mCLASS® Math

Activities Guide

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COUNTING

ADDITION

SUBTRACTION

CONTENTS

MULTIPLICATION

WRITTEN

MANIPULATIVES

COUNTING

Instructions

Introduce the activity by saying, "Today we're going to practice clapping and counting. Let's count and clap our hands at the same time. Ready?"

Wait for confirmation, then begin to count and clap: "One (clap), two (clap)..."

Choose a new movement each time to vary the routine. For example, as you count, hop, raise your right hand, or turn in circles, etc. The main goal is to make counting fun. Don't worry too much if a child does not clap exactly once for each number word.

Items

For kindergartners, count until around 10 at first, and then gradually increase to 19.

COUNT TO 20 (#1)

Objective

To learn the counting words to 20

Materials

None

Participants

Individual, small group, and/or whole class

Norms

Most children can achieve the activity's goal by about 5.

NCTM Standard

Count with understanding and recognize "how many" in sets of objects. (Number and Operations: Understand numbers, ways of representing numbers, relationships among numbers, and number systems.)

COUNT TO 100 (#2)

Objective

To extend the counting sequence gradually over time

Materials

None

Participants

Individual, small group, and/or whole class

Norms

On average, 5-year-olds can count up to 20 in the correct sequence and 5 1/2-year-olds can count up to 42.

NCTM Standards

Count with understanding and recognize "how many" in sets of objects. (Number and Operations: Understand numbers, ways of representing numbers, relationships among numbers, and number systems.)

Instructions

Introduce the activity by saying, "Today we're going to practice counting as high as we can. Let's start with one and count as high as we can: One, two… (let your voice fade out)."

If the children count accurately to 19, stop at that point. Then say, "Let's think. What comes after 19? What comes next? Twenty. That's right." Repeat the pause and questioning at the end of each decade, up to 99.

Stop if and when a child makes an error. For example, if the children say "four" after "two," say, "Oops, I think we forgot a number. Let's start again." Begin the counting sequence from one and count with the group up through twenty.

If a child makes an error within a decade, such as skipping 27, say, "Oops, I think we forgot a number. Let's start again," but start counting at the beginning of the decade and stop counting at the end of the next decade. So, if a child skipped 27, say, "Oops, I think we forgot a number. Let's start again with 20. Like this: 20, 21 … 39, 40."

Try a new movement or sound for each new decade. For example, hop from 20 to 29. "Let's think. What happens after 29, 39, 49, etc." For example, jump from 30 to 39 and wiggle your arm from 40 to 49, etc.

Items

Set a number that is a challenging yet attainable as a goal. Children may need many repetitions of this activity. It will take months for them to learn the sequence to 100.

Instructions

Introduce the activity by saying, "Today we're going to play a game. I'll say two numbers and, as fast as you can, you tell me which number is bigger (or smaller). If you're right, you get a point. The person with the most points at the end of the game wins. OK? Let's begin."

"Which is bigger: [m] or [n]?"

"Which is smaller: [m] or [n]?"

Alternate between the two types of questions.

Items

Use familiar numbers (i.e., if children can accurately count only up to 25, exclude numbers above 25). Alternate the position of the larger number (e.g., Which is bigger 13 or 19? Which is smaller 17 or 12?).

2 or 5
3 or 1
4 or 7
5 or 2
3 or 8
5 or 9
7 or 4
8 or 5
11 or 14
13 or 9
15 or 21
16 or 19
13 or 11
24 or 42
31 or 29
27 or 32
41 or 36

COMPARE MAGNITUDE (#3)

Objective

To understand the relative size of the counting numbers, to understand which numbers are "bigger" or "smaller" than others in the counting sequence

Materials

None

Participants

Individual, small group, and/or whole class

Norms

On average, 5 1/2-year-olds can compare magnitudes of numbers up to 100.

NCTM Standards

Develop an understanding of the relative position and magnitude of whole numbers as well as ordinal and cardinal numbers and their connections. (Number and Operations: Understand numbers, ways of representing numbers, relationships among numbers, and number systems.)

COMPARE MAGNITUDE, CONT'D. (#3)

38 or 53
73 or 32
65 or 98
87 or 49
76 or 94
83 or 72

CATCH MY MISTAKE (#4)

Objective

To identify and correct common counting errors

Materials

None

Participants

Individual, small group, and/or whole class.

Administer only to children who cannot accurately count to 19.

Norms

On average, 5-year-olds can count up to 20 in the correct sequence.

NCTM Standards

Count with understanding and recognize "how many" in sets of objects. (Number and Operations: Understand numbers, ways of representing numbers, relationships among numbers, and number systems.)

Instructions

Introduce the activity by saying, "Today we're going to practice counting. I want you to listen carefully because I may make a mistake. If you hear a mistake, I want you to raise your hand so we can fix it together. OK? Let's begin."

- If an error is made and the children do not raise their hands say, "Let's try that again. Remember, I want you to listen carefully for mistakes," then repeat the error.
- If several children raise their hand upon hearing an error, say, "Did I make a mistake, [child's name]?" Guide the child to identifying and correcting the error. Enlist the help of other children if he or she has difficulty.

Repeat this exercise as often as necessary.

Items

Follow the directions for each error type.

Skip: Skip one or two numbers in a sequence while counting. For example, say, "One, two, four, five, six, seven."

Repeat/Double-Count: Repeat a number more than once while counting. For example, say, "One, two, three, four, four, five, six, seven."

Reverse: Reverse the order of numbers while counting. For example, say, "One, three, two, four, five, six, seven."

Decade: When counting from one decade to the next (e.g., 29–30), make a rule-governed mistake. For example:

- When counting from 9–10, say, "Eight, nine, ten, ten-one, ten-two…"
- When counting from 19–20, say, "Eighteen, nineteen, ten-teen, eleven-teen, twelve-teen..."
- When counting from 29–30, say, "Twenty-eight, twenty-nine, twenty-ten, twenty-eleven, twenty-twelve…"

NUMBER AFTER (#5)

Objective

To continue the counting sequence with a number one more than the target number

Materials

None

Participants

Individual, small group, and/or whole class

Norms

On average, 5-year-olds can count after a specified count term for numbers between one and nine; 5 1/2-year-olds can count after a specified count term for numbers between 10 and 40; and 6-year-olds can count after a specified count term for numbers between 29 and 99.

NCTM Standards

Develop a sense of whole numbers and represent and use them in flexible ways, including relating, composing, and decomposing numbers. (Number and Operations: Understand numbers, ways of representing numbers, relationships among numbers, and number systems.)

Instructions

Introduce the activity by saying, "I'm going to say a number and, as fast as you can, I want you to tell me what number comes after. So if I say five, what comes after? You say six as fast as you can. OK? Let's begin."

"If I say…"

Repeat as often as necessary.

Items

Be sure to begin with a goal that is challenging yet attainable. The items used in this game depend on how high the group of children can count. For example, if the children can accurately count only up to 20, then the start number should not be higher than 19. Similarly, if children can accurately count only up to 37, the start number should not be higher than 36.

Instructions

Introduce the activity by saying, “I'm going to say a number and, as fast as you can, I want you to tell me what number comes before. So if I say five, what comes before? You say four as fast as you can. OK? Let's begin.”

“If I say…”

Repeat as often as necessary.

Items

Be sure to begin with a goal that is challenging yet attainable. The items used in this game depend on how high the group of children can count. For example, if the children cannot accurately count past 10, do not ask what comes before 11. Similarly, if the children cannot accurately count past 40, do not ask the children what comes before 41.

NUMBER BEFORE (#6)

Objective

To continue the counting sequence with a number one less than the target number

Materials

None

Participants

Individual, small group, and/or whole class

Norms

On average, 5-year-olds can flexibly cite the number before a specified count term for numbers between two and ten, and 5 1/2-years-olds can do the same for numbers between 11 and 29.

NCTM Standards

Develop a sense of whole numbers and represent and use them in flexible ways, including relating, composing, and decomposing numbers. (Number and Operations: Understand numbers, ways of representing numbers, relationships among numbers, and number systems.)

COUNT BACKWARD (#7)

Objective

To learn the backward counting words from 20

Materials

None

Participants

Individual, small group, and/or whole class.

Administer only to children who can count in sequence forward by ones.

Norms

On average, 5 1/2-year-olds can count backward from 10 and 6 1/2-year-olds can count backward from 20.

NCTM Standards

Count with understanding and recognize "how many" in sets of objects. (Number and Operations: Understand numbers, ways of representing numbers, relationships among numbers, and number systems.)

Instructions

Introduce the activity by saying, "Today we're going to practice counting backward, like when a rocket blasts off: Three, two, one, blast off! Now let's do it together. Count backward from five. Ready?"

Wait for confirmation.

Begin counting backward from 5 then gradually increase to 7, 9, 13, 15, 17, 20, 30, etc. To vary the routine, you may want to introduce a movement such as clapping, hopping, or walking backward one step each time you count backward.

Children may need many repetitions of this activity. Repeat as often as necessary.

Items

Be sure to start with a goal that is challenging yet attainable.

Instructions

Introduce the activity by saying, "Today we're going to practice counting by tens. This is just like counting from one to ten, only the numbers are bigger. Can anyone think of a reason why mathematicians sometimes count by tens?"

Listen to children's answers and respond accordingly.

"Sometimes mathematicians count by tens because they can count really high faster. If I was going to count to 100 by ones it would take me a really long time: One (pause), two (pause), three (pause), four (pause). It would take sooooo long, I'm not even going to try to finish right now. But if I counted by tens, I could do it really fast: 10, 20, 30, 40, 50, 60, 70, 80, 90, 100!"

"Let's see how high we can count from 10: 10, 20, 30, keep on counting until I tell you to stop."

Stop after a child makes a decade error and count through that error again, making sure that all children hear the correct sequence.

Repeat as often as necessary. Each time you repeat the activity, remind the children of their highest count and encourage them to count higher.

Items

None

COUNT BY TENS FROM 10 (#8)

Objective

To learn to count by decades

Materials

None

Participants

Individual, small group, and/or whole class

Norms

On average, 5 1/2-year-olds can successfully skip count by tens to 100.

NCTM Standards

Use multiple models to develop initial understandings of place value and the Base 10 number system. (Number and Operations: Understand numbers, ways of representing numbers, relationships among numbers, and number systems.)

COUNT BY TENS, RUNNING START (#9)

Objective

To learn to count by decades

Materials

None

Participants

Individual, small group, and/or whole class

Norms

On average, 5 1/2-year-olds can successfully skip count by tens to 100.

NCTM Standards

Use multiple models to develop initial understandings of place value and the Base 10 number system. (Number and Operations: Understand numbers, ways of representing numbers, relationships among numbers, and number systems.)

Instructions

Introduce the activity by saying, “Today we’re going to practice counting by tens. We’re still going to count as high as we can, but this time we may not start with the number 10. We may start with 50, 30, or some other number. So if I say let’s count from 30, you say 30, 40, 50 and keep going until I tell you to stop. OK?”

“Let’s see how high we can count from 20: 20, 30, keep on counting until I tell you to stop.”

Stop after a child makes a decade error and count through that error again, making sure that all children hear the correct sequence.

Repeat as often as necessary.

Items

None

Instructions

Introduce the activity by saying, "Now we're going to play another counting game. I'm going to select two children. One child will be the "Ones" child and the other will be the "Tens" child. First, the Ones child will select a single-digit number and the Tens child has to say the double-digit number that is like that single-digit number."

"For example, if the Ones child says two, the Tens child says twenty. If the Ones child says eight, the Tens child says eighty. OK? Let's begin."

Repeat as often as necessary.

Next, select two other children and say, "OK, now the Tens child will go first. The Tens child will select a double-digit number and the Ones child has to say the single-digit number that goes with that double-digit number."

"For example, if the Tens child says forty, the Ones child says four."

Repeat as often as necessary.

In both cases, if an error is made, ask, "How can we check to be sure?"

Items

None

IDENTIFY TENS AND ONES (#10)

Objective

To discover decade patterns by identifying the units and tens while counting by tens

Materials

None

Participants

Individual, small group, and/or whole class.

Administer only to children who can count by tens to 100.

Norms

None

NCTM Standards

Use multiple models to develop initial understandings of place value and the Base 10 number system. (Number and Operations: Understand numbers, ways of representing numbers, relationships among numbers, and number systems.)

ENUMERATE CHIPS (#11)

Objective

To say counting words while counting objects and tracking what has been counted

Materials

Chips

Participants

Individual, small group, and/or whole class. (Most beneficial for small groups.)

Norms

On average, 5 1/2-year-olds can accurately produce 10 objects and 6-year-olds can accurately produce 20 objects.

NCTM Standards

Count with understanding and recognize "how many" in sets of objects. (Number and Operations: Understand numbers, ways of representing numbers, relationships among numbers, and number systems.)

Instructions

Introduce the activity by saying, "Today we're going to play a counting game with chips. I'll show one of you some chips, and that child has to tell me how many chips there are altogether. So if I put out a bunch of chips, like this (randomly distribute three chips on the table), you count the chips."

Demonstrate counting the chips: One (push aside a chip), two (push aside another chip), three (push aside the last chip).

"There are three chips in the set. OK?" Wait for the children to respond.

"After that, I'm going to ask another child to come up and check the answer. Let's begin."

- If the first child counts correctly, say, "OK, let's check to be sure." Select another child to check.
- If the first child's response is incorrect, say, "Let's try again. This time count slowly and carefully, and make sure you touch each chip only once. Go ahead." If the child counts incorrectly again, say, "Now let's do it together," and push aside each chip as it is counted.

Repeat the activity as often as necessary.

Items

None

GIVE ME (#12)

Objective

To carefully use strategies to count out a given number of objects and to check the result

Materials

Chips

Participants

Individual, small group, and/or whole class

Norms

On average, 5 1/2-year-olds can accurately produce 10 objects and 6-year-olds can accurately produce 20 objects.

NCTM Standards

Count with understanding and recognize "how many" in sets of objects. (Number and Operations: Understand numbers, ways of representing numbers, relationships among numbers, and number systems.)

Instructions

Introduce the activity by saying, "Today we're going to help each other practice counting with these chips. I'm going to say a number and you have to count out the same number of chips."

Distribute 30 chips to each child.

"If I say 'Give me two chips,' I want you to count out two chips like this (dramatically push aside the two chips, one by one). Then I'll ask you to check your partner's work to see if you agree."

"If I say 'Give me four chips,' I want you to count out four chips like this (dramatically push aside the four chips, one by one). Then I'll ask you to check your partner's work. OK?"

"Each time you and your partner count out the correct number of chips, your team gets a point. The team with the most points wins the game."

"OK, let's begin with…"

After each problem, ask children to check their partner's answer. Encourage the use of a systematic strategy (e.g., pushing aside as modeled above). Say, "OK, time to check your partner's answer. If you think your partner is correct, put your thumb up. If you disagree with your partner, raise your hand."

- If one or both children in a pair count(s) incorrectly, take a moment to interview them about their different answers.
- If the children in the pair have different answers, enlist one or more child "arbitrators" or "judges" to help resolve the disagreement.
- If one or both children in a pair count(s) incorrectly, guide the child(s) toward carefully counting the chips by saying, "This time, make sure you push aside each chip as you count it. Go ahead." If the children's response is still incorrect, say, "Now let's do it together" and push aside each chip together.

As children become more proficient, offer the option of checking by saying, "Should we check to make sure?" If the children are correct and confident in their answer, move on to the next item.

Repeat as often as necessary.

Items

None

ENUMERATE DOTS (#13)

Objective

To coordinate number words with counting objects, developing counting strategies

Materials

Two sets of dot cards, with the number of dots ranging 1–10. One set of cards displays the dots in an organized manner and the other randomly.

Participants

Individual, small group, and/or whole class. (Most beneficial for small groups.)

Norms

On average, 5-year-olds can accurately enumerate sets of 10 objects and 6 1/2-year-olds can accurately enumerate sets of 20 objects.

NCTM Standards

Connect number words and numerals to the quantities they represent, using various physical models and representations. (Number and Operations: Understand numbers, ways of representing numbers, relationships among numbers, and number systems.)

Instructions

Introduce the activity by saying, "Today we're going to play a game with dots. I'll show you a card with dots on it (show the children an example) and you tell me how many dots there are on the card."

"Whoever answers the fastest, with the correct number, gets to hold the card. Whoever has the most cards at the end, wins the game. OK? Let's begin."

After each problem, ask the children to check the answer.

- If the answer is correct, say, "OK, let's check to be sure" and have the group count the dots together. Give the card to the child who answered correctly the fastest.
- If a child calls out the incorrect number of dots, call on that child and ask, "How can we check to be sure?" Guide him or her to carefully count the dots on the card by saying, for instance, "This time, make sure you touch only one dot at a time. Go ahead." If the child's response is still incorrect, say, "Let's do it together" and point to each dot together.

As children become more proficient, offer the option of checking by saying, "Should we check to make sure?" If the children are correct and confident in their answer, move on to the next item.

Repeat as often as necessary.

Items

None

ADDITION

Instructions

Introduce the activity by saying, "Today we're going to practice adding numbers together. Does anyone know what it means to add?

Smart mathematicians like you add things together all the time. They combine different numbers into one number or combine groups of things into one larger group. Let's try it together." To reduce reliance on memory, you may want to project or write the addends on the board.

"I'm going to tell you a story and you can use the chips to help you figure out the answer." Distribute 15 chips to each child, then present the problems.

There are three story themes in which to present the problems. Model the first problem with chips.

1. "Pretend you have three cats (count while simultaneously pushing aside three chips) and you get two more (count while simultaneously pushing aside two more chips). How many cats do you have altogether?" (Push the chips together and then count all the chips before announcing the sum.)

   To allow the children enough time to count and group the chips, pause after saying each number in the problem and again before requesting the sum.

1. "Pretend you have four treasures and you find three more. How many treasures do you have altogether?"
2. "Pretend two friends are at your house and five more friends come over. How many friends are at your house altogether?"

All children benefit from developing checking skills, so after all the children have determined a sum, ask several to share their answer and then ask the group to check the answer. Specifically, ask, "How can we check to make sure the answer is right?" Ask several children to share their checking procedures, then say, "OK, let's all check each of our answers."

Items

Small Number

0 + 1
1 + 1
2 + 0

ADD BY COUNTING ALL, CHIPS (#14)

Objective

To use counting strategies to solve and check addition problems

Materials

Chips

Participants

Individual, small group, and/or whole class

Norms

On average, 5-year-olds and 7-year-olds use the Count All strategy to solve addition problems with sums to 10 and 18 respectively.

NCTM Standards

Model situations that involve the addition and subtraction of whole numbers, using objects, pictures, and symbols. (Algebra: Use mathematical models to represent and understand quantitative relationships.)

ADD BY COUNTING ALL, CHIPS CONT'D. (#14)

2 + 1
0 + 3
3 + 1
4 + 0
4 + 1
0 + 5
5 + 1
6 + 1
7 + 1
8 + 1
9 + 1
1 + 2
3 + 2
4 + 2
2 + 5
6 + 2

Large Number

2 + 7
8 + 2
2 + 9
3 + 4
5 + 4
6 + 3
6 + 4
6 + 0
7 + 4
0 + 7

5 + 3
6 + 5
8 + 0
7 + 3
8 + 4
9 + 3
0 + 9

ADD BY COUNTING ALL, CHIPS CONT'D. (#14)

LARGE NUMBER COUNT ON, CHIPS (#15)

Objective

To develop advanced addition strategies

Materials

Styrofoam cups, numbered 0–9

Chips

Participants

Individual, small group, and/or whole class

Norms

On average, 6 1/2-year-olds successfully Count On to solve addition problems with sums to 18.

NCTM Standard

Develop and use strategies for whole-number computations, with a focus on addition and subtraction. (Number and Operations: Compute fluently and make reasonable estimates.)

Instructions

Introduce the activity by saying, "Today we're going to investigate a strategy that mathematicians use to add numbers. Sometimes, they add numbers together really fast by counting on from the larger number. Let's think about this. Try to think of a really smart way to add nine and one together. Hmmm, I know. I could count like this, 'One (pause), two (pause), three, (pause).' Wow, this is going to take a long time. Maybe I could try something a smart mathematician might do. I could start with nine and add one. Let's see, nine, and one more is…ten! That's so much faster! Right? Can someone come up and check my work?"

"Let's try this together now." Place chips on the table, followed by Styrofoam cups with numbers written on the front of each cup that every child in the room can see.

Say, "Let's try three plus one," then take the #3 cup and add three chips to the cup, one at a time, counting out loud. Say, "The number on the front of the cup tells how many chips are inside the cup. Let's count on starting from three to find the total. Like this three (whisper)…" Drop one more chip into the cup, "…four." You may need to illustrate counting on several times.

"Let's try (see items). Use these chips to help you count on." To encourage counting on from the larger number, distribute the maximum number of chips that can be used to represent the smallest addend in the set of items (four chips for small items and three for larger items).

Ask several children to share their answer. If even one child responds incorrectly say, "Let's check to be sure" and ask that child to check the answer. If all the answers are correct say, "Should we check to make sure?" If the children are correct and confident in their answer, move on to the next item.

Items

Small Number

0 + 3
1 + 3
2 + 1
5 + 0
4 + 2

3 + 4
0 + 7
6 + 1
5 + 1
9 + 0
7 + 2
7 + 1
4 + 1
9 + 1
6 + 2
5 + 3
6 + 3
8 + 2

Large Number

9 + 3
8 + 3
7 + 2
10 + 0
7 + 3
9 + 2
10 + 1
10 + 2
10 + 3
0 + 11
11 + 1
11 + 2
11 + 3

LARGE NUMBER COUNT ON, CHIPS, CONT'D. (#15)

LARGE NUMBER COUNT ON, CHIPS, CONT'D. (#15)

12 + 1
12 + 0
12 + 2
13 + 1
0 + 13
13 + 2
14 + 1
0 + 14
14 + 2
14 + 3

Instructions

Introduce the activity by saying, “OK, mathematicians. Today I have a challenge for you. We’re going to practice adding different kinds of numbers, and we’re going to do it all in our heads. We’re not going to use paper, chips, or our fingers; we’re going to use only our minds to figure out the answer. OK?”

“Let’s do it together first. If I say, ‘How much is 2 + 1,’ you count on, like this: ‘Two, three...’ So if I say how much is 20 + 10, you can count on by tens from 20, like this: 20 (whisper), 30, the answer is 30.”

“Let’s begin. How much is…”

After each decade item, ask, “How did you know that?” The following prompts may be used to guide the discussion:

- If the response indicates that the children relied on Base 10 knowledge, move on (e.g., “I just counted on from 20 by tens”).
- If the response indicates anything other than Base 10 knowledge, ask, “How could we have counted on by tens to figure out the answer?”

After solving several single- and double-digit problems, ask, “Do you see a pattern?” Focus discussion on recognizing the similarities between single-digit addends and their decade counterparts in addition.

Items

Basic

5 + 1	50 +10
6 + 1	60 + 10
7 + 1	70 + 10
8 + 1	80 + 10
9 + 1	90 + 10
2 + 1	20 + 10
3 + 1	30 +10
4 + 1	40 + 10

ADD BY TENS (#16)

Objective

To use tens to develop mental calculation skills

Materials

None

Participants

Individual, small group, and/or whole class

Norms

On average, 7 1/2-year-olds can add a decade plus 10 successfully.

NCTM Standard

Use multiple models to develop initial understandings of place value and the Base 10 number system. (Number and Operations: Understand numbers, ways of representing numbers, relationships among numbers, and number systems.)

ADD BY TENS, CONT'D. (#16)

Advanced

2 + 3	20 + 30
2 + 2	20 + 20
4 + 2	40 + 20
3 + 5	30 + 50
3 + 6	30 + 60
7 + 3	70 + 30
2 + 5	20 + 50
6 + 6	60 + 60
4 + 3	40 + 30
5 + 4	50 + 40
4 + 4	40 + 40
3 + 9	30 + 90
7 + 4	70 + 40
5 + 8	50 + 80
4 + 9	40 + 90
8 + 7	80 + 70

Instructions

Shuffle the cards.

Introduce the activity by saying, "Today we're going to play another game with dots. I'll show you two cards and, as fast as you can, tell me which card has more dots. So if I show you these two cards (show the children two cards, one with more dots than the other), you would say this one has more dots (point to the card with more dots). Then we'll check by counting the dots on each card. If you're right, you get a point. The person with the most points at the end of the game wins. OK? Let's begin."

Hold up the next two cards in the pile.

As children become more proficient, offer the option of checking by saying, "Should we check to make sure?" If the children are correct and confident in their answer, move on to the next item.

Items

Limit to numbers up to 15.

Difficulty may be increased by using cards with dots numbered in the teens.

COMPARE MAGNITUDE, DOTS (#17)

Objective

To determine the relative magnitude of sets and to learn to check by counting

Materials

Two sets of dot cards, with the number of dots ranging 1–10. One set of cards displays the dots in an organized manner and the other randomly.

Participants

Individual, small group, and/or whole class

Norms

On average, 3-year-olds can determine which side has more up to 10.

NCTM Standard

Develop understanding of the relative position and magnitude of whole numbers and of ordinal and cardinal numbers and their connections. (Number and Operations: Understand numbers, ways of representing numbers, relationships among numbers, and number systems.)

PREDICT, COUNT ALL (#18)

Objective

To learn to add mentally and to check the result by counting sets

Materials

Chips

Participants

Individual, small group, and/or whole class

Norms

On average, 5-year-olds can use concrete strategies to solve problems with sums to 18.

NCTM Standard

Develop and use strategies for whole-number computations, with a focus on addition and subtraction. (Number and Operations: Compute fluently and make reasonable estimates.)

Instructions

Distribute 15 chips to each child.

Introduce the activity by saying, "Today we're going to practice adding numbers. Can anyone tell me what it means to add?"

"Smart mathematicians like you add things together all the time. They combine different numbers into one number or combine groups of things into one larger group. Let's try it together. I'm going to tell you a story and I want you to predict the answer. Does everyone know what *predict* means? Predict is another way of saying guess. Mathematicians and scientists use the word *predict* to describe how they think really hard in their minds about what the answer to a question might be. Then they check to see if they are right. We will use chips to check our predictions."

Present the problem in a story theme. For example, say, "Pretend you have three treasures and then you get one more treasure. Predict how many treasures there are altogether?" Verbally and pictorially present each problem either on the blackboard, flannel board, or any other mode that allows all the children in the group or class to see the transformation.

If the children begin to count or use their chips, say, "Let's not count (or use the chips) just yet. Let's make a prediction. How many treasures do you think there are altogether?"

After several children have generated an answer, say, "Now let's see how good we are at predicting the answer. How can we check to see if our predictions were right?" Ask several children to share their "checking" procedures then say, "OK, let's check."

Items

1 + 2
3 + 1
4 + 1
1 + 5
6 + 1
1 + 7
8 + 1
1 + 9

10 + 1
2 + 2
3 + 2
4 + 2
5 + 2
6 + 2
7 + 2
8 + 2
9 + 2
10 + 2
11 + 1
11 + 2
3 + 3
3 + 4
5 + 3
6 +3
3 + 7
8 + 3
3 + 9
10 + 3
3 + 11
12 + 1
2 + 12
12 + 3

PREDICT, COUNT ALL, CONT'D.
(#18)

SMALL NUMBER ADDITION, CHIPS (#19)

Objective

To develop advanced addition strategies by practicing addition combinations

Materials

Cards numbered 1–9

Chips

Participants

Individual, small group, and/or whole class. If used with small group or whole class, increase the number of cards used (e.g., two number card sets for a small group and four sets for whole class instruction). If used individually, place the cards face down on the table.

Norms

On average, 6 1/2-year-olds use advanced counting strategies such as Counting On to solve addition problems with sums to 18 and 7 1/2-year-olds display fluency on single-digit addition combinations.

NCTM Standard

Develop fluency with basic number combinations for addition and subtraction. (Number and Operations: Compute fluently and make reasonable estimates.)

Instructions

If used individually, place the cards face down on the table.

If used in a small group or with the whole class, you may hold selected cards up or project them onto a more visible surface.

Distribute 20 chips to each child.

Introduce the activity by saying, "Today we're going to play another game. I'll show you two cards with numbers on them and I want you to add the numbers together. If you're right, and you tell me how you figured out the answer, you get to keep the cards. The person with the most cards at the end of the game wins. OK? Let's begin."

Select a child.

Before presenting the first problem, say, "You can use the chips to help you figure out the answer."

Hold up two cards (numbers facing toward the children) and ask, "How much is (the number on the first card) plus (the number on the second card)?"

After each child solves a problem, ask, "How did you know that?"

To get credit for solving the tasks, the children must correctly add the numbers on the face of the cards and articulate the method used to solve the problem. Only if the child correctly adds the numbers on the first try does he or she hold the card.

Repeat this process until the children have collected all the cards. Before moving on to the next activity, have each child count his or her cards, then announce the winner of the game.

Items

None

Instructions

If used individually, place the cards face down on the table.

If used in a small group or with the whole class, you may hold selected cards up or project them onto a more visible surface.

Do not provide any manipulatives for this activity.

Introduce the activity by saying, "Today we're going to play another game. I'll show you two cards with numbers on them and, as fast as you can, add the numbers from the two cards together. If you're right and you tell me how you figured out the answer, you get to keep the cards. The person with the most cards at the end of the game wins. OK? Let's begin."

Select a child.

Hold up two cards (numbers facing toward the children) and ask, "How much is (the number on the first card) plus (the number on the second card)?"

After each child solves a problem, ask, "How did you know that?"

To get credit for solving the tasks, the children must correctly add the numbers on the face of the cards and articulate the method used to solve the problem. Only if the child correctly adds the numbers on the first try does he or she hold the card.

Repeat this process until the children have collected all the cards. Before moving on to the next activity, have each child count his or her cards, then announce the winner of the game.

Items

None

SMALL NUMBER ADDITION (#20)

Objective

To develop advanced addition strategies through extended practice with addition combinations

Materials

Cards numbered 1–9

Participants

Individual, small group, and/or whole class. If used with a small group or whole class, increase the number of cards used (e.g., two number card sets for a small group and four sets for whole class instruction).

Norms

On average, 6 1/2-year-olds use advanced counting strategies such as Counting On to solve addition problems with sums to 18 and 7 1/2-year-olds display fluency with single-digit addition combinations.

NCTM Standard

Develop fluency with basic number combinations for addition and subtraction. (Number and Operations: Compute fluently and make reasonable estimates.)

RETRIEVE DOUBLES (#21)

Objective

To master doubles combinations

Materials

Cards numbered 1–9

Chips

Participants

Individual, small group, and/or whole class. If used with a small group or whole class, increase the number of cards used (e.g., two sets of cards for a small group and four sets for whole class instruction).

Norms

On average, 7 1/2-year-olds have mastered large addition doubles (e.g., 9 + 9).

NCTM Standard

Recognize, describe, and extend patterns such as sequences of sounds and shapes or simple numeric patterns and translate from one representation to another. (Algebra: Understand patterns, relations and functions.)

Instructions

Distribute 20 chips to each child.

Introduce the activity by saying, "Today we are going to learn about doubles. When you double a number, you add that number to itself."

Hold the number cards facing away from the children. "These cards have numbers on the other side. I'll show you a card with a number on it and, as fast as you can, tell me what the double of that number is."

Select a child and hold up the #4 card.

Say, "For example, here's the card. What is the double of four?"

After each child gives an answer, ask, "How did you know that?"

- If the child says, "I added 4 + 4," then give him or her the card.
- If the child produces an incorrect answer, ask him or her to check the answer using chips. After the child gives a correct answer, return the card to your pile and shuffle the deck.

Repeat this process until the children have collected all the cards. Before moving on to the next activity, have each child count his or her cards, then announce the winner of the game.

Items

None

Instructions

Distribute 20 chips to each child.

Introduce the activity by saying, "Today we are going to learn a new way of adding numbers by using doubles." Hold the number cards facing away from the children. "These cards have numbers on the other side."

Select a child and hold up the #2 card.

Say, "Here's the number two. How much is 2 + 2?" If the child correctly doubles the number, say, "OK, if 2 + 2 = 4, then how much is 2 + 3?"

After the child produces an answer, ask, "How did you know that?"

- If the child states anything other than the use of doubles and counting on, ask, "How can knowing 2 + 2 help you solve 2 + 3?" If used in a small group or whole class setting, allow other children to respond. Place the card back in your pile and shuffle.
- If the child correctly solved the non-double addition problem and articulates the use of doubles and counting on, you then give the card to the child.
- If the child produces an incorrect answer, ask him or her to check the answer using the chips. Say, "Let's use the chips to check the answer." After the child generates a correct answer, place the card back in your pile and shuffle.

Repeat this process until the children have collected all the cards. Before moving on to the next activity, have each child count his or her cards, then announce the winner of the game.

Items

1 + 1	2 + 1
2 + 2	2 + 3
3 + 3	3 + 4
4 + 4	4 + 5
5 + 5	5 + 6
6 + 6	6 + 7
7 + 7	7 + 8
8 + 8	8 + 9
9 + 9	9 + 10

DOUBLES TO SOLVE DOUBLES (#22)

Objective

To use knowledge of addition doubles to solve problems containing one or two more or less than a double (e.g., 7 + 6)

Materials

Cards numbered 1–9

Chips

Participants

Individual, small group, and/or whole class. Use two sets of cards for a small group and four sets for whole class instruction. The child should be able to accurately produce all doubles before starting this activity.

Norms

On average, 6 1/2-year-olds use advanced counting strategies such as Counting On to solve addition problems with sums to 18 and 7 1/2-year-olds display fluency with these addition combinations.

NCTM Standard

Develop and use strategies for whole-number computations, with a focus on addition and subtraction. (Number and Operations: Compute fluently and make reasonable estimates.)

ADDING & THE ZERO RULE (#23)

Objective

To learn the zero principle in addition

Materials

Chips

Participants

Individual, small group, and/or whole class

Norms

On average, 5 1/2-year-olds can apply the additive zero principle to determine unknown sums to 18.

NCTM Standard

Illustrate general principles and properties of operations, such as commutativity, using specific numbers. (Algebra: represent and analyze mathematical situations and structures using algebraic models.)

Instructions

Introduce the activity by asking, "Does anyone know what zero means?

Zero is a special number smart mathematicians use to represent the absence of any other number.

Today we're going to play a thinking game. We're going to add some really big numbers in our heads. We're not going to use paper, chips, or our fingers. We're going to use our minds to break the problem into smaller parts so we can figure out the answer. Smart mathematicians do this all the time.

OK mathematicians, let's investigate what zero means.

We start out with three chips (pause to slowly count out three chips). Does everyone have three chips? And then we add (pause for effect while adding a make-believe but dramatic nothing) zero chips. How many chips do we have altogether?" (Pause and re-count the chips before answering.)

After several children have shared answers, ask, "How can we check to make sure?" Guide the discussion toward the realization that counting the chips is unnecessary since the answer to addition problems involving zero is always the same as the addend. The following questions can be used to help guide the discussion

- "Do you see a pattern?" The response should be a unanimous "Yes," to which you ask, "Why?" or "How did that happen?"
- "Is there one number that is the same in all of these problems?"
- "Is the answer the same as any other number in the problem?"

Draw their attention to the zeros in each problem. If the children respond with "Yes, zero is in each problem," say, "What else do you notice about these problems?"

If the children articulate the zero principle, move on to Generalize the Zero Rule, Addition.

Items

1 + 0
2 + 0
3 + 0

4 + 0
5 + 0
6 + 0
7 + 0
8 + 0
9 + 0
10 + 0
11 + 0
12 + 0
13 + 0
14 + 0
15 + 0
16 + 0
17 + 0
18 + 0

ADDING & THE ZERO RULE, CONT'D. (#23)

GENERALIZE THE ZERO RULE, ADDITION (#24)

Objective

To generalize the zero principle in addition

Materials

None

Participants

Individual, small group, and/or whole class

Norms

On average, 5 1/2-year-olds can apply the additive zero principle to determine unknown sums to 18.

NCTM Standard

Illustrate general principles and properties of operations, such as commutativity, using specific numbers. (Algebra: represent and analyze mathematical situations and structures using algebraic models.)

Instructions

Introduce the activity by asking, "Does anyone know what zero means?" Listen to the children's responses.

"Zero is a special number smart mathematicians use to represent the absence of any other number."

Next say, "Today we're going to play a thinking game. We're going to add some really big numbers in our heads. We're not going to use paper, chips, or our fingers. We're going to use our minds to investigate adding zero to really big numbers. Let's start with…"

If the child struggles with this activity, go back to Adding & the Zero Rule.

Items

27 + 0
35 + 0
37 + 0
32 + 0
43 + 0
48 + 0
54 + 0
51 + 0
59 + 0
62 + 0
67 + 0
63 + 0
70 + 0
76 + 0
89 + 0
99 +0

Instructions

Distribute 20 chips to each child.

Introduce the activity by saying, "Today we are going to solve some addition problems. You may already know the answer, but we're going to use the chips to help us think about the numbers. OK?

Let's pretend we have three cookies (count out three chips and wait for all children to do the same). Then we get two more cookies (slide two chips to the side and pause until all children do the same). How many cookies do we have altogether?"

After several children share their answers, say, "OK, let's leave these chips on our desks and start the next problem. If you use chips for the next problem, be sure to place them above your other chips. Pretend you have two cookies and you get three more cookies. How many cookies do you have altogether?"

After several children share their answers, ask, "How did you know that?"

Guide the discussion toward the realization that the second addition problem contained the same numbers as the first, but in a different order. The following questions can be used to help guide the discussion:

- "How can we use the numbers in the first problem to help us figure out the answer to the second addition problem?" Wait for the children to respond.
- "Are the numbers in the problems the same or different?" Wait for the children to respond.

Draw attention to the numbers in the problems. Saying, for example, that you can shift around the chips in the second problem to show that they are just the same as the chips in the first problem.

Repeat this line of questioning for each item until you can move on to Generalize the Order Rule, Addition.

Items

2 + 3	3 + 2
4 + 3	3 + 4
2 + 5	5 + 2
4 + 5	5 + 4

ADDING & THE ORDER RULE (#25)

Objective

To identify the order principle

Materials

Chips

Participants

Individual, small group, and/or whole class

Norms

On average, 6 1/2-year-olds can apply the additive commutativity principle to determine unknown sums to 18.

NCTM Standard

Illustrate general principles and properties of operations, such as commutativity, using specific numbers. (Algebra: represent and analyze mathematical situations and structures using algebraic models.)

ADDING & THE ORDER RULE, CONT'D. (#25)

5 + 6	6 + 5
9 + 5	5 + 9
4 + 8	8 + 4
7 + 5	5 + 7
5 + 3	3 + 5
8 + 7	7 + 8
9 + 6	6 + 9
9 + 8	8 + 9
7 + 6	6 + 7

Instructions

Introduce the activity by saying, "You are all such smart mathematicians. I think you are ready to try some addition problems with really big numbers. These are going to be really, really big problems, but I think you can figure out the answers if you think hard."

Divide the chalkboard or overhead projector into two columns. In the first column, write out 24 + 17 = 41. In the second column write out the commuted pair without the answer: 17 + 24 = ?

Direct the children's attention to the item in the first column. Say, "The problem in this first column says 24 + 17 = 41. Can the answer to this problem help us figure out the answer to the next problem, 17 + 24?"

If the children do not respond, say, "Let's look at the numbers. Are the numbers in the problems the same or different?" Wait for the children to respond.

- If children say the numbers are different, say, "What's different about these numbers?"
- If the children's responses relate to order, say, "Does the order change the sum?"
- If children say that the numbers are the same, just "turned" or "switched around," ask, "How can knowing the answer to the first problem in the first box help you figure out the answer to the second problem in the first box?" (Point to each problem in the box.)

Repeat the activity as often as necessary.

Items

16 + 24 = 40	24 + 16 = ?
27 + 12 = 39	12 + 27 = ?
22 + 34 = 56	34 + 22 = ?
29 + 31 = 60	31 + 29 = ?
39 + 43 = 82	43 + 39 = ?
45 + 33 = 78	33 + 45 = ?
56 + 47 = 103	47 + 56 = ?
52 + 49 = 101	49 +52 = ?

GENERALIZE THE ORDER RULE, ADDITION (#26)

Objective

To generalize the order principle using large numbers

Materials

Chalkboard or overhead projector

Participants

Individual, small group, and/or whole class

Norms

On average, 6 1/2-year-olds can apply the additive commutativity principle to determine unknown sums to 18.

NCTM Standard

Illustrate general principles and properties of operations, such as commutativity, using specific numbers. (Algebra: represent and analyze mathematical situations and structures using algebraic models.)

MAKE TENS IN ADDITION, CHIPS (#27)

Objective

To learn to add to make tens from single-digit numbers

Materials

Chips

Participants

Individual, small group, and/or whole class

Norms

On average, 7 1/2-year-olds can make a 10 to solve addition problems with sums to 18.

NCTM Standard

Use multiple models to develop initial understandings of place value and the Base 10 number system. (Number and Operations: Understand numbers, ways of representing numbers, relationships among numbers, and number systems.)

Instructions

Distribute 10 chips to each child.

Introduce the activity by saying, "Today we're going to practice making tens. Can anyone think of a reason why smart mathematicians like to work with the number 10?" Listen to the children's responses.

"Smart mathematicians do this so that they can solve really difficult problems without getting confused by lots of little numbers. Let's practice making tens together. I'm going to say a number and I want you tell me what number needs to be added to it so that the answer is 10."

"For example, if I say the first number is eight, we'll take eight chips out of the main pile like this. We can use the chips to help us." Count out eight chips.

"To figure out the second number, we can count on from eight like this, eight, nine (push aside one chip), ten (push aside another chip)."

"If we start out with eight, we need two more chips to make ten. So then, eight plus two equals ten. OK?" If even one child appears confused, model more items.

Next say, "OK, let's begin. If I have [n], how can I make 10?" If the children do not respond, ask, "How many more chips would I need to make 10, if I start out with [n]?"

Repeat the activity as often as necessary.

Items

8
3
6
5
7

Instructions

Introduce the activity by saying, "Today we're going to practice making tens in our heads. I'll say a number and you tell me what the other number needs to be so that the numbers add up to 10. So if I say eight, you say two: eight plus two equals ten. Let's begin…"

Repeat the activity as often as necessary.

Items

0
10
4
9
2

MAKE TENS IN ADDITION (#28)

Objective

To add to make tens from single-digit numbers

Materials

None

Participants

Individual, small group, and/or whole class

Norms

On average, 7 1/2-year-olds can make a 10 to solve addition problems with sums to 18.

NCTM Standard

Use multiple models to develop initial understandings of place value and the Base 10 number system. (Number and Operations: Understand numbers, ways of representing numbers, relationships among numbers, and number systems.)

USE TENS IN ADDITION, CHIPS (#29)

Objective

To use knowledge of making tens to simplify difficult problems

Materials

Chips

Participants

Individual, small group, and/or whole class

Norms

On average, 7 1/2-year-olds can make a 10 to solve addition problems with sums to 18.

NCTM Standard

Use multiple models to develop initial understandings of place value and the Base 10 number system. (Number and Operations: Understand numbers, ways of representing numbers, relationships among numbers, and number systems.)

Instructions

Distribute 20 chips to each child.

Introduce the activity by saying, "Today we're going to practice using tens to make adding easier. We talked about this a bit earlier. Can anyone remember why we mathematicians like to work with the number 10?" Listen to responses.

"Smart mathematicians like to combine numbers into tens so they don't have to think about lots and lots of little numbers when they're solving really difficult problems. For example, let's figure out nine plus four together by making tens.

First, let's take out nine chips (count and push aside nine chips).

Next, let's take out four chips (count and push aside four chips).

What happens if we take one chip away from the set of four and add it to the set of nine?"

Guide children toward recognizing that the problem has now been transformed into 10 + 3.

"See, it's now ten plus three, which is easier than nine plus four."

Be sure all children understand the procedure. If even one child appears confused, model more items.

Next say, "OK, now you try the next one on your own…" After children have solved the problem, ask several of them to share their answers and explain how they used their knowledge of tens to transform the problem. Specifically, after an answer is provided, ask, "How did you know that?" If the child uses any method other than Using Tens say, "That was great. But let's think about how you could have used 10 to make it easy." Open the discussion up to the class and ask, "How could we have made the problem easier by using 10?" Guide the children toward using tens.

Repeat the activity as often as necessary.

Items

9 + 6
9 + 8
9 + 7
9 + 4

9 + 5
8 + 7
6 + 8
8 + 4
8 + 5

USE TENS IN ADDITION, CHIPS, CONT'D. (#29)

USE TENS IN ADDITION (#30)

Objective

To use knowledge of making tens to simplify difficult problems

Materials

Pencil

Paper divided into three columns

Overhead projector or chalkboard

Participants

Individual, small group, and/or whole class

Norms

On average, 7 1/2-year-olds can make a 10 to solve addition problems with sums to 18.

NCTM Standard

Use multiple models to develop initial understandings of place value and the Base 10 number system. (Number and Operations: Understand numbers, ways of representing numbers, relationships among numbers, and number systems.)

Introductions

Introduce the activity by saying, "Today we're going to practice using tens to help us solve other addition problems in our heads. First we're going to write some problems on this paper and solve them together."

Distribute the paper with three columns to each child. On the chalkboard (or overhead projector), draw three columns.

"OK, let's write 3 + 9 in the first column." Write 3 + 9 in the first box on the transparency or chalkboard.

"In the second column, let's change it into 10 plus another number." Write 10 + 2 in the second box.

"What's the answer to both of these problems?" Write 12 in the third box.

Follow the same format for the balance of the items. After solving several problems, ask, "What pattern do you notice?" If the children do not respond, ask, "How can this tens problem (point to 10 + 2) help us figure out the answer to 3 + 9?" The discussion should emphasize the efficiency of transforming problems into combinations of 10.

Items

Initial problem	Use tens	Answer
11 + 4	10 + 5	15
11 + 5	10 + 6	16
12 + 3	10 + 5	15
12 + 4	10 + 6	16
12 + 5	10 + 7	17
13 + 6	10 + 9	19
14 + 1	10 + 5	15
15 + 4	10 + 9	19
17 + 2	10 + 9	19

Instructions

Introduce the activity by saying, "Today we're going to play a thinking game. We're going to add some really big numbers in our heads. We're not going to use paper, chips, or our fingers. We're going to use our minds to break the problem down into smaller parts so we can figure out the answer. Smart mathematicians do this all the time. They try to think of ways to make really difficult problems a little bit easier. For example, if I say how much is 24 plus 10, first we add 20 plus 10 to get 30 and then we add in the 4 to get 34. OK? (Wait for confirmation. You may want to provide additional examples.) Let's begin.

How much is…?"

After each item, ask, "How did you know that?" The following prompts may be used to guide the discussion:

- If responses indicate reliance on tens and ones, move on (e.g., "I just counted on from 20 by tens and added the 4").
- If responses indicate anything other than the above, ask, "How can we break the problem into smaller parts to figure out the answer?"
- If even one child produces an incorrect answer, ask, "How can we check the answer?"
- If responses indicate simplifying the problem (e.g., "Let's separate the tens from the ones"), follow their lead and do not introduce the unifix cubes.
- If responses indicate using the unifix cubes, say, "Show me how you would use the cubes to check the answer." Help children to group the cubes by tens and ones and then ask, "How could we have solved the problem in our heads without the cubes?" Guide the discussion toward separating and adding the tens first and then the ones. After the answer has been checked, say, "OK, now we're going to put the cubes away and get back to our *thinking* game."

Items

31 + 20
42 + 30
17 + 20
47 + 30
45 + 30

ADD MULTIPLES OF 10 (#31)

Objective

To use decomposition to mentally add simple two-digit numbers

Materials

Unifix cubes

Participants

Individual, small group, and/or whole class.

Administer only to children who can simplify two-digit numbers. To assess this, say, "Let's begin today's lesson by breaking large numbers into smaller ones. For example, 23 can be broken down into 20 plus 3. Now you try. How can we break down 46?"

Items: 29, 36, 41, 53, 68

Norms

On average, 8 1/2-year-olds can mentally decompose two-digit numbers to determine the sum.

NCTM Standard

Develop a sense of whole numbers and represent and use them in flexible ways, including relating, composing, and decomposing numbers. (Number and Operations: Understand numbers, ways of representing numbers, relationships among numbers, and number systems.)

ADD MULTIPLES OF 10, CONT'D.

(#31)

37 + 40
55 + 70
39 + 50
35 + 20
64 + 10
37 + 10
33 + 20
48 +20
47 + 30
74 + 20
85 + 20
52 + 40
67 + 10
75 + 10
64 + 20
19 + 30

SIMPLIFY TWO-BY-TWO ADDITION (#32)

Objective
To use decomposition to mentally add complex two-digit numbers

Materials
Unifix cubes

Participants
Individual, small group, and/or whole class

Norms
On average, 8 1/2-year-olds can mentally decompose two-digit numbers to determine the sum.

NCTM Standard
Develop a sense of whole numbers and represent and use them in flexible ways, including relating, composing, and decomposing numbers. (Number and Operations: Understand numbers, ways of representing numbers, relationships among numbers, and number systems.)

Instructions

Introduce the activity by saying, "Let's continue thinking like mathematicians. I think you are ready to try adding problems with really big numbers, using only your minds. We're not going to use paper, chips, or our fingers. We're going to use our mathematical minds to break the problems into smaller parts so we can figure out the answers to some difficult problems. For example, if I say how much is 24 plus 13, first we add 20 plus 10 to get 30, then we add 4 plus 3 to get 7, and then we add 30 plus 7 to get 37. OK?" Wait for confirmation. You may want to provide additional examples.

"Let's begin. How much is… "

After each item, ask, "How did you know that?" The following prompts may be used to guide the discussion:

- If responses indicate reliance on tens and ones (e.g., "I just counted on from 20 by tens to get 30 and then added 4 plus 3 to get 7, and when I put it all together, I got 37"), move on.
- If responses indicate anything other than the above, ask, "How can we break the problem into smaller parts to figure out the answer?"
- If even one child produces an incorrect answer, ask, "How can we check the answer?"
- If responses indicate simplifying the problem into smaller parts (e.g., "Let's separate the tens from the ones"), follow their lead and do not introduce the cubes.
- If child responses indicate using the unifix cubes to check, say, "Show me how you would use the cubes to check the answer." Once children grouped the cubes by tens and ones, ask, "How could we have solved the problem in our heads without the cubes?" Guide the discussion toward separating and adding the tens first and then the ones. After checking the answer, say, "OK, now we're going to put the cubes away and get back to our *thinking* game."

Items

Non-carrying

26 + 13
33 + 24

SIMPLIFY TWO-BY-TWO ADDITION, CONT'D. (#32)

52 + 15
25 + 44
41 + 18
53 + 34
84 + 13
47 +21
53 + 27

Carrying

35 + 27
46 + 24
39 + 28
84 + 17
49 + 57
58 + 46
87 + 36
76 + 59
56 + 39

SUBTRACTION

SEPARATE FROM, CHIPS (#33)

Objective

To use counting strategies to solve and check subtraction problems

Materials

Chips

Participants

Individual, small group, and/or whole class

Norms

On average, 5-year-olds and 7-year-olds use separate-from to solve subtraction problems with differences to 10 and 18 respectively.

NCTM Standard

Develop and use strategies for whole-number computations, with a focus on addition and subtraction. (Number and Operations: Compute fluently and make reasonable estimates.)

Instructions

Introduce the activity by saying, "Today we're going to practice subtracting numbers. I'm going to tell you a story and you can use the chips to help you figure out the answer."

To reduce reliance on memory you may want to project or write the problem on the board.

Distribute 15 chips to each child.

There are three story themes in which to present the problems. Model the first one using chips.

1. "Pretend you had two cookies (count while pushing aside two chips) and you ate one cookie (push aside one chip). How many cookies did you have left?" (Count the remaining chips in the main pile before announcing the difference.)

   Allow the children enough time to count the chips by pausing after the presentation of each number in the problem and again before requesting the difference.
2. "Pretend you had three treasures and a monster took two of them. How many treasures do you have left?"
3. "Pretend four friends were at your house and then one friend went home. How many friends were still there?"

Every child can benefit from developing checking skills. After all the children have determined the difference, ask several to share their answer and then ask the group to check the answer. Specifically, ask, "How can we check to make sure the answer is right?" Ask several children to share their checking procedures, then say, "OK, let's all check our answers."

Items

4 – 3
5 – 3
6 – 4
6 – 5
7 – 4
7 – 5
7 – 6

SEPARATE FROM, CHIPS, CONT'D.

(#33)

8 – 4
8 – 5
8 – 6
8 – 7
9 – 4
9 – 5
9 – 6
9 – 7
9 – 8
10 – 4
10 – 7
10 – 8
10 – 9
11 – 4
11 – 5
11 – 6
11 – 7
11 – 8
11 – 9
11 – 10
12 – 4
12 – 5
12 – 6
12 – 7
12 – 8
12 – 9
12 – 10
12 – 11

Instructions

Introduce the activity by saying, "Today we're going to practice subtracting numbers. I'm going to tell you a story and I want you to predict the answer. Does everyone know what *predict* means? Predict is another way of saying guess. Mathematicians and scientists use the word *predict* to describe how they think really hard in their minds about what the answer to a question might be. Then they check to see if they are right. We will use chips to check our predictions."

Distribute 15 chips to each child.

Verbally and pictorially present each problem, using any mode that allows all the children in the group to see the transformation.

Present the problem in a story theme. For example, say, "Pretend there are three children on the playground and one child leaves to go home and have dinner. Predict how many children are left on the playground."

If the children begin to count or use their chips say, "Let's not count or use the chips just yet. Let's be mathematicians and make predictions, how many children do you think are left on the playground?" After several children have generated answers, say, "Now let's see how good we are at predicting the answer. How can we check to see if our predictions were right?" Ask several children to share their checking procedures, then say, "OK, let's check."

Affirm each child's checking procedures by saying something like, "Wow, _______ sounds just like a mathematician. He or she thought about _______ in his or her mind, then checked the answer with chips to see if he or she was right."

Items

10 – 1
9 – 1
8 – 1
7 – 1
6 – 1
5 – 1
12 – 1
12 – 3

PREDICT, SEPARATE FROM, CHIPS (#34)

Objective

To mentally subtract and to check the result by counting sets

Materials

Chips

Participants

Individual, small group, and/or whole class

Norms

On average, 5-year-olds can use concrete strategies to solve problems with differences to 18.

NCTM Standard

Develop and use strategies for whole-number computations, with a focus on addition and subtraction. (Number and Operations: Compute fluently and make reasonable estimates.)

PREDICT,
SEPARATE FROM,
CHIPS, CONT'D.
(#34)

11 – 2
9 – 3
9 – 2
7 – 1
6 – 2
4 – 3
15 – 5
10 – 4
6 – 3
3 – 2
15 – 2
13 – 3
11 – 5

SMALL NUMBER SUBTRACTION, CHIPS (#35)

Objective

To develop advanced subtraction strategies through extended practice with addition combinations

Materials

Cards numbered 0–18

Chips

Participants

Individual, small group, and/or whole class. If used with a small group or whole class, increase the number of cards used (e.g., two number card sets for small group and four sets for whole class instruction).

Norms

On average, 7 1/2-year-olds use advanced counting strategies such as Counting Up to solve subtraction problems with differences to 18 and 8 1/2-year-olds display fluency on single-digit subtraction combinations.

NCTM Standard

Understand various meanings of addition and subtraction of whole numbers and the relationship between the two operations. (Number and Operations: Understand meanings of operations and how they relate to one another.)

Instructions

If used individually, place the cards face down on the table. If used in a small group or with the whole class, hold selected cards up or project cards onto a more visible surface.

Distribute 20 chips to each child.

Introduce the activity by saying, "Today we're going to play another game. I'll show you two cards with numbers on them and I want you to subtract the numbers on them. If you're right and you tell me how you figured out the answer, you get to keep the cards. The person with the most cards at the end of the game wins. OK? Let's begin."

Select a child.

Before presenting the first problem, say, "You can use the chips to help you figure out the answer."

Hold up two cards (numbers facing toward the children) and ask, "How much is (the number on the first card) minus (the number on the second card)?"

After each child solves a problem, ask, "How did you know that?"

To get credit for solving the tasks, the children must correctly subtract the numbers on the face of the cards and articulate the method used to solve the problem. If the child correctly subtracts the numbers on the first try only, he or she holds the card.

Time permitting, this process is repeated until the children collect all the cards. Before moving on to the next activity, ask the children to count the number of cards in their possession and declare the child with the most cards the winner of the game.

Items

None

SMALL NUMBER SUBTRACTION (#36)

Objective

To develop advanced subtraction strategies through extended practice with addition combinations

Materials

Cards numbered 0–18

Participants

Individual, small group, and/or whole class. If used with a small group or whole class, increase the number of cards used (e.g., two number card sets for small group and four sets for whole class instruction).

Norms

On average, 7 1/2-year-olds use advanced counting strategies such as Counting Up to solve subtraction problems with differences to 18 and 8 1/2-year-olds display fluency on single-digit subtraction combinations.

NCTM Standard

Understand various meanings of addition and subtraction of whole numbers and the relationship between the two operations. (Number and Operations: Understand meanings of operations and how they relate to one another.)

Instructions

If used individually, place the cards face down on the table. If used in a small group or with the whole class, hold selected cards up or project cards onto a more visible surface.

Distribute 20 chips to each child.

Introduce the activity by saying, "Today we're going to play another game. I'll show you two cards with numbers on them and I want you to subtract the numbers on them. If you're right and you tell me how you figured out the answer, you get to keep the cards. The person with the most cards at the end of the game wins. OK? Let's begin."

Select a child.

Before presenting the first problem, say, "You can use the chips to help you figure out the answer."

Hold up two cards (numbers facing toward the children) and ask, "How much is (the number on the first card) minus (the number on the second card)?"

After each child solves a problem, ask, "How did you know that?"

To get credit for solving the tasks, the children must correctly subtract the numbers on the face of the cards and articulate the method used to solve the problem. If the child correctly subtracts the numbers on the first try only, he or she holds the card.

Time permitting, this process is repeated until the children collect all the cards. Before moving on to the next activity, ask the children to count the number of cards in their possession and declare the child with the most cards the winner of the game.

Items

None

Instructions

Before beginning the activity, model the zero principle, using manipulatives.

Distribute 20 chips to each child.

Introduce the activity by saying, "Today we are going to play a thinking game. We are going to subtract some numbers."

"We start out with three chips (pause and slowly count out three chips). Does everyone have three chips? And then we subtract (pause for effect while subtracting a make-believe but dramatic nothing) zero chips. How many chips do we have left?" (Pause and re-count the chips before answering.)

After several children have shared answers ask, "How can we check to make sure?" Guide the discussion toward the realization that counting the chips is unnecessary since the answer to subtraction problems involving 0 is always the same as the minuend. The following questions can be used to help guide the discussion:

- "Do you see a pattern?" The response should be a unanimous "Yes" to which you ask, "Why?" or "How did that happen?"
- "Is there one number that is the same in all of these problems?"
- "Is the answer the same as any other number in the problem?"

Draw their attention to the zeros in each problem. If the children respond with "Yes, zero is in each problem," ask, "What else do you notice about these problems?"

Repeat the activity until you can move on to Generalize the Zero Rule, Subtraction.

Items

1 – 0
2 – 0
3 – 0
4 – 0
5 – 0
6 – 0

SUBTRACTING & THE ZERO RULE (#37)

Objective

To learn the zero principle in subtraction

Materials

Chips

Participants

Individual, small group, and/or whole class

Norms

On average, 5 1/2-year-olds can apply the subtractive zero principle to determine unknown differences to 18.

NCTM Standard

Illustrate general principles and properties of operations, such as commutativity, using specific numbers. (Algebra: represent and analyze mathematical situations and structures using algebraic models.)

SUBTRACTING & THE ZERO RULE, CONT'D. (#37)

7 – 0
8 – 0
9 – 0
10 – 0
11 – 0
12 – 0

Instructions

Introduce the activity by asking, "Does anyone know what zero means?" Listen to responses. "Zero is a special number smart mathematicians use to represent the absence of any other number."

Next say, "Today we're going to play a thinking game. We're going to subtract some really big numbers in our heads. We're not going to use paper, chips, or our fingers. We're going to use our minds to investigate subtracting zero from really big numbers. Let's start with…"

If the children struggle with this activity, go back to Subtracting & the Zero Rule.

Items

13 – 0
14 – 0
15 – 0
16 – 0
17 – 0
18 – 0
19 – 0
20 – 0
23 – 0
27 – 0
31 – 0
35 – 0
42 – 0
46 – 0
45 – 0
58 – 0
50 – 0
64 – 0

GENERALIZE THE ZERO RULE, SUBTRACTION (#38)

Objective

To generalize the zero principle in subtraction

Materials

None

Participants

Individual, small group, and/or whole class

Norms

On average, 5 1/2-year-olds can apply the subtractive zero principle to determine unknown differences to 18.

NCTM Standard

Illustrate general principles and properties of operations, such as commutativity, using specific numbers. (Algebra: represent and analyze mathematical situations and structures using algebraic models.)

GENERALIZE THE ZERO RULE, SUBTRACTION, CONT'D. (#38)

67 – 0
71 – 0
72 – 0
84 – 0
88 – 0
91 – 0
93 – 0
99 – 0
100 – 0
129 – 0
139 – 0
183 – 0
248 – 0
265 – 0

Instructions

Before beginning the activity below, model the same number principle with manipulatives.

Distribute 20 chips to each child.

Introduce the activity by saying, "Today we are going to play a thinking game. We are going to subtract some numbers."

"We start out with four chips (pause and count out four chips very carefully). Does everyone have four chips? And then we subtract (pause for effect) four chips (dramatically remove the chips). How many chips do we have left?"

After several children have shared answers ask, "How can we check to make sure?" Guide the discussion toward the realization that checking is unnecessary as the answer to subtraction problems involving same numbers is always 0. The following questions can be used to help guide the discussion:

- "Do you see a pattern?" The response should be a unanimous "Yes" to which you ask, "Why?" or "How did that happen?"
- "Are the numbers in the problem the same or different?"
- "Are the answers to the different problems the same?"

Draw their attention to the numbers in each problem. If the children respond with "Yes, the numbers are the same in each problem," ask, "What else do you notice about these problems?"

If all children articulate the principle, move on to Generalize the Same Number Rule.

Items

1 – 1
2 – 2
3 – 3
4 – 4
5 – 5
6 – 6
7 – 7
8 – 8

FIND THE SAME NUMBER RULE (#39)

Objective

To learn the same number principle in subtraction

Materials

Chips

Participants

Individual, small group, and/or whole class

Norms

Most children can achieve the activity's goal by about 5.

NCTM Standard

Count with understanding and recognize "how many" in sets of objects. (Number and Operations: Understand numbers, ways of representing numbers, relationships among numbers, and number systems.)

FIND THE SAME NUMBER RULE, CONT'D. (#39)

9 – 9
10 – 10
11 – 11
12 – 12
13 – 13
14 – 14
15 – 15
16 – 16
17 – 17
18 – 18

Instructions

Introduce the activity by saying, "Today we're going to play a thinking game. We're going to subtract some really big numbers in our heads. We're not going to use paper, chips, or our fingers. Let's start with…"

If the child struggles with this activity, go back to Find the Same Number Rule.

Items

12 – 12
19 – 19
21 – 21
24 – 24
32 – 32
39 – 39
43 – 43
49 – 49
52 – 52
56 – 56
64 – 64
67 – 67
71 – 71
74 – 74
85 – 85
89 – 89
93 – 93
99 – 99
105 – 105
113 – 113
128 – 128

GENERALIZE THE SAME NUMBER RULE (#40)

Objective

To learn the same number principle in subtraction

Materials

None

Participants

Individual, small group, and/or whole class

Norms

On average, 6 1/2-year-olds can apply the negation principle to determine unknown differences to 18.

NCTM Standard

Illustrate general principles and properties of operations, such as commutativity, using specific numbers. (Algebra: Represent and analyze mathematical situations and structures using algebraic models.)

GENERALIZE THE SAME NUMBER RULE, CONT'D. (#40)

132 – 132
144 – 144
157 – 157
161 – 161
176 – 176
185 – 185
192 – 192

DISCOVER THE INVERSE RULE (#41)

Objective

To discover the inverse principle

Materials

Chips

Participants

Individual, small group, and/or whole class

Norms

On average, 7-year-olds can apply the inverse principle to determine related addition sums.

NCTM Standard

Illustrate general principles and properties of operations, such as commutativity, using specific numbers. (Algebra: Represent and analyze mathematical situations and structures using algebraic models.)

Instructions

Before beginning the activity, model the inverse principle with manipulatives.

Distribute 20 chips to each child.

Introduce the activity by saying, "Today we are going to solve addition and subtraction problems. You may already know the answer to some of these problems, but we're going to use the chips to help us think about the numbers. OK?

Let's pretend there are five children on the playground (count out five chips and wait for all children to do the same). Three of the children need to go home for lunch (slide three chips to the side and pause until all children do the same). How many children are left on the playground? How much is five minus three?"

After several children share their answers, say, "OK, so we know that five minus three equals two. For the next part of the problem you can look at the chips, but try not to move them. Now what happens if the three children that went home for lunch come back to the playground? How much is three plus two?"

After several children share their answers, ask, "How did you know that?" Guide the discussion toward the realization that the addition problem was just the inverse of the subtraction problem that was just solved. The following questions can be used to help guide the discussion:

- "How can we use the subtraction problem we just solved to help figure out the answer to the addition problem?" Wait for the children to respond.
- "Are the numbers in the problems the same or different?" Wait for the children to respond.

Direct their attention toward the components of the inverse principle. For example, discuss how the minuend in the subtraction problem is actually the sum of the addends in the addition problem and that the subtrahend in the subtraction problem is an addend in the addition problem.

Repeat this line of questioning for each item until the children can move on to Generalize the Inverse Rule.

Items

3 – 1	2 + 1

DISCOVER THE INVERSE RULE, CONT'D. (#41)

4 – 1	3 + 1
6 – 2	4 + 2
7 – 3	4 + 3
6 – 5	5 + 1
12 – 7	6 + 7
13 – 5	8 + 5
14 – 6	8 + 6
17 – 5	12 + 5
16 – 9	7 + 9
13 – 6	6 + 7
15 – 7	7 + 8
16 – 8	8 + 8
17 – 8	8 + 9
17 – 9	8 + 9

Instructions

Introduce the activity by saying, "I've been very impressed with all of you mathematicians. I think you are all ready to work with some very big numbers. These are going to be really, really big problems, but I think you can figure out the answers."

Divide the chalkboard or overhead projector into two columns. In the first column, write out the initial problem with the answer (i.e. 56 - 23 = 33) and in the second column write out the inversed pair without the answer (i.e. 33 + 23 = ?).

Direct the children's attention to the items in the first column. Say, "The first problem in the first column says that 56 minus 23 equals 33. How can we use this subtraction problem to help us figure out the answer to the addition problem?" If there is no response, ask specific questions about the problems. The following questions can be used to help guide the discussion:

- "Let's look at the numbers in the first column. How can we use the subtraction problem to help figure out the addition problem?" Wait for the children to respond.
- "Are the numbers in the problems the same or different?" Wait for the children to respond.

Direct their attention toward the components of the inverse principle. For example, discuss how the minuend in the subtraction problem is actually the sum of the addends in the addition problem and that the subtrahend in the subtraction problem is an addend in the addition problem. The first and largest number in the subtraction problem equals the sum of the two other numbers.

Items

26 – 19 = 7	7 + 19 = ?
54 – 36 = 18	18 + 36 = ?
42 – 27 = 15	15 + 27 = ?
37 – 23 = 14	14 + 23 = ?
69 – 29 = 40	40 + 29 = ?

GENERALIZE THE INVERSE RULE (#42)

Objective

To learn to generalize the inverse principle

Materials

Chalkboard or overhead projector

Participants

Individual, small group, and/or whole class

Norms

On average, 7-year-olds can apply the inverse principle to determine related addition sums.

NCTM Standard

Illustrate general principles and properties of operations, such as commutativity, using specific numbers. (Algebra: Represent and analyze mathematical situations and structures using algebraic models.)

MAKE TENS IN SUBTRACTION, CHIPS (#43)

Objective

To subtract by making tens

Materials

Chips

Participants

Individual, small group, and/or whole class.

Administer only to children who are familiar with single-digit combinations that add to make 10.

Norms

On average, 7 1/2-year-olds can make a 10 to solve subtraction problems with differences to 18.

NCTM Standard

Use multiple models to develop initial understandings of place value and the Base 10 number system. (Number and Operations: Understand numbers, ways of representing numbers, relationships among numbers, and number systems.)

Instructions

Distribute 20 chips to each child.

Introduce the activity by saying, "Today we're going to practice solving subtraction problems by making tens. Smart mathematicians do this so that they can solve really difficult problems without getting confused by lots of little numbers. I'll say a subtraction problem and we'll think and talk about how to make a 10 to help us solve the problem."

"We can use the chips to help us if we want. For example, let's solve 13 – 7 by making tens. If we start with seven and figure out how much we need to add to reach thirteen, whatever we add will be the answer. So how much do we need to add to seven to get thirteen?"

"Let's use the chips to help us:

- Seven plus three (push aside three chips) is ten.
- Ten plus three (push aside another three chips) is thirteen.

So we added one, two, three, four, five, six chips to the seven to get thirteen. The answer then is six. In this case we used the chips to help us track how much was added to the seven to get to thirteen."

Model the procedure as many times as necessary.

Next say, "OK, now you try the next one on your own. How much is …"

After children have solved the problem, ask several to share their answers and explain how they used their knowledge of tens to transform the problem.

After a child answers, ask, "How did you know that?" If the child uses any method other than 10-facts say, "That was great. But let's think about how we could use our knowledge of 10-facts to help us." Open the discussion up to the class and ask, "How could we have transformed the problem using 10-facts?" Guide the children toward identifying useful 10-facts.

Repeat the activity as often as necessary.

Items

13 – 7
12 – 8
11 – 4

13 – 8
12 – 5
11 – 7
12 – 7
16 – 9
14 – 9
13 – 9
15 – 8
17 – 9
13 – 7
11 – 8

MAKE TENS IN SUBTRACTION, CHIPS, CONT'D. (#43)

MAKE TENS IN SUBTRACTION (#44)

Objective

To subtract by making tens

Materials

None

Participants

Individual, small group, and/or whole class.

Administer only to children who are familiar with single-digit combinations that add to make 10.

Norms

On average, 7 1/2-year-olds can make a 10 to solve subtraction problems with differences to 18.

NCTM Standard

Use multiple models to develop initial understandings of place value and the Base 10 number system. (Number and Operations: Understand numbers, ways of representing numbers, relationships among numbers, and number systems.)

Instructions

Introduce the activity by saying, "Today we're going to practice solving subtraction problems by making tens. Smart mathematicians do this so that they can solve really difficult problems without getting confused by lots of little numbers. I'll say a subtraction problem and we'll think and talk about how to make a 10 to help us solve the problem.

For example, let's solve 13 – 7 by making tens.

We know that 13 minus 3 is 10.

We were supposed to subtract seven but only subtracted three to make ten. This means we have four more to take away.

Now all we need to figure out is how much 10 – 4 is. Ten minus four is six, so what is the answer to 13 – 7?

Model the procedure as necessary. Next say, "OK, now you try the next one on your own." After children have solved the problem ask several to share their answers and explain how they used their knowledge of tens to transform the problem.

After an answer is provided, ask, "How did you know that?" If the child uses any method other than 10-facts say, "That was great. But let's think about how we could use our knowledge of 10-facts to help us." Open the discussion up to the class and ask, "How could we have transformed the problem using 10-facts?" Guide the children toward identifying useful 10-facts.

Repeat the activity as often as necessary.

Items

13 – 7
12 – 8
11 – 4
13 – 8
12 – 5
11 – 7
12 – 7
16 – 9
14 – 9

13 – 9
15 – 8
17 – 9
13 – 7
11 – 8

MAKE TENS IN SUBTRACTION, CONT'D. (#44)

COUNT BACKWARD BY TENS (#45)

Objective

To count backward by decades

Materials

None

Participants

Individual, small group, and/or whole class.

Administer only to children who can count forward by tens to 100.

Norms

On average, 5 1/2-year-olds can successfully skip count by tens to 100.

NCTM Standard

Use multiple models to develop initial understandings of place value and the Base 10 number system. (Number and Operations: Understand numbers, ways of representing numbers, relationships among numbers, and number systems.)

Instructions

Introduce the activity by saying, "Today we're going to practice counting backward by tens. This is just like counting backward from 10 to 1, only the numbers are much bigger. Let's see how far back we can count from 50, like this: 50, 40, keep counting backward."

Repeat as often as necessary. After all children successfully count backward from 50, begin counting backward from a larger decade (e.g., 70).

Items

None

Instructions

Introduce the activity by saying, "Today we're going to practice subtracting numbers. Sometimes we use chips or paper or other things to figure out the answers to problems, but today we're going to try to do it all in our heads. We're not going to use paper, chips, or our fingers. We're going to use our minds to help us figure out the answer. OK?" Wait for confirmation.

Let's do it together first. "If I say, 'How much is three minus one,' you count backward by one, like this: Three, two... So, if I say, 'How much is 30 minus 10,' you count backward by tens, like this: 30 (whisper), 20... The answer is 20."

"OK. Now you try it. How much is…"

After each decade item, ask, "How did you know that?"

- If responses indicate that they relied on Base 10 knowledge, move on (e.g., "I just counted backward from 30 by tens").
- If responses indicate anything other than the above, ask, "How could we have counted backward to figure out the answer?"

After several single- and double-digit items have been solved, ask, "Do you see a pattern?" Focus discussion on recognizing the similarities between single-digit problems and their decade counterparts in subtraction.

Items

2 – 1	20 – 10
3 – 1	30 – 10
4 – 1	40 – 10
5 – 1	50 – 10
6 – 1	60 – 10
7 – 1	70 – 10
8 – 1	80 – 10
9 – 1	90 – 10
3 – 2	30 – 20
4 – 2	40 – 20
5 – 3	50 – 30

SUBTRACT BY TENS (#46)

Objective

To use patterns to subtract by tens

Materials

None

Participants

Individual, small group, and/or whole class

Norms

On average, 7 1/2-year-olds can add a decade plus 10 and 8 1/2-year-olds can add multiples of ten successfully.

NCTM Standard

Recognize, describe, and extend patterns such as sequences of sounds and shapes or simple numeric patterns and translate from one representation to another. (Algebra: Understand patterns, relations, and functions.)

SUBTRACT BY TENS, CONT'D.

(#46)

6 – 3	60 – 30
7 – 3	70 – 30
7 – 2	70 – 20
5 – 2	50 – 20
4 – 3	40 – 30
5 – 4	50 – 40
9 – 3	90 – 30
7– 4	70 – 40
8 – 5	80 – 50
9 – 4	90 – 40
8 – 7	80 – 70

SUBTRACT MULTIPLES OF 10 (#47)

Objective

To use decomposition to mentally subtract simple two-digit numbers

Materials

Unifix cubes

Participants

Individual, small group, and/or whole class

Norms

Beginning around the age of seven and a half, children can mentally decompose two-digit numbers to determine the difference.

NCTM Standard

Develop a sense of whole numbers and represent and use them in flexible ways, including relating, composing, and decomposing numbers. (Number and Operations: Understand numbers, ways of representing numbers, relationships among numbers, and number systems.)

Instructions

Introduce the activity by saying, "Today we're going to play a thinking game. We're going to subtract some really big numbers in our heads. We're not going to use paper, chips, or our fingers. We're going to use our minds to break the problem into smaller parts so we can figure out the answer. Smart mathematicians do this all the time. They try to think of ways to make really difficult problems a little bit easier.

For example, if I say 'How much is 24 minus 10,' we can break the problem down into pieces. First, we know 20 minus 10 is 10. Second, we know four minus zero is four. Finally, we know that 10 plus 4 is 14. So, 24 minus 10 is 14. OK?"

Wait for confirmation. You may want to provide additional examples.

"Let's begin. How much is…"

After each item, ask, "How did you know that?"

- If responses indicate reliance on tens and ones, move on (e.g., "First I subtracted the tens by counting backward. Then I was going to subtract the ones, but four minus zero is four, so I just put the tens and ones together to get fourteen").
- If responses indicate anything other than the above, ask, "How can we break the problem into smaller parts to figure out the answer?"
- If even one child produces an incorrect answer, ask, "How can we check the answer?"
- If the child's response indicates simplifying the problem into smaller parts (e.g., "Let's separate the tens from the ones"), follow his or her lead and do not introduce the cubes.
- If the child's response indicates using the unifix cubes to check, say, "Show me how you would use the cubes to check the answer." Allow children to group the cubes by tens and ones and then ask, "How could we have solved the problem in our heads without the cubes?" Guide the discussion toward separating and adding the tens first and then the ones. After the answer has been checked say, "OK. Now we're going to put the cubes away and get back to our *thinking* game."

Items

(Next page)

SUBTRACT MULTIPLES OF 10, CONT'D. (#47)

31 – 20
42 – 30
47 – 30
87 – 30
75 – 50
50 – 30
35 – 20
64 – 10
37 – 10
33 – 20
48 – 20
74 – 20
85 – 20
52 – 40
67 – 10
75 – 30
64 – 20
58 – 20
63 – 40
56 – 20
93 – 20
85 – 40
67 – 30
98 – 30
76 – 40

Instructions

Introduce the activity by saying, "Let's continue thinking like mathematicians. I think you are ready to try subtraction problems with really big numbers in your heads. We're not going to use paper, chips, or our fingers. We're going to use our mathematical minds to break the problems into smaller parts so we can figure out the answers to some difficult problems.

First let's break the numbers down into tens and ones. Twenty-four has two tens and four ones. Thirteen has one ten and three ones. If I subtract the one ten in thirteen from the two tens in forty-one, I have one ten left over. Then, if I subtract the three in thirteen from the four in twenty-four, I have one left over. Finally, if I add the leftovers together, I get 10 + 1 = 11." You may want to provide additional examples.

"Let's try some more. How much is…" After each item, ask, "How did you know that?"

- If responses indicate reliance on tens and ones move on (e.g., "I just separated the tens and ones").
- If responses indicate anything other than the above, ask, "How can we break the problem into smaller parts to figure out the answer?"

If even one child produces an incorrect answer ask, "How can we check the answer?"

- If a child's response indicates simplifying the problem into smaller parts (e.g., "Let's separate the tens from the ones"), follow his or her lead and do not introduce the cubes.
- If responses indicate using the unifix cubes to check, say, "*Show* me how you would use the cubes to check the answer." Once children have grouped the cubes by tens and ones, ask, "How could we have solved the problem in our heads without the cubes?" Guide the discussion toward separating and adding the tens first and then the ones. After the answer has been checked say, "OK. Now we're going to put the cubes away and get back to our *thinking* game."

Items

Non-carrying

48 – 22
37 – 14
86 – 31

SIMPLIFY TWO-BY-TWO SUBTRACTION (#48)

Objective

To use decomposition to mentally subtract complex two-digit numbers

Materials

Unifix cubes

Participants

Individual, small group, and/or whole class.

Administer only to children who can simplify two-digit numbers. To assess child knowledge, say, "We talked a little bit before about how smart mathematicians think about ways to make really difficult problems easier to solve by breaking large numbers into smaller ones. For example, 23 can be broken down into two tens and three ones. Now you try. How can we break down 26?" Repeat with 29, 36, 41, 53, and 68.

Norms

Beginning around the age of seven and a half, children can mentally decompose two-digit numbers to determine the difference.

(continued on next page)

SIMPLIFY TWO-BY-TWO SUBTRACTION, CONT'D. (#48)

NCTM Standard

Develop a sense of whole numbers and represent and use them in flexible ways, including relating, composing, and decomposing numbers. (Number and Operations: Understand numbers, ways of representing numbers, relationships among numbers, and number systems.)

76 – 43
73 – 12
67 – 21
26 – 13
52 – 31
48 – 25
45 – 31
57 – 24
47 – 12
58 – 13
93 – 61

Carrying

35 – 27
46 – 29
34 – 28
84 – 57
62 – 57
52 – 46
83 – 36
76 – 59
44 – 27
53 – 27
51 – 34
84 – 59

MULTIPLICATION

MULTIPLY BY COUNTING ALL, CHIPS (#49)

Objective

To solve and check multiplication problems using counting strategies

Materials

Chips

Participants

Individual, small group, and/or whole class

Norms

On average, 6 1/2-year-olds use repeated addition to solve simple multiplication problems with concrete objects.

NCTM Standard

Understand situations that entail multiplication and division, such as equal groupings of objects and sharing equally. (Number and Operations: Understand meanings of operations and how they relate to one another.)

Instructions

Introduce the activity by saying, "Today we're going to practice multiplying numbers together. Does anyone know what it means to multiply?" Listen to explanations volunteered by children.

"Instead of adding a number to itself a certain number of times, smart mathematicians use a strategy called multiplication. Let's try it. I'm going to tell you a story and you can use the chips to help you figure out the answer."

To reduce reliance on memory you may want to project or write the multiplicand and multiplier on the board.

Distribute 20 chips to each child.

There are two story themes in which to present the problems. Model the first one using chips.

1. "Pretend you have two boxes and you want to put three cookies in each box. Let's put three cookies in the first box (count while pushing aside three chips). Now, let's put three cookies in the second box (count while pushing aside three more chips). How many cookies do you have altogether?" (Count the chips in both piles together, then announce the product.)

   Allow the children enough time to count the chips by pausing after the presentation of each number in the problem and again before requesting the product.
2. "Pretend you are baking muffins for four friends and each friend wants to eat two muffins. How many muffins do you have to make so that each friend can eat two muffins?"

Pause after the presentation of each number in the problem and again before requesting the product to allow the children enough time to count out and group the chips.

All children benefit from developing checking skills. After all the children have determined the product, ask several to share their answers and then ask the group to check the answer. Specifically, ask, "How can we check to make sure the answer is right?" Ask several children to share their checking procedures, then say, "OK, let's all check our answers."

Items

2 x 3
3 x 3

MULTIPLY BY COUNTING ALL, CHIPS, CONT'D.
(#49)

2 x 4
2 x 5
2 x 6
2 x 7
2 x 8
2 x 9
3 x 4
3 x 5
3 x 6
4 x 5

Instructions

Distribute 20 chips to each child.

Hold the number cards (facing away from the children) and tell the children that there are numbers on the other side of the cards.

Introduce the activity by saying, "Today we're going to use our knowledge of addition to solve multiplication problems. I'll show you a number and then you'll tell me what happens to that number when you add the same number to it. For example, here is two, what happens when we add two plus two?"

After it is clear that all children understand the activity, say, "Now instead of asking what is a number added to itself when I hold up a card I'll ask, what is the number times two, which is really just a shortcut for asking you to add a number to itself. OK?" Provide additional examples if necessary.

"Let's try another."

Hold up the #4 card and say, "What is four times two? Remember, this is just a different way of saying what is four plus four."

"OK, let's do some more. How much is [n] times two?"

- If the child provides the correct answer, give the card to the child.
- If the child produces an incorrect answer, ask to check his or her answer using the chips. Say, "Let's use the chips to check the answer." After the child generates a correct answer, place the card back in your pile and shuffle the deck.

Children may use the chips to help figure out the answer. Time permitting, this process is repeated until the children collect all of the cards. Before moving on to the next activity, ask the children to count the number of cards in their possession and declare the child with the most cards the winner of the game.

Repeat this activity until all the children in the class have accurately and consistently produced each double.

Items

None

LINK ADDITION TO MULTIPLICATION (#50)

Objective

To understand that adding a number to itself is the same as multiplying the number by two

Materials

Cards numbered 2–9

Chips

Participants

Individual, small group, and/or whole class. If used with small group or whole class, increase the number of cards used (e.g., 2 sets of cards for small group and 4 sets for whole class instruction).

Norms

On average, 8-year-olds can use repeated addition to mentally solve multiplication combinations.

NCTM Standard

Recognize equivalent representations for the same number and generate them by decomposing and composing numbers. (Number and Operations: Understand numbers, ways of representing numbers, relationships among numbers, and number systems.)

SMALL NUMBER MULTIPLICATION, CHIPS (#51)

Objective

To master multiplication combinations

Materials

Cards numbered 0–9

Chips

Participants

Individual, small group, and/or whole class. If used with small group or whole class, increase the number of cards used (e.g., two number card sets for small group and four sets for whole class instruction).

Norms

At approximately 7 1/2, children begin to develop fluency with single-digit multiplication combinations.

NCTM Standard

Develop fluency with basic number combinations for multiplication and division and use these combinations to mentally compute related problems, such as 30 x 50. (Number and Operations: Compute fluently and make reasonable estimates.)

Instructions

If used individually, place the cards face down on the table. If used in a small group or with the whole class, you may hold selected cards up or project cards onto a more visible surface.

Distribute 20 chips to each child.

Introduce the activity by saying, “Today we’re going to play another game. I’ll show you two cards with numbers on them and I want you to multiply the numbers on them. If you’re right and you tell me how you figured out the answer, you get to keep the cards. The person with the most cards at the end of the game wins. OK? Let’s begin.”

Select a child.

Before presenting the first problem, say, “You can use the chips to help you figure out the answer.”

Hold up two cards (numbers facing toward the children) and ask, “How much is (the number on the first card) times (the number on the second card)?”

After each child solves a problem, ask, “How did you know that?”

To get credit for solving the tasks, the children must correctly subtract the numbers on the face of the cards and articulate the method used to solve the problem. If the child correctly subtracts the numbers on the first try only, he or she holds the card.

If the child produces an incorrect answer, another child may be selected to answer the problem but place the cards back in your pile and shuffle the deck. Children may only collect cards if their answer is correct on the first try.

Time permitting, repeat this process until the children collect all the cards. Before moving on to the next activity, ask the children to count the number of cards in their possession and declare the child with the most cards the winner of the game.

Items

None

Instructions

If used individually, place the cards face down on the table. If used in a small group or with the whole class, you may hold selected cards up or project cards onto a more visible surface.

Introduce the activity by saying, "Today we're going to play another game. I'll show you two cards with numbers on them and I want you to multiply the numbers on them. If you're right and you tell me how you figured out the answer, you get to keep the cards. The person with the most cards at the end of the game wins. OK? Let's begin."

Select a child.

Hold up two cards (numbers facing toward the children) and ask, "How much is (the number on the first card) times (the number on the second card)?"

After each child solves a problem, ask, "How did you know that?"

To get credit for solving the tasks, the children must correctly subtract the numbers on the face of the cards and articulate the method used to solve the problem. If the child correctly subtracts the numbers on the first try only, he or she holds the card.

If the child produces an incorrect answer, another child may be selected to answer the problem but place the cards back in your pile and shuffle the deck. Children may only collect cards if their answer is correct on the first try.

Time permitting, repeat this process until the children collect all the cards. Before moving on to the next activity, ask the children to count the number of cards in their possession and declare the child with the most cards the winner of the game.

Do not provide the children with manipulatives but encourage them to multiply the numbers on the face of the cards as fast as possible.

Items

None

SMALL NUMBER MULTIPLICATION (#52)

Objective

To master multiplication combinations

Materials

Cards numbered 0–9

Participants

Individual, small group, and/or whole class. If used with small group or whole class, increase the number of cards used (e.g., two number card sets for small group and four sets for whole class instruction).

Norms

At approximately 7 1/2, children begin to develop fluency with single-digit multiplication combinations.

NCTM Standard

Develop fluency with basic number combinations for multiplication and division and use these combinations to mentally compute related problems, such as 30 x 50. (Number and Operations: Compute fluently and make reasonable estimates.)

MULTIPLYING & THE ZERO RULE (#53)

Objective

To discover the zero principle in multiplication

Materials

Chips

Medium-sized bags

Participants

Individual, small group, and/or whole class

Norms

On average, 8 1/2-year-olds can apply the multiplicative zero principle to problems.

NCTM Standard

Illustrate general principles and properties of operations, such as commutativity, using specific numbers. (Algebra: Represent and analyze mathematical situations and structures using algebraic models.)

Instructions

Before beginning the activity below, model the zero principle with manipulatives.

Distribute 20 chips and several bags to each child.

Introduce the activity by saying, "If we have one bag here and we put three chips in it (place three chips in a bag), how many chips are in the bag?" (Wait for confirmation.)

"That's right, we have three chips in the bag. Here's another bag. Let's put three chips in this bag as well" (Place three chips in the other bag).

"How many chips do we have altogether?" (Wait for the children to answer.)

"That's right, we have six chips. So if we have two bags each with three chips in them then we have six chips altogether. Another way of saying that is two times three is six."

Empty the bags of all chips and say, "Now let's see what happens if we have two bags with no chips, that is two times zero? If we have two empty bags then we have zero chips, so two times zero is zero.

Now let's try it with five bags. Put zero chips in each bag. How many chips do you have? That's right. None. So five times zero is zero. Now let's do it with more bags…."

Guide the discussion toward the realization that anything times zero is always zero. The following questions can be used to help guide the discussion:

- "Do you see a pattern?"
- "Is there one number that is the same in all of these problems?"

Draw their attention to the zeros in each problem.

- If the children respond with "Yes, zero is in each problem," ask, "What else do you notice about these problems?"
- If the children do not respond, ask, "Is the answer the same as any other number in the problem?" The response should be a unanimous "Yes" to which you ask, "Why?" or "How did that happen?"

If all children articulate the principle, move on to Generalize the Zero Rule, Multiplication.

Items

1 x 0
2 x 0
3 x 0
4 x 0
5 x 0
6 x 0
7 x 0
8 x 0
9 x 0
10 x 0
11 x 0
12 x 0

MULTIPLYING & THE ZERO RULE, CONT'D. (#53)

GENERALIZE THE ZERO RULE, MULTIPLICATION (#54)

Objective

To generalize the zero principle in multiplication

Materials

None

Participants

Individual, small group, and/or whole class

Norms

On average, 8 1/2-year-olds can apply the multiplicative zero principle to problems.

NCTM Standard

Illustrate general principles and properties of operations, such as commutativity, using specific numbers. (Algebra: Represent and analyze mathematical situations and structures using algebraic models.)

Instructions

Introduce the activity by asking, "Does anyone know what zero means?" (Listen to responses.)

"Zero is a special number smart mathematicians use to represent the absence of any other number."

Next say, "Today we're going to play a thinking game. We're going to multiply some really big numbers in our heads. We're not going to use paper, chips, or our fingers. We're going to use our minds to investigate multiplying really big numbers by zero. Let's start with…"

If the child struggles with this activity, go back to Multiplying & the Zero Rule.

Items

13 x 0
14 x 0
15 x 0
16 x 0
17 x 0
18 x 0
19 x 0
21 x 0
24 x 0
28 x 0
32 x 0
34 x 0
43 x 0
47 x 0
49 x 0
56 x 0
52 x 0
63 x 0

66 x 0
72 x 0
76 x 0
83 x 0
86 x 0
92 x 0
94 x 0
99 x 0
101 x 0
127 x 0
131 x 0
176 x 0
269 x 0
284 x 0

GENERALIZE THE ZERO RULE, MULTIPLICATION, CONT'D. (#54)

FIND THE IDENTITY RULE IN MULTIPLICATION (#55)

Objective

To learn the identity principle in multiplication

Materials

Chips

Medium-sized bags

Participants

Individual, small group, and/or whole class

Norms

On average, 8 1/2-year-olds can apply the multiplicative identity principle to problems.

NCTM Standard

Illustrate general principles and properties of operations, such as commutativity, using specific numbers. (Algebra: Represent and analyze mathematical situations and structures using algebraic models.)

Instructions

Before beginning the activity below, model the identity principle with manipulatives.

Distribute 20 chips and 5 bags to each child.

Introduce the activity by saying, "If we have one bag here and we put four chips in it, how many chips are in the bag?" (Place four chips in the bag.)

"Four. That's right. We have four chips in the bag. So one times four is four."

Empty the bags of all chips and say, "Now let's see what happens if we have one bag with six chips, that is one times six. How many chips are in the bag?"

Guide the discussion toward the realization that any number times one is always that number. The following questions can be used to help guide the discussion:

- "Do you see a pattern?" "Is there one number that is the same in all of these problems?" The response should be a unanimous "Yes" to which you ask, "Why?" or "How did that happen?"
- "Is the answer the same as any other number in the problem?" If the children respond with "Yes. One is in each problem," ask, "What else do you notice about these problems?"

Draw the children's attention to the ones in each problem.

If all children articulate the principle, move on to the generalization activity.

Items

1 x 1
2 x 1
3 x 1
4 x 1
5 x 1
6 x 1
7 x 1

8 x 1
9 x 1
10 x 1
11 x 1
12 x 1

FIND THE IDENTITY RULE IN MULTIPLICATION, CONT'D. (#55)

GENERALIZE THE IDENTITY RULE IN MULTIPLICATION (#56)

Objective
To generalize the identity principle in multiplication

Materials
None

Participants
Individual, small group, and/or whole class

Norms
On average, 8 1/2-year-olds can apply the multiplicative identity principle to problems.

NCTM Standard
Illustrate general principles and properties of operations, such as commutativity, using specific numbers. (Algebra: Represent and analyze mathematical situations and structures using algebraic models.)

Instructions
Introduce the activity by saying, "Today we're going to play a thinking game. We're going to multiply some really big numbers in our heads. We're not going to use paper, chips, or our fingers. We're going to use our minds to investigate multiplying really big numbers by one. Let's start with…"

If the child struggles with this activity, go back to Find the Identity Rule in Multiplication.

Items

13 x 1
14 x 1
15 x 1
16 x 1
17 x 1
18 x 1
19 x 1
23 x 1
26 x 1
33 x 1
36 x 1
43 x 1
49 x 1
51 x 1
57 x 1
67 x 1
73 x 1
77 x 1
85 x 1
93 x 1
99 x 1

121 x 1
172 x 1
189 x 1

GENERALIZE THE IDENTITY RULE IN MULTIPLICATION, CONT'D. (#56)

FIND THE ORDER RULE IN MULTIPLICATION (#57)

Objective

To learn the order principle in multiplication

Materials

Overhead projector

Chips

Participants

Individual, small group, and/or whole class

Norms

On average, 9-year-olds can apply the multiplicative commutativity principle to problems.

NCTM Standard

Understand and use properties of operations, such as the distributivity of multiplication over addition. (Number and Operations: Understand meanings of operations and how they relate to one another.)

Instructions

Before beginning the activity below, model the order principle with manipulatives.

Distribute 20 chips to each child.

Introduce the activity by saying, “Today we are going to solve some multiplication problems. You may already know the answer, but we’re going to use the chips to help us think about the problems. OK? Let’s try it together. Let’s first take out two groups of three chips.” (Pause until all children comply.)

“Now let’s figure out how much two times three equals without moving the chips. Just leave the chips in the separate piles.”

After several children share their answers, say, “OK, let’s leave these chips on our desks and start the next problem. If you use chips for the next problem, be sure to place them above the first problem. How much is three times two?”

After several children have shared answers, ask, “How can we check to make sure?” Guide the discussion toward the realization that calculating the commuted pair is unnecessary. The following questions can be used to help guide the discussion:

- “Do you see a pattern?”
- “Are the numbers in the problems the same?”
- “Is the answer to both problems the same or different?” The response should be a unanimous “Yes” to which you ask, “Why?” or “How did that happen?”

Draw the children’s attention to the numbers in each problem. If the children respond with “Yes, the numbers are the same,” ask, “What else do you notice about these problems?”

If all children articulate the principle, move on to Generalize the Order Rule, Multiplication

Items

2 x 1	1 x 2
2 x 3	3 x 2
3 x 4	4 x 3
2 x 5	5 x 2

3 x 5	5 x 3
3 x 1	1 x 3
4 x 1	1 x 4
1 x 5	5 x 1
6 x 2	2 x 6
6 x 1	1 x 6

FIND THE ORDER RULE IN MULTIPLICATION, CONT'D. (#57)

GENERALIZE THE ORDER RULE, MULTIPLICATION (#58)

Objective

To generalize the order principle in multiplication

Materials

Chalkboard or overhead projector

Participants

Individual, small group, and/or whole class

Norms

On average, 9-year-olds can apply the multiplicative commutativity principle to problems.

NCTM Standard

Understand and use properties of operations, such as the distributivity of multiplication over addition. (Number and Operations: Understand meanings of operations and how they relate to one another.)

Instructions

Introduce the activity by saying, "I'm going to show you some multiplication problems. These are going to be really, really big problems, but since you are all wonderful mathematicians, I think you can figure out the answers."

Divide the chalkboard or overhead projector into two columns. In the first column, write out the initial problem with the answer (i.e., 27 x 18 = 486) and in the second write out the commuted pair without the answer (i.e., 18 x 27 = ?).

Direct the children's attention to the items in the first column. Say, "The problem in the first column says that 27 times 18 equals 486. Can the answer to this problem help us figure out the answer to the problem in the second column?"

- If a child does not respond, say, "Let's look at the numbers in the first column. Are the numbers in the problems the same or different?" Wait for the children to respond.
- If the children say the numbers are different, ask, "What's different about these numbers?"
- If the responses relate to order, ask, "Does the order change the product?"
- If the children say that the numbers are the same just "turned" or "switched around," ask, "How can knowing the answer to the problem in the first column help you figure out the answer to the problem in the second column?" (Point to each problem.)

Repeat this line of questioning for each item until the children generalize the principle.

Items

13 x 16 = 208	16 x 13 = ?
15 x 23 = 345	23 x 15 = ?
17 x 21 = 357	21 x 17 = ?
22 x 29 = 638	29 x 22 = ?
32 x 26 = 832	26 x 32 = ?

MULTIPLY BY TENS (#59)

Objective

To identify patterns of tens in multiplication

Materials

None

Participants

Individual, small group, and/or whole class

Norms

By about 8, children begin to identify tens patterns.

NCTM Standard

Describe, extend, and make generalizations about geometric and numeric patterns. (Algebra: Understand patterns, relations, and functions.)

Instructions

Introduce the activity by saying, "Today we're going to practice multiplying. Sometimes we are going to multiply single-digit numbers by other single-digit numbers and sometimes we're going to multiply single-digit numbers by decades. Does any one know what a decade is?" (Wait for children to respond.)

"A decade is a group of ten. Sometimes, smart mathematicians think of big numbers as groups of 10. Can anyone think of why that might be a good idea?" (Wait for children to respond.)

"Sometimes it is easier to think of big numbers by putting them into groups of numbers, like groups of tens. For example, if I say, 'How much is 20 times 2?' You could count on from 20 like this, 20 (whisper), 30, 40. The answer is 40. Or you could just multiply two times two to get four and add a zero, so the answer is forty.

Let's try this strategy together. Let's try to do it in our heads! We're not going to use paper, chips, or our fingers. We're going to use our minds to help us figure out the answers. OK? (Wait for confirmation.) Let's begin. How much is…"

After each item, ask, "How did you know that?" The following prompts may be used to guide the discussion:

- If responses indicate that they relied on Base 10 knowledge, move on (e.g., "I just counted on from 20 by tens").
- If the children describe a pattern (e.g., "You just add a zero when multiplying by tens"), say, "That's right. But if we did not know this rule how else could we have solved the problem?"
- If responses indicate anything other than the above, ask, "How could we have counted on to figure out the answer?"

The goal is for children to identify a pattern and be able to apply the Using Tens strategy to simple multiplication problems.

Items

Basic

2 x 1	2 x 10
3 x 1	3 x 10
4 x 1	4 x 10

MULTIPLY BY TENS, CONT'D. (#59)

5 x 1	5 x 10
6 x 1	6 x 10
7 x 1	7 x 10
8 x 1	8 x 10
9 x 1	9 x 10

Advanced

2 x 3	2 x 30
2 x 2	2 x 20
4 x 2	4 x 20
3 x 5	3 x 50
3 x 6	3 x 60
7 x 3	7 x 30
2 x 5	2 x 50
6 x 6	6 x 60
4 x 3	4 x 30
3 x 9	3 x 90
7 x 4	7 x 40
5 x 8	5 x 80
4 x 9	4 x 90
8 x 7	8 x 70

Instructions

Introduce the activity by saying, "Today we're going to continue playing our thinking game but this time the numbers are going to be really big and we're still going to multiply them in our heads. We're not going to use paper, chips, or our fingers. We're going to use our minds and our math knowledge to break the problems into smaller parts so we can figure out the answer. For example, if I say, 'How much is 24 times 2?' First we multiply 20 times 2 to get 40, then 4 times 2 to get 8, and then we add 40 plus 8 to get 48. So 24 times 2 is 48. OK? (Wait for confirmation. You may want to provide additional examples.)

Let's begin. How much is…"

After each item, ask, "How did you know that?" The following prompts may be used to guide the discussion:

- If responses indicate reliance on tens and ones, move on (e.g., "First I multiplied 20 times 2, then 4 times 2, and then added 40 plus 8 to get 48").
- If responses indicate anything other than the above, ask, "How can we break the problem into smaller parts to figure out the answer?"

If even one child produces an incorrect answer, ask, "How can we check the answer?"

- If responses indicate simplifying the problem into smaller parts (e.g., "Let's multiply the by tens and ones"), follow their lead and do not introduce the cubes.
- If responses indicate using the unifix cubes to check, say, "Show me how you would use the cubes to check the answer." Once children have grouped the cubes by tens, ask, "How could we have solved the problem in our heads without the cubes?" Guide the discussion toward multiplying the tens first and then the ones. After the answer has been checked, say, "OK, now we're going to put the cubes away and get back to our thinking game."

Items

Non-carrying

12 x 3
13 x 3
14 x 2

MENTALLY SIMPLIFY TWO-BY-ONE MULTIPLICATION (#60)

Objective

To decompose simple two-digit numbers to mentally solve multiplication problems

Materials

None

Participants

Individual, small group, and/or whole class

Norms

At approximately 9, children begin to simplify two-digit multiplication problems.

NCTM Standards

Develop fluency with basic number combinations for multiplication and division and use these combinations to mentally compute related problems, such as 30 x 50. (Number and Operations: Compute fluently and make reasonable estimates.)

MENTALLY SIMPLIFY TWO-BY-ONE MULTIPLICATION, CONT'D. (#60)

21 x 2
22 x 3
22 x 4
23 x 2
23 x 3
32 x 2
33 x 3
42 x 2
53 x 2
112 x 2
112 x 4
102 x 3
102 x 4
120 x 3
121 x 2
121 x 3
122 x 3
123 x 3
124 x 2

Carrying

17 x 7
18 x 4
19 x 6
24 x 5
27 x 9
35 x 5
38 x 3

44 x 3
49 x 2
53 x 8
112 x 6
117 x 2
127 x 3
129 x 2
132 x 6
142 x 4

MENTALLY SIMPLIFY TWO-BY-ONE MULTIPLICATION, CONT'D. (#60)

WRITTEN

WRITE ONE-DIGIT NUMERALS (#61)

Objective

To accurately write numerals

Materials

Paper

Pencil

Cards numbered 0–9

Colored pencils (optional)

Blank sheets (write common errors)

Participants

Individual, small group, and/or whole class

NCTM Standard

Connect number words and numerals to the quantities they represent, using various physical models and representations (Number and Operations: Understand numbers, ways of representing numbers, relationships among numbers, and number systems).

Instructions

Children must be able to correctly read the numbers presented to them.

"Today we are going to practice writing numerals. Does anyone know what a numeral is? (Listen to responses.) A numeral is how we write a number. First I'll pick a card and show it to you. Then you tell me what numeral is on the card and write that numeral on your paper. We'll check to make sure we all wrote down the correct numeral. You can write the numerals in different colors or you can use the same color. OK, let's begin with …. " Shuffle the cards before and after each item in the activity.

If the numeral written is incorrect, ask, "What numeral is that?" After the child responds, say, "Actually, you wrote the numeral _ ____. This is how to write the numeral ____. Now you try. Write the numeral ____ next to mine."

Items

The numerals 0–9.

IDENTIFY ONE-DIGIT ERRORS (#62)

Objective

To accurately write numerals

Materials

Paper

Pencil

Cards numbered 0–9

Colored pencils (optional)

Blank sheets (write common errors)

Participants

Individual, small group, and/or whole class

Norms

On average, 4 1/2-year-olds can recognize and read single-digit numerals.

NCTM Standard

Connect number words and numerals to the quantities they represent, using various physical models and representations (Number and Operations: Understand numbers, ways of representing numbers, relationships among numbers, and number systems).

Instructions

"Today we are all teachers. We are going to check the work someone did with numerals. Our job is to figure out if the numerals are written correctly or incorrectly."

Write a zero on the board or a piece of paper.

"The first numeral should be a zero. Raise your hand if you think this numeral is written correctly."

Call on a few children and ask, "How can you tell this zero is written correctly?" If some children do not believe the numeral is written correctly, ask, "How would you write it differently?"

"Now, using your best teacher handwriting, write a zero as neatly and carefully as you can on your paper."

Follow the same format for the remaining numbers.

Items

The numerals 0–9.

Instructions

Children must be able to correctly read the numbers presented to them.

"I'm going to say a number, and I want you to write down the number I say."

Items

17
23
29
31
33
36
40
42
54
58
65
70
73
87
99

WRITE TWO-DIGIT NUMERALS (#63)

Objective

To accurately write large numbers

Materials

Paper

Pencil

Participants

Individual, small group, and/or whole class

Norms

On average, 6-year-olds can write double-digit numbers up to 99 and 7 1/2-year-olds can write three-digit numbers up to 999.

NCTM Standard

Connect number words and numerals to the quantities they represent, using various physical models and representations (Number and Operations: Understand numbers, ways of representing numbers, relationships among numbers, and number systems).

WRITE THREE-DIGIT NUMERALS (#64)

Objective

To accurately write large numbers

Materials

Paper

Pencil

Participants

Individual, small group, and/or whole class

Norms

On average, 6-year-olds can write double-digit numbers up to 99 and 7 1/2-year-olds can write three-digit numbers up to 999.

NCTM Standard

Connect number words and numerals to the quantities they represent, using various physical models and representations (Number and Operations: Understand numbers, ways of representing numbers, relationships among numbers, and number systems).

Instructions

Children must be able to correctly read the numbers presented to them.

"I'm going to say a number, and I want you to write down the number I say."

Items

159
241
328
432
597
683
707
864
950

Instructions

Children must be able to correctly read the numbers presented to them.

"I'm going to say a number, and I want you to write down the number I say."

Items

1091
3291
5382
7381
9468
2576
4056
6309
8456

WRITE FOUR-DIGIT NUMERALS (#65)

Objective

To accurately write large numbers

Materials

Paper

Pencil

Participants

Individual, small group, and/or whole class

Norms

On average, 6-year-olds can write double-digit numbers up to 99 and 7 1/2-year-olds can write three-digit numbers up to 999.

NCTM Standard

Connect number words and numerals to the quantities they represent, using various physical models and representations (Number and Operations: Understand numbers, ways of representing numbers, relationships among numbers, and number systems).

IDENTIFY TWO-DIGIT ERRORS (#66)

Objective

To accurately write large numbers

Materials

Paper

Pencil

Participants

Individual, small group, and/or whole class

Norms

On average, 6-year-olds can read double-digit numbers up to 99, 6 1/2-year-olds can read three-digit numbers up to 999, and 7-year-olds can read four-digit numbers up to 5000.

NCTM Standard

Connect number words and numerals to the quantities they represent, using various physical models and representations (Number and Operations: Understand numbers, ways of representing numbers, relationships among numbers, and number systems).

Instructions

Children must be able to correctly read the numbers presented to them to engage in this activity.

"Today we are all teachers. We are going to check the work someone did with numerals. Our job is to figure out if the numerals are written correctly or incorrectly."

"The first numeral this person should have written is …. Raise your hand if you think this numeral is written correctly."

Call on a few children and ask, "How can you tell if this (refer to worksheet and say number) is written correctly?" If some children do not believe the numeral is written correctly, ask, "How would you write it differently?

Now, using your best teacher handwriting, write the numeral as neatly and carefully as you can on your paper."

Follow the same format for the remaining numbers.

Items

Say	Write
17	70
23	203
29	29
31	301
33	33
36	36
40	14
42	42
54	154
87	78

Instructions

Children must be able to correctly read the numbers presented to them to engage in this activity.

"Today we are all teachers. We are going to check the work someone did with numerals. Our job is to figure out if the numerals are written correctly or incorrectly.

The first numeral this person should have written is …. Raise your hand if you think this numeral is written correctly."

Call on a few children and ask, "How can you tell if this (refer to worksheet and say number) is written correctly?" If some children do not believe the numeral is written correctly, ask, "How would you write it differently?

Now, using your best teacher handwriting, write the numeral as neatly and carefully as you can on your paper."

Follow the same format for the remaining numbers.

Items

Say	Write
159	1509
241	241
328	300028
432	432
597	597
683	683
707	707
864	864
950	950
999	999

IDENTIFY THREE-DIGIT ERRORS (#67)

Objective

To accurately write large numbers

Materials

Paper

Pencil

Participants

Individual, small group, and/or whole class

Norms

On average, 6-year-olds can read double-digit numbers up to 99, 6 1/2-year-olds can read three-digit numbers up to 999, and 7-year-olds can read four-digit numbers up to 5000.

NCTM Standard

Connect number words and numerals to the quantities they represent, using various physical models and representations (Number and Operations: Understand numbers, ways of representing numbers, relationships among numbers, and number systems).

IDENTIFY FOUR-DIGIT ERRORS (#68)

Objective

To accurately write large numbers

Materials

Paper

Pencil

Participants

Individual, small group, and/or whole class

Norms

On average, 6-year-olds can read double-digit numbers up to 99, 6 1/2-year-olds can read three-digit numbers up to 999, and 7-year-olds can read four-digit numbers up to 5000.

NCTM Standard

Connect number words and numerals to the quantities they represent, using various physical models and representations (Number and Operations: Understand numbers, ways of representing numbers, relationships among numbers, and number systems).

Instructions

Children must be able to correctly read the numbers presented to them to engage in this activity.

"Today we are all teachers. We are going to check the work someone did with numerals. Our job is to figure out if the numerals are written correctly or incorrectly."

"The first numeral this person should have written is … Raise your hand if you think this numeral is written correctly."

Call on a few children and ask, "How can you tell if this (refer to worksheet and say number) is written correctly?" If some children do not believe the numeral is written correctly, ask, "How would you write it differently?

Now, using your best teacher handwriting, write the numeral as neatly and carefully as you can on your paper."

Follow the same format for the remaining numbers.

Items

Say	Write
8456	8456
6309	6309
4056	400056
2576	2576
9468	9000400608
7381	7381
5382	53802
3291	32091
1091	1091
9000	900

CREATE & IDENTIFY GROUPS OF 10 (#69)

Objective

To develop Base 10 knowledge by identifying 10 objects as a group of 10

Materials

Ten-frame

Chips

Cups

Participants

Individual or small group

Norms

On average, 5 1/2-year-olds can concretely group objects into tens.

NCTM Standard

Use multiple models to develop initial understandings of place value and the Base 10 number system (Number and Operations: Understand numbers, ways of representing numbers, relationships among numbers, and number systems).

Instructions

Children must be able to count by tens to participate in this activity.

Distribute one ten-frame and 30 chips to each child, or group of children.

Introduce the activity by saying, "Today we are going to practice figuring out how many chips there are by creating groups of 10. Can anyone think of a reason why smart mathematicians like to work with the number 10? (Listen to responses.) Smart mathematicians try to make tens out of numbers so that they can solve really difficult problems without getting confused by lots of little numbers. Your job is to figure out how many tens there are and how many ones are left over. You can use this ten-frame to help you. See, it holds 10 chips. I'll give you a bunch of chips and you will figure out how many tens and ones make up that number. Let's try one together using the ten-frame."

"Here are some chips (place 11 chips in the center of the table). Place one chip in the frame (wait until the child has completed the task). How many tens do we have?" After the child provides the correct response, ask, "And how many ones do we have left over?" Before administering the next item, ask, "How many chips did we have altogether?" To discourage counting by ones, place each group of ten in a cup.

Observe how the child determines the total number of chips on the table. If he or she counts by tens and ones, move on to the next item. If the child uses any other method (e.g., counting by ones only), say, "Good job. Now let's try counting by tens and ones together."

Items

Any number between 10 and 30. Increase the size of the numbers by increasing the number of chips distributed to each child.

COUNT GROUPS (#70)

Objective

To use unifix cubes to develop Base 10 place value skills and concepts

Materials

Unifix cubes

Numbered cards

Participants

Individual, small group, and/or whole class

Norms

On average, 7-year-olds recognize that one 10 equals 10 ones and 7 1/2-year-olds recognize that one 100 equals ten tens or one-hundred ones.

NCTM Standard

Use multiple models to develop initial understandings of place value and the Base 10 number system (Number and Operations: Understand numbers, ways of representing numbers, relationships among numbers, and number systems).

Instructions

Introduce the activity by saying, "Today we are going to practice counting by groups of 10. Does anyone remember why smart mathematicians like to work with the number 10? (Listen to responses.) Smart mathematicians like to work with tens because it helps them solve really difficult problems more quickly without getting confused by lots of little numbers. Let's start by using these blocks to show the number 17, like this (count out 17 blocks). Now, let's group the blocks by tens (link 10 blocks to create one group of 10). So now we have one group of ten and seven units, right? That's 17." Wait for the group to confirm.

Ask a child to pick a card, then say, "This card will tell us how many more units to add. For example, if the card with the number four is selected, we'll add four units to the ones group. See what happens, we get another group of 10 (link 10 blocks together); so now we have two groups of 10 or 20 and one unit left over, so that makes 10, 20, 21 altogether. Let's do some more."

Choose a number to begin with then choose a card to see how many units to add. When the activity is first introduced, begin with small numbers such as 12, 16, 19, 23 and distribute 40 unifix cubes. As children become more familiar with the activity and develop Base 10 skills begin with a larger number like 31, 33, 37, 39, etc.

When children have mastered grouping by tens and ones, have them try grouping by hundreds as well (using a second set of unifix cubes).

Items

None

EXPLORE PLACE VALUE (#71)

Objective

To recognize numbers in the ones, tens, hundreds, and thousands place

Materials

Cards numbered 0–9

Participants

Small group

Norms

On average, 7-year-olds recognize numbers in the ones and tens place and 8-year-olds recognize numbers in the ones, tens, hundreds, and thousands place.

NCTM Standard

Use multiple models to develop initial understandings of place value and the Base 10 number system (Number and Operations: Understand numbers, ways of representing numbers, relationships among numbers, and number systems).

Instructions

Shuffle the deck of cards and say, "Today we're going to practice our place value skills. Does anyone know what place value means? (Listen to responses.) Smart mathematicians like you can look at any number, no matter how big or how small, and figure out how many ones and tens and hundreds and thousands make up the number."

"In this game, each of you will pick two cards from the deck and arrange them any way you want. The goal is to arrange your cards so that they make the biggest number possible, then you win a point for your team. But to win the point you have to explain why your number is the biggest. That is, you have to say how many groups of tens and ones make up your number. For example, the biggest number possible with these two cards (hold up a 2 and a 1 card) is 21. The two is in the tens place and is made up of two tens and the one is in the ones place."

After the child masters the activity with two-digit numerals, move on to three- and four-digit numerals by adding additional cards. You may need to include additional decks.

Items

None

TWO-DIGIT PLACE VALUE CONCEPTS (#72)

Objective

To understand the concept of place value by recognizing that digits in different positions represent different values

Materials

None

Participants

Small group

Norms

On average, 7-year-olds recognize numbers in the ones and tens place and 8-year-olds recognize numbers in the ones, tens, hundreds, and thousands place.

NCTM Standard

Use multiple models to develop initial understandings of place value and the Base 10 number system (Number and Operations: Understand numbers, ways of representing numbers, relationships among numbers, and number systems).

Instructions

Does anyone remember what place value means? (Listen to responses.) Smart mathematicians like you can look at any number, no matter how big or how small, and figure out how many ones and tens and hundreds and thousands make up the number. Today, mathematicians, I'm going to ask you to decipher the place value of digits in different numbers. I'll say a number and then I'll ask you some questions about how many tens and ones make up a number. For example, 23 is made up of two tens and three ones. The two in 23 stands for two groups of 10."

"Let's try some together:

- How many tens are in 12? How many ones are in 12?
- How many tens are in 43? How many ones are in 43?"

Items

17
28
21
30
37
35
49
43
56
50
67
63

THREE-DIGIT PLACE VALUE CONCEPTS (#73)

Objective

To understand the concept of place value by recognizing that digits in different positions represent different values

Materials

None

Participants

Small group

Norms

On average, 7-year-olds recognize numbers in the ones and tens place and 8-year-olds recognize numbers in the ones, tens, hundreds, and thousands place.

NCTM Standard

Use multiple models to develop initial understandings of place value and the Base 10 number system (Number and Operations: Understand numbers, ways of representing numbers, relationships among numbers, and number systems).

Instructions

Does anyone remember what place value means? (Listen to responses.) Smart mathematicians like you can look at any number, no matter how big or how small, and figure out how many ones and tens and hundreds and thousands make up the number. Today, mathematicians, I'm going to ask you to decipher the place value of digits in different numbers. I'll say a number and then I'll ask you some questions about how many hundreds, tens, and ones make up a number. For example, 203 is made up of two hundreds and three ones. The two in 203 stands for two groups of 100."

"Let's try some together:

- How many hundreds are in 125? How many tens are in 125? How many ones are in 125?
- How many hundreds are in 138? How many tens are in 138? How many ones are in 138?"

Items

117
124
132
149
153
167
174
183
196

FOUR-DIGIT PLACE VALUE CONCEPTS (#74)

Objective

To understand the concept of place value by recognizing that digits in different positions represent different values

Materials

None

Participants

Small group

Norms

On average, 7-year-olds recognize numbers in the ones and tens place and 8-year-olds recognize numbers in the ones, tens, hundreds, and thousands place.

NCTM Standard

Use multiple models to develop initial understandings of place value and the Base 10 number system (Number and Operations: Understand numbers, ways of representing numbers, relationships among numbers, and number systems).

Instructions

Does anyone remember what place value means? (Listen to responses.) Smart mathematicians like you can look at any number, no matter how big or how small, and figure out how many ones and tens and hundreds and thousands make up the number. Today, mathematicians, I'm going to ask you to decipher the place value of digits in different numbers. I'll say a number and then I'll ask you some questions about how many hundreds, tens, and ones make up a number. For example, 2603 is made up of two thousands, six hundreds, and three ones. The two in 2603 stands for two groups of 1000."

"Let's try some together:

- How many thousands are in 1275? How many hundreds are in 125? How many tens are in 1275? How many ones are in 1275?
- How many thousands are in 1382? How many hundreds are in 138? How many tens are in 1382? How many ones are in 1382?"

Items

1284
2714
3167
4925
5417
6892
7365
8641
9764

Instructions

Introduce the activity by saying, "Today we are going to practice writing problems. Can anyone think of why mathematicians need to write problems carefully? (Listen to responses.) There are so many reasons. Mathematicians need to write neatly so they can read what they wrote and mathematicians need to write problems in specific ways so that other mathematicians can understand them, too. Some of the numbers in the problems may be really big, but we don't have to worry about that because we are not going to solve these problems, we are just going to practice writing them on paper in mathematical language. So if I say write down 23 plus 4, you write it on the paper like this:

$$\begin{array}{r} 23 \\ \underline{+\ \ 4} \end{array}$$

What if I wrote it like this?

$$\begin{array}{l} \ \ 23 \\ +4 \end{array}$$

What's wrong with that?" Guide the discussion toward aligning based on place value.

"We always have to make sure the numbers are lined up according to place value and that you include an operation sign and line under the second number. OK? (Wait for confirmation.) Let's try some more."

Observe how the children write the problems. If even one child makes an alignment, line, or operational sign error, ask another child to write the correct answer on the board and then discuss all of the elements necessary for correctly setting up the problem (i.e., proper alignment, operational sign, line, and numbers).

Follow the same procedure for all of the items.

Items

21 + 3
14 – 7
17 + 5
15 – 6
29 + 5

SETUP PROBLEMS (+, –) (#75)

Objective

To properly align numbers

Materials

Paper

Pencil

Participants

Individual or small group

Norms

On average, 8-year-olds can properly align numbers in problems.

NCTM Standard

Create and use representations to organize, record, and communicate mathematical ideas (Representation).

SETUP PROBLEMS (+, –), CONT'D.

(#75)

27 – 6
27 + 3
24 – 3
32 + 6
46 – 3
36 + 4
43 – 8
68 + 7
76 – 3
87 + 4
65 – 9
79 + 3
58 – 6
95 + 8
87 – 9
127 + 9
182 – 7
145 + 4
138 – 7
167 + 6
332 – 62
176 + 21
876 – 53
198 + 24
398 – 84
450 + 89
494 – 98
809 + 76

Instructions

Introduce the activity by saying, "Today we are going to practice writing problems. Can anyone think of why mathematicians need to write problems carefully? (Listen to responses.) There are so many reasons. Mathematicians need to write neatly so they can read what they wrote and mathematicians need to write problems in specific ways so that other mathematicians can understand them, too. Some of the numbers in the problems may be really big, but we don't have to worry about that because we are not going to solve these problems, we are just going to practice writing them on paper in mathematical language. So if I say write down 23 plus 4, you write it on the paper like this:

$$\begin{array}{r} 37 \\ \underline{+\ \ 4} \end{array}$$

What if I wrote it like this?

$$\begin{array}{l} \ 37 \\ \text{x}4 \end{array}$$

What's wrong with that?" Guide the discussion toward aligning based on place value.

"We always have to make sure the numbers are lined up according to place value and that you include an operation sign and line under the second number. OK? (Wait for confirmation.) Let's try some more."

Observe how the children write the problems. If even one child makes an alignment, line, or operational sign error, ask another child to write the correct answer on the board and then discuss all of the elements necessary for correctly setting up the problem (i.e., proper alignment, operational sign, line, and numbers).

Follow the same procedure for all of the items.

Items

439 + 78
448 – 95
7840 + 542
5653 – 264
4981 + 987

SETUP PROBLEMS (+, –, x) (#76)

Objective

To properly align numbers

Materials

Paper

Pencil

Participants

Individual or small group

Norms

On average, 8-year-olds can properly align numbers in problems.

NCTM Standard

Create and use representations to organize, record, and communicate mathematical ideas (Representation).

SETUP PROBLEMS (+, –, x), CONT'D. (#76)

9876 – 654
5457 + 598
9843 – 389
23 x 5
37 x 2
49 x 4
58 x 3
61 x 9
73 x 7
80 x 7
96 x 7

TRADE IN ADDITION (#77)

Objective

To understand the Base 10 concepts underlying carrying

Materials

Place value mats

Unifix cubes (Several longs and cubes)

Participants

Individual, small group, and/or whole class

Norms

Beginning at 6, children can solve double-digit addition problems with concrete manipulatives, beginning at 6 1/2, children can solve three-digit problems with concrete manipulatives, and beginning at 8, children can solve problems with sums to 1000.

NCTM Standard

Create and use representations to organize, record, and communicate mathematical ideas (Representation).

Instructions

Introduce the activity by saying, "Today we're going to practice adding large numbers. We'll use these place value mats and blocks to help us. Can anyone think of a reason why these ten-frames might help us when we're adding really large numbers? (Listen to responses.) Smart mathematicians like to work with tens when they're adding really big numbers. Whenever they can combine little numbers into tens, they do so because it makes the problem easier. Does everyone see these blocks? (Hold up a block.) Each block represents one unit. Does everyone see these longs? (Hold up a long.) Each long represents ten units/blocks see (count the individual units one, two … ten) so a long is made up of ten units/blocks. Does anyone know how many units one of these flats represents? (Hold up a flat. Listen to responses.) A flat represents 100 units. Can anyone tell me how many longs are represented in a flat? (Listen to responses.) A flat represents 10 longs. OK? (Wait for confirmation.) Let's use the blocks to solve 17 plus 15 together. Seventeen is made up of one long and seven units/blocks. Place the number of blocks representing the ones on the place value mat and the number of longs representing the tens next to the mat."

Guide the child toward the correct representation of the first number and say, "Good, now let's do the same for the number 15." After the child has successfully represented both numbers, position the mats and longs horizontally and say, "OK, now we need to add the numbers together. Do we have enough ones to trade for a 10?" If the child does not respond, say, "Let's use some of the blocks on the second mat to fill up the first." After filling the first mat, say, "OK, now we can trade these ten units/blocks for a long (empty the place value mat and hand the child a long). We cannot make any more trades, so let's count the longs and units/blocks to see how much 17 plus 15 is altogether." Count the longs out loud by tens and ones (i.e., "10, 20, 30, 31, 32") and say, "Good job. Let's try some more … "

Items

18 + 15
26 + 17
39 + 14
45 + 19
54 + 27
38 + 24

TRADE IN ADDITION, CONT'D. (#77)

66 + 29
37 + 16
327 + 148
296 + 437
538 + 246
529 + 413
214 + 387
855 + 470
496 + 897

MODEL ALGORITHMS IN ADDITION (#78)

Objective

To understand the Base 10 concepts underlying carrying by explicitly linking the concrete model to the written algorithm

Materials

Paper

Pencil

Unifix cubes (several flats, longs, and cubes)

Participants

Individual, small group, and/or whole class

Norms

On average, 8-year-olds can relate written procedures to concrete models with double-digit problems, 8 1/2-year-olds can do so with three-digit problems.

NCTM Standard

Create and use representations to organize, record, and communicate mathematical ideas (Representation).

Instructions

Introduce the activity by saying, "Today we're going to practice adding large numbers. We'll use these place value mats and blocks to help us. Does anyone remember how many units one of these blocks represents? (Hold up a block. Listen to responses.) Each block represents one unit. Does anyone remember how many units one long represents? (Listen to responses.) Each long represents 10 units. See? (Count the individual units: one, two … ten.) So, a long is made up of ten units. Does anyone know how many units represent a flat? (Hold up a flat. Listen to responses.) A flat represents 100 units. Does anyone know how many longs a flat represents? (Hold up a flat.) A flat represents 10 longs. OK?" Wait for confirmation.

"Let's use the blocks to solve 28 plus 16 together. Twenty-eight is made up of two longs and eight one-units. Place the number of longs representing the tens next to the units representing the ones." Guide the child toward the correct representation of the first number and say, "Good, now let's do the same for the number 16. Sixteen is made up of one long and six one-units. Place the number of longs representing the tens next to the units representing the ones." After the child has successfully represented both numbers, say, "Write the number corresponding to each model here" (point to a spot next to the model). Next, say, "OK, now we need to add the numbers together. First we are going to add with the blocks." Allow child to spontaneously solve the problem. If the child does not respond, say, "Start by adding up all the units." Allow the child the opportunity to count the units, then ask, "How many do you get?" If the child made a counting error, help him or her to accurately re-count the blocks. Next ask, "Can we make any trades? Do we have enough ones to make a trade for a ten?" If the child says no, say, "Let's count the units to see if we can make a trade." Guide the child toward trading 10 units for one long. If the child says "Yes, we can trade," be sure he or she places the long on top of the tens. Next say, "How many tens and ones do we have altogether?" Guide the child toward correctly counting the tens and ones.

After the child generates the correct response, say, "We just added 28 plus 16, using the blocks. Now let's solve the same problem using the algorithm. First we add the ones: that is eight plus six just like we did with the model (point to the ones in the model). That makes 14, so we put the four under the line in the ones column and we place a one over the two in the tens column. Show me how we carried the 10 to solve the problem when we were using the model." Guide the discussion toward the grouping of ten ones and the carried long, then say, "Good job, let's try some more."

MODEL ALGORITHMS IN ADDITION, CONT'D. (#78)

Items

18 + 15
26 + 17
39 + 14
45 + 19
54 + 27
38 + 24
66 + 29
37 + 16
128 + 39
134 + 27
174 + 82
182 + 19
132 + 86
146 + 391
139 + 187
176 + 432

Instructions

Introduce the activity by saying, "OK mathematicians, today we're going to practice adding really big numbers. We're going to use our minds, math knowledge, and paper to help us figure out the answer. First we'll solve the problem and then I'll ask you some questions."

Execute the procedures necessary for successful completion of the problem.

	Hundreds	Tens	Ones
	1 **3**	1 **5**	**6**
+		**6**	**7**
	4	2	3

Discuss the place value concepts underlying the carried numbers. For example, say, "What does the one above the five mean?" If the child does not respond, say, "Did we trade, or regroup, ten ones for one ten?" If the child still does not respond, follow the checking procedures below. If the child responds yes, say, "How do you know?" Follow the same line of questioning for the hundreds (e.g., "Did we trade or regroup ten tens for one 100?") and thousands ("Did we trade or regroup ten hundreds for one 1000?"), when applicable.

After solving each problem (regardless of accuracy), say, "Show me how we can use the blocks to check the answer." If the child does not respond, say, "Show ___ with the hundreds, tens, and ones blocks." After the child represents the first number, say, "Good. Now show _____ with the hundreds, tens, and ones blocks." After checking the answer, say, "OK, now we're going to put the cubes away and get back to solving problems on paper."

Items

57 + 36
24 + 37
52 + 67
89 + 16
47 + 55
38 + 19

REGROUP IN ADDITION (#79)

Objective

To practice regrouping in addition

Materials

Paper

Pencil

Unifix cubes (several flats, longs, and cubes

Participants

Individual, small group, and/or whole class

Norms

On average, 8-year-olds can relate written procedures to concrete models with double-digit problems, 8 1/2-year-olds can do so with three-digit problems, and beginning at 7 1/2 children can do so with four-digit numbers.

NCTM Standard

Understand the place-value structure of the Base 10 number system and be able to represent and compare whole numbers and decimals (Number and Operations: Understand numbers, ways of representing numbers, relationships among numbers, and number systems).

REGROUP IN ADDITION, CONT'D. (#79)

49 + 52
186 + 329
549 + 438
206 + 918
726 + 108
215 + 837
481 + 372
916 + 487
1316 + 4802
5369 + 2357
1473 + 924
6872 + 598
4072 + 7683
5240 + 2691
2431 + 6351

TRADE IN SUBTRACTION (#80)

Objective

To understand the Base 10 concepts underlying borrowing

Materials

Unifix cubes (several longs and cubes)

Participants

Individual, small group, and/or whole class

Norms

Beginning at 6, children can solve double digit subtraction problems with concrete manipulatives, beginning at 6 1/2, children can solve three-digit problems with concrete manipulatives, and beginning at 8, children can solve problems with differences less than 1000.

NCTM Standard

Understand the place-value structure of the Base 10 number system and be able to represent and compare whole numbers and decimals (Number and Operations: Understand numbers, ways of representing numbers, relationships among numbers, and number systems).

Instructions

Introduce the activity by saying, "Today we're going to practice subtracting large numbers. We'll use these blocks to help us. Each block represents one unit. Each long represents 10 units/blocks." Show the children each unit/block of the long.

"A flat represents 100 units or 10 longs. OK? (Wait for confirmation.) Let's use these blocks to solve 35 minus 12 together. Here are 35 blocks (give each child 35 blocks). Take away 12 to figure out the answer to 35 minus 12." If the child does not respond, say, "Push aside 12 blocks." If the child is correct, say, "That was great. Now let's try it another way. Here are three longs and five blocks, that makes 35. Now take away 12."

Once the child answers, say, "Good job. Let's try one more together."

Solve a problem that involves regrouping. For example, "Here's 35 again. Now take away 19." If the child does not respond, say, "Do we need to regroup, or break, one long apart?" Guide the child toward regrouping one ten as ten ones by breaking apart one long. Next say, "Good. Now we can subtract nine ones from the 15." Wait for the child to remove nine ones and say, "How many ones do we have left now?" If the child responds correctly, move on to subtracting the tens. If the child does not respond, say, "Count the units." After the child responds with the correct number of units, say, "OK, now let's subtract the tens." After the child has removed the appropriate number of tens say, "How much is 35 minus 19?" If the child does not respond, say, "Count the left over tens and ones." If the child generates the correct answer, say, "Good job. Let's try some more … "

Model the approach as many times as necessary.

Items

75 – 9
43 – 8
56 – 29
80 – 28
70 – 7
53 – 36
52 – 46

TRADE IN SUBTRACTION, CONT'D. (#80)

24 – 7
84 – 7
73 – 5
91 – 24
35 – 17
50 – 35
412 – 35
642 – 351
45 – 28
593 – 234
217 – 186
784 – 236
954 – 328
562 – 159

MODEL ALGORITHMS IN SUBTRACTION (#81)

Objective

To understand the Base 10 concepts underlying borrowing by explicitly linking the concrete model to the written algorithm

Materials

Paper with recording charts

Pencil

Unifix cubes (several flats, longs, and cubes)

Participants

Individual, small group, and/or whole class

Norms

Beginning around 7 1/2, children can relate written procedures to concrete models with two- and three-digit subtraction problems.

NCTM Standard

Create and use representations to organize, record, and communicate mathematical ideas (Representation).

Instructions

Introduce the activity by saying, "OK mathematicians, today we're going to practice subtracting large numbers. We'll use these blocks to help us. Each block represents how many units? (Listen to responses.) Each block represents one unit. Each long represents how many units? (Listen to responses.) Each long represents 10 units/blocks, see … (count the individual units one, two … 10). So a long is made up of 10 units or blocks. A flat represents how many units? (Listen to responses.) A flat represents 100 units. And how many longs does one flat represent? (Listen to responses.) A flat represents 10 longs. OK?" Wait for confirmation.

Let's use the blocks to solve 35 minus 19 together. Thirty-five is made up of three longs/tens and five units/blocks. Place the number of longs representing the tens next to the units representing the ones." If the child is experiencing difficulty, guide him or her toward the correct representation of the first number by saying, "Good. Now let's do the same for the number 19. Nineteen is made up of one long and nine units. Place the number of longs representing the tens next to the blocks representing the units."

After the child has successfully represented both numbers, say, "Write the number corresponding to each model here (point to a spot next to each model)." Next say, "OK, let's figure out how much 35 minus 19 is by using the blocks first." Allow the child to spontaneously solve the problem. If the child does not respond, say, "Start by subtracting all the units." If the child makes a counting error, help him or her accurately re-count the blocks.

Next say, "Do we need to regroup one ten as ten ones?" After the child says yes, say, "Show me how we would do that." Guide the child toward regrouping one ten as ten ones. Be sure to place the long that was regrouped on top of the five units that make up 35.

Next, say, "Good. Now subtract nine units from the 15." Wait for the child to remove nine units and ask, "How many units do we have left now?" If the child responds correctly (i.e., 6), move on to subtracting the tens. If the child does not respond, say, "Count the unit cubes."

After the child responds with the correct number of units, say, "OK, now let's subtract the tens." After the child has removed the appropriate number of tens, say, "How much is 35 minus 19?" If the child does not respond, say, "Count the leftover tens and ones." After the child provides the correct answer say, "Good job. We just solved 35 minus 19 using the blocks. Now let's solve the same problem using the written instructions. Do

MODEL ALGORITHMS IN SUBTRACTION , CONT'D. (#81)

we need to regroup one ten as ten ones again?" After the child says yes, say, "That's right. Can you tell me why?" If the child does not respond, say, "We cannot take nine away from five, so what should we do?" If the child still does not respond, say, "We need to borrow 10 from 30 like we did with the model (cross out the three and place a two over it then place a one over the five or cross out the five and replace it with 15). Now we can subtract nine from 15 to get six. Where do we put the six?" If the child does not respond, say, "We put the six under the line in the ones column. Next we subtract the numbers in the tens column. Twenty minus 10 is 10, so 35 minus 19 is 16."

After writing the difference on the chart, say, "Good job. Let's try some more."

Model the approach as many times as necessary.

Hundreds	Tens	Ones
	$^{2}\cancel{3}$	$^{1}5$
–	1	9
	1	6

Items

30 – 17
96 – 9
71 – 37
45 – 27
35 – 17
61 – 37
45 – 37
26 – 8
48 – 39
50 – 17
28 – 19
55 – 8
66 – 49

98 – 25
75 – 46
61 – 36
70 – 26
481 – 236
674 – 368
540 – 261
284 – 152
829 – 306
908 – 823
813 – 267
412 – 169
600 – 350

MODEL ALGORITHMS IN SUBTRACTION , CONT'D. (#81)

REGROUP IN SUBTRACTION (#82)

Objective

To practice regrouping in subtraction

Materials

Paper

Pencil

Unifix cubes (several flats, longs, and cubes)

Participants

Individual, small group, and/or whole class

Norms

Beginning around age 7 1/2, children can relate written procedures to concrete models with two- and three-digit subtraction problems.

NCTM Standard

Create and use representations to organize, record, and communicate mathematical ideas (Representation).

Instructions

Introduce the activity by saying, "OK mathematicians, Today we're going to practice subtracting really big numbers. We're going to use our mind, math knowledge, and paper to help us figure out the answer. We'll solve the problem first and then I'll ask you some questions."

Execute the procedures necessary for successful completion of the problem.

	Hundreds	Tens	Ones
	$^{2}\cancel{3}$	$^{13}\cancel{4}$	**15**
			5
–		**5**	**7**
	2	8	8

Next, discuss the place value concepts underlying the carried numbers. For example, say, "Why did we cross out the five and write 15 on top of it?" If the child does not respond, say, "Did we regroup one ten as ten ones?" If the child still does not respond follow the checking procedures. If the child responds with yes, say, "Tell me how you did that." Follow the same line of questioning for the hundreds (e.g., "Did we trade, or regroup, one one-hundred for ten tens?") and thousands (e.g., "Did we trade, or regroup, one 1000 for ten hundreds?"), when applicable.

After solving each problem (regardless of accuracy), say, "Show me how we can use the blocks to check the answer." If the child does not respond, say, "Show ___ with the hundreds, tens, and ones blocks." After the child represents the minuend, say, "Good. Now let's think about whether we need to regroup one ten for ten ones. After the answer has been checked, say, "OK now we're going to put the blocks away and get back to solving problems on paper."

Items

45 – 26
78 – 59
52 – 47
208 – 199

496 – 197
561 – 248
63 – 27
352 – 169
60 – 24
482 – 247
80 – 49
986 – 438
57 – 39
71 – 35
36 – 29
537 – 213
813 – 289
529 – 378
476 – 299
772 – 529
6201 – 547
8726 – 6987
8035 – 3048
7350 – 1827
716 – 197
700 – 266
301 – 163
506 – 177
699 – 198
708 – 398

REGROUP IN SUBTRACTION, CONT'D. (#82)

MODEL ALGORITHMS IN MULTIPLICATION (#83)

Objective

To understand the Base 10 concepts underlying multiplication

Materials

Paper

Pencil

Unifix cubes (several flats, longs, and cubes)

Participants

Individual, small group, and/or whole class

Norms

On average 6 1/2-year-olds can use concrete manipulatives to solve simple multiplication problems.

NCTM Standard

Create and use representations to organize, record, and communicate mathematical ideas (Representation).

Instructions

Introduce the activity by saying, "Today we're going to practice multiplying large numbers. Does anyone know what it means to multiply? (Listen to responses.) The procedure of adding a number to itself a specified number of times is called multiplication. We'll use these blocks and paper to help us. Each block represents how many units? (Listen to responses.) One unit. Each long represents how many units? (Listen to responses.) Ten units/blocks, see … (count the individual units one, two … 10). So a long is made up of 10 units/blocks. A flat represents how many units? (Listen to responses.) One hundred units. And how many longs does a flat represent? (Listen to responses.) Ten longs. OK?" Wait for confirmation.

"Let's solve three times 15. Fifteen is made up of one long and five units. Show three groups of 15." Guide the child toward correctly modeling the multiplicand with the blocks and say, "Good. Next let's see if we can regroup. Do we have enough units to trade for a 10?" Guide the child toward regrouping ten ones as one ten. If he or she does not respond, say, "Count the units to see if we have 10." After the child counts 15 blocks, say, "OK, let's trade 10 units for a long, leaving us with five units." Be sure to place the traded long in the appropriate column. Next say, "Do we have enough longs to trade for a flat?" If even one child says yes, say, "Remember a flat is made up of 10 longs. Let's count the longs to check." Next say, "How much is three times 15 altogether?" If the child does not respond, say, "Let's count up all the blocks. How much is three times 15?" After the child produces the correct answer, say, "Now let's record what we did on our chart."

After the child says the correct response, you say, "We just multiplied three times 15, using the blocks. Now let's solve the same problem using the algorithm. First we multiply the ones: that is five times three, just like we did with the model (point to the ones in the model). That makes 15, so we put the five under the line in the ones column and we place a one over the one in the tens column. Tell me how we carried the 10 in 15 when we were using the model to solve the problem." Guide the discussion toward the grouping of 10 units and the carried long, then say, "Good job. Let's try some more."

	Tens	Ones
	^{1}1	5
x		3
	4	5

} 15 ones

Items

13 x 5
14 x 7
11 x 6
17 x 5
15 x 4
18 x 5
3 x 22
24 x 4
25 x 3
26 x 9
31 x 3
32 x 4
34 x 4
36 x 5
24 x 5
48 x 5
42 x 8
43 x 5
46 x 5
39 x 3
59 x 7
52 x 4
57 x 3
64 x 5
69 x 4
53 x 3
55 x 9
54 x 6
63 x 7

MODEL ALGORITHMS IN MULTIPLICATION, CONT'D. (#83)

SIMPLIFY TWO-BY-ONE NON-CARRYING MULTIPLICATION PROBLEMS (#84)

Objective

To learn the Base 10 place value concepts underlying written algorithms

Materials

Paper

Pencil

Overhead projector or board

Participants

Individual, small group, and/or whole class

Norms

Beginning around 9, children can simplify large multiplication problems to determine the product.

NCTM Standard

Create and use representations to organize, record, and communicate mathematical ideas (Representation).

Instructions

Introduce the activity by saying, "Today we're going to practice multiplying really big numbers. Does anyone remember what it means to multiply? (Listen to responses.) When mathematicians multiply, we add a number to itself a specified number of times. We're going to use our minds and our math knowledge to break the problems into smaller parts, so we can figure out the answer. For example, if I say how much is 27 times 3, first we multiply the tens by the multiplier. So 20 times 3 is 60. Then we multiply the ones by the multiplier, so seven times 3 is 21. Next we added 60 plus 21 to get 81. So we separated the problem by tens and ones to figure out the answer. OK? (Wait for confirmation.) Let's try another one. How much is … "

You may want to provide multiple examples.

Next say, "Now write down the way you solved the problem." If the child partitioned, say, "Tell me how you solved the problem." If the child did not partition, say, "Let me teach you another way" and model partitioning to solve the problem.

Items

11 x 3
13 x 2
14 x 2
21 x 2
22 x 3
32 x 2
33 x 3
42 x 2
53 x 2
112 x 2
112 x 3
112 x 4
102 x 2
102 x 3
120 x 3

121 x 2
122 x 3
123 x 3
124 x 2

SIMPLIFY TWO-BY-ONE NON-CARRYING MULTIPLICATION PROBLEMS, CONT'D. (#84)

SIMPLIFY TWO-BY-ONE CARRYING MULTIPLICATION PROBLEMS (#85)

Objective

To learn the Base 10 place value concepts underlying written algorithms

Materials

Paper

Pencil

Overhead projector or board

Participants

Individual, small group, and/or whole class

Norms

Beginning around 9, children can simplify large multiplication problems to determine the product.

NCTM Standard

Create and use representations to organize, record, and communicate mathematical ideas (Representation).

Instructions

Introduce the activity by saying, "Today we're going to practice multiplying really big numbers. Does anyone remember what it means to multiply? (Listen to responses.) When mathematicians multiply, we add a number to itself a specified number of times. We're going to use our minds and our math knowledge to break the problems into smaller parts, so we can figure out the answer. For example, if I say how much is 27 times 3, first we multiply the tens by the multiplier. So 20 times 3 is 60. Then we multiply the ones by the multiplier, so seven times 3 is 21. Next we added 60 plus 21 to get 81. So we separated the problem by tens and ones to figure out the answer. OK? (Wait for confirmation.) Let's try another one. How much is … "

You may want to provide multiple examples.

Next say, "Now write down the way you solved the problem." If the child partitioned, say, "Tell me how you solved the problem." If the child did not partition, say, "Let me teach you another way" and model partitioning to solve the problem.

Items

18 x 6
19 x 5
17 x 8
23 x 6
25 x 8
33 x 8
32 x 9
47 x 5
48 x 6
57 x 5
117 x 8
119 x 3
124 x 6
127 x 4
137 x 3
149 x 5

Instructions

Introduce the activity by saying, "Today we're going to practice adding really big numbers. We're going to use our minds and our math knowledge to break the problems into smaller parts so we can figure out the answer. For example, if I say how much is 37 plus 46, first we add 30 plus 40 to get 70. Then we add seven plus six to get 13. We're left with 70 plus 13, so we add 70 plus 10 to get 80 and then 80 plus three to get 83. So the answer is 83, and we figured it out by first adding all the tens and then the ones. OK? (Wait for confirmation.) Let's try another one. How much is … "

You may want to provide multiple examples.

Next say, "Now write down the way you solved the problem." If the child partitioned, say, "Tell me how you solved the problem." If the child did not partition, say, "Let me teach you another way" and model partitioning to solve the problem.

Items

35 + 27
46 + 24
39 + 28
84 + 37
49 + 57
58 + 46
87 + 36
76 + 59
64 + 28
80 + 49
76 + 38
98 + 46
227 + 46
239 + 58
368 + 74
443 + 98

SIMPLIFY ADDITION PROBLEMS (#86)

Objective

To use partitioning to add complex two-, three-, and four-digit numbers

Materials

Paper

Pencil

Overhead projector or board

Participants

Individual, small group, and/or whole class

Norms

On average, 8 1/2-year-olds can simplify two-digit numbers to determine the sum. Beginning around 8, children can simplify problems containing three-digit numbers to determine the sum.

NCTM Standard

Select appropriate methods and tools for computing with whole numbers from among mental computation, estimation, calculators, and paper and pencil according to the context and nature of the computation and use the selected method or tools (Number and Operations: Compute fluently and make reasonable estimates).

SIMPLIFY ADDITION PROBLEMS, CONT'D. (#86)

507 + 32
635 + 27
798 + 63
872 + 92
976 + 65
493 + 28
237 + 49
827 + 59
739 + 432
345 + 765
567 + 873
984 + 598
899 + 274
932 + 640
723 + 589
524 + 369
429 + 326
746 + 282
539 + 376
843 + 629

SIMPLIFY SUBTRACTION PROBLEMS (#87)

Objective

To use decomposition to mentally subtract complex two-digit numbers

Materials

Paper

Pencil

Overhead projector or board

Participants

Individual, small group, and/or whole class

Norms

Beginning around 7 1/2, children can simplify two and three digit numbers to determine the difference.

NCTM Standard

Select appropriate methods and tools for computing with whole numbers from among mental computation, estimation, calculators, and paper and pencil according to the context and nature of the computation and use the selected method or tools (Number and Operations: Compute fluently and make reasonable estimates).

Instructions

Introduce the activity by saying, "Today we're going to practice subtracting really big numbers. We're going to use our minds and our math knowledge to break the problems into smaller parts so we can figure out the answer in the same way as smart mathematicians. For example, if I say how much is 24 minus 13, first we subtract 20 minus 10 to get 10. Then we subtract four minus three to get one. Then we figure out what 24 minus 13 is by adding 10 plus one to get 11. So the answer is 11, and we figured it out by simplifying the problem by tens and then ones. OK? (Wait for confirmation.) Let's begin. How much is … "

You may want to provide multiple examples.

Next say, "Now write down the way you solved the problem." If the child partitioned, say, "Tell me how you solved the problem." If the child did not partition, say, "Let me teach you another way" and model partitioning to solve the problem.

Items

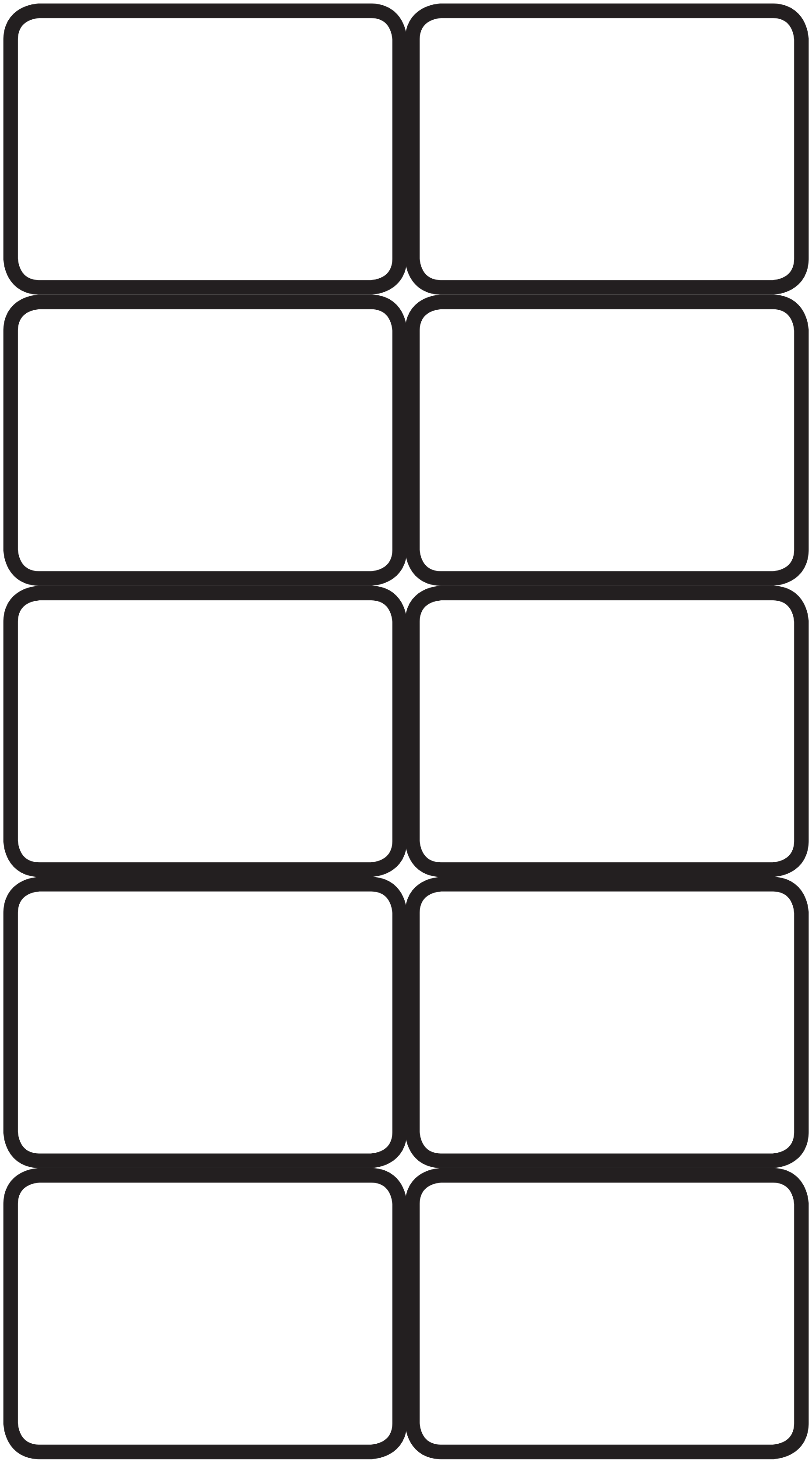
36 – 29
73 – 28
54 – 27
40 – 32
48 – 29
35 – 27
33 – 28
50 – 34
61 – 38
90 – 46
88 – 49
61 – 32
268 – 50
308 – 46
472 – 54
236 – 29

SIMPLIFY SUBTRACTION PROBLEMS, CONT'D. (#87)

740 – 31
443 – 68
873 – 59
647 – 76
558 – 72
562 – 49
237 – 85
583 – 67
636 – 274
324 – 243
462 – 247
348 – 272
529 – 304
634 – 244
374 – 290
316 – 252
457 – 329
429 – 348
609 – 437
235 – 227

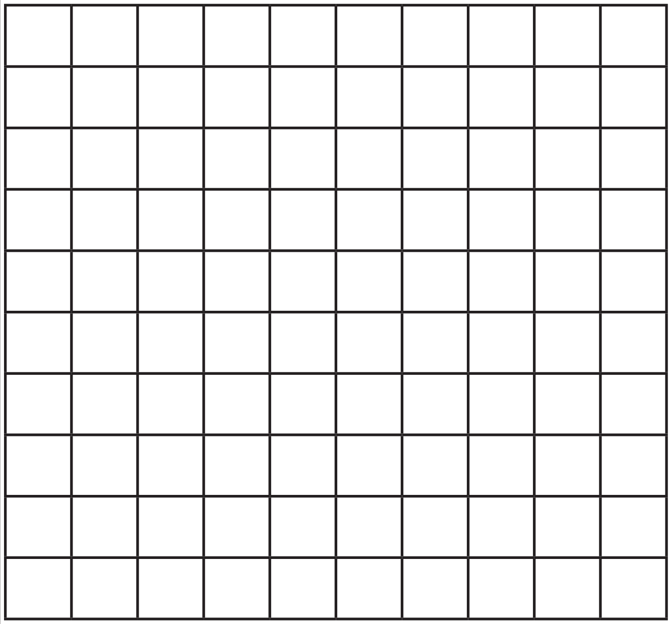
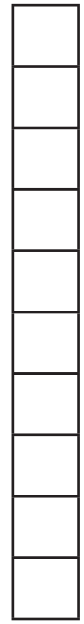
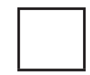
MANIPULATIVES

mCLASS:Math Ten-Frame



mCLASS:Math Place Value Mat

For best results, copy the mCLASS:Math Place Value Mat on the next page onto 11 x 17 paper at 129%.

		
Hundreds	Tens	Ones

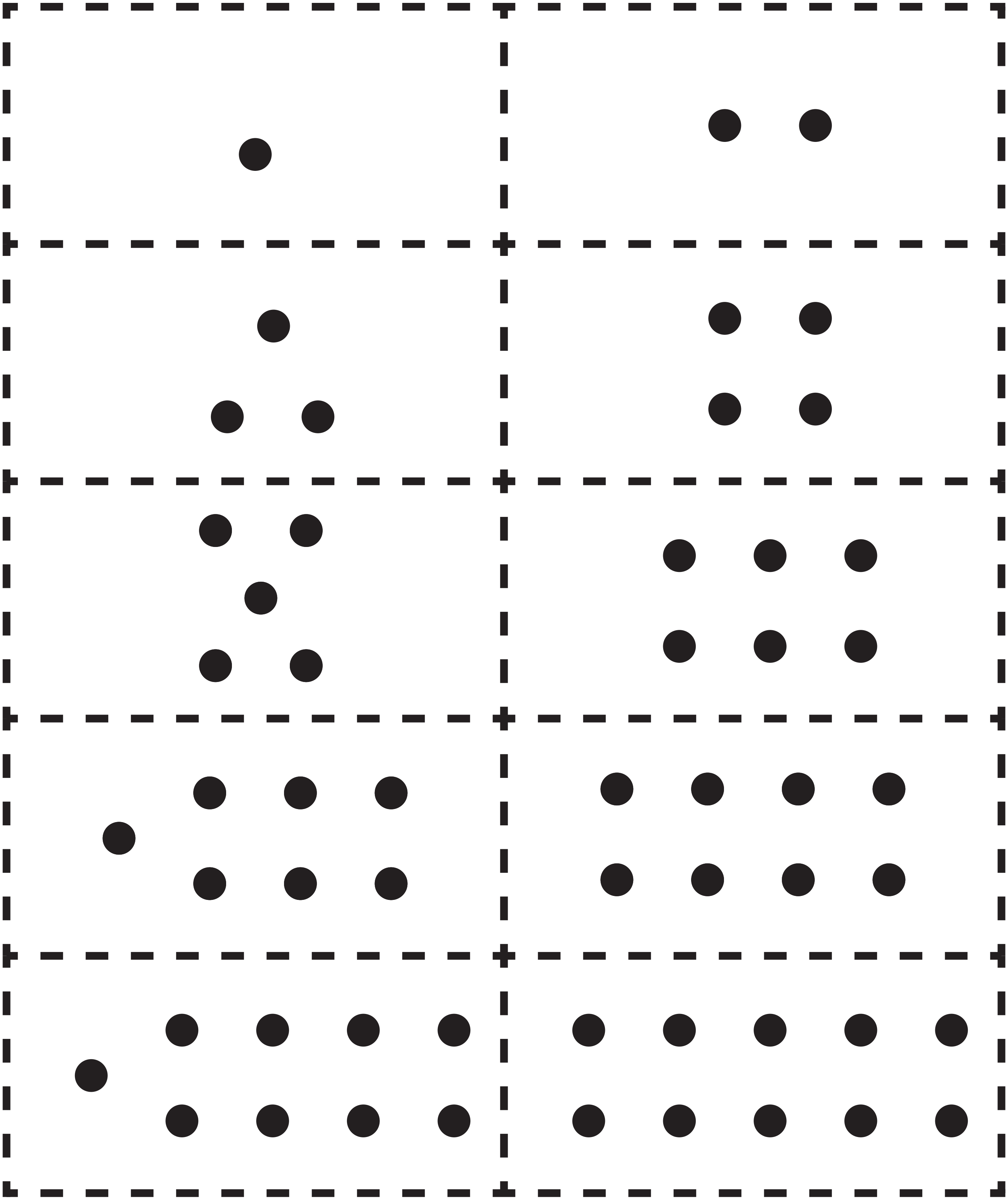
mCLASS:Math Number Cards

0	1
2	3
4	5
6	7
8	9

mCLASS:Math Number Cards, cont'd.

10	11
12	13
14	15
16	17
18	19

mCLASS:Math Organized Dot Cards



mCLASS:Math Randomized Dot Cards