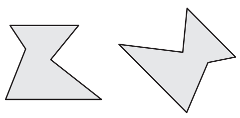
coefficient A constant by which a variable is multiplied, written in front of the variable. For example, in the expression $3x + 2y$, 3 is the coefficient of x.

cone A three-dimensional solid that consists of a circular base connected by a curved surface to a single point.



congruent Two figures are "congruent" to each other if one figure can be mapped onto the other by a sequence of rigid transformations.

congruent Two figures are *congruent* to each other if one figure can be mapped onto the other by a sequence of rigid transformations.



constant A value that does not change, meaning it is not a variable.

constant of proportionality The number in a proportional relationship that the value of one quantity is multiplied by to get the value of the other quantity.

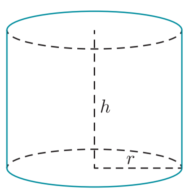
coordinate plane A two-dimensional plane that represents all the ordered pairs (x, y), where x and y can both represent on values that are positive, negative, or zero.

corresponding parts Parts of two scaled copies that match up, or "correspond" with each other. These corresponding parts could be points, segments, angles, or lengths.

counterclockwise A rotation in the opposite direction as the way hands on a clock move is called a *counterclockwise* rotation.

cube root The cube root of a positive number p is a positive solution to equations of the form $x^3 = p$. Write the cube root of p as $\sqrt[3]{p}$.

cylinder A three-dimensional solid that consists of two parallel, circular bases joined by a curved surface.

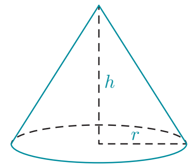


Español

agrupación Una agrupación representa valores de datos que se agrupan de manera cercana entre ellos.

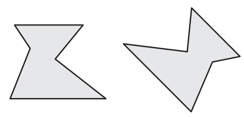
coeficiente Constante por la cual una variable es multiplicada, escrita frente a la variable. Por ejemplo, en la expresión $3x + 2y$, 3 es el coeficiente de x.

cono Sólido tridimensional compuesto de una base circular conectada a un solo punto por medio de una superficie curva.



congruente Dos figuras son "congruentes" si una de las figuras puede mapearse con la otra mediante una secuencia de transformaciones rígidas.

congruente Dos figuras son *congruentes* entre sí, si una figura puede adquirir la forma de la otra figura mediante una secuencia de transformaciones rígidas.



constante Valor que no cambia, lo que significa que no es una variable.

constante de proporcionalidad En una relación proporcional, el número por el cual el valor de una cantidad es multiplicado para obtener el valor de otra cantidad.

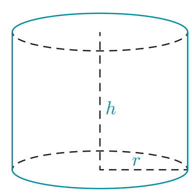
plano de coordenadas Plano bidimensional que representa todos los pares ordenados (x, y), donde tanto x como y pueden representar valores positivos, negativos o cero.

partes correspondientes Partes de dos copias a escala que coinciden, o "se corresponden", entre sí. Estas partes correspondientes pueden ser puntos, segmentos, ángulos o longitudes.

en el sentido contrario a las agujas del reloj Una rotación en la dirección opuesta a la forma en que las agujas de un reloj se mueven es llamada una rotación *en el sentido contrario a las agujas del reloj*.

raíz cúbica La raíz cúbica de un número positivo p es una solución positiva a las ecuaciones de la forma $x^3 = p$. Escribimos la raíz cúbica p como $\sqrt[3]{p}$.

cilindro Sólido tridimensional compuesto por dos bases paralelas y circulares unidas por una superficie curva.



English

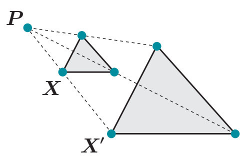
D

dependent variable The dependent variable represents the output of a function.

diagonal A line segment connecting two vertices on different sides of a polygon or polyhedra.

diameter The distance across a circle through its center. The line segment with endpoints on the circle, that passes through its center.

dilation A transformation defined by a fixed point P (called the *center of dilation*) and a scale factor k. The dilation moves each point X to a point X' along ray PX, such that its distance from P changes by the scale factor.



Distributive Property A property relating addition and multiplication: $a(b + c) = ab + ac$.

E

equation A mathematical statement that two expressions are equal.

equivalent If two mathematical objects (especially fractions, ratios, or expressions) are equal in any form, then they are equivalent.

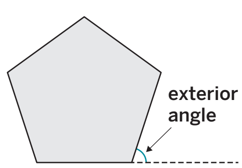
equivalent equations Equations that have the same solution or solutions.

equivalent expressions Two expressions whose values are equal when the same value is substituted into the variable for each expression.

exponent The number of times a factor is multiplied by itself.

expression A quantity that can include constants, variables, and operations.

exterior angle An angle between a side of a polygon and an extended adjacent side.



Español

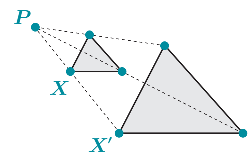
D

variable dependiente La variable dependiente representa el resultado, o salida, de una función.

diagonal Segmento de línea que conecta dos vértices que están en lados diferentes de un polígono o de un poliedro.

diámetro Distancia que atraviesa un círculo por su centro. El segmento de línea cuyos extremos se ubican en el círculo y que pasa a través de su centro.

dilatación Transformación definida por un punto fijo P (llamado *centro de dilatación*) y un factor de escala k. La dilatación mueve cada punto X a un punto X' a lo largo del rayo PX, de manera tal que su distancia con respecto a P es cambiada por el factor de escala.



Propiedad distributiva Propiedad que relaciona la suma con la multiplicación: $a(b + c) = ab + ac$.

E

ecuación Declaración matemática de que dos expresiones son iguales.

equivalente Si dos objetos matemáticos (especialmente fracciones, razones o expresiones) son iguales de cualquier manera, entonces son equivalentes.

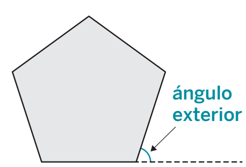
ecuaciones equivalentes Ecuaciones que tienen la misma solución o soluciones.

expresiones equivalentes Dos expresiones cuyos valores son iguales cuando se sustituye el mismo valor en la variable de cada expresión.

exponente Número de veces que un factor es multiplicado por sí mismo.

expresión Cantidad que puede incluir constantes, variables y operaciones.

ángulo exterior Ángulo que se encuentra entre un lado de un polígono y un lado extendido adyacente.



Glossary/Glosario

English

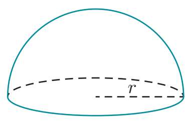
F

function A function is a rule that assigns exactly one output to each possible input.

H

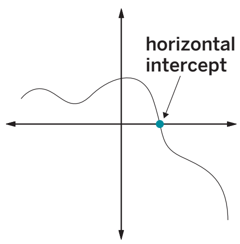
hanger diagram A model in which quantities are represented as weights attached to either side of a hanger. When the hanger is balanced, the sum of the quantities on either side must be equal.

hemisphere Half of a sphere.

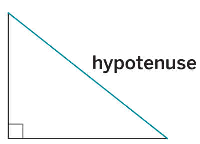


horizontal Running straight from left to right (or right to left).

horizontal intercept A point where a graph intersects the horizontal axis. Also known as the x-intercept, it is the value of x when y is 0.



hypotenuse In a right triangle, the side opposite the right angle is called the hypotenuse.



I

image A new figure that is created from an original figure (called the *preimage*) by a transformation.

independent variable The independent variable represents the input of a function.

initial value The starting amount in a context.

input The independent variable of a function.

integers Whole numbers and their opposites. For example, -4, 0, and 15 are whole numbers.

interior angle An angle between two adjacent sides of a polygon.

irrational number A number that is not rational. That is, an irrational number cannot be written as a fraction.

Español

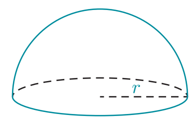
F

función Una función es una regla que asigna exactamente un resultado, o salida, a cada posible entrada.

H

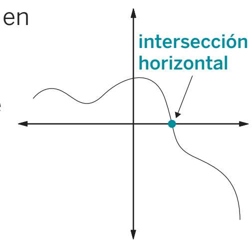
diagrama de colgador Modelo en el cual ciertas cantidades son representadas como pesos sujetos a cada lado de un colgador. Cuando el colgador está en equilibrio, la suma de las cantidades en cualquiera de los lados debe ser igual.

hemisferio La mitad de una esfera.

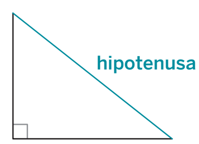


horizontal Que corre en línea recta de izquierda a derecha (o de derecha a izquierda).

intersección horizontal Punto en que una gráfica se interseca con el eje horizontal. Conocida también como intersección x, se trata del valor de x, cuando y es 0.



hipotenusa En un triangulo rectángulo, el lado opuesto al ángulo recto se llama la hipotenusa.



I

imagen Nueva figura que se crea a partir de una figura original (llamada la *preimagen*) por medio de una transformación.

variable independiente La variable independiente representa la entrada de una función.

valor inicial Monto inicial en un contexto.

entrada La variable independiente de una función.

enteros Números completos y sus opuestos. Por ejemplo, -4, 0 y 15 son números enteros.

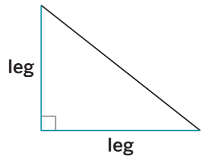
ángulo interior Ángulo que se encuentra entre dos lados adyacentes de un polígono.

número irracional Número que no es racional. Es decir, un número irracional no puede ser escrito como fracción.

English

L

legs The two sides of a right triangle that form the right angle.



like terms Parts of an expression that have the same variables and exponents. *Like terms* can be added or subtracted into a single term.

line of reflection See the definition for *reflection*.

linear association If a straight line can model the data, the data have a linear association.

linear function A linear relationship which assigns exactly one output to each possible input.

linear model A linear equation that models a relationship between two quantities.

linear relationship A relationship between two quantities in which there is a constant rate of change. When one quantity increases by a certain amount, the other quantity increases or decreases by a proportional amount.

long division A way to show the steps for dividing base ten whole numbers and decimals, dividing one digit at a time, from left to right.

$$\begin{array}{r} 0.375 \\ 8\overline{)3.000} \\ -2\;4 \\ \hline 60 \\ -56 \\ \hline 40 \\ -40 \\ \hline 0 \end{array}$$

N

negative association A negative association is a relationship between two quantities where one tends to decrease as the other increases.

nonlinear association If a straight line cannot model the data, the data have a nonlinear association.

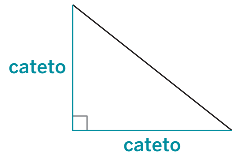
nonlinear function A function that does not have a constant rate of change. Its graph is not a straight line.

nonproportional relationship A relationship between two quantities in which the corresponding values do not have a constant ratio. (In other words, a relationship that is not a proportional relationship.)

Español

L

catetos Los dos lados de un triángulo rectángulo que componen el ángulo recto.



términos similares Partes de una expresión que tienen las mismas variables y exponentes. Los *términos similares* pueden ser reducidos a un solo término mediante su suma o resta.

línea de reflexión Ver *reflexión*.

asociación lineal Si una línea recta puede modelar los datos, los datos tienen una asociación lineal.

función lineal Relación lineal que asigna exactamente un resultado, o salida, a cada entrada posible.

modelo lineal Ecuación lineal que modela una relación entre dos cantidades.

relación lineal Relación entre dos cantidades en la cual existe una tasa de cambio constante. Cuando una cantidad aumenta un cierto monto, la otra cantidad aumenta o disminuye en un monto proporcional.

división larga Forma de mostrar los pasos necesarios para dividir números enteros en base diez y decimales, por medio de la división de un dígito a la vez, de izquierda a derecha.

$$\begin{array}{r} 0.375 \\ 8\overline{)3.000} \\ -2\;4 \\ \hline 60 \\ -56 \\ \hline 40 \\ -40 \\ \hline 0 \end{array}$$

N

asociación negativa Una asociación negativa es una relación entre dos cantidades, en la cual una tiende a disminuir a medida que la otra aumenta.

asociación no lineal Si una línea recta no puede modelar los datos, los datos tienen una asociación no lineal.

función no lineal Función que no tiene un índice constante de cambio. Su gráfica no es una línea recta.

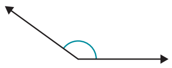
relación no proporcional Relación entre dos cantidades, en la cual los valores correspondientes no tienen una razón constante. (En otras palabras, una relación que no es una relación proporcional.)

Glossary/Glosario

O

English

obtuse angle An angle that measures more than 90 degrees.



order of operations When an expression has multiple operations, they are applied in a consistent order (the *order of operations*) so that the expression is evaluated the same way by everyone.

ordered pair Two values x and y, written as (x, y), that represent a point on the coordinate plane.

orientation The arrangement of the vertices of a figure before and after a transformation. A figure's orientation changes when it is reflected across a line.

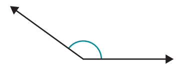
origin The point represented by the ordered pair $(0, 0)$ on the coordinate plane. The *origin* is where the x- and y-axes intersect.

outlier Outliers are points that are far away from their predicted values.

output The dependent variable of a function.

Español

ángulo obtuso Ángulo que mide más de 90 grados.



orden de las operaciones Cuando una expresión tiene múltiples operaciones, estas se aplican en cierto orden consistente (el *orden de las operaciones*) de manera que la expresión sea evaluada de la misma manera por todas las personas.

par ordenado Dos valores x y y, escritos como (x, y), que representan un punto en el plano de coordenadas.

orientación El arreglo de los vertices de una figura antes y después de una transformación. La orientación de una figura cambia cuando esta es reflejada con respecto de una línea.

origen Punto representado por el par ordenado $(0, 0)$ en el plano de coordenadas. El *origen* es donde los ejes x y y se intersecan.

valor atípico Los valores atípicos son puntos que están muy lejos de sus valores predichos.

resultado o salida Variable dependiente de una función.

P

English

perfect cube A number that is the cube of an integer. For example, 8 is a perfect cube because $2^3 = 8$.

perfect square A number that is the square of an integer. For example, 16 is a perfect square because $4^2 = 16$.

pi The ratio of the circumference of a circle to its diameter. It is usually represented by π.

piecewise function A function that is defined by two or more equations. Each equation is valid for some interval.

polygon A closed, two-dimensional shape with straight sides that do not cross each other.

Español

cubo perfecto Número que es el cubo de un número entero. Por ejemplo, 8 es un cubo perfecto porque $2^3 = 8$.

cuadrado perfecto Número que es el cuadrado de un número entero. Por ejemplo, 16 es un cuadrado perfecto porque $4^2 = 16$.

pi Razón entre la circunferencia y el diámetro de un círculo. Usualmente se representa como π.

función por partes Función definida por dos o más ecuaciones. Cada ecuación es válida para alguno de los intervalos.

polígono Forma cerrada y bidimensional de lados rectos que no se entrecruzan.

English	Español
positive association A positive association is a relationship between two quantities where one tends to increase as the other increases.	**asociación positiva** Una asociación positiva es una relación entre dos cantidades, en la cual una tiende a aumentar a medida que la otra disminuye.
preimage See the definition of *image*.	**preimagen** Ver *imagen*.
prime notation A labeling notation that uses a tick mark. *Prime notation* is typically applied to an image, to tell it apart from its preimage.	**notación prima** Notación para etiquetar que usa un signo de prima. Una *notación prima* usualmente se aplica a una imagen, para distinguirla de su preimagen.
Properties of Equality Rules that apply to all equations. These include properties of addition, subtraction, multiplication, and division, which state that if an equation is true, then applying the same operation to both sides will give a new equation that is also true.	**Propiedades de igualdad** Reglas que son aplicables a todas las ecuaciones. Incluyen las propiedades de suma, resta, multiplicación y división, las cuales señalan que si una ecuación es verdadera, al aplicar la misma operación a ambos lados se obtendrá una nueva ecuación que también es verdadera.
proportional relationship A relationship in which the values for one quantity are each multiplied by the same number (the *constant of proprtionality*) to get the values for the other quantity.	**relación proporcional** Relación en la que los valores de una cantidad se multiplican cada uno por el mismo número (la *constante de proporcionalidad*) para obtener los valores de la otra cantidad.
Pythagorean Theorem The Pythagorean Theorem states that, for any right triangle, leg^2 + leg^2 = hypotenuse^2. Sometimes this can be presented as $a^2 + b^2 = c^2$, where a and b represent the length of the legs and c represents the length of the hypotenuse.	**Teorema de Pitágoras** El Teorema de Pitágoras establece que para todo triángulo rectángulo: cateto^2 + cateto^2 = hipotenusa^2. A veces puede ser también presentado como $a^2 + b^2 = c^2$, donde a y b representan las longitudes de los catetos y c representa la longitud de la hipotenusa.
Pythagorean triple Three positive integers a, b, and c, such that $a^2 + b^2 = c^2$.	**Triplete pitagórico** Tres enteros positivos a, b y c, tales como $a^2 + b^2 = c^2$.

Q

English	Español
quadrilateral A polygon with exactly four sides.	**cuadrilátero** Polígono de exactamente cuatro lados.

Glossary/Glosario

English

R

radius A line segment that connects the center of a circle with a point on the circle. The term can also refer to the length of this segment.

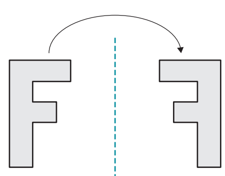
rate of change The amount one quantity (often y) changes when the value of another quantity (often x) increases by 1. The *rate of change* in a linear relationship is also the slope of its graph.

ratio A comparison of two quantities by multiplication or division.

rational numbers The set of all the numbers that can be written as positive or negative fractions.

rectangular prism A polyhedron with two congruent and parallel bases, whose faces are all rectangles.

reflection A transformation that flips each point on a preimage across a *line of reflection* to a point on the opposite side of the line.

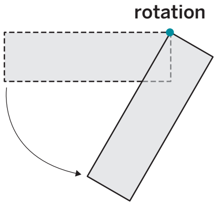


relative frequency The relative frequency is the ratio of the number of times an outcome occurs in a set of data. It can be written as a fraction, a decimal, or a percentage.

repeating decimal A decimal in which there is a sequence of non-zero digits that repeat indefinitely.

rigid transformation A move that does not change any of the measurements of a figure. Translations, rotations, and reflections are all examples of *rigid transformations* (as well as any sequence of these).

rotation A transformation that turns a figure a certain angle (called the *angle of rotation*) about a point (called the *center of rotation*).



Español

radio Segmento de línea que conecta el centro de un círculo con cualquier punto del círculo. El término puede también referirse a la longitud de este segmento.

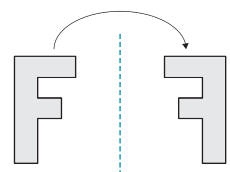
tasa de cambio Monto en que una cantidad (usualmente y) cambia cuando el valor de otra cantidad (usualmente x) aumenta en un factor de 1. La *tasa de cambio* en una relación lineal es también la pendiente de su gráfica.

razón Comparación de dos cantidades a través de una multiplicación o una división.

números racionales Conjunto de todos los números que pueden ser escritos como fracciones positivas o negativas.

prisma rectangular Poliedro con dos bases congruentes y paralelas, cuyas caras son todas rectángulos.

reflexión Transformación que hace girar cada punto de una preimagen a lo largo de una *línea de reflexión* hacia un punto en el lado opuesto de la línea.

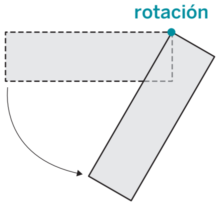


frecuencia relativa La frecuencia relativa es la razón del número de veces que ocurre un resultado en un conjunto de datos. Se puede escribir como una fracción, un decimal o un porcentaje.

número decimal periódico Decimal que tiene una secuencia de dígitos diferentes de cero que se repite de manera indefinida.

transformación rígida Movimiento que no cambia medida alguna de una figura. Traslaciones, rotaciones y reflexiones son ejemplos de *transformaciones rígidas* (como también cualquier secuencia de estas transformaciones).

rotación Transformación que hace girar una figura en cierto ángulo (llamado *ángulo de rotación*) alrededor de un punto (llamado *centro de rotación*).



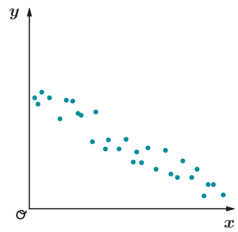
English

S

scale factor The value that side lengths are multiplied by to produce a certain scaled copy.

scaled copy A copy of a figure where every length in the original figure is multiplied by the same value to produce corresponding lengths in the copy.

scatter plot A scatter plot is a graph that shows the values of two variables on a coordinate plane. It allows us to investigate connections between the two variables.

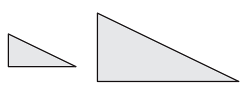


scientific notation A way of writing very large or very small numbers. When a number is written in scientific notation, the first factor is a number greater than or equal to one, but less than ten. The second factor is an integer power of ten. For example, $23000 = 2.3 \times 10^4$ and $0.00023 = 2.3 \times 10^{-4}$.

segmented bar graph A segmented bar graph compares two categories within a data set. The whole bar represents all the data within one category. Then, each bar is separated into parts (segments) that show the percentage of each part in the second category.

sequence of transformations Two or more transformations that are performed in a particular order.

similar Two figures are *similar* if they can be mapped onto each other by a sequence of transformations, including dilations.



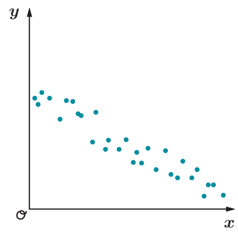
slope The numerical value that represents the ratio of the vertical side length to the horizontal side length in a slope triangle. The rate of change in a linear relationship is also the slope of its graph.

Español

factor de escala Valor por el cual las longitudes de cada lado son multiplicadas para producir una cierta copia a escala.

copia a escala Copia de una figura donde cada longitud de la figura original es multiplicada por el mismo valor, para producir longitudes correspondientes en la copia.

diagrama de dispersión Un diagrama de dispersión es una gráfica que muestra los valores de dos variables en un plano de coordenadas. Nos ayuda a investigar relaciones entre las dos variables.

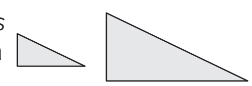


notación científica Manera de escribir números muy grandes o números muy pequeños. Cuando un número es escrito en notación científica, el primer factor es un número mayor o igual a uno, pero menor que diez. El segundo factor es un número entero que es potencia de diez. Por ejemplo, $23000 = 2.3 \times 10^4$ y $0.00023 = 2.3 \times 10^{-4}$.

gráfica de barras segmentada Una gráfica de barras segmentada compara dos categorías dentro de una serie de datos. La barra completa representa la totalidad de los datos dentro de una categoría. Entonces, cada barra es separada en partes (llamadas segmentos) que muestran el porcentaje de cada parte en la segunda categoría.

secuencia de transformaciones Dos o más transformaciones que se llevan a cabo en un orden particular.

similar Dos figuras son *similares* si pueden ser imagen la una de la otra, mediante una secuencia de transformaciones que incluyen las dilataciones.

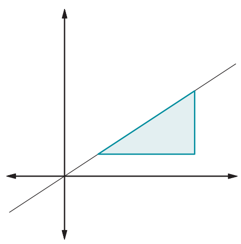


pendiente El valor numérico que representa la razón entre la longitud del lado vertical y la longitud del lado horizontal en un triángulo de pendiente. Dada una línea, todo triángulo de pendiente tiene la misma pendiente.

Glossary/Glosario

English

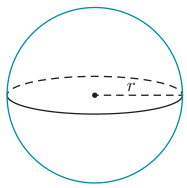
slope triangle A right triangle whose longest side is part of a line, and whose other sides are horizontal and vertical. *Slope triangles* can be used to calculate the slope of a line.



solution A value that makes an equation true.

solution to a system of equations An ordered pair that makes every equation in a system of equations true.

sphere A three-dimensional figure that consists of the set of points, in space, that are the same distance from a given point called the center.



square root The square root of a positive number p is a positive solution to equations of the form $x^2 = p$. Write the square root of p as $\sqrt{p}$.

straight angle An angle that forms a straight line. A straight angle measures 180 degrees.

substitution Replacing an expression with another expression that is known to be equal.

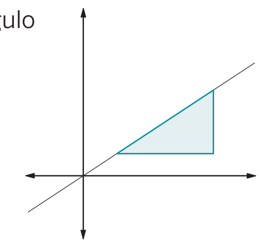
supplementary angles Two angles whose measures add up to 180 degrees.

symmetry When a figure can be transformed in a certain way so that it returns to its original position, it is said to have *symmetry*, or be *symmetric*.

system of equations A set of two equations with two variables. (In a later course, you will see systems with more than two equations and variables.)

Español

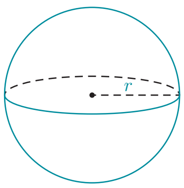
triángulo de pendiente Triángulo rectángulo cuyo lado más largo es parte de una línea, y cuyos otros lados son horizontales y verticales. Los *triángulos de pendiente* pueden ser usados para calcular la pendiente de una línea.



solución Valor que hace verdadera a una ecuación.

solución al sistema de ecuaciones Par ordenado que hace verdadera cada ecuación de un sistema de ecuaciones.

esfera Figura tridimensional que consiste en una serie de puntos en el espacio que están a la misma distancia de un punto específico, llamado centro.



raíz cuadrada La raíz cuadrada de un número positivo p es una solución positiva a las ecuaciones de la forma $x^2 = p$. Escribimos la raíz cuadrada de p como $\sqrt{p}$.

ángulo llano Ángulo que forma una línea recta. Un ángulo llano mide 180 grados.

sustitución Reemplazo de una expresión por otra expresión que se sabe es equivalente.

ángulos suplementarios Dos ángulos cuyas medidas suman 180 grados.

simetría Cuando una figura puede ser transformada de manera tal que regrese a su posición original, se dice que tiene *simetría* o que es *simétrica*.

sistema de ecuaciones Conjunto de dos ecuaciones con dos variables. (En un curso posterior verán sistemas con más de dos ecuaciones y variables.)

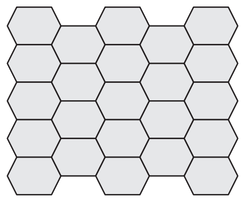
English

T

term An expression with constants or variables that are multiplied or divided.

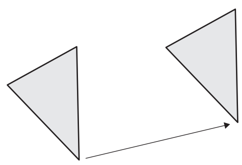
terminating decimal A decimal that ends in 0s.

tessellation A pattern made of repeating shapes that completely covers a plane, without any gaps or overlaps.

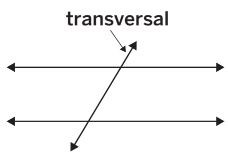


transformation A rule for moving or changing figures on the plane. Transformations include translations, reflections, and rotations.

translation A transformation that slides a figure without turning it. In a *translation*, each point of the figure moves the same distance in the same direction.



transversal A line that intersects two or more other lines.



Triangle Sum Theorem A theorem that states the sum of of the three interior angles of any triangle is 180 degrees.

two-way table A two-way table provides a way to compare two categorical variables. It shows one of the variables across the top and the other down one side. Each entry in the table is the frequency or relative frequency of the category shown by the column and row headings.

U

unit rate How much one quantity changes when the other changes by 1.

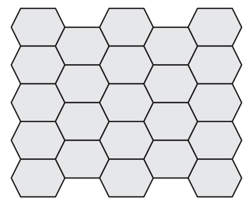
Español

T

término Expresión con constantes o variables que son multiplicadas o divididas.

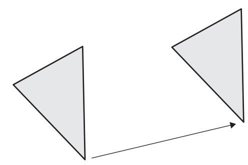
decimal exacto Un decimal que termina en ceros.

teselado Patrón compuesto por formas repetidas que cubren por completo un plano, sin dejar espacios vacíos ni superposiciones.

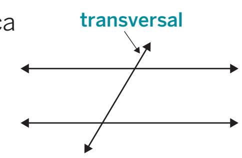


transformación Regla que se aplica al movimiento o al cambio de figuras en el plano. Traslaciones, rotaciones y reflexiones son ejemplos de transformaciones.

traslación Transformación que desliza una figura sin hacerla girar. En una *traslación* cada punto de la figura se mueve la misma distancia en la misma dirección.



transversal Línea que se interseca con dos o más líneas distintas.



Teorema de la suma del triángulo Teorema que afirma que la suma de los tres ángulos interiores de cualquier triángulo es 180 grados.

tabla de dos entradas Una tabla de dos entradas provee una forma de comparar dos variables categóricas. Muestra una de las variables de forma horizontal y la otra de forma vertical. Cada entrada en la tabla es la frecuencia o frecuencia relativa de la categoría mostrada en los encabezados de la columna y la fila.

U

tasa unitaria Cuánto cambia cierta cantidad cuando la otra cambia por un factor de 1.

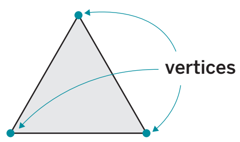
Glossary/Glosario

V

English

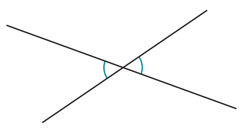
variable A quantity that can take on different values, or that has a single unknown value. Variables are typically represented using letters.

vertex A point where two sides of a two-dimensional shape or two or more edges of a three-dimensional figure intersect. (The plural of *vertex* is *vertices*.)

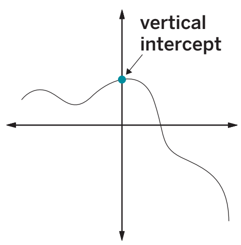


vertical Running straight up or down.

vertical angles Opposite angles that share the same vertex, formed by two intersecting lines. Vertical angles have equal measures.



vertical intercept A point where a graph intersects the vertical axis. Also known as the y-intercept, it is the value of y when x is 0.

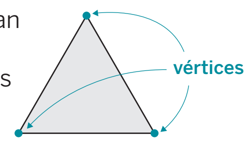


volume The number of unit cubes needed to fill a three-dimensional figure without gaps or overlaps.

Español

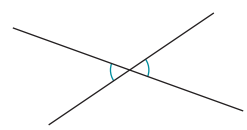
variable Cantidad que puede asumir diferentes valores o que tiene un solo valor desconocido. Las variables usualmente son representadas por letras.

vértice Punto donde se intersecan dos lados de una forma bidimensional, o dos o más aristas de una figura tridimensional.

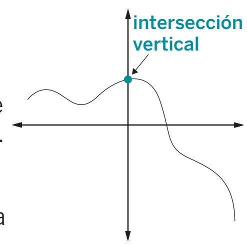


vertical Que corre en línea recta hacia arriba o hacia abajo.

ángulos verticales Ángulos opuestos que comparten el mismo vértice, conformado por dos líneas que se intersecan. Los ángulos verticales tienen las mismas medidas.



intersección vertical Punto en que una gráfica se interseca con el eje vertical. También conocida como intersección y, se trata del valor de y cuando x es 0.



volumen Número de unidades cúbicas necesario para llenar una figura tridimensional sin dejar espacios vacíos ni superposiciones.

X

x-intercept See the definition for *horizontal intercept*.

intersección x Ver *intersección horizontal*.

Y

y-intercept See the definition for *vertical intercept*.

intersección y Ver *intersección vertical*.

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