

Keeping the Balance

Let’s walk some dogs.

Focus	Standards
Goals: - Generalize (orally) that doing the same thing to each side of a dog walker model keeps it balanced. - Explain (orally) how to use a balanced dog walker model to solve an equation of the form $px + q = r$. Coherence Today: Students learn how a dog walker model can be used to illustrate balance and the concept of equality (MP2). This understanding is essential in solving for unknown values in equations. Previously: In Grade 6, students wrote and solved one-step equations to represent word problems. Coming Soon: In the next few lessons, students will work with hanger diagrams to expand their toolbox of strategies for solving multi-step equations.	Addressing: 7.EE.B.4 Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities. Building On: 7.EE.A.1 7.EE.A.2 7.EE.B.3 Building Toward: 7.EE.B.4.A 7.EE.B.4.B

Warm-up	Activity 1	Activity 2	Activity 3	Summary	Exit Ticket
5 min	10 min	10 min	10 min	5 min	5 min
Pairs	Pairs	Pairs	Pairs	Whole Class	Independent
7.EE.B.4	7.EE.B.4	7.EE.B.4	7.EE.B.4	7.EE.B.4	7.EE.B.4
MP2			MP2		
Desmos Activity and Presentation Slides					
Slide(s) X-X	Slide(s) X-X	Slide(s) X-X	Slide(s) X-X	Slide(s) X-X	Slide(s) X-X

Vocabulary

New words

- Addition property of equality
- Division property of equality
- Multiplication property of equality
- Subtraction property of equality

Review words

- Equation

Materials

- Exit Ticket PDF
 - Additional Practice Book
- * indicates materials not provided

Desmos Digital Classroom



Digital Activity Slide 2

Dynamic Dog Leashes

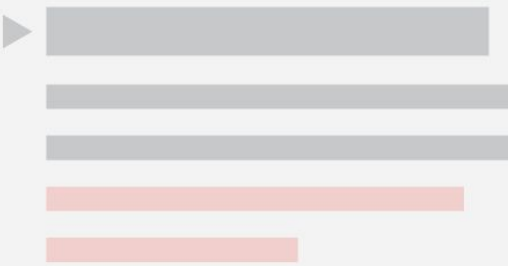
Students can manipulate the position of the dogs on either side of the leashes while trying to balance them.

Warm-up: Keeping the Balance

Students look at two pictures representing the dog walker model to gain a sense of balanced and unbalanced situations.

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SE Activity Inset Page



Launch

Have students work in pairs. Suggest that students share their thinking with their partner before writing an answer.

Monitor

Help students get started by saying “Tell me about what you see happening in the first picture.”

Look for points of confusion:

- **Thinking the size of the dog matters.** “Can a smaller dog be stronger than a larger dog?”
- **Not noticing that the top picture represents imbalance.** “What does it mean if the walker is being pulled in one direction?”
- **Saying Champ is the strongest because he is pulling Bobby and Ace.** “Bobby is pulling the same direction as Champ. Does that work for or against Champ?”

Connect

Have students share their observations about the diagrams.

Highlight that the picture on the bottom represents a balanced dog walker. Balance and equality will be important themes throughout this unit.

Ask:

- “What is an example of something in your own life that you feel you must ‘balance’?” *[Answers will vary, but students may refer to balancing responsibilities, or friendships.]*
- “How do we represent balance in math?” (**MP2**) *[Using an equal sign.]*

Alternate Warm-up Activity	Differentiated Support	
N/A	English Language Learners: N/A	Students with Disabilities: N/A

Activity 1: Walking Dogs like a Pro

Students balance dogs of varying strengths on opposite sides of the dog walker, connecting this model with balance in equations.

*Do not delete. Design to replace.

SE Activity Inset Page



Launch

Give students a few minutes to work independently, and then have them compare their work with a partner. Explain that this activity lends itself well to trial and error. Suggest students begin by using rough-draft thinking.

Monitor

Help students get started by suggesting they write the strengths of the dogs on small pieces of paper and manipulate them by hand.

Look for points of confusion:

- **Not using all five dogs.** "The questions asks us to balance all five dogs."
- **Finding a combination that is not precisely balanced.** "If this dog walker were on roller skates, what would happen if even one side was a little stronger than the other?"

Connect

Have students share where they placed the dogs. Chart this so the class can see. Be sure to include a student who reversed the sides of the dogs, which is equally valid

Display the equality statement that represents each student-shared answer.

Highlight that "This is also true about equations, if we think of the dog walker as our equal sign. Both sides must be balanced in order for it to be an equation. The equal sign tells us it is balanced."

Differentiated Support

Students Who Need Help N/A	Students Ready for More Complete the Extension problems.	English Language Learners <i>MLR3 Clarify, Critique, Correct.</i> Present an incorrectly balanced set of dogs. Prompt students to identify the error, and then fix it. This helps students evaluate, and improve on, the written mathematical arguments of others.	Students with Disabilities N/A
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Activity 2: Learning About Your New Customers

Students find the strength of an unknown dog in a balanced dog walker model to reintroduce the concept of variables in equations.

*Do not delete. Design to replace.

SE Activity Inset Page

Launch

Say, "Let's read this scenario together first, and then you will continue to work with a partner on the task."

Monitor

Help students get started by asking, "What is the strength of the dogs on the left? What do we know must be true about the dogs on the right?"

Look for points of confusion:

- Thinking each new dog has a strength of 5: "Try adding the values on the right side and make sure they are balanced with the values on the left."

Look for productive strategies:

- Using trial and error to find missing values. Note this as strategy 1.
- Using counting on from 9 to find that the missing combined value is 5. Note this as strategy 2.
- Subtracts 9 from 14 to find the difference that accounts for the unknown combined value. Note this as strategy 3.

Connect

Have students share strategies. Depending on the strategy most students use, you may choose to share either Strategy 2 or Strategy 3 from above. Strategy 3 is ideal because it will prepare students for the work later on in the unit.

Ask, "What equation could we write to match this diagram?" [$6 + 8 = 9 + x + x$]

Highlight the connection between the dog walker scenario and equations.

Differentiated Support

Students Who Need Help Remove one of the unknown dogs for students to find the value of. Then, reintroduce it and ask if they know what each dog's strength must be.	Students Ready For More Complete the Extension problems.	English Language Learners N/A	Students with Disabilities N/A
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Activity 3: Inventing Your Own Terminology

Students engage with new vocabulary by using their own language to create new names for the properties of equality. This will help make these concepts “stickier.”

*Do not delete. Design to replace.

SE Activity Inset Page



Launch

Do the first one together, as a whole class. Give students 1 minute to brainstorm a new name for the first arrangement. Encourage them to use whatever wording they'd like. Suggest "Add the same thing to both sides" as an example for the first.

Monitor

Help students get started by suggesting they give the dogs names so they can more easily recognize their positioning.

Look for points of confusion:

- **Tries using words from the "Old Name":** Cover up the old name. Have students describe the picture in their own words.
- **Thinks new names need to be short:** "Don't be afraid to have multiple drafts of your new names. Shorter is not necessarily better."

Connect

Display the pictures on a poster, showing both the "old name" and a "new name" for each property.

Highlight "rough-draft thinking" as a part of the mathematical reasoning process. Say, "We can think of addition and subtraction as similar properties, and multiplication and division as similar properties."

Define:

- *Addition property of equality*
- *Division property of equality*
- *Multiplication property of equality*
- *Subtraction property of equality*

Differentiated Support

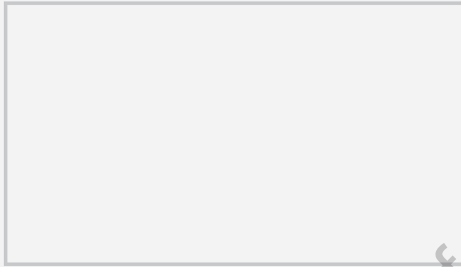
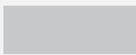
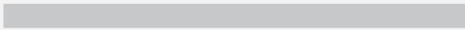
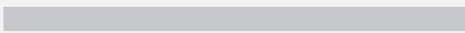
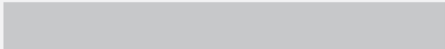
Students Who Need Help N/A	Students Ready for More Challenge students to come up with names that use alliteration or are especially catchy.	English Language Learners N/A	Students with Disabilities N/A
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Summary

Review and synthesize the big ideas of the lesson, that dog walkers (like equations) prefer to stay in balance.

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SE Summary Inset Page



Synthesize

Display the Summary BLM page with two dog walker diagrams.

Highlight that these diagrams illustrate the subtraction property of equality. Discuss how else we could add or remove dogs to demonstrate this property, and other properties.

Review vocabulary:

- Addition property of equality
- Division property of equality
- Multiplication property of equality
- Subtraction property of equality

Exit Ticket

Students demonstrate their understanding by transferring their understanding of the properties of equality to equations.

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Exit Ticket Inset PDF



Success looks like...

1. Generalize (orally) that doing the same thing to each side of a dog walker model keeps it balanced.
2. Explain (orally) how to use a balanced dog walker model to solve an equation of the form $px + q = r$.

Suggested next steps

If students struggle to identify which arrangements work, consider:

- Having them find the total strength on each side, and compare these strengths to determine if the dog walker is balanced.
- Assigning Practice Problem 2.

If students struggle to identify which property of equality applies to Problems 1 and 3, consider:

- Taking another look at the examples in Activity 3, and seeing if they can use them to identify the properties shown in the Exit Ticket.

Practice

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SE Practice Inset Page

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SE Practice Inset Page



Practice Question Analysis

Question	Refer to	Standards	DOK
1	Activity 1	7.EE.B.4	2
2	Activity 2	7.EE.B.4	1
3	Activity 2	7.EE.B.4	3
4 (spiral)	6th grade	6.EE.B.7	2
5 (spiral)	6th grade	6.EE.B.7	1
6 (formative)	Unit 6 Lesson 2	7.NS.A	2

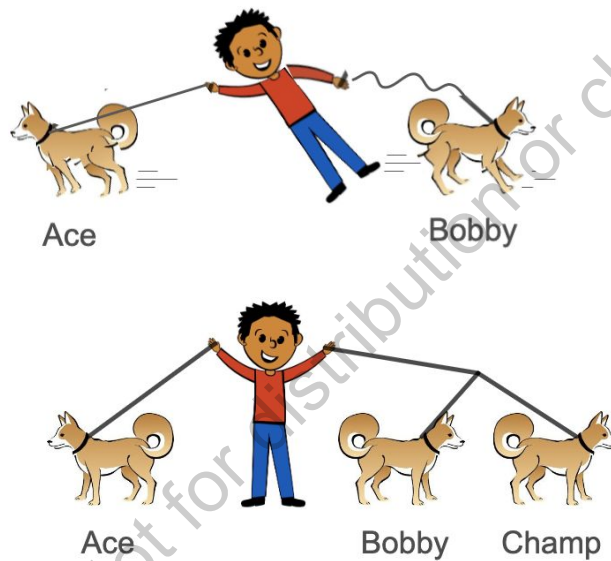
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Keeping the Balance

Let's walk some dogs.

Warm-up: Notice and Wonder

Look at the pictures below.



1. What do you notice?
2. What do you wonder?
3. What could be true about the dogs?
4. What cannot be true?

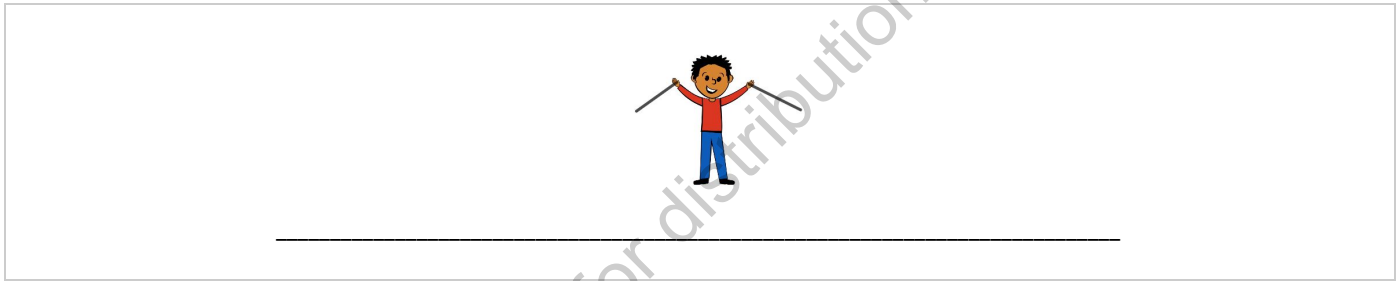
Activity 1: Walking Dogs like a Pro

Welcome to Pawston University! We've produced some of the nation's finest dog walkers. Your first challenge is to balance some leashes.

How can you balance all five dogs on your leashes?



Write the names and strengths of the dogs you would put on the left side and the right side.



Extension

Directions: Which equations are true? Explain your thinking.

1. $2 + 7 + 9 = 3(4 + 2)$

2. $10.5 - 8 + 0.5 = 11 - 3 \cdot 3$

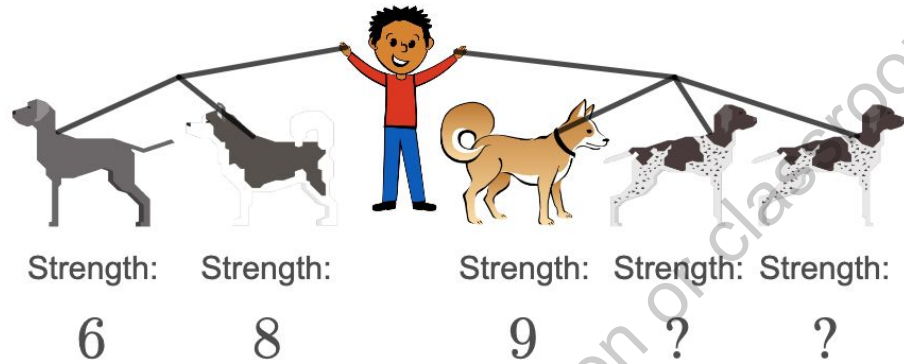
3. $\frac{3}{5} \cdot \frac{2}{9} = \frac{1}{6} \div \frac{5}{4}$

4. $9 - 15 - 7 = -5(3) - 2$

Activity 2: Two New Customers

Congratulations! You've learned how to balance your leashes. But what happens when you don't know the strength of a dog?

Let's figure out the strength of the new dogs in the diagram, whose strengths have not been labeled. Assume that the dog walker feels an equal pull in both directions, and that dogs who look the same have the same strength.



What is the strength of the new dogs? Explain your thinking.

Extension

For each equation, fill in the boxes with a single value that makes each equation true.

1. $9 + 14 = 7 + 2 \cdot \square$ 8

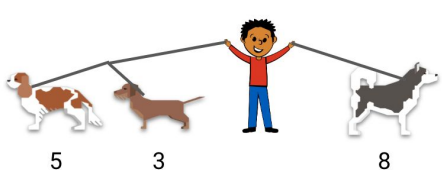
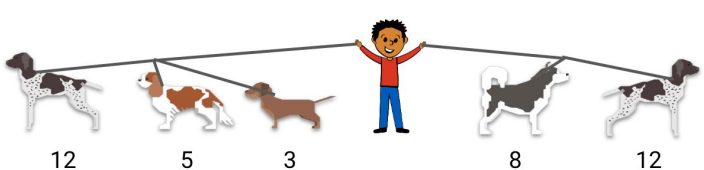
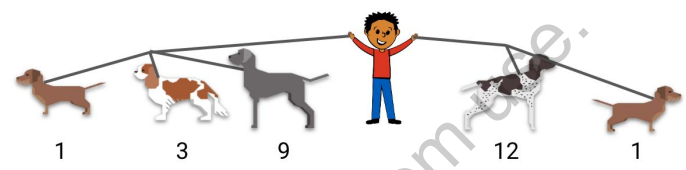
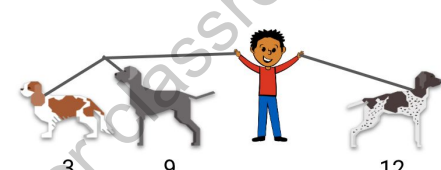
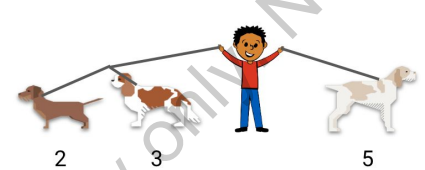
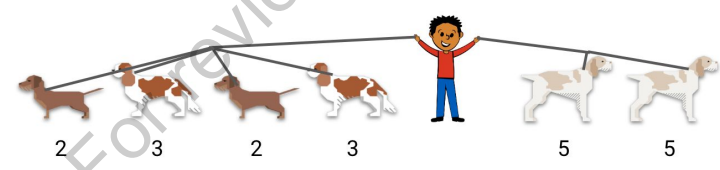
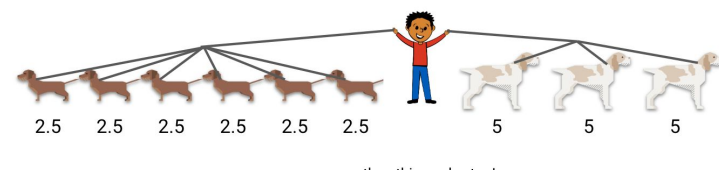
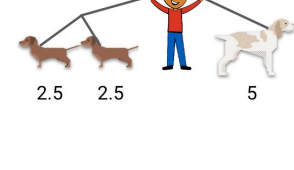
2. $5(\square + 8) = 9 \cdot 7 - 3$ 4

3. $4 + \square = 3 \cdot \square$ 2

Activity 3: Inventing Your Own Terminology

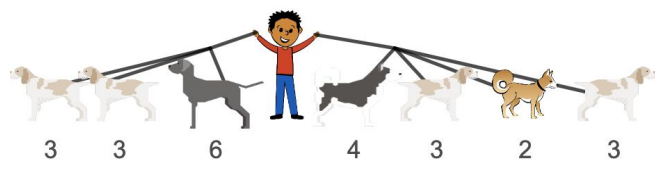
As leaders in our field, Pawston University is constantly innovating new techniques in the field of dog walking.

We need your help in renaming some advanced dog-walking techniques.

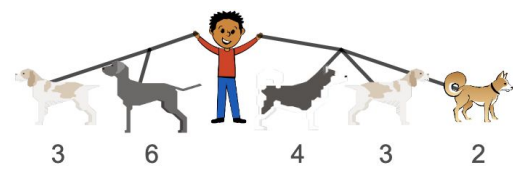
If this works...  5 3 8 ...then this works, too!  12 5 3 8 12	If this works...  1 3 9 12 1 ...then this works, too!  3 9 12
Old name: Addition property of equality New name:	Old name: Subtraction property of equality New name:
If this works...  2 3 5 ...then this works, too!  2 3 2 3 5 5	If this works...  2.5 2.5 2.5 2.5 2.5 2.5 5 5 5 ...then this works, too!  2.5 2.5 5
Old name: Multiplication property of equality New name:	Old name: Division property of equality New name:

Exit Ticket: Keeping the Balance

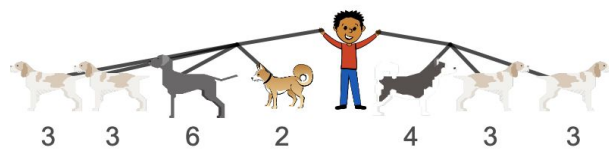
If this works, then which other arrangements also work? Explain how you know using the properties of equality.



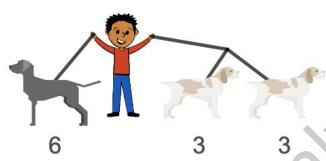
1.



2.



3.



Self-Assess

- A. I can recognize when a dog walking diagram is balanced.
- B. I can find the value of an unknown in a dog walking diagram by using the concept of balance.
- C. I can identify the properties of equality that can be used to maintain the balance of a dog walking diagram.

Key: (No content needed here, already in the design template)

- 1. I'm lost
- 2. I don't really get it

3. I'm starting to get it
4. I got it
5. I could teach it

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Summary

OK, let's be real. When you see someone walking their dog, chances are you're not thinking, "Oh, what an adorable math problem!"

You're thinking, "Look at the *pwetty* puppies!"

...or "Who's a good boy?"

...or "Someone better clean that up."

But believe it or not, that dog is made up of numbers. It's inside how that dog walks down the street, how it wags its tail, how it chows down on its kibble and chases squirrels.

In fact, everything we can observe can be expressed in the language of mathematics.

Like Mandarin and English, and Hindustani, we can use math to describe how things are and how they work. Where spoken languages use words, math uses numbers, symbols, operators and diagrams.

From getting tugged by a labradoodle to building the pyramids, all the stories of the human experience can be told in the language of mathematics.

Welcome to Unit 6

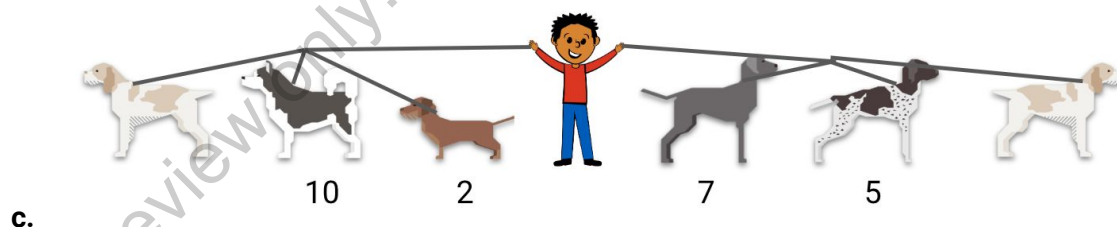
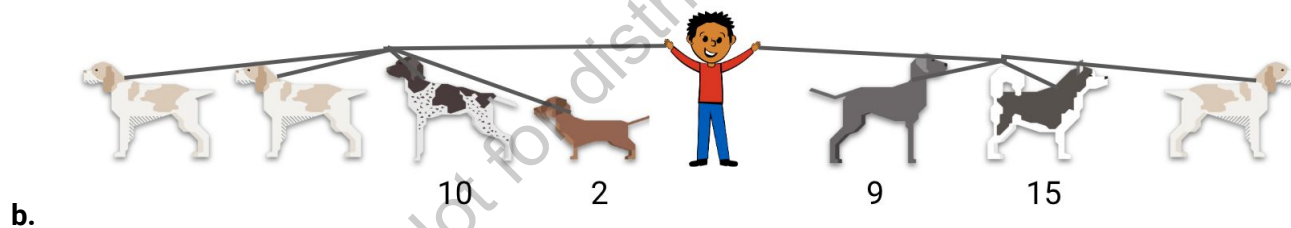
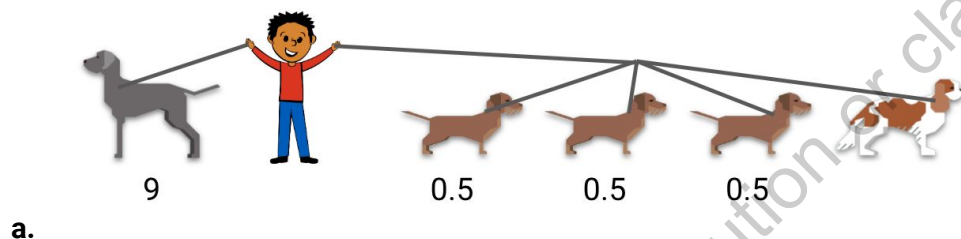
My Notes

Practice

- Using each of the numbers from 0 to 9 at most once, find as many different ways to make the equation below true as you can.

$$\square + \square = \square + \square + \square$$

- Each diagram has some dogs with unknown strengths. Find their strengths.



- Create a list of four dogs, including their strengths, that could not be balanced in any way on opposite sides of the dog walker.

Practice (continued)

4. Which question **cannot** be answered by the solution to the equation $3x = 27$?
- A. Elena read three times as many pages as Noah. She read 27 pages. How many pages did Noah read?
 - B. Lin has 27 stickers. She gives 3 stickers to each of her friends. With how many friends did Lin share her stickers?
 - C. Diego paid \$27 for a concert ticket. What will be the cost of 3 such tickets?
 - D. The coach splits a team of 27 students into 3 groups to practice skills. How many students are in each group?

5. Solve each equation.

a. $8.5 \cdot (-3) = a$

b. $-7 + b = -11$

c. $c - (-3) = 15$

d. $d \cdot (-4) = 32$

6. Circle the true statements.

A. $9 - 5 = -2 \cdot (-2)$

B. $-10 \div \left(-\frac{1}{10}\right) = 1$

C. $0.5 - 1 + 2.5 = 2$

D. $-\frac{2}{3} > 2 - 3$

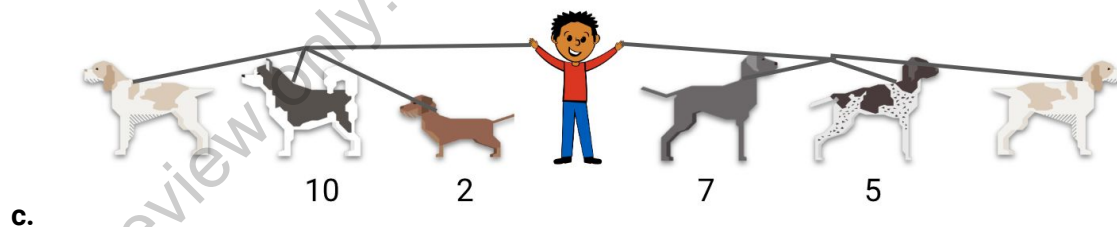
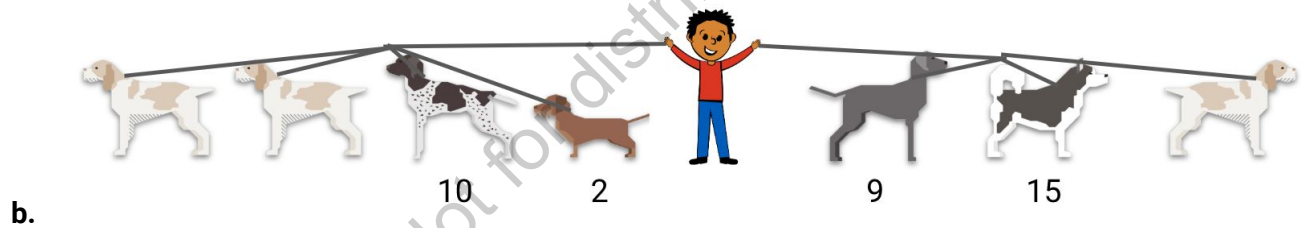
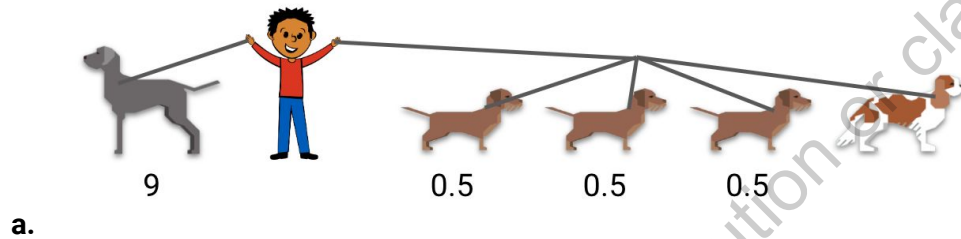
E. $7 + 7 < -14$

Practice

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Practice (continued)

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C. $0.5 - 1 + 2.5 = 2$

D. $-\frac{2}{3} > 2 - 3$

E. $7 + 7 < -14$

Balanced and Unbalanced

Let’s see how hanger diagrams can represent balanced relationships.

Focus	Standards
Goal(s): - Generalize (orally) that doing the same thing to both sides of a hanger diagram keeps it balanced. - Find a missing weight on a hanger diagram. Coherence Today: Students find unknown values on balanced hanger diagrams that model two-step equations. They use the properties of equality to manipulate the diagrams, while ensuring they remain balanced. Previously: In Lesson 1, students used a dog walker model to review the properties of equality. Coming Soon: In Lesson 3, students will use hanger diagrams to solve two-step equations specifically of the form $px + q = r$.	Addressing: 7.EE.B.4 Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities. Building On: 6.EE.B.5 6.EE.B.6 6.EE.B.7 Building Toward: 7.EE.B.4.A 7.EE.B.4.B

Warm-up	Activity 1	Activity 2	Activity 3	Summary	Exit Ticket
5 min	10 min	10 min	10 min	5 min	5 min
<i>Independent</i>	<i>Pairs</i>	<i>Pairs</i>	<i>Pairs</i>	<i>Whole Class</i>	<i>Independent</i>
7.EE.B.4	7.EE.B.4	7.EE.B.4	7.EE.B.4	7.EE.B.4	7.EE.B.4
		MP2			
Desmos Activity and Presentation Slides					
Slide(s) X-X	Slide(s) X-X	Slide(s) X-X	Slide(s) X-X	Slide(s) X-X	Slide(s) X-X

Vocabulary

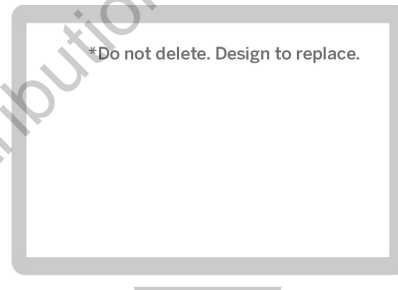
Review words

- Equation
- Hanger diagram
- Addition property of equality
- Division property of equality
- Multiplication property of equality
- Subtraction property of equality

Materials

- Exit Ticket PDF
 - Additional Practice Book
 - Lesson 2 Summary BLM
- * indicates materials not provided

Desmos Digital Classroom



Activities 2 and 3: Dynamic Hanger Diagrams

When students remove weights from a balanced hanger diagram, the hanger will animate, giving real-time feedback that shows whether the hanger is balanced.

3

*Do not delete. Design to replace.

SE Activity Inset Page

Diagram illustrating a list of six horizontal bars. The first three bars are grey, and the last three bars are red. A grey triangle points to the first bar.

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Highlight that carrying poles are another representation of balance. When the weights on both sides are equal, the pole is balanced. When one side is heavier than the other, the pole is unbalanced.

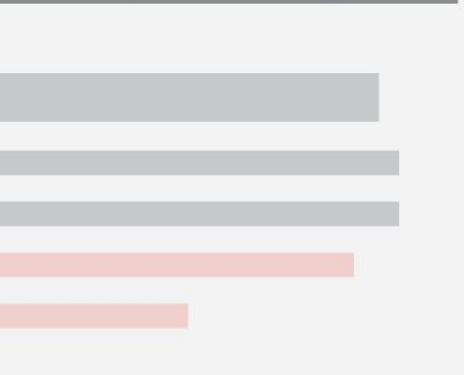
Grade 7 | Lesson 2: Balanced and Unbalanced 3

Activity 1: Hanging Out

Students investigate relationships shown in hanger diagrams, to prepare for solving for missing weights on a hanger diagram.

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SE Activity Inset Page



The diagram shows two hanger diagrams. The top hanger has a gray bar on the left and a red bar on the right. The bottom hanger has a gray bar on the left and a red bar on the right. The red bars are longer than the gray bars, indicating they are heavier.

Launch

Give students 2 minutes to consider the diagrams and complete the problems. Then have them share their thinking with a partner.

Monitor

Help students get started by asking, “Which diagram is balanced? Which diagram is not balanced? What does this tell you about the weights of the shapes?” Also consider discussing the meaning of “must” (always, for any weights), “could” (sometimes), and “cannot” (there is no example) in this context.

Look for productive strategies:

- Comparing the weights of the shapes (e.g., saying that a triangle is heavier than a square.)

Connect

Display the two hanger diagrams.

Have students share what “must”, “could”, and “cannot” be true about the shapes and the diagrams. (**MP2**)

Highlight how students justify their conclusions about the diagrams.

Ask, “How do you know that must/could/cannot be true?”

Differentiated Support

Students Who Need Help N/A	Students Ready for More Have them assign possible weights to the triangle, square, and circle, based on the relationships shown in the diagram.	English Language Learners N/A	Students with Disabilities N/A
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Activity 2: Manipulating a Hanger Diagram (Part 1)

Students manipulate a balanced hanger diagram that models an equation of the form $px + q = r$ to determine an unknown weight.

*Do not delete. Design to replace.

SE Activity Inset Page



Launch

Have students work in pairs, and suggest they mark up the hanger diagram (or sketch new ones) when considering Problems 1 and 2.

Monitor

Help students get started by asking, “If we remove one triangle from both sides, will the hanger stay balanced? What if we remove one triangle from one side and two triangles from the other side?”

Look for points of confusion:

- **Removing unequal amounts of weight from each side.** Remind students that they must do the same thing to both sides in order to keep the hanger balanced.

Look for productive strategies:

- Marking up the diagram or drawing a new one to show the weights being removed.
- Labeling the triangles with 4 (Problem 2).
- Writing and solving the equation $4x = 8$ to find the weight of a square.

Connect

Display the hanger diagram shown in the problem.

Have students share how they removed weights. Ask a student to explain how they found the weight of a square, and see if anyone else used a different strategy.

Highlight the connection between Problem 1 and the properties of equality. For example, removing two triangles from each side is allowed by the subtraction property of equality, while the division property of equality allows us to remove half the shapes (for each type of shape) from each side.

Ask, “How could we use the addition and multiplication properties of equality to create new balanced hanger diagrams?” *[Add the same number of triangles or squares to both sides. Double or triple the amount of shapes on both sides.]*

Differentiated Support

Students Who Need Help	Students Ready For More	English Language Learners	Students with Disabilities
Have students only find one answer for Problem 1 instead of “as many as possible.”	Complete the Extension problem.	N/A	N/A

Activity 3: Manipulating a Hanger Diagram (Part 2)

Students manipulate a balanced hanger diagram that models an equation of the form $p(x + q) = r$ to determine an unknown weight.

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SE Activity Inset Page



Launch

Discuss that the arrangement of shapes on this hanger diagram is slightly different from the one in Activity 2, and note that the weight on each side doesn't depend on the relative arrangement of the shapes.

Monitor

Help students get started by suggesting they consider which moves from Activity 2 will also work with this hanger diagram.

Look for productive strategies:

- Removing three squares on each side, to see that three circles equal six squares.
- Splitting both sides into three equal groups, and then removing all but one group.

Connect

Display the hanger diagram shown in the problem.

Have students share their strategies for removing weights. Try to find examples of students using combinations of moves.

Highlight two distinct strategies for removing weights: 1) removing three squares from each side and then dividing the result into three groups; 2) dividing both sides into three equal groups, and then removing one square from each group. Discuss the properties of equality that prove these moves keep the hanger balanced. Note that both strategies result in seeing that one circle weighs the same as two squares, and use that to answer Problem 2.

Differentiated Support

Students Who Need Help $\frac{1}{2}$ Replace “$\frac{1}{2}$” with 1 in Problem 2.	Students Ready for More Complete the Extension problem.	English Language Learners N/A	Students with Disabilities N/A
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Summary

Review and synthesize how we can use the properties of equality to manipulate hanger diagrams.

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SE Summary Inset Page

Synthesize

Display the Lesson 2 Summary BLM.

Ask students to compare the two hanger diagrams with a partner. “What is the same?” *[Both diagrams show that eight squares weigh the same as four squares and two triangles.]* “What is different?” *[The arrangement of the shapes on the right.]* Then have them figure out how many squares equal the weight of a triangle.

Have students share their thinking with a partner.

Highlight that the arrangement of Diagram A makes it easier to remove four squares from each side (subtraction property of equality) and then divide both sides into two groups (division property of equality), to see that one triangle equals one-and-a-half squares. Meanwhile, the arrangement of Diagram B makes it easier to start by dividing both sides into two groups (division property of equality) and then remove two squares from each side (subtraction property of equality), to see that one triangle equals one-and-a-half squares. Note that both strategies are valid and result in the same equivalency.

Exit Ticket

Students demonstrate their understanding by reasoning about the relationships shown in a hanger diagram.

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Exit Ticket Inset PDF



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Success looks like...

1. Generalize (orally) that doing the same thing to both sides of a hanger diagram keeps it balanced.
2. Find a missing weight on a hanger diagram.

Suggested next steps

If students give a different weight for the square than the circle, consider:

- Reviewing how they reasoned about removing weights in the hanger diagram to arrive at that conclusion.
- Assigning Practice Problems 1 and 2.

If students give only one possible weight for the square and circle, consider:

- Asking if this is the only possible weight for these shapes. If they say it is, demonstrate why, as long as the square and circle have the same weight, the diagram will be balanced.
- Assigning Practice Problem 3.

Practice

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SE Practice Inset Page

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SE Practice Inset Page



Practice Problem Analysis

Problems	Refer to	Standards	DOK
1	Activity 2	7.EE.B.4	1
2	Activity 2	7.EE.B.4	2
3	Activity 2	7.EE.B.4	2
4 (<i>spiral</i>)	Unit 4	7.RP.A.1	2
5 (<i>formative</i>)	Unit 6 Lesson 3	6.EE.B.7	1

(Space to show Additional Practice)

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Balanced and Unbalanced

Let's see how hanger diagrams can represent balanced relationships.

Warm-up: Carry On

Since early civilization, humans have used carrying poles to transport heavy loads over long distances.



Use the following diagram to answer the questions.

Diagram A

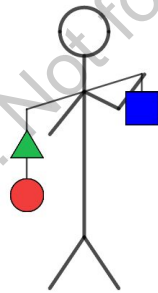
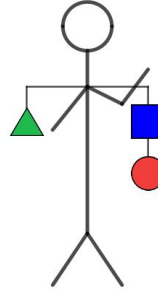


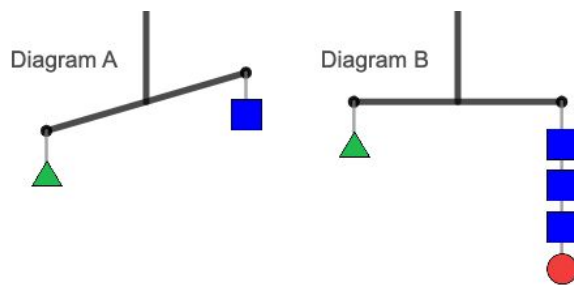
Diagram B



1. What do Diagrams A and B represent?
2. If you had to carry a heavy load, would you rather carry it as shown in Diagram A or Diagram B? Explain your thinking.

Activity 1: Hanging out

In the two hanger diagrams, all the triangles weigh the same as each other, and all the squares weigh the same as each other.

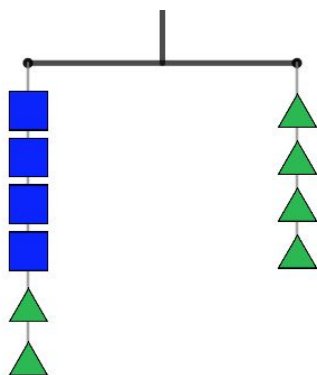


Based on the diagrams, what is...

1. One thing that *must* be true?
2. One thing that *could* be true?
3. One thing that *cannot* possibly be true?

Activity 2: Manipulating a Hanger Diagram (Part 1)

This picture represents a hanger diagram that is balanced because the weight on both sides is the same.



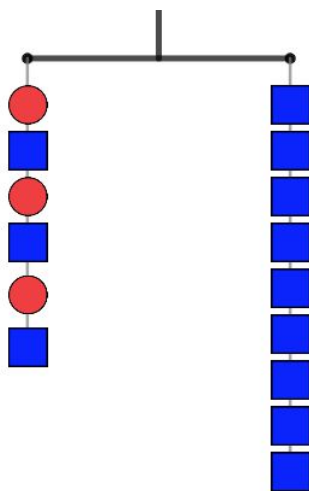
1. Which weights can be removed so that the hanger would still be balanced? Find as many answers as possible.
2. If a triangle weighs 4 grams, how much does a square weigh? Explain your thinking.

Extension

If the weight of a square is x grams and the weight of a triangle is 4 grams, what equation would represent this hanger diagram?

Activity 3: Manipulating a Hanger Diagram (Part 2)

This picture represents a hanger diagram that is balanced, since the weight on both sides is the same.



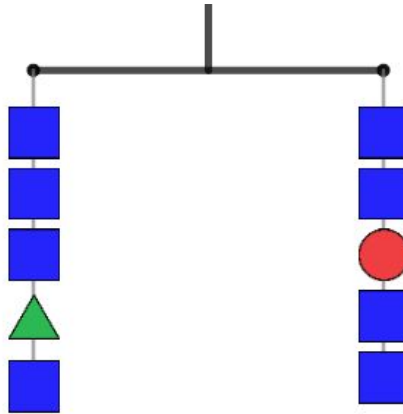
1. Which weights can be removed so that the hanger diagram would still be balanced? Find as many answers as possible.
2. If a square weighs $\frac{1}{2}$ lb, how much does a circle weigh? Explain your thinking.

Extension

If the weight of a circle is x lb and the weight of a square is $\frac{1}{2}$ lb, what equation would represent this hanger diagram?

Exit Ticket

The weight of the square is 2.8 grams. What could the weights of the triangle and the circle be? Explain your thinking.



Self-Assess

- I can find the value of a missing weight on a hanger diagram.
- I can manipulate a hanger diagram according to the properties of equality to produce a new balanced hanger diagram.

Key: (No content needed here, already in the design template)

- I'm lost
- I don't really get it
- I'm starting to get it
- I got it
- I could teach it

Summary

Today, we used hanger diagrams to represent balanced relationships. We saw that we can use the properties of equality to reason with and manipulate hanger diagrams to find the weight of different shapes on the diagram.

My Notes:

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Practice

- Look at the following hanger diagrams, and determine what was done to create the second hanger diagram from the first. Then, name the property (or properties) of equality that tell us if the first hanger is balanced, then so is the second hanger.

First hanger	Second hanger	What was done?	What property?
a.			
b.			
c.			

- Each hanger diagram from Problem 1 is balanced. Determine the weight of each lettered shape and explain how you know. You may draw on the diagrams if it will help.

a. $z = \underline{\hspace{2cm}}$

b. $w = \underline{\hspace{2cm}}$

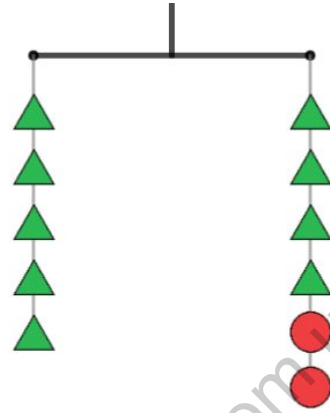
c. $y = \underline{\hspace{2cm}}$

Practice (continued)

3. Look at the balanced hanger diagram.

Find the weight of a circle if each triangle weighs:

- a. 2 pounds
- b. 1 pound
- c. 0.5 pounds



4. A car is traveling at a constant speed. Find the number of miles the car would travel in 1 hour at each given rate.

a. 135 miles in 3 hours

b. 22 miles in $\frac{1}{2}$ hour

c. 7.5 miles in $\frac{1}{4}$ hour

d. $\frac{100}{3}$ miles in $\frac{2}{3}$ hour

5. Solve each equation. Show your work or explain your thinking.

a. $21 = x + 9$

b. $x - 7.5 = 18.5$

c. $3x = 57$

d. $15 = \frac{5}{8}x$

Practice

1. Look at the following hanger diagrams, and determine what was done to create the second hanger diagram from the first. Then, name the property (or properties) of equality that tell us if the first hanger is balanced, then so is the second hanger.

First hanger	Second hanger	What was done?	What property?
a.			
b.			
c.			

2. Each hanger diagram from Problem 1 is balanced. Determine the weight of each lettered shape and explain how you know. You may draw on the diagrams if it will help.

a. $z = \underline{\hspace{2cm}}$

b. $w = \underline{\hspace{2cm}}$

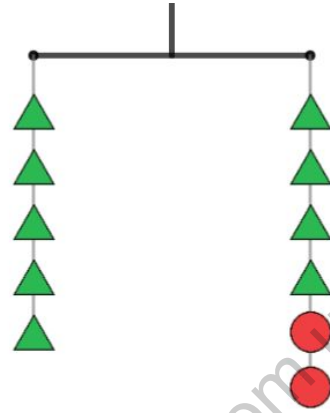
c. $y = \underline{\hspace{2cm}}$

Practice (continued)

3. Look at the balanced hanger diagram.

Find the weight of a circle if each triangle weighs:

- a. 2 pounds
- b. 1 pound
- c. 0.5 pounds



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a. $21 = x + 9$

b. $x - 7.5 = 18.5$

c. $3x = 57$

d. $15 = \frac{5}{8}x$

Reasoning about Solving Equations (Part 1)

Let’s see how a balanced hanger diagram is like an equation and how moving its weights is like solving an equation.

Focus	Standards
Goals: - Interpret a balanced hanger diagram, and write an equation of the form $px + q = r$ to represent the relationship shown. - Explain (orally and in writing) how to solve an equation of the form $px + q = r$.	Addressing: 7.EE.B.4 Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities. 7.EE.B.4.a Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. Building On: 6.EE.B.5 6.EE.B.6 6.EE.B.7 Building Toward: 7.EE.B.4.A
Coherence Today: Students connect hanger diagrams and two-step equations of the form $px + q = r$. Students match hanger diagrams with corresponding equations, and use them to reason about solutions. Previously: In Grade 6, students wrote and solved one-step equations. Students review finding solutions to one-step equations during the warm-up. Coming Soon: In Lesson 4 students will continue working with hanger diagrams to understand and solve different two-step equations in the form $p(x + q) = r$.	

Warm-up	Activity 1	Activity 2	Activity 3	Activity 4	Summary	Exit Ticket
7 min	15 min	13 min	N/A	N/A	5 min	5 min
<i>Independent</i>	<i>Pairs</i>	<i>Pairs</i>	N/A	N/A	<i>Whole Class</i>	<i>Independent</i>
6.EE.B.5, 6.EE.B.7	7.EE.B.4, 7.EE.B.4.A	7.EE.B.4.A	N/A	N/A	7.EE.B.4.A	7.EE.B.4.A
Desmos Activity and Presentation Slides						
Slide(s) X-X	Slide(s) X-X	Slide(s) X-X	N/A	N/A	Slide(s) X-X	Slide(s) X-X

Vocabulary

Review words

- Constant
- Coefficient
- Equation
- Hanger diagram
- Properties of equality
- Solution to an equation
- Variable

Materials

- Exit Ticket PDF
 - Additional Practice Book
 - Summary BLM
- * indicates materials not provided

Desmos Digital Classroom

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Warm-up

Dynamic Hanger Diagrams

When students enter a weight for a variable in a hanger diagram, the hanger will animate, giving real-time feedback that shows whether the hanger is balanced.

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Activity 1

Digital Card Sort

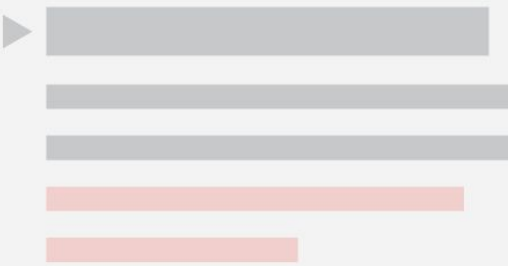
Students match hanger diagrams with their corresponding equations by dragging and connecting them on screen.

Warm-up: Hanger Diagrams and Equations

Students solve two pairs of one-step equations using hanger diagrams in preparation for solving two-step equations using hanger diagrams.

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SE Activity Inset Page



Launch

Explain that we can write equations to represent hanger diagrams. Suggest that while students answer the questions they consider *how* the equations represent the hanger diagrams.

Monitor

Help students get started by asking “How can we make changes to the hangers while still keeping them balanced? Which values of the variables will make each hanger balanced and each equation true?”

Look for productive strategies:

- **Using guess and check to solve the diagrams.** Review how to manipulate a hanger diagram to find the solution.
- **Using guess and check to solve the equations.** Review solving one-step equations using inverse operations.

Connect

Have students share how they solved each diagram, and how they solved each equation.

Highlight how the expressions $2y$ and $x + 2$ are represented in the diagrams, and remind students that $2y = 2 \cdot y = y + y$. Discuss why we would subtract to solve an addition equation and divide to solve a multiplication equation, illustrating using diagrams.

Ask “How is finding the missing weight on the diagram similar to solving the equation?”

Alternate Warm-up Activity	Differentiated Support	
For students who need additional support with solving one-step equations (from Lesson 2 Practice Problem 6): Have a discussion after students answer the first problem. Ask students to explain how they used the diagrams to find the values of the variables. Then show how those moves correspond to how we solve an equation. (ex. For the first problem, if a student says “I removed two weights on each side and saw that x weighs 4.”, show them that removing two weights on both sides is the same as subtracting two from both sides of the equation.)	English Language Learners: N/A	Students with Disabilities: Help struggling students explain their thinking by providing the following sentence prompts: • To find the value of x, first I... • To find the value of y, first I...

Activity 1: Matching Hanger Diagrams and Equations

Students match equations to diagrams, analyzing the relationship between each pair to find the unknown values.

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SE Activity Inset Page



Launch

Explain that each equation has a box where the variable will go. Say that on the diagrams, each shape labeled with a letter has an unknown weight, and shapes labeled with the same letter have the same weight.

Monitor

Look for points of confusion:

- **Confusing which parts of the diagram model addition or multiplication.** Refer to equations and diagrams from the Warm-up.
- **Being unsure how to work with the diagram pieces labeled with numbers other than 1.** Demonstrate splitting pieces, to remove the same amount from both sides.

Look for productive strategies:

- **Find the value of the variables using guess and check strategies.** Check that they know how to manipulate the hanger diagram to find the unknown values.
- **Solve the equations by reasoning about the hanger diagrams.** Note students who did this.

Connect

Have students share strategies for matching the equations and diagrams, and how they found the values of w and x .

Highlight the structure of these equations ($px + q = r$ where p , q , and r are specific given numbers), and compare them to the equations in the Warm-up. Referring to the diagram, generalize that to solve these equations, subtract the constant from both sides and then divide each side by coefficient. Demonstrate how the moves made in the diagram can be written algebraically as steps to solve the equation.

Differentiated Support

Students Who Need Help Replace diagrams in C and D with diagrams where each block is either labeled with a variable or the number 1 (see the diagrams A and B for an example).	Students Ready for More Challenge students to write out the steps for solving a 2-step equation on their own based on how they solved the diagram and the procedure for solving 1-step equations.	English Language Learners <i>MLR7 Compare and Connect.</i> Help students make connections between the diagram and the equations with questions like “Where do you see division in both the hanger diagram and the equation?” This will help students reason about how to find unknown values and explain the meaning of a solution to an equation.	Students with Disabilities <i>Representation: Internalize Comprehension.</i> Demonstrate and encourage students to use color coding and annotations to highlight connections between representations. For example, use the same color to highlight the variables in the hanger with the same variables in its corresponding equation. <i>Supports accessibility for: Visual-spatial processing</i>
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SE Activity Inset Page

The diagram illustrates a sequence of activity steps represented by horizontal bars. The sequence begins with a grey triangle pointing to the first grey bar. This is followed by three more grey bars. Then, there are two red bars. This is followed by five grey bars, and the sequence ends with a final red bar. A large, diagonal watermark across the bottom right corner of the page reads "view only. Not for dis".

Highlight how to draw a diagram from an equation. Review the steps for solving equations in the form $px + q = r$. Say, “For some equations (e.g., see **Are you ready for more?**), drawing a hanger diagram is impractical, which is why we use the properties of equality and algebraic steps to solve.” Review how to check that a value is the solution to an equation.

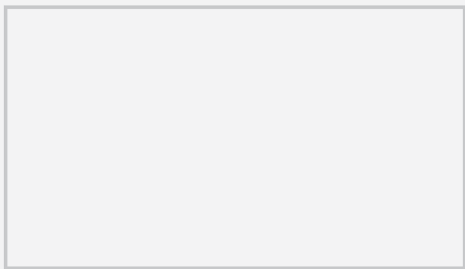
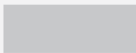
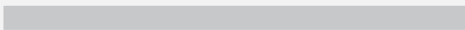
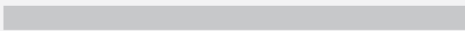
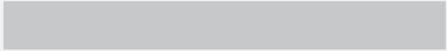
Students Who Need Help N/A	Students Ready For More Complete the Extension problems.	English Language Learners MLR8 Discussion Supports Use sentence frames to support students' explanations in the class discussion. For example, provide the frame "First, I ___ because . . .", "Then I ___ because . . ." Verbalize mathematical language in the students' explanations (e.g., "subtracting the constant", and "dividing by the coefficient") to help students explain their reasoning with the diagram and the equation.	Students with Disabilities <i>Representation: Develop Language and Symbols.</i> Create a display of important terms and vocabulary. Maintain the display for reference throughout the unit. For example, display an example of a balanced hanger. With class participation, create step-by-step instructions on how to write and solve an equation based on the hanger. <i>Supports accessibility for: Memory; Language</i>
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Summary

Review and synthesize how to solve an equation of the form $px + r = q$.

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SE Summary Inset Page



Synthesize

Display the Lesson 3 Summary BLM.

Highlight each step for solving the equation, both algebraically and using the diagram. Highlight the connections between the methods. Review the words *constant* and *coefficient* and how to check that solution is correct.

Ask the following, “Compare the two strategies we’ve used to solve equations and hanger diagrams. How are they similar and how are they different?”

Exit Ticket

Students demonstrate their understanding by solving an equation of the form $px + q = r$ algebraically.

*Do not delete. Design to replace.

Exit Ticket Inset PDF



Success looks like...

1. Interpret a balanced hanger diagram, and write an equation of the form $px + q = r$ to represent the relationship shown.
2. Explain (orally and in writing) how to solve an equation of the form $px + q = r$.

Suggested next steps

If students find an incorrect solution with an incorrect or missing hanger diagram, consider:

- Practicing drawing a hanger diagram to represent the equation. Then seeing if students can find the solution using the diagram and reviewing how to solve an equation in the form $px + q = r$ by subtracting the constant from both sides and then dividing both sides by the coefficient.
- Assigning Practice Question 3.

If students find an incorrect solution with the correct hanger diagram, consider:

- Referring to the hanger diagram for reference, reviewing how to solve an equation in the form $px + q = r$ by subtracting the constant from both sides and then dividing both sides by the coefficient.
- Assigning Practice Question 2.

Practice

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Practice Problem Analysis

Problems	Refer to	Standards	DOK
1	Activity 1	7.EE.B.4	1
2	Activity 1	7.EE.B.4	2
3	Activity 2	7.EE.B.4.a	3
4 (spiral)	Unit 3	7.G.B.4	3
5 (spiral)	Unit 5	7.NS.A.3	2
6 (formative)	Unit 6 Lesson 4	6.EE.A.4	1

(Space to show Additional Practice)

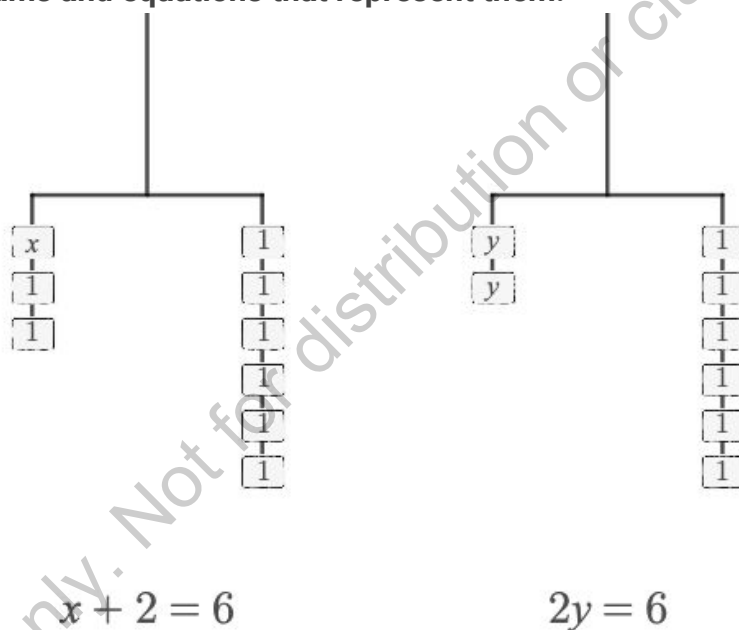
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Reasoning about Solving Equations (Part 1)

Let's see how a balanced hanger diagram is like an equation and how moving its weights is like solving an equation.

Warm-up: Hanger Diagrams and Equations

Look at the hanger diagrams and equations that represent them.



1. Explain how to reason with the *diagrams* to find the values of x and y .
2. Explain how to reason with the *equations* to find the values of x and y .

Activity 1: Matching Hanger Diagrams and Equations

1. Each of these equations represents one of the hanger diagrams.

$2\Box + 3 = 5$	$6 = 2\Box + 3$	$7 = 3\Box + 1$	$3\Box + 2 = 3$
-----------------	-----------------	-----------------	-----------------

Write the equation below its matching hanger diagram. Replace the box in each equation with either w , x , y , or z .

2. Use the hanger diagram to help you find the solution to each equation.

$$w = 2$$

$$z = 1$$

$$x = \frac{1}{3}$$

$$y = 1.5$$

Activity 2: Solving Equations

Solve the equations. Show all work. Draw a hanger diagram if needed.

1. $3x + 1 = 7$

2. $4w + \frac{3}{2} = \frac{17}{2}$

Extension

Directions: Solve each equation without using a hanger diagram.

a. $2.3z + 2.2 = 6.8$

b. $\frac{3}{4}w + \frac{1}{4} = \frac{19}{4}$

Exit Ticket

Solve the equation. Draw a hanger diagram if needed.

$$5x + 4 = 61$$

Self-Assess
a. I can explain how a hanger diagram and an equation represent the same situation. b. I can find an unknown weight on a hanger diagram and solve an equation that represents the diagram.
Key: (No content needed here, already in the design template) 1. I'm lost 2. I don't really get it 3. I'm starting to get it 4. I got it 5. I could teach it

Summary

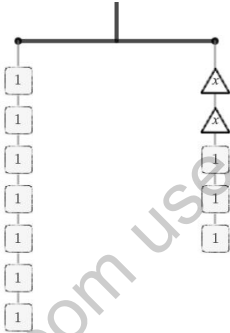
In this lesson, we showed that two amounts are equal using hanger diagrams and equations. We can use a hanger diagram to think about steps for finding an unknown amount in an equation. We can also write the steps for finding an unknown amount in an equation, without a hanger diagram.

My Notes:

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Practice

1. Explain how the parts of the hanger diagram shown to the right compare to the parts of the equation, $7 = 2x + 3$.

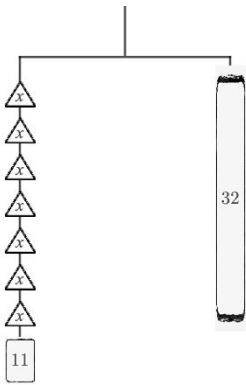


2. Clare used a hanger diagram to find the solution of the equation $8 = 2x + 4$. Write an equation to represent each step.

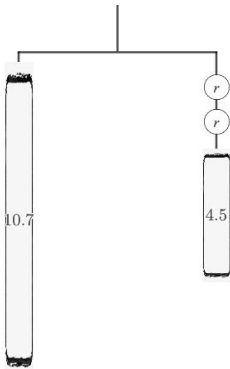
$8 = 2x + 4$				

3. Solve the equations. Use the hanger diagrams provided or draw your own if needed.

a. $7x + 11 = 32$



b. $2r + 4.5 = 10.7$



Practice (continued)

4. Lin and Tyler are drawing circles. Tyler's circle has twice the diameter of Lin's circle. Tyler thinks that his circle will have twice the area of Lin's circle as well. Do you agree with Tyler?

5. Jada and Priya are trying to solve the equation $\frac{2}{3} + x = 4$.

- Jada says, "I think we should multiply each side by $\frac{3}{2}$ because that is the reciprocal of $\frac{2}{3}$."
- Priya says, "I think we should add $-\frac{2}{3}$ to each side because that's the opposite of $\frac{2}{3}$."

- a. Which person's strategy should they use? Why?
- b. Write an equation that can be solved using the other person's strategy.

6. Use the distributive property to simplify each expression.

a. $4(x + 9)$

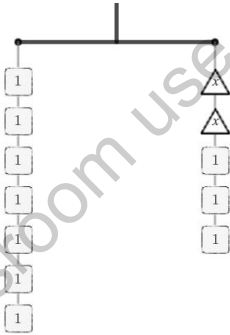
b. $6(a + b - c)$

c. $-2(2x + 3)$

d. $8(y - 2z)$

Practice

1. Explain how the parts of the hanger diagram shown to the right compare to the parts of the equation, $7 = 2x + 3$.

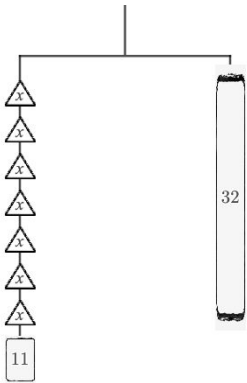


2. Clare used a hanger diagram to find the solution of the equation $8 = 2x + 4$. Write an equation to represent each step.

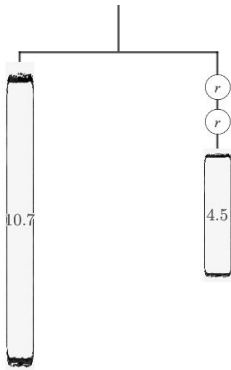
$8 = 2x + 4$				

3. Solve the equations. Use the hanger diagrams provided or draw your own if needed.

a. $7x + 11 = 32$



b. $2r + 4.5 = 10.7$



Practice (continued)

4. Lin and Tyler are drawing circles. Tyler's circle has twice the diameter of Lin's circle. Tyler thinks that his circle will have twice the area of Lin's circle as well. Do you agree with Tyler?

5. Jada and Priya are trying to solve the equation $\frac{2}{3} + x = 4$.

- Jada says, "I think we should multiply each side by $\frac{3}{2}$ because that is the reciprocal of $\frac{2}{3}$."
- Priya says, "I think we should add $-\frac{2}{3}$ to each side because that's the opposite of $\frac{2}{3}$."

- a. Which person's strategy should they use? Why?
- b. Write an equation that can be solved using the other person's strategy.

6. Use the distributive property to simplify each expression.

a. $4(x + 9)$

b. $6(a + b - c)$

c. $-2(2x + 3)$

d. $8(y - 2z)$
