Unit 5 | Lesson 1

A Historic Epidemic

Let's examine an epidemic.

Focus

Goal(s):

1. Explain and give an example of an epidemic.
2. Recognize the growth pattern of an epidemic given a table.
3. Recognize a graphical representation of an epidemic model.

Coherence

Today:

This unit launches with a close look at epidemics and the spread of disease. Here, students are informally introduced to representations of exponential growth by simulating a flu epidemic.

Previously:

In previous units, students were introduced to functions, and had to distinguish between linear and nonlinear functions.

Coming Soon:

Students will develop the concept of exponential growth and decay, before formally defining exponential functions.

Standards

- **Addressing:**
  HSF.LE.2
  Construct and compare linear, quadratic, and exponential models and solve problems.
  HSF.LE.5
  Interpret expressions for functions in terms of the situation they model.
- **Building On:**
  - 8.F.A.2
  - 8.F.B.4
  - 8.F.B.5
- **Building Towards:**
  - HSF.IF.B.4
  - HSF.IF.A.3

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Pacing Guide

Suggested Total Lesson Time | ~55 min

Warm-Up	Activity 1	Activity 2	Activity 3	Activity 4	Summary	Exit Ticket
5 min	25 min	20 min	10 min	N/A	5 min	5 min
Independent	*Whole Class*	*Whole Class*	*Groups*	*N/A*	*Whole Class*	*Independent*
NA	*HSF.LE.2*	*HSF.LE.2*	*HSF.LE.5*	*Zero-two standard(s)*	*N/A*	*N/A*
MP7	*MP 4*	*MP5*	*MP1*			
Desmos Activity Slides						
Slide 1	Slides 2	Slides 3–7	Slides 8–12	N/A	Slide 13	Slide 14

Vocabulary

Review

- **linear**
- **nonlinear**

Materials

- Exit Ticket PDF
- Additional Practice Book
- Black Line Masters
- Additional graph paper/ t-chart paper*
- Spreadsheet Technology*

* indicates materials not provided

Desmos Digital Classroom

Warm-up: Digital Feature Notice and Wonder
The warm-up provides the opportunity for students to visually observe a pattern of growth that doubles.

Activity 1: Simulating an Epidemic
After the class simulates the spread of the flu, students record their data in tables which interact dynamically with a graph.

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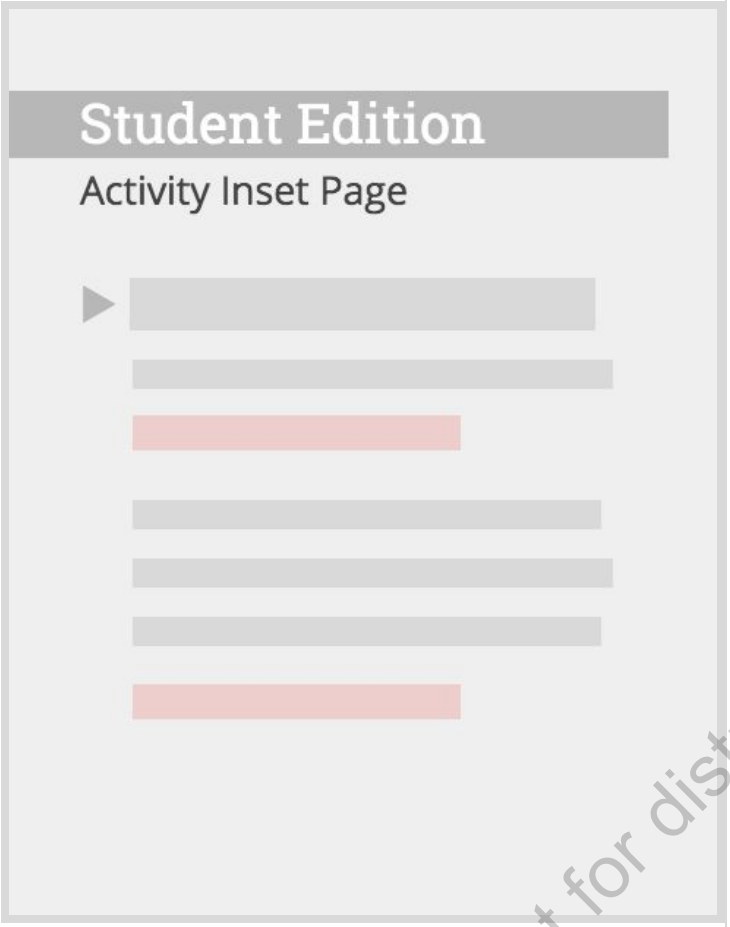
Warm-Up: Notice and Wonder

Students examine an image that depicts an increasing pattern. (It's exponential, but we're not calling it that just yet.)



Launch

Display the visual representation of (exponential) growth. Have students work independently for X minutes.

Monitor

Help students get started by asking what they see or don't see changing within the image.

Look for points of confusion:

- **Misinterprets growth as linear.** Ask the student, "Does the pattern add a constant number of dots each time?"
- **Does not realize the growth factor is constant.** Ask students to write the number of dots above each set and consider how they change.

Look for productive strategies:

- Describes the image *qualitatively* (e.g., the image begins with one dot and ends with many dots).
- Describes the image *quantitatively* (e.g., counting to determine how the number of dots is increasing).
- Makes real-world connections.

Connect

- **Display** the visual representation of (exponential) growth .
- **Have students share** their qualitative and quantitative responses.
-
- Ask, "How does this growth pattern compare to linear growth in earlier units?"
- **Define** *nonlinear*, a word students will have encountered before. (But don't introduce *exponential* just yet -- that will come late on in this unit.)

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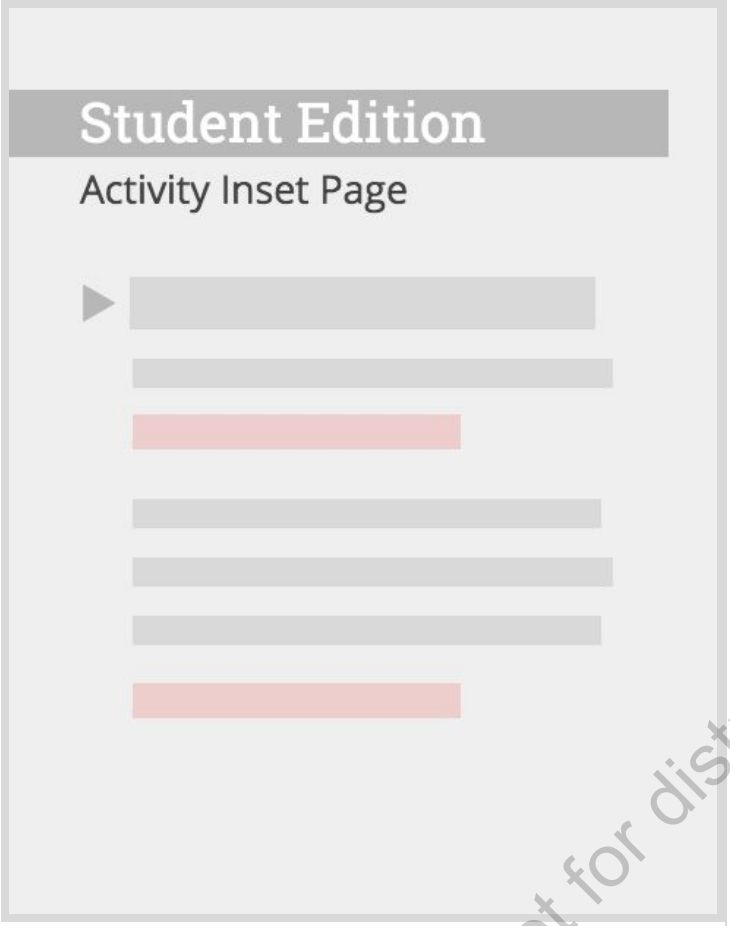
Alternate Warm-Up	Differentiated Support	
N/A	**English Language Learners** *MLR 8 Compare and Connect* Use this to amplify mathematical uses of language to communicate about doubling, rate of change, common factor. Invite students to use these words when stating their ideas, and restating ideas of others. Ask students to chorally repeat the phrases that include these words in context.	**Students with Disabilities** Consider using manipulatives to model the increase/decrease in each image, presenting only part of image at a time to prevent sensory overload, create lists of what might be happening (i.e., possible events) with the dots in each part of the image.

Activity 1: Simulating an Epidemic

Students simulate a flu epidemic and are introduced to its mathematical representations.



Launch

Have students read the instructions on the BLM. Then, perform multiple simulations of the "flu," randomly and confidentially selecting a number of students to be initially infected before each simulation. Students should record the data from the spread over several rounds, until the entire class is infected.

Monitor

Look for points of confusion:

- **Identifying the variables**. Ask which variable is independent: the round or the number of infected students?
- **Identifying a pattern**. Use arrows, with operations and numbers, to show changes in the tables.
- **Choosing a scale for the graphs:** Ask about smallest and largest values? Does it make sense to count by 1s, 2s, 5s..?

Look for productive strategies:

- Analyzing the tables to see what is happening from one round to the next.
- Noticing that each person infects another, so that the number of infected doubles each round.
- Discussing with their group if the growth is linear, and then verifying that it is not.

Connect

Display completed tables from each simulation.

Have students share patterns they noticed, along with their graphs. Choose groups that mentioned "doubling." Look for graphs that are exponential, but note that students have not been formally introduced to exponential functions. Have students describe the graphs in their own words.

Define *common factor*

Ask, "How does the pattern of epidemics differ from previous functions?"

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Differentiated Support

Students Who Need Help	Students Ready For More	English Language Learners	Students with Disabilities
Provide pre-populated tables of data for simulations. Direct students to complete the activity for each simulation.	Have students complete the Extension problem.		Provide specific roles when working within groups, i.e. data recorder, grapher, reporter.

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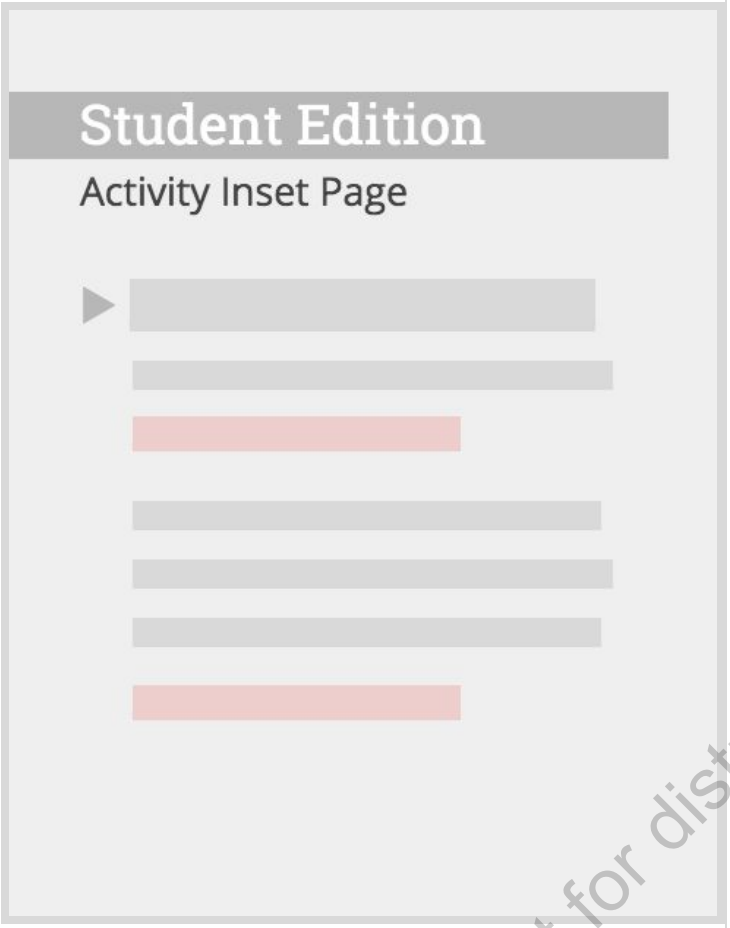
Activity 2: Global Implications

Students predict how the data from their simulations would change, given real-world constraints.



Launch

Keep students in their groups. Provide them with additional graph paper or chart paper, and give them the option to use technology. (MP5)

Monitor

Help students get started by having them prepare a table or draw models for Problem 1.

Look for points of confusion:

- **Difficulty visualizing changes to the graph.** Provide students with blank graphs with different scales to help process each change.
-
- **Not understanding prior contact or immunity.** Define these terms for students, and provide blank tables if helpful. Funnel questions to help students fill in the table or have them draw a model.
-

Connect

Display: Table and graph of the number of doubling cases, alongside first simulation graph,.

Have students share how they thought each constraint would affect the spread of disease.

Highlight that the disease spreads more slowly as more people are ultimately infected, and if more people are naturally immune.

Ask, "How do you think we could model an epidemic algebraically? How about with an equation?"

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Differentiated Support

Students Who Need Help	Students Ready for More	English Language Learners	Students with Disabilities
Provide a partially filled table for Problem 1. Direct students to skip Problem 3.			

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Summary

Review and synthesize how epidemics can be modeled nonlinearly.

Synthesize

Display the Summary, which introduces the students to the kinds of change and growth they will encounter in Unit 5, and have them read it.

Have students share what they found most interesting about today's lesson and what more would they like to learn.

Review vocabulary:

- Linear
- Nonlinear
- Immunity
- Epidemic

Ask, "We've seen what the spread of a disease looks like. What do you think happens to a graph when everyone takes medicine to prevent the spread of a disease?"

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Exit Ticket

Students demonstrate their understanding by explaining the advantages of using tables and graphs to model an epidemic.

Success looks like...

1. Students can explain and give an example of an epidemic.
2. Students can recognize the growth pattern of an epidemic given a table.
3. Students can recognize a graphical representation of an epidemic model.

Suggested next steps

If students use repeated addition to solve Problem 1b, consider:

- Reviewing multiplicative strategies from Activity 2
- Assigning Practice Problem 3
- Asking, "How would you solve this if, instead of multiplying by 3, you were multiplying by 103?"

Next Steps

- Students can explain what an epidemic is.
- Students can explain some advantages and disadvantages between using models, tables, and graphs.
- Students can explain why technology may be most appropriate to model an epidemic.

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Practice

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Practice Problem Analysis

Problems	Refer to	Standards	DOK
1	Activity 2	HSF.IF.A.3	*1*
2	Activity 2	HSF.IF.B.4	*2*
3	Unit 4 Lesson 10	HSF-IF.A HSF.IF.A.2	*1*
4	Unit 4 Lesson 4	HSF-IF.A HSF-IF.C	*2*
5	Unit 2 Lesson 19	HSA.REI.B.3	*1*
6 *(formative)*	Unit 5 Lesson 2	HSF-LE.A 8.F.B.4	*2*

Lesson A1.U5.1

Notice and Wonder

Let's look at some growth.

Warm Up

What do you notice? What do you wonder?

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Activity 1: Simulating an Epidemic

Record the data from the class simulation. Then answer each question.

1. For each simulation, complete each data table so that it shows the number of infected students after each round. (Some cells in the tables may be left empty, depending on the number of students in your class.)

Simulation 1:

x						
y						

Simulation 2:

x						
y						

Simulation 3:

x						
y						

2. What patterns do you notice?

3. How did you define your variables x and y?

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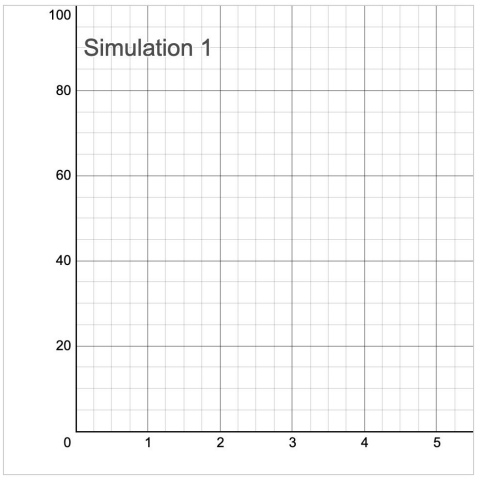
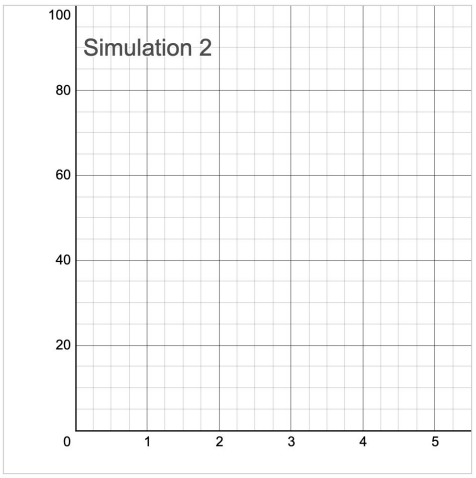
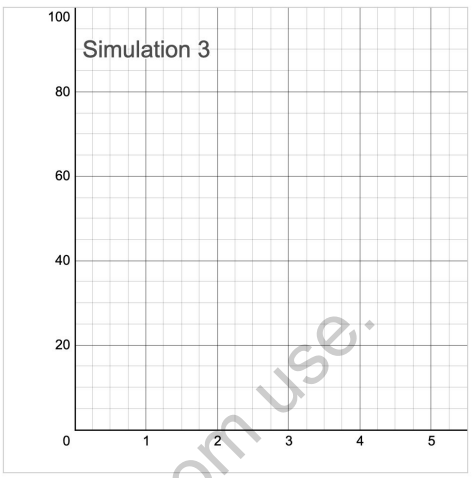
4. Create a graph for each of your tables.







5. For each simulation, determine the number of rounds do you think are needed until $1,000$ students would be infected?

6. Are these graphs linear or nonlinear? Explain how you know.

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Are you ready for more?

Directions: Suppose a simulation were conducted in a school with $2,000$ students, and started with 4 infected students. Each student can infect one other student. How many students would be infected after Round 8? Use drawings, models, or words to explain your reasoning.

x_3	y_3
0	4
1	16
2	64
3	256
4	1024
5	4096

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Activity 2: Global Implications

In your groups, answer each question.

1. Predict how your graphs would change if every sick person infected 2 new people (rather than just 1). What do you think would happen to the shape of the graph?

2. In real life, infected people might come in contact with other infected people (rather than always infecting someone new). How would this change the outcome of the simulation?

3. In real life, some people might be immune. Predict how this could change the outcome of the simulation.

4. Name other real-world phenomena that could result in these types of patterns.

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Exit Ticket: Representing an Epidemic

1. What are some advantages and disadvantages of tables and graphs when representing an epidemic?

	Tables	**Graphs**
Advantages		
Disadvantages		

2. How can technology be used for modeling an epidemic?

Self-Assess
A. I can explain an epidemic. B. I can recognize an epidemic using data in a table. C. I can recognize an epidemic on a graph.
Key: 1. I'm lost 2. I don't really get it 3. I'm starting to get it 4. I got it 5. I could teach it

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Summary

Have you ever seen one of those zombie movies? What makes zombies scary (besides the brain-munching) is their *speed*. No, not their famously slow shuffle. Zombies are speediest when it comes to population growth. Just one nibble in a busy food court, and pretty soon the whole mall is packed to the gills with the undead!

Sure, these chomp-happy ghouls may be fiction, but the explosive, violent growth they exhibit is not. They're based on the very real, very lethal epidemics that have ravaged human populations throughout history.

Smallpox, polio, measles, bubonic plague–the explosive spread of each of these diseases was followed by an immense human cost. When the number of infected keeps doubling, then doubling again, and again and again– things can get pretty overwhelming pretty fast.

In these next lessons, not only will we look at how to map out this kind of growth, but think about how to manage and predict it. We'll hear stories about the toll these diseases took on us as a species, as well as stories of perseverance and hope – and what we did to fight back.

Welcome to Unit 5.

My Notes:

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Practice

1. For the two following patterns, describe how their growth is similar or different.
   a. $1, 4, 7, 10, 13, 16, 19, 22, 25,$
   b. $1, 3, 9, 27, 81, 243,$

2. One student isn't feeling well, and transforms into a zombie while at a school dance. The table represents what happens afterwards, where x represents the number of minutes that have passed since the first student turned into a zombie, and y is the total number of zombies at the dance.. The input is in minutes and the output is the number of zombies when a certain number of minutes have passed.

x	0	5	10	15	20	25	30	35	40	45	50	55
y	1	3	9	27	81	243	729	732	732	732	732	732

   What does this table tell you about what happened? How do you know?

3. Select the correct input-output pairs for the function $f(x) = x^2$.
   a. (-1, 1)
   b. $(3, 9)$
   c. $(4, 8)$
   d. $(3, 6)$
   e. $(\frac{1}{2}, \frac{1}{4})$
   f. $(\frac{1}{2}, 1)$

4. Match each function rule to its correct corresponding verbal description.

   a. $(f(x) = 3x - 2$
   b. $(g(x) = 3(x - 2)$
   c. $h(x) = 2x - 3$
   d. $h(x) = 2(x - 3)$

   1. To get the output, subtract 3 from the input, then multiply the result by 2.
   2. To get the output, multiply the input by 3, then subtract 2 from the result.
   3. To get the output, subtract 2 from the input, then multiply the result by 3.
   4. To get the output, multiply the input by 2, then subtract 3 from the result.

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5. Solve for the solution set of the following inequality: $5x - 10 >= 3(2 - x) + 4$

6. Label each table as linear or nonlinear. For tables that could represent a linear function write the function rule.

a.

x	0	1	2	3	4	5
$f(x)$	0.5	1	1.5	2	2.5	3

b.

x	0	1	2	3	4	5
$g(x)$	0.5	1.5	4.5	13.5	40.5	121.5

c.

x	-4	-2	0	2	4	6
$h(x)$	-64	-8	0	8	64	216

d.

x	-4	-2	0	2	4	6
$k(x)$	12	6	0	-6	-12	-18

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Practice

1. For the two following patterns, describe how their growth is similar or different.
   a. $1, 4, 7, 10, 13, 16, 19, 22, 25,$
   b. $1, 3, 9, 27, 81, 243,$

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   What does this table tell you about what happened? How do you know?

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   1. To get the output, subtract 3 from the input, then multiply the result by 2.
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$f(x)$	0.5	1	1.5	2	2.5	3

b.

x	0	1	2	3	4	5
$g(x)$	0.5	1.5	4.5	13.5	40.5	121.5

c.

x	-4	-2	0	2	4	6
$h(x)$	-64	-8	0	8	64	216

d.

x	-4	-2	0	2	4	6
$k(x)$	12	6	0	-6	-12	-18

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TE Lesson A1.5.2

Unit 5 | Lesson 2

Patterns of Growth

Let's compare different patterns of growth.

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Focus

Goals:

- Students can describe patterns in tables that represent linear and exponential (here, referred to as "nonlinear") relationships.
- Students can create tables and write expressions given descriptions of linear and exponential (here, referred to as "nonlinear") relationships.

Coherence

Today:

Students study linear and exponential patterns by examining tables, observing common differences and common factors. Students match tables with corresponding expressions, and construct a table given a description of an exponential relationship.

Previously:

In Grade 8, students were informally introduced to exponential relationships.

Coming Soon:

Students will continue to build on their conceptual understanding of linear and exponential relationships by exploring tables and graphs showing different growth patterns.

Standards

- **Addressing:**
  HSF.BF.A.1.a
  Determine an explicit expression, a recursive process, or steps for calculation from a context.

  HSF.IF.B.4
  For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.

- **Building Towards:**
  - HSF.BF.A
  - HSF.LE.A.1
  - HSF.LE.A.2

Pacing Guide

Suggested Total Lesson Time | ~55 min

Warm-Up	Activity 1	Activity 2	Summary	Exit Ticket
8 min	20 min	15 min	7 min	5 min
Independent	*Pairs*	*Pairs*	*Whole Class*	*Independent*
HSF.BF.A, HSF.LE.A.2	*HSF.BF.A.1.a, HSF-IF.B.4*	*HSF.BF.A.1a*	*HSF.BF.A.1.a*	*HSF.BF.A.1.a*
Desmos Activity and Presentation Slides				
Slide 1	Slides 2-8	Slides 9-11	Slide 12	Slide 13-14

Vocabulary

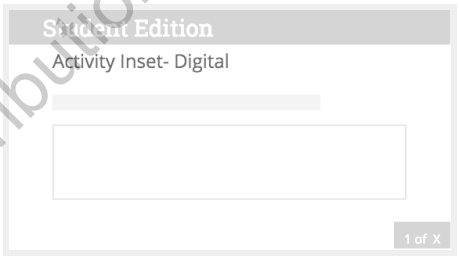
New

- **common difference**
- **common factor**

Materials

- A1.5.2 Exit Ticket PDF
- A1.5.2 Additional Practice Book
- Vocabulary List
- A1.5.2 Black Line Master

Desmos Digital Classroom



Slide 9

Activity 2: Digital Card Sort

Students match tables and expressions to the scenarios that they represent.

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Warm-Up: Which One Doesn't Belong?

Students compare tables of linear and nonlinear data to determine which table doesn't belong.

Launch

Display the four tables. Have students work individually for X minutes, and then allow 30 seconds for them to turn and talk to their neighbor about which table they selected and why.

Monitor

Help students get started by prompting them to first find a pattern for each table.

Look for points of confusion:

- **Struggling to find a pattern.** Encourage students to use addition or multiplication to find a pattern in the y column.

Look for productive strategies:

- Drawing lines or arcs between consecutive terms in the y columns of each table to understand the rate of change. Encourage students to label them with an operation and number to highlight the difference between the patterns.

Connect

Have students share their partner's choice of table and reasoning with the class.

Highlight that Tables A, B, and D are linear and Table C is nonlinear.

Define *common difference* as the difference between two consecutive terms in a linear sequence.

Ask, "What pattern do you notice in Table C? How does this differ from the other tables?"

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Alternate Warm-up Activity

How are these tables similar? How are they different?

Table C	x	0	1	2	3	4
	y	1	4	16	64	256

Table D	x	0	1	2	3	4

Differentiated Support

English Language Learners

Students with Disabilities

	y	4	8	12	16	20

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Activity 1: Pharmacy Expansion

Students compare two plans (one linear, one nonlinear) for the expansion of a pharmacy, determining which results in more growth.

Launch

Read the scenario and the two plans with the class. Then have students work through the problems in pairs.

Monitor

Help students get started by having them find the precise number of stores that open each year. Refer them to the strategies they used in the warm-up.

Look for points of confusion:

- **Struggling to describe Plan B in words.** Encourage students to use the term *nonlinear* to describe patterns that do not have a constant rate of change.

Look for productive strategies:

- Using repeated calculations in tables or written expressions.

Connect

Display the table from Problem 2.

Have students share how the tables are similar and how they are different. Model how to annotate the tables as students describe the patterns they found.

Highlight that a pattern generated by a *common difference* is linear, while a pattern generated by a *common factor* is not linear.

Ask, "Which plan resulted in the greatest number of pharmacies over 10 years? Is this what you predicted? How has your perspective changed?"

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Differentiated Support

Students Who Need Help	Students Ready for More		Students with Disabilities
To support struggling students, have them complete the table from Question 2 up to Year 5 and then move on to answering the questions that follow.	Have students complete the Extension problems.		*Representation: Internalize Comprehension* Allow students to use calculators to ensure inclusive participation in the activity.

Activity 2: Friends and Followers

Students will read descriptions of two different scenarios and determine which representations (tables and expressions) match each description.

Launch

Read both scenarios with the class. Draw attention to phrases indicating operations.

Monitor

Help students get started by having them generate a table for each scenario, and then match these to the given tables.

Look for points of confusion:

- **Unsure about expressions A and C.** Ask, "Which scenario describes repeated addition? Which describes repeated multiplication?"

Look for productive strategies:

- Writing equivalent expressions for D and F to determine which is equivalent to A, and which is equivalent to C.

Connect

Display the two scenarios, as well as the six expressions and tables.

Have students share their answers. Then use Problems 1–3 to facilitate a discussion about the patterns described in Scenarios 1 and 2.

Highlight that, in Scenario 1, the constant factor of 3 means that the number of Tyler's followers gets 3 times bigger every year.

Define *common factor:* the factor we multiply each term by to generate a (type of) nonlinear pattern.

Ask, "What does the common difference mean in Scenario 2?"

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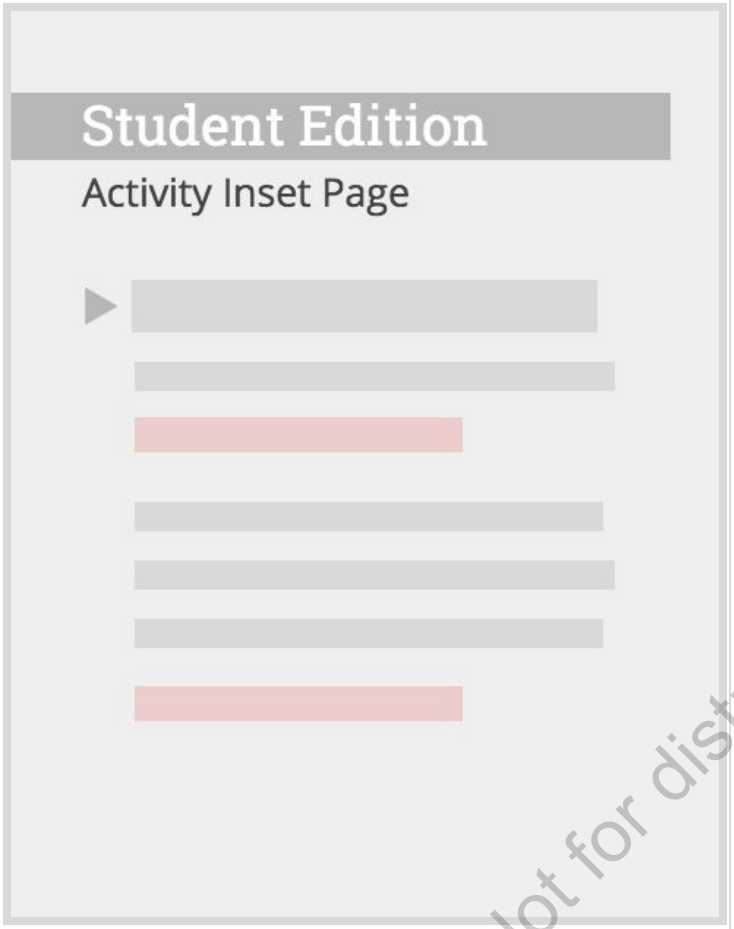
Differentiated Support

Students Who Need Help	Students Ready for More	English Language Learners	Students with Disabilities
If students need further help connecting expressions and scenarios, have them use calculators to evaluate each expression and determine which tables (B or D) those values belong to.		*MLR8: Discussion Supports*For each observation shared, ask students to restate what they heard using precise mathematical language. Consider providing students time to restate what they hear to a partner before selecting students to share with the class.	*Representation: Internalize Comprehension* Use color coding and annotations to highlight connections between the text of each scenario and the corresponding tables and expressions. Have students identify the common difference or factor in each representation.

Summary

Students will observe tables with two different growth patterns and summarize the key features of both patterns. Students may use the space on this page to capture key concepts, questions, and ideas from this lesson.



Synthesize

Display: The two tables from Slide X.

Have students share: Allow students 2 minutes to look at the tables and jot down the patterns they see.

Ask:

- "How do the values in each table grow?"
- "Which table of values show a common difference? Which one shows a common factor?"
- "What expression can we write to find the next term in the first table? the 6th term? The 7th term?"
- "What expression can we write to find the next term in the second table? the 6th term? the 7th term?"

Highlight: Explicitly state the two patterns of growth covered in today's lesson, using each vocabulary term.

Define vocabulary: *common difference* and *common factor* when identifying the growth patterns in the tables displayed. Annotate the tables to help students visually see how the common difference and common factor determine the pattern of growth for each table.

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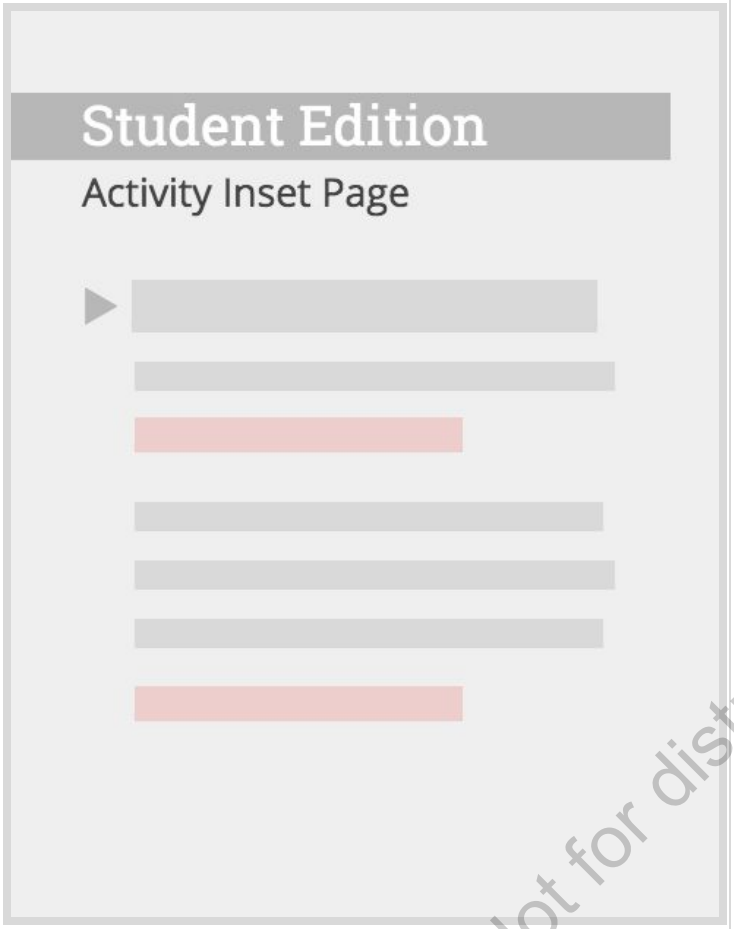
Exit Ticket

Students demonstrate their understanding by determining patterns of growth given two scenarios and contextualizing the common difference and factor, with suggested next steps.



Success looks like

1. Students can describe linear and nonlinear patterns of growth.
2. Students can tell if a table has a common difference or common factor.
3. Students can write an expression to find the next term in a linear or nonlinear pattern.

Next Steps

If students struggle to describe linear and nonlinear patterns of growth in Question 1, consider:

- Revisiting the Warm-up activity

If students are unable to identify a common difference or factor from a table of values in Question 2, consider:

- Reviewing tables of values showing repeated addition and repeated multiplication
- Revisiting the differences between Plans A and B in Activity 1

If students write incorrect or incomplete expressions to represent the patterns of growth in Questions 3 and 4, consider:

- Revisiting Activity 2

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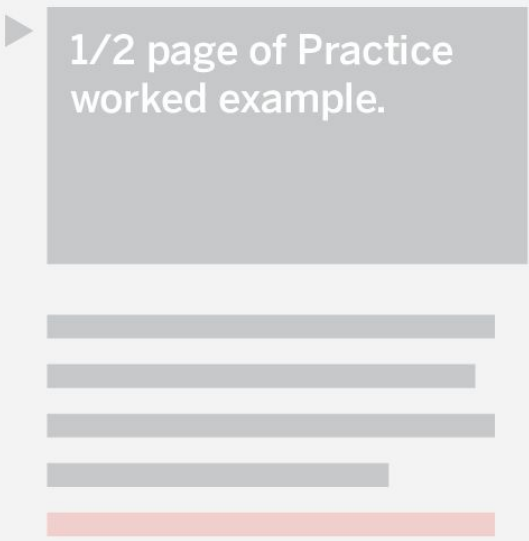
Practice

MP8

*Do not delete. Design to replace.

SE Practice Inset Page

1/2 page of Practice worked example.



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SE Practice Inset Page

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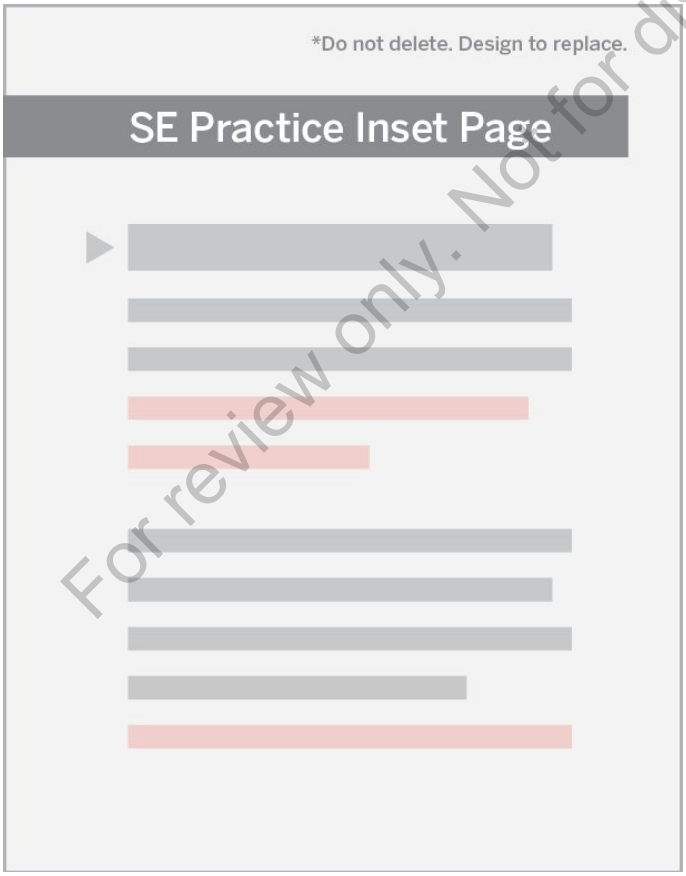
SE Practice Inset Page



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Practice Problem Analysis

Question	Refer to	Standards	DOK
1	5.2.1	HSF.LE.A.2	*2*
2	5.2.2	HSF.IF.B.4	*1*
3	5.2.1	HSF.LE.A.1	*2*
4	5.2.1	HSF.LE.A.1	*2*
5	5.2	HSF.LE.A.2	*3*
6	4.2	HSA.SSE.A.1	*1*

Unit 5 | Lesson 2

Patterns of Growth

Let's compare different patterns of growth.

Warm-Up: Which One Doesn't Belong?

Which table does not belong? Justify your answer.

Table A

x	y
1	8
2	16
3	24
4	32
8	64

Table B

x	y
0	0
2	16
4	32
6	48
8	64

Table C

x	y
0	1
1	4
2	16
3	64
4	256

Table D

x	y
0	4
1	8
2	12
3	16
4	20

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Activity 1: Pharmacy Expansion

PharmAid, a retail pharmacy company, has 5 locations in Orlando, Florida. Due to reports of a recent outbreak of an infectious disease in other states, PharmAid is considering two plans for expanding their chain of pharmacies nationwide.

PLAN A: Open 20 new pharmacies each year.
PLAN B: Double the number of pharmacies each year.

1. Which plan do you think will result in a greater number of pharmacies over the next 10 years?

2. Complete the table for each plan.

Number of PharmAid pharmacies		
Year	**Plan A**	**Plan B**
0	5	5
1	25	10
2		
3		
4		
5		
6		
7		
8		
9		
10		

3. Compare and contrast **Plans A** and **B**. What pattern(s) did you notice as you were filling in the table from one year to the next?

4. Does either plan have a *common difference*? If so, what is it?

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5. If you know how many pharmacies there are in a certain year, how could you determine the number of pharmacies there will be 3 years later according to **Plan A**? What about **Plan B**?

6. Which plan will result in more pharmacies over the next 10 years? Does this match your prediction?

Extension

Directions: Suppose PharmAid decides to expand from the 5 pharmacies it has now, so that it will have between 600 and 800 pharmacies 5 years from now.

1. Come up with a plan for the company to achieve this, so that it adds the same number of pharmacies each year.

2. Come up with a plan for the company to achieve this, so that the number of stores is multiplied by the same factor each year. (You may need to round the outcome to the nearest whole number for some years.)

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Activity 2: Friends and Followers

Read the two scenarios. Then match each table or expression with its corresponding scenario.

SCENARIO 1: Tyler has 80 followers on The Gram. His number of followers triples each year. How many followers will he have after 4 years?

SCENARIO 2: Priya currently has 80 friends on a social media app. Everyday she adds 3 new friends. How many social media friends will she have after 4 days?

A. $80 \cdot 3 \cdot 3 \cdot 3$

D. $80 + 4 \cdot 3$

B.

x	y
0	80
1	240
2	720
3	2160
4	6480

E.

x	y
0	80
1	83
2	86
3	89
4	92

C. $80 + 3 + 3 + 3 + 3$

F. $80 \cdot 81$

1. Which scenario represents a linear pattern? Explain your reasoning.
2. Which scenario represents a nonlinear pattern? Explain your reasoning.
3. Which scenario has a constant factor? Explain what the constant factor means in the scenario.

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Exit Ticket: Linear or Not?

Directions: Use the tables to answer each question.

Table A

Time (minutes)	**Height, in centimeters,** of the amount of water in a swimming pool that is being filled up
0	50
1	53
2	56
3	59
4	62

Table B

Time (months)	**Height, in inches,** of rainwater that has been collected in a barrel
1	2
2	4
3	8
4	16
5	32

1. How are the values growing in **Table A**? How are they growing in **Table B**?

2. Which table shows a common difference? Which shows a common factor?

3. Write an expression for the height of the water in the pool after 10 minutes.

4. Write an expression for the height of rainwater in the barrel after 7 months.

Self-Assess

A. I can describe patterns of growth in a table of values in my own words.
B. I can tell if a table has a common difference or common factor.
C. I can write an expression to find the next term in a linear or nonlinear pattern.

Key:
1. I'm lost
2. I don't really get it
3. I'm starting to get it
4. I got it
5. I could teach it

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Summary

In this lesson we looked at tables and expressions that represent two different patterns of growth.

x	y
1	50
2	150
3	250
4	

- This pattern increases at a constant rate of 100. This is a *linear* pattern.
- 100 is the *common difference*. We can add 100 to any term in this pattern to find the next one. We can determine any term in the pattern by repeated addition of the common difference.

In the second table…

x	y
1	50
2	150
3	450
4	1350

- This pattern grows by a factor of 3. This pattern is *nonlinear*.
- 3 is the *common factor*. We can multiply any term in this pattern by 3 to find the next term. We can determine any term in the pattern by repeated multiplication of the common factor.

My Notes:

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SE Practice

Directions: Answer the questions below.

1. The population of a colony of ants is $10,000$ at the start of April. Since then, it triples each month.

Months since April	Ant population
0	$10,000$
1	
2	
3	
4	

a. Complete the table.

b. What do you notice about the population from month to month?

c. If there are n ants in one month, how many ants will there be a month later?

2. A swimming pool contains 500 gallons of water. A hose is turned on, and it fills the pool at a rate of 24 gallons per minute. Which expression represents the amount of water in the pool, in gallons, after 8 minutes?

A. $500 \cdot 24 \cdot 8$

B. $500 + 24 + 8$

C. $500 + 24 \cdot 8$

D. $500 \cdot 24^8$

3. The population of a city is $100,000$. It doubles each decade for 5 decades. Select **all** expressions that represent the population of the city after 5 decades.

A. $32,000$

B. $320,000$

C. $100,000 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$

D. $100,000 \cdot 5^2$

E. $100,000 \cdot 2^5$

4. This table shows the height, in centimeters, of the water in a swimming pool at different times since the pool started to be filled.

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Minutes	Height
0	150
1	150.5
2	151
3	151.5

a. Does the height of the water increase by the same amount each minute? Explain how you know.

b. Does the height of the water increase by the same factor each minute? Explain how you know.

5. Bank **account C** starts with $\$10$, and doubles each week. Bank account D starts with $\$1000$, and grows by $500 each week. When will **account C** contain more money than **account D**? Explain your reasoning.

6. Suppose *C* is a rule that takes time as the input and gives your class on Monday as the output. For example, *C*(10:15) = Biology.

a. Write three sample input-output pairs for *C*.

b. Does each input for *C* have exactly one output? Explain how you know.

c. Explain why *C* is a function.

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Unit 5 | Lesson 2

Practice

Directions: Answer the questions below.

1. The population of a colony of ants is $10,000$ at the start of April. Since then, it triples each month.

Months since April	Ant population
0	$10,000$
1	
2	
3	
4	

a. Complete the table.

b. What do you notice about the population from month to month?

c. If there are n ants in one month, how many ants will there be a month later?

2. A swimming pool contains 500 gallons of water. A hose is turned on, and it fills the pool at a rate of 24 gallons per minute. Which expression represents the amount of water in the pool, in gallons, after 8 minutes?

A. $500 \cdot 24 \cdot 8$

B. $500 + 24 + 8$

C. $500 + 24 \cdot 8$

D. $500 \cdot 24^8$

3. The population of a city is $100,000$. It doubles each decade for 5 decades. Select **all** expressions that represent the population of the city after 5 decades.

A. $32,000$

B. $320,000$

C. $100,000 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$

D. $100,000 \cdot 5^2$

E. $100,000 \cdot 2^5$

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4. This table shows the height, in centimeters, of the water in a swimming pool at different times since the pool started to be filled.

Minutes	Height
0	150
1	150.5
2	151
3	151.5

a. Does the height of the water increase by the same amount each minute? Explain how you know.

b. Does the height of the water increase by the same factor each minute? Explain how you know.

5. Bank **account C** starts with $\$10$, and doubles each week. Bank account D starts with $\$1000$, and grows by $500 each week. When will **account C** contain more money than **account D**? Explain your reasoning.

6. Suppose *C* is a rule that takes time as the input and gives your class on Monday as the output. For example, *C*(10:15) = Biology.

a. Write three sample input-output pairs for *C*.

b. Does each input for *C* have exactly one output? Explain how you know.

c. Explain why *C* is a function.

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Unit 5 | Lesson 3

Growing and Growing

Let's connect different patterns to their tables and graphs.

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Focus

Goals:

1. Compare linear and exponential relationships by performing calculations and by interpreting graphs that show two growth patterns.

Coherence

Today:

Students continue to build a conceptual understanding of linear and exponential relationships by exploring tables and graphs that show different linear and nonlinear growth.Students increase their understanding of diseases in Activity 2, looking at the spread of flu (MP2, MP4).

Previously:

Students observed patterns characterized by common differences and factors by examining tables. Students matched tables with expressions and constructed a table of an exponential relationship.

Coming Soon:

Students will formally define exponential growth connecting its equation and graph. They will write and interpret equations representing exponential growth.

Standards

- **Addressing:**
  HSF.IF.B.4
  For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.

  HSF.LE.A.3
  Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.

- **Building On:**
  - 6.EE.A.1
  - 7.EE.B.3
  - 8.F.B.5

- **Building Towards:**
  - HSF.BF.A.1.a
  - HSF.LE.A.3

Pacing Guide

Suggested Total Lesson Time | ~ 60 min

Warm-Up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	15 min	30 min	5 min	5 min
Independent	*Pairs*	*Pairs*	*Whole Class*	*Independent*
8.F.B.5, HSF.LE.A.3	*HSF.LE.A.3*	*HSF.IF.B.4*	*HSF.LE.A.3*	*HSF.LE.A.3*
MP4	*MP4*	*MP2*	*MP4*	*MP4*
Desmos Activity and Presentation Slides				
Slide 1	Slides 2–6	Slides 7–11	Slide 12	Slide 13

Vocabulary

Review words

- **Common difference**
- **Common factor**

Materials

- Exit Ticket PDF
- Additional Practice Book

* indicates materials not provided

Desmos Digital Classroom

Activity Warm-up
Overlaid Sketches
Students sketch the growth of bacteria, which you can overlay to visualize collective thinking.

Activity 2
Interactive Graphs
Using tabular data, students move points on an interactive graph to model exponential growth.

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Warm-Up: Splitting Bacteria

Students draw a nonlinear growth pattern, connecting the pattern to repeated multiplication.

Launch

Pair students. Discuss cell division produces new cells.

Monitor

Help students get started by prompting to make a tree with two new branches growing from each old branch.

Look for points of confusion:

- **Not divide bacteria into two**: Remind students that each cell divides into two new cells.
- **Not remember how to convert from minutes to hours**: Remind them there are 60 minutes in 1 hour.

Look for productive strategies:

- Make a tree diagram showing the bacteria splits into two new bacteria every 20 minutes.

Connect

Display the Warm-Up narrative and incomplete diagram.

Have students share their sketch with their partner. Select student pairs to share their drawings and observations about how the bacteria produces new cells..

Highlight the growth pattern is nonlinear because we are *multiplying* (not *adding*) by 2 new cells each time.

Ask: "What pattern do you see in the bacteria's growth?"

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Alternate Warm-up Activity

Alternative Warm Up:
Provide students with a prefilled diagram of bacteria growth.

1. The diagram shows the number of bacteria at 40 and 60 minutes. What pattern do you notice? [*Each bacterium divides into two new bacteria every 20 minutes so the number of bacteria is multiplied by 2 each time.*]
2. How many bacteria are there after the first hour? Explain your thinking. [*Total of 8. The bacterium divide in two every 20 minutes and there will be three divisions in one hour.*
3. *That means there will be $2 \cdot 2 \cdot 2 = 8$ total bacteria. Some students may make the connection $2^3 = 8$.*]

Differentiated Support

English Language Learners:
MLR2: Collect and Display. Capture students' words and phrases while drawing the bacteria division and scribe on the board. Organize, revoice, and connect this to another language in a display that students can refer to and make connections. This provides feedback and meta-awareness of language.

Students with Disabilities:
Representation: Internalize Comprehension. Represent the cell division through different modalities using manipulatives. Have students sketch the diagram using models from manipulatives model.

Activity 1: Viral Memes

Students explore linear and nonlinear relationships by interpreting tables and graphs that show each growth pattern in order to understand nonlinear growth.

Launch

In pairs or trios, have students read scenarios aloud and discuss the reflection questions. Students may answer questions independently or by collaborating with a partner.

Monitor

Help students get started by modeling finding common differences and common factors given a table. Discuss the change in the dependent variable over equal increments in the independent variable.

Look for points of confusion:

- **Not remember graphing on the coordinate plane:** Spiral back to review reading a graph in the coordinate plane. Invite students to demonstrate finding and describing slopes.
- **Shift their calculations by one hour ahead or behind:** Have them double check their work in the table, cell by cell.

Look for productive strategies:

- Look for common differences and then common factors.
- Identify ordered pairs from a table of values and graph.

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Activity 1: Viral Memes continued

Students explore linear and nonlinear relationships by interpreting tables and graphs that show each growth pattern in order to understand nonlinear growth.

Connect

Display the tables of meme shares.

Have students share their observations about the relationship between the tables and graphs of each the meme shares.

Highlight "We can find a common factor by using either a table of values or a graph."

Define *common factor* as the factor we multiply each term by to generate a (type of) nonlinear pattern

Ask: "How do the growth patterns compare between the scenarios?" [Andre's shares are linear. Jada's shares are nonlinear.]

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Differentiated Supports

Students Who Need Help
Focus on questions 1, 4, and 5. Provide support when finding common difference and common factor, helping them connect both to linear and nonlinear growth on the graphs.

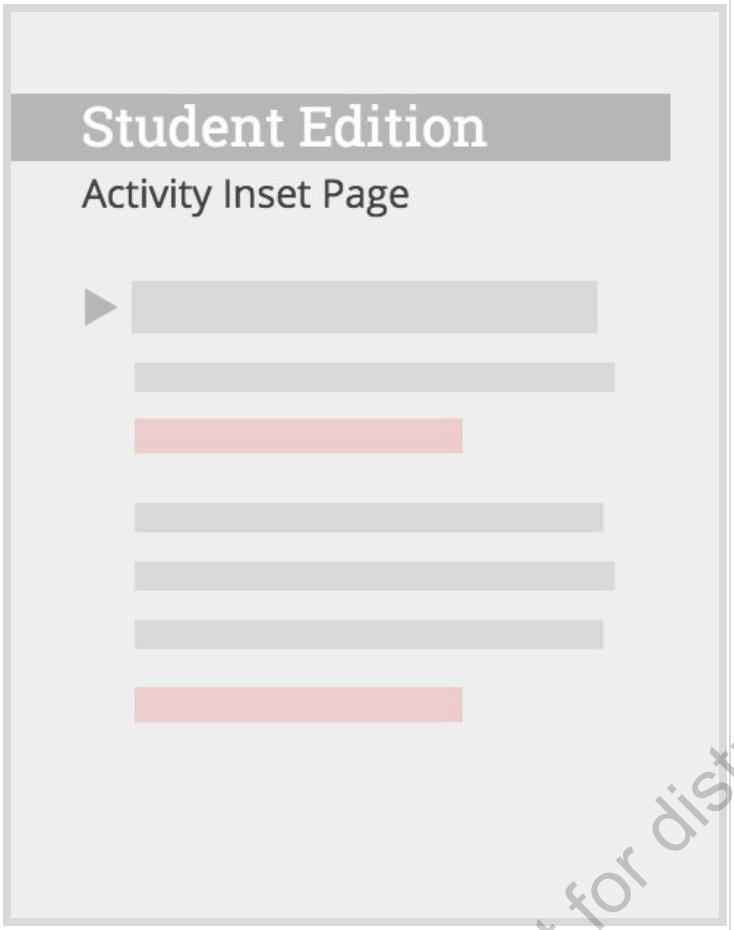
Students Ready for More
Have students complete "Are You Ready for More?", in which they will write a rule for each growth pattern.

English Language Learners
MLR6 Three Reads. Read first to understand the scenarios. The second reading identifies quantities and relationships. After the third read, students brainstorm strategies to answer the questions. Students connect the language in the word problem and the reasoning needed to solve the problem.

Students with Disabilities
Representation: Internalize Comprehension. Students sketch diagrams that show the number of meme shares for each scenario. Use color coding for each person, annotate/ highlight connections between the representations for each scenario.

Activity 2: Viral Reproduction Number

Students compare linear and exponential relationships by completing tables and graphs that show two growth patterns.



Launch

In pairs or trios, have students read scenarios aloud and discuss the reflection questions. Students may answer questions independently or by collaborating with a partner.

Monitor

Help students get started by explaining the subscripts and variables. Help students connect an $R_0 = 1$ to linear growth, while values greater than 1 are nonlinear.

Look for points of confusion:

- **Unable to find the total number of cases:** draw a tree diagram and count the number of cases, connect their calculations for the table of values.
- **Struggle to label the axes on their graphs:** label the horizontal axis as time and the vertical axis as total number of infections.

Look for productive strategies:

- Look for common differences and then common factors.
- Apply common factors to complete a table of values.
- Identify ordered pairs from a table of values and graph.

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Activity 2: Viral Reproduction Number continued

Students compare linear and exponential relationships by completing tables and graphs that show two growth patterns.

Connect

Display examples of student graphs.

Have students share emerging patterns in the total number of cases depending on the value of R_0.

Highlight the connections between the value of R_0 and its growth pattern shown in the table and graph.

Ask: How do we see nonlinear growth patterns in the tables? In the graphs? [A nonlinear growth pattern has a common factor in the table and graph.]

Which basic reproduction number is associated with the least number of infections? [$R_0 = 1$]

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Differentiated Support

Students Who Need Help:
Provide struggling students with prelabeled graphs that have the first three points plotted. Have them graph only points for the values they found in the tables.

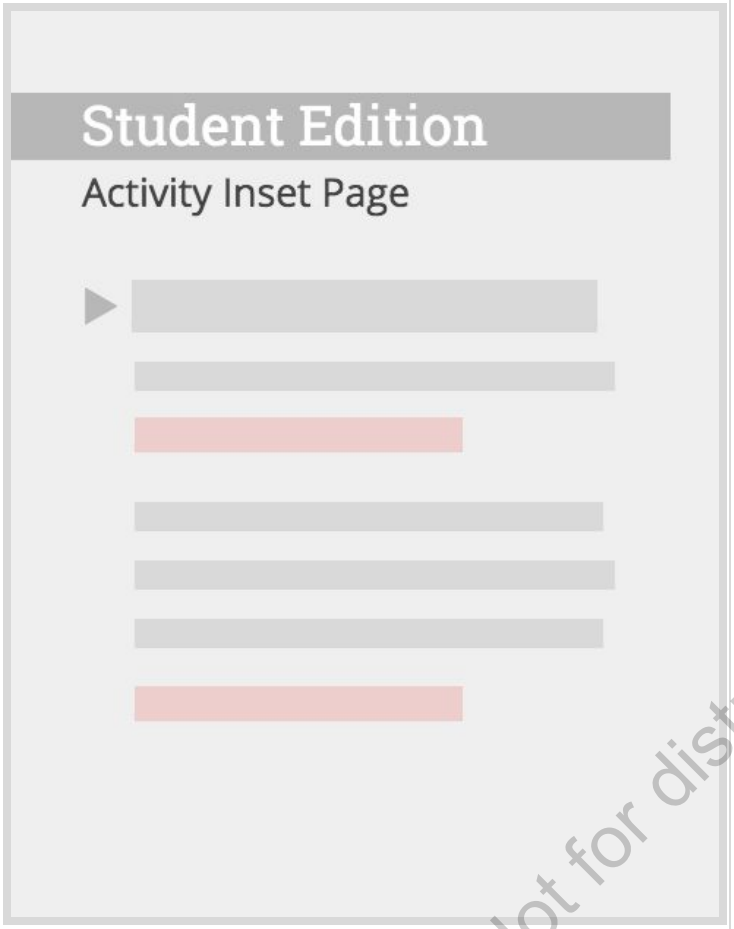
Students Ready for More
Students can use graphing calculators or create spreadsheets, to identify patterns, and predict values.

English Language Learners
MLR6 Three Reads. First read to understand how viruses spread for each value of R_0. Then read to identify quantities and relationships. Read again to brainstorm strategies and connect the language in the word problem and the reasoning needed to solve the problem.

Students with Disabilities
Representation: Internalize Comprehension. Represent the same information using diagrams. Students sketch diagrams showing how the virus spreads for each value of R_0. Use color coding and annotations highlighting connections between the representations for each R_0 value.

Summary

Students make connections between multiple representations of nonlinear growth patterns and exponential expressions. Students may use the space on the notes page to capture key concepts, questions, and ideas from this lesson.



Synthesize

Display the table and graph.

Have students share patterns they see after observing the table and graph. Then have them record the patterns and observations.

Highlight the growth patterns explored in this lesson by explicitly stating them and make the connection with nonlinear growth exceeding linear growth.

Review vocabulary:

- Common difference
- Common factor

Ask:

- "How are the values in the table growing?" [nonlinear growth pattern]
- "What is the common factor?" [2]
- "How could we find the next term when $x = 5$?"[Multiply $0.016 \cdot 2$ or $0.001 \cdot (2)^5$]
- "Does this expression match the graph?"[Yes, as the x value increases by 1, the y value doubles.]

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Exit Ticket: Genie in a Bottle

Students compare linear and exponential relationships by interpreting graphs that show the two growth patterns.

Success looks like...

- Students can identify linear and nonlinear growth given a graph.
- Students can explain the difference between linear and nonlinear growth given a table of values.
- Students can explain why nonlinear growth will exceed linear growth.

Suggested next steps

- The vertical intercept for Purse B is lower, so Purse B starts off with less money than Purse A.
- The values in Purse B stay lower than the values in Purse A until day 10, when they appear to be the same.
- Purse A is better up until day 10, and after that point Purse B becomes a better choice.

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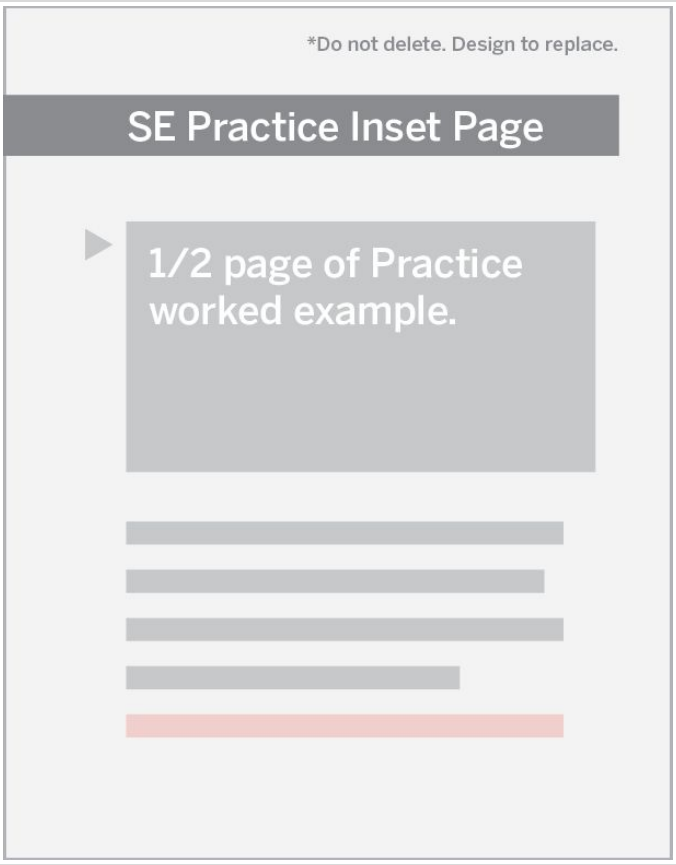
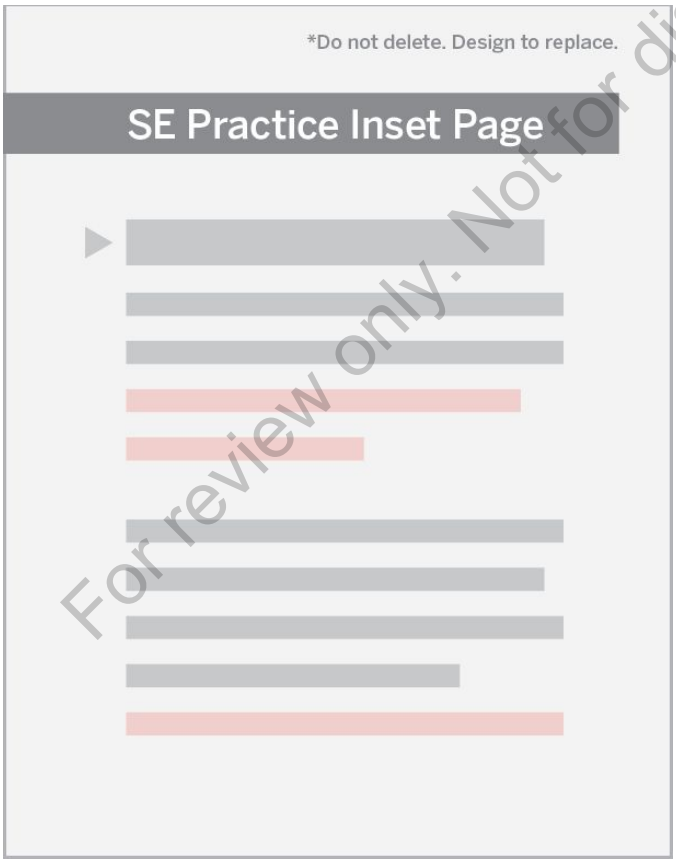
Practice





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Practice Problem Analysis

Problem	Refer to	Standards	DOK
1	Unit 6 Lesson 15	6.EE.A.1	*1*
2	Unit Lesson	7.EE.B.3	*1*
3a, b, c, d	Activity 1	HSF.IF.B.4 HSF.LE.A.3	*2*
4	Unit Lesson	6.EE.A.3	*3*
5 a, b, c	Activity 2	HSF.LE.A.3	*2*
6 a, b	Activity 2	HSF.IF.B.4 HSF.LE.A.3	*2*

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Lesson A1.5.03

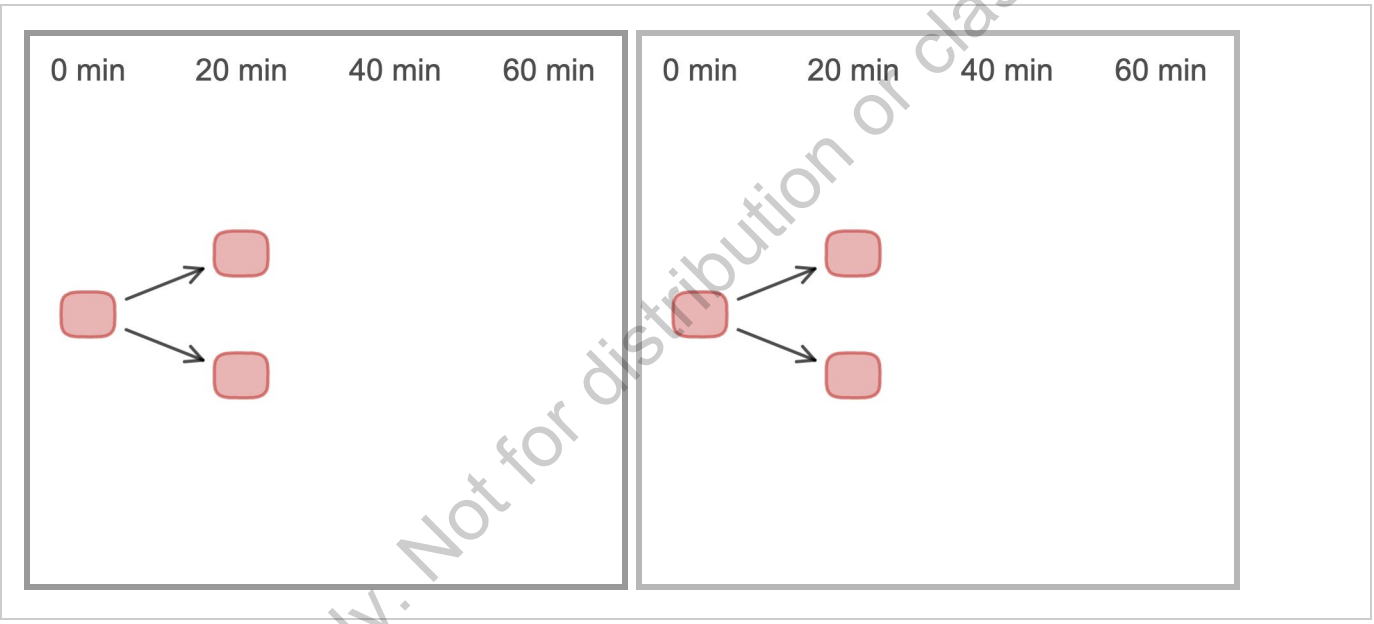
Growing and Growing

Let's connect different patterns to their tables and graphs.

Warm-up: Splitting Bacteria

Under ideal conditions, *E. coli* bacteria in a laboratory petri dish divide every 20 minutes. This diagram shows a single bacterium dividing into two bacteria in the first 20 minutes.

Answer box



1. Complete the diagram by sketching the number of bacteria at 40 and 60 minutes. What pattern do you notice?

2. How many bacteria are there after the first hour? Explain your thinking.

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Activity 1: Viral Memes

Read each scenario, then answer each question.

Scenario 1: Andre shares a meme with 20 followers. One of his followers shares the meme with 10 more followers per hour, for the next 5 hours.

Andre's shares						
Hour	0	1	2	3	4	5
Shares	20	30	40	50	60	70

Scenario 2: Jada shares a meme with 3 followers. Each follower shares with another 3 followers each hour. This continues for the next 5 hours.

Jada's shares						
Hour	0	1	2	3	4	5
Shares	3	9	27	81	243	729

1. Compare Andre's shares to Jada's shares. Describe the growth patterns of shares per hour.

2. What is the common difference for the table that has linear growth? What is the common factor for the table that has nonlinear growth?

3. How many shares will there be in both scenarios after 6 hours have passed? Explain your thinking.

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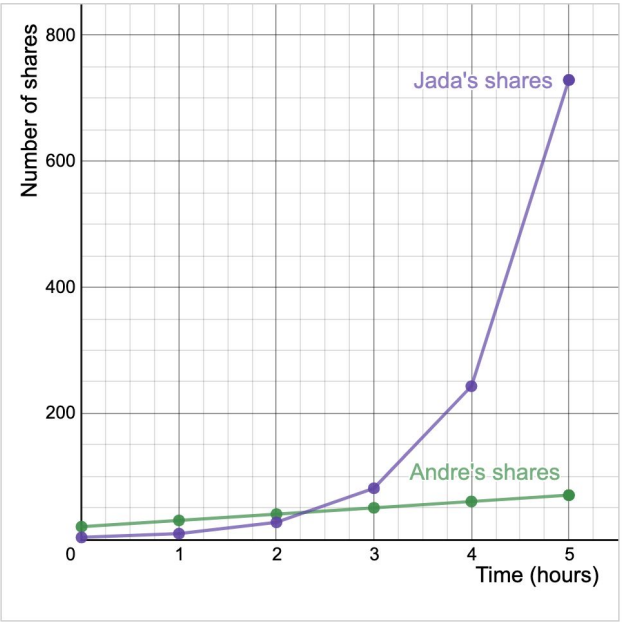

4. Compare the graphs of the memes shared. Do they appear to match your descriptions from Question 1?

5. Use the graph to determine when Jada's meme shares will exceed Andre's meme shares?

Are you ready for more?

Directions: Use the table of values for Scenarios 1 and 2, to write a rule for each growth pattern.

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Activity 2: Viral Reproduction Number

The basic reproduction number of an infectious disease is called R_0. If $R_0 = 1$, then each infected person is likely to infect one new person, who will infect one other person, and so on. If $R_0 = 2$, then each infected person is likely to infect two new people, who also each infect two new people, and so on. Let's explore the spread of viruses using tables and graphs.

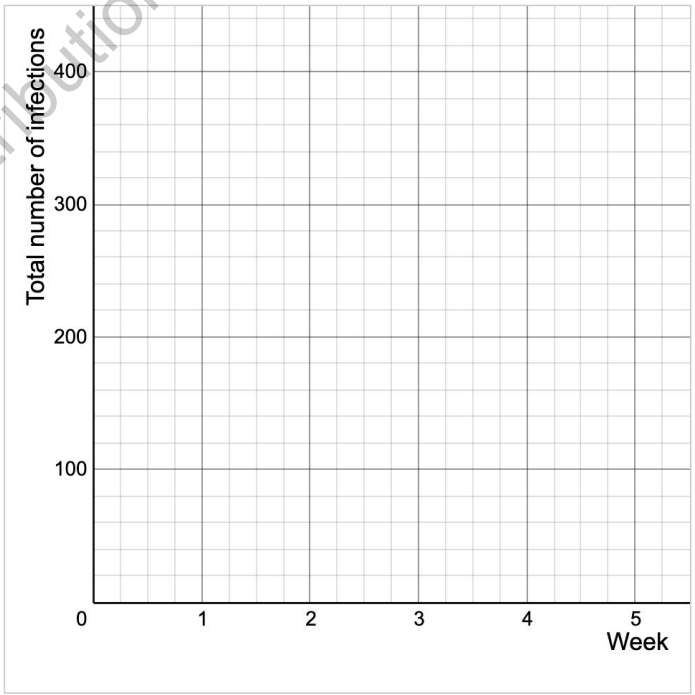
1. Measles is a particularly infectious disease. What would you guess its value of R_0 is? In other words, in a population without immunity to measles, how many people would each person infected with measles also infect?

2. A particular strain of the flu has a basic reproduction number of $R_0 = 3$, and is infectious for a week. Complete the table and graph for the total number of infections one person with this flu strain will cause.

Scenario 1:

$R_0 = 3$

Week	Number of new infections	Total number of infections
0	1	1
1	3	4
2	9	13
3		
4		
5		



3. After the Centers for Disease Control issues a warning, advising people to avoid public spaces during the outbreak, the basic reproduction number falls to $R_0 = 2$. Complete the table and graph for the total number of infections one person with this flu strain will now cause.

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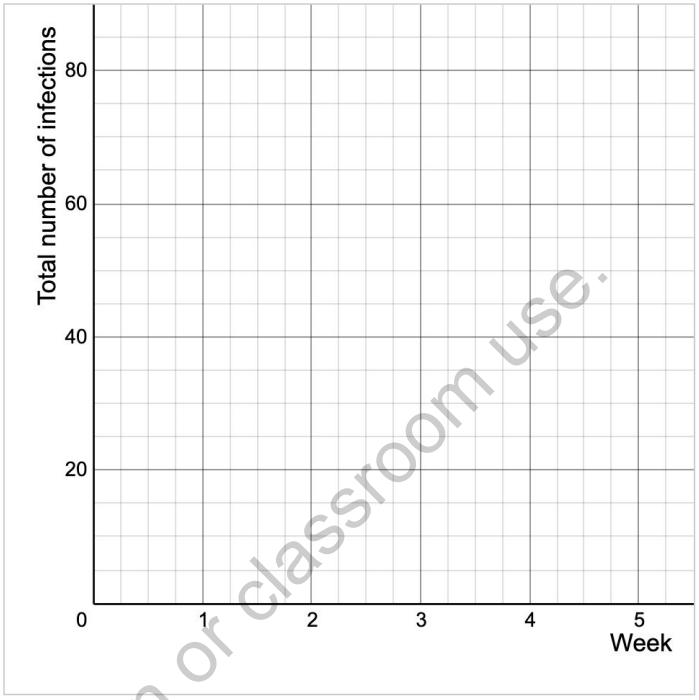
Scenario 2

$R_0 = 2$

Week	Number of new infections	Total number of infections
0	1	1
1	2	3
2	4	7
3		
4		
5		

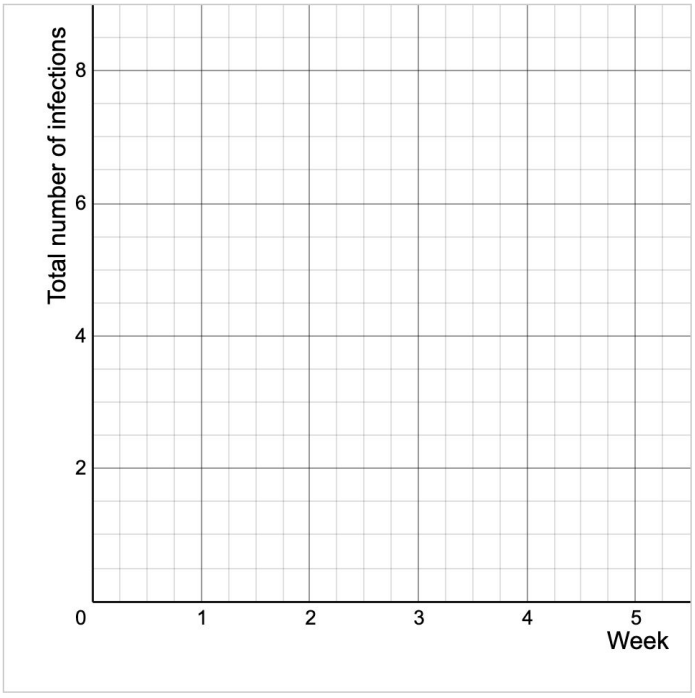


4. A vaccine for this strain of flu has been made available to the public, so that the basic reproduction number is now $R_0 = 1$. Complete the table and graph for the total number of infections one person with this flu strain will now cause.

Scenario 3

$R_0 = 1$

Week	Number of new infections	Total number of infections
0	1	1
1	1	2
2	1	3
3		
4		
5		



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5. Compare the graphs of the three different scenarios. Which graphs are linear, and which are nonlinear?

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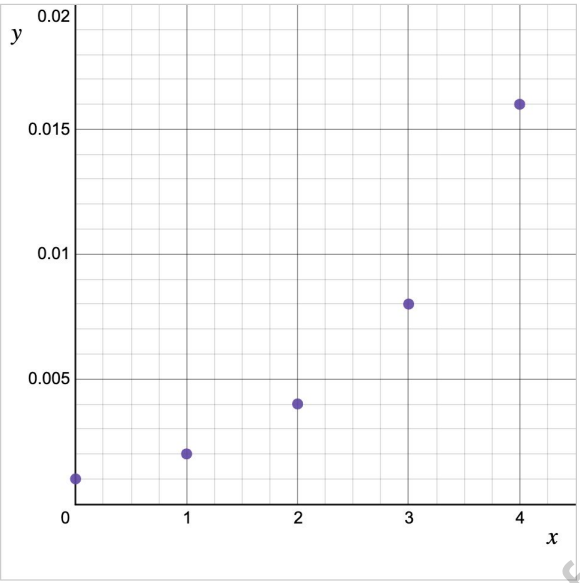
Summary

In this lesson, we compared tables and graphs of linear and nonlinear growth patterns. When we repeatedly double (or triple, or quadruple, etc.) a positive number, it eventually becomes *very* large. The table and graph show what happens when we begin to double a small number, 0.001:

Expression: $0.001 \cdot 2^x$					
x	0	1	2	3	4
$y = 0.001 \cdot 2^x$	0.001	0.002	0.004	0.008	0.016



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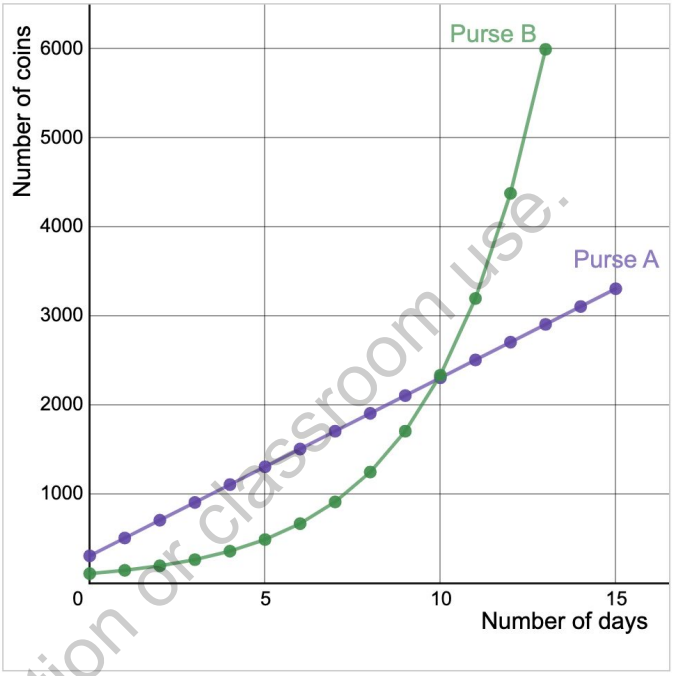
Exit Ticket: Genie in a Bottle

Directions: Use the graph to determine the best offer.

At the beach, you discover a magic genie in a bottle. The genie offers two purses, to choose from. He gives you a graph to show how the money in each purse grows.

The genie is still deciding how many days he will let the money in the purses grow. Plan a strategy for picking the purse with the most money while the genie thinks.



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Self-Assess
A. I can describe patterns of growth on a graph in my own words. B. I can tell if a graph shows a common difference or common factor. C. I can write an expression to find the next term in a linear or nonlinear pattern.
Key: 1. I'm lost 2. I don't really get it 3. I'm starting to get it 4. I got it 5. I could teach it

Practice A1.5.3

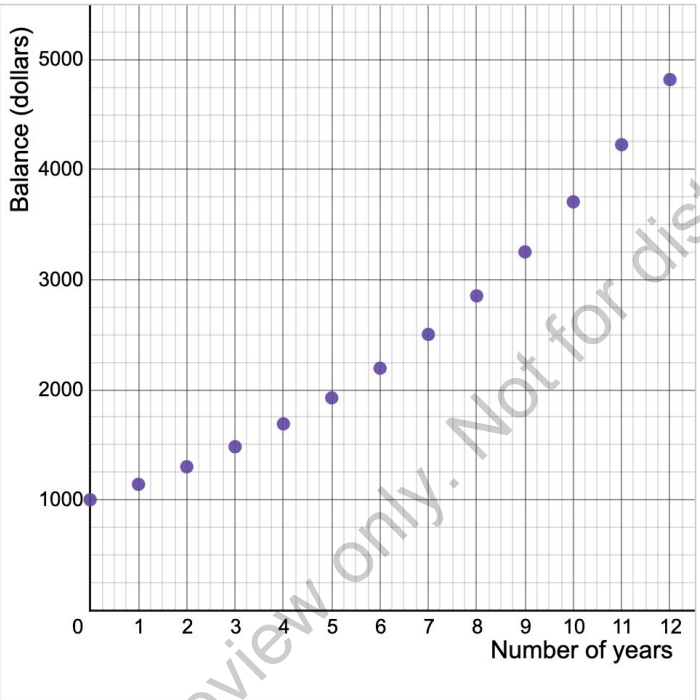
Directions: An

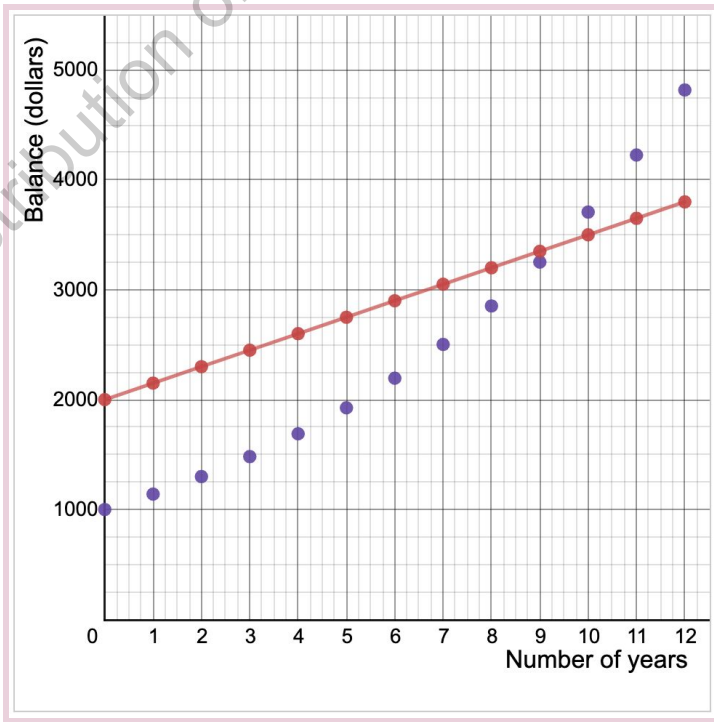
1. Which expression is equivalent to 2^7?

   A. $2+2+2+2+2+2+2$
   B. $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$
   C. $2 \cdot 7$
   D. $2+7$

2. Evaluate the expression $3 \cdot 5^x$, when x is 2.

3. The graph shows the yearly balance, in dollars, in an investment account.





   a. What is the initial balance in the account?

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b. Is the account growing by the same number of dollars each year? Explain your thinking.

c. A second investment account starts with $\$2,000$ and grows by $\$150$ each year. Sketch the values of this account on the graph.

d. How do the balances in each account compare?

4. Jada rewrites $5 \cdot 3^x$ as $15x$. Do you agree with Jada that these are equivalent expressions? Explain your thinking.

5. Ali's account has an initial balance of $\$200$ and doubles every year. Tyler's account has an initial balance of $\$1,000$ and increases by $\$100$ each year.

a. How long does it take for each account to double?

b. When will the value of Ali's account be greater than Tyler's account?

c. How does the growth in these two accounts compare? Explain your thinking.

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6. A study of 100 recent high school graduates showed a link between their childhood reading habits and achievement in high school. Participants were asked if another person read them books daily as a young child, as well as their high school GPA. The results are represented in the table.

	Was read books daily as a child	**Was *not* read books daily as a child**
A average	16	10
B average	21	14
C average	12	16
D average	3	8

a. What does the 21 mean in this context?

b. What does the 10 represent?

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A1.5.3

Practice

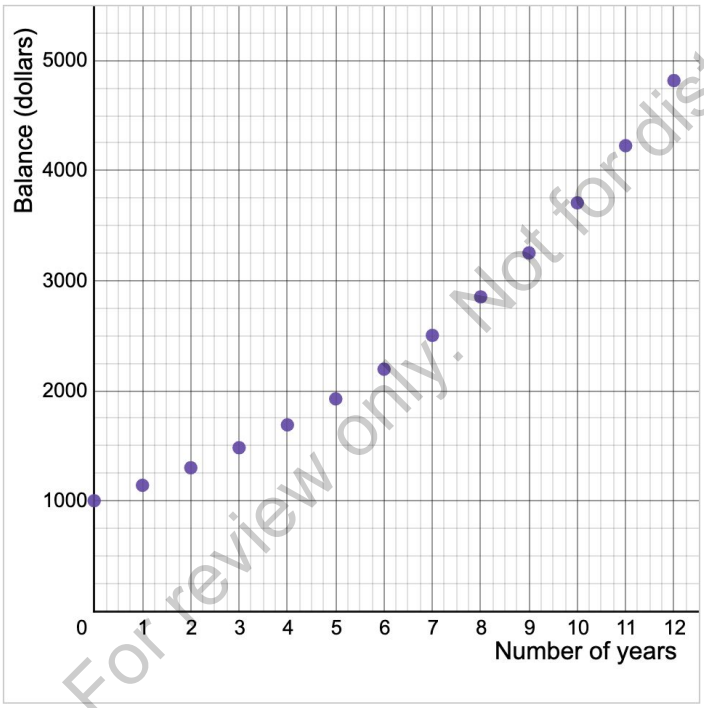
Directions: Answer the questions.

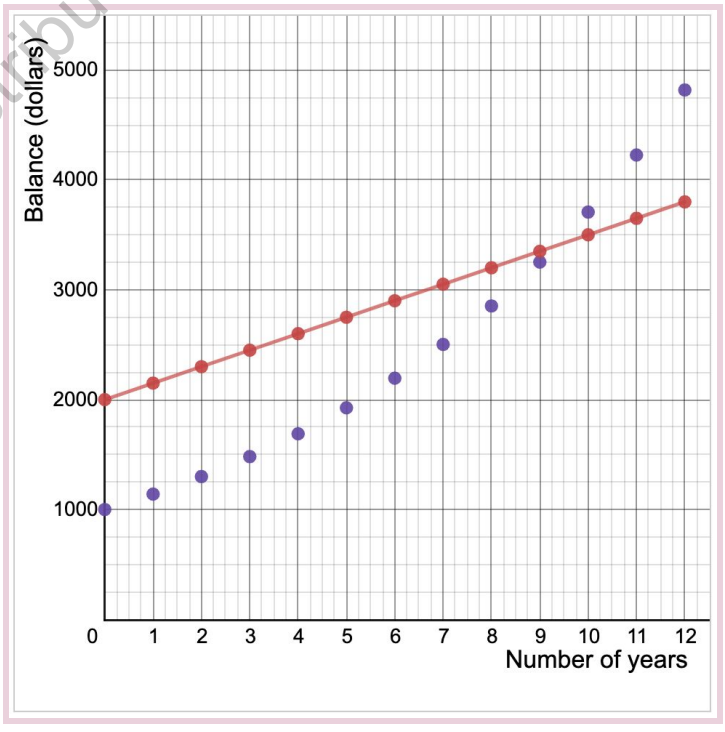
1. Which expression is equivalent to 2^7?

   A. $2+2+2+2+2+2+2$
   B. $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$
   C. $2 \cdot 7$
   D. $2+7$

2. Evaluate the expression $3 \cdot 5^x$, when x is 2.

3. The graph shows the yearly balance, in dollars, in an investment account.





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