Unit 6 | Lesson 1

Detecting Counterfeit Coins

How can a balance scale help you detect a counterfeit coin?

Focus

Goal:

1. Understand how the concept of balance relates to math, both concretely and abstractly.

Coherence

Today:
Students explore the physical concept of balance as they model weighing different numbers of coins to identify a counterfeit among them. All students will be able to engage with these tasks by applying some repeated reasoning, taking the form of a combination of guess-and-check, brute force, or higher-order thinking and logic (**MP1**, **MP8**). This lesson sets the stage for the important concept of balance as it relates to algebraic expressions and equations, which will be carried into all the lessons that follow.

Previously:
In Grades 3-5, students wrote and evaluated numerical expressions involving positive rational numbers, and used the equal sign to indicate when two expressions, or an expression and a number, represented the same value.

Coming Soon:
In Lessons 2-9, students will expand their understanding of balance as an important mathematical concept as it relates to numerical and algebraic expressions, and also to solving equations with variables representing unknown values. In particular, the balances seen here will connect with the hanger diagrams in Lessons 5-6.

Standards

- **Building Toward:**
  - 6.EE.B.6

Pacing Guide

Suggested Total Lesson Time | ~45-60 min

Warm-up	Activity 1	Activity 2	*Optional Activity 3*	Summary	Exit Ticket
5 min	10 min	20 min	*15 min*	5 min	5 min
Independent	*Pairs*	*Small Groups*	*Small Groups*	*Whole Class*	*Independent*
	MP1, MP8	*MP1, MP8*	*MP1, MP8*		
Desmos Activity Slides					
Slide 1	Slides 2–3	Slide 4	Slides 5–6	Slide 7	Slide 8

Vocabulary

Materials

- Exit Ticket PDF
- Additional Practice Book

Desmos Digital Classroom

Warm-up: Launch with a Narrative
Explain why teachers should be aware of a Desmos digital feature in the Lesson. Highlight any engaging features that make this moment shine

Warm-up: Which One Doesn't Belong?

Students look at three images of currency and identify one they believe does not belong, in order to begin thinking about the context of money and the word *counterfeit*.

Launch

Display the images for students and allow 2-3 minutes of quiet think time, followed by a whole-class discussion.

Monitor

Help students get started by leveraging part of another familiar routing and asking them, "What do you notice?" Consider asking additional probing questions such as, "How are the images the same? How are any of the images different?"

Connect

Have students share their selection of which image does not belong and the rationale for their choice.

Highlight that the denomination of a piece of currency indicates its value, but only real money is actually worth its stated value. Money that has been created for the purpose of a board game for example, has no real value, but also was not intended to. However, pieces of currency created with the intention of being passed off as real, trying to imitate the real thing identically in every way, is called *counterfeit*.

Alternate Warm-up Activity	Differentiated Support	
	English Language Learners:	**Students with Disabilities:**

Activity 1: Three Coins

In this low-floor, high-ceiling activity, students figure out how many weighings are needed to determine which of three coins is counterfeit.

Launch

Arrange students in pairs. Give them time to work individually and then have them share and compare answers with their partner, followed by a whole-class discussion. Make sure all students are familiar with how a balance scale works.

Monitor

Help students get started by asking, "What would you put on the scale first? What will the result of that weighing tell you?"

Look for points of confusion:

- **Not distinguishing a single trial from a generalization.** Have students recount and record their weighings that lead to an identification. Ask, "What if one of those weighings went the other way? Could you have still identified the counterfeit coin after the same number of weighings?"
- **Double counting different possible weighings.** Each weighing has two possible results (balanced or unbalanced), each of which leads to a different next weighing. But only one result will happen at any given time, so only one next weighing should be counted.

Connect

Display a model of a balance scale and three coins labeled A, B, and C.

Have students share their strategies for finding the counterfeit coin. Start with students who used more weighings (**MP1**), and work toward students with more efficient strategies (**MP8**). Ask, "Was anyone able to identify the counterfeit coin in fewer weighings?"

Ask, "What does a balanced weighing tell you? An unbalanced weighing? How does knowing whether the counterfeit coin is heavier or lighter affect your weighings?"

Differentiated Support

Students Who Need Help	Students Ready for More	English Language Learners	Students with Disabilities
Suggest students draw diagrams, either as representations of the physical scenario or abstract ways of recording results.			Provide actual coins, counters, or concrete objects to be used for simulating the weighings.

Activity 2: More Coins

Students move beyond three coins, again identifying strategies for determining the counterfeit coin.

Launch

Arrange students in groups of 4. Allow the groups time to work first, followed by a whole-class discussion.

Explore

Help students get started by asking, "What would you put on the scale first? What will the result of that weighing tell you?"

Look for points of confusion:

- **Determining the counterfeit coin in more than three weighings.** Students might only weigh two coins at a time. Ask, "What if I told you it could be done in fewer weighings? Can you think of a different way to eliminate more coins at the same time?"

Look for productive strategies:

- Systematically and simultaneously considering the results and implications of each weighing (**MP8**).

Connect

Display a model of a balance scale and each total number of coins, labeled using the letters A, B, C, etc.

Have students share the number of weighings it took for each number of coins and their strategies for finding the counterfeit. Start with students who used more weighings (**MP1**), and work toward students with more efficient strategies (**MP8**).

Ask, "What was the same about your strategy for each different number of coins? Do you think that strategy would still work for, say, 24 coins? How would you change your strategy for an odd number of coins?"

Differentiated Support

Students Who Need Help	Students Ready for More	English Language Learners	Students with Disabilities
If 8 and 12 coins prove too complex for some students, consider letting them attempt the same problem for 5 and 6 coins instead.	"For all of these sets of coins, the counterfeit could be determined with certainty in just three weighings. How many coins would there have to be for the number of weighings needed to go up to four?" [13]		Provide actual coins, counters, or concrete objects to be used to simulate the weighings.

Optional Activity 3: *A Cruel Twist*

Students grapple with thirteen coins, figuring out how many weighings are needed to determine which, if any, is counterfeit.

Launch

NOTE: This is an optional activity, and would extend the necessary class time beyond 45 minutes. If working on this activity in class, keep students in their same groups of 4, and give them 10 minutes to work, followed by a whole-class discussion.

Explore

Help students get started by asking, "To figure out if there is even a counterfeit coin at all, what would you need to know? Would you weigh the coins any differently than before?"

Look for points of confusion:

- **Adding 1 to the number of weighings it took in previous examples without trying.** Remind them that they will need to justify their answers.

Connect

Display a model of a balance scale and each total number of coins, labeled using the letters A, B, C, etc.

Have students share the number of weighings needed to determine whether there was a counterfeit, and how those looked. Then have them share the number of weighings needed to determine which coin was counterfeit when there was one. Again, start with strategies that used more weighings (**MP1**), and move toward more efficient strategies (**MP8**).

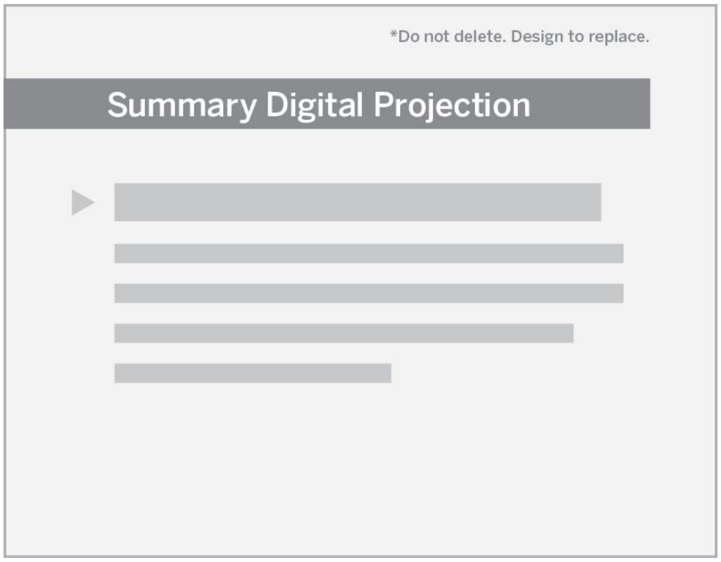
Differentiated Support

Students Who Need Help	**Students Ready for More**	**English Language Learners**	**Students with Disabilities**
For students that have been struggling with organizing and recording their trials and results in order to reach generalizations, consider reducing the number of coins to 7, but keeping the rest of the scenario the same.			

Summary

5 min

Synthesize the critical aspect of these puzzles as the concept of balance, which will play a significant role in the unit.



Synthesize

Ask students how the different variations of the counterfeit coin problem changed their thinking and strategies, using questions such as:

- "What changes to the scenario had the greatest impact on the number weighings needed?"
- "Were there any aspects of your strategies that were used consistently no matter the number of coins or given information?"

Highlight that *balance* is an important mathematical concept, which was very obvious with weighing the coins, but also as a more abstract representation of the concept of equality. In either case, there are many possible moves, as well as some natural laws guiding what can be done or makes sense.

Exit Ticket

Students express their initial understandings of balance, representative of how math is not just about numbers and calculations, but is also a general way of thinking.

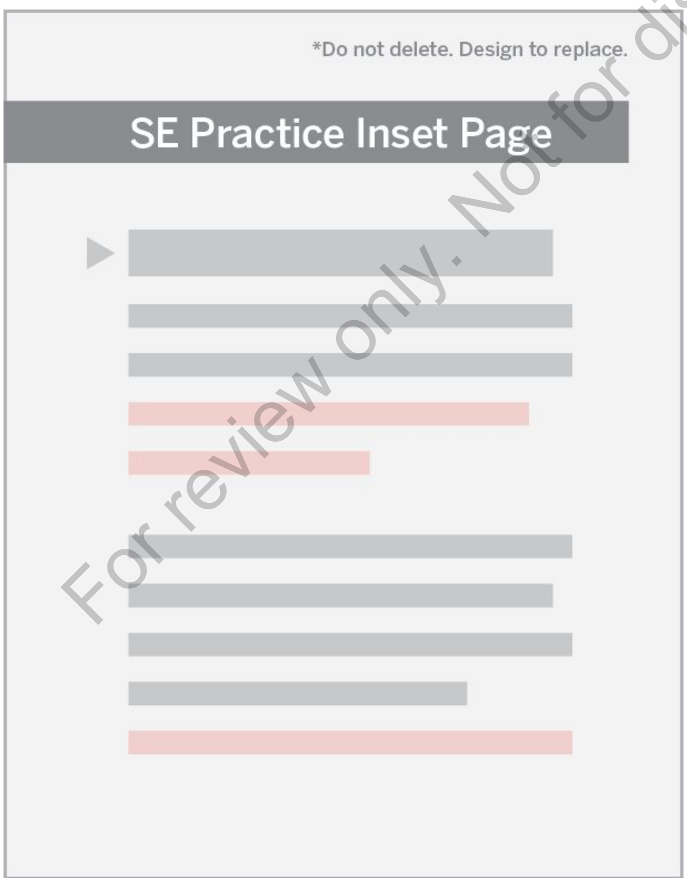
Success looks like...

1. Understand how the concept of balance relates to math, both concretely and abstractly.

Suggested next steps

- There are no right or wrong answers, but students should recognize the partitioning of coins into equal groups as relating to math. Exemplary responses would be those that reference repeated reasoning and logic, such as “if, then” thinking, and using words like “equal” or “even.”
- Draw attention back to the general notion of balance, as both a physical representation and a mathematical principle, as it relates to the concepts presented in subsequent lessons. Hanger diagrams will be presented in Lessons 5-6, offering a direct connection to the balances seen here.

Practice



Practice Question Analysis

Question	Refer to	Standards	DOK
1 *(spiral)*	Unit 3 Lesson 13	6.RP.A.3.c	2
2 *(spiral)*	Unit 5 Lesson 8	6.NS.B.3	*1*
3 *(spiral)*	Unit 5 Lesson 13	6.NS.B.3	2
4 *(spiral)*	Unit 3 Lesson 12	6.RP.A.3.c	2
5 *(formative)*	Unit 6 Lesson 2	2.OA.A.1	2

Detecting Counterfeit Coins

How can a balance scale help you detect a counterfeit coin?

Warm-up: Which One Doesn't Belong?

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Activity 1: Three Coins

There are 3 coins that look completely identical, but exactly 1 of the coins is counterfeit, and is slightly heavier than the 2 real coins. You cannot feel the difference in weight when you hold them. But you have a balance scale that is sensitive enough to detect it.

1. If you weigh 2 of the coins, you might get lucky and find that one of them is heavier than the other. But you might not be so lucky. Will you always know which coin is the counterfeit coin after just one weighing? Why or why not?

2. Now imagine you again have 3 coins and 1 of them is counterfeit. But this time, you don't know if the counterfeit coin is slightly heavier or slightly lighter than the other 2 coins.

   a. Is it possible to know which coin is counterfeit after just one weighing?

   b. Will you *always* know which coin is the counterfeit after just one weighing?

   c. What is the smallest number of weighings you would need to make so you could *always* say which coin is counterfeit?

Activity 2: More Coins

For each number of coins, assume one coin is counterfeit, and that the counterfeit coin is either slightly heavier or slightly lighter than the other coins. Determine the smallest number of weighings you would need to make so you could *always* say which coin is counterfeit.

1. 4 coins

2. 8 coins

3. 12 coins

Optional Activity 3: A Cruel Twist

Consider the same setup as the previous examples, with 13 total coins, EXCEPT now you only know that either exactly 1 coin is counterfeit (and could be slightly heavier or slightly lighter than the other coins) <u>or</u> all of the coins are real.

1. What is the smallest number of weighings it would take to always be able to determine whether:

    a. any of the coins is actually counterfeit?

    b. which coin is counterfeit (if one of them is)?

Exit Ticket

Reflect on how you thought mathematically (not just with numbers) as you tried to find the smallest number of weighings needed to detect counterfeit coins. What did you do the same, and what did you do differently as you attempted new scenarios?

Self-Assess
A. I can see how the idea of balance relates to math.
Key: 1. I'm lost 2. I don't really get it 3. I'm starting to get it 4. I got it 5. I could teach it

Summary

As long as there have been coins, there have been coin counterfeiters.

It got so bad in 375 BCE that the Greek city-state of Athens installed a *dokimastes* – an official coin-tester – in the marketplace. Their job was to do what you just did – use a balance to sniff out the fakes.

Like counterfeit coins, not everything is always what it seems. In math, as in life, we encounter uncertainty and unknowns.

So what do we do? Hide under our tunics?

No! We do as the dokimastes do. Whether it's unknowns in trade, history, or different ecosystems, the method is still the same. If we can use what we know about *balance*, we can, step-by-step, strip away the mystery, until all that's left is the truth.

Welcome to Unit 6.

My Notes:

Practice

1. Write at least two different mathematical expressions or equations about the image. Include either a fraction, a decimal number, or a percentage in each.

2. Find each product.

   a. $(21.2) \cdot (0.02)$

   b. $(2.05) \cdot (0.004)$

Practice (continued)

3. Calculate $141.75 \div 2.5$ using a method of your choice. Show or explain your reasoning.

4. There are 90 students in the band. Most band members rent their instruments, but the remaining 20% of the band members own their instruments.

   a. How many students in the band own their instruments?

   b. How many students in the band rent their instruments?

   c. What percentage of students rent their instruments?

5. Mai brought 12 apples to school in the morning and then shared 6 with friends at lunch. Write an expression to represent the number of apples Mai had left after lunch.

Unit 6 | Lesson 2

Write Expressions Where Letters Stand for Numbers

Let's use expressions with variables to describe scenarios.

Focus

Goals:

1. Write an expression with a variable to generalize the relationship between quantities in a situation.
2. Comprehend that a "variable" is a letter standing in for a number, and recognize that a coefficient next to a variable indicates multiplication (orally and in writing).
3. Describe (orally) a situation that could be represented by an expression of the form $x + p$ or px, for rational number p and unknown x.

Coherence

Today:

Students begin to formally cross from arithmetic to algebra. They write expressions that record operations with numbers and letters (now defined as variables), which stand in for unknown numbers and can take on different values (**MP8**). They connect stories relating two quantities using expressions, and identify which quantity the variable represents (**MP2**).

Previously:

In Grades 3-5, students worked strictly with numerical expressions and equations. They informally used symbols and letters for missing values, and informally determined these values using properties of operations and/or relationships between operations.

Coming Soon:

In Lessons 3-6, students will work with algebraic equations, as they progress toward understanding and applying properties of operations to more efficiently solve for variables.

Standards

- **Addressing:**
  6.EE.A.2.A
  Write expressions that record operations with numbers and with letters standing for numbers.

  6.EE.B.6
  Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.
- **Also Addressing:**
  6.EE.A.2.C
- **Building On:**
  - 5.OA.A.2
  - 4.OA.A.3
- **Building Towards:**
  - 6.EE.B.5
  - 6.EE.B.6
  - 6.EE.B.7

Pacing Guide

Suggested Total Lesson Time | ~45 min

Warm-Up	Activity 1	Activity 2	Summary	Exit Ticket
10 min	10 min	15 min	5 min	5 min
Independent	*Independent, Pairs*	*Pairs*	*Whole Class*	*Independent*
6.EE.A.2.c	*6.EE.A.2.a* *6.EE.B.6*	*6.EE.A.2.a* *6.EE.B.6*		*6.EE.A.2.a* *6.EE.B.6*
	MP2, MP8	*MP2*		
Desmos Activity and Presentation Slides				
Slides 1-2	Slides 3-6	Slides 7-8	Slide 9	Slides 10-11

Vocabulary

New words

- **Variable**
- **Coefficient**

Review words

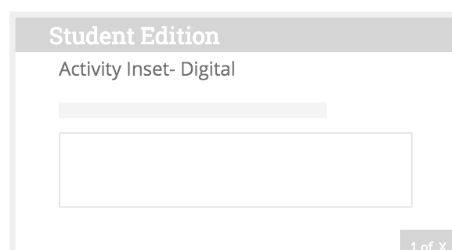
- **Expression**

Materials

- Exit Ticket PDF
- Additional Practice Book

Desmos Digital Classroom

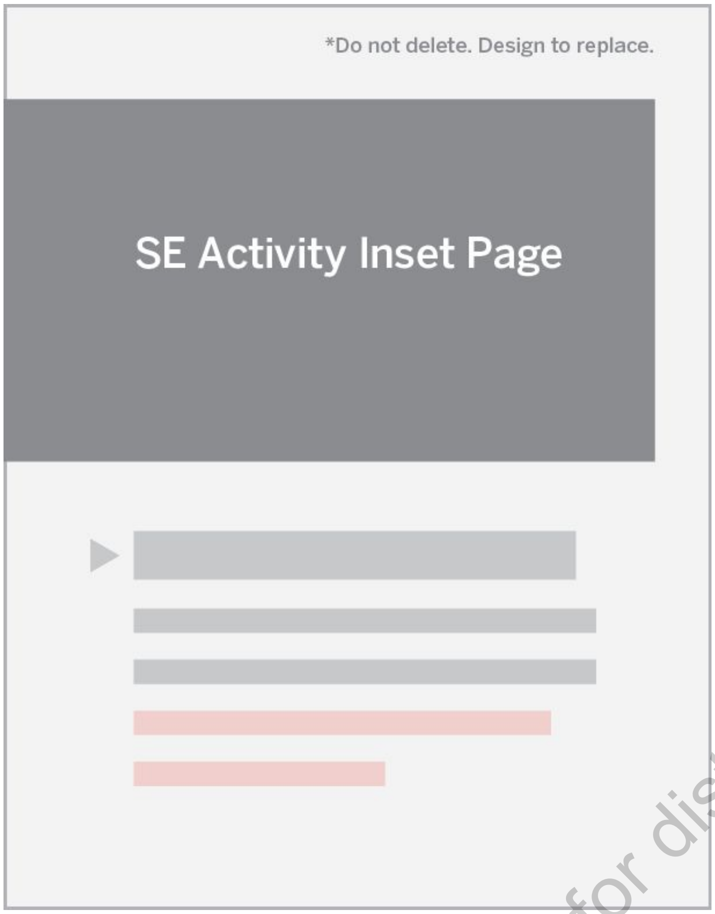
Warm-up: Launch with a Narrative
Launch your lesson with details about the African salt trade, which will weave in and out over the next several lessons.



Exit Ticket: Real-Time Results
Check in real time if your students can turn situations into expressions with variables using a digital Exit Ticket.

Warm-Up: The African Salt Trade

Students review translating verbal statements to numerical expressions, using letters for unknown values.



Launch

Allow students 5 minutes of quiet work time, followed by a whole-class discussion.

Monitor

Help students get started by asking,"What number words do you see? What operation words do you see? How would you write an unknown number?"

Look for points of confusion:

- **Trying to evaluate expressions.** This activity requires the students to create the expressions only, not find the solution.
- **Writing $>$ for "more than"**. Focus on the phrases "more than" (meaning addition) versus "is more than" (an inequality).
- **Not recalling "one-half of" means multiplication**. Consider asking "What is $\frac{1}{2}$ of 6? What operation did you use to come up with 3?"
- **Picking an actual number instead of a letter**. The phrase "a number" indicates an unknown, and should be represented using a letter (for example, $a+7$, not $1+7$).

Look for productive strategies:

- Recognizing the underlined text represents the "answer," but expressions should not be evaluated here.
- Knowing which operation to use for each scenario.

Connect

Display correct expressions for all four statements.

Have students share the different ways they wrote their expressions and which are equivalent (or not).

Highlight that the phrase "a number" indicates an "unknown" (the term *variable* will be introduced in the next Activity). Any letter could be used, and they could be the same or different, even if they represent the same or different values, across scenarios.

Alternate Warm-up Activity	Differentiated Support	
For students who need additional support with writing an expression to represent a problem (from Lesson 1 Practice Problem 5): "Can you figure out how many apples Mai would have left after lunch? What calculation did you do to determine that? Now can you write it as an expression?"	**English Language Learners**	**Students with Disabilities**

Activity 1: Known, Known, Unknown

Students write expressions describing situations with an unknown quantity, and recognize how variables can take on multiple values.

Launch

Arrange students in pairs. Provide 5 minutes of work time, followed by a whole-class discussion.

Monitor

Help students get started by asking, “For each scenario, what are the two quantities involved, and what operation relates them?”

Look for points of confusion:

- **Using incorrect operations**. Have students identify key words, and draw a diagram to check the reasonableness of solutions.
- **Writing multiplication as repeated addition**. Acknowledge this as correct, but inefficient, and ask them to write it another way.

Look for productive strategies:

- Applying formulas consistently by continuing the pattern throughout each example given.
- Testing different variable values to check if formula is accurate.

Connect

Have students share each of their expressions, identifying the information that helped them determine the operation (**MP2**).

Highlight that the “next to” notation represents multiplication by a variable without an operation symbol. There is a repeated structure of common terms and operations among all of the expressions within each scenario **(MP8).**

Define a *variable* as a letter that represents an unknown number in expressions and equations, and a *coefficient* as a number that is being multiplied by a variable, typically written in front of and “next to” the variable without an operation symbol.

Ask, “For the tables, if we knew the expressions first, how could they have been used to determine specific values?”

Differentiated Support

Students Who Need Help	Students Ready for More	English Language Learners	Students with Disabilities
Begin with more accessible values. Extend the given tables for the weight of salt and number of shops, and explore values of 1, 2, 3, 4, and 10 in each scenario. Draw students’ attention to what changes and what stays the same for each set of calculations.	How would the table from Problem 2 change if the top row were instead labeled “How many fewer shops than Niani”? [Gao: 36, Timbuktu: 41, Djenne: $43 - d$]	*MLR8: Discussion Supports* Provide sentence frames, such as: “To determine the amount of bolts of cloth in Priya’s market, I would ____ because ____.” Encourage students to revoice or press for more explanation using, “So what I heard you say...” or “Can you tell me more about...?”.	Connect a new concept to one with which students have experienced success. For example, invite students to draw a picture or create a table to help as an intermediate step before writing an expression.

Activity 2: Storytime

Students invent scenarios that could correspond to expressions with unknown amounts.

Launch

Keep students in the same pairs. Clarify that for every equation, each pair should come up with one story to be recorded. Allow 5 minutes for students to work with their partner, followed by a whole-class discussion.

Monitor

Help students get started by asking, "How many quantities should be in your story? Is one always greater than the other? How?"

Look for points of confusion:

- **Describing different operations; confusing the variable and the expression**: Have students substitute values for the variable to assess the expression's meaning and reasonableness.
- **Choosing unrealistic quantities:** While there are no incorrect scenarios, discuss the appropriateness of chosen values in the context of their stories.

Look for productive strategies:

- Writing stories that correctly represent each term in the equation.

Connect

Have students share their group's story for each expression, what quantity x represents, and what quantity the expression represents (**MP2**).

Highlight that the same variable, x, was used in each expression, but does not represent the same quantity, or value, across expressions.

Ask, "How do the values 30 and 12 fit into some of the stories shared? What do they represent?"

Differentiated Support

Students Who Need Help	**Students Ready for More**	**English Language Learners**	**Students with Disabilities**
For students who may be hung up on 30 and 12 holding realistic or reasonable meanings in their stories, allow them to change the two expressions to $1+x$ and $2x$.	Challenge students to come up with a new scenario that involves three related quantities, where x represents the same quantity throughout, and the other two quantities make sense as $30+x$ and $12x$. [*Sample answer: x represents dozens of eggs bought, $12x$ represents total eggs bought, and $30+x$ represents the total dozens of eggs in inventory.*]	*MLR1: Stronger and Clearer Each Time* Have students move through successive pair-shares to provide opportunities to revise and refine their writing. Provide prompts for feedback to help strengthen ideas and clarify language (e.g., "How are the parts of the expression represented in your story? How does the operation fit into your story?"). Allow time to complete a final draft based on feedback.	

Summary

Review how to use variables to represent unknowns in expressions and stories.

Synthesize

Display the expressions $p + 3$ and $7x$ to support the discussion.

Highlight that when two quantities are related mathematically, a variable can be used as a placeholder when writing an expression for that relationship.You can choose different numbers for the quantity represented by the variable, and then evaluate the expression to find the corresponding value for the other quantity.

Ask, “What is the coefficient of p in the expression $p + 3$?” “What is the value of $7x$ when x is 5? What operation did you use to figure it out?”

Review vocabulary:

- Variable
- Coefficient

Exit Ticket

Students demonstrate their understanding by matching an algebraic expression to a given scenario, and describing a scenario for a given expression.

Success looks like...

1. Write an expression with a variable to generalize the relationship between quantities in a situation.
2. Comprehend that a “variable” is a letter standing in for a number and recognize that a coefficient next to a variable indicates multiplication (orally and in writing).
3. Describe (orally) a situation that could be represented by an expression of the form $x + p$ or px, for rational number p and unknown x.

Suggested next steps

If students select the incorrect expression for Problem 1, consider:

- Reviewing critical mathematical cue words in both statements from Activity 1 Problem 1.
- Assigning Practice Problem 3.

If students use additive (not multiplicative) language in their scenario for Problem 2, consider:

- Reviewing critical mathematical cue words in Activity 1 Problem 2.
- Asking, “In the term $8x$, 8 is the coefficient of x. What operation does that represent? How can your story show a multiplicative action?”

SE Practice

Practice Problem Analysis

Problems	Refer to	Standards	DOK
1	Activity 2	6.EE.A.2.A	2
2	Activity 2	6.EE.A.2.A	2
3	Activity 3	6.EE.B.6	2
4 *(spiral)*	Unit 3 Lesson 11	6.RP.A.3.C	2
5 *(spiral)*	Unit 3 Lesson 14	6.RP.A.3.C	2
6 *(formative)*	Unit 6 Lesson 3	5.OA.A.2	2

Write Expressions Where Letters Stand for Numbers

Let's use expressions with variables to describe scenarios.

Warm-up: The African Salt Trade

Write an expression to represent the underlined quantity from each statement. Use a letter to represent any unknown numbers.

Statement	Expression
The <u>total distance a caravan traveled</u>, if it went 5 kilometers before sunrise, and 3 more kilometers since	
The <u>total kilograms of salt</u> a small caravan can carry, if each camel can carry a load of 9 kilograms, and there are 3 camels	
The <u>high temperature of the day</u>, if it is 37 degrees warmer than the overnight low temperature	
The <u>total number of blocks of salt traded</u> for bars of gold, if the number of blocks of salt is equal to one-half the number of bars of gold	

Activity 1: Known, Known, Unknown

Read each scenario below and complete the tables.

1. A gold bar is worth the same as 12.5 kilograms of salt. Show how much salt, in kilograms, is worth the same as each number of gold bars.

Number of gold bars	2	12	b
Salt (kilograms)			

2. There are 43 shops in the city market of Niani. The cities of Gao, Timbuktu, and Djenne each have more shops than Niani. Show how many shops each city has.

	Gao	**Timbuktu**	**Djenne**
How many more shops than Niani	7	2	d
Number of shops			

Write an expression to represent each of the following scenarios.

3. Two-fifths of the goods Priya is selling at her market stand are bolts of cloth. How many bolts are there if Priya's stand has:

   a. 20 goods?

   b. x goods?

4. Diego had 31.25 grams of salt on Monday. On Tuesday, he received x more grams of salt. Write an expression to show how much salt Diego had on Tuesday.

Activity 2: Storytime

Think of a story that might be represented by each expression. For each, state what quantity x represents and what quantity the expression represents.

1. $30 + x$

   a. My story:

   b. In my story, the x represents:

   c. In my story, the expression, $30 + x$, represents:

2. $12x$

   a. My story:

   b. In my story, the x represents:

   c. In my story, the expression $12x$ represents:

Exit Ticket

Directions: A plant was 8 inches tall last week, and grew x inches this week.

1. Circle the expression that represents how tall, in inches, the plant is now.

    - $x+8$
    - $8x$

    Explain how you know.

2. For the expression not chosen, describe a scenario the expression might represent.

Self-Assess
A. I can write an expression with a variable to represent a calculation where I do not know one of the numbers. B. I can match an expression to a real life scenario that it could represent. C. When I see an expression, I can make up a story that it might represent, and explain what the variable represents in the story.
Key: 1. I'm lost 2. I don't really get it 3. I'm starting to get it 4. I got it 5. I could teach it

Summary

We often use a letter, such as x or a, as a placeholder for an unknown number in expressions. This letter is called a **variable**. For example, in the expression $u+1$, the variable is u.

Variables make it possible to write an expression for a calculation, even when we don't know all the values in the calculation. You can choose different numbers for the value of the variable.

Also, when a number is written “next to” a variable without an operation symbol, the number and the variable are being multiplied. A number written next to a variable in this way is called a **coefficient**. For example, $7x$ means the same thing as $7 \cdot x$. The variable is x, and the coefficient is 7.

And if no coefficient is written, the coefficient is 1. For example, in the expression $p+3$, the coefficient of p is 1. You could also write $1p$, but it’s not necessary, since $1 \cdot p = p$.

My Notes:

Practice

1. Instructions for a craft project say that the length of a piece of red ribbon should be 7 inches less than the length of a piece of blue ribbon. How long is the blue ribbon if the length of the red ribbon is:

    a. 10 inches?

    b. 27 inches?

    c. x inches?

2. Tyler has 3 times as many books as Mai.
How many books does Tyler have if Mai has:

    a. 15 books?

    b. 21 books?

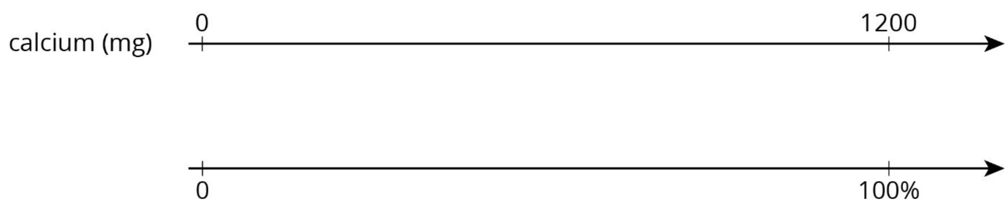
    c. x books?

3. A bottle holds 24 ounces of water. It has x ounces of water in it.

    a. What does $24 - x$ represent in this situation?

    b. Write a question about this situation that has $24 - x$ for the answer.

4. The daily recommended allowance of calcium for a sixth grader is $1,200$ mg.
   One cup of milk has 25% of the recommended daily allowance of calcium.
   How many milligrams of calcium are in a cup of milk?
   Consider using the double number line to help with your thinking.



5. A trash bin has a capacity of 50 gallons. What percentage of its capacity is each amount? Show your reasoning.

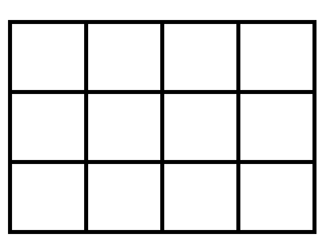
   a. 5 gallons

   b. 30 gallons

   c. 45 gallons

   d. 100 gallons

6. What is the total area of the figure, if each small square has an area of 1 square unit?



Unit 6 | Lesson 3

Tape Diagrams and Equations

Let's see how tape diagrams and equations can show relationships between amounts.

Focus

Goals:

1. Draw tape diagrams to represent equations of the forms $x+p=q$ and $px=q$.
2. Interpret (orally and in writing) tape diagrams that represent equations of the form $p+x=q$ or $px=q$.

Coherence:

Today:
Students build on previous work with tape diagrams to represent operations with letters standing in for numbers (**MP2**). These tape diagrams serve as a tool to help students visualize the relationships between quantities represented by expressions and equations. Students interpret the structure of tape diagrams and create their own, based on the relationships in given story problems (**MP4, MP7**).

Previously:
Since Grade 3, students have used tape diagrams to represent operations with numbers. In Lesson 2 of this unit, students wrote numerical and algebraic expressions with letters standing in for numbers, to represent scenarios and to relate operations with numbers.

Coming Soon:
In Lesson 4, students will continue to leverage tape diagrams for determining and recognizing solutions to equations with unknown values. Later, in Lessons 5-9, students will solve a variety of real-world and mathematical problems by writing and solving equations in which tape diagrams may still prove to be a useful strategy.

Standards

- **Addressing:**
  6.EE.B.6
  Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.
- **Building On:**
  - 5.OA.A.2
  - 6.EE.A.2
- **Building Towards:**
  - 6.EE.B.5
  - 6.EE.B.7
  - 7.EE.B.4

Pacing Guide

Suggested Total Lesson Time | ~45 min

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	10 min	20 min	5 min	5 min
Independent	*Pairs* *MLR2*	*Independent, Pairs* *MLR7*	*Whole Class*	*Independent*
6.EE.B.6	*6.EE.B.6*	*6.EE.B.5, 6.EE.B.7*	*6.EE.B.6*	*6.EE.B.6*
MP7	*MP7*	*MP2, MP4, MP7*	*MP7*	*MP7*
Digital Activity and Presentation Slides				
Slides 1–4	Slides 5-6	Slides 7-9	Slide 10	Slides 8-9

Vocabulary

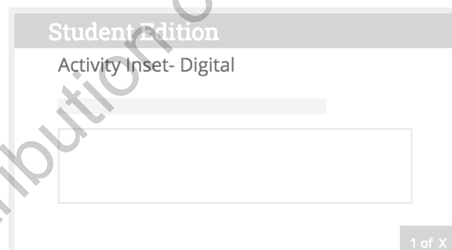
Review words

- **Tape diagram**
- **Expression**
- **Equation**
- **Variable**
- **Coefficient**

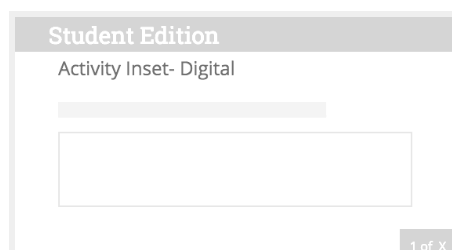
Materials

- Exit Ticket PDF
- Additional Practice book

Desmos Digital Classroom



Activity 2: Dynamic Tape Diagrams
Students can create digital tape diagrams, and you can overlay them all to see similarities and differences at a glance.



Activity 2: See Student Thinking
Students are asked to explain their thinking behind matching equations to tape diagrams, and these explanations are passed to you.

Warm-up: Moving from Expressions to Equations

Students recall how tape diagrams can be used to represent addition and multiplication relationships.

Launch

Give students 2 minutes of quiet think time, followed by a whole-class discussion.

Monitor

Help students get started by asking, "How is the first pair of tape diagrams different from the second pair? How are they similar?"

Look for points of confusion:

- **Not identifying the 5 in the first diagram**. Ask, "How many 2s do you see?"
- **Not making the jump from expressions to equations:** Ask, "What changed from the first diagram to the third?

Look for productive strategies:

- Using repeated addition. Ask, "Is there a more efficient way you could represent this sum?"
- Representing repeated addition as multiplication (**MP7**).

Connect

Have students share expressions that represent each diagram; then equations that represent each diagram.

Ask:

- "How do you see the 5 represented in the first diagram?"
- "How did you find the total length for the first diagram in Problem 2? Where does that total length appear in the equation?"
- "How do the expressions from Problem 1 relate to the equations from Problem 2? How are they alike? How are they different?"

Highlight that boxes within a tape diagram can always be interpreted as addends whose sum is equal to the total length, and that same size boxes can also be represented by multiplication.

Alternate Warm-up Activity	Differentiated Support	
For students who need additional support creating an expression representing a diagram (from Lesson 2 Practice Problem 6) "Each smaller square has an area of 1. How many smaller squares make up the whole rectangle? What operation could we use to determine the total area? What would the expression look like?"	**English Language Learners**	**Students with Disabilities** Consider providing counters for students to create physical models related to the tape diagrams.

Activity 1: Tape Diagrams with Variables

Students use relationships between operations to construct multiple equations represented by the same tape diagram.

Launch

Remind students that x is a **variable**, standing in for an unknown number. Students should work independently for 2 minutes, then share their answers with a partner, followed by a whole-class discussion.

Monitor

Help students get started by asking, "What does the 12 represent? Where does 12 fit into an equation representing the diagram?"

Look for points of confusion:

- **Not identifying the 4 in the first diagram.** Ask, **"**How many xs do you see?"
- **Not yet using "next to" notation**. Remind students that $4 \cdot x$ can be written as a variable with a coefficient, $4x$.
- **Thinking only one equation is possible**: Remind students to think about inverse and related operations.
- **Thinking x must represent the same value in all of the equations**. Students are not expected to "solve" these equations yet, but note that "the same variable can be used to represent different values in different scenarios."

Look for productive strategies:

- Using repeated addition. Ask, "Is there a more efficient way we could represent the sum $x + x + x + x$?"

Connect

Display the tape diagrams.

Have students share one equation at a time, until all possible valid equations have been recorded for each diagram.

Highlight that a multiplicative relationship can also be expressed using division, and an additive relationship can also be expressed using subtraction (**MP7**).

Ask:

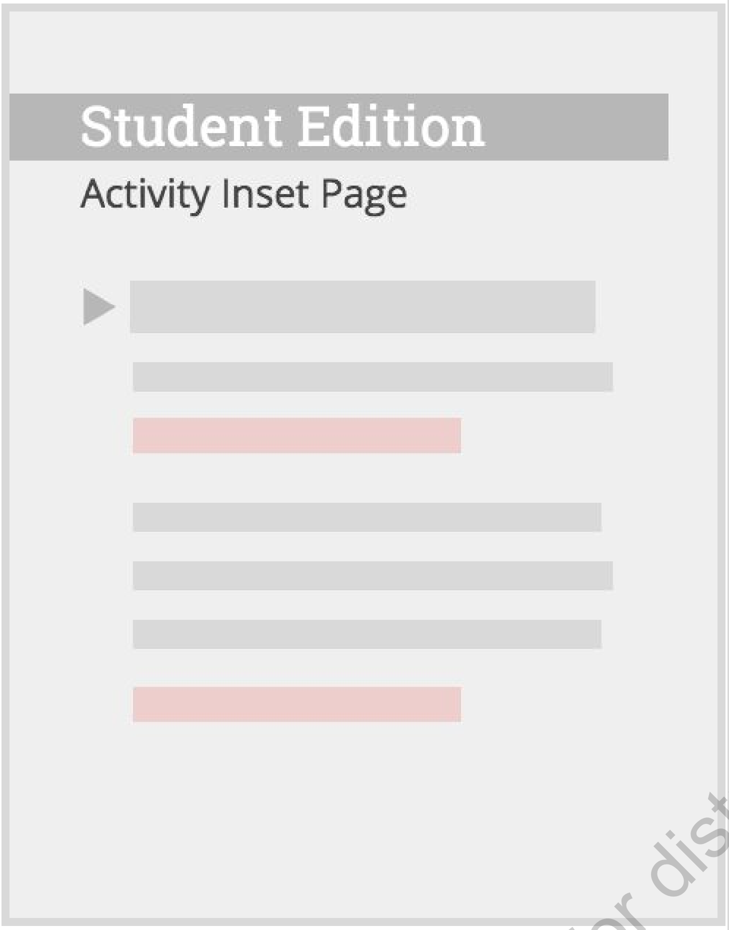
- "How are all of the equations for each tape diagram related?"
- "How are the two diagrams the same? And different?"

Differentiated Support

Students Who Need Help	**Students Ready for More**	**English Language Learners**	**Students with Disabilities**
Refer back to the equations from the Warm-up and guide students in noticing similarities.	Have students draw similar diagrams to those given, in which all xs become 12s, and vice versa. Challenge them to write one or more equations for each new diagram. [$4 \cdot 12 = x$ *and* $4 + 12 = x$]	***MLR2: Collect and Display*** Circulate and listen to students discuss their reasoning. Ask, "How do the parts of the equations match the parts of the tape diagrams?" Consider displaying relevant mathematical vocabulary.	Provide students with equation templates, such as "__ + __ = __".

Activity 2: Storytime with Tape Diagrams

Students connect scenarios with an unknown amount to tape diagrams and equations.



Launch

Give students 5 minutes of quiet work time for Part A. Then, have students compare answers with a partner, and work with their partner on Part B, followed by a whole-class discussion.

Monitor

Help students get started by asking, "What quantity is not known in the first story?"

Look for points of confusion:

- **Drawing unreasonably-sized parts.** Refer to previous diagrams as examples. Ask, "Which part should be larger here, the x or the 5?"
- **Being misled by key words**. If students interpret "total" to mean they need to add 5 and 20, or "five times as many" to mean they need to multiply 20 by 5, suggest they act out the scenarios or draw literal pictures to represent each scenario.
- **Not knowing how to interpret** $5x$. Explain that $5x$ is the same as $5 \cdot x$, and is read as "5 times x."
- **Not knowing what the** x **represents.** Explain that "The variable x represents the unknown value." And ask, " In this case, is it a part or the total?"

Look for productive strategies:

- Drawing tape diagrams that represent the equations (**MP4**). Students may draw all four tape diagrams first, and then match them to the scenarios before writing equations.

Connect

Have students share their tape diagrams, equations, and thinking for each story in Part A; then their story problem for Part B, and by which equation each can be represented (**MP2**).

Ask, "All of the equations and scenarios involve the same three numbers. Did you rule out any equations right from the start? Which ones? Why?"

Highlight the connections among each scenario, a corresponding tape diagram, and an equation (**MP7**).

Differentiated Support

Students Who Need Help	Students Ready for More	English Language Learners	Students with Disabilities
For Part B Problem 1, provide students with a sentence frame to represent $x = 20 + 5$ that mirrors Problem 1, but has the values and variables replaced with blanks to be filled in.		***MLR7: Compare and Connect*** Ask questions to draw attention to the connections among each scenario, a corresponding tape diagram, and an equation, to help strengthen students' mathematical language use and reasoning with multiple representations.	For each equation, provide students with a blank template of a tape diagram for students to complete. Also consider providing rulers to help students partition their diagrams.

Summary

Review and synthesize the connections between the visual appearance of tape diagrams and the equations they represent.

Synthesize

Display the summary table showing two tape diagrams.

Ask:

- "How are tape diagrams useful in visualizing a relationship between two quantities?"
- "How can you tell whether a tape diagram represents an addition or multiplication relationship?"
- "Where in a tape diagram do we see the equal sign that is in the equation it represents?"
- "Why can a diagram be represented by more than one equation?"
- "The top diagram can be seen as either addition or multiplication. Why might multiplication be the better operation here?"

Exit Ticket: Tape Diagrams and Equations

Students demonstrate their understanding of how tape diagrams and equations can show relationships between values.

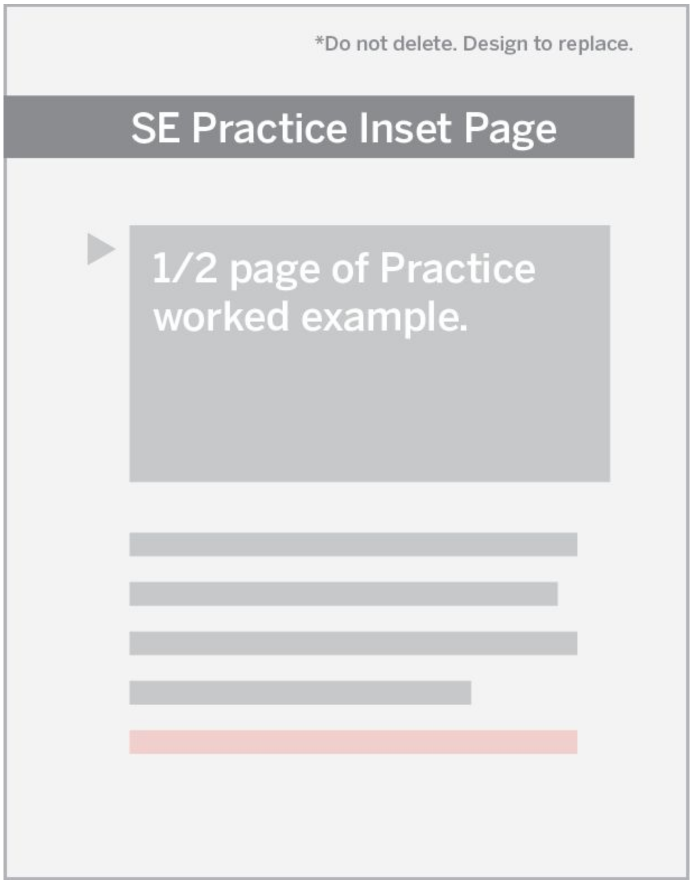
Success looks like...

1. Draw tape diagrams to represent equations of the forms $x+p=q$ and $px=q$.
2. Interpret (orally and in writing) tape diagrams that represent equations of the form $p+x=q$ or $px=q$.

Suggested next steps

- If a student is unclear about how to divide the tape diagrams based of the operations in each equation, consider:
  - Referring to Activity 1 and how the operation was identified based on how the diagrams were divided. Ask, "How did we figure out that the top one was multiplication and the bottom one was addition? How can that help you divide these tape diagrams?"
- If students are having difficulty with the variable, consider:
  - Allowing the student to solve the equation by asking "What times 5 gives us 15? Where would we put the 3 in the tape diagram? Now, let's take the 3 out and replace it with the variable, x."

Practice



Practice Problem Analysis

Problems	Refer to	Standards	DOK
1	Activity 2	6.EE.B.6 6.EE.B.7	2
2	Activity 2	6.EE.B.6 6.EE.B.7	2
3	Activity 1	6.EE.B.6	2
4 *(spiral)*	Unit 5 Lesson 13	6.NS.B.2 6.NS.B.3	3
5 *(spiral)*	Unit 3 Lesson 14	6.RP.A.3.c	3
6 *(formative)*	Unit 6 Lesson 4	6.EE.B.5	1

Unit 6 | Lesson 3

Tape Diagrams and Equations

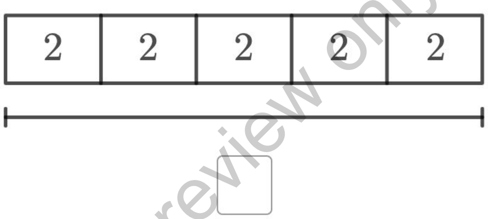
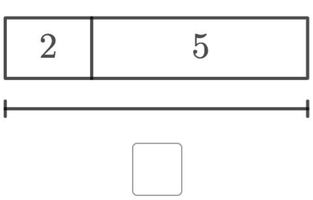
Let's see how tape diagrams and equations can show relationships between amounts.

Warm-up: Moving from Expressions to Equations

1. **Write an expression that each tape diagram could represent.**

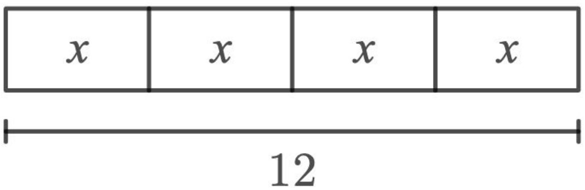
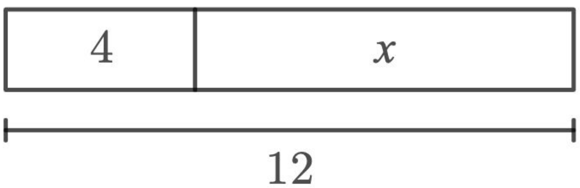
Tape Diagram	Expression
	
	

2. **Label the length of each tape diagram. Write as many equations as you can that each diagram could represent.**

Tape Diagrams	Equations
	
	

Activity 1: Tape Diagrams with Variables

Write as many different equations as you can that each tape diagram could represent.

Tape Diagrams	Equations
	
	

Activity 2: Storytime with Tape Diagrams

Part A

Draw a tape diagram to represent each story. Then use your drawings to determine which of the four equations best represents each story.

$$x + 5 = 20 \qquad x = 20 + 5$$

$$5 \cdot 20 = x \qquad 5x = 20$$

1. After Amara sold 5 kg of salt at the market in Gao on Friday, she had sold a total of 20 kg of salt for the week. She sold x kg before Friday.

Tape Diagram	Equation

2. Kofi traded some salt for 20 gold bars at the market in Timbuktu, which is 5 times as many as he was able to trade in Djenne. He got x gold bars when he traded in Djenne.

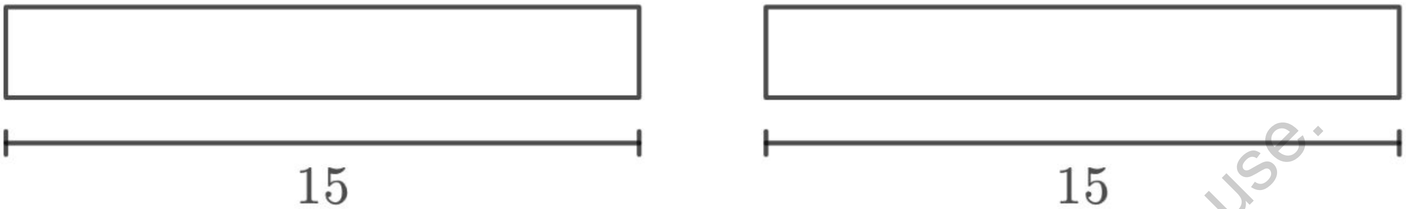
Tape Diagram	Equation

Part B

1. Choose one of the remaining equations not used in Part A, and create your own story that goes with the equation. Then draw a tape diagram for the equation.

Exit Ticket

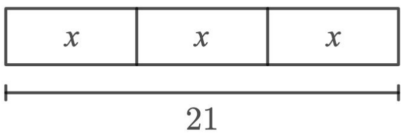
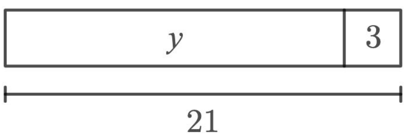
Finish the diagrams so that the first diagram represents $5 \cdot x = 15$, and the second diagram represents $5 + y = 15$.



Self-Assess
A. I can tell whether or not a tape diagram could represent an equation. B. I can use a tape diagram to represent a story.
Key: 1. I'm lost 2. I don't really get it 3. I'm starting to get it 4. I got it 5. I could teach it

Summary

Tape diagrams help us visualize relationships between quantities. Here are two examples:

Tape Diagram	Interpretations	Equations
	• as 3 equal parts whose sum is 21 • as a number multiplied by 3, and for which the product is 21.	$x+x+x=21$ $3 \cdot x = 21$ $3x = 21$
	• as two unequal parts, 3 and another number, and the sum is 21 • as 3 taken away from the total of 21 • as a number taken away from the total of 21, and whose difference is 3	$y+3=21$ $21-3=y$ $21-y=3$

My Notes:

Practice

1. Here is an equation: $x + 4 = 17$

    a. Draw a tape diagram to represent the equation. All three parts of the equation should be identified.

    b. How does your diagram show that $x + 4$ has the same value as 17?

2. Diego is trying to find the value of x in $5 \cdot x = 35$. He draws this diagram, but is not certain how to proceed.

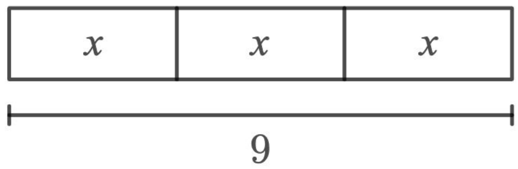
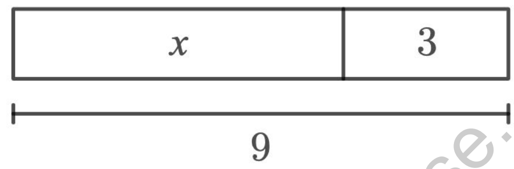
Complete the tape diagram so it represents the equation $5 \cdot x = 35$.

a. Find the value of x. $x = 7$

3. Match each equation to one of the two tape diagrams.

$x + 3 = 9$ $\quad 3 \cdot x = 9$ $\quad 9 = 3 \cdot x$ $\quad x + x + x = 9$

$3 + x = 9$ $\quad x = 9 - 3$ $\quad x = 9 \div 3$

4. A shopper paid $\$2.52$ for 4.5 pounds of potatoes, $\$7.75$ for 2.5 pounds of broccoli, and $\$2.45$ for 2.5 pounds of pears. What was the unit price of each item she bought? Explain your thinking.

5. A sports drink bottle contains 16.9 fluid ounces. Andre drank 80% of the bottle. How many fluid ounces did Andre drink? Explain your thinking.

6. Is the following equation true or false? Explain your thinking.

$a + 6 = 11$, when $a = 5$